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Q1. Two nuclei have mass number in the ratio 1 : 3. What is the ratio of their nuclear densities?

Q2. In the Rutherford scattering experiment, the distance of closest approach for an α-particle is d_0. If α-particle is replaced by a proton, how much kinetic energy in comparison to α-particle will be required to have the same distance closest approach, d_0?

Q3. Out of the two characteristics, the mass number (A) and the atomic number (Z) of a nucleus, which one does not changes during nuclear β-decay?

Q4. The radius of innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. What is the radius of orbit in the second excited state?

Q5. What is the ratio of radii of the orbits corresponding to first excited state and ground state, in a hydrogen atom?

Q6. Define ionization energy. What is its value for a hydrogen atoms?

Q7. Why is the classical (Rutherford) model for an atom of electron orbiting around the nucleus-out able to explain the atomic structure?

Q8. Choose the correct alternative from the clues given at the end of the statement:

The size of the atom in Thomson's model is .... the atomic size in Rutherford's model. (much greater than/no different from/much less than.)

Q9. Choose the correct alternative from the clues given at the end of the statement:

In the ground state of .... electrons are in stable equilibrium, while in .... electrons always experience a net force. (Thomson's model/ Rutherford's model.)

Q10. Choose the correct alternative from the clues given at the end of the statement:

A classical atom based on .... is doomed to collapse. (Thomson's model/ Rutherford's model.)

Q11. Choose the correct alternative from the clues given at the end of the statement:

An atom has a nearly continuous mass distribution in a .... but has a highly nonuniform mass distribution in .... (Thomson's model/ Rutherford's model.)

Q12. Choose the correct alternative from the clues given at the end of the statement:

The positively charged part of the atom possesses most of the mass in .... (Rutherford's model/both the models.)

Q13. Answer the following questions, which help you understand the difference between Thomson's model and Rutherford's model better.

(a) Is the average angle of deflection of α-particles by a thin gold foil predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

(b) Is the probability of backward scattering (*i.e.*, scattering of á-particles at angles greater than 90°) predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

Q14. Why is that the mass of the nucleus does not enter the formula for impact parameter, but its charge does?

Q15. Answer the following questions, which help you understand the difference between Thomson's model and Rutherford's model better.

(a) Keeping other factors fixed, it is found experimentally that for small thickness *t*, the number of α-particles scattered at moderate angles is proportional to *t*. What clue does this linear dependence on t provide?

(b) In which model is it completely wrong to ignore multiple scattering for the calculation of average angle of scattering of α-particles by a thin foil?

Q16. Draw a labeled schematic arrangement showing Rutherford's α-particle scattering experiment.

Q17. Nuclear atom model is required to explain the results of Rutherford experiment. Why?

Q18. How is the size of a nucleus experimentally determined? Write the relation between the radius and mass number of the nucleus. Show that the density of nucleus is independent of its mass number.

Q19. The nuclear radius of ${}_{8}O^{15}$ is 3×10^{-15} m. What is the nuclear radius of ${}_{82}Pb^{205}$?

Q20. In a head on collision between the α-particle and gold nucleus, the closest distance of approach is 41.3 fermi. Calculate the energy of α-particle.

Q21. The total energy of an electron in the first excited state of the hydrogen atom is – 3.4 eV. What are the values of potential and kinetic energies of the electron in this state?

Q22. Calculate the radius of the smallest orbit of H-atom.

Q23. Explain, how Rutherford's experiment of scattering of a-particles led to the estimation of the size of the nucleus?

Q24. Write the main postulates of Rutherford's atomic model and the cause of failure of this model.

Q25. Explain Rutherford's experiment on the scattering of alpha particles from a gold foil and state the significance of the results.

Q26. State basic assumptions of the Rutherford's model of the atom. Explain in brief, why this model cannot account for the stability of an atom.

Q27. Draw a schematic arrangement of the Geiger-Marsden experiment. How did the scattering of α-particles by a thin foil of gold provide an important way to determine an upper limit on the size of the nucleus? Explain briefly.

Q28. (a) Using Bohr's second postulate of quantization of orbital angular momentum show that the circumference of the electron in the r^{th} orbital state in hydrogen atom is n times the de-Broglie wavelength associated with it.

(b) The electron in hydrogen atom is initially in the third excited state. What is the maximum number of spectral lines which can be emitted when it finally moves to the ground state?

Q29. (a) Explain, how Rutherford's experiment on scattering of α-particles led to the estimation of the size of the nucleus?

(b) The ground state energy of hydrogen atom is –13.6 eV. If an electron makes a transition from an energy level – 0.85 eV to – 1.51 eV, calculate the wavelength of the spectral line emitted. To which series of hydrogen spectrum does this wavelength belong?

Q30. The ground state energy of hydrogen atom is – 13.6 eV. The photon emitted during the transition of electron from $n = 3$ to $n = 1$ state, is incident on a photosensitive material of unknown work function.

The photoelectrons are emitted from the materials with a maximum kinetic energy of 9eV. Calculate the threshold wavelength of the material used.

Q31. Using the postulates of Bohr's model of hydrogen atom, obtain an expression for the frequency of radiation emitted when the atom makes a transition from the higher energy state with quantum number n_i to the lower energy state with quantum number $n_f (n_f < n_i)$.

Q32. (a) What is distance of closest approach?

(b) State the basic assumption of the Rutherford model of the atom. Explain in brief why this model cannot account for the stability of an atom?

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S1. Nuclear density is independent of mass number.

S2. When α-particle is replaced by proton then there will change in atomic number and mass of the particle .

$\because$ For given distance of closest approach kinetic energy $\propto$ Z(atomic number)

$$\Rightarrow \qquad \frac{K_{\text{proton}}}{K_\alpha} = \frac{Z_{\text{proton}}}{Z_\alpha} = \frac{1}{2}$$

$$\Rightarrow \qquad K_{\text{proton}} : K_\alpha = 1 : 2$$

S3. The mass number, A of a nucleus does not change during nuclear β-decay.

S4. The radius of atom whose principal quantum number is n, is given by

$$r = n^2 r_0$$

where, r_0 = radius of innermost electron orbit for hydrogen atom and $r_0 = 5.3 \times 10^{-11}$ m

For second excited state, $n = 3$

$$\because \qquad r = 3^2 \times r_0 = 9 \times 5.3 \times 10^{-11}$$

$$r = 4.77 \times 10^{-10} \text{ m.}$$

S5. For first excited states $n = 2$

Ground state occurs for $n = 1$

$$\because \qquad r_n = r_1 n^2$$

$$\because \qquad r \propto n^2$$

$$\Rightarrow \qquad \frac{r_1}{r_2} = \left(\frac{n_1}{n_2}\right)^2 = \left(\frac{2}{1}\right)^2$$

$$\because \qquad r_1 : r_2 = 4 : 1$$

where r_1 and r_2 are radii corresponding to first excited state and ground state of the atom.

S6. Ionisation energy: The minimum amount of energy required to remove an electron from the ground state of the atom, is known as ionisation energy.

Ionisation energy for hydrogen atom

$$= 13.6 \text{ eV}$$

S7. The classical method could not explain the atomic structure as the electron revolving around the nucleus are accelerated and emits energy as the result, the radius of the circular path goes on decreasing. Ultimately electrons fall into the nucleus, which is not in practical.

S8. The sizes of the atoms taken in Thomson's model and Rutherford's model have the same order of magnitude.

S9. In the ground state of Thomson's model, the electrons are in stable equilibrium. However, in Rutherford's model, the electrons always experience a net force.

S10. A classical atom based on Rutherford's model is doomed to collapse.

S11. An atom has a nearly continuous mass distribution in Thomson's model, but has a highly non-uniform mass distribution in Rutherford's model.

S12. The positively charged part of the atom possesses most of the mass in both the models.

S13. (a) About the same.

The average angle of deflection of α-particles by a thin gold foil predicted by Thomson's model is about the same size as predicted by Rutherford's model. This is because the average angle was taken in both models.

(b) Much less.

The probability of scattering of α-particles at angles greater than 90° predicted by Thomson's model is much less than that predicted by Rutherford's model.

S14. The scattering takes place due to the electrostatic field of the nucleus and not due to its gravitational field. For this reason, charge of the nucleus enters into the expression for the impact parameter and mass does not.

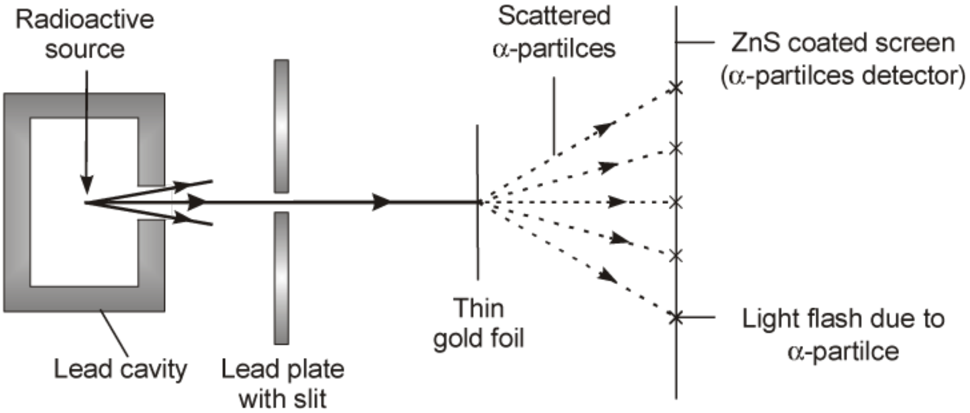
S15. (a) Scattering is mainly due to single collisions. The chances of a single collision increases linearly with the number of target atoms. Since the number of target atoms increase with an increase in thickness, the collision probability depends linearly on the thickness of the target.

(b) Thomson's model.

It is wrong to ignore multiple scattering in Thomson's model for the calculation of average angle of scattering of a.particles by a thin foil. This is because a single collision causes very little deflection in this model. Hence, the observed average scattering angle can be explained only by considering multiple scattering.

S16. The α-particles form $_{83}Bi^{214}$ (a radioactive source) contained in a lead cavity are collimated into a narrow beam with the help of a lead plate having a narrow slit as shown in the figure below.

The α-particles scattered in different directions were detected with the help of an α-particle detector. The whole apparatus was arranged inside a vacuum chamber to prevent the scattering of α-particles from air molecules.



S17. In Rutherford experiment, a few α-particles are scattered through large angles. Therefore, massive and positive part of atom should be concentrated in a small portion of the atom. This small positive part having almost the entire mass of the atom was called nucleus. Therefore, the main feature of any atom model should require the presence of the central positive and massive nucleus.

S18. The size of the nucleus can be determined by the Rutherford experiments on α ray scattering. The distance of the nearest approach is approximately the size of the nucleus. Here, it is assumed that only coulomb repulsive force caused scattering. With α rays of 5.5 MeV the size of the nucleus was found to be less than 4×10^{-14} m.

Scattering experiments performed with fast electrons bombarding targets of different elements, the size of the nuclei of various elements have been determined accurately.

The required relation is

$$R = R_0 A^{1/3}, \quad \text{where} \quad R_0 = 1.2 \times 10^{-15}\,\text{m}$$

Density of a nucleus of mass number A and radius R is given by

$$R = \frac{M}{V} = \frac{A \times 1.6 \times 10^{-27}}{\frac{4}{3}\pi \times R_0^3 A}$$

$$= \frac{1.6 \times 10^{-27} \times 3}{4 \times 3.142 \times (1.2 \times 10^{-15})^3}$$

$$= \frac{1.6 \times 3}{4 \times 3.14 \times 1.2 \times 1.2 \times 1.2} \times 10^{18}\ \text{kgm}^{-3}$$

$= 2.3 \times 10^{17}\,\text{kg m}^{-3}$, which is independent of the mass number A.

S19. Radius R of a nucleus is

$$R = R_0 A^{1/3}.$$

If two elements having atomic weights A_1 and A_2 have radii R_1 and R_2 respectively, then

$$R_1 = R_0 A_1^{1/3}$$

and $$R_2 = R_0 A_2^{1/3}$$

$$\frac{R_2}{R_1} = \left(\frac{A_2}{A_1}\right)^{1/3}$$

Here $R_1 = 3 \times 10^{-15}$ m, $A_1 = 16$

$R_2 = ?$, $A_2 = 205$

$$R_2 = R_1\left(\frac{A_2}{A_1}\right)^{1/3} = 3 \times 10^{-15} \times \left(\frac{205}{16}\right)^{1/3}$$

$$= \mathbf{7.531 \times 10^{-15}\, m}.$$

S20. Given: $r_0 = 41.3$ fermi $= 41.3 \times 10^{-15}$ m, $E = ?$

For gold, $Z = 79$

Energy, $$E = \frac{Ze(2e)}{4\pi\varepsilon_0 r_0} \quad \text{(Here, } r = r_0\text{)}$$

$$= \frac{9 \times 10^9 \times 79 \times 2 \times (1.6\ 10^{-19})^2}{41.3 \times 10^{-15}}$$

$$= 8.814 \times 10^{-13}\ \text{J} = \frac{8.814 \times 10^{-13}}{1.6 \times 10^{-13}}\ \text{MeV}$$

$$= \mathbf{5.51\ MeV}.$$

S21. Kinetic energy = – Total energy

$= -(-3.4\ \text{eV}) = +3.4\ \text{eV}$

Potential energy = 2 (total energy)

$= 2 \times (-3.4\ \text{eV})$

$= \mathbf{-6.8\ eV}$.

S22. The radius of smallest orbit in H-atom corresponds to the orbit $n = 2$. It is given by

$$r = 4\pi\varepsilon_0 \cdot \frac{1^2 \times h^2}{4\pi^2 me^2} \quad \text{(for } n = 1\text{)}$$

or $$r = 4\pi\varepsilon_0 \cdot \frac{h^2}{4\pi^2 me^2}$$

Now, $h = 6.62 \times 10^{-34}$ J s; $m = 9.1 \times 10^{-31}$ kg; $e = 1.6 \times 10^{-19}$ C and $4\pi\varepsilon_0 = 1/(9 \times 10^9)\ \text{Nm}^2\text{C}^{-2}$.

In the above equation, setting the values of h, m, e and $4\pi\varepsilon_0$, it can be obtained that

$$r = 0.53 \times 10^{-10}\,\text{m} = \mathbf{0.53\ Å}.$$

S23. As most of the α-particles passed straight through the gold foil and a very few were scattered back, it indicated that the central part of the atom must be positive and massive. As only 1 in about 8,000 α-particles retraced its path, it indicated that this central part should be only a very small portion of the whole atom. The size of this central part, called nucleus, was estimated to be about 1/1,00,000 th of the size of the atom.

S24. As most of the α-particle passed straight through the gold foil and a very few were scattered back, it indicated that the central part atom must be positive and massive. As only one in about 8,000 α-particles retread its path, it indicate that this central part should be only a very small portion of the whole atom. The size of this central part called nucleus was estimated to about 1/1,00,000 of the size of the atom.

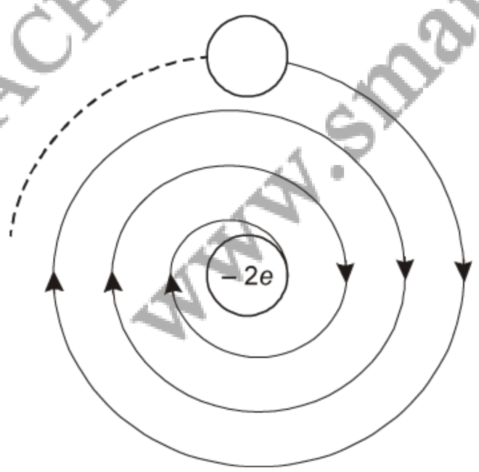
Drawbacks of Rutherford's models:

(a) The electron can revolve round the nucleus without following into it. Thus, Rutherford's model can't explain the stability of the atom.

(b) The electron can revolve in orbits of all possible radii, hence it should be emit continuous energy spectrum. However the atom like hydrogen passes line spectrum.

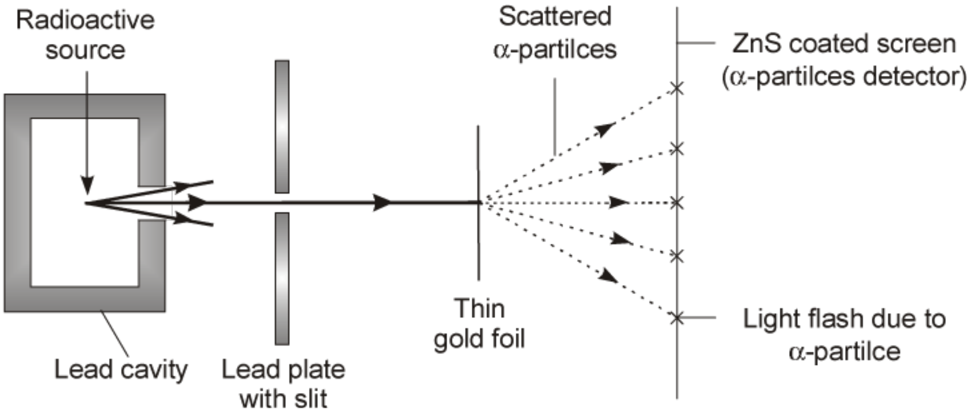
S25. As most of the α-particle passed straight through the gold foil and a very few were scattered back, it indicated that the central part atom must be positive and massive. As only one in about 8,000 α-particles retread its path, it indicate that this central part should be only a very small portion of the whole atom. The size of this central part called nucleus was estimated to about 1/1,00,000 of the size of the atom.

