PHYSICS

Q.1 A 2.00-MHz sound wave travels through a pregnant woman's abdomen and is reflected from the fetal heart wall of her unborn baby. The heart wall is moving toward the sound receiver as the heart beats. The reflected sound is then mixed with the transmitted sound, and 85 beats per second are detected. The speed of sound in body tissue is 1500 m/s. Calculate the speed of the fetal heart wall at the instant this measurement is made.

Sol. Let f_0 = 2.00 MHz be the frequency of the generated wave. The frequency with which the heart wall receives this wave is $f_H = \frac{v+v_H}{v} f_0$, and this is also the frequency with which the heart wall re-emits the wave. The detected frequency of this reflected wave is

$f' \frac{v}{v-v_H}$, f_H, with the minus sign indicating that the heart wall, acting now as a source of waves, is moving toward the receiver. Combining, $f' \left(\frac{v+v_H}{v-v_H}\right) f_0$, and the beat frequency is

$$f_{beat} = f' - f_0 = \left(\frac{v+v_H}{v-v_H} - 1\right) f' = \frac{2v_H}{v-v_H} f_0.$$

Solving for v_H,

$$v_H = v\left(\frac{f_{beat}}{2f_0 + f_{beat}}\right) = (1500 \text{ m/s}) \left(\frac{85\,\text{Hz}}{2(2.00\times10^6\,\text{Hz})+(85\,\text{Hz})}\right)$$

Note that in the denominator in the final calculation, f_{beat} is negligible compared to f_0.

Q.2 The sound source of a ship's sonar system operates at a frequency of 22.0 kHz. The speed of sound in water (assumed to be at a uniform 20ºC) is 1482 m/s.

a) What is the wavelength of the waves emitted by the source?

b) What is the difference in frequency between the directly radiated waves and the waves reflected from a whale traveling directly toward the ship at 4.95 m/s? The ship is at rest in the water.

Sol. **a)** $\lambda = v/f = (1482 \text{ m/s})/(22.0 \times 10^3 \text{ Hz}) = 6.74 \times 10^{-2}$ m.

b) See Problem 1 or Problem 5; the difference in frequencies is

$$\Delta f = f_s\left(\frac{2v_W}{v-v_W}\right) = (22.0 \times 10^3 \text{ Hz}) \frac{2(4.95 \text{ m/s})}{(1482 \text{ m/s})-(4.95 \text{ m/s})} = 147 \text{ Hz}.$$

The reflected waves have higher frequency.

Q.3 A police siren of frequency f_{siren} is attached to a vibrating platform. The platform and siren oscillate up and down in simple harmonic motion with amplitude A_p and frequency f_p.

a) Find the maximum and minimum sound frequencies that you would hear at a position directly above the siren.

b) At what point in the motion of the platform is the maximum frequency heard? The minimum frequency? Explain.

Sol. **a)** The maximum velocity of the siren is $\omega_p A_p = 2\pi f_p A_p$. You hear a sound with frequency $f_L = f_{siren}\, v/(v + v_S)$, where v_S varies between $+2\pi f_p A_p$ and $-2\pi f_p A_p$.

So $f_{L-max} = f_{siren}\, v/(v - 2\pi f_p A_p)$ and $f_{L-min} = f_{siren}\, v/(v + 2\pi f_p A_p)$.

b) The maximum (minimum) frequency is heard when the platform is passing through equilibrium and moving up (down).

Q.4 Horseshoe bats (genus Rhinolophus) emit sounds from their nostrils, then listen to the frequency of the sound reflected from their prey to determine the prey's speed. (The "horseshoe" that gives the bat its name is a depression around the nostrils that acts like a focusing mirror, so that the bat emits sound in a narrow beam like a flash-light.) A Rhinolophus flying at speed v_{bat} emits sound of frequency f_{bat}; the sound it hears reflected from an insect flying toward it has a higher frequency f_{refl}.

a) Show that the speed of the insect is

$$v_{insect} = v\left[\frac{f_{refl}(v - v_{bat}) - f_{bat}(v + v_{bat})}{f_{refl}(v - v_{bat}) + f_{bat}(v + v_{bat})}\right]$$

where v is the speed of sound.

b) If f_{bat} = 80.7 kHz, f_{refl} = 83.5 kHz, and v_{bat} = 3.9 m/s, calculate the speed of the insect.

