Date: 10/10/2021

Q1. Expand the following $(1 - x + x^2)^4$.

Q2. Find r^{th} term in the expansion of $\left(x + \frac{1}{x}\right)^{2r}$.

Q3. Find the middle term (terms) in the expansion of $\left(\frac{p}{x} + \frac{x}{p}\right)^9$.

Q4. Find the coefficient of x in the expansion of $(1 - 3x + 7x^2)(1 - x)^{16}$.

Q5. Find the coefficient of x^{15} in the expansion of $(x - x^2)^{10}$.

Q6. Find the term independent of x, $x \neq 0$, in the expansion of $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^{15}$.

Q7. Find the term independent of x in the expansion of $\left(3x - \frac{2}{x^2}\right)^{15}$.

Q8. Find the value of r, if the coefficients of $(2r + 4)^{\text{th}}$ and $(r - 2)^{\text{th}}$ terms in the expansion of $(1 + x)^{18}$ are equal.

Q9. Show that the middle term in the expansion of $\left(x - \frac{1}{x}\right)^{2n}$ is $\frac{1\ 3\ 5\ \ldots (2n-1)}{\lfloor n}\times(-2)^n$.

Q10. If p is a real number and if the middle term in the expansion of $\left(\frac{p}{2} + 2\right)^8$ is 1120, find p.

Q11. Find the middle term (terms) in the expansion of:

(i) $\left(\frac{x}{a} - \frac{a}{x}\right)^{10}$ (ii) $\left(3x - \frac{x^3}{6}\right)^9$

Q12. Find the sixth term in the expansion of $\left(y^{\frac{1}{2}} + x^{\frac{1}{3}}\right)^n$, if the binomial coefficient of third term from the end is 45.

Q13. Find the middle term in the expansion of $\left(2ax - \frac{b}{x^2}\right)^{12}$.

Q14. Find the coefficients of $\frac{1}{x^{17}}$ in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$.

Q15. Which of the following is larger?

$$99^{50} + 100^{50} \quad \text{or} \quad 101^{50}.$$

Q16. Find the term independent of x in the expansion of $\left(\frac{\sqrt{x}}{\sqrt{3}} + \frac{\sqrt{3}}{2x^2}\right)^{10}$.

Q17. Show that $2^{4n+4} - 15n - 16$, where $n \in N$ is divisible by 225.

Q18. Find the term independent of x in the expansion of $(1 + x + 2x^3)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$.

Q19. If the term free from x in the expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, find the value of K.

Q20. Find the coefficient of x^{50} after simplifying and collecting the like terms in the expansion of $(1 + x)^{1000} + x(1 + x)^{999} + x^2(1 + x)^{998} + ... + x^{1000}$.

Q21. If n is a positive integer, find the coefficient of x^{-1} in the expansion of $(1+x)^n\left(1+\frac{1}{x}\right)^n$.

Q22. Determine whether the expansion of $\left(x^2-\frac{2}{x}\right)^{18}$ will contain a term containing x^{10}?

Q23. Find the coefficients of x^{11} in the expansion of $\left(x^3-\frac{2}{x^2}\right)^{12}$.

Q24. If x^p occurs in the expansion of $\left(x^2+\frac{1}{x}\right)^{2n}$, prove that its coefficient is

$$\frac{\lfloor 2n}{\lfloor \frac{4n-p}{3} \lfloor \frac{2n+p}{3}}.$$

Q25. Find n in the binomial $\left(\sqrt[3]{2}+\frac{1}{\sqrt[3]{3}}\right)^n$, if the ratio of 7^{th} term from the beginning to the 7^{th} term from the end is $\frac{1}{6}$.

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S1. Put $1 - x = y$

Then, $(1 - x + x^2)^4 = (y + x^2)^4$

$$= {}^4C_0\, y^4(x^2)^0 + {}^4C_1\, y^3(x^2)^1 + {}^4C_2\, y^2(x^2)^2 + {}^4C_3\, y\,(x^2)^3 + {}^4C_4(x^2)^4$$

