Date: 9/10/2021

Q1. A circular wire of radius 3 cm is cut and bent so as to lie along the circumference of a hoop whose radius is 48 cm. Find the angle in degrees which is subtended at the centre of hoop.

Q2. Prove that: $\dfrac{\tan A + \sec A - 1}{\tan A - \sec A + 1} = \dfrac{1 + \sin A}{\cos A}$.

Q3. Prove that: $\sin 4A = 4 \sin A \cos^3 A - 4 \cos A \sin^3 A$.

Q4. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then prove that $m^2 - n^2 = 4\sin\theta\tan\theta$.

Q5. If $\tan(A + B) = p$, $\tan(A - B) = q$, then show that $\tan 2A = \dfrac{p+q}{1-pq}$.

Q6. If $\sin\theta + \cos\theta = 1$, then find the general value of θ.

Q7. Find the most general value of θ satisfying the equation $\tan\theta = -1$ and $\cos\theta = \dfrac{1}{\sqrt{2}}$.

Q8. If $\cot\theta + \tan\theta = 2\,\text{cosec}\,\theta$, then find the general value of θ.

Q9. If $2\sin^2\theta = 3\cos\theta$, where $0 < \theta < 2\pi$, then find the value of θ.

Q10. If $\sec x \cos 5x + 1 = 0$, where $0 < x \le \pi/2$, then find the value of x.

Q11. Find the general solution of the equation: $5\cos^2\theta + 7\sin^2\theta - 6 = 0$.

Q12. If $A = \cos^2\theta + \sin^4\theta$ for all values of θ, then prove that $\dfrac{3}{4} \le A \le 1$.

Q13. If $m\sin\theta = n\sin(\theta + 2\alpha)$, then prove that $\tan(\theta + \alpha)\cot\alpha = \dfrac{m+n}{m-n}$.

Q14. If $\sin(\theta + \alpha) = a$ and $\sin(\theta + \beta) = b$, then prove that:

$$\cos 2(\alpha - \beta) - 4ab\cos(\alpha - \beta) = 1 - 2a^2 - 2b^2.$$

Q15. If $\cos(\theta + \phi) = m\cos(\theta - \phi)$, then prove that:

$$\tan\theta = \frac{1-m}{1+m}\cot\phi.$$

Q16. If θ lies in the first quadrant and $\cos\theta = \dfrac{8}{17}$, then find the value of $\cos(30° + \theta) + \cos(45° - \theta) + \cos(120° - \theta)$.

Q17. Find the value of the expression: $\cos^4\dfrac{\pi}{8} + \cos^4\dfrac{3\pi}{8} + \cos^4\dfrac{5\pi}{8} + \cos^4\dfrac{7\pi}{8}$.

Q18. Find the value of $\sqrt{3}\,\text{cosec}\,20° - \sec 20°$.

Q19. Find the value of $\tan 9° - \tan 27° - \tan 63° + \tan 81°$.

Q20. Show that $2\sin^2\beta + 4\cos(\alpha + \beta)\sin\alpha\sin\beta + \cos 2(\alpha + \beta) = \cos 2\alpha$.

Q21. Find the value of: $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$.

Q22. Prove that:

$$\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{\tan 8\theta}{\tan 2\theta}.$$

Q23. If $a\cos 2\theta + b\sin 2\theta = c$ has α and β as its roots, then prove that:

$$\tan\alpha + \tan\beta = \frac{2b}{a+c}.$$

Q24. Find the value of the expression

$$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4\left(\frac{3\pi}{2}+\alpha\right)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi+\alpha)\right].$$

Q25. If $\frac{\sin(x+y)}{\sin(x-y)}=\frac{a+b}{a-b}$, then show that $\frac{\tan x}{\tan y}=\frac{a}{b}$.

Q26. If $\tan\theta = \frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}$, then show that $\sin\alpha + \cos\alpha = \sqrt{2}\cos\theta$.

Q27. Prove that: $\cos\theta\cdot\cos\frac{\theta}{2}-\cos 3\theta\cos\frac{9\theta}{2}=\sin\frac{7\theta}{2}\sin 4\theta$.

Q28. If $\cos(\alpha+\beta)=\frac{4}{5}$ and $\sin(\alpha-\beta)=\frac{5}{13}$ where α lies between 0 and $\frac{\pi}{4}$, find the value of $\tan 2\alpha$.

Q29. Solve the equation: $\sin\theta + \sin 3\theta + \sin 5\theta = 0$.

Q30. Find the general solution of the equation: $(\sqrt{3}-1)\cos\theta+(\sqrt{3}+1)\sin\theta=2$.

Q31. Find the general solution of the equation, $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$.

Q32. Solve $\sqrt{3}\cos\theta + \sin\theta = \sqrt{2}$.