Sol. **a)** Let v_b be the speed of the bat, v_i the speed of the insect and f_i the frequency with which the sound waves both strike and are reflected from the insect. The frequencies at which the bat sends and receives the signals are related by

$$f_L = f_i\left(\frac{v + v_b}{v - v_i}\right) = f_S\left(\frac{v + v_i}{v - v_b}\right)\left(\frac{v + v_b}{v - v_i}\right).$$

Solving for v_i,

$$v_i = v\left[\frac{1 - \frac{f_S}{f_L}\left(\frac{v + v_b}{v - v_b}\right)}{1 + \frac{f_S}{f_L}\left(\frac{v + v_b}{v - v_b}\right)}\right] = v\left[\frac{f_L(v - v_b) - f_S(v + v_b)}{f_L(v - v_b) + f_S(v + v_b)}\right].$$

Letting $f_L = f_{refl}$ and $f_s = f_{bat}$ gives the result.

b) If f_{bat} = 80.7 kHz, f_{refl} = 83.5 kHz, and v_{bat} = 3.9 m/s, v_{insect} = 2.0 m/s.

Q.5 A sound wave with frequency f_0 and wavelength λ_0 travels horizontally toward the right. It strikes and is reflected from a large, rigid, vertical plane surface, perpendicular to the direction of propagation of the wave and moving toward the left with a speed v_1.

a) How many positive wave crests strike the surface in a time interval t?

b) At the end of this time interval, how far to the left of the surface is the wave that was reflected at the beginning of the time interval?

c) What is the wavelength of the reflected waves in terms of λ_0?

d) What is the frequency of the reflected waves in terms of f_0?

e) A listener is at rest at the left of the moving surface. How many beats per second does she detect as a result of the combined effect of the incident and reflected waves?

Sol. **a)** In a time t, the wall has moved a distance $v_1 t$ and the wavefront that hits the wall at time t has traveled a distance vt, where $v = f_0\lambda_0$, and the number of wavecrests in the total distance is $\frac{(v + v_1)t}{\lambda_0}$.

b) The reflected wave has traveled vt and the wall has moved $v_1 t$, so the wall and the wavefront are separated by $(v - v_1)t$.

c) The distance found in part (b) must contain the number of reflected waves found in part (a), and the ratio of the quantities is the wavelength of the reflected wave, $\lambda_0 \frac{v - v_1}{v + v_1}$.

d) The speed v divided by the result of part (c), expressed in terms of f_0 is $f_0 \frac{v - v_1}{v + v_1}$.

e) $f_0 \frac{v + v_1}{v - v_1} - f_0 = f_0 \frac{2v_1}{v - v_1}$.

Q.6 **a)** Show that Eq. $f_R = \sqrt{\frac{c-v}{c+v}}\, f_S$ (Doppler effect for light) can be written as

$$f_R = f_S\left(1-\frac{v}{c}\right)^{1/2}\left(1+\frac{v}{c}\right)^{-1/2}$$

b) Use the binomial theorem to show that if v << c, this is approximately equal to

$$f_R = f_S\left(1-\frac{v}{c}\right)$$

c) A pilotless reconnaissance aircraft emits a radio signal with a frequency of 243 MHz. It is flying directly toward a test engineer on the ground. The engineer detects beats between the received signal and a local signal also of frequency 243 MHz. The beat frequency is 46.0 Hz. What is the speed of the aircraft? (Radio waves travel at the speed of light, $c = 3.00 \times 10^8$ m/s.)

Sol. **a)** $f_R = f_L\sqrt{\frac{c-v}{c+v}} = f_S\frac{\sqrt{1-\frac{v}{c}}}{\sqrt{1+\frac{v}{c}}} = f_S\left(1-\frac{v}{c}\right)^{1/2}\left(1+\frac{v}{c}\right)^{-1/2}$.

b) For small x, the binomial theorem gives $(1-x)^{1/2} \approx 1 - x/2$, $(1+x)^{-1/2} \approx 1 - x/2$, so

$$f_L \approx f_S\left(1-\frac{v}{2c}\right)^2 \approx f_S\left(1-\frac{v}{c}\right)$$

where the binomial theorem has been used to approximate $(1 - x/2)^2 \approx 1 - x$.

The above result may be obtained without resort to the binomial theorem by expressing f_R in terms of f_S as

$$f_R = f_S\frac{\sqrt{1-(v/c)}}{\sqrt{1+(v/c)}}\frac{\sqrt{1-(v/c)}}{\sqrt{1-(v/c)}} = f_S\frac{1-(v/c)}{\sqrt{1-(v/c)^2}}.$$

To first order in v/c, the square root in the denominator is 1, and the previous result is obtained.

c) For an airplane, the approximation v << c is certainly valid, and solving the expression found in part (b) for v,

$$v = c\,\frac{f_S - f_R}{f_S} = c\,\frac{f_{beat}}{f_S} = (3.00 \times 10^8 \text{ m/s})\,\frac{46.0\text{Hz}}{2.43\times10^8\text{Hz}} = 56.8 \text{ m/s},$$

and the approximation v << c is seen to be valid. Note that in this case, the frequency difference is known to three figures, so the speed of the plane is known to three figures.

