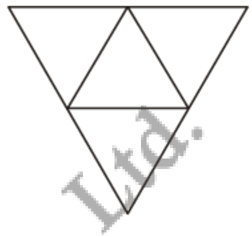
Date: 10/10/2021

Q1. In a certain city, all telephone numbers have six digits, the first two digits always being 41 or 42 or 46 or 62 or 64. How many telephone numbers have all six digits distinct?

Q2. In a class, there are 27 boys and 14 girls. The teacher wants to select 1 boy and 1 girl to represent the class for a function. In how many ways can the teacher make this selection?

Q3. How many numbers are there between 99 and 1000 having 7 in the units place?

Q4. In how many ways can this diagram be coloured subject to the following two conditions?



(i) Each of the smaller triangle is to be painted with one of three colours: red, blue or green.

(ii) No two adjacent regions have the same colour.

Q5. Eight chairs are numbered 1 to 8. Two women and 3 men wish to occupy one chair each. First the women choose the chairs from amongst the chairs 1 to 4 and then men select from the remaining chairs. Find the total number of possible arrangements.

Q6. In how many ways can 5 children be arranged in a line such that (i) two particular children of them are always together (ii) two particular children of them are never together.

Q7. There are 10 persons named P_1, P_2, P_3, ..., P_{10}. Out of 10 persons, 5 persons are to be arranged in a line such that in each arrangement P_1 mush occur whereas P_4 and P_5 do not occur. Find the number of such possible arrangements.

Q8. A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected from the lot.

Q9. How many committee of five persons with a chairperson can be selected from 12 persons?

Q10. We wish to select 6 persons from 8, but if the person *A* is chosen, then *B* must be chosen. In how many ways can selections be made?

Q11. Find the number of permutations of *n* different things taken *r* at a time such that two specific things occur together.

Q12. In how many ways 3 mathematics books, 4 history books, 3 chemistry books and 2 biology books can be arranged on a shelf so that all books of the same subjects are together.

Q13. A boy has 3 library tickets and 8 books of his interest in the library. Of these 8, he does not want to borrow Mathematics Part II, unless Mathematics Part I is also borrowed. In how many ways can he choose the three books to be borrowed?

Q14. If 20 lines are drawn in a plane such that no two of them are parallel and no three are concurrent, in how many points will they intersect each other?

Q15. A student has to answer 10 questions, choosing atleast 4 from each of Parts *A* and *B*. If there are 6 questions in Part *A* and 7 in Part *B*, in how many ways can the student choose 10 questions?

Q16. In an examination, a student has to answer 4 questions out of 5 questions; questions 1 and 2 are however compulsory. Determine the number of ways in which the student can make the choice.

Q17. Find the number of integers greater than 7000 that can be formed with the digits 3, 5, 7, 8 and 9 where no digits are repeated.

Q18. A bag contains six white marbles and five red marbles. Find the number of ways in which four marbles can be drawn from the bag if (i) they can be of any colour (ii) two must be white and two red and (iii) they must all be of the same colour.

Q19. 18 mice were placed in two experimental group and one control group, with all groups equally large. In how many ways can the mice be placed into three groups?

Q20. Out of 18 points in a plane, no three are in the same line except five points which are collinear. Find the number of lines that can be formed joining the point.

Q21. Find the number of different words that can be formed from the letters of the word 'TRIANGLE' so that no vowels are together.

Q22. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$ and ${}^nC_{r+1} = 126$, then find ${}^rC_2$.

Q23. If the letters of the word RACHIT are arranged in all possible ways as listed in dictionary. Then what is the rank of the word RACHIT?

Q24. Three married couples are to be seated in a row having six seats in a cinema hall. If spouses are to be seated next to each other, in how many ways can they be seated? Find also the number of ways of their seating if all the ladies sit together.

Q25. In a small village, there are 87 families, of which 52 families have atmost 2 children. In a rural development programme 20 families are to be chosen for assistance, of which atleast 18 families must have at most 2 children. In how many ways can the choice be made?

Q26. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has: (i) no girls, (ii) at least one boy and one girl, (iii) at least three girls.

Q27. A sports team of 11 students is to be constituted, choosing at least 5 from Class XI and atleast 5 from Class XII. If there are 20 students in each of these classes, in how many ways can the team be constituted?

Q28. In how many ways can a football team of 11 players be selected from 16 players? How many of them will

(i) include 2 particular players? (ii) exclude 2 particular players?