Q33. Solve $2\tan^2 x + \sec^2 x = 2$ for $0 \le x \le 2\pi$.

Q34. If α and β are the solutions of the equation $a\tan\theta + b\sec\theta = c$, then show that

$$\tan(\alpha+\beta)=\frac{2ac}{a^2-c^2}.$$

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S1. Given that circular wire is of radius 3 cm, so when it is cut then its length = $2\pi \times 3 = 6\pi$ cm. Again, it is being placed along a circular hoop of radius 48 cm. Here, $s = 6\pi$ cm is the lengh of arc and $r = 48$ cm is the radius of the circle. Therefore, the angle θ, in radian, subtended by the arc at the centre of the circle is given by

$$\theta = \frac{\text{Arc}}{\text{Radius}} = \frac{6\pi}{48} = \frac{\pi}{8} = 22.5^\circ.$$

S2.

$$\text{LHS.} = \frac{\tan A + \sec A - 1}{\tan A - \sec A + 1}$$

$$= \frac{\tan A + \sec A - (\sec^2 A - \tan^2 A)}{\tan A - \sec A + 1}$$

$$= \frac{(\tan A + \sec A)(1 - \sec A + \tan A)}{\tan A - \sec A + 1}$$

$$= \frac{\sin A}{\cos A} + \frac{1}{\cos A}$$

$$= \frac{\sin A + 1}{\cos A} = \text{R.H.S.}$$

S3. Let

$$\text{R.H.S.} = 4 \sin A \cos^3 A - 4 \cos A \sin^3 A$$

$$= 4 \sin A \cos A (\cos^2 A - \sin^2 A)$$

$$= 2 (2 \sin A \cos A) \cos 2A$$

$$= 2 \sin 2A \cos 2A$$

$$= \sin 4A = \text{L.H.S.}$$

S4. $\because$

$$\tan\theta + \sin\theta = m$$

$$\tan\theta - \sin\theta = n$$

$\therefore$

$$2\tan\theta = m + n, \quad 2\sin\theta = m - n$$

Now,

$$\text{L.H.S.} = m^2 - n^2$$

$$= (m + n)(m - n)$$

$$= 2\tan\theta \, 2\sin\theta$$

$$= 4\sin\theta \tan\theta = \text{R.H.S.}$$

S5. $\because$

$$2A = (A + B) + (A - B)$$

$$\text{L.H.S.} \quad \tan 2A = \frac{\tan(A + B) + \tan(A - B)}{1 - \tan(A + B)\tan(A - B)}$$

$$= \frac{p+q}{1-pq} = \text{R.H.S.}$$

S6. Given, $\sin\theta + \cos\theta = 1$

$\Rightarrow \quad \sin\theta \cdot \frac{1}{\sqrt{2}} + \cos\theta \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \sin\theta \cos\frac{\pi}{4} + \cos\theta \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \sin\left(\theta + \frac{\pi}{4}\right) = \sin\frac{\pi}{4}$

$\Rightarrow \quad \theta + \frac{\pi}{4} = n\pi + (-1)^n \frac{\pi}{4}, \quad n \in I$

Hence, $\theta = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{4}, \quad n \in I.$

S7. Given, $\tan\theta = -1$ and $\cos\theta = \frac{1}{\sqrt{2}}$

$\tan\theta$ is negative and $\cos\theta$ is positive in IVth quadrant.

$\therefore \quad \theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

Hence, general solution is $\theta = 2n\pi + \frac{7\pi}{4}, n \in I.$

S8. Given, $\cot\theta + \tan\theta = 2 \operatorname{cosec}\theta$

$\Rightarrow \quad \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = \frac{2}{\sin\theta}$

$\Rightarrow \quad \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} = \frac{2}{\sin\theta}$

$\Rightarrow \quad \frac{1}{\sin\theta\cos\theta} = \frac{2}{\sin\theta}$

$\Rightarrow \quad \frac{1}{\cos\theta} = 2$

$\Rightarrow \quad \cos\theta = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$

Hence, $\theta = 2n\pi \pm \frac{\pi}{3}, n \in I$ (General solution).

S9. Given, $2\sin^2\theta = 3\cos\theta$

$\Rightarrow \quad 2(1 - \cos^2\theta) = 3\cos\theta$

$\Rightarrow \quad 2 - 2\cos^2\theta = 3\cos\theta$

$\Rightarrow \quad 2\cos^2\theta + 3\cos\theta - 2 = 0$

$\Rightarrow \quad 2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0$

$\Rightarrow \quad 2\cos\theta(\cos\theta + 2) - 1(\cos\theta + 2) = 0$

$\Rightarrow \quad (2\cos\theta - 1)(\cos\theta + 2) = 0$

$\Rightarrow \quad 2\cos\theta - 1 = 0 \quad \text{or} \quad \cos\theta = -2 \quad [\text{Impossible}]$

$\Rightarrow \quad \cos\theta = \frac{1}{2}$

As $0 < \theta < 2\pi$, then $\theta = \frac{\pi}{3}, 2\pi - \frac{\pi}{3} = \frac{\pi}{3}, \frac{5\pi}{3}$.

