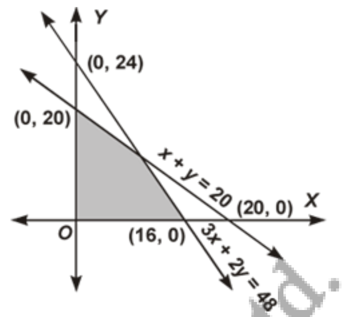
Date: 9/10/2021

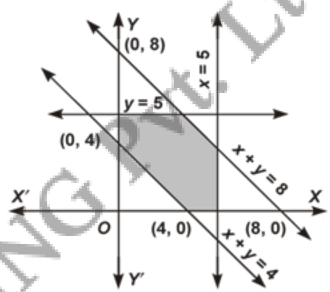
**Q1.** Solve : $|3 - 4x| \geq 9$.

**Q2.** Solve : $1 \leq |x - 2| \leq 3$.

**Q3.** Find the linear inequalities for which the shaded region in the given figure is the solution set.




**Q4.** Find the linear inequalities for which the shaded region in the given figure is the solution set.




**Q5.** A company manufactures cassettes. Its cost and revenue functions are $C(x) = 26{,}000 + 30x$ and $R(x) = 43x$, respectively, where $x$ is the number of cassettes produced and sold in a week, How many cassettes must be sold by the company to realise some profit?

**Q6.** The longest side of a triangle is twice the shortest side and the third side is 2 cm longer than the shortest side. If the perimeter of the triangle is more than 166 cm, then find the minimum length of the shortest side.

**Q7.** In drilling world's deepest hole, it was found that the temperature $T$ in degree celcius, $x$ km below the Earth's surface was given by $T = 30 + 25(x - 3)$, $3 \leq x \leq 15$. At what depth will the temperature be between 155°C and 205°C?

**Q8.** The cost and revenue functions of a product are given by $C(x) = 20x + 4000$ and $R(x) = 60x + 2000$, respectively, where x is the number of items produced and sold. How many items must be sold to realise some profit?

**Q9.** Solve the following system of linear inequalities:

$$3x + 2y \geq 24, \quad 3x + y \leq 15, \quad x \geq 4.$$

**Q10.** Show that the following system of linear inequalities has no solution:

$$x + 2y \leq 3, \quad 3x + 4y \geq 12, \quad x \geq 0, \quad y \geq 1.$$

**Q11.** Solve the following system of inequalities

$$\frac{x}{2x+1} \geq \frac{1}{4}, \quad \frac{6x}{4x-1} < \frac{1}{2}$$

**Q12.** Solve the following system of inequalities

$$\frac{2x+1}{7x-1} > 5, \quad \frac{x+7}{x-8} > 2$$

**Q13.** Solve for $x$, in the following inequalities:

$$-5 \le \frac{2-3x}{4} \le 9$$

**Q14.** Solve for $x$, in the following inequalities: $|x-1| \le 5$, $|x| \ge 2$.

**Q15.** Solve for $x$, in the following inequalities:

$$\frac{4}{x+1} \le 3 \le \frac{6}{x+1}, \qquad (x > 0)$$

**Q16.** Solve for $x$ : $\frac{|x+3|+x}{x+2} > 1$.

**Q17.** Solve for $x$, $|x+1| + |x| > 3$.

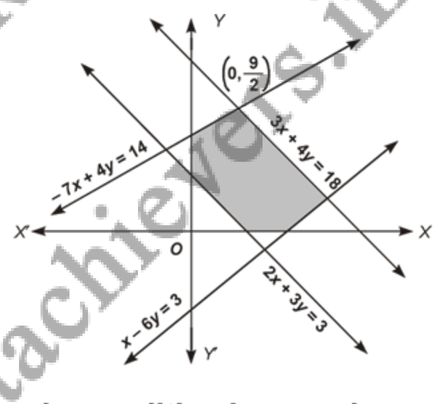
**Q18.** Solve the inequality, $3x - 5 < x + 7$, when

(i) $x$ is a natural number
(ii) $x$ is a whole number
(iii) $x$ is an integer
(iv) $x$ is a real number

**Q19.** Solve for $x$ : $\frac{x-2}{x+5} > 2$.

**Q20.** Solve for $x$: $\frac{1}{|x|-3} \le \frac{1}{2}$.

**Q21.** Find the linear inequalities for which the shaded region in the given figure is the solution set.




**Q22.** Show that the solution set of the following system of linear inequalities is an unbounded region

$$2x + y \ge 8, \quad x + 2y \ge 10, \quad x \ge 0, \quad y \ge 0.$$

**Q23.** A solution of 9% acid is to be diluted by adding 3% acid solution to it. The resulting mixture is to be more than 5% but less than 7% acid. If there is 460 litres of the 9% solution, how many litres of 3% solution will have to be added?

