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Q1. Define nuclear fusion.

Q2. Upto what distances nuclear forces operate?

Q3. How nuclear forces are dependent of charge?

Q4. Name the strongest known force.

Q5. How does the nuclear (mass) density depend on the size of the nucleus?

Q6. Name the quantities that are conserved during the course of any nuclear reaction.

Q7. Write the relation connecting the radius R of a nucleus and its mass number.

Q8. All protons is an atom remain packed in a small nucleus inspite of the electrostatic repulsive force among them. Why?

Q9. Two nuclei have mass numbers in the ratio 1 : 2. What is the ratio of the nuclear densities?

Q10. What will be the ratio of the radii of two nuclei of mass numbers A_1 and A_2?

Q11. How many electrons, protons and neutrons are there in a nucleus of atomic number 11 and mass number 24?

Q12. Given the mass of iron nucleus as 55.85 u and $A = 56$, find the nuclear density?

Q13. Calculate the energy equivalent of 1 g of substance.

Q14. Differentiate between isotopes and isobars with suitable examples.

Q15. The radius of ${}_{12}Al^{27}$ nucleus is estimated to be 6 fermi. Find the radius of ${}_{52}Te^{125}$ nuclei.

Q16. The radius of oxygen nucleus (${}_{8}O^{16}$) is 2.8×10^{-15} m. Calculate the radius of the lead nucleus (${}_{82}Pb^{205}$).

Q17. Why is the density of nucleus more than that of the atom?

Q18. The three stable isotopes of neon: ${}^{20}_{10}Ne$, ${}^{21}_{10}Ne$ and ${}^{22}_{10}Ne$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of the three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon.

Q19. Why heavy stable nuclei must contain more neutrons than protons?

Q20. (a) Two stable isotopes of lithium ${}^{6}_{3}Li$ and ${}^{7}_{3}Li$ have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u, respectively. Find the atomic mass of lithium.

(b) Boron has two stable isotopes, ${}^{10}_{5}B$ and ${}^{11}_{5}B$. Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is 10.811 u. Find the abundances of ${}^{10}_{5}B$ and ${}^{11}_{5}B$.

Q21. Answer the following, giving reasons

(a) Why is the binding energy per nucleon found to be constant for nuclei in the range of mass number (*A*) lying between 30 and 170?

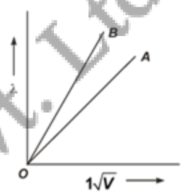
(b) When a heavy nucleus with mass number *A* = 240, breaks into two nuclei, *A* = 120, energy is related in the process.

(c) In β-decay the experimental detection of neutrinos (or antineutrinos) is found to be exactly different nuclei. Briefly state, how nuclear fusion and nuclear fission can be explained on the basic of this graph?

Q22. Assuming that protons and neutrons have equal masses, calculate how many times nuclear matter is denser than water. Given that nuclear radius is given by $R = 1.2 \times 10^{-15} A^{1/3}$ metre and mass of a nucleon = 1.67×10^{-27} kg.

Q23. A nucleus of mass number 225 splits into two fresh nuclei having mass numbers in the ratio 3 : 2. If the nuclear radius is given by $R = 1.1 \times 10^{-15} A^{1/3}$ m, find the radii of the new nuclei formed.

Q24. The two lines marked *A* and *B* in figure below shows a plot of de-Broglie wavelength (λ) as a function of $1\sqrt{V}$ (*V* is the accelerating potential) for two nuclei $_1H^2$ and $_1H^3$.



(a) What does the slope of the lines represent?

(b) Identify, which liens correspond to these nuclei?

Q25. In a periodic table the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on Earth. The three isotopes and their masses are $^{24}_{12}Mg$ (23.98504 u), $^{25}_{12}Mg$ (24.98584 u) and $^{26}_{12}Mg$ (25.98259 u). The natural abundance of $^{24}_{12}Mg$ is 78.99% by mass. Calculate the abundances of other two isotopes.

Q26. Distinguish between nuclear fission and fusion. In a fusion reaction

$$^2_1H + ^3_1H + \rightarrow ^4_2He + n,$$

calculate the amount of energy, in MeV released. Given,

$m(^2_1H)$ = 2.014102u; $m(^3_1H)$ = 3.016049 u; $m(^4_2He)$ = 4.002603*u*; m_n = 1.00867 u;

1u = 931.5 MeV

Q27. If the nucleons of a nucleus are separated far apart from each other, the sum of masses of all these nucleus is larger than the mass of the nucleus. Where does this mass difference come from?

Calculate the energy released, if ^{238}U nucleus emits an α-particle.

Given,

Atomic mass of ^{238}U = 238.0508 u   Atomic mass of ^{234}U = 234.04363 u

Atomic mass of α-particle = 4.00260 u and 1 u = 931 MeV

Q28. Calculate the amount of energy released during the α-decay of

$$^{238}_{92}U \rightarrow ^{234}_{90}Th + ^4_2He$$

Given,

(a) Atomic mass $^{238}_{92}U$ = 238.05079 u   (b) Atomic mass $^{234}_{90}Th$ = 234.04363 u

(c) Atomic mass of ^4_2He = 4.00260 u

1u = 931.5 MeV/c^2 Is this decay spontaneous? Given reason.

Q29. (a) Discuss the basic principle working of a nuclear reactor with labeled diagram.

(b) The fission properties of $^{239}_{94}Pu$ are very similar to those of $^{235}_{92}U$. The average energy released per fission is 180 MeV. How much energy in MeV, is released if all the atoms is 1 kg of pure $^{239}_{94}Pu$ undergo fission?

Q30. (a) What is nuclear fission and nuclear fusion? Give one example, each of fission and fusion.

(b) The fission of one nucleus of $_{92}U^{235}$ releases 200 MeV energy. How many fissions should occur per second for producing a power of 1 MW ($1\ eV = 1.6 \times 10^{-19}\ J$).

Q31. What is a nuclear reactor? With the help of a labelled diagram, describe the construction and working of a nuclear reactor.

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S1. The phenomenon of fusion of two or more lighter nuclei to form a single heavier nucleus is called ***nuclear fusion***.

S2. A few Fermi.

S3. Nuclear forces are independent of charge.

S4. Nuclear force.

S5. The nuclear density is independent of the size of the nucleus.

S6. Momentum, charge and energy (K.E. + rest mass energy) are conserved during the course of any nuclear reaction.

S7.
$$R = R_0 \,.\, A^{1/3}$$

Where, $R_0 = 1.1 \times 10^{-15}$ m.

S8. Inside the nucleus, the electrostatic force of repulsion between the protons is extremely large. They are held inside the nucleus due to nuclear force between them, which is basically a strong attractive force.

S9. Nuclear density is independent of the mass number of nucleus. Since the two nuclei having mass numbers in the ratio 1 : 2 (or 1 : 3) have the same nuclear density, ratio of their nuclear densities is 1.

S10. Let R_1 and R_2 be radii of the nuclei of mass numbers A_1 and A_2 respectively. Then, from the relation $R = R_0 A^{1/3}$, it follows that

$$\frac{R_1}{R_2} = \frac{R_0 A_1^{1/3}}{R_0 A_2^{1/3}} = \left(\frac{A_1}{A_2}\right)^{1/3}.$$

S11. Number of electrons or protons, Z = **11** and number of neutrons, $A - Z = 24 - 11 =$ **13**.

S12.
$$m_{Fe} = 55.85, \quad u = 9.27 \times 10^{-26} \text{ kg}$$

$$\text{Nuclear density} = \frac{\text{Mass}}{\text{Volume}}$$

$$= \frac{9.27 \times 10^{-26}}{(4\pi/3)(1.2 \times 10^{-15})^3} \times \frac{1}{36}$$

$$= 2.29 \times 10^{17} \text{ kg m}^{-3}$$

The density of matter in neutron stars (an astrophysical object) is comparable to this density. This shows that matter in these objects has been compressed to such an extent that they resemble a *big nucleus*.

S13. Energy, $E = 10^{-3} \times (3 \times 10^8)^2$ J $\quad [\because \; E = \Delta mc^2]$

$$E = 10^{-3} \times 9 \times 10^{16} = 9 \times 10^{13} \text{ J}$$

Thus, if one gram of matter is converted to energy, there is a release of enormous amount of energy.

S14. Differentiate between isotopes and isobars

Isotopes: The atoms of an element, which have the same atomic number but different mass numbers, are called ***isotopes***.

Examples: (a) ${}_8O^{16}, {}_8O^{17}, {}_8O^{18}$, (b) ${}_{17}Cl^{35}, {}_{17}Cl^{37}$ (c) ${}_{82}Pb^{206}, {}_{82}Pb^{207}, {}_{82}Pb^{208}$

Isobars: The atoms, which have the same mass number but different atomic numbers, are called ***isobers***.

Examples: (a) ${}_1H^3$ and ${}_2He^3$, (b) ${}_3Li^7$ and ${}_4Be^7$ (c) ${}_{18}Ar^{40}$ and ${}_{20}Ca^{40}$

S15. Let R_1 and R_2 be radii of the ${}_{13}Al^{27}$ and ${}_{52}Te^{125}$ nuclei of mass numbers $A_1(= 27)$ and $A_2(= 125)$ respectively. Then,

$$R_2 = R_1 \times \left(\frac{A_2}{A_1}\right)^{1/3}$$

$$= 6 \times \left(\frac{27}{125}\right)^{1/3} = 6 \times \frac{3}{5} = \mathbf{3.6\ fermi}.$$

S16. **For oxygen:**

$$R = R_0 A^{1/3}$$

or $$R_0 = \frac{R}{A^{1/3}} = \frac{2.8 \times 10^{-15}}{(16)^{1/3}} = \frac{2.8 \times 10^{-15}}{2.52} = \mathbf{1.11 \times 10^{-15}\ m}$$

For lead:

$$R = R_0 A^{1/3} = 1.11 \times 10^{-15} \times (205)^{1/3}$$
$$= 1.11 \times 10^{-15} \times 5.90 = \mathbf{6.55 \times 10^{-15}\ m}.$$

S17. The size of nucleus is of the order of 10^{-15} m and it is made of protons and neutrons only.

On the other hand, size of atom is of the order of 10^{-10} m. Whereas, the size of atom increases by a very large factor (10^5 times as large as the nucleus), its mass increases only by a small amount (additional mass is the mass of orbital electrons only). For the reason, the density of nucleus is very large as compared to that of the atom.

S18. Atomic mass of $^{20}_{10}Ne$, $m_1 = 19.99\,u$

Abundance of $^{20}_{10}Ne$, $\eta_1 = 90.51\%$

Atomic mass of $^{21}_{10}Ne$, $m_2 = 20.99\,u$

Abundance of $^{21}_{10}Ne$, $\eta_2 = 0.27\%$

Atomic mass of $^{22}_{10}Ne$, $m_3 = 21.99\,u$

Abundance of $^{22}_{10}Ne$, $\eta_3 = 9.22\%$

The average atomic mass of neon is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2 + m_3\eta_3}{\eta_1 + \eta_2 + \eta_3}$$

$$= \frac{19.99 \times 90.51 + 20.99 \times 0.27 + 21.99 \times 9.22}{90.51 + 0.27 + 9.22}$$

$$= 20.1771\,u.$$

S19. The heavy nuclei tend to become unstable due to large number of protons inside it. For heavy nuclei to be stable, the electrostatic repulsive force between protons must be dominated by the attractive nuclear force resulting from neutron-neutron, neutron-proton and proton-proton interactions. So that it can happen, the number of neutrons must be greater than the number of protons.

S20. (a) Mass of lithium isotope $^{6}_{3}Li$, $m_1 = 6.01512\,u$

Mass of lithium isotope $^{7}_{3}Li$, $m_2 = 7.01600\,u$

Abundance of $^{6}_{3}Li$, $\eta_1 = 7.5\%$

Abundance of $^{7}_{3}Li$, $\eta_2 = 92.5\%$

The atomic mass of lithium atom is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2}$$

$$= \frac{6.01512 \times 7.5 + 7.01600 \times 92.5}{92.5 + 7.5}$$

$$= 6.940934\,u$$

(b) Mass of boron isotope $^{10}_{5}B$, $m_1 = 10.01294\,u$

Mass of boron isotope $^{11}_{5}B$, $m_2 = 11.00931\,u$

Abundance of $^{10}_{5}B$, $\eta_1 = x\,\%$

Abundance of $^{11}_{5}B$, $\eta_2 = (100 - x)\,\%$

Atomic mass of boron, $m = 10.811\,u$

The atomic mass of boron atom is given as:

$$m = \frac{m_1\eta_1 + m_2\eta_2}{\eta_1 + \eta_2}$$

$$10.811 = \frac{10.01294 \times x + 11.00931 \times (100 - x)}{x + 100 - x}$$

$$10.811 = 10.01294x + 1100.931 - 11.00931x$$

$\therefore$ $$x = \frac{19.821}{0.99637} = 19.89\%$$

And $100 - x = 80.11\%$

Hence, the abundance of ${}^{10}_{5}B$ is 19.89% and that of ${}^{11}_{5}B$ is 80.11%.

S21. (a) The *BE* per nucleon for nucleus of range, $30 < A < 170$ is close to its maximum value. So, the nucleus belongs to this region is highly stable and does not shows radioactivity.

(b) After breaking the nucleus nuclei with $A = 120$ is formed, when these nuclei fuses to for more tightly bounded nucleus.

(c) In β-decay, the detection of a neutrino is found to be difficult. A neutrino hits a proton in H-atom and collision occurred may be as of the three kinds. Neutrino do not have any change which implies that they are unaffected in the region of electromagnetic forces.

S22. Given: $R = 1.2 \times 10^{-15} A^{1/3}$ m; mass of one nucleon = 1.67×10^{-27} kg.

Let us calculate the density of the nucleus of mass number *A*.

Then, mass of the nucleus = $1.67 \times 10^{-27} \times A$ kg.

Volume of the nucleus = $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(1.2 \times 10^{-15} A^{1/3})^3$ m^3

Therefore, density of the nucleus

$$\rho_{nuc} = \frac{1.67 \times 10^{-27} \times A}{\frac{4}{3}\pi(1.2 \times 10^{-15} A^{1/3})^3} = \frac{3 \times 1.67 \times 10^{18}}{4\pi \times (1.2)^3}$$

$$= 2.307 \times 10^{17} \text{ kg m}^{-3}.$$

Now, density of water, $\rho_{wat} = 10^3$ kg m^{-3}

$\therefore$ $$\frac{\rho_{nuc}}{\rho_{wat}} = \frac{2.307 \times 10^{17}}{10^3} = \mathbf{2.307 \times 10^{14}}.$$

S23. Given: $A = 225$; $R = 1.1 \times 10^{-15} A^{1/3}$ m

Let A_1 and A_2 be the respective mass numbers of the two new nuclei formed. Then,

$$A_1 = \frac{3}{3+2} \times A = \frac{3}{5} \times 225 = 135$$

and $$A_2 = \frac{2}{3+2} \times A = \frac{2}{5} \times 225 = 90$$

Let R_1 and R_2 be the respective radii of the two new nuclei formed. Then,

$$R_1 = 1.1 \times 10^{-15} A_1^{1/3} = 1.1 \times 10^{-15} \times 135^{1/3}$$

$$= 1.1 \times 10^{-15} \times 5.13 = \mathbf{5.643 \times 10^{-15}\,m}$$

$$R_2 = 1.1 \times 10^{-15} A_2^{1/3} = 1.1 \times 10^{-15} \times 90^{1/3}$$

$$= 1.1 \times 10^{-15} \times 4.48 = \mathbf{4.93 \times 10^{-15}\,m}.$$

S24. In terms of accelerating potential V, the de-Broglie wavelength of a charged particle is given by

$$\lambda = \frac{h}{\sqrt{2meV}}, \quad \text{... (i)}$$

where e is charge and m, mass of the particle.

The equation (i) represents a straight line, whose slope is $h/\sqrt{2me}$.

(a) The slope of the line is inversely proportional to $\sqrt{m}$.

(b) Since the slope of line A is lesser, it represents the particle of heavier mass.

S25. Average atomic mass of magnesium, $m = 24.312\,u$

Mass of magnesium isotope $^{24}_{12}Mg$, $m_1 = 23.98504\,u$

Mass of magnesium isotope $^{25}_{12}Mg$, $m_2 = 24.98584\,u$

Mass of magnesium isotope $^{26}_{12}Mg$, $m_3 = 25.98259\,u$

Abundance of $^{24}_{12}Mg$, $\eta_1 = 78.99\%$

Abundance of $^{25}_{12}Mg$, $\eta_2 = x\,\%$

Hence, abundance of $^{26}_{12}Mg$, $\eta_3 = 100 - x - 78.99\% = (21.01 - x)\,\%$

We have the relation for the average atomic mass as:

$$m = \frac{m_1\eta_1 + m_2\eta_2 + m_3\eta_3}{\eta_1 + \eta_2 + \eta_3}$$

$$24.312 = \frac{23.98504 \times 78.88 + 24.98584 \times x + 25.98259 \times (21.01 - x)}{100}$$

$$24.312 = 1894.5783096 + 24.98584x + 545.8942159 - 25.98259x$$

$$0.99675x = 9.2725255$$

$\therefore$ $x \approx 9.3\%$

And $21.01 - x = 11.71\%$

Hence, the abundance of ${}^{25}_{12}Mg$ is 9.3% and that of ${}^{26}_{12}Mg$ is 11.71%.

S26.

S. No.	Nuclear fission	Nuclear fusion
1	Heavy nucleus get split up into two smaller nucleus of comparable mass.	Two lighter nuclei combined together to form a heavier nucleus.
2	The physical condition viz, high temperature and pressure are not essentiall for fission.	The conditions of high pressure and temperature are essential are essential for nuclear fusion.

Given fusion reaction,

$${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + n$$

Total mass of the reactants

$$= 2.014102 + 3.016049$$
$$= 5.030151 \text{ u}$$

Total mass of the product

$$= 4.002603 + 1.00867$$
$$= 5.011273 \text{ u}$$

$\Rightarrow$ Mass defect = 0.018878 u

$\therefore$ Energy release $= 0.018878 \times 931.5$ MeV

$$= 17.584857 \text{ MeV}$$

S27. The energy consumed in separating the nucleus far apart from each other in against of binding forces, energy is converted into the mass as per Einstein's mass energy relation, $E = mc^2$ and hence mass difference occurs.

a-decay of ${}^{238}U$

$${}^{238}U \xrightarrow{\alpha} {}^{234}U + \alpha$$

The mass of parent nuclei ${}^{238}U$, $m_1 = 238.0.508$ u

The mass of product nuclei m_2 = mass of ${}^{234}U$ + mass of a particle

$$= 234.04363 + 4.00260 = 238.04623$$

Mass defect $m = m_1 - m_2$

$$= 238.0508 - 238.04623$$
$$= 0.00457$$

$\because$ Energy released $= 0.00457 \times 931$ meV

$$= 4.25 \text{ meV.}$$

S28. ${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$

The mass of parent nucleus $m_1 = 238.05079$ u

The mass of product nuclei m_2 = mass of ${}^{234}_{90}Th$ + mass of ${}^{4}_{2}He$.

