Date: 20/10/2021

Q1. Why is of electric charge not important when dealing with macroscopic charge?

Q2. What is quantization of electric charge?

Q3. Charges reside only on the outer surface of a charged conductor. why?

Q4. Can a body have a charge of 0.8×10^{-19}C?

Q5. A conductor has a cavity in it and is given a charge + Q. what will be the total charge on its surface it another conductor carrying charge + q is placed in the cavity without touching the out conductor?

Q6. What kind of charge are produced on each, when (a) a glass rod is rubbed with silk and (b) an ebonite rod is rubbed with wool?

Q7. What does $q_1 + q_2 = 0$ signify electrostatics?

Q8. Find number of electrons present in – 1 C Charge?

Q9. If $q_1 q_2 > 0$, what is the nature of force between the two charge?

Q10 State two basic properties of electronic charge.

Q11 What is the basic cause of quantisation of charge?

Q12 What is the least possible value of charge?

Q13 When a polythene piece is rubbed with wool, it acquires negative charge. Is there a transfer of mass from wool to polythene?

Q14. Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other.

Is the magnitude of electrostatic force between them exactly given by $Q_1Q_2/4\pi\varepsilon_0 r^2$, where r is the distance between their centres?

Q15. A polythene piece rubbed with wool is found to have a negative charge of 6×10^{-7} C. Find the number of electrons transferred to polythene from wool?

Q16. What do you mean conservation of charge?

Q17. If Coulomb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss's law be still true?

Q18. We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there?

Q19. A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the field line passing through that point?

Q20. If 10^9 electrons move out of a body to another body every second. How much time is required to get a total charge of 1 C on the other body?

Q21. How much positive and negative charge is there in a cup of water?

Q22. A cup contains 250 g of water. Find the number of positive charges present in the cup of water.

Q23. Estimate the number of free electrons in 1 *g* of water and the negative charge possessed by them. Given that Avogadro number = 6.02×10^{23} and molecular weight of water = 18.

Q24. Calculate the total positive or negative charge on a 3.11 g copper penny. Given Avogadro number = 6.02×10^{23}, atomic number of copper = 29 and atomic mass of copper = 63.5.

Q25. When you touch a metal object like door handle on a dry winter day, you are likely to see a spark and feel a definite shock. We usually explain this by saying that we have built up a static charge. Why does this not happen on a humid day in the summer?

Q26. Two identical metallic spherical shells *A* and *B* having charges $+4Q$ and $-10Q$ are kept a certain distance apart. *A* third identical uncharged sphere *C* is first placed in contact with sphere *A* and then with sphere *B*, then spheres *A* and *B* are brought in contact and then separated. Find the charge on the spheres *A* and *B*.

Q27. Two identical metallic spheres exactly equal masses are taken. One is given a positive charge *Q* coulomb and the other an equal negative charge by friction. Are their masses after charging equal?

Q28. Give six properties of electric charge.

Q29. Explain the following terms:

(a) Linear charge density (b) Surface charge density (c) Volume charge density

Q30. A glass rod rubbed with silk is brought close to two unchanged spheres in contact with each other inducing charges on as shown in given figure. Show with diagram in each case, what happens when

(a) The spheres are slightly separated?

(b) The glass rod is subsequently removed?

(c) The spheres are separated far apart?

Date: 20/10/2021

S1. $q = ne$. For macroscopic charges, because e is very small, n become so large that "ne" will be equal to "$(n + 1)e$" even to an accuracy to the thousandth place of decimal.

S2. The quantisation of charge means that all charges must be an integral multiple of some elementary charge *i.e.*,

$$q = ne \qquad \text{where} \quad n = 1, 2, 3, \ldots$$

S3. Because if there is any net charge inside a conductor, it will cause some electric field inside a conductor, which is not possible. This is called skin or surface effect.

S4. No, because given charge is one-half of the charge of an electron. A fraction of e other than quarks is not possible.

S5. $Q + q$. (By induction charge $+ q$ will also come on the surface), because charge are shows additive nature.

S6. (a) On glass rod : Positive charge and
On silk : negative charge

(b) On ebonite rod : negative charge and
On wool : Positive charge.

S7. The charges q_1 and q_2 are equal and opposite.

S8. Here, charge (q) = -1 C

Where charge of electron = 1.6×10^{-19}

From the quantisation of charge,

$$q = \text{ne}$$

or $$n = \frac{q}{e} = \frac{-1}{-(1.6 \times 10^{-19})} = 6 \times 10^{18} \text{ electrons}$$

S9. When $q_1 q_2 > 0$, it implies that both the charge are positive in nature. Hence, the force between the two charges will be repulsive.

S10. (a) Electric charge is quantized.

(b) Electric charge is always additive in nature.

S11. During the charging process, only integral number of electrons is transferred from one body to the other. Therefore, charge possessed by a body is always an integral multiple of $\pm 1.6 \times 10^{-19}$C.

S12. A positively charged particle can posses positive charge equal to the charge on a proton ($+1.6 \times 10^{-19}$ C) and a negatively charged particle equal to the charge on an electron (-1.6×10^{-19} C). A particle cannot have a fractional part of this elementary amount of charge *i.e.*, 1.6×10^{-19}C.

S13. The polythene piece acquires negative charge due to transfer of electrons from wool to it. Since electrons are material particles, there is a transfer of mass from wool to polythene.

S14. The force between two conducting spheres is not exactly given by the expression, $Q_1Q_2/4\pi\varepsilon_0 r^2$, because there is a non-uniform charge distribution on the spheres.

S15. Here, $q = -6 \times 10^{-7}$ C

Charge on one electron, $e = -1.6 \times 10^{-19}$ C

$\therefore$ Number of electrons transferred from wool to polythene,

$$n = \frac{q}{e} = \frac{-6 \times 10^{-7}}{-1.6 \times 10^{-19}} = 3.75 \times 10^{12}.$$

S16. It states that for an isolated system, the net charge always remains constant. In any physical process, the charge may get transferred from one part of the system to another, but net charge will always remain the same.

Or

Charge can neither be created nor destroyed. Its transferred one body to another body.

S17. Gauss's law will not be true, if Coulomb's law involved $1/r^3$ dependence, instead of$1/r^2$, on r.

S18. No.

Electric field is discontinuous across the surface of a charged conductor. However, electric potential is constant.

S19. Yes,

If a small test charge is released at rest at a point in an electrostatic field configuration, then it will travel along the field lines passing through the point, only if the field lines are straight. This is because the field lines give the direction of acceleration and not of velocity.

S20. In one second 10^9 electrons move out of the body. Therefore the charge given out in one second is $1.6 \times 10^{-19} \times 10^9$ C = 1.6×10^{-10} C. The time required to accumulate a charge of 1 C can then be estimated to be 1 C $\div$ (1.6×10^{-10} C/s) = 6.25×10^9 s = $6.25 \times 10^9 \div (365 \times 24 \times 3600$ yers) = 198 years. Thus to collect a charge of one coulomb, from a body from which 10^9 electrons move out every second, we will need approximately 200 years. One coulomb is, therefore, a very large unit for many practical purposes.

S21. Let us assume that the mass of one cup of water is 250 g. The molecular mass of water is 18 g. Thus, one mole (= 6.02×10^{23} molecules) of water is 18 g. Therefore the number of water molecules in one cup of water is $(250/18) \times 6.02 \times 10^{23}$.

Each molecule of water contains two hydrogen atoms and one oxygen atom, *i.e.*, 10 electrons and 10 protons. Hence the total positive and total negative charge has the same magnitude. It is equal to $(250/18) \times 6.02 \times 10^{23} \times 10 \times 1.6 \times 10^{-19}$ C = 1.34×10^{7} C.

S22. Mass of water = 250 g

Molecular mass of water = 18

Number of molecules in 18 g of water

(Avogadro's Number) = 6.02×10^{23}

$\therefore$ Number of molecules in one cup of water

$$= \frac{250}{18} \times 6.02 \times 10^{23}$$

Each molecule of water contains two hydrogen atoms and one oxygen atom, i.e. 10 electrons and10 protons.

$\therefore$ Total positive and negative charge has the same magnitude and is

$$= \frac{250}{18} \times 6.02 \times 10^{23} \times 10 \times 1.6 \times 10^{-19} \text{C} = 1.34 \times 10^{7} \text{C}$$

S23. One molecule of water (H_2O) contains 2 + 8 *i.e.*, 10 electrons.

Number of molecules in 1 g of water,

$$n = \frac{\text{Avogardro number}}{\text{Atomic weight}} = \frac{6.02 \times 10^{23}}{18} = 3.344 \times 10^{22}$$

Therefore, number of electrons in 1 g of water,

$$n' = n \times 10 = 3.344 \times 10^{22} \times 10 = 3.344 \times 10^{23}$$

The negative charge possessed by 1 g of water,

$$q = n' e = 3.344 \times 10^{23} \times 1.6 \times 10^{-19}$$

$$= \mathbf{5.35 \times 10^{4} \ C}$$

S24. Given, mass of the copper penny, m = 3.11 g

Z = 29; A = 63.5 and $N = 6.02 \times 10^{23}$

Number of atoms in the copper penny, m = 3.11 g

$$= \frac{6.02 \times 10^{23} \times 3.11}{63.5} = 2.95 \times 10^{22}$$

Number of protons or electrons in the copper penny

$$= n \times Z = 2.95 \times 10^{22} \times 29$$

Therefore, total positive or negative charge on the copper penny,

$$q = 2.95 \times 10^{22} \times 29 \times 1.6 \times 10^{-19} = \mathbf{1.37 \times 10^5\ C}$$

S25. When we build up a sizeable static charge and touch a metal object (door handle), the transfer of charge is in the form of a spark.

The static charge can build up on a substance, when it is well insulated from its surroundings. On a humid day in well insulated from its surroundings. On a humid day in summer, the static charge can not build up. In fact, the charge leaks away as fast as it builds up. As on a humid day in the summer, we do not have static charge built on us; no spark occurs, when we touch a metal object.

S26. When two identical conducting charged spheres are brought in contact, then redistribution of charge takes place *i.e.*, charge is equally divided on both the spheres.

When C and A are placed in contact, charge of A equally divides in two spheres. Therefore charge on each A and $C = +2Q$.

Now, C is placed in contact with B, then charge on each B and C becomes

$$\frac{2Q + (-10\,Q)}{2} = -4Q$$

When A and B are placed in contact then charge on each, A and B becomes

$$\frac{2Q + (-4\,Q)}{2} = -Q.$$

S27. When two bodies are rubbed together, electrons get removed from one body and these electrons so removed get removed positively charged and the body to which the electrons get transferred become negatively charged. Since an electron is a material particle, the mass of the positively charged body decreases and that of the negatively charged body increases.

Hence, when one of the two identical metallic spheres is charged positive and the other negative, their masses after charging will not be equal.

S28. The following are a few important properties of electric charge:

(a) Like charges repel each other and unlike charges attract each other.

(b) The magnitude of elementary negative or positive charge is same and is equal to 1.6×10^{-19} C.

(c) The electric charge is ***additive*** in nature. It implies that total charge on an object is algebraic sum of the charges located at different points in the object.

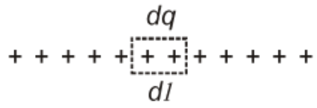
(d) The charge is ***quantized*** *i.e.*, charge carried by a charged object is equal to $\pm\, ne$, where n is an integer.

(e) The electric charge of a system is always ***conserved***.

(f) Unlike mass, the electric charge on an object is not affected by the motion of the object.

S29. (a) **Linear charge density:** If the charge is distributed over a straight line or over the circumference of a circle or over the edge of a cuboid, etc, then the distribution is called 'linear charge distribution'.

$$\lambda = \frac{q}{l} \quad \text{or} \quad \lambda = \frac{dq}{dl}$$



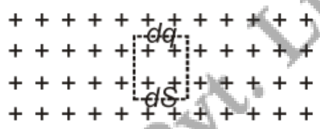
Total charge on line l,

$$q = \int_l \lambda \, dl$$

Linear charge density is the charge per unit length. Its SI unit is C/m.

(b) **Surface charge density:** If the charge is distributed over a surface area, then the distribution is called 'surface charge distribution'.

$$\sigma = \frac{q}{S} \quad \text{or} \quad \sigma = \frac{dq}{dS}$$



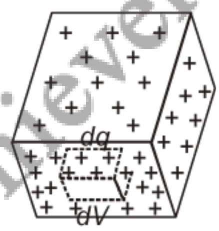
Total charge on surface S,

$$q = \int_S \sigma \, dS$$

Surface charge density is the charge per unit area. Its SI unit is C/m^2.

(c) **Volume charge density:** If the charge is distributed over a volume, then the distribution is called 'volume charge distribution'.

$$\rho = \frac{q}{V} \quad \text{or} \quad \rho = \frac{dq}{dV}$$

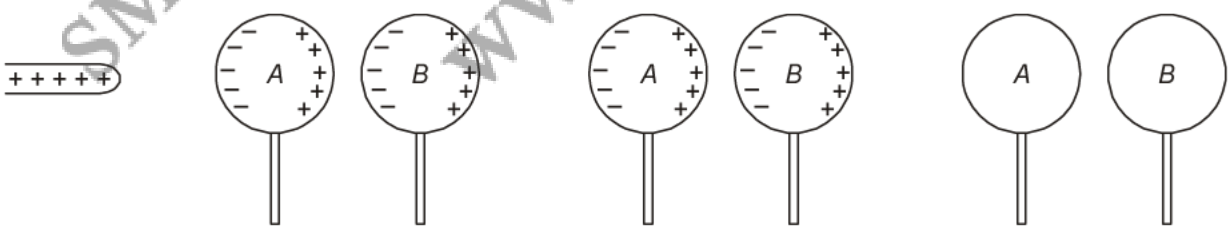


Total charge on volume V,

$$q = \int_V \rho \, dV$$

Volume charge density is the charge per unit volume. Its SI unit is C/m^3.

S30. (a) When the spheres are slightly separated, redistribution of charges take place as shown in figure (a). On the right side of sphere A positive charges are induced while on the left side of sphere B facing the sphere A negative charges are induced.



(a) (b) (c)

(b) When the glass rod is removed subsequently and the separation between the sphere is small, the charge distribution remains unaltered as shown in figure (a) and (b).

(c) When the spheres are separated far apart so that they do no feel the influence of one another the positive and negative charges on each other flow and neutralise leaving the spheres uncharged as shown in figure (a) and (c).

Date: 20/10/2021

Q1. Write the principle of superposition. What is its application?

Q2. Electrostatic force between two charges is called central force. Why?

Q3. When the distance between two equal point charges is half and their individual charges are remains same, what would happen to the force between them?

Q4. Force between two point electric charges kept at a distance *d* apart in air is *F*. If these charges are kept at the same distance in water, how does the force between them change?

Q5. Two point charge q_1 and q_2 are placed close to each other. What is the nature of force between the charges when $q_1 q_2 < 0$?

Q6. In an electric field an electron kept freely. If the electron is replaced by a proton, what will be the relationship between the forces experienced by them?

Q7. If the dielectric constant of a medium is unity, what is its permittivity?

O8. Give the dielectric constant of a conductor.

Q9. How does the force between two point charges change, if the dielectric constant of the medium in which they are kept, increases?

Q10. What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?

Q11. Point out right or wrong for the following statements

(a) The mutual forces between two charges do not get affected by the presence of other charges.

(b) The potential, due to a dipole, at any point on its axial line, is zero.

Q12. The force on an electron kept in an electric field in a particular direction is *F*. What will be the magnitude and direction of the force experienced by a proton kept at the same point in the field? Mass of proton is about 1836 times the mass of the electron.

Q13. Two point charges having equal charges separated by 10 cm distance experience a force of 5 N. What will be the force experienced by them, if they are held in water, at the same distance? (Given : $K_{water} = 80$)

Q14. Four point charges $q_A = 2\ \mu C$, $q_B = -5\ \mu C$, $q_C = 2\mu C$ and $q_D = -5\mu C$ are located at the corners of a square *ABCD* of side of 10 cm. What is the force on charge of $1\mu C$ placed at the centre of the square?

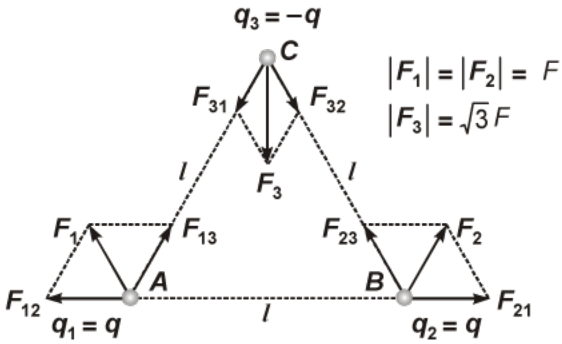
Q15. Two electrons have been removed from each atom. Find the distance between two such atoms, if they repel each other with a force of 8.8×10^{-9} N, when placed in free space.

Q16. Force of attraction between two point electric charge placed at a distance *r* in a medium is *F*. What distance apart should these be kept in the same medium, so that force between them becomes $F/4$.

Q17. The electrostatic force 0.2 N on a small sphere of charge 0.4 μC due to another small sphere of 0.8 μC charge: (a) Find the distance between charge; (b) What is the force on the second sphere due to the first?

Q18. The electrostatic force of repulsion between two equal positively charged ions is 3.7×10^{-9} N, when they are separated by a distance of 5 Å. How many electrons are missing from each ion?

Q19. Consider the charges q, q, and $-q$ placed at the vertices of an equilateral triangle, as shown in figure. What is the force on each charge?



Q20. Calculate the Coulomb's force between two α-particles separated by a distance of 3.2×10^{-15} m.

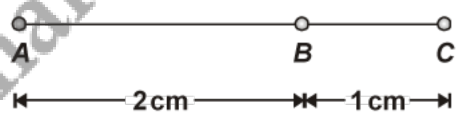
Q21. What equal charges would have to be placed on earth and moon to neutralize their gravitational force of attraction? Given that mass of earth = 10^{25} kg and mass of moon = 10^{23} kg.

Q22. State the limitations of coulomb's law in electrostatics.

Q23. Two equal small spheres each weighing 1 g are hung by equal silk threads attached to the same point, the spheres are charged when in contact and come to rest with their centers 2 cm apart and 20 cm vertically below the point of support. Find the charge on each sphere, if g = 980 cm s^{-2}. How will the force of repulsion be affected, if the plate of glass of K = 8 is placed between the spheres?

Q24. Two point charges of +2 μC and +6 μC repel each other with a force of 12 N. If each is given additional charge of -4μC, what will be the new force?

Q25. Three equally charged objects are located as shown in figure. The electric force exerted by the object A on B is 3.0×10^{-6} N.



(a) What electric force C exert upon B?

(b) What is the net electric force on B?

Q26. Two opposite corners of a square carry Q charge each and the other two opposite corners of the square carry q charge each. If the resultant force on Q is zero, how are Q and q related?

Q27. The charges q and $-3q$ are placed fixed on x-axis separated by distance d. Where should a third charge $2q$ be placed such that it will not experience any force?

Q28. Find the time taken by a particle of mass 1.5×10^{-18} kg and carrying a charge 6.4×10^{-19} C to fall through a distance of 8 m in a uniform electric field of intensity 7.2×10^2 N C^{-1}.

Q29. A particle of mass 2×10^{-3} kg and charges 4 μC is thrown at a speed 24 ms^{-1} against a uniform electric field of strength 4×10^5 NC^{-1}. How much distance will it travel before coming to rest momentarily?

Q30. Two fixed point charges 4 *Q* and 2 *Q* are separated by a distance *x*. Where should a third point charge *q* be placed for it to be equilibrium?

Q31. Two small sphere each of mass 2.5×10^{-4} kg are suspended from the same point by silk threads 0.12 m long. Equal charges are given to the balls, which separate, until the threads enclose an angle of 30°. Calculate the charge on each small sphere (g = 9.8 ms^{-2}).

Q32. Plot a graph showing the variation of Coulomb's force (*F*) versus $1/r^2$, where *r* is the distance between the two charges of each pair of charges : (1 μC, 2 μC) and (1 μC, – 3 μC). Interpret the graphs obtained.

Q33. The sum of two point charges in 7 μC. They repel each other with a force of 1 N when kept 30 cm apart in free space. Calculate the value of each charge.

Q34. Coulomb's law for electrostatic force between two point charges and Newton's law for gravitational force between two stationary point masses, both have inverse-square dependence on the distance between the charges/masses.

(a) Compare the strength of these forces by determining the ratio of their magnitudes (i) for an electron and a proton and (ii) for two protons.

(b) Estimate the accelerations of electron and proton due to the electrical force of their mutual attraction when they are 1 Å (= 10^{-10} m) apart? (m_p = 1.67×10^{-27} kg, m_e = 9.11×10^{-31} kg)

Q35. A free pith-ball of 8 g carries a positive charge of 5×10^{-8} C. What must be the nature and magnitude of charge that should be given to a second pith-ball fixed 5 cm vertically below the former pith-ball so that the upper pith-ball is stationary?

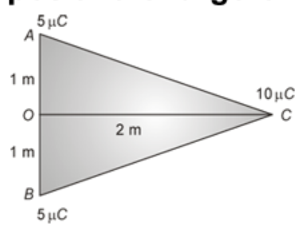
Q36. Three charges 10 μC, 5 μC and – 5μC are placed in air at the three corners *A*, *B* and *C* of an equilateral triangle of side 0.1 m. Find the resultant force experienced by charge placed at corner *A*.

Q37. Two pith-balls each weighing 10^{-3} kg are suspended from the same point by means of silk threads 0.5 m long. On charging the pith-balls equally, they are found to repel each other to a distance of 0.2 m. Calculate the charge on each ball.

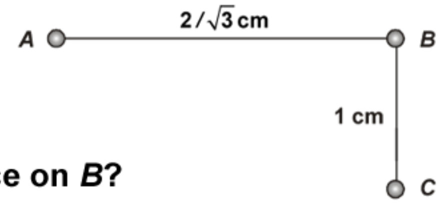
Q38. Three equal charges, each having a charge of $+2 \times 10^{-6}$ C are placed at the three corners of a right angled triangle of sides 3 cm, 4 cm and 5 cm. Find the force on the charge at the right-angled corner.

Q39. Two equal positive charges each of 5 μC interact with a third positive charge of 10 μC situated as shown in figure below.

Find the magnitude and direction of the force experienced by the charge of 10 μC.



Q40. Three small charged spheres, with equal charges on them are placed as shown in figure below.



A and *C* are fixed in position and *B* can move.

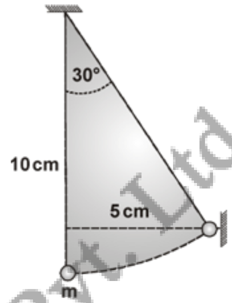
C exerts force of 4×10^{-6} N on *B* due to A ?

(a) What force *A* exerts on *B*? (b) What is the net force on *B*?

Q41. Two point electric charges values *q* and 2*q* are kept at a distance d apart from each other in air. A third charge *Q* is to be kept along the same line in such a way that the net force acting on *q* and 2*q* is zero. Calculate the position of charge *Q* in terms of *q* and *d*.

Q42. A small cork ball with of mass 0.58 g is suspended from a thread 10 cm long. Another ball is fixed at a distance 10 cm from the point of suspension and at a distance of 5 cm from the thread as shown in Fig.

What should the magnitude of the like and equal charges on the balls be the balls be to deflect the thread through 30°?

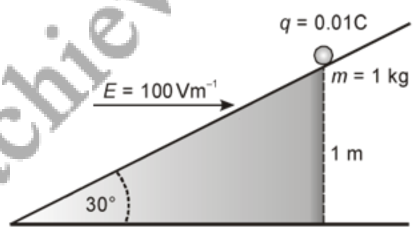


Q43. Four charges + *q*, + *q*, –*q* and –*q* are placed respectively at the four corners *A*, *B*, *C* and *D* respectively of a square of side *a*. Calculate the force on a charge *Q* placed at the centre of the square.

Q44. Suppose the spheres *A* and *B* have identical sizes of each having 6.5×10^{-7} C charge. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second and finally removed from both. What is the new force of repulsion between *A* and *B*?

Q45. An inclined plane making an angle of 30° with the horizontal is placed in a uniform horizontal electric field of 100 V m^{-1} as shown in Figure.

A particle of mass 1 kg and charge 0.01 C is allowed to slide down from rest from a height of 1 m. If the coefficient of friction is 0.2, find the time it will take the particle to reach the bottom.



Q46. A charge particle of mass 1g is suspended through a thread of length 40 cm in an electric field of 4.0×10^4 NC^{-1}. If the particle stays at a distance of 24 cm from the wall equilibrium, find the charge on the particle.

Q47. A pendulum bob of mass 80 mg and carrying a charge of 2×10^{-8} coulomb is at rest in a horizontal uniform electric field of 20,000 V m^{-1}. Find the tension in thread of the pendulum and the angle it makes with the vertical.

Q48. An oil drop of 12 excess electrons is held stationary under constant electric field of 2.55×10^4 Vm^{-1} in Millikan's oil drop experiment. The density of the oil is 1.26 g cm^{-3}. Estimate the radius of the drop. Given that $g = 9.81$ m s^{-2} and $e = 1.6 \times 10^{-19}$ C.

Q49. Two free point charge +4*q* and + *q* are at a distance r apart, where should a third point charge *Q* be placed between them so that the entire system is in equilibrium? What will be the magnitude and sign of the charge *Q*?

Q50. To point charges q_1 and q_2 at a separation r in vacuum exert a force F on each other. What should be their separation in an oil of a relative permittivity 64 so that the force between them remains F only?

Q51. Two point charges of +3 μC and +5 μC repel each other with a force of 24 N. If each is given additional charge of – 7 μC, what will be the new force?

Q52. Two point electric charges of values q and $4q$ are kept at a distance r apart from each other in air. A third charge Q is to be kept along the same line in such a way that the net force acting on q and $4q$ is zero. Calculate the position of charge Q in terms of q and r.

Q53. Two charges each of + Q units are placed along a line. A electron 'e' is placed between them. At what position and for what value of 'e', will the system be in equilibrium?

Q54. Consider the charges $3e$, $3e$ and $-3e$ placed at the vertices of an equilateral triangle of each side 'a'. Find the force on each charge?

Q55. Two identical point charges Q are kept at a distance r from each other. A third point charge q is placed on the line joining the above two charges, such that all the three charges are in equilibrium. What is the magnitude, sign and position of the third charge?

Date: 20/10/2021

S1. The principle of superposition states that when a number of charges are interacting, the total force on a given charge is vector sum of the forces exerted on it by all other charges.

It is used to find force on a charge, when a group of charges are interacting.

S2. The force between two electric charges always acts along the line joining the two charges. For this reason, it is called central force.

