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Q1. Write the position vector of mid-point of the vector joining points A (3, 4, – 2) and B (1, 2, 4).

Q2. If A, B and C are the vertices of a $\triangle ABC$, then what is the value of $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$?

Q3. Find the scalar components of $\overrightarrow{AB}$ with initial point $\vec{A}$ (2, 1) and terminal point $\vec{B}$ (–5, 7).

Q4. If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ are two equal vectors, then write the value of $x + y + z$.

Q5. Find a unit vector in the direction of $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$.

Q6. Find a unit vector in the direction of vector $\vec{b} = 6\hat{i} - 2\hat{j} + 3\hat{k}$.

Q7. Find a unit vector in the direction of $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$.

Q8. Find a unit vector in the direction of vector $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$.

Q9. Write a unit vector in the direction of the sum of vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 3\hat{k}$.

Q10. Find sum of vectors $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$.

Q11. Find the magnitude of the vector $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$.

Q12. Find $|\vec{x}|$ if $\vec{a}$ is unit vector such that $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 15$.

Q13. Write the direction cosines of vector $-2\hat{i} + \hat{j} - 5\hat{k}$.

Q14. Find a vector in the direction of $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ which has magnitude 6 units.

Q15. Write a vector of magnitude 15 units in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$.

Q16. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ be the position vectors of points A, B, C, D respectively and $\vec{b} - \vec{a} = 2(\vec{d} - \vec{c})$, show that the point of intersection of straight lines AD and BC divide these lines in the ratio 2 : 1.

Q17. Write a unit vector in the direction of $\vec{b} = 2\hat{i} + \hat{j} + 2\hat{k}$.

Q18. P and Q are two points with position vectors $3\vec{a} - 2\vec{b}$ and $\vec{a} + \vec{b}$, respectively. Write the position vector of a point R which divides the line segment PQ in the ratio 2:1 externally.

Q19. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a} + \vec{b}| = |\vec{a}|$, then prove that vector $2\vec{a} + \vec{b}$ is perpendicular to vector $\vec{b}$.

Q20. L and M are two points with position vectors $2\vec{a} - \vec{b}$ and $\vec{a} + 2\vec{b}$, respectively. Write the position vector of a point N which divides the line segment LM in the ratio 2:1 externally.

Q21. Write the position vector of mid-point of the vector joining points P (2, 3, 4) and Q (4, 1, –2).

Q22. $\vec{A}$ and $\vec{B}$ are two points with position vectors $2\vec{a}-3\vec{b}$ and $6\vec{b}-\vec{a}$, respectively. Write the position vector of a point $\vec{P}$ which divides the line segment $\overrightarrow{AB}$ internally in the ratio 1 : 2.

Q23. Write a unit vector in the direction of $\vec{a}=2\hat{i}-6\hat{j}+3\hat{k}$.

Q24. Find a unit vector in the direction of vector $\vec{a}=-2\hat{i}+\hat{j}+2\hat{k}$.

Q25. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}, \vec{b}=4\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$. Find a vector of magnitude 6 units which is parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$.

Q26. Find a vector of magnitude 5 units and parallel to the resultant of $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$.

Q27. Find the position vector of a point $\vec{R}$ which divides the line joining two points $\vec{P}$ and $\vec{Q}$ whose position vectors are $2\vec{a}+\vec{b}$ and $\vec{a}-3\vec{b}$ respectively, externally in the ratio 1:2. Also show that $\vec{P}$ is the mid-point of line segment $\overrightarrow{RQ}$.

Q28. The position vectors of the points, *P, Q, R* and *S* are respectively, $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+5\hat{j}$, $3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$, prove that the lines *PQ* and *RS* are parallel and the ratio of their length is $\frac{1}{2}$.

Q29. The two adjacent sides of a parallelogram are $2\hat{i}+4\hat{j}-5\hat{k}$ and $\hat{i}+2\hat{j}+3\hat{k}$. Find the unit vectors along the diagonals of the parallelogram.