$$= y^4 + 4y^3x^2 + 6y^2x^4 + 4yx^6 + x^8$$

$$= (1 - x)^4 + 4x^2(1 - x)^3 + 6x^4(1 - x)^2 + 4x^6(1 - x) + x^8$$

$$= 1 - 4x + 10x^2 - 16x^3 + 19x^4 - 16x^4 - 16x^5 + 10x^6 - 4x^2 + x^8.$$

S2. We have

$$T_r = {}^{2r}C_{r-1}(x)^{2r-r+1}\left(\frac{1}{x}\right)^{r-1}$$

$$= \frac{\lfloor 2r}{\lfloor r-1 \lfloor r+1} x^{r+1-r+1}$$

$$= \frac{\lfloor 2r}{\lfloor r-1 \lfloor r+1} x^2.$$

S3. Since, the power of binomial is odd. Therefore, we have two middle terms which are 5th and 6th. These are given by

$$T_5 = {}^9C_4\left(\frac{p}{x}\right)^5\left(\frac{x}{p}\right)^4 = {}^9C_4\,\frac{p}{x} = \frac{126p}{x}$$

and

$$T_6 = {}^9C_5\left(\frac{p}{x}\right)^4\left(\frac{x}{p}\right)^5 = {}^9C_5\,\frac{x}{p} = \frac{126x}{p}.$$

S4. Let $(1 - 3x + 7x^2)(1 - x)^{16} = (1 - 3x + 7x^2)\,[{}^{16}C_0 + {}^{16}C_1(-x) + {}^{16}C_2(-x)^2 + \ldots]$

$$= (1 - 3x + 7x^2)\,[1 - 16x + 120x^2 + \ldots]$$

$\therefore$ Coefficient of $x = -16 - 3 = \mathbf{-19}$.

S5. Let term containing x^{15} occur as T_{r+1}.

$$T_{r+1} = {}^{10}C_r\,(x)^{10-r}\,(-x^2)^r$$

$$= {}^{10}C_r\,(-1)^r\,x^{10-r+2r}$$

Let $10 + r = 15$

$\Rightarrow$ $r = 5$

$\therefore$ $T_{5+1} = {}^{10}C_5\,(-1)^5\,x^{15}$

$$= -252\,x^{15}$$

$\therefore$ Coefficient of $x^{15} = \mathbf{-252}$.

S6. Let the term independent of x be T_{r+1}.

$$T_{r+1} = {}^{15}C_r \left(\frac{3x^2}{2}\right)^{15-r} \left(-\frac{1}{3x}\right)^r$$

$$= {}^{15}C_r \left(\frac{3}{2}\right)^{15-r} \left(-\frac{1}{3}\right)^r x^{30-2r-r}$$

Let $30 - 3r = 0 \Rightarrow r = 10$

$$\therefore \quad T_{10+1} = T_{11} = {}^{15}C_{10} \left(\frac{3}{2}\right)^5 \left(-\frac{1}{3}\right)^{10}$$

$$= \mathbf{{}^{15}C_{10} \left(\frac{1}{6}\right)^5}.$$

S7. Let the term independent of x be T_{r+1}.

$$T_{r+1} = {}^{15}C_r \,(3x)^{15-r} \left(-\frac{2}{x^2}\right)^r$$

$$= {}^{15}C_r \,(3)^{15-r} \cdot (-2)^r \,x^{15-r-2r}$$

Let $15 - 3r = 0 \Rightarrow r = 5$

$$\therefore \quad T_{5+1} = T_6 = {}^{15}C_5 \,(3)^{15-5} (-2)^5$$

$$= -\,{}^{15}C_5 \,(3)^{10} (2)^5$$

$$= -\mathbf{3003\,(3)^{10}\,(2)^5}.$$

S8. Now,

$$T_{2r+4} = T_{(2r+3)+1}$$
$$= {}^{18}C_{2r+3}\,(x)^{2r+3}$$

and

$$T_{r-2} = T_{(r-3)+1}$$
$$= {}^{18}C_{r-3}\,(x)^{r-3}$$

Now,

$${}^{18}C_{2r+3} = {}^{18}C_{r-3}$$

$\therefore \quad 2r + 3 + r - 3 = 18$

$\Rightarrow \quad 3r = 18$

$\Rightarrow \quad r = \mathbf{6}$.

S9. Given Binomial .

