Date: 21/10/2021

Q1. Define the capacitive reactance. Give its SI unit.

Q2. What do you the mean value of an alternating current ?

Q3. A reactive element, in an a.c. circuit, in an a.c. circuit, causes the current flowing (a) to lead in phase by $\pi/2$ (b) to lag in phase by $\pi/2$ w.r.t. the applied voltage. Identify the element in each case.

Q4. The instantaneous current from an a.c. source is $I = 6 \sin 314\, t$. What is the r.m.s. value of the current?

Q5. The instantaneous current from a.c. source is $I = 5 \sin 314\, t$. What is the peak value of current?

Q6. Why a capacitor blocks d.c. but allows a.c. to pass through it?

Q7. What is the frequency of direct current?

Q8. What is meant by the statement that the current through an inductor lags e.m.f. across it by $\pi/2$?

Q9. The current flowing through a pure inductor of inductance 4 mH is $I = 12 \cos 300\, t$ ampere. What is (a) r.m.s. and (b) average value of the current for a complete cycle?

Q10. Discuss the behaviour of an inductor in d.c. and high frequency a.c. circuits.

Q11. The frequency of a.c. is doubled. What happens to X_L?

Q12. What is the mean value of an alternating current over a complete cycle?

Q13. What is the function of a step up transformer?

Q14. Why a capacitor behaves like a perfect conductor for high frequency a.c.?

Q15. The peak value of e.m.f. in a.c. is E_0. Write its (a) r.m.s. and (b) average value over a complete cycle.

Q16. Why an inductor acts as a conductor for d.c.?

Q17. What is the phase relationship between current and voltage in an inductor?

Q18. Which value of the current do you measure with an a.c. ammeter?

Q19. Peak value of e.m.f. of an a.c. source is E_0. What is its r.m.s. value?

Q20. A d.c. voltmeter and d.c. ammeter cannot read a.c. Why?

Q21. Determine the reactance of a capacitance C at f Hz.

Q22. What is the relation between peak value and root mean square value of alternating current?

Q23. What is the root mean square value of an alternating current ?

Q24. Why the divisions marked on the scale of an a.c. ammeter are not equally spaced?

Q25. Why a galvanometer connected in an a.c. circuit does not show any deflection?

Q26. A 100 Ω resistor is connected to a 220 V, 50 Hz a.c. supply.

(a) What is the rms value of current in the circuit?

(b) What is the net power consumed over a full cycle?

Q27. With reference to alternating currents and voltages, state any one fundamental difference between 'resistance' and 'reactance'.

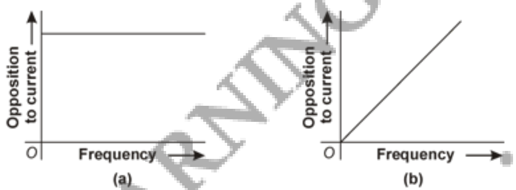
Q28. (a) The peak voltage of an a.c. supply is 300 V. What is the rms voltage?

(b) The r.m.s. value of current in an a.c. circuit is 10 A. What is the peak current?

Q29. In any a.c. circuit, is the applied instantaneous voltage equal to the algebraic sum of the instantaneous voltages across the series elements of the circuit? Is the same true for r.m.s. voltage?

Q30. A light bulb is rated at 100 W for a 220 V supply. Find (a) the resistance of the bulb; (b) the peak voltage of the source; and (c) the r.m.s. current through the bulb.

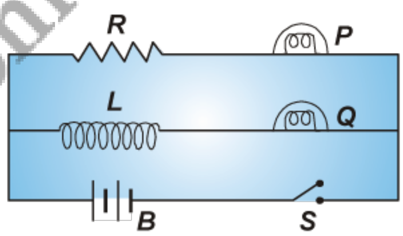
Q31. (a) Write the expression for the impedance offered by the series combinations of the two elements (below) connected across the a.c. Which will be ahead in phase in this circuit, voltage or current



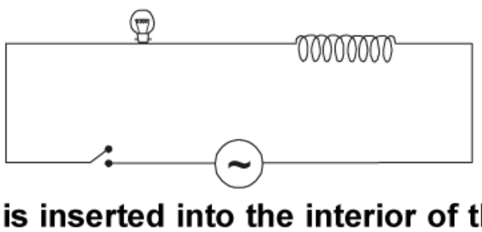
(b) The above graphs (a) and (b) represent the variation of the opposition offered by the circuit element to the flow of a.c. with frequency of the applied e.m.f. Identify the circuit element corresponding to each graph.

Q32. Show mathematically that the average value of a.c. over half cycle is 0.637 I_0.

Q33. The given figure shows and inductor *L* and resistor *R* connected in parallel to a battery *B* through a switch *S*. The resistance of *R* is the same as that of the coil that makes *L*. Two identical bulbs, *P* and *Q* are put in each arm of the circuit as shown in the figure. When *S* is closed, which of the two bulbs will light up earlier? Justify your answer.



Q34. A light bulb and an open coil inductor are connected to an a.c. source through a key as shown in the figure below.



The switch is closed and after sometime, an iron rod is inserted into the interior of the inductor. The glow of the light bulb (a) increase; (b) decreases; (c) is unchanged, as the iron rod is inserted. Give your answer with reason. What will be your answer if a.c. source is replaced by a d.c. source?

Q35. Draw the graph showing the variation of reactance of (a) a capacitor and (b) an inductor with the frequency of an a.c. circuit.

Q36. Explain the terms reactance and impedance as applied to components of an a.c. circuit.

Q37. An electric lamp connected in series with a variable capacitor and an a.c. source is glowing with some brightness. How will the brightness change on increasing the value of capacitance and why?

Q38. A capacitor blocks d.c. Why?

Q39. Prove mathematically that the average value of alternating current over one complete cycle is zero.

Q40. Distinguish between 'average value' and 'r.m.s.' value of an alternating current.

Q41. When an inductor L and resistor R in series are connected across a 12 V, 50 Hz supply, a current of 0.5 A flows in the circuit. The current differs in phase from applied voltage by $\pi/3$ radian. Calculate the value of R.

Q42. An alternating current of frequency f is applied across a series LCR-circuit. Let f_r be resonance frequency for the circuit. Will the current in the circuit lag, lead or remain in phase with the applied voltage, when (a) $f > f_r$ and (b) $f < f_r$? Explain the answer in each case.

Q43. When a capacitor is connected in series with a series LR-circuit, the alternating current flowing in the circuit increases. Explain, why

Q44. The total impedance of a circuit decreases, when a capacitor is added in series with L and R. Explain, why?

Q45. An air core coil and an electric bulb are connected in series across a 220 V – 50 Hz a.c. source. The bulb glows with some brightness. How will the glow of the bulb be affected on introducing a capacitor in series in the circuit? Explain your answer.

Q46. A resistor of 200 Ω and a capacitor of 15 μF are connected in series to a 220 V-50 Hz a.c. source. Calculate the current in the circuit and the r.m.s voltage across the resistor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

Q47. An a.c. source of 10 V (r.m.s.), 50 Hz is connected across a 20 ohm resistance and a 50 mH inductor in series. Calculate (a) impedance of the circuit and (b) r.m.s. current in the circuit.

Q48. A bulb of resistance 10 Ω, connected to an inductor of inductance L, is in series with an a.c. source marked 100 V-50 Hz. If the phase angle between the voltage and current is $\pi/4$ radian, calculate the value of L.

Q49. An alternating voltage given by $E = 280 \sin 50\pi t$ is connected across a pure resistor of 40 Ω. Find (a) frequency of the source and (b) r.m.s. current through the bulb.

Q50. When an alternating voltage of 220 V is applied across a device X, a current of 0.5 A flows through the circuit and is in phase with the applied voltage. When the same voltage is applied across another device Y, the same current again flows through the circuit, but it lags behind the applied voltage by $\pi/2$ radians. (a) Name the devices X and Y. (b) Calculate the current flowing in the circuit, when same voltage is applied across the series combination of X and Y.

Q51. A 30 μF capacitor is connected to a 220 V, 50 Hz source. Find its (a) capacitive reactance; (b) r.m.s. current and peak current; (c) impedance of the circuit.

Q52. Find the time required for a 50 Hz alternating current to change its value from zero to the r.m.s. value.

Q53. When a series combination of inductance and resistance is connected with a 10 V-50 Hz supply, a current of 1 A flows through the circuit. The voltage leads the current by a phase angle of π/3 radian. Calculate the values of inductance and resistance.

Q54. A light bulb is rated 200 W for a 220 V supply of 50 Hz. Calculate (a) resistance of the bulb and (b) r.m.s. current through the bulb.

Q55. In a series *R*-Circuit, $R = 30\ \Omega$, $C = 0.25\ \mu F$, $E = 100$ V and $\omega = 10{,}000\ \text{rad s}^{-1}$. Find the current in the circuit and calculate the voltage across the resistor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

Q56. An inductance coil has a reactance of 100 Ω. When a.c. signal of frequency 1,000 Hz is applied to the coil, the applied voltage leads the current by 45°. Calculate the self inductance of the coil.

Q57. The current through a coil of inductance 2 mH is represented by $I = 0.2 \sin 200\,t$ ampere. Calculate the maximum value of the induced current.

Q58. A sinusoidal voltage $E = 200 \sin 314\,t$ is applied to a resistor of 10 Ω resistance. Calculate (a) r.m.s. value of the voltage (b) r.m.s. value of the current and (c) power dissipated as heat in watt.

Q59. A current of 11 A flows through a coil, when connected to a 110 V d.c. When 110 V a.c. of 50 Hz is applied to the same coil, only 0.5 A current flows. Calculate the (a) resistance, (b) impedance and (c) inductance of the coil.

Q60. When a series combination of a coil of inductance *L* and a resistor of resistance *R* is connected across a 12 V-50 Hz supply, a current of 0.5 A flows through the circuit. The current differs in phase from applied voltage by π/3 radian. Calculate the value of inductance and resistance

Q61. Explain the term 'inductive reactance'. Show graphically the variation of inductive reactance with frequency of the applied alternating voltage.
An a.c. voltage $E = E_0 \sin \omega t$ is applied across a pure inductor of inductance *L*. Show mathematically that the current flowing through it lags behind the applied voltage by a phase angle of π/2.

Q62. Explain the term 'capacitive reactance'. Show graphically the variation of capacitive reactance with frequency of the applied alternating voltage.
An a.c. voltage $E = E_0 \sin \omega t$ applied across a pure capacitor of capacitance *C*. Show mathematically that the current flowing through it leads the applied voltage by a phase angel of π/2.

Q63. (a) Distinguish between the terms resistance, reactance and impedance of an a.c. circuit.
(b) A 100 μF capacitor in series with a 40 Ω resistance is connected to a 100 V, 60 Hz supply. Calculate (i) the reactance, (ii) the impedance, and (iii) maximum current in the circuit.

Date: 21/10/2021

S1. The resistance offered by a capacitor in an a.c. circuit is called capacitive reactance.

Its SI unit is **ohm**.

S2. It is that steady current, which when passed through a circuit for half the time period of the alternating current, sends the same amount of charge as is done by the alternating current in the same time through the same circuit.

S3. (a) Capacitor (b) Inductor

S4. Given: $I = 6 \sin 314\, t$ ... (i)

$I = I_0 \text{ sit } \omega t$ ... (ii)

Compare the Eq. (i) and (ii), we get

$I_0 = 6\text{A}$

$\therefore \quad I_{\text{r.m.s}} = \frac{I_0}{\sqrt{2}} = \frac{6}{\sqrt{2}} = \mathbf{3\sqrt{2}\ A}$

S5. Given: $I = 5 \sin 314\, t$ ... (i)

We know, $I = I_0 \sin \omega t$ ... (ii)

Compare the Eq. (i) and (ii), we get

$I_0 = \mathbf{5\ A}$

S6. Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}.$$

It follows that capacitive reactance is infinite for d.c. ($f = 0$) and has a finite value for a.c. In other words, a capacitor serves as a block for d.c. and offers an easy path to a.c.

S7. Zero.

S8. It means that if at any instant, the e.m.f is maximum (or minimum) in the circuit, then the current will become so after a time $T/4$, where T is the period of a.c.

S9. Given: $L = 4\text{ mH} = 4 \times 10^{-3}\text{ H}$

$I = 12 \cos 300\, t$

$I = I_0 \cos 300\, t$

$I_0 = 12$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{12}{\sqrt{2}} = 8.49 \text{ A}$$

I_{vag} = zero A (For one complete cycle)

S10. Inductive reactance,

$$X_L = \omega L = 2\pi f L$$

It follows that X_L is zero for d.c. ($f = 0$) and has a finite value for a.c. Hence, an inductor offers an easy path to d.c. and a resistive path to a.c.

S11. We know, $X_L = 2\pi f L$

Hence, on doubling the frequency of a.c. supply, X_L also becomes double.

S12. It is zero.

S13. The function of step-up transformer to make low voltage to high voltage.

S14. We know,

$$X_C = \frac{1}{2\pi f C}.$$

When frequency of a.c. supply is high, X_C will be approximately zero *i.e.*, capacitor will behave as a conductor.

S15. Given peak value of e.m.f E_0

$$I_{rms} = \frac{E_0}{\sqrt{2}}$$

The average value of complete cycle is zero

$$E_{avg} = 0$$

S16. For d.c. $f = 0$.

Hence, $X_L = 2\pi f L = 2\pi (0) L = 0$.

S17. The current lags behind the voltage by phase angle $\pi/2$.

S18. Root mean square value of the current.

S19. $E_{r.m.s} = \dfrac{E_0}{\sqrt{2}}$.

S20. It is because, average value of a.c. is zero over a complete cycle.

S21. Given: Capacitive (C) and frequency (f),

We know, capacitance reactance

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}.$$

S22. $I_{\text{r.m.s.}} = \frac{I_0}{\sqrt{2}}$

Where I_{rms} is root mean square value of current and I_0 is the peak value of the current.

S23. It is that steady current, which when passed through a resistance for a given time will produce the same amount of heat as the alternating current does in the same resistance and in the same time.

S24. An a.c. ammeter is constructed on the basis of heating effect of electric current. Since the heat produced varies as the square of current (and not directly with the current), the divisions marked on the scale are not equally spaced.

S25. The galvanometer measures mean value of alternating current, which is zero over a complete cycle.

S26. Resistance of the resistor, $R = 100\,\Omega$

Supply voltage, $V = 220\,\text{V}$

Frequency, $\nu = 50\text{ Hz}$

(a) The r.m.s. value of current in the circuit is given as

$$I = \frac{V_{rms}}{R} = \frac{220}{100} = 2.20\,\text{A}$$

(b) The net power consumed over a full cycle is given as:

$$P = E_v I_v \cos\phi$$

$\because$ $\cos\phi = 1$ case of resistor

$\therefore$ $P = VI = 220 \times 2.2 = 484\,\text{W}$

S27. An Ohmic resistor has fixed value of resistance, while reactance changes with the change in frequency of a.c. supply.

S28. (a) Peak voltage of the ac supply, $V_0 = 300$ V

R.m.s. voltage is given as:

$$V = \frac{V_0}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 212.1\,\text{V}$$

(b) The r.m.s. value of current is given as:

$$I = 10\,\text{A}$$

Now, peak current is given as:

$$I_0 = \sqrt{2}I = 10\sqrt{2} = 14.1\,\text{A}.$$

S29. Yes; the statement is not true for r.m.s. voltage.

It is true that in any ac circuit, the applied voltage is equal to the average sum of the instantaneous voltages across the series elements of the circuit. However, this is not true for rms voltage because voltages across different elements may not be in phase.

S30. (a) We are given $P = 100$ W and $V_{rms} = 220$ V.

The resistance of the bulb is

$$R = \frac{V^2}{P} = \frac{(220\text{ V})^2}{100\text{ W}} = 484\,\Omega$$

(b) The peak voltage of the source is

$$\nu_m = \sqrt{2}\,V_{rms} = 311\text{ V}$$

(c) Since, $P = I\,V$

$$I = \frac{P}{V} = \frac{100\text{ W}}{220\text{ V}} = 0.450\text{ A.}$$

S31. (a) In above figure (a), the opposition to the flow of current does not depend upon frequency, the circuit element is a resistor. In above figure (b), the opposition increase with frequency, the current element in an inductor.

(b) When the resistor R and the inductance L are connected in series across an a.c. source, then impedance Z of the circuit is given by

$$Z = \sqrt{R^2 + X_L^2}\,, \quad \text{where } X_L \text{ is inductive reactance.}$$

S32. Let an alternating current be represented by

$$I = I_0 \sin \omega t$$

Assuming the strength of current to be constant for a small time dt, the amount of charge that flows through the circuit during this time dt is

$$dq = I\,dt$$

The total charge (q) flowing through the circuit during the first half cycle (0 to $T/2$) is given by

$$q = \int_0^{\frac{T}{2}} I\,dt = \int_0^{\frac{T}{2}} I_0 \sin \omega t\,.\,dt$$

$$= I_0 \left| -\frac{\cos \omega t}{\omega} \right|_0^{\frac{T}{2}} = -\frac{I_0}{\omega}\left(\cos \frac{2\pi}{T}\cdot\frac{T}{2} - \cos 0^\circ\right)$$

$$= -\frac{I_0}{\omega}(\cos \pi - \cos 0)\,\frac{I_0}{\omega}(-1-1) = \frac{2I_0}{\omega}$$

If I_{av} is the average or mean value of current during first half cycle then

$$I_{av} = \frac{q}{t} = 2.\frac{q}{T} = 2.\frac{2I_0}{T}.\frac{T}{2\pi} = \frac{2}{\pi} I_0 \qquad \left(\because \quad \omega = \frac{2\pi}{T}\right)$$

or $$I_{av} = \frac{2}{\pi} I_0 = \mathbf{0.637}\, \boldsymbol{I_0}.$$

S33. When switch S is closed, bulb P will light up earlier. Bulb P is connected in series with a resistor. So the current in bulb P will instantly rise to its steady value. On the other hand, bulb Q will grow exponentially to its steady value which will be the same as for bulb P. This is due to the production of induced e.m.f. in the inductor. However, the steady state value of current will be the same in both the bulbs.

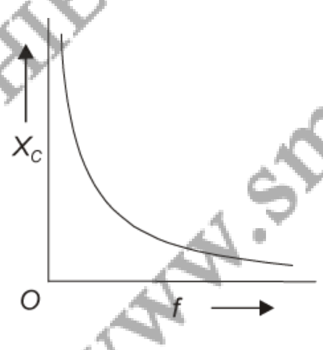
S34. As the iron rod is inserted, the magnetic field inside the coil magnetised the iron increasing the magnetic field inside it. Therefore, the inductance of the coil increases. Consequently, the inductive reactance of the coil increases. As a result, a larger fraction of the applied a.c. voltage appears across the inductor, leaving less voltage across the bulb. Hence, the glow of the light bulb decreases.

S35. (a) Reactance of a capacitor,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

or $$X_C \propto \frac{1}{f} \qquad (\because \; 2\pi C \text{ is constant for particular value of } C)$$

Graph between f and X_C will be as shown below

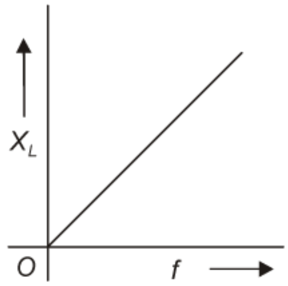


(b) Reactance of an inductor,

$$X_L = \omega L = 2\pi f L \qquad (\because \; 2\pi L \text{ is constant for particular value of } L)$$

$$X_L \propto f$$

Graph between f and X_L will be as shown below



S36. **Reactance:** The reactance of an a.c. circuit is the resistance offered by an inductor or a capacitor connected in the circuit. It arises because of the fact that the alternating e.m.f. and the current differ in phase by $\pi/2$, when a.c. flows through an inductor or a capacitor.

Impedance: The impedance of an a.c. circuit is the effective resistance offered by the *LR*-circuit or *CR*-circuit or *LCR* circuit.

S37. Capacitive reactance is given by,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}.$$

$$X_C \propto \frac{1}{C} \qquad (\because\ 2\pi f \text{ is constant})$$

When the capacitance (C) is increased, the capacitive reactance (X_C) will decrease. Due to decrease in the value of X_C, the current in the circuit will increase and hence brightness of the source will also increase.

S38. The capacitive reactance in any circuit is given by

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

For d.c., $f = 0$

$\therefore$ $X_C \text{ (for d.c.)} = \frac{1}{2\pi(0)C} = \infty\,\Omega.$

Hence, a capacitor offers infinite resistance to d.c. and hence acts as a perfect block for it.

S39. The, average value of alternating current over one complete cycle,

$$I = I_0 \sin \omega t$$

$$I_{avg} = \frac{1}{T}\int_0^T I_0 \sin \omega t\, dt = \frac{I_0}{T}\int_0^T \sin \omega t\, dt$$

$$= \frac{I_0}{T}\left[-\frac{\cos\omega t}{\omega}\right]_0^T = -\frac{I_0}{\omega T}[\cos\omega t]_0^T$$

$$= -\frac{I_0}{\omega T}\left[\cos\frac{2\pi}{T}\times T - \cos\frac{2\pi}{T}\times 0\right]$$

$$= -\frac{I}{\omega T}(\cos 2\pi - \cos 0) = -\frac{I}{\omega T}(1-1) = 0\,.$$

S40. **Average value** of an alternating current over any half cycle is defined as that value of steady (or direct) current which would send same amount of charge through a circuit when passed for the same time (*i.e.*, $\frac{T}{2}$) as is sent by the alternating current. It is given by

$$I_{av} = \frac{2}{\pi} I_0 = 0.637\, I_0$$

That is, average or mean value of a.c. over first (or positive) half cycle is 0.637 times its peak value. During the next half cycle (or negative half cycle, $\frac{T}{2}$ to T) the average value $I_{av} = -0.637\, I_0$. Therefore, average value of an a.c. over one complete cycle (= $0.637\, I_0 - 0.637\, I_0$) is zero.

R.M.S. Value of an alternating current is defined as that value of steady (or direct) current which would produce the same amount of heat in a given resistor in a given time, as is done by the a.c. when passed for the same time through the same resistor. It is equal to

$$I_{\text{r.m.s.}} = \frac{I_0}{\sqrt{2}} = 0.7070\, I_0.$$

That is the r.m.s, value of an a.c. is 0.707 times its peak value or 70.7% of its peak value.

It is known as *virtual value* or *effective value* of a.c.

S41. Given: $E_v = 12$ V; $f = 50$ Hz; $I = 0.5$ A ; $\phi = \pi/3$ rad

Now, current in the circuit, $I_v = \frac{E_v}{Z} = \frac{E_v}{\sqrt{R^2 + X_L^2}}$

or $$\sqrt{R^2 + X_L^2} = \frac{E}{I_v} = \frac{1.2}{0.5} = \mathbf{24\,\Omega} \qquad \text{... (i)}$$

Also, $$\tan\phi = \frac{X_L}{R}$$

or $$X_L = R\tan\phi = R\tan\pi/3 = R\times\sqrt{3} = \sqrt{3}\,R$$

In the Eqn. (i), substituting for X_L, we have

$$\sqrt{R^2 + (\sqrt{3}R)^2} = 24 \quad \text{or} \quad \sqrt{4R^2} = 24$$

or $$R = \mathbf{12\,\Omega}.$$

S42. We know that

$$X_L = 2\pi f L$$

i.e., $$X_L \propto f$$

and $$X_C = \frac{1}{2\pi f C}$$

i.e., $$X_C \propto \frac{1}{f}$$

As the frequency f is increased, X_L increases and X_C decreases. At the resonance frequency f_r, X_L becomes equal to X_C.

(a) **When $f > f_r$:** Then, X_L will be greater than X_C *i.e.*, the circuit will be inductive in nature. Therefore, current will **lag behind** the voltage.

(b) **When $f < f_r$:** The value of X_L is less than X_C *i.e.*, the circuit will be capacitive in nature. Hence current will **lead** the voltage.