Q30. A triangle has vertices (1, 2, 4), (– 2, 2, 1) and (2, 4, – 3), prove that the triangle is right-angled and find its other angles.

Q31. If the point *A* (2, β, 3), *B* (α, – 5, 1) and *C* (– 1, 11, 9) are collinear, find the values of α and β by vector method.

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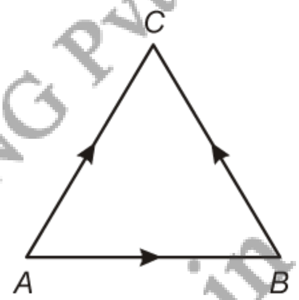
S1. Position vector of mid-point of line segment AB, where A (3, 4, –2) and B (1, 2, 4) is given as follows.

Let $\overrightarrow{OA} = 3\hat{i} + 4\hat{j} - 2\hat{k}$ and $\overrightarrow{OB} = \hat{i} + 2\hat{j} + 4\hat{k}$

$$\therefore \text{ Position vector of mid-point} = \frac{OA + OB}{2}$$
$$= \frac{(3\hat{i} + 4\hat{j} - 2\hat{k}) + (\hat{i} + 2\hat{j} + 4\hat{k})}{2}$$
$$= \frac{4\hat{i} + 6\hat{j} + 2\hat{k}}{2} = 2\hat{i} + 3\hat{j} + \hat{k}$$

S2. Let ΔABC be the given triangle.



Now, by triangle law of vector addition, we get

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$
$$\Rightarrow \quad \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{CA} + \overrightarrow{AC}$$
$$\Rightarrow \quad \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{CA} - \overrightarrow{CA} \qquad [\because \overrightarrow{AC} = -\overrightarrow{CA}]$$
$$\therefore \quad \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0$$

S3. $\because$ Scalar components of $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ = Position vector of $\vec{B} -$ Position vector of $\vec{A}$

Given, points $\vec{A}$ (2, 1) and $\vec{B}$ (–5, 7) scalar component of $\overrightarrow{AB}$ are $x_2 - x_1$ and $y_2 - y_1$.

$$= -5 - 2 = -7 \text{ and } 7 - 1 = 6.$$

S4. Two vectors are equal, if its coefficient of the components are equal.

Given that,

$$\vec{a} = \vec{b}$$
$$\Rightarrow \quad x\hat{i} + 2\hat{j} - z\hat{k} = 3\hat{i} - y\hat{j} + \hat{k}$$
$$\therefore \quad x = 3, \quad y = -2, \quad z = -1$$
$$\therefore \quad x + y + z = 3 - 2 - 1 = 0$$

S5. Unit vector in the direction of $\vec{a}$ is $\hat{a}$ given by $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$

$$= \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{(1)^2 + (1)^2 + (2)^2}} = \frac{\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{6}}$$

$$= \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$$

S6. Unit vector in direction of $\vec{b} = \frac{\vec{b}}{|\vec{b}|}$

$$= \frac{6\hat{i} - 2\hat{j} + 3\hat{k}}{\sqrt{(6)^2 + (-2)^2 + (3)^2}} = \frac{6\hat{i} - 2\hat{j} + 3\hat{k}}{\sqrt{49}}$$

$$= \frac{6}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{3}{7}\hat{k}$$

S7. Unit vector in the direction of $\vec{a} = \frac{\vec{a}}{|\vec{a}|}$

$$= \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{(2)^2 + (-3)^2 + (6)^2}} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{49}}$$

$$= \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$$

S8. Unit vector in the direction of $\vec{a} = \frac{\vec{a}}{|\vec{a}|}$

$$= \frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{\sqrt{(2)^2 + (3)^2 + (6)^2}}$$

$$= \frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{\sqrt{49}}$$

$$= \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$$

S9. Let $\vec{c} = \vec{a} + \vec{b} = (2\hat{i} - \hat{j} + 2\hat{k}) + (-\hat{i} + \hat{j} + 3\hat{k}) = \hat{i} + 0\hat{j} + 5\hat{k}$