$= 234.04363 + 4.00260$

$= 238.04623$ u

Mass defect $\Delta m = m_1 - m_2$

$= 238.05079 - 238.04623$

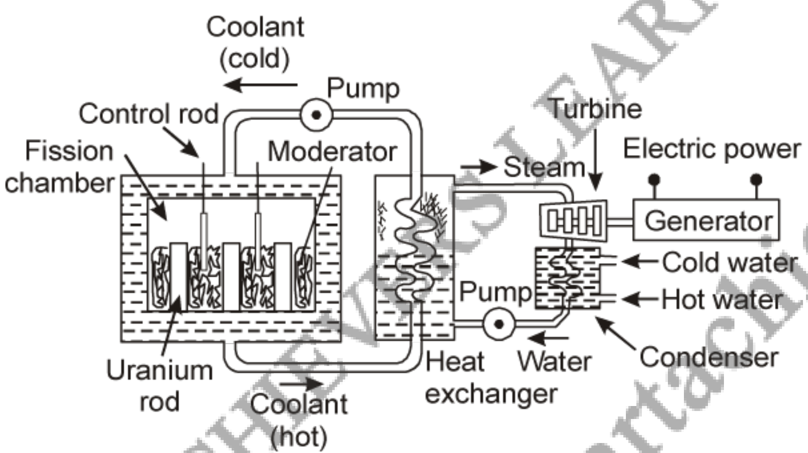
$= 0.00456$u

Energy released $= 0.00456 \times 931$ MeV

$= 4.25$ MeV.

The above decay is spontaneous, as the nucleus has the tendency to convert as the more stable nucleus.

S29. (a) A nuclear reactor is a device used to generate electric energy besides other applications. It works on the principle of controlled chain reaction of nuclear fission process.



Control rods: To control the chain reaction, rods of neutron absorbing material like boron or cadmium are inserted into the reactor core. As a result of it, the desired number of neutrons are absorbed and only limited number of neutrons are left to produce further fission. The depth of control rod inside the reactor control the number of neutrons absorbed.

Working: In the beginning, some neutrons are produced by the action of α-particles on beryllium. These neutrons are slowed down and are used to initiate fission of U^{235} nuclei.

Fast neutrons liberated are slowed down by the moderator. These thermal neutrons causes fission of more U^{235} nuclei and controlled chain reaction build up with help of control rods. To stop chain reaction control rods are inserted deep inside the core.

(b) 239 gms of Pu contain 6.023×10^{23} atoms

1000 gm of Pu contain $\frac{6.023}{239} \times 1000 \times 10^{23} = 2.52 \times 10^{24}$ atoms

Fission of each atom releases 180 MeV.

$\therefore$ Energy released due to fission of 1 kg ${}^{239}_{94}Pu$ or 2.52×10^{24} atoms

$$= 2.52 \times 10^{24} \times 180 \text{ MeV} = \mathbf{4.53 \times 10^{26} \text{ MeV}}.$$

S30. (a) ***Nuclear fission:*** Nuclear fission is the phenomenon of splitting of a heavy nucleus (usually of mass number greater than 230) into two (or more) lighter nuclei. For example, nuclear fission of ${}_{92}U^{235}$ when it is hit by a neutron is represented as

$${}_{92}U^{235} + {}_{0}n^{1} \longrightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_{0}n^{1} + Q$$

Where Q is the energy released in the process. In the process, certain mass disappears *i.e.*, sum of the masses of final products is found to be slightly less than the sum of the masses of the reactant components. This difference in masses is called mass defect (Δm). The mass defect appears in the form energy in accordance with Einstein mass-energy relation, $E = \Delta m.c^2$.

Nuclear fusion: Nuclear fusion is the phenomenon in which two or more lighter nuclei fuse to form a single heavy nucleus. The nuclear fusion takes place under the conditions of very high temperature ($\approx 10^7$ K) and pressure.

$${}_{1}H^{2} + {}_{1}H^{2} \longrightarrow {}_{2}He^{4} + Q$$

where Q is the energy released in the process. The mass of the product nucleus is slightly less than the sum of the masses of the lighter nuclei fusion together. This difference in masses (called mass defect Δm) results in the release of tremendous amount of energy, in accordance with Einstein's mass energy relation, $E = \Delta m.c^2$.

(b) Energy in one fission = 200 MeV

$$= 200 \times 10^6 \text{ eV}$$

$$= 200 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

Energy to be produced = 1 MW = 10^6 W = 10^6 J/s

$\therefore$ Required number of fission per second

$$= \frac{10^6}{200 \times 10^6 \times 1.6 \times 10^{-19}} \text{ sec}^{-1}$$

$$= 3.12 \text{ X } 10^{16} \text{ sec}^{-1}$$

S31. A nuclear reactor is a device used to generate electric energy besides other applications. It works on the principle of controlled chain reaction of nuclear fission process.

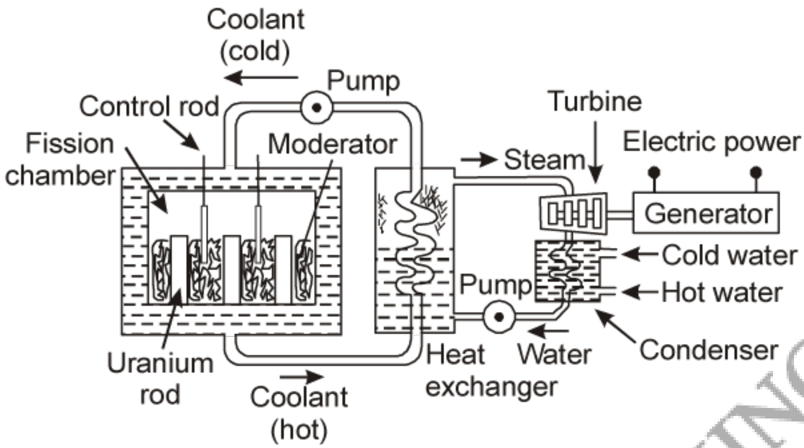
A labeled diagram of a nuclear reactor is shown in figure. the main components of a nuclear reactor are:

(a) ***Nuclear fuel:*** It is a fissionable material *e.g.*, U^{235}, Th^{232}, Pu^{239}, etc. The fuel is sealed in aluminum cylinders. This cylinders are arranged in fission chamber inside reactor. When slow neutrons bombard on the fuel, fission starts.

(b) ***Moderator:*** The neutrons released by the fission of uranium have high energies of the order of 2 MeV. However, for a controlled chain reaction, slowly moving neutrons are required. Moderators are the substances (rich in protons) used to slow down the fast moving neutrons.

Heavy water, graphite, deuterium, paraffin, etc. are some of the commonly used moderators.

When fast moving neutrons collide with the protons of moderator, their energies are interchanged and thus the neutrons are slowed down. The slowed down neutrons are called *thermal neutrons*.



(c) ***Control rods:*** To control the chain reaction, rods of neutron absorbing material like boron or cadmium are inserted into the reactor core. As a result of it, the desired number of neutrons are absorbed and only limited number of neutrons are left to produce further fission. The depth of control rod inside the reactor control the number of neutrons absorbed.

(d) ***Coolant:*** A large amount of heat is produced due to nuclear fission inside the reactor. The material which is used to remove the heat produce and transfer it from the core of the nuclear reactor to the surrounding is called coolant. At ordinary temperatures, liquid sodium is used as a coolant. Air at high pressure is used as coolant.

The coolant releases the heat energy to the water in a heat exchanger. Thus, superheated steam is produced which drives the turbines of generator.

(e) **Shielding:** Whole nuclear reactor is enclosed in thick concrete walls called protective shield so that nuclear radiations do not produce harmful effects on the people in the surrounding area.

Working: In the beginning, some neutrons are produced by the action of α-particles on beryllium. These neutrons are slowed down and are used to initiate fission of U^{235} nuclei.

Fast neutrons liberated are slowed down by the moderator. These thermal neutrons causes fission of more U^{235} nuclei and controlled chain reaction build up with help of control rods. To stop chain reaction control rods are inserted deep inside the core.

Date: 22/10/2021

Q1. Define binding energy of a nucleus.

Q2. A nucleus of mass number A has a mass defect Δm. Give the formula for the binding energy per nucleon of this nucleus.

Q3. Define binding energy of a nucleus.

Q4. State Einstein's mass-energy relation.

Q5. What is meant by binding energy per nucleons?

Q6. What is the maximum value of binding energy per nucleon? Also, name the element.

Q7. What is the average binding energy per nucleon for elements having mass number in the range 30 to 120?

Q8. Obtain the binding energy (in MeV) of a nitrogen nucleus ($^{14}_{7}N$), given $m(^{14}_{7}N)$ =14.00307 u.

Q9. Find the energy equivalent of one atomic mass unit, first in Joules and then in MeV. Using this, express the mass defect of $^{16}_{8}O$ in MeV/c^2.

Q10. We are given the following atomic masses:

$^{238}_{92}U$ = 238.05079 u $^{4}_{2}He$ = 4.00260 u $^{234}_{90}Th$ = 234.04363 u

$^{1}_{1}H$ = 1.00783 u $^{237}_{91}Pa$ = 237.05121 u

Here the symbol Pa is for the element protactinium (Z = 91).

(a) Calculate the energy released during the alpha decay of $^{238}_{92}U$.

(b) Show that $^{238}_{92}U$ can not spontaneously emit a proton.

Q11. What is mass defect? What light does it throw as the binding energy of a nucleus?

Q12. Why the mass of a nucleus always less than the sum of the masses of the consituents, neutrons and protons?

Q13. Calculate the binding energy per nucleon of $_{17}Cl^{35}$ nucleus. Given that mass of $_{17}Cl^{35}$ nucleus = 34.98000 a.m.u., mass of proton = 1.007825 a.m.u., mass of neutron = 1.008665 a.m.u. and 1 a.m.u = 931.5 MeV.

Q14. Express one atomic mass unit in energy units, first in joule and then in MeV. Using this expression, find the mass defect of $_{8}O^{16}$ in MeV. Given that mass of $_{8}O^{16}$ is 15.9994 a.m.u.

Q15. Calculate the energy released in MeV in the following nuclear reaction:

$$_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4} + Q$$

Mass of $_{92}U^{238}$ = 238.05079 a.m.u.; Mass of $_{90}Th^{234}$ = 234.04363 a.m.u.

and Mass of $_{2}He^{4}$ = 4.00260 a.m.u.;

Q16. Obtain the binding energy of the nuclei $^{56}_{26}Fe$ and $^{209}_{83}Bi$ in units of MeV from the following data:

$$m(^{56}_{26}Fe) = 55.934939 \text{ u} \qquad m(^{209}_{83}Bi) = 208.980388 \text{ u}$$

Q17. A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of $^{63}_{29}Cu$ atoms (of mass 62.92960 u).

Q18. The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei $^{41}_{20}Ca$ and $^{27}_{13}Al$ from the following data:

$$m(^{40}_{20}Ca) = 39.962591 \text{ u} \qquad m(^{41}_{20}Ca) = 40.962278 \text{ u}$$

$$m(^{26}_{13}Al) = 25.986895 \text{ u} \qquad m(^{27}_{13}Al) = 26.981541 \text{ u}$$

Q19. Under certain circumstances, a nucleus can decay by emitting a particle more massive than an α-particle. Consider the following decay processes:

$$^{223}_{88}Ra \longrightarrow {}^{209}_{82}Pb + {}^{14}_{6}C \qquad {}^{223}_{88}Ra \longrightarrow {}^{219}_{86}Rn + {}^{4}_{2}He$$

Calculate the Q-values for these decays and determine that both are energetically allowed.

Q20. An electromagnetic wave of wavelength λ is incident on a photosensitive surface of negligible work function. If the photo-electrons emitted from this surface have de-Broglie wavelength λ_1, prove that

$$\lambda = \frac{2mc}{h}\lambda_1^2$$

Q21. The de-Broglie wavelength of a photon is same as the wavelength of an electron. Show that the K.E. of photon is $2\lambda mc/h$ times the K.E. of the electron, where m is the mass of electron and c is the velocity of light.

Q22. An electron and a photon each have a de-Broglie wavelength of 1 nm. Write the ratio of their linear momenta. Compare the energy of the photon with the kinetic energy of the electron.

Q23. An electron and alpha particle have the same de-Broglie wavelength associated with them. How are their kinetic energies related to each other.

Q24. Assume that the de-Broglie wave associated with an electron can form a standing wave between the atoms arranged in a one dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed, if the distance d between the atoms of the array is 2 Å. A similar standing wave is again formed, if the distance d is increased to 2.5 Å, but not for any intermediate value of d. Find the energy of the electrons in eV and the least value of d for which the standing wave of the type described above can form.

Q25. The de-Broglie wavelength of a particle of kinetic energy K is λ. What would be the wavelength of the particle, if its kinetic energy were $K/4$?

Q26. Draw a graph showing the variation of binding energy per nucleon with mass number of different nuclei. Mark the regions, where nuclei are (a) prone to fusion (b) prone to fission and (c) most stable.

Q27. Draw the graph showing the variation of binding energy per nucleon with mass number. Give the reason for the decrease of binding energy per nucleon for nuclei with high mass number.

Q28. Draw a plot of the binding energy per nucleon as a function of mass number for a large number of nuclei, $2 \le A \le 240$. How do you explain the constancy of binding energy per nucleon in the range $30 \le A \le 170$ using the property that nuclear force is short-ranged.

Q29. Draw the graph showing variation of binding energy per nucleon with mass number. Write two main inferences drawn from the graph.

Q30. The binding energies per nucleon for deutron ($_1H^2$) and helium ($_2He^4$) are 1.1 MeV and 7.0 MeV respectively. Calculate the energy released, when two deutrons fuse to form a helium nucleus ($_2He^4$).

Q31. (a) Draw a graph showing the variation of potential energy of a pair of nucleons as a function of their separation. Indicate the regions in which nuclear force is (i) attractive and (ii) repulsive.

(b) Write two characteristic features of nuclear force, which distinguish it from the Coulomb force.

Q32. Define binding energy. How does binding energy per nucleon vary with mass number? What is its significance?

Q33. (a) Draw the plot of binding energy per nucleon (BE/A) as a function of mass number A. Write two important conclusions that can be drawn regarding conclusions of nuclear force.

(b) Use this graph to explain the release of energy in both the processes of nuclear fusion and fission.

(c) Write the basic nuclear process of neutron undergoing β-decay. Why is the detection of neutrinos found very difficult?

Q34. Draw a plot of binding energy per nucleon (BE/A) vs mass number (A), for a large number of nuclei lying between $2 < A < 240$. Using this graph, explain clearly how the energy is released in both the process of nuclear fission and fusion?

Q35. Draw the graph to show variation of binding energy per nucleon with mass number of different atomic nuclei. Calculate binding energy/nucleon of $_{20}Ca^{40}$ nucleus.

Given, mass of $_{20}Ca^{40}$ = 39.962589 u; mass of proton = 1.007825 u; mass of neutron = 1.008665 u and 1u = 931 MeV/C^2.

Date: 22/10/2021

S1. The binding energy of a nucleus is defined as the energy required to break up the nucleus into its constituent nucleons apart upto a distance so that they do not interact with each other.

Alternatively, the binding energy of a nucleus is the energy with which nucleons are bound in the nucleus.

S2. Binding energy per nucleon = $\dfrac{\Delta m \times c^2}{A}$.

S3. The binding energy of a nucleus is equal to the amount of work done to separate the nucleons an infinite distance apart from each other, so that they no longer interact with each other.

S4. In a process, when a mass m is completely converted into energy, the energy obtained is given by

$$E = mc^2.$$

S5. Binding energy per nucleon is the average energy required to remove a nucleon from a nucleus to a distance outside the influence of the nucleus. It is equal to total binding energy divided by the mass number of the nucleus.

S6. 8.8 MeV for ${}_{26}Fe^{56}$.

S7. 8.5 MeV.

S8. Atomic mass of nitrogen (${}_{7}N^{14}$), m = 14.00307 u

A nucleus of nitrogen ${}_{7}N^{14}$ contains 7 protons and 7 neutrons.

Hence, the mass defect of this nucleus, $\Delta m = 7m_H + 7m_n - m$

Where,

Mass of a proton, $m_H = 1.007825$ u

Mass of a neutron, $m_n = 1.008665$ u

$\therefore$ $\Delta m = 7 \times 1.007825 + 7 \times 1.008665 - 14.00307$

$= 7.054775 + 7.06055 - 14.00307$

$= 0.11236$ u

But $1\text{ u} = 931.5\text{ MeV}/c^2$

$\therefore$ $\Delta m = 0.11236 \times 931.5\text{ MeV}/c^2$

Hence, the binding energy of the nucleus is given as:

$$E_b = \Delta m c^2$$

Where, c = Speed of light

$\therefore$ $$E_b = 0.11236 \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) \times c^2$$

$$= 104.66334 \text{ MeV}$$

Hence, the binding energy of a nitrogen nucleus is 104.66334 MeV.

S9. $$1\text{u} = 1.6605 \times 10^{-27} \text{ kg}$$

To convert it into energy units, we multiply it by c^2 and find that energy equivalent

$$= 1.6605 \times 10^{-27} \times (2.9979 \times 10^8)^2 \text{ kg m}^2/\text{s}^2$$

$$= 1.4924 \times 10^{-10} \text{ J}$$

$$= \frac{1.4924 \times 10^{-10}}{1.602 \times 10^{-19}} \text{ eV}$$

$$= 0.9315 \times 10^9 \text{ eV}$$

$$= 931.5 \text{ MeV}$$

or, $$1\text{u} = 931.5 \text{ MeV}/c^2$$

For ${}^{16}_{8}\text{O}$, $$\Delta M = 0.13691 \text{ u}$$

$$= 0.13691 \times 931.5 \text{ MeV}/c^2$$

$$= 127.5 \text{ MeV}/c^2$$

The energy needed to separate ${}^{16}_{8}\text{O}$ into its constituents is thus 127.5 MeV/c^2.

S10. (a) The alpha decay of ${}^{238}_{92}\text{U}$ is given by Eq. (13.20). The energy released in this process is given by

$$Q = (M_\text{U} - M_\text{Th} - M_\text{He})\, c^2$$

Substituting the atomic masses as given in the data, we find

$$Q = (238.05079 - 234.04363 - 4.00260)\text{u} \times c^2$$

$$= (0.00456 \text{ u})\, c^2$$

$$= (0.00456 \text{ u})\, (931.5 \text{ MeV/u})$$

$$= 4.25 \text{ MeV.}$$

(b) If ${}^{238}_{92}\text{U}$ spontaneously emits a proton, the decay process would be

$${}^{238}_{92}\text{U} \longrightarrow {}^{237}_{91}\text{P} + {}^{1}_{1}\text{H}$$

The Q for this process to happen is

$$= (M_\text{U} - M_\text{Pa} - M_\text{H})\, c^2$$

$$= (238.05079 - 237.05121 - 1.00783) \text{ u} \times c^2$$

$$= (-\,0.00825 \text{ u})\, c^2$$

$= -(0.00825\ u)(931.5\ MeV/u)$

$= -7.68\ MeV$

Thus, the *Q* of the process is negative and therefore it cannot proceed spontaneously. We will have to supply an energy of 7.68 MeV to a $^{238}_{92}U$ nucleus to make it emit a proton.

S11. **Mass defect:** The difference between the sum of the masses of the nucleons constituting a nucleus and the rest mass of the nucleus is known as ***mass decfect***. It is denoted by Δm.

The binding energy of a nucleus is measured as the enrrgy equivalent to the mass defect.

S12. To make a nucleus, protons and neutrons have to come together within a very small space of the order of 10^{-15} m. The energy required to do so is provided by the nucleons at the expense of their masses. Due to this, the mass of the nucleus formed is always less than the sum of the masses of the constituent nucleons.

