Date: 20/10/2021

Q1. The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words, what keeps the atmosphere charged?

Q2. A man fixes outside his house one evening a two metre high insulating slab carrying on its top a large aluminium sheet of area 1 m^2. Will he get an electric shock if he touches the metal sheet next morning?

Q3. The top of the atmosphere is at about 400 kV with respect to the surface of the Earth, corresponding to an electric field that decreases with altitude. Near the surface of the Earth, the field is about 100 Vm^{-1}. Why then do we not get an electric shock as we step out of our house into the open? (Assume the house to be a steel cage so there is no field inside!)

Q4. A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q_2 on the shell is.

Q5. Can electric potential any point in space be zero while intensity of electric field at that point is not zero?

Q6. Two spherical conductors A and B of radii r_A and r_B ($r_A > r_B$) have been given equal charges. In which direction will the charge flow when these spheres brought in contact? Given reason for your answer.

Q7. Why do charges reside on the surface of the conductor?

Q8. What will be the electric potential of a charge at infinite?

Q9. (a) Calculate the potential at a point P due to a charge of 4×10^{-7} C located 9 cm away.

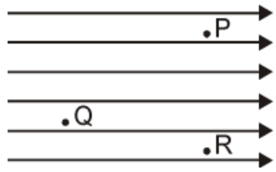
(b) Hence obtain the work done in bringing a charge of 2×10^{-9} C from infinity to the point P. Does the answer depend on the path along which the charge is brought?

Q10. What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning? (Hint: The earth has an electric field of about 100 Vm^{-1} at its surface in the downward direction, corresponding to a surface charge density = -10^{-9} $C m^{-2}$. Due to the slight conductivity of the atmosphere up to about 50 km (beyond which it is good conductor), about + 1800 C is pumped every second into the earth as a whole. The Earth, however, does not get discharged since thunderstorms and lightning occurring continually all over the globe pump an equal amount of negative charge on the Earth.)

Q11. Find the electric field between two metal plates 3 mm apart, connected to a 12 V battery.

Q12. Why the electric field at the outer surface of a hollow charged conductor is normal to the surface?

Q13. As shown in figure three points *P*, *Q* and *R* in a uniform electrostatic field. At which point will be electric potential be maximum?

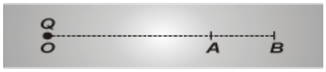


Q14. Mark two points As *A* and *B* around a point charge *Q* at which electric potential is the same.

Q15. For the system of two point charges $+q$ and $-q$ kept a distance *d* apart, draw any two equipotential surfaces.

Q16. Why is it that a man sitting in an insulated metal cage does not receive any shock when it is connected to a high voltage supply?

Q17. A point charge *Q* is placed at point *O* as shown in Figure.



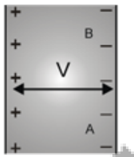
Is the potential difference $V_A - V_B$ positive, negative or zero, if *Q* is (a) positive (b) negative?

Q18. Draw an equipotential surface in a uniform electric field.

Q19. What is the shape of equipotential surface for a given point charge?

Q20. Define electric potential.

Q21. Two protons A and B are placed between two parallel plates having a potential difference V as shown in the figure.



Will these protons experience equal or unequal force?

Q22. A charge of 2 *C* moves between two plates maintained at a potential difference of 1 volt. What is the energy acquired the charge?

Q23. Define the unit of electric potential.

Q24. Define electric potential at a point in an electric field.

Q25. Define electric potential difference between two points. Is it scalar or vector ?

Q26. Name the physical quantity, whose SI unit is JC^{-1}. Is it scalar or vector?

Q27. How is electric field at a point related to potential gradient?

Q28. Given the dependence of electrostatic potential due to a small electric dipole at a far off point lying on (a) the axial line and (b) the equatorial line.

Q29. Potential difference between two given points, 15 cm apart, is 40 V. What is the value of electric field?

Q30. What is the shape of equipotential surfaces for a uniform electric field?

Q31. What is the shape of equipotential surfaces for a given point charge?

Q32. If the electrostatic field at a given point is zero, must the electrostatic potential be also zero at that point?

Q33. The electric potential is constant in a region, What can you say about electric field there?

Q34. Name the physical quantity, whose SI unit is volt metre^{-1}.

Q35. Why must electrostatic field be normal to the surface at every point of a charged conductor.

Q36. A tiny sphere carrying a negative charge of 2*e* is suspended in equilibrium between the horizontal metal plates 15 cm apart, having a potential difference of 3.0×10^3V across them. What is the mass of the particle.

Q37. A regular hexagon of side 10 cm has a charge 5 μC at each of its vertices. Calculate the potential at the centre of the hexagon.

Q38. A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7} C distributed uniformly on its surface. What is the electric field: (a) Inside the sphere; (b) Just outside the sphere; (c) At a point 18 cm from the centre of the sphere?

Q39. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge *Q*.

(a) A charge *q* is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?

(b) Is the electric field inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape? Explain.

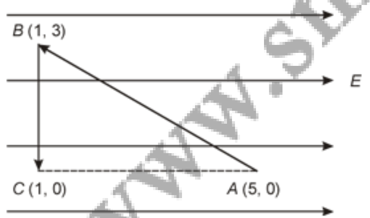
Q40. A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (*i.e.*, bending of field lines at the ends).

Q41. (a) Determine the electrostatic potential energy of a system consisting of two charges 7 μC and – 2 μC (and with no external field) placed at (– 9 cm, 0, 0) and (9 cm, 0, 0) respectively.

(b) How much work is required to separate the two charges infinitely away from each other?

(c) Suppose that the same system of charges is now placed in an external electric field $E = A(1/r^2)$; $A = 9 \times 10^5$ C m^{-2}. What would the electrostatic energy of the configuration be?

Q42. A test charge *q* is moved without acceleration in electric field as shown in figure.



(a) Calculate the potential difference between *A* and *C*.

(b) At which point (of the two) is the electric potential more and why?

Q43. Define electric potential at a point, When kept in an electric field, does a proton move from lower to higher potential of from higher to lower potential region?

Q44. An electric dipole of length 4 cm, when placed with its axis making an angle of 60° with a uniform electric field experiences a torque of $4\sqrt{3}\ N\ m$. Calculate the (a) magnitude of the electric field and (b) potential energy of the dipole, if the dipole has charges of $\pm$8nC.

Q45. A metal wire is bent into a circle of radius 10 cm. It is given a charge of 200μC, Which spreads on it uniformly. Calculate the electric potential at its centre.

Q46. Calculate the potential at the centre of a square *ABCD* of each side $\sqrt{2}\ m$ due to charges 2, – 2, – 3 and 6 μC at four corners of it.

Q47. Draw a plot showing the variation of (a) electric field *E* and (b) electric potential *V* with distance *r* due to a point charge *q*.

Q48. What is an equipotential surface? Show that the electric field is always directed perpendicular to an equipotential surface.

Q49. Two point charges +*q* and –*q* are separated by a distance *d*. where besides at infinity is the electric potential zero?

Q50. Show mathematically that the potential at a point on the equatorial line of an electric dipole is zero.

Q51. Can two equipotential surfaces intersect each other? Justify your answer.

Q52. Show that the amount of work done in moving a test charge along the equipotential surface is zero.

Q53. Draw one equipotential surface (a) in a uniform electric field and (b) for a charge ($Q < 0$).

Q54. Two point charges +5 μC and –5 μC are placed at a distance 5 cm apart.
(a) Draw the equipotential surface of the system.
(b) Why do the equipotential surfaces get closer to each other near the point charges?

Q55. (a) Draw equipotential surface due to a point charge $Q > 0$.
(b) Are these surface equidistance from each other? if not, explain why.

Q56. The electric field at a point due to a point charge is 20 NC^{-1} and the electric potential at that point is 10 JC^{-1}. Calculate the distance of the point from the charge and the magnitude of the range.

Q57. A dipole, with its charges, –*q* and +*q*, are located at the points (0, –*b*, 0) and (0, +*b*, 0), is present in uniform electric field *E*. The equipotential surface of this field are planes to the *YZ*-planes.
(a) What is the direction of the electric field *E*?
(b) How much torque would the dipole experience in this field?

Q58. A conductor charged to a potential of 40 V is connected to an uncharged conductor of 10 F capacitance. The common potential developed is 25 V. Calculate the capacitance of first conductor.

Q59. Two point charges $q_1 = 10 \times 10^{-8}$ C and $q_2 = -2 \times 10^{-8}$ C are separated by a distance of 60 cm in air.
(a) Find at what distance from the charge, would the electric potential be zero.
(b) Also calculate the electrostatic potential energy of the system

Q60. The electric potential $V(x)$ in a region along the X-axis varies with the distance x (in metre) according to the relation $V(x) = 4x^2$. Calculate the force experienced by a 1 μC charge placed at point $x = 1$ m.

Q61. What is the potential gradient (Vm^{-1}) at a distance of 10^{-12} m from the centre of the platinum nucleus? What is the potential gradient at the surface of the nucleus? Atomic number of platinum is 78 and the radius of platinum nucleus may be taken as 5×10^{-15} m.

Q62. Four point charges 16 μC, – 16 μC, 16 μC and – 16 μC are located at the corners of a square of each side 10 cm. Find the value of electric field intensity and electric potential at the centre of the square.

Q63. Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential and electric field at the mid-point of the line joining the two charges.

Q64. The charges equal to + 20 μC and – 10 μC are placed at points 6 cm apart. Find the value of the potential at a point distant 4 cm on the right bisector of the line joining the two charges.

Q65. *A* and *B* are two conducting spheres of same radius, *A* being solid and *B* hollow. Both are charged to the same potential. What will be the relation between charges on the two spheres?

Q66. A conductor has a charge of 0.5 μC and at potential of 20 V. Another uncharged conductor, whose capacity is 0.015 μF is momentarily connected to the first and then separated. Calculate the charges carried by each after contact and what is the value of the common potential.

Q67. Show that surface of a conductor is an equipotential surface.

Q68. Can a metal of sphere 1 cm hold a charge of 1 coulomb? justify your answer.

Q69. Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential and electric field: (a) at the mid-point of the line joining the two charges, and (b) at a point 10 cm from this midpoint in a plane normal to the line and passing through the mid-point.

Q70. A cube of side *b* has a charge *q* at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.

Q71. Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Q72. (a) Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by

$$(\vec{E_2} - \vec{E_1}) \cdot \hat{n} = \frac{\sigma}{\varepsilon_0}$$

Where $\hat{n}$ is a unit vector normal to the surface at a point and σ is the surface charge density at that point. (The direction of $\hat{n}$ is from side 1 to side 2.) Hence show that just outside a conductor, the electric field is $\sigma \hat{n} / \varepsilon_0$.

(b) Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another. [Hint: For (a), use Gauss's law. For, (b) use the fact that work done by electrostatic field on a closed loop is zero.]

Q73. Two charges 3×10^{-8} C and -2×10^{-8} C are located 15 cm apart. At what point on the line joining the two charges is the electrical potential zero? Take the potential at infinity to be zero.

Q74. (a) Calculate the potential at a point *P* due to a to charge of 4×10^{-7} C located 9 cm away.

(b) Hence, obtain the work done in bringing a charge of 2×10^{-9} C from infinity to the point *P*. Does the answer 2×10^{-9} C from infinity to the point *P*. Does the answer depend on the path along which the charge is brought?

Q75. A parallel plate capacitor is charged by a battery. After sometime the battery is disconnected and a dielectric slab of dielectric constant *K* is inserted between the plates. How would (a) the capacitance, (b) the electric field between the plates and (c) the energy stored in the capacitor, be affected? Justify your answer.

Q76. A conducting bubble of radius, a thickness *t* ($t << a$) has potential *V*. Now the bubble collapses into a droplet. Find the potential of the droplet.

Q77. An electron in a circulating around the nucleus of a hydrogen atom in a circular orbit of radius 5.3×10^{-11} m. Calculate (a) the electric potential at this radius and (b) the electric potential energy of the atom in eV. What would be the electric potential due to a helium nucleus at the same radius. Given, that $(4\pi \varepsilon_0)^{-1} = 9 \times 10^9$ m F^{-1} and $e = 1.6 \times 10^{-19}$C.

Q78. Calculate the voltage needed to balance an oil drop carrying 10 electrons, when located between plates of a capacitor, which are 5 mm apart. Given mass of the drop = 3×10^{-16} kg, charge on electron = 1.6×10^{-19} C and $g = 9.8$ m s^{-2}.

Q79. Two charges $-q$ and $+q$ are located at points (0, 0, – *a*) and (0, 0, *a*), respectively.

(a) What is the electrostatic potential at the points (0, 0, *z*) and (*x*, *y*, 0)?

(b) Obtain the dependence of potential on the distance *r* of a point from the origin when $r/a >> 1$.

(c) How much work is done in moving a small test charge from the point (5, 0, 0) to (– 7, 0, 0) along the *x*-axis? Does the answer change if the path of the test charge between the same points is not along the *x*-axis?

Q80. Describe schematically the equipotential surfaces corresponding to

(a) a constant electric field in the *z*-direction,

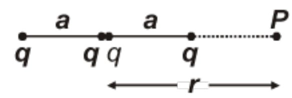
(b) a field that uniformly increases in magnitude but remains in a constant (say, *z*) direction

(c) a single positive charge at the origin, and

(d) a uniform grid consisting of long equally spaced parallel charged wires in a plane.

Q81. Deduce an expression for the electric potential due to an electric dipole at any point on its axis. Mention one contrasting feature of electric potential of a dipole at a point as compared to that due to a single charge.

Q82. The figure shows a charge array known as an *electric quadrupole*. For a point on the axis of the quadrupole, obtain the dependence of potential on *r* for $r/a >> 1$, and contrast your results with that due to an electric dipole, and an electric monopole (*i.e.*, a single charge).

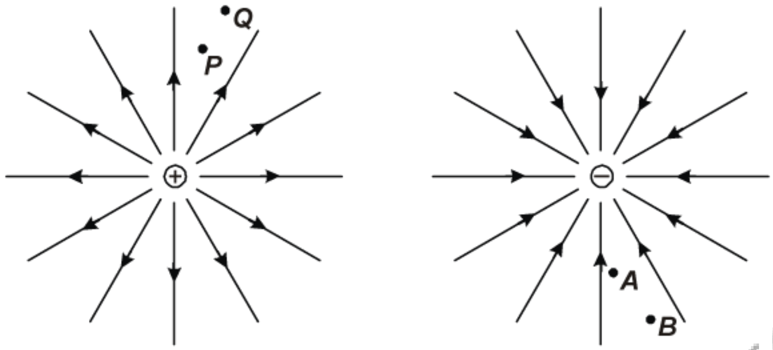


Q83. **Define 'electric potential'. Deduce an expression for the electric potential at a point distance 'r' from a point charge ($q > 0$).**

Q84. **A small sphere of radius, a carrying a positive charge, q. Is placed concentrically inside a larger hollow conducting shell of radius b ($b > a$). This outer shell has charge Q on it. Show that if these spheres are connected by a conducting wire, charge will always flow from the inner sphere to the outer sphere, irrespective of the magnitude of the two charges.**

Q85. **Define the electrostatic potential. Derive an expression for the electric potential at a point due to an electric dipole. Show that potential at any point on the perpendicular bisector of the dipole is zero.**

Q86. **Figures (a) and (b) show the field lines of a positive and negative point charge respectively.**



(a) (b)

(a) **Give the signs of the potential difference $V_P - V_Q$, $V_B - V_A$.**

(b) **Give the sign of the potential energy difference of a small negative charge between the points Q and P, A and B.**

(c) **Give the sign of the work done by the field in moving a small positive charge from Q to P.**

(d) **Give the sign of the work done by the external agency in moving a small negative charge from B to A.**

(e) **Does the kinetic energy of a small negative charge increase or decrease in going from B to A?**

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S1. The occurrence of thunderstorms and lightning charges the atmosphere continuously. Hence, even with the presence of discharging current of 1800 A, the atmosphere is not discharged completely. The two opposing currents are in equilibrium and the atmosphere remains electrically neutral.

S2. Yes, the man will get an electric shock if he touches the metal slab next morning. The steady discharging current in the atmosphere charges up the aluminium sheet. As a result, its voltage rises gradually. The raise in the voltage depends on the capacitance of the capacitor formed by the aluminium slab and the ground.

S3. We do not get an electric shock as we step out of our house because the original equipotential surfaces of open air changes, keeping our body and the ground at the same potential.

S4. According to Gauss's law, the electric field between a sphere and a shell is determined by the charge q_1 on a small sphere. Hence, the potential difference, V, between the sphere and the shell is independent of charge q_2. For positive charge q_1, potential difference V is always positive.

S5. Yes.

$$V = -\vec{E}.\vec{r}$$

S6. As $r_A > r_B \Rightarrow V_A < V_B$ As $V = \frac{kq}{r}$

$\therefore$ Charge will flow from B to A.

S7. Charges lie at the ends of lines of forces. These lines of force tend to contract in length, and thus pull the charges from inside a conductor to its outer surface.

S8. Zero, because electric field strength at infinite is zero.

S9. (a) $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = 9\times10^9\ \text{Nm}^2\ \text{C}^{-2}\times\frac{4\ 10^{-7}\ \text{C}}{0.09\ \text{m}}$

$= 4\times10^4\ \text{V}$

(b) $W = qV = 2\times10^{-9}\ \text{C}\times4\times10^4\ \text{V}$

$= 8\times10^{-5}\ \text{J}$

No, work done will be path independent. Any arbitrary infinitesimal path can be resolved into two perpendicular displacements: One along ***r*** and another perpendicular to ***r***. The work done corresponding to the later will be zero.

S10. During lighting & thunderstrom, light energy, heat energy and sound energy are dissipated in the atmosphere.

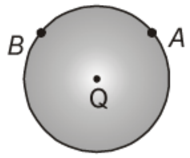
S11.

$$E = \frac{V}{d} = \frac{12}{3\times10^{-3}} = \mathbf{4\times10^3\ volts/m.}$$

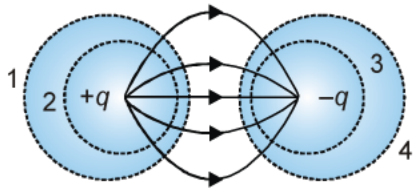
S12. Field in any direction other than normal will make the charge move on the surface of the conductor. Since surface is equipotential and no charge moves on it, field is normal to the surface.

S13. At point Q, the electric potential will be maximum. Because electric field string is more than other two points .

S14. Any two point *A* and *B* on any spherical surface with its centre at the given point charge.



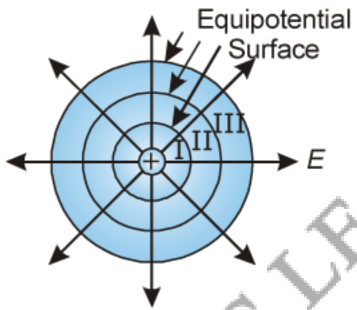
S15. 1, 2, 3, 4 are equipotential surfaces



S16. The charge in the cage goes to its outer surface. There is no electric charge and field inside the cage to give a shock to the man.

S17. (a) Positive (b) Negative

S18. I, II, III are equipotential surfaces



S19. Spherical

S20. The electric potential at a point in an electric field is the work done by an external force per unit charge to bring a small positive test charge from infinity to that point without imparting any acceleration to it.

S21. Field at A = Field at $B = \frac{\sigma}{\varepsilon_0}$

$\therefore$ Force on proton at *A* = force on proton at *B*

S22. Energy acquired by charge = q ΔV = 2 x 1 = 2 J.

S23. If 1 joule of work is done in carrying a test-charge of 1 coulomb from one point to the other in an electric field, then the potential difference between these points will be 1 volt.

S24. The electric potential at a point in an electric field is the work done in carrying a test-charge of unit magnitude from infinity to that point.

S25. The work done by an external agent in carrying a test-charge of unit magnitude from one point to the other in an electric field is called the potential difference between those points.

Electric potential difference is a scalar quantity.

S26. Electric potential difference. It is a scalar quantity.

S27. Electric field at any point is equal to the negative of potential gradient at that point *i.e.*

$$E = -\frac{dV}{dr}$$

S28. (a) The electrostatic potential due to a small electric dipole at a point on axial line varies inversely as the square of the distance of the points the dipole.

(b) At any point on the equatorial line, the electrostatic potential is zero.

S29. Given, $dV = 40\,\text{V}; \quad dr = 15\,\text{cm} = 0.15\,\text{m}$

Now, $$E = -\frac{dV}{dr} = -\frac{40}{0.15} = -266.67\ \text{Vm}^{-1}$$

The negative sign indicates that the direction of electric field is always in the direction of decrease of electric potential.

S30. For a uniform electric field, equipotential surfaces are planes at right angle to the direction of electric field.

S31. For a point charge, the equipotential surface are concentric spherical shells, whose centers are located at the given point charge.

S32. No. Electric potential may be zero or constant at that point.

S33. We know, $$E = -\frac{dV}{dr}$$

Since V is constant, electric field is zero.

S34. Electric field intensity.

S35. The electrostatic field at the surface of the conductor cannot have tangential component. Had it been so, the free charge on the surface would experience force and move *i.e.* will not remain static. It would lead to the flow of surface current. Therefore, the field lines must enter or leave the conductor at right angles to its surface.

S36.

$$E = \frac{V}{d} = \frac{3000}{5 \times 10^{-2}} = 6 \times 10^{4}\ V\ m^{-1}$$

As the charge particle remains suspended,

$$qE = m\,g$$

or $$m = \frac{qE}{g} = \frac{1.6 \times 10^{-19} \times 6 \times 10^{4}}{9.8}$$
$$= \mathbf{9.8 \times 10^{-16}\ kg}$$

S37. The given figure shows six equal amount of charges q, at the vertices of a regular hexagon.

Where,

Charge, $q = 5\ \mu C = 5 \times 10^{-6}\ C$

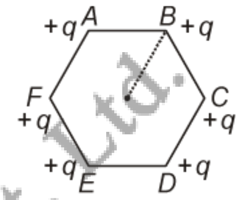
Side of the hexagon, $l = AB = BC = CD = DE = EF = FA = 10\ cm$

Distance of each vertex from centre O,

$$d = 10\ cm$$

Electric potential at point O,

$$V = \frac{6 \times q}{4\pi\varepsilon_0 d}$$



Where, ε_0 = Permittivity of free space

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9}\ N\ C^{-2}\ m^{2}$$

$$\therefore \quad V = \frac{6 \times 9 \times 10^{9} \times 5 \times 10^{-6}}{0.1} = 2.7 \times 10^{6}\ V$$

Therefore, the potential at the centre of the hexagon is 2.7×10^{6} V.

S38. Given, Radius of the spherical conductor, $r = 12\ cm = 0.12\ m$

Charge is uniformly distributed over the conductor, $q = 1.6 \times 10^{-7}\ C$

(a) Electric field inside a spherical conductor is zero. According to Gauss law.

$$E \,.\, \Delta S = \frac{q_{\text{inside}}}{\epsilon_0}$$

$\because$ q_{inside} is 0.

$\therefore$ $E = 0$.

(b) Electric field E just outside the conductor is given by the relation,

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

Where, ε_0 = Permittivity of free space

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^{9}\ N\ m^{2}\ C^{-2}$$

$$\therefore \quad E = \frac{1.6 \times 10^{-7} \times 9 \times 10^{-9}}{(0.12)^2} = 10^{5}\ N\ C^{-1}$$

Therefore, the electric field just outside the sphere is $10^{5}\ N\ C^{-1}$.

(c) Electric field at a point 18 m from the centre of the sphere = E_1

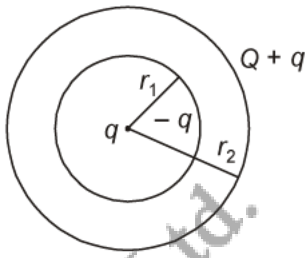
Distance of the point from the centre, d = 18 cm = 0.18 m

$$E_1 = \frac{q}{4\pi\varepsilon_0 d^2}$$

$$= \frac{9 \times 10^9 \times 1.6 \times 10^{-7}}{(18 \times 10^{-2})^2} = 4.4 \times 10^4 \text{ N/C}$$

Therefore, the electric field at a point 18 cm from the centre of the sphere is 4.4×10^4 N/C.

S39. (a) Charge placed at the centre of a shell is $+q$. Hence, a charge of magnitude $-q$ will be induced to the inner surface of the shell. Therefore, total charge on the inner surface of the shell is $-q$.



Surface charge density at the inner surface of the shell is given by the relation,

$$\sigma_1 = \frac{\text{Total charge}}{\text{Inner surface area}} = \frac{-q}{4\pi r_1^2} \quad \text{... (i)}$$

A charge of $+q$ is induced on the outer surface of the shell. A charge of magnitude Q is placed on the outer surface of the shell. Therefore, total charge on the outer surface of the shell is $Q + q$. Surface charge density at the outer surface of the shell,

$$\sigma_2 = \frac{\text{Total charge}}{\text{Outer surface area}} = \frac{Q+q}{4\pi r_2^2} \quad \text{... (ii)}$$

(b) Yes.

The electric field intensity inside a cavity is zero, even if the shell is not spherical and has any irregular shape. Take a closed loop such that a part of it is inside the cavity along a field line while the rest is inside the conductor. Net work done by the field in carrying a test charge over a closed loop is zero because the field inside the conductor is zero.

Hence, electric field is zero, whatever is the shape.

S40. Length of a co-axial cylinder, l = 15 cm = 0.15 m

Radius of outer cylinder, r_1 = 1.5 cm = 0.015 m

Radius of inner cylinder, r_2 = 1.4 cm = 0.014 m

Charge on the inner cylinder, q = 3.5 μC = 3.5×10^{-6} C

Capacitance of a co-axial cylinder of radii r_1 and r_2 is given by the relation,

$$C = \frac{2\pi\varepsilon_0 l}{\log_e \frac{r_1}{r_2}}$$

Where, ε_0 = Permittivity of free space

$= 8.85 \times 10^{-12}\ \text{N}^{-1}\,\text{m}^{-2}\,\text{C}^2$

$\therefore$
$$C = \frac{2\pi \times 8.85 \times 10^{-12} \times 0.15}{2.3026 \log_{10}\left(\frac{0.15}{0.14}\right)}$$

$$= \frac{2\pi \times 8.85 \times 10^{-12} \times 0.15}{2.3026 \times 0.0299} = 1.2 \times 10^{-10}\,\text{F}$$

Potential difference of the inner cylinder is given by,

$$V = \frac{q}{C} = \frac{3.5 \times 10^{-6}}{1.2 \times 10^{-10}} = 2.92 \times 10^{4}\,\text{V.}$$

S41. (a) $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = 9 \times 10^9 \times \frac{7 \times (-2) \times 10^{-12}}{0.18} = -0.7$ J.

(b) $W = U_2 - U_1 = 0 - U = 0 - (-0.7) = 0.7$ J.

(c) The mutual interaction energy of the two charges remains unchanged. In addition, there is the energy of interaction of the two charges with the external electric field. We find,

$$q_1 V(\mathbf{r}_1) + q_2 V(\mathbf{r}_2) = A\frac{7\,\mu\text{C}}{0.09\,\text{m}} + A\frac{-2\,\mu\text{C}}{0.09\,\text{m}}$$

and the net electrostatic energy is

$$q_1 V(\mathbf{r}_1) + q_2 V(\mathbf{r}_2) +$$

$$q_1 V(\mathbf{r}_1) + q_2 V(\mathbf{r}_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}} = A\frac{7\,\mu\text{C}}{0.09\,\text{m}} + A\frac{-2\,\mu\text{C}}{0.09\,\text{m}} - 0.7\text{ J}$$

$$= 70 - 20 - 0.7 = 49.3\text{ J.}$$

S42. (a) The work done in moving a test charge between two points in an electric field depends only on the initial and final positions of the charge. Let dV be the potential difference between A and C. If $AC = dr$, then

Here $dr = 5 - 1 = 4$ units

$$dV = (5 - 1)\,E = 4\,E$$

(b) The potential at C is greater than that at A *i.e.* $V_C > V_A$. It is because, the direction of electric field is in the direction of decrease of potential difference.

S43. The electric potential at a point in an electric field is the work done in carrying a test-charge of unit magnitude from infinity to the point, without any acceleration.

In an electric field, a proton (positive charged particle) will move in the direction of electric field *i.e.* from the higher to lower potential region.

S44. Given: $q = \pm 8\text{nC} = \pm 8 \times 10^{-9}$ C; $2a = 4$ cm $= 0.04$ m; $\tau = 4\sqrt{3}$ N m and $\theta = 60°$

(a) Now, $p = q\,(2a) = 8 \times 10^{-9} \times 0.04 = 3.2 \times 10^{-10}$ Cm

Torque on the on the electric dipole,

$$\tau = p\,E\sin\theta$$

or $$E = \frac{\tau}{p\sin\theta} = \frac{4\sqrt{3}}{3.2\times10^{-10}\times\sin 60^\circ}$$

$$= \frac{4\sqrt{3}\times2}{3.2\times10^{-10}\times\sqrt{3}} = 2.5\times10^{10}\,NC^{-1}$$

(b) Potential energy of the electric dipole,

$$U = -pE\cos\theta = -3.2\times10^{-10}\times2.5\times10^{10}\times\cos 60^\circ$$

$$= -3.2\times2.5\times0.5 = -4\text{ J}$$

S45. Here $r = 10$ cm $= 0.1$ m

and $q = 200\ \mu C = 200 \times 10^{-6}$ C

Consider an elementary portion of the circular wire. Suppose that it possesses a small amount of charge dq over it. we have assume that the charge on elementary portion behave as point charge, then potential due to elementary portion at the centre,

$$dV = \frac{1}{4\pi\varepsilon_0}.\frac{dq}{r}$$

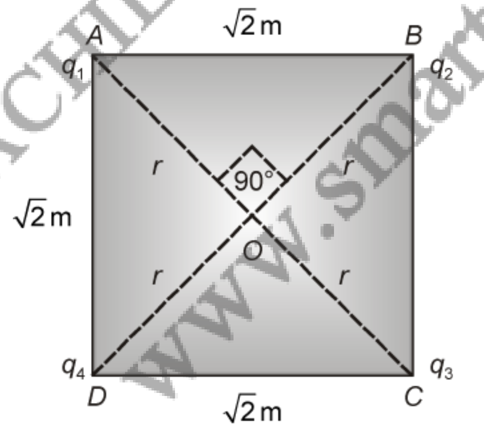
The potential due to the whole circular wire,

$$V = \Sigma dV = \Sigma\frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\Sigma\, dq = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$= 9\times10^9\times\frac{200\times10^{-6}}{0.1} = \mathbf{1.8\times10^7\,V}$$

S46. Given: $q_1 = 2\mu C = 2 \times 10^{-6}$ C; $q_2 = -2\mu C = -2 \times 10^{-6}$ C; $q_3 = -3\mu C = -3 \times 10^{-6}$ C; $q_4 = 6\mu C = 6 \times 10^{-6}$ C; and $AB = BC = CD = AD = \sqrt{2}\ m$

Four charges q_1, q_2, q_3 and q_4 are placed at the four corners of the square $ABCD$ as shown in figure.



Let r be the distance of each charge from the centre O of the square,

Then, $\sqrt{r^2 + r^2} = \sqrt{2}$ or $r = 1\ m$

Potential at point O due to charge at the four corners,

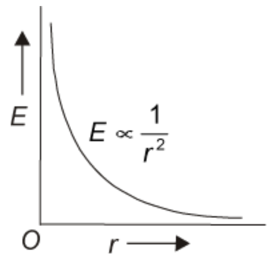
$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right) = \frac{1}{4\pi\varepsilon_0}.\frac{1}{r}(q_1 + q_2 + q_3 + q_4)$$

$$= \frac{9\times10^9}{1}\left(2\times10^{-6} + (-2\times10^{-6}) + (-3\times10^{-6}) + 6\times10^{-6}\right)$$

$$= \mathbf{2.7 \times 10^4\ V}$$

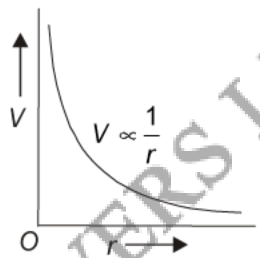
S47. (a) Given, $E = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r^2}$ or $E \propto \frac{1}{r^2}$

The variation of electric field *E* with distance *r* is as shown in figure below.



(b) Here, $V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$ or $V \propto \frac{1}{r}$

The variation of electric potential *V* with distance *r* is as shown in Figure.



S48. **Equipotential surface:** Any surface which the electric potential is same everywhere is called an equipotential surface.

Let $\vec{E}$ be electric field at a point on equipotential surface. Small displacement $\vec{dr}$ along the surface,

$$dW = \vec{F}.\vec{dr} = (-q_0\vec{E}).\vec{dr}$$

Since work done in moving a test charge along an equipotential surface is always zero,

$$(-q_0\vec{E}).\vec{dr} = 0$$

$$\vec{E}.\vec{dr} = 0$$

Hence, electric field is directed perpendicular to the surface.