Now, $|\vec{c}| = |\vec{a} + \vec{b}| = \sqrt{1^2 + 5^2} = \sqrt{26}$

$\therefore \quad \hat{c} = \frac{(\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} \qquad \left[\because \hat{a} = \frac{\vec{a}}{|\vec{a}|}\right]$

$$= \frac{1}{\sqrt{26}}\hat{i} + \frac{5}{\sqrt{26}}\hat{k}$$

S10. Sum of the vectors *a*, *b* and *c* is

$$\vec{a} + \vec{b} + \vec{c} = \left(\hat{i} - 2\hat{j} + \hat{k}\right) + \left(-2\hat{i} + 4\hat{j} + 5\hat{k}\right) + \left(\hat{i} - 6\hat{j} - 7\hat{k}\right)$$

$$= -4\hat{j} - \hat{k}$$

S11. Magnitude of a vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is $|r| = \sqrt{x^2 + y^2 + z^2}$.

Magnitude of $\vec{a} = |\vec{a}| = \sqrt{(3)^2 + (-2)^2 + (6)^2}$

$$= \sqrt{9 + 4 + 36} = \sqrt{49} = 7.$$

S12. Given that, $\vec{a}$ is a unit vector.

Then, $|\vec{\boldsymbol{a}}| = 1$

Now, we have

$$(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 15$$

$\Rightarrow \quad \vec{x} \times \vec{x} - \vec{a} \times \vec{x} + \vec{x} \times \vec{a} - \vec{a} \times \vec{a} = 15$ [$\because$ Scalar product is commutative]

$\Rightarrow \quad |\vec{x}|^2 - |\vec{a}|^2 = 15$ $\left[\because z \cdot z = |z|^2\right]$

$\Rightarrow \quad |\vec{x}|^2 - 1 = 15$

$\Rightarrow \quad |\vec{x}|^2 = 16$

$\Rightarrow \quad |\vec{x}| = 4$

S13. Direction cosines of the vector $a\hat{i} + b\hat{j} + c\hat{k}$ is

$$\frac{a}{\sqrt{a^2 + b^2 + c^2}}, \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

Let $\vec{a} = -2\hat{i} + \hat{j} - 5\hat{k}$

$\therefore$ Direction cosines of *a* are

$$\frac{-2}{\sqrt{(-2)^2 + (1)^2 + (-5)^2}}, \frac{1}{\sqrt{(-2)^2 + (1)^2 + (-5)^2}} \text{ and } \frac{-5}{\sqrt{(-2)^2 + (1)^2 + (-5)^2}}$$

$$= \frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$$

S14. Firstly, we find a unit vector in the direction of $\vec{a} = \frac{\vec{a}}{|\vec{a}|}$

$$= \frac{2\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{(2)^2 + (-1)^2 + (2)^2}}$$

$$= \frac{2\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{9}}$$

$$= \frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$$

Now, vector of magnitude 6 units $= 6\left[\frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}\right]$

$$= 4\hat{i} - 2\hat{j} + 4\hat{k}$$

S15. Let $a = \hat{i} - 2\hat{j} + 2\hat{k}$

Unit vector in the direction of $\vec{a} = \frac{\vec{a}}{|\vec{a}|}$

$$= \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{(1)^2 + (-2)^2 + (2)^2}} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{9}}$$

$$= \left(\frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}\right)$$

$\therefore$ Vector of magnitude 15 units $= 15\left(\frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}\right) = 5\hat{i} - 10\hat{j} + 10\hat{k}$

S16. Let P and Q be the points which divide AD and BC respectively in the ratio 2 : 1.