S13. We know, mass defect of $_{17}Cl^{35}$

$$\Delta m = (17\ m_p + 18\ m_n) - 34.9800$$

$$\Delta m = 17\ (1.007825) + 18\ (1.008665) - 34.9800$$

$$\Delta m = 0.308995$$

$$B.E. = 0.3088 \times 931 = 297.67\ MeV$$

S14. 1 a.m.u $=$ $\mathbf{1.4925 \times 10^{-10}\ J}$

$= \mathbf{931.5\ MeV}$

We know, mass defect of $_8O^{16}$,

$$\Delta m = (8m_n + 8m_p) - m(_8O^{16})$$

Here, $m(_8O^{16}) = 15.9994$ a.m.u.,

$m_n = 1.0086665$ a.m.u.

and $m_p = 1.007825$ a.m.u

$\therefore$ $\Delta m = (8 \times 1.0086665 + 8 \times 1.007825) - 15.9994$

$= 0.13252 \times 931.5 = \mathbf{123.44\ MeV}$.

S15.

$$Q = [m(_{90}Th^{234}) + m(_2He^4) - m(_{92}U^{238})] \times 931$$

$$= [238.05079 - 234.04363 - 4.00260] \times 931$$

$$= 0.0456 \times 931 = \mathbf{4.245\ MeV}.$$

S16. Atomic mass of ${}^{56}_{26}Fe$, $m_1 = 55.934939\,u$

${}^{56}_{26}Fe$ nucleus has 26 protons and (56 – 26) = 30 neutrons

Hence, the mass defect of the nucleus, $\Delta m = 26 \times m_H + 30 \times m_n - m_1$

Where, Δm = [Mass of proton + mass of neutrons – mass of nucleus]

Mass of a proton, $m_H = 1.007825\,u$

Mass of a neutron, $m_n = 1.008665\,u$

$$\therefore \quad \Delta m = 26 \times 1.007825 + 30 \times 1.008665 - 55.934939$$
$$= 26.20345 + 30.25995 - 55.934939$$
$$= 0.528461\,u$$

But $1\,u = 931.5\,\text{MeV}/c^2$

$$\therefore \quad \Delta m = 0.528461 \times 931.5\,\text{MeV}/c^2$$

The binding energy of this nucleus is given as:

$$E_{b1} = \Delta mc^2$$

Where, c = Speed of light

$$E_{b1} = 0.528461 \times 931.5$$
$$= 492.26\,\text{MeV}$$

Average binding energy per nucleon = $\dfrac{492.26}{56}$ = 8.79 MeV

Atomic mass of ${}^{209}_{83}Bi$, $m_2 = 208.980388\,u$

${}^{209}_{83}Bi$ nucleus has 83 protons and (209 – 83) 126 neutrons.

Hence, the mass defect of this nucleus is given as:

$$\Delta m' = 83 \times m_H + 126 \times m_n - m_2$$

Where,

Mass of a proton, $m_H = 1.007825\,u$

Mass of a neutron, $m_n = 1.008665\,u$

$$\therefore \quad \Delta m' = 83 \times 1.007825 + 126 \times 1.008665 - 208.980388$$
$$= 83.649475 + 127.091790 - 208.980388$$
$$= 1.760877\,u$$

But $1\,u = 931.5\,\text{MeV}/c^2$

$$\Delta m' = 1.760877 \times 931.5\,\text{MeV}/c^2$$

Hence, the binding energy of this nucleus is given as:

$$E_{b2} = \Delta m' c^2$$
$$= 1.760877 \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) \times c^2$$

$= 1640.26$ MeV

Average bindingenergy per nucleon = $\frac{492.26}{56}$ = 7.848 MeV.

S17. Mass of a copper coin, $m' = 3$ g

Atomic mass of ${}_{29}Cu^{63}$ atom, $m = 62.92960$ u

The total number of ${}_{29}Cu^{63}$ atoms in the coin,

$$N = \frac{N_A \times m}{\text{Mass number}}$$

Where, N_A = Avogadro's number = 6.023×10^{23} atoms/g

Mass number = 63 g

$$\therefore \quad N = \frac{6.023 \times 10^{23} \times 3}{63} = 2.868 \times 10^{22} \text{ atoms}$$

${}_{29}Cu^{63}$ nucleus has 29 protons and (63 – 29) 34 neutrons

∴ Mass defect of this nucleus,

$$\Delta m' = 29 \times m_H + 34 \times m_n - \text{m}$$

Where,

Mass of a proton, $m_H = 1.007825$ u

Mass of a neutron, $m_n = 1.008665$ u

$$\therefore \quad \Delta m' = 29 \times 1.007825 + 34 \times 1.008665 - 62.9296$$

$$= 0.591935 \text{ u}$$

Mass defect of all the atoms present in the coin,

$$\Delta m = 0.591935 \times 2.868 \times 10^{22}$$

$$= 1.69766958 \times 10^{22} \text{ u}$$

But $1 \text{ u} = 931.5 \text{ MeV}/c^2$

$$\therefore \quad \Delta m = 1.69766958 \times 10^{22} \times 931.5 \text{ MeV}/c^2$$

Hence, the binding energy of the nuclei of the coin is given as:

$$E_b = \Delta mc^2$$

$$= 1.69766958 \times 10^{22} \times 931.5 \left(\frac{\text{MeV}}{c^2}\right) \times c^2$$

$$= 1.581 \times 10^{25} \text{ MeV}$$

But $1 \text{ MeV} = 1.6 \times 10^{-13}$ J

$$E_b = 1.581 \times 10^{25} \times 1.6 \times 10^{-13}$$
$$= 2.5296 \times 10^{12}\,\text{J}$$

This much energy is required to separate all the neutrons and protons from the given coin.

S18. For ${}^{41}_{20}\text{Ca}$: Separation energy = 8.363007 MeV

For ${}^{27}_{13}\text{Al}$: Separation energy = 13.059 MeV

A neutron (${}_0n^1$) is removed from a ${}^{41}_{20}\text{Ca}$ nucleus. The corresponding nuclear reaction can be written as:

$${}^{41}_{20}\text{Ca} \longrightarrow {}^{40}_{20}\text{Ca} + {}^{1}_{0}n$$

It is given that:

$$\text{Mass } m({}^{40}_{20}\text{Ca}) = 39.962591\,\text{u}$$
$$\text{Mass } m({}^{41}_{20}\text{Ca}) = 40.962278\,\text{u}$$
$$\text{Mass } m({}^{1}_{0}n) = 1.008665\,\text{u}$$

The mass defect of this reaction is given as:

$$\Delta m = m({}^{40}_{20}\text{Ca}) + ({}^{1}_{0}n) - m({}^{41}_{20}\text{Ca})$$
$$= 39.962591 + 1.008665 - 40.962278 = 0.008978\,\text{u}$$

But $1\,\text{u} = 931.5\,\text{MeV}/c^2$

$$\Delta m = 0.008978 \times 931.5\,\text{MeV}/c^2$$

Hence, the energy required for neutron removal is calculated as:

$$E = \Delta mc^2$$
$$= 0.008978 \times 931.5 = 8.363007\,\text{MeV}$$

For ${}^{27}_{13}\text{Al}$, the neutron removal reaction can be written as:

$${}^{27}_{13}\text{Al} \longrightarrow {}^{26}_{13}\text{Al} + {}^{1}_{0}n$$

It is given that:

$$\text{Mass } m({}^{27}_{13}\text{Al}) = 26.981541\,\text{u}$$
$$\text{Mass } m({}^{26}_{13}\text{Al}) = 25.986895\,\text{u}$$

The mass defect of this reaction is given as:

$$\Delta m = m({}^{26}_{13}\text{Al}) + ({}^{1}_{0}n) - m({}^{27}_{13}\text{Al})$$
$$= 25.986895 + 1.008665 - 26.981541 = 0.014019\,\text{u}$$
$$= 0.014019 \times 931.5\,\text{MeV}/c^2$$

Hence, the energy required for neutron removal is calculated as:

$$E = \Delta mc^2$$
$$= 0.014019 \times 931.5 = 13.059\,\text{MeV}.$$

S19. Take a ${}^{14}_{6}C$ emission nuclear reaction:

$$ {}^{223}_{88}Ra \longrightarrow {}^{209}_{82}Pb + {}^{14}_{6}C $$

We know that:

Mass of ${}^{223}_{88}Ra$, $m_1 = 223.01850\,u$

Mass of ${}^{209}_{82}Pb$, $m_2 = 208.98107\,u$

Mass of ${}^{14}_{6}C$, $m_3 = 14.00324\,u$

Hence, the Q-value of the reaction is given as:

$$ Q = \Delta mc^2 $$

$$ Q = (m_1 - m_2 - m_3)\,c^2 $$

$$ = (223.01850 - 208.98107 - 14.00324)\,c^2 $$

$$ = (0.03419\,c^2)\,u $$

But $1\,u = 931.5\,MeV/c^2$

$\therefore$ $Q = 0.03419 \times 931.5 = 31.848\,MeV$

Hence, the Q-value of the nuclear reaction is 31.848 MeV. Since the value is positive, the reaction is energetically allowed.

Now take a ${}^{4}_{2}He$ emission nuclear reaction:

$$ {}^{223}_{88}Ra \longrightarrow {}^{219}_{86}Rn + {}^{4}_{2}He $$

We know that:

Mass of ${}^{223}_{88}Ra$, $m_1 = 223.01850$

Mass of ${}^{219}_{86}Rn$, $m_2 = 219.00948$

Mass of ${}^{4}_{2}He$, $m_3 = 4.00260$

Q-value of this nuclear reaction is given as:

$$ Q = (m_1 - m_2 - m_3)\,c^2 $$

$$ = (223.01850 - 219.00948 - 4.00260)\,c^2 $$

$$ = (0.00642\,c^2)\,u $$

$$ = 0.00642 \times 931.5 = 5.98\,MeV $$

Hence, the Q value of the second nuclear reaction is 5.98 MeV. Since the value is positive, the reaction is energetically allowed.

S20. Since the photosensitive surface of negligible work function,

K.E. of emitted electron = energy of the incident photon

i.e., $\frac{1}{2} m v^2 = h\nu$

or $$\frac{p^2}{2m} = \frac{hc}{\lambda}$$

or $$p = \sqrt{\frac{2mhc}{\lambda}}$$

Therefore, de-Broglie wavelength of the emitted photoelectrons,

$$\lambda_1 = \frac{h}{p} = h \times \sqrt{\frac{\lambda}{2mhc}}$$

or $$\boldsymbol{\lambda = \frac{2mc}{h}\lambda_1^2}.$$

S21. If the photon and an electron having mass m and moving with velocity v possess the same de-Broglie wavelength λ,

the energy of photon, $\frac{hc}{\lambda} = mvc$.

and the energy of electron = mc^2.

The rest mass energy of photon is zero. Therefore, energy of photon refers to kinetic energy of photon. However, the energy of the electron *i.e.*, mc^2 is the sum of its rest mass energy (m_0c^2) and kinetic energy. If E_1 and E_2 are kinetic energies of photon and electron, then

$$\frac{E_1}{E_2} = \frac{mvc}{\frac{1}{2}mv^2} = \frac{2c}{v} \qquad \text{... (i)}$$

Now, de-Broglie wavelength,

$$\lambda = \frac{h}{mv} \quad \text{or} \quad v = \frac{h}{m\lambda}$$

In the equation (i), substituting for v, we get

$$\frac{E_1}{E_2} = 2c\Big/\frac{h}{m\lambda} = \frac{2\lambda mc}{h}$$

or $$E_1 = \frac{2\lambda mc}{h}E_2$$

S22. Now, $$p = \frac{h}{\lambda}.$$

Since the electron and photon have the same de-Broglie wavelength of 1 nm, both the particles will have the same momentum. Hence, the ratio of their momenta will be 1.

For comparison of the energy of the photon with the kinetic energy of the electron. It will be found that the ratio of their energies is 824.75.

S23. If λ is the de-Broglie wavelength of a particle having kinetic energy K, then

$$\lambda = \frac{h}{\sqrt{2mK}}$$

$\therefore$ $$\lambda_e = \frac{h}{\sqrt{2 m_e K_e}}$$

and $$\lambda_\alpha = \frac{h}{\sqrt{2 m_\alpha K_\alpha}}$$

Since $\lambda_e = \lambda_\alpha$, we get

$$\frac{h}{\sqrt{2 m_e K_e}} = \frac{h}{\sqrt{2 m_\alpha K_\alpha}}$$

or $$m_e K_e = m_\alpha K_\alpha$$

or $$\frac{K_e}{K_\alpha} = \frac{m_\alpha}{m_e}$$

S24. When the standing wave is formed in a distance 2 Å:

Suppose that in the distance 2 Å, n loops of the standing wave are formed. Then,

$$n \times (\lambda / 2) = 2 \text{ Å} \quad \text{... (i)}$$

When the standing wave is formed in a distance 2.5 Å:

Now, in the distance 2.5 Å, $(n + 1)$ loops of the standing wave will be formed. Therefore,

$$(n + 1) \times (\lambda / 2) = 2.5 \text{ Å} \quad \text{... (ii)}$$

Subtracting the equation (i) from (ii), we get

$$(n + 1) \times (\lambda / 2) - n \times (\lambda / 2) = 2.5 - 2$$

or $$\lambda = 2 \times 0.5 = 1 \text{ Å} = 10^{-10} \text{ m}$$

Now, de-Broglie wavelength of the electrons is given by

$$\lambda = \frac{h}{\sqrt{2mE}}$$

or $$E = \frac{h^2}{2m\lambda^2} = \frac{(6.62 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (10^{-10})^2}$$

$$= 2.44 \times 10^{-17}\, J = \frac{2.44 \times 10^{-17}}{1.6 \times 10^{-19}} = \mathbf{152.50\ eV}.$$

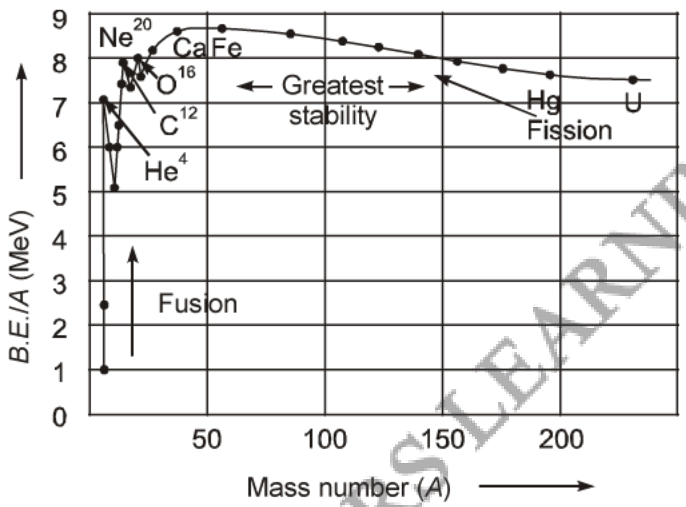
S25. If λ is the de-Broglie wavelength of the particle of kinetic energy K, then

$$\lambda = \frac{h}{\sqrt{2mK}}.$$

Suppose that the de-Broglie wavelength of the particle becomes λ', when its kinetic energy is $K/4$.

Then $$\lambda' = \frac{h}{\sqrt{2mK/4}} = 2\left(\frac{h}{\sqrt{2mK}}\right) = \mathbf{2\lambda}.$$

S26. For graph between $B.E./A$ and A,

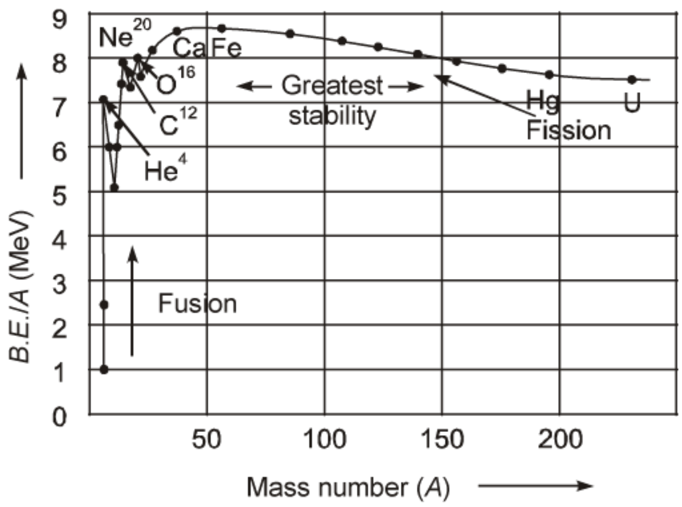


(a) In the region $A < 20$, the $B.E./A$ and A of the nuclei is quite low except for the nuclei ${}_2He^4$, ${}_6C^{12}$ and ${}_8O^{16}$. In an attempt to have greater value of $B.E./A$ and A, the nuclei in the region $A < 20$ unite to form a heavier nucleus and therefore, the nuclei in this region are prone to nuclear fusion.

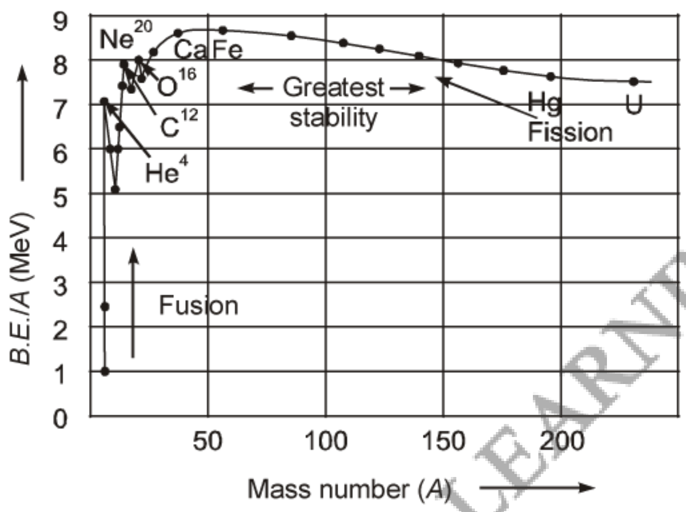
(b) In the region $A > 210$, the $B.E./A$ and A of the nuclei is again quite low. The nuclei in this region have a tendency to split so as to improve the value of their $B.E./A$. Hence, in region $A > 210$, the nuclei are prone to nuclear fission.

(c) In the region $40 < A > 210$, the nuclei are most stable, It is indicated by the flat shape of the graph. The value of the $B.E./A$ in this region is maximum ($\approx$ 8.8 MeV per nucleon).

S27. The binding energy per nucleon for nuclei with high mass number decreases due to increase Coulomb's repulsive force with increase in the number of protons.