S10. Given, $\sec x \cos 5x + 1 = 0$

$\Rightarrow \quad \frac{1}{\cos x}\cos 5x + 1 = 0$

$\Rightarrow \quad \cos 5x + \cos x = 0$

$\Rightarrow \quad 2\cos 3x \cos 2x = 0$

$\Rightarrow \quad \cos 3x = 0, \quad \cos 2x = 0$

$\Rightarrow \quad 3x = \frac{\pi}{2}, \quad 2x = \frac{\pi}{2}$

$\Rightarrow \quad x = \frac{\pi}{6}, \quad x = \frac{\pi}{4}$.

S11. Given, $5\cos^2\theta + 7\sin^2\theta - 6 = 0$

or $\quad 5\cos^2\theta + 7(1 - \cos^2\theta) - 6 = 0$

or $\quad 5\cos^2\theta + 7 - 7\cos^2\theta - 6 = 0$

or $\quad 1 - 2\cos^2\theta = 0$

or $\quad \cos^2\theta = \frac{1}{2} = \left(\frac{1}{\sqrt{2}}\right)^2 = \cos^2\frac{\pi}{4} = \cos^2\left(\frac{\pm\pi}{4}\right)$

or $\quad \theta = n\pi \pm \frac{\pi}{4}, \quad n \in I.$

S12. We have $A = \cos^2\theta + \sin^4\theta$

$= \cos^2\theta + \sin^2\theta \sin^2\theta$

$\leq \cos^2\theta + \sin^2\theta$

Therefore, $A \leq 1$

Also, $A = \cos^2\theta + \sin^4\theta$

$$= (1 - \sin^2\theta) + \sin^4\theta$$

$$= \left(\sin^2\theta - \frac{1}{2}\right)^2 + \left(1 - \frac{1}{4}\right)$$

$$= \left(\sin^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4} \geq \frac{3}{4}$$

Hence, $\frac{3}{4} \leq A \leq 1$.

S13. Let $m \sin\theta = n \sin(\theta + 2\alpha)$

$$\therefore \quad \frac{\sin(\theta + 2\alpha)}{\sin\theta} = \frac{m}{n}$$

Applying componendo-dividendo, we get

$$\frac{\sin(\theta + 2\alpha) + \sin\theta}{\sin(\theta + 2\alpha) - \sin\theta} = \frac{m+n}{m-n}$$

$$\Rightarrow \quad \frac{2\sin\left(\frac{\theta + 2\alpha + \theta}{2}\right)\cos\left(\frac{\theta + 2\alpha - \theta}{2}\right)}{2\cos\left(\frac{\theta + 2\alpha + \theta}{2}\right)\sin\left(\frac{\theta + 2\alpha - \theta}{2}\right)} = \frac{m+n}{m-n}$$

$$\Rightarrow \quad \frac{\sin(\theta + \alpha)\cos\alpha}{\cos(\theta + \alpha)\sin\alpha} = \frac{m+n}{m-n}$$

Hence, $\tan(\theta + \alpha)\cot\alpha = \frac{m+n}{m-n}$. **(Proved)**

S14. Let $\cos(\alpha - \beta) = \cos[(\theta + \alpha) - (\theta + \beta)]$

$$= \cos(\theta + \alpha)\cos(\theta + \beta) + \sin(\theta + \alpha)\sin(\theta + \beta)$$

$$= \cos(\theta + \alpha)\cos(\theta + \beta) + ab$$

$$= ab + \sqrt{1 - \sin^2(\theta + \alpha)}\sqrt{1 - \sin^2(\theta + \beta)}$$

$$\cos(\alpha - \beta) = ab + \sqrt{1 - a^2}\sqrt{1 - b^2}$$

$$[\cos(\alpha - \beta) - ab]^2 = (1 - a^2)(1 - b^2)$$

$$\cos^2(\alpha - \beta) + a^2b^2 - 2ab\cos(\alpha - \beta) = 1 - a^2 - b^2 + a^2b^2$$

$$\frac{1 + \cos 2(\alpha - \beta)}{2} - 2ab\cos(\alpha - \beta) = 1 - a^2 - b^2$$

$$1 + \cos 2(\alpha - \beta) - 4ab\cos(\alpha - \beta) = 2 - 2a^2 - 2b^2$$

$$\cos 2(\alpha - \beta) - 4ab\cos(\alpha - \beta) = 1 - 2a^2 - 2b^2.$$

Hence, the result.

