## PHYSICS

Q. 1 A uniform cylinder of length $L$ and mass $M$ having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half-submerged in a liquid of density $\rho$ at equilibrium position When the cylinder is given a small downward push and released it starts oscillating vertically with a small amplitude. If the force constant of the spring is k , the frequency of oscillation of the cylinder is -
(A) $\frac{1}{2 \pi}\left(\frac{\mathrm{k}-\mathrm{A} \rho \mathrm{g}}{\mathrm{M}}\right)^{1 / 2}$
(B) $\frac{1}{2 \pi}\left(\frac{\mathrm{k}+\mathrm{A} \rho \mathrm{g}}{\mathrm{M}}\right)^{1 / 2}$
(C) $\frac{1}{2 \pi}\left(\frac{\mathrm{k}+\rho g \mathrm{~L}^{2}}{\mathrm{M}}\right)^{1 / 2}$
(D) $\frac{1}{2 \pi}\left(\frac{\mathrm{k}+\mathrm{A} \rho \mathrm{g}}{\mathrm{A} \rho \mathrm{g}}\right)^{1 / 2}$

Sol. $\quad \mathrm{F}=-\{\mathrm{kY}+\mathrm{AY} \rho \mathrm{g}\} \Rightarrow \mathrm{Ma}=-\{\mathrm{k}+\mathrm{A} \rho \mathrm{g}\} \mathrm{Y}$ $a=-\left\{\frac{k+A \rho g}{M}\right\} Y \Rightarrow f=\frac{1}{2 \pi} \sqrt{(K+A \rho g) / M}$

Q. 2 Two liquids which do not react chemically are placed in a bent tube a shown in figure. The heights of the liquids above their surface of separation are -

(A) directly proportional to their densities
(B) inversely proportional to their densities
(C) directly proportional to square of their densities
(D) equal

Sol. The pressure at the interface must be same, calculated via either tube. Since, both tube are open to the atmosphere, we must have
$h_{1} \rho_{1} g=h_{2} \rho_{2} g$
or $h_{1} \rho_{1}=h_{2} \rho_{2}$
or $\mathrm{h} \rho=$ constant
or $\mathrm{h} \propto \frac{1}{\rho}$
Q. 3 There are two identical small holes of area of cross-section a on the opposite sides of a tank containing a liquid of density $\rho$. The difference in height between the holes is $h$. Tank is resting on a smooth horizontal surface. Horizontal force which will have to be applied on the tank to keep it in equilibrium is -

(A) ghpa
(B) $\frac{2 \mathrm{gh}}{\rho \mathrm{a}}$
(C) $2 \rho a g h$
(D) $\frac{\rho g h}{a}$
[C]

Sol. Thrust force


$$
\begin{aligned}
& \mathrm{F}=\mathrm{F}_{1}-\mathrm{F}_{2}=\rho \mathrm{a} v_{1}^{2}-\rho a v_{2}^{2} \\
& =\rho \mathrm{a}\left(2 \mathrm{gh}_{1}\right)-\rho \mathrm{a}\left(2 \mathrm{gh}_{2}\right) \\
& =2 \rho \mathrm{ag}\left(\mathrm{~h}_{1}-\mathrm{h}_{2}\right) \\
& =2 \rho \mathrm{agh}
\end{aligned}
$$

Q. 4 A block of iron is kept at the bottom of a bucket full of water at $2^{\circ} \mathrm{C}$. The water exerts bouyant force on the block. If the temperature of water is increased by $1^{\circ} \mathrm{C}$ the temperature of iron block also increased by $1^{\circ} \mathrm{C}$. The buoyant force on the block by water.
(A) will increase
(B) will decrease
(C) will not change
(D) may decrease or increase depending on the values of their coefficient of expansion

Sol. Increasing the temperature of water from $2^{\circ} \mathrm{C}$ to $3^{\circ} \mathrm{C}$ increases it's density while decreases the density of iorn.
Hence the bouyant force increases.
Q. 5 A cylinder of radius $R$ is floating in a liquid as shown. The work done in submerging the cylinder completely in the liquid of density $p$ is