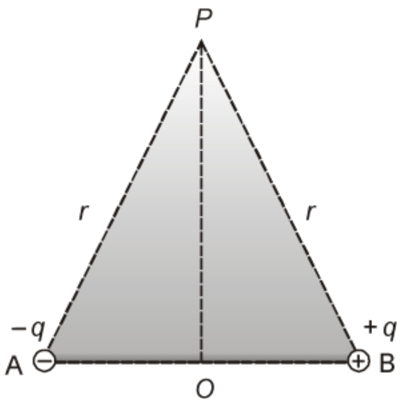
S49. Suppose that the electric potential is zero at the point, which is at a distance r_1 from the charge $+q$ and r_2 from the charge $-q$. Then,

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r_1}+\frac{1}{4\pi\varepsilon_0}\cdot\frac{-q}{r_2}=0$$

or $$r_1 = r_2$$

All such points for which $r_1 = r_2$ lie on a plane, which is right bisector of the line joining the two charges.

S50. Figure shows an electric dipole consisting of charge $-q$ at point A and $+q$ at point B.



Let P be point on the equatorial line of the dipole.

Then, $$PA = PB = r \text{ (say)}$$

Electrical potential at point P due to the dipole,

$$V=\frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)}{PA}+\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{PB}$$

$$=-\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}+\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r}=0\text{ V}.$$

S51. No, because an equipotential surface is normal to the electric field. If two equipotential surfaces intersect each other, then at the point of intersection there will be two directions of electric field, which is impossible.

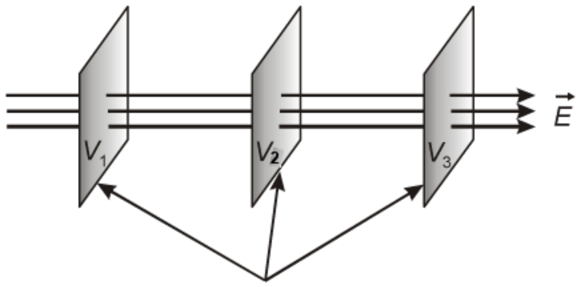
S52. $$V_B - V_A = \frac{W_{AB}}{q_0}$$

Since the two points A and B are on the same equipotential surface,

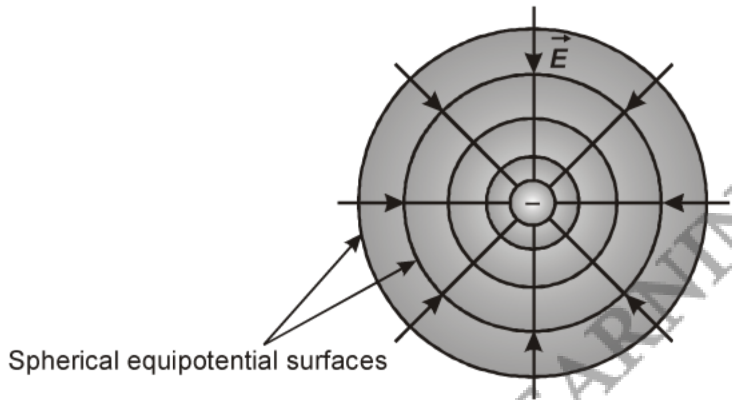
$$V_B - V_A = 0$$

$$\therefore \quad \frac{W_{AB}}{q_0} = 0 \quad \text{or} \quad W_{AB} = 0 \qquad (\because\ q_0 \neq 0)$$

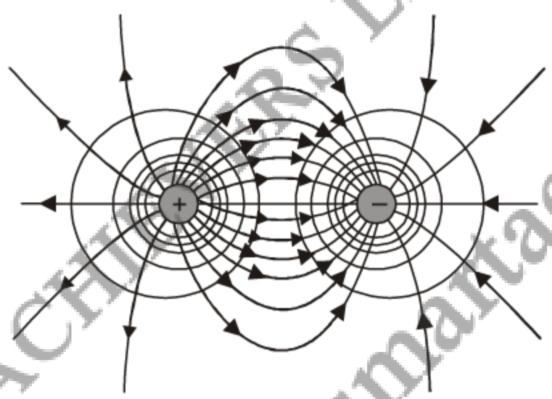
S53. (a) In a uniform electric field, the equipotential surface is a plane right angle to direction field.



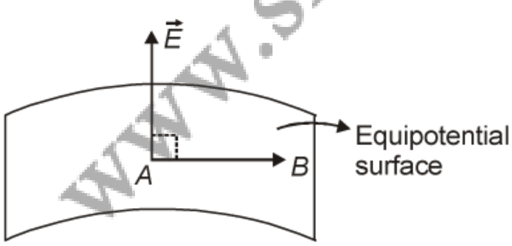
(b) Figure shows the equipotential surface for a point charge $Q > 0$. For a charge $Q < 0$, the surface will be same but the arrows representing the direction of electric field will be direction inwards.



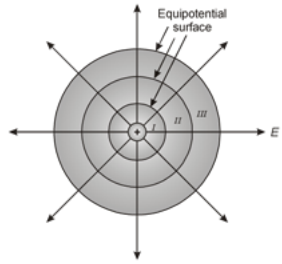
S54. (a)



(b)



S55. (a) **The equipotential surface when Q > 0.**



(b) We know that

$$E = -\frac{dV}{dr} \quad \text{or} \quad dr = -\frac{dV}{E}$$

For same change in value of dV = constant, we have

$$dr \propto \frac{1}{E}$$

i.e., the spacing between the equipotential surface will be lesser in the region, where the electric field is stronger and vice-versa. Therefore, the equipotential surfaces are closer together, where the electric field is stronger and farther apart, where the field is weaker.

S56. Given: Electric field $E = 20\ \text{NC}^{-1}$ and electric potential $V = 10\ \text{JC}^{-1}$ let distance of point charge is r.

Now, $$V = -E \cdot r \Rightarrow r = \frac{V}{E} = \frac{10}{20} = 0.5 \text{ metre.}$$

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} \Rightarrow q = 4\pi\varepsilon_0 \times V \times r$$

$$q = \frac{1}{9 \times 10^9} \times 10 \times 0.5 = 5.56 \times 10^{-10}$$

S57. (a) Since the electric field is always at right angles to the equipotential surface, will be along X-axis.

(b) Now, $\tau = p\,E \sin\theta$

Here, $p = q \times 2b$ and $\theta = 90°$

$\therefore$ $\tau = (q \times 2b)\,E \sin 90° = \mathbf{2\,q\,b\,E}$

S58. Let C_1 be the capacitance of the first conductor,

Given: $V_1 = 40\,\text{V}$; $C_2 = 10\,\text{F}$; $q_2 = 0$

Now, $q_1 = C_1 V_1 = C_1 \times 40$

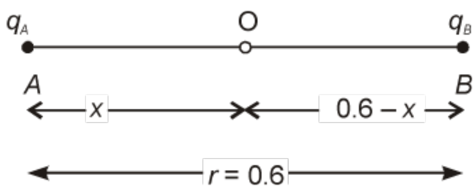
Common potential,

$$V = \frac{q_1 + q_2}{C_1 + C_2} = \frac{C_1 \times 40 + 0}{C_1 + 10} = \frac{C_1 \times 40}{C_1 + 10}$$

Thus, $\dfrac{C_1 \times 40}{C_1 + 10} = 25$

or $C_1 = \mathbf{16.67\,F}$.

S59. Given: $q_A = 10 \times 10^{-8}$ C; $q_B = -2 \times 10^{-8}$ C $r = 60$ cm = 0.6 m



Now $V_A = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q_A}{x}$

$= 9 \times 10^9 \dfrac{(10 \times 10^{-8})}{x} = \dfrac{900}{x}$

S60. Given: $V(x) = 4x^2$

Now, $E = -\dfrac{dV(x)}{dx} = -8x$

$\therefore$ E (at x = 1 m) = $8 \times 1 = -8\,NC^{-1}$ (or Vm^{-1})

Force experienced by the μC charge,

$$F = qE = 10^{-6} \times (-8) = \mathbf{-8 \times 10^{-6}\,N}.$$

S61. Given: $q = +Ze = 78 \times 1.6 \times 10^{-19} C$; $r = 5 \times 10^{-15}$ m

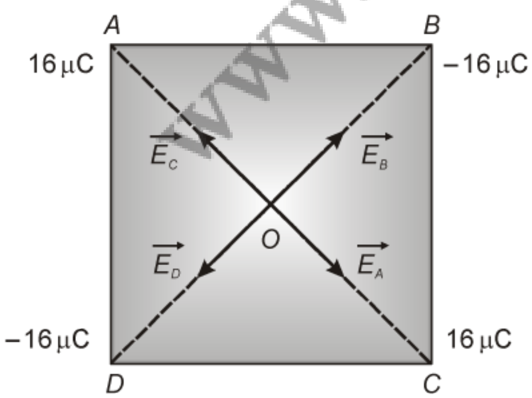
Potential gradient at a point is numerically equal to electric field at that point *i.e.*,

$$\frac{dV}{dr} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

$\therefore$ $\dfrac{dV}{dr}$ (at $r = 10^{-12}$ m) = $\dfrac{9 \times 10^9 \times 78 \times 1.6 \times 10^{-19}}{(10^{-12})^2}$ = $\mathbf{1.123 \times 10^{17}\,Vm^{-1}}$

Similarly, $\dfrac{dV}{dr}$ (at $r = 5 \times 10^{-15}$ m) = $\mathbf{4.493 \times 10^{21}\,Vm^{-1}}$.

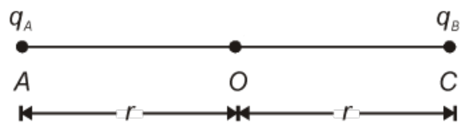
S62. The figure shows the directions in which the electric fields due to the four charges will be directed.



Since all the four charges are of same magnitude and are at the same distance from the point O, the electric fields due to them will also be of same magnitude. Further, as two charges are positive and the other two negative, the electric fields will cancel each other in pairs. Hence, the net electric field at the point O will be ***zero***.

For the same reason, the net electric potential will also be ***zero***.

S63. Given: $q_A = 1.5\,\mu C = 1.5 \times 10^{-6}\,C$, $q_B = 2.5\,\mu C = 2.5 \times 10^{-6}\,C$



Potential at point O,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q_A}{OA} + \frac{q_B}{OB}\right) = 9 \times 10^9 \left(\frac{1.5 \times 10^{-6}}{1.5 \times 10^{-2}} + \frac{2.5 \times 10^{-6}}{1.5 \times 10^{-2}}\right) = \mathbf{2.4 \times 10^6\,V}.$$

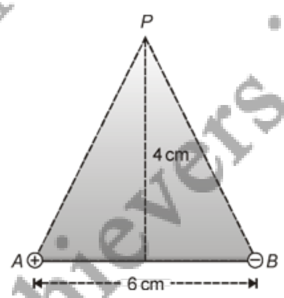
Electric field at point O, $E = E_B - E_A$ $\qquad (\because\ E_B > E_A)$

or
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q_B}{OB^2} - \frac{q_A}{OA^2}\right) = \mathbf{4.0 \times 10^7\,NC^{-1}} \quad \textbf{(along } \boldsymbol{OA}\textbf{)}.$$

S64. Given: $AB = 6\,cm$; $OP = 4\,cm$; $q_A = +\,20\,\mu C = 20 \times 10^{-6}\,C$;

$q_B = -\,10\,\mu C = -\,10 \times 10^{-6}\,C$,

$\therefore \qquad AP = BP = \sqrt{3^2 + 4^2} = 5\,cm = 0.05\,m$



Potential at the point P,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A}{AP} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_B}{BP}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A + q_B}{AP}$$

$$= \frac{9 \times 10^9 \times (20 \times 10^{-6} - 10 \times 10^{-6})}{0.5} = \mathbf{1.8 \times 10^5\,V}.$$

S65. Let Q_A and Q_B be the charges on the solid and the hollow conducting spheres respectively and R be the radius of each sphere. When charge is given to a solid conducting sphere, it appears on the outer surface. However, for the calculation of electric field or potential due to a sphere (Whether hollow or solid) at a point on or outside the sphere, the charge behaves as if it is concentrated at the centre of the sphere. If V_A and V_B are potentials of the two sphere, then

$$V_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_A}{R} \quad \text{and} \quad V_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q_B}{R}$$

Since $V_A = V_B$, it follows that

$\boldsymbol{Q_A = Q_B}$.

S66. Given: $q_1 = 0.5\ \mu C$; $V_1 = 20$ V; $C_2 = 0.015\ \mu F$; $q_2 = 0$

Now, $$C_1 = \frac{q_1}{V_1} = \frac{0.5}{20} = 0.025\ \mu\text{F}$$

when the capacitor C_2 is connected momentarily to C_1 (in parallel),

total capacitance,

$$C = C_1 + C_2 = 0.025 + 0.015 = 0.04\ \mu F$$

total charge,

$$q = q_1 + q_2 = 0.5 + 0 = 0.5\ \mu C$$

common potential,

$$V = \frac{q}{C} = \frac{0.5}{0.04} = 12.5\ \text{V}$$

Now, $q_1' = C_1 V = 0.025 \times 12.5 = 0.3125\ \mu\text{C}$

and $q_2' = C_2 V = 0.015 \times 12.5 = 0.1875\ \mu\text{C}$

S67. We know that the electric field at any point is equal to the negative of the potential gradient *i.e.*,

$$E = -\frac{dV}{dr}$$

Inside a conductor electric field is zero *i.e.*,

$$E = 0$$

or $$-\frac{dV}{dr} = 0$$

or $$\frac{dV}{dr} = 0$$

or $$V = \text{constant}$$

Thus, the potential at all points inside and on the conductor must be the same. In other words, the surface of a conductor is an equipotential surface.

S68. Given: $q = 1C$, $r = 1$ cm $= 1 \times 10^{-2}$ m

$\therefore$ Potential on such a sphere, $$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = \frac{9\times10^9\times1}{1\times10^{-2}} = 9\times10^{11}V$$

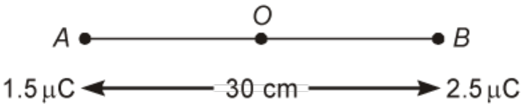
This potential is very large and causes ionisation of air surrounding the sphere. The charge on the metal sphere therefore will leak into the atmosphere.

S69. Two charges placed at points A and B are represented in the given figure. O is the mid-point of the line joining the two charges.

Magnitude of charge located at A, $q_1 = 1.5\,\mu C$

Magnitude of charge located at B, $q_2 = 2.5\,\mu C$

Distance between the two charges, d = 30 cm = 0.3 m



(a) Let V_1 and E_1 are the electric potential and electric field respectively at O.

V_1 = Potential due to charge at A + Potential due to charge at B

$$V_1 = \frac{q_1}{4\pi\varepsilon_0\left(\frac{d}{2}\right)} + \frac{q_2}{4\pi\varepsilon_0\left(\frac{d}{2}\right)} = \frac{1}{4\pi\varepsilon_0\left(\frac{d}{2}\right)}(q_1 + q_2)$$

Where, ε_0 = Permittivity of free space

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\,\text{N C}^2\,\text{m}^{-2}$$

$$\therefore \quad V_1 = \frac{9 \times 10^9 \times 10^{-6}}{\left(\frac{0.30}{2}\right)}(2.5 + 1.5) = 2.4 \times 10^5\,\text{V}$$

E_1 = Electric field due to q_2 – Electric field due to q_1

$$= \frac{q_2}{4\pi\varepsilon_0\left(\frac{d}{2}\right)^2} - \frac{q_1}{4\pi\varepsilon_0\left(\frac{d}{2}\right)^2}$$

$$= \frac{9 \times 10^9}{\left(\frac{0.30}{2}\right)^2} \times 10^6 \times (2.5 + 1.5) = 4 \times 10^5\,\text{V m}^{-1}.$$

Therefore, the potential at mid-point is 2.4×10^5 V and the electric field at mid-point is $4 \times 10^5\,\text{V m}^{-1}$. The field is directed from the larger charge to the smaller charge.

(b) Consider a point Z such that normal distance OZ = 10 cm = 0.1 m, as shown in the following figure.

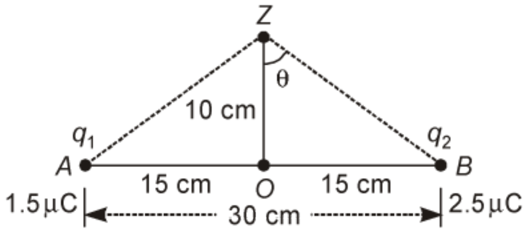
V_2 and E_2 are the electric potential and electric field respectively at Z.

It can be observed from the figure that distance,

$$BZ = AZ = \sqrt{(0.1)^2 + (0.15)^2} = 0.18\,\text{m}$$

V_2 = Electric potential due to A + Electric Potential due to B

$$= \frac{q_1}{4\pi\varepsilon_0(AZ)} + \frac{q_2}{4\pi\varepsilon_0(BZ)}$$



$$= \frac{9 \times 10^9 \times 10^{-6}}{0.18}(1.5 + 2.5)$$

$$= 2 \times 10^5 \text{ V}.$$

Electric field due to q at Z,

$$E_A = \frac{q_1}{4\pi\varepsilon_0(AZ)^2}$$

$$= \frac{9 \times 10^9 \times 1.5 \times 10^{-6}}{(0.18)^2} = 0.416 \times 10^6 \text{ V/m}.$$

Electric field due to q_2 at Z,

$$E_B = \frac{q_2}{4\pi\varepsilon_0(BZ)^2}$$

$$= \frac{9 \times 10^9 \times 2.5 \times 10^{-6}}{(0.18)^2} = 0.69 \times 10^6 \text{ V m}^{-1}.$$

The resultant field intensity at Z,

$$E = \sqrt{E_A^2 + E_B^2 + 2E_AE_B \cos 2\theta}$$

Where, 2θ is the angle, $\angle AZB$

From the figure, we obtain

$$\cos\theta = \frac{0.10}{0.18} = \frac{5}{9} = 0.5556$$

$$\theta = \cos^{-1} 0.5556 = 56.25$$

$\therefore$ $$2\theta = 112.5°$$

$$E = \sqrt{(0.416 \times 10^6)^2 + (0.69 \times 10^6)^2 + 2 \times 0.416 \times 0.69 \times 10^{12} \times (-0.38)}$$

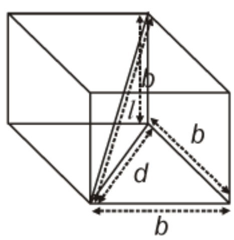
$$= 6.6 \times 10^5 \text{ V m}^{-1}.$$

Therefore, the potential at a point 10 cm (perpendicular to the mid-point) is 2.0×10 V and electric field is 6.6×10^5 V m^{-1}.

S70. Length of the side of a cube = b.

Charge at each of its vertices = q.

A cube of side b is shown in the following figure.



d = Diagonal of one of the six faces of the cube

$$d^2 = \sqrt{b^2 + b^2} = \sqrt{2b^2}$$

$$d = b\sqrt{2}$$

l = Length of the diagonal of the cube

$$l^2 = \sqrt{d^2 + b^2}$$

$$= \sqrt{(\sqrt{2}b)^2 + b^2} = \sqrt{2b^2 + b^2} = \sqrt{3b^2}$$

$$l = b\sqrt{3}$$

$$r = \frac{l}{2} = \frac{b\sqrt{3}}{2}$$

is the distance between the centre of the cube and one of the eight vertices.

The electric potential (V) at the centre of the cube is due to the presence of eight charges at the vertices.

$$V = \frac{8q}{4\pi\varepsilon_0 r} = \frac{8q}{4\pi\varepsilon_0\left(b\frac{\sqrt{3}}{2}\right)} = \frac{4q}{\sqrt{3}\pi\varepsilon_0 b}$$

Therefore, the potential at the centre of the cube is $\frac{4q}{\sqrt{3}\pi\varepsilon_0 b}$.

The electric field at the centre of the cube, due to the eight charges, gets cancelled. This is because the charges are distributed symmetrically with respect to the centre of the cube. Hence, the electric field is zero at the centre.

S71. There are two charges,

$$q_1 = 5 \times 10^{-8}\,\text{C}$$

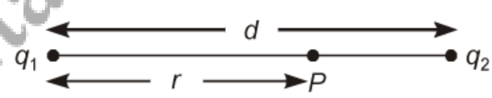
$$q_2 = -3 \times 10^{-8}\,\text{C}$$

Distance between the two charges, d = 16 cm = 0.16 m

Case I: Consider a point P on the line joining the two charges, as shown in the given figure.

r = Distance of point P from charge q_1

Let the electric potential (V) at point P be zero.



Potential at point P is the sum of potentials caused by charges q_1 and q_2 respectively.

$$\therefore \quad V = \frac{q_1}{4\pi\varepsilon_0 r} + \frac{q_2}{4\pi\varepsilon_0 (d - r)} \quad \text{... (i)}$$

Where,

ε_0 = Permittivity of free space

For $V = 0$, equation (i) reduces to

$$\frac{q_1}{4\pi\varepsilon_0 r} = -\frac{q_2}{4\pi\varepsilon_0 (d - r)}$$

$$\frac{q_1}{r} = \frac{-q_2}{d - r}$$

$$\frac{5\times10^{-8}}{r} = \frac{(3\times10^{-8})}{(0.16-r)}$$

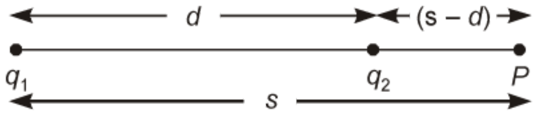
$$\frac{0.16}{r} - 1 = \frac{3}{5}$$

$$\frac{0.16}{r} = \frac{8}{5}$$

$\therefore$ $r = 0.1$ m = 10 cm

Therefore, the potential is zero at a distance of 10 cm from the positive charge between the charges.

Case II: Suppose point P is outside the system of two charges at a distance s from the negative charge, where potential is zero, as shown in the following figure.



For this arrangement, potential is given by,

$$V = \frac{q_1}{4\pi\varepsilon_0 s} + \frac{q_2}{4\pi\varepsilon_0 (s-d)} \qquad \text{... (iI)}$$

For $V = 0$, equation (ii) reduces to

$$\frac{q_1}{4\pi\varepsilon_0 s} = -\frac{q_2}{4\pi\varepsilon_0 (s-d)}$$

$$\frac{q_1}{s} = \frac{-q_2}{s-d}$$

$$\frac{5\times10^{-8}}{s} = \frac{(-3\times10^{-8})}{(s-0.16)}$$

$$1 - \frac{0.16}{s} = \frac{3}{5}$$

$$\frac{0.16}{s} = \frac{2}{5}$$

$\therefore$ $s = 0.4$ m = 40 cm

Therefore, the potential is zero at a distance of 40 cm from the positive charge outside the system of charges.

S72. (a) Electric field on one side of a charged body is E_1 and electric field on the other side of the same body is E_2. If infinite plane charged body has a uniform thickness, then electric field due to one surface of the charged body is given by,

$$\vec{E_1} = -\frac{\sigma}{2\varepsilon_0}\hat{n} \qquad \text{... (i)}$$

Where,

$\hat{n}$ = Unit vector normal to the surface at a point

σ = Surface charge density at that point

Electric field due to the other surface of the charged body,

$$\vec{E_2} = -\frac{\sigma}{2\varepsilon_0}\hat{n} \quad \text{... (ii)}$$

Electric field at any point due to the two surfaces,

$$\vec{E_2} - \vec{E_1} = \frac{\sigma}{2\varepsilon_0}\hat{n} + \frac{\sigma}{2\varepsilon_0}\hat{n} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

$$(\vec{E_2} - \vec{E_1}) \cdot \hat{n} = \frac{\sigma}{\varepsilon_0} \quad \text{... (ii)}$$

(b) (i) Since inside a closed conductor,

$$\vec{E_1} = 0$$

$$\therefore \quad \vec{E} - \vec{E_2} = \frac{\sigma}{\varepsilon_0}$$

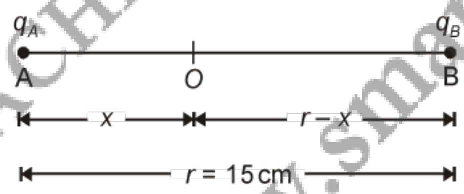
Therefore, the electric field just outside the conductor is $\frac{\sigma}{\varepsilon_0}\hat{n}$.

(ii) When a charged particle is moved from one point to the other on a closed loop, the work done by the electrostatic field is zero. Hence, the tangential component of electrostatic field is continuous from one side of a charged surface to the other.

S73. Given, $q_A = 3 \times 10^{-8}$ C; $q_B = -2 \times 10^{-8}$ C;

r = 15 cm = 0.15 m

Let O be the point, where the electric potential is zero due to the two charges.



Suppose that the distance $AO = x$. Then,

$$BO = r - x = 0.15 - x$$

Electric potential at point O due to q_A,

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{q_A}{AO}$$

$$= 9 \times 10^9 \times \frac{3 \times 10^{-8}}{x} = \frac{270}{x}$$

Electric potential at point O due to q_B,

$$V_B = \frac{1}{4\pi\varepsilon_0} \frac{q_B}{BO}$$

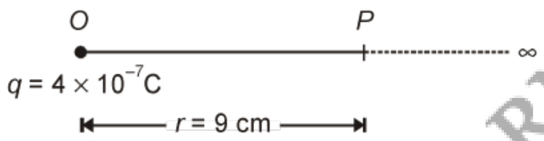
$$= 9 \times 10^9 \times \frac{(-2 \times 10^{-8})}{0.15 - x} = -\frac{180}{0.15 - x}$$

Since the electric potential at point O is zero, we have

or $\quad \frac{270}{x} + \left(-\frac{180}{0.15 - x}\right) = 0$ or $\frac{270}{x} = \frac{180}{0.15 - x}$

or $\quad x = 0.09$ m = **9 cm (from charge of 3×10^{-8} C)**

S74. Given, $q = 4 \times 10^{-7}$ C; $r = 9$ cm $= 0.09$ m



(a) Potential at point P due to charge q,

$$V_P = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

$$= 9 \times 10^9 \times \frac{4 \times 10^{-7}}{0.09} = 4 \times 10^4 \text{ V}$$

(b) From the definition, the potential at point P equals the work done in bringing a unit positive charge from infinity to point P. Therefore, the work done in bringing a charge of 2×10^{-9} C from infinity to point P,

Since, $\quad W = qV_P$

$W = 2 \times 10^{-9} \times$ Potential at point P

$= 2 \times 10^{-9} \times 4 \times 10^4$

$= 8 \times 10^{-5}$ J

The work done in bring the charge from infinity to Point P done not depend on the path along which charge is brought.

S75. Let C be the capacitance and V be the potential difference. The charge on the capacitor plates will then be, $Q = CV$.

The electric field between the plates $E = \frac{V}{d}$ and the energy stored = $E_n = \frac{Q^2}{2C}$ or $\frac{1}{2}CV^2$.

As the dielectric (K) is introduced after disconnecting the battery, we have, the new values of

Charge $\quad Q' = Q$

Capacitance $\quad C' = KC$

Potential $\quad V' = \frac{Q}{KC} = \frac{V}{K}$

(a) New capacitance is K time its original.

(b) New electric field E = $\frac{V'}{d} = \frac{V}{Kd} = \frac{E}{K}$ *i.e.*, $\frac{1}{K}$ time the original field.

(c) New energy = $\frac{Q^2}{2C'} = \frac{Q^2}{2KC} = \frac{1}{K}(E_n)$ *i.e.*, $\frac{1}{K}$ times the original energy.

S76. Before the bubble collapses droplet:

Here, radius of the bubble = a;

thickness of the bubble = t

Let q be charge on the bubble. Then potential of the bubble,

$$V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{a}$$

or $\quad q = 4\pi\varepsilon_0 V a$ ... (i)

After the bubble collapses into droplet:

Let r be the radius of the droplet formed. Then, volume of the droplet = volume of the bubble

or $\quad \frac{4}{3}\pi r^3 = 4\pi a^2 \times t$ ... (ii)

or $\quad r = (3a^2 t)^{1/3}$

If V' is potential of the droplet, then

$$V' = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r} \quad \text{... (iii)}$$

In the equation (iii), substituting the value of q and r, we have

$$V' = \frac{1}{4\pi\varepsilon_0}.\frac{4\pi\varepsilon_0 Va}{(3a^2 t)^{(1/3)}} = V\left(\frac{a}{3t}\right)^{(1/3)}$$

S77. Given, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \text{ m F}^{-1}$; $e = 1.6 \times 10^{-19}$C

(a) Radius of electron orbit in the hydrogen atom,

$$r = 5.3 \times 10^{-11} \text{ m}$$

The nucleus of hydrogen atom contains one proton. Therefore, charge on the nucleus of hydrogen atom,

$$q_1 = + e = 1.6 \times 10^{-19} \text{ C}$$

Electric potential the electron orbit of hydrogen atom,

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r} = \frac{9 \times 10^9 \times 1.6 \times 10^{-19}}{5.3 \times 10^{-11}} = \mathbf{27.17\ V}$$

(b) Charge on the electron, $q_2 = - e = -1.6 \times 10^{-19}$ C

Therefore, potential energy of H-atom (proton-electron system),

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$$

$$= \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times (-1.6 \times 10^{-19})}{5.3 \times 10^{-11}}$$

$$= -27.17 \times 1.6 \times 10^{-19} \text{ J}$$

Now, $1 \text{ eV} = 1.6 \times 10^{-19}$ J

$\therefore$ $$U = -\frac{27.17 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-19}} = \mathbf{-27.17\ eV}$$

A helium nucleus contains two protons. Therefore, charge on the helium nucleus,

$$q = +2e = 2 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-19} \text{C}$$

Hence, electric potential at distance r = (= 5.3×10^{-11} m),

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} = \frac{9 \times 10^9 \times 3.2 \times 10^{-19}}{5.3 \times 10^{-11}} = \mathbf{54.34\ V}$$

S78. Given, charge on the oil drop,

$$q = 10e = 10 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-18} \text{ C}$$

Mass of the oil drop, $m = 3 \times 10^{-16}$ kg

Distance between the plates of the capacitor,

$$r = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$$

The electric field at a point is numerically equal to the gradient of electric potential at that point. Since electric field between the plates of a capacitor is uniform, it is given by

$$E = \frac{V}{r}$$

Here, V is potential difference between the plates of the capacitor held at distance r apart.

So that the oil drop is balanced,

$$q\,E = m\,g \text{ or } q\left(\frac{V}{r}\right) = m\,g$$

or $$V = \frac{m\,g\,r}{q} = \frac{3\times10^{-16}\times9.8\times5\times10^{-3}}{1.6\times10^{-18}} = \mathbf{9.19\ V}$$

S79. (a) Zero at both the points

Charge $-q$ is located at (0, 0, $-a$) and charge $+q$ is located at (0, 0, a). Hence, they form a dipole. Point (0, 0, z) is on the axis of this dipole and point (x, y, 0) is normal to the axis of the dipole. Hence, electrostatic potential at point (x, y, 0) is zero. Electrostatic potential at point (0, 0, z) is given by,

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{z-a}\right) + \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{z+a}\right)$$

$$= \frac{q(z+a-z+a)}{4\pi\varepsilon_0(z^2-a^2)}$$

$$= \frac{2qa}{4\pi\varepsilon_0(z^2-a^2)} = \frac{p}{4\pi\varepsilon_0(z^2-a^2)}$$

Where, ε_0 = Permittivity of free space

p = Dipole moment of the system of two charges = $2qa$

(b) Distance r is much greater than half of the distance between the two charges. Hence, the potential (V) at a distance r is inversely proportional to square of the distance *i.e.*,

$$V \propto \frac{1}{r^2}$$

Zero. The answer does not change if the path of the test is not along the x-axis.

A test charge is moved from point (5, 0, 0) to point (–7, 0, 0) along the x-axis. Electrostatic potential (V_1) at point (5, 0, 0) is given by,

$$V_1 = \frac{-q}{4\pi\varepsilon_0}\frac{1}{\sqrt{(5-0)^2+(-a)^2}} + \frac{q}{4\pi\varepsilon_0}\frac{1}{(5-0)^2+a^2}$$

$$= \frac{-q}{4\pi\varepsilon_0\sqrt{25^2+a^2}} + \frac{q}{4\pi\varepsilon_0\sqrt{25^2+a^2}} = 0$$

(c) Electrostatic potential, V_2, at point (– 7, 0, 0) is given by,

$$V_2 = \frac{-q}{4\pi\varepsilon_0}\frac{1}{\sqrt{(-7)^2+(-a)^2}} + \frac{q}{4\pi\varepsilon_0}\frac{1}{(-7)^2+(a)^2}$$

$$= \frac{-q}{4\pi\varepsilon_0\sqrt{49+a^2}} + \frac{q}{4\pi\varepsilon_0\sqrt{49+a^2}} = 0$$

Hence, no work is done in moving a small test charge from point (5, 0, 0) to point (– 7, 0, 0) along the *x*-axis.

