Date: 9/10/2021

Q1. If $P = \{x : x < 3, x \in N\}$, $Q = \{x : x \le 2, x \in W\}$. Find $(P \cup Q) \times (P \cap Q)$, where W is the set of whole numbers.

Q2. Is the following relation a function? Justify your answer.

(i) $R_1 = \left\{(2,3), \left(\frac{1}{2}, 0\right), (2,7), (-4,6)\right\}$ (ii) $R_2 = \{(x, |x|) \mid x \text{ is a real number}\}$.

Q3. Find the domain for which functions $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal.

Q4. Find the range of the following functions given by $\frac{|x-4|}{x-4}$.

Q5. Redefine the function which is given by

$$f(x) = |x - 1| + |x + 1|, \quad -2 \le x \le 2.$$

Q6. Find the domain of the function f given by:

$$f(x) = \frac{1}{\sqrt{[x]^2 - [x] - 6}}.$$

Q7. Find the domain of the following function given by: $f(x) = x|x|$.

Q8. Find the domain of the following function given by: $f(x) = \frac{1}{\sqrt{1 - \cos x}}$.

Q9. Find the value of x for which the functions $f(x) = 3x^2 - 1$ and $g(x) = 3 + x$ are equal.

Q10. If $R_3 = \{(x, |x|) : x \text{ is a real number}\}$ is a relation. Then find domain and range of R_3.

Q11. If $R_1 = \{(x, y) \mid y = 2x + 7, \text{ where } x \in R_1 \text{ and } -5 \le x \le 5\}$ is a relation, then find the domain and range of R_1.

Q12. Let f and g be real functions defined by $f(x) = 2x + 1$ and $g(x) = 4x - 7$.

(i) For what real numbers x, $f(x) = g(x)$ (ii) For what real numbers x, $f(x) < g(x)$

Q13. Find the domain of the following function given by: $f(x) = \frac{1}{\sqrt{x - 5}}$.

Q14. Find the range of the following function given by: $f(x) = \frac{3}{2 - x^2}$.

Q15. Find the domain of the following function given by: $\frac{3x}{28 - x}$.

Q16. Find the domain of the following function given by: $f(x) = \frac{x^2 - x + 3}{x^2 - 1}$.

Q17. Let $A = \{1, 2, 3, 4)$ and $B = \{5, 7, 9\}$. Determine

(i) $A \times B$ (ii) $B \times A$ (iii) Is $A \times B = B \times A$? (iv) Is $n(A \times B) = n(B \times A)$?

Q18. Let $A = \{-1, 2, 3\}$ and $B = \{1, 3)$. Determine:

(i) $A \times B$ (ii) $B \times A$ (iii) $B \times B$ (iv) $A \times A$

Q19. Find x and y if:

(i) $(4x + 3, y) = (3x + 5, -2)$ (ii) $(x - y, x + y) = (6, 10)$

Q20. Find the domain and range of the relation R given by:

$$R = \left\{(x, y) : y = x + \frac{6}{x};\ \text{ where } x, y \in N \text{ and } x < 6\right\}.$$

Q21. If $A = \{x : x \in W, x < 2\}$, $B = \{x : x \in N, 1 < x < 5\}$ and $C = \{3, 5\}$, find

(i) $A \times (B \cap C)$ (ii) $A \times (B \cup C)$

Q22. In each of the following cases, find a and b:

(i) $(2a + b, a - b) = (8, 3)$ (ii) $\left(\frac{a}{4}, a - 2b\right) = (0, 6 + b)$

Q23. If f and g are real functions defined by $f(x) = x^2 + 7$ and $g(x) = 3x + 5$, find each of the following:

(i) $f(3) + g(-5)$ (ii) $f\left(\frac{1}{2}\right) \times g(14)$ (iii) $f(-2) + g(-1)$

(iv) $f(t) - f(-2)$ (v) $\frac{f(t) - f(5)}{t - 5}$

Q24. Is the given relation a function? Give reasons for your answer:

(i) $h = \{(4, 6), (3, 9), (-11, 6), (3, 11)\}$ (ii) $f = \{(x, x) \mid x$ is a real number$\}$

(iii) $g = \left\{\left(n, \frac{1}{n}\ n \text{ is a positive integer}\right)\right\}$ (iv) $S = \{(n, n^2) \mid n$ is a positive integer$\}$

(v) $t = \{(x, 3) \mid x$ is a real number$\}$

Q25. If $R_2 = \{(x, y) \mid x$ and y are integers and $x^2 + y^2 = 64\}$ is a relation. Then find R_2.