(A) $\frac{2}{9} \rho \pi R^{2} L^{2} g$
(B) $\frac{8}{18} \rho \pi R^{2} L^{2} g$
(C) $\frac{1}{3} \rho \pi R^{2} L^{2} g$
(D) $\frac{2}{9} \rho R^{2} L^{2} g$

Sol. At any submerged length $x$

$$
\begin{equation*}
F_{B}=\rho \pi R^{2} x g \tag{1}
\end{equation*}
$$

If density of cylinder is $\rho_{\mathrm{C}}$ then,

$$
\frac{\rho_{\mathrm{C}}}{\rho}=\frac{\frac{\mathrm{L}}{3}}{\mathrm{~L}}=\frac{1}{3} \Rightarrow \rho_{\mathrm{C}}=\frac{\rho}{3}
$$

then,

$$
\mathrm{W}=\int_{\mathrm{L} / 3}^{\mathrm{L}}\left(\rho \pi \mathrm{R}^{2} \mathrm{xg}-\frac{\rho}{3} \pi \mathrm{R}^{2} \mathrm{Lg}\right) \mathrm{dx}
$$

Q. 6 A large tank is filled with water to height H and there is a small hole at the bottom of tank. If in time $\mathrm{T}_{1}$ the height of water level decreases to height $\frac{H}{n}(n>1)$ and in the same time $T_{1}$ rest of water flows out, then the value of $n$ is -
(A) 2
(B) 3
(C) 4
(D) $2 \sqrt{2}$
[C]
Sol. $\frac{\mathrm{dV}}{\mathrm{dt}}=\mathrm{A} \sqrt{2 \mathrm{gh}}$
$\therefore \frac{\mathrm{dh}}{\sqrt{\mathrm{h}}}=\sqrt{2 \mathrm{~g}} \mathrm{dt}$
$\therefore \sqrt{\mathrm{H}}-\sqrt{\mathrm{h}}=\mathrm{Kt}$
Q. 7 A 20 cm long capillary tube is dipped in water. The water rises up to 15 cm . If the entire arrangement is put in a freely falling elevator the length of water column in the capillary tube will be
(A) 20 cm
(B) 4 cm
(C) 10 cm
(D) 18 cm

Sol, [A] In condition of weightless, water rises to the whole of the available length.
Q. 8 A large open tank has two holes in the wall. One is a square hole of side $L$ at a depth $y$ from the top and the other is a circular hole of radius $R$ at a depth $4 y$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same. The, R is equal to :
(A) $\mathrm{L} / \sqrt{2 \pi}$
(B) $2 \pi \mathrm{~L}$
(C) L
(D) L / $2 \pi$

Sol. Velocity of efflux at a depth $h$ is given by $v$ $=\sqrt{2 g h}$. Volume of water flowing out per second from both the holes are qual.
$\therefore \mathrm{a}_{1} \mathrm{~V}_{1}=\mathrm{a}_{2} \mathrm{~V}_{2}$
or $\left(L^{2}\right) \sqrt{2 g(y)}=\pi R^{2} \sqrt{2 g(4 y)}$
or $\mathrm{R}=\frac{\mathrm{L}}{\sqrt{2 \pi}}$
Q. 9 A block of wood floats in fresh water with two third of its volume submerged. In oil the block
floats with one fourth of is volume submerged. The density of oil is -
(A) $2666.7 \mathrm{~kg} / \mathrm{m}^{3}$
(B) $5333.3 \mathrm{~kg} / \mathrm{m}^{3}$
(C) $1333.3 \mathrm{~kg} / \mathrm{m}^{3}$
(D) $3333.3 \mathrm{~kg} / \mathrm{m}^{3}$
[A]
Sol. $\quad \rho_{\mathrm{w}} \frac{2}{3} \mathrm{~V}=\rho_{\mathrm{b}} \mathrm{V}$

$$
\begin{aligned}
& \rho_{\text {oil }} \frac{1}{4} \mathrm{~V}=\rho_{\mathrm{b}} \mathrm{~V} \\
& \therefore \frac{2}{3} \rho_{\mathrm{w}} \\
&=\frac{1}{4} \rho_{\text {oil }} \\
& \rho_{\text {oil }}=\frac{8}{3} \rho_{\mathrm{w}} \\
&=\frac{8000}{3}=2666.7
\end{aligned}
$$