S15. Let

$$\cos(\theta + \phi) = m \cos(\theta - \phi)$$

$$\frac{\cos(\theta+\phi)}{\cos(\theta-\phi)} = \frac{m}{1}$$

$$\Rightarrow \quad \frac{\cos(\theta-\phi)}{\cos(\theta+\phi)} = \frac{1}{m}$$

Applying componendo-dividendo, we get

$$\Rightarrow \quad \frac{\cos(\theta-\phi)-\cos(\theta+\phi)}{\cos(\theta-\phi)+\cos(\theta+\phi)} = \frac{1-m}{1+m}$$

$$\Rightarrow \quad \frac{2\sin\left(\frac{\theta-\phi+\theta+\phi}{2}\right)\sin\left(\frac{\theta+\phi-\theta+\phi}{2}\right)}{2\cos\left(\frac{\theta-\phi+\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi-\theta-\phi}{2}\right)} = \frac{1-m}{1+m}$$

$$\Rightarrow \quad \frac{\sin\theta}{\cos\theta}\times\frac{\sin\phi}{\cos\phi} = \frac{1-m}{1+m}$$

$$\Rightarrow \quad \tan\theta \times \tan\phi = \frac{1-m}{1+m}$$

$$\Rightarrow \quad \tan\theta = \frac{1-m}{1+m}\cot\phi$$

Hence, the result.

S16. Let

$$\cos\theta = \frac{8}{17} \quad \Rightarrow \quad \sin\theta = \frac{15}{17}$$

Now,

$$\cos(30° + \theta) + \cos(45° - \theta) + \cos(120° - \theta)$$

$$= \cos 30° \cos\theta - \sin 30° \sin\theta + \cos 45° \cos\theta + \sin 45° \sin\theta + \cos 120° \cos\theta + \sin 120° \sin\theta$$

$$= \frac{\sqrt{3}}{2}\times\frac{8}{17} - \frac{1}{2}\times\frac{15}{17} + \frac{1}{\sqrt{2}}\times\frac{8}{17} + \frac{1}{\sqrt{2}}\times\frac{15}{17} - \frac{1}{2}\times\frac{8}{17} + \frac{\sqrt{3}}{2}\times\frac{15}{17}$$

$$= \frac{8\sqrt{3} - 15 + 8\sqrt{2} + 15\sqrt{2} - 8 + 15\sqrt{3}}{2\times 17}$$

$$= \frac{23\sqrt{3} + 23\sqrt{2} - 23}{34} = \frac{23(\sqrt{3}+\sqrt{2}-1)}{34}.$$

S17. Given expression

$$\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8} = \left[\cos^2\frac{\pi}{8}\right]^2+\left[\cos^2\frac{3\pi}{8}\right]^2+\left[\cos^2\frac{5\pi}{8}\right]^2+\left[\cos^2\frac{7\pi}{8}\right]^2$$

$$= \left[\frac{1+\cos\frac{\pi}{4}}{2}\right]^2+\left[\frac{1+\cos\frac{3\pi}{4}}{2}\right]^2+\left[\frac{1+\cos\frac{5\pi}{4}}{2}\right]^2+\left[\frac{1+\cos\frac{7\pi}{4}}{2}\right]^2$$

$$= \left[\frac{1+\cos\frac{\pi}{4}}{2}\right]^2+\left[\frac{1-\cos\frac{\pi}{4}}{2}\right]^2+\left[\frac{1-\cos\frac{\pi}{4}}{2}\right]^2+\left[\frac{1+\cos\frac{\pi}{4}}{2}\right]^2$$

$$= \left[\frac{1+\frac{1}{\sqrt{2}}}{2}\right]^2+\left[\frac{1-\frac{1}{\sqrt{2}}}{2}\right]^2+\left[\frac{1-\frac{1}{\sqrt{2}}}{2}\right]^2+\left[\frac{1+\frac{1}{\sqrt{2}}}{2}\right]^2$$

$$= \left[\frac{\sqrt{2}+1}{2\sqrt{2}}\right]^2+\left[\frac{\sqrt{2}-1}{2\sqrt{2}}\right]^2+\left[\frac{\sqrt{2}-1}{2\sqrt{2}}\right]^2+\left[\frac{\sqrt{2}+1}{2\sqrt{2}}\right]^2$$

$$= 2\left[\frac{\sqrt{2}+1}{2\sqrt{2}}\right]^2+2\left[\frac{\sqrt{2}-1}{2\sqrt{2}}\right]^2$$

$$= 2\left[\frac{2+1+2\sqrt{2}}{8}\right]+2\left[\frac{2+1-2\sqrt{2}}{8}\right]$$

$$= \frac{3+2\sqrt{2}}{4}+\frac{3-2\sqrt{2}}{4}=\frac{6}{4}=\frac{3}{2}.$$

S18. We have $\sqrt{3}$ cosec 20° – sec 20° $= \frac{\sqrt{3}}{\sin 20^\circ}-\frac{1}{\cos 20^\circ}$