S3. Originally, force between the two charges,

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times q}{r^2}$$

When the distance between the charges is half, the force is given by

$$F' = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times q}{\left(\frac{r}{2}\right)^2} = 4\left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{q.q}{r^2}\right)$$

$$F' = 4F.$$

S4. When charges are placed at the same distance in water of dielectric constant K (say), the force between the charges reduces by a factor K *i.e.* it becomes

$$F' = F/K.$$

S5. The given charges are unlike charges, because $q_1q_2 < 0$ means q_1 or q_2 is a negative quantity. Thus the nature of force is attractive.

S6. Force experienced by proton = – Force experienced by electron

S7. Given $K = 1$

$$\varepsilon = K\varepsilon_0, = 1 \times 8.854 \times 10^{-12}\,\text{Nm}^{-2}$$

S8.

$$K = \frac{\text{Applied electric field}}{\text{Induced electric field}}$$

S9. Increase:

Explanation: Force is inversely proportional to the dielectric constant. Hence if dielectric constant of medium increases, the force between them decreases.

S10. Whenever the electron completes an orbit, either circular or elliptical, the work done by the field of a nucleus is zero. Since the displacement is zero.

S11. (a) Right, because mutual force acting between two point charges is proportion to the product of magnitude of charges and inversely proportional to the square of the distance between them, *i.e.*, independent of the other charges.

(b) Wrong, as potential due to an electric dipole is zero on equatorial line inspite of axial line.

Note: The potential due to a dipole at any point on equatorial line is zero, not an axial line.

S12. Magnitude of force remains same but direction of force gets reversed as nature of charge on proton is opposite to that of electron.

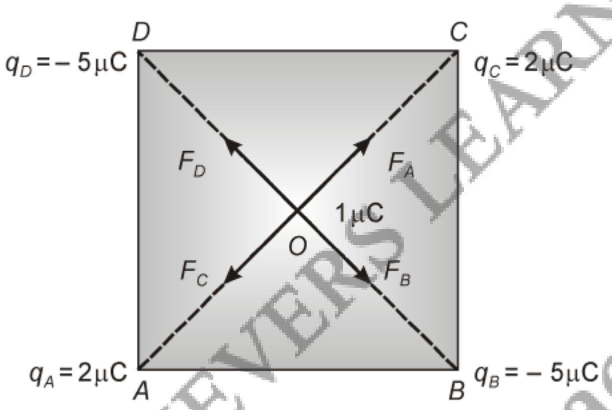
Note: Electrostatic force acting on a charge particle in electric field, is independent of the mass of the particle.

S13. Here, F_{air} = 5 N, K_{water} = 80

Suppose F' is the force in water.

$$F' = \frac{F_{air}}{K} = \frac{5N}{80} = 0.063\,N$$

S14. Consider the square *ABCD* of each side 10 cm and centre *O*. The charge of 1 μC is placed at the point *O*. Obviously, *OA* = *OB* = *OC* = *OD*.



Since $q_A = q_C$, the charge of 1 μC experiences equal and opposite forces due to the charge q_A and q_C. Similarly, the charge of 1 μC experiences equal and opposite forces F_B and F_D due to the charges q_B and q_D.

Therefore, the net force on the charge of 1 μC due to the given arrangement of the charges is zero.

S15. When two electrons are removed from an atom, charge on the atom

$$q = +2 \times 1.6 \times 10^{-19} = +3.2 \times 10^{-19}\text{ C};\ \ F = 8.8 \times 10^{-9}\text{ N}$$

For $q_1 = q$, we get

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q \times q}{r^2}$$

or
$$r^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{F}$$

$$= \frac{9 \times 10^9 \times (3.2 \times 10^{-19})^2}{8.8 \times 10^{-9}} = 1.047 \times 10^{-19}\,\text{m}^2$$

We get,

$$r = \mathbf{3.24 \times 10^{-10}\ m}.$$

S16. Let q_1 and q_2 be the two point charges. Then, force between the charges, when kept at a distance r apart,

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2} \quad \text{... (i)}$$

Suppose that force between the two charge becomes $F' = F/4$, when the charges are kept at a distance x apart. Then,

$$F' = F/4$$

From Eqn. (i), we get

$$\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{x^2}\right) = \frac{1}{4}\left(\frac{1}{\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}\right)$$

or
$$\frac{x}{2} = r$$

or
$$x = \mathbf{2r}.$$

S17. Given,
$q_1 = 0.4\ \mu C = 0.4 \times 10^{-6}\ C;$
$q_2 = -0.8\ \mu C = -0.8 \times 10^{-6}\ C$

(a) Force on charge q_1 due to q_2, $F_{12} = 0.2\ N$

In air,
$$F_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}$$

$$r^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{F_{12}}$$

$$\therefore \quad = \frac{9 \times 10^9 \times (0.4 \times 10^{-6}) \times (0.8 \times 10^{-6})}{0.2} = 0.0144$$

or
$$r = \mathbf{0.12\ m}.$$

(b) Force on charge q_2 due to q_1,

$$F_{21} = F_{12} = \mathbf{0.2\ N\ (attractive)}.$$

S18. Given, $F = 3.7 \times 10^{-9}$ N; $r = 5$ Å $= 5 \times 10^{-10}$ m

Suppose that n electrons have been removed to form each positive ion. Then, charge on each positive ion,

We know, $$q = ne$$

$$q_1 = q_2 = n \times 1.6 \times 10^{-19} \text{ C}$$

Now, $$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$\therefore$ $$3.7 \times 10^{-9} = \frac{9 \times 10^9 \times (n \times 1.6 \times 10^{-19})^2}{(5 \times 10^{-10})^2}$$

$\Rightarrow$ $$n^2 = \frac{3.7 \times 10^{-9} \times (5 \times 10^{-10})^2}{9 \times 10^9 \times (1.6 \times 10^{-19})^2} = 4$$

or $n = \mathbf{2}$

S19. The forces acting on charge q at A due to charges q at B and $-q$ at C are $\boldsymbol{F}_{12}$ along BA and $\boldsymbol{F}_{13}$ along AC respectively, as shown in figure. By the parallelogram law, the total force $\boldsymbol{F}_1$ on the charge q at A is given by

$$\boldsymbol{F}_1 = F\hat{\boldsymbol{r}}_1 \quad \text{where } \hat{\boldsymbol{r}}_1 \text{ is a unit vector along } BC.$$

The force of attraction or repulsion for each pair of charges has the same magnitude

$$F = \frac{q^2}{4\pi\varepsilon_0 l^2}$$

The total force $\boldsymbol{F}_2$ on charge q at B is thus $\boldsymbol{F}_2 = F\hat{\boldsymbol{r}}_2$, where $\hat{\boldsymbol{r}}_2$ is a unit vector along AC.

Similarly the total force on charge $-q$ at C is $\boldsymbol{F}_3 = \sqrt{3}F\hat{\boldsymbol{n}}$, where $\hat{\boldsymbol{n}}$ is the unit vector along the direction bisecting the $\angle BCA$.

It is interesting to see that the sum of the forces on the three charges is zero, *i.e.*,

$$\boldsymbol{F}_1 + \boldsymbol{F}_2 + \boldsymbol{F}_3 = 0$$

It follows straight from the fact that Coulomb's law is consistent with Newton's third law.

S20. Given, $r = 3.2 \times 10^{-15}$ m;

charge on α-particles, $q_1 = q_2 = 2 \times 1.6 \times 10^{-19} C$

According to Coulomb's law,

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$= 9 \times 10^9 \times \frac{(2 \times 1.6 \times 10^{-19})^2}{(3.2 \times 10^{-15})^2} = \mathbf{90\ N}.$$

S21. Given: $G = 6.6 \times 10^{-11}\, Nm^2\, kg^{-2}$; $M_{earth} = 10^{25}$ kg; $M_{moon} = 10^{23}\, kg$,

If q is charge placed on the earth and the moon, then

$$F_e = F_g$$

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times q}{r^2} = G\frac{M_{earth} \times M_{moon}}{r^2}$$

or $$q = \sqrt{4\pi\varepsilon_0 G M_{earth} \times M_{moon}}$$

$$= \sqrt{\frac{1}{9 \times 10^9} \times 6.6 \times 10^{-11} \times 10^{25} \times 10^{23}}$$

$$q = \mathbf{8.57 \times 10^{13} C}.$$

S22. Coulomb's law in electrostatics does not hold in all the situations. It is applicable only in the following situations:

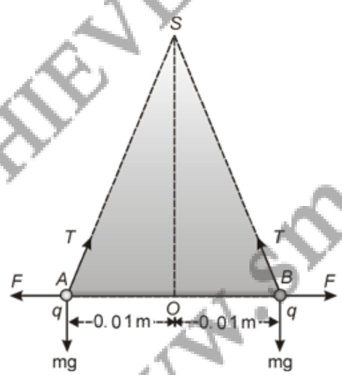
(a) The electric charges must be stationary.

(b) The electric charges must be points in size. Coulomb's law does not apply to two charged bodies of finite sizes, say two charged spheres. It is because, the distribution of charge does not remain uniform, when the two bodies are brought together.

S23. Given: $AB = 2$ cm; $SO = 20$ cm; $m = 1$ g $= 1 \times 10^{-3}$ kg; $g = 980$ cm s^{-2}; $K = 8$

According to triangle law of forces,

$$\frac{F}{OA} = \frac{mg}{SO} = \frac{T}{AS} \quad \text{... (i)}$$



$$\frac{q \times q}{(AB)^2} = mg \times \frac{OA}{SO}$$

Setting $m = 1\, g$, $g = 980$ cm s^{-2} and $OA = AB/2 = 1$ cm, we get

$$q = 14 \text{ stat C}$$

On introducing dielectric:

$$F = \frac{1}{K} \cdot \frac{q \times q}{(AB)^2}$$

Setting $K = 8$, we get

$F = \mathbf{6.125\ dyne}$

S24. Before giving additional charge:

Given: $q_1 = +2\mu C$; $q_2 = +6\mu C$; $F = 12$ N

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \quad \text{... (i)}$$

After giving additional charge:

Given, $q_1' = +2 + (-4) = -2\ \mu C$,

$q_2' = +6 + (-4) = 2\ \mu C$,

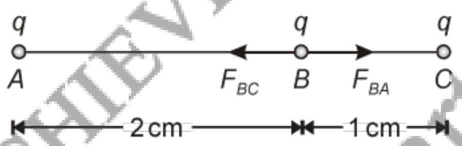
$$F' = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1' q_2'}{r^2} \quad \text{... (ii)}$$

From the equations (i) and (ii), we get

$$F' = \frac{q_1' q_2'}{q_1 q_2} F = \frac{2 \times (-2)}{2 \times 6} \times 12 = \mathbf{-4N}.$$

The negative sign shown that the force is attractive.

S25. Here, $AB = 2$ cm $= 0.02$ m and BC = 1 cm = 0.01 m



Let the charge on each object be q.

Now, $$F_{BA} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times q}{(AB)^2}$$

Setting $F_{BA} = 3.0 \times 10^{-6}$ N, we get

$q = 3.65 \times 10^{-10}$ C

(a) $$F_{BC} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times q}{(BC)^2}$$

$= 12.0 \times 10^{-6}$ N (along BA)

(b) Net force on the charge at B,

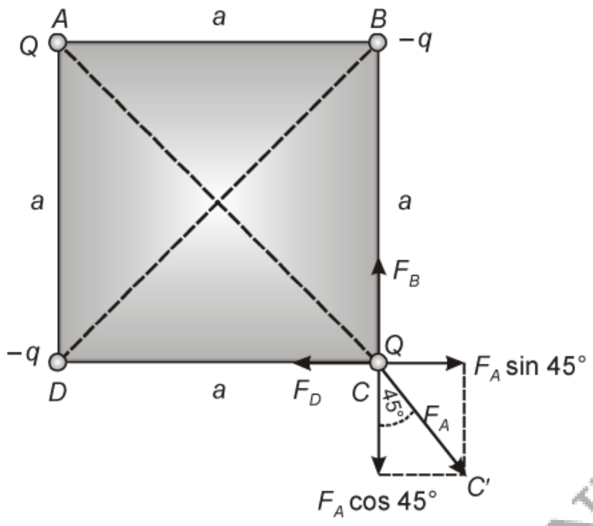
$$F = F_{BC} - F_{BA} = 12.0 \times 10^{-6} - 3.0 \times 10^{-6}$$

$$= \mathbf{9.0 \times 10^{-6}\ N} \quad \textbf{(along } BA\textbf{)}.$$

S26. Let each side of the square be of length a.

Then, $AC = \sqrt{a^2 + a^2} = \sqrt{2}\,a$

It follows that the resultant force on the charge at point C



can be zero, if charge at the points B and D are negative in nature. Then, the force F_A, F_B and F_D will act on the charge Q at point C as shown in the above figure.

So that the resultant force on the charge at point C is zero,

$$F_B = F_A \cos 45°$$

and $$F_D = F_A \sin 45°$$

From the equation (i), we have

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{(-q) \times Q}{BC^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \times Q}{AC^2} \cos 45°$$

or $$-\frac{q}{a^2} = \frac{Q}{(\sqrt{2}\,a)^2} \times \frac{1}{\sqrt{2}}$$

or $$Q = \mathbf{-2\sqrt{2}\,q}$$

S27. Let charge $2q$ be placed at P, at a distance l from A where charge q is placed, as shown in figure.

The charge $2q$ will not experience any force, when force of repulsion on it de to q is balanced by force of attraction on it due to $-3q$ at B where $AB = d$

or $$\frac{(2q)(q)}{4\pi\varepsilon_0 l^2} = \frac{(2q)(-3q)}{4\pi\varepsilon_0 (l+d)^2}$$

$$(l + d)^2 = 3l^2$$

or $$2l^2 - 2ld - d^2 = 0$$

$\therefore$ $$l = \frac{2d \pm \sqrt{4d^2 + 8d^2}}{4} = \frac{d}{2} \pm \frac{\sqrt{3}d}{2}$$

$$l = \frac{d + \sqrt{3}d}{2}.$$

S28. Given: $E = 7.2 \times 10^2$ NC^{-1}; $q = 6.4 \times 10^{-19}$C; $M = 1.5 \times 10^{-18}$ kg; $S = 8$ m

The acceleration produced by the electric field,

$$a = \frac{qE}{m} = \frac{6.4 \times 10^{-19} \times 7.2 \times 10^2}{1.5 \times 10^{-18}} = 307.2 \text{ m s}^{-2}$$

Now, $S = 8$ m, $u = 0$, $a = 307.2$ m s^{-2}

Using the relation: $S = ut + \frac{1}{2}at^2 \rightarrow S = \frac{1}{2}at^2$,
we get

$t =$ **0.23 s**.

S29. Given, m = 2.0×10^{-3} kg; $q = 4\ \mu C = 4 \times 10^{-6}\ C$;
$u = 24$ m s^{-1}; $E = 4 \times 10^5$ N C^{-1} and v = 0 ms^{-1}

Force on the charged particle,

$$F = qE = 4 \times 10^{-6} \times 4 \times 10^5 = 1.6 \text{ N}$$

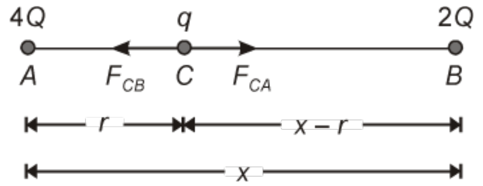
Since the particle is thrown against the electric field, acceleration of the particle,

$$a = -\frac{F}{m} = -\frac{1.6}{2.0 \times 10^{-3}} = -0.8 \times 10^3\ m\,s^{-2}$$

Form the relation : $v^2 = u^2 + 2aS$, we have

$$S = \frac{v^2 - u^2}{2a} = \frac{(0)^2 - (24)^2}{2 \times (-0.8 \times 10^3)} = 0.36 \textbf{ m}$$

S30. Suppose that charge q is placed at a distance $AC = r$ from the charge $4Q$ as shown in figure below.



Then, $$BC = x - r$$

For the charge q to be in equilibrium,

$$F_{CA} = F_{CB}$$

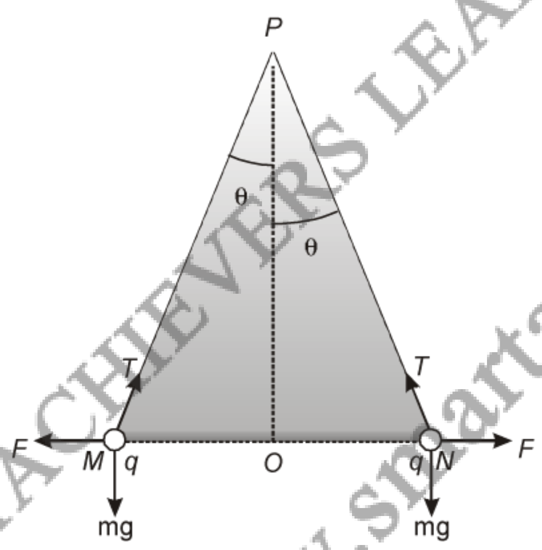
or $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{(4Q)q}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q(2Q)}{(x-r)^2}$$

or $$r = \mathbf{0.586\,x} \quad \text{or} \quad \mathbf{3.414\,x}$$

S31. Here, $MP = 0.12$ m and $\theta = 30°/2 = 15°$

According to the triangle law of forces,

$$\frac{F}{OM} = \frac{mg}{PO} = \frac{T}{MP}$$



or $$F = mg \times \frac{OM}{PO}$$

or $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q \times q}{MN^2} = mg \times \frac{OM}{PO} = mg\tan\theta$$

Putting $m = 2.5 \times 10^{-4}$ kg, $g = 9.8$ ms^{-2}

and $MN = 0.12 \times OM = 0.12 \therefore MP \sin\theta$, we get

$$\boldsymbol{q = 3.958 \times 10^{-8}\,C.}$$

S32. According to Coulomb's law, magnitude of force acting between two stationary point charges is given by

$$F = \left(\frac{q_1 q_2}{4\pi\varepsilon_0}\right)\left(\frac{1}{r^2}\right)$$

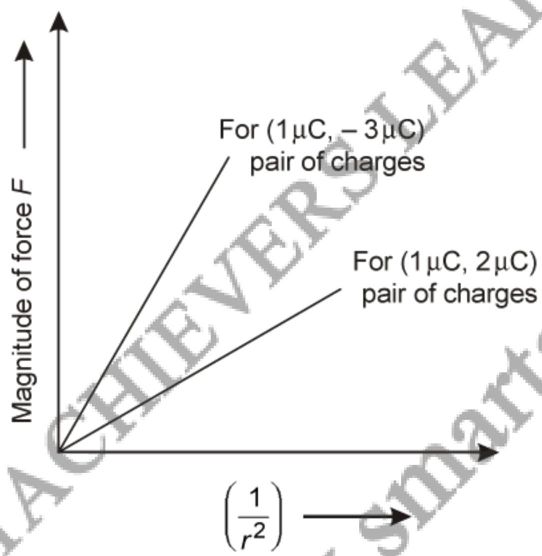
For given $q_1\, q_2$, $F \propto \left(\frac{1}{r^2}\right)$

The slope of $F - \frac{1}{r^2}$, graph depends on $q_1\, q_2$.

Magnitude of $q_1\, q_2$ is higher for second pair.

$\therefore$ Slope of $F - \frac{1}{r^2}$ graph.

Corresponding to second pair (1 μC, – 3 μC) is greater.



Higher the magnitude of product of charges q_1, q_2 higher he slope.

S33. Let one of to charges is $x\,\mu$C. Therefore, other charge will be $(7 - x)\,\mu$C.

By Coulomb's law,

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$1 = 9 \times 10^9 \times \frac{(x \times 10^{-6})(7 - x) \times 10^{-6}}{(0.3)^2}$$

$$9 \times 10^{-2} = 9 \times 10^{9-12}\, x(7 - x)$$

$$10 = x\,(7 - x)$$

$\Rightarrow$ $$x^2 - 7x + 10 = 0$$

$\Rightarrow$ $$(x - 2)(x - 5) = 0$$

$$x = 2\ \mu C,\ 5\ \mu C.$$

S34. (a) (i) The electric force between an electron and a proton at a distance r apart is:

$$F_e = \frac{-1}{4\pi\varepsilon_0} \frac{e^2}{r^2}$$

where the negative sign indicates that the force is attractive. The corresponding gravitational force (always attractive) is:

$$F_G = -G\,\frac{m_p m_e}{r^2}$$

where m_p and m_e are the masses of a proton and an electron respectively.

$$\left[\frac{F_e}{F_G}\right] = \frac{e^2}{4\pi\varepsilon_0 G m_p m_e} = 2.4 \times 10^{39}$$

(ii) On similar lines, the ratio of the magnitudes of electric force to the gravitational force between two protons at a distance r apart is:

$$\left[\frac{F_e}{F_G}\right] = \frac{e^2}{4\pi\varepsilon_0 G m_p m_p} = 1.3 \times 10^{36}$$

However, it may be mentioned here that the signs of the two forces are different. For two protons, the gravitational force is attractive in nature and the Coulomb force is repulsive . The actual values of these forces between two protons inside a nucleus (distance between two protons is ~ 10^{-15} m inside a nucleus) are F_e ~ 230 N whereas F_G ~ 1.9×10^{-34} N.

The (dimensionless) ratio of the two forces shows that electrical forces are enormously stronger than the gravitational forces.

(b) The electric force ***F*** exerted by a proton on an electron is same in magnitude to the force exerted by an electron on a proton; however the masses of an electron and a proton are different. Thus, the magnitude of force is

$$|\boldsymbol{F}| = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$
$$= 8.987 \times 10^9 \text{ Nm}^2/\text{C}^2 \times (1.6 \times 10^{-19}\text{ C})^2/(10^{-10}\text{ m})^2$$
$$= 2.3 \times 10^{-8}\text{ N}$$

Using Newton's second law of motion, $F = ma$, the acceleration that an electron will undergo is

$$a = 2.3 \times 10^{-8}\text{ N}/9.11 \times 10^{-31}\text{ kg} = 2.5 \times 10^{22}\text{ m/s}^2$$

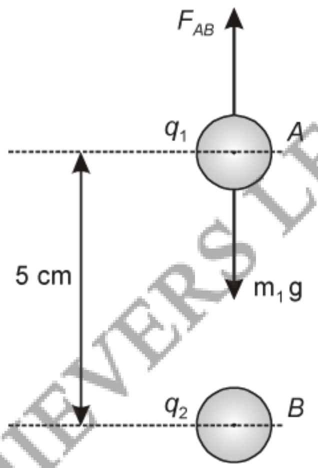
Comparing this with the value of acceleration due to gravity, we can conclude that the effect of gravitational field is negligible on the motion of electron and it undergoes very large accelerations under the action of Coulomb force due to a proton.

The value for acceleration of the proton is

$$2.3 \times 10^{-8}\text{ N}/1.67 \times 10^{-27}\text{ kg} = 1.4 \times 10^{19}\text{ m/s}^2.$$

S35. Given, AB = 5 cm = 0.05 m, charge on the pith-ball A, $q_1 = 5 \times 10^{-8}$C mass of the pith-ball A, $m_1 = 8\text{ g} = 8 \times 10^{-3}$ kg The weight m_1 g of the pith-ball A acts vertically downwards.

Let q_2 be charge on the pith-ball A remains stationary. It can be possible only, if the charges two pith-balls are of same signs *i.e.* if charge on the pith-ball A is positive, charge on B should also be positive. As such, the force on the pith-ball A due to B *i.e.* F_{AB} will act vertically upwards.



For charge q_1 to remain stationary,

$$F_{AB} = m_1 g$$

or
$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{AB^2} = m_1 g \Rightarrow q_2 = \frac{(4\pi\varepsilon_0)(m_1 g)(AB)^2}{q_1}$$

$$= \frac{1}{9\times10^9}\cdot\frac{(8\times10^{-3})\times 9.8\times(0.05)^2}{5\times10^{-8}} = \mathbf{4.36 \times 10^{-7}\text{ C (positive)}}$$

S36. Given: $q_A = 10\ \mu C = 10^{-5}\ C$; $q_B = 5 \times 10^{-6}$ C; $q_C = -5 \times 10^{-6}\ C$ and $AB = BC = AC = 0.1$ m

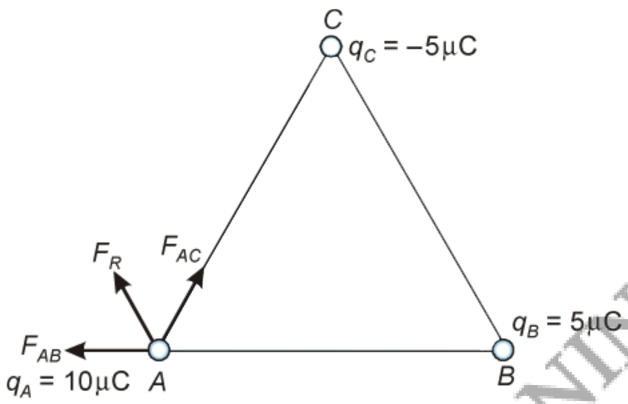
Now, $$F_{AB} = \frac{1}{4\pi\varepsilon_0}.\frac{q_A q_B}{AB^2} = \frac{9 \times 10^9 \times 10^{-5} \times 5 \times 10^{-6}}{(0.1)^2}$$

or $$F_{AB} = 45 \text{ N (repulsive)}$$

Also, $$F_{AC} = \frac{1}{4\pi\varepsilon_0}.\frac{q_A q_C}{AC^2} = \frac{9 \times 10^9 \times 10^{-5} \times 5 \times 10^{-6}}{(0.1)^2}$$

or $$F_{AC} = 45 \text{ N (Attractive)}$$

The forces F_{AB} and F_{AC} are inclined at an angle of 120° as shown in figure below.



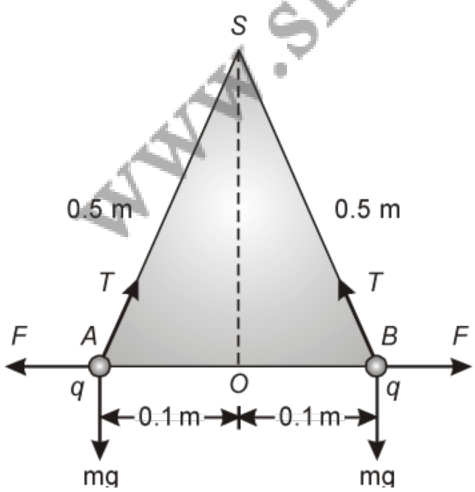
If F is the resultant force on the charge q_A, then

$$F = \sqrt{F_{AB}^2 + F_{AC}^2 + 2FA_{AB}.F_{AC}\cos 120^\circ}$$

$$= \sqrt{45^2 + 45^2 + 2(45 \times 45)(-0.5)} = 45\sqrt{1+1-1} = \mathbf{45\ N}$$

S37. Consider two pith-balls A and B each having charge q and mass 10^{-3} kg.

When the pith-balls are suspended from point S by two threads each 0.5 m long, they repel each other to the distance $AB = 0.2$ m as shown in figure below.