Q. 10 Calculate the velocity with which the liquid gushes out of the $4 \mathrm{~cm}^{2}$ outlet, if the liquid flowing in the tube is water and liquid in U tube has a specific gravity 12 . Velocity of liquid at point A is $\sqrt{20.2} \mathrm{~m} / \mathrm{s}$ -

(A) $2.5 \mathrm{~m} / \mathrm{s} \quad$ (B) $5.5 \mathrm{~m} / \mathrm{s}$ (C) $8 \mathrm{~m} / \mathrm{s}$ (D) 10 $\mathrm{m} / \mathrm{s}$
Sol.[B] $P_{A}+\frac{1}{2} \rho v_{A}^{2}=P_{B}+\frac{1}{2} \rho v_{B}^{2}$
$P_{A}-P_{B}=\frac{1}{2} \rho\left(v_{B}^{2}-v_{A}^{2}\right)$
$0.02 \times 12000 \times 10=\frac{1}{2} \times 1000\left(\mathrm{v}_{\mathrm{B}}^{2}-20.2\right)$
$4.8=\mathrm{v}_{\mathrm{B}}^{2}-20.2 \Rightarrow \mathrm{v}_{\mathrm{B}}=5 \mathrm{~m} / \mathrm{s}$
$\Rightarrow \mathrm{A}_{3} \cdot \mathrm{~V}_{\mathrm{B}}=\mathrm{A}_{1} \mathrm{~V}_{1}+\mathrm{A}_{2} \mathrm{~V}_{2}$
$\Rightarrow 30=4 \mathrm{v}_{1}+8$
$\Rightarrow 4 \mathrm{v}_{1}=22 \Rightarrow \mathrm{v}_{1}=\frac{22}{4}=5.5 \mathrm{~m} / \mathrm{s}$
Q. 11 The cross sectional area of a horizontal tube increases along its length linearly as moved in the direction of flow. The variation of pressure, as we move along its length in the direction of flow
(x-direction), can be best represented by the following graph -
(A)

(B)

(C)

(D)

Sol. As $\mathrm{Av}=\mathrm{k}_{1}$ (constant)
 also $P+\frac{1}{2} \rho v^{2}+\rho g h=k_{2}$ (constant)

$$
\therefore \quad \mathrm{P}=\mathrm{k}_{2}-\frac{1}{2}, \rho \frac{\mathrm{k}_{1}^{2}}{\mathrm{~A}^{2}}
$$

Q. 12 A smalliblock of wood of specific gravity 0.5 is submerged at a depth of 1.2 m in a vessel filled with water. The vessel accelerated upward with an acceleration of $\mathrm{a}=\mathrm{g} / 2$. Time taken by block to reach the surface is $\left(\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}\right)$ -
(A) 0.6 sec
(B) 0.4 sec
(C) 1.2 sec
(D) 1 sec

Sol.[C] Acceleration w.r.t vessel is $a=\left(\frac{1-S}{S}\right)(g+a)$.
Q. 13 The container is massless. Mass of liquid initially at $\mathrm{t}=0$ filled in the container is M . There is a hole in the container as shown. If there is no friction between container and ground, then -

(A) initial acceleration of system is less then $\mathrm{g} / 2$
(B) initial acceleration of system is equal to $\mathrm{g} / 2$
(C) initial acceleration of system is greater than $\mathrm{g} / 2$
(D) system remain at rest
[A]
Sol. Due to water flowing out from hole a force will acts on container towards left
Q. 14 A spherical air bubble of radius 2 cm is released 30 m below the surface of a pond at 280 K . Its volume when it reaches the surface, which is at 300 K assuming it is in thermal equilibrium the
whole time, is V . Ignore the size of the bubble compared to other dimensions like 30 m . Then V is equal to -
(A) $1.4 \times 10^{-4} \mathrm{~m}^{3}$
(B) $2.8 \times 10^{-4} \mathrm{~m}^{3}$
(C) $0.7 \times 10^{-4} \mathrm{~m}^{3}$
(D) $4.2 \times 10^{-4} \mathrm{~m}^{3}$