The answer does not change because work done by the electrostatic field in moving a test charge between the two points is independent of the path connecting the two points.

S80. (a) Equidistant planes parallel to the *x*-*y* plane are the equipotential surfaces.

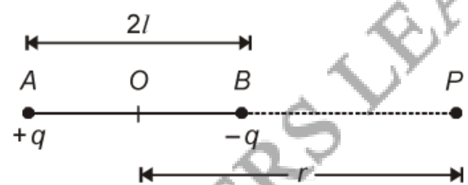
(b) Planes parallel to the *x*-*y* plane are the equipotential surfaces with the exception that when the planes get closer, the field increases.

(c) Concentric spheres centered at the origin are equipotential surfaces.

(d) A periodically varying shape near the given grid is the equipotential surface. This shape gradually reaches the shape of planes parallel to the grid at a larger distance.

S81. Potential at '*P*' due to charge at *A* is

$$V_{PA} = \frac{kq}{r+l}$$



Potential at '*P*' due to charge at *B* is

$$V_{PB} = \frac{kq}{r-l}$$

Net potential at '*P*' is

$$V_P = V_{PA} + V_{PB} = kq\left[\frac{1}{r-l} - \frac{1}{r+l}\right]$$

$$= kq\left[\frac{r+l-r+l}{r^2-l^2}\right]$$

$$= \frac{kq(2l)}{r^2-l^2} = \frac{kp}{r^2-l^2} \qquad [\therefore\ p = q(2l)]$$

In case $r >> l$

$$V_P = \frac{kp}{r^2}, i.e. V_P \propto \frac{1}{r^2}$$

Whereas, due to a single charge potential at a point is $V \propto \frac{1}{r}$.

S82. Four charges of same magnitude are placed at points X, Y, Y, and Z respectively, as shown in the following figure.

A point is located at P, which is r distance away from point Y.

The system of charges forms an electric quadrupole.

It can be considered that the system of the electric quadrupole has three charges.

Charge $+q$ placed at point X

Charge $-2q$ placed at point Y

Charge $+q$ placed at point Z

$$XY = YZ = a$$
$$YP = r$$
$$PX = r + a$$
$$PZ = r - a$$

Electrostatic potential caused by the system of three charges at point P is given by,

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{XP} - \frac{2q}{YP} + \frac{q}{ZP}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r+a} - \frac{2q}{r} + \frac{q}{r-a}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r(r-a) - 2(r+a)(r-a) + r(r+a)}{r(r+a)(r-a)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r^2 - ra - 2r^2 + 2a^2 + r^2 + ra}{r(r^2 - a^2)}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{2a^2}{r(r^2 - a^2)}\right]$$

$$= \frac{2qa^2}{4\pi\varepsilon_0 r^3\left(1 - \frac{a^2}{r^2}\right)}$$

Since , $\quad \frac{r}{a} >> 1$

$\therefore \quad \frac{a}{r} << 1$

$\frac{a^2}{r^2}$ is taken as negligible.

$\therefore$ $$V = \frac{2qa^2}{4\pi\varepsilon_0 r^3}$$

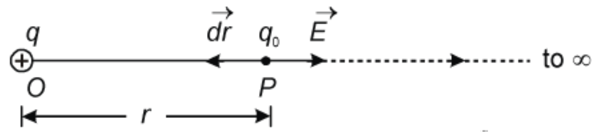
It can be inferred that potential, $V \propto \frac{1}{r^3}$

However, it is known that for a dipole, $V \propto \frac{1}{r^2}$

And, for a monopole, $V \propto \frac{1}{r}$.

S83. The electric potential (V) at a point is equal to the work done (W) in moving a point charge (q_0) from infinity to that point. That is,

$$V = \frac{W}{q_0}$$



The S.I. unit of potential is volt.

Let the point charged q be located at the origin O (see the above figure). Electric field at a point P at distance r from this point charge is

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

The field is directed away from the charge q. If a test charge q_0 is placed at P, it experiences a force $q_0\vec{E}$. Therefore, force $\vec{F} = -q_0\vec{E}$ has to be applied to move the test charge. Therefore, work done in moving the test charge from P through a small distance dr against the field $\vec{E}$ is

$$dW = F\,dr = -q_0 E\,dr$$

Total work done in moving the test charge q_0 from infinity to the point P is

$$W = \int_\infty^r dW = -q_0 \int_\infty^r E\,dr$$

$$= -q_0 \int_\infty^r \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}\,dr = -\frac{qq_0}{4\pi\varepsilon_0} \int_\infty^r \frac{1}{r^2}\,dr$$

or $$W = \frac{qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_\infty^r = \frac{qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

or $$W = \frac{qq_0}{4\pi\varepsilon_0} \cdot \frac{1}{r}$$

Therefore, the electric potential at *r* or the work done in bringing a unit positive test charge from infinity to point P is

$$V = \frac{W}{q_0} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

This potential is spherically symmetric because its value is same at all points on a spherical surface having its centre on the point charge.

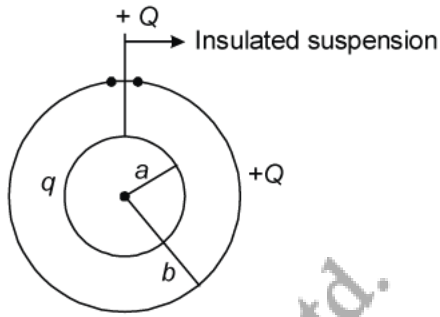
S84. Where,

q = charge on small sphere

a = radius of small sphere

Q = charge of outer shell

b = radius of outer shell



Let small sphere of charge q and radius a is placed inside a outer shell of charge + Q and radius b.

Electric potential on the small sphere due to its own charge q.

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{a} \quad \text{... (i)}$$

Similarly, electric potential on outer sphere due to its own charge

$$V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{b} \quad \text{... (ii)}$$

Also, same potential V_2 existat every point inside outer shell, due to its own charge, + Q. Now, net electric potential at inner sphere of radius a.

V_i = Electric potential due to its own charge

$$V_i = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{a} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{b} \quad \text{... (iii)}$$

Net electric potential at outer sphere, due to charge on the both spheres

$$V_o = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{b} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{b} \quad \text{... (iv)}$$

$$\therefore \quad V_i - V_o = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) \quad \text{[From Eq (iii) and (iv)]} \quad \text{... (v)}$$

$$\because \quad a < b \quad \text{hence} \quad \frac{1}{a} > \frac{1}{b}$$

$$\Rightarrow \quad V_i - V_o > 0$$

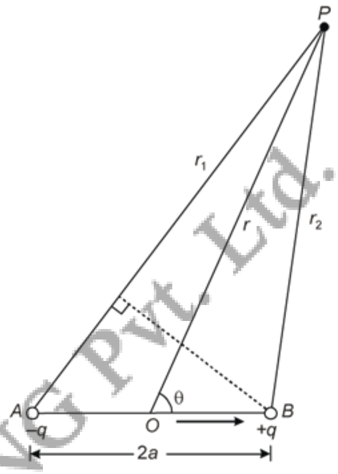
Thus, inner sphere has net potential higher than potential of outer sphere for every value of q and Q. Therefore, when they are connected by a wire, positive charge always flow from inner sphere (at lower potential) irrespective of the magnitude of charge.

S85. The electric potential (V) at a point is equal to the work done (W) in moving a point charge (q_0) from infinity to that point. That is,

$$V = \frac{W}{q_0}$$

Let us consider an electric dipole consisting of two point charges $-q$ and $+q$ separated by a distance $2a$, as shown in the given figure. We intend to calculate its potential at a point P at a distance r from the centre O of the dipole making an angle θ with the dipole moment $\vec{p}$. Let $AP = r_1$, and $BP = r_2$. The dipole has a dipole moment given by

$$\vec{p} = q \times 2\vec{a}$$



Net potential at a point P due to the dipole is

$$V = \frac{q}{4\pi\varepsilon_0 r_2} - \frac{q}{4\pi\varepsilon_0 r_1}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] = \frac{q}{4\pi\varepsilon_0}\left[\frac{r_1 - r_2}{r_1 r_2}\right]$$

When the point P lies far away from the dipole

$$r_2 - r_1 \simeq AB\cos\theta = 2a\cos\theta$$

and $\quad r_1 r_2 \simeq r_2$

$\therefore \quad V = \frac{q}{4\pi\varepsilon_0}\frac{2a\cos\theta}{r^2} = \frac{2aq\cos\theta}{4\pi\varepsilon_0 r^2}$

or $\quad V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}$

For the points lying on the perpendicular bisector of the dipole, $\theta = 90°$, $\cos 90° = 0$

$\therefore \quad V = 0$

i.e., the potential at any point on the perpendicular bisector of the dipole is zero.

S86. (a) As $\quad V \propto \frac{1}{r}$, $V_P > V_Q$ and $V_B > V_A$

Thus, $\quad V_P - V_Q$ is +ve

$V_B - V_A$ is +ve

(b) Negative charge moves from Q to P *i.e.*, from higher potential energy to lower potential energy side. Therefore,

$$(P.E.)_Q > (P.E.)_P > 0$$

Similarly, $(P.E.)_A > (P.E.)_B$

i.e., $(P.E.)_Q > (P.E.)_B > 0$

(c) In order to move a small positive charge from Q to P, work has to be done by an external agency against the electric field. So, the work done by the field is negative.

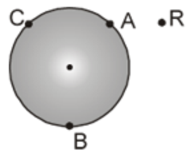
(d) Work is done by an external agency in moving a small negative charge from B to A. Therefore it is positive.

(e) Due to force of repulsion, velocity of the negative charge decreases. Hence K.E. decreases in going from B to A.

Date: 20/10/2021

Q1. Define electron-volt.

Q2. A charge q_0 has been taken from R to A, R to B and R to C (figure given below). In which condition work done is maximum?



Q3. A positive charge $+q$ is located at a point. What is the work done if a unit positive charge is carried once around this charge along a circle of radius r about this point.

Q4. A bird perches on a bare high power line, and nothing happens to the bird. A man standing on the ground touches the same line and gets a fatal shock. Why?

Q5. Vehicles carrying inflammable materials usually have metallic ropes touching the ground during motion. Why?

Q6. A comb run through one's dry hair attracts small bits of paper. Why?

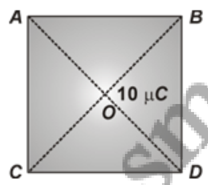
What happens if the hair is wet or if it is a rainy day? (Remember, a paper does not conduct electricity.)

Q7. Ordinary rubber is an insulator. But special rubber tyres of aircraft are made slightly conducting. Why is this necessary?

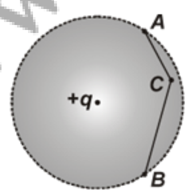
Q8. A 500 μC charge is at the centre of a square of side 10 cm. Find the work done in moving a charge of 10 μC between two diagonally opposite points on the square.

Q9. How much work is done in moving a 500 μC charge between two points on an equipotential surface?

Q10. What is the work done in moving a 2 μC Point charge from corner A to corner B of square $ABCD$ as shown in the given, When a 10 μC charge exists at the centre of the square?



Q11. If a point charge $+q$ is taken first from A to C and then from C to B of a circle drawn with another point charge $+q$ as centre as shown in the figure, then along which path more work will be done?



Q12. In the nucleus of $_{92}U^{238}$, two protons are at a distance of 6×10^{-15}m. Calculate their electrostatic potential energy.

Q13. The potential at a point 0.1 m from an isolated point charge is +100 volt. Find the nature and magnitude of the point charge.

Q14. When is the potential energy of an electric dipole maximum, when placed in uniform electric field?

Q15. A molecule of a substance has a permanent electric dipole moment of magnitude 10^{-29} C m. A mole of this substance is polarised (at low temperature) by applying a strong electrostatic field of magnitude 10^{6} V m^{-1}. The direction of the field is suddenly changed by an angle of 60°. Estimate the heat released by the substance in aligning its dipoles along the new direction of the field. For simplicity, assume 100% polarisation of the sample.

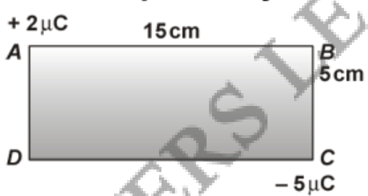
Q16. If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion H_2^+. In the ground state of an H_2^+, the two protons are separated by roughly 1.5 Å, and the electron is roughly 1 Å from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.

Q17. An electric dipole consists of two opposite charges each of 1 μC separated by 2 cm. The dipole is placed in an external uniform field of 10^{-5} NC^{-1} intensity. Find (a) maximum torque exerted by the field on the dipole and (b) the work done in rotating the dipole through 180° starting from the position $\theta = 0°$.

Q18. Two point charges equal to + 10 μC and + 20 μC are 1 m apart. What is the amount of work done to bring them closer to each other by 50 cm.

Q19. The potential difference between two point charges at 0.05 V. If the points are 4 cm apart, what is the component of the electric field parallel to the line joining the points *A* and *B*.

Q20. The sides of a rectangle *ABCD* are 15 cm and 5 cm. Two point charges of + 2 μC and – 5 μC are placed at the corners *A* and *C* respectively as shown in the figure below.



Calculate the work done in carrying a charge of 3 μC from point *B* to *D*.

Q21. What do you mean by potential energy of an electric dipole, when placed in electric field?

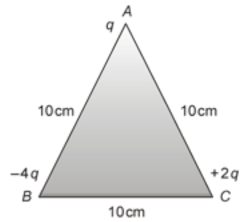
Q22. A positive charge + *q* is located at a point. What is work done, if a unit positive charge is carried once around this charge along a circle of radius *r* about this point?

Q23. Three charges – *q*, *Q* and – *q* are placed at equal distances on a straight line. If the potential energy of the system of three charges is zero, then what is the ratio of *Q* : *q*?

Q24. Three points charges *q* , 2*q* and *Q* are placed at the three vertices of an equilateral triangle. Find the value of charge *Q* (in terms of *q*), so that electric potential energy of the system is zero.

Q25. Two metallic spheres of same radii, one solid and the other hollow are charged to the same potential. Which of the two will hold more charge? Give reasons.

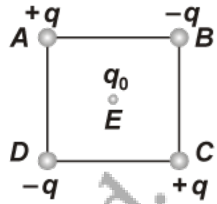
Q26. Calculate the work done to dissociate the system of three charges ($q = 1.6 \times 10^{-10}$ C) placed on the vertices of a triangle as shown in the fig.



Q27. Two electrons are moving towards each other, each with a velocity of 10^6 ms^{-1}. What will be closest distance of approach between them?

Q28. A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-9} C from a point P (0, 0, 3 cm) to a point Q (0, 4 cm, 0), via a point R (0, 6 cm, 9 cm).

Q29. Four charges are arranged at the corners of a square $ABCD$ of side d, as shown in figure. (a) Find the work required to put together this arrangement. (b) A charge q_0 is brought to the centre E of the square, the four charges being held fixed at its corners. How much extra work is needed to do this?



Q30. In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 Å:

(a) Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton.

(b) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (a)?

(c) What are the answers to (a) and (b) above if the zero of potential energy is taken at 1.06 Å separation?

Q31. (a) Depict the equipotential surfaces for a system of two identical positive point charges placed at a distance 'd' apart.

(b) Deduce the expression for the potential energy of a system of two point charges q_1 and q_2 brought from infinity to the points $\vec{r}_1$ and $\vec{r}_2$ respectively in the presence of external electric field $\vec{E}$.

Q32. A capacitor of 200 pF is charged by a 300 V battery. The battery is then disconnected and the charged capacitor is connected to another unchanged capacitor of 100 pF. Calculate the difference between the final energy stored in the combined system and the initial energy stored in the single capacitor.

Q33. Derive the expression for the energy stored in a parallel plate capacitor with air between the plates. How does the stored energy change if air is replaced by medium of dielectric constant K?

Q34. A capacitor has some dielectric between its plates, and the capacitor is connected to a DC source. The battery is now disconnected and then the dielectric is removed. State whether the capacitance, the energy stored in it, electric field, charge stored and the voltage will increase, decrease or remain constant.

Q35. A parallel plate capacitor of plate separation 'd' is charged to a potential difference ΔV. A dielectric slab of thickness 'd' and dielectric constant 'K' is introduced between the plates while the battery remains connected to the plates.

(a) Find the ratio of energy stored in the capacitor after and before the dielectric is introduced. Given the physical explanation for this change in stored energy.

(b) What happens to the charge on the capacitor?

Q36. What is potential Energy? Derive the potential energy of a system of two point charges.

Q37. Find out the expression for the potential energy of a system of three point charges q_1, q_2, and q_3 located at $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$ w.r.t the common origin *O*.

Date: 20/10/2021

S1. Electron-volt is the unit of energy. The amount of energy that would be imparted to an electron on being accelerated through a potential difference of one volt is called electron-volt.

S2. Work done will be same in all cases because *A*, *B*, *C* are on an equipotential surface.

S3. No work is done because potential difference is zero between the final and the initial points while

Work done = charge × potential difference.

S4. This is because bird is not in contact with the earth and the circuit is not complete however the circuit is complete in case of man.

S5. Reason similar to (b).

S6. This is because the comb gets charged by friction. The molecules in the paper gets polarised by the charged comb, resulting in a net force of attraction. If the hair is wet, or if it is rainy day, friction between hair and the comb reduces. The comb does not get charged and thus it will not attract small bits of paper.

S7. To enable them to conduct charge (produced by friction) to the ground; as too much of static electricity accumulated may result in spark and result in fire.

S8. $W = q \times$ Pot. difference between the two points $= q \times 0 =$ **0 J**

S9. Work done = charge × p.d

Work done in moving a charge of 500 μC between two points on an equipotential surface is *W* and p.d is zero, then

$$W = 500\ \mu C \times 0 = \mathbf{0\ J}$$

S10. The points *A* and *B* are at the same distance from 10 μC charge. Since $V_A = V_B$, no work will be done in moving a 2 μC charge from point *A* to *B*.

S11. $V_C - V_A = V_C - V_B$

Hence, work done in taking a point charge from *A* to *C* or from *C* to *B* will be the same.

S12.

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}$$

$$= \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{6 \times 10^{-15}}$$

$$= \mathbf{3.84 \times 10^{-14}\ J}$$

S13. Now $$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

or $$q = 4\,\pi\,\varepsilon_0\, r\, V$$

$$= \frac{0.1 \times 100}{9 \times 10^9} = \mathbf{1.11 \times 10^{-9}\,C\ (positive)}$$

S14. The potential energy of an electric dipole is maximum, when it is aligned antiparallel to electric field.

S15. Here, dipole moment of each molecules = 10^{-29} C m

As 1 mole of the substance contains 6×10^{23} molecules, total dipole moment of all the molecules,

$$p = 6 \times 10^{23} \times 10^{-29}\ \text{C m}$$
$$= 6 \times 10^{-29}\ \text{C m}$$

Initial potential energy, $U_i = -pE\cos\theta = -6 \times 10^{-6} \times 10^{6} \cos 0° = -6\ \text{J}$

Final potential energy (when $\theta = 60°$),

$$U_f = -6 \times 10^{-6} \times 10^{6} \cos 60° = -3\ \text{J}$$

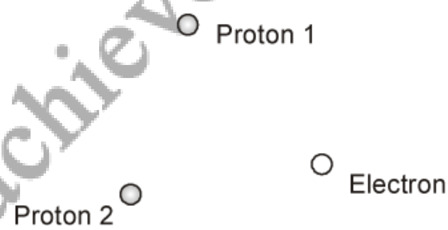
Change in potential energy = – 3 J – (– 6 J) = 3 J

So, there is loss in potential energy. This must be the energy released by the substance in the form of heat in aligning its dipoles.

S16. The system of two protons and one electron is represented in the given figure.

Charge on proton 1, $q_1 = 1.6 \times 10^{-19}$ C

Charge on proton 2, $q_2 = 1.6 \times 10^{-19}$ C



Charge on electron, $q_3 = -1.6 \times 10^{-19}$ C

Distance between protons 1 and 2,

$$d_1 = 1.5 \times 10^{-10}\ \text{m}$$

Distance between proton 1 and electron,

$$d_2 = 1 \times 10^{-10}\ \text{m}$$

Distance between proton 2 and electron,

$$d_3 = 1 \times 10^{-10}\ \text{m}$$

The potential energy at infinity is zero.

Potential energy of the system, $$V = \frac{q_1 q_2}{4\pi\varepsilon_0 d_1} + \frac{q_2 q_3}{4\pi\varepsilon_0 d_3} + \frac{q_3 q_1}{4\pi\varepsilon_0 d_2}$$

Substituting, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{Nm}^2\text{C}^{-2}$, we obtain

$$V = \frac{9\times10^{9}\times10^{-19}\times10^{-19}}{10^{-10}}\left[-(16)^2+\frac{(1.6)^2}{1.5}+-(1.6)^2\right]$$

$$= -30.7\times10^{-19}\,\text{J} = \mathbf{-19.2\,eV} \qquad [1\,\text{J} = 1.6\times10^{-19}\,\text{eV}]$$

Therefore, the potential energy of the system is – 19.2 eV.

S17. Given: $E = 10^{-5}\,\text{NC}^{-1}$; $q = 1\,\mu C = 10^{-6}\,C$; $2a = 2\,\text{cm} = 0.02\,\text{m}$

$\therefore$ $$p = q(2a) = 10^{-6}\times0.02 = 2\times10^{-8}\,\text{C m}$$

(a) The torque acting on a dipole is given by

$$\tau = p\,E\sin\theta$$

Torque is maximum, when $\sin\theta = 1$

$\therefore$ $$\tau = 2\times10^{-8}\times10^{5}\times1 = 0.002\,\text{Nm}$$

(b) Now, work done in rotating the dipole from $\theta = \theta_1$ to $\theta = \theta_2$ is given by

$$W = pE(\cos\theta_1 - \cos\theta_2)$$

Here, $\theta_1 = 0°, \theta_2 = 180°$

$\therefore$ $$W = 2\times10^{-8}\times10^{5}(\cos0° - \cos180°)$$

$$= 0.002(1-(-1)) = \mathbf{0.004\,J}$$

S18. Given: $q_1 = 10\,\mu\text{C} = 10\times10^{-6}\text{C}$ and $q_2 = 20\,\mu\text{C} = 20\times10^{-6}\,\text{C}$

Now, $$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r}$$

When at r = 1 m:

$$U_1 = \frac{9\times10^{9}\times10\times10^{-6}\times20\times10^{-6}}{1} = \mathbf{1.8\,J}$$

When brought closer by 50 cm *i.e.*, at $r = 1 - 0.5$ *i.e.*, 0.5 m:

$$U_2 = \frac{9\times10^{9}\times10\times10^{-6}\times20\times10^{-6}}{0.5} = \mathbf{3.6\,J}$$

Therefore, work done,

$$W = U_2 - U_1 = 3.6 - 1.8 = \mathbf{1.8\,J}$$

S19. Given: $dV = V_A - V_B = 0.05\,\text{V}$ and $dr = 4\,\text{cm} = 0.04\,\text{m}$

Now, $$E = \frac{dV}{dr} \quad \text{(in magnitude)}$$

$$= \frac{0.05}{0.04} = \mathbf{1.25\,Vm^{-1}}.$$

S20.

$$V_B = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q_A}{AB} + \frac{q_C}{CB}\right)$$

$$= 9 \times 10^9 \left(\frac{2 \times 10^{-6}}{0.15} + \frac{-5 \times 10^{-6}}{0.05}\right) = -78 \times 10^4\,\text{V}$$

$$V_D = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q_A}{AD} + \frac{q_C}{CD}\right)$$

$$= 9 \times 10^9 \left(\frac{2 \times 10^{-6}}{0.05} + \frac{-5 \times 10^{-6}}{0.15}\right) = +6 \times 10^4\,\text{V}$$

Work done to move the $3\,\mu$C charge from the point B to D,

$$W = (V_D - V_B) \times q$$

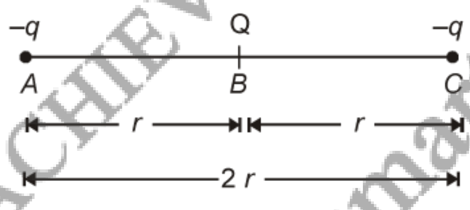
$$= [6 \times 10^4 - (-78 \times 10^4)] \times 3 \times 10^{-6} = \mathbf{2.52\,J}.$$

S21. An electric dipole always to orient itself along the direction of electric field. Work has to be done in rotating the dipole to some other orientation and this work done in rotating the dipole gets stored in the dipole in the form of its potential energy.

S22. The potential at each point on the circular path around the charge is same *i.e.*, potential difference between the initial and final position is zero.

Therefore, work done, $W = V \times q = 0 \times 10 =$ **0 J**

S23. Suppose that charges $-q$, Q and $-q$ are placed at points A, B and C on a straight line, such that $AB = BC = r$.



Then, $$\frac{1}{4\pi\varepsilon_0}\left[\frac{(-q)Q}{AB} + \frac{(-q)(-q)}{AC} + \frac{Q(-q)}{BC}\right] = 0$$

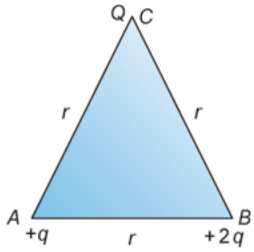
or $$-\frac{Q}{r} + \frac{q}{2r} - \frac{Q}{r} = 0$$

or $$\frac{2Q}{r} = \frac{q}{2r}$$

or $$\frac{Q}{q} = \frac{\mathbf{1}}{\mathbf{4}}.$$

S24. Suppose that charges $+q$, $+2q$ and Q are placed at the corners A, B and C of an equilateral ΔABC of side r. Then,

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{q(2q)}{r}+\frac{qQ}{r}+\frac{(2q)Q}{r}\right]=0$$



or $2q + Q + 2Q = 0$

or $Q = -\mathbf{2q/3\ C}$.

S25. Both the spheres will hold the same amount of charge. It is because, the two spheres possess equal capacitance. The capacitance of a sphere depends only on its radius. It does not matter, whether the sphere is hollow or solid.

S26. Given: $q_A = q$; $q_B = -4q$; $q_C = 2q$, where $q = 1.6 \times 10^{-10}$ C and $r = AB = BC = CA = 10$ cm $= 0.1$ m.

Work done to dissociate the system of three charges,

$$W = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_A q_B}{AB} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_B q_C}{BC} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_C q_A}{CA}$$

$$= \frac{1}{4\pi\varepsilon_0 r}\cdot[q_A q_B + q_B q_C + q_C q_A]$$

$$= \frac{1}{4\pi\varepsilon_0 r}\cdot[q\times(-4q) + (-4q)\times 2q + 2q\times q]$$

$$= \frac{1}{4\pi\varepsilon_0 r}\cdot[-4q^2 - 8q^2 + 2q^2] = -\frac{10q^2}{4\pi\varepsilon_0 r}$$

$$= -\frac{9\times10^9\times10\times(1.6\times10^{-10})^2}{0.1} = \mathbf{-2.304\times10^{-8}\ J}.$$

S27. The electrons will approach each other to a distance r_0 (closest distance), where their kinetic energy will become zero. It will appear as their potential energy. Therefore,

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e\times e}{r_0}$$

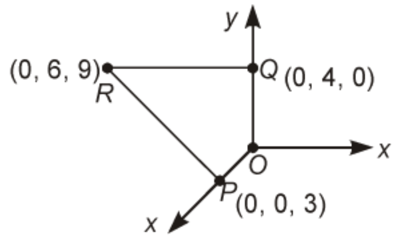
or $$r_0 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{\frac{1}{2}mv^2} = \frac{9\times10^9\times(1.6\times10^{-19})^2}{\frac{1}{2}(9.1\times10^{-31})\times(10^6)^2} = \mathbf{5.06\times10^{-10}\ m}.$$

S28. Charge located at the origin,

$$q = 8\text{ mC} = 8 \times 10^{-3}\text{ C}$$

Magnitude of a small charge, which is taken from a point P to point R to point Q,

$$q_1 = -2 \times 10^{-9}\text{ C}$$



All the points are represented in the given figure.

Point P is at a distance, $d_1 = 3$ cm, from the origin along z-axis.

Point Q is at a distance, $d_2 = 4$ cm, from the origin along y-axis.

Potential at point P, $\quad V_1 = \dfrac{q}{4\pi\varepsilon_0 d_1}$

Potential at point Q, $\quad V_2 = \dfrac{q}{4\pi\varepsilon_0 d_2}$

Work done (W) by the electrostatic force is independent of the path.

$$\therefore \quad W = q_1 [V_2 - V_1]$$

$$= q_1 \left[\frac{q}{4\pi\varepsilon_0 d_2} - \frac{q}{4\pi\varepsilon_0 d_1}\right]$$

$$= \frac{qq_1}{4\pi\varepsilon_0}\left[\frac{1}{d_2} - \frac{1}{d_1}\right] \quad \text{... (i)}$$

Where, $\quad \dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9\text{ Nm}^2\text{C}^{-2}$

$$\therefore \quad W = 9 \times 10^9 \times 8 \times 10^{-3} \times (-2 \times 10^{-9})\left[\frac{1}{0.04} - \frac{1}{0.03}\right]$$

$$= -144 \times 10^{-3} \times \left(\frac{-25}{3}\right)$$

$$= \mathbf{1.27\ J}$$

Therefore, work done during the process is 1.27 J.

S29. (a) Since the work done depends on the final arrangement of the charges, and not on how they are put together, we calculate work needed for one way of putting the charges at A, B, C and D. Suppose, first the charge $+q$ is brought to A, and then the charges $-q$, $+q$, and $-q$ are brought to B, C and D, respectively. The total work needed can be calculated in steps:

(i) Work needed to bring charge $+q$ to A when no charge is present elsewhere: this is zero.

(ii) Work needed to bring $-q$ to B when $+q$ is at A. This is given by (charge at B) $\times$ (electrostatic potential at B due to charge $+q$ at A)

$$= -q \times \left(\frac{q}{4\pi\varepsilon_0 d}\right) = -\frac{q^2}{4\pi\varepsilon_0 d}$$

(iii) Work needed to bring charge $+q$ to C when $+q$ is at A and $-q$ is at B. This is given by (charge at C) $\times$ (potential at C due to charges at A and B)

$$= +q\left(\frac{+q}{4\pi\varepsilon_0 d\sqrt{2}} + \frac{-q}{4\pi\varepsilon_0 d}\right)$$

$$= \frac{-q^2}{4\pi\varepsilon_0 d}\left(1 - \frac{1}{\sqrt{2}}\right)$$

(iv) Work needed to bring $-q$ to D when $+q$ at A, $-q$ at B, and $+q$ at C. This is given by (charge at D) $\times$ (potential at D due to charges at A, B and C)

$$= -q\left(\frac{+q}{4\pi\varepsilon_0 d} + \frac{-q}{4\pi\varepsilon_0 d\sqrt{2}} + \frac{q}{4\pi\varepsilon_0 d}\right)$$

$$= \frac{-q^2}{4\pi\varepsilon_0 d}\left(2 - \frac{1}{\sqrt{2}}\right)$$

Add the work done in steps (i), (ii), (iii) and (iv). The total work required is

$$= \frac{-q}{4\pi\varepsilon_0 d}\left\{(0) + (1) + \left(1 - \frac{1}{\sqrt{2}}\right) + \left(2 - \frac{1}{\sqrt{2}}\right)\right\}$$

$$= \frac{-q^2}{4\pi\varepsilon_0 d}(4 - \sqrt{2})$$

The work done depends only on the arrangement of the charges, and not how they are assembled. By definition, this is the total electrostatic energy of the charges.

(b) The extra work necessary to bring a charge q_0 to the point E when the four charges are at A, B, C and D is $q_0 \times$ (electrostatic potential at E due to the charges at A, B, C and D). The electrostatic potential at E is clearly zero since potential due to A and C is cancelled by that due to B and D. Hence no work is required to bring any charge to point E.

S30. The distance between electron-proton of a hydrogen atom, $d = 0.53\,\text{Å}$.

Charge on an electron, $q_1 = -1.6 \times 10^{-19}\,\text{C}$

Charge on a proton, $q_2 = +1.6 \times 10^{-19}\,\text{C}$

Potential at infinity is zero.

(a) Potential energy of the system, *p-e* = Potential energy at infinity – Potential energy at distance, *d*

$$= 0 - \frac{q_1 q_2}{4\pi\varepsilon_0 d}$$

Where, ε_0 is the permittivity of free space

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \, N\,m^2\,C^{-2}$$

$\therefore$ Potential energy = $\frac{-9 \times 10^9 \times (1.6 \times 10^{-19})^2}{0.53 \times 10^{10}} = -43.7 \times 10^{-19}$ J

Since 1.6×10^{-19} J = 1 eV.