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S1. When first two digits are 41 (or 42 or 46 or 62 or 64), then the remaining 4 digits can be selected in ways

$$= {}^8P_4 = 8 \times 7 \times 6 \times 5 = 1680$$

In a similar way we have for 42, 46, 62, 64.

Now, total telephone numbers are = 1680×5 = **8400**.

S2. Here, the teacher is to perform two operations:

(i) Selecting a boy from among the 27 boys and

(ii) Selecting a girl from 14 girls

The first of these can be done in 27 ways and second can be performed in 14 ways. By the fundamental principle of counting, the required number of ways is $27 \times 14 = 378$.

S3. First note that all these numbers have three digits. 7 is in the unit's place. The middle digit can be any one of the 10 digits from 0 to 9. The digit in hundred'splace can be any one of the 9 digits from 1 to 9. Therefore, by the fundamental principle of counting, there are $10 \times 9 = 90$ numbers between 99 and 1000 having 7 in the units place.

S4. These conditions are satisfied exactly when we do as follows: First paint the central triangle in any one of the three colours. Next paint the remaining 3 triangles, with any one of the remaining two colours.

By the fundamental principle of counting, this can be done in $3 \times 2 \times 2 \times 2 = 24$ ways.

S5. Let 2 women occupy the chairs, from 1 to 4 in ${}^4P_2$ ways and 3 men occupy the remaining chairs in ${}^6P_3$ ways.

Now total number of possible arrangements

$$= {}^4P_2 \times {}^6P_3$$

$$= \frac{4!}{(4-2)!} \times \frac{6!}{(6-3)!}$$

$$= \frac{4 \times 3.2!}{2!} \times \frac{6 \times 5 \times 4 \times 3!}{3!} = 12 \times 120 = \mathbf{1440}.$$

S6. (i) We consider the arrangements by taking 2 particular children together as one and hence the remaining 4 can be arranged in 4! = 24 ways. Again two particular children taken together can be arranged in two ways. Therefore, there are $24 \times 2 = 48$ (total) ways of arrangement.

(ii) Among the 5! = 120 permutations of 5 children, there are 48 in which two children are together. In the remaining 120 – 48 = 72 permutations, two particular children are never together.

S7. P_1 occurs in one way.

P_4 and P_5 does not occur.

Remaining 7 can be selected in ${}^7C_4$ ways.

Now, each group of 5 persons can be arranged in 5! ways.

Hence, required number of arrangements

$$= {}^7C_4 \times 5! \text{ ways.}$$

S8. 2 black balls out of 5 can be selected in ways = ${}^5C_2$.

3 red balls out of 6 can be selected in ways = ${}^6C_3$.

Required number of ways = ${}^5C_2 \times {}^6C_3$

$$= 10 \times \frac{6 \times 5 \times 4}{1 \times 2 \times 3}$$

$$= 10 \times 20 = \textbf{200 ways}.$$

S9. One chairperson can be selected in 12 ways.

Now, out of remaining 11 persons, remaining 4 can be selected in ${}^{11}C_4$ ways.

Total number of committee = $12 \times {}^{11}C_4$

$$= 12 \times \frac{11 \times 10 \times 9 \times 8}{1 \times 2 \times 3 \times 4}$$

$$= 12 \times 330 = \textbf{3960}.$$

S10. **Case (i):** When A is chosen, then B is also selected, then number of selections

$$= {}^6C_4 = \frac{6 \times 5}{1 \times 2} = 15$$

Case (ii): When A is not chosen, then number of selections

$$= {}^7C_6 = 7$$

Total number of selections = 15 + 7 = **22**.

S11. A bundle of 2 specific things can be put in r places in $(r - 1)$ ways and 2 things in the bundle can be arranged themselves into $\underline{|2}$ ways. Now $(n - 2)$ things will be arranged $(r - 2)$ places in ${}^{n-2}P_{r-2}$ ways.

Thus, using the fundamental principle of counting, the required number of permutations will be $\underline{|2} \cdot (r - 1) \cdot {}^{n-2}P_{r-2}$.

S12. First we take books of a particular subject as one unit. Thus there are 4 units which can be arranged in 4! = 24 ways. Now in each of arrangements, mathematics books can be arranged in 3! ways, history books in 4! ways, chemistry books in 3! ways and biology books in 2! ways. Thus the total number of ways = $24 \times 4! \times 3! \times 4! \times 2!$ = 165888.