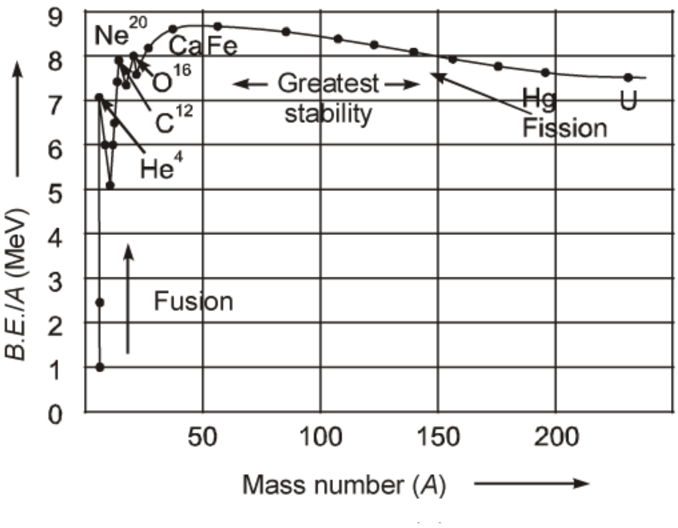


S28. For graph between $B.E./A$ and A,



From the graph, it follows that in the region $30 \le A \le 170$, the binding energy per nucleon of the nuclei is almost constant. It implies that a nucleon in the nucleus is not attracted by all the remaining nucleons. In case, a nucleon was attracted by all the remaining nucleons, the binding energy per nucleon would have increased with increasing mass number. Thus, a nucleon is attracted by only its some close neighbours, Hence, the nuclear force is short-ranged.

S29. For graph between $B.E./A$ and A,



The following inferences can be drawn from this graph

(a) The peaks corresponding to ${}_2He^4$, ${}_6C^{12}$ and ${}_8O^{16}$ indicate that these nuclei are more stable than those in their neighbourhood.

(b) The $B.E./A$ has a low value for both light nuclei ($A < 20$) and for very heavy nuclei ($A > 210$). The light nuclei tend to unite (by the process of fusions) and heavy nuclei tend to split (by the process of fission) in order to attain a greater value of the $B.E./A$

S30. The fusion reaction may be represented as

$$ {}_1H^2 + {}_1H^2 \longrightarrow {}_2He^4 + Q. $$

Given: $\frac{B.E.}{A}$ of ${}_1H^2$ nucleus = 1.1 MeV; $\frac{B.E.}{A}$ of ${}_2He^4$ nucleus = 7.0 MeV.

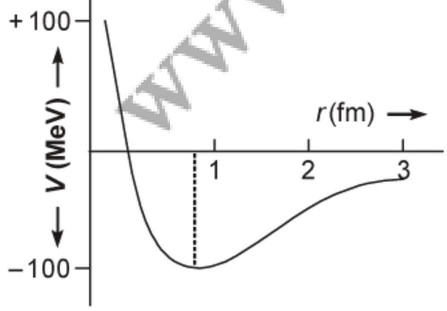
Therefore, total $B.E.$ of ${}_1H^2$ nucleus = $1.1 \times 2 = 2.2$ MeV

Hence, total initial $B.E.$ = 2.2 + 2.2 = 4.4 MeV

Also, total $B.E.$ of ${}_2He^4$ nucleus = $7.0 \times 4 = 28.0$ MeV

Therefore, energy released, $Q = 28.4 - 4.4 =$ **23.6 MeV**.

S31. (a) The plot of potential energy (V) of a pair of nucleons as a function of their (r) is as shown in the figure



Conclusions:

(i) At large distances between the two nucleons, the potential energy is small negative and becomes slowly more negative as the distance decreases. Since $F = -\frac{dV}{dr}$, it implies that over the large part of the nucleus, the nuclear force between two nucleons is attractive.

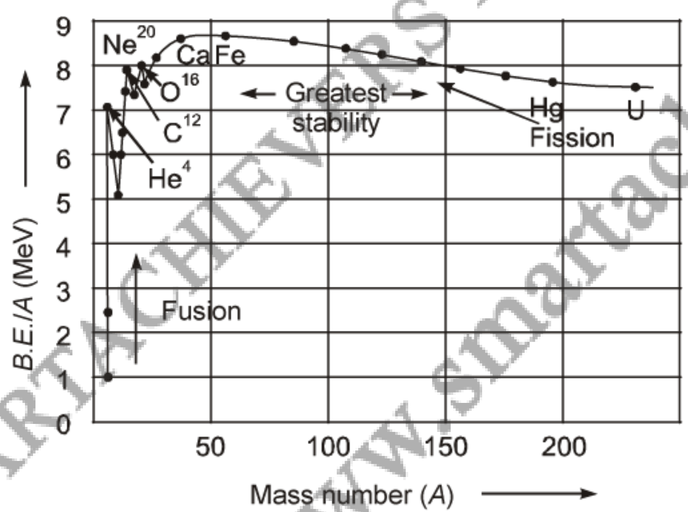
(ii) When the distance between the nucleons is 0.8 fm, the potential energy becomes maximum negative and then, as the distance decreases further, it becomes zero and then positive. It implies that the nuclear force is zero at the distance of 0.8 fm and strong repulsive, when the distance is very small.

(b) (i) **Nuclear forces are strongest forces in nature:** Nuclear forces are about 10^{36} times as strong as coulomb's forces.

(ii) **Nuclear forces are short range forces:** Nuclear force between two nucleons become negligible when distance between them is greater than 4.2 term or 4.2×10^{-15} m where the magnitude of nuclear force become maximum value. Thus, unlike electromagnetic forces, nuclear forces is short range force.

S32. **Binding energy:** The binding energy of a nucleus is defined as the energy required to break up the nucleus into its constituent nucleons apart upto a distance so that they do not interact with each other.

For graph between $B.E./A$ and A,

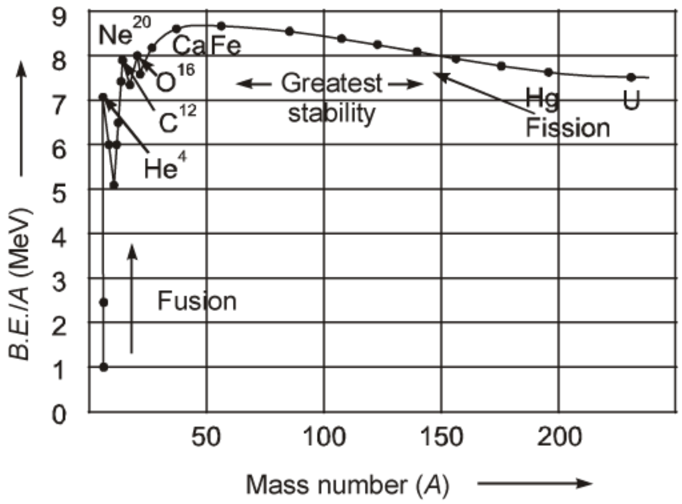


The following inferences can be drawn from this graph

(a) The peaks corresponding to ${}_2He^4$, ${}_6C^{12}$ and ${}_8O^{16}$ indicate that these nuclei are more stable than those in their neighbourhood.

(b) The $B.E./A$ has a low value for both light nuclei (A < 20) and for very heavy nuclei (A > 210). The light nuclei tend to unite (by the process of fusions) and heavy nuclei tend to split (by the process of fission) in order to attain a greater value of the $B.E./A$

S33. (a) Plot of binding energy per nucleon as the function of mass number A is given as below



Following are the two conclusion that can be drawn regarding the nature of the nuclear force

(i) The force is attractive and strong enough to produce a binding energy of few MeV per nucleon.

(ii) The constancy of the binding energy in the range 30 < A < 170 is a consequence of the fact that the nuclear force is short range force.

(b) ***Nuclear fission:*** A very heavy nucleus (say A = 240) has lower binding energy per nucleon as compared to the nucleus with A = 120. Thus if the heavier nucleus breaks to the lighter nucleon, nucleons are tightly bound. This implies that energy will be released in the process which justifies the energy release in fission reaction.

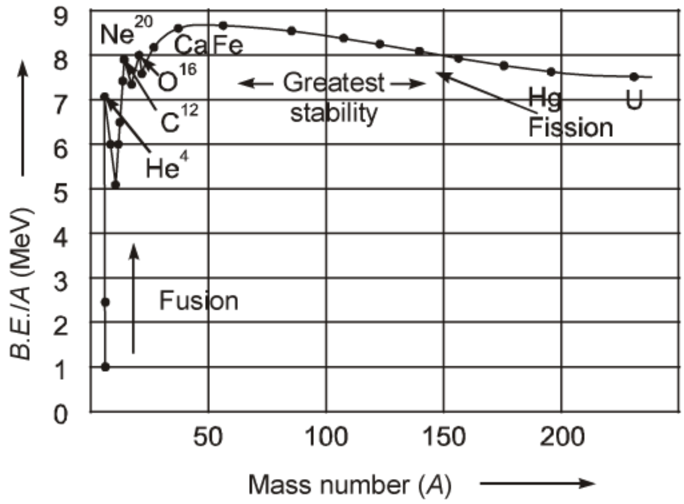
Nuclear fusion: When two light nuclei (A < 10) are combined to form a heavier nuclei, the binding energy of the fused heavier nuclei is more that the binding energy per nucleon of the lighter nuclei. Thus the final system is more tightly bound than the initial system. Again the energy will be released in fusion reaction.

(c) The basic nuclear process of neutron undergoing β-decay is given as

$$n \rightarrow p + e^- + \bar{\nu}$$

Neutrinos Interact very weakly with matter so, they have a very high penetrating power. That's why the detection of neutrinos is found very difficult.

S34. The binding energy curve per nucleon is shown below.

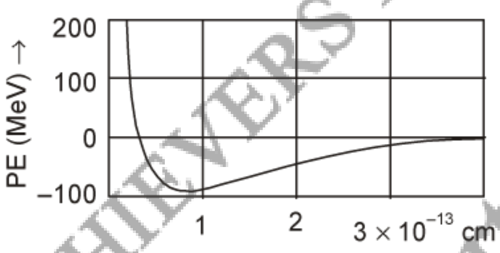


Explanation of release of energy in nuclear fission and fusion. The curve reveals that binding energy per nucleon is smaller for heavier nuclei than the middle level nuclei. This shows that heavier nuclei in nuclear fission, binding energy per nucleon of reactants (heavier nuclei) changes from nearly 7.6 MeV to 8.4 MeV (for nuclei of middle level mass).

Higher value of the binding energy of the nuclear product results in the liberation of energy during the phenomena of nuclear fission.

In nuclear fusion, binding energy per nucleon of lighter nuclei into heavier one changes from low value and release of energy take place in fusion e.g., two ${}_1H^2$ (Be $\simeq$ 1.5 MeV/nucleon) combine to form ${}_2He^4$ (Binding energy per nucleon $\simeq$ 7 MeV/nuclei) and therefore the energy is liberated during nuclear fusion.

S35.



Net interactive force is zero when P.E. is minimum i.e., nearly, r_0 = 1fm (in graph)

(a) The nuclear force is attractive when separation between the nuclei is greater than r_0 > 1 fm.

(b) Repulsive when r_0 < 1 fm.

Mass of ${}_{20}Ca^{40}$ nucleus : 39.962589 u

sum of masses of nucleons of

$${}_{20}Ca^{40} = 20\,m_p + 20\,m_n$$

$$= 20 \times 1.007825 + 20 \times 1.008665$$

$$= 40.329800\ \text{u}$$

$\therefore$ Mass defect = Sum of masses of nucleon – mass of nucleus

= 40.3298 – 39.962589

= 0.367211 amu

$\therefore$ Binding energy = $0.3672211 \times 931 MeV$

= 341.87344 MeV

$\therefore$ Binding energy per nucleon

= 8.546836 MeV.

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Q1. What characteristic property of nuclear force explains the constancy of binding energy per nucleon (B.E./A) in the range of mass number A lying $30 < A < 170$?

Q2. Two nuclei have mass numbers in the ratio 27 : 125. What is the ratio of their nuclear radii?

Q3. Write any two characteristic properties of nuclear force.

Q4. What is an endoergic nuclear reaction?

Q5. Give four characteristic properties of nuclear forces.

Q6. Calculate the height of the potential barrier for a head on collision of two deuterons. (Hint: The height of the potential barrier is given by the Coulomb repulsion between the two deuterons when they just touch each other. Assume that they can be taken as hard spheres of radius 2.0 fm.)

Q7. From the relation $R = R_0 A^{1/3}$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (*i.e.*, independent of A).

Q8. Give two important characteristics of nuclear forces.

Q9. Calculate the following nuclear reaction:

$$_5B^{10} + (?) \longrightarrow {}_3Li^7 + {}_2He^4$$

Q10. Define Q-value of a nuclear reaction.

Q11. A chain reaction dies out. State the reasons, why this could happen.

Q12. Distinguish between nuclear-fission and nuclear-fusion.

Q13. Draw a plot of potential energy of a pair of nucleons as a function of their separation. Write two important conclusions which you draw regarding the nature of nuclear forces.

Q14. Draw a plot of potential energy of a pair of nucleons, as a function of their separations. Mark the regions where the nuclear force is (a) attractive and (b) repulsive. Writhe any two characteristic features of nuclear forces.

Q15. State the law of radioactive decay.

Plot a graph showing the number (N) of undecayed nuclei as a function of time (t) for a given radioactive sample having half-life $T_{½}$. Depict in the plot, the number of undecayed nuclei at (a) $t = 3\,T_{½}$, (b) $t = 5\,T_{½}$.

Q16. What is the basic mechanism for the emission of β^- and β^+ particles in a nuclide? Give an example by writing explicitly a decay process for β^- emission. Is

(a) the energy of the emitted β-particles continuous or discrete?

(b) the daughter nucleus obtained through β-decay, an isotope or an isobar of the parent nucleus?

Q17. (a) What is meant by half-life of a radioactive element?

(b) The half-life of a radioactive substance is 30 s. Calculate

(i) the decay constant and

(ii) time taken for the sample to decay by $3/4^{th}$ of the initial value.

Q18. Sketch a graph showing the variation of potential energy of a pair of nucleons as function of their separation. Write three characteristic properties of nuclear force which distinguish it from the electrostatic force.

Q19. What are nuclear forces? State their properties. Explain the possible cause of these force.

Q20. (a) Draw a graph showing the variation of potential energy of a pair of nucleons as a function of their separation. Indicate the regions in which nuclear force is (i) attractive and (ii) repulsive.

(b) Write four characteristic features of nuclear force.

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S1. It is because nuclear forces are saturated forces.

S2. Since $R \propto A^{1/3}$, $R_1 : R_2 = 27^{1/3} : 125^{1/3} = 3 : 5$.

S3. Two characteristics of nuclear force:

(a) These are short range force

(b) These are strong force of attractive nature.

S4. The nuclear reaction, in which there is absorption of energy, is called endoergic nuclear reaction.

S5. Characteristic properties of nuclear forces as follows:

(a) It varies inversely with some higher power of distance.

(b) It is basically an attractive force.

(c) It is a short-range force and is operative only over the size of the nucleus ($\approx 10^{-15}$ m).

(d) The ***π-meson*** is its field particle.

S6. When two deuterons collide head-on, the distance between their centres, d is given as:
Radius of 1st deuteron + Radius of 2nd deuteron

Radius of a deuteron nucleus = 2 fm = 2×10^{-15} m

$$\therefore \quad d = 2 \times 10^{-15} + 2 \times 10^{-15} = 4 \times 10^{-15}\ \text{m}$$

Charge on a deuteron nucleus = Charge on an electron

$$= e = 1.6 \times 10^{-19}\ \text{C}$$

Potential energy of the two-deuteron system:

$$V = \frac{e^2}{4\pi\varepsilon_0 d}$$

Where, ε_0 = Permittivity of free space

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{N m}^2\,\text{C}^{-2}$$

$$\therefore \quad V = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{4 \times 10^{-15}}\ \text{J}$$

$$= \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{4 \times 10^{-15} \times (1.6 \times 10^{-19})} \text{ eV} = 360 \text{ keV}.$$

Hence, the height of the potential barrier of the two-deuteron system is 360 keV.

S7. We have the expression for nuclear radius as:

$$R = R_0 A^{1/3}$$

Where, R_0 = Constant.

A = Mass number of the nucleus

Nuclear matter density, $\rho = \frac{\text{Mass of the nucleus}}{\text{Volume of the nucleus}}$

Let m be the average mass of the nucleus.

Hence, Mass of the nucleus = mA

$$\therefore \quad \rho = \frac{mA}{\frac{4}{3}\pi R^3} = \frac{3mA}{4\pi(R_0 A^{\frac{1}{3}})^3} = \frac{3mA}{4\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

Hence, the nuclear matter density is independent of A. It is nearly constant.

S8. Two important characteristics of nuclear forces as follows:

(a) ***Nuclear forces are strongest forces in nature:*** Nuclear forces are about 10^{38} times as strong as gravitational forces.

(b) ***Nuclear forces are short range forces:*** The nuclear force between two nucleons becomes negligible, when the distance between them is greater than 4.2 fermi or 4.2×10^{-15} m.

However, the range of nuclear force is taken as 1.5 fermi, where the magnitude of the nuclear force becomes maximum value. Thus, unlike gravitational and electromagnetic forces (which extend upto infinity), nuclear force is a short range force.

S9. Let $_ZP^A$ be the unknown bombarding particle.

Now, $5 + Z = 3 + 2$ or $Z = 0$

Also, $10 + A = 7 + 4$ or $A = 1$

Therefore, the unknown bombarding particle is $_0n^1$ (neutron).

S10. Q-value of a nuclear reaction is equal to the energy equivalent of the difference in masses of the reacting nuclei and the product nuclei. Mathematically,

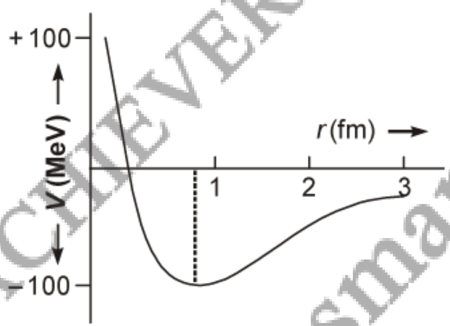
$$Q = [\text{mass of reacting nuclei} - \text{mass of product nuclei}] \times 931 \text{ MeV}.$$

S11. A nuclear chain reaction dies out, when the neutrons produced are so fast that they escape the uranium block without interacting or when the neutrons are lost due to the extremely small size of the uranium block.

S12.

Nuclear fission	Nuclear fusion
1. In nuclear fission, a heavy nucleus splits into two smaller nuclei of nearly same masses.	1. In nuclear fusion, two or more lighter nuclei fuse together to form a heavier nucleus.
2. It does not require extremely high temperature.	2. It takes places at extremely high temperature ($\approx 10^7$ K).
3. Comparatively lower amount of energy is released.	3. Extraordinary huge amount of energy is released in this process.
4. This process is carried out to generate electric power.	4. This process has not been successfully carried out anywhere in the world till now, in a controlled manner.
5. Fission products are generally radioactive.	5. Fusion products are non-radioactive and do not pose any hazard for life.
6. The sources of fissionable materials (e.g. uranium, thorium, etc.) are limited and exhaustible.	6. The sources of fusion reactions (*e.g.* hydrogen, etc.) are present in large quantity and inexhaustible.
7. This process is used in making atom bomb.	7. This process is used in hydrogen bomb.

S13. The plot of potential energy (V) of a pair of nucleons as a function of their (r) is as shown in the figure

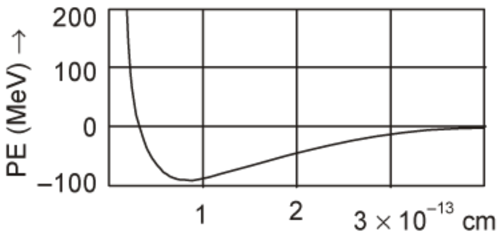


Conclusions:

(a) At large distances between the two nucleons, the potential energy is small negative and becomes slowly more negative as the distance decreases. Since $F = -\frac{dV}{dr}$, it implies that over the large part of the nucleus, the nuclear force between two nucleons is attractive.