Each of the two pith-balls is in equilibrium under the action of the following three forces:

(a) The electrostatic repulsive force *F*.

(b) The weight *mg* acting vertically downwards.

(c) The tension T in the string directed towards point *S*.

The three forces *mg*, *F* and *T* can be represented by the three sides *SO*, *OA* and *AS* of the $\Delta\, AOS$ taken in order.

Therefore, according to triangle law of forces,

$$\frac{F}{OA} = \frac{mg}{SO} = \frac{T}{AS} \qquad \text{... (i)}$$

Here, $$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{(AB)^2} = 9\times10^9\times\frac{q^2}{(0.2)^2}\,\text{N},$$

$$mg = 10^{-3}\times 9.8 = 9.8\times10^{-3}\ \text{N}$$

From the Eq. (i), we have

$$F = mg\times\frac{OA}{SO}$$

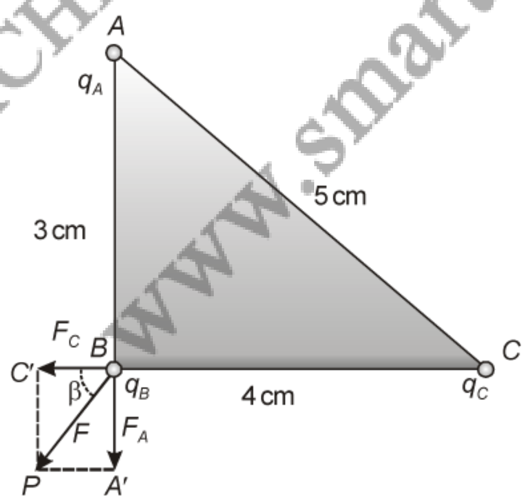
or $$9\times10^9\times\frac{q^2}{(0.2)^2} = 9.8\times10^{-3}\times\frac{0.1}{\sqrt{(0.5)^2-(0.1)^2}}$$

$$\Rightarrow \qquad q^2 = \frac{9.8\times10^{-3}\times0.1\times(0.2)^2}{9\times10^9\times\sqrt{0.25-0.01}}$$

$$q^2 = 8.891\times10^{-15}$$

or $$q = \mathbf{9.4\times10^{-8}\,C}$$

S38. Given: $q_A = q_B = q_C = +2\times10^{-6}\,\text{C}$; $AB = 3\,\text{cm} = 0.03\,\text{m}$, $BC = 4\,\text{cm} = 0.04\,\text{m}$ and $AC = 5\,\text{cm} = 0.05\,\text{m}$



Let F_A and F_B be the forces exerted by the charges q_A and q_C on the charge q_B at point B. Then,

$$F_A = \frac{1}{4\pi\varepsilon_0}.\frac{q_A \times q_B}{(AB)^2} = \mathbf{40\,N} \quad \textbf{(along } \boldsymbol{BA'}\textbf{)}$$

and $$F_C = \frac{1}{4\pi\varepsilon_0}.\frac{q_C \times q_B}{(BC)^2} = \mathbf{22.5\,N} \quad \textbf{(along } \boldsymbol{BC'}\textbf{)}$$

The resultant force on the charge at B,

$$F = \sqrt{F_A^2 + F_C^2} = \mathbf{45.9\ N} \quad \textbf{(along } \boldsymbol{BP}\textbf{)}$$

If the direction of F makes an angle β with BC, then

tan $$\tan\beta = \frac{F_A}{F_C} = 1.778 \quad \Rightarrow \quad \beta = \tan^{-1}(1.778)$$

or $$\beta = \mathbf{60.64°} \quad \textbf{(with side } \boldsymbol{BC}\textbf{)}.$$

S39. Here, $q_A = q_B = 5\ \mu C = 5 \times 10^{-6}$ C;

$q_C = 10\ \mu C = 10 \times 10^{-6}$ C $= 10^{-5}$ C

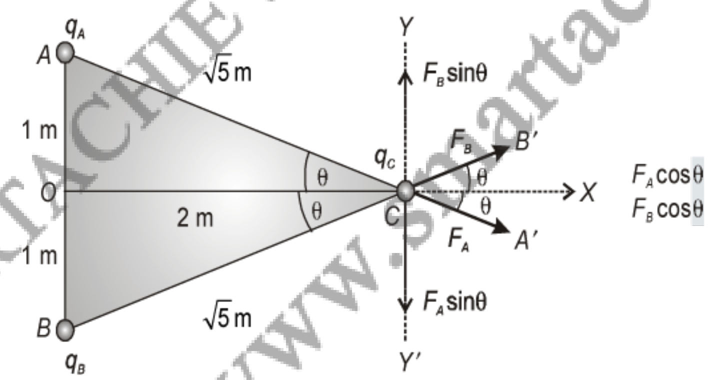
$AC = BC = \sqrt{1^2 + 2^2} = \sqrt{5}\ m$

Let F_A and F_B be the forces exerted by the charge q_A and q_B on q_C. Then,

$$F_A = F_B = \frac{1}{4\pi\varepsilon_0}.\frac{q_A \times q_C}{(\sqrt{5})^2}$$

$$= \frac{9 \text{ x } 10^9 \text{ x } 5 \text{ x } 10^{-6} \text{ x } 10^{-5}}{5} = 0.09\text{ N}$$

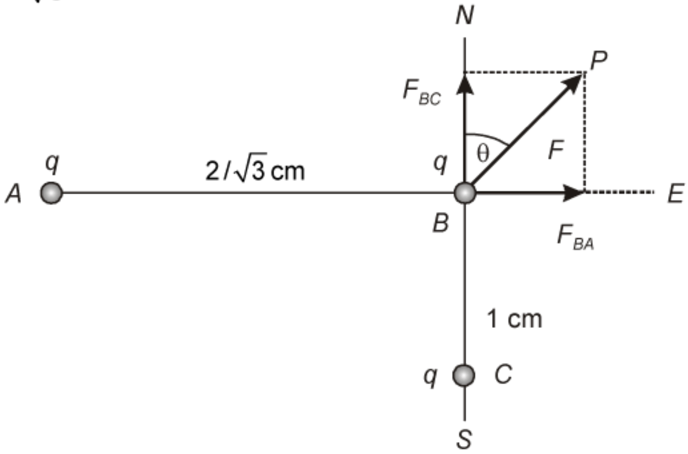
Resolve the forces F_A and F_B into rectangular components along X-axis and Y-axis.



The components along Y-axis are equal and opposite and hence cancel each other, while the components along X-axis are equal and in the same direction. Therefore, net force on the charge at C,

$$F = 2F_A \cos\theta = 2 \times 0.09 \times \frac{2}{\sqrt{5}} = \mathbf{0.161\ N} \quad \textbf{(along } \boldsymbol{CX}\textbf{)}$$

S40. Given: $AB = \frac{2}{\sqrt{3}}$ cm $= \frac{0.02}{\sqrt{3}} m$; $BC = 1\,\text{cm} = 0.01\,\text{m}$



Let q be the charge on each sphere.

$$F_{BC} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q \times q}{(BC)^2}$$

Given $\quad F_{BC} = 4 \times 10^{-6}$ N & BC = 1 cm = 0.01 m

$$q^2 = \frac{4 \times 10^{-6} \times (0.01)^2}{9 \times 10^9} = 4.44 \times 10^{-20}$$

$$q = 2.1 \times 10^{-10}$$

(a) $F_{BA} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q \times q}{(AB)^2}$

$= \mathbf{3 \times 10^{-6}}$ **N (along *BE*)**

(b) Net force on the charge at B,

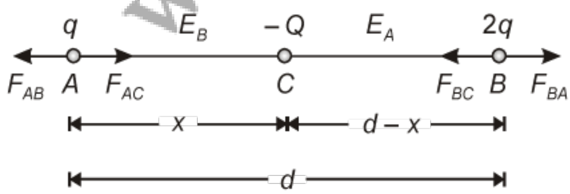
$$F = \sqrt{(F_{BA})^2 + (F_{BC})^2} = \mathbf{5 \times 10^{-6}\ N} \quad \textbf{(along } BP\textbf{)}.$$

If the direction of F makes an angle θ with NS-line, then

$$\tan\theta = \frac{F_{BA}}{F_{BC}} = 0.7500 \quad \Rightarrow \quad \theta = \tan^{-1}(0.75)$$

or $\quad \theta =$ **36.9° (east north)**

S41. Suppose that the charge q, $2q$ and Q are placed as shown in the figure.



It follows that the net force on charge q and $2q$ can be zero, only if the charge Q is of opposite to those of charges q and $2q$. Therefore, if charges q and $2q$ are positive, then charge Q (negative) from q be equal to x.

For force on charge q to be zero,

$$F_{AB} = F_{AC}$$

or $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q(2q)}{d^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{x^2}$$

or $$\frac{Q}{q} = \frac{2x^2}{d^2} \quad \text{... (i)}$$

For force on charge $2q$ to be zero,

or $$F_{BA} = F_{BC}$$

or $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q(2q)}{d^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(2q)Q}{(d-x)^2}$$

$$\frac{Q}{q} = \frac{(d-x)^2}{d^2} \quad \text{... (ii)}$$

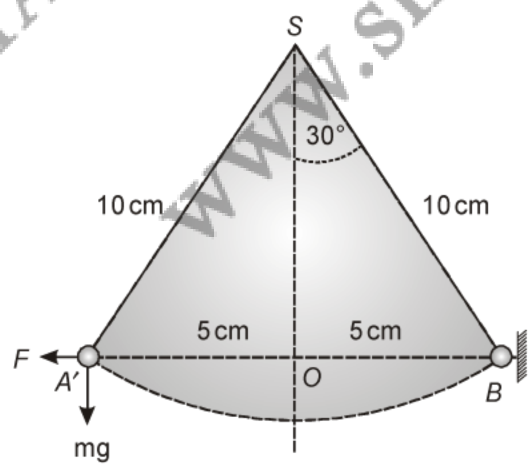
From the equations (i) and (ii), we get

$$x = \frac{\boldsymbol{d}}{\sqrt{\mathbf{2-1}}} \text{ or } -\frac{\boldsymbol{d}}{\sqrt{\mathbf{2-1}}}$$

S42. Let q be the charge on each ball. suppose that the suspended cork ball deflected to the position A′ so that ∠A′SO = 30°.

Then, $$OA' = A'S \sin 30° = 10 \times 0.5 = 5 \text{ cm}$$

It follows that $SA'B$ is an equilateral triangle. The electrostatic force of repulsion F and weight $m\,g$ of the cork ball will act as shown in Figure.



Since cork ball is in equilibrium,

$$\frac{F}{OA'} = \frac{mg}{SO}$$

or $$F = \text{m g} \times \frac{OA'}{SO} \quad \text{———— (i)}$$

Here, $$F = \frac{q \times q}{(A'B)^2} \text{ (in cgs system)},$$

$$\text{SO} = 10\sqrt{10^2 - 5^2} = 5\sqrt{3} \text{ cm}, m = 0.58\ g, g = 980 \text{ cm s}^{-2}$$

Setting for F, m, g, OA' and SO in the equation (i), we get

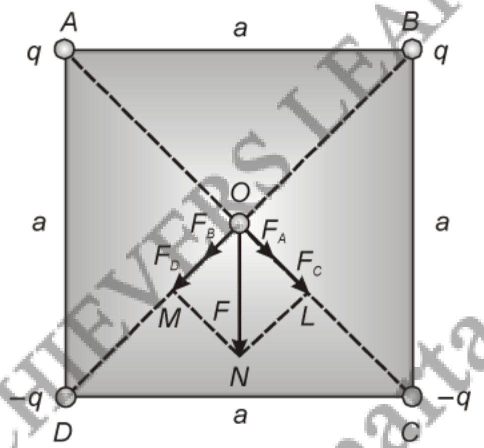
$$q = \mathbf{181.16\ StatC}$$

S43. Given, $$AB = BC = CD = DA = a$$

Obviously, $$AO = BO = CO = DO = a\cos 45° = a/\sqrt{2}$$

Let F_A, F_B, F_C and F_D be the forces exerted by the charges at the points A, B, C and D on the charge Q at point O. It follows that

$$F_A = F_B = F_C = F_D = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times Q}{\left(a/\sqrt{2}\right)^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2qQ}{a^2}$$



The resultant of the forces F_A and F_C,

$$F_1 = F_A + F_C = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2qQ}{a^2} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{2qQ}{a^2}$$

or $$F_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4qQ}{a^2} \quad \text{(along } OL\text{)}$$

Similarly, resultant of the forces F_B and F_D

$$F_2 = F_B + F_D = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4qQ}{a^2} \quad \text{(along } OM\text{)}$$

Therefore, resultant force on the charge Q,

$$F = \sqrt{F_1^2 + F_2^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4\sqrt{2}\, q\, Q}{a^2} \quad \textbf{(along } ON\textbf{)}$$

Since forces F_1 and F_2 are equal in magnitude, the resultant force F will act along the bisector of $\angle COD$ *i.e.* parallel to side AD or BC.

S44. The original charge on the sphere A and B,

$$q_1 = q_2 = 6.5 \times 10^{-7}\ \text{C}$$

The distance between the two spheres, $r = 0.5$ m.

Since all the spheres are of same size, they will possess equal charges on being brought in contact.

When uncharged sphere C ($q_3 = 0$) is brought in contact with A, charge left on the sphere A,

$$q_1' = \frac{q_1 + q_3}{2} = \frac{6.5 \times 10^{-7} + 0}{2} = 3.25 \times 10^{-7}\ \text{C}$$

and charge on the sphere C, $q_3' = 3.25 \times 10^{-7}$ C.

Therefore, when the sphere C is brought in contact with B, charge left on the sphere B,

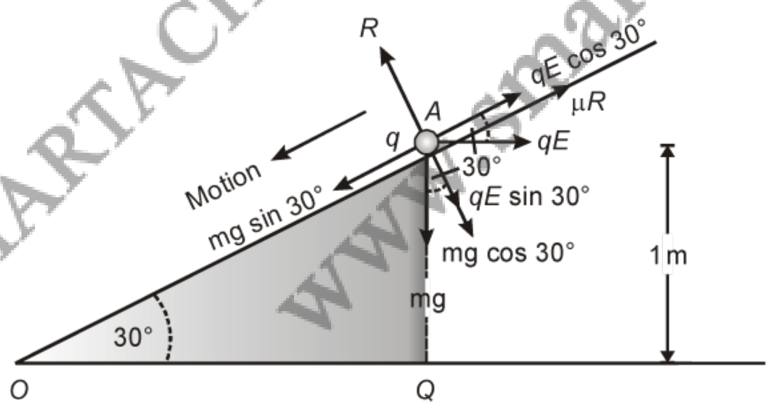
$$q_2' = \frac{q_2 + q_3'}{2} = \frac{6.5 \times 10^{-7} + 3.25 \times 10^{-7}}{2} = 4.875 \times 10^{-7}\ \text{C}$$

Therefore, new force of repulsion between the spheres A and B

$$F_{\text{air}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1' q_2'}{2}$$

$$= 9 \times 10^9 \times \frac{3.25 \times 10^{-7} \times 4.875 \times 10^{-7}}{(0.5)^2} = \mathbf{5.704 \times 10^{-3}\ N}$$

S45. The various forces act on the charge particle as shown in figure.



Net force on the charge particle along the along the inclined plane in downward direction,

$$F = m\, g \sin 30° - \mu\, R - q\, E \cos 30°$$

Now, $R = m\,g \cos 30° + q\,E \sin 30°$

$\therefore$ $F = m\,g\,(\sin 30° - \mu \cos 30°) - q\,E\,(\cos 30° + \mu \sin 30°)$

$$= 1 \times 9.8\,(0.5 - 0.2 \times 0.866) - 0.01 \times 100(0.866 + 0.2 \times 0.5)$$

$$= 2.327 \text{ N}$$

Acceleration of the charged particle along the inclined plane,

$$a = \frac{F}{m} = \frac{2.327}{1} = 2.327\ m\,s^{-2}$$

Let OA = S. Then,

$$\sin 30° = \frac{AQ}{OA} = \frac{1}{S}$$

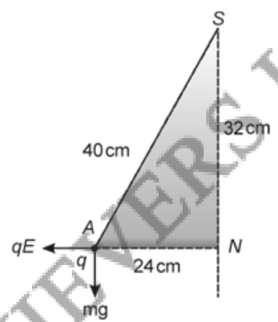
or $$S = \frac{1}{\sin 30°} = 2m$$

Initial velocity of the charge particle, $u = 0$

Using the relation : $S = u\,t + \frac{1}{2}a\,t^2$, we have

t = **1.31 s**

S46. Let q be the charge on the particle. The charged particle experiences a force qE along the horizontal, which takes it away from the wall as shown in Figure.



Since the charged particle is in equilibrium,

$$\frac{qE}{AN} = \frac{mg}{SN}$$

or $$q = \frac{mg}{E} \times \frac{AN}{SN} \quad \text{... (ii)}$$

Given, $m = 1 \text{ g} = 10^{-3} \text{ kg}$, $E = 4.0 \times 10^4 \text{ N C}^{-1}$,

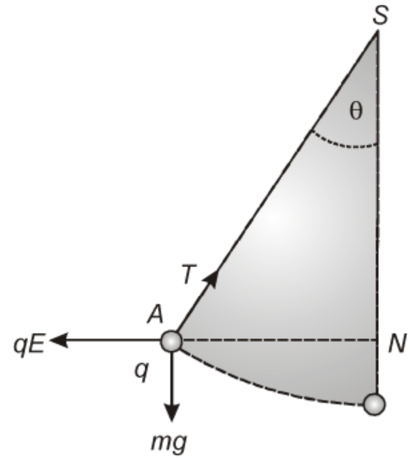
$g = 9.8 \text{ m s}^{-2}$, $AN = 24 \text{ cm} = 0.24 \text{ m}$

and $$SN = \sqrt{40^2 - 24^2} = 32cm = 0.32\ m$$

Setting the values in Eq. (ii) we get

$$q = \frac{10^{-3} \times 9.8 \times 0.24}{4.0 \times 10^4 \times 0.32} = \mathbf{1.838 \times 10^{-7} C}$$

S47. Consider that a pendulum bob of weight $m\,g$ and carrying a charge q is suspended in the electric field E. Suppose that it comes to rest at point A, so that it makes an angle θ with the vertical as shown in figure.



At point A, the bob is acted upon by the following three forces:

(a) Weight mg acting vertically downward.

(b) Tension T in the string along AS.

(c) Electrostatic force qE on the bob along horizontal.

Since the bob is in equilibrium under the action of the forces mg, T and qE, there forces can be represented by the sides SN, AS and NA of the triangle ANS. Therefore,

$$\frac{mg}{SN} = \frac{qE}{NA} = \frac{T}{AS} \qquad \text{... (i)}$$

Given: m = 80 mg = 80×10^{-6} kg; $q = 2 \times 10^{-8}$ C and E = 20, 000 Vm^{-1}

From the equation (i), we get

$$\frac{NA}{SN} = \frac{qE}{mg}$$

or $$\tan\theta = \frac{qE}{mg} = \frac{2 \times 10^{-8} \times 20{,}000}{80 \times 10^{-6} \times 9.8} = 0.51$$

or $$\theta = \mathbf{27.02°}$$

Again from the equation (i), we have

$$\frac{T}{AS} = \frac{qE}{NA}$$

$$T = \frac{qE}{NA/AS} = \frac{qE}{\sin\theta} = \frac{2\times10^{-8}\times20{,}000}{\sin(27.02^\circ)}$$

$$= \frac{2\times10^{-8}\times20{,}000}{\sin(27.02^\circ)} = \mathbf{9.7\times10^{-4}\ N}.$$

S48. Given, ρ = 1.26 g cm^{-3} = $1.26\times(10^{-3}\text{ kg})\times(10^{-2}\text{ m})^{-3} = 1.26\times10^{3}$ kg m^{-3}; g = 9.81 ms^{-2} and $E = 2.55\times10^{4}$ Vm^{-1}

Charge on the drop,

$$q = 12\,e = 12\times1.6\times10^{-19}$$
$$= 1.92\times10^{-18}\text{ C}$$

Force on the oil drop due to the electric field,

$$F_e = qE = 1.92\times10^{-18}\times2.55\times10^{4}\text{ N}$$

Force on the drop due to gravity,

$$F_g = mg = \frac{4}{3}\pi r^3\rho g$$

$$\therefore \qquad F_g = \frac{4}{3}\pi r^3\times1.26\text{ x}10^3\times9.81\text{N}$$

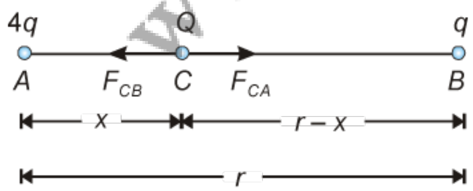
The drop will be held stationary, if F_g and F_e are equal and opposite *i.e.*

$$\frac{4}{3}\pi r^3\times1.26\times10^3\times9.81 = 1.92\times10^{-18}\times2.55\times10^4$$

or
$$r = \left(\frac{3\times1.92\times10^{-18}\times2.55\times10^4}{4\pi\times1.26\times10^3\times9.81}\right)^{\frac{1}{3}}$$

$$= (0.9456\times10^{-18})^{1/3}$$
$$= 9.82\times10^{-7}\text{ m} = \mathbf{9.82\times10^{-4}\ mm}$$

S49. For the system to be in equilibrium the third charge Q has to be negative in nature. Let x be the distance of the charge Q from the charge +4 q as shown in the figure.



For charge $-Q$ to be in equilibrium,

$$F_{CA} = F_{CB}$$

or $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{(4q)Q}{r^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{(r-x)^2}$$

$$4x^2 - 8rx + 3r^2 = 0$$

or $$x = \frac{2r}{3} \text{ or } \frac{r}{2}$$

Since the third charge has been placed between the charges + 4 q and + q,

$$x = \mathbf{\frac{2r}{3}}$$

For charge +4 q to be in equilibrium,

or $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{(4q)Q}{x^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(4q)q}{r^2}$$

$$Q = q\frac{x^2}{r^2}$$

Setting $x = 2r/3$, we get

$$Q = \mathbf{\frac{4}{9}q} \quad \textbf{(negative)}$$

S50. Here, $K = 64$,

$$F_{\text{vacuum}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2_{\text{vacuum}}}; \qquad \dots \text{(i)}$$

When medium of dielectric constant K is taken

$$F_{\text{oil}} = \frac{1}{4\pi\varepsilon_0 K}\cdot\frac{q_1q_2}{r^2_{\text{oil}}} \qquad \dots \text{(ii)}$$

From equation (i) and (ii), we get

$$F_{\text{vacuum}} = F_{\text{oil}}$$

or $$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2_{\text{vacuum}}} = \frac{1}{4\pi\varepsilon_0 K}\cdot\frac{q_1q_2}{r^2_{\text{oil}}}$$

or $$r^2_{\text{oil}} = \frac{r^2_{\text{vacuum}}}{K}$$

$$r_{\text{oil}} = \frac{r_{\text{vacuum}}}{\sqrt{K}}$$

$$r_{oil} = \frac{r_{vacuum}}{\sqrt{64}}$$

$$r_{oil} = \frac{r_{vacuum}}{8}$$

S51. Initial charge:

Here, $q_1 = +3\text{ mC},\ q_2 = +5\text{ mC};\ F = 24\text{ N}$

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \quad \dots \text{(i)}$$

After giving additional charge:

Here, $q_1' = +3 + (-7) = -4\ \mu\text{C},$

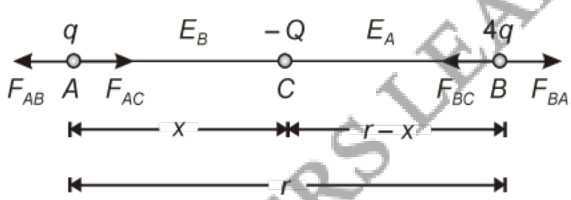
$q_2' = +5 + (-7) = -2\ \mu\text{C};$

$$F' = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1' q_2'}{r^2} \quad \dots \text{(ii)}$$

From the equation (i) and (ii), we get

$$F' = \frac{q_1' q_2'}{q_1 q_2} F = \frac{-4 \times (-2)}{3 \times 5} \times 24 = 12.8\text{ N (repulsive)}$$

S52. Suppose that the charge q, $4q$ and Q are placed as shown in the figure below



It follows that the net force on charge q and $4q$ can be zero, only if the charges Q is of opposite sign to those of charges q and $4q$. Therefore, if charges q and $4q$ are positive, then charge Q must be negative in nature. Let the distance of charge Q (negative) from q be equal to x.

For force on charge q to be zero,

$$F_{AB} = F_{AC}$$

or $$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q(4q)}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qQ}{x^2}$$

or $$\frac{Q}{q} = \frac{4x^2}{r^2} \quad \text{... (i)}$$

For force on charge $4q$ to be zero,

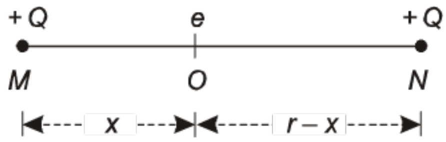
or $$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q(4q)}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(4q)Q}{(r-x)^2}$$

or $$\frac{Q}{q} = \frac{(r-x)^2}{r^2} \quad \text{... (ii)}$$

From the equations (i) and (ii), we get

$$x = -r \quad \text{or} \quad x = \frac{r}{3}$$

S53. Let the two charges + Q each be placed at points M and N at a distance r between them and electron 'e' be placed at point O, such that $MO = x$ and $ON = r - x$ as shown in Figure.



The system will be in equilibrium, if electron 'e' as well as either of the two charges + Q each placed at the points M and N experience zero net force.

(a) ***For electron 'e' to be in equilibrium:*** For this, force on the electron 'e' at point O due to the charge + Q at point M should be equal and opposite to that due to the charge + Q at the point N i.e.

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{Qe}{x^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qe}{(r-x)^2}$$

or $$x^2 = (r-x)^2$$

or $$x = \pm(r-x)$$

or $$x = \frac{r}{2}$$

(b) ***For charge + Q at M (or at N) to be in equilibrium:*** For this, force on the charge + Q at the point M due to the electron 'e' at O should be equal and opposite to that due to the charge + Q at N i.e.

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{Qe}{(r/2)^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{QQ}{r^2}$$

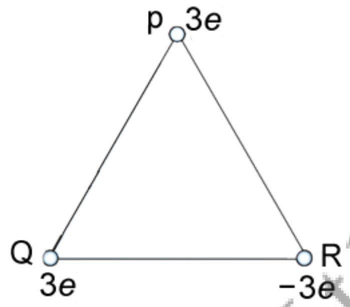
or $$e = \frac{Q}{4}$$

Therefore, the system of three charges will be in equilibrium, if electron '*e*' is equal **– Q/4** and is placed at the mid-point of the distance between the two charges of + *Q* each.