$\therefore$ Potential energy = $-43.7 \times 10^{-19} = \frac{-43.7 \times 10^{-19}}{1.6 \times 10^{-19}} = -27.2$ eV.

Therefore, the potential energy of the system is – 27.2 eV.

(b) Kinetic energy is half of the magnitude of potential energy.

$$\text{Kinecit energy} = \frac{1}{2} \times (-27.2) = 13.6 \, \text{eV}$$

$$\text{Total energy} = 13.6 - 27.2 = -13.6 \, \text{eV}$$

Therefore, the minimum work required to free the electron is 13.6 eV.

(c) When zero of potential energy is taken, d_1 = 1.06 Å.

$\therefore$ Potential energy of the system = Potential energy at d_1 – Potential energy at d

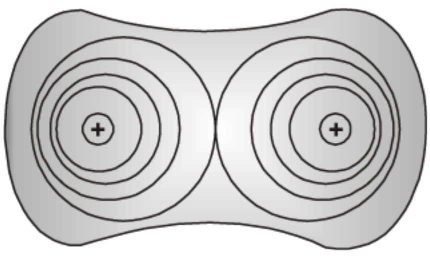
$$= \frac{q_1 q_2}{4\pi\varepsilon_0 d} - 27.2 \, \text{eV}$$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{1.06 \times 10^{-10}} - 27.2 \, \text{eV}$$

$$= 21.73 \times 10^{-19} \, \text{J} - 27.2 \, \text{eV}$$

$$= 13.58 \, \text{eV} - 27.2 \, \text{eV} = -13.6 \, \text{eV}.$$

S31. (a) Equipotential surface for a system of two identical positive point charges placed a distance '*d*' apart.



(b) Work done in bringing the charge q_1 from infinity to $\vec{r_1}$. Against the external electric field

$$W_1 = q_1 V(r_1) \quad \text{... (i)}$$

Work done in bringing the charge q_2 from infinity to $\vec{r_2}$.

$$W_2 = q_2 V(r_2) \quad \text{... (ii)}$$

Work done on q_2 against the field due q_1

$$W_3 = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}} \quad \text{... (iii)}$$

Add the Eq. (i) (ii) and (iii) we get

Potential energy of the system = $W_1 + W_2 + W_3$

Potential energy of the system

$$= q_1 V(r_1) + q_2 V(r_2) + \frac{q_1 q_2}{4\pi \epsilon_0 r_{12}}$$

S32. $C_1 = 200\ pF = 200 \times 10^{-12}\ F$

$V_1 = 3 \times 10^2\ V$

Initial energy,

$$E_{in} = \frac{1}{2} C_1 V_1^2 \quad E_{in} = \frac{1}{2}(200 \times 10^{-12}) \times 9 \times 10^4$$

$$= 9 \times 10^{-6}\ J$$

Common potential attained by the combination

$$V_2 = \frac{C_1 V_1}{C_1 + C_2} = 200\ V = 2 \times 10^2\ V$$

$$C_2 = 100\ pF = 100 \times 10^{-12}\ F$$

Final energy of the combination

$$E_f = \frac{1}{2}(C_1 + C_2) V_2^2 = \frac{1}{2} \times 300 \times 10^{-12} \times 4 \times 10^4$$

$$= 6 \times 10^{-6}\ J$$

Difference in energy

$$\Delta E = E_{in} - E_f$$
$$= 9 \times 10^{-6} - 6 \times 10^{-6}$$
$$= 3 \times 10^{-6}\ J$$

S33. Consider a capacitor of capacitance C. Let it be charge gradually. At any instant time if the charge on the capacitor is q.

$$V = \frac{q}{C}$$

To add further charge dq to the condenser amount of work done is $dW = \frac{q}{C} \times dq$

Total amount of work done in giving a charge Q to the condenser

$$W = \int_{q=0}^{q=Q} \frac{q}{C}\, dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q = \frac{1}{2}\frac{Q^2}{C}$$

$\therefore$ $Q = CV$

$$W = \frac{1}{2}CV^2$$

This work done gets stored in the condenser. Thus, energy stored in a parallel plate capacitor is

$$U = \frac{1}{2}CV^2$$

On replacing air with medium of dielectric constant K capacitance increases to KC_o.

$$U_i = \frac{1}{2}\frac{Q^2}{C_o}$$

Charge on the capacitor remain same

Thus, $U_f = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{KC_o}$ $(\therefore\ C = KC_o)$

$$U_f = \frac{U_i}{K}$$

Energy get reduced by a factor K.

S34. C will decrease

Energy stored = $\frac{1}{2}CV^2$ and hence will increase.

Electric field will increase.

Charge stored will remain the same. V will increase.

S35. Capacity of a parallel plate capacitor is

$$C_0 = \frac{\varepsilon_0 A}{d}$$

On charging up to a potential ΔV, the charge stored by it is

$$Q = C_0\,\Delta V = \frac{\varepsilon_0 A}{d}\Delta V$$

On introducing dielectric slab of thickness 'd' and dielectric constant 'K' between the capacitor plates, new capacitance is

$$C = K\frac{\varepsilon_0 A}{d}$$

Charge stored by the capacitor is

$$Q' = C\Delta V = K\left(\frac{\varepsilon_0 A}{d}\Delta V\right) = KQ$$

(a)

$$E_i = \frac{1}{2}C_0V^2$$

$$E_f = \frac{1}{2}CV^2 = \frac{1}{2}(KC_0)V^2$$

$$= K\left(\frac{1}{2}C_0V^2\right)$$

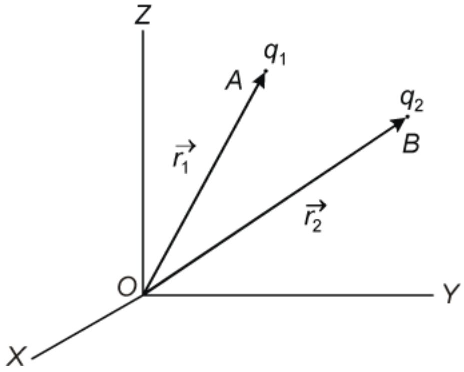
$$\frac{E_f}{E_i} = \frac{K}{1}$$

On introducing dielectric slab, capacitance of the capacitor increases. To charge the capacitor to same potential some more charge is supplied to it. The amount of work done to supply this extra charge to the capacitor is stored in the form of electrostatic energy.

(b) Charge also increases K times its initial value.

S36. The electrostatic potential energy of a system of point charges is defined as the work required to be done to bring the charges constituting the system to their respective locations from infinity.

Consider two point charges q_1 and q_2 lying at points A and B, whose locations are $\vec{r}_1$ and $\vec{r}_2$ respectively (see the given figure).



To calculate the electric potential energy of the two charges, remove the two charges to positions, such that they are at infinite distance from each other. First of all, bring the charge q_1 from infinity to its original position A. For this, no work is required. It is because, when charge q_1 is moved, no electrostatic force due to any other charge opposes it.

Now, move charge q_2 to its original position B. When charge q_2 is moved, the electric field due to the charge q_1 lying at point A, opposes it. Hence, work has to be done. The work done in moving charge q_2 from infinity to point B in the electric field of charge q_1 is given by

$$W = (\text{electric potential due to charge } q_1 \text{ at } B) \times q_2 = \left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{AB}\right) q_2$$

Since $AB = |\vec{r_1} - \vec{r_2}|$, we have

$$W = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|}$$

The work done in brining the two charges to their respective position is stored as the potential energy of the configuration of two charges *i.e.*,

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\vec{r_1} - \vec{r_2}|} \qquad \dots \text{(i)}$$

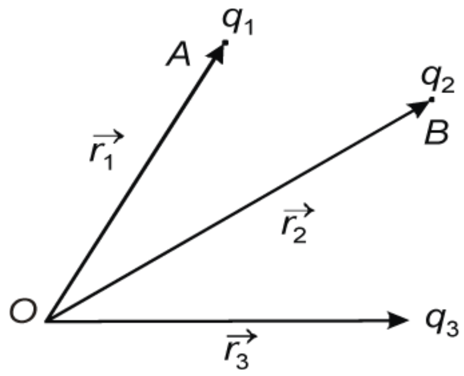
If distance $|\vec{r_1} - \vec{r_2}|$ is denoted as r_{12}, then

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}} \qquad \dots \text{(ii)}$$

The equations (i) and (ii) give electric potential energy of a system of two point charges.

S37. Consider that three point charges q_1, q_2 and q_3 are lying at locations $\vec{r_1}$, $\vec{r_2}$ and $\vec{r_3}$ respectively. Let us find the electric potential energy of the system of the three charges.

First of all, remove all the three charges to infinite distance from each other and then move the charge q_1 to its location $\vec{r_1}$. For this, no work has to be done.



Now, move the charge q_2 from infinity to its location $\vec{r}_2$.

$$W_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|} \quad \text{... (i)}$$

Now, move charge q_3 from infinity to its location $\vec{r}_3$. The work, say W_{13} has to be done for moving charge q_3 in the electric field of charge q_1 and work, say W_{23} has to be done for moving it in the electric field of charge q_2. The values of W_{13} and W_{23} are given by

$$W_{13} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_3}{|\vec{r}_1 - \vec{r}_3|} \quad \text{... (ii)}$$

and

$$W_{23} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2 q_3}{|\vec{r}_2 - \vec{r}_3|} \quad \text{... (iii)}$$

Adding Eqs (i), (ii) and (iii), we get

$$U = W_{12} + W_{13} + W_{23}$$

or

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_3}{|\vec{r}_1 - \vec{r}_3|} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2 q_3}{|\vec{r}_2 - \vec{r}_3|}$$

$$U = \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q_1 q_2}{|\vec{r}_1 - \vec{r}_2|} + \frac{q_1 q_3}{|\vec{r}_1 - \vec{r}_3|} + \frac{q_2 q_3}{|\vec{r}_2 - \vec{r}_3|} \right)$$

Date: 20/10/2021

Q1. What is a dielectric?

Q2. What are non-polar molecules and how are they polarised?

Q3. Distinguish between polar and non-polar dielectrics.

Q4. Define the term' dielectric constant' of a medium.

Q5. Define 'dielectric constant of a medium' in terms of force between electric charges.

Q6. The dielectric constant of a medium is unity. What will be its permittivity?

Q7. Write down the relation between dielectric constant and electric susceptibility.

Q8. Why does the electric field inside a dielectric decrease, when it is placed in an external electric field?

Q9. Define dielectric strength of a dielectric.

Q10. What is the function of a dielectric in a capacitor?

Q11. A parallel plate capacitor of capacitance *C* is charged to a potential difference *V* and then the battery's disconnected. Now a dielectric slab of the dimensions equal to the spacing between the plates is inserted between the plates. What are the changes, if any, in the capacitance, potential difference, charge, electric field and the energy stored?

Q12. A parallel plate capacitor has two plates of sides 0.055 m and 0.04 m of air. Their distance apart is 0.7 mm. The dielectric constant of the medium in between is 4. Find the capacitance of the capacitor.

Q13. A parallel plate capacitor has plates of area 0.02 m^2 and separation between the plates 1 mm. What potential difference will be developed, if a charge of 1 *n*C is given to the capacitor? If the plate separation is now increased to 2 mm. What will be the new potential difference?

Q14. A parallel plate capacitor having plate area 50 cm^2 and separation 1.0 mm holds a charge of 0.8 μC when connected to a 120 V battery. Find the dielectric constant of the material filling the gap.

Q15. The two plates of a parallel plate capacitor are 4 mm apart. A slab of dielectric constant 3 and thickness 3 mm is introduced between the plates with its faces parallel to them. The distance between the plates is so adjusted that the capacitance of the capacitor becomes (2/3)rd of its original value. What is the new distance between the plates?

Q16. (a) How is the electric field due to a charged parallel plate capacitor affected when a dielectric slab is inserted between the plates fully occupying the intervening region?

(b) A slab of material of dielectric constant *K* has the same area as the plates of a parallel plate capacitor but has thickness $\frac{1}{2}d$, where *d* is the separation between the plates. Find the expression for the capacitance when the slab is inserted between the plates.

Q17. A parallel plate capacitor is charged by a battery. After sometime the battery is disconnected and a dielectric slab with its thickness equal to the plate separation is inserted between the plates. How will

(a) the capacitances of the capacitor

(b) new Potential of the Capacitor

(c) the energy stored in the capacitors

Justify your answer in each case.

Q18. A parallel plate capacitor, each with plate area *A* and separation *d*, is charged to a potential difference *V*. The battery used to charge it is then disconnected. A dielectric slab of thickness *d* and dielectric constant *K* is now placed between the plates. What change, if any will take place in

(a) electric field intensity between the plates

(b) capacitance of the capacitor?

Justify your answer in each case.

Q19. (a) Definite the polarisation. Show that polarisation of a dielectric is equal to the induced surface charge density.

(b) A parallel plate capacitor is made of 201 plates separated by parafined paper 0.001 cm thick of relative permittivity 2.5. The effective size of each plate is 15 × 30 cm. What is the capacitance of this capacitor? $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^2$.

Q20. (a) What is a dielectrics? Describe what happens physically when a dielectric is placed in a uniform electric field. Hence define dielectric constant.

(b) Two metal plates form a parallel plate capacitor. The distance between the plates is *d*. A metal plate of thickness *d*/2 and of same area is inserted completely between the plates. What is the ratio of the capacitances in the two cases?

Q21. Explain the phenomenon of polarisation in dielectric with a suitable diagram. Obtain expression for polarisation vector.

Q22. Define the terms

(a) capacitance of a capacitor

(b) dielectric strength of a dielectric

(c) When a dielectric is inserted between the plates of a charged parallel plate capacitor, fully occupying the intervening region, how does the polarization of the dielectric medium affect the net electric field? For linear dielectric, show that the introduction of the dielectric increases its capacitance by a factor *K*, which is characteristic of the dielectric.

Q23. Define the electric susceptibility.
Show that dielectric constant and electric susceptibility is related by the equation $K = 1 + \chi$.

Date: 20/10/2021

S1. A material that does not conduct electricity but on applying electric field, induced charges are produced on its faces.

S2. The molecules, which do not possess permanent electric dipole moment, are called non-polar molecules.

When a non-polar molecule is subjected to an electric field, the positive nucleus gets pulled along the direction of electric field, while electrons get pulled in the opposite direction. As a result of stretching of the molecule, centre of gravity of positive nucleus does not coincide with that of negative electrons. Due to this, the molecule acquires a dipole moment and is said to be polarised.

S3. A dielectric, the molecules of which possess electric dipole moment even when electric field is not applied, is called polar dielectric and a dielectric, the molecules of which do not possess electric dipole movement are called non polar dielectrics.

S4. The dielectric constant (or specific inductive capacity) of a material is the ratio of the capacitance of a given capacitor completely filled with that material to the capacitance of the same capacitor in vacuum.

S5. $K = \frac{F'}{F}$ where F = Force between two charges with vacuum

F' = Force between tow charges with dielectric

S6. Permittivity is equal to $\varepsilon_0 = 8.86 \times 10^{-12}$

Since permittivity is constant, relative permittivity is constant, relative permittivity is constant, relative permittivity changes due to medium.

S7. The dielectric constant (K) and susceptibility (χ) of a dielectric are related to each other by the relation

$$K = 1 + \chi$$

S8. When a dielectric is placed in an external electric field, the electric field due to polarisation of the dielectric opposes it. Hence, the electric field gets reduced.

S9. The dielectric strength of a dielectric is the maximum value of applied electric field required to just break down of the dielectric material.

S10. When a dielectric slab is introduced between the two plates of a capacitor, the electric field between the plates gets reduced due to polarisation of the dielectric. The reduced value of the electric field is equivalent to a decreased value of potential difference between the plates. In order to make the potential difference again the same, more charge has to be given to the capacitor. *i.e.*, the capacitance of the capacitor increases.

S11. Let K be dielectric constant of the slab and q, E and U be charge on the plates of the capacitor, electric field between the plates and energy stored in the capacitor before inserting the slab.

On Inserting Dielectric Slab:

The capacitance of capacitor will become,

$$C' = \boldsymbol{KC} \qquad \textbf{(increases)}$$

Potential difference between the plates,

$$V' = \frac{q}{C'} = \frac{q}{KC} = \frac{\boldsymbol{V}}{\boldsymbol{K}} \qquad \textbf{(decreases)}$$

Charge on capacitor,

$$Q' = C'V' = KC \times \frac{V}{K} = CV = \boldsymbol{Q} \qquad \textbf{(unaffected)}$$

Electric field between the plates,

$$E' = \frac{V'}{d} = \left(\frac{V}{K}\right) \times \frac{1}{d} = \frac{\boldsymbol{E}}{\boldsymbol{K}} \qquad \textbf{(decreases)}$$

Energy stored in the capacitor,

$$U' = \frac{1}{2}C'V'^2 = \frac{1}{2}(KC)\left(\frac{V}{K}\right)^2 = \frac{1}{2}\left(\frac{1}{2}CV^2\right) = \frac{\boldsymbol{U}}{\boldsymbol{K}} \qquad \textbf{(decreases)}$$

S12. Given:

$$C = \frac{\varepsilon_0 K A}{d}$$

Here, $K = 4$, $A = 0.055 \times 0.04\,\text{m}^2$,

$d = 0.7\,\text{mm} = 0.7 \times 10^{-3}\,\text{m}$

$$\therefore \quad C = \frac{8.854 \times 10^{-12} \times 4 \times 0.055 \times 0.04}{0.7 \times 10^{-3}} = \mathbf{1.1 \times 10^{-10}\,F}.$$

S13.

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 0.02}{1 \times 10^{-3}} = 1.77 \times 10^{-10}\,\text{F}$$

Now,

$$V = \frac{q}{C} = \frac{10^{-9}}{1.77 \times 10^{-10}} = \mathbf{5.65\,V}.$$

On increasing the separation:

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 0.02}{2 \times 10^{-3}} = 8.854 \times 10^{-11}\,\text{F}$$

Now, $V = \frac{q}{C} = \frac{10^{-9}}{8.854 \times 10^{-11}} = \mathbf{11.3\,V}.$

S14. The capacitance of the capacitor is

$$C = \frac{Q}{V} = \frac{0.8\,\mu C}{120 V} = 6.66 \times 10^{-9}\,F$$

If ε_r is the dielectric constant, the capacitance is also equal to $\frac{\varepsilon_r \varepsilon_0 A}{d}$

$\therefore$ $\varepsilon_r = \frac{Cd}{\varepsilon_0 A}$

Here, $A = 50\text{ cm}^2 = 50 \times 10^{-4}\text{ m}^2$

$d = 1.00\text{ mm} = 1.0 \times 10^{-3}\text{ m}$

and taking $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, we have

$$\therefore \quad \varepsilon_r = \frac{6.66 \times 10^{-9} F \times 1.0 \times 10^{-3} m}{8.85 \times 10^{-12} F/m \times 50 \times 10^{-4} m^2}$$

$$= 150.5$$

S15. Before introduction of slab

$$d_1 = 4\text{ mm} = 4 \times 10^{-3}\text{ m}$$

Plate area = A (Say)

$\therefore$ Capacitance $C_1 = \frac{\varepsilon_0 A}{d_1}$

Suppose after introduction of slab, the new distance between the place is d_2.

Plate area of each plate = A

$K = 3$

Thickness of slab $t = 3\text{ mm} = 3 \times 10^{-3}\text{ m}$

Capacitance $C_2 = \frac{\varepsilon_0 A}{(d_2 - t + t/K)}$

According to the question,

$$\frac{2}{3} C_1 = C_2$$

$$\frac{2}{3} \cdot \frac{\varepsilon_0 A}{d_1} = \frac{\varepsilon_0 A}{d_2 - t + t/K}$$

Substituting the values (Given in question)

$$\frac{2}{3 \times 4 \times 10^{-3}} = \frac{1}{(d_2 - 3 + 3/3) \times 10^{-3}} \qquad [d_2 \text{ is in mm}]$$

$$\frac{1}{6} = \frac{1}{(d_2 - 2)}$$

$\Rightarrow$ $$d_2 - 2 = 6 \quad \Rightarrow \quad d_2 = 8 \text{ mm}$$

S16. (a) The total charge of the capacitor remain conserved on introduction of dielectric slab.

Also, the capacitance of capacitor increases to K times of original values.

$\therefore$ $$CV = C'V'$$

$$CV = (KC)V' \quad \Rightarrow \quad V' = \frac{V}{K}$$

$\therefore$ New electric field

$$E' = \frac{V'}{d} = \left(\frac{V/K}{d}\right) = \left(\frac{V}{d}\right)\frac{1}{K}$$

$$E' = \frac{E}{K}$$

$\therefore$ On introduction of dielectric medium new electric field E' becomes $\frac{1}{K}$ times of its original value.

(b) $\therefore$ Capacitance of a parallel plate capacitor partially filled with dielectric medium is given by

$$C = \frac{\varepsilon_0 A}{(d - t + t/K)}$$

where, t is the thickness of dielectric medium.

Here, $$t = \frac{d}{2}$$

$$C = \frac{\varepsilon_0 A}{d - \frac{d}{2} + \frac{d}{2K}} = \frac{\varepsilon_0 A}{\frac{d}{2}\left(1 + \frac{1}{K}\right)}$$

$\therefore$ $$C = \frac{2\varepsilon_0 AK}{(K+1)d}$$

S17. On introduction of dielectric slab in isolated charged capacitor.

(a) The capacitance (C') becomes K times of original capacitor as

$$C = \frac{\varepsilon_0 A}{d}$$

and $$C' = \frac{K\varepsilon_0 A}{d} = KC$$

(b) $\because$ Charge remain conserved in the phenomenon.

$\therefore$ $$CV = C'V'$$

$$V' = \frac{CV}{C'} = \frac{CV}{KC}$$

$\Rightarrow$ $$V' = \frac{V}{K}$$

Potential difference decreases and become $\frac{1}{K}$ times of original value.

(c) Energy stored initially $U = \frac{q^2}{2C}$

Energy stored later

$\therefore$ $$U' = \frac{q^2}{2(KC)} \qquad [\because C' = KC]$$

where, K = dielectric constant of medium

$\Rightarrow$ $$U' = \frac{1}{K}\left(\frac{q^2}{2C}\right)$$

$\Rightarrow$ $$U' = \frac{1}{K}(U)$$

$$U' = \frac{1}{K} \times U$$

The energy stored in capacitor decrease and become $\frac{1}{K}$ times of original energy.

S18. (a) Charge is conserved

$\therefore$ $$C'V' = CV$$

$$V' = \frac{CV}{C'} = \frac{CV}{KC} = \frac{V}{K}$$

$$V' = \frac{V}{K}$$

Potential difference decreases and become $1/K$ times of original value.

$\because$ d is same

$\therefore$ $$E' = \frac{V'}{d} = \left(\frac{V/K}{d}\right) = \left(\frac{V}{d}\right)\frac{1}{K}$$

$$E' = \frac{E}{K}$$

(b) When a dielectric slab of dielectric constant K is introduced between the plates of charged capacitor, then

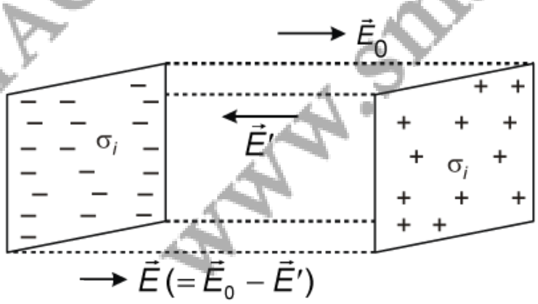
$$C = \frac{A\varepsilon_0}{d}$$

and $$C' = \frac{KA\varepsilon_0}{d}$$

$\therefore$ Capacitance $C' = KC$

S19. (a) Dielectric polarisation is the phenomenon in which the application of electric field results in the distortion of centres of positive and negative charges within each atom causing an induced electric-dipole moment. Dielectric, as a whole, remain neutral on getting polarised.

When dielectric gets polarised equal and opposite charge appear on the opposite sides. Therefore, for macroscopic purposes, uniformly polarised rectangular dielectric slab with uniform dielectric polarisation $\vec{P}$ behaves like, and thererfore can be replaced by, two surface charge densities (say) σ_i and $-\sigma_i$, at the two surfaces perpendicular to $\vec{P}$ and zero volume charge density inside the slab (see the given figure).



Let the dielectric slab be of thickness x and cross-sectional area A. If N is the number of atomic dipoles per unit volume, then the number atomic dipoles in the volume of dielectric slab is NAx. The induced charge in this volume is

$$Q = NAxq$$

where q is the charge on a single dipole.

Therefore, induced surface charge density is equal to

$$\sigma_i = \frac{\text{Charge}}{\text{Area}} = \frac{Q}{A} = Nxq$$

As, $\vec{P} = Nq\vec{x}$, therefore,

$$\sigma_i = P$$

or $\sigma_i = \vec{P}\,.\hat{n}$

where, $\hat{n}$ is a unit normal vector at the surface. Thus, the electric polarisation of a dielectric is equal to the induced surface charge density.

(b) In a parallel plate capacitor N plates make $(N - 1)$ individual capacitors. Hence No. of capacitors joined in parallel = 201 – 1 = 200

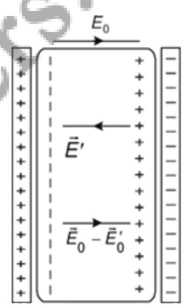
Now $\varepsilon_r = K = 2.5$, $d = 0.001$ cm $= 10^{-5}$ m; $A = 15 \times 30$ cm$^2 = 450 \times 10^{-4}$ m^2

$$C = \frac{K\varepsilon_0 A(N-1)}{d} = \frac{2.5 \times 8.85 \times 10^{-12} \times 450 \times 10^{-4} \times 200}{10^{-5}}$$

$$= 19.91 \times 10^{-3} \text{ F}$$

S20. (a) A material that does not conduct electricity but on applying electric field, induced charges are produced on its faces.

When a non-polar dielectric slab is placed in a uniform electric field E_0 of a parallel plate capacitor, each of its atom get polarised and becomes an individual dipole (see the given figure). The negative induced charges face the positively charged plate and the positive induced charges face the negatively charged. The net inside the slab is zero due to cancellation of positive and negative charges. There is a net negative charge towards the left of the slab and net positive charges on the right side of the



slab. These induced surface charges set up an electric field $\vec{E}$ in the dielectric in the opposite direction of the field due to parallel plates. The resultant field inside the dielectric is $\vec{E} = \vec{E}_0 - \vec{E}$, directed in the direction of the field $\vec{E}_0$.

The ratio of the original field to the resultant field $\vec{E}_0 - \vec{E}$ in the dielectric is called dielectric constant K of the slab, i.e.,

$$K = \frac{\vec{E}_0}{\vec{E}_0 - \vec{E}} = \frac{\vec{E}_0}{\vec{E}}$$

(b) If A is the area of each plate separated by a distance d, then with air as the intervening media,

Capacitance $$C = \frac{\varepsilon_0 A}{d} \quad \text{... (i)}$$

In now a metal plate of thickness $t < d$ and dielectric constant k is introduced, then

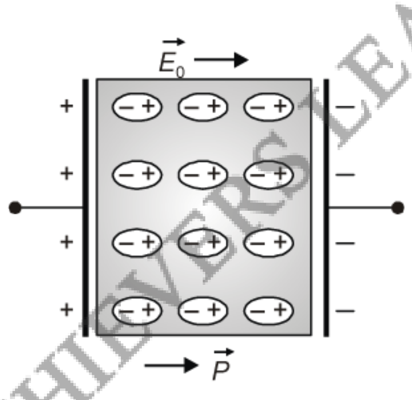
Capacitance $$C' = \frac{\varepsilon_0 A}{d - t + t/k} = \frac{\varepsilon_0 A}{d - d/2} \qquad \because \ K \text{ for metal is } \infty$$

or $$C' = \frac{2\varepsilon_0 A}{d}$$

$\therefore$ $$\frac{C'}{C} = \frac{2\varepsilon_0 A}{d} \times \frac{d}{\varepsilon_0 A} = \frac{2}{1}$$

or $$C' : C = 2 : 1$$

S21. When a non-polar dielectric is placed in an electric field, the electrons get pulled in a direction opposite to the applied electric field while the positively charged nucleus are pulled in the direction of the applied field. The separation between the charges stops when the force due to the external field is balanced by the restoring force due to internal fields in the molecule. The non-polar molecule thus develops an induced dipole moments and the dielectric is said to be **polarised.** The dielectric now has a net dipole moment.



A polarised dielectric slab

In case of polar dielectrics, which already have molecular dipole moment but randomly oriented, develops a net dipole moment due to alignment of individual dipole. Therefore, the dielectric develops a net dipole moment in the direction of the external field and it is said to be polarised. A polarised dielectric is shown in above figure.

Let the separation between the centres of positive and negative charges be x and $+q$ be the magnitudes of charges at these centres. Then the dipole moment of an atomic dipole

$$\vec{p} = q\vec{x} \quad \text{or} \quad p = qx$$

If N is the number atoms per unit volume, then dipole moment density (or dipole moment per unit volume) is given by

$$\vec{P} = N\vec{p} = Nq\vec{x} \quad \text{or} \quad P = Nqx$$

$\vec{P}$ is called electric polarisation vector or simply polarisation vector. Its unit is cm^{-2}. The magnitude of $\vec{P}$ indicates the extent to which the molecules of a dielectric becomes polarized by an electric field. The direction of $\vec{p}$ is same as that of individual atomic dipole moment $\vec{p}$. Thus, in a sense, $\vec{P}$ is a cumulative effect of individual atomic dipole moments due to external applied electric field.

S22. (a) For a given capacitor, the charge, q on the capacitor is proportional to the potential difference, V between the plates.

i.e., $$q \propto V$$

$$q = CV$$

The proportionality constant, C is called the capacitance of the capacitor.

It depends on the shape, size and geometry of capacitor.

(b) ***Dielectric strength:*** the dielectric strength of the dielectrics is equal to that maximum value of electric field that can exist in that dielectric without causing the dielectric breakdown of its insulating property. Its SI unit is V/m.

(c) ***Dielectric*** between the plates of capacitor.

As $$E_p \propto E_0$$

$\therefore$ $$E_p = \frac{1}{K} E_0$$

$\Rightarrow$ $$V = Ed = \frac{Qd}{\varepsilon_0 AK} = \frac{V_0}{K}$$

Thus capacitance is

$$C = \frac{Q}{\frac{V_0}{K}} = \frac{KQ}{V_0} = KC_0$$

$\Rightarrow$ $$\frac{C}{C_0} = K$$

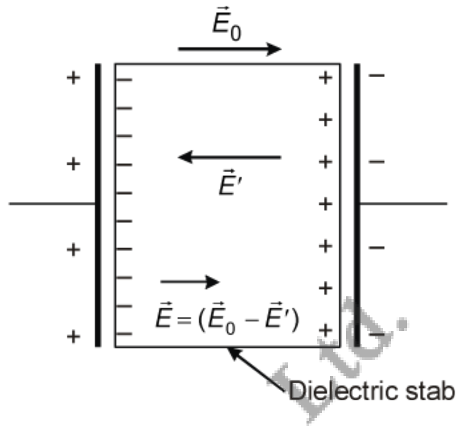
K is a factor (> 1) by which the capacitance is increased.

S23. The polarisation density of a dielectric slab is directly proportional to the reduced value of the electric field and may be expressed as

$$P = \chi \varepsilon_0 E$$

The constant of proportionality χ is called electric susceptibility of the dielectric slab. It is dimensionless constant.

Suppose a dielectric slab is placed in a uniform electric field $\vec{E}_0$. We assume that the dielectric completely fill the space between the plates. The dielectric gets polarised and charges ar induced on the opposite surfaces facing the plates. Let be the $\vec{E}'$ be the electric field produced by the induced surface charges. The induced electric field opposes the applied field $\vec{E}_0$ (see the given figure). Therefore, the resultant field $\vec{E}$, in the direction of $\vec{E}_0$ is given by



$$\vec{E} = \vec{E}_0 - \vec{E}'$$

or $$\vec{E} = \vec{E}_0 - \frac{\vec{P}}{\varepsilon_0} \qquad \left(\because\ \vec{E} = \frac{\vec{P}}{\varepsilon_0}\right)$$

Also, $$\vec{P} = \varepsilon_0 \chi \vec{E}$$

$\therefore$ $$\vec{E} = \vec{E}_0 - \chi \vec{E}$$

or $$\vec{E}_0 = (1 + \chi)\vec{E}$$

or $$\frac{\vec{E}_0}{\vec{E}} = 1 + \chi$$

We know, dielectric constant $K = \frac{\vec{E}_0}{\vec{E}}$

$\therefore$ $$K = 1 + \chi$$

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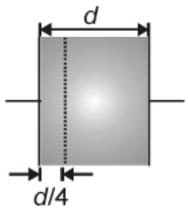
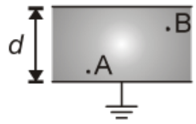
Q1. Two insulated charged sphere of radii 7 cm and 13 cm and having the same charge are connected by a conductor and then they are separated? Which of the two spheres will carry greater charge?