**Q24.** The water acidity in a pool is considered normal when the average pH reading of three daily measurements is between 8.2 and 8.5. If the first two pH readings are 8.48 and 8.35, find the range of pH value for the third reading that will result in the acidity level being normal.

**Q25.** A solution is to be kept between 40°C and 45°C. What is the range of temperature in degree fahrenheit, if the conversion formula is $F = \frac{9}{5}C + 32$?

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**S1.** We have, $|3-4x| \geq 9$

$\Rightarrow \quad 3-4x \leq -9 \quad \text{or} \quad 3-4x \geq 9 \quad$ (Since, $|x| \geq a \Rightarrow x \leq -a$ or $x \geq a$)

$\Rightarrow \quad -4x \leq -12 \quad \text{or} \quad -4x \geq 6$

$\Rightarrow \quad x \geq 3 \quad \text{or} \quad x \leq \frac{-3}{2} \quad$ (Dividing both sides by $-4$)

$\Rightarrow \quad x \in \left(-\infty, \frac{-3}{2}\right] \cup [3, \infty)$.

**S2.** We have, $1 \leq |x-2| \leq 3$

$\Rightarrow \quad |x-2| \geq 1 \quad \text{and} \quad |x-2| \leq 3$

$\Rightarrow \quad (x-2) \leq -1 \text{ or } x-2 \geq 1) \quad \text{and} \quad (-3 \leq x-2 \leq 3)$

$\Rightarrow \quad (x \leq 1 \text{ or } x \geq 3) \quad \text{and} \quad (-1 \leq x \leq 5)$

$\Rightarrow \quad x \in (-\infty, 1] \cup [3, \infty) \quad \text{and} \quad x \in [-1, 5]$

Combining the solutions of two inequalities, we have

$x \in [-1, 1] \cup [3, 5]$.

**S3.** Required inequalities are: $x + y \leq 20$

$3x + 2y \leq 48$

$x \geq 0$

$y \geq 0$.

**S4.** Let $x + y \leq 8$

$x + y \geq 4$

$x \leq 5$

$y \leq 5$

$x \geq 0$

$y \geq 0$.

**S5.** Given, $C(x) = 26,000 + 30x$

$R(x) = 43x$

$R(x) > C(x) \quad$ (For profit)

$\Rightarrow \quad 43x > 26,000 + 30x$

$\Rightarrow \qquad 13x > 26{,}000$

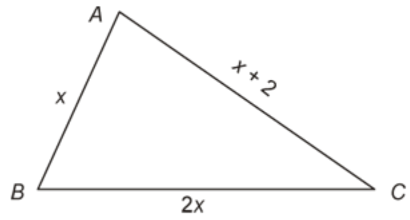
$\Rightarrow \qquad x > 2{,}000$

Hence, more than 2000 cassettes must be sold.

**S6.** Let Shortest side = $AB = x$

and Longest side = $BC = 2x$

and Third side = $AC = x + 2$




Perimeter = $x + 2x + x + 2$

$= 4x + 2$

Now, $4x + 2 > 166$

$\Rightarrow \qquad 4x > 164$

$\Rightarrow \qquad x > \dfrac{164}{4}$

$\Rightarrow \qquad x > 41$

Hence,, minimum length of shortest side = 41 cm.

**S7.** We know that $3 \le x \le 15$ and $T = 30 + 25\,(x - 3)$

Now, $155 < T < 205$

$\Rightarrow \qquad 155 < 30 + 25\,(x - 3) < 205$

$\Rightarrow \qquad 155 < 30 + 25x - 75 < 205$

$\Rightarrow \qquad 155 < 25x - 45 < 205$

$\Rightarrow \qquad 155 + 45 < 25x < 205 + 45$

$\Rightarrow \qquad 200 < 25x < 250$

$\Rightarrow \qquad 8 < x < 10$

Hence, for depth $x$, 8 km < $x$ < 10 km.