S54. Given Figure, force on q_1 (= 3*e*) at *P*

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} = F\hat{r}_1$$

Here, $F = \frac{9e^2}{4\pi\varepsilon_0}$ and $\hat{r}$, is the unit vector along *QR*



Force on q_2 (= 3*e*) at *Q*.

$$\vec{F}_2 = \vec{F}_{21} + \vec{F}_{23} = F\hat{r}_2$$

Here, $F = \frac{9e^2}{4\pi\varepsilon_0}$ and $\hat{r}_2$, is the unit vector along *PR*

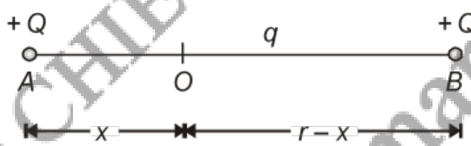
Force on q_3 (= – 3*e*) at *R*.

$$\vec{F}_3 = \vec{F}_{31} + \vec{F}_{32} = (\sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos 60^\circ})\,\hat{n} = \sqrt{3}\,F\hat{n}$$

Here, $\hat{n}$ = unit vector along the direction bisecting $\angle QRA$

or $$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0.$$

S55. Let the two charges of + *Q* each be placed at points *A* and *B* at a distance *r* apart as shown in the figure below.



Suppose that the third charge *q* (in magnitude) is placed at the point *O* on the line joining the other two charges, such that *AO* = *x* and *OB* = *r* – *x*. For the system of the three charges to be in equilibrium, net force on each of the three charges must be zero.

At first instance, if we assume that the charge *q* placed at point *AO* is positive in nature, then it will experience forces due to the other two charges in opposite directions and hence net force on the charge *q* may be zero. But as such (when *q* is taken positive), the force on the charge *Q* at point *A* or at point *B* will not be zero. It is because, the forces on a charge *Q* due to the other two charges will act in same direction. However, if the charge *q* placed at point *O* is taken negative, then on a charge *Q* forces due to the other two charges will act in opposite directions. Hence, the third charge *q* placed at point *O* must be negative in nature.

(a) **For charge – *q* to be in equilibrium:** For this, force on the charge $-q$ at point *O* due to the charge $+Q$ at point *A* should be equal and opposite to that due to that charge $+Q$ at point *B i.e.*,

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{x^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{(r-x)^2}$$

or $$x^2 = (r-x)^2 \quad \text{or} \quad x = \pm(r-x)$$

or $$x = \frac{r}{2}.$$

(b) **For charge + *Q* at *A* (or at *B*) to be in equilibrium:** For this, force on the charge $+Q$ at point *A* due to the charge $-q$ at *O* should be equal and opposite to that due to the charge $+Q$ at *B i.e.*,

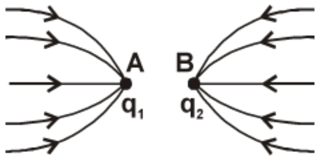
$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{(r/2)^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{QQ}{r^2}$$

or $$q = \frac{Q}{4}.$$

Therefore, the system of three charges will be in equilibrium, if charge $-q$ is equal to $-Q/4$ and is placed at the mid-point of the distance between the two charges of $+Q$ each.

Date: 20/10/2021

Q1. Figure shows electric lines of force due to two point charge q_1 and q_2 placed at points *A* and *B* respectively. Write the nature of charge on them.



Q2. What is an electric line of force? Sketch lines of force due to two equal positive charges placed at a small distance apart in air.

Q3. Electric field inside conductor is zero. Explain.

Q4. Draw lines of force to represent a uniform electric field.

Q5. Two point charges + *q* and – *q* are placed a distance '*d*' apart. Show the point at which the field is parallel to the line joining the two charges.

Q6. How does the electric field of an electric dipole change with distance?

Q7. Define electric field at a point.

Q8. Guess a possible reason why water has a much greater dielectric constant (= 80) than say, mica (= 6).

Q9. Why two electric lines of force cannot intersect each other?

Q10. A point charge *q* is placed at the origin. How does the electric field due to the charge vary with distance *r* from the origin?

Q11. A proton is placed in a uniform electric field along the positive *X*-axis. In which direction will it tend to move?

Q12. Define electric field intensity at a point.

Q13. The test charge used to measure electric field at a point should be vanishingly small. Why?

Q14. Is electric field intensity a scalar or a vector quantity. Given its SI units.

Q15. Why does the electric field inside a dielectric decrease when it is placed in an external electric field?

Q16. A force of 2.25 N acts on a charges of 15×10^{-4} C. Find intensity of electric field at that point?

Q17. Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude $16 \times 10^{-22}\ \text{Cm}^{-2}$. Find the electric field between the plates?

Q18. Why most electrostatic field be normal to the surface at every point of a charged conduction.

Q19. Why do the electric field lines not form closed loop.

Q20. Two point charges of 0.2×10^{-7} *C* and 0.1×10^{-7}C are1.5 cm apart. What is the magnitude of the field produced by either charge at the site of the other?
Use standard value of $1/4\pi\varepsilon_0$.

Q21. A charged particle is free to move in an electric field. Will always move along electric line of force?

Q22. Sketch the electric lines of force due to point charges (a) $q > 0$ and (b) $q < 0$.

Q23. Define electric line of force and give its three important properties.

Q24. Two point electric charges of unknown magnitude and sign are placed a distance apart. The electric field intensity is zero at a point not between the charge but on the line joining them. Write two essential conditions for this to happen.

Q25. The electric field *E* due to a point charge at any point near it is defined as

$$E = \underset{q_0 \to 0}{Lt} \frac{F}{q_0}$$

Where *q* is the test charge and F is the force acting on it. What is the physical significance of $q_0 \to 0$ in this expression?

Q26. A dipole of length 0.3m consists of two charges of $\pm$ 600μC. What is its electric dipole moment? Calculate the electric field due to the dipole at a point on the axis distant 0.2 m from one of the charges in air.

Q27. Two point charges of $+1.5 \times 10^{-19}$ C and $+ 15.5 \times 10^{-19}$ C are separated by a distance 2.4 m. Find the point on the line joining them at which electric field intensity is zero.

Q28. Draw the shapes of the suitable Gaussian surfaces, while applying Gauss's law to calculate the electric field due to

(a) a uniformly charged long straight wire.

(b) a uniformly charged infinite plane sheet.

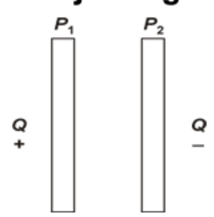
Q29. Two point charges $4Q$, Q are separated by 1 m in air. At what point on the line joining the charges is the electric field intensity zero?

Q30. Two charges $+Q$ and $-Q$ are kept at $(x_2, 0)$ and $(x_1, 0)$ respectively in the $X-Y$ plane. Find the magnitude and direction of the net electric field at the origin (0, 0).

Q31. Figure shows two large metal plates P_1 and P_2 tightly held against each other and placed between two equal and unlike point charges perpendicular to the line joining them.

(a) What will happen to the plates when they are released?

(b) Draw the pattern of the electric field linear for the system.



Q32. Deduce the expression for the electric field E due to a system of two charges q_1 and q_2 with position vectors r_1 and r_2 at a point *r* with respect to common origin.

Q33. The electric field in a region is radially outward and varies with distance r as

$$E = 250\, r\, V\, m^{-2}$$

Calculate the charge contained in a sphere of radius 0.2 m centred at the charge contained in a sphere of radius 0.2 m centred at the origin.

Q34. Two charges $\pm 10\ \mu C$ are placed 5.0 mm apart. Determine the electric field at (a) a point on the axis of the dipole 15 cm away from its centre O on the side of the positive charge, (b) a point 15 cm away from point O on a line passing through O and normal to the axis of the dipole.

Q35. Two charged conducting spheres of radii r_1 and r_2 connected to each other by a wire. Find the ratio of electric fields at the surfaces of the two spheres.

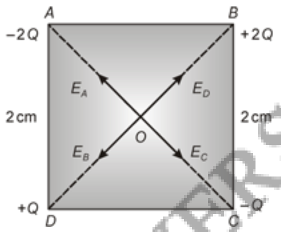
Q36. Sketch the pattern of electric field lines due to (a) a conducting sphere having negative charge on it and (b) an electric dipole.

Q37. Three charges of equal magnitude q is placed at the vertices of an equilateral triangle of side l. Find the force on a charge Q placed at the centroid of the triangle is?

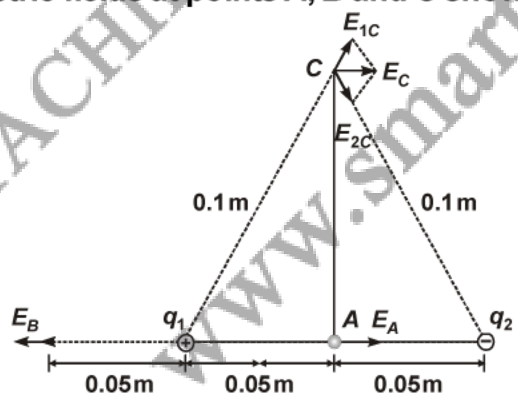
Q38. ABC is an equilateral triangle of side 5 cm. Charges of + 60 statC and –30 statC are placed at points A and B respectively. Calculate completely the electric field at point C. Given, $1C = 3 \times 10^9$ stat C.

Q39. Three charges, each equal to q, are placed at the three corners of a square of side a. Find the electric field at the fourth corner of the square.

Q40. Figure shown four point charges at the corners of a square of side 2 cm. Find the magnitude and direction of the electric field at the centre O of the square, if $Q = 0.02\ \mu C$.

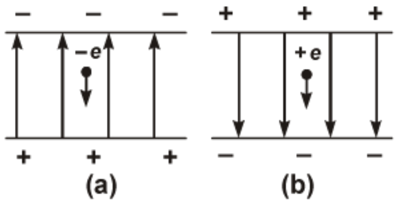


Q41. Two point charges q_1 and q_2, of magnitude $+10^{-8}$ C and -10^{-8} C, respectively, are placed 0.1 m apart. Calculate the electric fields at points A, B and C shown in figure.



Q42. Two point charge $q_A = 3\ \mu C$ and $q_B = -3\ \mu C$ are located 20 cm apart in vacuum. (a) what is the electric field at the mid-point O of the line AB joining the two charges? (b) if a negative test charge of magnitude 1.5×10^{-9} C is placed at this point, what is the force experienced by the test charge?

Q43. An electron falls through a distance of 1.5 cm in a uniform electric field of magnitude 2.0×10^4 N C^{-1} [fig. (a)]. The direction of the field is reversed keeping its magnitude unchanged and a proton falls through the same distance [fig. (b)]. Compute the time of fall in each case. Contrast the situation with that of 'free fall under gravity



Q44. Two point charge 3 μC and –3 μC are located 20 cm apart in vacuum.

(a) Calculate the electric field at the midpoint *O* of the line *AB* joining the two charges.

(b) What is the force experienced by a negative test charge of magnitude 1.5×10^{-9} C placed at this point?

Q45. A thin conducting spherical shell of radius *R* has charge *Q* spread uniformly over its surface. Using Gauss's law, derive an expression for an electric field at a point outside the shell. Draw a graph of electric field *E*(*r*) with distance *r* form the centre of the shell for $0 \leq r \leq \infty$.

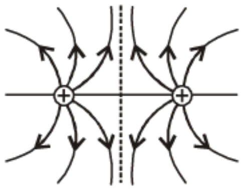
Q46. Define electric field. Calculate the field due to an electric dipole of length 5 cm and consisting of charges of $\pm$ 100 μC at a point 15 cm from each charge.

Q47. Derive an expression for the electric field at any point along the equatorial line of an electric dipole.

Date: 20/10/2021

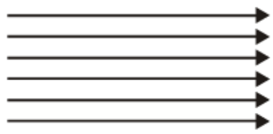
S1. Unlike charges.

S2. Electric line of force in an electric field is the path along which a unit positive charge tends to move if it is free to do so.



S3. Conductors (such as metals) posses free electrons. If a net electric field in the conductor, these free charges will experience a force which will set a current flow. When no current flows, the resultant force and the electric field must be zero. Thus, under electrostatic conditions, the value of $\vec{E}$ at all points within the conductor is zero.

S4.



S5. The resultant electric field at all points on the perpendicular bisector (equatorial line) is parallel to the line joining the charges + q and – q.

S6. $E \propto \frac{1}{r^3}$.

S7. Electric field defined as if we bring test charge q_0 near the charge q, then the charge q_0 experiences a force (attraction or repulsion) due to the charge q. The force experienced by q_0 is said to be due to the "electric field".

S8. Water has an unsymmetrical shape as compared to mica. Since it has a permanent dipole moment, it has a greater dielectric constant than mica.

S9. The tangent at a point on the line of force gives the direction of electric field at that point. If two lines of force intersect each other at a point, then electric field at that point will have two directions. As the same cannot be true, lines of force can never intersect each other.

S10. The electric field varies inversely as the square of the distance from the point charge.

S11. Along the direction of the electric field *i.e.*, along the positive X-axis.

S12. The intensity of electric field at a point in space is the force acting on a unit test-charge placed at that point in space.

S13. In case, test charge is not vanishingly small, it will produce its own electric field and the measured value of electric field will be different from the actual value of electric field at that point.

S14. Electric field intensity is a vector quantity. Its SI unit is NC^{-1} or $V\ m^{-1}$.

S15. An electric field ($\vec{E}_p$) is induced inside the dielectric in a direction opposite to the direction of external electric field ($\vec{E}_o$). Thus, net field is

$$\vec{E} = \vec{E}_p - \vec{E}_o$$

S16. Electric field, $E = \frac{F}{q} = \frac{2.25\,N}{15 \times 10^{-4}\,C} = 1500\,NC^{-1}$

S17. Here, $E = \frac{\sigma}{\varepsilon_0} = \frac{16 \times 10^{-22}}{8.854 \times 10^{-12}} = 1.8 \times 10^{-10} NC^{-1}$

S18. As electric field inside a conductor is always zero. The electric lines of forces exerts lateral pressure on each other leads to explain repulsion between like charges. Thus in order to stable spacing, the lines are normal to the surface.

S19. As electric field lines never goes through negative potential to positive potential. thus it cannot form the closed loops.

S20. Given, $q_1 = 0.2 \times 10^{-7}$ C; $q_2 = 0.1 \times 10^{-7}$ C and $r = 1.5$ cm $= 1.5 \times 10^{-2}$ m

The electric field at the site of a charge will be the electric field produced by the other charge.

Therefore, electric field due to the charge q_1 at the site of q_2,

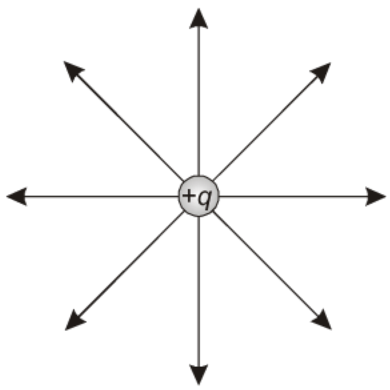
$$E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r^2} = \frac{9 \times 10^9 \times 0.2 \times 10^{-7}}{(1.5 \times 10^{-2})^2} = \mathbf{8.0 \times 10^5\ NC^{-1}}$$

Also, electric field due to the charge q_2 at the site of q_1,

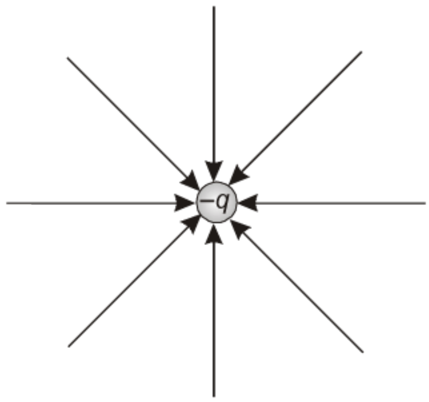
$$E_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r^2} = \frac{9 \times 10^9 \times 0.1 \times 10^{-7}}{(1.5 \times 10^{-2})^2} = \mathbf{4.0 \times 10^5\ NC^{-1}}$$

S21. The charged particle may or may not move along an electric line of force. If the charge particle was initially at rest, it will move along an electric line of force. In case the charged particle had some initial velocity making certain angle with a line of force, then its resultant path will not be along the line of force.

S22. (a) For a charge $q > 0$ *i.e.*, for a positive charge, the electric lines of force are as shown in Figure below.



(b) For a charge $q < 0$ *i.e.*, for a negative charge, the electric lines of force are is as shown in figure below.



S23. An electric line of force is that imaginary smooth curve drawn in an electric field along which a free, isolated positive charge moves. The tangent drawn at any point on the electric line of force gives the direction of the forces acting on a positive charge placed at that point.

(a) The electric line of forces are continuous lines/curves always originating from a positive charge and terminating on a negative charge.

(b) The tangent at any point to the line of force gives the directions of the field at the point.

(c) Two lines of force cannot cross at one point.

S24. (a) Both the charges cannot be of same sign.

(b) The observation point (where electric field intensity is zero) has to be closer to the smaller charge than to the bigger charge.

S25. The idea of taking the limit $q_0 \to 0$ is that on placing the test charge at the observation point, the source charge will not be disturbed. In case, test charge is not vanishingly small, it will produce its own electric field and the measured value of electric field will be different from the actual value of electric field that point.

S26. Given, $q = 600\ \mu C = 600 \times 10^{-6}$ C ; $2a = 0.3$ m

Therefore, electric dipole moment of the dipole,

$$p = q(2a) = 600 \times 10^{-6} \times 0.3 = 1.8 \times 10^{-4} \text{ Cm}$$

The electric field at a point on axial line at a distance r from the centre of the electric dipole is given by

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2pr}{(r^2 - a^2)^2}$$

Given, $p = 50 \times 10^{-6}\ C\,m$; $a = 0.1/2 = 0.05$ m

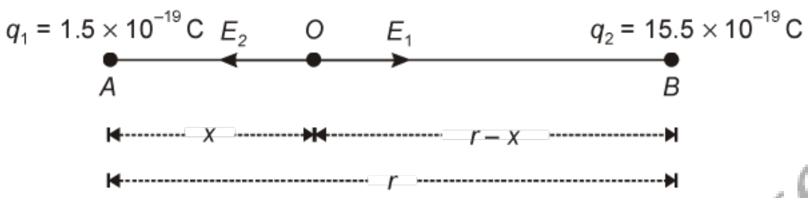
and $r = 0.2 + 0.05 = 0.25$ m

Setting values $E = 9 \times 10^9 \times \dfrac{2 \times 50 \times 10^{-6} \times 0.25}{(0.25^2 - 0.05^2)^2}$

$= \mathbf{6.25 \times 10^7\ NC^{-1}}$

S27. Given, $q_1 = +\,1.5 \times 10^{-19}$C; $q_2 = 15.5 \times 10^{-19}$ C and $r = 2.4$ m

Suppose that the resultant electric field is zero at point O on the line AB joining the two charges.



If $AO = x$, then $BO = r - x = 2.4 - x$

Let E_1 and E_2 be the electric fields at the points O due to the charges q_1 and q_2 respectively. Then, electric field will be zero at the point O, if E_1 and E_2 are equal in magnitude and opposite in direction *i.e.*,

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{(AO)^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{(BO)^2}$$

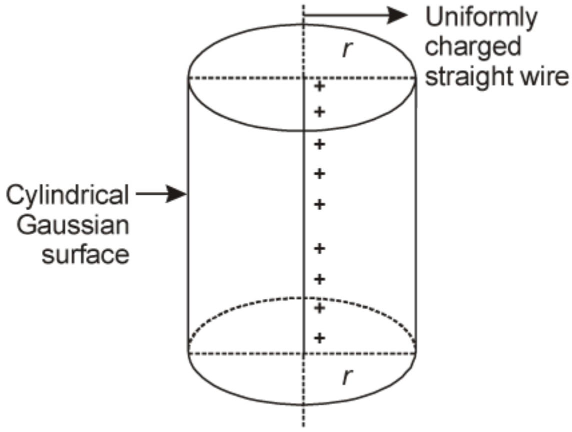
or $$\frac{1.5 \times 10^{-19}}{x^2} = \frac{15.5 \times 10^{-19}}{(2.4 - x)^2}$$

$$14x^2 + 4.8x - 8.64 = 0$$

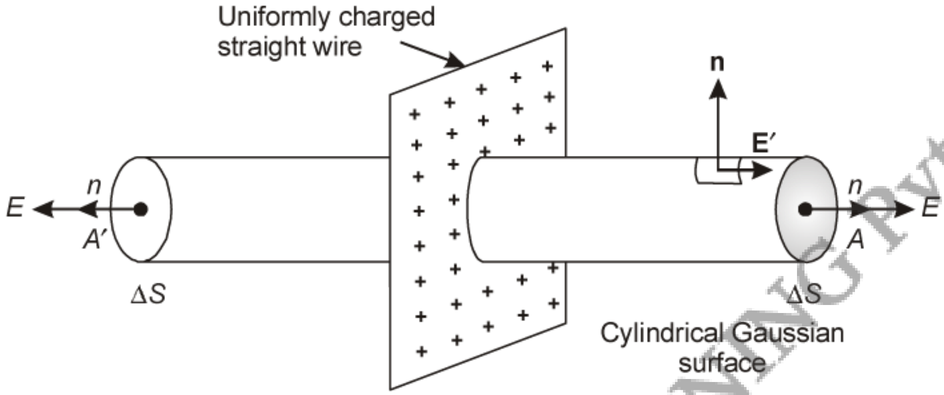
$$x = 0.63 \text{ m.}$$

S28. *The surface that we choose for application of Gauss's theorem is called Gaussian surface. We usually choose a spherical Gaussian surface.*

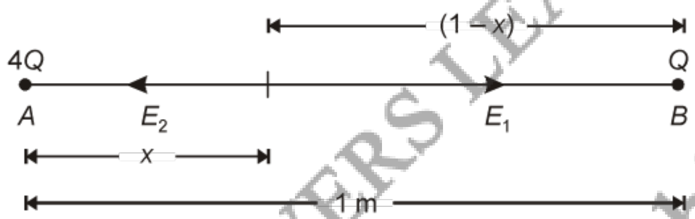
(a)



(b) Electric field due to a plane sheet of charge



S29. Let electric field intensity at any point P which lie at a distance x m from $4Q$ be zero.



$\therefore$ Electric field intensity (E_1) due to $4Q$ at P = Electric field intensity (E_2) due to $+Q$ at P

As direction of $\mathbf{E}_1$ and $\mathbf{E}_2$ are in opposite directions.

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\cdot\frac{4Q}{x^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{(1-x)^2}$$

$$\text{or} \quad \frac{4}{x^2} = \frac{1}{(1-x)^2}$$

$$\Rightarrow \quad \left(\frac{1-x}{x}\right)^2 = \frac{1}{4}$$

$$\frac{1-x}{x} = \frac{1}{2}$$

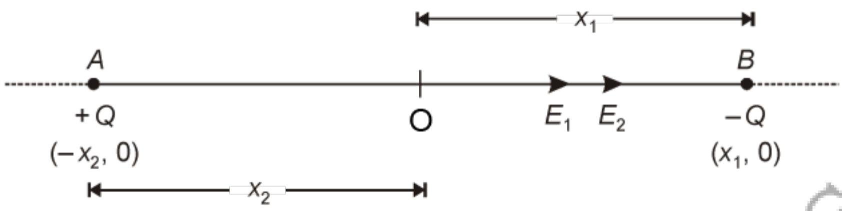
$$\frac{1}{x} - 1 = \frac{1}{2}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{3}{2}$$

$$x = \frac{2}{3}\text{ m}$$

Electric field intensity is zero at apoint which lie at a distance $x = \frac{2}{3}$ m from $+4Q$ charge on the line joining two charges.

S30. *To find the electric field intensity at a point due to two chartges, first of all find the individual electric fields due to both charges and then find the resultant field using vector addition.*



Electric field intensity at O due to $+Q$ charge

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(x_2)^2} \qquad \text{(Towards } B\text{)} \quad \ldots \text{(i)}$$

Electric field intensity at O due to $-Q$ charge

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(x_1)^2} \qquad \text{(Towards } B\text{)} \quad \ldots \text{(ii)}$$

$\because$ E_1 and E_2 act along the same direction.

$\therefore$ Net electric field intensity at O,

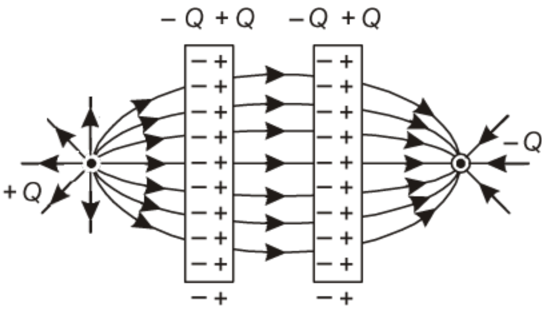
$$E = E_1 + E_2 \qquad \text{(Towards } B\text{)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{(x_2)^2} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{(x_1)^2}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{x_2^2} + \frac{1}{x_1^2}\right].$$

S31. (a) By electrostatic induction, charge induces on the plates and opposite nature of charge appears on the surface facing each other. Therefore, they start attracting towards each other.

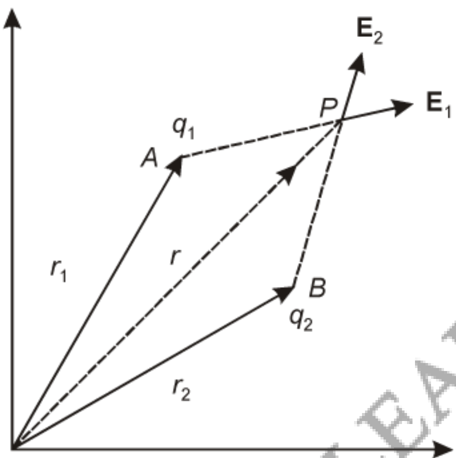
(b)



Note: Field lines must be perpendicular to the plates. Also, equispaced field lines exist between two plates as electric field between them is uniform.

S32. Let two point charges q_1 and q_2 situated at points A and B have position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$.

$\because$
$$\mathbf{AP} = \mathbf{r} - \mathbf{r}_1$$
$$\mathbf{BP} = \mathbf{r} - \mathbf{r}_2$$



Electric field intensity at point P due to q_1,

$$\mathbf{E}_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{|\mathbf{AP}|^3}\,\mathbf{AP}$$

Similarly,
$$\mathbf{E}_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{|\mathbf{BP}|^3}\,\mathbf{BP}$$

$\therefore$ Net electric field intensity at point P,

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2$$
$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{|\mathbf{r} - \mathbf{r}_1|^3}(\mathbf{r} - \mathbf{r}_1) + \frac{q_2}{|\mathbf{r} - \mathbf{r}_2|^3}(\mathbf{r} - \mathbf{r}_2)\right]$$

S33. Given: $E = 250\, r\ V\, m^{-2}$; radius of sphere, $a = 0.2\ m$

Let $E_{surface}$ be electric field at the surface of the sphere.