$$= \frac{\sqrt{3}\cos 20^\circ-\sin 20^\circ}{\sin 20^\circ\cos 20^\circ}$$

$$= 4\left(\frac{\frac{\sqrt{3}}{2}\cos 20^\circ-\frac{1}{2}\sin 20^\circ}{2\sin 20^\circ\cos 20^\circ}\right)$$

$$= 4\left(\frac{\sin 60^\circ\cos 20^\circ-\cos 60^\circ\sin 20^\circ}{\sin 40^\circ}\right)$$

$$= 4\left(\frac{\sin(60^\circ-20^\circ)}{\sin 40^\circ}\right)=4$$

S19. We have,

$$\tan 9° - \tan 27° - \tan 63° + \tan 81°$$
$$= \tan 9° + \tan 81° - \tan 27° - \tan 63°$$
$$= \tan 9° + \tan (90° - 9°) - \tan 27° - \tan (90° - 27°)$$
$$= \tan 9° + \cot 9° - (\tan 27° + \cot 27°) \quad \ldots \text{(i)}$$

Also, $$\tan 9° + \cot 9° = \frac{1}{\sin 9° \cos 9°} = \frac{2}{\sin 18°} \quad \ldots \text{(ii)}$$

Similarly, $$\tan 27° + \cot 27° = \frac{1}{\sin 27° \cos 27°} = \frac{2}{\sin 54°} = \frac{2}{\cos 36°} \quad \ldots \text{(iii)}$$

Using Eq. (ii) and (iii) in Eq. (i), we get

$$\tan 9° - \tan 27° - \tan 63° + \tan 81° = \frac{2}{\sin 18°} - \frac{2}{\cos 36°} = \frac{2 \times 4}{\sqrt{5} - 1} - \frac{2 \times 4}{\sqrt{5} + 1} = 4.$$

S20.

$$\text{L.H.S.} = 2 \sin^2 \beta + 4 \cos (\alpha + \beta) \sin \alpha \sin \beta + \cos^2 (\alpha + \beta)$$
$$= 2 \sin^2 \beta + 4 (\cos \alpha \cos \beta - \sin \alpha \sin \beta) \sin \alpha \sin \beta$$
$$+ (\cos 2\alpha \cos 2\beta - \sin 2\alpha \sin 2\beta)$$
$$= 2 \sin^2 \beta + 4 \sin \alpha \cos \alpha \sin \beta \cos \beta - 4 \sin^2 \alpha \sin^2 \beta$$
$$+ \cos 2\alpha \cos 2\beta - \sin 2\alpha \sin 2\beta$$
$$= 2 \sin^2 \beta + \sin 2\alpha \sin 2\beta - 4 \sin^2 \alpha \sin^2 \beta + \cos 2\alpha \cos 2\beta - \sin 2\alpha \sin 2\beta$$
$$= (1 - \cos 2\beta) - (2 \sin^2 \alpha)(2 \sin^2 \beta) + \cos 2\alpha \cos 2\beta$$
$$= (1 - \cos 2\beta) - (1 - \cos 2\alpha)(1 - \cos 2\beta) + \cos 2\alpha \cos 2\beta$$
$$= \cos 2\alpha. = \text{R.H.S.}$$

S21.

$$\left(1 + \cos \frac{\pi}{8}\right)\left(1 + \cos \frac{3\pi}{8}\right)\left(1 + \cos \frac{5\pi}{8}\right)\left(1 + \cos \frac{7\pi}{8}\right)$$
$$= \left(1 + \cos \frac{\pi}{8}\right)\left(1 + \cos \frac{3\pi}{8}\right)\left(1 + \cos \left(\pi - \frac{3\pi}{8}\right)\right)\left(1 + \cos \left(\pi - \frac{\pi}{8}\right)\right)$$
$$= \left(1 - \cos^2 \frac{\pi}{8}\right)\left(1 - \cos^2 \frac{3\pi}{8}\right)$$
$$= \sin^2 \frac{\pi}{8} \sin^2 \frac{3\pi}{8}$$
$$= \frac{1}{4}\left(1 - \cos \frac{\pi}{4}\right)\left(1 - \cos \frac{3\pi}{4}\right)$$
$$= \frac{1}{4}\left(1 - \cos \frac{\pi}{4}\right)\left(1 + \cos \frac{\pi}{4}\right)$$
$$= \frac{1}{4}\left(1 - \cos^2 \frac{\pi}{4}\right) = \frac{1}{4}\left(1 - \frac{1}{2}\right) = \frac{1}{8}.$$