There are $2n + 1$ terms in its exansion $(n + 1)^{th}$ term is middle term

$$T_{r+1} = {}^nC_r\,(a)^r\,b^{n-r}$$

$$T_{r+1} = {}^{2n}C_n\,(x)^n\left(-\frac{1}{x}\right)^n = \frac{(2n)!}{n!\,n!}(-1)^n$$

$$= \frac{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot\cdots\cdot(2n-1)(2n)}{n!\,n!}(-1)^n$$

$$= \frac{[1\cdot 3\cdot 5\cdot\cdots\cdot(2n-1)][2\cdot 4\cdot 6\cdot\cdots\cdot(2n)]}{n!\,n!}(-1)^n$$

$$= \frac{[1\cdot 3\cdot 5\cdot\cdots\cdot(2n-1)][(1\times 2)(2\times 2)(3\times 2)\cdots(n\times 2)]}{n!\,n!}(-1)^n$$

$$= \frac{[1\cdot 3\cdot 5\cdot\cdots\cdot(2n-1)][1\cdot 2\cdot 3\cdot\cdots\cdot n]2^n(-1)^n}{n!\,n!}$$

$$= \frac{[1\cdot 3\cdot 5\cdot\cdots\cdot(2n-1)](-2)^n}{n!}$$

Hence Proved.

S10. $\left(\frac{p}{2}+2\right)^8$

There are 9 terms in its expansion.

$\therefore$ 5^{th} term is the middle term.

Now, $T_{r+1} = {}^nC_r\, a^r\, b^{n-r}$

$$T_5 = T_{4+1} = {}^8C_4\left(\frac{p}{2}\right)^4(2)^4$$

Now, ${}^8C_4\left(\frac{p}{2}\right)^4(2)^4 = 1120$

$\Rightarrow$ $\frac{8\times 7\times 6\times 5}{1\times 2\times 3\times 4}\times p^4 = 1120$

$\Rightarrow$ $70p^4 = 1120$

$\Rightarrow$ $10p^4 = 160$

$\Rightarrow$ $p^4 = 16$

$\Rightarrow$ $p^2 = 4$

$\Rightarrow$ $p = \mathbf{\pm 2}$.

S11. (i) $\left(\frac{x}{a}-\frac{a}{x}\right)^{10}$

It has 11 terms. Thus 6^{th} term is the middle term.

$$T_6 = T_{5+1} = {}^{10}C_5 \left(\frac{x}{a}\right)^{10-5} \left(-\frac{a}{x}\right)^5 = -{}^{10}C_5$$

$$= -\frac{10 \times 9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4 \times 5} = \mathbf{-252}$$

(ii) $\left(3x - \frac{x^3}{6}\right)^9$

It has 10 terms. The middle terms are 5^{th} and 6^{th} terms.

$$T_5 = T_{4+1} = {}^9C_4\,(3x)^{9-4} \left(-\frac{x^3}{6}\right)^4$$

$$= {}^9C_4\, 3^5 \left(-\frac{1}{6}\right)^4 x^{5+12}$$

$$= {}^9C_5\, 3^5 \left(-\frac{1}{6}\right)^4 x^{17}$$

$$= \frac{9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4} \times \frac{3}{16} x^{17}.$$

$$= \mathbf{\frac{189}{8} x^{17}}$$

$$T_6 = T_{5+1} = {}^9C_5\,(3x)^{9-5} \left(\frac{-x^3}{6}\right)^5$$

$$= \frac{9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4} \times 3^4 \left(-\frac{1}{6}\right)^5 x^{15+4}$$

$$= \mathbf{\frac{-21}{16} x^{19}}.$$

S12. Binomial coefficient of third term from the end

= Binomial coefficient of third term from beginning

$= {}^nC_2$

Now, ${}^nC_2 = 45$

$\Rightarrow \quad \frac{n(n-1)}{2} = 45$

$\Rightarrow \quad n^2 - n - 90 = 0$

$\Rightarrow \quad (n-10)(n+9) = 0$

$\Rightarrow \quad n = 10$

Given: $\left(y^{\frac{1}{2}} + x^{\frac{1}{3}}\right)^n$

Now, $T_{r+1} = {}^nC_r a^{n-r} \cdot b^r$

$$T_6 = T_{5+1} = {}^{10}C_5 \left(y^{\frac{1}{2}}\right)^{10-5} \left(x^{\frac{1}{3}}\right)^5$$

$$= \frac{10 \times 9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4 \times 5} y^{\frac{5}{2}} x^{\frac{5}{3}}$$

$$= \mathbf{252\, y^{\frac{5}{2}}\, x^{\frac{5}{3}}}.$$

S13. Since the power of binomial is even, it has one middle term which is the $\left(\frac{12+2}{2}\right)^{\text{th}}$ term and it is given by

$$T_7 = {}^{12}C_6 \, (2ax)^6 \left(\frac{-b}{x^2}\right)^6$$

$$= {}^{12}C_6 \, \frac{2^6 a^6 x^6 \cdot (-b)^6}{x^{12}}$$

$$= {}^{12}C_6 \, \frac{2^6 a^6 x^6 \cdot b^6}{x^{12}} = \frac{591363 a^6 b^6}{x^6}.$$