Q2. A capacitor of capacitance *C* has distance between plate *d*. A very thin mesh wire is placed as shown. Estimate new capacitance.



Q3. In a parallel plate capacitor, the capacitance increase from 6 μF to 60 μF, on introducing a dielectric medium between the plates. What is the dielectric constant of the medium?

Q4. In a parallel plate capacitor, the potential difference of 10^2 V is maintained between the plates. What will be the electric field at points A & B?



Q5. Define capacitance.

Q6. On inserting dielectric between the plates of a capacitor, its capacitance is found to increase 5 times. What is the relative permittivity of the dielectric?

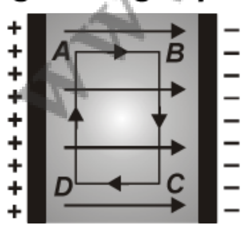
Q7. Define dielectric constant of a medium in terms of capacitance of a capacitor.

Q8. Can we give any desired amount of charge to a capacitor?

Q9. Sketch a graph to show how charge '*Q*' acquired by a capacitor of capacitance '*C*' varies with increase in potential difference between its plates.

Q10. Write the physical quantity which has as its unit coulomb volt^{-1}. Is it a vector or a scalar quantity?

Q11. A uniform electric field *E* exists between two charged plates as shown in the figure. What would be the work done in moving a charge '*q*' along the closed rectangular path *ABCDA*?



Q12. Two identical capacitor are joined parallel, charged to potential *V*, separated and then connected in series, the positive plate of one is connected to negative of other. What will effect on charges on the free plates?

Q13. The radius of circular plates of the parallel plates condenser is r. Air is there as dielectric. What should be distance between the plates if its capacitance is to be equal to that of an isolated sphere of radius r'?

Q14. A metal plate is introduced between the plates of a charged parallel plate capacitor. What is its effect on the capacitance of the capacitor?

Q15. A capacitor has been charged by a DC source. What are the the magnitude of conduction and displacement current, when it is fully charged?

Q16. Three capacitors each of capacitance 9 pF are connected in series.

(a) What is the total capacitance of the combination?

(b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

Q17. Explain what would happen if in the capacitor given in Exercise 2.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,

(a) While the voltage supply remained connected.

(b) After the supply was disconnected.

Q18. An electrical technician requires a capacitance of 2 μF in a circuit across a potential difference of 1 kV. A large number of 1 μF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors.

Q19. Two charged conducting spheres of radii *a* and *b* are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.

Q20. Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.

(a) What is the total capacitance of the combination?

(b) Determine the charge on each capacitor if the combination is connected to a 100 V supply.

Q21. A parallel plate capacitor with air between the plates has a capacitance of 8 pF ($1\text{ pF} = 10^{-12}\text{ F}$). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

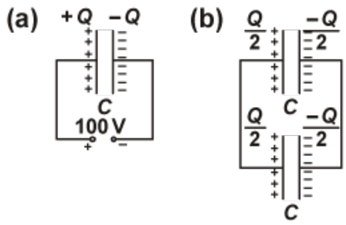
Q22. A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about 10^7 Vm^{-1}. (Dielectric strength is the maximum electric field a material can tolerate without breakdown, *i.e.*, without starting to conduct electricity through partial ionisation.) For safety, we should like the field never to exceed, say 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 pF?

Q23. A slab of material of dielectric constant *K* has the same area as the plates of a parallel-plate capacitor but has a thickness (3/4) *d*, where *d* is the separation of the plates. How is the capacitance changed when the slab is inserted between the plates?

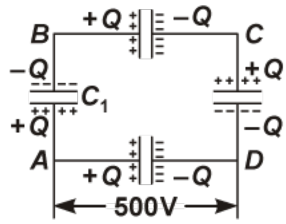
Q24. Calculate the capacitance of a spherical capacitor, if the diameter of inner sphere is 0.2 m and that of the outer sphere is 0.3 m, the space between them being filled with a liquid having dielectric constant 20.

Q25. (a) A 900 pF capacitor is charged by 100 V battery [figure (a)]. How much electrostatic energy is stored by the capacitor?

(b) The capacitor is disconnected from the battery and connected to another 900 pF capacitor [figure (b)]. What is the electrostatic energy stored by the system?

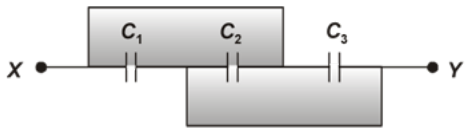


Q26. A network of four 10 μF capacitors is connected to a 500 V supply, as shown in figure. Determine (a) the equivalent capacitance of the network and (b) the charge on each capacitor. (Note, the *charge on a capacitor* is the charge on the plate with higher potential, equal and opposite to the charge on the plate with lower potential.)

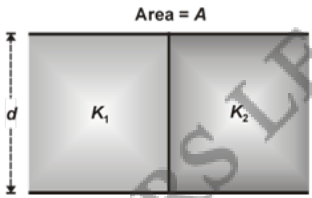


Q27. Capacitors *P*, *Q* and *R* have each a capacitance *C*. A battery can charge the capacitor *P* to a potential difference *V*. If after charging *P*, the battery is disconnected from it and the charged capacitor *P* is connected in following separate instances to *Q* and *R*: (a) to *Q* in parallel, and (b) to *R* in series, then what will be potential differences between the plates of *P* in the two instances?

Q28. Three capacitors C_1, C_2 and C_3 are connected as shown in figure. Find the equivalent capacitance of the combination between points *X* and *Y*.

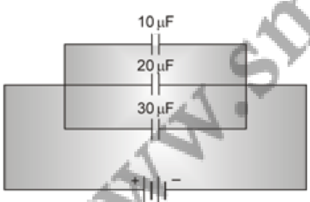


Q29. Two dielectric slabs of dielectric constant K_1 and K_2 are filled in between the two plates, each of area *A*, of the parallel plate capacitor as shown in the figure below. Find the capacitance of capacitor.



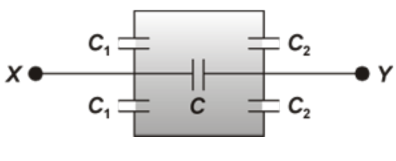
Q30. Find the ratio of the potential differences that must be applied across the parallel, series combination of two identical capacitors so that the energy stored in the two cases becomes the same.

Q31. Three capacitors of capacitances 10 μF, 20 μF and 30 μF are connected in parallel to a 100 V battery as shown in the figure. Calculate the total energy stored in the capacitors.

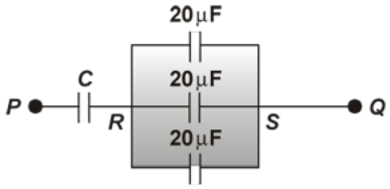


Q32. If there are *n* capacitors in parallel connected to *V* volt source, find the energy stored in the arrangement.

Q33. Find the equivalent capacitance of the system shown in the figure, between the points *X* and *Y*.



Q34. Calculate the capacitance of the capacitor C in the network as shown in the figure below.



The equivalent capacitance of the combination between P and Q is 30 μF.

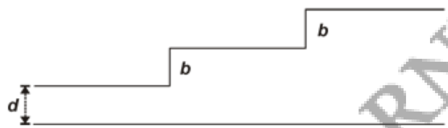
Q35. Three capacitors, each of capacitance 6 mF are connected together in series and are also connected in series with three capacitors of values 2 μF, 4 μF and 2 μF, which are grouped together in parallel. Calculate the total combined capacitance.

Q36. The capacitances of three capacitors are in the ratio 1 : 2 : 3. Their equivalent capacitance in parallel is greater than their equivalent capacitance in series by 60/11 μF. Calculate their individual capacitances.

Q37. At what distance should the two plates each of surface area 0.2 m × 0.1 m of an air capacitor be placed in order to have the same capacitance as a spherical conductor of radius 0.5 m?

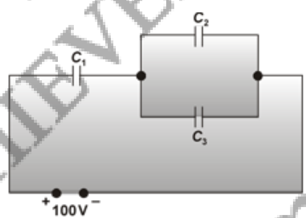
Q38. Twenty seven spherical droplets of radius 3 mm and each carrying 10^{-12} C of charge are combined to form a bigger drop. Find the capacitance of the bigger drop.

Q39. A capacitor is made of a flat plate of area A and a second plate having a stair-like structure as shown in the figure.



The height of first step of the stair is d, while those of subsequent steps is b as shown in the above figure.
If all the steps are of same width, calculate the capacitance of the structure.

Q40. The capacitors C_1, C_2 and C_3 are connected as shown in the figure, to a 100 V d.c. supply. If C_1 = 2 μF, C_2 = 3 μF and C_3 = 5 μF, find potential difference across each capacitor.



Q41. A capacitor, made of two parallel plates each of the plate A and separation d, is being charged by an external AC source. Show that the displacement current inside the capacitor is the same as the current changing the capacitor.

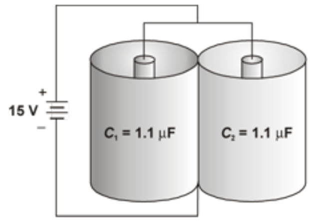
Q42. A slab of material of dielectric constant K has the same area as that of the plates of a parallel plate capacitor but has the thickness $d/2$, where d is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor.

Q43. What is the capacitance of arrangement of four plates of area *A* at distance *d* in air, as shown in figure.

Q44. How will you connect four capacitors, each of capacitance 1 μF, to obtain a net capacitance of 0.75 μF? Draw a diagram to show the combination.

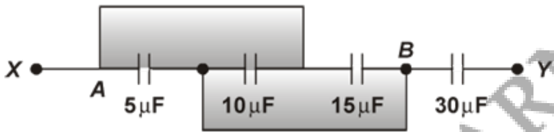
Q45. Assuming the earth's surface and upper level of stratosphere to constitute a spherical capacitor, estimate its capacitance.

Q46. The outer cylinders of two cylindrical capacitors of capacitance 1.1 μF each are kept in contact and the inner cylinders are connected through a wire. A battery of e.m.f. 15 V is connected as shown in the figure.



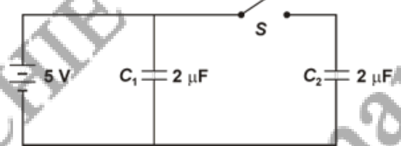
Find the total charge supplied by the battery to the inner cylinders.

Q47. Four capacitors are connected as shown in the figure



Calculate the equivalent capacitance between the points *X* and *Y*.

Q48. Figure shows two identical capacitors C_1 and and C_2, each of 2 μF capacitance, connected to a battery of 5 V. Initially switch '*S*' is closed. After some time *S* is left open and dielectric slabs of dielectric constant *K* = 5 are inserted to fill completely the space between the plates of the two capacitors. How will the (a) charge and (ii) potential difference between the plates of the capacitors be affected after the slabs are inserted?



Q49. Net capacitance of three identical capacitors in series is 1μF. What will be their net capacitance if connected in parallel?
Find the ratio of energy stored in the two configurations if they are both connected to the same source.

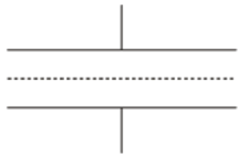
Q50. In a parallel plate capacitor with air between the plates, each plate has an area of 6×10^{-3} m^2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

Q51. Three capacitors of capacitances 5, 4 and 3 μF respectively are connected with the first and second in series and the third in parallel with them. Find the capacitance of the combination.

Q52. Figure shows a sheet of aluminium foil of negligible thickness placed between the plates of a capacitor. How will its capacitance be affected if

(a) the foil is electrically insulated?

(b) the foil is connected to the upper plate with a conducting wire?



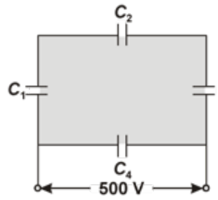
Q53. A parallel plate capacitor is charge to a potential difference V by a D.C. source. If the distance between the plates is doubled, state with reason how the following will changes;

(a) electric field between the plates,

(b) capacitance, and

(c) energy, stored in the capacitor.

Q54. A network of four capacitors each of 12 μF capacitance is connected to a 500 V supply as shown in the figure. Determine (a) equivalent capacitance of the network and (b) charge on each capacitor.



Q55. A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 μC. The space between the concentric spheres is filled with a liquid of dielectric constant 32.

(a) Determine the capacitance of the capacitor.

(b) What is the potential of the inner sphere?

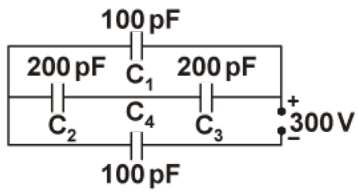
(c) Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain why the latter is much smaller.

Q56. A parallel plate capacitor having plate separation of 3 mm possesses a capacitance of 17.7 pF. The capacitor is connected to a 100 V supply. Explain what would happen, if a 3 mm thick mica sheet of dielectric constant 6 were inserted between the plates (a) while the voltage supply remains connected, (*b*) the supply was disconnected.

Q57. A conducting slab of thickness '*t*' is introduced without torching between the plates of a parallel plate capacitor, separated by a distance '*d*' ($t < d$). Derive an expression for the capacitance of the capacitor.

Q58. Deduce an expression for the capacitance of a parallel plate capacitor with air as the medium between the plates.

Q59. Obtain the equivalent capacitance of the network in (see figure). For a 300 V supply, determine the charge and voltage across each capacitor.



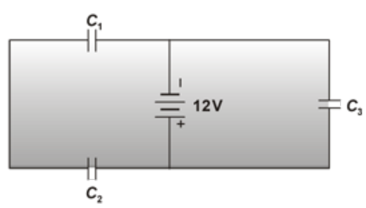
Q60. Three identical capacitors C_1, C_2 and C_3 of capacitance 6 μF each are connected to a 12 V battery as shown.

Find

(a) charge on each capacitor.

(b) equivalent capacitance of the network.

(c) energy stored in the network of capacitors.



Q61. The equivalent capacitance of the combination between *A* and *B* in the given figure is 4 μ*F*.

(a) Calculate capacitance of the capacitor *C*.

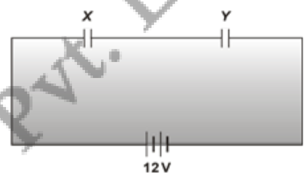
(b) Calculate charge on each capacitor if a 12 V battery is connected across terminals *A* and *B*.

(c) What will be the potential drop across each capacitor?

Q62. (a) How is the electric field due to a charged parallel plate capacitor affected when a dielectric slab is inserted between the plates fully occupying the intervening region?

(b) A slab of material of dielectric constant *K* has the same area as the plates of parallel plate capacitor has thickness $\frac{1}{2}d$, where *d* is the separation between the plates. Find the expression for the capacitance when the slab is inserted between the plates.

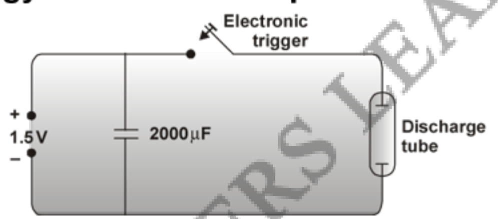
Q63. *X* and *Y* are two parallel plate capacitors having the same area of plates and same separation between the plates. X has air between the plates and Y contains a dielectric medium of $K = 5$.



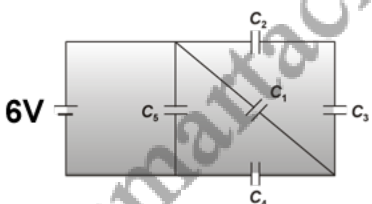
(a) Calculate the potential difference between the plates of *X* and *Y*.

(b) What is the ratio of electrostatic energy stored in *X* and *Y*?

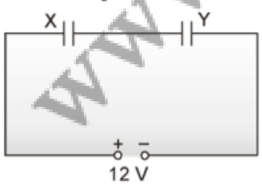
Q64. Figure gives a circuit for a camera flash. A 2,000 μ*F* capacitor is charged by a 1.5 V cell. When a flash is required, the energy stored in the capacitor is made to discharge (actuated by an electronic trigger) through the discharge tube in 0.1 millisecond giving a powerful flash. Calculate the energy stored in the capacitor and the power of the flash.



Q65. Find the total energy stored in the capacitors in the network shown in Figure. Given that $C_1 = C_5 = 1\ \mu F$, $C_2 = C_3 = C_4 = 2\mu F$.



Q66. Two parallel plate capacitor, *X* and *Y*, have the same area of plates and same separation between them. *X* has air between the plates while *Y* contains a dielectric medium of $\epsilon_r = 4$.

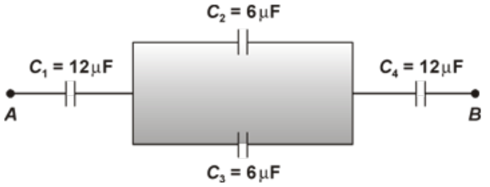


(a) Calculate capacitance of each capacitor if equivalent capacitance of the combination is 4 μ*F*.

(b) Calculate the potential difference between the plates of *X* and *Y*.

(c) What is the ratio of electrostatic energy stored in *X* and *Y*?

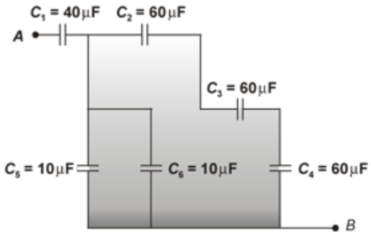
Q67. Find the resultant capacitance of the capacitors connected as shown in the figure.



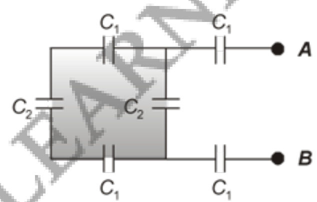
Q68. A 10 μF capacitor is charge by a 30 V d.c. supply and then connected across an uncharged 50μF capacitor. Calculate (a) the final potential difference across the combination and (b) the initial and final energies. How will you account for the difference in energy?

Q69. A capacitor of capacitance C_1 = 1 μF withstands the maximum voltage V_1 = 6.0 kV while for the capacitor of capacitance C_2 = 2.0 μF the maximum voltage is V_2 = 4.0 kV. What voltage will the system of these two capacitors withstand if they are connected series.

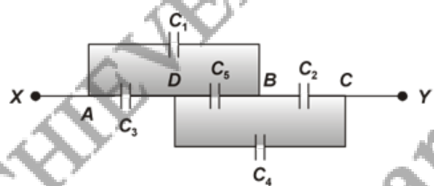
Q70. Find the equivalent capacitance of the combination of capacitors between the points *A* and *B* as shown in figure. Also calculate the total charge that flows in the circuit, when a 100 V battery is connected between the points *A* and *B*.



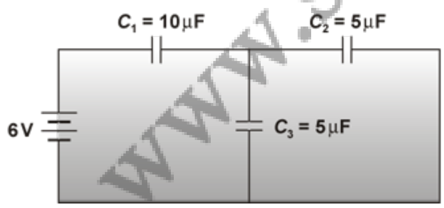
Q71. If $C_1 = 3\,pF$ and $C_2 = 2$ pF, calculate the equivalent capacitance of the network shown in figure between points *A* and *B*?



Q72. Find the effective capacitance between the terminals *X* and *Y* of the network shown in figure below. Given that C_1 = 5, μF, C_2 = 10 μ*F*, C_3 = 2 μ*F*, C_4, = 4 μ*F* and C_5 = 10 μ*F*.



Q73. Three capacitors C_1, C_2 and C_3 are connected to a battery of 6 V as shown in figure below. Find the charges on the three capacitors.



Q74. Obtain an expression for the capacitance of a parallel plate capacitor when a conducting slab is inserted between its plates. Assume, that the thickness of slab *t* is less than the separation *d* between the plates.

Q75. Define capacitance of a capacitor. Give its SI unit. Derive the expression for the capacitance of a parallel plate capacitor, whose plates are separated by a dielectric medium.

Q76. **Explain the principle of a capacitor. Derive an expression for the capacitance of a parallel plate capacitor.**

Q77. **(a)** **Show that in a parallel plate capacitor, if the medium between the plates of a capacitor is filled with an insulating substance of dielectric constant *K*, its capacitance increases.**

(b) **Deduce the expression for the energy stored in a capacitor of capacitance *C* with charge *Q*.**

Q78. **(a)** **Show that the effective capacitances, *C* of a series combination of three capacitors C_1, C_2 and C_3 is given by**

$$C = \frac{C_1 C_2 C_3}{(C_1 C_2 + C_2 C_3 + C_3 C_1)}$$

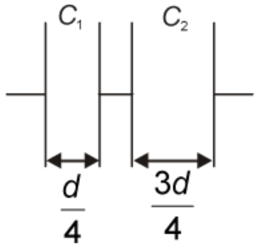
(b) **Calculate the capacitance of a spherical capacitor, if the diameter of inner sphere is 0.2 m and that of the outer sphere is 0.3 m, the space between them being filled with a liquid having dielectric constant 20.**

Date: 20/10/2021

S1. The sphere of 13 cm radius will carry greater charge because its capacitance is greater than the sphere of 7 cm radius.

S2. the new capacitance C_n is given by

$$C_n = \frac{C_1C_2}{C_1+C_2} = \frac{4C \times \frac{4C}{3}}{4C + \frac{4C}{3}} = C\,\text{F}$$



∴ The capacitance will not change.

S3. Dielectric constant of the medium,

$$\varepsilon_r = \frac{C_m}{C_a} = \frac{60}{6} = \mathbf{10}.$$

S4. Since the value of distance between the points is not given, we cannot calculate the actual value of this electric field.

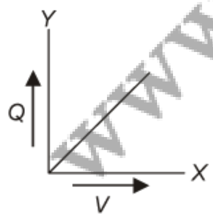
S5. Capacitance of conductor is defined as the ratio of the charge given to the rise in potential of the conductor.

S6. When dielectric of relative permittivity ε_r is inserted, the capacitance of the capacitor increases ε_r time. Since on inserting the dielectric, capacitance increases 5 times, the relative permittivity of the dielectric is 5.

S7. The dielectric constant of a dielectric medium is defined as the ratio of the capacitance of the capacitance of the capacitor, when the dielectric fills the space between the two plates of the capacitor to the capacitance of the capacitor when the air fills the space between the two plates of the capacitor.

S8. No the maximum charge that can be given to a capacitor is limited by the dielectric strength of the medium between the two plates of the capacitor.

S9. As $Q \propto V$, or $Q = CV$



S10. Capacitance, scalar quantity.

S11. Zero, no net work is done in moving a charge along a closed path in a uniform electric field.

S12. The charges on the free plates are enhanced.

S13.
$$4\pi\varepsilon_0 r' = \frac{\varepsilon_0 \pi r^2}{d} \quad \text{or} \quad d = \frac{r^2}{4r'}$$

S14. There will not be any effect on the capacitance of the capacitor.

S15. Electric flux through capacitor, $\phi_E = \frac{q}{\varepsilon_0}$. Here, q = constant, the capacitor becomes fully charged.

Displacement current, $$I_D = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d\left(\frac{q}{\varepsilon_0}\right)}{dt} = 0$$

Conduction current, $I = C\frac{dV}{dt} = 0$ as voltage becomes constant when the capacitor becomes fully charged.

S16. Capacitance of each of the three capacitors, $C = 9\,\text{pF}$.

(a) Equivalent capacitance (C') of the combination of the capacitors is given by the relation,

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$$

$$= \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{3}{9} = \frac{1}{3}$$

$\therefore$ $C' = 3\,\text{pF}$

Therefore, total capacitance of the combination is 3 pF.

Supply voltage, $V = 100\,\text{V}$

(b) Potential difference (V') across each capacitor is equal to one-third of the supply voltage.

$\therefore$ $V' = \frac{V}{3} = \frac{120}{3} = 40\,\text{V}$

Therefore, the potential difference across each capacitor is 40 V.

S17. Dielectric constant of the mica sheet, $k = 6$.

Initial capacitance, $C = 1.771 \times 10^{-11}\,\text{F}$

New capacitance, $C' = kC = 6 \times 1.771 \times 10^{-11} = 106\,\text{pF}$

(a) Supply voltage, $V = 100\,\text{V}$

New capacitance, $q' = C'V = 6 \times 1.771 \times 10^{-9} = 106 \times 10^{-8}\,\text{C}$

Potential across the plates remains 100 V.

Dielectric constant, $k = 6$

Initial capacitance, $C = 1.771 \times 10^{-11}\,\text{F}$

New capacitance, $C' = kC = 6 \times 1.771 \times 10^{-11} = 106\,\text{pF}$

(b) If supply voltage is removed, then there will be no effect on the amount of charge in the plates.

$$\text{Charge} = 1.771 \times 10^{-9}\,\text{C}$$

Potential across the plates is given by,

$$\therefore \qquad V' = \frac{q}{C'} = \frac{1.771 \times 10^{-9}}{106 \times 10^{-12}} = 16.7\,\text{V}.$$

S18. Total required capacitance, $C = 2\ \mu\text{F}$

Potential difference, $V = 1\ \text{kV} = 1000\ \text{V}$

Capacitance of each capacitor, $C_1 = 1\mu\text{F}$

Each capacitor can withstand a potential difference, $V_1 = 400\ \text{V}$

Suppose a number of capacitors are connected in series and these series circuits are connected in parallel (row) to each other. The potential difference across each row must be 1000 V and potential difference across each capacitor must be 400 V. Hence, number of capacitors in each row is given as

$$\frac{1000}{400} = 2.5$$

Hence, there are three capacitors in each row.

$$\text{Capacitance of each row} = \frac{1}{1+1+1} = \frac{1}{3}\,\mu\text{F}$$

Let there are n rows, each having three capacitors, which are connected in parallel. Hence, equivalent capacitance of the circuit is given as

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \ldots\ldots\ldots\ n \text{ terms}$$

$$= \frac{n}{3}$$

However, capacitance of the circuit is given as $2\ \mu\text{F}$.

$$\therefore \qquad \frac{n}{3} = 2$$

$$n = 6$$

Hence, 6 rows of three capacitors are present in the circuit. A minimum of 6×3 *i.e.*, 18 capacitors are required for the given arrangement.

S19. Let a be the radius of a sphere A, Q_A be the charge of the sphere, and C_A be the capacitance of the sphere. Let b be the radius of a sphere B, Q_B be the charge on the sphere, and C_B be the capacitance of the sphere. Since the two spheres are connected with a wire, their potential (V) will become equal.

Let E_A be the electric field of sphere A and E_B be the electric field of sphere B. Therefore, their ratio,

$$\frac{E_A}{E_B} = \frac{Q_A}{4\pi\varepsilon_0 \times a^2} \times \frac{b^2 \times 4\pi\varepsilon_0}{Q_B}$$

$$\frac{E_A}{E_B} = \frac{Q_A}{Q_B} \times \frac{b^2}{a^2} \qquad \dots \text{(i)}$$

However, $\frac{Q_A}{Q_B} = \frac{C_A V}{C_B V}$ $\quad C_A = 4\pi\varepsilon_0 a, \quad C_B = 4\pi\varepsilon_0 b$

$$\therefore \quad \frac{Q_A}{Q_B} = \frac{a}{b} \qquad \dots \text{(ii)}$$

Putting the value of (ii) in (i), we obtain

$$\therefore \quad \frac{E_A}{E_B} = \frac{a}{b}\frac{b^2}{a^2} = \frac{b}{a}$$

Therefore, the ratio of electric fields at the surface is $\frac{b}{a}$.

S20. Capacitances of the given capacitors are:

$$C_1 = 2\,\text{pF}, \quad C_2 = 3\,\text{pF} \quad \text{and} \quad C_3 = 4\,\text{pF}$$

(a) For the parallel combination of the capacitors, equivalent capacitor C' is given by the algebraic sum,

$$C' = 2 + 3 + 4 = 9\,\text{pF}$$

Therefore, total capacitance of the combination is 9 pF.

Supply voltage, $V = 100\,\text{V}$

The voltage through all the three capacitors is same = $V = 100\,\text{V}$.

(b) Charge on a capacitor of capacitance C and potential difference V is given by the relation,

$$q = VC \qquad \dots \text{(i)}$$

For $C = 2\,\text{pF}$,

$$\text{Charge} = VC = 100 \times 2 = 200\,\text{pC} = 2 \times 10^{-10}\,\text{C}.$$

For $C = 3\,\text{pF}$,

$$\text{Charge} = VC = 100 \times 3 = 300\,\text{pC} = 3 \times 10^{-10}\,\text{C}.$$

For $C = 4\,\text{pF}$,

$$\text{Charge} = VC = 100 \times 4 = 400\,\text{pC} = 4 \times 10^{-10}\,\text{C}.$$

S21. Capacitance between the parallel plates of the capacitor, $C = 8\,\text{pF}$

Initially, distance between the parallel plates was d and it was filled with air. Dielectric constant of air, $k = 1$.

Capacitance, C, is given by the formula,

$$C = \frac{k\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{d} \quad \text{... (i)}$$

Where,

A = Area of each plate

ε_0 = Permittivity of free space

If distance between the plates is reduced to half, then new distance, $d' = \frac{d}{2}$.

Dielectric constant of the substance filled in between the plates, $k' = 6$.

Hence, capacitance of the capacitor becomes

$$C' = \frac{k'\varepsilon_0 A}{d} = \frac{6\varepsilon_0 A}{\frac{d}{2}} \quad \text{... (ii)}$$

Taking ratios of equations (i) and (ii), we obtain

$$C' = 2 \times 6\, C$$

$$= 12\, C$$

$$= 12 \times 8 = 96\,\text{pF}.$$

Therefore, the capacitance between the plates is 96 pF.

S22. Potential rating of a parallel plate capacitor, $V = 1\,\text{kV} = 1000\,\text{V}$

Dielectric constant of a material, $\varepsilon_r = 3$

Dielectric strength = 10^7 V/m

For safety, the field intensity never exceeds 10% of the dielectric strength.

Hence, electric field intensity, E = 10% of $10^7 = 10^6$ V/m

Capacitance of the parallel plate capacitor, $C = 50\,\text{pF} = 50 \times 10^{-12}\,\text{F}$

Distance between the plates is given by,

$$d = \frac{V}{E} = \frac{1000}{10^6} = 10^{-3}\,\text{m}$$

Capacitance is given by the relation,

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

Where, A = Area of each plate

ε_0 = Permittivity of free space

$= 8.85 \times 10^{-12}\,\text{N}^{-1}\,\text{C}^2\,\text{m}^{-2}$

$$\therefore \quad A = \frac{Cd}{\varepsilon_0 \varepsilon_r}$$

$$= \frac{50 \times 10^{-12} \times 10^{-3}}{8.85 \times 10^{-12} \times 3} \approx 19\,\text{m}^2$$

Hence, the area of each plate is about 19 cm.

S23. Let $E_0 = V_0/d$ be the electric field between the plates when there is no dielectric and the potential difference is V_0. If the dielectric is now inserted, the electric field in the dielectric will be $E = E_0/K$. The potential difference will then be

$$V = E_0\left(\frac{1}{4}d\right) + \frac{E_0}{K}\left(\frac{3}{4}d\right)$$

$$= E_0 d\left(\frac{1}{4} + \frac{3}{4K}\right) = V_0\frac{K+3}{4K}$$

The potential difference decreases by the factor $(K + 3)/4K$ while the free charge Q_0 on the plates remains unchanged. The capacitance thus increases

$$C = \frac{Q_0}{V} = \frac{4K}{K+3}\frac{Q_0}{V} = \frac{4K}{K+3}C_0$$

S24. Given, $a = 0.2$ m; $b = 0.3$ m and K = 20;

we know, $$C = 4\pi\varepsilon_0\, K \cdot \frac{ab}{b-a}$$

$$= \frac{1}{9 \times 10^9} \times 20 \times \frac{0.2 \times 0.3}{0.3 - 0.2}$$

$$= 1.333 \times 10^{-9}\text{ F} = 1333\text{ pF}$$

S25. (a) The charge on the capacitor is

$$Q = CV = 900 \times 10^{-12}\,\text{F} \times 100\,\text{V} = 9 \times 10^{-8}\,\text{C}$$

The energy stored by the capacitor is

$$= (1/2)\,CV^2 = (1/2)\,QV$$
$$= (1/2) \times 9 \times 10^{-8}\,\text{C} \times 100\,\text{V} = 4.5 \times 10^{-6}\text{ J}$$

(b) In the steady situation, the two capacitors have their positive plates at the same potential, and their negative plates at the same potential. Let the common potential difference be V_2. The charge on each capacitor is then $Q' = CV'$. By charge conservation, $Q' = Q/2$. This implies $V' = V/2$. The total energy of the system is

$$= 2 \times \frac{1}{2}Q'V' = \frac{1}{4}QV = 2.25 \times 10^{-6}\,\text{J}$$

Thus in going from (a) to (b), though no charge is lost; the final energy is only half the initial energy. *Where has the remaining energy gone*?