S26. The basic assumption of Rutherford's model the α-particle scattered from the gold foil are very heavy particles (about 9000 heavier than electrons) and posses high initial speed.



When the electron revolve round the nucleus they are continuously accelerated towards the centres of the nucleus. According to Lorentz a charged particle should be radiate the energy continuously. Therefore, in the atom a revolving electron should be continuously emit energy and hence the radius of its path should go on decreasing and ultimately, it should be fall into the nucleus as shown in figure. However, electron revolve round the nucleus without falling into it. Thus, Rutherford's atom model can't explain the stability of the atom.

S27. Given figure shows a schematic diagram of Geiger-Marsden experiment.



Thus, if an α-particle has large impact parameter, it gets scattered through a very small angle and may practically go undirected and if the α-particle has small impact parameter, it will get scattered through a large angle.

Thus, if an α-particle has impact parameter zero (i.e. if the α-particle travels directly towards the centre of the nucleus), it will be scattered through 180°

S28. (a) Bohr's second postulate states that the electron revolves around the nucleus in certain privileged orbit which satisfy certain quantum condition that angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$, where h is Planck's constant,

i.e., $$L = mvr = \frac{nh}{2\pi}$$

where m = mass of electron, v = speed of electron and r = radius of orbit of electron

$$\Rightarrow \qquad 2\pi r = n\left(\frac{h}{mv}\right)$$

Circumference of electron n^{th} orbit = $n \times$ de-Broglie wavelength associated with electron.

(b) Given, the electron in hydrogen atom is initially in third excited state.

$$\therefore \qquad n = 4$$

And the total number of spectral lines of an atom can exists is given by the relation $\frac{n(n-1)}{2}$

Here, $n = 4$

So, number of spectral = $\frac{4(4-1)}{2}$

$$= \frac{4 \times 3}{2} = 6$$

S29. (a) As most of the α-particles passed straight through the gold foil and a very few were scattered back, it indicated that the central part of the atom must be positive and massive. As only 1 in about 8,000 α-particles retracted its path, it indicated that this central part should be only a very small portion of the whole atom. The size of this central part, called nucleus, was estimated to be about 1/1,00,000 th of the size of the atom.

(b) Photon is emitted when electron transits from higher energy state to lower energy state, the difference of energy of the state appear in form of energy of photon.

According to Bohr's theory of hydrogen atom, energy of photon released, $E_2 - E_1 = h\nu$

Given, $E_1 = -1.51$ eV

$E_2 = -0.85$

$$E_2 - E_1 = -0.85 - (-1.51)$$

$$= 1.51 - 0.85$$

$$E_2 - E_1 = 0.66 \text{ eV}$$

$\therefore$ $E = E_2 - E_1 = 0.66$ eV

So, the wavelength of emitted spectral line,

$$\lambda = \frac{1242 \text{ eV-nm}}{E(\text{in eV})} = \frac{1242 \text{ eV-nm}}{0.66 \text{ eV}}$$

As here, $\lambda = 1.88 \times 10^{-6}$m

$$\approx 18751 \times 10^{-10}\text{m}$$

The wavelength belongs to Paschen series of hydrogen spectrum.

S30. Given, ground state energy $E_1 = -13.6$eV

Energy of electron in nth orbit

$$E_n = \frac{13.6}{n^2} \text{eV}$$

For $n = 1$,

$$E_1 = -13.6 \text{ eV}$$

For $n = 3$,

$$E_3 = \frac{-13.6}{3^2} = -1.5 \text{ eV}$$

$\therefore$ The energy of photon released during the transition of electron from $n = 3$ to $n = 1$ is

$$E = E_3 - E_1$$

$$= -(1.5) - (-13.6)$$

$$= 12.1 \text{ eV}$$

Now, Einstein's photoelectric equation energy of photon (E) = $KE_{max} + \phi$

where, ϕ is work function of metal

$$12.1 \text{ eV} = 9 \text{ eV} + \phi$$

$$\phi = 12.1 - 9$$

$$= 3.1 \text{ eV}$$

$\therefore$ Work function $\phi = 3.1$ eV

$\therefore$ The wavelength correspond to 3.1 eV, threshold wavelength

$$\lambda_0 = \frac{1242 \text{ eV-nm}}{\phi(\text{in eV})}$$

$$= \frac{1242 \text{ eV-nm}}{3.1 \text{ eV}}$$

$$= 401 \text{ nm}$$

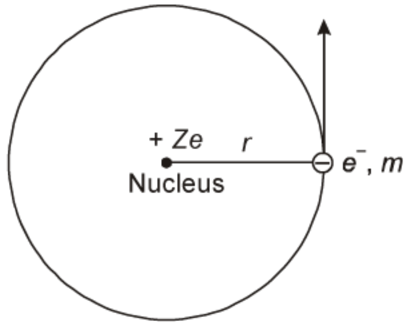
S31. Let an electron revolves around the nucleus of hydrogen atom. The necessary centripetal force is provided by electrostatic force of attraction.

$$\therefore \quad \frac{mv^2}{r} = \frac{ke^2}{r^2} \quad \Rightarrow \quad r = \frac{ke^2}{mv^2} \quad \text{... (i)}$$

Where, m mass of electron and v is its speed of a circular path of radius, r

By Bohr's second postulates,

$$mvr = \frac{nh}{2\pi}$$



where, $n = 1, 2, 3...$

$$r = \frac{nh}{2\pi mv} \qquad \text{(ii)}$$

Comparing Eqs. (i) and (ii),

$$\frac{ke^2}{mv^2} = \frac{nh}{2\pi mv} \Rightarrow v = \frac{2\pi ke^2}{nh}$$

Substituting in Eq. (ii)

$$r = \frac{n^2h^2}{4\pi^2 mke^2} \Rightarrow r \propto n^2 \qquad \text{... (iii)}$$

Now, kinetic energy of electron

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{ke^2}{2r}$$

Also, potential energy,

$$\text{P.E.} = -\frac{ke^2}{r}$$

Energy of electron in n^{th} orbit

$$E_n = \frac{-ke^2}{2r} = -\frac{ke^2}{2} \cdot \frac{4\pi^2 mke^2}{n^2h^2} \Rightarrow E_n = \frac{2\pi^2 mk^2e^4}{n^2h^2} \qquad \text{... (iv)}$$

where, $R = \frac{2\pi^2 mk^2e^4}{ch^3} \Rightarrow E_n = \frac{Rhc}{n^2}$ ... (v)

where, $n = n_i \Rightarrow E_n \propto \frac{1}{n^2}$ and $E_{n_i} = \frac{Rhc}{n_f^2}$

By Bohr's postulates,

$$E_{n_f} = \frac{Rhc}{n_f^2}$$

By Bohr's postulates,

$$E_{n_f} - E_{n_i} = h\nu$$

where, C = velocity of light

$$\nu = Rc\left[\frac{1}{n_i^2} - \frac{1}{n_f^2}\right]$$

This is required expression for frequency associated with photon.

S32. (a) At large distance from the nucleus, an α-particle has zero potential energy. As the α-particle moves towards the centre of the nucleus, its kinetic energy decreases and appears as the potential energy. It goes back on reaching up to a distance r_0 from the nucleus, where its kinetic energy is totally converted into the potential energy. This minimum distance r_0, to which the α-particle moves upto the nucleus, is called the distance of closest approach.

(b) Basic assumptions of Rutherford atomic model are given below

(i) Atom consists of small central core, called atomic nucleus in which whole mass and positive charge is assumed to be concentrated.

(ii) The size of the nucleus is much smaller than size of the atom.

(iii) The nucleus is surrounded by electrons. Atom are electrically neutral as total negative charge of electrons surrounding the nucleus is equal to total positive charge on the nucleus.

(iv) Electrons revolves around the nucleus in various circular orbits and necessary centripetal force is provided by electrostatic force of attraction between positively charged nucleus and negatively charged electrons.

Stability of atom: When an electron revolves around the nucleus, then it radiates electromagnetic energy and hence, radius of orbit of electron decreases gradually.

Thus, electron revolve on spiral path of decreasing radius and finally, it should fall into nucleus, but this does not happen. Thus Rutherford atomic model cannot account for stability of atom.

Date: 22/10/2021

Q1. **The ground state energy of the hydrogen atom is – 13.6 eV. What is the kinetic and potential energy of the electron in this state?**

Q2. **The total energy of an electron in the first excited state of the hydrogen atom is about – 3.4 eV. What is the kinetic energy of the electron in this state?**

Q3. **The total energy of an electron in the first excited state of the hydrogen atom is about – 3.4 eV. What is the potential energy of the electron in this state?**

Q4. **Energy of an electron in the n^{th} orbit hydrogen atom is given by**

$$E_n = -\frac{13.6}{n^2}\ \text{eV}.$$

How much energy is required to take an electron from the ground tate to the first excited state?

Q5. **The wavelengths of some of the spectral lines obtained in hydrogen spectrum are 9,546 Å, 6,463 Å and 1,216 Å. Which one of these wavelengths belongs to Lyman series?**

Q6. **Name the series of hydrogen spectrum, which has least wavelength.**

Q7. **Two nuclei have mass number in the ratio 2 : 5. What is the ration of their nuclear densities?**

Q8. **Explain, why the spectrum of hydrogen atom has many lines, although a hydrogen atom contains only one electron?**

Q9. **What causes the production of X-rays of minimum wavelength, when a tube is operated at a given accelerating potential?**

Q10. **Explain, why are electrons revolving around the nucleus of an atom?**

Q11. **The ground state energy of hydrogen atom is –13.6 eV. What is the kinetic and potential energy of electron in this state?**

Q12. **Write the empirical relation for Paschen series lines of Hydrogen spectrum.**

Q13. **What would happen, if the electrons in an atom were stationary?**

Q14. **Suppose you are given a chance to repeat the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14 K.) What results do you expect?**

Q15. **A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom makes a transition from the upper level to the lower level?**

Q16. **What is the shortest wavelength present in the Paschen series of spectral lines?**

Q17. **The ground state energy of hydrogen atom is – 13.6 eV. What are the kinetic and potential energies of the electron in this state?**

Q18. In the Rutherford's nuclear model of the atom, the nucleus (radius about 10^{-15} m) is analogous to the Sun about which the electron move in orbit (radius $\approx 10^{-10}$ m) like the Earth orbits around the Sun. If the dimensions of the solar system had the same proportions as those of the atom, would the Earth be closer to or farther away from the sun than actually it is? The radius of Earth's orbit is about 1.5×10^{11} m. The radius of Sun is taken as 7×10^{8} m.

Q19. According to the classical electromagnetic theory, calculate the initial frequency of the light emitted by the electron revolving around a proton in hydrogen atom.

Q20. In a Geiger-Marsden experiment, what is the distance of closest approach to the nucleus of a 7.7 MeV α-particle before it comes momentarily to rest and reverses its direction?

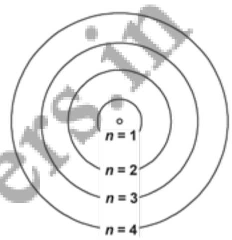
Q21. A 10 kg satellite circles earth once every 2 h in an orbit having a radius of 8000 km. Assuming that Bohr's angular momentum postulate applies to satellites just as it does to an electron in the hydrogen atom, find the quantum number of the orbit of the satellite.

Q22. The electron in hydrogen atom is initially in the third excited state, What is the maximum number of spectral lines which can be emitted, when it finally moves to the ground state?

Q23. What is the maximum possible number of spectral lines observed, when the hydrogen atom is in its second excited state? Justify your answer. Calculate ratio of the maximum and minimum wavelengths of the radiations emitted in this process.

Q24. Find the ratio of the energies of photons produced due to transition of an electron of the hydrogen atom from its (a) second permitted energy level to the first level and (b) the highest permitted energy level to the first permitted level.

Q25. In the figure below for the stationery orbits of the hydrogen atom, mark the transitions representing the Balmer and Lyman series.



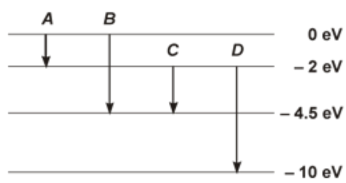
Q26. What are X-rays? How do they differ from electrons?

Q27. What is ionisation potential? Calculate it for hydrogen atom.

Q28. The energy levels of an atom are as shown in the figure below.

(a) Which one of these transitions will result in the emission of a photon of wavelength 275 mm?

(b) Which transition corresponds to emission of radiation of maximum wavelength?



Q29. Determine the speed of the electron in the $n = 3$ orbit of hydrogen atom.

Q30. In Geiger-Marsden experiment, a 5 MeV α-particle is scattered through 180° from the gold foil. If the atomic number of gold is 79, find the distance of closest approach to the gold nucleus.

Q31. The H_α-line of the Balmer series is obtained from the transition $n = 3$ (energy = -1.5 eV) to $n = 2$ (energy = -3.4 eV). Calculate the wave length of this line. Given that $h = 6.6 \times 10^{-34}$ J s; 1 eV = 1.66×10^{-19} J and $c = 3 \times 10^{8}$ m s^{-1}.

Q32. Calculate the frequency of revolution of electron in the first Bohr orbit of hydrogen atom, if the radius of the first Bohr orbit is 0.5 Å and the velocity of electron in the first orbit is $2.24 \times 10^6\, m s^{-1}$.

Q33. X-rays are produced in an X-ray tube operating at a given accelerating voltage. What is the wavelength range of the emitted continuous X-rays?

Q34. The ground state energy of hydrogen atom is – 13.6 V. If an electron makes a transition from an energy level – 0.85 to – 3.4 eV, calculate the wavelength of the spectral line emitted. To which series of hydrogen spectrum, does this wavelength belong.

Q35. A proton moves with a speed of 7.45×10^5 m/s directly towards a free proton originally at rest. Find the distance of closest approach for the two protons.

Q36. The wavelength of H_α line of the Balmer series is 6563 A°. Calculate the wavelength of the H_β line of Balmer series.

Q37. Obtain the range of wavelengths within which the Balmer series of hydrogen lies.

Q38. Explain, in brief, why Rutherford's model cannot account for the stability of an atom.

Q39. Photons, with a continuous range of frequencies, are made to pass through a sample of rarefied hydrogen. The transitions, shown here, indicate three of the spectral absorption lines in the continuous spectrum.

(a) Identify the spectral series, of the hydrogen emission spectrum, to which each of these three lines correspond.

(b) Which of these lines corresponds to the absorption of radiation of maximum wavelength?

Q40. Draw a neat labeled energy level diagram for hydrogen atom, showing Balmer and paschen series.

Q41. Draw a schematic arrangement of the Geiger-Marsden experiment. How did the scattering of α-particle by a thin foil of gold provide an important way to determine an upper limit on the size of the nucleus? Explain briefly.

Q42. The value of ground state energy of hydrogen atom is – 1.6 eV. (a) What does the negative sign signify? (b) How much energy is required to take an electron in this atom from the ground state to the second excited state.

Q43. Draw a labeled diagram of a Geiger-Marsden experiment on the scattering of alpha-particles. How is the size of the nucleus estimated in this experiment?

Q44. The wavelength of first member of the Lyman series is 1,216 Å. Calculate the wavelength of second member of the Balmer series.

Q45. The energy of the electron, in the ground state of hydrogen, is –13.6 eV. Calculate the energy of the photon that would be emitted if the electron where to make a transition corresponding to the emission of the first line of the

(a) Lyman series (b) Balmer series of the hydrogen spectrum.

Q46. The ground state energy of hydrogen atom is – 13.6 eV.

(a) What is kinetic energy of an electron in the 2^{nd} excited state?

(b) If the electron jumps to the ground state from the 2^{nd} excited state, calculate the wavelength of the spectral line emitted.

Q47. (a) Nuclear atom model is required to explain the results of Rutherford experiment. Why?

(b) In a Geiger-Marsden experiment, calculate the distance of closest approach to the nucleus of $Z = 80$, when an α-particle of 8 MeV energy impinges on it before it comes to momentarily rest and reverses its direction.

How will the distance of closest approach be affected when the kinetic energy of the α-particle is doubled?

Q48. Draw a schematic arrangement of the Geiger-Marsden experiment for studying α-particle scattering by a thin foil of gold. Describe briefly, by drawing trajectories of the scattered α-particles. How this study can be used to estimate the size of the nucleus?