S43. The impedance of *LR*-circuit,

$$Z_1 = \sqrt{R^2 + X_L^2}$$

So, current in *LR*-circuit,

$$I_1 = \frac{E_v}{Z_1} = \frac{E_v}{\sqrt{R^2 + X_L^2}}. \quad \text{... (i)}$$

When a capacitor is connected in series with *LR*-circuit, then

$$Z_2 = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

Therefore, current in the circuit,

$$I_2 = \frac{E_v}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \quad \text{... (ii)}$$

From the Eqn. (i) and (ii), it follows that $I_2 > I_1$. Hence, when a capacitor is connected in series to an *LR*-circuit, current in the circuit increases.

S44. The impedance of *LR*-circuit is given by

$$Z_1 = \sqrt{R^2 + X_L^2}$$

When capacitor is added in series with *L* and *R*, impedance of the circuit becomes

$$Z_2 = \sqrt{R^2 + \left(X_L - \frac{1}{X_C}\right)^2}$$

$$\because \qquad X_L > \left(X_L - \frac{1}{X_C}\right),$$

It follows that $Z_1 > Z_2$.

S45. The brilliance of the bulb depends upon the current through the bulb. Let *R* be the resistance of the filament of the bulb and *L*, the inductance of the coil. Then, current through the bulb,

$$I_{avg} = \frac{E}{\sqrt{R^2 + \omega^2 L^2}}$$

When capacitor is put in series, the current through the bulb becomes

$$I'_{avg} = \frac{E}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

Since $I'_{avg} > I_{avg}$, the bulb will glow more brightly on putting the capacitor in series.

S46. Given: $R = 200\,\Omega$; $C = 15\,\mu F = 15 \times 10^{-6}\,F$; $f = 50$ Hz; $E = 220$ V; $\omega = 2\pi \times 50 = 100\,\pi$.

The capacitive inductance

$$X_C = \frac{1}{2\pi f C} = \frac{1}{100\,\pi \times 15 \times 10^{-6}} = 2122.07\,\Omega$$

The impedance of the *RC*-circuit

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(200)^2 + (2122.07)^2} = 2131.47\,\Omega$$

Now, the current in *RC*-circuit given by

$$I = \frac{E}{Z} = \frac{220}{2131.4} = 0.103\,A$$

Now voltage across the resistor V_R is

$$V_R = IR = 0.103 \times 200 = 20.6\,V$$

Voltage across the capacitor is V_C

$$V_C = I \times X_C = 0.103 \times 2122.07 = \mathbf{218.57\,V}$$

$$V = V_C + V_R = 20.6 + 218.57 = \mathbf{239.17\,V}.$$

S47. Given: $E_v = 100\,\text{V}$; $f = 50\,\text{Hz}$; $R = 20\,\Omega$; $L = 50\,\text{mH} = 0.05\,\text{H}$

(a) $Z = \sqrt{R^2 + (2\pi f L)^2} = \sqrt{(20)^2 + (2\pi \times 50 \times 0.05)^2}$

$= \sqrt{400 + 246.74} = \mathbf{25.43\,\Omega}$

(b) $I_v = \dfrac{E_v}{Z} = \dfrac{100}{25.43} = \mathbf{3.93\,A}$.

S48. Given: $R = 10\,\Omega$ Inductance L; $\phi = \pi/4$; $f = 50\,\text{Hz}$; $V = 100\,\text{V}$.

We know $\tan\phi = \dfrac{X_L}{R} = \dfrac{\omega L}{R}$

$$L = \frac{\tan\phi \times R}{2\pi f} = \frac{\tan\left(\pi/4\right) \times 10}{2\pi \times 50}$$

or $L = \dfrac{1}{10\pi} = \mathbf{0.032\,H}$.

S49. Given; $E = 280 \sin 50\,\pi t$ ... (i)

$E = E_0 \sin \omega t$ ... (ii)

Compare the Eq. (i) w.r.t. to Eq. (ii), we get

$E_0 = 280\,\text{V}$; $\omega = 50\,\pi\,\text{Hz}$

$R = 40\,\Omega$.

(a) $f = \dfrac{\omega}{2\pi} = \dfrac{50\pi}{2\pi} = \mathbf{25\,Hz}$.

(b) $I_v = \dfrac{E_v}{Z} = \dfrac{280}{40} = 7$

$I_{\text{r.m.s.}} = \dfrac{I_v}{\sqrt{2}} = \dfrac{7}{\sqrt{2}} = \mathbf{4.95\,A}$.

S50. (a) The current and voltage are in phase with each other, when alternating voltage is applied across a resistor. Hence, the ***device X is a resistor***.

Obviously, $R = \frac{E_v}{I_v} = \frac{220}{0.5}$ = **440 Ω**.

The current lags behind the voltage by phase angle $\pi/2$, when alternating voltage is applied across an inductor. Therefore the ***device Y is an inductor***.

Obviously, $R = \frac{E_v}{I_v} = \frac{220}{0.5}$ = **440 Ω**.

(b) Given; $E_v = 220\,V$; $R = 440\,\Omega$; $X_L = 440\,\Omega$

If Z is impedance of the LR-circuit, then

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{440^2 + 440^2} = \mathbf{440\sqrt{2}\,\Omega}$$

Hence, current in the LR-circuit,

$$I_v = \frac{E_v}{Z} = \frac{220}{440\sqrt{2}} = \mathbf{0.354\,A}.$$

S51. Given: $C = 30\,\mu F = 30 \times 10^{-6} F$; $E_v = 220\,V$; $f = 50\,Hz$

(a) Capacitance reactance of the capacitor

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{100\pi \times 30 \times 10^{-6}} = 106.1 \text{ ohm}$$

(b) R.M.S. current I_v

$$I_v = \frac{E_v}{X_C} = \frac{220}{106.1} = 2.07\,A.$$

Peak current I_0

$$I_0 = \sqrt{2}\,I_v = \sqrt{2} \times 2.07 = 2.93\,A$$

(c) Impedance of the capacitor is

$$Z = X_C$$

$$Z = \mathbf{106.1\,\Omega}.$$

S52. Given, $I = I_0 \sin \omega t$ ... (i)

Suppose that when $t = t'$, $I = I_v = \frac{I_0}{\sqrt{2}}$

In the Eqn. (i), setting the above condition, we get

$$\frac{I_0}{\sqrt{2}} = I_0 \sin \omega t$$

or $$\sin \omega t = \frac{1}{\sqrt{2}} \quad \text{or} \quad \omega t' = \frac{\pi}{4}$$

or $$t' = \frac{\pi}{4\omega} = \frac{\pi}{4 \times 2\pi / T} = \frac{T}{8}$$

or $$t' = \frac{1}{8f} = \frac{1}{8 \times 50} = \mathbf{2.5 \times 10^{-3}\,s} \qquad \left(\because \; T = \frac{1}{f}\right)$$

S53. Given: $E_v = 10\,\text{V}$; $f = 50\,\text{Hz}$; $\omega = 100\,\pi$; $I = 1\,\text{A}$; $\phi = \pi/3$

Impedance of the circuit, $Z = \sqrt{R^2 + X_L^2}$

$$I_v = \frac{E_v}{Z} \Rightarrow Z = \frac{E_v}{I_V}$$

$$\sqrt{R^2 + (100\,\pi L)^2} = 10$$

$$R^2 + (100\,\pi L)^2 = 100 \qquad \text{...(i)}$$

$$\tan \phi = \frac{X_L}{R} \Rightarrow \tan\left(\frac{\pi}{3}\right) = \frac{100\,\pi L}{R}$$

$$\sqrt{3}\,R = 100\,\text{p}L \qquad \text{...(ii)}$$

$$R = \frac{100\,\pi L}{\sqrt{3}}$$

Put the value R in Eqn. (i), we get

$$\left(\frac{100\,\pi L}{\sqrt{3}}\right)^2 + (100\,\pi L)^2 = 100$$

$$L = 0.028\,\text{H}$$

Put the value L in Eqn. (ii), we get

$$R = 5\,\Omega.$$

S54. Given: $P = 200\,\text{W}$; $E_v = 220\,\text{V}$; $f = 50\,\text{Hz}$

$$P = VI \quad \Rightarrow \quad I = \frac{P}{V} = \frac{200}{220} = 0.91\,\text{A}$$

We know,

$$V = IR \quad \Rightarrow \quad R = \frac{V}{I} = \frac{220}{0.91}\,\Omega$$

$$R = \mathbf{242\,\Omega}.$$

S55. Given: $R = 30\,\Omega$, $C = 0.25\,\mu F = 0.25 \times 10^{-6}\,F$; $\omega = 10{,}000\ \text{rad s}^{-1}$ and $E = 100\,V$

The capacitive inductance,

$$X_C = \frac{1}{\omega C} = \frac{1}{10{,}000 \times 0.25 \times 10^{-6}} = 400\,\Omega$$

The impedance of R-C circuit,

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(30)^2 + (400)^2} = 401.12\,\Omega$$

Hence, current in the R-C circuit,

$$I = \frac{E}{Z} = \frac{100}{401.12} = 0.2493\,A$$

Now, voltage across R,

$$V_R = I \times R = 0.2493 \times 30 = 7.48\,V$$

and voltage across C,

$$V_C = I \times X_C = 0.2493 \times 400 = 99.72\,V$$

$\therefore$ $$V_R + V_C = 7.48 + 99.72 = 107.2\,V.$$

If follows that algebraic sum of V_R and V_C is more than the source voltage.

It is not correct to add V_R and V_C algebraically by treating them as scalars. In fact they are phasors. If the phase difference between them is taken into account, the sum of V_R and V_C will be equal to the source voltage.

S56. Since voltage applied across the inductance leads the current by 45° (and not by 90°), the given inductance is not a pure inductance.

The given inductance behaves as a series combination of L and R. If Z is impedance of the given inductance and ϕ is the phase angle by which applied voltage leads the current, then

$$Z = \sqrt{R^2 + X_L^2} \quad \text{... (i)}$$

and $$\tan\phi = \frac{X_L}{R} \quad \text{... (ii)}$$

Now, $Z = 100\,\Omega$; $\phi = 45°$

From, the Eqn. (i), we get

$$\sqrt{R^2 + X_L^2} = 100 \quad \text{... (iii)}$$

Again, from the Eqn. (ii), we get

$$\tan 45° = \frac{X_L}{R} \quad \text{or} \quad 1 = \frac{X_L}{R}$$

or $$R = X_L$$

Substituting for R in the Eqn. (iii), we get

$$\sqrt{X_L^2 + X_L^2} = 100 \quad \text{or} \quad \sqrt{2}\, X_L = 100$$

or $$X_L = 70.71\,\Omega$$

If L is self inductance of the coil, then

$$X_L = 2\pi f L$$

or $$L = \frac{X_L}{2\pi f} = \frac{70.71}{2\pi \times 1{,}000} = \mathbf{1.1254 \times 10^{-2}\,H}.$$

S57. Given: $L = 2\text{ mH} = 2 \times 10^{-3}\text{ H}$;

and $$I = 0.2 \sin 200\,t \quad \text{... (i)}$$

The instantaneous value of alternating current is given by

$$I = I_0 \sin \omega t \quad \text{... (ii)}$$

Comparing the Eqns. (i) and (ii), we get

$$\omega = 200\text{ rad s}^{-1}$$

The inductive reactance,

$$X_L = \omega L = 200 \times 2 \times 10^{-3} = 0.4\,\Omega$$

The induced e.m.f. (in magnitude) produced in the coil,

$$e = L\frac{dI}{dt} = 2 \times 10^{-3} \times \frac{d}{dt}(0.2 \sin 200\,t)$$

$$= 2 \times 10^{-3} \times 0.2 \times \cos 200\,t \times 200$$

$$= 0.08 \cos 200\,t$$

Hence, the maximum value of induced e.m.f.,

$$e_0 = 0.08\text{ V}$$

So, the maximum value of induced current,

$$I_0 = \frac{e_0}{X_L} = \frac{0.08}{0.4} = \mathbf{0.2\,A}.$$

S58. Given: $R = 10\ \Omega$

Instantaneous value of voltage,

$$E = 200 \sin \omega t \qquad \text{... (i)}$$

But instantaneous e.m.f. in an a.c. circuit,

$$E = E_0 \sin \omega t \qquad \text{... (ii)}$$

(a) Comparing the equations (i) and (ii), we get

$$E_0 = 200\,\text{V}$$

Hence, r.m.s value of voltage,

$$E_v = \frac{E_0}{\sqrt{2}} = \frac{200}{\sqrt{2}} = \mathbf{141.4\ V}$$

(b) The r.m.s value of current,

$$I_v = \frac{E_v}{R} = \frac{141.4}{10} = \mathbf{14.14\ A}$$

(c) Average power dissipated as heat,

$$P_{av} = E_v I_v = 141.4 \times 14.14 = \mathbf{2{,}000\ W}.$$

S59. Case I: When the coil applied d.c. voltage. Coil is behave as a resistor

We know, $V = IR$

$$R = \frac{V}{I} = \frac{110}{11} = \mathbf{10\ \Omega}.$$

Case II: When the a.c. source applied the circuit is

(a) given by

$$E_v = 110\,\text{V}; \qquad I_v = 0.5$$

$$Z = \frac{E_v}{I_v} = \frac{110}{0.5} = \mathbf{220\ \Omega}.$$

$\because$ $Z = X_L$.

(b) We know $X_L = 2\pi f L$

$\Rightarrow$ $$L = \frac{X_L}{2\pi f} = \frac{220}{100\pi} = \mathbf{0.74\ H}$$

S60.

$$I_v = \frac{E_v}{\sqrt{R^2 + (2\pi f L)^2}}$$

or $$0.5 = \frac{12}{\sqrt{R^2 + (2\pi \times 50 \times L)^2}}$$

or $$\sqrt{R^2 + (100\pi L)^2} = 24 \qquad \text{...( )}$$

Also, $$\tan\phi = \frac{\omega L}{R} = \frac{2\pi f L}{R}$$

or $$\tan\frac{\pi}{3} = \frac{2\pi \times 50 \times L}{R}$$

or $$\frac{100\pi L}{R} = \tan 60°$$

or $$\frac{100\pi L}{R} = \sqrt{3}$$

or $$100\,\pi L = \sqrt{3}\,R \qquad \text{... (ii)}$$

From the Eqn. (i) and (ii), we get

$$\sqrt{R^2 + (\sqrt{3}\,R)^2} = 24$$

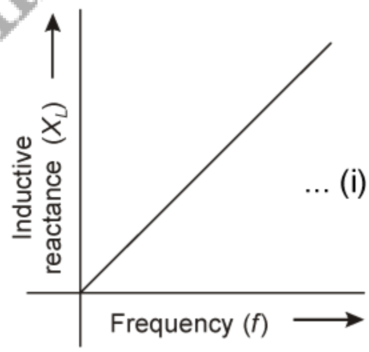
or $$R = \mathbf{12\,\Omega}.$$

From the Eqn. (ii), we get

$$L = \frac{\sqrt{3}\,R}{100\,\pi} = \frac{\sqrt{3} \times 12}{100\,\pi} = \mathbf{0.066\,H}.$$

S61. Inductive reactance is the opposition offered by an inductor towards the flow of current passing through it.

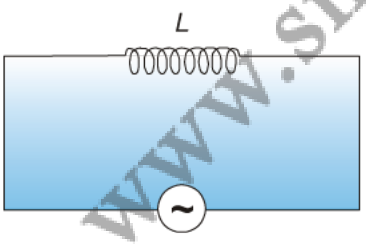
$$X_L = 2\pi f L$$



Applied a.c. voltage is

$$E = E_0 \sin \omega t \qquad \text{... (i)}$$

e.m.f. induced in the inductor is given by the relation



In order to maintain the flow of current through the inductor we must have

$$E = -E$$

i.e., $$E = L\frac{dI}{dt}$$

$$\frac{dI}{dt} = \frac{E}{L} = \frac{E_0}{L} \sin \omega t$$

$$\int dI = \frac{E_0}{L} \int \sin \omega t \, dt$$

$$I = \frac{E_0}{L\omega} (-\cos \omega t)$$

$$= \frac{E_0}{L\omega} \sin (\omega t - \pi/2)$$

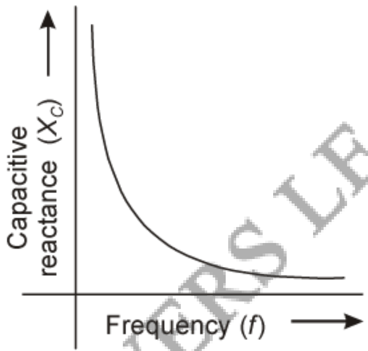
$$I = I_m \sin (\omega t - \pi/2) \quad \text{... (ii)}$$

where $$I_m = \frac{E_0}{L\omega} = \frac{E_0}{X_L} \quad (\because X_L = L\omega)$$

From equations (i) and (ii), we conclude that voltage leads the current by a phase angle $\pi/2$.

S62. Capacitive reactance is the opposition offered by a capacitor towards the flow of current passing through it.

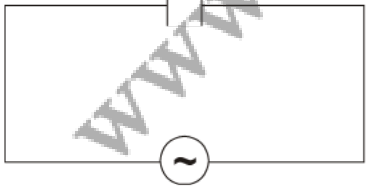
$$X_C = \frac{1}{2\pi f C}$$



Applied a.c. voltage is

$$E = E_0 \sin \omega t \quad \text{... (i)}$$

Let 'q' be the charge on the capacitor at any time 't'. The instantaneous voltage across the capacitor is



$$E = \frac{q}{C}$$

$$E_0 \sin \omega t = \frac{q}{C}$$

$$q = CE_0 \sin \omega t$$

$$\frac{dq}{dt} = CE_0 \omega \cos \omega t$$

$$I = \frac{E_0}{1/C\omega} \sin (\omega t + \pi/2)$$

$$= \frac{E_0}{X_C} \sin (\omega t + \pi/2)$$

$$I = I_0 \sin (\omega t + \pi/2) \qquad \text{... (ii)}$$

From equations (i) and (ii), we conclude that current leads the voltage by a phase angle $\pi/2$.

S63. (a) **Resistance** is the opposition towards the flow of current which does not depend upon frequency.

Reactance is the opposition offered by a component of the circuit which depends upon frequency of current.

Impedance is the combined effect of frequency dependent and frequency independent opposition offered by the components of a circuit towards the flow of current.

(b) Given: $C = 100\,\mu F = 10^{-4}\,F$; $R = 40\,\Omega$, $E_v = 100$ volt, $f = 60$ Hz

Formulas:

(i) $$X_C = \frac{1}{2\pi f C}$$

(ii) $$Z = \sqrt{R^2 + X_C^2}$$

(iii) $$I_0 = \sqrt{2}\left(\frac{E_v}{Z}\right)$$

Calculation:

(i) $$X_C = \frac{1}{2\pi \times 60 \times 10^{-4}}$$

$$X_C = 26.5\,\Omega.$$

(ii) $$Z = \sqrt{(40)^2 + (26.5)^2} = 48\,\Omega$$

(iii) $$I_0 = \frac{\sqrt{2} \times 100}{48} = 2.9\,A$$

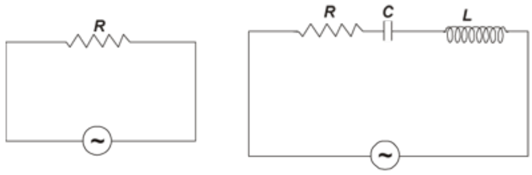
$$I_0 = \mathbf{3\,A}.$$

Date: 21/10/2021

Q1. Study the circuits (a) and (b) shown in the figure below and answer the following questions.

(a) Under which conditions would the r.m.s. currents in the two circuits be the same?

(b) Can the r.m.s. current in circuit (b) be larger than in (a)?



(a) (b)

Q2. What is the phase difference between the voltage across an inductor and a capacitor in an a.c. circuit?

Q3. A 15.0 μF capacitor is connected to a 220 V, 50 Hz source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and the current?

Q4. A lamp is connected in series with a capacitor. Predict your observations for d.c. and a.c. connections. What happens in each case if the capacitance of the capacitor is reduced?

Q5. A pure inductor of 25.0 mH is connected to a source of 220 V. Find the inductive reactance and r.m.s. current in the circuit if the frequency of the source is 50 Hz.

Q6. A 44 mH inductor is connected to 220 V, 50 Hz a.c. supply. Determine the r.m.s. value of the current in the circuit.

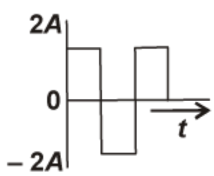
Q7. Sketch a graph showing the variation of inductive reactance with frequency of the applied voltage.

Q8. Define the phasor?

Q9. A bulb and a capacitor are connected in series to an a.c. source of variable frequency. How will the brightness of the bulb change on increasing the frequency of the a.c. source? Give reason.

Q10. What do you mean by the impedance of *LCR*-circuit?

Q11. Calculate the r.m.s value of the alternating current shown in the figure:



Q12. Sketch a graph showing the variation of reactance of a capacitor with frequency of the applied voltage.

Q13. A 60 μF capacitor is connected to a 110 V, 60 Hz a.c. supply. Determine the r.m.s. value of the current in the circuit.

Q14. An ideal resistor is connected across an a.c. source. Draw phasor diagram and show that current and voltage are in same phase.

Q15. **Obtain the answers (a) to (b) in Exercise 7.13 if the circuit is connected to a high frequency supply (240 V, 10 kHz). Hence, explain the statement that at very high frequency, an inductor in a circuit nearly amounts to an open circuit. How does an inductor behave in a dc circuit after the steady state?**

Q16. **A 100 V-50 Hz a.c. source is connected to a series combination of an inductance of 100 mH and a resistance of 25 ohm. Calculate the magnitude and phase of the current.**

Q17. **What is the value of current in the a.c. circuit containing $R = 10\ \Omega$, $C = 50\ \mu F$ in series across 200 V-50 Hz a.c. source?**

Q18. **Derive an expression for the impedance of a series *LCR*-circuit connected to an a.c. supply of variable frequency.**

Plot a graph showing variation of current with the frequency of the applied voltage. Explain briefly how the phenomenon of resonance in the circuit can be used in the tuning mechanism of a radio or a TV set.

Date: 21/10/2021

S1. [T].

S2. If we connect the capacitor in parallel in the main then power factor will be improve.

S3. It resides inside the inductor in the form of magnetic field.

S4. Magnetic energy analogous to kinetic energy and electrical energy analogous to potential energy.

S5. It is 1.

S6. The metal detector works on the principle of resonance in a.c. circuits. When you walk through a metal detector, you are, in fact, walking through a coil of many turns. The coil is connected to a capacitor tuned so that the circuit is in resonance. When you walk through with metal in your pocket, the impedance of the circuit changes – resulting in significant change in current in the circuit. This change in current is detected and the electronic circuitry causes a sound to be emitted as an alarm.

S7. A choke coil is needed in the use of fluorescent tubes with ac mains because it reduces the voltage across the tube without wasting much power. An ordinary resistor cannot be used instead of a choke coil for this purpose because it wastes power in the form of heat.

S8. Given:

Peak voltage of a.c. $V_m = ?$

$$V_m = \sqrt{2}\ V_{rms}$$
$$= \sqrt{2} \times 12 = \mathbf{16.97\ V}.$$

S9. Due to the resistance of the inductor, the LC-oscillations produced are damped one. It is because, during each oscillation, a part of electric energy is dissipated in the form of heat energy.

S10. Zero.

Explanation: $V = 230 \sin (\omega t + \pi/2)$

$I = 10 \sin (\omega t)$

Voltage is leading the current $\pi/2$

$\cos \pi/2 = 0$

Therefore, power dissipation is zero.

S11. Given power factor $\cos \phi = 0.5$

$\cos \phi = 1/2$

$$\phi = \cos^{-1}(1/2)$$

$$\boldsymbol{\phi = \pi/3}.$$

Phase difference $\pi/3$.

S12. No, power is dissipated across the resistance only.

S13. Given:

$$I = 10 \sin 314\,t\ \text{A}$$

$$V = 50 \sin (314 + {}^{\pi}/_{2})\ \text{V}$$

The phase difference between current and voltage is $^{\pi}/_{2}$.

Power dissipation is *zero* because phase difference between current and voltage is zero.

S14. The cosine of the phase angle between alternating current and e.m.f. in an a.c. circuit is called its power factor. Therefore,

power factor = $\cos \phi$.

S15. The maximum value of power factor is 1 and the minimum value is 0.