There is a transient period before the system settles to the situation (b). During this period, a transient current flows from the first capacitor to the second. Energy is lost during this time in the form of heat and electromagnetic radiation.

S26. (a) In the given network, C_1, C_2 and C_3 are connected in series. The effective capacitance C_2 of these three capacitors is given by

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

For $C_1 = C_2 = C_3 = 10\,\mu F$, $C_2 = (10/3)\,\mu F$. The network has C_2 and C_4 connected in parallel. Thus, the equivalent capacitance C of the network is

$$C = C' + C_4 = \left(\frac{10}{3} + 10\right)\mu F = 13.3\,\mu F$$

(b) Clearly, from the figure, the charge on each of the capacitors, C_1, C_2 and C_3 is the same, say Q. Let the charge on C_4 be Q'. Now, since the potential difference across AB is Q/C_1, across BC is Q/C_2, across CD is Q/C_3, we have

$$\frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = 500\,V$$

Also, $Q'/C_4 = 500\,V.$

This gives for the given value of the capacitances,

$$Q = 500\,V \times \frac{10}{3}\,\mu F = 1.7 \times 10^{-3}\,C$$

and $Q' = 500\,V \times 10\,\mu F = 5.0 \times 10^{-3}\,C$

S27. When the capacitor P is charge to a potential difference V, then charge a acquired by the capacitor, $q = CV$.

(a) **When capacitor *P* is connected to *Q* in parallel:** When the capacitors P and Q (each of capacitance C) are connected in parallel,

total capacitance = $C + C = 2C$.

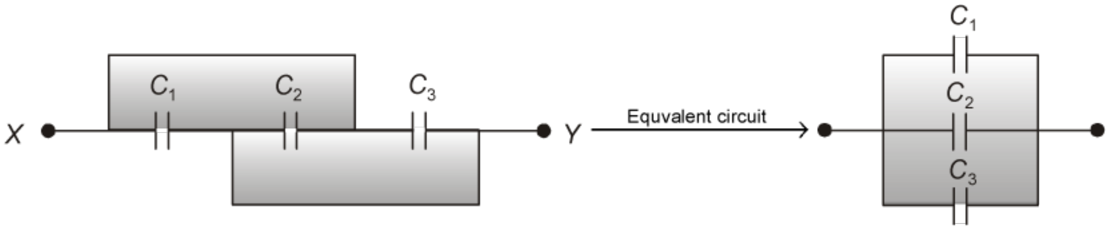
total charge on the two capacitors = $q + 0 = q = CV$.

Therefore, potential difference across each capacitor or the capacitor P will become

$$V' = \frac{CV}{2C} = \frac{\mathbf{V}}{\mathbf{2}}.$$

(b) **When capacitor *P* is connected to *R* in series:** When the capacitor P is connected to R in series, the circuit will not get completed and likewise the sharing of charges does not take place. Therefore, the potential difference across the capacitor P will remain unchanged *i.e.*, **V**.

S28. The given arrangement of the three capacitors is equivalent to their parallel combination as shown in the figure



Therefore, the equivalent capacitance,

$$C = C_1 + C_2 + C_3.$$

S29. The given arrangement is equivalent to parallel combination of two capacitors, each of plate area $A/2$, plate separation d, such that one has medium of dielectric constant K_1 and other of dielectric constant K_2. If C_1 and C_2 are capacitances of the two capacitors so formed, then

$$C_1 = \frac{\varepsilon_0 K_1 A/2}{d} = \frac{\varepsilon_0 K_1 A}{2d}$$

and $$C_2 = \frac{\varepsilon_0 K_2 A/2}{d} = \frac{\varepsilon_0 K_2 A}{2d}$$

If C is the capacitance of the capacitor, then

$$C = C_1 + C_2 = \frac{\varepsilon_0 K_1 A}{2d} + \frac{\varepsilon_0 K_2 A}{2d}$$

$$= \frac{\varepsilon_0 (K_1 + K_2) A}{2d}$$

S30. Let each capacitor be of capacitance C.

Then, $$C_p = C + C = 2C$$

and $$C_s = \frac{C \times C}{C + C} = \frac{C}{2}$$

Let V_p and V_s be the values of the potential difference applied across parallel and series combinations respectively, so that energy stored in the two cases becomes equal. Then

$$U_p = \frac{1}{2} C_p V_p^2 = \frac{1}{2} \times 2C \times V_p^2 = CV_p^2$$

and $$U_s = \frac{1}{2} C_s V_s^2 = \frac{1}{2} \times \frac{C}{2} \times V_s^2 = \frac{1}{4} CV_s^2$$

Since $U_p = U_s$, we get

$$CV_p^2 = \frac{1}{4}CV_s^2 \qquad \text{or} \qquad \frac{V_p^2}{V_s^2} = \frac{1}{4}$$

or $$\frac{V_p}{V_s} = \mathbf{\frac{1}{2}}.$$

S31. Given: $C_1 = 10\,\mu F$; $C_2 = 20\,\mu F$ and $C_3 = 30\,\mu F$

If C is their equivalent capacitance, then

$$C = C_1 + C_2 + C_3 = 10 + 20 + 30 = 60\,\mu F$$
$$= 60 \times 10^{-6}\,F$$

The total energy stored in the combination,

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 60 \times 10^{-6} \times (100)^2 = \mathbf{0.3\ J}.$$

S32. Let C' be equivalent capacitance of n capacitors, each of capacitance C, connected is parallel. Then,

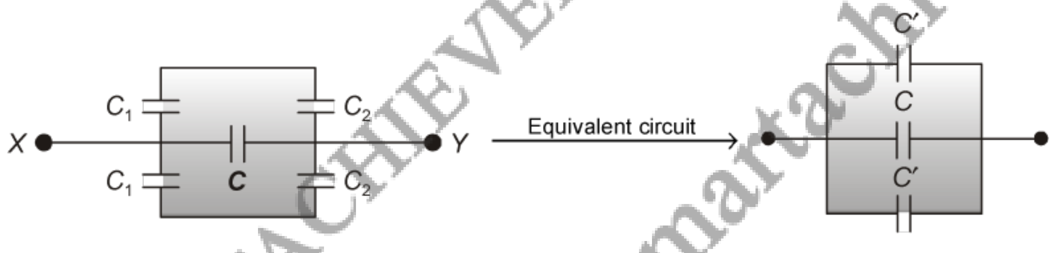
$$C' = nC$$

Therefore, energy stored in the network of n capacitors,

$$U = \frac{1}{2}C'V^2 = \mathbf{\frac{1}{2}nCV^2}.$$

S33. Let C' be the capacitance of the series combination of C_1 and C_2. Then,

$$C' = \frac{C_1 C_2}{C_1 + C_2}$$



Therefore, equivalent capacitance of the system between the point X and Y,

$$C_{eq.} = C' + C + C' = C + 2C' = C + \frac{2C_1C_2}{C_1 + C_2}$$

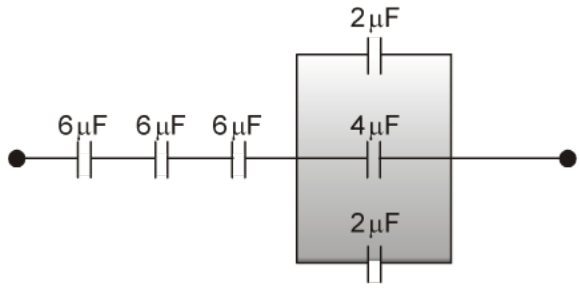
S34. The capacitance of the parallel combination of three capacitors, each of $20\,\mu F$,

$$C' = 20 + 20 + 20 = 60\,\mu F.$$

Now, $$\frac{1}{C} + \frac{1}{C'} = \frac{1}{30} \qquad \text{or} \qquad \frac{1}{C} = \frac{1}{30} - \frac{1}{60} = \frac{1}{60}$$

or $$C = \mathbf{60\,\mu F}.$$

S35. The capacitors are connected as shown in the figure.



The capacitance of the parallel combination of 2 μF, 4 μF and 2 μF capacitors,

$$C' = 2 + 4 + 2 = 8\,\mu F$$

The capacitance of the whole network is given by

$$\frac{1}{C} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{C'} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{8} = \frac{15}{24}$$

or $$C = \frac{24}{15} = \mathbf{1.6\,\mu F}.$$

S36. Let the capacitances of the three capacitors be C, $2C$ and $3C$

When connected in parallel:

$$C' = C + 2C + 3C = 6C$$

When connected in series:

$$\frac{1}{C''} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{3C}$$

or $$C'' = \frac{6C}{11}$$

According to the statement of the problem,

$$6C = \frac{6C}{11} + \frac{60}{11}$$

or $$C = 1\,\mu F.$$

Therefore, capacitances of the three capacitors are **1 μF**, **2 μF** and **3 μF**.

S37. Capacitance of the spherical conductor,

$$C = 4\pi\varepsilon_0 R$$

or $$C = 4\pi\varepsilon_0 \times 0.5 \quad \text{... (i)}$$

Let d be the distance between the two plates of air capacitor.

Area of each plate, $A = 0.2 \times 0.1 = 0.02\,\text{m}^2$

$$\therefore \quad C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \times 0.02}{d} \quad \text{... (ii)}$$

From the equations (i) and (ii), we get

$$\frac{\varepsilon_0 \times 0.02}{d} = 4\pi\varepsilon_0 \times 0.5$$

or $$d = \frac{0.02}{4\pi \times 0.5} = \mathbf{3.183 \times 10^{-3}\,m}.$$

S38. $$R = 3r = 3 \times 3 = 9\text{ mm} = 9 \times 10^{-3}\,\text{m}$$

$$\therefore \quad C = 4\pi\varepsilon_0 R = \frac{9 \times 10^{-3}}{9 \times 10^{9}} = \mathbf{10^{-12}\,F}.$$

S39. The given arrangement is equivalent to the parallel combination of three capacitors C_1, C_2 and C_3. The plate area of each of the three capacitors are is $A/3$, while the plate separations in the three capacitors are d, $d + b$ and $d + 2b$ respectively.

$$\therefore \quad C_1 = \frac{\varepsilon_0 A/3}{d},$$

$$C_2 = \frac{\varepsilon_0 A/3}{d + b}$$

and $$C_3 = \frac{\varepsilon_0 A/3}{d + 2b}$$

Hence, equivalent capacitance of the arrangement,

$$C = C_1 + C_2 + C_3$$

On substituting for C_1, C_2 and C_3, it can be obtained that

$$C = \frac{\varepsilon_0 \mathbf{A(3d^2 + 6bd + 2b^2)}}{\mathbf{3d(d + b)(d + 2b)}}.$$

S40. The equivalent capacitance of the parallel combination of C_2 and C_3,

$$C_{23} = C_2 + C_3 = 3 + 5 = 8\,\mu\text{F}$$

The equivalent capacitance of the whole network,

$$C = \frac{C_1 \times C_{23}}{C_1 + C_{23}} = \frac{2 \times 8}{2 + 8} = 1.6\,\mu F$$

The charge acquired by the network,

$$q = CV = 1.6 \times 10^{-6} \times 100 = 1.6 \times 10^{-4}\,\text{C}.$$

Since C_1 and C_{23} are in series, charge on each of them is same as on the network *i.e.*, equal 1.6×10^{-4} C.

Therefore, potential difference across C_1,

As C_2 and C_3 are in parallel,

$$V_2 = V_3 = \frac{q}{C_{23}} = \frac{1.6 \times 10^{-4}}{8 \times 10^{-6}} = \mathbf{20\,V}.$$

S41. Let the alternating emf charging the plates of capacitor be $E = E_0 \sin \omega t$

Charge on the capacitor, $q = EC = CE_0 \sin \omega t$

Instantaneous current is I.

$$I = \frac{dq}{dt} = \frac{d}{dt}(CE_0 \sin \omega t) = \omega CE_0 \cos \omega t = I_0 \cos \omega t$$

[where $I_0 = \omega CE_0$]

Displacement current, $I_D = \varepsilon_0 \frac{d\phi_E}{dt}$

$$\varepsilon_0 A \frac{d(E)}{dt} = \varepsilon_0 A \frac{d}{dt}\left(\frac{q}{\varepsilon_0 A}\right)$$

$$= \varepsilon_0 A \frac{d}{dt}\left(\frac{CE_0 \sin \omega t}{\varepsilon_0 A}\right)$$

$$= \frac{d}{dt}(CE_0 \sin \omega t)$$

$$= \omega CE_0 \cos \omega t = I_0 \cos \omega t$$

Thus, the displacement current inside the capacitor is the same as the current charging the capacitor.

S42. Initially dielectric when there is vacuum between the two plates, the capacitance of the plate is,

$$C_0 = \frac{\varepsilon_0 A}{d}$$

where A is the area of parallel plates.

Suppose that the capacitor is connected to a battery, an electric field E_0 is produced. Now if we insert the dielectric slab of thickness $t = d/2$, the electric field reduces to E.

Now the gap between plants is divided in two parts, for distance t there is electric field E and for the remaining distance $(d - t)$ the electric field is E_0.

If V be the potential difference between the plates of the capacitor, then $V = Et + E_0(d - t)$

$$V = \frac{Ed}{2} + \frac{E_0 d}{2} = \frac{d}{2}(E + E_0) \qquad \left[\because t = \frac{d}{2}\right]$$

$$\Rightarrow \qquad V = \frac{d}{2}\left(\frac{E_0}{K} + E_0\right)$$

$$= \frac{dE_0}{2K}(K+1) \qquad \left[\text{as } \frac{E_0}{E} = K\right]$$

Now, $$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{q}{\varepsilon_0 A}$$

$$\Rightarrow \qquad V = \frac{d}{2K}\frac{q}{\varepsilon_0 A}(K+1)$$

We know, $$C = \frac{q}{V} = \frac{2K\varepsilon_0 A}{d(K+1)}$$

S43. A shown in Figure, suppose the end A is connected to the positive terminal and the end to B to the negative terminal of a battery. Thus, the given arrangement becomes two capacitors I and II with the positive plates connected together and their negative plates connected together, *i.e.*, they are connected in parallel. As the separation and area of the plates are equal, their capacitance C will be equal. Therefore, their equivalent capacitance is

$$C_{eq} = C_{\mathrm{I}} + C_{\mathrm{II}} = C + C = 2C = \frac{2\varepsilon_0 A}{d}$$

S44. A possible combination is shown in figure. A parallel combination of three capacitors should be connected in series with the fourth capacitor.

Equivalent capacitance of parallel combination = 1+ 1 + 1 = 3 μF

Since this parallel combination is connected in series with the fourth capacitor, so the equivalent capacitance C of the four capacitor is

$$\frac{1}{C} = \frac{1}{3} + \frac{1}{1} = \frac{1+3}{3} = \frac{4}{3}$$

$$C = \frac{3}{4} = 0.75\,\mu\text{F}$$

S45. We know that capacitance of a spherical capacitor is

$$C = 4\pi\varepsilon_0 \frac{r_a r_b}{(r_b - r_a)}$$

Here, radius of earth, $r_a = 6.4 \times 10^6$ m

Upper level of stratosphere, $r_b = 6.4 \times 10^6 + 5 \times 10^4\ \text{m}^2$

$= 6.45 \times 10^6$ m (upper level $\simeq$ 50 km)

$$\therefore \quad C = 4 \times \frac{22}{7} \times 8.85 \times 10^{-12} \times \frac{6.4 \times 10^6\,\text{m} \times 6.45 \times 10^6}{(6.45 - 6.40) \times 10^6}$$

$\simeq 0.091853897$ F.

$\simeq 0.1$ F

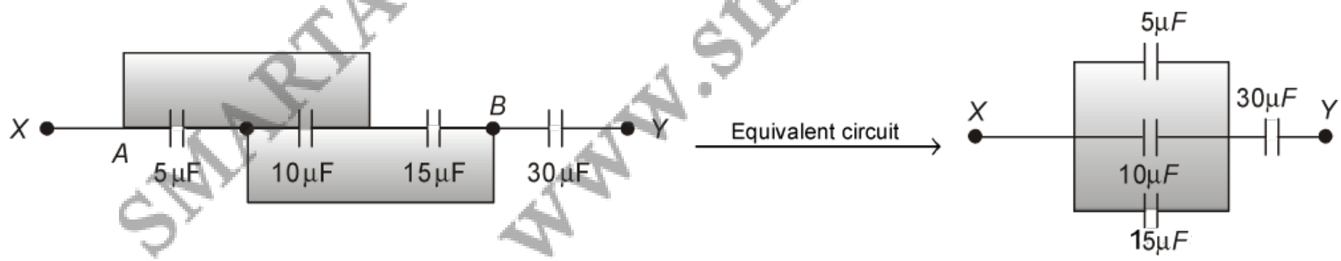
S46. The two capacitors are connected in parallel.

$\therefore \quad C = 1.1 + 1.\ 1 = 2.2\,\mu\text{F}$

Now, $q = CV = 2.2 \times 15 =$ **33 μC**.

S47. The three capacitors connected between the points A and B are in parallel. If C' is the capacitance of their combination, then

$$C' = 5 + 10 + 15 = 30\,\mu\text{F}.$$



Now, the capacitor C' and 30 μF capacitor are in series. Therefore, equivalent capacitance between the point X and Y.,

$$C = \frac{C' \times 30}{C' + 30} = \frac{30 \times 30}{30 + 30} = \mathbf{15\,\mu F}.$$

S48. When a dielectric medium of dielectric constant K is introduced.

(a) in an isolated (not connected with battery) capacitor, then total charge on capacitor remains same.

(b) in a capacitor connected with battery, then potential difference across the capacitor remains same as that of potential difference across battery

Two identical capacitors C_1 and C_2 gets fully charged with 5 V battery initially.

So, the charge and potential difference on both capacitors becomes

$$q = CV$$

$$= 2 \times 10^{-6} \times 5\text{V} = 10\ \mu\text{C}$$

and $$V = 5\text{ V}$$

On introduction of dielectric medium of $K = 5$.

For C_1 (Continue to be connected with battery) potential difference of C_1, $(V') = 5$ V

Capacitance of $C_1' = KC = 5 \times 2\ \mu\text{F} = 10\ \mu\text{F}$

Charge $q' = C'V' = (10\ \mu\text{F})\ (5\text{ V}) = 50\ \mu\text{C}$

For C_2 (Disconnected with battery) charge $q' = q = 10\ \mu\text{C}$

Potential difference $$V' = \frac{V}{K} = \frac{5}{5} = 1\text{ V}$$

S49. If n identical capacitors, each of capacitance C are connected in series combination gives equivalent capacitance,

$$C_s = \frac{C}{n}$$

and when connected in parallel, then equivalent capacitance,

$$C_p = nC$$

$$\Rightarrow \qquad \frac{C_p}{C_s} = \frac{nC}{C/n} = n^2$$

$$C_p = n^2 C_s$$

Also, for same voltage energy stored in capacitor is given by

$$U = \frac{1}{2}CV^2 \qquad \text{[for } V = \text{constant]}$$

$$U \propto C$$

$$C = 1\ \mu\text{F} \qquad n = 3$$

According to problem,

$$C = nC_s = 3 \times 1\ \mu\text{F} = 3\ \mu\text{F}$$

For each capacitor.

In parallel combination,

$$C_p = nC = 3 \times 3 = 9\ \mu F$$

$$C_p = 9\ \mu F$$

For same voltage, $U \propto C$

$$\Rightarrow \quad \frac{U_s}{U_p} = \frac{C_s}{C_p} = \frac{C/n}{nC} = \frac{1}{n^2}$$

$$\frac{1}{(3)^2} = \frac{1}{9}$$

$$\Rightarrow \quad \frac{U_s}{U_p} = \frac{1}{9}$$

or $U_s : U_p = 1 : 9$

S50. Area of each plate of the parallel plate capacitor,

$$A = 6 \times 10^{-3}\,m^2.$$

Distance between the plates,

$$d = 3\ mm = 3 \times 10^{-3}\,m.$$

Supply voltage, $V = 100\,V$

Capacitance C of a parallel plate capacitor is given by,

$$C = \frac{\varepsilon_0 A}{d}$$

Where,

$$\varepsilon_0 = \text{Permittivity of free space}$$
$$= 8.854 \times 10^{-12}\,N^{-1}\,m^{-2}\,C^{-2}$$

$$\therefore \quad C = \frac{8.854 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}}$$
$$= 17.71 \times 10^{-12}\,F$$
$$= 17.71\,pF$$

Potential V is related with the charge q and capacitance C as

$$V = \frac{q}{C}$$

$\therefore \qquad q = VC$

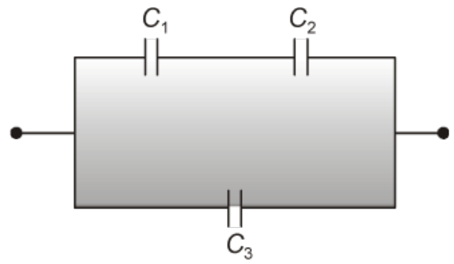
$$= 100 \times 17.71 \times 10^{-12}$$

$$= 1.771 \times 10^{-9}\,C$$

Therefore, capacitance of the capacitor is 17.71 pF and charge on each plate is 1.771×10^{-9} C.

S51. Here, $C_1 = 5\mu F$; $C_2 = 4\ \mu F$; $C_3 = 3\mu F$

The three capacitors have been connected as shown in figure.



Let C' be capacitance of the series combination of C_1 and C_2. Then,

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5} + \frac{1}{4} = \frac{4+5}{20} = \frac{9}{20}$$

or $\qquad C' = 2.22\ \mu F$

The series combination of C_1 and C_2 have been put in parallel with C_3.

If C is the effective capacitance of the arrangement, then

$$C = C' + C_3 = 2.22 + 3 = \mathbf{5.22\ \mu F}$$

S52. (a) The system will be equivalent to two identical capacitors connected in series combination in which two plates of each capacitor have separation half of the original separation.

Thus, new capacitance of each capacitor

$$C' = 2C \qquad \left[\because\ C \propto \frac{1}{d}\right]$$

$\because$ C' and C' are in series

$$\Rightarrow \qquad C_{net} = \frac{2C \times 2C}{2C + 2C} = C$$

$$C_{net} = C = \text{original capacitor}$$

(b) System reduces to a capacitor whose separation reduces to half of original one.

$\therefore$ New capacitance $C' = 2C$

S53. (a) Electric field between the plates

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A}$$

As surface charge density remains same E is constant.

(b) Capacitance $C = \dfrac{\epsilon_0 A}{d}$

If distance between the plates is doubled, then, capacitance reduces to half.

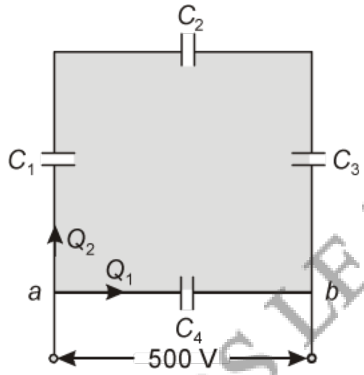
(c) Energy stored in the capacitor, $E = \dfrac{Q^2}{2C}$

As, capacitance reduces to half energy get doubled.

S54. (a) Equivalent capacitance of the network,

$$C_{123} = \frac{12\mu F}{3} = 4\mu F$$

$$C_{eq} = C_{123} + C_4 = (4 + 12)\ \mu F$$
$$C_{eq} = 16\ \mu F$$



(b) $$Q = CV$$
$$Q_1 = C_4 V = 500 \times 12 \times 10^{-6}$$
$$= 6000 \times 10^{-6} = 6 \times 10^{-3}\ C$$

Charge on each of the capacitors C_1, C_2 and C_3 is 2×10^{-3} C and charge on C_4 is 6×10^{-3} C.

S55. Radius of the inner sphere, $r_2 = 12\ cm = 0.12\ m$

Radius of the outer sphere, $r_1 = 13\ cm = 0.13\ m$

Charge on the inner sphere, $q = 2.5\ \mu C = 2.5 \times 10^{-6}\ C$

Dielectric constant of a liquid, $\varepsilon_r = 32$

(a) Capacitance of the sphere is given by the relation,

$$C = \frac{4\pi\varepsilon_0 \varepsilon_r r_1 r_2}{r_1 - r_2}$$

Where, ε_0 = Permittivity of free space

$$= 8.85 \times 10^{-12}\, C^2 N^{-1} m^{-2}$$

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\, N\, m^2\, C^{-2}$$

$$\therefore \quad C = \frac{32 \times 0.12 \times 0.13}{9 \times 10^9 \times (0.13 - 0.12)} = 5.5 \times 10^{-9}\, F$$

Hence, the capacitance of the capacitor is approximately 5.5×10^{-9} F.

(b) Potential of the inner sphere is given by,

$$V = \frac{q}{C} = \frac{2.5 \times 10^{-6}}{5.5 \times 10^{-9}} = 4.5 \times 10^2\, V.$$

Hence, the potential of the inner sphere is 4.5×10^2 V.

Radius of an isolated sphere, $r = 12 \times 10^{-2}$ m

Capacitance of the sphere is given by the relation,

$$C' = 4\pi\varepsilon_0 r$$
$$= 4\pi \times 8.85 \times 10^{-12} \times 12 \times 10^{-12}$$
$$= 1.33 \times 10^{-11}\, F$$

(c) The capacitance of the isolated sphere is less in comparison to the concentric spheres. This is because the outer sphere of the concentric spheres is earthed. Hence, the potential difference is less and the capacitance is more than the isolated sphere.

S56. Here, dielectric constant of mica sheet, $K = 6$; capacitance of the capacitor, $C = 17.7$ pF

(a) When the voltage supply remains connected. The capacitance of the capacitor will become K times.

Therefore, $C' = KC = 6 \times 17.7 =$ **106.2 pF**

The potential difference across the two plates of the capacitor will remain equal to supply voltage *i.e.*, 100 V.

The charge on the capacitor,

$$q' = C'V = 106.2 \times 10^{-12} \times 100$$
$$= \mathbf{1.06 \times 10^{-8}\, C}$$

(b) After the voltage supply is disconnected. As calculated above, the capacitance of the capacitor,

$$C' = \mathbf{106.2\ pF}$$

The potential difference will decrease on introducing mica sheet by a factor K, so that

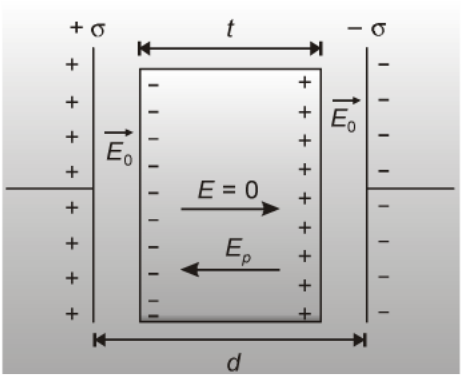
$$V' = \frac{V}{K} = \frac{100}{6} = \mathbf{16.67\ V}$$

The charge on the capacitor,

$$q' = C' V' = 106.2 \times 10^{-12} \times 16.67 = \mathbf{1.77 \times 10^{-9}\ C}$$

S57. Let σ be surface charge density on capacitor plates of area A.

Electric field between the plates in the air space is



As in case of conducting slab $E_p = E_0$. Net electric field inside the conducting slab is zero. Now potential. difference between the plates of capacitor is

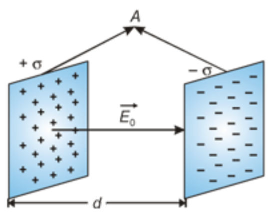
$$V = E_0(d-t) = \frac{\sigma}{\varepsilon_0}(d-t) \quad \text{... (i)} \qquad \because \quad E_0 = \frac{\sigma}{\varepsilon_0}$$

$$Q = \sigma A \quad \text{... (ii)}$$

From Eq. (i) and (ii), we get

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d-t} = \frac{C_0}{1-t/d} \qquad \because C_0 = \frac{\varepsilon_0 A}{d}$$

S58. Consider a capacitor with surface charge density σ on its plates. Suppose area of each plate is 'A' and separation between the plates is 'd'.



We know $\quad Q = CV$

$$C = \frac{Q}{V} \quad \text{... (i)}$$

Here, $\quad Q = \sigma A \quad$ ... (ii)

$$V = E_0 d$$

$$V = \frac{\sigma}{\varepsilon_0} d \quad \left(\therefore E_0 = \frac{\sigma}{\varepsilon_0}\right) \quad \text{... (iii)}$$

From Eq. (ii) and (iii) put the value Q and V in Eq (i), we get

$$C = \frac{\varepsilon_0 A}{d}.$$

S59. Capacitance of capacitor C_1 is 100 pF.

Capacitance of capacitor C_2 is 200 pF.

Capacitance of capacitor C_3 is 200 pF.

Capacitance of capacitor C_4 is 100 pF.

Supply potential, $V = 300\,V$

Capacitors C_2 and C_3 are connected in series. Let their equivalent capacitance be C'.

$$\therefore \quad \frac{1}{C'} = \frac{1}{200} + \frac{1}{200} = \frac{2}{200}$$

$$C' = 100\,pF$$

Capacitors C_1 and C' are in parallel. Let their equivalent capacitance be C''.

$$\therefore \quad C'' = C' + C_1$$

$$= 100 + 100 = 200\,pF$$

C'' and C_4 are connected in series. Let their equivalent capacitance be C.

$$\therefore \quad \frac{1}{C} = \frac{1}{C''} + \frac{1}{C_4}$$

$$= \frac{1}{200} + \frac{1}{100} = \frac{2+1}{200}$$

$$C = \frac{200}{3}\,pF$$

Hence, the equivalent capacitance of the circuit is $\frac{200}{3}$ pF .

Potential difference across $C'' = V''$

Potential difference across $C_4 = V_4$

$$\therefore \quad V'' + V_4 = V = 300\,V$$

Charge on C_4 is given by,

$$Q_4 = CV$$

$$= \frac{200}{3} \times 10^{-12} \times 300$$

$$= 2 \times 10^{-8}\,C$$

$$\therefore \quad V_4 = \frac{Q_4}{C_4} = \frac{2 \times 10^{-8}}{100 \times 10^{-12}} = 200\,V$$

$\therefore$ Voltage accross C_1 is given by,

$$V_1 = V - V_4$$

$$= 300 - 200 = 100\,V$$

Hence, potential difference, V_1, across C_1 is 100 V.

Charge on C_1 is given by,

$$Q_1 = C_1V_1$$
$$= 100 \times 10^{-12} \times 100 = 10^{-8}\,C$$

C_2 and C_3 having same capacitances have a potential difference of 100 V together. Since C_2 and C_3 are in series, the potential difference across C_2 and C_3 is given by,

$$V_2 = V_3 = 50\,V$$

Therefore, charge on C_2 is given by,

$$Q_2 = C_2V_2$$
$$= 200 \times 10^{-12} \times 50 = 10^{-8}\,C$$

And charge on C_3 is given by,

$$Q_3 = C_3V_3$$
$$= 200 \times 10^{-12} \times 50 = 10^{-8}\,C$$

Hence, the equivalent capacitance of the given circuit is $\frac{200}{3}$ pF with.

$$Q_1 = 10^{-8}\,C, \qquad V_1 = 100\,V$$
$$Q_2 = 10^{-8}\,C, \qquad V_2 = 50\,V$$
$$Q_3 = 10^{-8}\,C, \qquad V_3 = 50\,V$$
$$Q_4 = 2 \times 10^{-8}\,C, \qquad V_4 = 200\,V$$

S60. C_1 and C_2 in series, make C_4 using

$$\frac{1}{C_4} = \frac{1}{C_1} + \frac{1}{C_2} = \left(\frac{1}{6} + \frac{1}{6}\right) = \frac{2}{6}.$$

$$C_4 = 3\ \mu F$$

(a) 12 V of potential is available C_4 and C_3.

Charge in $C_3 = Q_3 = C_3V = 6 \times 10^{-6} \times 12 = 72\ \mu C$.

Charge in $C_4 = Q_4 = C_4\,V = 3 \times 10^{-6} \times 12 = 36\ \mu C$.

$\therefore$ Charge on C_1 and C_2 will also be $36\ \mu C$.

(b) C_4 and C_3 are in parallel to the source

So, $C_{eq} = 3 + 6 = 9\ \mu F$.

(c) Energy stored $= \frac{1}{2}C_{eq}.V^2$

$$= \frac{1}{2} \times 9 \times 10^{-6} \times 12^2$$

$$= 648\ \mu J.$$

S61. $C_{eq} = 4\ \mu F$

(a) Since 20 μF and C are in series, we have

$$\frac{1}{4} = \frac{1}{20} + \frac{1}{C}$$

$$\Rightarrow \quad \frac{1}{C} = \frac{1}{4} - \frac{1}{20} = \frac{5-1}{20}$$

$$\Rightarrow \quad C = \frac{20}{4} = 5\ \mu F$$

(b) Charge drawn from 12 V battery is

$$Q = C_{eq} \,.\, V = 4 \times 12 = 48\ \mu C.$$

So charge on each capacitor = 48 μC.