Q26. Find the domain of each of the following functios:

(i) $f(x) = \frac{x}{x^2 + 3x + 2}$ (ii) $f(x) = [x] + x$.

Q27. If f and g are two real valued functions defined as $f(x) = 2x + 1$, $g(x) = x^2 + 1$

(i) $f + g$ (ii) $f - g$ (iii) fg (iv) $\frac{f}{g}$

Q28. Let $f(x) = \sqrt{x}$ and $g(x) = x$ be two functions defined in the domain $R^+ \cup \{0\}$. Find

(i) $(f + g)(x)$ (ii) $(f - g)(x)$ (iii) $(fg)(x)$ (iv) $\left(\frac{f}{g}\right)(x)$

Q29. If $f(x) = \frac{x - 1}{x + 1}$, then show that:

(i) $f\left(\frac{1}{x}\right) = -f(x)$ (ii) $f\left(-\frac{1}{x}\right) = \frac{-1}{f(x)}$

Q30. **Express the following functions as set of ordered pairs and determine their range. $f : x \rightarrow R$, $f(x) = x^3 + 1$, where $x = \{-1, 0, 3, 9, 7\}$.**

Q31. **Is $g = \{(1, 1), (2, 3), (3, 5), (4, 7)$ is a function? Justify. If this is described by the relation, $g(x) = \alpha x + \beta$, then what values should be assigned to α and β?**

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S1. Let

$$P = \{1, 2\}$$
$$Q = \{0, 1, 2\}$$
$$P \cup Q = \{0, 1, 2\}$$
$$P \cap Q = \{1, 2\}$$
$$(P \cup Q) \times (P \cap Q) = \{(0, 1), (0, 2), (1, 1), (1, 2), (2, 1), (2, 2)\}.$$

S2. (i) Since (2, 3) and (2, 7) $\in R_1$

$\Rightarrow$ $R_1(2) = 3$ and $R_1(2) = 7$

So, $R_1(2)$ does not have a unique image. Thus R_1 is not a function.

(ii) $R_2 = \{(x, |x|) / |x| \in \boldsymbol{R}\}$

For every $x \in \boldsymbol{R}$ there will be unique image as $|x| \in \boldsymbol{R}$.

Therefore R_2 is a function.

S3. For $f(x) = g(x)$

$\Rightarrow$ $2x^2 - 1 = 1 - 3x$

$\Rightarrow$ $2x^2 + 3x - 2 = 0$

$\Rightarrow$ $2x^2 + 4x - x - 2 = 0$

$\Rightarrow$ $2x(x + 2) - 1(x + 2) = 0$

$\Rightarrow$ $(2x - 1)(x + 2) = 0$

Thus, domain for which the function f $(x) = g(x)$ is $\left\{\frac{1}{2}, -2\right\}$.

S4.

Let, $$f(x) = \frac{|x-4|}{x-4} = \begin{cases} \dfrac{x-4}{x-4} = 1, & x \geq 4 \\ \dfrac{-(x-4)}{x-4} = -1, & x < 4 \end{cases}$$

Thus, the range of $\frac{|x-4|}{x-4} = \{1, -1\}$.

S5. Given, $f(x) = |x - 1| + |x + 1|, \quad -2 \leq x \leq 2$

$$= \begin{cases} -x + 1 - 1 - x, & -2 \leq x < -1 \\ -x + 1 + x + 1, & -1 \leq x < 1 \\ x - 1 + 1 + x, & 1 \leq x \leq 2 \end{cases}$$

$$= \begin{cases} -2x, & -2 \leq x < -1 \\ 2, & -1 \leq x < 1 \\ 2x, & 1 \leq x \leq 2 \end{cases}.$$

S6. Given $f(x) = \dfrac{1}{\sqrt{[x]^2 - [x] - 6}}$

f is defined if $[x]^2 - [x] - 6 > 0$

or $([x] - 3)([x] + 2) > 0$

$\Rightarrow$ $[x] < -2$ or $[x] > 3$

$\Rightarrow$ $x < -2$ or $x \geq 4$

Hence, Domain = $(-\infty, -2) \cup [4, \infty)$.

S7. Given, $f(x) = x\,|x|$

Clearly, Domain = R (real numbers).