S22. We have,

$$\frac{\sec 8\theta - 1}{\sec 4\theta - 1} = \frac{(1 - \cos 8\theta)\cos 4\theta}{\cos 8\theta\,(1 - \cos 4\theta)}$$

$$= \frac{2\sin^2 4\theta \cos 4\theta}{\cos 8\theta\; 2\sin^2 2\theta}$$

$$= \frac{\sin 4\theta\,(2\sin 4\theta \cos 4\theta)}{2\cos 8\theta \sin^2 2\theta}$$

$$= \frac{\sin 4\theta \sin 8\theta}{2\cos 8\theta \sin^2 2\theta}$$

$$= \frac{2\sin 2\theta \cos 2\theta \sin 8\theta}{2\cos 8\theta \sin^2 2\theta}$$

$$= \frac{\tan 8\theta}{\tan 2\theta}.$$

S23. Given, $a\cos 2\theta + b\sin 2\theta = c$

$$a\left(\frac{1 - \tan^2\theta}{1 + \tan^2\theta}\right) + b\left(\frac{2\tan\theta}{1 + \tan^2\theta}\right) = c$$

$$a - a\tan^2\theta + 2b\tan\theta = c + c\tan^2\theta$$

$$(c + a)\tan^2\theta - 2b\tan\theta + c - a = 0$$

It is a quadratic equation in $\tan\theta$.

Now, $\tan\alpha$ and $\tan\beta$ are its roots,

$\therefore$ Sum of roots $= \tan\alpha + \tan\beta$

$$= -\left(\frac{-2b}{c + a}\right) = \frac{2b}{a + c}.$$

S24. Given expression $= 3\left[\sin^4\left(\frac{3\pi}{2} - \alpha\right) + \sin^4\left(\frac{3\pi}{2} + \alpha\right)\right] - 2\left[\sin^6\left(\frac{\pi}{2} + \alpha\right) + \sin^6(5\pi + \alpha)\right]$

$= 3[(-\cos\alpha)^4 + (-\sin\alpha)^4] - 2[(\cos\alpha)^6 + (\sin\alpha)^6]$

$= 3[\cos^4\alpha + \sin^4\alpha] - 2[(\cos\alpha)^6 + (\sin\alpha)^6]$

$= 3[(\cos^2\alpha + \sin^2\alpha)^2 - 2\sin^2\alpha\cos^2\alpha] - 2[(\cos^2\alpha)^3 + (\sin^2\alpha)^3]$

$= 3(1 - 2\sin^2\alpha\cos^2\alpha) - 2(\cos^2\alpha + \sin^2\alpha)(\cos^4\alpha + \sin^4\alpha - \sin^2\alpha\cos^2\alpha]$

$= 3(1 - 2\sin^2\alpha\cos^2\alpha) - 2 \times 1\,[(\cos^2\alpha + \sin^2\alpha)^2 - 3\sin^2\alpha\cos^2\alpha]$

$= 3 - 6\sin^2\alpha\cos^2\alpha - 2[1 - 3\sin^2\alpha\cos^2\alpha]$

$= 3 - 6\sin^2\alpha\cos^2\alpha - [2 - 6\sin^2\alpha\cos^2\alpha]$

$= 3 - 6\sin^2\alpha\cos^2\alpha - 2 + 6\sin^2\alpha\cos^2\alpha$

$= 1.$

S25. $\because$ $$\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$$

Using componendo-dividendo, we get

$$= \frac{\sin(x+y)+\sin(x-y)}{\sin(x+y)-\sin(x-y)} = \frac{a+b+a-b}{a+b-(a-b)}$$

$$\Rightarrow \quad \frac{2\sin x\cos y}{2\cos x\sin y} = \frac{a}{b}$$

$$\Rightarrow \quad \frac{\tan x}{\tan y} = \frac{a}{b}.$$ **(Hence Proved)**

S26. $\because$ $$\tan\theta = \frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}$$

$$\Rightarrow \quad \tan\theta = \frac{\tan\alpha-1}{\tan\alpha+1}$$

$$\Rightarrow \quad \tan\theta = \tan\left(\alpha-\frac{\pi}{4}\right)$$

$$\Rightarrow \quad \theta = \alpha-\frac{\pi}{4}$$

$$\Rightarrow \quad \cos\theta = \cos\left(\alpha-\frac{\pi}{4}\right)$$

$$\Rightarrow \quad \cos\theta = \cos\alpha\cos\frac{\pi}{4}+\sin\alpha\sin\frac{\pi}{4}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2}}\cos\alpha+\frac{1}{\sqrt{2}}\sin\alpha$$

$$\Rightarrow \quad \sin\alpha+\cos\alpha = \sqrt{2}\cos\theta.$$ **(Hence Proved)**