Q49. (a) Using postures of Bohr's theory of hydrogen atom, show that

(i) the radii of orbits increase as n^2 and

(ii) the total energy of the electron increases as $1/n^2$, where n is the principal quantum number of the atom.

(b) Calculate the wavelength of H_α line in Balmer series of hydrogen atom.

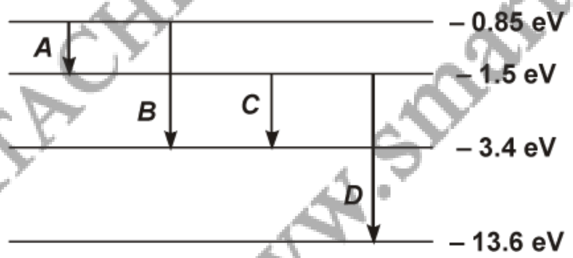
Given, Rydberg constant, $R = 1.097 \times 10^7$ m^{-1}.

Q50. Using Bohr's postulates, obtain the expression for the total energy of the electron in the stationary states of the hydrogen atom. Hence draw the energy level diagram showing how the line spectral corresponding to Balmer series occur due to transition between energy levels.

Q51. (a) What is ionisation potential? Calculate it for hydrogen atom.

(b) The energy level diagram of an element is given below. Identify, by doing necessary calculations, which transition corresponds to the emission of the spectral line of wavelength 102.7 nm.

(c) An α-particle having kinetic energy of 8.7 MeV is projected towards the nucleus of copper with atomic number $Z = 29$. Calculate its nearest distance of approach, $e = 1.6 \times 10^{-19}$ C and 1 eV = 1.6×10^{-19} J.



Date: 22/10/2021

S1. The potential energy (E_p) of the electron in an orbit is equal to twice its total energy (E).

$$\therefore \quad E_p = 2E$$

$$= -13.6 \times 2 = \mathbf{-27.2\,eV}.$$

Kinetic energy, $E_K = -E = 13.6$ eV

S2. The potential energy (E_p) of the electron in an orbit is equal to twice its total energy (E).

$$\therefore \quad E_k = -E$$

$$= -(-3.4) = \mathbf{3.4\,eV}.$$

S3. The potential energy (E_p) of the electron in an orbit is equal to twice its total energy (E).

$$\therefore \quad E_p = 2E$$

$$= -3.4 \times 2 = \mathbf{-6.8\,eV}.$$

S4. Energy required to take an electron from the ground state (n = 1) to the first excited state (n = 2) is given by

$$E_2 - E_1 = -\frac{13.6}{2^2} - \left(-\frac{13.6}{1^2}\right)$$

$$= -3.4 + 13.6 = \mathbf{10.2\ eV}.$$

S5. Lyman series lies in ultraviolet region. Therefore, wavelength of the spectral lines in Lyman series must be less than the wavelengths in visible region *i.e.*, 3,900 Å. Therefore, spectral line of wavelength 1,216 Å lies in Lyman series.

S6. Lyman series.

S7. The ratio of nuclear density will be 1 : 1, because mass density of neclear independent of mass number.

S8. A hydrogen atom contains only one electron, but this electron can be raised to higher energy states. The large number of spectral lines in hydrogen atom spectrum are due to the fact that a large number of transitions of the electron can take place between the different energy states.

S9. When the energy of the X-ray emitted is equal to the kinetic energy of the electron striking the target, X-rays of maximum energy and hence of minimum wavelength are produced.

S10. The structure of an atom can be stable, only if the electrons are revolving around the nucleus.

S11. In case of hydrogen atom, the kinetic energy is equal to the negative of total energy and potential energy is equal to twice to the energy.

Given, total ground state energy

$$(TE) = (-13.6 \text{ eV})$$

$\therefore$ Kinetic energy = – TE

$$= -(-13.6 \text{ eV}) = 13.6 \text{ eV}.$$

Potential energy = 2 x TE = 2 x (–13.6 eV) = –27.2 eV

S12. The wavelengths of the spectral lines in Paschen series are given by

$$\frac{1}{\lambda} = R_H\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right) \qquad \text{where} \quad n_i = 4, 5, 6, \ldots$$

S13. The electrons will be pulled into the nucleus due to Coulomb's attractive force.

S14. In the alpha-particle scattering experiment, if a thin sheet of solid hydrogen is used in place of a gold foil, then the scattering angle would not be large enough. This is because the mass of hydrogen (1.67×10^{-27} kg) is less than the mass of incident α-particles (6.64×10^{-27} kg). Thus, the mass of the scattering particle is more than the target nucleus (hydrogen). As a result, the α-particles would not bounce back if solid hydrogen is used in the α-particle scattering experiment.

S15. Separation of two energy levels in an atom,

$$E = 2.3 \text{ eV}$$
$$= 2.3 \times 1.6 \times 10^{-19}$$
$$= 3.68 \times 10^{-19} \text{ J}$$

Let ν be the frequency of radiation emitted when the atom transits from the upper level to the lower level.

We have the relation for energy as:

$$E = hv$$

Where, h = Planck's constant = 6.62×10^{-34} Js

$$\therefore \qquad \nu = \frac{E}{h} = \frac{3.68 \times 10^{-19}}{6.62 \times 10^{-32}} = 5.55 \times 10^{14} \text{Hz}$$

Hence, the frequency of the radiation is 5.6×10^{14} Hz.

S16. Rydberg's formula is given as:

$$\frac{hc}{\lambda} = 21.76 \times 10^{-19}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

Where, h = Planck's constant = 6.6×10^{-34} Js

c = Speed of light = 3×10^8 m/s

(n_1 and n_2 are integers)

The shortest wavelength present in the Paschen series of the spectral lines is given for values $n_1 = 3$ and $n_2 = \infty$.

$$\frac{hc}{\lambda} = 21.76 \times 10^{-19}\left(\frac{1}{(3)^2} - \frac{1}{(\infty)^2}\right)$$

$$\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8 \times 9}{21.76 \times 10^{-19}}$$

$$= 8.189 \times 10^{-7}\,\text{m} = 818.9\ \text{nm}$$

S17. Ground state energy of hydrogen atom, $E = -13.6\,\text{eV}$

This is the total energy of a hydrogen atom. Kinetic energy is equal to the negative of the total energy.

$$\text{Kinetic energy} = -E = -(-13.6) = 13.6\,\text{eV}$$

Potential energy is equal to the negative of two times of kinetic energy.

$$\text{Potential energy} = -2 \times (13.6) = -27.2\,\text{eV}.$$

S18. The ratio of the radius of electron's orbit to the radius of nucleus is $(10^{-10}\,\text{m})/(10^{-15}\,\text{m}) = 10^5$, that is, the radius of the electron's orbit is 10^5 times larger than the radius of nucleus. If the radius of the Earth's orbit around the Sun were 10^5 times larger than the radius of the Sun, the radius of the Earth's orbit would be $10^5 \times 7 \times 10^8\,\text{m} = 7 \times 10^{13}\,\text{m}$. This is more than 100 times greater than the actual orbital radius of Earth. Thus, the Earth would be much farther away from the Sun.

It implies that an atom contains a much greater fraction of empty space than our solar system does.

S19. From Example 12.3 we know that velocity of electron moving around a proton in hydrogen atom in an orbit of radius 5.3×10^{-11} m is 2.2×10^{-6} m/s. Thus, the frequency of the electron moving around the proton is

$$\nu = \frac{v}{2\pi r} = \frac{2.2 \times 10^{6}\,\text{m s}^{-1}}{2\pi(5.3 \times 10^{-11}\,\text{m})}$$

$$\approx 6.6 \times 10^{15}\,\text{Hz}.$$

According to the classical electromagnetic theory we know that the frequency of the electromagnetic waves emitted by the revolving electrons is equal to the frequency of its revolution around the nucleus. Thus the initial frequency of the light emitted is 6.6×10^{15} Hz.

S20. The key idea here is that throughout the scattering process, the total mechanical energy of the system consisting of an α-particle and a gold nucleus is conserved. The system's initial mechanical energy is E_i, before the particle and nucleus interact, and it is equal to its mechanical energy E_f when the α-particle momentarily stops. The initial energy E_i is just the kinetic energy K of the incoming α-particle. The final energy E_f is just the electric potential energy U of the system. The potential energy U can be calculated from Eq. (12.1).

Let d be the centre-to-centre distance between the α-particle and the gold nucleus when the α-particle is at its stopping point. Then we can write the conservation of energy $E_i = E_f$ as

$$K = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(Ze)}{d} = \frac{2Ze^2}{4\pi\varepsilon_0 d}$$

Thus the distance of closest approach d is given by

$$d = \frac{2Ze^2}{4\pi\varepsilon_0 K}$$

The maximum kinetic energy found in α-particles of natural origin is 7.7 MeV or 1.2×10^{-12} J. Since $1/4\pi\varepsilon_0 = 9.0 \times 10^9$ N m^2/C^2. Therefore with $e = 1.6 \times 10^{-19}$ C, we have,

$$d = \frac{(2)(9.0\times10^9\,\text{Nm}^2/\text{C}^2)(1.6\times10^{-19}\,\text{C})^2\,Z}{1.2\times10^{-12}\,\text{J}}$$
$$= 3.84 \times 10^{-16}\,Z\,\text{m}$$

The atomic number of foil material gold is $Z = 79$, so that

$$d\,(\text{Au}) = 3.0 \times 10^{-14}\,\text{m} = 30\,\text{fm}. \ (1\,\text{fm}\ (\textit{i.e.}, \text{fermi}) = 10^{-15}\,\text{m.})$$

The radius of gold nucleus is, therefore, less than 3.0×10^{-14} m. This is not in very good agreement with the observed result as the actual radius of gold nucleus is 6 fm. The cause of discrepancy is that the distance of closest approach is considerably larger than the sum of the radii of the gold nucleus and the α-particle. Thus, the α-particle reverses its motion without ever actually *touching* the gold nucleus.

S21. From Eq. (12.13), we have

$$m\,v_n r_n = nh/2\pi$$

Here, m = mass of satellite

v_n = Velocit of satellite

r = Orbital radius of satellite

Here m = 10 kg and $rn = 8 \times 10^6$ m. We have the time period T of the circling satellite as 2 h. That is T = 7200 s.

Thus, the velocity $v_n = 2\pi r_n/T$.

The quantum number of the orbit of satellite

$$n = (2\pi r_n)^2 \times m/(T \times h).$$

Substituting the values, $n = (2\pi \times 8 \times 10^6\,\text{m})^2 \times 10/(7200\,\text{s} \times 6.64 \times 10^{-34}\,\text{J s})$

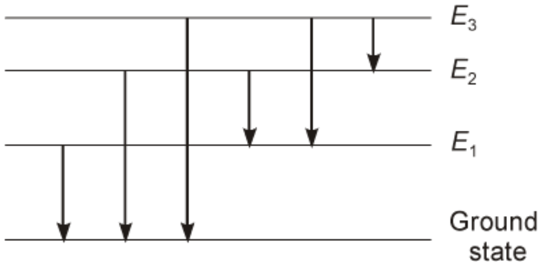
$$= 5.3 \times 10^{45}$$

Note that the quantum number for the satellite motion is extremely large! In fact for such large quantum numbers the results of quantisation conditions tend to those of classical physics.

S22. Total number of spectral lines emitted between the energy levels $n = 4$ (third excited state) and $n = 1$ (ground state) is given by

$$N = \frac{4\,(4-1)}{2} = \mathbf{6}.$$

As shown in figure below, the schematic emission of these spectral lines.



S23. When the electron is in the second excited state (n = 3) of the hydrogen atom, the following transitions an take place:

From the state $n = 3$ to $n = 2$ and $n = 1$ (two transitions)

and from the state $n = 2$ to $n = 1$ (one transitions)

Thus, the maximum possible number of spectral lines observe will be *three*.

The energy emitted is minimum, when the transition of electron takes place from the state $n = 2$ to $n = 1$. The energy emitted is given by

$$E_{\min} = E_2 - E_1$$
$$= -3.4 - (-13.6) = 10.2 \text{ eV}.$$

Likewise, the wavelength emitted will be maximum.

The energy emitted is maximum, when the transition of electron takes place from the state $n = 3$ to $n = 1$. The energy emitted is given by

$$E_{\max} = E_3 - E_1$$
$$= -1.51 - (-13.6) = 12.09 \text{ eV}.$$

Likewise, the wavelength emitted will be minimum.

Hence, ratio of the maximum and minimum wavelengths of the radiations emitted,

$$\frac{\lambda_{\min}}{\lambda_{\max}} = \frac{hc/E_{\min}}{hc/E_{\max}} = \frac{E_{\max}}{E_{\min}} = \frac{12.09}{10.2} = \mathbf{1.185}.$$

S24. (a) *When the electron goes from second energy level to the first level:* The energy of photon,

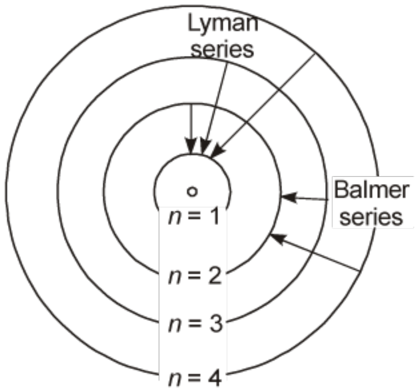
$$E = E_2 - E_1$$
$$= -3.4 - (-13.6) = 10.2 \text{ eV}.$$

(b) *When the electron goes from the highest energy level to the first level:* The energy of photon,

$$E' = E_{\infty} - E_1$$
$$= -0 - (-13.6) = 13.6 \text{ eV}$$

$$\therefore \quad \frac{E}{E'} = \frac{10.2}{13.6} = \mathbf{0.75}.$$

S25. The transitions representing the Balmer and Lyman series are as shown in the figure below.



S26. When fast moving electrons strike a target of high atomic weight, X-rays are produced. The X-rays are electromagnetic radiations of very short wavelength (≈ 1 Å) and are highly penetrating. On the other hand, the electrons are negatively charged atomic particles.

S27. The minimum accelerating potential required to knock out the electron from the outermost orbit of an atom is called its ionisation potential.

In case of hydrogen atom, the energy of electron in the first orbit of the atom is – 13.6 eV. Since to knock out this electron, an energy equal to 13.6 eV is required, the ionisation potential of H-atom is **13.6 V**.

S28. (a) Given, $\lambda = 275 \text{ mm} = 275 \times 10^{-9}\text{ m}$

Hence, energy of the emitted photon,

$$E = \frac{hc}{\lambda}$$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{275 \times 10^{-9}} = 7.22 \times 10^{-19}\text{ J}$$

$$= \frac{7.22 \times 10^{-19}}{1.6 \times 10^{-19}} = 4.5 \text{ eV.}$$

The photon of energy 4.5 eV (or of wavelength 275 mm) will be emitted, corresponding to the *transition B*.

(b) The *transition A* corresponds to emission of radiation of minimum energy and hence maximum wavelength.

S29. Velocity of electron in n^{th} orbit of hydrogen atom is given by

$$v_n = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\pi e^2}{nh}$$

Therefore, velocity of the electron in $n = 3$ orbit of hydrogen atom is given by

$$v_3 = \frac{9 \times 10^9 \times 2\pi \times (1.6 \times 10^{-19})^2}{3 \times 6.62 \times 10^{-34}}$$

$$= \mathbf{7.29 \times 10^5\, m\, s^{-1}}.$$

S30. The distance of closest approach is given by

$$r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{\frac{1}{2}mu^2}$$

Here, $Z = 79;\ \frac{1}{2}mu^2 = 5\,\text{MeV} = 5 \times 1.6 \times 10^{-13}\,\text{J}$

We know, $e = 1.6 \times 10^{-19}\,\text{C}$

$$\therefore \quad r_0 = \frac{9 \times 10^9 \times 2 \times 79 \times (1.6 \times 10^{-19})^2}{5 \times 1.6 \times 10^{-13}}$$

$$= \mathbf{4.55 \times 10^{-14}\, m}.$$

Aliter: At the distance of the closest approach, the kinetic energy (E_k) of the α-particle gets converted into the potential energy (E_p) of the α-particle and the gold nucleus.

Hence, $E_k = 5\,\text{MeV} = 5 \times 1.6 \times 10^{-12} = 8 \times 10^{-13}\,\text{J}$

and $$E_p = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2e \times Ze}{r_0} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{r_0}$$

$$= 9 \times 10^9 \times \frac{2 \times 79 \times (1.6 \times 10^{-19})^2}{r_0}$$

$$= \frac{3.64 \times 10^{-26}}{r_0}\, J$$

At the distance of the closest approach,

$$E_p = E_k$$

or $$\frac{3.64 \times 10^{-26}}{r_0} = 8 \times 10^{-13}$$

or $$r_0 = \frac{3.64 \times 10^{-26}}{8 \times 10^{-13}} = \mathbf{4.55 \times 10^{-14}\, m}.$$

S31. Given, $h = 6.6 \times 10^{-34}\,\text{J s};\ 1\,\text{eV} = 1.66 \times 10^{-19}\,J$ and $c = 3 \times 10^8\,\text{m s}^{-1}$.