Sol. $\quad[A] \frac{P_{i} V_{i}}{T_{i}}=\frac{P_{f} V_{f}}{T_{f}}$
$V_{f}=\left(\frac{P_{i} V_{i}}{P_{f}} \frac{T_{f}}{T_{i}}\right)$
$\mathrm{P}_{\mathrm{f}}=\mathrm{P}_{\mathrm{atm}}$
$P_{i}=P_{a t m}+\rho g h$
Q. 15 A ball of mass 10 kg and density $1 \mathrm{gm} / \mathrm{cm}^{3}$ is attached to the base of a container having a liquid of density
$1.1 \mathrm{gm} / \mathrm{cm}^{3}$, with the help of a spring as shown in the figure. The container is going up with an acceleration
$2 \mathrm{~m} / \mathrm{s}^{2}$. If the spring constant of the spring is 200 $\mathrm{N} / \mathrm{m}$, the elongation in the spring is -

(A) 2 cm
(B) 4 cm
(C) 6 cm (D) 8 cm

Sol. [C]

(1.1) $y(g+a)-1 \times V(g+a)=200 x$
$x=6 \mathrm{~cm}$
Q. 16 A cubical vessel open from top of side $L$ is filled with a liquid of density $\rho$ then the torque of hydrostatic force on a side wall about an axis passing through one of bottom edges is-
(A) $\frac{\rho g L^{4}}{4}$
(B) $\frac{\rho g L^{4}}{6}$
(C) $\frac{2 \rho g L^{4}}{3}$
(D) $\frac{\rho g L^{4}}{3}$

Sol. Force by hydrostatic pressure

$$
=\mathrm{P}_{\mathrm{av}} \times \text { Area }=\frac{1}{2} \rho g \mathrm{~L} \times \mathrm{L}^{2}=\frac{1}{2} \rho g \mathrm{~L}^{3}
$$

and centre of pressure is at height $\frac{L}{3}$.
Q. 17 A cylindrical vessel contains a liquid of density $\rho$ upto a height h . The liquid is closed by a piston of mass m and area of cross-section A . There is a small hole at the bottom of the vessel. The speed $v$ with which the liquid comes out of the hole is -


Sol.
Applying Bernoulli's theorem at 1 and 2 : difference in pressure energy between 1 and $2=$ difference in kinetic energy between 1 and 2

or $\rho g h+\frac{m g}{A}=\frac{1}{2} \rho v^{2}$
or $v=\sqrt{2 g h+\frac{2 m g}{\rho A}}=\sqrt{2\left(g h+\frac{m g}{\rho A}\right)}$
Q. 18 A wooden cube just floats inside water when a 200 g mass is placed on it. When the mass is removed, the cube is 2 cm above the water level. What is the size of each sides of the cube?
(A) 6 cm
(B) 8 cm
(C) 10 cm
(D) 12 cm

Let a be the size of each side of the cube. Then,

$$
\begin{array}{cc} 
& 200 \times \mathrm{g}=(2) \times\left(\mathrm{a}^{2}\right) \times 1 \times \mathrm{g} \\
\therefore & \mathrm{a}=10 \mathrm{~cm}
\end{array}
$$

Q. 19 A vessel contain oil ( $\mathrm{d}=0.8 \mathrm{~g} / \mathrm{cc}$ ) over mercury $(\mathrm{d}=13.6 \mathrm{~g} / \mathrm{cc})$. A homogeneous sphere floats with half its volume immersed in mercury and the other half in oil. The density of the material of the sphere in $\mathrm{g} / \mathrm{cc}$ is -
(A) 3.3
(B) 6.4
(C) 7.2
(D) 12.8
[C]
Sol. For equilibrium, the total upward pull will be equal to the gravitational force
$\frac{\mathrm{V}}{2} \times 13.6 \times \mathrm{g}+\frac{\mathrm{V}}{2} \times(0.8) \mathrm{g}=\mathrm{V} \rho \mathrm{g}$
$\rho=7.2 \mathrm{~g} / \mathrm{cc}$
Q. 20 Water is flowing through a tube of non uniform cross section. If the radii of the tube at the entrance and exit are in the ratio $3: 2$, then the ratio of velocity of liquid entering and leaving the tube is-
(A) $1: 1$
(B) $4: 9$
(C) $9: 4$
(D) $8: 27$