Let the position vector of P and Q be $\vec{\alpha}$ and $\vec{\beta}$, respectively, then

$$\vec{\alpha} = \frac{2\vec{d} + \vec{a}}{2+1} = \frac{2\vec{d} + \vec{a}}{3} \quad \text{... (i)}$$

and $$\vec{\beta} = \frac{2\vec{c} + \vec{b}}{2+1} = \frac{2\vec{c} + \vec{b}}{3} \quad \text{... (ii)}$$

Given, $\vec{b} - \vec{a} = 2(\vec{d} - \vec{c}) \quad \therefore \vec{b} = \vec{a} + 2\vec{d} - 2\vec{c}$ ... (iii)

Putting the value of $\vec{b}$ in (ii), we get

$$\vec{\beta} = \frac{2\vec{c} + (\vec{a} + 2\vec{d} - 2\vec{c})}{3} = \frac{2\vec{d} + \vec{a}}{3} = \vec{\alpha} \qquad \text{[From (iii)]}$$

Hence P and Q are the same points and point of intersection of AD and BC divides these lines in the ratio 2 : 1.

S17. Unit vector in the direction of $\vec{b} = \frac{\vec{b}}{|\vec{b}|}$

$$= \frac{2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{(2)^2 + (1)^2 + (2)^2}}$$

$$= \frac{2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{9}} = \frac{2\hat{i} + \hat{j} + 2\hat{k}}{3}$$

$$= \frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$$

S18. Given that, P and Q are two points with position vectors $3\vec{a} - 2\vec{b}$ and $\vec{a} + \vec{b}$ respectively. Also, a point R which divides the line segment PQ in the ratio 2 : 1 externally. [By section formula]

$\therefore$ Position vector of a point $R = \frac{2 \times (\vec{a} + \vec{b}) - 1 \times (3\vec{a} - 2\vec{b})}{2 - 1}$

$$= \frac{2\vec{a} + 2\vec{b} - 3\vec{a} + 2\vec{b}}{2 - 1}$$

$$= \frac{-\vec{a} + 4\vec{b}}{1} = -\vec{a} + 4\vec{b}$$

S19. To prove $(2\vec{a} + \vec{b}) \perp \vec{b}$

Given $|\vec{a} + \vec{b}| = |\vec{a}|$

Squaring on both sides, we get

$$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2$$

$\Rightarrow \quad |\vec{a}|^2 + 2\vec{a}.\vec{b} + |\vec{b}|^2 = |\vec{a}|^2$

$\Rightarrow \quad 2\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} = 0 \qquad \left[\because |\vec{x}|^2 = \vec{x} \cdot \vec{x}\right]$

$\Rightarrow \quad \vec{b} \cdot (2\vec{a} + \vec{b}) = 0$

$\Rightarrow \quad (2\vec{a} + \vec{b}) \perp \vec{b} \qquad [\text{If } \vec{a} \perp \vec{b} \Leftrightarrow \vec{a} \cdot \vec{b} = 0]$

Hence proved.

S20. Given that, L and M are two points with position vectors $2\vec{a} - \vec{b}$ and $\vec{a} + 2\vec{b}$, respectively. Also a point N divides the line segment LM in the ratio 2:1 externally.

$$\therefore \quad \text{Position vector of a point } N = \frac{2 \times (\vec{a} + 2\vec{b}) - 1 \times (2\vec{a} - \vec{b})}{2-1}$$

$$= \frac{2\vec{a} + 4\vec{b} - 2\vec{a} + \vec{b}}{1} = 5\vec{b}$$

S21. Mid-point of the position vectors

$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $b = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is $\dfrac{\vec{a} + \vec{b}}{2}$ or $\dfrac{(a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k}}{2}$

Let Position vector of mid-point of a vector joining points $P(2, 3, 4)$ and $Q(4, 1, -2)$ is R

$$\overrightarrow{OR} = \frac{\overrightarrow{OP} + \overrightarrow{OQ}}{2} \rightarrow = \frac{\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) + \left(4\hat{i} + \hat{j} - 2\hat{k}\right)}{2}$$

$$\therefore \qquad \overrightarrow{OR} = \frac{6\hat{i} + 4\hat{j} + 2\hat{k}}{2} = 3\hat{i} + 2\hat{j} + \hat{k}$$