(b) When the distance between the nucleons is 0.8 fm, the potential energy becomes maximum negative and then, as the distance decreases further, it becomes zero and then positive. It implies that the nuclear force is zero at the distance of 0.8 fm and strong repulsive, when the distance is very small.

S14.



Net interactive force is zero when P.E. is minimum i.e., nearly, r_0 = 1fm (in graph)

(a) The nuclear force is attractive when separation between the nuclei is greater than $r_0 > 1$ fm.

(b) Repulsive when $r_0 < 1$ fm.

Two characteristics of nuclear force:

(a) These are short range force

(b) These are strong force of attractive nature.

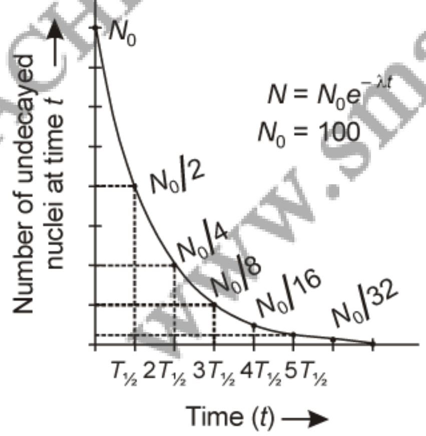
S15. (a) ***Law of radioactive decay:*** The rate of disintegration of radioactive sample at any instant is directly proportional to the number of undisintergated nuclei present in the sample at that instant *i.e.*,

$$\frac{dN}{dt} \propto N$$

$$\frac{dN}{dt} = -\lambda N$$

where, N = number of undisintegrated nuclei present in the sample at any instant t and $\frac{dN}{dt}$ is rate of disintegration.

(b) The required plot is shown below:



Decay curve for a radioactive element

S16. During β^--decay from the nucleus, nuclei undergoes a change in such a way that atomic number increases by one and mass number remains same.

In β^+-decay, the mass number of present radioactive nuclei remains same whereas atomic number decrease by one.

Example of β^--decay

$$^{32}_{15}P \rightarrow {}^{32}_{16}S + e^- + \bar{\nu} \qquad \text{[Half-life = 26 days]}$$

In β-decay, an electron and an antineutrino are created in following manner

$$n \rightarrow p + e^- + \bar{\nu}$$

(i) The energy of emitted β-particles is continuous.

(ii) As there is no change in mass number during β-decay. So, the daughter nucleus is isobar of the parent nucleus.

S17. (a) ***Half-life:*** Half-life of a radioactive element is the time taken by the sample to disintegrate upto half of its original amount.

Half-life period, $T_{½} = \dfrac{0.693}{\lambda}$

where, λ is decay constant.

(b) $T_{½} = 30$ s

(i) $\lambda = ?$

$\because \quad T_{½} = \dfrac{0.693}{\lambda}$

$\Rightarrow \quad \lambda = \dfrac{0.693}{T_{½}} = \dfrac{0.603}{30} = 0.0231\,\text{s}^{-1}$

(ii) $\because \quad N = N_0\left(\dfrac{1}{2}\right)^n$

where, n = number of half-lives

N = number of undisintegrated nuclei present in the sample.

N_0 = original number of undisintegrated atom.

Here, $N = N_0 - \dfrac{3}{4}N_0$

$$N = \frac{1}{4}N_0$$

$$N = N_0\left(\frac{1}{2}\right)^n$$

$$\frac{N_0}{4} = N_0 \left(\frac{1}{2}\right)^n$$

$$\Rightarrow \quad \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^n$$

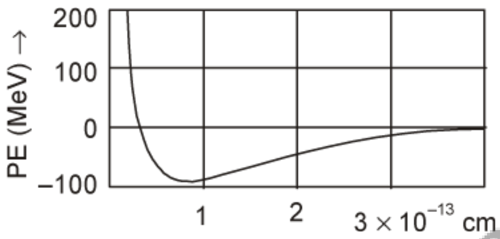
$$\Rightarrow \quad n = 2$$

But number of half-lives = $\dfrac{\text{Total time taken}}{\text{Half-life}}$

$$= \frac{\text{Total time taken}}{(30 \text{ s})}$$

Total time taken = 60 s = 1 min.

S18. Net interactive force is zero when P.E. is minimum *i.e.*, nearly, r_0 = 1 fm (in graph)

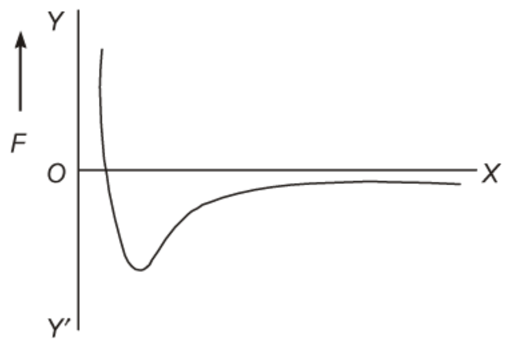


(a) The nuclear force is attractive when separation between the nuclei is greater than r_0 > 1 fm.

(b) Repulsive when r_0 < 1 fm.

S. No.	Nuclear force	Electrostatic force
1	strongest short range force which operate upto which operate upto distance of 2–3 fm	It is not very short range force neccessaritly
2	It does not obey inverse square law	It obey inverse square law
3	it exhibit charge independent character.	It depends on the nature of charge, like charge repel whereas opposite charge attracts, each other.

S19. Nuclear forces are strong force of attraction which hold together the nucleons (neutrons and protons) inside nucleus in spite of electrostatic repulsion between protons. Nuclear force is 100 times stronger than electrostatic force. The variation of nuclear force between two nucleons with the separation between them is shown in figure. It is very strong at an optimum separation between nucleons and decreases very rapidly both for increase and decrease in separation between nucleolus.



Important properties of nuclear forces:

(a) *Nuclear forces are charge independent.* Nuclear force between *p-p*, *n-n* and *n-p* pairs are same.

(b) *Nuclear forces are strongest force in nature.* They are 10^{38} times stronger than gravitational force and 10^2 times stronger than electrostatic force.

(c) *Nuclear forces are short range forces.* Nuclear force between two nucleons becomes negligibly small if the separation between them becomes greater than 4.2 fermi (4.2×10^{-15} m). The range of nuclear force is taken as 1.5 fermi.

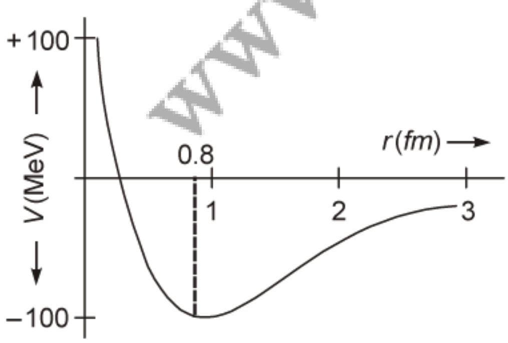
(d) *Nuclear force between two nucleons becomes repulsive if distance between them reduces to less than 0.5 fermi*.

(e) *Nuclear forces are exchange forces.* According to Yukawa, the nuclear force between the two nucleons is the result of the exchange of π mesons between them.

(f) *Nuclear forces exhibit saturation properties.* Each nucleon interacts with its immediate neighbors only.

(g) Nuclear forces are dependent on spin or angular momentum of nuclei.

S20. (a) The graph of potential energy (V) of a pair of nucleons as a function of their separation (r) is as shown in figure



(i) At large distances between the two nucleons, the potential energy is small negative and becomes slowly more negative as the distance decreases. Since $F = -\frac{dV}{dr}$, it implies that over the large part of the nucleus, the nuclear force between two nucleons is attractive.

(ii) When the distance between the nucleons is 0.8 fm, the potential energy becomes maximum negative and then, as the distance decreases further, it becomes zero and then positive. It implies that the nuclear force is zero at the distance of 0.8 fm and strong repulsive, when the distance is very small.

(b) ***Properties of nuclear forces:***

(a) *Nuclear forces are charge independent.* Nuclear force between *p-p*, *n-n* and *n-p* pairs are same.

(b) *Nuclear forces are strongest force in nature.* They are 10^{38} times stronger than gravitational force and 10^2 times stronger than electrostatic force.

(c) *Nuclear forces are short range forces.* Nuclear force between two nucleons becomes negligibly small if the separation between them becomes greater than 4.2 fermi (4.2×10^{-15} m). The range of nuclear force is taken as 1.5 fermi.

(d) *Nuclear force between two nucleons becomes repulsive if distance between them reduces to less than 0.5 fermi.*

Date: 22/10/2021

Q1. Write nuclear reaction equations for β^+-decay of ${}^{11}_{6}C$.

Q2. Write nuclear reaction equations for β^--decay of ${}^{210}_{83}Bi$.

Q3. Write nuclear reaction equations for β^+-decay of ${}^{32}_{15}P$.

Q4. Write nuclear reaction equations for α-decay of ${}^{242}_{94}Pu$.

Q5. Write nuclear reaction equations for α-decay of ${}^{226}_{88}Ra$.

Q6. Which has greater ionising power: alpha particle or beta particle?

Q7. Among alpha, beta and gamma radiations, which are the ones affected by a magnetic field?

Q8. Name two radioactive elements, which are not found in observable quantities in nature. Why is it so?

Q9. Why are neutrons used to initiate fission?

Q10. Why heavy water is generally used as a moderator in a nuclear reactor?

Q11. Tritium has a half-life of 12.5 y undergoing beta decay. What fraction of a sample of pure tritium will remain undecayed after 25 y.

Q12. Write nuclear reaction equations for Electron capture of ${}^{120}_{54}Xe$.

Q13. Write nuclear reaction equations for β^--decay of ${}^{97}_{43}Tc$.

Q14. A radioisotope of silver has half-life of 20 minutes. What fraction of the original mass would remain after one hour?

Q15. Give the relation between 'half-life' and decay constant of a radioactive material.

O16. Define half life of a radioactive material.

Q17. Arrange α-rays, β-rays and γ-rays in the ascending order of their penetrating power.

Q18. Why do alpha particles have a high ionising power?

Q19. Write symbolically the β-decay process of ${}_{15}P^{32}$.

Q20. Give the mass number and atomic number of elements on the right hand side of the decay process:

$$_{86}Rn^{220} \longrightarrow Po + He$$

Q21. Plot a graph showing the variation of activity of a given radioactive sample with time.

Q22. A radioactive substance having *N* nuclei has activity *R*. Write down an expression for its half-life in terms of *R* and *N*.

Q23. Define activity of radioactive material and write its SI units.

Q24. A radioactive nuclide has a decay constant equal to λ. Give the formula for the
(a) half life, (b) mean life of this nuclide.

Q25. Why is the energy variation of the electrons emitted during a β-decay continuous?

Q26. What is a radioisotope?

Q27. What is the difference between an electron and a β-particle?

Q28. Write the nuclear decay process for β-decay of $_{15}P^{32}$.

Q29. Explain, why "all radioactive substances seem to be identical"?

Q30. What are γ-rays?

Q31. Name two radioactive elements, which are not found in observable quantities in nature. Why is it so?

Q32. What do you mean by radioactive substance?

Q33. If $_3Li^7$ is bombarded with a certain particle, two alpha particles are produced. identify the bombarding particle.

Q34. Name the absorbing material used to control the reaction rate of neutrons in a nuclear reactor.

Q35. The mass of a nucleus is less than the sum of the masses of the nucleons forming it. Why?

Q36. Write any one equation representing nuclear fusion reaction.

Q37. Obtain the amount of $^{60}_{27}Co$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of $^{60}_{27}Co$ is 5.3 years.

Q38. The half-life of $^{90}_{38}Sr$ is 28 years. What is the disintegration rate of 15 mg of this isotope?

Q39. Obtain approximately the ratio of the nuclear radii of the gold isotope $^{197}_{79}Au$ and the silver isotope $^{107}_{47}Ag$.

Q40. The half-life of $^{238}_{92}U$ undergoing α-decay is 4.5×10^9 years. What is the activity of 1 g sample of $^{238}_{92}U$?

Q41. You are given two nuclides $_3X^7$ and $_3Y^4$.

(a) Are they the isotopes of the same element? Why?

(b) Which one of the two is likely to be more stable? Give reasons.

Q42. Why all the three types of rays *i.e.*, α, β and γ come out from a radioactive sample though a single radioactive sample obeys a particular decay mode?

Q43. The radioactive isotope *D* decays according to the sequence:

$$D \xrightarrow{\beta^-} D_1 \xrightarrow{\alpha} D_2$$

If the mass number and atomic number of D_2 are 176 and 71 respectively, what is (a) the mass number and (b) atomic number of *D*?

Q44. The sequence of stepwise decay of a radioactive nucleus is

$$D \xrightarrow{\alpha} D_1 \xrightarrow{\beta^-} D_2$$

If the mass number and atomic number of D_2 are 176 and 71 respectively, what are their corresponding values for D?

Q45. What fraction of tritium will remain undecayed after 25 years? Given, half life is 12.5 years.

Q46. State the laws of radioactivity.

Q47. A radioactive nucleus A decays are follows:

$$A \xrightarrow{\beta^-} A_1 \xrightarrow{\alpha} A_2$$

If the mass number and atomic number of A_2 are 176 and 71 respectively, what are the mass number and atomic number of A_1 and A? Which of these elements are isobars?

Q48. The radioactive isotope D decays according to the sequence:

$$D \xrightarrow{\alpha\, n^1} D_1 \xrightarrow{\alpha} D_2$$

If the mass number and atomic number of D_2 are 176 and 71 respectively, find the mass number and atomic number of D. Amongst D, D_1 and D_2, do we have any isobars or isotopes?

Q49. How many disintegrations per second will occur in one gram of ${}_{92}U^{238}$, if its half-life against alpha decay is 1.42×10^{17}s?

Q50. Why are α-particles emitted rather than say protons or ${}_2He^3$ nuclei?

Q51. A sequence of stepwise decays of radioactive nucleus is:

$$D \xrightarrow{\alpha} D_1 \xrightarrow{\beta} D_2 \xrightarrow{\alpha} D_3 \xrightarrow{\alpha} A_4$$

If the nucleon number and atomic number of D_2 are respectively 176 and 71, what are the corresponding values for D and D_4 nuclei. Justify your answers.

Q52. A radioactive nucleus undergoes a series of decays according to the sequence:

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

The mass number and atomic number of A are 180 and 72 respectively. What are these numbers for A_4.

Q53. A radioactive nucleus undergoes a series of decays according to the sequence:

$$A \xrightarrow{\beta} A_1 \xrightarrow{\alpha} A_2 \xrightarrow{\alpha} A_3$$

If the mass number and atomic number of A_3 are 172 and 69 respectively, what is the mass number and atomic number of A.

Q54. In the hypothetical fission reaction: $\quad {}_{92}X^{236} \longrightarrow {}_{a}Y^{141} + {}_{36}Z^{b} + 3\,{}_0n^1$

What are the values of the numbers a and b? Calculate the total energy released per nuclear fission in MeV units, when the masses in a.m.u. units are of neutron = 1.009, of X-nucleus = 235.891, of Y-nucleus = 140.673 and of Z nucleus 91.791.

Q55. Determine the amount of ${}_{84}Po^{210}$ necessary to provide a source of α-particles of 5 mci strength. The half-life of polonium is 138 days.

Q56. Calculate the half-life period of a radioactive substance, if its activity drops to 1/16th of its initial value in 30 years.

Q57. The half-life of a radioactive substance is 1672 years. If the initial mass of the substance is 1 g, after how many years will only 1 mg of it to be left behind?

Q58. 4 g of radioactive material of half-life 10 years is kept in a store for 15 years. How much material is disintegrated?

Q59. The half-life of radium is 1,590 years. In how many years will one gram of the pure element lose 1 centigram?

Q60. Differentiate between artificial and induced radioactivity.

Q61. ${}_{92}U^{238}$ decays successively to form ${}_{90}Th^{234}$, ${}_{91}Pa^{234}$, ${}_{92}U^{234}$, ${}_{90}Th^{230}$, ${}_{88}Ra^{226}$, and ${}_{86}Rn^{222}$. What are radioactive radiations emitted in each decay process?

Q62. How was neutron discovered? Explain briefly?

Q63. Define the term decay constant for a radioactive substance. Deduce the relation between decay constant and half-life period.

Q64. Briefly discuss the uses of nuclear reactor.

Q65. Neutrons produced in fission can be slowed down even by using ordinary water. Then, why is heavy water used for this purpose?

Q66. Explain the phenomenon of fusion. Give one representative equation.

Q67. Explain the phenomenon of fission. Give one representative equation.

Q68. Name the reaction which takes place when a slow neutron beam strikes ${}^{235}_{92}U$ nuclei. Write the nuclear reaction involved.

Q69. A radioactive isotope has a half-life of *T* years. How long will it take the activity to reduce to (a) 3.125%, (b) 1% of its original value?

Q70. The normal activity of living carbon-containing matter is found to be about 15 decays per minute for every gram of carbon. This activity arises from the small proportion of radioactive ${}^{14}_{6}C$ present with the stable carbon isotope ${}^{12}_{6}C$. When the organism is dead, its interaction with the atmosphere (which maintains the above equilibrium activity) ceases and its activity begins to drop. From the known half-life (5730 years) of ${}^{14}_{6}C$, and the measured activity, the age of the specimen can be approximately estimated. This is the principle of ${}^{14}_{6}C$ dating used in archaeology. Suppose a specimen from Mohenjodaro gives an activity of 9 decays per minute per gram of carbon. Estimate the approximate age of the Indus-Valley civilisation.

Q71. The radionuclide ${}^{11}C$ decays according to

$$ {}^{11}_{6}C \longrightarrow {}^{11}_{5}B + e^{+} + \nu: \qquad T_{1/2} = 20.3 \text{ min} $$

The maximum energy of the emitted positron is 0.960 MeV.

Given the mass values:

$$ m({}^{11}_{6}C) = 11.011434 \text{ u} \quad \text{and} \quad m({}^{11}_{6}B) = 11.009305 \text{ u} $$

calculate *Q* and compare it with the maximum energy of the positron emitted.

Q72. For the β^+ (positron) emission from a nucleus, there is another competing process known as electron capture (electron from an inner orbit, say, the *K*-shell, is captured by the nucleus and a neutrino is emitted).

$$e^+ + {}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + \nu$$

Show that if β^+ emission is energetically allowed, electron capture is necessarily allowed but not vice-versa.

Q73. In an experiment, the activity of 1.2 milligram (mg) of radioactive potassium chloride (chloride of isotope K-40) was found to be 170 s^{-1}. Taking molar mass of K-40 Cl to be 0.075 kg $mole^{-1}$, find the number of K-40 atoms in the sample and hence find the half-life of K-40. Given, Avogadro number = 6.0×10^{23} $mole^{-1}$.

Q74. An observer, in a laboratory, starts with N_0 nuclei of a radioactive sample and keeps on observing the number (N) of the left over nuclei at regular intervals of 10 minutes each. She prepares the following table on the basis of observation:

t (in min)	0	10	20	30	40
$\log_e \frac{N_0}{N}$	0	3.465	6.930	10.395	13.860

Use this data to plot a graph of loge (N_0/N) versus time (t) and calculate the decay constant and half life of the given sample.

Q75. Explain with an example, whether the neutron to proton ratio in a nucleus increases or decreases due to β-decay.