**S8.** We have, Profit = Revenue – Cost

$= (60x + 2000) - (20x + 4000)$

$= 40x - 2000$

To earn some profit, $40x - 2000 > 0$

$\Rightarrow \qquad x > 50$

Hence, the manufacturer must sell more than 50 items to realise some profit.

**S9.** $3x + 2y \ge 24$

Let $3x + 2y = 24$

$$y = \frac{24 - 3x}{2}$$

| $x$ | 0 | 4 | 8 |
|---|---|---|---|
| $y$ | 12 | 6 | 0 |

Also, (0, 0) does not lie in the required half plane:

Again, $3x + y \le 15$

Let $\Rightarrow$ $3x + y = 15$

$\Rightarrow$ $y = 15 - 3x$

Here, (0, 0) lies in the required half plane.

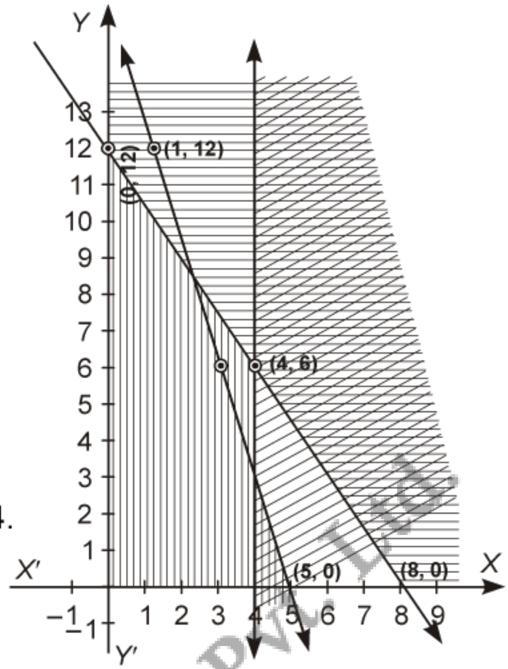
| $x$ | 5 | 3 | 1 |
|---|---|---|---|
| $y$ | 0 | 6 | 12 |

Now $x \ge 4$

It is the half plane which contains $x = 4$ and the points $x > 4$.

Triple shaded region is **Null Set**.

(There is no region which is Triple shaded)




**S10.** $x + 2y \le 3$

Let $x + 2y = 3$

$\Rightarrow$ $x = 3 - 2y$

| | A | B | C |
|---|---|---|---|
| $x$ | 3 | 1 | –1 |
| $y$ | 0 | 1 | 2 |

Now (0, 0) satisfy $x + 2y \le 3$ or makes it true.

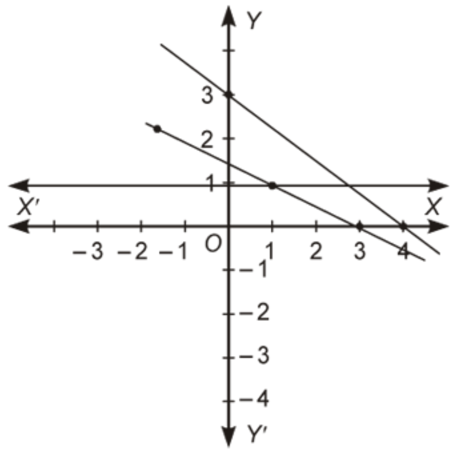
$\therefore$ Half plane containing (0, 0) is the solution.

Now, $3x + 4y \ge 12$

Let $3x + 4y = 12 \Rightarrow y = \dfrac{12 - 3x}{4}$

Now (0, 0) does not satisfy $3x + 4y \ge 12$

| $x$ | 0 | 4 |
|---|---|---|
| $y$ | 3 | 0 |




$\therefore$ The half plane of the solution does not contains (0, 0)

Now, $x \geq 0$ gives the solution where points lie on the $y$-axis and to the right of it.

Now, $y \geq 1$ gives us a line $y = 1$ which is parallel to $x - a = 0$ and lies on the upper side of it. ($a \in R$)

Since, the two shaded portions do not have any thing common, therefore the solution set is **Null Set**.