S14. Let $\frac{1}{x^{17}}$ occur in T_{r+1}.

$$T_{r+1} = {}^{15}C_r \, (x^4)^{15-r} \left(-\frac{1}{x^3}\right)^r$$

$$= {}^{15}C_r \, (-1)^r \, x^{60-4r-3r}$$

Let $60 - 4r - 3r = -17$

$-7r = -77$

$\Rightarrow$ $r = 11$

$\therefore$ $T_{11+1} = {}^{15}C_{11} \, (-1)^{11} \, x^{-17}$

$$= \frac{-15 \times 14 \times 13 \times 12}{1 \times 2 \times 3 \times 4} x^{17}$$

$$= -1365\, x^{-17}.$$

Hence, coefficient of $\frac{1}{x^{17}}$ $= \mathbf{-1365}$.

S15. We have $(101)^{50} = (100 + 1)^{50}$

$$= 100^{50} + 50\,(100)^{49} + \frac{50 \cdot 49}{2 \cdot 1}(100)^{48} + \frac{50 \cdot 49 \cdot 48}{3 \cdot 2 \cdot 1}(100)^{47} + \ldots \qquad \ldots \text{(i)}$$

Similarly, $99^{50} = (100 - 1)^{50}$

$$= 100^{50} - 50 \cdot 100^{49} + \frac{50 \cdot 49}{2 \cdot 1}(100)^{48} - \frac{50 \cdot 49 \cdot 48}{3 \cdot 2 \cdot 1}(100)^{47} + \ldots \quad \ldots \text{(ii)}$$

Subtracting Eq. (ii) from (i), we get

$$101^{50} - 99^{50} = 2\left[50 \cdot (100)^{49} + \frac{50 \cdot 49 \cdot 48}{3 \cdot 2 \cdot 1} 100^{47} + \ldots\right]$$

$$\Rightarrow \quad 101^{50} - 99^{50} = 100^{50} + 2\left(\frac{50 \cdot 49 \cdot 48}{3 \cdot 2 \cdot 1}\right) 100^{47} + \ldots$$

$$\Rightarrow \quad 101^{50} - 99^{50} > 100^{50}$$

Hence, $101^{50} > 99^{50} + 100^{50}$.

S16. Let $(r + 1)^{\text{th}}$ term be independent of x which is given by

$$T_{r+1} = {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r} \left(\frac{\sqrt{3}}{2x^2}\right)^r$$

$$= {}^{10}C_r \left(\frac{x}{3}\right)^{\frac{10-r}{2}} 3^{\frac{r}{2}} \left(\frac{1}{2^r x^{2r}}\right)$$

$$= {}^{10}C_r \, 3^{\frac{r}{2} - \frac{10-r}{2}} \, 2^{-r} \, x^{\frac{10-r}{2} - 2r}$$

Since, the term is independent of x, we have

$$\frac{10-r}{2} - 2r = 0 \quad \Rightarrow \quad r = 2$$

Hence, 3^{rd} term is independent of x and its value is given by

$$T_3 = {}^{10}C_2 \, \frac{3^{-3}}{4} = \frac{10 \times 9}{2 \times 1} \times \frac{1}{9 \times 12} = \mathbf{\frac{5}{12}}.$$

S17. We have $2^{4n+4} - 15n - 16 = 2^{4(n+1)} - 15n - 16$

$$= 16^{n+1} - 15n - 16$$

$$= (1 + 15)^{n+1} - 15n - 16$$

$$= {}^{n+1}C_0 \, 15^0 + {}^{n+1}C_1 \, 15^1 + {}^{n+1}C_2 \, 15^2 + {}^{n+1}C_3 \, 15^3 + \ldots + {}^{n+1}C_{n+1} (15)^{n+1} - 15n - 16$$

$$= 1 + (n + 1)\, 15 + {}^{n+1}C_2 \, 15^2 + {}^{n+1}C_3 \, 15^3 + \ldots + {}^{n+1}C_{n+1} (15)^{n+1} - 15n - 16$$

$$= 1 + 15n + 15 + {}^{n+1}C_2\, 15^2 + {}^{n+1}C_3\, 15^3$$
$$+ \ldots + {}^{n+1}C_{n+1}\,(15)^{n+1} - 15n - 16$$
$$= 15^2\,[{}^{n+1}C_2 + {}^{n+1}C_3\, 15 + \ldots \text{ so on}]$$

Thus, $2^{4n+4} - 15n - 16$ is divisible by 225.