Then,
$$E_{surface} = 250\, a\ V\, m^2 = 250 \times 0.2 = 50\ V\, m^{-2}$$

Electric flux crossing through the sphere,

$$\Phi = \oint \vec{E}_{\text{surface}} . \vec{dS} = E_{\text{surface}} \times 4\pi a^2$$

$$\Phi = 50 \times 4 \pi \times (0.2)^2 \text{ N m}^2 \text{ C}^{-1}$$

If q is charge contained in the sphere, then according to Gauss' theorem,

$$\Phi = \frac{q}{\varepsilon_0}$$

or

$$q = \varepsilon_0 \Phi = \varepsilon_0 \times 50 \times 4\pi \times (0.2)^2$$

$$= 4 \pi \varepsilon_0 \times 50 \times (0.2)^2$$

$$= \frac{1}{9 \times 10^9} \times 50 \times (0.2)^2 = \mathbf{2.22 \times 10^{-10} C}.$$

S34. Given

$$q = 10 \mu\text{C}$$

$$= 1.0 \times 10^{-5} \text{ C}$$

$$2a = 5 \text{ mm} = 5 \times 10^{-3} \text{ m},$$

$$r = 15 \text{ cm} = 15 \times 10^{-2} \text{ m}$$

$\therefore$

$$p = q(2a) = 10^{-5} \times 5 \times 10^{-3} = 5 \times 10^{-8} \text{ C m}$$

For an electric dipole of short length ($2a << r$),

(a)

$$E_{\text{axial}} = \frac{1}{4\pi\varepsilon_0} . \frac{2p}{r^3} = \frac{9 \times 10^9 \times 2 \times 5 \times 10^{-8}}{(0.15)^3} = 2.67 \times 10^5 \text{ N C}^{-1}$$

(b)

$$E_{\text{equi}} = \frac{1}{4\pi\varepsilon_0} . \frac{p}{r^3} = \frac{9 \times 10^9 \times 5 \times 10^{-8}}{(0.15)^3} \text{ or } E_{\text{equi}} = \frac{E_{axial}}{2} = \mathbf{1.33 \times 10^5 N C^{-1}}$$

S35. *When two charged conducting spheres are connected then charge flows between the two, till their potentials become same.*

Electric potential on the surface of connected charged conducting sphere would be equal.

i.e.,

$$V_1 = V_2$$

$$\frac{1}{4\pi\varepsilon_0} . \frac{q_1}{r_1} = \frac{1}{4\pi\varepsilon_0} . \frac{q_2}{r_2} \quad \Rightarrow \quad \frac{q_1}{q_2} = \frac{r_1}{r_2}$$

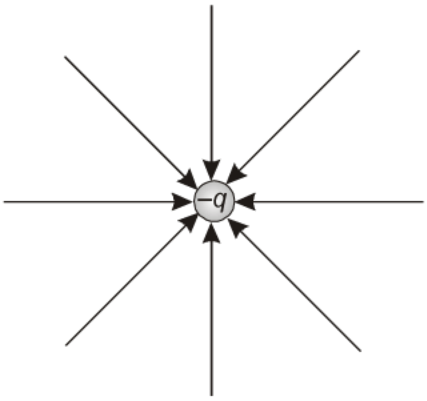
[Assuming q_1 and q_2 are charges on the spheres connected to each other and r_1, r_2 are their radii.]

Now, ratio of electric fields

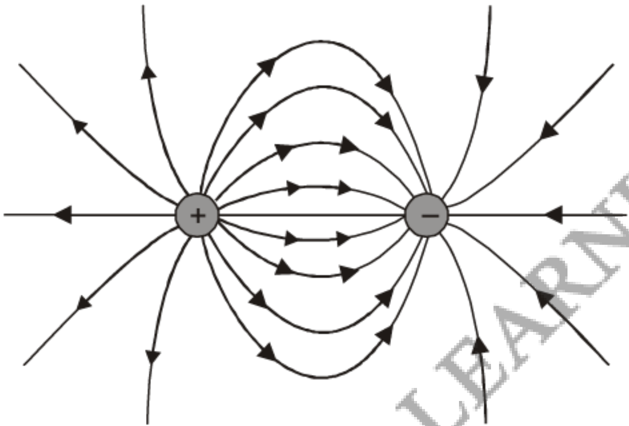
$$\frac{E_1}{E_2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_1^2} \times \frac{4\pi\varepsilon_0}{1} \cdot \frac{r_2^2}{q_2}$$

$$\frac{E_1}{E_2} = \frac{q_1}{q_2} \times \frac{r_2^2}{r_1^2} \Rightarrow \frac{E_1}{E_2} = \frac{r_2}{r_1}$$

S36. (a)



(b)

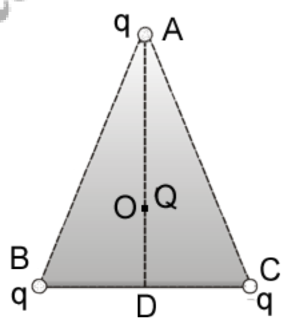


S37. As shown in figure draw $AD \perp BC$.

$\therefore$ $$AD = AB \cos 30° = \frac{l\sqrt{3}}{2}$$

Distance AO of the centroid O from A

$$= \frac{2}{3} AD = \frac{2l}{3} \frac{\sqrt{3}}{2} = \frac{l}{\sqrt{3}}$$



$\therefore$ Force on Q at O due to charge $q_1 = q$ at A

$$F_1 = \frac{1}{4\pi\varepsilon_0} \frac{Qq}{(l/\sqrt{3})^2} = \frac{3Qq}{4\pi\varepsilon_0 l^2} \quad \text{along AO}$$

Similarly, force on O due to charge $q_2 = q$ at B

$$F_2 = \frac{3Qq}{4\pi\varepsilon_0 l^2}, \text{ along } BO$$

and force on Q due to charge $q_3 = q$ at C

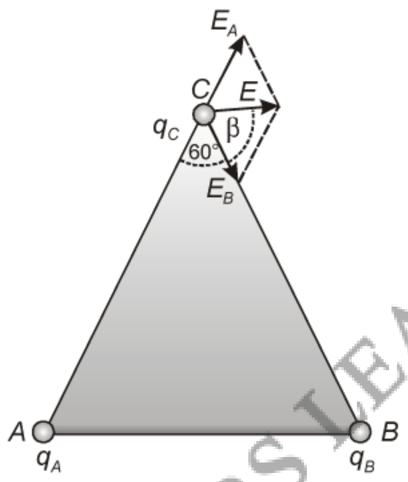
$$F_3 = \frac{3Qq}{4\pi\varepsilon_0 l^2}, \text{ along CO}$$

Angle between forces F_2 and $F_3 = 120°$

By parallelogram law, resultant of F_2 and $F_3 = \frac{3Qq}{4\pi\varepsilon_0 l^2}$, along OA

$\therefore$ Total force on $Q = \frac{3Qq}{4\pi\varepsilon_0 l^2} - \frac{3Qq}{4\pi\varepsilon_0 l^2} = 0$

S38. Given, $AB = BC = CA = 5 \text{ cm} = 5 \times 10^{-2}$ m



$$q_A = 60 \text{ StatC} = \frac{60}{3 \times 10^9} = 2 \times 10^{-8}\ C$$

$$q_B = -30 \text{ statC} = -10^{-8} \text{ C}$$

Now, $E_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A}{AC^2} = 7.2 \times 10^4\ N\ C^{-1}$ (along AC, when produced)

and $E_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_B}{AC^2} = 3.6 \times 10^4\ N\ C^{-1}$ (along CB)

Electric fields E_A and E_B are inclined at angle 180° – 60° i.e. 120°.

Therefore, resultant electric field at point C,

$$\text{E} = \sqrt{E_A^2 + E_B^2 + 2E_A E_B \cos 120°} = 6.24 \times 10^4\ N\ C^{-1}$$

If resultant electric field makes an angle β with CB, then

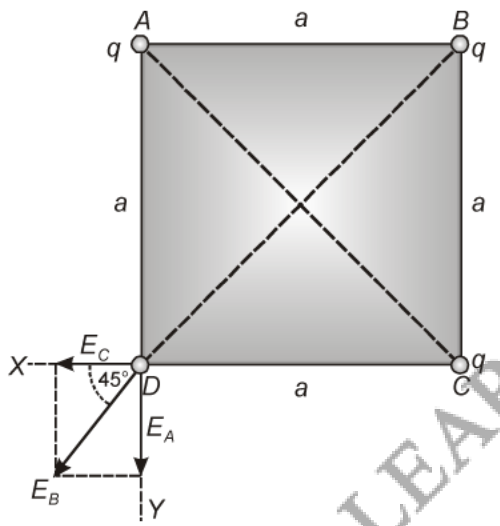
$$\tan\beta = \frac{E_A \sin 120^\circ}{E_B + E_A \cos 120^\circ} = \infty$$

or $$\beta = \mathbf{90^\circ}$$

S39. Given, $$AB = BC = CD = AD = a$$

$$BD = \sqrt{a^2 + a^2} = \sqrt{2}\, a$$

Now, $$E_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{(\sqrt{2}a)^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{2a^2}$$



Since E_A and E_C are equal, their resultant will be equally inclined to them *i.e.* it will act along BD.
If is resultant of E_A and E_C, them

$$E' = \sqrt{E_A^2 + E_C^2} = \sqrt{E_A^2 + E_A^2} = \sqrt{2}\, E_A \qquad \therefore \quad E_A = E_C$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{\sqrt{2}\,q}{2a^2} \quad \textbf{(along } BD\textbf{)}$$

Hence, the resultant of electric fields due to the three charges,

$$\text{E} = E' + E_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\sqrt{2}\,q}{2a^2} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{2a^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{2a^2}(\sqrt{2}+1) \quad \textbf{(along } BD\textbf{)}$$

S40. Given, $$Q = 0.02\ \mu\text{C} = 0.02 \times 10^{-6}\ \text{C}$$

and $$AB = BC = CD = AD = 2\ \text{cm}$$

$\therefore \qquad AO = BO = CO = DO = \frac{\sqrt{2^2 + 2^2}}{2} = \sqrt{2}\text{ cm} = \sqrt{2} \times 10^{-2}\text{ m}$

Now, $\qquad E_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Q}{(OA)^2} = \frac{1}{4\pi\varepsilon_0}.Q \times 10^4$

$$E_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Q}{(OB)^2} = \frac{1}{4\pi\varepsilon_0}.Q \times 10^4$$

$$E_C = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{(OC)^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2} \times 10^4$$

and $\qquad E_D = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{(OD)^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2} \times 10^4$

Net electric field along *OA*,

$$E_1 = E_A - E_C = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{2} \times 10^4$$

Also, net electric field along *OD*,

$$E_2 = E_B - E_D = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{2} \times 10^4$$

Hence, resultant electric field at the point *O*,

$$E = \sqrt{E_1^2 + E_2^2} = \frac{1}{4\pi\varepsilon_0}.\frac{Q}{\sqrt{2}} \times 10^4$$

$$= 9 \times 10^9 \times \frac{0.02 \times 10^{-6} \times 10^4}{\sqrt{2}}$$

$$= \mathbf{9\sqrt{2} \times 10^5 NC^{-1}} \textbf{(parallel to side } \boldsymbol{BA}\textbf{)}$$

S41. The electric field vector $\boldsymbol{E}_{1A}$ at *A* due to the positive charge q_1 points towards the right and has a magnitude

$$E_{1A} = \frac{(9 \times 10^9\text{ Nm}^2\text{C}^{-2}) \times (10^{-8}\text{ C})}{(0.05\text{ m})^2}$$

$$= 3.6 \times 10^4\text{ N C}^{-1}$$

The electric field vector $\boldsymbol{E}_{2A}$ at *A* due to the negative charge q_2 points towards the right and has the same magnitude. Hence the magnitude of the total electric field E_A at *A* is

$$E_A = E_{1A} + E_{2A} = 7.2 \times 10^4\text{ N C}^{-1}$$

$\boldsymbol{E}_A$ is directed toward the right.

The electric field vector $\boldsymbol{E}_{1B}$ at *B* due to the positive charge q_1 points towards the left and has a magnitude

$$E_{1B} = \frac{(9\times10^9\ \text{Nm}^2\text{C}^{-2})\times(10^{-8}\ \text{C})}{(0.05\ \text{m})^2}$$
$$= 3.6\times10^4\ \text{N C}^{-1}$$

The electric field vector $\boldsymbol{E}_{2B}$ at B due to the negative charge q_2 points towards the right and has a magnitude

$$E_{2B} = \frac{(9\times10^9\ \text{Nm}^2\text{C}^{-2})\times(10^{-8}\ \text{C})}{(0.15\ \text{m})^2}$$
$$= 4\times10^3\ \text{N C}^{-1}$$

The magnitude of the total electric field at B is

$$E_B = E_{1B} - E_{2B} = 3.2\times10^4\ \text{N C}^{-1}$$

$\boldsymbol{E}_B$ is directed towards the left.

The magnitude of each electric field vector at point C, due to charge q_1 and q_2 is

$$E_{1C} = E_{2C} = \frac{(9\times10^9\ \text{Nm}^2\text{C}^{-2})\times(10^{-8}\ \text{C})}{(0.10\ \text{m})^2}$$
$$= 9\times10^3\ \text{N C}^{-1}$$

The directions in which these two vectors point are indicated in figure. The resultant of these two vectors is

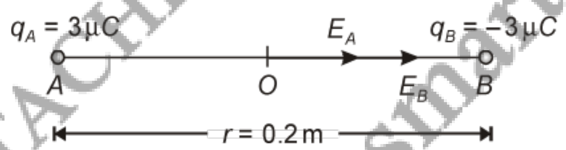
$$E_C = E_1\cos\frac{\pi}{3} + E_2\cos\frac{\pi}{3}$$
$$= 9\times10^3\ \text{N C}^{-1}$$

$\boldsymbol{E}_C$ points towards the right.

S42. Here, $q_A = 3\mu\text{C} = 3\times10^{-6}$ C;

$q_B = -3\mu\text{C} = -3\times10^{-6}$ C and $r = 20$ cm $= 0.2$ m

Let O be the mid-point of the line AB.



Then, $OA = OB = \frac{r}{2} = \frac{0.2}{2} = 0.1\text{m}$

(a) The electric field at point O due to q_A,

$$E_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_A}{(OA)^2} = 9\times10^9\times\frac{3\times10^{-6}}{(0.1)^2}$$

$$= 2.7\times10^6\ \text{N C}^{-1}\ \textbf{(along } \boldsymbol{OB}\textbf{)}$$

The electric field at point O due to q_B,

$$E_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_B}{(OB)^2} = 9\times10^9\times\frac{3\times10^{-6}}{(0.1)^2}$$

$$= 2.7\times10^6\ \text{N C}^1 \qquad \textbf{(along } \boldsymbol{OB}\textbf{)}$$

Therefore, net electric field at point O due to the charges q_A and q_B,

$$E = E_A + E_B = 2.7\times10^6 + 2.7\times10^6$$

$$= \mathbf{5.4\times10^6\ N\ C^{-1}}\ \textbf{(along } \boldsymbol{OB}\textbf{)}$$

(b) Force on a negative charge of magnitude 1.5×10^{-9} C placed at point O,

$$F = q\,E = 1.5\times10^{-9}\times5.4\times10^6$$

$$= \mathbf{8.1\times10^{-3}\ N}\ \textbf{(along } \boldsymbol{OA}\textbf{)}$$

The force on the negative charge acts in a direction opposite to that of the electric field.

S43. In figure (a) the field is upward, so the negatively charged electron experiences a downward force of magnitude eE where E is the magnitude of the electric field. The acceleration of the electron is

$$a_e = eE/m_e$$

where m_e is the mass of the electron.

Starting from rest, the time required by the electron to fall through a distance h is given by

$$t_e = \sqrt{\frac{2h}{a_e}} = \sqrt{\frac{2hm_e}{eE}}$$

For $e = 1.6\times10^{-19}$ C, $m_e = 9.11\times10^{-31}$ kg,

$E = 2.0\times10^4\ \text{N C}^{-1}$, $h = 1.5\times10^{-2}$ m,

$t_e = 2.9\times10^{-9}$ s

In figure (b), the field is downward, and the positively charged proton experiences a downward force of magnitude eE. The acceleration of the proton is

$$a_p = eE/m_p$$

where m_p is the mass of the proton; $m_p = 1.67\times10^{-27}$ kg. The time of fall for the proton is

$$t_e = \sqrt{\frac{2h}{a_p}} = \sqrt{\frac{2hm_p}{eE}} = 1.3\times10^{-7}\ \text{s}$$

Thus, the heavier particle (proton) takes a greater time to fall through the same distance. This is in basic contrast to the situation of 'free fall under gravity' where the time of fall is independent of the mass of the body. Note that in this example we have ignored the acceleration due to gravity in calculating the time of fall. To see if this is justified, let us calculate the acceleration of the proton in the given electric field:

$$a_p = \frac{eE}{m_p} = \frac{(1.6 \times 10^{-19}\ \text{C}) \times (2.0 \times 10^4\ \text{N C}^{-1})}{1.67 \times 10^{-27}\ \text{kg}}$$

$$= 1.9 \times 10^{12}\ \text{m s}^{-2}$$

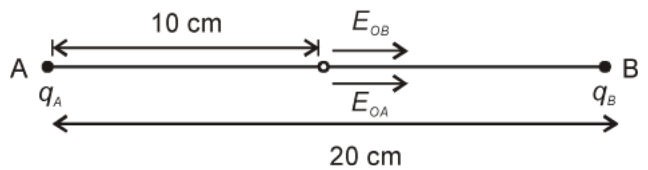
which is enormous compared to the value of $g\,(9.8\ \text{m s}^{-2})$, the acceleration due to gravity. The acceleration of the electron is even greater. Thus, the effect of acceleration due to gravity can be ignored in this example.

S44. Given

$$q_A = 3\ \mu\text{C}$$

$$q_B = -3\ \mu\text{C}$$

$$r = 20 \times 10^{-2}\ \text{m}$$



Electric field at 'O' due to charge at A is

$$E_{OA} = \frac{kq_A}{r^2} = \frac{9 \times 10^9 \times 3 \times 10^{-6}}{(0.1)^2} = 27 \times 10^5\ \text{NC}^{-1}$$

Similarly,

$$E_{OB} = \frac{kq_B}{r_{BO}^2} = \frac{9 \times 10^9 \times 3 \times 10^{-6}}{(0.1)^2} = 27 \times 10^5\ \text{NC}^{-1}$$

As $\qquad E_{OA} = E_{OB}$

Net electric field at mid-point is

$$E_O = E_{OA} + E_{OB}$$

$$= 2E_{OA} = 54 \times 10^5\ \text{NC}^{-1}$$

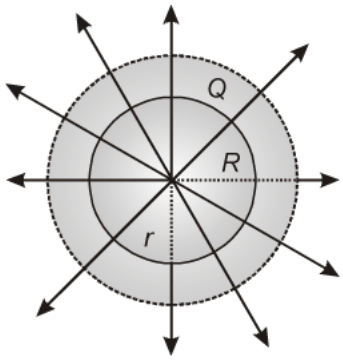
(b) $\qquad F_O = 1.5 \times 10^{-9} \times E_O$

$$= 1.5 \times 10^{-9} \times 54 \times 10^5$$

$$= 8.1 \times 10^{-3}\ \text{N}$$

Direction of force is along –*ve* x-axis.

S45. Consider the given spherical shell of radius R holding charge Q. Construct a Gaussian surface of radius r (concentric and symmetrical). The field lines will pass perpendicularly through the Gaussian surface in all the direction.



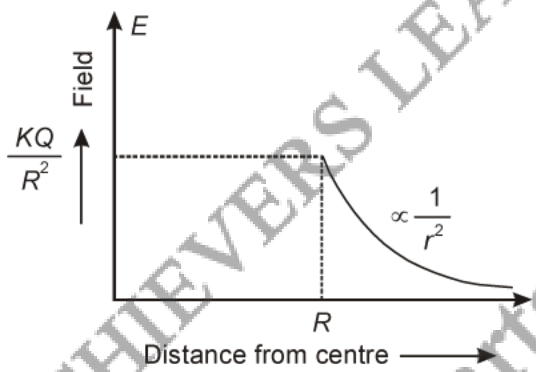
So, the effective Gaussian surface area having normal field will be $4\pi r^2$.

Using Gauss theorem, $\oint \vec{E}.\vec{ds} = \frac{q_{en}}{\epsilon_0}$ we get.

$$\int E\, ds \cos 0^\circ = \frac{Q}{\epsilon_0}$$

$\Rightarrow$ $$E \int ds = \frac{Q}{\epsilon_0}$$

$\Rightarrow$ $$E\, 4\pi r^2 = \frac{Q}{\epsilon_0}$$



$\therefore$ $$E = \frac{Q}{4\pi \epsilon_0 r^2}$$

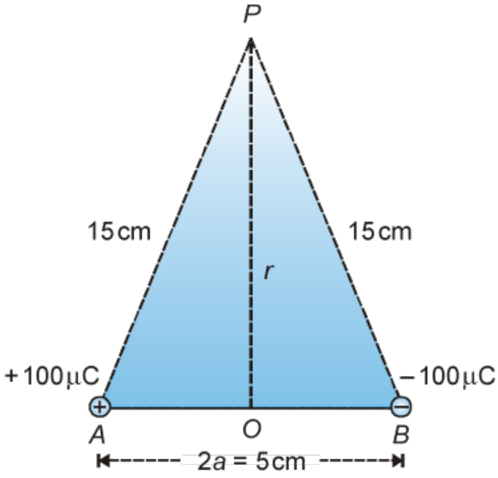
For all points inside the shall, since $q_{en} = 0$, $E = 0$. The graph showing the variation of E with distance r from the centre is shown.

S46. The force experienced by a unit positive test charge placed at a point gives the magnitude of the electric field at that point and its direction is the direction of the experienced force.

Let P be the point at a distance of 20 cm from either charge as shown in figure.

Then, $AP = BP = 15\,\text{cm} = 0.15\,\text{m}$.

Let E electric field at point P due to the electric dipole. The point P lies at a distance $OP = r$ on the equatorial line of the dipole. Therefore,



$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{P}{(r^2 + a^2)^{3/2}} \qquad \text{(i)}$$

Given, $q = 100\,\mu C = 10^{-4}\,C$

and $2a = 5\text{ cm} = 0.05\text{ m}$

$\therefore$ $p = q(2a) = 10^{-4} \times 0.05\text{ Cm}$

Also, $(OP^2 + OA^2)^{1/2} = AP$

or $(r^2 + a^2)^{1/2} = 0.15\text{ m}$

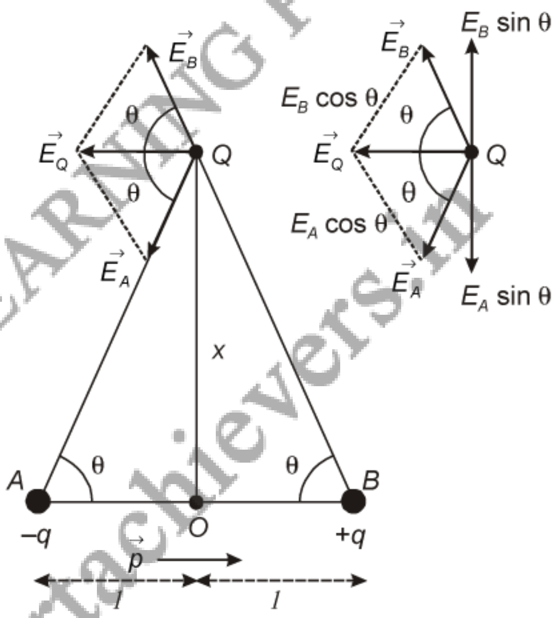
Setting these values in Eq. (i), we get

$$E = \frac{9\times10^9 \times 10^{-4} \times 0.05}{(0.15)^3} = \frac{9}{8}\times10^7 = \mathbf{1.33\times10^8\,NC^{-1}}.$$

S47. Resultant electric field intensity at the point Q is

$$\vec{E}_Q = \vec{E}_A + \vec{E}_B$$

The vectors $\vec{E}_A$ and $\vec{E}_B$ are acting at an angle 2θ.



$$\vec{E}_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{(x^2 + l^2)}\hat{i}$$

$$\vec{E}_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{(x^2 + l^2)}\hat{i}$$

The vectors $E_A \sin\theta$ and $E_B \sin\theta$ are opposite to each other and hence cancel out.

The vectors $E_A \cos\theta$ and $E_B \cos\theta$ are acting along the same direction and hence add up.

$\therefore$ $E_Q = E_A \cos\theta + E_B \cos\theta$

$$E_Q = \frac{2}{4\pi\varepsilon_0}\frac{q}{(x^2 + l^2)}\frac{l}{(x^2 + l^2)^{1/2}}$$

$$E_Q = \frac{1}{4\pi\varepsilon_0}\frac{q\cdot 2l}{(x^2 + l^2)^{3/2}}$$

$$E_Q = \frac{1}{4\pi\varepsilon_0}\frac{p}{(x^2 + l^2)^{3/2}}$$

$$\vec{E_Q} = \frac{1}{4\pi\varepsilon_0} \frac{q \cdot 2l}{(x^2 + l^2)^{3/2}} (-\hat{i})$$

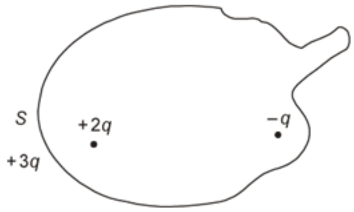
If $l \ll x$, then

$$E_Q = \frac{1}{4\pi\varepsilon_0} \frac{p}{x^3}$$

The direction of electric field intensity at a point on the equatorial line due to a dipole is parallel and opposite to the direction of the dipole moment.

Date: 20/10/2021

Q1. Figure shows three point charges, $+2q$, $-q$ and $+3q$. Two charges $+2q$ and $-q$ are enclosed within a surface S. What is the electric flux due to this configuration through the surface S?



Q2. Two charges of magnitudes –2Q and + Q are located at points (a, 0) and (4a, 0) respectively. What is the electric flux due to these charges through a sphere of radius '3a' with its centre at the origin?

Q3. A charge q is placed at the centre of a cube of side l. What is the electric flux passing through each face of the cube?

Q4. Write an expression for the flux $\Delta\Phi$, of the electric field $\vec{E}$, through an area element $\vec{\Delta S}$.

Q5. An area S is held inside electric field E, such that normal to the area S subtends angle θ with the direction of electric field. What is the electric flux linked with the surface area?

Q6. Define electric flux. write its SI units.

Q7. What is the relation between electric field intensity and electric flux?

Q8. A spherical rubber balloon carries a charge 2 that is uniformly distributed over its surface. As the balloon is blown up and increases in size, how does the total electric flux coming out of the surface change? Give reason.

Q9. Define electric flux. Write its SI unit.