**S11.** From the first inequality, we have

$$\frac{x}{2x+1} - \frac{1}{4} \geq 0$$

$$\Rightarrow \quad \frac{2x-1}{2x+1} \geq 0$$

$$\Rightarrow \quad (2x - 1 \geq 0 \text{ and } 2x + 1 > 0) \quad \text{or} \quad (2x - 1 \leq 0 \text{ and } 2x + 1 < 0) \quad [\text{Since, } 2x + 1 \neq 0]$$

$$\Rightarrow \quad \left(x \geq \frac{1}{2} \text{ and } x > -\frac{1}{2}\right) \quad \text{or} \quad \left(x \leq \frac{1}{2} \text{ and } x < -\frac{1}{2}\right)$$

$$\Rightarrow \quad x \geq \frac{1}{2} \quad \text{or} \quad x < -\frac{1}{2}$$

$$\Rightarrow \quad x \in \left(-\infty, -\frac{1}{2}\right) \cup \left[\frac{1}{2}, \infty\right) \quad \text{... (i)}$$

From the second inequality, we have

$$\frac{6x}{4x-1} - \frac{1}{2} < 0$$

$$\Rightarrow \quad \frac{8x+1}{4x-1} < 0$$

$$\Rightarrow \quad (8x + 1 < 0 \text{ and } 4x - 1 > 0) \quad \text{or} \quad (8x + 1 > 0 \text{ and } 4x - 1 < 0) \quad [\text{Since, } 4x - 1 \neq 0]$$

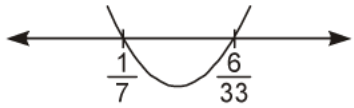
$$\Rightarrow \quad \left(x < -\frac{1}{8} \text{ and } x > \frac{1}{4}\right) \quad \text{or} \quad \left(x > -\frac{1}{8} \text{ and } x < \frac{1}{4}\right)$$

$$\Rightarrow \quad x \in \left(-\frac{1}{8}, \frac{1}{4}\right) \quad \text{(since the first is not possible)} \quad \text{... (ii)}$$

Since, common solution of Eq. (i) and (ii) is null set. Hence, the given system of inequalities has no solution.

**S12.** Let $\dfrac{2x+1}{7x-1} > 5, \quad \dfrac{x+7}{x-8} > 2$

$\Rightarrow \quad \dfrac{2x+1}{7x-1} > 5 \quad \Rightarrow \quad \dfrac{2x+1}{7x-1} - 5 > 0$




$\Rightarrow \quad \dfrac{2x+1-35x+5}{7x-1} > 0 \quad \Rightarrow \quad \dfrac{-33x+6}{7x-1} > 0$

$$x > \left(-\infty, \frac{1}{7}\right) \cup \left(\frac{6}{33}, \infty\right) \qquad \text{... (i)}$$

$\dfrac{x+7}{x-8} > 2 \quad \Rightarrow \quad \dfrac{x+7}{x-8} - 2 > 0$

$\Rightarrow \quad \dfrac{x+7-2x+16}{x-8} > 0 \quad \Rightarrow \quad \dfrac{-x+23}{x-8} > 0$


$$x > (-\infty, 8) \cup (23, \infty) \qquad \text{... (ii)}$$

Combining (i) and (ii), we get

$$x < \frac{1}{7} \quad \text{or} \quad x > 23.$$

**S13.** Now, $-5 \leq \dfrac{2-3x}{4}$

$\Rightarrow \quad -20 \leq 2 - 3x$

$\Rightarrow \quad 3x \leq 2 + 20$

$\Rightarrow \quad x \leq \dfrac{22}{3}$ ... (i)

Again $\dfrac{2-3x}{4} \leq 9$

$\Rightarrow \quad 2 - 3x \leq 36$

$\Rightarrow \quad -3x \leq 34$

$\Rightarrow \quad 3x \geq -34$

$x \geq \dfrac{-34}{3}$ ... (ii)

From Eq. (i) and (ii), we get the solution:

$$\left[\frac{-34}{3}, \frac{22}{3}\right].$$

**S14.** We have, $|x-1| \leq 5, \quad |x| \geq 2$

$\Rightarrow \quad -(x-1) \leq 5 \quad$ (for $x < 1$) and $x - 1 \leq 5$ (for $x \geq 1$)

$\Rightarrow \quad -x + 1 \leq 5$

and $x \le 6$

$\Rightarrow$ $-x \le 4$

and $1 \le x \le 6$

$\Rightarrow$ $x > -4$

and $1 \le x \le 6$

$\Rightarrow$ $-4 \le x \le 1$

and $1 \le x \le 6$ ... (i)