Energy of $n = 2$ level, $E_2 = -3.4\,\text{eV}$

Energy of n = 3 level, $E_3 = -1.5\,\text{eV}$

Therefore, energy of the photon emitted during transition from n = 3 to n = 2 level,

$$h\nu = E_3 - E_2 = -1.5 - (-3.4) = 1.9 \text{ eV}$$

or $$\frac{hc}{\lambda} = 1.9\,\text{eV} = 1.9 \times 1.6 \times 10^{-19}\,J$$

$\therefore$ $$\lambda = \frac{hc}{1.9 \times 1.6 \times 10^{-19}} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{1.9 \times 1.6 \times 10^{-19}}$$

$$= 6{,}513 \times 10^{-10}\,\text{m} = \mathbf{6{,}513\,Å}.$$

S32. Given: $r_1 = 0.5\,Å = 0.5 \times 10^{-10}\,\text{m}$ and $v_1 = 2.24 \times 10^{6}\,\text{ms}^{-1}$

Therefore, time period of electron in the first orbit,

$$T = \frac{2\pi r_1}{v_1} = \frac{2\pi \times 0.5 \times 10^{-10}}{2.24 \times 10^{6}} = 1.4 \times 10^{-16}\,s$$

So, frequency of revolution of the electron in the first Bohr orbit of hydrogen atom,

$$\nu = \frac{1}{T} = \frac{1}{1.4 \times 10^{-16}} = \mathbf{7.143 \times 10^{15}\,Hz}.$$

S33. For the given accelerating voltage, the wavelength of the continuous X-rays can vary from a certain minimum value to infinity (theoretically). Thus, the wavelength range of the continuous X-rays emitted will be from λ_{min} to ∞, where $\lambda_{min} > 0$.

S34. Energy emitted

$$\Delta E = E_2 - E_1$$

$$= -3.4\,\text{eV} - (-0.85\,\text{eV})$$

$$= -2.55\,\text{eV}$$

$$= -2.55 \times 1.6 \times 10^{-19}\,\text{J}$$

As $$\Delta E = h\nu = \frac{hc}{\lambda}$$

$\therefore$ Wavelength of emitted spectral line $(\lambda) = \dfrac{hc}{\Delta E}$

$$= \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{2.55 \times 1.6 \times 10^{-19}} m$$

$$= 4.868 \times 10^{-7} \text{ m} = 4867 \text{ Å}$$

Because, $$E_2 = -3.4 = -\frac{13.6}{n_1^2} = -\frac{13.6}{2^2} \Rightarrow n_1 = 2$$

this wavelength belongs to balmer series of hydrogen spectrum.

S35. Let initial velocity of the proton moving towards the free proton at rest be v_1 and let v be the velocity of each proton after impact, then applying the law of conservation of momentum, we have

$$mv_i + 0 = mv + mv$$

or $$mv_i = 2mv$$

or $$v = \frac{v_i}{2}$$

By law of conservation of energy, we have

$$\frac{1}{2} mv_i^2 = \frac{1}{2} mv^2 + \frac{1}{2} mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$\frac{1}{2} mv_i^2 = \frac{1}{4} mv_i^2 + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

or $$\frac{1}{4} mv_i^2 = \frac{e^2}{4\pi\varepsilon_0 r}$$

ordistances of the closest approach, $r = \left(\frac{4}{mv_i^2}\right)\left(\frac{e^2}{4\pi\varepsilon_0}\right)$

or $$r = \frac{4 \times (1.6 \times 10^{-19})^2 \times 9 \times 10^9}{1.66 \times 10^{-27} \times (7.45 \times 10^5)^2} \qquad \left(\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\right)$$

$$\cong \mathbf{10^{-12}\ m}.$$

S36. The Balmer series is represented by the relation

$$\frac{1}{\lambda} = \vec{v} = R\left(\frac{1}{2} - \frac{1}{n^2}\right)$$

The H_α line corresponds to the transition $n = 3$ whereas H_β line corresponds to $n = 4$. Let λ_1 and λ_2 denote thewavelengths of H_α and H_β lines of the Balmer series.

$$\frac{1}{\lambda_1} = R\left(\frac{1}{2^2}-\frac{1}{3^2}\right) = R\left(\frac{1}{4}-\frac{1}{9}\right) = \frac{5}{36}R \qquad \text{... (i)}$$

and
$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2}-\frac{1}{4^2}\right) = R\left(\frac{1}{4}-\frac{1}{16}\right) = \frac{3}{16}R \qquad \text{... (ii)}$$

Dividing Eqn. (i) by (ii), we get

$$\frac{\lambda_2}{\lambda_1} = \frac{5}{36}\times\frac{16}{3} = \frac{20}{27}$$

or
$$\lambda_2 = \frac{20}{27}\times\lambda_1 = \frac{20}{27}\times 6563\ A^\circ = 4761.5\ A^\circ.$$

S37. For the Balmer series, the base or reference level is $n = 2$ level. Hence the short wavelength limit (λ_s) and the longwavelength limit (λ_l) are given by

$$\frac{1}{\lambda_s} = R\left(\frac{1}{2^2}-\frac{1}{\infty^2}\right) = \frac{R}{4} \quad \text{or} \quad \lambda_s = \frac{4}{R}$$

and
$$\frac{1}{\lambda_l} = R\left(\frac{1}{2^2}-\frac{1}{3^2}\right) = \frac{5}{36}R \quad \text{or} \quad \lambda_l = \frac{36}{5R}$$

As,
$$\frac{1}{R} = 912\ \text{A}^\text{o}$$

$\therefore$
$$\lambda_s = 4\times 912\ \text{A}^\text{o} = 3648\ \text{A}^\text{o}$$

and
$$\lambda_l = \frac{36}{5}\times 912\ \text{A}^\circ = 6566\ \text{A}^\circ$$

S38. The following are the drawbacks of Rutherford's model

(a) Ruther suggested that "On revolving in the orbits, electron radiates energy and strikes consequently towards the nucleus" 2-e, the radius followed by the electrons, gradually decreases based on it, electron should fall into nucleus and atom should be destroyed.

(b) According to it we should obtain radiation to it we should obtain radiation of all possible wavelength but in actual practice atomic spectrum is line spectrum.

S39. Hydrogen spectrum continuous and different series are obtained from transition of electron from obtained energy state to other. Also, higher the difference between the state will be the energy of the spectrum obtained.

(a) Lyman series is obtained when an electron jumps from the first orbit to any outer orbit.

So , Ist spectrum represent Lyman series because in this spectrum, electron jumps from $n = 1$ to $n = 2$.

Balmer series is obtained when an electron jumps from the second orbit to any outer orbit

So, IInd spectrum represent Balmer series because in this spectrum, electron jumps from $n = 2$, to $n = 3$.

Similarly, in paschen series, electron jumps from the third orbit to any outer orbit.

So, IIIrd spectrum represent paschen series.

(b) Spectral lines IIIrd *i.e.*, paschen series corresponds to the absorption of radiation of maximum wavelength.

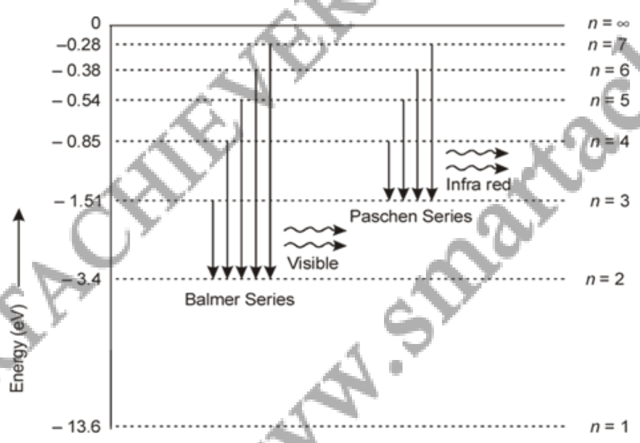
S40.

$$E_n = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \cdot \frac{2\pi^2 me^4}{n^2 h^2}$$

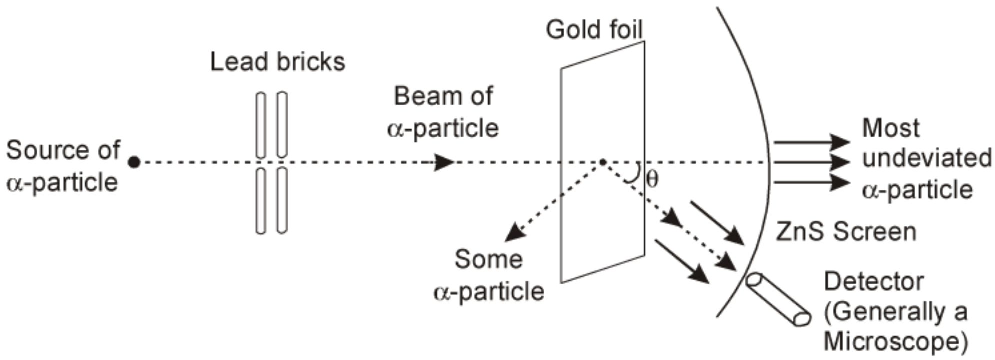
$$= \frac{-(9\times10^9)^2\times2\times9.87\times9.1\times10^{-31}\times(1.6\times10^{-19})^4}{n^2(6.62\times10^{-34})^2}$$

$$= \frac{-21.76\times10^{-19}}{n^2}\,J$$

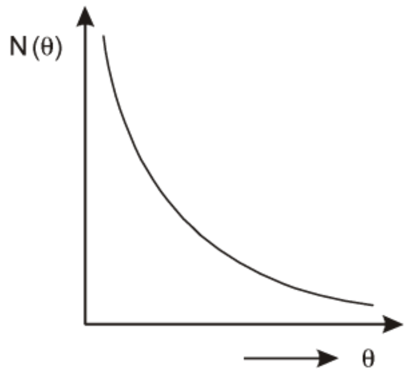
$$E_n = \frac{\mathbf{-13.6}}{\boldsymbol{n^2}}\,\mathbf{eV}.$$



S41. Arrangement of the Geiger-Marsden Experiment



With angle of scattering the number of scattered particles detected change as shown in the adjoining diagram. In the scattering experiment,



(a) Beam of α-particles get deviated at various angles with different probabilities.

(b) α-particles with least impact parameter suffers larger scattering – rebounding on head on collision.

(c) For larger impact parameter, the particle remains almost undeviated.

The fact that the number of particles rebounding back is less means that head on collision is less-indicating the concentration of the atom at its centre. This confirms that the nucleus of the atom has a size limit.

S42. The negative sign tells that an electron and the nucleus a band system.

Given: Ground state energy – 13.6 eV

We know,

$$E_n = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \cdot \frac{2\pi^2 me^2}{n^2 h^2} \qquad \text{... (i)}$$

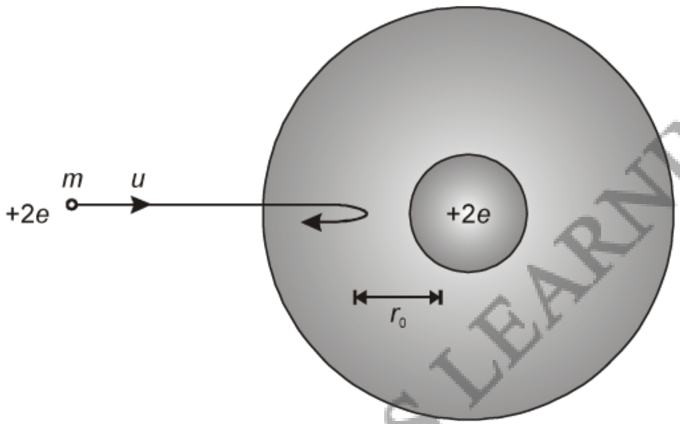
$$E_1 = -\left(\frac{1}{4\pi\varepsilon_0}\right)^4 \cdot \frac{2\pi me^4}{h^2} \qquad \text{... (ii)}$$

$$E_2 = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \cdot \left(\frac{2\pi me^2}{4h^2}\right)$$

$$= \frac{1}{4}\left\{\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \left(\frac{2\pi me^2}{h^2}\right)\right\}$$

$$E_2 = \frac{\mathbf{1}}{\mathbf{4}}\boldsymbol{E}_1.$$

S43. An α-particle travelling directly towards the centre of the nucleus slows down as it approaches the nucleus. At a certain distance, say r_0 from the nucleus, the α-particle comes to rest and its initial kinetic energy is completely covered into electrostatic potential energy. The distance r_0 is called the distance of closest approach as shown in the figure below. The value of the distance of closest approach gives an estimate of the size of the nucleus.



Consider that an α-particle of mass m possess initial velocity u, when it is at a large distance from the nucleus of an atom having atomic number Z. At the distance of closest approach, the kinetic energy of the α-particle is completely converted into potential energy. Mathematically,

$$\frac{1}{2}mu^2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2e(Ze)}{r_0}$$

$$r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{\frac{1}{2}mu^2}$$

In Geiger-Marsden experiment, α-particles of kinetic energy 5.5 MeV were directed towards the gold nucleus ($Z = 79$). By calculating the distance of closest approach r_0, an estimate of the size of the nucleus can be made. The calculations show that r_0 comes out be 4.13×10^{-14} m. Thus size of the nucleus is of the order of 10^{-14} m.

S44. The wavelengths of the different members of Lyman series are given by

$$\frac{1}{\lambda} = R_H\left(\frac{1}{1^2}-\frac{1}{n_i^2}\right), \qquad \text{\where} \quad n_i = 2, 3, 4, ...$$

Let λ_1 be the wavelength of first member of the Lyman series (n_i = 2). Then,

$$\frac{1}{\lambda_1} = R_H\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = \frac{3R_H}{4} \qquad \text{... (i)}$$

The wavelengths of different members of the Balmer series are given by

$$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2}-\frac{1}{n_i^2}\right), \qquad \text{where} \quad n_i = 3, 4, 5, ...$$

Let λ_2' be the wavelength of the second member of the Balmer series (n_i = 4). Then,

$$\frac{1}{\lambda_2'} = R_H\left(\frac{1}{2^2}-\frac{1}{4^2}\right) = \frac{3R_H}{16} \qquad \text{... (ii)}$$

Dividing the Eqn. (i) by (ii), we get

$$\frac{\lambda_2'}{\lambda_1} = \frac{3R_H}{4}\times\frac{16}{3R_H} = 4$$

or $\qquad \lambda_2' = 4\lambda_1$

Therefore, $\qquad \lambda_1 = 1,216$ Å

$\therefore \qquad \lambda_2' = 4 \times 1,216 =$ **4,864 Å**.

S45. Given, the energy of the electron, in the ground state of hydrogen is –13.6 eV.

$\therefore \qquad E_1 = -13.6$ eV

$\Rightarrow \qquad$ For n = 2, $E_2 = -3.4$ eV

For n = 3, $E_3 = -1.5$ eV. $\qquad \left[\because E_n = -\frac{13.6}{n^2}\text{eV}\right]$

$\therefore$ Energy of photon corresponding to the first line of the

(a) Lyman series, $\qquad E = E_2 - E_1$

$= (-3.4) - (-13.6)$

$= 10.2$ eV

(b) Balmer series, $E = E_3 - E_2$

$$= (-1.5\text{eV}) - (-3.4)$$

$$= 1.9 \text{ eV}$$

S46. Ground state energy, $E_i = -13.6$ eV

As, $$E_n = \frac{E_i}{n^2}$$

$\therefore$ $$E_n = -\frac{13.6}{n^2} \text{ eV}$$

(a) K.E. of an electron = – Total energy of electron

In second excited state, $n = 3$

Total energy, $$E_n = -\frac{13.6}{3^2} = -1.5 \text{ eV}$$

$\Rightarrow$ K.E. of the electron in 2nd excited state = E_3

$$= -(-1.5 \text{ eV}) = 1.5 \text{ eV}$$

(b) Transition of electron from $n_2 = 3$ to $n_1 = 1$

We have wavelength of emitted radiation

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$= R\left[\frac{1}{1^2} - \frac{1}{3^2}\right]$$

$$\frac{1}{\lambda} = \frac{8R}{9} = \frac{9}{8 \times 1.09 \times 10^7}$$

$$\lambda = 1.03 \times 10^{-7} \text{m}$$

$$\lambda = 1030 \text{Å}$$

The wavelength of emitted electron is 1030 Å.

S47. (a) In Rutherford experiment, a few α-particles are scattered through large angles. Therefore, massive and positive part of atom should be concentrated in a small portion of the atom.

This small positive part having almost the entire mass of the atom was called nucleus. Therefore, the main feature of any atom model should require the presence of the central positive and massive nucleus.