(c) Potential drop across

$$20\ \mu F = V_{20} = \frac{Q}{C} = \frac{48 \mu C}{20 \mu F} = 2.4 \text{ volt}$$

$$5 \mu F = V_5 = \frac{48\ \mu C}{5\ \mu C} = 9.6 \text{ volt.}$$

S62. (a) Electric field will decrease. On inserting dielectric slab, due to polarisation of charges, an electric field is developed inside the slab in a direction opposite to the electric field due to capacitor plates. Thus, a reduced value of electric field is obtained.

(b) In the absence of dielectric slab, let the electric field between the capacitor plates be

$$E_0 = \frac{V_0}{d}$$

where V_0 is potential difference between the plates.

when the dielectric is inserted electric field in it will be

$$E = \frac{E_0}{K}$$

Now potential difference between capacitor plates will be

$$V = E_0\left(\frac{d}{2}\right) + \frac{E_0}{K}\left(\frac{d}{2}\right)$$

$$= E_0 d\left[\frac{1}{2} + \frac{1}{2K}\right] = \frac{E_0 d}{2}\left[\frac{K+1}{K}\right]$$

$$V = \frac{V_0(K+1)}{2K} \qquad \left[\because E_0 = \frac{V_0}{d}\right]$$

As the charge on capacitor plates remains same.

$$C = \frac{Q_0}{V} = \frac{2KQ_0}{V_0(K+1)} = \frac{2K}{K+1}C_0$$

S63. Since that capacitor X has air and the capacitor Y has dielectric medium of $K = 5$, their capacitances of the capacitors X and Y respectively, then

$$C_1 = \text{C} \quad \text{and} \quad \text{C}_2 = 5\ \text{C}$$

(a) When the two capacitors connected in series the charge on them is always equal. Let q be charge on each of the two capacitors. If V_1 and V_2 are potential difference across the plates of the capacitors X and Y, then

$$V_1 + V_2 = 12 \qquad \text{... (i)}$$

Now, $V_1 = \frac{q}{C_1} = \frac{q}{C}$ and $V_2 = \frac{q}{C_2} = \frac{q}{5C}$

In the equation (i), substituting for V_1 and V_2, we have

$$\frac{q}{C} + \frac{q}{5C} = 12$$

or $\frac{6q}{5C} = 12$ or $\frac{q}{C} = 10$

$\therefore$ $V_1 = \frac{q}{C} = \mathbf{10\ V}$

and $V_2 = \frac{q}{5C} = \frac{10}{5} = 2\text{V}$

(b) Let U_1 and U_2 be the electrostatic energy stored in the capacitors X and Y respectively.

Then, $U_1 = \frac{1}{2}C_1 V_1^2 = \frac{1}{2} \times C \times (10)^2 = 50\,C$ J and $$U_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2} \times 5C \times (2)^2 = 10C \text{ J}$$

$\therefore$ $$\frac{U_1}{U_2} = \frac{50\,C}{10\,C} = \mathbf{5}.$$

S64. Given, V = 1.5 volt; C = 2,000 μF = 2, 000 × 10^{-6} F

Therefore, energy stored in the capacitor.

$$U = \frac{1}{2}C\,V^2 = \frac{1}{2} \times 2{,}000 \times 10^{-6} \times (1.5)^2$$

$$= \mathbf{2.25 \times 10^{-3}\ J}$$

The time in which the capacitor discharges in order to produce the flash,

$$t = 0.1 \text{ millisecond} = 0.1 \times 10^{-3} = 10^{-4} \text{ s}$$

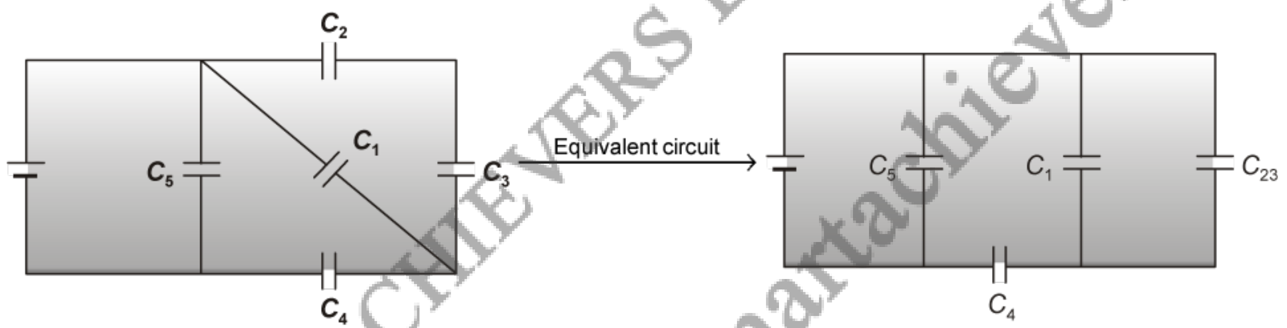
Hence, power of the flash,

$$P = \frac{U}{t} = \frac{2.25 \times 10^{-3}}{10^{-4}} = \mathbf{22.5\ W}$$

S65. The capacitors C_2 and C_3 are in series. Their effective capacitance,

$$C_{23} = \frac{C_2 \times C_3}{C_2 + C_3} = \frac{2 \times 2}{2 + 2} = 1\,\mu\text{F}$$

The given arrangement of capacitors is equivalent to the arrangement as shown in figure below.



Now, the capacitors C_1 and C_{23} are in parallel. Let C_{123} be their effective capacitance. Then,

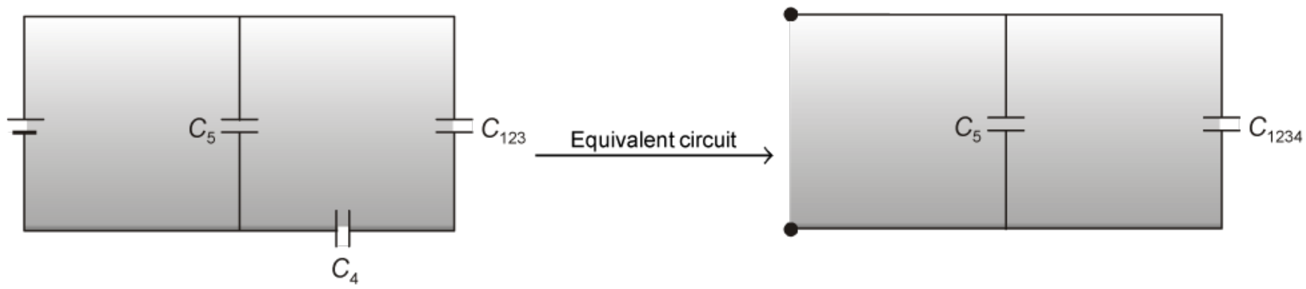
$$C_{123} = C_1 + C_{23} = 1 + 1 = 2\mu\text{F}$$

The arrangement of the capacitors shown in above figure is now equivalent to the arrangement shown in the figure below.

Since the capacitors C_{123} and C_4 are in series, their equivalent capacitance is given by

$$C_{1234} = \frac{C_{123} \times C_4}{C_{123} + C_4} = \frac{2 \times 2}{2 + 2} = 1\,\mu F$$

The arrangement of the capacitors shown in above figure is equivalent to the arrangement shown in the figure below.



$$C = C_{1234} + C_5 = 1 + 1 = 2\ \mu F$$

Therefore, energy stored in the network of the capacitors,

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 2 \times 10^{-6} \times (6)^2 = \mathbf{3.6 \times 10^{-5}\ J}$$

S66. Since area A and separation d are same, if C_x = C then $C_y = 4C$.

(a) Since they are in series,

$$\frac{1}{4} = \frac{1}{C} + \frac{1}{4C}$$

$$\Rightarrow \qquad 4 = \frac{4C}{5} \text{ or } C = 5\ \mu F$$

The two capacitors are therefore 5 μF and 20 μF.

(b) Since the capacitance of capacitors are in the ratio 1 : 4, the potential drop across them should be in the ratio 4 : 1 making them $4 \times \frac{12}{5} : \frac{12}{5}$. Therefore, V_x = 9.6 Volt and V_y = 2.4 volt.

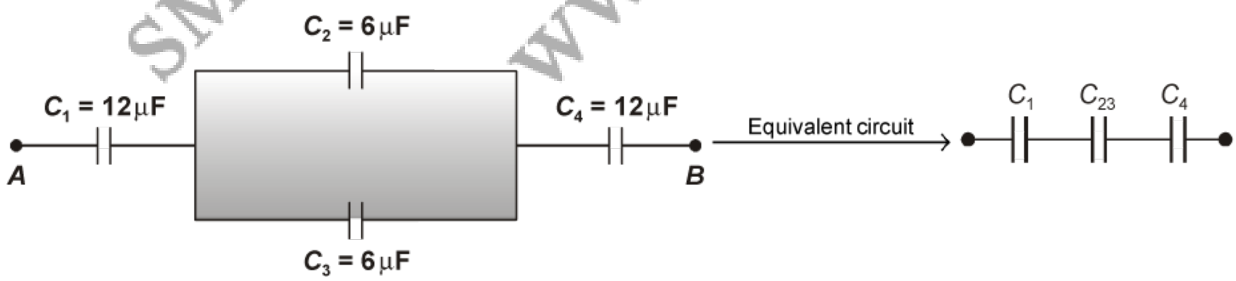
(c) Since they carry same charge, the ratio of the electrostatic is $\frac{Q^2}{2C_x} : \frac{Q^2}{2C_y}$

i.e., $\qquad C_y : C_x = 4 : 1$

S67. Let C_{23} be the equivalent capacitance of the capacitors C_2 and C_3 connected in parallel. Then,

$$C_{23} = C_2 + C_3 = 6 + 6 = 12\ \mu F$$

The given network will be equivalent to series arrangement of the capacitors as shown below in figure.



If C is equivalent capacitance of the series combination, then

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_4} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{1}{4}$$

or $C = 4\ \mu F$

S68. Here, $C_1 = 10\ \mu F = 10 \times 10^{-6}$ F ;

$C_2 = 50\ \mu F = 50 \times 10^{-6}\ F$;

(a) Let q_1 and q_2 be the charge on the two capacitors initially. Then,

$$q_1 = C_1 V_1 = 10 \times 10^{-6} \times 30 = 3 \times 10^{-4}\ \text{C}$$

and $q_2 = C_2 V_2 = 50 \times 10^{-6} \times 0 = 0$

When the charged capacitor C_1 is connected across the uncharged capacitor C_2, the two capacitors from a parallel combination.

If C is capacitance of the combination, then

$$C = C_1 + C_2 = 10 \times 10^{-6} + 50 \times 10^{-6} = 60 \times 10^{-6}\text{F}$$

The final potential difference across the combination,

$$V = \frac{q_1 + q_2}{C} = \frac{3 \times 10^{-4} + 0}{60 \times 10^{-6}} = \mathbf{5V}$$

(b) The initial energy stored in the two capacitors,

$$U_1 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_1V_2^2$$

$$= \frac{1}{2} \times 10 \times 10^{-6} \times (30)^2 + \frac{1}{2} \times 50 \times 10^{-6} \times (0)^2$$

$$= 45 \times 10^{-4} + 0 = 45 \times 10^{-4}\ \text{J}$$

The final energy stored in the combination,

$$U_2 = \frac{1}{2}(C_1 + C_2)V^2$$

$$= \frac{1}{2} \times 60 \times 10^{-6} \times (5)^2 = 7.5 \times 10^{-4} J$$

The difference in energy,

$$U_1 - U_2 = 45 \times 10^{-4} - 7.5 \times 10^{-4} = 37.5 \times 10^{-4}\ \text{J}$$

The difference in energy is lost in the form of heat and electromagnetic radiation, when the two capacitors share charge with each other.

S69. The maximum charges that the two capacitor can store are

$$Q_1 = C_1V_1 = 1.0 \times 10^{-6} \times 6.0 \times 10^3 = 6 \times 10^{-3}\ \text{C}$$
$$Q_2 = C_2V_2 = 2.0 \times 10^{-6} \times 4.0 \times 10^3 = 8 \times 10^{-3}\ \text{C}$$

When the two capacitors are connected in series, their equivalent capacitance is

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1} + \frac{1}{2} = \frac{3}{2}$$

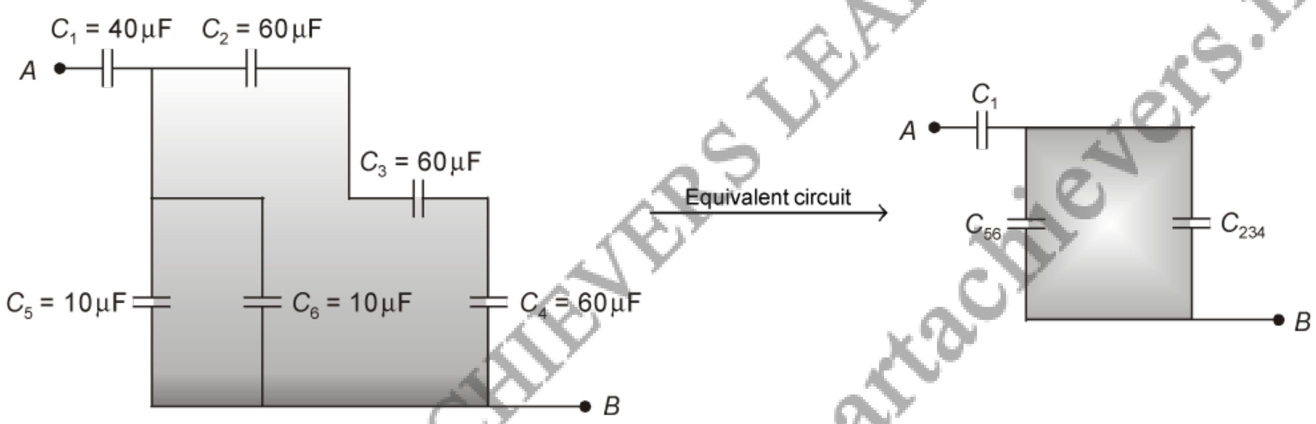
$$C = \frac{2}{3}\mu F$$

In series combination the charge on each capacitor is same, so the maximum should not exceed 6.0×10^{-3} C. Therefore, maximum voltage

$$V_m = \frac{Q}{C} = \frac{6.0 \times 10^{-3} C}{\frac{2}{3} \times 10^{-6}\ F} = 9.0 \times 10^3 \text{ Volt} = 9.0 \text{ kV}.$$

S70. Figure shows the arrangement of the capacitors. The capacitors C_5 and C_6 are in parallel. Their effective capacitance,

$$C_{56} = C_5 + C_6 = 10 + 10 = 20\ \mu\text{F}$$



Also, the capacitors C_2, C_3 and C_4 are in series, so that their effective capacitance is given by

$$\frac{1}{C_{234}} = \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}$$

$$= \frac{1}{60} + \frac{1}{60} + \frac{1}{60} = \frac{1}{20}$$

or $$C_{234} = 20\ \mu F$$

The given arrangement of the capacitors is equivalent to the arrangement as shown in figure below.

Now, the capacitors C_{56} and C_{234} are in parallel. Let C' be their effective capacitance. Then,

$$C' = C_{56} + C_{234} = 20 + 20 = 40\ \mu F$$

The arrangement of the capacitors shown in figure above, is now equivalent to the arrangement shown in figure below.

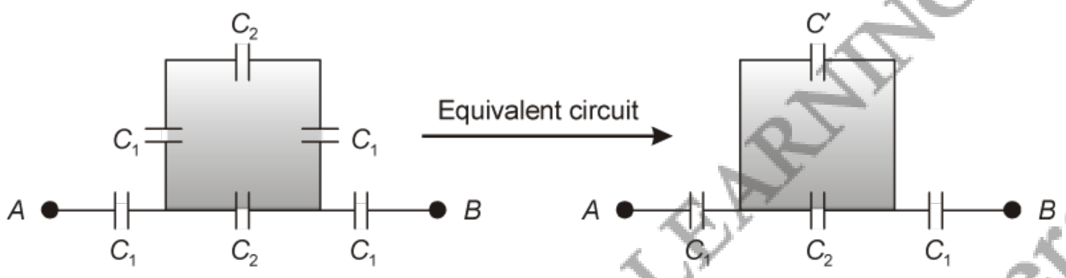
Since the capacitors C_1 and C' are in series, the equivalent capacitance of the combination between the points A and B is given by

$$C = \frac{C_1 \times C'}{C_1 + C'} = \frac{40 \times 40}{40 + 40} = 20\ \mu F$$

Therefore, total charge that flows in the circuit, when a 100 V battery is connected between the point A and B is given by

$$q = C\,V = 20 \times 10^{-6} \times 100 = 2 \times 10^{-3}\ C$$

S71. The given network shown in the above figure is equivalent to the arrangement shown in figure (a), which is further equivalent to the arrangement shown in figure (b).



(a) (b)

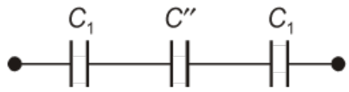
Here, C' is capacitance of the series combination of C_1, C_2 and C_1. Therefore,

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{1}{3} + \frac{1}{2} + \frac{1}{3} = \frac{7}{6}$$

or $$C' = \frac{6}{7}\ pF$$

The arrangement shown in above figure (b) is equivalent to the network drawn in figure.



Here, C'' is capacitance of the parallel combination of C' and C_2. Therefore,

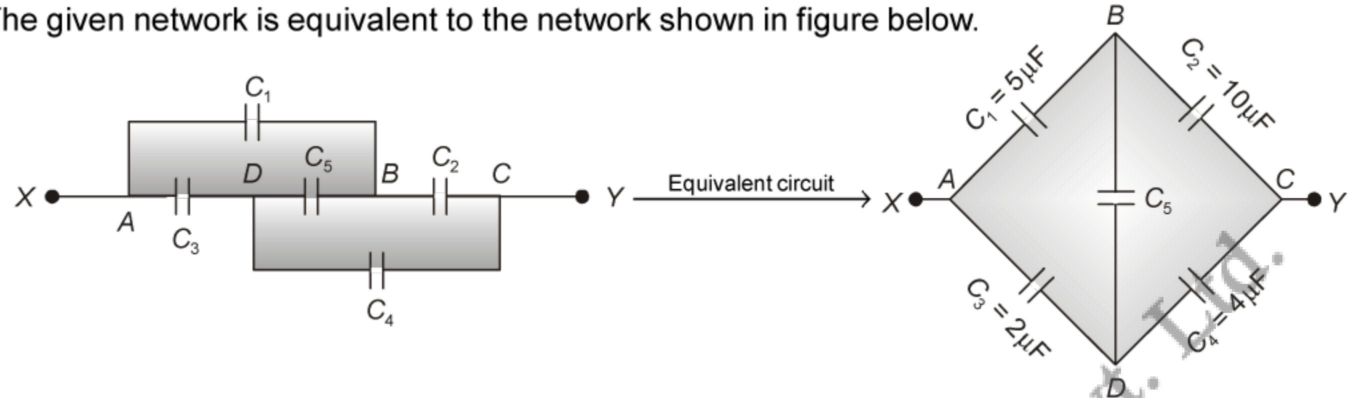
$$C'' = C' + C_2 = \frac{6}{7} + 2 = \frac{20}{7}\ pF$$

If C_{equi} is capacitance of the series combination of C_1, C'' and C_1, then

$$\frac{1}{C_{equi}} = \frac{1}{C_1} + \frac{1}{C''} + \frac{1}{C_1} = \frac{1}{3} + \frac{7}{20} + \frac{1}{3} = \frac{61}{30}$$

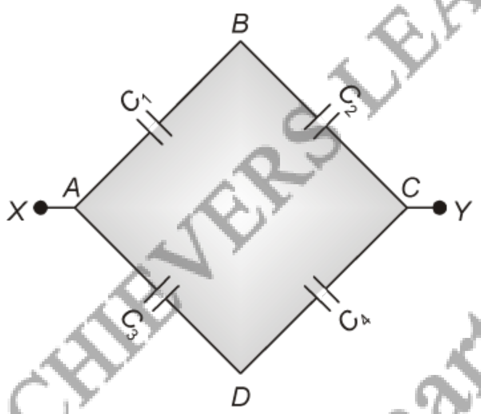
$$C_{equi} = \frac{60}{61} \text{ pF}$$

S72. The given network is equivalent to the network shown in figure below.



We find that $\frac{C_1}{C_2} = \frac{C_3}{C_4}$

Therefore, the bridge is balanced and hence when some potential difference is applied across the terminals X and Y, no current will flow across the path BD. Thus, the path BD behaves as an open path and the given network is equivalent to the network as shown in below.



Let C_{12} and C_{34} be the capacitance of the series combination of C_1, C_2 and C_3, C_4 respectively.

Then, $$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$$

or $$C_{12} = \frac{10}{3} \mu\text{F}$$

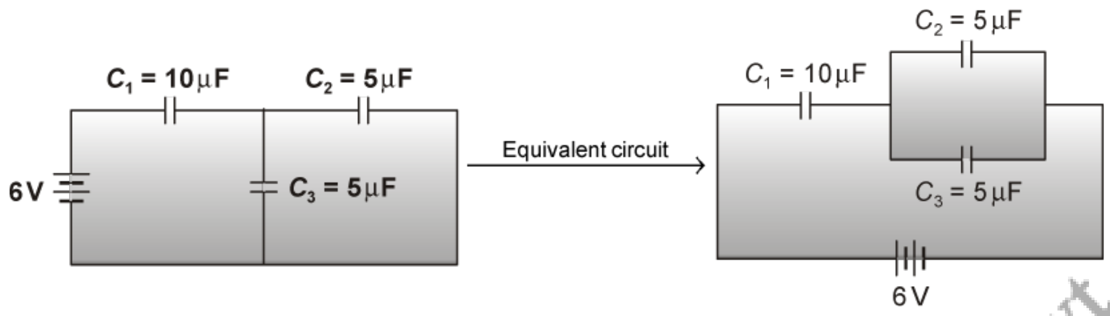
Also, $$\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

or $$C_{34} = \frac{4}{3}\,\mu F$$

Now, C_{12} and C_{34} form a parallel combination. If C is net capacitance of the network, then

$$C = C_{12} + C_{34} = \frac{10}{3} + \frac{4}{3} = \frac{14}{3} = \mathbf{4.67\,\mu F}$$

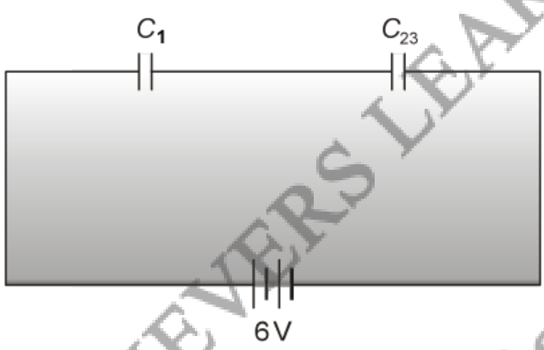
S73. The given arrangement of the three capacitors connected to a battery is equivalent to the arrangement shown in figure below.



Since the capacitors C_2 and C_3 are in parallel, their equivalent capacitance,

$$C_{23} = 5 + 5 = 10\ \mu F$$

The arrangement of the capacitors shown in figure above is now equivalent to the network shown in figure below.



As C_1 and C_{23} are equal, potential difference across each of them will be

$$V = \frac{6}{2} = 3 \text{ volt}$$

Therefore, charge on the capacitor C_1,

$$q_1 = C_1 V = 10 \times 3 = 30\ \mu C$$

Also, charge on the parallel combination of C_2 and C_3,

$$q_{23} = C_{23}\, V = 10 \times 3 = 30\ \mu C$$

As C_2 and C_3 are equal, the charge q_{23} is shared equally by the two capacitors. Therefore,

$$q_2 = q_3 = \frac{30}{2} = \mathbf{15\ \mu C}$$

S74. Let us consider a parallel plate capacitor of plate area A and plate separation d. If the space between the plates is vacuum, its capacitance is given by

$$C_0 = \frac{\varepsilon_0 A}{d}$$

If initially the charges on the plates are $\pm Q$, then the uniform electric field between the capacitor plates is given by

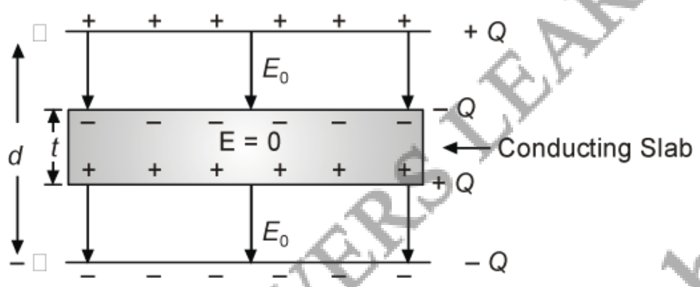
$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

where $\sigma = \frac{Q}{A}$ is the surface charge density of capacitor plate.

$\therefore$ Potential difference across the capacitor plates is

$$V_0 = E_0 d = \frac{Qd}{A\varepsilon_0}$$

when a conducting slab of thickness $t < d$ is introduced between the plates of the capacitor, the free electrons within the conducting slab rearrange themselves so as to reduce the field inside the slab to zero. Induced charges $-Q$ and $+Q$ appear on the upper and lower faces of the slab (as shown in the figure). Thus, electric field exist now only in a distance $(d - t)$ instead of d. The potential difference between the capacitor plates now reduces to



$$V = E_0(d - t) = \frac{Q}{A\varepsilon_0}(d - t)$$

$\therefore$ Capacitance of the capacitor in the presence of conducting slab becomes

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{(d-t)} = \frac{\varepsilon_0 A}{d} \cdot \frac{d}{(d-t)}$$

or $$C = C_0 \cdot \frac{d}{(d-t)}$$

As $$\frac{d}{d-t} > 1 \quad \Rightarrow \quad C > C_0$$

Thus the introduction of a conducting slab increases the capacitance of a capacitor.

S75. Capacitance of a capacitor may be defined as charge required to be supplied to either of the plates of the capacitor so as to raise the potential difference between them by unit amount (or one Volt). That is,

$$\text{Capacitance} = \frac{\text{Charge on either plate}}{\text{Potential diff. between the two plates}} \quad \text{or} \quad C = \frac{Q}{V}$$

The SI unit of capacitance is Farad (F).

$$1 \text{ Farad} = \frac{1\,\text{Coulomb}}{1\,\text{Volt}} \quad \text{or} \quad 1\text{ F} = \frac{1\text{ C}}{1\text{ V}}$$

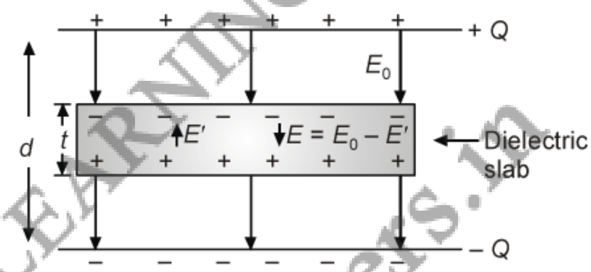
Capacitance of a parallel plate capacitor with a dielectric slab between plates: Let us consider a parallel plate capacitor having plates of area A and separated by a distance d with vacuum between the plates. Its capacitance is given by

$$C_0 = \frac{\varepsilon_0 A}{d}$$

Suppose initially the charges on the capacitor plates are $\pm Q$. Then the uniform electric field between the plates of the capacitor is

$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \qquad \left[\because \sigma = \frac{Q}{A}\right]$$

Now, suppose a dielectric slab of thickness $t (t < d)$ and dielectric constant K is placed between the plates of the capacitor. The field E_0 induces charges on the faces on the slab opposite to those on the capacitor plates. The induced charges produce a field E' in a direction opposite to E_0. Therefore, the net field inside the dielectric is



$$E = E_0 - E' = \frac{E_0}{K} \qquad \left[\because \frac{E_0}{E} = K\right]$$

Thus, field E_0 exists in the region between capacitor plates and dielectric while field exists inside the dielectric. Hence the potential difference between the capacitor plates is given by

$$V = E_0(d - t) + Et = E_0\,(d - t) + \frac{E_0}{K}t = E_0\left(d - t + \frac{t}{K}\right)$$

or $$V = \frac{Q}{A\varepsilon_0}\left(d - t + \frac{t}{K}\right)$$

$\therefore$ Capacitance of the capacitor becomes

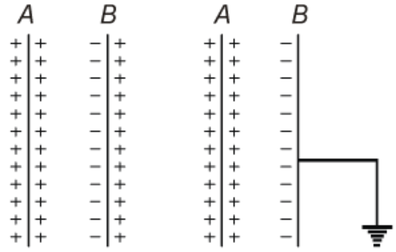
$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$

It $t = d$ *i.e.* the dielectric slab completely fills the entire space between the plates

$$C = \frac{\varepsilon_0 A}{d}\,.\,K \quad \text{or} \quad C = KC_0$$

Thus the capacitance of a parallel plate capacitor increases K times when entire space between the plates is filled by a dielectric of dielectric constant K.

S76. Consider a positively charged metallic plate A, as shown in figure. Suppose another metallic uncharged plate B is placed near the plate A. Due to induction, negative charges appear on the plate B towards the side facing A and positive charges on the other side of B. The negative charge on B tends to reduce the potential of A while its positive charge tends to increase the potential on A. As the negative charge of B is closer to A than positive charge, the net potential of plate A decreases. Plate A now holds same charge at lower potential than before. Thus the capacitance $\left(C = \frac{Q}{V}\right)$ of plate A has increased.



If the plate B is now earthed, the positive charge on B disappears and therefore the potential on plate A gets further lowered. Thus, the capacitance of plate A is further raised. Hence the potential of an isolated conductor (or metallic plate) can be considerably decreased and hence its capacitance can be considerably increased when an earthed conductor is placed near it raising its charge holding capacity. This is the basic principle of a capacitor. Such an arrangement is called a capacitor.

Capacitance of a parallel plate capacitor: A parallel plate capacitor consists of two plane parallel conducting plates separated by a small distance. As shown in figure, let us consider a parallel plate capacitor having each plate of area A and separated by a small distance d. The charge density on the two plates is $+\sigma$ C/m^2 $-\sigma$ C/m^2 respectively so that positively charges plate carries a charge $Q = \sigma A$ and negatively charged plate $Q = -\sigma A$. Electric field outside the plates is zero and that between the plates $E = \frac{\sigma}{\varepsilon_0}$, where ε_0 is the permittivity of free space.

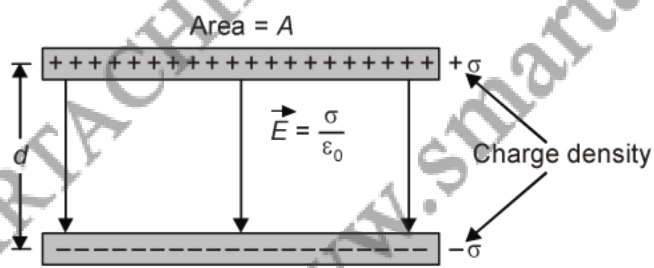


Fig. 3.

∴ Potential difference between the plates,

$$V = Ed = \frac{\sigma d}{\varepsilon_0}$$

$\therefore$ Capacitance of the parallel plate capacitor is given by

$$C = \frac{Q}{V} = \frac{\sigma A}{\frac{\sigma d}{\varepsilon_0}} \quad \text{or} \quad C = \frac{\varepsilon_0 A}{d}$$

S77. (a) On introduction of dielectric slab between the plates of charged capacitor, an additional electrical field due to induced charge produces inside the dielectric medium. Hence, net applied external electrical field gets reduced inside the dielectric. This decreases potential difference between the plates of capacitor. This in turn increases the capacitance of capacitor

as $C \propto \frac{1}{V}$.

(b) Let charge and potential difference across the capacitor at any instant during charging of capacitor are q and V respectively.

$\Rightarrow$ $$q = CV$$

where, C is the capacitance of capacitor.

$\Rightarrow$ $$V = \frac{q}{C} \qquad \text{... (i)}$$

Let dq additional charge be transferred by the battery at potential difference V and work done is given by

$$dW = Vdq$$

$\Rightarrow$ $$dW = \frac{q}{C}\, dq \qquad \text{[From Eq. (i)]}$$

$\therefore$ Total work done over capacitor is charging the capacitor from 0 to q is

$$W = \int_0^q \frac{q}{C} dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^q$$

$$W = \frac{q^2}{2C}$$

$\therefore$ This work done is stored in the form of electrostatic potential energy in capacitor.

$\therefore$ $$U = \frac{q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}qV \qquad \text{[Using } q = CV\text{]}$$

The stored energy on introduction of dielectric medium of dielectric constant K.