Again $|x| \ge 2$

$\Rightarrow$ $-x \ge 2$ (for $x < 0$)

and $x \ge 2$ (for $x \ge 0$)

$\Rightarrow$ $x \le -2$

and $x \ge 2$ ... (ii)

From Eq. (i) and (ii), we get the solution:

$$[-4, -2] \cup [2, 6].$$

**S15.** Let $\frac{4}{x+1} \le 3$

$\Rightarrow$ $4 \le 3x + 3$

$\Rightarrow$ $3x + 3 \ge 4$

$\Rightarrow$ $3x \ge 1 \Rightarrow x \ge \frac{1}{3}$

Again, $3 \le \frac{6}{x+1}$

$\Rightarrow$ $3x + 3 \le 6$

or $3x \le 3$

$\Rightarrow$ $x \le 1$

Hence, solution set is $\frac{1}{3} \le x \le 1$.

**S16.** We have, $\frac{|x+3|+x}{x+2} > 1$

$\Rightarrow$ $\frac{|x+3|+x}{x+2} - 1 > 0$

$\Rightarrow$ $\frac{|x+3|-2}{x+2} > 0$

Now two cases arise:

**Case 1 :** When $x + 3 \ge 0$, *i.e.*, $x \ge -3$.

Then $\dfrac{|x+3|-2}{x+2} > 0 \quad \Rightarrow \quad \dfrac{x+3-2}{x+2} > 0$

$\Rightarrow \quad \dfrac{x+1}{x+2} > 0$

$\Rightarrow \quad \{(x + 1) > 0 \text{ and } x + 2 > 0\} \quad \text{or} \quad \{x + 1 < 0 \text{ and } x + 2 < 0\}$

$\Rightarrow \quad \{x > -1 \text{ and } x > -2\} \quad \text{or} \quad \{x < -1 \text{ and } x < -2\}$

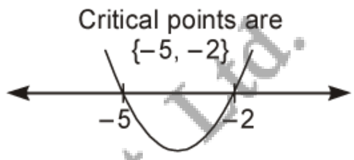
$\Rightarrow \quad x > -1 \quad \text{or} \quad x < -2$

$\Rightarrow \quad x \in (-1, \infty) \quad \text{or} \quad x \in (-\infty, -2)$

$\Rightarrow \quad x \in (-3, -2) \cup (-1, \infty) \quad [\text{Since, } x \geq -3]$

**Case 2:** When $x + 3 < 0$, *i.e.*, $x < -3$.

Then $\dfrac{|x+3|-2}{x+2} > 0 \quad \Rightarrow \quad \dfrac{-x-3-2}{x+2} > 0$




$\Rightarrow \quad \dfrac{-(x+5)}{x+2} > 0 \quad \Rightarrow \quad \dfrac{x+5}{x+2} < 0$

$\Rightarrow \quad x \in (-5, -2)$

Combining the results of case (1) and (2), the required solution is

$$x \in (-5, -2), \cup (-1, \infty).$$

**S17.** On L.H.S. of the given inequality, we have two terms both containing modulus. By equating the expression within the modulus to zero, we get $x = -1, 0$ as critical points. These critical points divide the real line in three parts as $(-\infty, -1)$, $[-1, 0)$, $[0, \infty)$.

**Case 1:** When, $-\infty < x < -1$

$|x + 1| + |x| > 3 \quad \Rightarrow \quad -x - 1 - x > 3 \quad \Rightarrow \quad x < -2.$

**Case 2:** When, $-1 \leq x < 0$

$|x + 1| + |x| > 3 \quad \Rightarrow \quad x + 1 - x > 3 \quad \Rightarrow \quad 1 > 3.$ (Not possible)

**Case 3:** When, $0 \leq x < \infty$

$|x + 1| + |x| > 3 \quad \Rightarrow \quad x + 1 + x > 3 \quad \Rightarrow \quad x > 1.$

Combining the results of case (1), (2) and (3), we get

$$x \in (-\infty, -2) \cup (1, \infty).$$

**S18.** We have, $3x - 5 < x + 7$

$\Rightarrow \quad 3x < x + 12$ (Adding 5 to both sides)

$\Rightarrow \quad 2x < 12$ (Subtracting $x$ from both sides)

$\Rightarrow \quad x < 6$ (Dividing by 2 on both sides)

(i) Solution set is {1, 2, 3, 4, 5}

(ii) Solution set is {0, 1, 2, 3, 4, 5}

(iii) Solution set is {..., –3, –2, –1, 0, 1, 2, 3, 4, 5}

(iv) Solution set is $\{x : x \in \boldsymbol{R} \text{ and } x < 6\}$, *i.e.*, any real number less than 6.