(b) Here don't confuse with the word Geiger-Marsden experiment, that the question is same way as to find the distance of closest approach in case of Rutherford experiment.

$$Z = 80,\ KE = 8 \text{ MeV}$$
$$= 8 \times 10^6 \times 1.6 \times 10^{-19} \text{J}$$

$\therefore$ Energy conservation law,

$$K = \frac{(Ze)(2e)}{4\pi\varepsilon_0 r_0}$$

where, r_0 = distance of closest approach.

$$r_0 = \frac{2Ze^2}{4\pi\varepsilon_0 (K)}$$

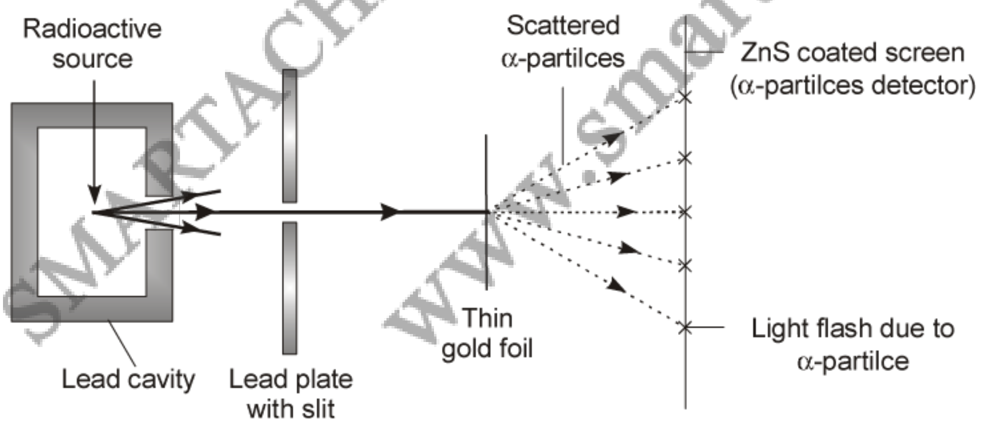
$$r_0 = \frac{9 \times 10^9 \times 2 \times 80 \times (1.6 \times 10^{-19})^2}{8 \times 10^6 \times 1.6 \times 10^{-19}}$$

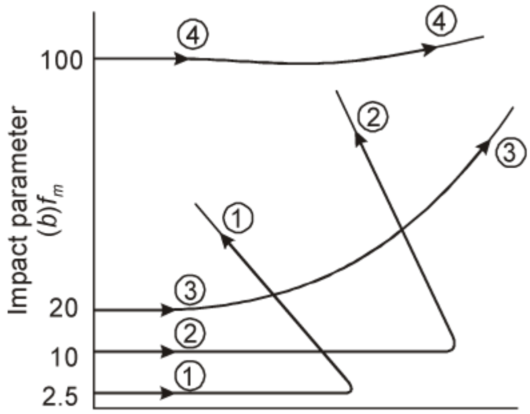
$$r_0 = 2.88 \times 10^{-19} \text{ m}$$

$$r_0 \propto \frac{1}{K}.$$

If K.E. gets doubles, distance of closest approach reduces to half.

S48. Given figure shows a schematic diagram of Geiger-Marsden experiment.





The size of the nucleus can be obtained by finding impact parameter b using trajectories of α-particle. The impact parameter is the perpendicular distance of the initial velocity vector of α-particle from the central line of nucleus, when it is far away from the atom. Rutherford calculated impact parameter as

$$b = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2 \cot(\theta/2)}{E}$$

Where, E = KE of α-particle

θ = Scattering angle

Z = Atomic number of atom

The size of the nucleus is smaller than the impact parameter.

The idea of size of nucleus can also be obtained by finding the distance of closest approach.

S49. (a) (i) Let an electron revolves around the nucleus of hydrogen atom. The necessary centripetal force is provided by electrostatic force of attraction.

$$\therefore \quad \frac{mv^2}{r} = \frac{ke^2}{r^2} \quad \Rightarrow \quad r = \frac{ke^2}{mv^2} \quad \text{... (i)}$$

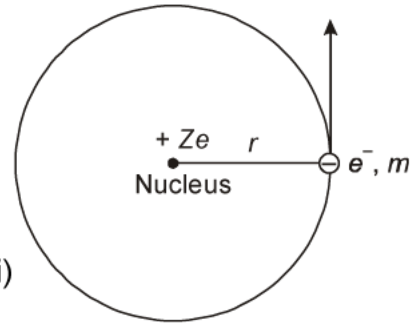
Where, m mass of electron and v is its speed of a circular path of radius, r

By Bohr's second postulates,

$$mvr = \frac{nh}{2\pi}$$

where, n = 1, 2,3...

$$r = \frac{nh}{2\pi mv} \quad \text{... (ii)}$$



Comparing Eqs. (i) and (ii),

$$\frac{ke^2}{mv^2} = \frac{nh}{2\pi mv} \quad \Rightarrow \quad v = \frac{2\pi ke^2}{nh}$$

Substituting in Eq. (ii)

$$r = \frac{n^2h^2}{4\pi^2mke^2} \qquad \text{... (iii)}$$

$$\Rightarrow \qquad r \propto n^2$$

(ii) Now, kinetic energy of electron

$$KE = \frac{1}{2}mv^2 = \frac{ke^2}{2r}$$

Also, potential energy, $PE = -\frac{ke^2}{r}$

Energy of electron in n th orbit

$$E_n = \frac{-ke^2}{2r} = -\frac{ke^2}{2} \cdot \frac{4\pi^2mke^2}{n^2h^2} \quad \Rightarrow \quad E_n = \frac{2\pi^2mk^2e^4}{n^2h^2} \qquad \text{... (iv)}$$

where, $R = \frac{2\pi^2mk^2e^4}{ch^3} \quad \Rightarrow \quad E_n = \frac{Rhc}{n^2}$ ... (v)

(b) For Balmer series, α-line, wavelength is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

where, $n = 3$

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{1}{4} - \frac{1}{9}\right) = \left(\frac{9-4}{36}\right)R$$

$$\frac{1}{\lambda} = \frac{5R}{36}$$

$$\lambda = \frac{36}{5R} = \frac{36}{5 \times 1.097 \times 10^7} = 6.563 \times 10^{-7} \text{ m} = 6563 \text{ Å}.$$

S50. According to Bohr's postulates, in a hydrogen atom, a single electron revolves around a nucleus of charge $+e$. For an electron moving with a uniform speed in a circular orbit of a given radius, the centripetal force is provided by coulomb force of attraction between the electron and the nucleus. The gravitational attraction may be neglected as the mass of electron and proton is very small.

So, $\frac{mv^2}{r} = \frac{ke^2}{r^2}$ or $mv^2 = \frac{ke^2}{r}$ ... (i) where, m = mass of electron; r = radius of electronic orbit; v = velocity of electron

Again, By Bohr's second postulates

$$mvr = \frac{nh}{2\pi} \quad \text{or} \quad v = \frac{nh}{2\pi mr}$$

putting the value of n in Eq.(i)

$$m\left(\frac{nh}{2\pi mr}\right)^2 = \frac{ke^2}{r} \quad \Rightarrow \quad r = \frac{n^2h^2}{4\pi^2 kme^2} \qquad \text{... (ii)}$$

kinetic energy of electron,

$$E_k = \frac{1}{2}mv^2 = \frac{ke^2}{2r} \qquad \left(\frac{mv^2}{r} = \frac{ke^2}{r^2}\right)$$

using Eq. (ii), we get

$$E_k = \frac{ke^2}{2}\frac{4\pi^2kme^2}{n^2h^2} = \frac{2\pi^2k^2me^4}{n^2h^2}$$

Potential energy

$$E_p = \frac{-k(e)\times(e)}{r} = \frac{-ke^2}{r}$$

Using Eq. (ii), we get

$$E_p = -ke^2 \times \frac{4\pi^2kme^2}{n^2h^2} = \frac{-4\pi^2kme^2}{n^2h^2}$$

Hence, total energy of the electron in the n^{th} orbit

$$E = E_p + E_k = \frac{-4\pi^2k^2me^2}{n^2h^2} + \frac{2\pi^2k^2me^4}{n^2h^2} = -\frac{2\pi^2k^2me^4}{n^2h^2} = -\frac{13.6}{n^2}eV$$

When the electron in a hydrogen atom jumps from higher energy level to the lower energy level, the difference of energies of the two energy levels is emitted as a radiation of particular wavelength. It is called a spectral line.

In H-atom, when an electron jumps from the orbit n_i to orbit n_f, the wavelength of the emitted radiation is given by,

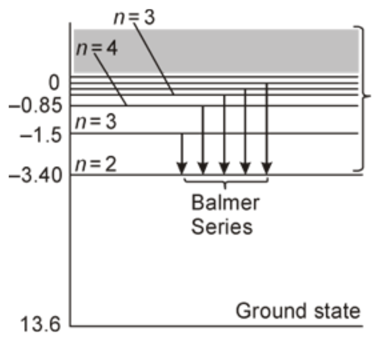
$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

For Balmer series, $n_f = 2$ and n_i = 3, 4, 5,.....

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$$

where, n_i = 3, 4, 5,.....

These spectral lines lie in the visible region.



S51. (a) The minimum accelerating potential required to knock out the electron from the outermost orbit of an atom is called its ionisation potential.

In case of hydrogen atom, the energy of electron in the first orbit of the atom is – 13.6 eV. Since to knock out this electron, an energy equal to 13.6 eV is required, the ionisation potential of H-atom is **13.6 V**.

(b) Energy and wavelength associated with photon is given by,

$$E\,(\text{in eV}) = \frac{1242 \text{ eV-nm}}{\lambda(\text{in nm})}$$

Given, λ = 102.7 nm

$$E = \frac{1242}{102.7} \simeq 12.09 \text{ eV}$$

i.e., This energy $E \simeq 12.09$ eV may possess by the photon emitted during the transition, D as the photon emitted during the transition, D as $(-1.5) - (-13.6) \cong 12.1$ eV.

(c) K.E. = 87 MeV = $8.7 \times 10^6 \times 1.6 \times 10^{-19} = 13.92 \times 10^{-13}$ J

Charge on α-particle $2e$, $Z = 29$.

$$\text{P.E.} = \frac{1}{4\pi\varepsilon_0}\frac{Ze.2e}{x} = \frac{9 \times 10^9 \times 2 \times 29 \times (1.6 \times 10^{-19})^2}{x}$$

As α-particle approaches the nucleus it comes to rest at a distance x from the nucleus and then is repelled. Hence at this point K.E. = P.E.

$$\therefore \quad 8.7 \times 1.6 \times 10^{-13} = \frac{9 \times 10^9 \times 29 \times (1.6 \times 10^{-19})^2 \times 2}{x}$$

or

$$x = \frac{9 \times 10^9 \times 29 \times 1.6 \times 1.6 \times 10^{-38} \times 2}{8.7 \times 1.6 \times 10^{-13}}$$

$$= 96 \times 10^{-16} \text{ m} = 9.6 \times 10^{-15} \text{ m}.$$

Date: 22/10/2021

Q1. Write the expression for Bohr's radius in hydrogen atom.

Q2. What is the ratio of radii of the orbits corresponding to first excited state and ground state in a hydrogen atom?

Q3. The radius of the first electron orbit of the hydrogen atom is 5.3×10^{-11} m. What is the radius of the second orbit?

Q4. What is the ratio of radii of the orbits corresponding to second excited state and ground a state of the hydrogen atom?

Q5. In derivation of frequency of radiation from the hydrogen atom, what do we mean by the equation

$$mvr = n\frac{h}{2\pi}?$$

Q6. What is Bohr's quantisation condition for angular momentum of electron in an atom?

Q7. State any two postulates of Bohr's theory of hydrogen atom.

Q8. State Bohr's quantisation condition for defining stationary orbits.

Q9. In hydrogen atom, if the electron is replaced by a particle which is 200 times heavier but has the same charge, how would its radius changes?

Q10. The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. What are the radii of the $n = 2$ and $n = 3$ orbits?

Q11. In accordance with the Bohr's model, find the quantum number that characterises the Earth's revolution around the Sun in an orbit of radius 1.5×10^{11} m with orbital speed 3×10^4 m/s. (Mass of Earth = 6.0×10^{24} kg.)

Q12. If Bohr's quantisation postulate (angular momentum = $nh/2\pi$) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do we never speak of quantisation of orbits of planets around the sun?

Q13. It is found experimentally that 13.6 eV energy is required to separate a hydrogen atom into a proton and an electron. Compute the orbital radius and the velocity of the electron in a hydrogen atom.

Q14. Find the ratio of energies of photons produced due to transition of an electron of hydrogen atom from its
(a) second permitted energy level to the first level and
(b) the highest permitted energy level to the first permitted level.

Q15. State Bohr postulates for hydrogen atom.

Q16. Show that the ionisation energy for the hydrogen atom is 13.6 eV.

Q17. Using Bohr's second postulate of quantisation of orbital angular momentum, show that the circumference of the electron in the n^{th} orbital state in hydrogen atom is n times the de-Broglie wavelength associated with it.

Q18. The ionisation energy of hydrogen atom is given to be 1.6 eV. A photon falls on the hydrogen atom, which is initially in the ground state and executes it to the $n = 4$ state. Calculate the wavelength of the photon.

Q19. Using the relevant Bohr's Postulates derive the expression for the radius of the electron in the n^{th} orbit of the electron in hydrogen atom.

Q20. (a) In hydrogen atom, an electron undergoes transition from 2nd excited state to the first excited state and then to the ground state. Identify the spectral series to which these transition belong.

(b) Find out the ratio of the wavelengths of the emitted radiations in the two cases.

Q21. How many times does the electron go round the first Bohr orbit in 1 second?

Q22. The ground state energy of hydrogen atom is – 13.6 eV

(a) What is the kinetic energy of an electron in the second excited state?

(b) If the electron jumps to the ground state from the second excited state, calculate the wavelength of the spectral line emitted.

Q23. The energy of the electron in the ground state of hydrogen atom is – 13.6 eV.

(a) What does the negative sign signify?

(b) How much energy is required to take an electron in this atom from the ground state to the first excited state?

Q24. In Bohr's theory of hydrogen atoms, calculate the energy of the photon emitted during a transition of the electron from the first excited state to its ground state. Write in which region of the electromagnetic spectrum this transition lies.

Given Rydberg constant $R = 1.03 \times 10^7 \text{ m}^{-1}$.

Q25. The ground state energy of hydrogen atom is –13.6 eV. The photon emitted during the transition of electron from $n = 2$ to $n = 1$ state, is incident on the photosensitive material of unknown work function. The photoelectrons are emitted from materials with a maximum kinetic energy of 8 eV,. Calculate the threshold wavelength of the material used.

Q26. The energy of the electron in the hydrogen atom is known to be expressible in the form of

$E_n = \frac{-13.6 \text{ eV}}{n^2}$, [where, $n = 1, 2, 3...$] Use this expression to show that the

(a) electron in the hydrogen atom cannot have an energy of –2 eV.

(b) spacing between the lines (consecutive energy levels) within the given set of the observed hydrogen spectrum decreases as n increases.

Q27. A hydrogen atom initially in the ground level absorbs a photon, which excites it to the $n = 4$ level. Determine the wavelength and frequency of the photon.

Q28. (a) Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the $n = 1, 2$, and 3 levels.

(b) Calculate the orbital period in each of these levels.

Q29. A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?

Q30. Obtain the first Bohr's radius and the ground state energy of a *muonic hydrogen atom* [*i.e.*, an atom in which a negatively charged muon (μ^-) of mass about 207 m_e orbits around a proton].

Q31. Obtain an expression for the frequency of radiation emitted when a hydrogen atom de-excites from level *n* to level (*n* – 1). For large *n*, show that this frequency equals the classical frequency of revolution of the electron in the orbit.

Q32. The gravitational attraction between electron and proton in a hydrogen atom is weaker than the coulomb attraction by a factor of about 10^{-40}. An alternative way of looking at this fact is to estimate the radius of the first Bohr orbit of a hydrogen atom if the electron and proton were bound by gravitational attraction. You will find the answer interesting.

Q33. Classically, an electron can be in any orbit around the nucleus of an atom. Then what determines the typical atomic size? Why is an atom not, say, thousand times bigger than its typical size? The question had greatly puzzled Bohr before he arrived at his famous model of the atom that you have learnt in the text. To simulate what he might well have done before his discovery, let us play as follows with the basic constants of nature and see if we can get a quantity with the dimensions of length that is roughly equal to the known size of an atom (~ 10^{-10} m).

(a) Construct a quantity with the dimensions of length from the fundamental constants *e*, m_e, and *c*. Determine its numerical value.

(b) You will find that the length obtained in (a) is many orders of magnitude smaller than the atomic dimensions. Further, it involves *c*. But energies of atoms are mostly in non-relativistic domain where *c* is not expected to play any role. This is what may have suggested Bohr to discard *c* and look for 'something else' to get the right atomic size. Now, the Planck's constant *h* had already made its appearance elsewhere. Bohr's great insight lay in recognising that *h*, m_e, and *e* will yield the right atomic size. Construct a quantity with the dimension of length from *h*, m_e, and e and confirm that its numerical value has indeed the correct order of magnitude.