S13. Let us make the following cases:

Case (i) : Boy borrows Mathematics Part II, then he borrows Mathematics Part I also. So the number of possible choices is ${}^6C_1 = 6$.

Case (i) : Boy does not borrows Mathematics Part II, then the number of possible choices is $^7C_3 = 35$.

Hence, the total number of possible choice is 35 + 6 = 41.

S14. Let n be the number of points of intersection.

For one point of intersection, we required two lines.

$\therefore$ Number of points of intersection

$$= {}^{20}C_2 = \frac{20 \times 19}{2 \times 1} = \mathbf{190}.$$

S15. The possibilities are: 4 from Part A and 6 from Part B

or 5 from Part A and 5 from Part B

or 6 from Part A and 4 from Part B

Therefore, the required number of ways is

$$= {}^6C_4 \times {}^7C_6 + {}^6C_5 \times {}^7C_5 + {}^6C_6 \times {}^7C_4$$

$$= 105 + 126 + 35 = 266.$$

S16. questions 1 and 2 are compulsory, they can be selected in one way.

Now, we choose 2 questions out of 3 in

$^3C_2$ ways = 3

Hence, total number of ways = $3 \times 1 = \mathbf{3}$.

S17. We can use the digits 3, 5, 7, 8 and 9.

No digit is repeated.

Besides 4 digit integers greater than 7000, five digit integers are always greater than 7000.

All five digit integers = $5 \times 4 \times 3 \times 2 \times 1 = \mathbf{120}$.

Again for 4 digit numbers: Thousandth place can be filled in by 7, 8, 9 in 3 ways.

Hundredth place can be filled in 4 ways.

Tenth place cvan be filled in 3 ways.

Unit place can be filled in 2 ways.

Total such numbers = $3 \times 4 \times 3 \times 2 = \mathbf{72}$.

Hence, Total integers = 120 + 72 = **192**.

S18. Here, Total number of marbles = 6 white marbles + 5 red marbles

= 11 marbles.

(i) Four marbles can be selected out of 11 marbles in ways = $\mathbf{^{11}C_4}$.

(ii) Required number of selections = $\mathbf{^6C_2 \cdot {}^5C_2}$.

(iii) Required number of way of selecting = $\mathbf{^6C_4 + {}^5C_4}$.

S19. 18 mice were placed equally in two experimental groups and one control group *i.e.*, three groups.

Thus, each group has 6 mice.

Now, the six mice of one group are alike.

Thus, Number of ways = $\dfrac{18!}{6!\,6!\,6!} = \dfrac{(18)!}{(6!)^3}$.

S20. Total number of points = 18.

Only five points are in a line.

Number of straight lines = ${}^{18}C_2 - {}^{5}C_2 + 1$

$= \dfrac{18 \times 17}{1 \times 2} - \dfrac{5 \times 4}{1 \times 2} + 1$

= 153 – 10 + 1 = **144**.

S21. TRIANGLE

Total number of letters = 8

Number of vowels (I, A, E) = 3

Number of consonants = 5

Place of C and V

V C V C V C V C V C V

Number of ways of placing the consonants

= 5! = 120

Number of ways of placing the vowels

= ${}^{6}P_3$ = 120

Total number of ways

= 120 × 120 = **14400**.

S22. Let $\dfrac{{}^nC_r}{{}^nC_{r+1}} = \dfrac{84}{126}$ and $\dfrac{{}^nC_r}{{}^nC_{r+1}} = \dfrac{84}{36}$

$\Rightarrow \quad \dfrac{n!}{r!(n-r)!} \times \dfrac{(r+1)!(n-r-1)!}{n!} = \dfrac{84}{126}$

$\Rightarrow \quad \dfrac{r+1}{n-r} = \dfrac{2}{3}$

$\Rightarrow \quad 3r + 3 = 2n - 2r$

$\Rightarrow \quad 2n - 5r = 3$ ... (i)