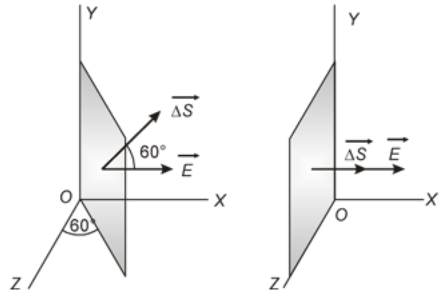
A charge q is enclosed by a spherical surface of radius R. If the radius is reduced to half, how will the electric flux through the surface change?

Q10. A thin straight infinitely long conduction wire having charge density λ is enclosed by a cylindrical surface of radius r and length, its axis coinciding with the length of the wire. Find the expression for the electric flux through the surface of the cylinder.

Q11. A cubical Gaussian surface encloses a charge of 8.85×10^{-10} C in vacuum at centre. Calculate the electric flux through the one of its faces.

Q12. Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is 4.0×10^3 N m^2 C^{-1}. (a) What is the net charge inside the box? (b) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? why or why not?

Q13. Consider a uniform electric field $\vec{E} = 3 \times 10^3 \hat{i}\ NC^{-1}$. (a) What is the flux of this field through a square of 10 cm on a side, whose plane is parallel to the *YZ*-plane? (b) What is the flux through the some square, if the normal to its plane makes a 60° angle with the *X*-axis?



Q14. A rectangular surface of sides 10 cm and 15 cm is placed inside a uniform electric field of 25 Vm^{-1} making an angle 60° with the direction of electric field. Find the flux of the electric field through the rectangular surface.

Q15. Calculate the number of electric lines of force originating from a charge of 5 C.

Given, $\varepsilon_0 = 8.854 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$.

Q16. The flux of the electrostatic field, through the closed surface *S′*, is found to be four times that through the closed spherical surface *S*. Find the magnitude of the charge *Q*. Given $q_1 = 1\ \mu C$, $q_2 = -2\ \mu C$, and $q_3 = 9.854\ \mu C$.

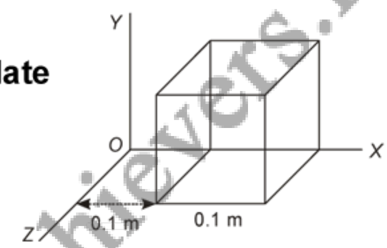
Q17. A large plane sheet of charge having surface charge density $5.0 \times 10^{-6}\ C\ m^{-2}$ lies in the *XY*-plane. Find the electric flux through a circular area of radius 0.1 m, if the normal to the circular area makes an angle of 60° with the *Z*-axis. Given that $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$.

Q18. The electric field components due to a charge inside the cube of side 0.1 m are shown in the figure.

$E_x = \alpha, x$, where $\alpha = 500\ N C^{-1}\ mm^{-1}$, $E_y = 0$ and $E_z = 0$. Calculate

(a) the electric flux through the cube and

(b) the charge inside the cube.



Q19. (a) A rectangular frame of wire of 25 cm × 15 cm is placed in a uniform electric field of strength $2 \times 10^4\ N\ C^{-1}$, such that the plane of the coil is normal to field. Find the electric flux linked with the rectangular frame.

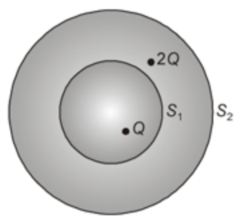
(b) Calculate the electric flux linked with the frame, when it is converted into (i) a square and (ii) a circular frame.

(c) In which case is the electric flux maximum?

Q20. S_1 and S_2 are two parallel concentric spheres enclosing charges *Q* and 2*Q* respectively as shown in Figure.

(a) What is the ratio of the electric flux through S_1 and S_2?

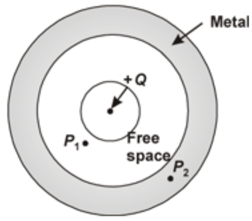
(b) How will the electric flux through the sphere S_1 change, if a medium of dielectric constant 5 is induced in the space inside S_1 placed of air?



Q21. A uniform electric field E = $E_x \hat{i} N/C$ for $x > 0$ and E = $E_x \hat{i} N/C$ for $x < 0$ are given.
A right circular cylinder of length cm and radius r cm has its centre at the origin and its axis along the x-axis. Find out the net outward flux. Using the Gauss's law write the expression for the net charge within the cylinder.

Q22. (a) A small metal sphere carrying charge $+Q$ is located at the centre of a spherical cavity inside a large uncharged metallic spherical shell as shown in the figure. Use Gauss's law to find the expressions for the electric field at points P_1 and P_2

(b) Draw the pattern of electric field lines in this arrangement.



Q23. There is uniform electric field of $3 \times 10^3 \hat{i} NC^{-1}$. What is the net flux of the uniform electric field through a cube of side 20 cm oriented so that its faces are parallel to the co-ordinate plane?

Date: 20/10/2021

S1. Electric flux through the closed surface S is

$$\phi_S = \frac{\Sigma q}{\varepsilon_0} = \frac{+2q - q}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

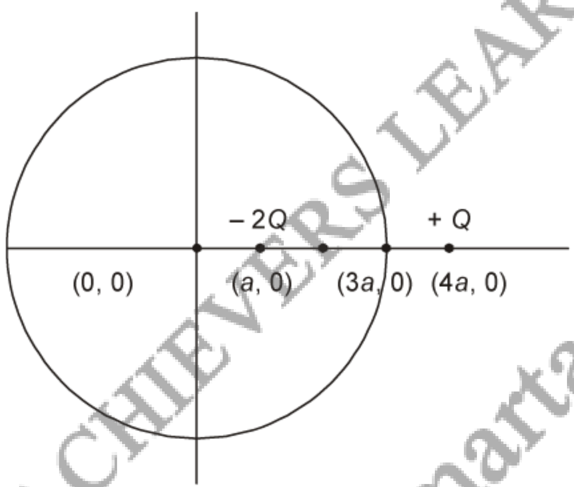
$\Rightarrow$ $$\phi_S = \frac{q}{\varepsilon_0}.$$

Note: Charge + 3q is outside the closed surface S, therefore, it would not be taken into consideration in applying Gauss's theorem.

S2. Gauss's theorem states that the total electric flux linked with closed surface s is

$$\phi_E = \mathbf{E}\,.\,d\mathbf{S} = \frac{q}{E_0}$$

where q is the total charge enclosed by the closed Gaussian (imaginary) surface.



The sphere enclose charge $= -2Q + Q$

$= -Q$

Therefore, $$\phi = \frac{Q}{\varepsilon_0}.$$

S3. By Gauss's theorem, total electric flux linked with a closed surface is given by

$$\phi = \frac{q}{\varepsilon_0}$$

where q is the total charge enclosed by the closed surface

$\therefore$ Total electric flux linked i^{th} cube $\frac{q}{\varepsilon_0}$.

As charge is at centre, therefore, electric flux is symmetrically distributed through all 6 faces.

$$\text{Flux linked with each face} = \frac{1}{6}\phi = \frac{1}{6} \times \frac{q}{\varepsilon_0}$$

$$= \frac{q}{6\varepsilon_0}.$$

S4. If an elementary surface area ΔS is held inside the electric field E and E_n is the component of electric flux through the surface,

$$\Delta\Phi = E_n \Delta S$$

In vector notation,

$$\Delta\Phi = \vec{E} \cdot \Delta\vec{S}$$

S5. The electric flux linked with the surface is given by

$$\Phi = ES\cos\theta$$

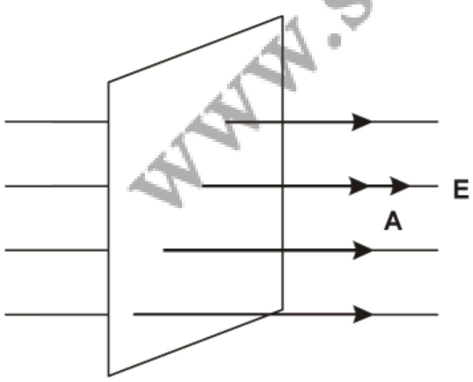
S6. Total number of electric lines of force passing through a given area normally is called electric flux through that area.

Its SI unit is $NC^{-1}m^2$.

S7. $d\Phi = \vec{E} \cdot \vec{dA}$ where $d\Phi$ is electric flux, E is the electric field dA is small area of the surface.

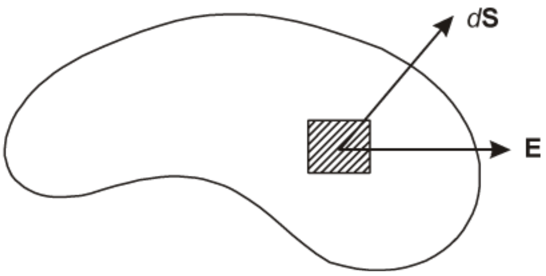
S8. The total electric flux coming out of the surface remains unchanged because it depends only on the charge enclosed by the surface.

S9. The total electric flux linked with a surface is equal to the total number of electric lines of force passes through the surface when surface is held normal to the direction on electric field.



$$\phi = EA$$

If surface is placed in non-uniform, then electric field



Total electric flux linked with the closed surface

$$\phi = \int_S \mathbf{E} \cdot d\mathbf{S}$$

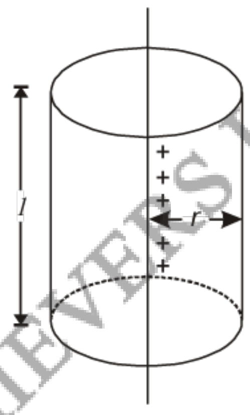
SI unit: N-m^2/C

The total electric flux coming out of the surface remains unchanged because it depends only on the charge enclosed by the surface.

S10. *A thin straight conducting wire will be a uniform linear charge distribution.*

Let q charge be enclosed by the cylindrical surface

$\therefore$ Linear charge density $\lambda = \dfrac{q}{l}$



Charge enclosed by the cylindrical surface

$\therefore$ $q = \lambda l$ ... (i)

By Gauss's theorem,

$\therefore$ Total electric flux through the surface of cylinder

$$\phi = \frac{q}{\varepsilon_0}$$ [Gauss's theorem]

$\therefore$ $$\phi = \frac{\lambda l}{\varepsilon_0}$$ [From Eq. (i)]

S11. $$q = 8.85 \times 10^{-19}\,\text{C}$$

Total electric flux through cubical Gaussian surface

$$\phi = \frac{q}{\varepsilon_0} = \frac{8.85 \times 10^{-10}}{8.85 \times 10^{-12}} = 10^2 \text{ N-m}^2/\text{C} \qquad \text{.... (i)}$$

As charge is present at the centre of the cube, therefore, total flux is equallized through each of 6 faces

$\therefore$ Electric flux through each face = $\frac{1}{6} \times \phi$

$$= \frac{1}{6} \times 10^2$$

$$= \frac{100}{6} = 16.67 \text{ N-m}^2/\text{C}.$$

S12. (a) Given: $\Phi = 4.0 \times 10^3 \text{ N m}^2 \text{ C}^{-1}$; $\varepsilon_0 = 8.854 \times 10^{-12}\, C^2 N^{-1} m^{-2}$

Suppose that the net charge inside the box is q. Then, according to Gauss′ theorem,

$$\Phi = \frac{q}{\varepsilon_0} \quad \text{or} \quad q = \varepsilon_0 \Phi$$

$$= 8.854 \times 10^{-12} \times 4.0 \times 10^3$$

$$= \mathbf{2.542 \times 10^{-8}\ C}.$$

(b) If the net outward flux through the surface of box were zero. Then, according to Gauss's theorem

$$\Phi = \frac{q}{\varepsilon_0} \quad \text{or} \quad q = \varepsilon_0 \Phi$$

$$\Phi = 0 \quad \text{therefore, } q = 0.$$

Hence, net charge inside the box is zero

S13. (a) The area of a surface can be represented as a vector along normal to the surface.

Here, $\vec{E} = 3 \times 10^3\, \hat{i}\, NC^{-1}$

Area of the square, $\Delta S = 10 \times 10 = 100 \text{ cm}^2 = 10^{-2}\, m^2$

Since normal to the square is along X-axis, we have

$$\vec{\Delta S} = 10^{-2}\, \hat{i}\, m^2$$

Electric flux through the square,

$$\phi = \vec{E} . \vec{\Delta S} = (3 \times 10^3\, \hat{i}).(10^{-2}\, \hat{i})$$

$$= \mathbf{30\ Nm^2\ C^{-1}}$$

(b) Here, angle between normal to the square *i.e*. area vector and the electric field is 60°. Therefore,

$$\phi = \vec{E}.\Delta\vec{S} = E\,\Delta S\cos 60^\circ = 3\times 10^3\times 10^{-2}\times 0.5$$

$$= \mathbf{15\ N\ m^2\ C^{-1}}$$

S14. The flux through the rectangular surface is given by

$$\phi = \vec{E}.\Delta\vec{S} = E\,\Delta S\cos\theta$$

Here, $E = 25\text{ V } m^{-1}$;

$$\Delta S = 10\times 15 = 150\text{ cm}^2 = 150\times 10^{-4}\text{ m}^2,\ \theta = 60^\circ$$

$\therefore$ $\phi = 25\times 150\times 10^{-4}\cos 60^\circ$

$$= 0.1875\text{ N m}^2\text{ C}^{-1}$$

S15. Here, $q = 5\text{ C};\ \varepsilon_0 = 8.854\times 10^{-12}\text{ C}^2\text{ N}^{-1}\text{ m}^{-2}$

From Gauss' theorem, electric flux *i.e.*, electric lines of force through a surface enclosing a charge q is given by

$$\Phi = \frac{q}{\varepsilon_0}$$

Therefore, electric lines of force originating from a charge of 5 C,

$$\Phi = \frac{5}{8.854\times 10^{-12}}$$

$$= 5.65\times 10^{11}\text{ Nm}^2\text{C}^{-1}$$

S16. Let Φ and Φ' be the flux of electrostatic field through the closed surface S and S' respectively.

According to Gauss' theorem,

$$\Phi = \frac{q_1+q_2+q_3}{\varepsilon_0}$$

$$= \frac{1\times 10^{-6}+(-2\times 10^{-6})+9.854\times 10^{-6}}{\varepsilon_0}$$

or $\Phi = \dfrac{8.854\times 10^{-6}}{\varepsilon_0}$

$$\Phi' = \frac{Q + q_1 + q_2 + q_3}{\varepsilon_0}$$

$$= \frac{Q + 1\times 10^{-6} + (-2\times 10^{-6}) + 9.854\times 10^{-6}}{\varepsilon_0}$$

or $$\Phi' = \frac{Q + 8.854\times 10^{-6}}{\varepsilon_0}$$

Since $\Phi' = 4\,\Phi$, we have

$$\frac{Q + 8.854\times 10^{-6}}{\varepsilon_0} = 4\times\frac{8.854\times 10^{-6}}{\varepsilon_0}$$

or $Q = (4 - 1)\times 8.854\times 10^{-6}$ C = $\mathbf{26.562\ \mu C}$

S17. The electric flux through a surface,

$$\Phi = \vec{E}.\vec{\Delta S} = E\ \Delta S\cos\theta$$

We know that electric field near the plane sheet of charge,

$$E = \frac{\sigma}{2\varepsilon_0}$$

and circular area, $\Delta S = \pi r^2$

$$\therefore \quad \Phi = \frac{\sigma}{2\varepsilon_0}\times\pi r^2\cos\theta$$

Here, $\sigma = 5.0\times 10^{-6}$ C m^{-2};

$\varepsilon_0 = 8.85\times 10^{-12}$ C^{2}N^{-1} m^2; $r = 0.1$ m

and $\theta = 60°$

Setting these values, $$\Phi = \frac{5.0\times 10^{-6}\times\pi\times(0.1)^2\cos 60°}{2\times 8.85\times 10^{-12}}$$

$$= \mathbf{4.44\times 10^3\ N\ m^2\ C^{-1}}$$

S18. Given: $E_x = 500$ N C^{-1}, $E_y = 0$ and $E_z = 0$.

(a) Since $E_y = E_z = 0$, the electric flux is only through the left and right faces of the cube. Let each side of the cube be of length *a*.

The electric field at the left face,

$$E_{\text{left}} = 500\ a\,\text{N C}^{-1}$$

The electric flux through the left face,

$$\Phi_{\text{left}} = \vec{E}_{left} \cdot \vec{\Delta S}_{left} = E_{left}\,\Delta S_{left} \cos 180^\circ$$

$$= 500\ a \times a^2 \times (-1) = -500\ a^3$$

The electric field at the right face,

$$E_{\text{right}} = 500 \times 2\ a = 1,000\ a\ N\ C^{-1}$$

The electric flux through the right face,

$$\Phi_{\text{right}} = \vec{E}_{right} \cdot \vec{\Delta S}_{right} = E_{right}\ \Delta S_{right} \cos 0^\circ$$

$$= 1{,}000a \times a^2 \times 1 = 1,000\ a^3$$

Therefore, net electric flux through the cube,

$$\Phi = \Phi_{\text{left}} + \Phi_{\text{right}} = -500\ a^3 + 1{,}000\ a^3 = 500\ a^3$$

$$= 500 \times (0.1)^3 = 0.5\ \text{N m}^2\text{C}^{-1}$$

(b) According to Gauss' law,

$$q = \varepsilon_0\Phi = 8.854 \times 10^{-12} \times 0.5 = \mathbf{4.425 \times 10^{-12}\ C}$$

S19. Given $A = 25 \times 15 = 375\ \text{cm}^2 = 375 \times 10^{-4}\ m^2$;

$E = 2 \times 10^4\ \text{N C}^{-1}$

(a) $$\Phi = E\,A = 2 \times 10^4 \times 375 \times 10^{-4}$$

$$= \mathbf{750\ N\ m^2\ C^{-1}}$$

(b) (i) When rectangular frame is converted into a square frame:

One side of the square = $\dfrac{2(25+15)}{4} = 20$ cm

Area of square, $A_1 = 20 \times 20 = 400\ \text{cm}^2 = 400 \times 10^{-4}\ m^2$

Now, $\Phi_1 = E\,A_1 = 2 \times 10^4 \times 400 \times 10^{-4} = \mathbf{800\ N\ m^2\ C^{-1}}$

(ii) When rectangular frame is converted into a circular frame:

Radius of circle, $r = \dfrac{2(25+15)}{2\pi} = \dfrac{40}{\pi}$ cm

Area of circle, $A_2 = \pi r^2 = \pi\left(\dfrac{40}{\pi}\right)^2$

$$= 509.3\ cm^2 = 509.3 \times 10^{-4}\ m^2$$

Now, $$\Phi_2 = E A_2 = 2 \times 10^4 \times 509.3 \times 10^{-4}$$

$$= \mathbf{1,018.6\ Nm^2C^{-1}}$$

(*c*) Since Φ_2 is greater than both Φ and Φ_1, the electric flux is maximum in case of circular frame.

S20. (a) Let Φ_1 and Φ_2 be the electric flux through the spheres S_1 and S_2 respectively. Then,

$$\Phi_1 = \frac{Q}{\varepsilon_0} \quad \text{... (i)}$$

and $$\Phi_2 = \frac{Q + 2Q}{\varepsilon_0} = \frac{3Q}{\varepsilon_0} \quad \text{... (ii)}$$

From Eq. (i) divided by (ii)

and $$\frac{\Phi_1}{\Phi_2} = \frac{Q}{\varepsilon_0} \Big/ \frac{3Q}{\varepsilon_0} = \frac{1}{3}$$

(b) Suppose E be electric field on the surface of the sphere S_1 due to the charge Q placed inside the sphere. Then, by Gauss's theorem,

$$\Phi_2 = \oint \vec{E} \cdot \vec{dS} = \frac{Q}{\varepsilon_0} \quad \text{... (iii)}$$

On introducing medium of dielectric constant K inside the sphere S_1, suppose that electric field on the surface of sphere becomes $\vec{E}'$. Then,

$$\vec{E} = \frac{1}{K}\vec{E} \quad \text{... (iv)}$$

If the electric flux through the sphere is now Φ_1. then

$$\Phi_1 = \oint \vec{E}' \cdot \vec{ds} = \oint \frac{1}{K}\vec{E} \cdot \vec{ds} = \frac{1}{K}\frac{Q}{\varepsilon_0}$$ [from the equations (iii) and (iv)]

Here, $K = 5$

$\therefore$ $$\Phi_1' = \frac{Q}{5\varepsilon_0}.$$

S21. Electric flux linked with curved surface

$$\mathbf{E} \cdot \mathbf{dS} = E\,dS \cos 90° = 0$$

According to problem

$$\mathbf{E}_R = E_x \hat{\mathbf{i}} \qquad [\text{For } x > 0]$$

$$\mathbf{E}_L = -E_x \hat{\mathbf{i}} \qquad [\text{For } x < 0]$$

$\therefore$ Toala flux linked with the right circular cylinder

$$\phi = \phi_R + \phi_L$$

where, $\phi_R = \mathbf{E}_R . \mathbf{A} = E_A A \cos 0°$

$$= E_x \left(\frac{\pi r^2}{10^4}\right) \qquad [\because A = \pi r^2]$$

$$\phi_L = \mathbf{E}_L \mathbf{A} = E_L A \cos 0°$$

$$= E_x \left(\frac{\pi r^2}{10^4}\right) \qquad [1 \text{ cm} = 10^{-2} \text{ m}]$$

$\therefore$ $\phi = \phi_R + \phi_L$

$$= [E_x (\pi r^2) + E_x (\pi r^2)] \times 10^{-4}$$
$$= 2\pi r^2 E_x \times 10^{-4} \text{ N-m}^2/\text{C}$$

By Gauss's law theorem $\phi = \dfrac{q}{\varepsilon_0}$

$\therefore$ Net charge $q = \phi\varepsilon_0 = 2E_x \pi r^2 \varepsilon_0 \times 10^{-4}$ C.

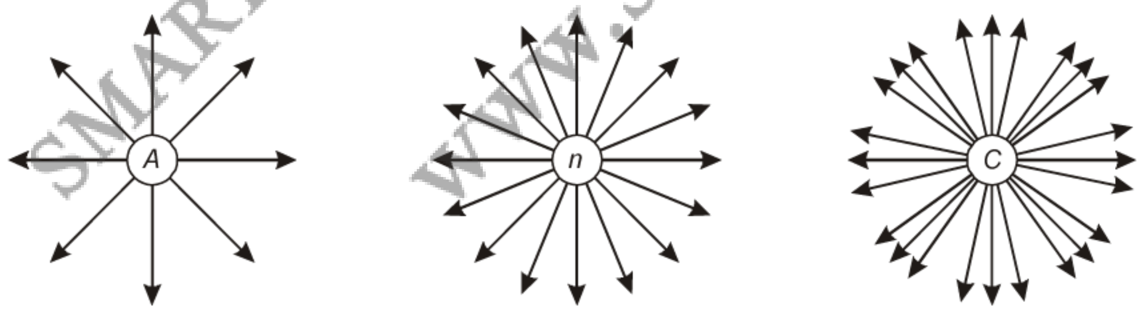
S22.(a) Using Gauss's theorem

$$E \times 4\pi_1^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r_1^2}$$

Field at $P_2 = 0$, because the electric field inside the conductor is zero.

(b)



S23. Given: $\vec{E} = 3 \times 10^3 \hat{i}\, N C^{-1}$; $A = 20 \times 20 = 400 \text{cm}^2 = 4 \times 10^{-2} \text{ m}^2$

Through a face parallel to XY-plane:

$$\vec{A}_1 = 4 \times 10^{-2}\, \hat{k}\, m^2 ;$$

$$\Phi_1 = \vec{E}.\vec{A}_1 = (3 \times 10^3\, \hat{i}).(4 \times 10^{-2}\, \hat{k}) = 0$$

Through a face parallel to *YZ*-plane:

$$\vec{A}_2 = 4 \times 10^{-2}\, \hat{i}\, m^2$$

$$\therefore \quad \Phi_2 = \vec{E}.\vec{A}_2 = (3 \times 10^3\, \hat{i})\,(4 \times 10^2\, \hat{i})$$

$$= 120 \text{ N m}^2\text{ C}^{-1}$$

Through a face parallel to *ZX*-plane:

$$\vec{A}_3 = 4 \times 10^{-2}\, \hat{j}\, m^2$$

$$\therefore \quad \vec{\Phi}_3 = \vec{E}.\vec{A}_3 = (3 \times 10^3\, \hat{i})\,(4 \times 10^{-2}\, \hat{j}) = 0$$

In each plane, there is a set of two faces of the cube. Through one face, electric flux enters and through the other face, an equal flux leaves.

Therefore, net flux through the cube

$$\Phi = (\Phi_1 - \Phi_1) + (\Phi_2 - \Phi_2) + (\Phi_3 - \Phi_3) = \mathbf{0}\,.$$

Date: 20/10/2021

Q1. When is the torque acting on an electric dipole maximum, when placed in uniform electric field?

Q2. Does an electric dipole always experience a torque, when placed in a uniform electric field?

Q3. What is the direction of electric field at a point on the equatorial of an electric dipole?

Q4. What is the direction of electric field at a point on axial line of an electric dipole?

Q5. Is it correct to write the unit of electric dipole moment as mC?

Q6. An electric dipole of dipole moment 20×10^{-6} cm is enclosed by a closed surface. What is the net flux coming out the surface?

Q7. Which orientation of an electric dipole in a uniform electric field would correspond to stable equilibrium?

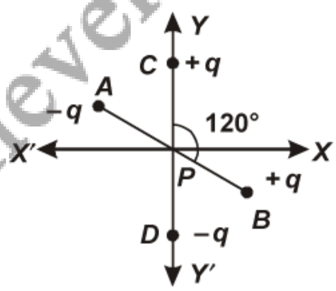
Q8. Define the term electric dipole moment of a dipole. State its S.I unit.

Q9. What is the net force on an electric dipole placed in a uniform electric field?

Q10. In what orientation does an electric dipole experience a zero torque and non-zero force in a non-uniform electric field?

Q11. What meaning would you give to the capacitance of a single conductor?

Q12. Two small identical dipoles *AB* and *CD*, each of dipole moment '*p*' are kept at an angle of 120° as shown in the figure. What is the resultant dipole moment of this combination? If this system is subjected to electric field (E) directed along $+X$-direction, what will be the magnitude and direction of the torque acting on this?



Q13. In which orientation, a dipole placed in a uniform electric field is in (a) stable, (b) unstable equilibrium?

Q14. An electric dipole of dipole moment $\vec{p}$ is present in a uniform electric field $\vec{E}$. Write the value of the angle between $\vec{p}$ and $\vec{E}$ for which the torque experienced by the dipole is minimum.

Q15. What is an ideal electric dipole? Give the examples.

Q16. The distance of the field point on the field on the equatorial plane of a small electric dipole is halved. By what factor will the electric field due to the dipole change?