Q76. With the help of one example, explain how the neutron to proton ratio changes during alpha decay of a nucleus.

Q77. A source contains two phosphorous radio nuclides ${}^{32}_{15}P$ ($T_{1/2} = 14.3d$) and ${}^{33}_{15}P$ ($T_{1/2} = 25.3d$). Initially, 10% of the decays come from ${}^{33}_{15}P$. How long one must wait until 90% do so?

Q78. Two radioactive nuclei *X* and *Y* initially contain an equal number of atoms. Their half life is 1 hour and 2 hours respectively. Calculate the ratio of their rates of disintegration after two hours.

Q79. Define the term decay constant for a radioactive substance. Deduce the relation between decay constant and half life period.

Q80. Define disintegration constant and mean life of a radioactive substance. Give the unit for each.

Q81. Calculate the amount of energy released during the ×-decay of

$${}_{90}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4}$$

Given atomic mass of ${}_{92}U^{238}$ = 238.05079 a.m.u. Atomic mass of ${}_{2}He^{4}$ = 4.00260 a.m.u.
Atomic mass of ${}_{90}Th^{234}$ = 234.04363 a.m.u. and 1 a.m.u. $\equiv$ 931.5 MeV

Q82. One gram of ${}_{88}Ra^{226}$ has activity of one curie. Find its half-life period. Given that a kilo mole of ${}_{88}Ra^{226}$ contains 6.025×10^{26} atoms.

Q83. The half-life of a radioactive sample is 5,500 years. Its initial activity is found to be 15 decays per minute per gram. In how much time would its activity reduce to 10 decays per minute per gram? Take $\log_e 3 = 1.0986$ and $\log_e 2 = 0.693$.

Q84. What is the basic mechanism for the emission of β^- or β^+-particles in a nuclide? Give an example by writing explicitly a decay process for β^--emission.

Is (a) the energy of the emitted β^--particles continuous or discrete; (b) the daughter nucleus obtained through β^--decay, an isotope or an isobar of the parent nucleus?

Q85. 'The half-life of ${}^{14}_{6}C$ is 57000 years.' What does it mean?
Two radioactive nuclei X and Y initially contain an equal number of atoms. Their half-life is 1 hour and 2 hours respectively. Calculate the ratio of their rates of disintegration after two hours.

Q86. Define the term decay constant for a radioactive substance. Deduce the relation between decay constant and half-life period. Find the half-life period of a radio active material if its activity drops to $\frac{1}{32}$ of its initial value in 25 years.

Q87. State the laws of radioactive disintegration. What is half-life period of a radioactive substance? Find an expression for it.

Q88. State the laws of radioactive decay. Plot a graph showing the number (N) of undecayed nuclei as a function of time (t) .

Q89. What is radioactivity? State the laws of radioactive decay. Show that radioactive decay is exponential in nature.

Q90. What is meant by radioactivity? What type of radiations are emitted? Explain briefly the nature of these radiations.

Q91. Derive the relation $N = N_0e^{-\lambda t}$ for radioactive decay. Obtain the relation between disintegration constant and half-life.

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S1. α is a nucleus of helium ($_2He^4$) and β is an electron (e^- for β^- and e^+ for β^+). In every α-decay, there is a loss of 2 protons and 4 neutrons. In every β^+-decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every β^--decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus.

For the given cases, the various nuclear reactions can be written as:

$$^{11}_{6}C \longrightarrow {}^{11}_{5}C + e^+ + \nu$$

S2. α is a nucleus of helium ($_2He^4$) and β is an electron (e^- for β^- and e^+ for β^+). In every α-decay, there is a loss of 2 protons and 4 neutrons. In every β^+-decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every β^--decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus.

For the given cases, the various nuclear reactions can be written as:

$$^{210}_{83}B \longrightarrow {}^{210}_{84}PO + e^- + \overline{\nu}$$

S3. α is a nucleus of helium ($_2He^4$) and β is an electron (e^- for β^- and e^+ for β^+). In every α-decay, there is a loss of 2 protons and 4 neutrons. In every β^+-decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every β^--decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus.

For the given cases, the various nuclear reactions can be written as:

$$^{32}_{15}P \longrightarrow {}^{32}_{16}S + e^- + \overline{\nu}$$

S4. α is a nucleus of helium ($_2He^4$) and β is an electron (e^- for β^- and e^+ for β^+). In every α-decay, there is a loss of 2 protons and 4 neutrons. In every β^+-decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every β^--decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus.

For the given cases, the various nuclear reactions can be written as:

$$_{88}Ra^{226} \longrightarrow {}_{86}Rn^{222} + {}_2He^4$$

S5. α is a nucleus of helium ($_2He^4$) and β is an electron (e^- for β^- and e^+ for β^+). In every α-decay, there is a loss of 2 protons and 4 neutrons. In every β^+-decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every β^--decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus.

For the given cases, the various nuclear reactions can be written as:

$$_{88}Ra^{226} \longrightarrow {}_{86}Rn^{222} + {}_2He^4$$

S6. Alpha particle.

S7. Alpha and beta radiations are affected by the magnetic field.

S8. Plutonium and tritium are two radioactive elements, which are not found in observable quantities in nature. It is because of their very small values of half lives.

S9. If the nucleons bound in a nucleus are separated apart from each other energy equal to the binding energy of the nucleus need to be transferred to these particles. As result, the sum of their masses becomes greater than mass of nucleus according to Einstein's mass-energy equivalence.

S10. Lesser absorption probability of neutrons more effective in slowing down neutrons to thermal energies.

S11. By definition of half-life, half of the initial sample will remain undecayed after 12.5 y. In the next 12.5 y, one-half of these nuclei would have decayed. Hence, one fourth of the sample of the initial pure tritium will remain undecayed.

S12. α is a nucleus of helium ($_2He^4$) and β is an electron (e^- for β^- and e^+ for β^+). In every α-decay, there is a loss of 2 protons and 4 neutrons. In every β^+-decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every β^--decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus.

For the given cases, the various nuclear reactions can be written as:

$$^{120}_{54}Xe + e^+ \longrightarrow {}^{120}_{53}I + \nu$$

S13. α is a nucleus of helium ($_2He^4$) and β is an electron (e^- for β^- and e^+ for β^+). In every α-decay, there is a loss of 2 protons and 4 neutrons. In every β^+-decay, there is a loss of 1 proton and a neutrino is emitted from the nucleus. In every β^--decay, there is a gain of 1 proton and an antineutrino is emitted from the nucleus.

For the given cases, the various nuclear reactions can be written as:

$$^{97}_{43}Tc \longrightarrow {}^{97}_{42}MO + e^+ + \nu$$

S14. Here, $T_{1/2}$ = 20 min; t = 1 hour = 60 min.

Now, $$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}} = \left(\frac{1}{2}\right)^{\frac{60}{20}} = \left(\frac{1}{2}\right)^3 = \mathbf{\frac{1}{8}}.$$

S15. Relation between half-life and decay constant:

$$T_{1/2} = \frac{0.693}{\lambda}.$$

S16. The half-life of a radioactive substance is defined as the time during which the nuclei of half of the atoms of the radioactive substance will disintegrate.

S17. α-rays, β-rays and γ-rays.

S18. Being massive and of large nuclear cross-section, α-particle possesses high ionising power.

S19. $_{15}P^{32}$ undergoes β-decay as given below:

$$_{15}P^{32} \longrightarrow {}_{16}S^{32} + {}_{-1}e^{0} + \overline{\nu} + E_{\beta}.$$

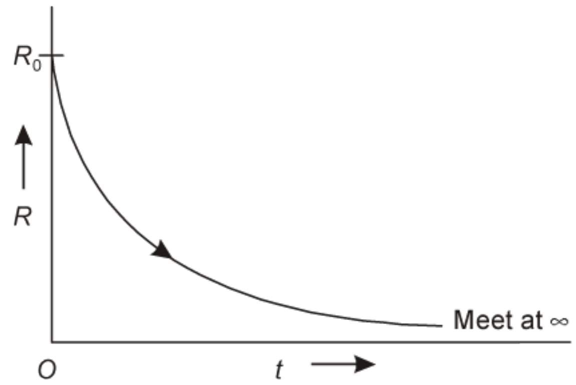
S20. The nucleus of *He* contains 2 protons and 2 neutrons. Therefore, for the nucleus of *He*,

$$Z = \mathbf{2} \quad \text{and} \quad A = 2 + 2 = \mathbf{4}.$$

Now, for the nucleus of *Po*,

$$A = 220 - 4 = \mathbf{216} \quad \text{and} \quad Z = 86 - 2 = \mathbf{84}.$$

S21. The variation of activity (R) of a given radioactive sample with time (t) is as shown in figure below



S22. Half life, $$T_{1/2} = \frac{0.693}{\lambda}$$

But $$R = N\lambda$$

or $$\lambda = \frac{R}{N}$$

$\therefore$ $$T_{1/2} = 0.693 \times \frac{N}{R}.$$

S23. The activity of a radioactive substance is defined as the rate at which the nuclei of its atoms in the sample disintegrate.

Thus, activity of a substance,

$$R = -\frac{dN}{dt}$$

Si unit of the activity of a radioactive material is **Becquerel (Bq)**. It is equal to 1 decay s^{-1}. The practical unit is **curie (Ci)**.

S24. (a) Half life $T_{1/2} = \dfrac{0.693}{\lambda}$

(b) Mean life = $\dfrac{1}{\lambda}$

S25. β-decay is the result of conversion of a neutron into a proton, electron and an antineutrino ($\overline{\nu}$). Since the energy available in β-decay is shared by the electron and the antineutrino in all possible proportions as they some out of nucleus; the β-ray spectrum is continuous in nature.

S26. The isotope of an element, capable of emitting radiation in the same manner as a radioactive substance does, is called a radioisotope. Usually, radioisotopes are artificially produced.

S27. An electron and a β-particle are essentially the same. The electron of nuclear origin is called a β-particle.

S28. ${}_{15}P^{32} \longrightarrow {}_{16}S^{32} + {}_{-1}e^{0}$.

S29. A radioactive sample requires infinite time to disintegrate completely and in this sense, all the radioactive substances are identical.

S30. γ-rays are electromagnetic waves of very short wavelength (65×10^{-13} m to 10^{-11} m).

S31. Plutonium and tritium are two radioactive elements, which are not found in observable quantities in nature. It is because of their very small values of half lives.

S32. A radioactive substance is one, whose atoms have unstable nuclei. The nuclei of radioactive substances emit α, β, and γ-rays.

S33. Let ${}_{Z}P^{A}$ unknown particle. Then,

$${}_{3}Li^{7} + {}_{Z}P^{A} \longrightarrow {}_{2}He^{4} + {}_{2}He^{4}$$

$$3 + Z = 2 + 2, \quad \text{or} \quad Z = 1$$

$$7 + A = 4 + 4 \quad \text{or} \quad A = 1$$

Therefore, the unknown bombarding particle is ${}_{1}\mathbf{H}^{1}$. **(proton)**.

S34. Cadmium.

S35. In the formation of the nucleus, the neutrons and protons have to collect in a very small space (of radius $\simeq 10^{-15}$ m). In the process the nucleons give out energy at the cost of their masses *i.e.*,

conversion of mass to energy take place. As a result, mass of the nucleus formed becomes less than the sum of the masses of the nucleons.

S36. Example of fusion reaction:

$${}_{1}H^{2} + {}_{1}H^{2} \longrightarrow {}_{2}He^{4} + 24\,\text{MeV}.$$

S37. The strength of the radioactive source is given as:

$$\frac{dN}{dt} = 8.0 \text{ mCi}$$

$$= 8 \times 10^{-3} \times 3.7 \times 10^{10}$$
$$= 29.6 \times 10^{7} \text{ decay/s}$$

Where, N = Required number of atoms

Half-life of $^{60}_{27}Co$, $T_{1/2}$ = 5.3 years

$$= 5.3 \times 365 \times 24 \times 60 \times 60$$
$$= 1.67 \times 10^{8}\,s$$

For decay constant λ, we have the rate of decay as:

$$\frac{dN}{dt} = \lambda N = \frac{0.693}{T_{1/2}} = \frac{0.693}{1.67 \times 10^{8}}\,s^{-1}$$

Where, λ

$$\therefore \quad N = \frac{1}{\lambda}\frac{dN}{dt} = \frac{29.6 \times 10^{7}}{\frac{0.693}{1.67 \times 10^{8}}} = 7.33\ \ 1016 \text{ atoms}$$

For $_{27}Co^{60}$:

Mass of 6.023×10^{23} (Avogadro's number) atoms = 60 g

$$\therefore \quad \text{Mass of } 7.133 \times 10^{16} \text{ atoms} = \frac{60 \times 7.133 \times 10^{16}}{6.023 \times 10^{23}} = 7.106 \times 10^{-6}\text{ g}$$

Hence, the amount of $_{27}Co^{60}$ necessary for the purpose is 7.106×10^{-6} g.

S38. Half life of $^{90}_{38}Sr$, $t_{1/2}$ = 28 years

$$= 28 \times 365 \times 24 \times 60 \times 60$$
$$= 8.83 \times 10^{8}\,s$$

Mass of the isotope, m = 15 mg

90 g of $^{90}_{38}Sr$ atom contains 6.023 × 10^{23} (Avogadro's number) atoms.

Therefore, 15 mg of $^{90}_{38}Sr$ contains:

$$\frac{6.023 \times 10^{23} \times 15 \times 10^{-3}}{90},\quad \textit{i.e.}, 1.0038 \times 10^{20} \text{ number of atoms}$$

Rate of distintegration, $\frac{dN}{dt} = \lambda N$

Where, λ = Decay constant = $\frac{0.693}{8.83 \times 10^{8}}\,s^{-1}$

$$\therefore \quad \frac{dN}{dt} = \frac{0.693 \times 1.0038 \times 10^{20}}{8.83 \times 10^{8}} = 7.878 \times 10^{10} \text{ atom/s}$$

Hence, the disintegration rate of 15 mg of the given isotope is 7.878×10^{10} atoms/s.

S39. Nuclear radius of the gold isotope $_{79}Au^{197} = R_{Au}$

Nuclear radius of the silver isotope $_{47}Ag^{107} = R_{Ag}$

Mass number of gold, $A_{Au} = 197$

Mass number of silver, $A_{Ag} = 107$

The ratio of the radii of the two nuclei is related with their mass numbers as:

$$\frac{R_{Au}}{R_{Ag}} = \left(\frac{R_{Au}}{R_{Ag}}\right)^{\frac{1}{3}} = \left(\frac{197}{107}\right)^{\frac{1}{3}} = 1.2256$$

Hence, the ratio of the nuclear radii of the gold and silver isotopes is about 1.23.

S40. Here,

$$T_{1/2} = 4.5 \times 10^9\,\text{y}$$
$$= 4.5 \times 10^9\,\text{y} \times 3.16 \times 10^7\,\text{s/y}$$
$$= 1.42 \times 10^{17}\,\text{s}$$

One k mol of any isotope contains Avogadro's number of atoms, and so 1g of $^{238}_{92}U$ contains

$$= \frac{1}{238 \times 10^{-3}} \times k\,\text{mol} \times 6.025 \times 10^{26}\,\text{atoms}/k\,\text{mol}$$

$$= 25.3 \times 10^{20}\,\text{atoms.}$$

The decay rate R is $R = \lambda N$

$$= \frac{0.693}{T_{1/2}} N = \frac{6.693 \times 25.3 \times 10^{20}}{1.42 \times 10^{17}}\,\text{s}^{-1}$$

$$= 1.23 \times 10^4\,\text{s}^{-1}$$
$$= 1.23 \times 10^4\,\text{Bq.}$$

S41. (a) Yes, ${}_3X^7$ and ${}_3Y^4$ are isotopes of the same elements. It is because, an element is characterised by its atomic number. Since both X and Y have atomic number 3, they represent the same element *i.e.*, *Li*.

(b) ${}_3X^7$ (or ${}_3Li^7$) is more stable than ${}_3X^4$ (or ${}_3Li^4$) It is because, the greater number of neutrons in ${}_3Li^7$ results in greater attractive force between the nucleons so as to win over the Coulomb's repulsive force between the protons.

S42. The decay of a radioactive element is not a single stage process. The daughter element formed further decays into some other element and so on, till stable lead is formed. In the complete decay scheme (from parent element to the stable lead), a few radioactive elements undergo α-decay, while others β-decay. The γ-ray is emitted in cases, when the nucleus after an α or a β-decay is left in the excited state.

S43. Since the nucleus D_2 is ${}_{71}D_2{}^{176}$, β^- is an electron (${}_{-1}e^0$) and α-particle is ${}_2He^4$ (nucleus of helium), the given decay series may be represented as

$${}_{72}D^{180} \xrightarrow{\beta^-} {}_{73}D_1^{180} \xrightarrow{\alpha} {}_{71}D_2{}^{176}$$

i.e., the mass number and atomic number of the radioactive isotope D are **180** and **72** respectively.

S44. Since the nucleus D_2 is ${}_{71}D^{176}$, β^- is an electron (${}_{-1}e^{\beta}$) and α-particle is ${}_2He^4$ (nucleus of helium), the given decay series may be represented as

$$_{72}D^{180} \xrightarrow{\alpha} {}_{70}D_1^{176} \xrightarrow{\beta^-} {}_{71}D_2^{176}$$

i.e., the mass number and atomic number of the radioactive isotope D are **180** and **72** respectively.

S45. Now, $$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}},$$

where, $T_{1/2}$ is half life and N/N_0 is the fraction of atoms left after time t.

Here, $T_{1/2}$ = 12.5 years; t = 25 years

$\therefore$ $$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{25}{12.5}} = \left(\frac{1}{2}\right)^2 = \mathbf{\frac{1}{4}}$$

S46. (a) Radioactivity is a spontaneous phenomenon and one cannot predict, when a particular atom (when the nucleus of a particular atom will undergo disintegration) in a given radioactive sample will undergo disintegration.

(b) When a radioactive atom disintegrates, wither an α-particle (nucleus of helium) or a β-particle (electron) is emitted.

(c) The emission of an α-particle by a radioactive atom results in a daughter atom, whose atomic number is 2 units less and mass number is 4 units less than that of the parent atom.

$$_{Z}X^{A} \xrightarrow{\alpha-\text{decay}} {}_{Z-2}Y^{A-4}$$

(d) The emission of a β-particle by a radioactive atom results in a daughter atom, whose atomic number is 1 unit more but mass number is same as that of the parent atom.

$$_{Z}X^{A} \xrightarrow{\beta-\text{decay}} {}_{Z+1}Y^{A}$$

S47. Since the nucleus A_2 is $_{71}A_2^{176}$, β^- is an electron ($_{-1}e^{\beta}$) and α-particle is $_2He^4$ (nucleus of helium), the given decay series may be represented as

$$_{72}A^{180} \xrightarrow{\beta^-} {}_{73}A_1^{180} \xrightarrow{\alpha} {}_{71}A_2^{176}$$

i.e., the mass number and atomic number of the radioactive isotope A are **172** and **68** respectively.

Mass number of A_1 = **180**, mass number of A = **180**.

Atomic number A_1 = **73**, atomic number A = **72.**

The elements ***A_1* and *A* are the isobars**.

S48. α-particle is nucleus of helium ($_2He^4$),

Therefore, the above mentioned radioactive decays will proceed as below:

$$_{73}D^{181} \xrightarrow{\alpha^{n^1}} {}_{73}D_1^{180} \xrightarrow{_2He^4} {}_{71}D_2^{176}$$

Therefore, the mass number and atomic number of D are **181** and **73** respectively.