Also, $$\frac{^nC_r}{^nC_{r-1}} = \frac{84}{36}$$

$$\Rightarrow \quad \frac{n!}{r!(n-r)!} \times \frac{(r-1)!(n-r+1)!}{n!} = \frac{7}{3}$$

$$\Rightarrow \quad \frac{n-r+1}{r} = \frac{7}{3}$$

$$\Rightarrow \quad 3n - 3r + 3 = 7r$$

$$\Rightarrow \quad 3n - 10r = -3 \qquad \text{... (ii)}$$

Now multiplying (i) by 2, we get

$$4n - 10r = 6 \qquad \text{... (iii)}$$

Eq. (iii) – Eq. (ii), we get

$$4n - 10r = 6$$
$$3n - 10r = -3$$
$$- \quad + \quad +$$
$$n = 9$$

$\therefore$ From (i)

$$2 \times 9 - 5r = 3$$

$$\Rightarrow \quad 18 - 5r = 3$$

$$\Rightarrow \quad r = 3$$

Now, $$^rC_2 = {^3C_2} = \mathbf{3}.$$

S23. For RACHIT, the letter in alphabet order are A, C, H, I, R, T.

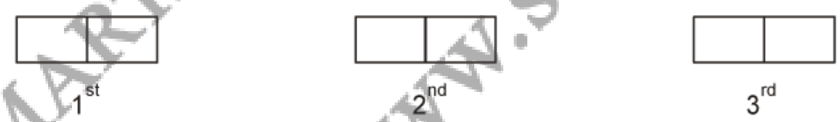
Now, in each case the number of words beginning with A, C, H, I are 5!

Now, next word will be RACHIT.

Hence, rank of the required word = $4 \times 5! + 1$

$$= 4 \times 120 + 1 = \mathbf{481}.$$

S24. Let us denote married couples by S_1, S_2, S_3, where each couple is considered to be a single unit as shown in the following figure:



Then the number of ways in which spouses can be seated next to each other is 3! = 6 ways.

Again each couple can be seated in 2! ways. Thus the total number of seating arrangement so that spouses sit next to each other = $3! \times 2! \times 2! \times 2! = 48$.

Again, if three ladies sit together, then necessarily three men must sit together. Thus, ladies and men can be arranged alltogether among themselves in 2! ways. Therefore, the total number of ways where ladies sit together is $3! \times 3! \times 2! = 72$.

S25. It is given that out 87 families, 52 families have at most 2 children so other 35 families are of other type. According to the question, for rural development programme, 20 families are to be chosen for assistane, of which at least 18 families must have atmost 2 children. Thus, the following are the number of possible choices:

$^{52}C_{18} \times {}^{35}C_2$ (18 families having atmost 2 children and 2 selected from other type of families)

$^{52}C_{19} \times {}^{35}C_1$ (19 families having atmost 2 children and 1 selected from other type of families)

$^{52}C_{20}$ (All selected 20 families having atmost 2 children)

Hence, the total number of possible choices is

$$^{52}C_{18} \times {}^{35}C_2 + {}^{52}C_{19} \times {}^{35}C_1 + {}^{52}C_{20}.$$

S26. Here, Number of girls = 4

Number of boys = 7

Total number of members = 11

(i) Number of teams when no girl is selected

$$= {}^7C_5$$

$$= {}^7C_2 = \frac{7 \times 6}{1 \times 2} = \mathbf{21}.$$

(ii) When atleast one boy and one girl is selected.

Boys	1	2	3	4
Girls	4	3	2	1

Number of such teams = $^7C_1 \times {}^4C_4 + {}^7C_2 \times {}^4C_3 + {}^7C_3 \times {}^4C_2 + {}^7C_4 \times {}^4C_1$

$= 7 \times 1 + 21 \times 4 + 35 \times 6 + 35 \times 4$

$= 7 + 84 + 210 + 140 = \mathbf{441}.$

(iii) When atleast three girls are included. Such cases are

Girls	3	4
Boys	2	1

Such number of teams = $^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1$

$$= 4 \times \frac{7 \times 6}{1 \times 2} + 1 \times 7$$

$= 84 + 7 = \mathbf{91}.$

S27. We can select sports team of 11 student in the following ways:

Class XI	5	6
Class XII	6	5

Number of ways of selecting teams

$$= {}^{20}C_5 \times {}^{20}C_6 + {}^{20}C_6 \times {}^{20}C_5$$

$$= \mathbf{2 \times ({}^{20}C_5 \times {}^{20}C_6)}.$$

S28. We have to select 11 players out of 16.

(i) When 2 particular players are selected, then we have to select 9 players of of 14.

∴ Number of teams = $\mathbf{{}^{14}C_9}$.

(ii) When we exclude 2 particular players, then we have to select 11 out of 14 players.

∴ Number of teams = $\mathbf{{}^{14}C_{11}}$.