S16. The power consumed is zero in both the cases.

S17. In the inductive circuit,

R.m.s. value of current, $I = 15.92$ A

R.m.s. value of voltage, $V = 220$ V

Hence, the net power absorbed can be obtained by the relation,

$$P = VI \cos \Phi$$

Where,

Φ = Phase difference between V and I.

For a pure inductive circuit, the phase difference between alternating voltage and current is 90° *i.e.*, $\Phi = 90°$.

Hence, $P = 0$ *i.e.*, the net power is zero.

In the capacitive circuit,

R.m.s. value of current, $I = 2.49$ A

R.m.s. value of voltage, $V = 110$ V

Hence, the net power absorbed can be obtained as:

$$P = VI \cos \Phi$$

For a pure capacitive circuit, the phase difference between alternating voltage and current is 90° *i.e.*, $\Phi = 90°$.

Hence, $P = 0$ *i.e.*, the net power is zero.

S18. (a) The frequency at which the resonance occurs is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{25.48 \times 10^{-3} \times 796 \times 10^{-6}}}$$

`

$$= 222.1 \text{rad/s}$$

$$\nu_r = \frac{\omega_0}{2\pi} = \frac{211.1}{2 \times 3.14} \text{Hz} = 25.4 \text{ Hz}$$

(b) The impedance Z at resonant condition is equal to the resistance:

$$Z = R = 3\,\Omega$$

The r.m.s. current at resonance is

$$= \frac{V}{Z} = \frac{V}{R} = \left(\frac{283}{\sqrt{2}}\right)\frac{1}{3} = 66.7 \text{ A}$$

The power dissipated at resonance is

$$P = I^2 \times R = (66.7)^2 \times 3 = 13.35 \text{ kW}$$

You can see that in the present case, power dissipated at resonance is more than the power dissipated in Example 7.8.

S19. Let q_0 be the initial charge on a capacitor. Let the charged capacitor be connected to an inductor of inductance L. This LC circuit will sustain an oscillation with frequency

$$\omega = 2\pi f = \frac{1}{\sqrt{LC}}$$

At an instant t, charge q on the capacitor and the current I are given by:

$$q(t) = q_0 \cos \omega t$$

$$I(t) = \frac{dq(t)}{dt}$$

$$I(t) = -q_0 \omega \sin \omega t$$

Energy stored in the capacitor at time t is

$$U_E = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{q_0^2}{2C}\cos^2(\omega t)$$

Energy stored in the inductor at time t is

$$U_M = \frac{1}{2}LI^2$$

$$= \frac{1}{2}L q_0^2 \omega^2 \sin^2(\omega t)$$

$$= \frac{q_0^2}{2C} \sin^2 \omega t \qquad (\because \ \omega = 1/\sqrt{LC})$$

Sum of energies

$$U_E + U_M = \frac{q_0^2}{2C} (\cos^2 \omega t + \sin^2 \omega t) \qquad (\because \ \cos^2 \omega t + \sin^2 \omega t = 1)$$

$$= \mathbf{\frac{q_0^2}{2C}}.$$

This sum is constant in time as q_0 and C, both are time-independent.

S20. (a) Impedance,

$$Z = \frac{e_0}{I} = \frac{150}{10} = 15\,\Omega$$

(b) Phase angle between current and voltage is

$$\phi = \pi/8 - (-\pi/8) = \pi/4$$

Power factor = $\cos \phi = \cos \pi/4 = \frac{1}{\sqrt{2}}$.

S21. Average power of an a.c. circuit,

$$P_{av} = E_v I_v \cos \phi$$

For a.c. circuit containing an ideal capacitor, $\phi = -\pi/2$

$\therefore \qquad P_{av} = E_v I_v \cos(-\pi/2) = E_v I_v (0) = 0.$

S22. Average power of an a.c. circuit,

$$P_{av} = E_v I_v \cos \phi$$

For a.c. circuit containing an ideal inductor, $\phi = \pi/2$

$\therefore \qquad P_{av} = E_v I_v \cos \pi/2 = E_v I_v (0) = 0$

S23. Given: $I = 10 \sin 300\,t$; $I = I_0 \sin \omega t$; $I_0 = 10$; $E = 200 \sin 300\,t$; $E = E_0 \sin \omega t$; $E_0 = 200\,\text{V}$.

Average power of an a.c. circuit,

$$P_{av} = E_v I_v \cos \phi = \frac{E_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos \phi = \frac{E_0 I_0}{2} \cos \phi$$

Hence, $E_0 = 200\,\text{V}$, $I_0 = 10\,\text{A}$ and $\phi = 0°$

$\therefore$ $$P_{av} = \frac{200 \times 10}{2} \cos 0^\circ = 1,000 \times 1 = \mathbf{1,000\,W}.$$

S24. **Advantage:** The transmission of a.c. at higher voltage is economical as well as the loss of electric energy across the transmission line is low.

Disadvantage: The a.c. supply at higher voltage is more fatal and dangerous.

S25. Inductance of the inductor, $L = 0.50\,\text{H}$

Resistance of the resistor, $R = 100\,\Omega$

Potential of the supply voltage, $V = 240\,\text{V}$

Frequency of the supply, $\nu = 50\,\text{Hz}$

(a) Peak voltage is given as:

$$V_0 = \sqrt{2}V$$

$$= \sqrt{2} \times 240 = 339.41\,\text{V}$$

Angular frequency of the supply,

$$\omega = 2\,\pi\nu = 2\pi \times 50 = 100\pi \text{ rad/s}$$

Maximum current in the circuit is given as:

$$I_0 = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}}$$

$$= \frac{339.41}{\sqrt{(100)^2 + (100\pi)^2(0.50)^2}} = 1.82\,\text{A}$$

(b) Equation for voltage is given as:

$$V = V_0 \cos \omega t$$

Equation for current is given as:

$$I = I_0 \cos(\omega t - \Phi)$$

Where,

Φ = Phase difference between voltage and current

At time, $t = 0$.

$V = V_0$ (voltage is maximum)

For $\omega t - \Phi = 0$ *i.e.*, at time $t = \frac{\phi}{\omega}$,

$I = I_0$ (current is maximum)

Hence, the time lag between maximum voltage and maximum current is $\frac{\phi}{\omega}$.

Now, phase angle Φ is given by the relation,

$$\tan\phi = \frac{\omega L}{R} = \frac{2\pi \times 50 \times 0.5}{100} = 1.57$$

$$\phi = 57.5^\circ = \frac{57.5\pi}{180} \text{ rad}$$

$$\omega t = \frac{57.5\pi}{180}$$

$$t = \frac{57.5}{180 \times 2\pi \times 50} = 3.19 \times 10^{-3} \text{ s} = 3.2 \text{ ms}$$

Hence, the time lag between maximum voltage and maximum current is 3.2 ms.

S26. Average power transferred to the resistor = 788.44 W

Average power transferred to the capacitor = 0 W

Total power absorbed by the circuit = 788.44 W

Inductance of inductor, $L = 80 \text{ mH} = 80 \times 10^{-3} \text{ H}$

Capacitance of capacitor, $C = 60\ \mu\text{F} = 60 \times 10^{-6} \text{ F}$

Resistance of resistor, $R = 15\,\Omega$

Potential of voltage supply, $V = 230 \text{ V}$

Frequency of signal, $\nu = 50 \text{ Hz}$

Angular frequency of signal, $\omega = 2\pi\nu = 2\pi \times (50) = 100\text{À}$ rad/s

The elements are connected in series to each other. Hence, impedance of the circuit is given as:

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)}$$

$$= \sqrt{(15)^2 + \left(100\pi(80 \times 10^{-3}) - \frac{1}{(100\pi \times 60 \times 10^{-6})}\right)^2}$$

$$= \sqrt{(15)^2 + (25.12 - 53.08)^2} = 31.728\,\Omega$$

Current flowing in the circuit, $I = \frac{V}{Z} = \frac{230}{31.728} = 7.25 \text{ A}$.

Average power transferred to resistance is given as:

$$P_R = I_2 R = (7.25)^2 \times 15 = 788.44 \text{ W}$$

Average power transferred to capacitor,

P_C = Average power transferred to inductor,

$P_L = 0$

Total power absorbed by the circuit:

$$P_R + P_C + P_L = 788.44 + 0 + 0 = 788.44 \text{ W}$$

Hence, the total power absorbed by the circuit is 788.44 W.

S27. (a) To find the impedance of the circuit, we first calculate X_L and X_C.

Inductive reactance, $X_L = 2\pi\nu L$

$= 2 \times 3.14 \times 50 \times 25.48 \times 10^{-3}\,\Omega = 8\,\Omega$

Capacitive reactance, $X_C = \dfrac{1}{2\pi\nu C}$

$= \dfrac{1}{2 \times 3.14 \times 50 \times 796 \times 10^{-6}} = 4\,\Omega$

Therefore, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{3^2 + (8-4)^2} = 5\,\Omega$

(b) Phase difference, $\phi = \tan^{-1}\dfrac{X_C - X_L}{R}$

$= \tan^{-1}\left(\dfrac{4-8}{3}\right) = -53.1°$

Since ϕ is negative, the current in the circuit lags the voltage across the source.

(c) The power dissipated in the circuit is

$P = I^2 R$

Now, $I = \dfrac{i_m}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}\left(\dfrac{283}{5}\right) = 40\text{ A}$

Therefore, $P = (40\text{ A})^2 \times 3\,\Omega = 4800\text{ W}$

(d) Power factor = $\cos\phi = \cos 53.1° = 0.6$.

S28. Given: $E_v = 100\text{ V}$; $f = 50\text{ Hz}$; $C = 10\,\mu\text{F} = 10 \times 10^{-6} = 10^{-5}\text{ F}$; $R = 100\,\Omega$

(a) Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 10^{-5}} = \mathbf{318.31\,\Omega}.$$

(b) Current through the *CR*-circuit,

$$I_v = \frac{E_v}{\sqrt{R^2 + X_C^2}}$$

$$= \frac{100}{\sqrt{100^2 + (318.31)^2}} = \frac{100}{333.65} = \mathbf{0.3\,A}.$$

(c) Average power supplied,

$$P_{av} = E_v I_v \cos\phi = E_v I_v \cdot \frac{E_v}{\sqrt{R^2 + X_C^2}}$$

or $$P_{av} = 100 \times 0.3 \times \frac{100}{\sqrt{100^2 + (318.31)^2}} = \frac{0.3 \times 10^4}{333.65} = \mathbf{9\,W}.$$

S29. Given: $E_v = 200\,\text{V}$; $L = 5\,\text{H}$; $C = 80\,\mu\text{F} = 80 \times 10^{-6}\,\text{F}$ and $R = 40\,\Omega$

(a) For maximum current (or resonance) in the circuit,

$$X_L = X_C$$

Let ω_0 be the angular frequency of the source that provides maximum current in the circuit. Then,

$$\omega_0 L = \frac{1}{\omega_0 C} \quad \text{or} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

or $$\omega_0 = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = \mathbf{50\,rad\,s^{-1}}.$$

(b) At resonance, the impedance of *LCR*-circuit is equal to the ohmic resistance in the circuit. Hence, r.m.s. value of current at resonance,

$$I_v = \frac{E_v}{R} = \frac{200}{40} = \mathbf{5\,A}.$$

Therefore, current amplitude at resonance.

$$I_0 = \sqrt{2}\,I_v = \sqrt{2} \times 5 = \mathbf{7.07\,A}.$$

(c) Now $$P_{av} = E_v I_v \cos\phi.$$

Since at resonance, the circuit is purely resistive, $\phi = 0°$. Hence, average power consumed in the circuit,

$$P_{av} = 200 \times 5 \times \cos 0° = \mathbf{1{,}000\,W}.$$

S30. At resonance $Z = R$, and $\phi = 0°$

Thus, $Z = 20\,\Omega$

$$P_{av} = E_v I_v \cos\phi$$

$$= E_v \times \frac{E_v}{Z} \qquad [\because \cos\phi = 1]$$

$$= \frac{(200)^2}{20} = 2000\,\text{W}$$

$$P_{av} = \mathbf{2\,kW}.$$

S31. Given: $R = 12\,\Omega$; $X_C = 14\,\Omega$; $X_L = 30\,\Omega$, $E_v = 230\,\text{V}$; $f = 50\,\text{Hz}$.

Now, $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(12)^2 + (30-14)^2} = \mathbf{20\,\Omega}.$$

(a) $$I_v = \frac{E_v}{z} = \frac{230}{20} = \mathbf{11.5\,A}.$$

(b) $$\tan\phi = \frac{X_L - X_C}{Z} = \frac{30-14}{12} = \mathbf{1.33}$$

or $$\phi = \mathbf{53.13°}.$$

(c) Power factor,

$$\cos\phi = \frac{R}{Z} = \frac{12}{20} = \mathbf{0.6}. \quad \text{or} \quad \phi = \cos^{-1}(0.6) = \mathbf{53.13°}.$$

S32. Given: $C = 5\,\mu\text{F} = 5 \times 10^{-6}\,\text{F}$; $R = 10\,\Omega$; $f = 50\,\text{Hz}$; $\cos\phi = 1$.

The power factor of an a.c. circuit is unity, when the circuit is purely resistive *i.e.*,

i.e., $$X_L = X_C \quad \text{or} \quad 2\pi f L = \frac{1}{2\pi f C}$$

or $$L = \frac{1}{4\pi^2 f^2 C}$$

$$= \frac{1}{4\pi^2 \times (50)^2 \times 5 \times 10^{-6}} = \mathbf{2.026\ H.}$$

S33. Given: $R = 10\,\Omega$; $L = 800\,\text{mH} = 0.8\,\text{H}$ and $E = 200 \sin 300\,t$

$\therefore$ $$E_0 = 200\,\text{V} \quad \text{and} \quad \omega = 300\,\text{rad s}^{-1}$$

(a) Now, $$X_L = \omega L = 300 \times 0.8 = 240\,\Omega$$

$\therefore$ $$Z = \sqrt{R^2 + X_L^2} = \sqrt{(10)^2 + (240)^2} = \mathbf{240.21\,\Omega}.$$

(b) $$I_0 = \frac{E_0}{Z} = \frac{200}{240.21} = \mathbf{0.833\,A}.$$

(c) Power factor,

$$\cos\phi = \frac{R}{Z} = \frac{10}{240.21} = \mathbf{0.042}.$$

S34. (a) Let an alternating current of $I = I_m \sin \omega t$ be passing through a network of L, C and R creating a potential difference of $V = V_m \sin(\omega t \pm \phi)$ where ϕ is the phase difference. Then the power consumed is

$$P = VI$$

$$= V_m I_m \sin(\omega t \pm \phi) \sin \omega t$$

$$\therefore \quad P = V_m I_m (\sin \omega t \cos \phi \pm \cos \omega t \sin \phi) \sin \omega t$$

$$= V_m I_m \left(\sin^2 \omega t \cos \phi \pm \frac{1}{2} \sin 2\omega t \sin \phi\right)$$

$$P_{av} = \frac{\int_0^T P dt}{\int_0^T dt}$$

$$= \frac{V_m I_m}{T}\left[\int_0^T \sin^2 \omega t \cos \phi \, dt + \frac{1}{2}\int_0^T \sin \phi \sin 2\omega t \, dt\right]$$

$$= \frac{V_m I_m}{T}\left[\frac{T}{2}\cos \phi + 0\right] \quad \left[\because \int_0^T \sin^2 \omega t \, dt = \frac{T}{2} \text{ and } \int_0^T \sin 2\omega t \, dt = 0\right]$$

$$= \frac{V_m I_m}{2} \cos \phi$$

$$P_{av} = \boldsymbol{V_{rms} I_{rms} \cos \phi}.$$

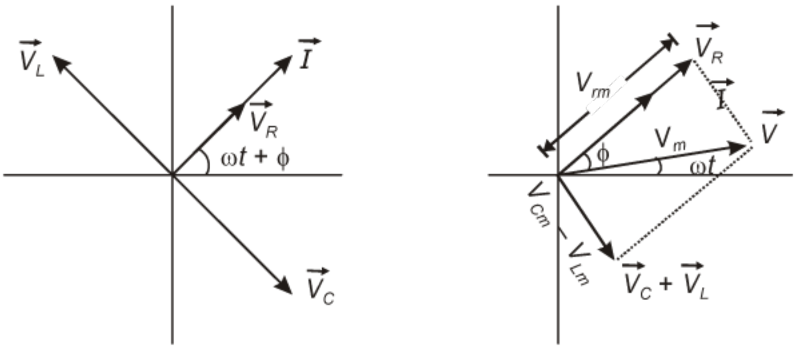
(b) Quality factor should be high to have the current corresponding to a particular frequency to be more and to avoid the other unwanted frequencies. Q-factor depends on f, L, R and C.

Sharpness of resonance is determined by Quality factor (Q) of the circuit i.e.,

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

Larger the value of Q, sharper is the resonance, *i.e.*, sharper peak in the current.

S35. (a) It is the combined effect of frequency dependent and frequency independent opposition offered by the components of LCR-circuit towards the flow of current



Mathematical expression for impedance:

Let the circuit be capacitive in nature

$$V_m^2 = V_{Rm}^2 + (V_{Cm} - V_{Lm})^2$$

$$V_m^2 = (I_0 R)^2 + I_m^2 (X_C - X_L)^2$$

$$V_m = \sqrt{R^2 + (X_C - X_L)^2}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}} = \frac{V_m}{Z} \qquad \left[\because Z = \sqrt{R^2 + (X_C - X_L)^2}\right]$$

Where Z is called impedance of the circuit.

(b) Given: $L = 2.0\,\text{H}$; $C = 32 \times 10^{-6}\,\text{F}$; $R = 10\,\Omega$

Formula: $\omega_r = \dfrac{1}{\sqrt{LC}}$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Calculation: $f_r = \dfrac{1}{2\pi\sqrt{2 \times 32 \times 10^{-6}}}$

$f_r =$ **19.0 Hz**.

S36. (a) **Resistance** is the opposition towards the flow of current which does not depend upon frequency.

Reactance is the opposition offered by a component of the circuit which depends upon frequency of current.

Impedance is the combined effect of frequency dependent and frequency independent opposition offered by the components of a circuit towards the flow of current.

(b) Given: $C = 100\,\mu F = 10^{-4}$ F; $R = 40\,\Omega$, $E_v = 100$ volt, $f = 60$ Hz

Formulas:

(i) $$X_C = \frac{1}{2\pi f C}$$

(ii) $$Z = \sqrt{R^2 + X_C^2}$$

(iii) $$I_0 = \sqrt{2}\left(\frac{E_v}{Z}\right)$$

Calculation:

(i) $$X_C = \frac{1}{2\pi \times 60 \times 10^{-4}}$$

$$X_C = 26.5\,\Omega.$$

(ii) $$Z = \sqrt{(40)^2 + (26.5)^2} = 48\,\Omega$$

(iii) $$I_0 = \frac{\sqrt{2} \times 100}{48} = 2.9\,\text{A}$$

$$I_0 = \mathbf{3\,A}.$$

S37. **Reactance:** The resistance of an a.c. circuit is the ohmic resistance offered by a conductor connected in the circuit. It is due to the nature of the material (resistivity) of the conductor.

Impedance: The impedance of an a.c. circuit is the effective resistance offered by the *LR*-circuit or *CR*-circuit or *LCR*-circuit.

Numerical: Given: For an *LR*-circuit

$$V_{rms} = 12\text{ volt}, \quad \phi = 50\text{ Hz},$$

$$I_{rms} = 0.5\text{ A} \quad \text{and} \quad \phi = \pi/3 \text{ radian}.$$

To find: *L* and *R*.

We know that $$\cos\phi = \frac{R}{Z}$$

Here, $$Z = \frac{V_{rms}}{I_{rms}} = \frac{12}{0.5}\,\Omega = 24\,\Omega$$

So, $$R = Z\cos\phi = 24\cos\left(\frac{\pi}{3}\right)\Omega = 12\,\Omega$$

$$Z^2 = R^2 + X_L^2$$

$$X_L^2 = Z^2 - R^2 = (24)^2 - (12)^2 = 432$$

$$X_L = 20.7\,\Omega$$
$$X_L = 2\pi f L$$
$$L = \frac{X_L}{2\pi f} = \frac{20.7}{2 \times 3.14 \times 50} = 0.06\ H$$

S38. (a) Given: $L = 80\text{ mH} = 80 \times 10^{-3}\text{ H}$; $C = 60\,\mu\text{F} = 60 \times 10^{-6}\text{ F}$; $f = 50\text{ Hz}$; $E_v = 230\text{ V}$.

$$I_v = \frac{E_v}{\left|\omega L - \frac{1}{\omega C}\right|}$$

$$= \frac{230}{|25.13 - 53.05|} = 8.24\text{ A}$$

–ve sign appears if $\omega L - \frac{1}{\omega C}$ and +ve sign

(b) $V_{C_{\text{r.m.s.}}} = 437\text{ V}$ (Use $V_{C_{\text{r.m.s.}}} = X_C I_{\text{r.m.s.}}$

$V_{L_{\text{r.m.s.}}} = 207\text{ V}$ $V_{L_{\text{r.m.s.}}} = X_L I_{\text{r.m.s.}}$)

The voltage across L and C gets subtracted because they are 180° out of phase and it is equal to applied r.m.s. voltage.

(c) In conductor, e.m.f. leads the current by $\pi/2$

$$P_{av} = E_v I_v \cos\frac{\pi}{2} = 0$$

Power transformed to the inductor = **0**.

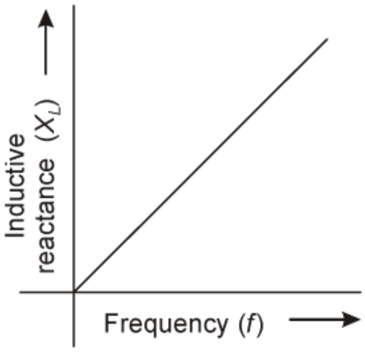
(d) As in capacitor, e.m.f. large behind the current by $\pi/2$

$$P_{av} = E_v I_v \cos\left(-\frac{\pi}{2}\right) = 0.$$

(e) Total average power absorbed = **0**.

S39. Inductive reactance is the opposition offered by an inductor towards the flow of current passing through it.

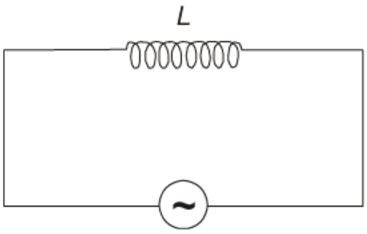
$$X_L = 2\pi f L$$



Applied a.c. voltage is

$$E = E_0 \sin \omega t \qquad \text{... (i)}$$

e.m.f. induced in the inductor is given by the relation



In order to maintain the flow of current through the inductor we must have

$$E = -E$$

i.e.,

$$E = L\frac{dI}{dt}$$

$$\frac{dI}{dt} = \frac{E}{L} = \frac{E_0}{L}\sin \omega t$$

$$\int dI = \frac{E_0}{L}\int \sin \omega t \, dt$$

$$I = \frac{E_0}{L\omega}(-\cos \omega t)$$

$$= \frac{E_0}{L\omega}\sin(\omega t - \pi/2)$$

$$I = I_m \sin(\omega t - \pi/2) \qquad \text{... (ii)}$$

where

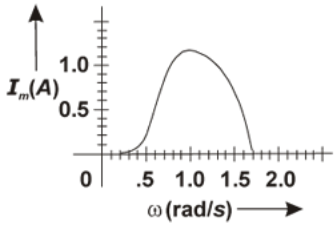
$$I_m = \frac{E_0}{L\omega} = \frac{E_0}{X_L} \qquad (\because\ X_L = L\omega)$$

From equations (i) and (ii), we conclude that voltage leads the current by a phase angle $\pi/2$.

Date: 21/10/2021

Q1. Why the algebraic sum of potential drops across the various elements in *LCR*-circuit is not equal to the applied voltage?

Q2. In series *LCR* circuit, the plot of I_{max} vs ω is shown in the figure below. Find the bandwidth and mark in the figure.