Q17. An electric dipole consists of two equal and opposite charges placed 2 cm apart. When the dipole is placed in a uniform electric field of strength 10^5 NC^{-1}, it experiences a maximum torque of 0.2×10^{-3} N m. Find the magnitude of each charge.

Q18. A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at the points A (0, 0 –15 cm) and B (0, 0 + 15 cm). What is the total charge and electric dipole moment?

Q19. What is the ratio of the strength of electric field at a point on axial line and at a point at same distance of equatorial line of an electric dipole of very small length?

Q20. An electric dipole, when held at 30° with respect to a uniform electric field of 10^4 NC^{-1} experiences a torque of 9×10^{-26} Nm. Calculate dipole moment of the dipole.

Q21. The define intensity of electric field at a point. At what points is the electric dipole field intensity parallel to the line joining the charges?

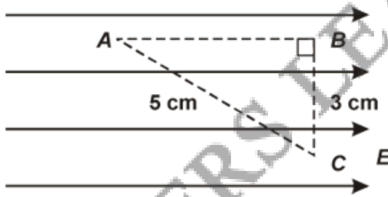
Q22. A dipole, with a dipole moment of magnitude p, is in stable equilibrium in an electrostatic field of magnitude E. Find the work done in rotating this dipole to its position of unstable equilibrium.

Q23. A dipole is present in an electrostatic field of magnitude 10^6 N/C. If the work done in rotating it, from its position of stable equilibrium to its position of unstable equilibrium, 2×10^{-23} J, find the magnitude of the dipole moment of this dipole.

Q24. Draw the equipotential surfaces due to an electric dipole. Locate the points where the potential due to the dipole is zero.

Q25. An electric dipole is held in a uniform electric field. Using suitable diagram, show that it does not undergo any translatory motion.

Q26. A test charge, q is moved without acceleration from A to C along the path from A to B and then from B to C in electric field E as shown in the figure. (a) Calculate the potential difference between A and C. (b) At which point (of the tow) is the electric potential more and why?



Q27. Obtain an expression for electric field at a point on (a) the axial line and (b) the equatorial line of an electric dipole. (c) Compare the two expressions. (d) How these expressions differ from that of a point charges?

Q28. An electric dipole is held at an angle θ in a uniform externally applied electric field E. (a) what is net force acting on the dipole (b) what is torque acting on it (c) Does it possess any potential energy?

Q29. Define the term dipole moment. Derive expression for the total work done in rotating the dipole through an angle θ in a uniform electric field and find the P.E.

Date: 20/10/2021

S1. The torque is maximum, when the electric dipole is placed perpendicular to the direction of electric field.

S2. No. It does not experience a torque, when it is placed along the direction of electric field.

S3. Opposite to the electric dipole moment vector.

S4. Same as that of electric dipole moment vector.

S5. No, it is not correct to write the unit of electric dipole moment as mC. The symbol mC represents milli-coulomb *i.e.*, unit of electric charge. The correct unit Cm.

S6. Zero because the total charge of the surface is zero.

S7. When an electric dipole is situated with its axis parallel to the uniform electric field, it is in stable equilibrium.

S8. Dipole moment is the product of charge 'q' & the separation between the pair of charge q & $-q$ or $p = q \times 2a$ S.I. unit: Cm.

S9. It is zero because dipole has equal and opposite charges which have equal force and opposite direction it is cancel each other.

S10. When the dipole is placed parallel to the non-uniform electric field.

S11. The capacitance of a single conductor is considered as a parallel plate capacitor with one of its two plates at infinity.

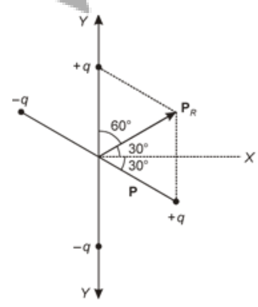
S12. Consider the figure,

$$|\mathbf{P}_A| = \mathbf{P}_C = P$$

The resultant, $\mathbf{P}_R$ is magnitude

$$|\mathbf{P}_R| = 2P\cos\frac{\theta}{2} = 2P\cos\frac{120^\circ}{2} = P$$

$\mathbf{P}_R$ will subtend an angle of 30 with X-axis.



Now, torque acting on the system,

$$\tau = \mathbf{P}_R \times \mathbf{E} = P_R E \sin\theta = \frac{1}{2} PE$$

Torque will work to align the dipole in direction of electric field **E**.

S13. (a) The dipole is in stable equilibrium, when $\vec{p}$ and $\vec{E}$ are parallel.

(b) The dipole is in unstable equilibrium, when $\vec{p}$ and $\vec{E}$ are antiparallel.

S14. 0°

S15. An electric dipole is a pair of equal and opposite point-charges placed at a short distance.

Example of electric dipole HCl, H_2O.

S16. Since $E \propto 1/r^3$, the electric field point will become 8 times on decreasing the distance of the field point to one half.

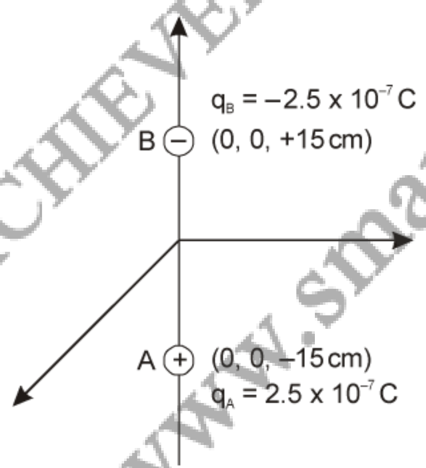
S17. Given: $2a$ = 2 cm = 0.02 m; $\tau_{max} = 0.2 \times 10^{-3}$ Nm and $E = 10^5$ N C^{-1}

Now, maximum torque on the dipole, when $\theta = 90°$

$$\tau_{max} = p\,E = q(2\,a) \times E$$

$$\therefore \qquad q = \frac{\tau_{max}}{(2a)\,E} = \frac{0.2 \times 10^{-3}}{0.02 \times 10^5} = \mathbf{10^{-7}\ C}$$

S18. The charges q_A and q_B are located at the points A (0, 0, – 15 cm) and B (0, 0, + 15 cm). The points A and B lie on Z-axis as shown in Figure below:



It follows that $AB = OA = OB$ = 15 + 15 = 30 cm = 0.3 cm

Total charge, $q = q_A + q_B$

$$= 2.5 \times 10^{-7} + (-2.5 \times 10^{-7}) = 0$$

Electric dipole moment, p = either charge $\times AB$

$$= 2.5 \times 10^{-7} \times 0.3$$

$$= 7.5 \times \mathbf{10^{-8}\,Cm}$$

The electric dipole moment is direction always opposite direction of electric field.

Therefore, the electric dipole moment is directed from A to B along Z-axis.

S19. Electric field at distance r on axial line and equatorial line due to an electric dipole of very small length are given by

$$E_{\text{axal}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3}$$

and $$\mathrm{E_{equi}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^3}$$

$\therefore$ $$\frac{E_{axial}}{E_{equi}} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3}\bigg/\frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^3} = \mathbf{2}$$

S20. Given: $E = 10^4$ N C^{-1}; $\tau = 9 \times 10^{-26}$ Nm and $\theta = 30°$

When electric dipole is placed at an angle θ with the direction of the electric field, torque acting on the dipole,

$$\tau = p\,E\sin\theta$$

Therefore, electric moment of the dipole,

$$p = \frac{\tau}{E\sin\theta} = \frac{9 \times 10^{-26}}{10^4 \times 0.5}$$

$$= \mathbf{1.8 \times 10^{-29}\ C\ m}$$

S21. The intensity of electric field at a point in space is the ratio of the force acting on a unit test charge placed at that point in space.

Electric field intensity due to an electric dipole is parallel to the line joining the charge at points

(a) On axial line of the dipole and

(b) On the plane, which is right bisector of the dipole.

S22. For stable equilibrium, angle between **p** and **E**, $\theta_1 = 0°$.

For unstable equilibrium $\theta_1 = 180°$.

Work done in rotating the dipole from angle θ_1 to θ_2

$$W = pE(\cos\theta_1 - \cos\theta_2)$$

$$= pE(\cos 0° - \cos 180°) = 2pE$$

$$W = \mathbf{2pE}.$$

S23. Electric field intensity $E = 10^6$ N/C

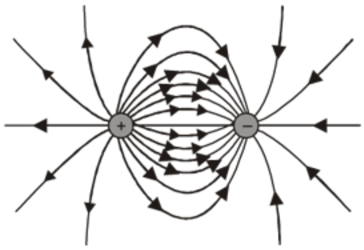
Work done $(W) = 2 \times 10^{-23}$ J

Work done in rotating the dipole from stable equilibrium position to unstable equilibrium position.

$$W = pE(\cos 0° - \cos 180°) = 2pE$$

$$\therefore \quad p = \frac{W}{2E} = \frac{2 \times 10^{-23}}{2 \times 10^6} = \mathbf{10^{-29}\ Cm.}$$

S24.

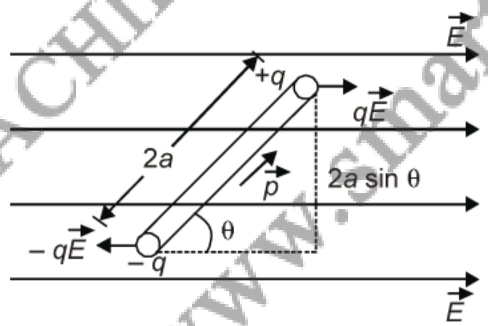


S25. The electric dipole moment of a dipole is equal to the product of either charge (q) and the vector $2\vec{a}$ (separation) between the two charges. It is denoted by $\vec{p}$. Thus,

$$\vec{p} = q \times 2\vec{a}$$

It is directed from negative to positive charge.

Let us consider a dipole placed in a uniform electric field making an angle θ with the field direction, as shown in figure below.



Two equal and opposite forces $+q\vec{E}$ and $-q\vec{E}$ act on its two ends and constitute a couple. Hence, it does not undergo any translatory motion

S26. (a) $\because$ Electric field intensity and potential difference are related as

$$E = -\frac{\Delta V}{\Delta r}$$

$\Rightarrow$ $$\Delta V = -E\Delta r$$

$\Rightarrow$ $$V_A - V_C = -(2+2)E$$

$$V_A - V_C = -4E$$

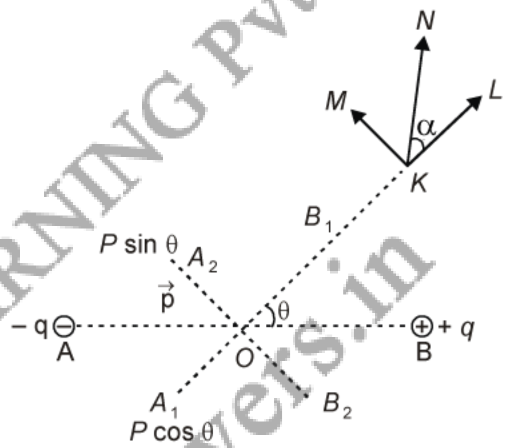
$$V_C - V_A = 4E$$

(b) As $V_C - V_A = 4E$, is positive

$\therefore$ $$V_C > V_A$$

Potential is greater at point C than point A, as potential decreases along the direction of electric field.

S27. Consider an electric dipole (as shown in the figure) of dipole moment $\vec{p}$ along $\overrightarrow{AB}$. With the point O being the centre of the dipole, K is the point where the field due to the dipole is to be calculated, where $OK = r$ and $\angle BOK = \theta$.



The dipole moment $\vec{p}$ can be resolved into two rectangular components $p\cos\theta$ (along, A_1, B_1) and $p\sin\theta$ (along $A_2B_2 \perp A_1B_1$)

Electric field at K, along OK, represented by $\overrightarrow{KL}$, is given by $E_1 = \dfrac{2p\cos\theta}{4\pi\varepsilon_0 r^3}$

Electric field at K, perpendicular to OK, represented by d, is given by $E_2 = \dfrac{P\sin\theta}{4\pi\varepsilon_0 r^3}$

when rectangle $KLMN$ is represented, according to $\parallel$ gm law, $\overrightarrow{KN}$ represents the resultant electric field d at K due to the dipole.

As $$KN = \sqrt{KL^2 + KM^2}$$

$\therefore$ $$|\vec{E}| = \sqrt{E_1^2 + E_2^2} = \sqrt{\left(\frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}\right)^3 + \left(\frac{p\sin\theta}{4\pi\varepsilon_0 r^3}\right)^3} = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{3\cos^2\theta + (\cos^2\theta + \sin^2\theta)} = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{3\cos^2\theta + 1}$$

or $$E = \frac{p\sqrt{3\cos^2\theta + 1}}{4\pi\varepsilon_0 r^3} \quad \text{... (i)}$$

Let $\angle LKN = \alpha$, In ΔKLN, $\tan\alpha = \dfrac{LN}{KL} = \dfrac{KM}{KL} = \dfrac{p\sin\theta}{4\pi\varepsilon_0 r^3}\cdot\dfrac{4\pi\varepsilon_0 r^3}{2p\cos\theta}$

or $$\tan\alpha = \frac{1}{2}\tan\theta \quad \Rightarrow \quad \alpha = \tan^{-1}\left(\frac{1}{2}\tan\theta\right) \qquad \text{... (ii)}$$

Important Cases:

(a) When the point K on *axial line of* dipole, then ($\theta = 0° \Rightarrow \cos 0° = 1$)

We put the value $\theta = 0°$ in Eq. (i), we get $\alpha = 0°$. *i.e.*, the resultant field is along the axial line.

(b) When the point K lies on the *equatorial line* of dipole, then ($\theta = 90° \Rightarrow \cos 90° = 0$)

We put the value $\theta = 90°$ in Eq. (i), we get $\alpha = 90°$. *i.e.*, the resultant field is perpendicular to the equatorial line and hence parallel to the axial line of the dipole.

(c) On comparison we see that the magnitude of the field in axial case is double than that in equatorial line case and the direction of the field in the two cases is opposite.

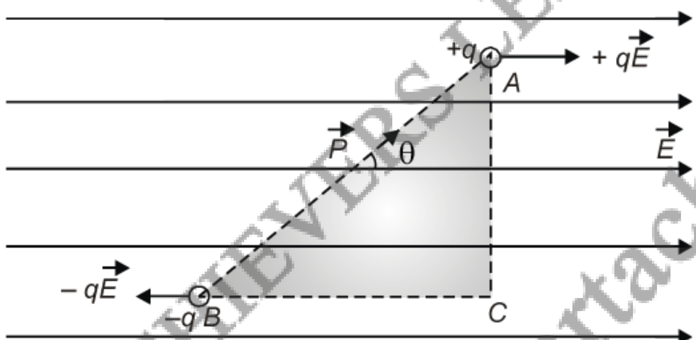
(d) In case of dipole, field has $\frac{1}{r^3}$ dependence unlike in case of point charge $\frac{1}{r^2}$.

S28. Let us consider an electric dipole consisting of two charges + q and – q separated by distance $2a$. Its dipole moment is $\vec{p} = q \times 2\vec{a}$.

(a) The dipole is placed in a uniform electric field $\vec{E}$ making an angle θ with the field (as shown in the figure).

Force experienced by the positive charge (+q) = + $q\vec{E}$, in the direction of the field $\vec{E}$

Force experienced by the negative charge (– q) = – $q\vec{E}$, in the direction on opposite to that of field $\vec{E}$.



Therefore, the net force on a dipole in a uniform field = + $q\vec{E}$ – $q\vec{E}$ = 0.

(b) Although, the two forces on the ends of the dipole are equal and opposite and cancel each other, however, the two forces act at different points and form a couple exerting a torque.

The torque τ, is given by

Torque = Either force × perpendicular distance between

$$\tau = qE \times 2a\sin\theta$$
$$= q \times 2a\,E\sin\theta$$

$$= pE \sin\theta$$

or $$\vec{\tau} = \vec{p} \times \vec{E}$$

The torque, or tuning effect, acting on the dipole reduce the angle θ to zero making the dipole moment vector parallel to the field $\vec{E}$. In other words, the dipole align itself parallel to the field.

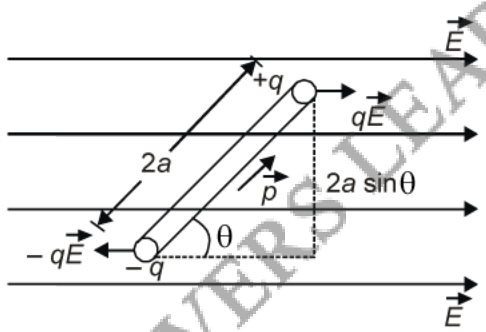
(c) Yes. When the orientation is some non-zero angle θ there must be potential energy stored in the dipole by the virtue of torque (or turning effect). If one wants to rotate the dipole from orientation $\theta = 0°$ (in which a dipole orient itself if left free) to some non-zero θ, he has to do some work against the torque due to the applied electric field, which is stored in the form of its potential energy.

S29. The electric dipole moment of a dipole is equal to the product of either charge (q) and the vector $2\vec{a}$ (separation) between the two charges. It is denoted by $\vec{p}$. Thus,

$$\vec{p} = q \times 2\vec{a}$$

It is directed from negative to positive charge.

Potential energy of an electric dipole placed in a uniform electric field. Let us consider a dipole placed in a uniform electric field making an angle θ with the field direction, as shown in figure below.



Two equal and opposite forces $+q\vec{E}$ and $-q\vec{E}$ act on its two ends and constitute a couple. The torque exerted by the couple is given by

$$\tau = qE \times 2a \sin\theta$$
$$= pE \sin\theta \quad (\because \; p = q \times 2a)$$

This torque tends to align the dipole in the direction of the field.

If the dipole is rotated through a small angle $d\theta$ against the torque acting on it, a small work dW is required to be done, which is given by

$$dW = \tau d\theta = pE \sin\theta \, d\theta$$

If the dipole is rotated from initial angle θ_i to a final orientation of angle θ_f, then the total work done in rotating the dipole is given by

$$E = \int_{\theta_i}^{\theta_f} dW = \int_{\theta_i}^{\theta_f} pE \sin\theta \, d\theta = pE \int_{\theta_i}^{\theta_f} \sin\theta \, d\theta$$

$$= pE \left|-\cos\right|_{\theta_i}^{\theta_f} = pE(\cos\theta_i - \cos\theta_f)$$

This work done is stored as the potential energy *U* of the system.

$\therefore$ $$U = pE (\cos \theta_i - \cos \theta_f)$$

If initially the dipole is perpendicular to the direction of the field *i.e.* $\theta_i = 90°$ and finally brought to the orientation making an angle θ *i.e.*, $\theta_f = \theta$ with the field direction, then potential energy of the dipole is given by

$$U = pE (\cos 90° - \cos \theta) = pE(0 - \cos \theta)$$

$$= - pE \cos \theta$$

or $$U = \vec{p} . \vec{E}$$

Date: 20/10/2021

Q1. What is the difference between a sheet of charge and a plane conductor having charge?

Q2. Does the strength of electric field due to an infinitely long line charge depend upon the distance of the observation point from line charge?

Q3. What is the application of Gaussian surface?

Q4. A charge q is placed at the centre of a cube of side l. what is the electric flux passing through two opposite faces of the cube?

Q5. A box encloses an electrical dipole consisting of charge $-5\ \mu C$ and of length 10 cm. What is the total electric flux through the box?

Q6. State Gauss' theorem in electrostatics.

Q7. How does electric field at a point change with distance r from an infinitely long charged wire?

Q8. A long charged cylinder of linear charged density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders?

Q9. An infinite line charge produces a field of $3 \times 10^4\ N\ C^{-1}$ at a distance of 4 cm. Calculate the linear charge density.

Q10. Find the electric flux through each face of a hallow cube of side 10 cm, if a charge of $8.854\ \mu C$ is placed at its centre.

Q11. A uniformly charged conducting sphere of 2.5 m in diameter has a surface charge density of $100 \mu C\ m^{-2}$, flux passing through the sphere.

Q12. A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q. A charge q is placed at the centre of the shell.

(a) What is the surface charge density on the (i) inner surface, (ii) outer surface of the shell?

(b) Write the expression for the electric field at a point $x > r_2$ from the centre of the shell.

Q13. An electric flux of $-6 \times 10^3\ Nm^2C^{-1}$ passes through a spherical Gaussian surface of radius 10 cm due to a point charge placed at the centre. (a) What is the charge enclosed by the Gaussian surface? (b) If the radius of the Gaussian surface is doubled, how much flux would pass through the surface?

Q14. Assume that earth has a charge of surface density 1 electron per metre^2. Calculate the earth's (a) electric field (b) electric potential outside the earth's surface.

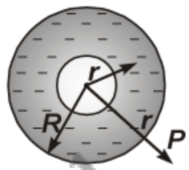
Q15. Two large parallel thin metallic plates are placed close to each other. The plates have charge densities of opposite signs and of magnitude 2.0×10^{-12} C/m^2. Calculate the electric field intensity

(a) in the outer region of the plates and

(b) in the interior region between the plates.

Q16. Two uniformly large parallel thin plates having charge densities $+\sigma$ and $-\sigma$ are kept in the $X - Z$ plane at a distance d apart. Sketch an equipotential surface due to electric field between the plates. If a particle of mass m and charge $-q$ remains stationary between the plates. What is the magnitude and direction of this field?

Q17. An early model for an atom considered it to have a positively charged point nucleus of charge Ze, surrounded by a uniform density of negative charge up to a radius R. The atom as a whole is neutral. For this model, what is the electric field at a distance r from the nucleus?



Q18. An electric field is uniform, and in the positive x direction for positive x, and uniform with the same magnitude but in the negative x direction for negative x. It is given that $E = 200\,\hat{i}$ N/C for $x > 0$ and $E = -200\,\hat{i}$ N/C for $x < 0$. A right circular cylinder of length 20 cm and radius 5 cm has its centre at the origin and its axis along the x-axis so that one face is at $x = +10$ cm and the other is at $x = -10$ cm (figure). (a) What is the net outward flux through each flat face? (b) What is the flux through the side of the cylinder? (c) What is the net outward flux through the cylinder? (d) What is the net charge inside the cylinder?

Q19. Two large conducting plates are placed parallel to each other with a separation of 2 cm between them. An electron starting from rest near on of the plates reaches the other plate in 2 μS. Find the surface charge density of the plates.

Q20. The radius of gold nucleus ($Z = 79$) is about 7.0×10^{-15} m. Assuming that the positive charge is distributed uniformly throughout the nuclear volume, find the strength of the electric field (a) at the surface of the nucleus and (b) at the middle point of the radius.

Q21. Using Gauss's law obtain the expression for the electric field due to uniformly charged spherical shell of radius R at a point outside the shell. Draw a graph showing the variation of electric field with r, for $r > R$ and $r < R$.

Q22. State Gauss' Law. A thin conducting spherical shell of radius R has charge Q spread uniformly over its surface. Using Gauss's law, derive an expression for an electric field at a point outside the shell.

Q23. Use Gauss's law to derive the expression for the electric field between two uniformly charge parallel sheets with surface charge densities σ and $-\sigma$ respectively.

Q24. State Gauss's theorem. Give its mathematical expression. Derive an expression for the electric field intensity at any point due to an infinite plane sheet of charge density σ C/m^2.

Q25. State Gauss's theorem. Use it is derive an expression for the electric field of a thin uniformly long straight line of charge, with a uniform charge density of 1 C/m.

Q26. Two infinite parallel planes have uniform charge densities σ_1 and σ_2 C/m^2. What is the electric field at points (a) to the left of the sheets, (b) between them and (c) to the right of the sheets?

Q27. State Gauss's theorem in electrostatics. Using this theorem show that for a spherical shell, the electric field inside the shell vanishes, whereas outside it, the field is as if all the charge had been concentrated at the centre.

Q28. (a) Define electric flux. Write its SI units.

(b) Using Gauss's law, prove that the electric field at a point due to a uniformly charged infinite plane sheet is independent of the distance from it.

(c) How is the field directed if

(i) the sheet is positively charged,

(ii) negatively charged?

Date: 20/10/2021

S1. On a sheet of charge, the same charge shows up on its two sides; whereas in case of a charged plane conductor the charges showing up on the two surface are not the same.

S2. Yes, the electric field due to an infinitely long line charge depends upon the distance of the observation point from the line charge.

S3. In order to find electric field due to a charge distribution by applying Gauss's theorem, one is required to evaluate a surface integral. The surface integral can be evaluated easily by choosing a suitable Gaussian surface for the charge distribution.

S4. The electric flux due to the charge q through all the six faces of the cube,

$$\Phi = \frac{q}{\varepsilon_0}$$

Therefore, electric flux through the two opposite faces,

$$\Phi = \frac{2}{6} \times \Phi = \frac{2}{6} \times \frac{q}{\varepsilon_0} = \frac{q}{3\varepsilon_0}$$

S5. Since net charge enclosed by the box is zero, electric flux through the box is also zero.

S6. The electric flux Φ_E through any closed surface is equal to $1/\varepsilon_0$ time the 'net' charge q enclosed by the surface.

S7. The electric field due to a line charge fall off with distance as $1/r$.

S8. Charge density of the long charged cylinder of length L and radius r is λ.

Another cylinder of same length surrounds the pervious cylinder. The radius of this cylinder is R.

Let E be the electric field produced in the space between the two cylinders.

Electric flux through the Gaussian surface is given by Gauss's theorem as,

$$\phi = E(2\pi d)L$$

Where, d = Distance of a point from the common axis of the cylinders

Let q be the total charge on the cylinder.

It can be written as

$$\therefore \quad \phi = E(2\pi dL) = \frac{q}{\varepsilon_0}$$

Where, q = Charge on the inner sphere of the outer cylinder

ε_0 = Permittivity of free space

$$E(2\pi dL) = \frac{\lambda L}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 d}$$

Therefore, the electric field in the space between the two cylinders is $\frac{\lambda}{2\pi\varepsilon_0 d}$.