S27. Let, $$\text{L.H.S.} = \cos\theta\cos\frac{\theta}{2}-\cos3\theta\cos\frac{9\theta}{2}$$

$$= \frac{1}{2}\left[2\cos\theta\cos\frac{\theta}{2}-2\cos3\theta\cos\frac{9\theta}{2}\right]$$

$$= \frac{1}{2}\left[\cos\left(\theta+\frac{\theta}{2}\right)+\cos\left(\theta-\frac{\theta}{2}\right)\right]-\left[\cos\left(3\theta+\frac{9\theta}{2}\right)+\cos\left(\frac{9\theta}{2}-3\theta\right)\right]$$

$$= \frac{1}{2}\left[\cos\frac{3\theta}{2}+\cos\frac{\theta}{2}-\cos\frac{15\theta}{2}-\cos\frac{3\theta}{2}\right]$$

$$= \frac{1}{2}\left[\cos\frac{\theta}{2} - \cos\frac{15\theta}{2}\right]$$

$$= \frac{1}{2}\left[2\sin\left(\frac{\frac{\theta}{2}+\frac{15\theta}{2}}{2}\right)\sin\left(\frac{\frac{15\theta}{2}-\frac{\theta}{2}}{2}\right)\right]$$

$$= \sin 4\theta \sin\frac{7\theta}{2} = \text{R.H.S.}$$

S28. $\because$ $\cos(\alpha+\beta) = \frac{4}{5}$

$\Rightarrow$ $\tan(\alpha+\beta) = \frac{3}{4}$

and $\sin(\alpha-\beta) = \frac{5}{13}$

$\Rightarrow$ $\tan(\alpha-\beta) = \frac{5}{12}$

Now, $\tan 2\alpha = \tan[(\alpha+\beta)+(\alpha-\beta)]$

$$= \frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$= \frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\times\frac{5}{12}}$$

$$= \frac{36+20}{48-15} = \frac{56}{33}.$$

S29. We have, $\sin\theta + \sin 3\theta + \sin 5\theta = 0$

or $(\sin\theta + \sin 5\theta) + \sin 3\theta = 0$

or $2\sin 3\theta \cos 2\theta + \sin 3\theta = 0$

or $\sin 3\theta(2\cos 2\theta + 1) = 0$

or $\sin 3\theta = 0$ or $\cos 2\theta = -\frac{1}{2}$

When $\sin 3\theta = 0$

then $3\theta = \pi$ or $\theta = \frac{2\pi}{3}$

When $\cos 2\theta = -\frac{1}{2} = \cos\frac{2\pi}{3}$

then $2\theta = 2n\pi \pm \frac{2\pi}{3}$ or $\theta = n\pi \pm \frac{\pi}{3}$

which gives $\theta = (3n + 1)\,\dfrac{\pi}{3}$ or $\theta = (3n - 1)\,\dfrac{\pi}{3}$

All these values of θ are contained in $\theta = \dfrac{n\pi}{3}$, $n \in Z$. Hence, the required solution set is given by $\left\{\theta : \theta = \dfrac{n\pi}{3}, n \in \boldsymbol{Z}\right\}$.

S30. Given, $(\sqrt{3} - 1)\cos\theta + (\sqrt{3} + 1)\sin\theta = 2$

Let $\sqrt{3} - 1 = r\cos\alpha$ and $(\sqrt{3} + 1) = r\sin\alpha$

$$\therefore \quad \tan\alpha = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$$

$$\tan\alpha = \tan\left(\frac{\pi}{4}+\frac{\pi}{6}\right) \Rightarrow \alpha = \frac{5\pi}{12}$$

and
$$r^2 = (\sqrt{3} - 1)^2 + (\sqrt{3} + 1)^2$$
$$r^2 = 3 + 1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3}$$
$$= 8 \quad \Rightarrow \quad r = 2\sqrt{2}$$

$$\therefore \quad r\cos\alpha\cos\theta + r\sin\alpha\sin\theta = 2$$
$$2\sqrt{2}\cos(\theta - \alpha) = 2$$
$$\cos(\theta - \alpha) = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$$

$$\therefore \quad \theta - \alpha = 2n\pi \pm \frac{\pi}{4}, \quad n \in I$$
$$\theta = 2n\pi \pm \frac{\pi}{4} + \frac{5\pi}{12}, \quad n \in I.$$