Sol.[B] By equation of continuity
$\mathrm{A}_{1} \mathrm{v}_{1}=\mathrm{A}_{2} \mathrm{v}_{2} \Rightarrow \pi \mathrm{r}_{1}^{2} \mathrm{v}_{1}=\pi \mathrm{r}_{2}^{2} \mathrm{v}_{2}$
$\Rightarrow \frac{\mathrm{v}_{1}}{\mathrm{v}_{2}}=\frac{\mathrm{r}_{2}^{2}}{\mathrm{r}_{1}^{2}}=\left(\frac{2}{3}\right)^{2}=\frac{4}{9}$
Q. 21 A tank is filled with water to a height H . A hole is punched in one of the walls at a depth $h$ below the water surface. Then the distance $x$ from the foot of the wall at which the stream strikes the floor is -
(A) $2 \sqrt{\mathrm{Hb}}$
(B) $\sqrt{2(\mathrm{H}-\mathrm{h}) \mathrm{H}}$
(C) $2 \sqrt{(\mathrm{H}-\mathrm{h}) \mathrm{h}}$
(D) $2 h\left(\frac{\mathrm{H}-\mathrm{h}}{\mathrm{H}}\right)$
[C]
Sol. Using Bernoulli's equation, $\mathrm{h} \rho \mathrm{g}=1 / 2 \rho \mathrm{v}^{2}$
$\mathrm{v}=\sqrt{2 \mathrm{gh}}=$ horizontal speed of water jet from the hole.
Time of fall $=\sqrt{\frac{2(\mathrm{H}-\mathrm{h})}{\mathrm{g}}}, \mathrm{x}=\mathrm{vt}$
Q. 22 A cubical sealed vessel with edge $L$ is placed on a cart, which is moving horizontally with an
acceleration ' $a$ ' as shown in figure. The cube is filled with an ideal fluid having density $d$. The gauge pressure at the centre of the cubical vessel is -

(A) $\frac{\mathrm{Ldg}}{2}$
(B) $\left.\frac{L d}{2}(g)+a\right)$
(C) $\frac{\text { Lda }}{2}$
(D) $\frac{L d}{2}(g-a)$
[B]
Sol.


$$
\mathrm{P}_{1}=\mathrm{P}_{\mathrm{atm}}
$$

$$
\mathrm{P}_{2}=\mathrm{P}_{\mathrm{atm}}+\frac{\mathrm{Ldg}}{2}
$$

$$
\mathrm{P}_{3}=\mathrm{P}_{2}+\frac{\mathrm{Lda}}{2}
$$

$$
\therefore \mathrm{P}_{3}-\mathrm{P}_{\mathrm{atm}}=\frac{\mathrm{Ld}}{2}(\mathrm{~g}+\mathrm{a})
$$

Q. 23 When a loaded boat enters into the sea from a river, it rises because -
(A) there is more water in the sea than in river
(B) sea water is denser than river
(C) there is difference of temperature
(D) sea is deeper than river

Sol. Since the density of sea-water is more than that of river therefore lesser volume of sea water is required to be displaced to balance the weight of the boat.
Q. 24 A beaker containing water is placed on the platform of a spring balance. The balance reads 1.5 kg . A stone of mass 0.5 kg and density $500 \mathrm{~kg} / \mathrm{m}^{3}$ is completely immersed in water without touching the walls of beaker. Now the balance reading will be -
(A) 2 kg
(B) 1 kg
(C) 2.5 kg
(D) 3 kg
[C]
Sol. $\quad \mathrm{R}=1.5 \mathrm{~g}+\mathrm{B}$

$$
\begin{aligned}
& =1.5 \mathrm{~g}+\left(\frac{0.5}{500}\right) 1000 \mathrm{~g} \\
& =2.5 \mathrm{~g}
\end{aligned}
$$

Q. 25 A homogenous aluminium ball of volume V is suspended on a weightless theread from one end of a homogeneous rod of mass M. Rod is placed on the edge of a tumbler so that half of the ball is submerged in water when system is in equilibrium. The densities of aluminium and water are $\rho_{\mathrm{A}}$ and $\rho_{\mathrm{w}}$ respectively then -