Q.7 An acoustical motion detector emits a 50 kHz signal and receives the echo signal. If the echoes have Doppler shift frequency components departing from 50 kHz by more than 100 Hz, a "moving object" is registered. For a sound velocity in air of 330 m/sec, calculate the speed with which an object must move toward (or away from) the detector in order to be registered as a "moving object".

Sol. Consider a source emitting sound of frequency ν. The Doppler effect has it that if an observer moves with velocity v toward the source he will detect the frequency as

$$\nu' = \left(\frac{c+v}{c}\right)\nu,$$

c being the speed of sound propagation. On the other hand, if the source moves with velocity v toward the observer, who is stationary, then

$$\nu' = \left(\frac{c}{c-v}\right)\nu.$$

Thus the object, moving toward the detector, receives a signal of frequency

$$\nu' = \left(\frac{c+v}{c}\right)\nu,$$

and the signal after reflection by the object is detected by the detector as having frequency

$$\nu'' = \left(\frac{c}{c-v}\right)\nu' = \left(\frac{c+v}{c-v}\right)\nu.$$

For the moving object to be registered, we must have $\nu'' = \nu \pm \Delta\nu$, where $\Delta\nu \geq 10^2$ Hz. Then

$$\nu \pm \Delta\nu = \left(\frac{c+v}{c-v}\right)\nu,$$

or

$$v = \pm\frac{c\Delta\nu}{2\nu \pm \Delta\nu} \approx \pm\frac{c\Delta\nu}{2\nu},$$

as $\Delta\nu << \nu$. Hence the object must be moving toward or receding from the detector at

$$v \geq \frac{330\times10^2}{2\times5\times10^4} = 0.330 \text{ m/s}$$

for it to be registered.

Q.8 A student near a railroad track hears a train's whistle when the train is coming directly toward him and then when it is going directly away. The two observed frequencies are 250 and 200 Hz. Assume the speed of sound in air to be 360 m/s. what is the train's speed?

Sol. Let ν_0, ν_1, ν_2 be respectively the frequency of the whistle emitted by the train, and the frequencies heard by the student when the train is coming and when it is moving away. The Doppler effect has it that

$$\nu_1 = \left(\frac{c}{c-v}\right)\nu_0,$$

$$\nu_2 = \left(\frac{c}{c+v}\right)\nu_0,$$

Where c is the speed of sound and v is the speed of the train, and thus

$$\frac{\nu_1}{\nu_2} = \frac{c+v}{c-v}.$$

Putting in the data, we have

$$1.25 = \frac{360+v}{360-v},$$

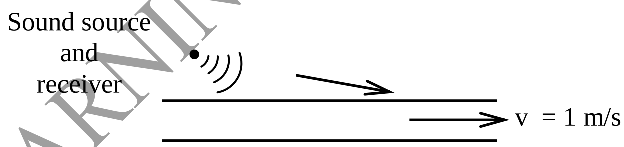
or

$$\frac{2.25}{0.25} = \frac{720}{2v},$$

and thus

$$v = \frac{360}{9} = 40 \text{ m/s}.$$

Q.9 The velocity of blood flow in an artery can be measured using Doppler shifted ultrasound. Suppose sound with frequency 1.5×10^6 Hz is reflected straight back by blood flowing at 1 m/s. Assuming the velocity of sound in tissue is 1500 m/s and that the sound is incident at a very small angle as shown in Fig., calculate the frequency shift between the incident and reflected waves.



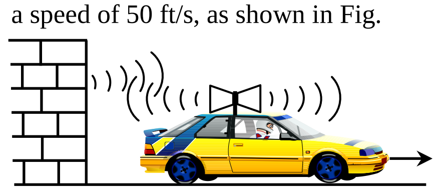
Sol. As the sound is incident at a very small angle, the blood can be considered to be flowing directly away. Then the results of Problem 7 can be applied with v replaced by –v.

$$\nu'' = \left(\frac{c-v}{c+v}\right)\nu.$$

The frequency shift is then

$$\nu'' - \nu = -\frac{2v\nu}{c+v} \approx -\frac{2v\nu}{c} = -2 \times 10^3 \text{ Hz}.$$

Q.10 A car has front- and back-directed speakers mounted on its roof, and drives toward you with a speed of 50 ft/s, as shown in Fig.



If the speakers are driven by a 1000 Hz oscillator, what beat frequency will you hear between the direct sound and the echo off a brick building behind the car? (Take the speed of sound as 1000 ft/s.)