S8. Given $f(x) = \dfrac{1}{\sqrt{1 - \cos x}}$

$f(x)$ is not defined for $1 - \cos x = 0$

$\cos x = 1$

$\cos x = \cos 0°$

$x = 2n\pi, \quad n \in I$

Thus, Domain of $f(x) = R - \{2n\pi : n \in I\}$

S9. Let $f(x) = g(x)$

$3x^2 - 1 = 3 + x$

$\Rightarrow$ $3x^2 - x - 4 = 0$

$\Rightarrow$ $3x^2 - 4x + 3x - 4 = 0$

$\Rightarrow$ $x(3x - 4) + 1(3x - 4) = 0$

$\Rightarrow$ $(3x - 4)(x + 1) = 0$

$\Rightarrow$ $x = \dfrac{4}{3}, -1.$

S10. Here, Domain of $R_3 = R$

Range of $R_3 = R^+ \cup \{0\}$.

S11. Given, $R_1 = \{(x, y) \mid y = 2x + 7, \text{ where } x \in R_1 \text{ and } -5 \leq x \leq 5\}$

Domain = $\{-5, 5\}$

Range = $\{-3, 17\}$.

S12. (i) Let $f(x) = g(x)$

$\Rightarrow$ $(2x + 1) = 4x - 7$

$\Rightarrow$ $2x - 4x = -7 - 1$

$\Rightarrow$ $-2x = -8$

$\Rightarrow$ $x = 4.$

(ii) Let $f(x) < g(x)$

$\Rightarrow$ $(2x + 1) < 4x - 7$

$\Rightarrow$ $4x - 7 > 2x + 1$

$\Rightarrow$ $2x > 8$

Hence, $x > 4$.

S13. Given $f(x) = \dfrac{1}{\sqrt{x-5}}$

For function to be defined,

$$\sqrt{x-5} > 0$$

$\Rightarrow$ $x - 5 > 0 \Rightarrow x > 5$

Domain of $f = (5, \infty)$

Range of $f = R^+$.

S14. Given $f(x) = \dfrac{3}{2-x^2}$

$f(x)$ is not defined for $2 - x^2 = 0$

$\Rightarrow$ $x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

$$\text{Range} = \left[\frac{3}{2}, \infty\right].$$

S15. Given $f(x) = \dfrac{3x}{28-x}$

Now, let $28 - x = 0 \Rightarrow x = 28$

Thus, Domain $= R \sim \{28\}$

S16. Given $f(x) = \dfrac{x^2 - x + 3}{x^2 - 1}$

$f(x)$ is not defined for $x^2 - 1 = 0$

$$x = \pm 1$$

Hence, Domain $= R - \{-1, 1\}$.

S17. Since, $A = \{1, 2, 3, 4)$ and $B = \{5, 7, 9\}$. Therefore

(i) $\therefore$ $A \times B = \{(1, 5), (1, 7), (1, 9), (2, 5), (2, 7), (2, 9)$
$(3, 5), (3, 7), (3, 9), (4, 5), (4, 7), (4, 9)\}$

(ii) $\therefore$ $B \times A = \{(5, 1), (5, 2), (5, 3), (5, 4), (7, 1), (7, 2),$
$(7, 3), (7, 4), (9, 1), (9, 2), (9, 3), (9, 4)\}$

(iii) No, $A \times B \neq B \times A$. Since, $A \times B$ and $B \times A$ do not have exactly the same ordered pairs.

(iv) $\because$ $n(A \times B) = n(A) \times n(B) = 4 \times 3 = 12$

$n(B \times A) = n(B) \times n(A) = 4 \times 3 = 12$

Hence, $n(A \times B) = n(B \times A)$

S18. Given $A = \{-1, 2, 3\}$ and $B = \{1, 3\}$

(i) $A \times B = \{(-1, 1), (-1, 3), (2, 1), (2, 3), (3, 1), (3, 3)\}$

(ii) $B \times A = \{(1, -1), (1, 2), (1, 3), (3, -1), (3, 2), (3, 3)\}$

(iii) $B \times B = \{(1, 1), (1, 3), (3, 1), (3, 3)\}$

(iv) $A \times A = \{(-1, -1), (-1, 2), (-1, 3), (2, -1), (2, 2), (2, 3), (3, -1), (3, 2), (3, 3)\}$