Since atomic number of D_1 and D are same, D and D_1 are isotopes.

S49.

$$\lambda = \frac{0.693}{1.42 \times 10^{17}} s^{-1} = 4.88 \times 10^{-18} s^{-1}$$

Number of atoms in 1g of U^{238},

$$N = \frac{6.02 \times 10^{23}}{238} = 2.62 \times 10^{21}$$

We know that: $$\frac{dN}{dt} = \lambda N, \quad \text{... (i)}$$

Setting the values (λ & N) in Eq. (i), we get

$$\frac{dN}{dt} = 4.88 \times 10^{-18} \times 2.62 \times 10^{21} = \mathbf{1.278 \times 10^4 s^{-1}}.$$

S50. It is because of the high value of binding energy of the α-particle. Therefore, on emission of α-particle, the binding energy per nucleon of the residual nucleus increases appreciably. On the other hand, the emission of a proton or $_2He^3$ nuclei may not be energetically possible.

S51. An, α-particle is the nucleus of helium ($_2He^4$) and a β-particle is an electron ($_{-1}e^0$). Therefore, the above mentioned radioactive decays will proceed as below:

$$_{72}D^{180} \xrightarrow{_2He^4} {}_{70}D_1^{176} \xrightarrow{-1e^0} {}_{71}D_2^{176} \xrightarrow{_2He^4} {}_{69}D_3^{172} \xrightarrow{_2He^4} {}_{67}D_4^{168}$$

The nucleon number and atomic number corresponding to D are **180** and **72**; while those corresponding to D_4 are **168** and **67**.

S52. Since the nucleus A is $_{72}A^{180}$, α-particle is $_2He^4$ (nucleus of helium), β-particle is an electron ($_{-1}e^0$) and γ-particle is merely an energy photon, the given decay series may be represented as

$$_{72}A^{180} \xrightarrow{\alpha} {}_{70}A_1^{176} \xrightarrow{\beta} {}_{71}A_1^{176} \xrightarrow{\alpha} {}_{69}A_3^{172} \xrightarrow{\gamma} {}_{69}A_4^{172}$$

i.e., the mass number and atomic number of the radioactive nucleus A_4 are **172** and **69** respectively.

S53. Since the nucleus A_3 is $_{69}A_3^{172}$, β-particle is an electron ($_{-1}e^0$) and α-particle is $_2He^4$ (nucleus of helium), the given decay series may be represented as

$$_{72}A^{180} \xrightarrow{\beta} {}_{73}A_1^{180} \xrightarrow{\alpha} {}_{71}A_2^{176} \xrightarrow{\alpha} {}_{69}A_3^{172}$$

i.e., the mass number and atomic number of the radioactive nucleus A are **180** and **72** respectively.

S54. $$_{92}X^{236} \longrightarrow {}_aY^{141} + {}_{36}Z^b + 3\,{}_0n^1$$

Obviously, $92 = a + 36$

or $a = \mathbf{56}$

Also, $236 = 141 + b + 3 \times 1$

or $b = \mathbf{92}$

$$Q = [m_X - (m_Y + m_Z + 3m_n)] \times 931$$

$$= [235.891 - (140.673 + 91.791 + 3 \times 1.009)] \times 931$$

$$= 0.4 \times 931 = \mathbf{372.4\ MeV}.$$

S55. $$R = \frac{dN}{dt} = 5 \text{ mci} = 5 \times 3.7 \times 10^7 \text{ s}^{-1}$$

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{138} \text{ day}^{-1} = \frac{0.693}{138 \times 24 \times 60 \times 60}$$

$$= 5.812 \times 10^{-8} \text{ s}^{-1}$$

Using the relation: $\frac{dN}{dt} = \lambda N,$

it can be obtained that

$$N = 3.183 \times 10^{15}$$

$$m = \frac{210 \times N}{\text{Avogadro number}} = \frac{210 \times 3.183 \times 10^{15}}{6.02 \times 10^{23}}$$

$$= \mathbf{1.11 \times 10^{-6}\ g}.$$

S56. Here, $\frac{R}{R_0} = \frac{1}{16}$ and $t = 30$ years

Now, $\frac{R}{R_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}}$ or $\frac{1}{16} = \left(\frac{1}{2}\right)^{30/T_{1/2}}$

or $$\left(\frac{1}{2}\right)^{30/T_{1/2}} = \left(\frac{1}{2}\right)^4 \quad \text{or} \quad \frac{30}{T_{1/2}} = 4$$

or $$T_{1/2} = 30/4 = \mathbf{7.5\ years}.$$

S57. $m_0 = 1\,\text{g}; \quad m = 1\,\text{mg} = 10^{-3}\,\text{g};$

$$\lambda = \frac{0.693}{1672} = 4.1447 \times 10^{-4}\,\text{year}^{-1}$$

We know, $$t = \frac{2.303}{\lambda}\log_{10}\frac{m_0}{m} = \frac{2.303}{4.1447 \times 10^{-4}}\log_{10}\frac{1}{10^{-3}}$$

$$= \frac{2.303 \times 3}{4.1447 \times 10^{-4}} = \mathbf{16{,}669.5\ years}.$$

S58. $$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{10} = 0.0693\ \text{year}^{-1}$$

Now, $$m = m_0 e^{-\lambda t} = 4e^{-0.0693 \times 15} \qquad (\because\ N \propto m)$$

$$= 4e^{-1.0395} = 4 \times 0.3536 = \mathbf{1.4144\ g}$$

Therefore, material disintegrated

$$= 4 - 1.4144 = \mathbf{2.5856\ g}.$$

S59. $$\lambda = \frac{0.693}{1590} = 4.3585 \times 10^{-4}\,\text{year}^{-1}; \quad m_0 = 1\,\text{g}; \quad m_0 - m = 0.01\,\text{g}$$

$\therefore$ $$m = m_0 - 0.01 = 1 - 0.01 = 0.9\,\text{g}$$

Using the relation: $$t = \frac{2.303}{\lambda}\log_{10}\frac{m_0}{m}$$

it can be obtained that

$$t = \mathbf{23.25\ years}.$$

S60. **Artificial radioactivity:** It is the phenomenon of disintegration of an otherwise stable nucleus by bombarding it with a suitable high energy particle. For example, when a nitrogen nucleus is bombarded by an alpha particle, a proton is released with the formation of an oxygen nucleus.

$${}_7N^{14} + {}_2He^4 \longrightarrow {}_8O^{17} + {}_1H^1$$

Induced radioactivity: It is the phenomenon in which the daughter nucleus produced by artificial radioactivity continues disintegrating even after the bombardment is stopped.

For example: When boron is bombarded by α-particles, it continue to disintegrate. It is represented as

$$_2He^4 + {}_5B^{10} \longrightarrow {}_7N^{14} \longrightarrow {}_7N^{13} + {}_0n^1$$

$$_7N^{13} \longrightarrow {}_6C^{13} + {}_{+1}e^0 .$$

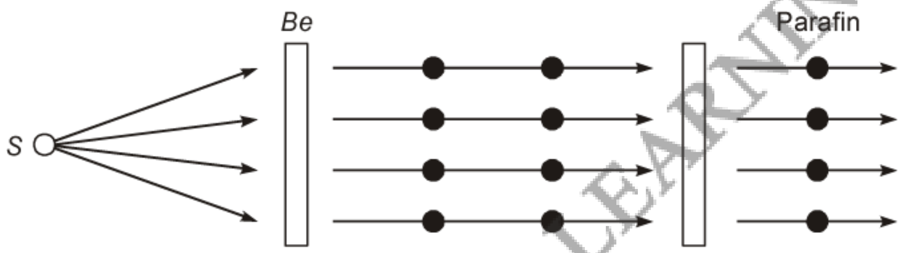
S61. The given successive decays can be represented as

$$_{92}U^{238} \longrightarrow {}_{90}Th^{234} \longrightarrow {}_{91}Pa^{234} \longrightarrow {}_{92}U^{234} \longrightarrow {}_{90}Th^{230} \longrightarrow {}_{88}Ra^{226} \longrightarrow {}_{86}Rn^{222}$$

Thus, the radioactive radiations emitted in successive decays are α-, β^--, β^--, α-, α-, α-particles.

S62. Chadwick, in 1932, performed an experiment in which α-particles coming from a polonium source were allowed to strike beryllium metal, as shown in figure below. It was observed that penetrating rays come out of beryllium metals and they were the particles without charge, called neutrons. The reaction is represented as

$$_2He^4 + {}_4Be^9 \longrightarrow {}_6C^{12} + {}_0n^1$$



A neutron ($_0n^1$) has no charge (therefore, its atomic number $Z = 0$) and its mass is nearly equal to that of a proton (therefore, its mass number $A = 1$).

S63. For radioactive nuclei, $N = N_0 e^{-\lambda t}$ ... (i)

If $t = \dfrac{1}{\lambda}$, then $N = N_0 e^{-\lambda/\lambda} = N_0 e^{-1} = \dfrac{N_0}{e}$

Thus, the radioactive decay constant of a radioactive element is the reciprocal of time during which the number of atoms left in a radioactive sample reduces to $\left(\dfrac{1}{e}\right)$ times the original number of atoms (N_0) in the sample.

At $T = T_{1/2}$

$$N = \frac{N_0}{2}$$

Setting the values (N & T) in Eq. (i), we get

$$\therefore \qquad \frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

$$\text{or} \qquad \frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$\text{or} \qquad 2 = e^{\lambda T_{1/2}}$$

$$\text{or} \qquad \lambda T_{1/2} = \log_e 2 = 0.693$$

$$\therefore \qquad T_{1/2} = \frac{0.693}{\lambda}.$$

S64. Nuclear reactor are primarily used to generate power from controlled nuclear fission. However, in addition to this, nuclear reactors are also used for other purposes. Some of them are mentioned below:

(a) Nuclear reactors are used to generate electric power.

(b) They are used to produced radioactive isotopes which are used in the fields of medicine, agriculture, industry, etc.

(c) Nuclear reactors are used for the propulsion of ships, submarines and aircrafts.

(d) They are used to produced neutron beam of high intensity which is used in nuclear research.

S65. Neutrons produced during fission is slowed down if they collide with a nucleus of the same mass. As ordinary water contains hydrogen atoms (having mass approximately equal to a neutron), therefore it can be used as a moderator. However, it absorbs neutrons at a fast rate according to reaction

$$N + p \longrightarrow d + \gamma$$

Where d is deuteron. It is due to this difficulty that heavy water is used as moderator which has negligible cross-section for neutron absorption.

S66. Nuclear fusion is the phenomenon in which two or more lighter nuclei fuse to form a single heavy nucleus. The nuclear fusion takes place under the conditions of very high temperature ($\approx 10^7$ K) and pressure.

For example, when two deuterium nuclei fuse together, a helium nucleus is formed. This is represented as

$${}_1H^2 + {}_1H^2 \longrightarrow {}_2He^4 + Q$$

Where Q is the energy released in the process. The mass of the product nucleus is slightly less than the sum of the masses of the lighter nuclei fusing together. This difference in masses (called mass defect Δm) results in the release of tremendous amount of energy, in accordance with Einstein's mass energy relation, $E = \Delta m.c^2$.

S67. Nuclear fission is the phenomenon of splitting of a heavy nucleus (usually of mass number greater than 230) into two (or more) lighter nuclei. For example, nuclear fission of ${}_{92}U^{235}$ when it is hit by a neutron is represented as

$$ {}_{92}U^{234} + {}_{0}n^{1} \longrightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3{}_{0}n^{1} + Q $$

Where Q is the energy released in the process. In the process, certain mass disappears *i.e.*, sum of the masses of final products is found to be slightly less than the sum of the masses of the reactant components. This difference in masses is called mass defect (Δm). The mass defect appears in the form energy in accordance with Einstein mass-energy relation, $E = \Delta m.c^2$.

S68. The reaction that takes place when a slow neutron beam strikes ${}^{235}_{92}U$ nuclei is called nuclear fission. The following nuclear reaction takes place

$$ {}^{1}_{0}n + {}^{235}_{92}U \longrightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2{}^{1}_{0}n + Q $$

where Q is the energy released in the process.

S69. Half-life of the radioactive isotope = T years

Original amount of the radioactive isotope = N_0

(a) After decay, the amount of the radioactive isotope = N

It is given that only 3.125% of N_0 remains after decay. Hence, we can write:

$$ \frac{N}{N_0} = 3.125\% = \frac{3.125}{100} = \frac{1}{32} $$

But $$ \frac{N}{N_0} = e^{-\lambda t} $$

Where, λ = Decay constant

t = Time

$\therefore$ $$ -\lambda t = \frac{1}{32} $$

$$ -\lambda t = \ln 1 - \ln 32 $$

$$ -\lambda t = 0 - 3.4657 $$

$$ t = \frac{3.4657}{\lambda} $$

Since, $$ \lambda = \frac{0.693}{T} $$

$\therefore$ $$t = \frac{3.466}{\frac{0.693}{T}} \approx 5T \text{ years}$$

Hence, the isotope will take about $5T$ years to reduce to 3.125% of its original value.

(b) After decay, the amount of the radioactive isotope = N

It is given that only 1% of N_0 remains after decay. Hence, we can write:

$$\frac{N}{N_0} = 1\% = \frac{1}{100}$$

But $$\frac{N}{N_0} = e^{-\lambda t}$$

$\therefore$ $$-\lambda t = \frac{1}{100}$$

$$-\lambda t = \ln 1 - \ln 100$$

$$-\lambda t = 0 - 4.6052$$

$$t = \frac{4.6052}{\lambda}$$

Since, $$\lambda = 0.693/\text{T}$$

$\therefore$ $$t = \frac{4.6052}{\frac{0.693}{T}} = 6.645T \text{ years}$$

Hence, the isotope will take about 6.645T years to reduce to 1% of its original value.

S70. Decay rate of living carbon-containing matter, R = 15 decay/min

Let N be the number of radioactive atoms present in a normal carbon-containing matter.

Half life of $^{14}_{6}C$, $T_{½}$ = 5730 years

The decay rate of the specimen obtained from the Mohenjodaro site:

$$R' = 9 \text{ decays/min}$$

Let N' be the number of radioactive atoms present in the specimen during the Mohenjodaro period.

Therefore, we can relate the decay constant, λ and time, t as:

$$\frac{N}{N'} = 1\% = \frac{R}{R'} = e^{-\lambda t}$$

$$e^{-\lambda t} = \frac{9}{15} = \frac{3}{5}$$

$$-\lambda t = \log_e \frac{3}{5} = -0.5108$$

$\therefore \quad t = \dfrac{0.5108}{\lambda}$

But $\quad \lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{5730}$

$\therefore \quad t = \dfrac{0.5108}{\dfrac{0.693}{5730}} = 4223.5$ years

Hence, the approximate age of the Indus-Valley civilisation is 4223.5 years.

S71. The given nuclear reaction is:

$$^{11}_{6}C \longrightarrow {}^{11}_{5}B + e^{+} + \nu$$

Half life $^{11}_{6}C$ nuclei, $T_{1/2} = 20.3$ min

Atomic mass of $m(^{11}_{6}C) = 11.011434$ u

Atomic mass of $m(^{11}_{6}B) = 11.009305$ u

Maximum energy possessed by the emitted positron = 0.960 MeV

The change in the Q-value (ΔQ) of the nuclear masses of the $^{11}_{6}C$ nucleus given as:

$$\Delta Q = [m'({}_6C^{11}) - [m'(^{11}_{5}B) + m_e]]\,c^2 \qquad \text{... (i)}$$

Where, m_e = Mass of an electron or positron = 0.000548 u

c = Speed of light

m' = Respective nuclear masses

If atomic masses are used instead of nuclear masses, then we have to add 6 m_e in the case of ^{11}C and 5 m_e in the case of ^{11}B.

Hence, Eq. (i) reduces to:

$$\Delta Q = [m({}_6C^{11}) - m(^{11}_{5}B) - 2m_e]\,c^2$$

Here, $m({}_6C^{11})$ and $m(^{11}_{5}B)$ are the atomic masses.

$\therefore \quad \Delta Q = [11.011434 - 11.009305 - 2 \times 0.000548]\,c^2$

$= (0.001033\,c^2)$ u

But $\quad 1\text{ u} = 931.5\text{ Mev}/c^2$

$\therefore \quad \Delta Q = 0.001033 \times 931.5 \approx 0.962$ MeV

The value of Q is almost comparable to the maximum energy of the emitted positron.

S72. Let the amount of energy released during the electron capture process be Q_1. The nuclear reaction can be written as:

$$e^{+} + {}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + \nu + Q_1 \qquad \text{... (i)}$$

Let the amount of energy released during the positron capture process be Q_2. The nuclear reaction can be written as:

$$^{A}_{Z}X \longrightarrow \ ^{A}_{Z-1}Y + e^{+} + \nu + Q_2 \qquad \text{... (ii)}$$

$m_N(^{A}_{Z}X)$ = Nuclear mass of $^{A}_{Z}X$

$m_N(^{A}_{Z-1}Y)$ = Nuclear mass of $^{A}_{Z-1}Y$

$m(^{A}_{Z}X)$ = Atomic mass of $^{A}_{Z}X$

$m(^{A}_{Z-1}Y)$ = Atomic mass of $^{A}_{Z-1}Y$

m_e = Mass of an electron

c = Speed of light

Q-value of the electron capture reaction is given as:

$$Q_1 = [m_N(^{A}_{Z}X) + m_e - m_N(^{A}_{Z-1}Y)]\,c^2$$

$$= [m(^{A}_{Z}X) - Zm_e + m_e - m(^{A}_{Z-1}Y) + (Z-1)\,m_e]\,c^2$$

$$= [m(^{A}_{Z}X) - m(^{A}_{Z-1}Y)]\,c^2 \qquad \text{... (iii)}$$

Q-value of the positron capture reaction is given as:

$$Q_2 = [m_N(^{A}_{Z}X) - m_N(^{A}_{Z-1}Y) - m_e]\,c^2$$

$$= [m(^{A}_{Z}X) - Zm_e - m(^{A}_{Z-1}Y) + (Z-1)\,m_e - m_e]\,c^2$$

$$= [m(^{A}_{Z}X) - m(^{A}_{Z-1}Y) - 2m_e]\,c^2 \qquad \text{... (iv)}$$

It can be inferred that if $Q_2 > 0$, then $Q_1 > 0$; Also, if $Q_1 > 0$, it does not necessarily mean that $Q_2 > 0$.

In other words, this means that if β^+ emission is energetically allowed, then the electron capture process is necessarily allowed, but not vice-versa. This is because the Q-value must be positive for an energetically-allowed nuclear reaction.

S73. Here, molar mass of K-40 Cl, $M = 0.075$ kg mole^{-1} mass of K-40 Cl, $m = 1.2$ mg $= 1.2 \times 10^{-6}$ kg

Since 1 mole of substance contains molecules equal to Avogadro number (= 6.0×10^{23}), the number of molecules in the K-40 Cl sample,

$$N = \frac{6.0 \times 10^{23}}{0.075} \times 1.2 \times 10^{-6} = \mathbf{9.6 \times 10^{18}}.$$

Also, activity of the given sample, $\dfrac{dN}{dt} = 170\,s^{-1}$.