S22. Given that, $\vec{A}$ and $\vec{B}$ are two points with position vectors $2\vec{a} - 3\vec{b}$ and $6\vec{b} - \vec{a}$, respectively. Also, a point $\vec{P}$ which divides the line segment $\overrightarrow{AB}$ in the ratio 1:2. [Now by section formula]

$$\therefore \quad \text{Position vector of a point } P = \frac{1 \times (6\vec{b} - \vec{a}) + 2 \times (2\vec{a} - 3\vec{b})}{1+2}$$

$$= \frac{6\vec{b} - \vec{a} + 4\vec{a} - 6\vec{b}}{3}$$

$$= \frac{3\vec{a}}{3} = \vec{a}$$

S23. Unit vector in the direction of $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|}$

$$= \frac{2\hat{i} - 6\hat{j} + 3\hat{k}}{\sqrt{(2)^2 + (-6)^2 + (3)^2}} = \frac{2\hat{i} - 6\hat{j} + 3\hat{k}}{\sqrt{49}}$$

$$= \frac{2}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{3}{7}\hat{k}$$

S24. Unit vector in the direction of $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|}$

$$= \frac{-2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{(-2)^2 + (1)^2 + (2)^2}}$$

$$= \frac{-2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{9}} = \frac{-2\hat{i} + \hat{j} + 2\hat{k}}{3}$$

$$= -\frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$$

S25. Firstly, find the vector $2\vec{a} - \vec{b} + 3\vec{c}$, then find the unit vector in the direction of $2\vec{a} - \vec{b} + 3\vec{c}$

Given that, $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$

Firstly, we find the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

$\therefore \quad 2\vec{a} - \vec{b} + 3\vec{c} = 2(\hat{i} + \hat{j} + \hat{k}) - (4\hat{i} - 2\hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k})$

$$= 2\hat{i} + 2\hat{j} + 2\hat{k} - 4\hat{i} + 2\hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}$$

$\therefore \quad 2\vec{a} - \vec{b} + 3\vec{c} = \hat{i} - 2\hat{j} + 2\hat{k}$

Now, we find a unit vector in the direction of vector $2\vec{a} - \vec{b} + 3\vec{c}$ which is equal to $\dfrac{2\vec{a} - \vec{b} + 3\vec{c}}{|2\vec{a} - \vec{b} + 3\vec{c}|}$.

$$= \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{(1)^2 + (-2)^2 + (2)^2}} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{9}}$$

$$= \frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$$

$\therefore$ Vector of magnitude 6 units parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$ is $= 6\left(\frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}\right)$

$$= 2\hat{i} - 4\hat{j} + 4\hat{k}$$

S26. First find resultant of the vectors $\vec{a}$ and $\vec{b}$ which is $\vec{a} + \vec{b}$. Then find a unit vector in the direction of $\vec{a} + \vec{b}$

Given that, $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Now, resultant of above vectors $= \vec{a} + \vec{b}$

$$= (2\hat{i} + 3\hat{j} - \hat{k}) + (\hat{i} - 2\hat{j} + \hat{k})$$

$$= 3\hat{i} + \hat{j}$$

Let $\vec{a} + \vec{b} = \vec{c}$

$$\therefore \qquad \vec{c} = 3\hat{i} + \hat{j}$$

Now, we find unit vector in the direction of $\vec{c}$

$$= \frac{\vec{c}}{|\vec{c}|} = \frac{3\hat{i} + \hat{j}}{\sqrt{(3)^2 + (1)^2}} = \frac{3\hat{i} + \hat{j}}{\sqrt{10}}$$

$$= \frac{3}{\sqrt{10}}\hat{i} + \frac{1}{\sqrt{10}}\hat{j}$$

Finally, vector of magnitude 5 units and parallel to resultant of $\vec{a}$ and $\vec{b}$ is given by