S19. (i) Since, $(4x + 3, y) = (3x + 5, -2)$

So, $4x + 3 = 3x + 5$

or $x = 2$ and $y = -2$

(ii) Let $x - y = 6$ $x + y = 10$

$\therefore$ $2x = 16$ or $x = 8$

$8 - y = 6$

$\therefore$ $y = 2.$

S20. When $x = 1$, $y = 7 \in N$, so $(1, 7) \in R$. Again for

$x = 2$, $y = 2 + \frac{6}{2} = 2 + 3 = 5$ N, so $(2, 5) \in R$. Again for

$x = 3$, $y = 3 + \frac{6}{3} = 3 + 2 = 5 \in N$, $(3, 5) \in R$. Similarly for $x = 4$

$y = 4 + \frac{6}{4} \notin N$ and for $x = 5$, $y = 5 + \frac{6}{5} \notin N$.

Thus, $R = \{(1, 7), (2, 5), (3, 5)\}$

where, Domain of $R = \{1, 2, 3\}$

Range of $R = \{7, 5\}$.

S21. Let $A = \{0, 1\}$, $B = \{2, 3, 4\}$, $C = \{3, 5\}$

(i) $A = \{0, 1\}$

$B \cap C = \{3\}$

$A \times (B \cap C) = \{(0, 3), (1, 3)\}$

(ii) $A = \{0, 1\}$

$B \cup C = \{2, 3, 4, 5\}$

$A \times (B \cup C) = \{(0, 2), (0, 3), (0, 4), (0, 5), (1, 2), (1, 3), (1, 4), (1, 5)\}$

S22. (i) Let $2a + b = 8$

$a - b = 3$

Adding we get $3a = 11 \Rightarrow a = \frac{11}{3}$

$\therefore \quad \frac{11}{3} - b = 3 \quad \Rightarrow \quad b = \frac{11}{3} - 3 = \frac{2}{3}$

Hence, $\quad a = \frac{11}{3}, \quad b = \frac{2}{3}$

(ii) $\quad \frac{a}{4} = 0 \quad \Rightarrow \quad a = 0$

and $\quad a - 2b = 6 + b$

$\Rightarrow \quad 0 - 2b = 6 + b$

$-3b = 6 \quad \Rightarrow \quad b = -2$

Hence, $\quad a = 0, \quad b = -2.$

S23. Let $\quad f(x) = x^2 + 7 \quad$ and $\quad g(x) = 3x + 5$

(i)
$$\begin{aligned} f(3) + g(-5) &= [(3)^2 + 7)] + [3(-5) + 5] \\ &= (9 + 7) + (-15 + 5) \\ &= 16 - 10 = 6 \end{aligned}$$

(ii)
$$\begin{aligned} f\left(\frac{1}{2}\right) g(14) &= \left[\left(\frac{1}{2}\right)^2 + 7\right][3 \times 14 + 5] \\ &= \left(\frac{1}{4} + 7\right)(42 + 5) \\ &= \frac{29}{4} \times 47 = \frac{1363}{4} \end{aligned}$$

(iii)
$$\begin{aligned} f(-2) + g(-1) &= [(-2)^2 + 7] + [3(-1) + 5] \\ &= (4 + 7) + (-3 + 5) \\ &= 11 + 2 = 13. \end{aligned}$$

(iv)
$$\begin{aligned} f(t) - f(-2) &= (t^2 + 7) - [(-2)^2 + 7] \\ &= t^2 + 7 - (4 + 7) \\ &= t^2 + 7 - 11 = t^2 - 4 \end{aligned}$$

(v)
$$\begin{aligned} \frac{f(t) - f(5)}{t - 5} &= \frac{[t^2 + 7] - [(5)^2 + 7]}{t - 5} \\ &= \frac{t^2 - (5)^2}{t - 5} = \frac{(t + 5)(t - 5)}{t - 5} \\ &= t + 5. \end{aligned}$$

S24. (i) Given $h = \{(4, 6), (3, 9), (-11, 6), (3, 11)\}$

h is a function if element of set A has one and only one image in set B. In this case 3 has two images 9 and 11.

Hence, h is **not a function**.

(ii) Given $f = \{(x, x) \mid x \text{ is a real number}\}$

In this case every distinct element of domain has distinct image in Range.

Hence, f is a **function**.

(iii) Given $g = \left\{\left(n, \frac{1}{n} \middle| n \text{ is a positive integer}\right)\right\}$

Here again every element of domain has a distinct image in Range.

Hence, g is a **function**.

(iv) Given $S = \{(n, n^2) \mid n \text{ is a positive integer}\}$

We know that every positive integer of domain has its square as its image in the range.