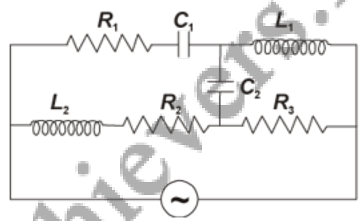
Q3. An applied voltage signal consists of a superposition of a d.c. voltage and an a.c. voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the d.c. signal will appear across *C* and the ac signal across *L*.

Q4. When does a series *LCR*-circuit have minimum impedance?

Q5. In a series *LCR* circuit, the voltages across an inductor, a capacitor and a resistor are 30 V, 30 V and 60 V respective. What is the phase difference between the applied voltage and the current in the circuit?

Q6. How the resonant frequency of a series *LCR*-circuit defined?

Q7. Draw the effective equivalent circuit of the circuit shown in the figure below, at very high frequencies and find the effective impedance.



Q8. In a series *LCR* circuit, the voltage across an inductor, capacitor and resistor are 20 V, 20 V and 40 V respectively. What is the phase difference between the applied voltage and the current in the circuit?

Q9. Give the phase difference between the applied a.c. voltage and the current in an *LCR*-circuit at resonance.

Q10. If the frequency of the a.c. source in a *LCR*-series circuit is increased, how does the current in the circuit change?

Q11. When are the voltage and current in *LCR*-circuit in same phase?

Q12. Suppose the initial charge on the capacitor in Exercise 7.7 is 6 mC. What is the total energy stored in the circuit initially? What is the total energy at later time?

Q13. Obtain the answers to (a) and (b) in Exercise 7.15 if the circuit is connected to a 110 V, 12 kHz supply? Hence, explain the statement that a capacitor is a conductor at very high frequencies. Compare this behaviour with that of a capacitor in a dc circuit after the steady state.

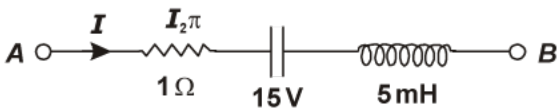
Q14. A 100 μF capacitor in series with a 40 Ω resistance is connected to a 110 V, 60 Hz supply. (a) What is the maximum current in the circuit? (b) What is the time lag between the current maximum and the voltage maximum?

Q15. A series *LCR* circuit with *R* = 20 Ω, *L* = 1.5 H and *C* = 35 μF is connected to a variable frequency 200 V a.c. supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Q16. Obtain the resonant frequency and *Q*-factor of a series *LCR* circuit with *L* = 3.0 H, *C* = 27 μF, and *R* = 7.4 Ω. It is desired to improve the sharpness of the resonance of the circuit by reducing its 'full width at half maximum' by a factor of 2. Suggest a suitable way.

Q17. A resistor of 200 Ω and a capacitor of 15.0 μF are connected in series to a 220 V, 50 Hz a.c. source. (a) Calculate the current in the circuit; (b) Calculate the voltage (r.m.s.) across the resistor and the capacitor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

Q18. The network shown in the figure is part of a complete circuit. If at a certain instant of time the current (*I*) is 5 A and is decreasing at a rate of 10^3 A s^{-1}, find out $V_B - V_A$.



Q19. An inductor 200 mH, a capacitor *C* and a resistor 10 ohm are connected in series with a 100 V, 50 s^{-1} a.c. source. If the current and voltage are in phase with each other, calculate the capacitance of the capacitor.

Q20. Mention the factors on which the resonant frequency of a series *LCR*-circuit depends. Plot a graph showing variation of impedance of a series *LCR*-circuit with the frequency of the applied a.c. source

Q21. (a) For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain.

(b) Power factor can often be improved by the use of a capacitor of appropriate capacitance in the circuit. Explain.

Q22. Calculate the quality factor of a series *LCR* circuit with *L* = 2.0 H, *C* = 2 μF and *R* = 10 Ω. Mention the significance of quality factor in *LCR* circuit.

Q23. State the condition under which the phenomenon of resonance occurs in a series *LCR* circuit. Plot a graph showing variation of current with frequency of a.c. source in a series *LCR* circuit.

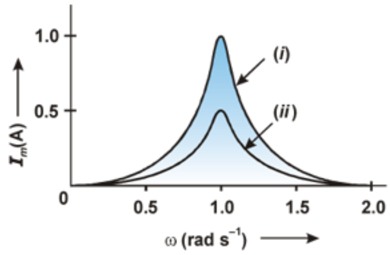
Q24. Obtain the resonant frequency ω_r of a series *LCR* circuit with *L* = 2.0 H, *C* = 32 μF and *R* = 15 Ω. What is the *Q*-value of this circuit?

Q25. (a) Draw the graphs showing variation of inductive reactance and capacitive reactance with frequency of applied a.c. source.

(b) Can the voltage drop across the inductor or the capacitor in a series *LCR* circuit be greater than the applied voltage of the a.c. source? Justify your answer.

Q26. What is the quality factor (*Q*) in an a.c. circuit?

Q27. The graphs shown the depict the variation of mean current I_m vs angular frequency ω for two different series *LCR*-circuits.



Observe the graphs carefully.

(a) State the relation between the *L* and *C* values of the two circuits, when the current in the two circuits is maximum.

(b) Indicate the circuit for which (i) power factor is higher and (ii) quality factor (*Q*) is higher.

Q28. A resistor of 50 ohm, an inductor of $20/\pi$ Henry and a capacitor of $5/\pi$ microfarad are connected in series to a voltage source 230 V-50 Hz. Find the impedance of the circuit.

Q29. A series *LCR*-circuit with $L = 0.12$ H, $C = 0.48 \times 10^{-7}$ F, and $R = 23$ ohm is connected to a variable frequency supply. At what frequency is the current maximum?

Q30. A resistor, a capacitor of 100 μF capacitance and an inductor are in series with an a.c. source of frequency 50 Hz. If the current in the circuit is in phase with the voltage, calculate the inductance of the inductor used.

Q31. A radio can tune over the frequency range of a portion of MW broadcast band: (800 kHz to 1200 kHz). If its *LC* circuit has an effective inductance of 200 μH, what must be the range of its variable capacitor?

[*Hint:* For tuning, the natural frequency *i.e.*, the frequency of free oscillations of the *LC* circuit should be equal to the frequency of the radio wave.]

Q32. Keeping the source frequency equal to the resonating frequency of the series *LCR* circuit, if the three elements, *L*, *C* and *R* are arranged in parallel, show that the total current in the parallel *LCR* circuit is minimum at this frequency. Obtain the current rms value in each branch of the circuit for the elements and source specified in Exercise 7.11 for this frequency.

Q33. A series *LCR*-circuit having $L = 10$ mH, $C = (400/\pi^2)$ μF and $R = 55\,\Omega$ is connected to a 220 V variable frequency a.c. supply.

(a) Find the frequency of the source, for which the average power absorbed by the circuit is maximum.

(b) Calculate the value of maximum current amplitude.

Q34. When an electric derive *X* is connected to a 220 V-50 Hz a.c. supply, the current is 0.5 A and is in same phase as the applied voltage. When another device *Y* is connected to the same supply, the electric current is again 0.5 A, but it leads the potential difference by $\pi/2$. (a) What are the devices *X* and *Y*? (b) When *X* and *Y* are connected in series across the same source, what will be the current?

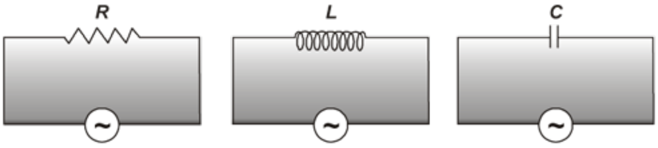
Q35. An inductor *L*, a capacitor 20 μF and a resistor 10 Ω are connected in series with an a.c. source of frequency 50 Hz. If the current is in phase with the voltage. Calculate the inductance of the inductor.

Q36. A 100 mH inductor, a 25 μF capacitor and a 15 Ω resistor are connected in series to a 120 V-50 Hz a.c. source. Calculate (a) impedance of the circuit at resonance; (b) current at resonance and (c) resonant frequency.

Q37. An alternating e.m.f. of 110 V is applied to a circuit containing a resistance of 40 Ω and an inductance *L* in series. The current is found to lag behind the voltage by an angle $\phi = \tan^{-1} 3/4$. Find the (a) inductive reactance; (b) impedance of the circuit and (c) current flowing in the circuit. If the inductance has a value of 0.1 H, find the frequency of the applied e.m.f.

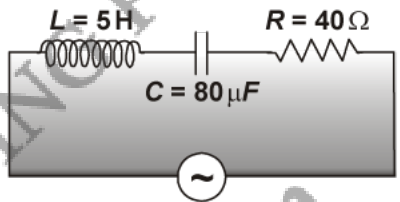
Q38. Define the term impedance of an a.c. circuit. How does the total impedance of a series *LCR*-circuit change if the frequency of the applied a.c. supply is increased and why?

Q39. (a) What do you understand by sharpness of resonance in series *LCR*-circuit? Derive an expression for *Q*-factor of the circuit



(b) Three electrical circuits having a.c. sources of variable frequency are shown in the figure. Initially the current flowing in each of these is same. If the frequency of the applied a.c. source is increased, how will the current flowing in these circuits be affected? Five reason for your answer.

Q40. The circuit as shown in the figure, a series *LCR*-circuit (*L* = 5 H, *C* = 80 μF and *R* = 40 Ω) connected to a variable frequency 230 V source.



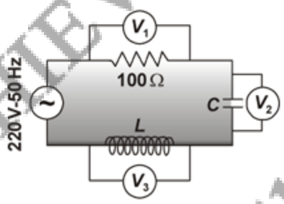
(a) Determine the source frequency, which drives the circuit in resonance.

(b) Obtain the impedance of the circuit and the amplitude of current at the resonating frequency.

(c) Determine the r.m.s. potential drop across the three elements of the circuit.

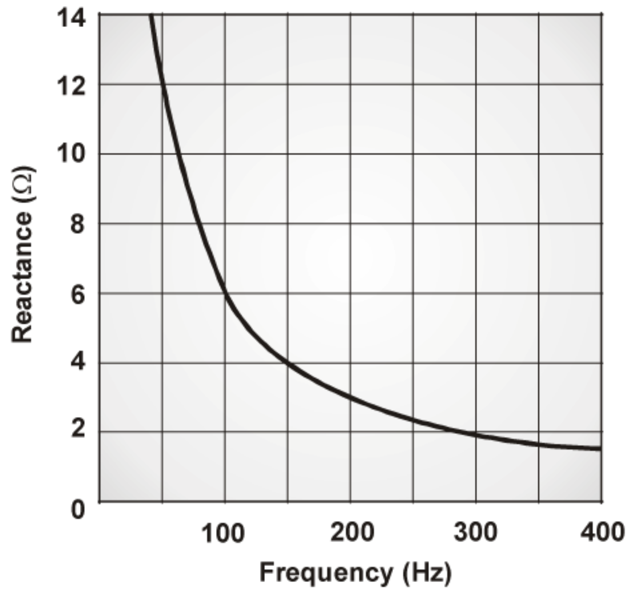
(d) How do you explain the observation that the algebraic sum of the voltages across the three elements obtained in (c) is greater than the supplied voltage?

Q41. A series *LCR*-circuit is connected to an a.c. source (220 V-50 Hz) as shown in figure below.



If the readings of the three voltmeter V_1, V_2 and V_3 are 65 V, 415 V and 204 V respectively, calculate (a) the current in the circuit; (b) the value of the inductor *L*; (c) the value of the capacitor *C*; (d) the value of C (for the same *L*) required to produce resonance.

Q42. Figure given below shows how the reactance of a capacitor varies with frequency.



(a) Use the information on graph to calculate the value of capacity of the capacitor.

(b) An inductor of inductance '*L*' has the same reactance as the capacitor at 100 Hz. Find the value of *L*.

(c) Using the same axes, draw a graph of reactance against frequency for the inductor given in part (b).

(d) If this capacitor and inductor were connected in series to a resistor of 10 Ω, what would be the impedance of the combination at 300 Hz?

Date: 21/10/2021

S1. The voltage across the different elements of the *LCR*-circuit are not in same phase.

S2. Bandwidth corresponds to frequencies at which

$$I_m = \frac{1}{\sqrt{2}} I_{\max} \approx 0.7\, I_{\max}.$$

S3. The d.c. signal will appear across capacitor *C* because for dc signals, the impedance of an inductor (L) is negligible while the impedance of a capacitor (*C*) is very high (almost infinite). Hence, a d.c. signal appears across *C*. For an a.c. signal of high frequency, the impedance of *L* is high and that of *C* is very low. Hence, an a.c. signal of high frequency appears across *L*.

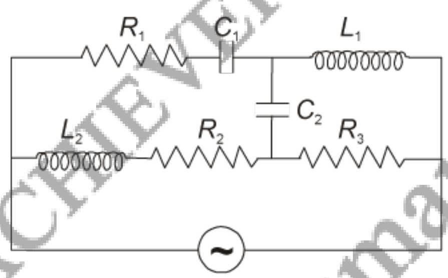
S4. At resonance.

S5. The phase difference of between voltage and current is zero $V_L = V_C$ is given it means circuit will be pure resistive nature.

S6. The frequency of the a.c. source, at which a series *LCR*-circuit, admits maximum current, is called the **resonant frequency**.

At the resonant frequency, the reactance of the circuit is zero. In other words, the impedance of the circuit is equal to *R*.

S7. At high frequencies, capacitor ≈ short circuit (low reactance) and inductor ≈ open circuit (high reactance). Therefore, the equivalent circuit $Z \approx R_1 + R_3$ as shown in the figure below.



S8. Zero.

Explanation: Given

$V_L = 20, \quad V_C = 20, \quad V_R = 40$

$V_L = V_C$

It means circuit will be resistive

Therefore, phase difference is **Zero**.

S9. It is zero.

S10. With increase in frequency, current in a.c. first increases, attains a maximum value (at resonant frequency) and then decreases.

S11. The voltage and current in an *LCR*-circuit are in same phase, when $X_L = X_C$.

S12. Capacitance of the capacitor, $C = 30\text{ pF} = 30\times10^{-6}\text{ pF}$

Inductance of the inductor, $L = 27\text{ mH} = 27\times10^{-3}\text{ pH}$

Charge on the capacitor, $Q = 6\text{ mC} = 6\times10^{-3}\text{ C}$

Total energy stored in the capacitor can be calculated as:

$$E = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\times\frac{(6\times10^{-3})^2}{30\times10^{-6}} = \frac{6}{10} = 0.6\text{ J}$$

Total energy at a later time will remain the same because energy is shared between the capacitor and the inductor.

S13. Capacitance of the capacitor, $C = 100\ \mu\text{F} = 100\times10^{-6}\text{ F}$

Resistance of the resistor, $R = 40\ \Omega$

Supply voltage, $V_{rms} = 110\text{ V}$

Frequency of the supply, $\nu = 12\text{ kHz} = 12\times10^3\text{ Hz}$

Angular Frequency, $\omega = 2\pi\nu = 2\times\pi\times12\times10^3 03$

$= 24\pi\times10^3\text{ rad/s}$

Peak voltage, $V_0 = V\sqrt{2} = 110\sqrt{2}\text{ V}$

Maximum current, $$I_0 = \frac{V_0}{\sqrt{R^2+\frac{1}{\omega^2C^2}}}$$

$$= \frac{110\sqrt{2}}{\sqrt{(40)^2+\frac{1}{(24\pi\times10^3\times100\times10^{-6})^2}}}$$

$$= \frac{110\sqrt{2}}{\sqrt{1600+\left(\frac{10}{24\pi}\right)^2}} = 3.9\text{ A}$$

For an *RC* circuit, the voltage lags behind the current by a phase angle of Φ given as:

$$\therefore \qquad \tan\phi = \frac{\frac{1}{\omega C}}{R} = \frac{1}{\omega CR}$$

$$= \frac{1}{24\pi\times10^3\times100\times10^{-6}\times40}$$

$$\tan\ \phi = \frac{1}{96\pi}$$

$\therefore$ $$\phi = 0.2° = \frac{0.2\pi}{180}\text{ rad}$$

$\therefore$ $$\text{Time lag} = \frac{\phi}{\omega} = \frac{0.2\pi}{180 \times 24\pi \times 10^3}$$

$$= 1.55 \times 10^{-3}\text{ s} = 0.04\ \mu\text{s}$$

Hence, Φ tends to become zero at high frequencies. At a high frequency, capacitor *C* acts as a conductor.

In a d.c. circuit, after the steady state is achieved, $\omega = 0$. Hence, capacitor *C* amounts to an open circuit.

S14. Capacitance of the capacitor, $C = 100\ \mu\text{F} = 100 \times 10^{-6}\text{ F}$

Resistance of the resistor, $R = 40\ \Omega$

Supply voltage, $V = 110\text{ V}$

(a) Frequency of oscillations, $\nu = 60\text{ Hz}$

Angular frequency, $\omega = 2\pi\nu = 2\pi \times 60\text{ rad/s}$

For a *RC* circuit, we have the relation for impedance as:

$$Z = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

Peak voltage, $V_0 = V\sqrt{2} = 110\sqrt{2}\text{ V}$

Maximum current is given as:

$$I_0 = \frac{V_0}{Z}$$

$$= \frac{V_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}}$$

$$= \frac{110\sqrt{2}}{\sqrt{(40)^2 + \frac{1}{(120\pi)^2 \times (10^{-4})^2}}}$$

$$= \frac{110\sqrt{2}}{\sqrt{1600 + \frac{10^8}{(120\pi)^2}}} = 3.24\text{ A}$$

(b) In a capacitor circuit, the voltage lags behind the current by a phase angle of Φ. This angle is given by the relation:

$\therefore$ $$\tan\phi = \frac{\frac{1}{\omega C}}{R} = \frac{1}{\omega CR}$$

$$= \frac{1}{120\pi \times 10^{-4} \times 40} = 0.6635$$

$$\phi = \tan^{-1}(0.6635) = 33.56°$$

$$= \frac{33.56\pi}{180}\text{ rad}$$

$\therefore$ $$\text{Time lag} = \frac{\phi}{\omega} = \frac{33.56\pi}{180 \times 120\pi}$$

$$= 1.55 \times 10^{-3}\text{ s} = 1.55\text{ ms}$$

Hence, the time lag between maximum current and maximum voltage is 1.55 ms.

S15. At resonance, the frequency of the supply power equals the natural frequency of the given *LCR* circuit.

Resistance, $R = 20\,\Omega$

Inductance, $L = 1.5\text{ H}$

Capacitance, $C = 35\,\mu\text{F} = 35 \times 10^{-6}\text{ F}$

A.C. supply voltage to the *LCR* circuit, $V = 200\text{ V}$

Impedance of the circuit is given by the relation,

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

At resonance, $\omega L = \frac{1}{\omega C}$

$\therefore$ $Z = R = 20\,\Omega$

Current in the circuit can be calculated as:

$$I = \frac{V}{Z} = \frac{200}{20} = 10\text{ A}$$

Hence, the average power transferred to the circuit in one complete cycle:

$$VI = 200 \times 10 = 2000\text{ W}.$$

S16. Inductance, $L = 3.0\text{ H}$

Capacitance, $C = 27\,\mu\text{F} = 27 \times 10^{-6}\text{ F}$

Resistance, $R = 7.4\,\Omega$

At resonance, angular frequency of the source for the given *LCR* series circuit is given as:

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{3\times 27\times 10^{-6}}} = \frac{10^3}{9} \text{ rad s}^{-1}$$

Q-factor of the series:

$$Q = \frac{\omega_r L}{R} = \frac{111.11\times 3}{7.4} = 45.0446$$

To improve the sharpness of the resonance by reducing its 'full width at half maximum' by a factor of 2 without changing ω_r, we need to reduce *R* to half *i.e.*,

$$\text{Resistance} = \frac{R}{2} = \frac{7.4}{2} = 3.7\,\Omega.$$

S17. Given, $R = 200\ \Omega$, $C = 15.0\ \mu\text{F} = 15.0\times 10^{-6}\,\text{F}$

$V = 220\,\text{V}$, $\nu = 50\text{Hz}$

(a) In order to calculate the current, we need the impedance of the circuit. It is

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + (2\pi\nu C)^{-2}}$$

$$= \sqrt{(200\ \Omega)^2 + (2\times 3.14\times 50\times 10^{-6}\text{F})^{-2}}$$

$$= \sqrt{(200\ \Omega)^2 + (212\ \Omega)^{-2}} = 291.5\,\Omega$$

Therefore, the current in the circuit is

$$I = \frac{V}{Z} = \frac{220\ \text{V}}{291.5\ \Omega} = 0.755\ \text{A}$$

(b) Since the current is the same throughout the circuit, we have

$$V_R = IR = (0.755\ \text{A})(200\,\Omega) = 151\ \text{V}$$

$$V_C = IX_C = (0.755\ \text{A})(212.3\,\Omega) = 160.3\,\text{V}$$

The algebraic sum of the two voltages, V_R and V_C is 311.3 V which is more than the source voltage of 220 V. How to resolve this paradox? As you have learnt in the text, the two voltages are not in the same phase. Therefore, *they cannot be added like ordinary numbers.* The two voltages are out of phase by ninety degrees. Therefore, the total of these voltages must be obtained using the Pythagorean theorem:

$$V_{R+C} = \sqrt{V_R^2 + V_C^2} = 220\ \text{V}.$$

Thus, if the phase difference between two voltages is properly taken into account, the total voltage across the resistor and the capacitor is equal to the voltage of the source.

S18. As current through the inductor is decreasing at a rate $\frac{dI}{dt} = 10^3\ A/s$. So, the e.m.f. is induced in the inductor with a polarity to support the current flowing through it. The e.m.f. induced in the inductor is

$$|e| = L\frac{dI}{dt} = 5\times 10^{-3}\times 10^3 = 5 \text{ volt}$$

Now, $V_A - 5 + 15 + 5 = V_B$

$$V_A - V_B = -15$$

or $V_B - V_A = 15$ volt.

S19. Given: $L = 200 \times 10^{-3}$ H; $R = 10\,\Omega$; $E_{r.m.s.} = 100$ V; $\omega_r = 50\,s^{-1}$.

As current and voltage are in phase with each other circuit is resistive in nature

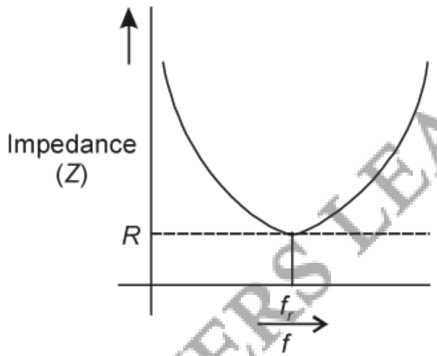
i.e., $Z = R$

and $X_L = X_C$

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$C = \frac{1}{L\omega_r^2} = \mathbf{2 \times 10^{-3}\,F}.$$

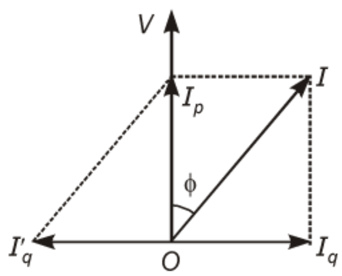
S20. Resonant frequency depends upon the value of inductor (L) and capacitor(C)



S21. (a) We know that $P = IV \cos \phi$ where $\cos \phi$ is the power factor. To supply a given power at a given voltage, if $\cos \phi$ is small, we have to increase current accordingly. But this will lead to large power loss (I^2R) in transmission.

(b) Suppose in a circuit, current I lags the voltage by an angle ϕ. Then power factor $\cos \phi = R/Z$.

We can improve the power factor (tending to 1) by making Z tend to R. Let us understand, with the help of a phasor diagram (see figure) how this can be achieved. Let us resolve $\boldsymbol{I}$ into two components. $\boldsymbol{I}_p$ along the applied voltage $\boldsymbol{V}$ and $\boldsymbol{I}_q$ perpendicular to the applied voltage. $\boldsymbol{I}_q$ as you have learnt in Section 7.7, is called the wattless component since corresponding to this component of current, there is no power loss. $\boldsymbol{I}_P$ is known as the power component because it is in phase with the voltage and corresponds to power loss in the circuit.



It's clear from this analysis that if we want to improve power factor, we must completely neutralize the lagging wattless current $\boldsymbol{I}_q$ by an equal leading wattless current $\boldsymbol{I}'_q$. This can be done by connecting a capacitor of appropriate value in parallel so that $\boldsymbol{I}_q$ and $\boldsymbol{I}'_q$ cancel each other and P is effectively $I_p V$.