Q34. The total energy of an electron in the first excited state of the hydrogen atom is about –3.4 eV.

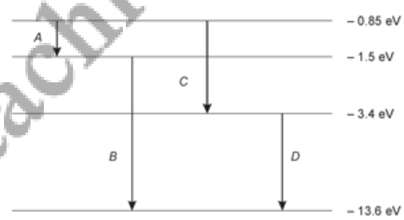
(a) What is the kinetic energy of the electron in this state?

(b) What is the potential energy of the electron in this state?

(c) Which of the answers above would change if the choice of the zero of potential energy is changed?

Q35. The energy levels of an element are given below:

Identify, using necessary calculations, the transition, which corresponds to the emission of a spectral line of wavelength 482 nm:



Q36. Using the Rydberg formula, calculate the wavelengths of the first four spectral lines in the Lyman series of the hydrogen spectrum.

Q37. The ground state energy of hydrogen atom is – 13.6 eV.

(a) What is the potential energy of an electron in the 3rd excited state?

(b) If the electron jumps to the ground state from the 3rd excited state, calculate the wavelength of the photon emitted.

Q38. Using Bohr's postulate for hydrogen atom, show that the total energy (*E*) of the electron in the stationary states can be expressed as the sum of kinetic energy (*K*) and potential energy (*U*), where *K* = – 2U. Hence, deduce the expression for the total energy in the n^{th} energy level of hydrogen atom.

Q39. Calculate the radius of the third Bohr's orbit of hydrogen atom and the energy of the electron in that orbit. Given that $h = 6.625 \times 10^{-34}$ Js; $e = 1.6 \times 10^{-19}$ C; $m = 9.1 \times 10^{-31}$ kg; $c = 3 \times 10^{8}$ ms^{-1} and $\varepsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1}.

Q40. State the two postulates of Bohr's atomic model. Obtain Bohr's quantisation condition of angular momentum on the basis of the wave picture of electron.

Q41. The electron in a given Bohr orbit has a total energy of – 1.5 eV. Calculate its

(a) Kinetic energy

(b) potential energy

(c) wavelength of radiation emitted when this electron makes a transition of the ground state.

[Given Energy in the ground state = – 13.6 eV and Rydberg's constant = 1.09×10^{7} m^{-1}]

Q42. Using postulates of Bohr's theory of hydrogen atom, show that

(a) radii of orbits increases as n^2, and

(b) the total energy of electron increases as $\frac{1}{n^2}$, where n is the principal quantum number of the atom.

Q43. Using the relevant Bohr's postulate, derive the expressions for the

(a) speed of the electron in the n^{th} orbit and

(b) radius of the n^{th} orbit of the electron in hydrogen atom.

Q44. By using Bohr's postulates of atomic model, derive mathematical expressions for (a) kinetic energy and (b) potential energy of an electron revolving in an orbit of radius r. How does the potential change with increase in the principal quantum number (n) for the electron and why?

Q45. (a) Using postulates of Bohr's theory of hydrogen atom, show that (i) the radii of orbits increases as n^2, and (ii) the total energy of the electron increases as $1/n^2$, where n is the principal quantum number of the atom.

(b) Calculate the wavelength of H_α line in Balmer series of hydrogen atom, given Rydberg constant $R = 1.0947 \times 10^{7}$ m^{-1}.

Date: 22/10/2021

S1. Expression for Bohr's radius in hydrogen atom,

$$r = \frac{n^2h^2}{4\pi^2 mkZe^2}$$

where n = principal quantum number,

m = mass of electron

$$k = \frac{1}{4\pi\varepsilon} = 9 \times 10^9 \text{ N - m}^2/\text{C}^2$$

Z = atomic number of atom = 1

h = planck's constant.

S2. We know,

$$r \propto h^2$$

$$\frac{r_2}{r_1} = \frac{4}{1}$$

$r_1 : r_2 =$ **4 : 1**.

S3. Radius of n^{th} orbit, $r_n \propto n^2$

$\therefore$ $\frac{r_2}{r_1} = \frac{2^2}{1^2}$ or $\frac{r_2}{r_1} = 4$

or $r_2 = 4\,r_1 = 4 \times 5.3 \times 10^{-11}$

$= \mathbf{2.12 \times 10^{-10}}$ **m**.

S4. Radius of n^{th} orbit, $r_n \propto n^2$

The ground state corresponds to $n = 1$ and the first excited state corresponds to $n = 3$.

$\therefore$ $\frac{r_2}{r_1} = \frac{3^2}{1^2}$ or $\frac{r_2}{r_1} = 9 : 1$.

S5. The given equation *implies that an electron can revolve round the nucleus only in those circular orbits in which the angular momentum of the electron is integral multiple of* $h/2\pi$, *where h is planck's constant*.

It is called *Bohr's frequency condition*.

S6. Bohr's quantisation condition:

$$m\,v\,r = n\frac{h}{2\pi} \qquad \text{where } n = 1, 2, 3, \ldots$$

S7. ***Postulates of Bohr's Atom:***

(a) In a hydrogen atom, the negatively charged electron revolves in a circular orbit around the heavy positively charged nucleus. The centripetal force required by the electron is provided by the attractive force exerted by the nucleus on it.

(b) The electron can revolve round the nucleus only in those circular orbits in which the angular momentum of an electron is integral multiple of $h/2\pi$, where h is Planck's constant $(= 6.62 \times 10^{-34}\text{ J s})$.

S8. According to Bohr's quantisation condition, electrons are permitted to revolve in only those orbits in which the angular momentum of electron is an integral multiple of $\frac{h}{2\pi}$ *i.e.*,

$$mvr = \frac{nh}{2\pi}, \qquad \text{where,} \quad n = 1, 2, 3, \ldots\ldots$$

m, v, r are mass, speed and radius of electron and h being planck's constant.

S9. $\because$ Radius of hydrogen atom $\propto \frac{1}{m}$

where, m = mass of particle

$$\frac{r_2}{r_1} = \frac{m_2}{m_1} = \frac{m_1}{200m_1} = \frac{1}{200}$$

[$\because$ The new particle is 200 times heavier, then first one]

$$r_2 = \frac{1}{200}r_1$$

New radius would become $\frac{1}{200}$ time to that of original radius.

S10. The radius of the innermost orbit of a hydrogen atom,

$$r_1 = 5.3 \times 10^{-11}\text{ m}.$$

Let r_2 be the radius of the orbit at $n = 2$. It is related to the radius of the innermost orbit as:

$$r_2 = (n)^2 r_1$$
$$= 4 \times 5.3 \times 10^{-11} = 2.12 \times 10^{-10} \text{ m}$$

For $n = 3$, we can write the corresponding electron radius as:

$$r_3 = (n)^3 r_1$$
$$= 9 \times 5.3 \times 10^{-11} = 4.77 \times 10^{-10} \text{ m}$$

Hence, the radii of an electron for $n = 2$ and $n = 3$ orbits are 2.12×10^{-10} m and 4.77×10^{-10} m respectively.

S11. Radius of the orbit of the Earth around the Sun,

$$r = 1.5 \times 1011 \text{ m}$$

Orbital speed of the Earth, $\nu = 3 \times 10^4$ m/s

Mass of the Earth, $m = 6.0 \times 10^{24}$ kg

According to Bohr's model, angular momentum is quantized and given as:

$$m\nu r = \frac{nh}{2\pi}$$

Where, h = Planck's constant = 6.62×10^{-34} Js

n = Quantum number

$$\therefore \qquad n = \frac{m\nu r 2\pi}{h}$$

$$= \frac{2\pi \times 6 \times 10^{24} \times 3 \times 10^4 \times 1.5 \times 10^{11}}{6.62 \times 10^{-34}}$$

$$= 25.61 \times 10^{73} = 2.6 \times 10^{74}$$

Hence, the quanta number that characterizes the Earth' revolution is 2.6×10^{74}.

S12. We never speak of quantization of orbits of planets around the Sun because the angular momentum associated with planetary motion is largely relative to the value of Planck's constant (h). The angular momentum of the Earth in its orbit is of the order of $10^{70} h$. This leads to a very high value of quantum levels n of the order of 10^{70}. For large values of n, successive energies and angular momenta are relatively very small. Hence, the quantum levels for planetary motion are considered continuous.

S13. Total energy of the electron in hydrogen atom is

$$-13.6 \text{ eV} = -13.6 \times 1.6 \times 10^{-19} \text{ J}$$
$$= -2.2 \times 10^{-18} \text{ J}.$$

Thus from Eq. (12.4), we have

$$-\frac{e^2}{8\pi\varepsilon_0 r} = -2.2 \times 10^{-18} \text{ J}.$$

This gives the orbital radius

$$r = -\frac{e^2}{8\pi\varepsilon_0 E}$$

$$= \frac{(9\times10^9\ \text{Nm}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2}{(2)(-2.2\times10^{-18}\ \text{J})}$$

$$= 5.3\times10^{-11}\ \text{m}.$$

The velocity of the revolving electron can be computed from Eq. (12.3) with $m = 9.1\times10^{-31}$ kg,

$$\text{v} = \frac{e}{\sqrt{4\pi\varepsilon_0 mr}} = 2.2\times10^6\ \text{m/s}.$$

S14. (a) Since, the second permitted energy level to the first level = $E_2 - E_1$ = Energy to photon released

$$= (-3.4\ \text{eV}) - (-13.6\ \text{eV}) = 10.2\ \text{eV}$$

(b) The highest permitted energy level to the first permitted level

$$= E_\infty - E_1 = 0 - (-13.6) = 13.6\ \text{eV}$$

$\therefore$ Ratio of energies of photon

$$= \frac{10.2}{13.6} = \frac{3}{4} = 3:4\,.$$

S15. In a hydrogen atom, the negatively charged electron revolves in circular orbit around the heavy positively charged nucleus. The centripetal force required by electron is provided by the attractive force exerted by the nucleus on it.

The electron can revolve around the nucleus only in these circular orbits in which the angular momentum of an electron is integral multiple of $h/2\pi$, where h is plank's constant (6.62×10^{-34} Js)

S16. The energy of the electron in first orbit of H-atom,

$$E_1 = -13.6\,\text{eV}$$

When the H-atom is ionised, the energy of the electron,

$$E_\infty = 0$$

Hence, ionisation energy of the H-atom

$$= E_\infty - E_1 = 0 - (-13.6) = \mathbf{13.6\ eV}.$$

S17. Bohr's postulate for the permitted orbits. The electron can revolve round the nucleus only in those circular orbits in which the angular momentum of an electron is integral multiple of $h/2\pi$, where h is Planck's constant.

It m and v are mass and velocity of the electron in a permitted orbit of radius r, then

$$mvr = n\frac{h}{2\pi},$$

where n is called the principal quantum number and it has the integral values 1, 2, 3, ...

From the above Bohr's condition, circumference of the permitted orbit is given by

$$2\pi r = n\frac{h}{mv} \qquad \text{... (i)}$$

According to de-Broglie hypothesis, an electron of mass m moving with speed v is associated with a wave of wavelength,

$$\lambda = \frac{h}{mv} \qquad \text{... (ii)}$$

From the equations (i) and (ii), we have

$$2\pi r = n\lambda$$

Thus, n^{th} permitted orbit for the electron can contain exactly n wave lengths of the de-Broglie wave associated with the electron in that orbit. In the figure given below, it has been shown as to how the four complete de-Broglie wave associated with the electron in that orbit.

S18. Energy of electron in the first orbit of the H-atom,

$$E_1 = -13.6\,\text{eV}$$

and energy of electron in the fourth orbit of the H-atom,

$$E_4 = -\frac{13.6}{4^2} = -0.85\ \text{eV}$$

Hence, energy required to excite an electron from first orbit of the H-atom to the fourth orbit,

$$E = E_4 - E_1 = -0.85 - (-1.36) = 12.75\,\text{eV}$$

$$= 12.75 \times 1.6 \times 10^{-19} = 20.4 \times 10^{-19}\,\text{J}$$

The wavelength of the incident photon,

$$\lambda = \frac{hc}{E} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{20.4 \times 10^{-19}} = 973.5 \times 10^{-10}\text{m}$$

$$= \mathbf{973.5\ Å}.$$

S19. Electron revolves in a stable orbit the centripetal force is provided by electrostatic force of attraction acting on it, due to positive charges in the nucleus.

Hence,

$$\frac{mv_n^2}{r_n} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r_n^2}$$

$$v_n^2 = \frac{e^2}{4\pi\varepsilon_0 m r_n} \qquad \text{... (i)}$$

and from Bohr's quantum condition, we have

$$mv_n r_n = \frac{nh}{2\pi} \quad \text{or} \quad v_n = \frac{nh}{2\pi m r_n} \qquad \text{... (ii)}$$

Squaring Eq. (ii) and then equating it with Eq. (i), we get

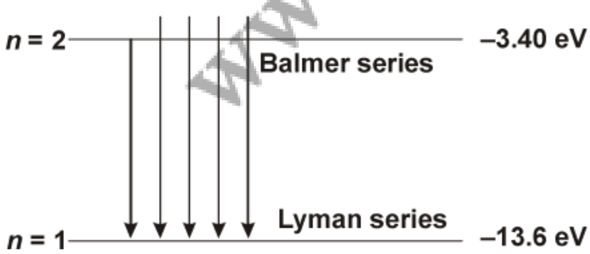
$$\frac{n^2h^2}{4\pi^2 m^2 r_n^2} = \frac{e^2}{4\pi\varepsilon_0 m r_n}$$

$$r_n = \frac{n^2h^2}{4\pi^2 m^2} \times \frac{4\pi\varepsilon_0 m}{e^2}$$

$$= \frac{\varepsilon_0 h^2}{\pi m e^2} . n^2$$

S20. (a) An electron undergoes transition from 2nd excited state to the first excited state is Balmer series and then to the ground state is Lyman series.

(b) The wavelength of the emitted radiations in the two cases.



For $n_2 \xrightarrow{\lambda} n_1$

$$\Delta E = (-3.40 + 13.6) = 10.20 \text{ eV}$$

$$\lambda = \frac{12.43 \times 10^{-7}}{10.2}$$
$$= 1.218 \times 10^{-7}\ \text{m}$$
$$\lambda = 1218\ \text{Å}$$

For $n_3 \xrightarrow{\lambda} n_2$

$$\Delta E = (-1.51 + 3.40) = 1.89\ \text{eV}$$
$$\lambda = \frac{12.43 \text{ x } 10^{-7}}{1.89} = 6.576 \text{ x } 10^{-7}\ \text{m} = 6576\ \text{Å}$$

S21. The distance travelled by electron in traversing the first Bohr orbit is $2\pi r$ where r is the radius of the first Bohr orbit.

Radius of first Bohr orbit $\quad r = \dfrac{h^2(4\pi\varepsilon_0)}{4\pi^2 me^2} = 0.5292 \times 10^{-10}\ m$

If T is the time taken by the electron to traverse the first bohr orbit, then

$$T = \frac{2\pi r}{v}$$

where velocity v is given by

$$v = \frac{2\pi e^2}{(4\pi\varepsilon_0)t} = 2.18 \times 10^6\ \text{m/s}$$

$\therefore$ Number of revolutions made by the electron per second

$$\nu = \frac{1}{T} = \frac{v}{2\pi r} = \frac{2.18 \times 10^6}{2 \times 3.14 \times 0.5292 \times 10^{-10}}\ \text{Hz}$$

$$= \mathbf{6.57 \times 10^{15}\ Hz}.$$

S22. (a) The second excited state corresponds to $n = 3$.

Now, $\quad E_3 = -\dfrac{13.6}{n^2} = -\dfrac{13.6}{3^2} = -\dfrac{13.6}{9} = -1.51\,\text{eV}$

The kinetic energy of an electron in the second excited state

$$= -(-1.51) = \mathbf{1.51\ eV}.$$

(b) Energy emitted, when the electron jumps from the second excited state to the ground state,

$$E = -1.51 - (-13.6) = 12.09\ \text{eV}$$

$$= 12.09 \times 1.6 \times 10^{-19}\,\text{J}$$

The wavelength of the spectral line emitted,

$$\lambda = \frac{hc}{E} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{12.09 \times 1.6 \times 10^{-19}}$$

$$= 1.027 \times 10^{-7}\,\text{m} = \mathbf{1{,}027\ \text{Å}}.$$

S23. (a) The negative sign imply that electrons are bound to the nucleus by means of electrostatic force of attraction.

(b) Energy of electron in nth orbit of hydrogen atom,

$$E_n = \frac{-13.6}{n^2}\,eV$$

For first excited state $n = 2$

$$E_2 = \frac{-13.6}{2^2}\,eV$$

$$= \frac{-13.6}{2^2} = -3.4\,\text{eV}$$

$$\text{energy required} = E_2 - E_1 = -3.4 + 13.6 = 10.2\ \text{eV}$$

S24. To calculate, the energy of the photon released we have to calculate the energy of each state between which transition takes place. This difference of energy of the states is equal to the energy of photon emitted.