**S19.** We have, $\frac{x-2}{x+5} > 2$

$\Rightarrow \quad \frac{x-2}{x+5} - 2 > 0$ [Subtracting 2 from each side]

$\Rightarrow \quad \frac{-(x+12)}{x+5} > 0$

$\Rightarrow \quad \frac{x+12}{x+5} < 0$ [Multiplying both sides by, – 1]

$\Rightarrow \quad x + 12 > 0$ and $x + 5 < 0$ [Since $\frac{a}{b} < 0 \Rightarrow a$ and $b$ are of opposite signs]

or $\quad x + 12 < 0$ and $x + 5 > 0$

$\Rightarrow \quad x > -12$ and $x < -5$

or $\quad x < -12$ and $x > -5$ [Not possible]

Therefore, $-12 < x < -5$ *i.e.*, $x \in (-12, -5)$.

**S20.** Let $|x| - 3 \geq 2$

$\Rightarrow \quad -x - 3 \geq 2,$ $(x < 0)$

and $\quad x - 3 \geq 2,$ $(x \geq 0)$

$\Rightarrow \quad -x - 3 \geq 2$

and $\quad x \geq 5$

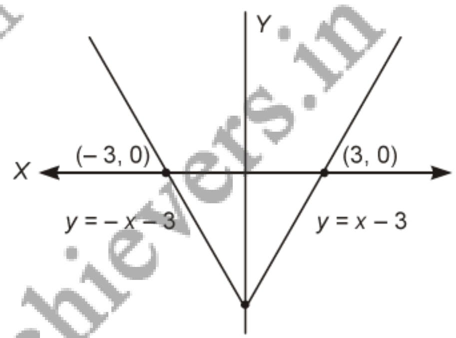
$\Rightarrow \quad -x \geq 5$

and $\quad x \geq 5$

$\Rightarrow \quad x \leq -5$

and $\quad x \geq 5$

Also, $\quad |x| - 3 \neq 0$




$x \in (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$ [From graph]

Now, the solution set is

$(-\infty, -5] \cup (-3, 3) \cup [5, \infty)$.

**S21.** (i) Consider $2x + 3y = 3$. We observe that the shaded region and the origin lie on opposite side of this line and (0, 0) satisfies $2x + 3y \leq 3$. Therefore, we must have $2x + 3y \geq 3$ as linear inequality corresponding to the line $2x + 3y = 3$.

(ii) Consider $3x + 4y = 18$. We observe that the shaded region and the origin lie on the same side of this line and (0, 0) satisfies $3x + 4y \leq 18$. Therefore, $3x + 4y \leq 18$ is the linear inequality corresponding to the line $3x + 4y = 18$.

(iii) Consider $-7x + 4y = 14$. It is clear from the figure that the shaded region and the origin lie on the same side of this line and (0, 0) satisfies the inequality $-7x + 4y \leq 14$. Therefore, $-7x + 4y \leq 14$ is the linear inequality corresponding to the line $-7x + 4y = 14$.

(iv) Consider $x - 6y = 3$. It may be noted that the shaded portion and origin lie on the same side of this line and (0, 0) satisfies $x - 6y \leq 3$. Therefore, $x - 6y \leq 3$ is the inequality corresponding to the line $x - 6y = 3$.