S22. Given: $L = 2.0\,\text{H}$; $C = 2\,\mu\text{F}$; $R = 10\,\mu\Omega$

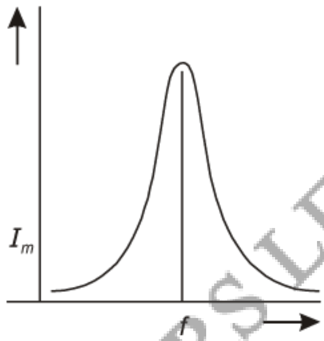
$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{10}\sqrt{\frac{2}{2\times 10^{-6}}} = \mathbf{100}.$$

It signifies the sharpness of the resonance.

S23. On increasing frequency inductive reactance (X_L) increases and capacitive reactance (X_C) decreases. For a particular value of frequency X_L and X_C become equal. This frequency is known as resonance frequency and LCR circuit is said to be in resonance.

As $\qquad X_L = X_C$



$$2\pi f_r L = \frac{1}{2\pi f_r L}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}.$$

S24. Inductance, $\qquad L = 2.0\,\text{H}$

Capacitance, $\qquad C = 32\,\mu\text{F} = 32\times 10^{-6}\,\text{F}$

Resistance, $\qquad R = 15\,\Omega$

Resonant frequency is given by the relation,

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{2 \times 32 \times 10^{-6}}} = \frac{1}{8 \times 10^{-3}} = \mathbf{125\ s^{-1}}$$

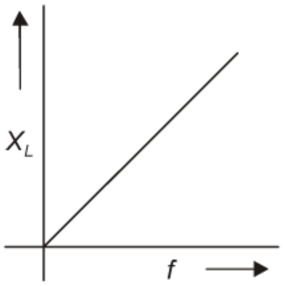
Now, Q-value of the circuit is given as:

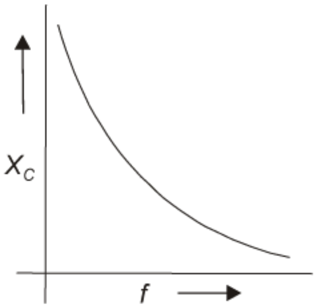
$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{15}\sqrt{\frac{2}{32 \times 10^{-6}}} = \frac{1}{15} \times \frac{1}{4 \times 10^{-3}} = \mathbf{16.67}$$

Hence, the Q-Value of this circuit is 16.67.

S25. (a)





(b) Yes, it can be. In series same amount of current passes through each component. We have $Z^2 = R^2 + (X_L - X_C)^2$. As it is the net reactance which is less than 'Z', individually X_L or X_C might be more than 'Z'.

S26. The Q-factor or quality factor of a resonant LCR-circuit is defined as ratio of the voltage drop across inductor (or capacitor) to the applied voltage.

$$Q = \frac{\text{Voltage across } L \text{ (or } C\text{)}}{\text{Applied voltage}}$$

S27. (a) The current in the two circuits will be maximum, when $X_L = X_C$.

(b) (i) The value of maximum current in the circuit (*ii*) is less than that for the circuit (*i*). It indicates that the resistance in circuit (*ii*) is more than that for the circuit (*i*).

Now, power factor of a circuit is given by

$$\cos\phi = \frac{R}{Z}$$

Therefore, the power factor is higher for the **circuit (*ii*)**.

(ii) The quality factor of a circuit is given by

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

It follows that the quality factor is higher for the **circuit (*i*)**.

S28. Given: $R = 50\,\Omega$; $L = \frac{20}{\pi}\,H$; $C = \frac{5}{\pi}\,\mu F = \frac{5}{\pi} \times 10^{-6}\,F$; $E_v = 230\,V$ and $f = 50\,Hz$.

Now, $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Here, $$X_L = 2\pi f L = 2\pi \times 50 \times \frac{20}{\pi} = 2{,}000\,\Omega$$

and $$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times (5/\pi \times 10^{-6})} = 2{,}000\,\Omega$$

∴ $$Z = \sqrt{(50)^2 + (2{,}000 - 2{,}000)^2} = \mathbf{50\,\Omega}.$$

S29. Given: $L = 0.12\,H$; $C = 0.48 \times 10^{-7}\,F$; $R = 23\,\Omega$.

When the current is maximum at resonance condition

$$X_L = X_C$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{0.12 \times 0.48 \times 10^{-7}}}$$

$$f = \mathbf{663.15\,Hz}.$$

S30. Given: $C = 100\,\mu F = 10^{-4}\,F$; $f = 50\,Hz$

The current in *LCR*-circuit is in phase with the voltage, *i.e.*,

$$X_L = X_C$$

or $$2\pi f L = \frac{1}{2\pi f C}$$

or $$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 \times (50)^2 \times 10^{-4}} = \mathbf{0.1\,H}.$$

S31. The range of frequency (ν) of a radio is 800 kHz to 1200 kHz.

Lower tuning frequency, ν_1 = 800 kHz = 800×10^3 Hz

Upper tuning frequency, ν_2 = 1200 kHz = 1200×10^3 Hz

Effective inductance of circuit, L = 200 ¼H = 200×10^{-6} H

Capacitance of variable capacitor for ν_1 is given as:

$$C_1 = \frac{1}{\omega_1^2 L}$$

Where, ω_1 = Angular frequency for capacitor C_1

$= 2\pi\nu_1 = 2\pi \times 800 \times 10^3 \text{ rad s}^{-1}$

$$\therefore \quad C_1 = \frac{1}{(2\pi \times 800 \times 10^3)^2 \times 200 \times 10^{-6}}$$

$$= 1.9809 \times 10^{-10} \text{ F} = 198.1 \text{ pF}$$

Capacitance of variable capacitor for ν_2 is given as:

$$C_2 = \frac{1}{\omega_2^2 L}$$

Where, ω_2 = Angular frequency for capacitor C_2

$= 2\pi\nu_2 = 2\pi \times 1200 \times 10^3 \text{ rad s}^{-1}$

$$\therefore \quad C_2 = \frac{1}{(2\pi \times 1200 \times 10^3)^2 \times 200 \times 10^{-6}}$$

$$= 88.04 \text{ pF}$$

Hence, the range of the variable capacitor is from 88.04 pF to 198.1 pF.

S32. An inductor (L), a capacitor (C), and a resistor (R) is connected in parallel with each other in a circuit where,

$$L = 5.0 \text{ H}; \quad C = 80 \ \mu\text{F} = 80 \times 10^{-6} \text{ F}; \quad R = 40\ \Omega$$

Potential of the voltage source, V = 230 V

Impedance (Z) of the given parallel LCR circuit is given as:

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2}$$

Where, ω = Angular frequency

At resonance, $$\frac{1}{\omega L} - \omega C = 0$$

$$\therefore \quad \omega = \frac{1}{\sqrt{LC}} = \frac{1}{5 \times 80 \times 10^{-6}} = 50 \text{ rad/s}$$

Hence, the magnitude of Z is the maximum at 50 rad/s. As a result, the total current is minimum.

R.m.s. current flowing through inductor L is given as:

$$I_L = \frac{V}{\omega L} = \frac{230}{50 \times 5} = 0.92 \text{ A}$$

R.m.s. current flowing through capacitor C is given as:

$$I_C = \frac{V}{\frac{1}{\omega C}} = \omega C V$$

$$= 50 \times 80 \times 10^{-6} \times 230 = 0.92 \text{ A}$$

R.m.s. current flowing through resistor R is given as:

$$I_R = \frac{V}{R} = \frac{230}{40} = 5.75 \text{ A.}$$

S33. Given: $L = 10 \text{ mH} = 10 \times 10^{-3} \text{ H} = 10^{-2} \text{ H}$;

$C = (400/\pi^2)\ \mu\text{F} = (400/\pi^2) \times 10^{-6} \text{ F}$;

$R = 55\ \Omega$; $E_v = 220 \text{ V}$

(a) Average power will also be maximum at resonant frequency. The resonant frequency is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{10^{-2} \times (400/\pi^2) \times 10^{-6}}} = \mathbf{250\ Hz}$$

(b) Current amplitude is maximum at resonant frequency.

Now, $$I_v = \frac{E_v}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

or $$I_0 = \frac{\sqrt{2}\, E_v}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

$\therefore$ $$I_0 \text{ (at } \omega_0) = \frac{\sqrt{2} E_v}{R} = \frac{\sqrt{2} \times 220}{55} = \mathbf{4\sqrt{2}\ A}. \qquad (\because\ \omega_0 L - 1/\omega_0 C = 0)$$

S34. (a) When the device X is connected to a.c. supply, the current and voltage are in phase with each other. Therefore, the device X is resistor. Its resistance is given by

$$R = \frac{220}{0.5} = 440\,\Omega.$$

When the device Y is connected to a.c. supply, the current leads e.m.f. by phase angle $\pi/2$. Therefore, the device Y is capacitor. Its reactance is given by

$$X_C = \frac{220}{0.5} = 440\,\Omega.$$

(b) Here,
$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(440)^2 + (440)^2} = 440\sqrt{2}\,\Omega$$

$\therefore$
$$I_v = \frac{E_v}{Z} = \frac{220}{440\sqrt{2}} = \mathbf{0.354\,A}.$$

S35. Given: $C = 20\,\mu\text{F} = 20 \times 10^{-6}\,\text{F}$; $R = 10\,\Omega$; $f = 50\,\text{Hz}$;, $L = ?$

The current in LCR-circuit is in phase with voltage when

$$X_L = X_C$$

$$2\pi f L = \frac{1}{2\pi f C}$$

$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 \times (50)^2 \times 20 \times 10^{-6}} = \mathbf{0.507\,H}.$$

S36. Given: $L = 100\,\text{mH} = 0.1\,\text{H}$; $C = 25\,\mu\text{F} = 25 \times 10^{-6}\,\text{F}$; $R = 15\,\Omega$ and $E_v = 120\,\text{V}$

(a) At resonance,
$$X_L = X_C$$

$$Z = R = \mathbf{15\,\Omega}$$

(b) At resonance,
$$I_v = \frac{E_v}{Z} = \frac{120}{15} = \mathbf{8\,A}$$

(c) Resonant frequency,
$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.1 \times 25 \times 10^{-6}}} = \mathbf{100.66\,Hz}.$$

S37. Given: $E_v = 110\,\text{V}$; $R = 40\,\Omega$ and $\tan\phi = \frac{3}{4}$

(a)
$$\tan\phi = \frac{\omega L}{R} = \frac{X_L}{R}$$

or $\quad X_L = R \times \tan\phi = 40 \times \frac{3}{4} = 30\,\Omega$

(b) $\quad Z = \sqrt{R^2 + X_L^2} = \sqrt{(40)^2 + (30)^2} = 50\,\Omega$

(c) $\quad I_v = \frac{E_v}{Z} = \frac{110}{50} = \mathbf{2.2\,A}$

Now, $\quad X_L = 2\pi f L$

or $\quad f = \frac{X_L}{2\pi L} = \frac{30}{2\pi \times 0.1} = \mathbf{47.75\,Hz}$.

S38. **Impedance:** Total opposition offered by resistance and reactance towards the flow of current is called impedance. It also depends on frequency and the phase angle.

Impedance, $\quad Z = \sqrt{R^2 + (X_L - X_C)^2}$

On increasing the frequency of the applied a.c., inductive reactance increases and capacitive reactance decreases.

$$X_L = 2\pi f L$$

$$X_C = \frac{1}{2\pi f C}$$

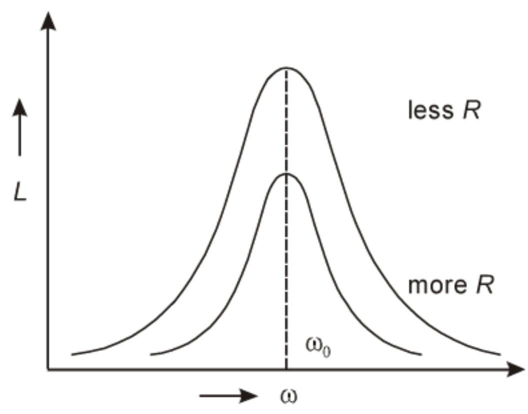
i.e., impedance is minimum.

For frequencies higher than resonance frequency $X_L - X_C$ again increases so is the value of Z.

S39. (a) In a series LCR-circuit the resonance occurs when the frequency of the applied a.c. is such that $X_L = X_C$ and the circuit is purely resistive. The current is maximum at $\omega = \omega_r$ such that

$$L\omega_r = \frac{1}{C\omega_r} \quad \text{or} \quad \omega_r^2 = \frac{1}{LC}$$

or $\quad \omega_r = \frac{1}{\sqrt{LC}}$



At a frequency less than or greater than ω_r, the current falls off. The maximum current is more if the resistance R is less.

A curve with low value of R falls very sharply. Resonance in this case is said to be sharper than the curve with a large R.

The sharpness of resonance is measured by Q-factor of the LCR-circuit. It is defined as the ratio of the voltage developed across the inductance (or capacitance) at resonance to the voltage developed across the resistance.

$$\therefore \qquad Q = \frac{L\omega_r I_{max}}{R I_{max}} = \frac{L\omega_r}{R}$$

Also,
$$Q = \frac{\frac{1}{C\omega_r} I_{max}}{R I_{max}} = \frac{1}{C\omega_r R}$$

Further
$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$\therefore \qquad Q = \frac{L}{R}\cdot\frac{1}{\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}\,.$$

Larger the Q value of the circuit, sharper is the resonance curve.

(b) (i) The first is a pure resistor. Since the resistance offered by a circuit having a resistor only does not change with frequency the current will also not change with frequency.

(ii) In this circuit, which has an inductance L the reactance $X_L = L\omega = 2\pi f L$.

Thus, $X_L \propto f$ *i.e.*, as f increases X_L also increases and the current $I_v = \frac{E_v}{X_L}$, I_v decrease.

(iii) In this circuit which contains only a pure capacitance, the resistance offered by it is given by $X_C = \frac{1}{C\omega} = \frac{1}{2\pi f C}$. Thus as f, the frequency increase X_C decreases and hence the current increases in the circuit.

S40. Given: $L = 5\,H$; $C = 80\,mF = 80 \times 10^{-6}\,F$; $R = 40\,\Omega$; $E_v = 230\,V$.

(a) Resonant frequency of the circuit,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{5\times 80\times 10^{-6}}} = \mathbf{7.96\ Hz}.$$

(b) At the resonant frequency, the inductive reactance cancels out the capacitive reactance. The impedance of the circuit is merely equal to the resistance in the circuit.

$$\therefore \qquad Z = \mathbf{40\,\Omega}.$$

The current amplitude at resonant frequency,

$$I_0 = \frac{E_0}{Z} = \frac{\sqrt{2}\,E_v}{R} = \frac{\sqrt{2}\times 230}{40} = \mathbf{8.13\,A}.$$

(c) The r.m.s. value of current in the circuit,

$$I_v = \frac{I_0}{\sqrt{2}} = \frac{8.13}{\sqrt{2}} = \mathbf{5.75\,A}.$$

Let V_R, V_L and V_C be the r.m.s. potential drops across the resistor, inductor and capacitor respectively.

Then, $V_R = I_v R = 5.75 \times 40 = 230\,\text{V}$;

$V_L = I_v X_L = I_v \times 2\pi f L$

$= 5.75 \times 2\pi \times 7.96 \times 5 = \mathbf{1,437.9\,V}$

and $V_C = I_v X_C = \frac{1}{2\pi f C}$

$$= 5.75 \times \frac{1}{2\pi \times 7.96 \times 80 \times 10^{-6}} = \mathbf{1,437.9\,V}.$$

(d) The voltages across R, L and C can not be added algebraically. It is because, they are not in phase with each other. It can be checked that

$$\sqrt{V_R^2 + (V_L - V_C)^2} = \mathbf{230\,V}.$$

S41. Given: $E_v = 220$ V; $f = 50$ Hz; $R = 100\,\Omega$

$V_R = 65$ V; $V_C = 415$ V; $V_L = 204$ V

(a) Let I_v be the r.m.s value of current in the circuit. Then,

$V_R = I_v R$

or $I_v = \frac{V_R}{R} = \frac{65}{100} = \mathbf{0.65\,A}$.

(b) Also, $V_L = I_v X_L$

or $X_L = \frac{V_L}{I_v} = \frac{204}{0.65} = 313.85\,\Omega$

Now, $X_L = 2\pi f L$

$\therefore$ $L = \frac{X_L}{2\pi f} = \frac{313.85}{2\pi \times 50} = \mathbf{1.0\,H}$.

(c) Again, $V_C = I_v X_C$

or $X_C = \frac{V_C}{I_v} = \frac{415}{0.65} = 638.46\,\Omega$

We know, $X_C = \frac{1}{2\pi f C}$

$\therefore$ $$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 638.46}$$

$$= 5 \times 10^{-6} = \mathbf{5\,\mu F}.$$

(d) If C' is capacitance of the capacitor that will produce resonance with inductor (L = 1.0 H), then

$$f = \frac{1}{2\pi\sqrt{LC'}}$$

or $$C' = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 \times (50)^2 \times 1.0}$$

$$= 10.1 \times 10^{-6}\,\text{F} = \mathbf{10.1\,\mu F}.$$

S42. (a) We know the reactance of the capacitor is

$$X_C = \frac{1}{2\pi f C}$$

$$C = \frac{1}{2\pi f X_C}$$

From the graph we take frequency ν = 100 Hz. The capacitive reactance corresponding to this frequency is $X_C = 6\,\Omega$.

Thus, $$C = \frac{1}{2\pi \times 100 \times 6} = \frac{1}{1200\pi}$$

$$C = 2.65 \times 10^{-4}\,\text{F}$$

(b) According to question $X_L = X_C$

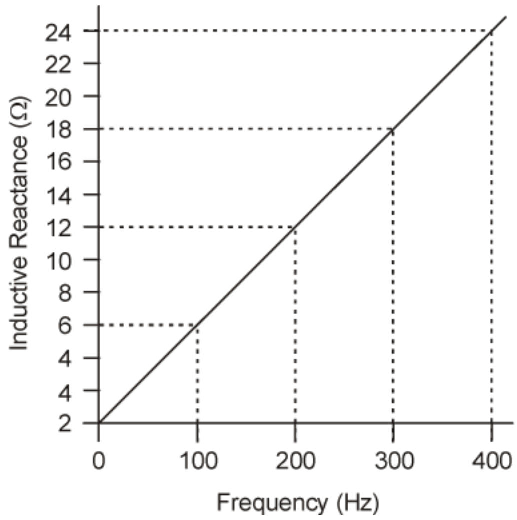
$$2\pi f L = X_C \quad or \quad L = \frac{X_c}{2\pi f}$$

$$L = \frac{6}{2\pi(100)} \qquad \left[\because \begin{array}{r} X_C = 6\,\Omega \\ f = 100\,\text{Hz} \end{array}\right]$$

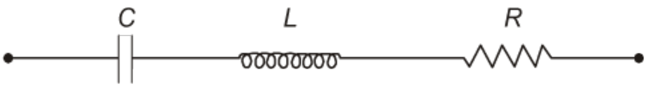
$$L = 9.55 \times 10^{-3}\text{ H.}$$

(c) $$X_L = 2\pi f L$$

f	X_L
100	$5.99 \approx 6$
200	$11.98 \approx 12$
300	$17.98 \approx 18$
400	$23.97 \approx 24$



(d)



$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Hence $$R = 10\,\Omega; \quad L = 9.54 \times 10^{-3}\text{ H}$$

$$C = 2.65 \times 10^{-4}\text{ F}; \quad f = 300\text{ Hz}$$

$$X_L = 2\pi f L = 600\pi \times 9.54 \times 10^{-3}\,\Omega = 17.98\,\Omega = 18\,\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{600\,\pi \times 2.65 \times 10^{-4}} = 2\,\Omega$$

$$Z = \sqrt{(10)^2 + (18-2)^2}$$

$$Z = \sqrt{100 + 256} = \sqrt{356} = 18.87\,\Omega$$

$$\mathbf{Z = 18.9\,\Omega \approx 19\,\Omega.}$$

Date: 21/10/2021

Q1. What are the dimensions of $\sqrt{LC}$?

Q2. How can the power factor of a series *LCR* circuit be improved? Suggest any one method.

Q3. Where does the energy reside in an inductor through which current has attained its maximum value?

Q4. If a *LC* circuit is considered analogous to a harmonically oscillating spring block system, which energy of the *LC* circuit would be analogous to potential energy and which one analogous to kinetic energy?

Q5. In a series *LCR* circuit, what is the value of power factor at resonance?

Q6. At an airport, a person is made to walk through the doorway of a metal detector, for security reasons. If she/he is carrying anything made of metal, the metal detector emits a sound. On what principle does this detector work?

Q7. Why is choke coil needed in the use of fluorescent tubes with ac mains? Why can we not use an ordinary resistor instead of the choke coil?

Q8. When a lamp is connected to an alternating voltage supply, it lights with the same brightness as when connected to a 12 V DC battery. What is the peak value of alternating voltage source?

Q9. What role does the resistance of inductor play in *LC*-circuit?

Q10. What is the power dissipated in an a.c. circuit in which voltage and current are given by

$$V = 230 \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{and} \quad I = 10 \sin \omega t$$

Q11. The power factor of an a.c. circuit is 0.5. What will be the phase difference between voltage and current in this circuit?

Q12. Is power dissipated across each element of an a.c. circuit containing *L*, *C* and *R*?

Q13. The instantaneous current and voltage of an a.c. circuit are given by $I = 10 \sin 314\, t$ A and $V = 50 \sin (314\, t + \frac{\pi}{2})$ V. What is the power of dissipation in the circuit?

Q14. What is the power factor ?

Q15. What are the maximum and minimum values of power factor of a.c. circuit?

Q16. How much power is consumed in a (a) purely inductive and (b) purely capacitive a.c. circuit?

Q17. In Exercises 7.3 and 7.4, what is the net power absorbed by each circuit over a complete cycle. Explain your answer.

Q18. Suppose the frequency of the source in the previous example can be varied. (a) What is the frequency of the source at which resonance occurs? (b) Calculate the impedance, the current, and the power dissipated at the resonant condition.

Q19. Show that in the free oscillations of an LC circuit, the sum of energies stored in the capacitor and the inductor is constant in time.

Q20. An alternating e.m.f., $e = 150 \sin(\omega t - \pi/8)$ is applied to an a.c. circuit. If the current in the circuit is $I = 10 \sin(\omega t + \pi/8)$, calculate the impedance of the circuit and the power factor of the circuit.

Q21. Prove that in an a.c. circuit, an ideal capacitor does not dissipate power.

Q22. In an a.c. circuit, there is no power consumption in an ideal inductor. Explain.

Q23. The instantaneous current and voltage in an a.c. circuit are given by

$$I = 10 \sin 300\,t \text{ (in } A) \quad \text{and} \quad E = 200 \sin 300\,t \text{ (in } V)$$

What is the average power dissipated in the circuit?

Q24. In India, domestic power supply is at 220 V-50 Hz, while in USA, it is 110 V-50 Hz. Give one advantage and one disadvantage of 220 V supply over 110 V supply.

Q25. A coil of inductance 0.50 H and resistance 100 Ω is connected to a 240 V, 50 Hz a.c. supply.

(a) What is the maximum current in the coil?

(b) What is the time lag between the voltage maximum and the current maximum?

Q26. Suppose the circuit in Exercise 7.18 has a resistance of 15 Ω. Obtain the average power transferred to each element of the circuit, and the total power absorbed.

Q27. A sinusoidal voltage of peak value 283 V and frequency 50 Hz is applied to a series LCR circuit in which $R = 3\,\Omega$, $L = 25.48$ mH, and $C = 796\,\mu$F. Find (a) the impedance of the circuit; (b) the phase difference between the voltage across the source and the current; (c) the power dissipated in the circuit; and (d) the power factor.

Q28. An alternating e.m.f. of 100 V (r.m.s.), 50 Hz is applied across a capacitor of 10 μF and a resistor of 100 Ω in series. Calculate (a) the reactance of the capacitor; (b) the current flowing; (c) the average power supplied.