Hence, S is a **function**

(v) Given $t = \{(x, 3) \mid x \text{ is a real number}\}$

Hence, $t(x) = 3$

Thus, t is a **constant function**.

S25. For $x^2 + y^2 = 64$

$\Rightarrow \quad x^2 + y^2 = (8)^2$

$\therefore \quad x = 0, 8, -8$

When $x = 0, \quad y = 8$

When $x = 0, \quad y = -8$

When $x = 8, \quad y = 0$

When $x = -8, \quad y = 0$

Hence, $R_2 = \{(0, 8), (0, -8), (8, 0), (-8, 0)\}$.

S26. (i) f is a rational function of the form $\frac{g(x)}{h(x)}$, where $g(x) = x$ and $h(x) = x^2 + 3x + 2$.

Now, $h(x) \neq 0 \Rightarrow x^2 + 3x + 2 \neq 0 \Rightarrow (x + 1)(x + 2) \neq 0$ and hence domain of the given function is $R - \{-1, -2\}$.

(ii) Let $f(x) = [x] + x \quad i.e., \quad f(x) = h(x) + g(x)$

where, $h(x) = [x] \quad \text{and} \quad g(x) = x$

The domain of h = $\boldsymbol{R}$ and the domain of $g = R$.

Therefore, Domain of f = $\boldsymbol{R}$.

S27. (i)

$$f + g = (f + g)(x)$$
$$= f(x) + g(x)$$
$$= 2x + 1 + x^2 + 1$$
$$= x^2 + 2x + 2.$$

(ii)

$$f - g = f(x) - g(x)$$
$$= (2x + 1) - (x^2 + 1)$$
$$= 2x + 1 - x^2 - 1$$
$$= -x^2 + 2x.$$

(iii)

$$fg = f(x) \cdot g(x)$$
$$= (2x + 1)(x^2 + 1)$$
$$= 2x^3 + x^2 + 2x + 1.$$

(iv)

$$\frac{f}{g} = \frac{2x+1}{x^2+1}.$$

S28. (i)

$$(f + g)(x) = f(x) + g(x)$$
$$= \sqrt{x} + x$$

(ii)

$$(f - g)(x) = f(x) - g(x)$$
$$= \sqrt{x} - x$$

(iii)

$$(fg)(x) = f(x) \cdot g(x)$$
$$= \sqrt{x}\,(x)$$
$$= x^{3/2}$$

(iv)

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}}.$$

S29. (i) Given

$$f(x) = \frac{x-1}{x+1}$$

$\therefore$

$$\text{LHS.} = f\left(\frac{1}{x}\right) = \frac{\frac{1}{x} - 1}{\frac{1}{x} + 1} = \frac{-x+1}{x+1}$$

$$= -\left(\frac{x-1}{x+1}\right) = -f(x) = \text{R.H.S.}$$

(ii) Let

$$f\left(-\frac{1}{x}\right) = \frac{-\frac{1}{x} - 1}{-\frac{1}{x} + 1} = \frac{-x-1}{x-1}$$

$$= -\left(\frac{x+1}{x-1}\right) = -\left(\frac{1}{\frac{x-1}{x+1}}\right)$$

$$= -\frac{1}{f(x)} = \text{R.H.S.}$$

S30. Let $f(x) = x^3 + 1$

$\therefore$ $f(-1) = (-1)^3 + 1 = 0$

$f(0) = 0^3 + 1 = 1$

$f(3) = (3)^3 + 1 = 28$

$f(9) = (9)^3 + 1 = 730$

$f(7) = (7)^3 + 1 = 344$

$f = \{(-1, 0), (0, 1), (3, 28), (9, 730), (7, (344)\}$

Hence, Range of $f = \{0, 1, 28, 344, 730\}$.

S31. Yes, g is a function.

Also, $g(x) = \alpha x + \beta$

$\therefore$ $g(1) = 1 \Rightarrow \alpha + \beta = 1$ ... (i)

$g(2) = 3 \Rightarrow 2\alpha + \beta = 3$ ... (ii)

From Eq, (i) and (ii) on subtraction,

$(2\alpha + \beta) - (\alpha + \beta) = 3 - 1$

$\Rightarrow$ $\alpha = 2$

$\Rightarrow$ $2 + \beta = 1 \Rightarrow \beta = -1$

Hence, $\alpha = 2, \quad \beta = -1$.