S31. Given, $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$

$\Rightarrow \quad \sin 3x + \sin x - 3\sin 2x = \cos 3x + \cos x - 3\cos 2x$

$\Rightarrow \quad 2\sin 2x\cos x - 3\sin 2x = 2\cos 2x\cos x - 3\cos 2x$

$\Rightarrow \quad \sin 2x\,[2\cos x - 3] = \cos 2x\,[2\cos x - 3]$

$\Rightarrow \quad \sin 2x = \cos 2x$

$\Rightarrow \quad \tan 2x = 1 = \tan\left(\dfrac{\pi}{4}\right)$

Hence, $2x = n\pi + \dfrac{\pi}{4}$

$\Rightarrow \quad x = \dfrac{n\pi}{2} + \dfrac{\pi}{8}, \quad n \in I$

S32. Divide the given equation by 2, we get

$$\frac{\sqrt{3}}{2}\cos\theta + \frac{1}{2}\sin\theta = \frac{1}{\sqrt{2}} \quad \text{or} \quad \cos\frac{\pi}{6}\cos\theta + \sin\frac{\pi}{6}\sin\theta = \cos\frac{\pi}{4}$$

or $$\cos\left(\frac{\pi}{6} - \theta\right) = \cos\frac{\pi}{4} \quad \text{or} \quad \cos\left(\theta - \frac{\pi}{6}\right) = \cos\frac{\pi}{4}$$

Thus, the solution are given by, *i.e.*,

$$\theta = 2m\pi \pm \frac{\pi}{4} + \frac{\pi}{6}$$

Hence, the solution are $\theta = 2m\pi + \frac{\pi}{4} + \frac{\pi}{6}$ and $2m\pi - \frac{\pi}{4} + \frac{\pi}{6}$

i.e., $\theta = 2m\pi + \frac{5\pi}{12}$ and $\theta = 2m\pi - \frac{\pi}{12}$.

S33. Here, $2\tan^2 x + \sec^2 x = 2$

which gives $\tan x = \pm\frac{1}{\sqrt{3}}$

If we take $\tan x = \frac{1}{\sqrt{3}}$, then $x = \frac{\pi}{6}$ or $\frac{7\pi}{6}$

Again, if we take $\tan x = \frac{-1}{\sqrt{3}}$, then $x = \frac{5\pi}{6}$ or $\frac{11\pi}{6}$

Therefore, the possible solutions of above equations are

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6} \text{ and } \frac{11\pi}{6}$$

where $0 \le x \le 2\pi$.

S34. Given that $a\tan\theta + b\sec\theta = c$ or $a\sin\theta + b = c\cos\theta$

Using the identities, $\sin\theta = \dfrac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$ and $\cos\theta = \dfrac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$

We have, $$\frac{a\left(2\tan\frac{\theta}{2}\right)}{1+\tan^2\frac{\theta}{2}} + b = \frac{c\left(1-\tan^2\frac{\theta}{2}\right)}{1+\tan^2\frac{\theta}{2}}$$

or $$(b+c)\tan^2\frac{\theta}{2} + 2a\tan\frac{\theta}{2} + b - c = 0$$

Above equation is quadriatic in $\frac{\theta}{2}$ and hence $\frac{\alpha}{2}$ and $\frac{\beta}{2}$ are the roots of this equation. Therefore

$$\tan\frac{\alpha}{2}+\tan\frac{\beta}{2} = \frac{-2a}{b+c} \quad \text{and} \quad \tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{b-c}{b+c}$$

Using the identity $\tan\left(\frac{\alpha}{2}+\frac{\beta}{2}\right) = \dfrac{\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}}{1-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}}$

We have, $$\tan\left(\frac{\alpha}{2}+\frac{\beta}{2}\right) = \frac{\frac{-2a}{b+c}}{1-\frac{b-c}{b+c}} = \frac{-2a}{2c} = \frac{-a}{c} \qquad \text{... (i)}$$

Again, using another identity

$$\tan 2\left(\frac{\alpha+\beta}{2}\right) = \frac{2\tan\frac{\alpha+\beta}{2}}{1-\tan^2\frac{\alpha+\beta}{2}}$$

We have $$\tan(\alpha+\beta) = \frac{2\left(-\frac{a}{c}\right)}{1-\frac{a^2}{c^2}} = \frac{2ac}{a^2-c^2} \quad \text{[From Eq. (i)]}$$

Alternatively: Given that $a\tan\theta + b\sec\theta = c$.

$\Rightarrow \quad (a\tan\theta - c)^2 = b^2(1+\tan^2\theta)$

$\Rightarrow \quad a^2\tan^2\theta - 2ac\tan\theta + c^2 = b^2 + b^2\tan^2\theta$

$\Rightarrow \quad (a^2-b^2)\tan^2\theta - 2ac\tan\theta + c^2 - b^2 = 0 \qquad \text{... (i)}$

Since, α and β are the roots of the Eq. (i), so

$$\tan\alpha + \tan\beta = \frac{2ac}{a^2-b^2} \quad \text{and} \quad \tan\alpha\cdot\tan\beta = \frac{c^2-b^2}{a^2-b^2}$$

Therefore, $$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$= \frac{\frac{2ac}{a^2-b^2}}{1-\frac{c^2-b^2}{a^2-b^2}} = \frac{2ac}{a^2-c^2}.$$