Q29. A 200 V variable frequency a.c. source is connected to a series combination of $L = 5$ H, $C = 80\,\mu$F and $R = 40\,\Omega$. Calculate (a) angular frequency of the source to get maximum current in the circuit, (b) the current amplitude at resonance and (c) the power dissipation in the circuit.

Q30. A series LCR-circuit with $R = 20\,\Omega$, $L = 1.5$ H and $C = 35\,\mu$F is connected to a variable-frequency 200 V a.c. supply. When the frequency of the supply equals the natural frequency of the circuit, what is the average power transferred to the circuit in one complete cycle?

Q31. A resistor of 12 Ω, a capacitor of reactance 14 Ω and an inductor of reactance 30 Ω are joined in series and placed across a 230 V, 50 Hz supply. Calculate (a) the current in circuit, (b) the phase angle between the current and the voltage and (c) power factor.

Q32. Calculate the value of an inductance, which should be connected in series with a capacitance of 5 μF, resistance of 10 ohm and a.c. source of 50 Hz, so that the power factor of the circuit is unity.

Q33. An alternating voltage $E = 200 \sin 300\,t$ is applied across a series combination of resistance of 10 Ω and an inductor of 800 mH. Calculate (a) impedance of the circuit; (b) peak value of current in the circuit and (c) power factor of the circuit.

Q34. (a) Derive an expression for the average power consumed in a series *LCR*-circuit connected to a.c. source in which the phase difference between the voltage and the current in the circuit is ϕ.

(b) Define the quality force in an a.c. circuit. Why should the quality factor have high value in receiving circuits? Name the factors on which it depends.

Q35. (a) Define the term 'impedance of series *LCR*-circuit'. Derive a mathematical expression for it using phasor diagram.

(b) Obtain the resonant frequency of a series *LCR*-Circuit with $L = 2.0$ H, $C = 32\ \mu$F and $R = 10\ \Omega$.

Q36. (a) Distinguish between the terms resistance, reactance and impedance of an a.c. circuit.

(b) A 100 μF capacitor in series with a 40 Ω resistance is connected to a 100 V, 60 Hz supply. Calculate (i) the reactance, (ii) the impedance, and (iii) maximum current in the circuit.

Q37. Distinguish between reactance and impedance. When a series combination of a coil of inductance *L* and a resistor of resistance *R* is connected across a 12 V, 50 Hz supply, a current of 0.5 A flows through the circuit. The current differs in phase from applied voltage by $\pi/3$ radian. Calculate the value of *L* and *R*.

Q38. A circuit containing a 80 mH inductor and a 60 μF capacitor in series is connected to a 230 V-50 Hz supply. The resistance of the circuit is negligible.

(a) Obtain the current amplitude and r.m.s. values.
(b) Obtain the r.m.s. values of potential drops across each element.
(c) What is the average power transferred to the inductor?
(d) What is the average power transferred to the capacitor?
(e) What is the total average power absorbed by the circuit? (Average implies 'average over one cycle'.)

Q39. Explain the term 'inductive reactance'. Show graphically the variation of inductive reactance with frequency of the applied alternating voltage.

An a.c. voltage $E = E_0 \sin \omega t$ is applied across a pure inductor of inductance *L*. Show mathematically that the current flowing through it lags behind the applied voltage by a phase angle of $\pi/2$.

Date: 21/10/2021

S1. [T].

S2. If we connect the capacitor in parallel in the main then power factor will be improve.

S3. It resides inside the inductor in the form of magnetic field.

S4. Magnetic energy analogous to kinetic energy and electrical energy analogous to potential energy.

S5. It is 1.

S6. The metal detector works on the principle of resonance in a.c. circuits. When you walk through a metal detector, you are, in fact, walking through a coil of many turns. The coil is connected to a capacitor tuned so that the circuit is in resonance. When you walk through with metal in your pocket, the impedance of the circuit changes – resulting in significant change in current in the circuit. This change in current is detected and the electronic circuitry causes a sound to be emitted as an alarm.

S7. A choke coil is needed in the use of fluorescent tubes with ac mains because it reduces the voltage across the tube without wasting much power. An ordinary resistor cannot be used instead of a choke coil for this purpose because it wastes power in the form of heat.

S8. Given:

Peak voltage of a.c. $V_m = ?$

$$V_m = \sqrt{2}\ V_{rms}$$
$$= \sqrt{2} \times 12 = \mathbf{16.97\ V}.$$

S9. Due to the resistance of the inductor, the LC-oscillations produced are damped one. It is because, during each oscillation, a part of electric energy is dissipated in the form of heat energy.

S10. Zero.

Explanation: $V = 230 \sin(\omega t + \pi/2)$

$I = 10 \sin(\omega t)$

Voltage is leading the current $\pi/2$

$\cos \pi/2 = 0$

Therefore, power dissipation is zero.

S11. Given power factor $\cos\phi = 0.5$

$$\cos\phi = 1/2$$

$$\phi = \cos^{-1}(1/2)$$

$$\boldsymbol{\phi = \pi/3}.$$

Phase difference $\pi/3$.

S12. No, power is dissipated across the resistance only.

S13. Given:

$$I = 10 \sin 314\, t \text{ A}$$

$$V = 50 \sin (314 + {}^{\pi}/_{2}) \text{ V}$$

The phase difference between current and voltage is $^{\pi}/_{2}$.

Power dissipation is *zero* because phase difference between current and voltage is zero.

S14. The cosine of the phase angle between alternating current and e.m.f. in an a.c. circuit is called its power factor. Therefore,

power factor = $\cos \phi$.

S15. The maximum value of power factor is 1 and the minimum value is 0.

S16. The power consumed is zero in both the cases.

S17. In the inductive circuit,

R.m.s. value of current, $I = 15.92$ A

R.m.s. value of voltage, $V = 220$ V

Hence, the net power absorbed can be obtained by the relation,

$$P = VI \cos \Phi$$

Where,

Φ = Phase difference between V and I.

For a pure inductive circuit, the phase difference between alternating voltage and current is 90° *i.e.*, $\Phi = 90°$.

Hence, $P = 0$ *i.e.*, the net power is zero.

In the capacitive circuit,

R.m.s. value of current, $I = 2.49$ A

R.m.s. value of voltage, $V = 110$ V

Hence, the net power absorbed can be obtained as:

$$P = VI \cos \Phi$$

For a pure capacitive circuit, the phase difference between alternating voltage and current is 90° *i.e.*, $\Phi = 90°$.

Hence, $P = 0$ *i.e.*, the net power is zero.

S18. (a) The frequency at which the resonance occurs is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{25.48 \times 10^{-3} \times 796 \times 10^{-6}}}$$

`

$$= 222.1 \text{rad/s}$$

$$\nu_r = \frac{\omega_0}{2\pi} = \frac{211.1}{2 \times 3.14} \text{Hz} = 25.4 \text{ Hz}$$

(b) The impedance Z at resonant condition is equal to the resistance:

$$Z = R = 3\,\Omega$$

The r.m.s. current at resonance is

$$= \frac{V}{Z} = \frac{V}{R} = \left(\frac{283}{\sqrt{2}}\right)\frac{1}{3} = 66.7 \text{ A}$$

The power dissipated at resonance is

$$P = I^2 \times R = (66.7)^2 \times 3 = 13.35 \text{ kW}$$

You can see that in the present case, power dissipated at resonance is more than the power dissipated in Example 7.8.

S19. Let q_0 be the initial charge on a capacitor. Let the charged capacitor be connected to an inductor of inductance L. This LC circuit will sustain an oscillation with frequency

$$\omega = 2\pi f = \frac{1}{\sqrt{LC}}$$

At an instant t, charge q on the capacitor and the current I are given by:

$$q(t) = q_0 \cos \omega t$$

$$I(t) = \frac{dq(t)}{dt}$$

$$I(t) = -q_0 \omega \sin \omega t$$

Energy stored in the capacitor at time t is

$$U_E = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{q_0^2}{2C}\cos^2(\omega t)$$

Energy stored in the inductor at time t is

$$U_M = \frac{1}{2}LI^2$$

$$= \frac{1}{2}L q_0^2 \omega^2 \sin^2(\omega t)$$

$$= \frac{q_0^2}{2C} \sin^2 \omega t \qquad (\because \ \omega = 1/\sqrt{LC})$$

Sum of energies

$$U_E + U_M = \frac{q_0^2}{2C} (\cos^2 \omega t + \sin^2 \omega t) \qquad (\because \ \cos^2 \omega t + \sin^2 \omega t = 1)$$

$$= \mathbf{\frac{q_0^2}{2C}}.$$

This sum is constant in time as q_0 and C, both are time-independent.

S20. (a) Impedance,

$$Z = \frac{e_0}{I} = \frac{150}{10} = 15\,\Omega$$

(b) Phase angle between current and voltage is

$$\phi = \pi/8 - (-\pi/8) = \pi/4$$

Power factor = $\cos \phi = \cos \pi/4 = \frac{1}{\sqrt{2}}$.

S21. Average power of an a.c. circuit,

$$P_{av} = E_v I_v \cos \phi$$

For a.c. circuit containing an ideal capacitor, $\phi = -\pi/2$

$\therefore$ $$P_{av} = E_v I_v \cos(-\pi/2) = E_v I_v (0) = 0.$$

S22. Average power of an a.c. circuit,

$$P_{av} = E_v I_v \cos \phi$$

For a.c. circuit containing an ideal inductor, $\phi = \pi/2$

$\therefore$ $$P_{av} = E_v I_v \cos \pi/2 = E_v I_v (0) = 0$$

S23. Given: $I = 10 \sin 300\,t$; $I = I_0 \sin \omega t$; $I_0 = 10$; $E = 200 \sin 300\,t$; $E = E_0 \sin \omega t$; $E_0 = 200\,\text{V}$.

Average power of an a.c. circuit,

$$P_{av} = E_v I_v \cos \phi = \frac{E_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos \phi = \frac{E_0 I_0}{2} \cos \phi$$

Hence, $E_0 = 200\,\text{V}$, $I_0 = 10\,\text{A}$ and $\phi = 0°$

$\therefore$ $$P_{av} = \frac{200 \times 10}{2} \cos 0^\circ = 1{,}000 \times 1 = \mathbf{1{,}000\,W}.$$

S24. **Advantage:** The transmission of a.c. at higher voltage is economical as well as the loss of electric energy across the transmission line is low.

Disadvantage: The a.c. supply at higher voltage is more fatal and dangerous.

S25. Inductance of the inductor, $L = 0.50\,H$

Resistance of the resistor, $R = 100\,\Omega$

Potential of the supply voltage, $V = 240\,V$

Frequency of the supply, $\nu = 50\,Hz$

(a) Peak voltage is given as:

$$V_0 = \sqrt{2}V$$
$$= \sqrt{2} \times 240 = 339.41\,V$$

Angular frequency of the supply,

$$\omega = 2\pi\nu = 2\pi \times 50 = 100\pi \text{ rad/s}$$

Maximum current in the circuit is given as:

$$I_0 = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}}$$
$$= \frac{339.41}{\sqrt{(100)^2 + (100\pi)^2 (0.50)^2}} = 1.82\,A$$

(b) Equation for voltage is given as:

$$V = V_0 \cos \omega t$$

Equation for current is given as:

$$I = I_0 \cos(\omega t - \Phi)$$

Where,

Φ = Phase difference between voltage and current

At time, $t = 0$.

$V = V_0$ (voltage is maximum)

For $\omega t - \Phi = 0$ *i.e.*, at time $t = \frac{\phi}{\omega}$,

$I = I_0$ (current is maximum)

Hence, the time lag between maximum voltage and maximum current is $\frac{\phi}{\omega}$.

Now, phase angle Φ is given by the relation,

$$\tan\phi = \frac{\omega L}{R} = \frac{2\pi \times 50 \times 0.5}{100} = 1.57$$

$$\phi = 57.5° = \frac{57.5\pi}{180} \text{ rad}$$

$$\omega t = \frac{57.5\pi}{180}$$

$$t = \frac{57.5}{180 \times 2\pi \times 50} = 3.19 \times 10^{-3}\text{ s} = 3.2 \text{ ms}$$

Hence, the time lag between maximum voltage and maximum current is 3.2 ms.

S26. Average power transferred to the resistor = 788.44 W

Average power transferred to the capacitor = 0 W

Total power absorbed by the circuit = 788.44 W

Inductance of inductor, $L = 80 \text{ mH} = 80 \times 10^{-3}\text{ H}$

Capacitance of capacitor, $C = 60\ \mu\text{F} = 60 \times 10^{-6}\text{ F}$

Resistance of resistor, $R = 15\,\Omega$

Potential of voltage supply, $V = 230\text{ V}$

Frequency of signal, $\nu = 50$ Hz

Angular frequency of signal, $\omega = 2\pi\nu = 2\pi \times (50) = 100$À rad/s

The elements are connected in series to each other. Hence, impedance of the circuit is given as:

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)}$$

$$= \sqrt{(15)^2 + \left(100\pi(80 \times 10^{-3}) - \frac{1}{(100\pi \times 60 \times 10^{-6})}\right)^2}$$

$$= \sqrt{(15)^2 + (25.12 - 53.08)^2} = 31.728\,\Omega$$

Current flowing in the circuit, $I = \frac{V}{Z} = \frac{230}{31.728} = 7.25\text{ A}$.

Average power transferred to resistance is given as:

$$P_R = I_2R = (7.25)^2 \times 15 = 788.44 \text{ W}$$

Average power transferred to capacitor,

P_C = Average power transferred to inductor,

$P_L = 0$

Total power absorbed by the circuit:

$$P_R + P_C + P_L = 788.44 + 0 + 0 = 788.44 \text{ W}$$

Hence, the total power absorbed by the circuit is 788.44 W.

S27. (a) To find the impedance of the circuit, we first calculate X_L and X_C.

Inductive reactance, $X_L = 2\pi\nu L$

$= 2 \times 3.14 \times 50 \times 25.48 \times 10^{-3}\,\Omega = 8\,\Omega$

Capacitive reactance, $X_C = \dfrac{1}{2\pi\nu C}$

$= \dfrac{1}{2 \times 3.14 \times 50 \times 796 \times 10^{-6}} = 4\,\Omega$

Therefore, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{3^2 + (8-4)^2} = 5\,\Omega$

(b) Phase difference, $\phi = \tan^{-1}\dfrac{X_C - X_L}{R}$

$= \tan^{-1}\left(\dfrac{4-8}{3}\right) = -53.1°$

Since ϕ is negative, the current in the circuit lags the voltage across the source.

(c) The power dissipated in the circuit is

$P = I^2R$

Now, $I = \dfrac{i_m}{\sqrt{2}} = \dfrac{1}{\sqrt{2}}\left(\dfrac{283}{5}\right) = 40\text{ A}$

Therefore, $P = (40\text{ A})^2 \times 3\,\Omega = 4800\text{ W}$

(d) Power factor = $\cos\phi = \cos 53.1° = 0.6$.

S28. Given: $E_v = 100\text{ V}$; $f = 50\text{ Hz}$; $C = 10\,\mu\text{F} = 10 \times 10^{-6} = 10^{-5}\text{ F}$; $R = 100\,\Omega$

(a) Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 10^{-5}} = \mathbf{318.31\,\Omega}.$$

(b) Current through the *CR*-circuit,

$$I_v = \frac{E_v}{\sqrt{R^2 + X_C^2}}$$

$$= \frac{100}{\sqrt{100^2 + (318.31)^2}} = \frac{100}{333.65} = \mathbf{0.3\,A}.$$

(c) Average power supplied,

$$P_{av} = E_v I_v \cos\phi = E_v I_v \cdot \frac{E_v}{\sqrt{R^2 + X_C^2}}$$

or $$P_{av} = 100 \times 0.3 \times \frac{100}{\sqrt{100^2 + (318.31)^2}} = \frac{0.3 \times 10^4}{333.65} = \mathbf{9\,W}.$$

S29. Given: $E_v = 200\,\text{V}$; $L = 5\,\text{H}$; $C = 80\,\mu\text{F} = 80 \times 10^{-6}\,\text{F}$ and $R = 40\,\Omega$

(a) For maximum current (or resonance) in the circuit,

$$X_L = X_C$$

Let ω_0 be the angular frequency of the source that provides maximum current in the circuit. Then,

$$\omega_0 L = \frac{1}{\omega_0 C} \quad \text{or} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

or $$\omega_0 = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = \mathbf{50\,rad\,s^{-1}}.$$

(b) At resonance, the impedance of *LCR*-circuit is equal to the ohmic resistance in the circuit. Hence, r.m.s. value of current at resonance,

$$I_v = \frac{E_v}{R} = \frac{200}{40} = \mathbf{5\,A}.$$

Therefore, current amplitude at resonance.

$$I_0 = \sqrt{2}\,I_v = \sqrt{2} \times 5 = \mathbf{7.07\,A}.$$

(c) Now $$P_{av} = E_v I_v \cos\phi.$$

Since at resonance, the circuit is purely resistive, $\phi = 0°$. Hence, average power consumed in the circuit,

$$P_{av} = 200 \times 5 \times \cos 0° = \mathbf{1{,}000\,W}.$$

S30. At resonance $Z = R$, and $\phi = 0°$

Thus, $Z = 20\,\Omega$

$$P_{av} = E_v I_v \cos\phi$$

$$= E_v \times \frac{E_v}{Z} \qquad [\because \cos\phi = 1]$$

$$= \frac{(200)^2}{20} = 2000\,\text{W}$$

$$P_{av} = \mathbf{2\,kW}.$$

S31. Given: $R = 12\,\Omega$; $X_C = 14\,\Omega$; $X_L = 30\,\Omega$, $E_v = 230\,V$; $f = 50\,Hz$.

Now, $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(12)^2 + (30-14)^2} = \mathbf{20\,\Omega}.$$

(a) $$I_v = \frac{E_v}{z} = \frac{230}{20} = \mathbf{11.5\,A}.$$

(b) $$\tan\phi = \frac{X_L - X_C}{Z} = \frac{30-14}{12} = \mathbf{1.33}$$

or $$\phi = \mathbf{53.13°}.$$

(c) Power factor,

$$\cos\phi = \frac{R}{Z} = \frac{12}{20} = \mathbf{0.6}. \quad \text{or} \quad \phi = \cos^{-1}(0.6) = \mathbf{53.13°}.$$

S32. Given: $C = 5\,\mu F = 5 \times 10^{-6}\,F$; $R = 10\,\Omega$; $f = 50\,Hz$; $\cos\phi = 1$.

The power factor of an a.c. circuit is unity, when the circuit is purely resistive *i.e.*,

i.e., $$X_L = X_C \quad \text{or} \quad 2\pi f L = \frac{1}{2\pi f C}$$

or $$L = \frac{1}{4\pi^2 f^2 C}$$

$$= \frac{1}{4\pi^2 \times (50)^2 \times 5 \times 10^{-6}} = \mathbf{2.026\ H.}$$

S33. Given: $R = 10\,\Omega$; $L = 800\,mH = 0.8\,H$ and $E = 200 \sin 300\,t$

$\therefore$ $$E_0 = 200\,V \quad \text{and} \quad \omega = 300\,rad\,s^{-1}$$

(a) Now, $$X_L = \omega L = 300 \times 0.8 = 240\,\Omega$$

$\therefore$ $$Z = \sqrt{R^2 + X_L^2} = \sqrt{(10)^2 + (240)^2} = \mathbf{240.21\,\Omega}.$$

(b) $$I_0 = \frac{E_0}{Z} = \frac{200}{240.21} = \mathbf{0.833\,A}.$$

(c) Power factor,

$$\cos\phi = \frac{R}{Z} = \frac{10}{240.21} = \mathbf{0.042}.$$

S34. (a) Let an alternating current of $I = I_m \sin \omega t$ be passing through a network of L, C and R creating a potential difference of $V = V_m \sin(\omega t \pm \phi)$ where ϕ is the phase difference. Then the power consumed is

$$P = VI$$

$$= V_m I_m \sin(\omega t \pm \phi) \sin \omega t$$

$$\therefore \quad P = V_m I_m (\sin \omega t \cos \phi \pm \cos \omega t \sin \phi) \sin \omega t$$

$$= V_m I_m \left(\sin^2 \omega t \cos \phi \pm \frac{1}{2} \sin 2\omega t \sin \phi\right)$$

$$P_{av} = \frac{\int_0^T P dt}{\int_0^T dt}$$

$$= \frac{V_m I_m}{T}\left[\int_0^T \sin^2 \omega t \cos \phi \, dt + \frac{1}{2}\int_0^T \sin \phi \sin 2\omega t \, dt\right]$$

$$= \frac{V_m I_m}{T}\left[\frac{T}{2}\cos \phi + 0\right] \quad \left[\because \int_0^T \sin^2 \omega t \, dt = \frac{T}{2} \text{ and } \int_0^T \sin 2\omega t \, dt = 0\right]$$

$$= \frac{V_m I_m}{2}\cos \phi$$

$$P_{av} = \mathbf{V_{rms} I_{rms} \cos \phi}.$$

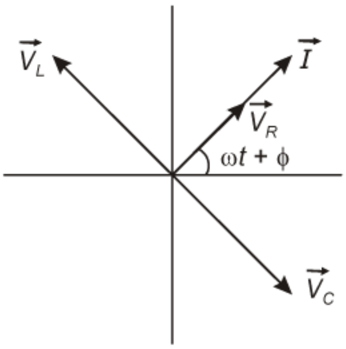
(b) Quality factor should be high to have the current corresponding to a particular frequency to be more and to avoid the other unwanted frequencies. Q-factor depends on f, L, R and C.

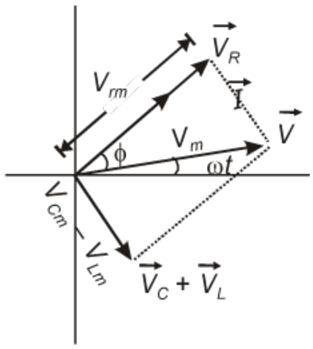
Sharpness of resonance is determined by Quality factor (Q) of the circuit i.e.,

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR} = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

Larger the value of Q, sharper is the resonance, *i.e.*, sharper peak in the current.

S35. (a) It is the combined effect of frequency dependent and frequency independent opposition offered by the components of LCR-circuit towards the flow of current





Mathematical expression for impedance:

Let the circuit be capacitive in nature

$$V_m^2 = V_{Rm}^2 + (V_{Cm} - V_{Lm})^2$$

$$V_m^2 = (I_0 R)^2 + I_m^2 (X_C - X_L)^2$$

$$V_m = \sqrt{R^2 + (X_C - X_L)^2}$$

$$I_m = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}} = \frac{V_m}{Z} \qquad \left[\because Z = \sqrt{R^2 + (X_C - X_L)^2}\right]$$

Where Z is called impedance of the circuit.

(b) Given: $L = 2.0\,\text{H}$; $C = 32 \times 10^{-6}\,\text{F}$; $R = 10\,\Omega$

Formula: $$\omega_r = \frac{1}{\sqrt{LC}}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Calculation: $$f_r = \frac{1}{2\pi\sqrt{2 \times 32 \times 10^{-6}}}$$

f_r = **19.0 Hz**.

S36. (a) **Resistance** is the opposition towards the flow of current which does not depend upon frequency.

Reactance is the opposition offered by a component of the circuit which depends upon frequency of current.

Impedance is the combined effect of frequency dependent and frequency independent opposition offered by the components of a circuit towards the flow of current.

(b) Given: $C = 100\,\mu F = 10^{-4}$ F; $R = 40\,\Omega$, $E_v = 100$ volt, $f = 60$ Hz

Formulas:

(i) $$X_C = \frac{1}{2\pi f C}$$

(ii) $$Z = \sqrt{R^2 + X_C^2}$$

(iii) $$I_0 = \sqrt{2}\left(\frac{E_v}{Z}\right)$$

Calculation:

(i) $$X_C = \frac{1}{2\pi \times 60 \times 10^{-4}}$$

$$X_C = 26.5\,\Omega.$$

(ii) $$Z = \sqrt{(40)^2 + (26.5)^2} = 48\,\Omega$$

(iii) $$I_0 = \frac{\sqrt{2} \times 100}{48} = 2.9\,A$$

$$I_0 = \mathbf{3\,A}.$$

S37. **Reactance:** The resistance of an a.c. circuit is the ohmic resistance offered by a conductor connected in the circuit. It is due to the nature of the material (resistivity) of the conductor.