Sol. The sound from the back-directed speaker has Doppler frequency

$$\nu_b = \left(\frac{c}{c+v}\right)\nu,$$

Where c and v are the speeds of sound and the car respectively, and ν is the frequency of the sound emitted. As the wall is stationary with respect to the observer, ν_b is also the frequency as heard by the latter. The sound from the front-directed speaker has Doppler frequency

$$\nu_f = \left(\frac{c}{c-v}\right)\nu.$$

Hence the beat frequency is

$$\nu_f - \nu_b = c\nu\left(\frac{1}{c-v}-\frac{1}{c+v}\right) = \frac{2vc\nu}{c^2-v^2} \approx \frac{2v\nu}{c} =$$

100 Hz.

Q.11 A physics student holds a tuning fork vibrating at 440 Hz and walks at 1.2 m/s away from a wall. Does the echo from the wall have a higher or lower pitch than the tuning fork? What beat frequency does he hear between the fork and the echo? The speed of sound is 330 m/s.

Sol. As the tuning fork, which emits sound of frequency ν, moves away from the wall at speed v, the sound that is incident on the wall has frequency

$$\nu' = \left(\frac{c}{c+v}\right)\nu.$$

Then the student, who is moving away from the wall at speed v, hears the reflected frequency

$$\nu'' = \left(\frac{c-v}{c}\right)\nu' = \left(\frac{c-v}{c+v}\right)\nu.$$

As

$$\nu'' - \nu = -\frac{2v\nu}{c+v} < 0,$$

The echo has a lower frequency. The beat frequency between the fork and the echo is

$$\frac{2v\nu}{c+v} \approx \frac{2v\nu}{c} = 3.2 \text{ Hz}.$$

Q.12 Two trains are travelling towards each other both at a speed of 90 km/h. If one of the trains sounds a whistle at 500 Hz, what will be the apparent frequency heard in the other train ? Speed of sound in air is 350 m/s.

Ans. 577 Hz

Q.13 The driver of a car approaching a vertical wall notices that the frequency of his car's horn changes from 440 Hz to 480 Hz when it gets reflected from the wall. Find the speed of the car if that of the sound is 330 m/s.

Ans. 52 km/h

Q.14 A person riding a car moving at 72 km/h sounds a whistle emitting a wave of frequency 1250 Hz. What frequency will be heard by another person standing on the road (a) in front of the car (b) behind the car ? Speed of sound in air = 340 m/s.

Ans. (a) 1328 Hz, (b) 1181 Hz

Q.15 Microwaves which travel with the speed of light are reflected from a distant aeroplane approaching the wave source radar. It is found that when the reflected waves are beat against the waves radiated from the source, the beat frequency is 990 Hz. If the microwaves are 0.1 m in wavelength, what is the approaching speed of the aeroplane ?

Ans. 49.5 m/s

Q.16 A bullet passes past a person at a speed of 220 m/s. Find the fractional change in the frequency of the whistling sound heard by the person as the bullet crosses the person. Speed of sound in air = 330 m/s.

Ans. 0.8

Q.17 A bat emitting an ultrasonic wave of frequency 4.5×10^4 Hz flies at a speed of 6 m/s between two parallel walls. Find the two frequencies heard by the bat and the beat frequency between the two. The speed of sound is 330 m/s.

Ans. 4.67×10^4 Hz, 4.34×10^4 Hz, 3270 Hz

Q.18 A source and a detector move away from each other, each with a speed of 10 m/s with respect to the ground with no wind. If the detector detects a frequency 1950 Hz of the sound coming from the source, what is the original frequency of the source ? Speed of sound in air = 340 m/s.

Ans. 2070 Hz

Q.19 A traffic policeman sounds a whistle to stop a car-driver approaching towards him. The car-driver does not stop and takes the plea in court that because of the Doppler shift, the frequency of the whistle reaching him might have gone beyond the audible limit of 20 kHz and he did not hear it. Experiments showed that the whistle emits a sound with frequency close to 16 kHz. Assuming that the claim of the driver is true, how fast was he driving the car ? Take the speed of sound in air to be 330 m/s. Is this speed practical with today's technology ?

Ans. 297 km/h

Q.20 A siren is fitted on a car going towards a vertical wall at a speed of 36 km/h. A person standing on the ground, behind the car, listens to the siren sound coming directly from the source as well as that coming after reflection from the wall. Calculate the apparent frequency of the wave (a) coming directly from the siren to the person and (b) coming after reflection. Take the speed of sound to be 340 m/s. Assume frequency of siren is 500 Hz.

Ans. (a) 472.27 Hz (b) 531.25 Hz