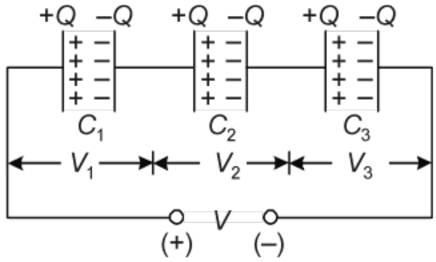
becomes K times if capacitor remains connected with battery as

$$U \propto C \qquad \left[U = \frac{1}{2}CV^2\right]$$

S78. (a) In series combination of capacitors, same charge lie on each capacitor for any value of capacitances. Also, potential difference across the combination is equal to the algebraic sum of potential differences across each capacitor *i.e.*,

$$V = V_1 + V_2 + V_3 \quad \text{... (i)}$$

where, V_1, V_2, V_3 and V are the potential differences across C_1, C_2, C_3 and equivalent capacitor respectively.



Capacitors in series combination

$\therefore$ $$q = C_1V_1$$

$\Rightarrow$ $$V_1 = \frac{q}{C_1}$$

Similarly, $$V_2 = \frac{q}{C_2}$$

$$V_3 = \frac{q}{C_3}$$

[From Eq. (i)]

$\therefore$ Total potential difference

$$V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\frac{V}{q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

[$\because$ $\frac{V}{q} = \frac{1}{C}$, where, C is equivalent capacitance of combination]

or $$\frac{1}{C} = \frac{C_2C_3 + C_3C_1 + C_1C_2}{C_1C_2C_3}$$

$\Rightarrow$ $$C = \frac{C_1C_2C_3}{C_1C_2 + C_2C_3 + C_3C_1}$$

(b) Given, $a = 0.2$ m; $b = 0.3$ m and K = 20;

we know, $$C = 4\pi\varepsilon_0 K \cdot \frac{ab}{b-a}$$

$$= \frac{1}{9\times10^9}\times 20 \times \frac{0.2\times0.3}{0.3-0.2} = 1.333 \times 10^{-9} \text{ F} = \mathbf{1333\ pF}.$$

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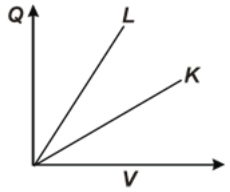
Q1. The following graph shows the variation of charge Q with voltage V, for two capacitors K and L. In which capacitor is more electrostatic energy stored?



Q2. What should be the capacitance of a capacitor capable storing on joule of energy, when used with a 100 V d.c. supply?

Q3. A capacitor is charged through a potential difference of 200 V, when 0.1 C charge is stored in it. How much energy will it release, when it is discharged?

Q4. A charge 5 μC is placed at a point. What is the work required to carry 1 C of charge once round it in a circle of 12 cm radius?

Q5. What will happen if the plates of a charged capacitor are suddenly connected by a metallic wire? Why does the wire become hot?

Q6. In what form of the energy stored in a charged capacitor?

Q7. How much energy will be stored by a capacitor of 470 μF when charged by a battery of 20 V ?

Q8. A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor?

Q9. The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply.

(a) How much electrostatic energy is stored by the capacitor?

(b) View this energy as stored in the electrostatic field between the plates, and obtain the energy per unit volume u. Hence arrive at a relation between u and the magnitude of electric field E between the plates.

Q10. A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

Q11. Why should circuits containing capacitor be handled cautiously, even when there is no current?

Q12. A capacitor charged from a 50 V d.c. supply is found to have a charge of 10 μC. What is the capacitance of the capacitor and how much energy is stored in it?

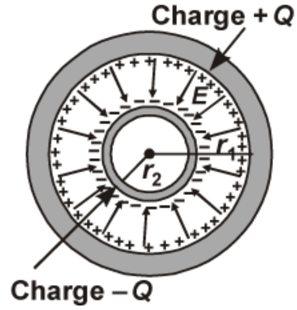
Q13. A parallel plate capacitor is charged to a potential difference V by a d.c. source. The capacitor is then disconnected from the source. If the distance between the plates is doubled, state with reason how the following will change:

(a) electric field between the plates,

(b) capacitance and

(c) energy stored in the capacitor.

Q14. A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports (see figure). Show that the capacitance of a spherical capacitor is given by



$$C = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$$

where r_1 and r_2 are the radii of outer and inner spheres, respectively.

Q15. A 4 μF capacitor is charged by a 200 V supply. It is then disconnected from the supply, and is connected to another uncharged 2 μF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation?

Q16. Show that the force on each plate of a parallel plate capacitor has a magnitude equal to (½) QE, where Q is the charge on the capacitor, and E is the magnitude of electric field between the plates. Explain the origin of the factor ½.

Q17. Define the dielectric constant of medium. Briefly explain the capacitance of a parallel plate capacitor increases, on introducing dielectric medium between the plates.

Q18. A parallel plate capacitor has a capacitance of 2 μF. A slab of dielectric constant 5 is inserted between the plates and the capacitor is charged to 100 V and then isolated,

(a) What is the new potential difference, if the dielectric slab is removed?

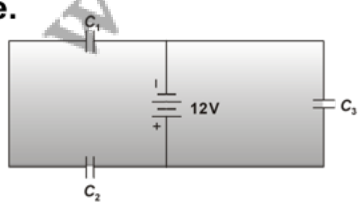
(b) How much work is required to remove the dielectric slab?

Q19. A parallel plate capacitor has capacitance 20 μF. A potential difference of 220 V is applied across it. Calculate the charge on the plates and energy stored in the capacitor. If a dielectric slab of dielectric constant 55 is introduced between its plates, calculate the new value of potential difference and the energy stored.

Q20. A capacitor with a capacitance of 50 μF is connected to a battery of 10 V. Find the charge on it in coulomb and its energy in joule.

Q21. Derive the expression for the energy stored in a parallel plate capacitor with the air between the plates. How does the stored energy change if air is replaced by a medium of dielectric constant K?

Q22. Three identical capacitors C_1, C_2 and C_3 of capacitance 6 μF each are connecte to a 12 V battery as shown in the figure.



Find (a) charge on each capacitor, (b) equivalent capacitance of the network and (c) the energy stored in the network of capacitor

Q23. A capacitor of 200 pF is charged by a 300 V battery. The battery is then disconnected and the charge capacitor is connected to another uncharged capacitor of 100 pF. Calculate the difference between the final energy stored in the combined system and the initial energy stored in the single capacitor.

Q24. (a) Plot a graph comparing the variation of potential, *V* and electric field *E* due to a point charge *Q* as a function of distance *R* from the point charge.

(b) Find the ratio of the potential differences that must be applied across the parallel and the series combination of two capacitors, C_1 and C_2 with their capacitances in the ratio 1 : 2 so that the energy stored, in the two cases, becomes the same.

Q25. Define capacitance of a capacitor. Give its SI unit. For a parallel plate capacitor, prove that the total energy stored in a capacitor is $1/2\ CV^2$ and hence derive the expression for the energy density in a capacitor.

Q26. (a) A parallel plate capacitor is charged by a battery to a potential. The battery is disconnected and dielectric slab is inserted to completely fill the space between the plates. How will (i) its capacitance; (ii) electric field between the plates and (iii) energy stored in the capacitor be affected? Justify your answer giving necessary mathematical expressions for each case.

(b) Sketch the pattern of electric field lines due to (i) a conducting sphere having negative charge on it. (ii) an electric dipole

Q27. Deduce an expression for the total energy stored in a parallel plate capacitor. If several capacitors are connected in series or in parallel, show that the energy would be additive in either case.

Q28. Derive an expression for the energy stored in a capacitor of capacitance *C* when it is charged by connecting it to a battery of potential difference *V*. Now the capacitor is disconnected from the battery. What will be the energy stored in the capacitor, when (a) separation between plates is doubled and (b) an uncharged and identical capacitor is connected across it ?

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S1. The slope of Q-V graph of capacitor gives capacitance. Therefore, capacitor L have greater capacitance. Since, two capacitors are given same voltage V, therefore, energy stored in capacitor will be given by formula.

$$U = \frac{1}{2}CV^2 \text{ and for same } V$$

$$U \propto C$$

$\because$ $C_L > C_K$

$\Rightarrow$ $U_L > U_K$

$\Rightarrow$ Capacitor L will store more electrostatic energy.

S2. Here, $U = 1$ J ; $V = 100$ volt

If C capacitance of the capacitor, then

$$U = \frac{1}{2}CV^2$$

or $$C = \frac{2U}{V^2} = \frac{2 \times 1}{(100)^2} = 2 \times 10^{-4} F$$

S3. Given, $V = 200$ Volt; $q = 0.1$ C

The energy stored,

$$U = \frac{1}{2}qV \times \frac{1}{2} \times 0.1 \times 200 = 10 \text{ J}$$

On being discharged, the capacitor will release the same amount of energy *i.e.*, **10 J**.

S4. It is zero. It is because, all the points on the circular path are at same potential.

S5. The capacitor will be discharged. Current flows form +ve plate to –ve plate, which heats the wire, The energy of the capacitor is converted into heat.

S6. The energy is stored in the capacitor in the form of electric field.

S7. $$E = \frac{1}{2}CV^2 = \frac{1}{2} \times 470 \times 10^{-6} \times 20 \times 20$$

$$= \mathbf{9.4 \times 10^{-2} \text{ J.}}$$

S8. Capacitance of the capacitor,

$$C = 12\,\text{pF} = 12 \times 10^{-12}\,\text{F}$$

Potential difference, $V = 50\,\text{V}$

Electrostatic energy stored in the capacitor is given by the relation,

$$E = \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times 12 \times 10^{-12} \times (50)^2$$

$$= 1.5 \times 10^{-8}\,\text{J}$$

Therefore, the electrostatic energy stored in the capacitor is 1.5×10^{-8} J.

S9. Area of the plates of a parallel plate capacitor, $A = 90\text{ cm}^2 = 90 \times 10^{-4}\,\text{m}^2$

Distance between the plates, $d = 2.5\text{ mm} = 2.5 \times 10^{-3}\,\text{m}$

Potential difference across the plates, $V = 400\,\text{V}$

(a) Capacitance of the capacitor is given by the relation,

$$C = \frac{\varepsilon_0 A}{d}$$

Electrostatic energy stored in the capacitor is given by the relation,

$$E_1 = \frac{1}{2}CV^2$$

$$= \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2$$

Where,

ε_0 = Permittivity of free space

$= 8.85 \times 10^{-12}\,\text{C}^2\,\text{N}^{-1}\,\text{m}^{-2}$

$$\therefore \quad E_1 = \frac{1 \times 8.85 \times 10^{-12} \times 90 \times 10^{-4} \times (400)^2}{2 \times 2.5 \times 10^{-3}}$$

$$= 2.55 \times 10^{-6}\,\text{J}$$

Hence, the electrostatic energy stored by the capacitor is 2.55×10^{-6} J.

Volume of the given capacitor,

$$V' = A \times d$$

$$= 90 \times 10^{-4} \times 25 \times 10^{-3}$$

$$= 2.25 \times 10^{-4}\,\text{m}^3$$

Energy stored in the capacitor per unit volume is given by,

$$u = \frac{E_1}{V'} = \frac{2.55 \times 10^{-6}}{2.25 \times 10^{-4}} = 0.113 \text{ J m}^{-3}$$

(b) Again, $u = \dfrac{E_1}{V'}$

$$= \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{\varepsilon_0 A}{2d}V^2}{Ad} = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2$$

Where, $\dfrac{V}{d}$ = Electric intensity = E

$\therefore$ $u = \dfrac{1}{2}\varepsilon_0 E^2$.

S10. Capacitance of the capacitor, $C = 600\text{ pF}$

Potential difference, $V = 200\text{ V}$

Electrostatic energy stored in the capacitor is given by,

$$E = \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times (600 \times 10^{-12}) \times (200)^2$$

$$= 1.2 \times 10^{-5}\text{ J}$$

If supply is disconnected from the capacitor and another capacitor of capacitance $C = 600\text{ pF}$ is connected to it, then equivalent capacitance (C') of the combination is given by,

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{C}$$

$$= \frac{1}{600} + \frac{1}{600} = \frac{2}{600} = \frac{1}{300}$$

$\therefore$ $C' = 300\text{ pF}$

New electrostatic energy can be calculated as

$$E' = \frac{1}{2} \times C' \times V^2$$

$$= \frac{1}{2} \times 300 \times (200)^2$$

$$= 0.6 \times 10^{-5}\text{ J}$$

Loss in electrostatic energy = $E - E'$

$$= 1.2 \times 10^{-5} - 0.6 \times 10^{-5}$$

$$= 0.6 \times 10^{-5}$$

$$= 6 \times 10^{-6}\ \text{J}$$

Therefore, the electrostatic energy lost in the process is 6×10^{-6} J.

S11. A capacitor does not discharge itself. In case the capacitor is connected in a circuit containing a source of high voltage, the capacitor charges itself to a very high potential. If some person handles such a capacitor without discharging it first, he may get a severe shock.

S12. Given: $V = 50\,\text{V};\quad q = 10\,\mu\text{V}$

$\therefore \qquad C = \dfrac{q}{V} = \dfrac{10}{50} = \mathbf{0.2\,\mu F}$

Energy stored,

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 0.2 \times 10^{-6} \times (50)^2 = \mathbf{2.5 \times 10^{-4}\ J}.$$

S13. (a) Electric field between the plates,

$$E = \frac{V}{d}$$

Therefore, if the distance between the plates is doubled, the electric field will reduce to one-half.

(b) Capacitance of the capacitor,

$$C = \frac{\varepsilon_0 A}{d}$$

Therefore, if the distance between the plates is double, the capacitance will reduce to one-half.

(c) Energy stored in the capacitor,

$$U = \frac{Q^2}{2C}$$

When the distance between the plates is double, the capacitance reduces to one-half. Therefore, energy stored in the capacitor will also **become double**.

S14. Radius of the outer shell = r_1

Radius of the inner shell = r_2

The inner surface of the outer shell has charge + Q.

The outer surface of the inner shell has induced charge – Q.

Potential difference between the two shells is given by,

$$V = \frac{Q}{4\pi\varepsilon_0 r_2} - \frac{Q}{4\pi\varepsilon_0 r_1}$$

Where, ε_0 = Permittivity of free space

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] = \frac{Q(r_1 - r_2)}{4\pi\varepsilon_0 r_1 r_2}$$

Capacitance of the given system is given by,

$$C = \frac{\text{Charge } (Q)}{\text{Potential difference } (V)} = \frac{4\pi\varepsilon_0 r_1 r_2}{r_1 - r_2}$$

Hence, proved.

S15. Capacitance of a charged capacitor,

$$C_1 = 4\,\mu\text{F} = 4 \times 10^{-6}\,\text{F}$$

Supply voltage, $V_1 = 200\text{ V}$

Electrostatic energy stored in C_1 is given by,

$$E_1 = \frac{1}{2} C_1 V_1^2$$

$$= \frac{1}{2} \times 4 \times 10^{-6} \times (200)^2$$

$$= 8 \times 10^{-2}\text{ J}$$

Capacitance of an uncharged capacitor,

$$C_2 = 2\,\mu\text{F} = 2 \times 10^{-6}\,\text{F}$$

When C_2 is connected to the circuit, the potential acquired by it is V_2.

According to the conservation of charge, initial charge on capacitor C_1 is equal to the final charge on capacitors, C_1 and C_2.

$\therefore$ $$V_2(C_1 + C_2) = C_1 V_1$$

$$V_2 \times (4 + 2) \times 10^{-6} = 4 \times 10^{-6} \times 200$$

$$V_2 = \frac{400}{3}\text{ V}$$

Electrostatic energy for the combination of two capacitors is given by,

$$E_1 = \frac{1}{2}(C_1 + C_2)V_2^2$$

$$= \frac{1}{2}(2 \times 4) \times 10^{-6} \times \left(\frac{400}{3}\right)^2$$

$$= 5.33 \times 10^{-2}\text{ J}$$

Hence, amount of electrostatic energy lost by capacitor C_1

$$= E_1 - E_2$$

$$= 0.08 - 0.0533 = 0.0267$$

$$= 2.67 \times 10^{-2}\ \text{J}.$$

S16. Let F be the force applied to separate the plates of a parallel plate capacitor by a distance of Δx. Hence, work done by the force to do so = $F\Delta x$.

As a result, the potential energy of the capacitor increases by an amount given as $uA\Delta x$.

Where,

u = Energy density

A = Area of each plate

d = Distance between the plates

V = Potential difference across the plates

The work done will be equal to the increase in the potential energy *i.e.*,

$$F\Delta x = uA\Delta x$$

$$F = uA = \left(\frac{1}{2}\varepsilon_0 E^2\right)A$$

Electric intensity is given by,

$$E = \frac{V}{d}$$

$$\therefore \quad F = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)EA = \frac{1}{2}\left(\varepsilon_0 A\frac{V}{d}\right)E$$

$$C = \frac{\varepsilon_0 A}{d}$$

However, capacitance,

$$\therefore \quad F = \frac{1}{2}(CV)E$$

Charge on the capacitor is given by,

$$Q = CV$$

$$\therefore \quad F = \frac{1}{2}QE$$

The physical origin of the factor, $\frac{1}{2}$, in the force formula lies in the fact that just outside the conductor, field is E and inside it is zero. Hence, it is the average value, $\frac{E}{2}$, of the field that contributes to the force.

S17. (a) When a dielectric medium is subjected to an external uniform electric field E_0 then, and electric field E_p due to induced charge develops inside the dielectric in a direction opposite to external field E_0 consequently electrical field gets reduced to $E_0 - E_p$.

The ratio of applied field to reduced electrical field inside the dielectric is known as dielectric constant K of the medium *i.e.*,

$$K = \frac{\text{Applied external field}}{\text{Reduced electrical field}}$$

$$K = \frac{E_0}{E_0 - E_p}$$

(b) On introducing of dielectric slab between the plates of charged capacitor, an additional electrical field due to induced charge produces inside the dielectric medium. Hence, net applied external electrical field gets reduced inside the dielectric. This decreases potential difference between the plates of capacitor. This in turn increases the capacitance of capacitor as $C \propto \frac{1}{V}$.

S18. The capacitance of capacitor on introducing dielectric slab,

$$C = 5 \times 2 = 10\,\mu\text{F}$$

$$\therefore \quad q = CV = 10 \times 10^{-6} \times 100 = 10^{-3}\,\text{C}$$

On removing the dielectric slab:

(a) The capacitance of capacitor, $C' = 2\,\mu\text{F}$

$$\therefore \quad V' = \frac{q}{C'} = \frac{10^{-3}}{2 \times 10^{-6}} = 500\text{ V}.$$

(b) Initial energy stored in the capacitor,

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 10 \times 10^{-6} \times (100)^2 = 0.05\text{ J}.$$

Final energy stored in the capacitor

$$U' = \frac{1}{2}C'V'^2 = \frac{1}{2} \times 2 \times 10^{-6} \times (500)^2 = 0.25\text{ J}.$$

Therefore, work done to move the dielectric slab,

$$W = U' - U = 0.25 - 0.05 = \mathbf{0.20\text{ J}}.$$

S19. Given: $q = CV = 20 \times 10^{-6} \times 220 = \mathbf{4.4 \times 10^{-3}\text{ C}}$.

Also, $U = \frac{1}{2}CV^2 = \frac{1}{2} \times 20 \times 10^{-6} \times (220)^2 = \mathbf{0.484\text{ J}}$.

On inserting dielectric:

$$C' = KC = 55 \times 20 \times 10^{-6} = 1.1 \times 10^{-3}\,\text{C}$$

$\therefore$ $$V' = \frac{q}{C'} = \frac{4.4 \times 10^{-3}}{1.1 \times 10^{-3}} = \mathbf{4\,V}$$

and $$U' = \frac{1}{2}C'V'^2 = \frac{1}{2} \times 1.1 \times 10^{-3} \times (4)^2 = \mathbf{8.8 \times 10^{-3}\,J}$$

S20. Given: $C = 50\,\mu\text{F}; \quad V = 10\,\text{V}$

$\therefore$ $$q = CV = 50 \times 10 = \mathbf{500\,\mu C}$$

Energy stored,

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 50 \times 10^{-6} \times (10)^2 = \mathbf{2.5 \times 10^{-3}\,J}.$$

S21. Let charge and potential difference across the capacitor at any instant during charging of capacitor are q and V respectively.

$\Rightarrow$ $$q = CV$$

where, C is the capacitance of capacitor.

$\Rightarrow$ $$V = \frac{q}{C} \quad \text{... (i)}$$

Let dq additional charge be transferred by the battery at potential difference V and work done is given by

$$dW = Vdq$$

$\Rightarrow$ $$dW = \frac{q}{C}dq \quad \text{[From Eq. (i)]}$$

$\therefore$ Total work done over capacitor is charging the capacitor from 0 to q is

$$W = \int_0^q \frac{q}{C}\,dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^q$$

$$W = \frac{q^2}{2C}$$

$\therefore$ This work done is stored in the form of electrostatic potential energy in capacitor.

$\therefore$ $$U = \frac{q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}qV \quad \text{[Using } q = CV\text{]}$$

The stored energy on introduction of dielectric medium of dielectric constant K.

becomes K times if capacitor remains connected with battery as

$$U \propto C \quad \left[U = \frac{1}{2}CV^2\right]$$

S22. Given: $C_1 = C_2 = C_3 = 6\,\mu\text{F}$ and $V = 12\,\text{V}$.

(a) Charge on capacitor C_3,

$$q_3 = C_3V = 6 \times 12 = \mathbf{72\,\mu C}.$$

Since capacitors C_1 and C_2 are of equal capacitance, potential difference a across each of them will be 12/2 = 6 V.

$$\therefore \quad q_1 = q_2 = 6 \times 6 = \mathbf{36\,\mu C} \qquad (\because\ q = C \times V)$$

(b) The equivalent capacitance of the network,

$$C = \frac{C_1C_2}{C_1 + C_2} + C_3 = \frac{6 \times 6}{6 + 6} + 6 = 3 + 6 = \mathbf{9\,\mu F}$$

(c) Energy stored in the network,

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times 9 \times 10^{-6} \times (12)^2 = \mathbf{6.48 \times 10^{-4}\,J}.$$

S23. Given, $C = 200\text{ pF} = 200 \times 10^{-12}\text{ F}$

$V = 300\text{ V}$

The energy (initial) stored by the capacitor is

$$U_i = CV^2$$

$$= \frac{1}{2} \times 200 \times 10^{-12} \times 300 \times 300$$

$$= 9 \times 10^{-6}\text{ J}$$

When two capacitors are connected, they have their positive plates at the same potential and negative plates also at the same potential. Let V' be the common potential difference. By charge conservation, charge would distribute but total charge would remain constant.

Thus, $Q = q + q'$

$$\frac{q}{C} = \frac{q'}{C'}$$

$$\frac{q}{200} = \frac{q'}{100}$$

$$q = 2q'$$

Thus, $Q = 2q' + q' = 3q'$

So, $q' = \frac{Q}{3} = \frac{60\,\text{nC}}{3} = 20$ nC

and $q = 2q' = 40$ nC

Thus, final energy

$$U_f = \frac{q^2}{2C} + \frac{q'^2}{2C'}$$

$$= \frac{1}{2} \times \frac{(40 \times 10^{-9})^2}{200 \times 10^{-12}} + \frac{1}{2} \times \frac{(20 \times 10^{-9})^2}{100 \times 10^{-12}}$$

$$= 4 \times 10^{-6} + 2 \times 10^{-6} = 6 \times 10^{-6} \text{ J}$$

Difference in energy = final energy – initial energy

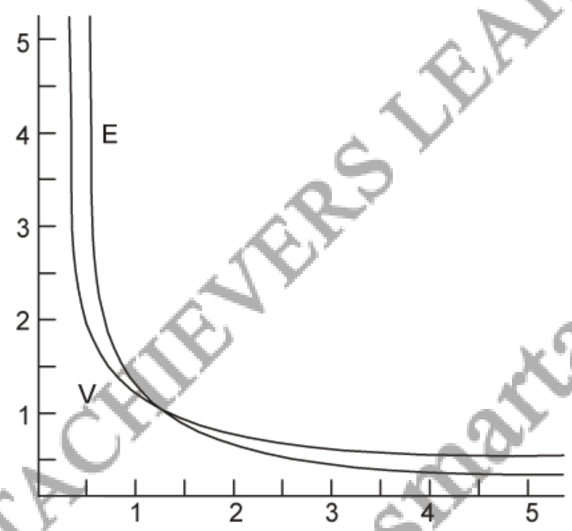
$$= U_f - U_i$$

$$= 6 \times 10^{-6} - 9 \times 10^{-6}$$

$$= -3 \times 10^{-6}$$

Thus, difference in energy is 3×10^{-6} J.

S24. (a) The graph comparing the variation of potential V and electric field refer to



(b) Let common factor of capacitor is C. $C_1 = 1C$, $C_2 = 2C$

$\therefore$ Equivalent capacitance

In series, $C_s = \frac{2C \times 1C}{2C + 1C} = \frac{2C}{3}$

In parallel, $C_p = 2C + 1C = 3C$

$\therefore$ V_p and V_s are potential difference across the final capacitor in parallel and series combination respectively, to have same potential energy.

$$U_p = U_s$$

$$\frac{1}{2}C_pV_p^2 = \frac{1}{2}C_sV_s^2$$

$$\frac{V_p}{V_s} = \sqrt{\frac{C_s}{C_p}}$$

$$= \sqrt{\frac{(2C/3)}{(3C)}} = \sqrt{\frac{2}{9}}$$

$$V_p : V_s = \sqrt{2} : 3$$

S25. Capacitance is a property of a capacitor which has storing the charge, and SI units farad (F).

Energy stored in a capacitor: Let us consider a capacitor of capacitance C. If the charges on the two plates are Q and $-Q$ when the capacitor is connected to a battery of potential V volts, then

$$V = \frac{Q}{C}$$

Suppose, the battery supplies a charge dQ to the capacitor at constant potential V, then the small amount of work done by the battery is given by

$$dW = V\,dQ = \frac{Q}{C}.dQ$$

The total work done in delivering a charge Q is given by

$$W = \int dW = \int_0^Q \frac{Q}{C}.dQ = \frac{1}{C}\int_0^Q Q\,dQ = \frac{1}{C}\left|\frac{Q^2}{2}\right|_0^Q$$

or $$W = \frac{1}{2}\frac{Q^2}{C}$$

This work done is stored as electrostatic potential energy U in the capacitor.

$\therefore$ Energy stored $$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}.\frac{(CV)^2}{C} = \frac{1}{2}CV^2$$

Energy density: In a parallel plate capacitor of plate area A and plate separation d, the capacitance is equal to

$$C = \frac{\varepsilon_0 A}{d}$$

If σ is the surface charge density on the capacitor plates then the electric field E and total charge Q is given by

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{and} \quad Q = \sigma A$$

or $$Q = \varepsilon_0 EA$$

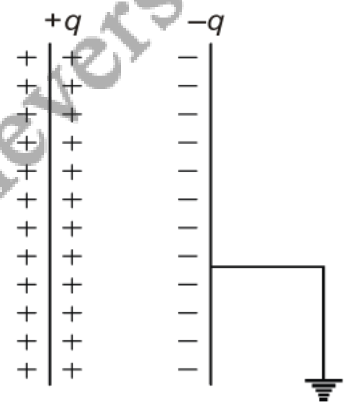
$\therefore$ Energy stored in the capacitor is

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(\varepsilon_0 EA)^2}{\frac{\varepsilon_0 A}{d}} = \varepsilon_0 E^2 Ad$$

As $Ad = V$, volume of capacitor

$\therefore$ Energy density $$U = \frac{U}{V} = \frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2$$

S26. (a) **Principle of parallel plate capacitor:** The capacitance of an insulated conductor increases significantly by bringing an uncharged near to it. This combination/set-up forms parallel plate capacitor.



(i) $\because$ Charge on capacitor = magnitude of charge on any of the two plates

$\therefore$ $$q = \sigma A$$

and potential difference, $V = \dfrac{\sigma d}{\varepsilon_0}$

$\therefore$ Capacitance, $$C = \frac{q}{V} = \frac{\sigma A}{\left(\frac{\sigma d}{\varepsilon_0}\right)} = \frac{\varepsilon_0 A}{d}$$

(ii) The electric field at a point between the plates

$$E = \frac{\sigma}{\varepsilon_0}$$

and directed from positive plate to negative plates.

(iii) Energy stored intially

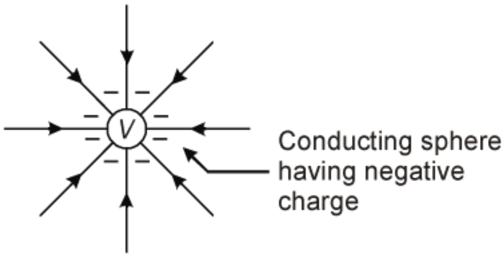
$$U = \frac{q^2}{2C}$$

Energy stored later

$\therefore \qquad U' = \frac{q^2}{2(KC)} \qquad [\because \quad C' = KC]$

where, K = dielectric constant of medium

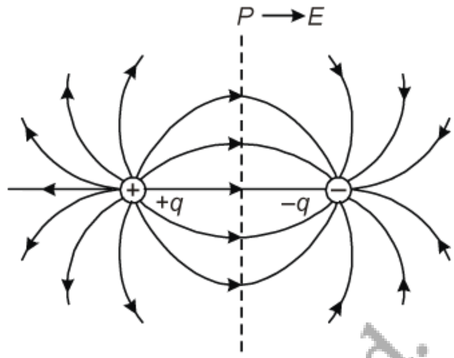
$$\Rightarrow \qquad U' = \frac{1}{K}\left(\frac{q^2}{2C}\right) = \frac{1}{K}(U) = \frac{1}{K} \times U$$



The energy stored in capacitor decrease and become $\frac{1}{K}$ times of original energy.

(b) (i) Electric field lines due to a conducting sphere as shown in the figure

(ii) Electric field lines due to an electric dipole are shown in figure.



Field lines of and electric dipole

S27. **Energy stored in a capacitor:** Let us consider a capacitor of capacitance C. If the charges on the two plates are Q and $-Q$ when the capacitor is connected to a battery of potential V volts, then

$$V = \frac{Q}{C}$$

Suppose, the battery supplies a charge dQ to the capacitor at constant potential V, then the small amount of work done by the battery is given by

$$dW = V\,dQ = \frac{Q}{C}\,.\,dQ$$

The total work done in delivering a charge Q is given by

$$W = \int dW = \int_0^Q \frac{Q}{C}\,.\,dQ = \frac{1}{C}\int_0^Q Q\,dQ = \frac{1}{C}\left|\frac{Q^2}{2}\right|_0^Q$$

or $\qquad W = \frac{1}{2}\frac{Q^2}{C}$

This work done is stored as electrostatic potential energy U in he capacitor.

$\therefore$ Energy stored $\qquad U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}.\frac{(CV)^2}{C} = \frac{1}{2}CV^2$

Total energy of capacitors in series combination.

For a series combination

$$Q = \text{constant (same in all capacitors)}$$

and $$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$$

∴ Total energy $$U = \frac{Q^2}{2C} = \frac{1}{2}.Q^2\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...\right]$$

$$= \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3} + ...$$

or $$U = U_1 + U_2 + U_3 + ...$$

Thus, total energy in series combination of capacitors is equal to the sum of the energies of individual capacitors.

Total energy of capacitors in parallel combination.

For a parallel combination

V = constant (same for all capacitors)

and $$C = C_1 + C_2 + C_3 + ...$$

$$U = 1/2\ C_1V^2 + 1/2\ C_2V^2 + 1/2\ C_3V^2 + ...$$

$$U = U_1 + U_2 + U_3 + ...$$

Thus, total energy in parellel combination of capacitors is equal to the sum of the energies of individual charges.

S28. Let charge and potential difference across the capacitor at any instant during charging of capacitor are q and V respectively.

⇒ $$q = CV$$

where, C is the capacitance of capacitor.

⇒ $$V = \frac{q}{C} \quad ...\text{(i)}$$

Let dq additional charge be transferred by the battery at potential difference V and work done is given by

$$dW = Vdq$$

⇒ $$dW = \frac{q}{C}dq \quad \text{[From Eq. (i)]}$$

∴ Total work done over capacitor is charging the capacitor from 0 to q is

$$W = \int_0^q \frac{q}{C}\,dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^q$$

$$W = \frac{q^2}{2C}$$

∴ This work done is stored in the form of electrostatic potential energy in capacitor.

$$\therefore \qquad U = \frac{q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}qV \qquad \text{[Using } q = CV\text{]}$$

The stored energy on introduction of dielectric medium of dielectric constant K.
becomes K times if capacitor remains connected with battery as

$$U \propto C \quad \left[U = \frac{1}{2}CV^2\right]$$