$\therefore$ Energy of electron in n^{th} orbit of hydrogen atom,

$$E_n = \frac{13.6}{n^2}\,\text{eV}$$

For ground state, $n = 1$

$$E_1 = \frac{13.6}{1^2} = 13.6\,\text{eV}$$

For 1st excited state, $n = 2$

$$E_2 = \frac{13.6}{2^2} = 3.4\,\text{eV}$$

$\therefore$ Energy of photon related,

$$E_2 - E_1 = (-3.4\ \text{eV}) - (-13.6\ \text{eV}) = 10.2\ \text{eV}$$

For Lyman series,

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n^2}\right], \quad \text{where, } n = 2, 3, 4, \ldots$$

Here, $n = 2$

$\therefore$ $$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3}{4}R$$

or $$\frac{1}{\lambda} = \frac{3}{4}R$$

$$\lambda = \frac{4}{3 \times 1.03 \times 10^7}$$

$$= 1.3 \times 10^{-7}\text{m}$$

$\therefore$ $\lambda = 1300$ Å

This wavelength falls in the region of Lyman series.

S25. Energy of electron in *n*th orbit of hydrogen atom

$$E_n = \frac{13.6}{n^2}\text{eV}$$

For $n = 1$,

$\Rightarrow$ $E_1 = 13.6$ eV

For $n = 2$,

$\Rightarrow$ $$E_2 = \frac{13.6}{n^2} = -3.4\,\text{eV}$$

Energy of photon released $= E_2 - E_1$

$= (-3.4) - (-13.6)$

$= 10.2 \text{ eV} = h\nu$

Also, $KE_{\max} = 8$ eV

According to Einstein equation

$$KE_{\max} = h\nu - \phi_0$$

$$8 \text{ eV} = 10.2 \text{ eV} - \phi_0$$

Wave function $(\phi) = 2.2$ eV

$\therefore$ Threshold wavelength

$$\lambda = \frac{1242\,\text{eV - nm}}{\text{wave function}\,(\phi)} = \frac{1242\,\text{eV - nm}}{2.2\,\text{eV}}$$

$$\lambda = 564.5\text{ nm}$$

S26. (a) For, $E_n = -2\text{ eV}$

$\Rightarrow$ $$-2\text{ eV} = \frac{-13.6}{n^2}\text{eV}$$

$\Rightarrow$ $$13.6 = 2n^2$$

$$n^2 = 6.8$$

$\Rightarrow$ n is necessarily non-integral value whereas it should be integer to satisfy quantization condition. Therefore, –2 eV energy of electron is not possible.

(b) Energy of the electron, $E_n = \dfrac{-13.6}{n^2}$ eV

$\Rightarrow$ For $n = 1$,

$$E_1 = \frac{-13.6}{1} = -13.6\,\text{eV}$$

For $n = 2$,

$$E_2 = \frac{13.6}{(2)^2} = -3.4\,\text{eV}$$

For $n = 3$,

$$E_3 = \frac{13.6}{(3)^2} = -1.51\text{eV}$$

For $n = 4$,

$$E_4 = \frac{13.6}{(4)^2} = -0.85\,\text{eV}$$

$\Rightarrow$ $E_n < E_{n-1} < E_{n-2} \quad \forall\, n \in N$

So, from above result we can say that spacing between spectral lines, decreases.

S27. For ground level, $n_1 = 1$

Let E_1 be the energy of this level. It is known that E_1 is related with n_1 as:

$$E_1 = \frac{-13.6}{n_1^2}\text{eV} = \frac{-13.6}{1^2} = -13.6\text{ eV}$$

The atom is excited to a higher level, $n_2 = 4$.

Let E_2 be the energy of this level.

$$\therefore \qquad E_2 = \frac{-13.6}{n_2^2}\text{ eV} = \frac{-13.6}{4^2} = -\frac{13.6}{16}\text{ eV}$$

The amount of energy absorbed by the photon is given as:

$$E = E_2 - E_1$$

$$= \frac{-13.6}{16} - \left(-\frac{13.6}{1}\right)$$

$$= \frac{13.6 \times 15}{16}\text{ eV}$$

$$= \frac{13.6 \times 15}{16} \times 1.6 \times 10^{-19} = 2.04 \times 10^{-18}\text{ J}$$

For a photon of wavelength λ, the expression of energy is written as:

$$E = \frac{hc}{\lambda}$$

Where, h = Planck's constant = 6.6×10^{-34} Js

c = Speed of light = 3×10^8 m/s

$$\therefore \qquad \lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2.04 \times 10^{-18}}$$

$$= 9.7 \times 10^{-8}\text{ m} = 97\text{ nm}$$

And, frequency of a photon is given by the relation,

$$\therefore \qquad \nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{9.7 \times 10^{-8}} \approx 3.1 \times 10^{15}\text{ Hz}$$

Hence, the wavelength of the photon is 97 nm while the frequency is 3.1×10^{15} Hz.

S28. (a) Let ν_1 be the orbital speed of the electron in a hydrogen atom in the ground state level, $n_1 = 1$. For charge (e) of an electron, ν_1 is given by the relation,

$$\nu_1 = \frac{e^2}{n_1 4\pi\varepsilon_0 \left(h/2\pi\right)} = \frac{e^2}{2\varepsilon_0 h}$$

Where, $e = 1.6 \times 10^{-19}$ C

ε_0 = Permittivity of free space

$= 8.85 \times 10^{-12}\text{ N}^{-1}\text{ C}^2\text{ m}^{-2}$

h = Planck's constant

$= 6.62 \times 10^{-34}$ Js

$$\therefore \quad \nu_1 = \frac{(1.6 \times 10^{-19})^2}{2 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}}$$

$$= 0.0218 \times 10^8 = 2.18 \times 10^6 \text{ m/s}$$

For level $n_2 = 2$, we can write the relation for the corresponding orbital speed as:

$$\nu_2 = \frac{e^2}{n_2 2\varepsilon_0 h}$$

$$= \frac{(1.6 \times 10^{-19})^2}{2 \times 2 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}}$$

$$= 1.09 \times 10^6 \text{ m/s}$$

And, for $n_3 = 3$, we can write the relation for the corresponding orbital speed as:

$$\nu_3 = \frac{e^2}{n_3 2\varepsilon_0 h}$$

$$= \frac{(1.6 \times 10^{-19})^2}{3 \times 2 \times 8.85 \times 10^{-12} \times 6.62 \times 10^{-34}}$$

$$= 7.27 \times 10^5 \text{ m/s}$$

Hence, the speed of the electron in a hydrogen atom in $n = 1$, $n = 2$, and $n = 3$ is 2.18×10^6 m/s, 1.09×10^6 m/s, 7.27×10^5 m/s respectively.

(b) Let T_1 be the orbital period of the electron when it is in level $n_1 = 1$.

Orbital period is related to orbital speed as:

$$T_1 = \frac{2\pi r_1}{\nu_1}$$

Where, r_1 = Radius of the orbit

$$= \frac{n_1^2 h^2 \varepsilon_0}{\pi m e^2}$$

h = Planck's constant = 6.62×10^{-34} Js

e = Charge on an electron = 1.6×10^{-19} C

ε_0 = Permittivity of free space = $8.85 \times 10-{}^{12}$ $N^{-1} C^2 m^{-2}$

m = Mass of an electron = 9.1×10^{-31} kg

$$\therefore \quad T_1 = \frac{2\pi r_1}{\nu_1}$$

$$= \frac{2\pi \times (1)^2 \times (6.62 \times 10^{-34})^2 \times 8.85 \times 10^{-12}}{2.18 \times 10^6 \times \pi \times 9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2}$$

$$= 15.27 \times 10^{-17} = 1.527 \times 10^{-16} \text{ s}$$

For level $n_2 = 2$, we can write the period as:

$$\therefore \qquad T_2 = \frac{2\pi r_2}{\nu_2}$$

Where, r_2 = Radius of the electron in $n_2 = 2$

$$= \frac{(n_2)^2 h^2 \varepsilon_0}{\pi m e^2}$$

$$\therefore \qquad T_2 = \frac{2\pi r_2}{\nu_2}$$

$$= \frac{2\pi \times (2)^2 \times (6.62 \times 10^{-34})^2 \times 8.85 \times 10^{-12}}{1.09 \times 10^6 \times \pi \times 9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2}$$

$$= 1.22 \times 10^{-15}\,\text{s}$$

And, for level $n_3 = 3$, we can write the period as:

$$T_3 = \frac{2\pi r_3}{\nu_3}$$

Where, r_3 = Radius of the electron in $n_3 = 3$

$$= \frac{(n_3)^2 h^2 \varepsilon_0}{\pi m e^2}$$

$$\therefore \qquad T_3 = \frac{2\pi r_3}{\nu_3}$$

$$= \frac{2\pi \times (3)^2 \times (6.62 \times 10^{-34})^2 \times 8.85 \times 10^{-12}}{1.09 \times 10^5 \times \pi \times 9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2}$$

$$= 4.12 \times 10^{-15}\,\text{s}$$

Hence, the orbital period in each of these levels is 1.52×10^{-16} s, 1.22×10^{-15} s, and 4.12×10^{-15} s respectively.

S29. It is given that the energy of the electron beam used to bombard gaseous hydrogen at room temperature is 12.5 eV. Also, the energy of the gaseous hydrogen in its ground state at room temperature is – 13.6 eV.

When gaseous hydrogen is bombarded with an electron beam, the energy of the gaseous hydrogen becomes – 13.6 + 12.5 eV *i.e.*, – 1.1 eV.

Orbital energy is related to orbit level (n) as:

$$E = \frac{-13.6}{(n)^2}\,\text{eV}$$

For $n = 3$, $$E = \frac{-13.6}{9} = -1.5\,\text{eV}$$

This energy is approximately equal to the energy of gaseous hydrogen. It can be concluded that the electron has jumped from $n = 1$ to $n = 3$ level.

During its de-excitation, the electrons can jump from $n = 3$ to $n = 1$ directly, which forms a line of the Lyman series of the hydrogen spectrum.

We have the relation for wave number for Lyman series as:

$$\frac{1}{\lambda} = R_y \left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

Where, R_y = Rydberg constant = $1.097 \times 10^7\ m^{-1}$

λ = Wavelength of radiation emitted by the transition of the electron

For $n = 3$, we can obtain λ as:

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{3^2}\right)$$

$$= 1.097 \times 10^7 \left(1 - \frac{1}{9}\right) = 1.097 \times 10^7 \times \frac{8}{9}$$

$$\lambda = \frac{9}{8 \times 1.097 \times 10^7} = 102.55\ nm$$

If the electron jumps from $n = 2$ to $n = 1$, then the wavelength of the radiation is given as:

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$= 1.097 \times 10^7 \left(1 - \frac{1}{4}\right) = 1.097 \times 10^7 \times \frac{3}{4}$$

$$\lambda = \frac{4}{3 \times 1.097 \times 10^7} = 121.54\ nm$$

If the transition takes place from $n = 3$ to $n = 2$, then the wavelength of the radiation is given as:

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$= 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{9}\right) = 1.097 \times 10^7 \times \frac{5}{36}$$

$$\lambda = \frac{36}{5 \times 1.097 \times 10^7} = 656.33\ nm$$

This radiation corresponds to the Balmer series of the hydrogen spectrum.

Hence, in Lyman series, two wavelengths *i.e.*, 102.5 nm and 121.5 nm are emitted. And in the Balmer series, one wavelength *i.e.*, 656.33 nm is emitted.

S30. Mass of a negatively charged muon, $m_\mu = 207\ m_e$

According to Bohr's model,

Bohr radius, $r_e \propto \left(\frac{1}{m_e}\right)$

And, energy of a ground state *electronic hydrogen atom*, $E_e \propto m_e$

Also, energy of a ground state *muonic hydrogen atom*, $E_\mu \propto m_\mu$

We have the value of the first Bohr orbit,

$$r_e = 0.53\ \text{Å} = 0.53 \times 10^{-10}\ \text{m}$$

Let r_μ be the radius of *muonic hydrogen atom*.

At equilibrium, we can write the relation as:

$$m_\mu r_\mu = m_e r_e$$

$$207\ m_e\ r_\mu = m_e r_e$$

$$\therefore \qquad r_\mu = \frac{0.53 \times 10^{-10}}{207} = 2.56 \times 10^{-13}\ \text{m}$$

Hence, the value of the first Bohr radius of a *muonic hydrogen atom* is 2.56×10^{-13} m.

We have, $E_e = -13.6\ \text{eV}$

Take the ratio of these energies as:

$$\frac{E_e}{E_\mu} = \frac{m_e}{m_\mu} = \frac{m_e}{207\ m_e}$$

$$E_\mu = 207\ E_e$$

$$= 207 \times (-13.6) = -2.81\ \text{keV}$$

Hence, the ground state energy of a muonic hydrogen atom is -2.81 keV.

S31. It is given that a hydrogen atom de-excites from an upper level (n) to a lower level ($n-1$).

We have the relation for energy (E_1) of radiation at level n as:

$$E_1 = h\nu_1 = \frac{hme^4}{(4\pi)^3 \varepsilon_0^2 \left(\frac{h}{2\pi}\right)^3} \times \left(\frac{1}{n^2}\right) \qquad \text{... (i)}$$

Where, ν_1 = Frequency of radiation at level n

h = Planck's constant

m = Mass of hydrogen atom

e = Charge on an electron

ε_0 = Permitivity of free space

Now, the relation for energy (E_2) of radiation at level ($n-1$) is given as:

$$E_2 = h\nu_2 = \frac{hme^4}{(4\pi)^3 \varepsilon_0^2 \left(\frac{h}{2\pi}\right)^3} \times \frac{1}{(n-1)^2} \qquad \text{... (ii)}$$

Where, ν_2 = Frequency of radiation at level $(n-1)$

Energy (E) released as a result of de-excitation:

$$E = E_2 - E_1$$
$$h\nu = E_2 - E_1 \quad \text{... (iii)}$$

Where, ν_1 = Frequency of radiation emitted

Putting values from Eqs. (i) and (ii) in equation (iii), we get:

$$\nu = \frac{me^4}{(4\pi)^3 \varepsilon_0^2 \left(\frac{h}{2\pi}\right)^3}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$= \frac{me^4(2n-1)}{(4\pi)^3 \varepsilon_0^2 \left(\frac{h}{2\pi}\right)^3 n^2(n-1)^2}$$

For large n, we can write $(2n-1) \simeq 2n$ and $(n-1) \simeq n$.

$$\nu = \frac{me^4}{32\pi^3 \varepsilon_0^2 \left(\frac{h}{2\pi}\right)^3 n^3} \quad \text{... (iv)}$$

Classical relation of frequency of revolution of an electron is given as:

$$\nu_c = \frac{v}{2\pi r} \quad \text{... (v)}$$

Where,

Velocity of the electron in the n^{th} orbit is given as:

$$v = \frac{e^2}{4\pi\varepsilon_0 \left(\frac{h}{2\pi}\right) n} \quad \text{... (vi)}$$

And, radius of the n^{th} orbit is given as:

$$r = \frac{4\pi\varepsilon_0 \left(\frac{h}{2\pi}\right)^2}{me^2} n^2 \quad \text{... (vii)}$$

Putting the values of equations (vi) and (vii) in equation (v), we get:

$$\nu_c = \frac{me^4}{32\pi^3 \varepsilon_0^2 \left(\frac{h}{2\pi}\right)^3 n^3} \quad \text{... (vii)}$$

Hence, the frequency of radiation emitted by the hydrogen atom is equal to its classical orbital frequency.

S32. Radius of the first Bohr orbit is given by the relation,

$$r_1 = \frac{4\pi\varepsilon_0\left(\frac{h}{2\pi}\right)^2}{m_e e^2} \quad \text{... (i)}$$

Where, ε_0 = Permittivity of free space

h = Planck's constant = 6.63×10^{-34} Js

m_e = Mass of an electron = 9.1×10^{-31} kg

e = Charge of an electron = 1.9×10^{-19} C

m_p = Mass of a proton = 1.67×10^{-27} kg

r = Distance between the electron and the proton

Coulomb attraction between an electron and a proton is given as:

$$F_C = \frac{e^2}{4\pi\varepsilon_0 r^2} \quad \text{... (ii)}$$

Gravitational force of attraction between an electron and a proton is given as:

$$F_G = \frac{Gm_p m_e}{r^2} \quad \text{... (iii)}$$

Where, G = Gravitational constant = 6.67×10^{-11} N m^2/kg^2

If the electrostatic (Coulomb) force and the gravitational force between an electron and a proton are equal, then we can write:

$$\therefore \quad F_G = F_C$$

$$\frac{Gm_p m_e}{r^2} = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

$$\therefore \quad \frac{e^2}{4\pi\varepsilon_0} = Gm_p m_e \quad \text{... (iv)}$$

Putting the value of Eq. (iv) in Eq. (i), we get:

$$r_1 = \frac{\left(\frac{h}{2\pi}\right)^2}{Gm_p m_e}$$

$$= \frac{\left(\frac{6.63 \times 10^{-34}}{2 \times 3.14}\right)^2}{6.67 \times 10^{-11} \times 1.67^{-27} \times (9.1 \times 10^{-11})^2} \approx 1.21 \times 10^{29} \text{ m}$$

It is known that the universe is 156 billion light years wide or 1.5×10^{27} m wide. Hence, we can conclude that the radius of the first Bohr orbit is much greater than the estimated size of the whole universe.