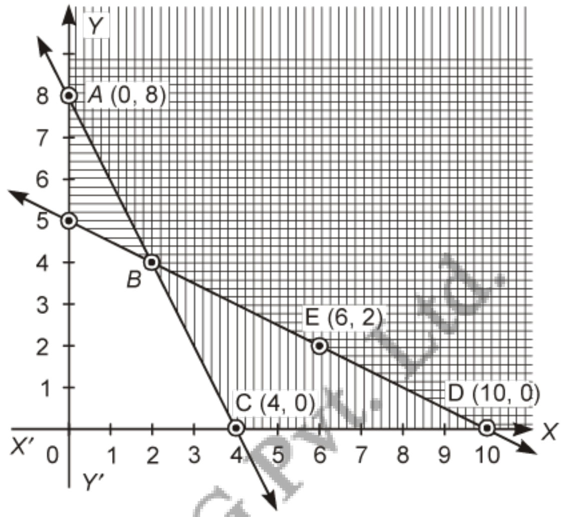
(v) Also the shaded region lies in the first quadrant only. Therefore, $x \geq 0$, $y \geq 0$. Hence, in view of (i) (ii), (iii) and (iv) above, the linear inequalities corresponding to the given solution set are:

$$2x + 3y \geq 3, \quad 3x + 4y \leq 18, \quad -7x + 4y \leq 14, \quad x - 6y \leq 3, \quad x \geq 0, \quad y \geq 0.$$

**S22.** As $x \geq 0$, $y \geq 0$, we shall plot the other inequalities in the first quadrant. Consider $2x + y \geq 8$.

Let $2x + y = 8 \Rightarrow y = 8 - 2x$

| | A | B | C |
|---|---|---|---|
| $x$ | 0 | 2 | 4 |
| $y$ | 8 | 4 | 0 |




For (0, 0) $2x + y \geq 8$ is false, therefore, the half plane does not contain (0, 0).

Again, $x + 2y \geq 10$

Let $x + 2y = 10 \Rightarrow x = 10 - 2y$

| | D | E | F |
|---|---|---|---|
| $x$ | 10 | 6 | 0 |
| $y$ | 0 | 2 | 5 |

Now for (0, 0) also $x + 2y \geq 10$ is false, therefore the half plane does not contain (0, 0).

Now, region *DBAY* is **unbounded**.

**S23.** The volume of 9% acid solution = 460 litres.

Let the volume of added solution of 3% solution = $x$ litres

Presence of acid in the two solution

$$= \left(\frac{9}{100} \times 460 + \frac{3}{100} \times x\right) \text{ litres}$$

Now, $\frac{5}{100}(460 + x) < \frac{9}{100} \times 460 + \frac{3}{100} \times x < \frac{7}{100} \times (460 + x)$

$\Rightarrow$ $5(460 + x) < 9 \times 460 + 3x < 7(460 + x)$

$\Rightarrow$ $2300 + 5x < 4140 + 3x < 3220 + 7x$

$\Rightarrow$ $5x < 1840 + 3x < 920 + 7x$

Now, $5x < 1840 + 3x$

or $1840 + 3x < 920 + 7x$

$\Rightarrow$ $2x < 1840$ or $920 + 7x > 1840 + 3x$

$\Rightarrow \quad x < 920 \quad \text{or} \quad 4x > 920$

$\Rightarrow \quad x < 920 \quad \text{or} \quad x > 230$

Hence, the required volume is **greater than 230 litres** and **less than 920 litres**.

**S24.** Given, First pH value = 8.48

and Second pH value = 8.35

Let Third pH value = $x$

Now, Average pH value = $\frac{8.48 + 8.35 + x}{3}$

$\therefore \quad 8.2 < \frac{8.48 + 8.35 + x}{3} < 8.5$

$\Rightarrow \quad 3 \times 8.2 < 16.83 + x < 3 \times 8.5$

$\Rightarrow \quad 24.6 < 16.83 + x < 25.5$

$\Rightarrow \quad 24.6 - 16.83 < x < 25.5 - 16.83$

$\Rightarrow \quad 7.77 < x < 8.67$

For third pH value the range is between **7.77** and **8.67**.

**S25.** Let the required temperature = $x$

$\therefore \quad 40°C < x < 45°C$

Now, $F = \frac{9}{5}C + 32$

$\Rightarrow \quad 5F = 9C + 160$

$\Rightarrow \quad C = \frac{5F - 160}{9}$

$\therefore \quad 40° < \frac{5F - 160}{9} < 45°$

$\Rightarrow \quad 360 < 5F - 160 < 405$

$\Rightarrow \quad 360 + 160 < 5F < 405 + 160$

$\Rightarrow \quad 520 < 5F < 565$

$\Rightarrow \quad \frac{520}{5} < F < \frac{565}{5}$

$\Rightarrow \quad 104 < F < 113$

hence, **104° < F < 113°**.