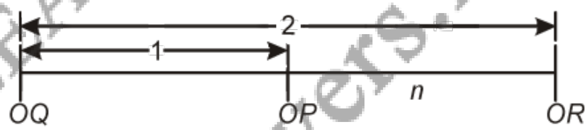
$$\Rightarrow \qquad 5\left(\frac{3}{\sqrt{10}}\hat{i} + \frac{1}{\sqrt{10}}\hat{j}\right) = \frac{15}{\sqrt{10}}\hat{i} + \frac{5}{\sqrt{10}}\hat{j}$$

S27. Given that, $\overrightarrow{OP}$ = Position vector of $\vec{P} = 2\vec{a} + \vec{b}$, $\overrightarrow{OQ}$ = Position vector of $\vec{Q} = \vec{a} - 3\vec{b}$

To find $\overrightarrow{OR}$ = Position vector of point $\vec{R}$ which divides $\overrightarrow{PQ}$ in ratio 1:2.

We know that, position vector of point $\vec{R}$ which divides line $\overrightarrow{PQ}$ externally in the ratio 1 : 2 is given by

$$\overrightarrow{OR} = \frac{m(\overrightarrow{OQ}) - n(\overrightarrow{OP})}{m - n} \qquad \ldots \text{(i)}$$



where, $\overrightarrow{OP}$ = Position vector of point P and $\overrightarrow{OQ}$ = Position vector of point $\vec{Q}$ given that $\overrightarrow{OP} = 2\vec{a} + \vec{b}$ and $\overrightarrow{OQ} = \vec{a} - 3\vec{b}$ and $m = 1$, $n = 2$.

Putting above values in eq. (i), we get

$$\overrightarrow{OR} = \frac{1(\vec{a} - 3\vec{b}) - 2(2\vec{a} + \vec{b})}{1 - 2} = \frac{\vec{a} - 3\vec{b} - 4\vec{a} - 2\vec{b}}{-1} = \frac{-3\vec{a} - 5\vec{b}}{-1} = 3\vec{a} + 5\vec{b}$$

Hence, $\qquad \overrightarrow{OR} = 3\vec{a} + 5\vec{b}$

Also, we have to show that $\vec{P}$ is the mid-point of $\overrightarrow{RQ}$.

$\therefore$ We have, to show that $\overrightarrow{OP} = \dfrac{\overrightarrow{OR} + \overrightarrow{OQ}}{2}$

We have, $\qquad \overrightarrow{OR} = 3\vec{a} + 5\vec{b}, \overrightarrow{OQ} = \vec{a} - 3\vec{b}$

$$\therefore \qquad \frac{\overrightarrow{OR} + \overrightarrow{OQ}}{2} = \frac{(3\vec{a} + 5\vec{b}) + (\vec{a} - 3\vec{b})}{2}$$

$$= \frac{4\vec{a}+2\vec{b}}{2} = \frac{2(2\vec{a}+\vec{b})}{2}$$

$$= 2\vec{a} + \vec{b} = \overrightarrow{OP} \qquad [\because \text{ Given } \overrightarrow{OP} = 2\vec{a} + \vec{b}]$$

Hence, we have shown that

$$\frac{\overrightarrow{OR} + \overrightarrow{OQ}}{2} = \overrightarrow{OP}$$

$\therefore$ P is the mid-point of line segment R.

S28. Let O *be* the origin.

$$\overrightarrow{OP} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{OQ} = 2\hat{i} + 5\hat{j}$$

$$\overrightarrow{OR} = 3\hat{i} + 2\hat{j} - 3\hat{k} \text{ and } \overrightarrow{OS} = \hat{i} - 6\hat{j} - \hat{k}$$