S33. (a) Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg

Speed of light, $c = 3 \times 10^8$ m/s

Let us take a quantity involving the given quantities as $\left(\frac{e^2}{4\pi\varepsilon_0 m_e c^2}\right)$.

Where, ε_0 = Permittivity of free space

And, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N m^2 C^{-1}

The numerical value of the taken quantity will be:

$$\frac{1}{4\pi\varepsilon_0} \times \frac{e^2}{m_e c^2} = 9 \times 10^9 \times \frac{(1.6 \times 10^{-19})^2}{9.1 \times 10^{-31} \times (3 \times 10^8)^2} = 2.81 \times 10^{-15}\text{ m}$$

Hence, the numerical value of the taken quantity is much smaller than the typical size of an atom.

(b) Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg

Planck's constant, $h = 6.63 \times 10^{-34}$ Js

Let us take a quantity involving the given quantities as $\frac{4\pi\varepsilon_0 \left(\frac{h}{2\pi}\right)^2}{m_e c^2}$.

Where, ε_0 = Permittivity of free space

And, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N m^2 C^{-2}

The numerical value of the taken quantity will be:

$$\frac{4\pi\varepsilon_0 \left(\frac{h}{2\pi}\right)^2}{m_e c^2} = \frac{\left(\frac{6.63 \times 10^{-34}}{2 \times 3.14}\right)^2}{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2} = 0.53 \times 10^{-10}\text{ m}$$

Hence, the value of the quantity taken is of the order of the atomic size.

S34. (a) Total energy of the electron, $E = -3.4$ eV

Kinetic energy of the electron is equal to the negative of the total energy.

$\Rightarrow$ $K = -E$

$= -(-3.4) = +3.4$ eV

Hence, the kinetic energy of the electron in the given state is + 3.4 eV.

(b) Potential energy (U) of the electron is equal to the negative of twice of its kinetic energy.

$\Rightarrow$ $U = -2K$

$$= -2 \times 3.4 = -6.8\,\text{eV}$$

Hence, the potential energy of the electron in the given state is -6.8 eV.

(c) The potential energy of a system depends on the reference point taken. Here, the potential energy of the reference point is taken as zero. If the reference point is changed, then the value of the potential energy of the system also changes. Since total energy is the sum of kinetic and potential energies, total energy of the system will also change.

S35. Given,
$$h\nu = E_1 - E_2$$
$$\frac{hc}{\lambda} = E_1 - E_2$$
$$\lambda = \frac{hc}{E_1 - E_2}$$

For the spectral line C, we get
$$E_1 - E_2 = -0.85 - (-3.4)$$
$$= 2.55\,\text{eV}$$
$$= 2.55 \times 1.6 \times 10^{-19}\,\text{J}$$
$$\lambda = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{2.55 \times 1.6 \times 10^{-19}}$$
$$\lambda = \mathbf{4.872 \times 10^{-7}\,cm}$$

This confirms transition C.

S36. The Rydberg formula is
$$hc/\lambda_{if} = \frac{me^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

The wavelengths of the first four lines in the Lyman series correspond to transitions from $n_i = 2, 3, 4, 5$ to $n_f = 1$. We know that
$$\frac{me^4}{8\varepsilon_0^2 h^2} = 13.6\,\text{eV} = 21.76 \times 10^{-19}\,\text{J}$$

Therefore,
$$\lambda_n = \frac{hc}{21.76 \times 10^{-19}\,\frac{1}{1} - \frac{1}{n_i^2}}\,\text{m}$$

$$= \frac{6.625 \times 10^{-34} \times 3 \times 10^{8} \times n_i^2}{21.76 \times 10^{-19} \times (n_i^2 - 1)} \text{ m}$$

$$= \frac{0.9134 \times n_i^2}{(n_i^2 - 1)} \times 10^{-7} \text{ m}$$

$$= 913.4\, n_i^2/(n_i^2 - 1) \text{ Å}$$

Substituting n_i = 2, 3, 4, 5, we get λ_{21} = 1218 Å, λ_{31} = 1028 Å, λ_{41} = 974.3 Å, and λ_{51} = 951.4 Å.

S37. (a) Total energy in 3rd excited state is

$$E_4 = \frac{13.6}{4^2} = -0.85 \text{ eV}$$

$$E_p = 2E_4$$

$$E_p = -1.7 \text{ eV}$$

(b)

$$\Delta E = E_4 - E_1$$

$$= -0.85 - (-13.6) = 12.75 \text{ eV}$$

$$\Delta E = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{12.75 \times 1.6 \times 10^{-19}}$$

λ = **970 Å**.

S38. According to Bohr's postulates for hydrogen atom, electron revolves in a circular orbit around the heavy positively charged nucleus.

These are the stationary (orbits) states of the atom.

For a particular orbit, electron moves there, so it has kinetic energy.

Also, there is potential energy due to charge on electron and heavy positively charged nucleus.

Hence, total energy (E) of atom is sum of kinetic energy (K) and potential energy (U).

i.e., $E = K + U$

We know, $r = \dfrac{Ze^2}{4\pi\varepsilon_0 mv^2}$

According to Bohr's quantisation condition, angular momentum of the electron,

$$m v r = n \frac{h}{2\pi} \qquad \text{... (i)}$$

Putting the value of v in the Eqn. (i), we get

$$mv\left(\frac{Ze^2}{4\pi\varepsilon_0 mv^2}\right) = n\frac{h}{2\pi} \quad \text{or} \quad v = \frac{Ze^2}{2\varepsilon_0 hn} \qquad \text{... (ii)}$$

Now, $$K = \frac{1}{2}mv^2 = \frac{mZ^2e^4}{8\varepsilon_0^2 h^2 n^2}$$

Potential energy of the atom

$$U = \frac{Ze^2}{4\pi\varepsilon_0 r}$$

$$r = \frac{Ze^2}{4\pi\varepsilon_0 r\,\frac{(Ze^2)^2}{(2\varepsilon_0 hn)^2}} = \frac{4\pi\varepsilon_0^2 h^2 n^2}{(4\pi\varepsilon_0)m\,Ze^2} = \frac{\varepsilon_0 h^2 n^2}{\pi\,m\,Ze^2}$$

Using value of r, in Eq. (ii)

$$U = \frac{-Ze^2}{4\pi\varepsilon_0\left(\frac{\varepsilon_0 h^2 n^2}{\pi\,m\,Ze^2}\right)} = \frac{-Z^2e^4 m}{4\,\varepsilon_0 h^2 n^2}$$

So, the total energy,

$$E = K + U = +\frac{mZ^2e^4}{8\,\varepsilon_0^2 h^2 n^2} - \frac{mZ^2e^4}{4\,\varepsilon_0^2 h^2 n^2} = -\frac{mZ^2e^4}{8\,\varepsilon_0^2 h^2 n^2}$$

S39. Given: $h = 6.625 \times 10^{-34}$ Js; $e = 1.6 \times 10^{-19}$ C; $m = 9.1 \times 10^{-31}$ kg; $c = 3 \times 10^8$ ms^{-1} and $\varepsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1}

The radius of the n^{th} orbit of hydrogen atom is given by

$$r_n = 4\pi\varepsilon_0 \cdot \frac{n^2h^2}{4\pi^2 me^2}$$

$\therefore$ $$r_3 = 4\pi \times 8.85 \times 10^{-12} \times \frac{3^2 \times (6.625 \times 10^{-34})^2}{4\pi^2 \times 9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2}$$

$= 4.777 \times 10^{-10}$ m = **4.777 Å**.

Energy of the electron in n^{th} Bohr's orbit is given by

$$E_n = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \cdot \frac{2\pi^2 me^4}{n^2h^2}$$

$\therefore \quad E_3 = -\left(\frac{1}{4\pi \times 8.85 \times 10^{-12}}\right)^2 \times \frac{2\pi^2 \times 9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4}{3^2 \times (6.625 \times 10^{-34})^2}$

$= -2.41 \times 10^{-19}$

$= \frac{2.41 \times 10^{-19}}{1.6 \times 10^{-19}} = \mathbf{-1.51\,eV}.$

S40. **Postulates of Bohr's atom model:**

(a) In a hydrogen atom, the negatively charged electron revolves in a circular orbit around the heavy positively charged nucleus. The centripetal force required by the electron is provided by the attractive force exerted by the nucleus on it

(b) The electron can revolve round the nucleus only in those circular orbits in which the angular momentum of an electron is integral multiple of $h/2\pi$, where h is Planck's constant ($= 6.62 \times 10^{-34}$ J s).

If m and v are mass and velocity of the electron in a permitted orbit of radius r, then

$$m v r = n \frac{h}{2\pi},$$

where n is called the principal quantum number and it has the integral values 1, 2, 3, ...

S41. (a) The kinetic energy (E_k) of the electron in an orbit is equal to negative of its total energy (E)

$$E_k = -E$$
$$= -(-1.5) = 1.5 \text{ eV}$$

(b) The potential energy (E_p) of the electron in an orbit is equal to twice its total energy (E).

i.e., $E_p = -2E$

$= -1.5 \times 2 = -3$ eV

(c) As a result of transition of electron from excited state to ground state. Energy of radiation = 1.5 – (–13.6) [$\because$ Ground state energy of H-atom = –13.6 eV]

$\Rightarrow \quad \frac{hc}{\lambda} = 12.1$ eV = energy of radiation

$\therefore \quad \lambda = \frac{12.1 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34} \times 3 \times 10^{8}}$

$\Rightarrow \quad \lambda = 1.025 \times 10^{-7}$ m = 1025 Å

S42. (a) As, radius of electron's n^{th} orbit in hydrogen atom

$$r_n = \frac{\varepsilon_0 h^2}{\pi m e^2} n^2$$

$$\Rightarrow \qquad r_n \propto n^2$$

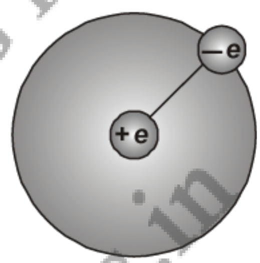
(b) Also, the total energy of an electron belonging to n^{th} orbital's,

$$E_n = \frac{me^2}{8\varepsilon_0^2 n^2 h^2}$$

$$\Rightarrow \qquad |E_n| \propto \frac{1}{n^2}$$

i.e., total energy of electron increases as $\frac{1}{n^2}$.

S43. In a hydrogen atom, an electron having charge $-e$ revolves round the nucleus having charge $+e$ in a circular orbit of radius r as shown in figure below.



The electrostatic force of attraction between the nucleus and the electron is given by

$$F_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e \times e}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2}. \qquad \text{... (i)}$$

If m and v are mass and orbital velocity of the electron, then the centripetal force required by the electron to move in circular orbit or radius r is given by

$$F_c = \frac{mv^2}{r}. \qquad \text{... (ii)}$$

The electrostatic force of attraction (F_e) between the electron and the nucleus provides the necessary centripetal force (F_c) to the electron.

Hence, from the Eqns. (i) and (ii), we get

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2}$$

or

$$m v^2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r} \qquad \text{... (iii)}$$

According to Bohr's quantisation condition, angular momentum of the electron,

$$m v r = n\frac{h}{2\pi} \quad \text{or} \quad v = \frac{nh}{4\pi m r} \qquad \text{... (iv)}$$

Putting the value of v in the Eqn. (iv), we get

$$m\left(\frac{nh}{4\pi mr}\right)^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r}$$

or $$r = 4\pi\varepsilon_0 \cdot \frac{n^2h^2}{4\pi^2me^2} \qquad . \text{(v)}$$

Since n = 1, 2, 3, 4, ..., from the Eqn. (v), it follows that the radii of the stationary orbits are proportional to n^2 and the radii increase in the ratio 1 : 4 : 9 : 16, ..., from the first orbit.

In the Eqn. (iv), substituting the value of r, we get

$$v = \frac{nh}{2\pi m}\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{4\pi^2me^2}{n^2h^2}\right)$$

or $$v = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\pi e^2}{nh} \qquad \text{... (vi)}$$

Hence, Eqn. (vi) gives velocity of electron in the n^{th} orbit.

S44. **Energy of electron:** Let E_k and E_p be respectively the kinetic and the potential energies of the electron in the n^{th} orbit.

Now, $$E_k = \frac{1}{2}mv^2$$

We know, $$E_k = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{2r}$$

The electrostatic potential energy of the electron (having charge $-e$) revolving in a circular orbit of radius r round the nucleus (having charge $+e$) is given by

$$E_p = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(+e)(-e)}{r} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r}$$

The total energy of electron revolving round the nucleus in the orbit of radius r is given by

$$E = E_k + E_p = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{2r} + \left(-\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r}\right)$$

or $$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(+e)(-e)}{2r} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{2r}$$

It may be pointed out that in the above equation, r represents the radius of the n^{th} orbit. Hence, the above equation is the expression for the energy of the electron in the n^{th} orbit. We have

$$E_n = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{2}\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{4\pi^2me^2}{n^2h^2}\right)$$

or $$E_n = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \cdot \frac{2\pi^2 me^4}{n^2h^2} = -\frac{21.76\times10^{-19}}{n^2} J = -\frac{21.76\times10^{-19}}{1.6\times10^{-19}\times n^2} = -\frac{13.6}{n^2}\text{ eV}$$

For $n = 1$, $E_1 = -\frac{13.6}{1^2} = -13.6$ eV

It is called ***ground state energy*** of the hydrogen atom.

For $n = 2$, $E_2 = -\frac{13.6}{2^2} = -3.4$ eV (Energy of first excited state)

For $n = 3$, $E_3 = -\frac{13.6}{3^2} = -1.51$ eV (Energy of second excited state)

For $n = 4$, $E_4 = -\frac{13.6}{4^2} = -0.85$ eV (Energy of third excited state)

For $n = 5$, $E_5 = -\frac{13.6}{5^2} = -0.54$ eV (Energy of fourth excited state)

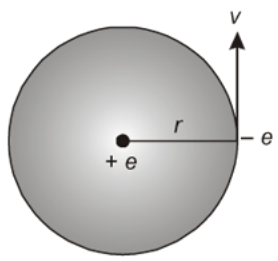
....

....

and for $n = \infty$ $E_\infty = -\frac{13.6}{\infty^2} = \mathbf{0}$.

Here, we find as n increases, the energy associated with a state becomes less negative and approaches closer and closer to the maximum value zero corresponding to $n = \infty$. It may be pointed out that for very large values are so close values of the principal quantum number n, the energy values are so close that they form an ***energy continuum***.

S45. (a) (i) The hydrogen atom consists of a nucleus having a charge $+e$ around which an electron of mass m is revolving in a circle of radius r. Since the electrostatic force supplies the necessary centripetal force supplies the necessary centripetal force.



$$\therefore \quad \frac{mv^2}{r} = k\frac{e^2}{r^2}, \quad \text{where} \quad k = \frac{1}{4\pi\varepsilon_0}$$

$$\therefore \quad r = \frac{ke^2}{mv^2}. \quad \text{... (i)}$$

Also, from Bohr's second postulate angular momentum is quantized.

$$\therefore \quad mvr = n\frac{h}{2\pi}, \quad \text{where} \quad n = 1,2,3,.....$$

$$\therefore \quad v = \frac{nh}{2\pi mr}. \quad \text{... (ii)}$$

Putting this value of v in Eqn. (i), we have

$$r = \frac{ke^2}{mn^2h^2}4\pi^2m^2r^2$$

which gives $r_n = \dfrac{n^2h^2}{4\pi^2 mke^2}$ ... (iii)

Clearly, $R \propto n^2$.

(ii) Total energy E_n of the electron in the n^{th} orbit is

$$E_n = E_k + E_p = \frac{1}{2}mv^2 - k\frac{e \times e}{r} = \frac{1}{2}\frac{ke^2}{r} - \frac{ke^2}{r} \qquad \left(\because \ mv^2 = \frac{ke^2}{r}\right)$$

$$E_n = -\frac{ke^2}{2r_n} = -\frac{ke^2}{2n^2h^2} \times 4\pi^2 mke^2$$

$$E_n = \frac{2\pi^2 mk^2e^4}{n^2h^2}$$

Clearly, $E \propto 1/n^2$.

(b) We know that wave number $\bar{v}$ is given by

$$\bar{v} = \frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Where R is called the Rydberg's constant.

For H_2 line in Balmer series

$$n_1 = 2, \quad n_2 = 3$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \times \left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \times \frac{5}{36}$$

$$\lambda = \frac{36}{1.097 \times 5} \times 10^{-7}\,\text{m} = \frac{36 \times 1000}{1.097 \times 5} \times 10^{-10}\,\text{m} = \mathbf{6563.4\,\text{Å}}.$$