S9. Given, $E = 3 \times 10^4\ N\ C^{-1}$; $r = 4$ cm = 0.04 m

Electric field at a distance r from a line charge having linear charge density λ is given by

$$E = \frac{1}{2\pi\varepsilon_0}.\frac{\lambda}{r}$$

$$\lambda = 2\pi\varepsilon_0\, E\, r = 4\pi\varepsilon_0.\frac{E\, r}{2}$$

$$= \frac{1}{9\times10^9}\times\frac{3\times10^4\times0.04}{2}$$

$$= \mathbf{2 \times 10^{-7}\ C\ m^{-1}}$$

S10. Given: $q = 8.854\ \mu C = 8.854\times10^{-6}\,C$

Total electric flux through the cube (all the six faces),

$$\Phi = \frac{q}{\varepsilon_0} = \frac{8.854\times10^{-6}}{8.854\times10^{-12}} = 10^6\ Nm^2\ C^{-1}$$

Therefore, electric flux through each face of the cube,

$$\Phi = \frac{1}{6}\Phi = \frac{10^6}{6} = 1.67\times10^5\ Nm^2\ C^{-1}.$$

S11. Given: $\sigma = 100\ \mu C\,m^{-2} = 10^{-4}\ Cm^{-2}$; $R = \frac{2.5}{2} = 1.25\,m$

Now $q = 4\pi R^2\sigma$

$= 4\pi\times(1.25)^2\times10^{-4} = \mathbf{1.96\times10^{-3}\,C}$

$$\Phi = \frac{q}{\varepsilon_0} = \frac{1.96\times10^{-3}}{8.854\times10^{-12}} = \mathbf{2.21\times10^8\ Nm^2\ C^{-1}}$$

S12. (a) (i) Surface charge density on the inner surface of the shell,

$$\sigma_{\text{inner}} = \frac{q}{4\pi r_1^2}$$

(ii) Surface charge density on the outer surface of the shell,

$$\sigma_{\text{outer}} = \frac{Q+q}{4\pi r_2^2}$$

(b) Electric field at a point $x > r_2$ from the centre of the shell,

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q+q}{x^2}$$

S13. (a) $q = \varepsilon_0\Phi = 8.854 \times 10^{-12} \times (-6 \times 10^3) = -5.31 \times 10^{-8}$ C.

(b) There will be no change in flux, when the radius of Gaussian surface is doubled.

S14. Given: $\sigma = 1$ electron $m^{-2} = -1.6 \times 10^{-19}$ C m^{-2}; Radius of the earth, $R = 6400$ km $= 6.4 \times 10^6$ m

Therefore, total charge on the earth, $q = 4\pi R^2 \sigma$

(a)
$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4\pi R^2\sigma}{R^2} = \frac{\sigma}{\varepsilon_0}$$

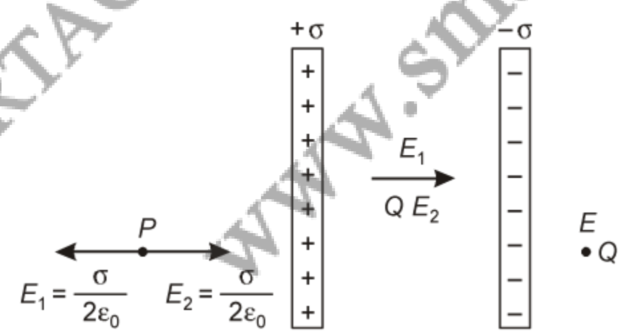
$$= \frac{(-1.6\times 10^{-19})}{8.854\times 10^{-12}}$$

$$= \mathbf{-0.18 \times 10^{-7}\ N/C.}$$

(b)
$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4\pi R^2\sigma}{R} = \frac{R\sigma}{\varepsilon_0} = \frac{6.4 \times 10^6 \times (-1.6 \times 10^{-19})}{8.854\times 10^{-12}}$$

$$= \mathbf{-0.1157\ V}$$

S15. The electric field due to positive charge is directed away from it and electric field due to negative charge is directed towards it.



(a) Electric field intensity due to thin large charged metallic plate at any nearby point is

$$E = \frac{\sigma}{2\varepsilon_0}$$

Electrical field at point P (in the outer region)

$$\mathbf{E} = \mathbf{E} + \mathbf{E}$$

$$= \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

∴ Net field at point P is zero as field due to +ve and –ve plate is equal and opposite.

(b) Electric field intensity at any point in the interior region between the plates in

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

from positive plate to negative plate.

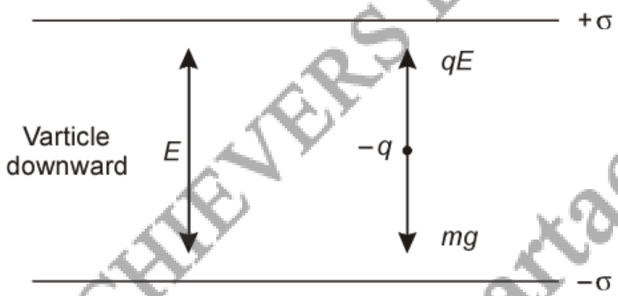
$$E = \frac{\sigma}{\varepsilon_0} = \frac{2 \times 10^{-12}}{8.85 \times 10^{-12}} = \mathbf{0.226\ N/C}.$$

S16. $-q$ charge experiences force in a direction opposite to the direction of electric field.

∴ $-q$ charge balance when

$$qE = mg$$

$$E = \frac{mg}{q}$$



The direction of electric field along vertically downward direction.

Note: The $X - Z$ plane is so chosen that the direction of electric field due to two plates is along vertically downward direction, otherwise weight (mg) of charge particle could not be balances.

S17. The charge distribution for this model of the atom is as shown in figure. The total negative charge in the uniform spherical charge distribution of radius R must be $-Ze$, since the atom (nucleus of charge Ze + negative charge) is neutral. This immediately gives us the negative charge density Á, since we must have

$$\frac{4\pi R^3}{3}\rho = 0 - Ze$$

or $$\rho = -\frac{3Ze}{4\pi R^3}$$

To find the electric field $\boldsymbol{E}(\boldsymbol{r})$ at a point P which is a distance r away from the nucleus, we use Gauss's law. Because of the spherical symmetry of the charge distribution, the magnitude of the electric field $\boldsymbol{E}(\boldsymbol{r})$ depends only on the radial distance, no matter what the direction of $\boldsymbol{r}$. Its direction is along (or opposite to) the radius vector $\boldsymbol{r}$ from the origin to the point P. The obvious Gaussian surface is a spherical surface centred at the nucleus. We consider two situations, namely, $r < R$ and $r > R$.

(i) $r < R$: The electric flux ϕ enclosed by the spherical surface is

$$\phi = E(r) \times 4\pi r^2$$

where $E(r)$ is the magnitude of the electric field at r. This is because the field at any point on the spherical Gaussian surface has the same direction as the normal to the surface there, and has the same magnitude at all points on the surface.

The charge q enclosed by the Gaussian surface is the positive nuclear charge and the negative charge within the sphere of radius r,

i.e., $$q = \frac{4\pi R^3}{3}\rho$$

Substituting for the charge density Á obtained earlier, we have

$$q = Ze - Ze\frac{r^3}{R^3}$$

Gauss's law then gives,

$$E(r) = \frac{Ze}{4\pi\varepsilon_0}\left(\frac{1}{r^2} - \frac{r}{R^3}\right); \quad r < R$$

The electric field is directed radially outward.

(ii) $r > R$: In this case, the total charge enclosed by the Gaussian spherical surface is zero since the atom is neutral. Thus, from Gauss's law,

$$E(r) \times 4\pi r^2 = 0 \quad \text{or} \quad E(r) = 0; \quad r > R$$

At $r = R$, both cases give the same result: $E = 0$.

S18. (a) We can see from the figure that on the left face $\boldsymbol{E}$ and $\Delta\boldsymbol{S}$ are parallel. Therefore, the outward flux is

$$\phi_L = \boldsymbol{E}\cdot\Delta\boldsymbol{S} = -200\ \hat{\boldsymbol{i}}\cdot\Delta\boldsymbol{S}$$

$$= +200\,\Delta S, \quad \text{since } \hat{\boldsymbol{i}}\cdot\Delta\boldsymbol{S} = -\Delta S$$

$$= +200 \times \pi(0.05)^2 = +1.57\ \text{N m}^2\,\text{C}^{-1}$$

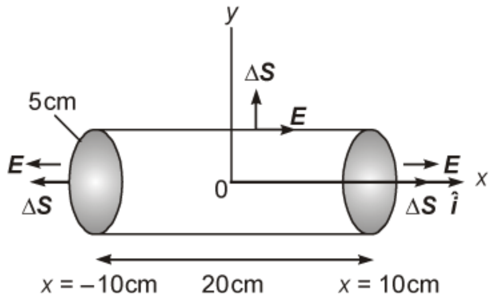
On the right face, $\boldsymbol{E}$ and $\Delta\boldsymbol{S}$ are parallel and therefore

$$\phi_R = \boldsymbol{E} \cdot \Delta\boldsymbol{S} = +\,1.57\ \text{N m}^2\,\text{C}^{-1}.$$

(b) For any point on the side of the cylinder $\boldsymbol{E}$ is perpendicular to $\Delta\boldsymbol{S}$ and hence $\boldsymbol{E} \cdot \Delta\boldsymbol{S} = 0$. Therefore, the flux out of the side of the cylinder is zero.

(c) Net outward flux through the cylinder

$$\phi = 1.57 + 1.57 + 0$$
$$= 3.14\ \text{N m}^2\,\text{C}^{-1}$$



(d) The net charge within the cylinder can be found by using Gauss's law which gives

$$q = \varepsilon_0 \phi$$
$$= 3.14 \times 8.854 \times 10^{-12}\ \text{C}$$
$$= 2.78 \times 10^{-11}\ \text{C}.$$

S19. Let σ be the surface charge density of the conducting plates. Then, electric field between the conducting plates,

$$E = \frac{2\sigma}{\varepsilon_0}$$

Force on electron, $$F = e \times \frac{2\sigma}{\varepsilon_0} \quad \text{...(i)}$$

If m is mass of the electron, then acceleration produced,

$$a = \frac{F}{m} = \frac{e}{m} \times \frac{2\sigma}{\varepsilon_0}$$

Since the electron is initially at rest,

$$S = 0 \times t + \frac{1}{2} a t^2 = \frac{1}{2} \times \frac{e}{m} \times \frac{2\sigma}{\varepsilon_0} \times t^2$$

or $$\sigma = \frac{\varepsilon_0\, mS}{e\, t^2} \quad \text{...(ii)}$$

Given, $m = 9.1 \times 10^{-31}$ kg; S = 2 cm = 0.02 m ;

$e = 1.6 \times 10^{-19}$ C and $t = 2\mu\text{s} = 2 \times 10^{-6}$ s

Setting the values in Eq. (ii) we get

$$\sigma = \frac{8.854 \times 10^{-12} \times 9.1 \times 10^{-31} \times 0.02}{1.6 \times 10^{-19} \times (2 \times 10^{-6})^2}$$

$$= \mathbf{2.52 \times 10^{-13}\ Cm^{-2}}$$

S20. Given: $q = +Ze = 79 \times 1.6 \times 10^{-19}$ C, $R = 7.0 \times 10^{-15}$ m

(a) Electric field at the surface of gold nucleus,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2} = \frac{9 \times 10^9 \times 79 \times 1.6 \times 10^{-19}}{(7.0 \times 10^{-15})^2}$$

$$= 2.32 \times 10^{21}\ \text{N C}^{-1}$$

(b) Despite the fact that gold is a conductor, it has been assumed that the positive charge is uniformly distributed over the entire nuclear volume. To find electric field at the middle point of the radius, draw a spherical shell of radius $R/2$ as the Gaussian surface. Therefore, charge enclosed by Gaussian surface,

$$q' = \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \times \rho$$

$$= \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \times \frac{q}{\frac{4}{3} \times \pi R^3} = \frac{q}{8}$$

Therefore, electric field at the middle point of the radius,

$$E' = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q'}{(R/2)^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q/8}{(R/2)^2}$$

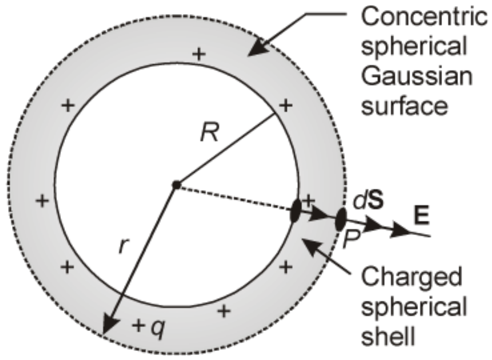
$$= \frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R^2}\right) = \frac{1}{2}E$$

$$= \frac{1}{2} \times 2.32 \times 10^{21}$$

$$= \mathbf{1.16 \times 10^{21}\ N\ C^{-1}}$$

S21. Let us consider charge $+q$ is uniformly distributed over a spherical shell of radius R. Let E is to be obtained at P lies outside of spherical shell.

$\because$ **E** at any point is radially outward (if charge q is +ve) and has same magnitude at all points which lies at the same distance (r) from centre of spherical such that $r > R$. Therefore, Gaussian surface is concentric sphere of radius r such that $r > R$.



$\because$ Gaussian surface enclosed charge q inside it. By Gauss's theorem

$$\oint \mathbf{E}.d\mathbf{S} = \frac{q}{\varepsilon_0}$$

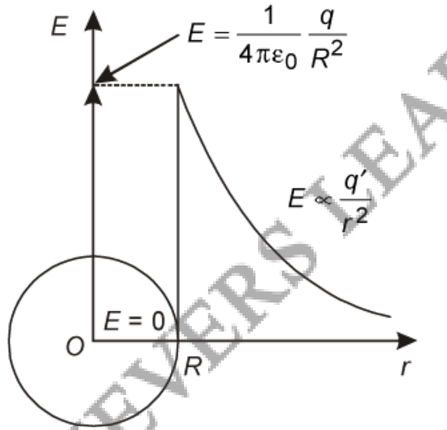
$$\oint \mathbf{E}.dS \cos 0^\circ = \frac{q}{\varepsilon_0} \qquad [\because \mathbf{E} \text{ and } d\mathbf{S} \text{ are along the same direction}]$$

$$E \oint dS = \frac{q}{\varepsilon_0} \qquad [\because \text{Magnitude of } E \text{ is same at every point on Gaussian surface}]$$

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r_1^2}$$

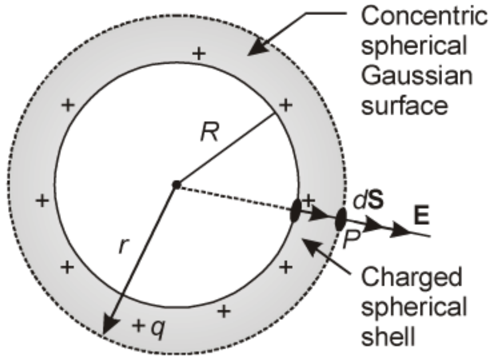
Now, graph,



Variation of E with r for as spherical shell of charge.

S22. Let us consider charge $+q$ is uniformly distributed over a spherical shell of radius R. Let E is to be obtained at P lies outside of spherical shell.

$\because$ **E** at any point is radially outward (if charge q is +ve) and has same magnitude at all points which lies at the same distance (r) from centre of spherical such that $r > R$. Therefore, Gaussian surface is concentric sphere of radius r such that $r > R$.



$\because$ Gaussian surface enclosed charge q inside it. By Gauss's theorem

$$\oint \mathbf{E}.d\mathbf{S} = \frac{q}{\varepsilon_0}$$

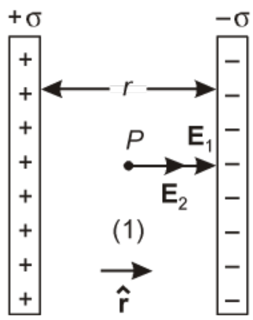
$$\oint \mathbf{E}.dS \cos 0^\circ = \frac{q}{\varepsilon_0}$$ [∵ **E** and *d***S** are along the same direction]

$$E \oint dS = \frac{q}{\varepsilon_0}$$ [∵ Magnitude of *E* is same at every point on Gaussian surface]

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r_1^2}$$

S23. Let us consider two uniformly charge large parallel sheets carrying charge densities +s and – s respectively are separated by a small distance from each other.



By Gauss's law, it can be proved that electric field intensity due to a uniformly charged infinite plane sheet at any nearby is given

$$E = \frac{\sigma}{2\varepsilon_0} \quad \text{... (i)}$$

The electric field is directed normally outward from the plane sheet, if nature of charge on sheet is +ve and normally inward, if charge is of –ve nature.

Let $\hat{\mathbf{r}}$ represents unit vector directed from +ve plate to –ve plate.

Now, Electric Field Intensity (EFI) at any point *P* between the two plates is given by

(a) $$\mathbf{E}_1 = +\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}}$$ [Due to +ve plate]

(b) $$\mathbf{E}_2 = +\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}}$$ [Due to –ve plate]

∴ New EFI at *P*,

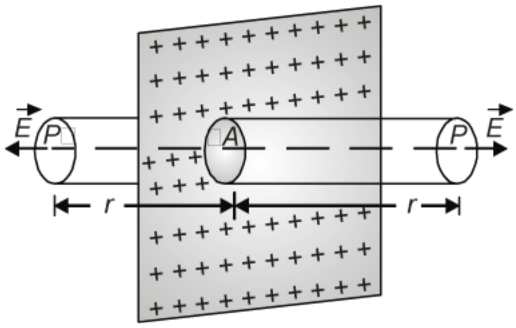
$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}} + \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{r}}$$

$$\mathbf{E} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{r}}$$

Thus, uniform electric field is produced between the two infinite parallel plane sheet of charge which is directed from +ve plate to –ve plate.

S24. The electric flux Φ_E through any closed surface is equal to $1/\varepsilon_0$ times the 'net' charge q enclosed by the surface.

As shown in figure below, consider a thin, infinite plane sheet of charge having a uniform surface charge density σ. We intend to obtain electric field at a point P at a distance r from it.



By symmetry, electric field points outward normal to the sheet. Also, it must have same magnitude and opposite direction at two points P and P' equidistant from the sheet and on opposite sides. We choose cylindrical Gaussian surface of cross sectional area A and length $2r$ with its axis perpendicular to the sheet.

The lines of forces being parallel to the curved surface of the cylinder (Gaussian surface), the flux through the curved surface is zero. The flux through the flat surfaces at the two ends of the cylinder is

$$\Phi_E = EA + EA = 2EA \qquad \dots (i)$$

The charge enclosed by the Gaussian surface

= charge density × cross-sectional area of Gaussian surface

= σA

According to Gauss's theorem, flux is

$$\Phi_E = \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \qquad \dots (ii)$$

Equating (i) and (ii), we get

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

or $$E = \frac{\sigma}{2\varepsilon_0}.$$

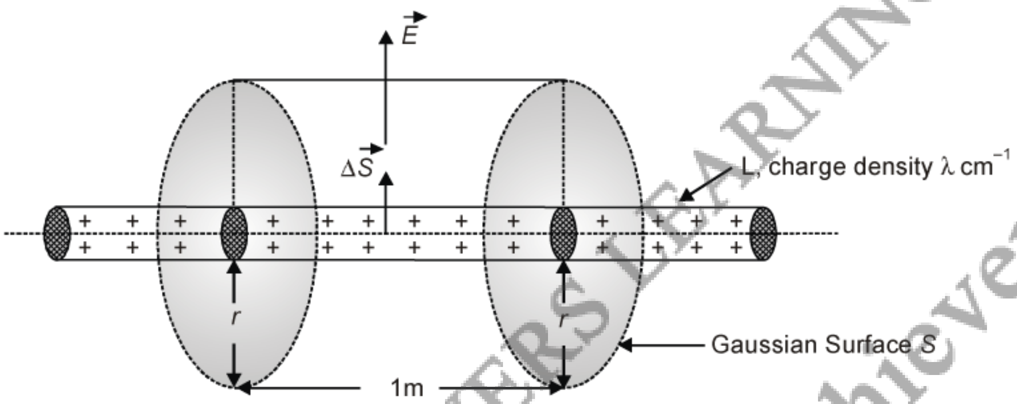
S25. According to Gauss's theorem the total outward electric flux through a closed surface S is given by

$$\oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

where q is charge enclosed inside the surface S and ε_0 the permittivity of the vacuum (or free space).

We consider a thin infinitely long straight line L of charge (As shown in the figure) having a uniform linear charge density λ Cm^{-1}. A circular cylinder of radius r with L as its axis and of unit length is a proper Gaussian surface. On the flat surfaces the electric field $\vec{E}$ over each small area element $\Delta\vec{S}$ are perpendicular to one another so that there is no electric flux ($\phi_E = E \cdot \Delta S \cos\theta$) through these faces. On the sides, *i.e.,* curved surface, $\vec{E}$ and $\Delta\vec{S}$ are parallel to each other for positive charge (antiparallel for negative charge). Therefore, total outward flux is given by

$$\Phi_E = 2\pi r\, E(r)$$

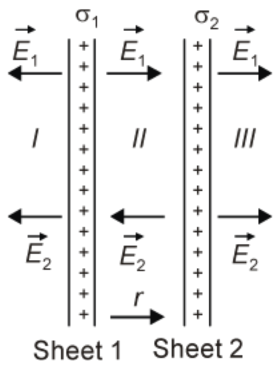


According to Gauss's theorem

$$2\pi r\, E(r) = \frac{\lambda}{\varepsilon_0}$$

$$\therefore \qquad E(r) = \frac{\lambda}{2\pi\varepsilon_0 r}$$

S26. Two plane parallel infinite sheets having charge densities σ_1 and σ_2 are shown in figure below. Let us assume $\sigma_1 > \sigma_2 > 0$. Say $\hat{r}$ is a unit vector pointing from left to right. We consider the three regions marked as I, II and III.



In the region I: Electric field due to charge sheet 1,

$$\vec{E}_1 = \frac{\sigma_1}{2\varepsilon_0}\hat{r}$$

where ε_0 is the permittivity of free space.
Electric field due to charge sheet 2,

$$\vec{E}_2 = \frac{\sigma_2}{2\varepsilon_0}\hat{r}$$

Using the principle of superposition, the total electric field at any point of region I is given by

$$\vec{E}_I = \vec{E}_1 + \vec{E}_2 = -\frac{(\sigma_1 - \sigma_2)}{2\varepsilon_0}\hat{r}$$

In the region II: Electric field due to charge sheet 1,

$$\vec{E}_1 = \frac{\sigma_1}{2\varepsilon_0}\hat{r}$$

Electric field due to charge sheet 2,

$$\vec{E}_2 = \frac{\sigma_2}{2\varepsilon_0}\hat{r}$$

$\therefore$ Total field $E_{II} = \vec{E}_1 + \vec{E}_2 = \frac{\sigma_1}{2\varepsilon_0}\hat{r} - \frac{\sigma_2}{2\varepsilon_0}\hat{r} = \frac{(\sigma_1 - \sigma_2)}{2\varepsilon_0}\hat{r}$

In the region III: Field due to sheet 1,

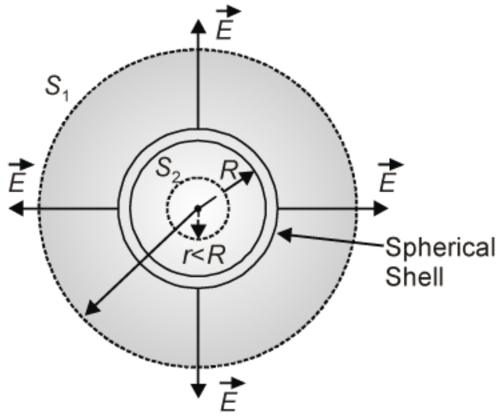
$$\vec{E}_1 = \frac{\sigma_1}{2\varepsilon_0}\hat{r}$$

Field due to sheet 2, $\vec{E}_2 = \frac{\sigma_2}{2\varepsilon_0}\hat{r}$

$\therefore$ Total field $\vec{E}_{III} = \vec{E}_1 + \vec{E}_2 = \frac{(\sigma_1 + \sigma_2)}{2\varepsilon_0}\hat{r}$

S27. The electric flux Φ_E through any closed surface is equal to $1/\varepsilon_0$ times the 'net' charge q enclosed by the surface.

"Electric field of a uniformly charged spherical shell". Let us consider a thin spherical shell of radius R having a uniform surface charge density s C/m^2 (as shown in the figure). From symmetry, it is clear that the electric field d at any point is radial and has same magnitude at points equidistant from the centre of the shell. To determine electric field at any point P at distance r from Q, we choose concentric sphere of radius r as the Gaussian surface. Depending upon the potential P, three cases of interest arise.



(a) **When point P lies outside the spherical shell:** The total charge q inside the Gaussian surface S_1 is equal to the charge on the spherical shell.

$$\therefore \quad q = \text{surface area} \times \text{surface charge density} = 4\pi R^2\sigma$$

Flux through the Gaussian surface S_1 is

$$\Phi_E = 4\pi R^2 E$$

By Gauss's theorem, $\Phi_E = \dfrac{q}{\varepsilon_0}$

or $$4\pi r^2 E = \frac{q}{\varepsilon_0}$$

or $$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2} \qquad (r > R)$$

(b) **When point P lies on the spherical shell:** In this case charge enclosed by the Gaussian surface S_1 is equal to $4\pi R^2\sigma$.

$$4\pi R^2 E = \frac{q}{\varepsilon_0}$$

or $$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2} \qquad (r = R)$$

or $$E = \frac{\sigma}{\varepsilon_0} \qquad (\text{as } 4\pi R^2\sigma = q)$$

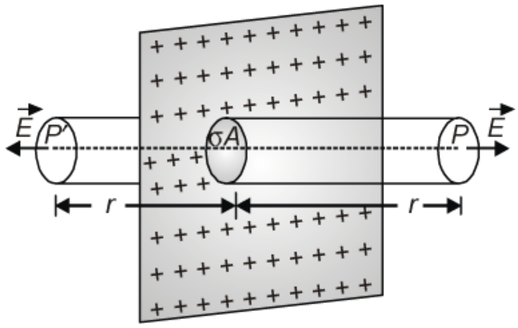
(c) **When point P lies inside the spherical shell:** Clearly the charge enclosed by the Gaussian surface $S_2 = 0$. Therefore, electric field at any point inside the shell is zero, *i.e.*,

$$E = 0 \qquad (r < R)$$

S28. (a) **Electric Flux:** Electric Flux over an area in an electric field represents the total number of electric lines of force crossing the area in a direction normal to the plane of the area.

Its Si unit of electric flux is N-m^2/C.

(b) The electric flux Φ_E through any closed surface is equal to $1/\varepsilon_0$ times the 'net' charge q enclosed by the surface.



As shown in figure, consider a thin, infinite plane sheet of charge having a uniform surface charge density σ. We intend to obtain electric field at a point P at a distance r from it.

By symmetry, electric field points outward normal to the sheet. Also, it must have same magnitude and opposite direction at two points P and P' equidistant from the sheet and on opposite sides. We choose cylindrical Gaussian surface of cross sectional area A and length $2r$ with its axis perpendicular to the sheet.

The lines of forces being parallel to the curved surface of the cylinder (Gaussian surface), the flux through the curved surface is zero. The flux through the flat surfaces at the two ends of the cylinder is

$$\Phi_E = EA + EA = 2EA \qquad \ldots (i)$$

The charge enclosed by the Gaussian surface

= charge density × cross-sectional area of Gaussian surface

$= \sigma A$

According to Gauss's theorem, flux is

$$\Phi_E = \frac{q}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \qquad \ldots (ii)$$

Equating (i) and (ii), we get

$$2EA = \frac{\sigma A}{\varepsilon_0} \quad \text{or} \quad E = \frac{\sigma}{2\varepsilon_0}.$$

(c) The field directed

(i) Normally away from the sheet when sheet is positively charged.

(ii) Normally inward, towards the sheet when plane sheet is negatively charged.