Now $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = (2\hat{i} + 5\hat{j}) - (i + \hat{j} + \hat{k}) = \hat{i} + 4\hat{j} - \hat{k}$

$$\overrightarrow{RS} = \overrightarrow{OS} - \overrightarrow{OR}$$

$$= (\hat{i} - 6\hat{j} - \hat{k}) - (3\hat{i} + 2\hat{j} - 3\hat{k})$$

$$= -2\hat{i} - 8\hat{j} + 2\hat{k} = -2(\hat{i} + 4\hat{j} - \hat{k})$$

Thus, $\overrightarrow{RS} = 2\overrightarrow{PQ}$ $\therefore PQ \parallel RS$ and

$$|\overrightarrow{RS}| = |-2\overrightarrow{PQ}| = 2|\overrightarrow{PQ}|$$

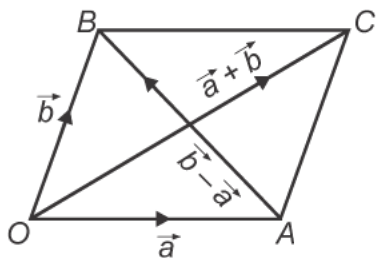
$\therefore$
$$\frac{|\overrightarrow{PQ}|}{|\overrightarrow{RS}|} = \frac{1}{2}$$

Hence ratio of length PQ and RS is $\frac{1}{2}$

S29. Let two adjacent sides OA, OB of parallelogram $OACB$ be represented by $\vec{a}$ and $\vec{b}$ respectively.

Then $\vec{a} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$



The vectors along the two diagonals are

$$\vec{d}_1 = \vec{a} + \vec{b} = 3\hat{i} + 6\hat{j} - 2\hat{k}$$

and $\vec{d}_2 = \vec{b} - \vec{a} = -\hat{i} - 2\hat{j} + 8\hat{k}$

The required unit vectors are

$$\hat{n}_1 = \frac{\vec{d}_1}{|\vec{d}_1|} = \frac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(3)^2+(6)^2+(-2)^2}} = \frac{3\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{9+36+4}}$$

$$= \frac{3}{7}\hat{i}+\frac{6}{7}\hat{j}-\frac{2}{7}\hat{k}$$

$$\hat{n}_2 = \frac{\vec{d}_2}{|\vec{d}_2|} = \frac{-\hat{i}-2\hat{j}+8\hat{k}}{\sqrt{(-1)^2+(-2)^2+(8)^2}} = \frac{-\hat{i}-2\hat{j}+8\hat{k}}{\sqrt{1+4+64}}$$

$$= -\frac{1}{\sqrt{69}}\hat{i}-\frac{2}{\sqrt{69}}\hat{j}+\frac{8}{\sqrt{69}}\hat{k}$$

S30. Let the vertices (1, 2, 4), (– 2, 2, 1) and (2, 4, – 3) of the triangle be *A, B, C* respectively and *O* be the origin.

Then $\overrightarrow{OA} = \hat{i}+2\hat{j}+4\hat{k},\ \overrightarrow{OB} = -2\hat{i}+2\hat{j}+\hat{k}$

and $\overrightarrow{OC} = 2\hat{i}+4\hat{j}-3\hat{k}$

Now $\overrightarrow{AB} = \overrightarrow{OB}-\overrightarrow{OA} = (-2\hat{i}+2\hat{j}+\hat{k})-(\hat{i}+2\hat{j}+4\hat{k}) = -3\hat{i}+0\hat{j}-3\hat{k} = -3\hat{i}-3\hat{k}$

$\overrightarrow{BC} = \overrightarrow{OC}-\overrightarrow{OB} = (2\hat{i}+4\hat{j}-3\hat{k})-(-2\hat{i}+2\hat{j}+\hat{k}) = 4\hat{i}+2\hat{j}-4\hat{k}$

$\overrightarrow{AC} = \overrightarrow{OC}-\overrightarrow{OA} = (2\hat{i}+4\hat{j}-3\hat{k})-(\hat{i}+2\hat{j}+4\hat{k}) = \hat{i}+2\hat{j}-7\hat{k}$

Now $AB = |\overrightarrow{AB}| = \sqrt{(-3)^2+0^2+(-3)^2} = \sqrt{9+0+9} = \sqrt{18} = 3\sqrt{2}$