S18. Let $(1 + x + 2x^3)\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

Consider $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

Let us find the term containing constant term,

$$\frac{1}{x} \quad \text{and} \quad \frac{1}{x^3}$$

Now, $T_{r+1} = {}^nC_r\, a^r\, b^{n-r}$

Now, $T_{r+1} = {}^9C_r \left(\frac{3}{2}x^2\right)^r \left(-\frac{1}{3x}\right)^{9-r}$

$$= {}^9C_r \left(\frac{3}{2}\right)^r \left(-\frac{1}{3}\right)^{9-r} x^{2r-9+r}$$

For constant term, let

$$3r - 9 = 0 \quad \Rightarrow \quad r = 3$$

$\therefore$ $T_{3+1} = {}^9C_3 \left(\frac{3}{2}\right)^3 \left(-\frac{1}{3}\right)^6$

$$= \frac{9 \times 8 \times 7}{1 \times 2 \times 3} \times \frac{27}{8} \times \frac{1}{27 \times 27} = \frac{7}{18}$$

For Coefficient of $\frac{1}{x} = x^{-1}$.

Let $3r - 9 = -1 \quad \Rightarrow \quad 3r = 8 \quad \Rightarrow \quad r = \frac{8}{3}$ (not possible)

coefficient of x^{-3}.

$$3r - 9 = -3 \quad \Rightarrow \quad 3r = 6 \quad \Rightarrow \quad \mathbf{r = 2}$$

$$T_{2+1} = {}^9C_2 \left(\frac{3}{2}\right)^2 \left(-\frac{1}{3}\right)^7 x^{-3}$$

$\therefore$ Coefficient of $x^{-3} = \frac{-9 \times 8}{1 \times 2}\left(\frac{9}{4}\right)\left(\frac{1}{27 \times 27} \times \frac{1}{3}\right) = \frac{1}{27}$

$\therefore$ Required coefficient $= \frac{7}{18} - 2 \times \frac{1}{27} = \frac{7}{18} - \frac{2}{27}$

$= \frac{21-4}{54} = \mathbf{\frac{17}{54}}.$

S19. Let the term independent of x be T_{r+1}.

$$T_{r+1} = {}^{10}C_r\ (\sqrt{x})^{10-r} \cdot \left(\frac{-K}{x^2}\right)^r$$

$$= {}^{10}C_r\ (-K)^r\ x^{5 - \frac{1}{2}r - 2r}$$

Let $5 - \frac{1}{2}r - 2r = 0$

$\Rightarrow$ $\frac{5}{2}r = 5 \quad \Rightarrow \quad r = 2$

$\therefore$ $T_{2+1} = {}^{10}C_2\,(-K)^2 = {}^{10}C_2 K^2$

Now, ${}^{10}C_2 K^2 = 405$

$\Rightarrow$ $\frac{10 \times 9}{1 \times 2} \times K^2 = 405$

$K^2 = \frac{810}{9 \times 10} = 9$

Hence, $K = \mathbf{\pm 3}$.

S20. Since, the above series is a geometric series with the common ratio $\frac{x}{1+x}$, its sum is

$$\frac{(1+x)^{1000}\left[1 - \left(\frac{x}{1+x}\right)^{1001}\right]}{\left[1 - \left(\frac{x}{1+x}\right)\right]} = \frac{(1+x)^{1000} - \frac{x^{1001}}{1+x}}{\frac{1+x-x}{1+x}} = (1+x)^{1001} - x^{1001}$$

Hence, coefficient of x^{50} is given by

$${}^{1001}C_{50} = \frac{\lfloor 1001}{\lfloor 50 \lfloor 951}.$$

S21. We have $(1+x)^n \left(1 + \frac{1}{x}\right)^n = (1+x)^n \left(\frac{x+1}{x}\right)^n = \frac{(1+x)^{2n}}{x^n}$

Now, to find the coefficient of x^{-1} in $(1+x)^n \left(1 + \frac{1}{x}\right)^n$, it is equivalent to finding coefficient of x^{-1} in $\frac{(1+x)^{2n}}{x^n}$ which in turn is equal to the coefficient of x^{n-1} in the expansion of $(1 + x)^{2n}$.