Impedance: The impedance of an a.c. circuit is the effective resistance offered by the *LR*-circuit or *CR*-circuit or *LCR*-circuit.

Numerical: Given: For an *LR*-circuit

$$V_{rms} = 12 \text{ volt}, \quad \phi = 50 \text{ Hz},$$

$$I_{rms} = 0.5\,A \quad \text{and} \quad \phi = \pi/3 \text{ radian}.$$

To find: *L* and *R*.

We know that $$\cos\phi = \frac{R}{Z}$$

Here, $$Z = \frac{V_{rms}}{I_{rms}} = \frac{12}{0.5}\,\Omega = 24\,\Omega$$

So, $$R = Z\cos\phi = 24\cos\left(\frac{\pi}{3}\right)\Omega = 12\,\Omega$$

$$Z^2 = R^2 + X_L^2$$

$$X_L^2 = Z^2 - R^2 = (24)^2 - (12)^2 = 432$$

$$X_L = 20.7\,\Omega$$

$$X_L = 2\pi fL$$

$$L = \frac{X_L}{2\pi f} = \frac{20.7}{2 \times 3.14 \times 50} = 0.06\ H$$

S38. (a) Given: $L = 80\text{ mH} = 80 \times 10^{-3}\text{ H}$; $C = 60\,\mu\text{F} = 60 \times 10^{-6}\text{ F}$; $f = 50\text{ Hz}$; $E_v = 230\text{ V}$.

$$I_v = \frac{E_v}{\left|\omega L - \frac{1}{\omega C}\right|}$$

$$= \frac{230}{|25.13 - 53.05|} = 8.24\text{ A}$$

–ve sign appears if $\omega L - \frac{1}{\omega C}$ and +ve sign

(b) $V_{C_{\text{r.m.s.}}} = 437\text{ V}$ (Use $V_{C_{\text{r.m.s.}}} = X_C I_{\text{r.m.s.}}$

$V_{L_{\text{r.m.s.}}} = 207\text{ V}$ $V_{L_{\text{r.m.s.}}} = X_L I_{\text{r.m.s.}}$)

The voltage across L and C gets subtracted because they are 180° out of phase and it is equal to applied r.m.s. voltage.

(c) In conductor, e.m.f. leads the current by $\pi/2$

$$P_{av} = E_v I_v \cos\frac{\pi}{2} = 0$$

Power transformed to the inductor = **0**.

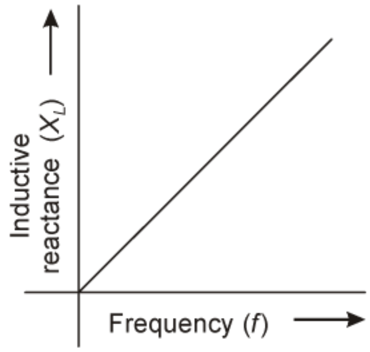
(d) As in capacitor, e.m.f. large behind the current by $\pi/2$

$$P_{av} = E_v I_v \cos\left(-\frac{\pi}{2}\right) = 0.$$

(e) Total average power absorbed = **0**.

S39. Inductive reactance is the opposition offered by an inductor towards the flow of current passing through it.

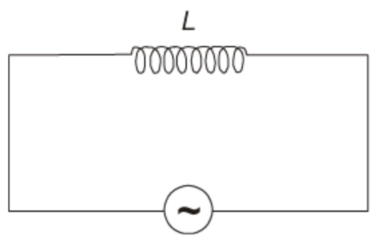
$$X_L = 2\pi fL$$



Applied a.c. voltage is

$$E = E_0 \sin \omega t \qquad \text{... (i)}$$

e.m.f. induced in the inductor is given by the relation



In order to maintain the flow of current through the inductor we must have

$$E = -E$$

i.e.,

$$E = L\frac{dI}{dt}$$

$$\frac{dI}{dt} = \frac{E}{L} = \frac{E_0}{L}\sin \omega t$$

$$\int dI = \frac{E_0}{L}\int \sin \omega t \, dt$$

$$I = \frac{E_0}{L\omega}(-\cos \omega t)$$

$$= \frac{E_0}{L\omega}\sin(\omega t - \pi/2)$$

$$I = I_m \sin(\omega t - \pi/2) \qquad \text{... (ii)}$$

where

$$I_m = \frac{E_0}{L\omega} = \frac{E_0}{X_L} \qquad (\because X_L = L\omega)$$

From equations (i) and (ii), we conclude that voltage leads the current by a phase angle $\pi/2$.

Date: 21/10/2021

Q1. **A charged 30 μF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit?**

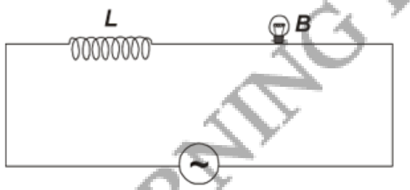
Q2. **Write two advantages of alternating current (a.c.) over direct current (d.c.).**

O3. **What is wattles current?**

Q4. **A coil of inductance *L*, a capacitor of capacitance *C* and a resistor of resistance *R* are all put in series with an alternating source of e.m.f. E (= $E_0 \sin \omega t$). Write an expression for the (a) total impedance of the circuit, (b) frequency of the source e.m.f. for which the circuit will show resonance.**

Q5. **A ratio can tune over the frequency range of a portion of MW broadcast band: (800 kHz to 1200 kHz). If its *LC* circuit has an effective inductance of 200 μH, what must be the range of its variable capacitor?**

Q6. **An inductor *L* of reactance X_L is connected in series with a bulb *B* to an a.c. source as shown in the figure**



Briefly explain how does the brightness of the bulb change, when

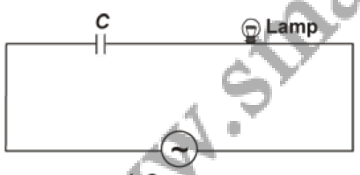
(a) number of turns of the inductor is reduced and

(b) a capacitor of reactance $X_C = X_L$ is included in series in the same circuit.

Q7. **Explain briefly, how the phenomenon of resonance in the circuit can be used in the tuning mechanism of a radio or a TV set?**

Q8. **A coil of inductance of 0.4 mH is connected to a capacitor of capacitance 250 pF. To what wavelength is the circuit tuned?**

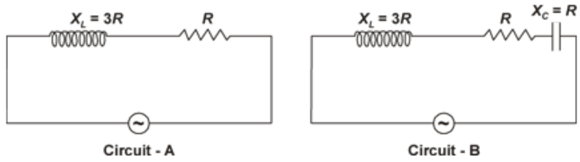
Q9. **As shown in the figure below, an electric lamp having coil of negligible inductance connected in series with a capacitor and an a.c. source is glowing with certain brightness.**



How does the brightness of the lamp change on reducing (a) the capacitance and (b) the frequency? Explain it.

Q10. **Why does an *LC*-circuit produce oscillation ?**

Q11. As shown in the figure below, two electric circuits *A* and *B*. Calculate the ratio of power factor of the circuit *B* to the power factor of the circuit *A*.



Q12. A capacitor of capacitance 100 μF is charged to a potential of 12 V and then connected to a 6.4 mH inductor to produce oscillations. What is the frequency of oscillations produced? Also find the value of maximum current in the circuit.

Q13. A variable frequency 230 V alternating voltage source is connected across a series combination L = 5.0 H, C = 80 μF and R = 40 Ω. Calculate (a) the angular frequency of the source which drives the circuit in resonance, (b) the amplitude of the current at resonant frequency and (c) r.m.s potential drop across the inductor at resonating frequency.

Q14. A series *LCR* circuit with L = 0.12 H, C = 480 nF, R = 23 Ω is connected to a 230 V variable frequency supply.

(a) What is the source frequency for which current amplitude is maximum? Obtain this maximum value.

(b) What is the source frequency for which average power absorbed by the circuit is maximum? Obtain the value of this maximum power.

(c) For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency? What is the current amplitude at these frequencies? (d) What is the *Q*-factor of the given circuit?

Q15. A capacitor, resistor of 5 Ω and an inductor of 50 mH are in series with an a.c. source marked 100 V-50 Hz. It is found that voltage is in phase with the current. Calculate the capacitance of the capacitor and the impedance of the circuit.

Q16. An *LC*-circuit contains a 20 mH inductor and a 50 μF capacitor with an initial charge of 10 mC. The resistance of the circuit is negligible. Let the instant the circuit is closed be $t = 0$.

(a) What is the total energy stored initially? Is it conserved during *LC* oscillations?

(b) What is the natural frequency of the circuit?

(c) At what time is the energy stored (i) completely electrical (*i.e.*, stored in the capacitor)? (ii) completely magnetic (*i.e.*, stored in the inductor)?

(d) At what times is the total energy shared equally between the inductor and the capacitor?

(e) If a resistor is inserted in the circuit, how much energy is eventually dissipated as heat?

Q17. 2 μF capacitor, 100 Ω resistor and 8 H inductor are connected in series with an a.c. source, What should be the frequency of this a.c. source, for which the current drawn in the circuit is maximum? If the peak value of e.m.f. of the source is 200 V, find for maximum current

(a) the inductive and capacitive reactance's of the circuit;

(b) total impedance of the circuit;

(c) peak value of current in the circuit;

(d) the phase relation between voltages across inductor and resistor;

(e) the phase difference between voltages across inductor and capacitor

Q18. A 200 V variable frequency a.c. source is connected to a series combination of $L = 5$ H, $C = 80\ \mu F$ and $R = 40\ \Omega$. Calculate

(a) angular frequency of the source to get maximum current in the circuit,

(b) the current amplitude at resonance and

(c) the power dissipation in the circuit.

Date: 21/10/2021

S1. Capacitance, $C = 30\,\mu F = 30 \times 10^{-6}$ F

Inductance, $L = 27$ mH $= 27 \times 10^{-3}$ H

Angular frequency is given as:

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{27 \times 10^{-3} \times 30 \times 10^{-6}}} = \frac{1}{9 \times 10^{-4}}$$

$$= 1.11 \times 10^3 \text{ rad/s}$$

Hence, the angular frequency of free oscillations of the circuit is 1.11×10^3 rad/s.

S2. (a) The alternating voltages can be easily stepped up or stepped down by using a transformer.

(b) The alternating currents can be regulated by using a choke coil without any significant wastage of electrical energy.

S3. If the phase angle between e.m.f. (E_0) and current (I_v) is ϕ, then the component $I_v \sin\phi$ is called wattles current. It is because, the component $I_v \sin\phi$ does not contribute to the electric power of the circuit.

S4. (a) $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$ (b) $f = \dfrac{1}{2\pi\sqrt{LC}}$

S5. The range of frequency (f) of a radio is 800 kHz to 1200 kHz.

Lower tuning frequency, $f_1 = 800$ kHz $= 800 \times 10^3$ Hz

Upper tuning frequency, $f_2 = 1200$ kHz $= 1200 \times 10^3$ Hz

Effective inductance of circuit $L = 200\,\mu H = 200 \times 10^{-6}$ H

Capacitance of variable capacitor for f_1 is given as:

$$C_1 = \frac{1}{\omega_1^2 L}$$

Where,

ω_1 = Angular frequency for capacitor C_1

$= 2\pi f_1 = 2\pi \times 800 \times 10^3 \text{ rad s}^{-1}$

$\therefore \qquad C_1 = \dfrac{1}{(2\pi \times 800 \times 10^3)^2 \times 200 \times 10^{-6}}$

$= \mathbf{1.9809 \times 10^{-10}\ F = 198.1\ pF}.$

Capacitance of variable capacitor for f_2,

$$C_2 = \frac{1}{\omega_2^2 L}$$

Where,

ω_2 = Angular frequency for capacitor C_2

$= 2\pi f_2 = 2\pi \times 1200 \times 10^3\ \text{rad s}^{-1}$

$\therefore \qquad C_2 = \dfrac{1}{(2\pi \times 1200 \times 10^3)^2 \times 200 \times 10^{-6}}$

$= \mathbf{88.04\ pF}.$

Hence, the range of the variable capacitor is from 88.04 pF to 198.1 pF.

S6. (a) If R is resistance of the filament of bulb, then the impedance of the circuit,

$$Z = \sqrt{R^2 + X_L^2}$$

When the number of turns of the inductor is reduced, X_L and hence Z will decrease. This result in increase of current in the circuit and hence the bulb will glow more brightly.

(b) When a capacitor of reactance X_C is connected in series, then impedance of the circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Since $X_L = X_C$, $Z = R$ *i.e.*, impedance of the circuit will be minimum (merely equal to the resistance of the filament of the bulb). Hence, brightness of the bulb will become maximum.

S7. In a radio or a TV set, the antenna picks up the signals from all the broadcasting/telecasting stations. The tuning circuit in a radio/TV set, is basically an *LC*-circuit having variable capacitor. By varying the capacitance of the capacitor, when the frequency of the tuning circuit is changed to the frequency of the station, which we desire to tune in, the resonance occurs between the signal of the tuned frequency and the signal of this frequency present in the signals picked up by the antenna.

S8. Given: $L = 0.4\,\text{mH} = 0.4 \times 10^{-3}\,\text{H}$; $C = 250\,\text{pF} = 250 \times 10^{-12}\,\text{F}$

We know, $$f = \frac{1}{2\pi\sqrt{LC}}$$

$$f = \frac{1}{2\pi\sqrt{0.4 \times 10^{-3} \times 250 \times 10^{-12}}} = 5.03 \times 10^5$$

$\therefore$ $$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{5.03 \times 10^5} = \mathbf{596.42\,m}.$$

S9. Now, capacitive reactance,

$$X_C = \frac{1}{2\pi f C}$$

If E is e.m.f. of the source, then current through the lamp,

$$I = \frac{E}{X_C} = \frac{E}{1/2\pi f C} = 2\pi f C E\,.$$

(a) When the capacitance (C) is decreased, it follows that current through the lamp will decrease and hence the brightness of the lamp will **decrease**.

(b) When the frequency of the source (f) is decreased, it will also decrease the current through the lamp and hence the brightness of the lamp will **decrease**.

S10. When a charged capacitor in an LC-circuit discharges through the inductor, the electric energy stored between the plates of capacitor appears as the magnetic energy inside the inductor. When the capacitor has discharged, the magnetic field linked with the inductor starts collapsing.

Due to this, induced e.m.f. is produced in the inductor and the capacitor again states charging. However, the polarity of the charge on the two plates of the capacitor is opposite. Ultimately, the magnetic energy appears as the electric energy across the capacitor. This process repeats again and again, giving rise to LC-oscillations.

S11. Power factor of circuit A,

$$\cos\phi_A = \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{R}{\sqrt{R^2 + (3R)^2}} = \frac{1}{\sqrt{10}}$$

Power factor of circuit B,

$$\cos\phi_A = \frac{R}{\sqrt{R^2+(X_L-X_C)^2}} = \frac{R}{\sqrt{R^2+(3R-R)^2}} = \frac{1}{\sqrt{5}}$$

$\therefore$
$$\frac{\cos\phi_B}{\cos\phi_A} = \frac{1/\sqrt{5}}{1/\sqrt{10}} = \sqrt{\mathbf{2}}\,.$$

S12. Given: $L = 6.4\,\text{mH} = 6.4\times10^{-3}\,\text{H}$: $C = 100\,\mu\text{F} = 100\times10^{-6}\,\text{F}$ and $V = 12\,\text{V}$

The frequency of oscillations produced by the circuit,

$$f = \frac{1}{2\pi\sqrt{6.4\times10^{-3}\times100\times10^{-6}}} = \mathbf{198.94\ Hz}.$$

The current in the circuit will be maximum, when the energy stored in the capacitor becomes zero and it gets stored in the inductor. If I_{max} is the maximum value of the current in the circuit, then

$$\frac{1}{2}LI_{max}^2 = \frac{1}{2}CV^2$$

or
$$I_{max} = V\sqrt{\frac{C}{L}} = 12\times\sqrt{\frac{100\times10^{-6}}{6.4\times10^{-3}}} = \mathbf{1.5\ A}.$$

S13. Given: $E_v = 230\,\text{V}$; $L = 5.0\,\text{H}$; $C = 80\,\mu\text{F} = 80\times10^{-6}\,\text{F}$ and $R = 40\,\Omega$

(a) For resonance in the circuit,

$$X_L = X_C$$

or
$$\omega L = \frac{1}{\omega C}$$

or
$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5.0\times80\times10^{-6}}} = \mathbf{50\ rad\ s^{-1}}.$$

(b) Current at resonance,

$$I_v = \frac{E_v}{R} = \frac{230}{40} = \mathbf{5.75\ A}.$$

(c) Now
$$V_L = I_v\times\omega L = 5.75\times50\times5 = \mathbf{1{,}437.5\ V}.$$

S14. Inductance, $L = 0.12\,\text{H}$

Capacitance, $C = 480\ \text{nF} = 480 \times 10^{-9}\,\text{F}$

Resistance, $R = 23\,\Omega$

Supply voltage, $V = 230\,\text{V}$

Peak voltage is given as: $V_0 = \sqrt{2} \times 230 = 325.22\,\text{V}$

(a) Current flowing in the circuit is given by the relation,

$$I_0 = \frac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

Where, I_0 = maximum at resonance

At resonance, we have Where,

$$\omega_R L - \frac{1}{\omega_R C} = 0$$

Where, ω_R = Resonance angular frequency

$$\therefore \qquad \omega_R = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{0.12 \times 480 \times 10^{-9}}} = 4166.67\ \text{rad/s}$$

$\therefore$ Resonant frequency, $\nu_R = \dfrac{\omega_R}{2\pi} = \dfrac{4166.67}{2 \times 3.14} = 663.48\ \text{Hz}$

And, maximum current $(I_0)_{\max} = \dfrac{V_0}{R} = \dfrac{325.22}{23} = 14.14\,\text{A}$

(b) Maximum average power absorbed by the circuit is given as:

$$(P_{av})_{\max} = \frac{1}{2}(I_o)^2_{\max} R$$

$$= \frac{1}{2} \times (14.14)^2 \times 23 = 2299.3\ \text{W}$$

Hence, resonant frequency (ν_R) is 663.48 Hz.

The power transferred to the circuit is half the power at resonant frequency.

Frequencies at which power transferred is half,

$$= \omega_R \pm \Delta\omega$$

$$= 2\pi (\nu_R \pm \Delta\omega)$$

Where, $$\Delta\omega = \frac{R}{2L} = \frac{23}{2 \times 0.12} = 95.83 \text{ rad/s}$$

Hence, change in frequency, $$\Delta\nu = \frac{1}{2\pi}\Delta\omega = \frac{95.83}{2\pi} = 15.26 \text{ Hz}$$

$\therefore$ $$\nu_R + \Delta\nu = 663.48 + 15.26 = 678.74 \text{ Hz}$$

And, $$\nu_R + \Delta\nu = 663.48 - 15.26 = 648.22 \text{ Hz}$$

Hence, at 648.22 Hz and 678.74 Hz frequencies, the power transferred is half.

At these frequencies, current amplitude can be given as:

$$I' = \frac{1}{\sqrt{2}} \times (I_o)_{max} = \frac{14.14}{\sqrt{2}} = 10 \text{ A}$$

Q-factor of the given circuit can be obtained using the relation,

$$Q = \frac{\omega_R L}{R} = \frac{4166.67 \times 0.12}{23} = 21.74$$

Hence, the *Q*-factor of the given circuit is 21.74.

S15. Here, $$C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2 \times (50)^2 \times 50 \times 10^{-3}}$$

$$= 2.0264 \times 10^{-4} \text{ F} = \mathbf{202.64\,\mu F}$$

Also, $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Since $$X_L = X_C$$

$$Z = R = \mathbf{5\,\Omega}.$$

S16. Given: $L = 20 \times 10^{-3}$ H; $C = 50 \times 10^{-6}$ F; $Q = 10$ mC $= 10 \times 10^{-3}$ C

(a) At $t = 0$, $E_n = \frac{Q^2}{2C} = \frac{10^{-4}}{2 \times 50 \times 10^{-6}} = 1.0$ J

Yes, sum of the energies stored in *L* and *C* is conserved if $R = 0$.

(b) $\omega_r = \frac{1}{\sqrt{LC}} = 10^3$ rad/s and $f_r = 159$ Hz.

(c) $q = q_0 \cos \omega t$

(i) energy stored is completely electrical at

$$t = 0, \frac{T}{2}, T, \frac{3T}{2}, \ldots$$

(ii) energy stored is completely magnetic at

$$t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \ldots$$

where, $T = \frac{1}{f} = 6.3 \times 10^{-3}$ s

(d) At $t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, \ldots$

Total energy is shared equally by capacitor and inductor, because.

$$q = q_0 \cos \omega \frac{T}{8}$$

$$q = q_0 \cos \frac{\pi}{4} = \frac{q_0}{\sqrt{2}}$$

Electrical energy,

$$E_n = \frac{q^2}{2C} = \frac{1}{2}, \left[\frac{q_0^2}{2C}\right]$$

which is half the total energy.

(e) R damps out the LC oscillations. The whole of the initial energy (1.0 J) is eventually dissipated as heat.

S17. Given: $C = 2\,\mu\text{F} = 2 \times 10^{-6}\,\text{F}$; $R = 100\,\Omega$; $L = 8\,\text{H}$; $E_0 = 200\,\text{V}$

The current in the circuit becomes maximum, when frequency of the a.c. source becomes equal to the resonant frequency f_0.

The resonant frequency,

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{8 \times 2 \times 10^{-6}}} = \mathbf{39.79\ Hz}.$$

(a) X_L (or (X_C) at resonant frequency

$$= 2\pi f_0 L = 2\pi \times 39.79 \times 8 = 2{,}000\ \Omega.$$

(b) Total impedance of the circuit at resonant frequency is simply equal to ohmic resistance in the circuit *i.e.*, **100 Ω**.

(c) Peak value of current,

$$I_0 = \frac{E_0}{z} = \frac{E_0}{R} = \frac{200}{100} = \mathbf{2\,A}.$$

(d) The voltages across inductor and resistor differ in phase by **90°**.

(e) The voltages across inductor and capacitor differ in phase by **180°**.

S18. Given: $E_v = 200\,\text{V}$; $L = 5\,\text{H}$; $C = 80\,\mu\text{F} = 80 \times 10^{-6}$ and $R = 40\,\Omega$.

(a) For maximum current (or resonance) in the circuit,

$$X_L = X_C$$

Let ω_0 be the angular frequency of the source that provides maximum current in the circuit. Then,

$$\omega_0 L = \frac{1}{\omega_0 C} \quad \text{or} \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

or $$\omega_0 = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = \mathbf{50\,rad\,s^{-1}}.$$

(b) At resonance, the impedance of *LCR*-circuit is equal to the ohmic resistance in the circuit. Therefore,

r.m.s. value of current at resonance,

$$I_v = \frac{E_v}{R} = \frac{200}{40} = 5\,\text{A}$$

Hence current amplitude at resonance,

$$I_0 = \sqrt{2} I_p = \sqrt{2} \times 5 = \mathbf{7.07\,A}.$$

(c) Now, $$P_{av} = E_v I_v \cos\phi.$$

Since at resonance, the circuit is purely resistive, $\phi = 0°$. Therefore, average power consumed in the circuit,

$$P_{av} = 200 \times 5 \times \cos 0° = \mathbf{1{,}000\,W}.$$

Date: 21/10/2021

Q1. Transformer cannot work on d.c. Why?

Q2. Write the principle of a transformer.

Q3. What is the function of a choke coil in a fluorescent tube?

Q4. What causes the core of a transformer to get heated up under operation?

Q5. A choke coil in series with a lamp is connected to a dc line. The lamp is seen to shine brightly. Insertion of an iron core in the choke causes no change in the lamp's brightness. Predict the corresponding observations if the connection is to an ac line.

Q6. A capacitor is used in the primary circuit of an induction coil.

Q7. Write any two factors responsible for energy losses in actual transformers?

Q8. Give two reasons for power loss in a transformer.

Q9. Why is the core of a transformer laminated?

Q10. What is copper loss in a transformer?