$BC = |\overrightarrow{BC}| = \sqrt{4^2+2^2+(-4)^2} = \sqrt{16+4+16} = \sqrt{36} = 6$

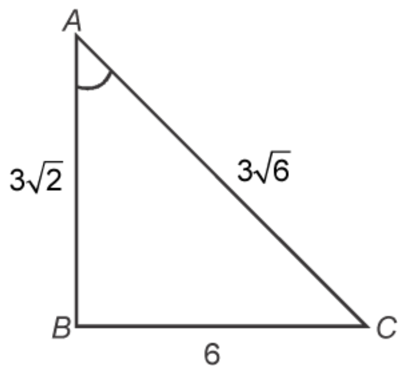
$AC = |\overrightarrow{AC}| = \sqrt{1^2+2^2+(-7)^2} = \sqrt{1+4+49} = \sqrt{54} = 3\sqrt{6}$

Thus $AB^2 = 18,\ BC^2 = 36,\ AC^2 = 54$

$\therefore$ $AC^2 = AB^2 + BC^2$

$\therefore$ $\angle B = 90^\circ$ and *AC* is the hypotenuse

Now, from ΔABC, $\sin A = \frac{BC}{AC} = \frac{6}{3\sqrt{6}} = \sqrt{\frac{2}{3}}$



$\therefore$ $A = \sin^{-1}\sqrt{\frac{2}{3}}$

and $\sin C = \frac{AB}{AC} = \frac{3\sqrt{2}}{3\sqrt{6}} = \frac{1}{\sqrt{3}}$

$\therefore$ $C = \sin^{-1}\sqrt{\frac{1}{3}}$

S31. Given :

$$\overrightarrow{OA} = 2\hat{i} + \beta\hat{j} + 3\hat{k}$$

$$\overrightarrow{OB} = \alpha\hat{i} - 5\hat{j} + \hat{k}$$

$$\overrightarrow{OC} = -\hat{i} + 11\hat{j} + 9\hat{k}$$

Here O is the origin.

$$\therefore \quad \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (\alpha\hat{i} - 5\hat{j} + \hat{k}) - (2\hat{i} + \beta\hat{j} + 3\hat{k})$$

$$= (\alpha - 2)\hat{i} - (5 + \beta)\hat{j} - 2\hat{k}$$

and

$$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = (-\hat{i} + 11\hat{j} + 9\hat{k}) - (2\hat{i} + \beta\hat{j} + 3\hat{k})$$

$$= -3\hat{i} + (11 - \beta)\hat{j} + 6\hat{k}$$

Given that point A, B and C are collinear

$\Rightarrow$ $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are collinear $\Rightarrow$ $\overrightarrow{AB} = \lambda\overrightarrow{AC}$ for some scalar $\lambda(\neq 0)$

$$\Rightarrow (\alpha - 2)\hat{i} - (5 + \beta)\hat{j} - 2\hat{k} = \lambda[-3\hat{i} + (11 - \beta)\hat{j} + 6\hat{k}]$$

$\therefore$ Equating the coefficient of $\hat{i}$, $\hat{j}$ and $\hat{k}$, we get

$$\alpha - 2 = -3\lambda, \; -5 - \beta = \lambda\,(11 - \beta) \text{ and } -2 = 6\lambda$$

$$\therefore \quad \alpha = 2 - 3\lambda, \; \beta = \frac{11\lambda + 5}{\lambda - 1} \text{ and } \lambda = -\frac{1}{3}$$

$$\Rightarrow \quad \alpha = 2 - 3.\left(-\frac{1}{3}\right) = 3, \; \beta = \frac{11\cdot\left(-\frac{1}{3}\right) + 5}{-\frac{1}{3} - 1} = \frac{-11 + 15}{-4} = -1$$

$$\therefore \quad \alpha = 3 \quad \text{and} \quad \beta = -1.$$