Since, $(1+x)^{2n} = {}^{2n}C_0\, x^0 + {}^{2n}C_1\, x^1 + {}^{2n}C_2\, x^2 + \ldots + {}^{2n}C_{n-1}\, x^{n-1} + \ldots + {}^{2n}C_{2n}\, x^{2n}$

Thus, the coefficient of x^{n-1} is

$${}^{2n}C_{n-1} = \frac{\lfloor 2n}{\lfloor n-1 \lfloor 2n-n+1} = \frac{\lfloor 2n}{\lfloor n-1 \lfloor n+1}.$$

S22. Let T_{r+1} contain x^{10}.

Then, $$T_{r+1} = {}^{18}C_r\,(x^2)^{18-r}\left(\frac{-2}{x}\right)^r$$
$$= {}^{18}C_r\, x^{36-2r}(-1)^r \cdot 2^r x^{-r}$$
$$= (-1)^r \cdot 2^r\, {}^{18}C_r\, x^{36-3r}$$

Thus, $36 - 3r = 10$, *i.e.*, $r = \frac{26}{3}$

Since r is a fraction, the given expansion cannot have a term containing x^{10}.

S23. Let the general term, *i.e.*, $(r+1)^{\text{th}}$ contain x^{11}.

We have $$T_{r+1} = {}^{12}C_r\,(x^3)^{12-r}\left(-\frac{2}{x^2}\right)^r$$
$$= {}^{12}C_r\, x^{36-3r-2r}(-1)^r 2^r$$
$$= {}^{12}C_r\,(-1)^r 2^r x^{36-5r}$$

Now, for this to contain x^{11}, we observe that

Let $36 - 5r = 11$ *i.e.*, $r = 5$

Thus, the coefficient of x^{11} is

$${}^{12}C_5\,(-1)^5\, 2^5 = -\frac{12\times 11\times 10\times 9\times 8}{5\times 4\times 3\times 2}\times 32 = -25344.$$

S24. Let x^p occurs as T_{r+1} in the expansion of $\left(x^2 + \frac{1}{x}\right)^{2n}$

$$T_{r+1} = {}^{2n}C_r\,(x^2)^r\left(\frac{1}{x}\right)^{2n-r}$$

$$= {}^{2n}C_r\, x^{2r-2n+r}$$
$$= {}^{2n}C_r\, x^{3r-2n}$$

Let $3r - 2n = p$

$\therefore$ $$r = \frac{p+2n}{3}$$

$\therefore$ Coefficient of $x^p = {}^{2n}C_r$

$$= \frac{(2n)!}{r!(2n-r)!}$$

$$= \frac{\lfloor 2n}{\left\lfloor \frac{p+2n}{3} \right. \left\lfloor 2n - \frac{p+2n}{3} \right.}$$

$$= \frac{\lfloor 2n}{\left\lfloor \frac{p+2n}{3} \right. \left\lfloor \frac{4n-p}{3} \right.}$$ **Proved.**

S25. Let, $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$

$$T_{r+1} = {}^nC_r\, x^r\, y^{n-r}$$

$$T_7 = T_{6+1} = {}^nC_6\, (\sqrt[3]{2})^6 \left(\frac{1}{\sqrt[3]{3}}\right)^{n-6}$$

T_7 from the end of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$ is T_7 from the beginning of $\left(\frac{1}{\sqrt[3]{3}} + \sqrt[3]{2}\right)^n$

$$\therefore \qquad \frac{{}^nC_6 \left(2^{\frac{1}{3}}\right)^{n-6} \left(3^{-\frac{1}{3}}\right)^6}{{}^nC_6 \left(3^{-\frac{1}{3}}\right)^{n-6} \left(2^{\frac{1}{3}}\right)^6} = \frac{1}{6}$$

$$\Rightarrow \qquad (2.3)^{\frac{n-6}{3} - \frac{6}{2}} = \frac{1}{6}$$

$$\Rightarrow \qquad 6^{\frac{n-12}{3}} = 6^{-1}$$

$$\therefore \qquad \frac{n}{3} - 4 = -1$$

$$\Rightarrow \qquad n = \mathbf{9}.$$