If λ is disintegration constant, then

$$\frac{dN}{dt} = \lambda N$$

or $$170 = \lambda \times 9.6 \times 10^{18}$$

or $$\lambda = \frac{170}{9.6 \times 10^{18}} = 1.77 \times 10^{-17}\, s^{-1}$$

Therefore, half life of the sample,

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{1.77 \times 10^{-17}} = 3.915 \times 10^{16}\, s^{-1}$$

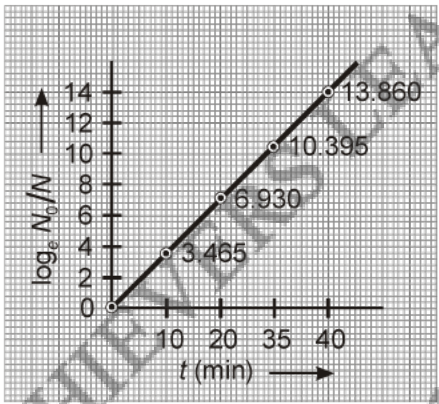
$$= \mathbf{1.241 \times 10^9\ years}.$$

S74. According to the radioactive decay law,

$$\log_e \frac{N}{N_0} = -\lambda t$$

or $$\log_e \frac{N_0}{N} = \lambda t$$

It follows that if we plot a graph between t (along X-axis) and $\log_e (N_0/N)$ (along Y-axis), it will be a straight line passing through the origin and having slope equal to λ as shown in the figure.



From the graph:

$$\lambda = \text{slope of the graph}$$

$$= \frac{13.860 - 3.465}{40 - 10} = \mathbf{0.3465\ min^{-1}}$$

Also, half life

$$T = \frac{0.693}{\lambda} = \frac{0.693}{0.3465} = \mathbf{2\ min}.$$

S75. ${}_{83}Bi^{210}$ decay into ${}_{82}Th^{210}$ by β-decay.

In ${}_{83}Bi^{210}$ nucleus, there are 83 protons and 210 – 83 *i.e.*, 127 neutrons.

Therefore, neutron to proton ratio,

$$\frac{N_n}{N_p} = \frac{127}{83} = 1.53$$

In ${}_{84}Po^{210}$ nucleus, there are 84 protons and 210 – 84 *i.e.*, 126 neutrons. Therefore,

$$\frac{N_n}{N_p} = \frac{126}{84} = 1.50$$

It follows that neutron to proton ratio ***decreases*** during β-decay.

S76. ${}_{92}U^{238}$ decay into ${}_{90}Th^{234}$ by α-decay.

In ${}_{92}U^{238}$ nucleus, there are 92 protons and 238 – 92 *i.e.*, 146 neutrons.

Therefore, neutron to proton ratio,

$$\frac{N_n}{N_p} = \frac{146}{92} = 1.587$$

In ${}_{90}U^{234}$ nucleus, there are 90 protons and 234 – 90 *i.e.*, 144 neutrons. Therefore,

$$\frac{N_n}{N_p} = \frac{146}{90} = 1.6$$

It follows that neutron to proton ratio ***increases*** during α-decay.

S77. Half life of ${}^{32}_{15}P$, $T_{1/2}$ = 14.3 days

Half life of ${}^{33}_{15}P$, $T'_{1/2}$ = 25.3 days

${}^{33}_{15}P$ nucleus decay is 10% of the total amount of decay.

The source has initially 10% of ${}^{33}_{15}P$ nucleus and 90% of ${}^{32}_{15}P$ nucleus.

Suppose after t days, the source has 10% of ${}^{32}_{15}P$ nucleus and 90% of ${}^{33}_{15}P$ nucleus.

Initially: Number of ${}^{33}_{15}P$ nucleus = N

Number of ${}^{32}_{15}P$ nucleus = 9 N

Finally: Number of ${}^{32}_{15}P$ nucleus = 9 N'

Number of ${}^{32}_{15}P$ nucleus = N'

For ${}^{32}_{15}P$ nucleus, we can write the number ratio as:

$$\frac{N'}{9N} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$$N' = 9N(2)^{\frac{-t}{14.3}} \qquad \ldots \text{(i)}$$

For ${}^{32}_{15}P$, we can write the number ratio as:

$$\frac{9N'}{N} = \left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}}$$

$$9N' = N(2)^{\frac{-t}{25.3}} \qquad \ldots \text{(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get:

$$\frac{1}{9} = 9 \times 2^{\left(\frac{t}{25.3} - \frac{t}{14.3}\right)}$$

$$\frac{1}{81} = 2^{-\left(\frac{11t}{25.3 \times 14.3}\right)}$$

$$\log 1 - \log 81 = \frac{-11t}{25.3 \times 14.3} \log 2$$

$$\frac{-11t}{25.3 \times 14.3} = \frac{0 - 1.908}{0.301}$$

$$t = \frac{25.3 \times 14.3 \times 1.908}{11 \times 0.301} \approx 208.5 \text{ days}$$

Hence, it will take about 208.5 days for 90% decay of ${}_{15}P^{33}$.

S78. Here, half life of sample X, T_1 = 1 h and half life of sample, Y, T_2 = 2 h.

Let N be the number of atoms of each of the two samples initially. Thus, number of atoms of the sample X after 2 h, $N_1 = N/4$ and number of atoms of the sample Y after 2 h. $N_2 = N/2$.

Let R_1 and R_2 be their rates of disintegration after 2 h. If λ_1 and λ_2 are their respective decay constant, then

$$\frac{R_1}{R_2} = \frac{\frac{N}{4}}{\frac{N}{2}} = \frac{1}{2} \quad \text{or } 1:2 \qquad \left[\because \ T_{1/2} = \frac{0.693}{\lambda}\right]$$

$$= \frac{2 \times N/4}{1 \times N/4}.$$

S79. According to radioactive decay law,

$$\frac{dN}{dt} = -\lambda N$$

or $$\lambda = \frac{-dN/dt}{N}$$

Radioactive decay constant of a substance (radioactive) may be defined as the rario of its instantaneous rate of disintegration to the number of atoms present at that time.

Again, $$N = N_0 e^{-\lambda t}$$

If $$t = \frac{1}{\lambda},$$

then $$N = N_0 e^{-\lambda \frac{1}{\lambda}} = \frac{1}{e} N_0 = \frac{N_0}{2.718} = 0.368 N_0 .$$

Radioactive decay constant of substance may also be defined as the reciprocal of the time, after which the number of atoms of a radioactive substance decreases to 0.368 (or 36.8%) of their number present initially.

Relation between $T_{1/2}$ and λ: Consider that a radioactive sample contains N_0 atoms at time $t = 0$. Then, the number of atoms left behind after time t is given by

$$N = N_0 e^{-\lambda t} \qquad \text{... (i)}$$

From the definition of half life, it follows that

when $t = T_{1/2}$ (half-life of the sample),

$$N = \frac{N_0}{2}$$

Setting the above condition in equation (i), we have

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

or $$e^{-\lambda T_{1/2}} = \frac{1}{2} \quad \text{or} \quad e^{\lambda T_{1/2}} = 2$$

or $$\lambda T_{1/2} = \log_e 2 = 2.303 \log_{10} 2 = 2.303 \times 0.3010 = 0.693$$

or $$T_{1/2} = \frac{0.693}{\lambda} \qquad \text{... (ii)}$$

S80. **Disintegration constant:** Radioactive decay constant of a substance (radioactive) may be defined as the ratio of its instantaneous rate of disintegration to the number of atoms present at that time.

Its unit is reciprocal of time *i.e.*, s^{-1} or $year^{-1}$.

Mean life: The average-life of a radioactive substance is defined as the average time of which the nuclei of the atoms of the radioactive substance exist.

Its unit is that of time *i.e.*, ***s*** or ***year***.

S81.

$$\lambda = \frac{0.693}{24.1} = 0.0288 \text{ day}^{-1}$$

Let t be the time in which 90% of the sample of UX_1 changes to UX_2.

Therefore, sample of UX_1 left behind after time t = 10%

or $$\frac{m}{m_0} = 10\% = 0.1$$

or $$\frac{m_0}{m} = 10$$

Using the relation: $t = \frac{2.303}{\lambda} \log_{10} \frac{m_0}{m}$, it can be obtained that

$$t = \mathbf{79.97 \text{ days}}.$$

S82. $R = \frac{dN}{dt} = 1 \text{ curie} = 3.7 \times 10^{10} \text{ s}^{-1}$

Number of atoms in 1 g of Ra^{226}.

$$N = \frac{6.025 \times 10^{26}}{1{,}000 \times 226}$$

Using the relation: $\frac{dN}{dt} = \lambda N$, it can be obtained that

$$\lambda = 1.388 \times 10^{-11} \text{ s}^{-1}$$

$\therefore$ $$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{1.388 \times 10^{-11}} \text{ s}$$

$$= \frac{0.693}{1.388 \times 10^{-11} \times 60 \times 60 \times 24 \times 365} = \mathbf{1{,}583.2 \text{ years}}.$$

S83. Given: R_0 = 15 decays per minute per gram

R = 10 decays per minute per gram

and $T_{1/2}$ = 5,500 years

Now, $$\frac{R}{R_0} = \left(\frac{1}{2}\right)^{t/T_{1/2}} \quad \text{or} \quad \frac{R_0}{R} = (2)^{t/T_{1/2}}$$

or $$\frac{15}{10} = (2)^{t/5,500} \quad \text{or} \quad \frac{3}{2} = (2)^{t/5,500}$$

or $$\frac{t}{5,500} \times \log_e 2 = \log_e 3 - \log_e 2$$

or $$t = 5,500 \times \left(\frac{\log_e 3}{\log_e 2} - 1\right) = 5,500 \times \left(\frac{1.0986}{0.693} - 1\right) = \mathbf{3,249.05\ years}.$$

S84. The β-decay occurs in case of nuclei, which possess a higher value of neutron to proton ratio than that for the stable nuclei. The β-particle is an electron of nuclear origin. Since an electron does not exist inside the nucleus, it is formed by the conversion of a neutron into a proton and electron.

Example: $$_{83}Bi^{210} \longrightarrow \ _{84}Po^{210} + \ _{-1}e^{0} + 1.17\ \text{MeV}$$

(a) The energy of the emitted β-particles is ***continuous***. It is because, the electron and the antineutrino come out of the nucleus and share the end point energy of β-emission in all proportions with each other.

(b) The parent and daughter nucleus obtained through β-decay are ***isobars***.

S85. The statement means that the numbers of (undecayed) atoms of $^{14}_{6}C$ will reduce to half after 5700 years due to the decay of other half.

Let initially the number of atoms of both X and Y be N_0.

After 1 hour number of atoms of X remaining undecayed = $N_0/2$.

After 2 hours number of atoms of X remaining undecayed = $\frac{1}{2} \cdot \frac{N_0}{2} = \frac{N_0}{4}$.

The number of atoms of Y remaining undecayed after 2 hours = $N_0/2$.

Since the rate of disintegration is proportional to the number of atoms present, the ratio of the rates of disintegration after 2 hours is given by

$$\frac{\text{Rate of disintegration of } X}{\text{Rate of disintegration of } Y} = \frac{\frac{N_0}{4}}{\frac{N_0}{2}} = \frac{1}{2} = 1:2$$

S86. Hence, radioactive decay constant of a substance (radioactive) may be defined as the ratio of its instantaneous rate of disintegration to the number of atoms present at that time.

Consider that a radioactive sample contains N_0 atoms at time $t = 0$. Then, the number of atoms left behind after time t is given by

$$N = N_0 e^{-\lambda t} \qquad \text{... (i)}$$

From the definition of half-life, it follows that

when $t = T_{1/2}$ (half-life of the sample),

$$N = \frac{N_0}{2}$$

Setting the above condition in equation (i), we have

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

or $$e^{-\lambda T_{1/2}} = \frac{1}{2} \quad \text{or} \quad e^{\lambda T_{1/2}} = 2$$

or $$\lambda T_{1/2} = \log_e 2 = 2.303 \log_{10} 2 = 2.303 \times 0.3010 = 0.693$$

or $$T_{1/2} = \frac{0.693}{\lambda}.$$

Here, $N = \frac{N_0}{32}$ when $t = 25$ years

No. of atoms left after n half-lives is given by

$$N = N_0 \left(\frac{1}{2}\right)^n \quad \text{or} \quad \frac{N}{N_0} = \left(\frac{1}{2}\right)^n \quad \text{or} \quad \frac{1}{32} = \left(\frac{1}{2}\right)^n$$

or $$2^n = 32 = 2^5 \quad \text{or} \quad n = 5$$

Half-life period $$T_{1/2} = \frac{t}{n} = \frac{25}{5} = 5 \text{ years}.$$

S87. ***Laws of radioactive decay:***

1. Radioactivity is a spontaneous phenomenon and one cannot predict, when a particular atom (It is better to say that 'when the nucleus of a particular atom will undergo disintegration'.) in a given radioactive sample will undergo disintegration.

2. *When a radioactive atom disintegrates, either an α-particle (nucleus of helium) or a β-particle (electron) is emitted.*

The new atom so formed (called daughter atom) may emit a γ-ray photon, in case the nucleus is left in exited state on emitting the α or β-particle. Further, both α and β-particles are ever emitted simultaneously. it may also be pointed out that a radioactive atom can never emit more than one α-particle or a β-particle at as time.

3. *The emission of an α-particle by a radioactive atom results in a daughter atom, whose atomic number is 2 units less and mass number is 4 units less than that of the parent atom.*

Due to α-decay, the transformation of the radioactive parent atom (${}_Z X^A$) into the daughter atom (${}_{Z-2}Y^{A-4}$) may be represented as below:

$${}_Z X^A \xrightarrow{\alpha\text{-decay}} {}_{Z-2}Y^{A-4}$$

4. *The emission of a β-particle by a radioactive atom results in a daughter atom, whose atomic number is 1 unit more but mass number is same as that of the parent atom.*

Due to β-decay, the transformation of the parent atom (${}_Z X^A$) into the daughter atom (${}_{Z+1}Y^A$) may be represented as below:

$${}_Z X^A \xrightarrow{\beta\text{-decay}} {}_{Z+1}Y^A$$

5. *The number of atoms disintegrating per second of a radioactive sample at any time is directly proportional to the number of atoms present at that time.*

The rate of disintegration of the sample cannot be altered by changing the external factors, such as pressure, temperature, etc. It is known as ***radioactive decay law***.

Consider that a radioactive sample contains N_0 atoms at time $t = 0$. Then, the number of atoms left behind after time t is given by

$$N = N_0 e^{-\lambda t} \quad \text{... (i)}$$

From the definition of half life, it follows that

when $t = T_{1/2}$ (half-life of the sample),

$$N = \frac{N_0}{2}$$

Setting the above condition in equation (i), we have

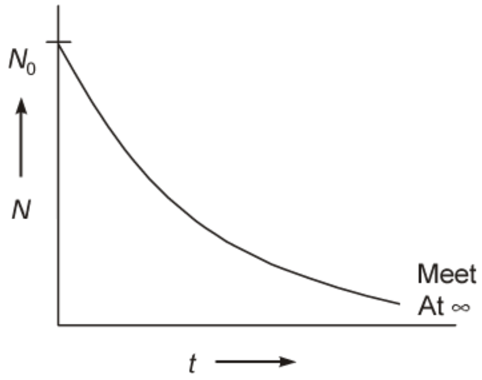
$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$$

or $$e^{-\lambda T_{1/2}} = \frac{1}{2} \quad \text{or} \quad e^{\lambda} T_{1/2} = 2$$

or $$\lambda T_{1/2} = \log_e 2 = 2.303 \log_{10} 2 = 2.303 \times 0.3010 = 0.693$$

or $$T_{1/2} = \frac{0.693}{\lambda}$$

S88. The spontaneous transformation of an element into another with the emission of some particle (or particles) or electromagnetic radiation is called ***natural radioactivity***.



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According to radioactive decay law,

$$\frac{dN}{dt} = -\lambda N$$

or $$\lambda = \frac{-dN/dt}{N}$$

Radioactive decay constant of a substance (radioactive) may be defined as the ratio of its instantaneous rate of disintegration to the number of atoms present at that time.

Again, $$N = N_0 e^{-\lambda t}$$

If $t = \frac{1}{\lambda}$,

then $N = N_0 e^{-\lambda \frac{1}{\lambda}} = \frac{1}{e} N_0 = \frac{N_0}{2.718} = 0.368\, N_0$.

S90. The spontaneous transformation of an element into another with the emission of some particle (or particles) or electromagnetic radiation is called ***natural radioactivity***.

The nature of the three types of radiation emitted:

(a) α-rays (b) β-rays (c) γ-rays

(a) ***Properties of α-rays:***

1. α-particles have identified as helium nuclei (atoms of helium that have lost two orbital electrons). Thus α-particles carry double the positive charge of proton and are four times as heavy.

2. The velocity of α-particles ranges from 1.4×10^7 ms^{-1} to 2.05×10^7 ms^{-1} depending upon the radioactive substance emitting them. In other words, energy of α-particles emitted from radioactive substance varies from 4.19 to 6.78 MeV.

3. Although α-particles processes large amount of energy, they are the least penetrating of the three types of rays. This is because, they are massive particles and hence are easily stopped by an aluminium sheet only 0.02 mm thick or even by an ordinary sheet of paper.

(b) ***Properties of β-rays:***

1. β-particles have been found to be streams of electrons.

2. The velocity of β-particles ranges from 33% to 99.8% of the velocity of light. These are the swiftest material particles known in physics. Although the velocities of these particles are enormous, their average energy is only 2 to 3 MeV. It is because, β-particles passes extremely small mass.

3. β-particles can easily pass through a few millimetre thick aluminium sheets.

(c) ***Properties of γ-rays:***

1. γ-rays are electromagnetic waves and have velocity equal to that of light.

2. γ-rays are highly penetrating. They can penetrate through several centimeters thick iron and lead blocks.

3. They have got small ionising power.

S91. Let us consider a radioactive element containing N_0 atoms at $t = 0$. Let this number reduces to N after time t due to disintegration of atoms. Suppose that a small number of atoms dN disintegrate in time interval between $t + dt$. Thus, rate of disintegration during this

Time integral is $\frac{dN}{N}$.

According to decay law (which states that the number of atoms undergoing disintegration per unit time at any instant is proportional to the number of atoms present at that instant),

$$\frac{dN}{N} \propto N$$

or $$\frac{dN}{N} = -\lambda N \quad \text{... (i)}$$

Here, λ is a constant of proportionality called disintegration (or decay) constant. The – ve sign here shows that N is decreasing with the increase in time.

From Eq. (i), we get

$$\frac{dN}{N} = -\lambda\, dt$$

Integrating both sides, we get

$$\int \frac{dN}{N} = -\lambda \int dt$$

or, $$\log_e N = -\lambda . t + c$$

Where c is constant of integration.

At $t = 0, \quad N = N_0$

$\therefore$ $$\log_e N_0 = c \quad \text{... (ii)}$$

Putting the value of c in the above equation, we have

$$\log_e N = -\lambda . t + \log_e N_0$$

or $$\log_e \left(\frac{N}{N_0}\right) = -\lambda . t$$

or $$\frac{N}{N_0} = e^{-\lambda t}$$

or $$N = N_0 e^{-\lambda t}. \quad \text{(iii)}$$

Put in Eq. (iii) $t = T_{1/2}$ and $N = \frac{N_0}{2}$

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \Rightarrow 2 = e^{\lambda T_{1/2}} \Rightarrow \lambda . T_{1/2} = \log_e 2 = 0.693$$

or $$T_{1/2} = \frac{0.693}{\lambda}$$