Q11. Mention the two characteristic properties of the material suitable for making core of a transformer.

Q12. Does a step up transformer contradict the principle of conservation of energy?

Q13. A transformer is used to step down a.c. voltage. What appliance will you use to step down a d.c. voltage?

Q14. What is iron loss in a transformer and how it can be reduced?

Q15. A power transmission line feeds input power at 2300 V to a stepdown transformer with its primary windings having 4000 turns. What should be the number of turns in the secondary in order to get output power at 230 V?

Q16. At a hydroelectric power plant, the water pressure head is at a height of 300 m and the water flow available is $100\ m^3 s^{-1}$. If the turbine generator efficiency is 60%, estimate the electric power available from the plant ($g = 9.8\ m s^{-2}$).

Q17. A transformer of 100% efficiency has 500 turns in the primary and 10,000 turns in the secondary coil. If the primary is connected to 220 V mains supply, what is the voltage across the secondary coil?

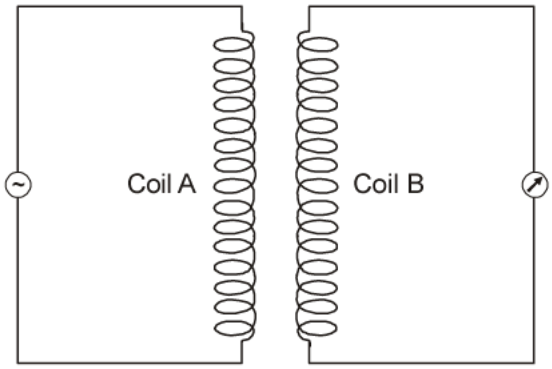
Q18. Write the working principle of a starter used along with a choke in a fluorescent tube.

Q19. A radio frequency choke is air-cored, whereas an audio frequency choke is iron-cored. Give reasons for this difference.

Q20. **The circuit arrangement given in figure below, when an a.c. passes through the coil *A*, the current starts flowing in the coil *B*.**

(a) State the underlying principle involved.

(b) Mention two factors on which the current produced in the coil *B* depends.



Q21. **A step-up transformer converts a low input voltage into a high output voltage. Does it violate law of conservation of energy? Explain.**

Q22. **In an ideal transformer, number of turns in the primary and secondary are 200 and 1000 respectively. If the power input to the primary is 10 kW at 200 V, calculate (a) output voltage and (b) current in primary**

Q23. **A transformer has an efficiency of 80%. It works at 4 kW and 100 volt. If the secondary voltage is 240 volt. Calculate the primary and secondary currents.**

Q24. **In an ideal transformer, number of turns in the primary and secondary are 200 and 1000 respectively. If the power input to the primary is 10 kW at 200 V. Calculate (a) output voltage and (b) current in primary.**

Q25. **Calculate the current drawn by the primary of a transformer, which steps down 200 V to 20 V to operate a device of resistance 20 Ω. Assume the efficiency of the transformer to be 80%.**

Q26. **A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric plant generating power at 440 V. The resistance of the two wire line carrying power is 0.5 Ω per km. The town gets power from the line through a 4000-220 V step-down transformer at a sub-station in the town.**

(a) Estimate the line power loss in the form of heat.

(b) How much power must the plant supply, assuming there is negligible power loss due to leakage?

(c) Characterise the step up transformer at the plant.

Q27. **Do the same exercise as above with the replacement of the earlier transformer by a 40,000 – 220 V step-down transformer (Neglect, as before, leakage losses though this may not be a good assumption any longer because of the very high voltage transmission involved). Hence, explain why high voltage transmission is preferred?**

Q28. **The primary coil of an ideal step-up transformer has 100 turns and the transformation ratio is also 100. The input voltage and power are 220 V and 1,100 W, respectively. Calculate (a) number of turns in the secondary; (b) the current in the primary; (c) voltage across the secondary; (d) the current in the secondary and (e) power in the secondary.**

Q29. 11 kW of electric power can be transmitted to a distant station at (a) 220 V or (b) 22,000 V. Which of the two modes of transmission should be preferred and why? Explain your answer with possible calculations.

Q30. Explain with the help of a labeled diagram the underlying principle and working of a step-up transformer. Why can not such a device be used to step-up d.c. voltage?

Q31. (a) State the principle of a step-up transformer. Explain, with the help of a labeled diagram, its working.

(b) Describe briefly any two energy losses, giving the reasons for their occurrence in actual transformers.

Q32. (a) With the help of a labeled diagram, describe briefly the underlying principle and working of a step up transformer.

(b) Write any two sources of energy loss in a transformer.

(c) A step-up transformer converts a low input voltage into a high output voltage. Does it violate law of conservation of energy?

Q33. (a) Draw a schematic diagram of a step-up transformer. Explain its working principle. Assuming the transformer to be 100% efficient, obtain the relation for (i) the current in the secondary in terms of the current in the primary, and (ii) the number of turns in the primary and secondary windings.

(b) Mention two important energy losses in actual transformers and state how these can be minimized.

Date: 21/10/2021

S1. If d.c. voltage is applied across the primary coil of a transformer, then the magnetic flux linked with the coil will not vary with time and hence no induced e.m.f. will be produced in the secondary coil.

S2. Mutual induction.

S3. The choke coil decreases current in the circuit without wasting electrical energy in the form of heat.

S4. Due to the production of eddy currents in the core of the transformer.

S5. If an iron core is inserted in the choke coil (which is in series with a lamp connected to the a.c. line), then the lamp will glow dimly. This is because the choke coil and the iron core increase the impedance of the circuit.

S6. High induced voltage is used to charge the capacitor.

A capacitor is used in the primary circuit of an induction coil. This is because when the circuit is broken, a high induced voltage is used to charge the capacitor to avoid sparks.

S7. (a) Iron. (b) Copper loss

S8. The magnetic flux loss and the hysteresis loss.

S9. The core of a transformer is laminated, so as to reduce the loss of energy due to eddy currents. The magnitude of the eddy currents set up is considerably minimised, when the core is laminated.

S10. Due to resistance of the windings of primary and secondary coils, some electrical energy is always converted into heat energy.

S11. (a) The material should have high permeability.

(b) The hysteresis loop of the material should be narrow.

S12. No, it does not.

S13. A pure resistance can be used to step down d.c. voltage.

S14. Due to production of eddy currents in the iron core, loss of electrical energy (in the form of heat energy) is known as iron loss. It is reduced by making use of a laminated iron core.

S15. Input voltage, $V_1 = 2300$

Number of turns in primary coil, $n_1 = 4000$

Output voltage, $V_2 = 230$ V

Number of turns in secondary coil $= n_2$

Voltage is related to the number of turns as:

$$\frac{V_1}{V_2} = \frac{n_1}{n_2}$$

$$\frac{2300}{230} = \frac{4000}{n_2}$$

$$n_2 = \frac{4000 \times 230}{2300} = 400$$

Hence, there are 400 turns in the second winding.

S16. Height of water pressure head, $h = 300\ \text{m}$

Volume of water flow per second, $V = 100\ \text{m}^3/\text{s}$

Efficiency of turbine generator, $n = 60\% = 0.6$

Acceleration due to gravity, $g = 9.8\ \text{m/s}^2$

Density of water, $\rho = 10^3\ \text{kg/m}^3$

Electric power available from the plant $= \eta \times h\rho g V$

$$= 0.6 \times 300 \times 10^3 \times 9.8 \times 100$$

$$= 176.4 \times 10^6\ \text{W}$$

$$= 176.4\ \text{MW}.$$

S17. For a 100% efficient transformer,

$$e_s = e_p \times \frac{N_s}{N_p}$$

Hence, $N_p = 500$; $N_s = 10{,}000$ and $e_p = 220$ V

$\therefore$ $$e_s = 220 \times \frac{10{,}000}{500} = \mathbf{4{,}400\ V}.$$

S18. A starter used along with a choke in a fluorescent tube is a thermal switch in the form of bimetallic strip. When the circuit is switched on, a high current passes through the thermal switch and raises the electrodes at the two ends of the Fluorescent tube to incandescence. Due to rise in temperature inside the starter, the bimetallic strip bends and the contact is instantaneously broken. Due to this, a high potential difference ($\approx 100\ \text{V}$) is induced between the electrodes of the tube, which allows a discharge to pass through the mercury vapours enclosed in the tube.

S19. The inductive reactance,

$$X_L = \omega L = 2\pi f(\mu_0 \mu_r n^2 l A).$$

It follows that at the radio frequency (f = very high), X_L will be very large, even when the choke is air-cored ($\mu_r = 1$). At the audio frequency (f = quite low), X_L will be large, provided the choke is iron-cored (μ_r = large). Therefore, a radio frequency choke is air-cored an audio frequency choke is iron-cored.

S20. (a) The phenomenon of mutual induction.

(b) The current produced in the coil B depends on the following two factors:

(i) Number of the turns of the two coils.

(ii) The manner in which the two coils are coupled. A large current will be produced, if the two coils are wound on each other using a soft iron core.

S21. No, it does not violate law of conservation of energy. In an ideal transformer, output power is always equal to the input power i.e.,

$$e_s I_s = e_p I_p.$$

Hence, in a step-up transformer, if we gain in voltage, we lose in current.

S22. Given: $N_p = 200$; $N_s = 1{,}000$; $e_p = 200\,V$

Power input to the primary, $e_p I_p = 10\,kW = 10^4\,W$

Now,

(a) $$e_s = \frac{N_S}{N_p} \times e_p = \frac{1{,}000}{200} \times 200 = 1{,}000\,V$$

(b) $$I_p = \frac{10^4}{e_p} = \frac{10^4}{200} = \mathbf{50\,A}.$$

S23. Given: $\eta = 80\%$; $e_p I_p = 4\,kW = 4{,}000\,W$; $e_p = 100\,V$; $e_s = 240\,V$

We know, $$I_p = \frac{e_p I_p}{e_p} = \frac{4{,}000}{100} = \mathbf{40\,A}$$

Also, $$\eta = \frac{e_s I_s}{e_p I_p} \Rightarrow I_s = \frac{\eta \times e_p I_p}{e_s}$$

or $$I_s = \frac{80 \times 4000}{240 \times 100} = \mathbf{13.33\,A}.$$

S24. Given: $N_p = 200$; $N_s = 1{,}000$; $e_p = 200\,V$

Power input to the primary,

$$e_p I_p = 10\,kW = 10^4\,W$$

(a) Now, $e_s = \frac{N_s}{N_p} \times e_p = \frac{1,000}{200} \times 200 = 1,000\text{ V}$

(b) $I_p = \frac{10^4}{e_p} = \frac{10^4}{200} = \mathbf{50\ A}$

S25. Given: $\eta = 80\%$; $e_p I_p = 4\text{ kW} = 4,000\text{ W}$; $e_p = 200\text{ V}$; $e_s = 20\text{ V}$

Now, $I_s = \frac{e_s}{Z} = \frac{20}{20} = \mathbf{1A}$

Also, $\eta = \frac{e_s I_s}{e_p I_p}$

or $\frac{80}{100} = \frac{20 \times 1}{200 \times I_p}$

or $I_p = \frac{20 \times 1 \times 100}{200 \times 80} = \mathbf{0.125\ A}$.

S26. Total electric power required, $P = 800\text{ kW} = 800 \times 10^3\text{ W}$

Supply voltage, $V = 220\text{ V}$

Voltage at which electric plant is generating power, $V' = 440\text{ V}$

Distance between the town and power generating station, $d = 15\text{ km}$

Resistance of the two wire lines carrying power = $0.5\ \Omega/\text{km}$

Total resistance of the wires, $R = (15 + 15)\ 0.5 = 15\ \Omega$

A step-down transformer of rating 4000 – 220 V is used in the sub-station.

Input voltage, $V_1 = 4000\text{ V}$

Output voltage, $V_2 = 220\text{ V}$

R.m.s. current in the wire lines is given as:

(a) Line power loss $= I^2R$

$= (200)^2 \times 15$

$= 600 \times 10^3\text{ W}$

$= 600\text{ kW}$

(b) Assuming that the power loss is negligible due to the leakage of the current:

Total power supplied by the plant = 800 kW + 600 kW

= 1400 kW

(c) Voltage drop in the power line $= IR = 200 \times 15 = 3000\text{ V}$

Hence, total voltage transmitted from the plant = 3000 + 4000 = 7000 V

Also, the power generated is 440 V.

Hence, the rating of the step-up transformer situated at the power plant is 440 V – 7000 V.

S27. The rating of a step-down transformer is 40000 V – 220 V.

Input voltage, $V_1 = 40000\,\text{V}$

Output voltage, $V_2 = 220\,\text{V}$

Total electric power required, $P = 800\text{ kW} = 800 \times 10^3\,\text{W}$

Source potential, $V = 220\,\text{V}$

Voltage at which the electric plant generates power,

$$V' = 440\,\text{V}$$

Distance between the town and power generating station,

$$d = 15\,\text{km}$$

Resistance of the two wire lines carrying power = 0.5 Ω/km

Total resistance of the wire lines, $R = (15 + 15)\,0.5 = 15\,\Omega$

$$P = V_1 I$$

R.m.s. current in the wire line is given as:

(a) Line power loss $= I^2R$

$$= (20)^2 \times 15 = 6\text{ kW}$$

(b) Assuming that the power loss is negligible due to the leakage of current.

Hence, power supplied by the plant = 800 kW + 6 kW = 806 kW

(c) Voltage drop in the power line $= IR = 20 \times 15 = 300$ V

Hence, voltage that is transmitted by the power plant

$$= 300 + 40000 = 40300\text{ V}$$

The power is being generated in the plant at 440 V.

Hence, the rating of the step-up transformer needed at the plant is 440 V – 40300 V.

Hence, power loss during transmission $= \dfrac{600}{1400} \times 100 = 42.8\%$.

In the previous exercise, the power loss due to the same reason is $\dfrac{6}{806} \times 100 = 0.744\%$.

Since the power loss is less for a high voltage transmission, high voltage transmissions are preferred for this purpose.

S28. Now, transformation ratio, $k = 100$; $N_p = 100$

(a) Given: $k = \dfrac{N_s}{N_p}$

$\therefore$ $N_s = k \times N_p = 100 \times 100 = \mathbf{10,000}$.

(b) Input power, $e_p I_p = 1,100$ watt; $e_p = 220$ volt

$$\therefore \quad I_p = \frac{1,100}{e_p} = \frac{1,100}{220} = \mathbf{5\ A}.$$

(c) Also, $k = \frac{e_s}{e_p}$

$$\therefore \quad e_s = ke_p = 100 \times 220 = \mathbf{22,000\ V}.$$

(d) Also, $\frac{e_s}{e_p} = \frac{I_p}{I_s}$

$$\therefore \quad I_s = \frac{e_p}{e_s} \times I_p = \frac{220}{22,000} \times 5 = \mathbf{0.05\ A}.$$

(e) Since the transformer is an ideal one,

power in the secondary = power in the primary

= **1,100 W**.

S29. Now, $e_p I_p = 11$ kW = 11,000 W.

(a) **When power is transmitted at e_s = 220 V:**

If I_s is current in the secondary coil, then

$$e_s I_s = e_p I_p$$

or $$I_s = \frac{e_p I_p}{e_s} = \frac{11,000}{220} = \mathbf{50\ A}.$$

If R is resistance of transmission lines, then electrical energy dissipated as heat,

$$I_s^2 R = 50^2 R = 2,500\, R \text{ watt}.$$

(c) **When power is transmitted at e_s' = 22,000 V:**

If I_s' is current in secondary coil, then

$$e_s' I_s' = e_s I_s$$

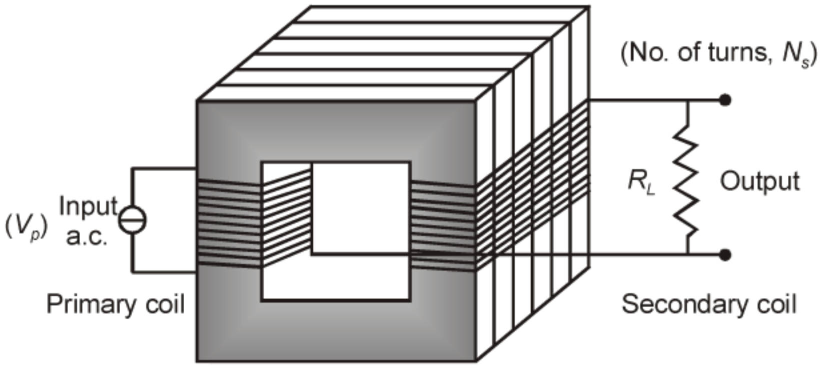
$$I_s' = \frac{e_p I_p}{e_s'} = \frac{11,000}{22,000} = \mathbf{0.5\ A}.$$

Therefore, electrical energy dissipated as heat,

$$(I_s')^2 R = (0.5)^2 R = 0.25\, R \text{ watt.}$$

Since electrical energy dissipated as heat is much less in the case (ii), transmission should be done at **22,000 V**.

S30. Step-up transformer is based on the principle of mutual induction.



An alternating potential (V_p) when applied to the primary coil induced an e.m.f. in it.

$$e_p = -N_p \frac{d\phi}{dt}$$

If resistance of primary coil is low $V_p = e_p$.

i.e., $$V_p = -N_p \frac{d\phi}{dt}$$

As same flux is linked with secondary coil with the help of soft iron core due to mutual induction e.m.f. is induced in it.

$$e_s = -N_s \frac{d\phi}{dt}$$

If output circuit is open

$$V_s = e_s$$

$$V_s = -N_s \frac{d\phi}{dt}$$

Thus, $$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

For step-up transformer $\frac{N_s}{N_p} > 1$.

In case of d.c. voltage flux does not change. Thus no e.m.f. is induced in the circuit.

S31. (a) **Principle:** It is based on the principle of metal induction *i.e.*, whenever there is a change in the current flowing through a coil, an e.m.f. is induced in the neighbouring coil.

Step-up transformer is based on the principle of mutual induction.

Working: When alternating voltage is applied to the primary, magnetic flux linked with it and with the secondary coil changes. This changing magnetic flux induces e.m.f. in it.

More is the flux linkage more is the e.m.f. induced *i.e.*, more are the number of turns more is the e.m.f. In case of set-up transformer number of turns in the secondary coil are more than the number of turns in the primary coil.

(b) **Energy Losses:** In practice, the power output of a transformer is less than the power input because of unavoidable energy losses. These losses are:

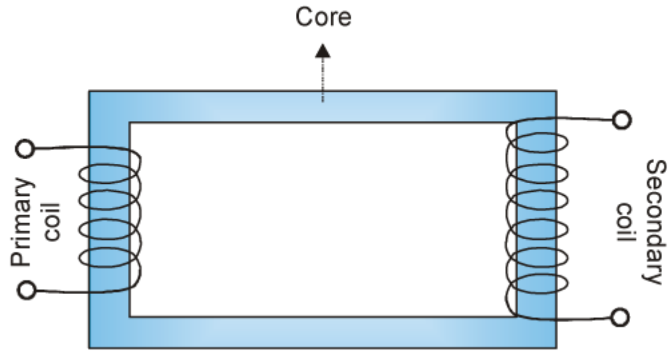
Copper Losses: As the alternating current flows through the primary and the secondary, heat is developed inside the copper turns. This waste of energy is known as 'copper losses'.

Eddy current Losses: Eddy current are set up in the iron core of the transformer and generate heat, with consequent loss of energy. To minimise these losses the iron core is laminated by making it of number of thin sheets of iron insulated from each other, instead of making it form one solid piece of iron.

Hysteresis Losses: During each cycle of A.C. the core is taken through a complete cycle of magnetisation. The energy expanded in this process is finally converted into heat and is therefore wasted. This loss minimised by using the core of a magnetic alloy for which the area of the hysteresis loop is a minimum.

Flux Losses: The coupling of the primary and the secondary coils is never perfect. Therefore, the whole of the magnetic flux generated in the primary does not pass through the secondary.

S32. (a) ***Principle:*** It is based on the principle of metal induction *i.e.*, whenever there is a change in the current flowing through a coil, an e.m.f. is induced in the neighbouring coil.
Step-up transformer is based on the principle of mutual induction.
Transformer is based on the principle of mutual induction.



Working: When alternating voltage is applied to the primary, magnetic flux linked with it and with the secondary coil changes. This changing magnetic flux induces e.m.f. in it.

More is the flux linkage more is the e.m.f. induced *i.e.*, more are the number of turns more is the e.m.f. In case of set-up transformer number of turns in the secondary coil are more than the number of turns in the primary coil.

(i) Input power = output power

(ii) Primary has negligible resistance

(iii) No flux loss

(iv) Secondary is an open circuit or resistance is very high.

Each e.m.f. in the primary is

$$e_p = -N_p \frac{d\phi}{dt} \qquad (\because N_p = \text{No. of turns in the primary})$$

Here input voltage $V_p = e_p$

So, $$V_p = -N_p \frac{d\phi}{dt} \qquad \text{... (i)}$$

Induced e.m.f. in the secondary is

$$e_s = -N_s \frac{d\phi}{dt}$$

Here output voltage $V_p = e_s$

So $$V_s = -N_s \frac{d\phi}{dt} \qquad \text{... (ii)}$$

Eqn. (i) ÷ (ii), we get

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

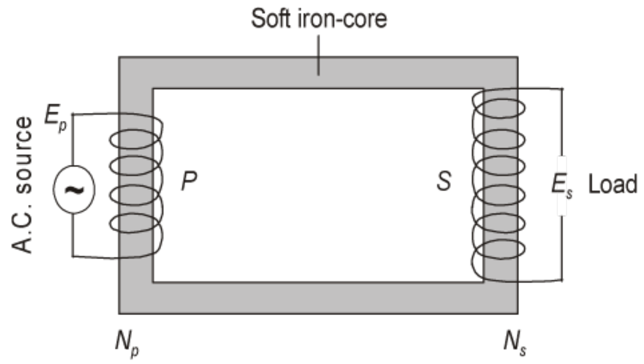
For step up transformer $\frac{N_s}{N_p} > 1.$

(b) **Energy Losses:** In practice, the power output of a transformer is less than the power input because of unavoidable energy losses. These losses are:

Copper Losses: As the alternating current flows through the primary and the secondary, heat is developed inside the copper turns. This waste of energy is known as 'copper losses'.

(c) For a given power supply high output voltage means low output current. As there is no gain in power law of conservation of energy is not violated.

S33. (a) The schematic diagram of a step-up transformer



A transformer works on the principle of mutual induction. When a varying current is passed through a coil, the magnetic flux with this coil will change. If another coil is placed near it, then the magnetic flux in the neighbouring coil will also change and hence an induced e.m.f. is produced in it also. The magnitude of the induced e.m.f if the neighbouring coil ill depend upon the rate of change of magnetic flux through the coil.

If E_p and E_s are induced e.m.f. produced in the primary and secondary coils of the transformer and $-(\frac{d\phi}{dt})$, the induced e.m.f. produced in the primary and secondary per turn, then

$$E_p = -N_p\left(\frac{d\phi}{dt}\right) \quad \text{... (i)}$$

$$E_s = -N_s\left(\frac{d\phi}{dt}\right) \quad \text{... (ii)}$$

$\therefore \; \frac{E_s}{E_p} = \frac{N_s}{N_p} = k$, where k is the turns ration.

In a 100% efficient transformer.

$E_s I_s = E_p I_p$ where I_s and I_p are the secondary and primary current.

$$\frac{E_s}{E_p} = \frac{I_p}{I_s} \quad \text{... (iii)}$$

From Eqn. (ii) and (iii), we get

$$\therefore \quad \frac{I_p}{I_s} = \frac{N_s}{N_p} = k$$

$$\therefore \quad I_s = \frac{I_p}{k}$$

$$\text{and} \quad N_p = \frac{N_s}{k}$$

(b) The two important energy losses in actual transformers are:

(i) **Hysteresis Losses:** During each cycle of A.C. the core is taken through a complete cycle of magnetisation. The energy expanded in this process is finally converted into heat and is therefore wasted. This loss minimised by using the core of a magnetic alloy for which the area of the hysteresis loop is a minimum.

(ii) **Flux Losses:** The coupling of the primary and the secondary coils is never perfect. Therefore, the whole of the magnetic flux generated in the primary does not pass through the secondary.