(A) $\frac{y}{x}=\frac{2 M g+2\left[\rho_{A} g V-\rho_{W} g \frac{V}{2}\right]}{M g}$
(B) $\frac{y}{x}=1+\frac{2\left[\rho_{A}-\frac{\rho_{W}}{2}\right] V}{M}$
(C) $\frac{y}{x}=1$
(D) $\frac{y}{x}=2$
[B]
Q. 26 Liquids A and B are at $30^{\circ} \mathrm{C}$ and $20^{\circ} \mathrm{C}$ respectively. When mixed in equal masses, the temperature of the mixture is found to be $26^{\circ} \mathrm{C}$, The specific heats of $A$ and $B$ are in the ratio of-
(A) $3: 2$
(B) $1: 1$
(C) $2: 3$
(D) $4: 3$

Sol.[A] $\mathrm{ms}_{\mathrm{A}}(30-26)=\mathrm{ms}_{\mathrm{B}}(26-20)$
$\Rightarrow 4 \mathrm{~s}_{\mathrm{A}}=6 \mathrm{~s}_{\mathrm{B}}$
$\therefore \frac{\mathrm{s}_{\mathrm{A}}}{\mathrm{S}_{\mathrm{B}}}=\frac{6}{4}=\frac{3}{2}$
Q.27 The force $F$ needed to support the liquid of density $\rho$ is -

Q. 28 A pipe ABCD of uniform cross-section is bent into three sections, viz., a horizontal section $A B$, a vertical section $B C$ with $C$ below $B$, and a horizontal section $C D$. Liquid flowing through the pipe has speed $v_{1}$ and pressure $p_{1}$ in section $A B$, and speed $v_{2}$ and pressure $p_{2}$ in section CD.
(A) $\mathrm{v}_{1}=\mathrm{v}_{2}, \mathrm{p}_{1}=\mathrm{p}_{2}$
(B) $\mathrm{v}_{1}=\mathrm{v}_{2}, \mathrm{p}_{2}>\mathrm{p}_{1}$
(C) $\mathrm{v}_{2}>\mathrm{v}_{1}, \mathrm{p}_{2}>\mathrm{p}_{1}$
(D) $\mathrm{v}_{2}>\mathrm{v}_{1}, \mathrm{p}_{1}=\mathrm{p}_{2}$

Sol. [B]

$v_{1}=v_{2}$ (equation of continuity)
Also; $\frac{\mathrm{v}_{1}^{2}}{2}+\mathrm{gh}+\frac{\mathrm{P}_{1}}{\rho}=\frac{\mathrm{v}_{2}^{2}}{2}+0+\frac{\mathrm{p}_{2}}{\rho}$
$\mathrm{P}_{2}-\mathrm{P}_{1}=\mathrm{h} \rho \mathrm{g} \therefore \mathrm{P}_{2}>\mathrm{P}_{1}$
Q. 29 Water and gasoline surfaces are open to atmosphere and at the same elevation. Specific gravity of gasoline and liquid are 0.6 and 1.6 respectively. The height of liquid is -

(A) 1 m
(B) 0.6
(C) 1.6 m
(D) 1.5 m
[C]
Q. 30 A uniform rod of length 2.0 m specific gravity 0.5 and mass 2 kg is hinged atone end to the bottom of a tank of water (specific gravity $=$ 1.0) filled upto a height of 1.0 m as shown in figure. Taking the case $\theta \neq 0^{\circ}$ the force exerted by the hinge on the rod is : $\left(\mathrm{g}=10 \mathrm{~m} / \mathrm{s}^{2}\right)-$

(A) 10.2 N upwards
(B) 4.2 N downwards
(C) 8.3 N downwards
(D) 6.2 N upwards

Sol. Length of rod inside the water $=1.0 \sec \theta=$ $\sec \theta$
Q. 31 A homogenous aluminium ball of volume V is suspended on a weightless thread from one end of a homogeneous rod of mass M. Rod is placed on the edge of a tumbler so that half of the ball is submerged in water when system is in equilibrium. The densities of aluminium and water are $\rho_{\mathrm{A}}$ and $\rho_{\mathrm{W}}$ respectively then -

Sol.
[B]
Taking torque,

$$
\left(V \rho_{A} g-\frac{V}{2} \rho_{\omega} g\right) x=M g \times\left[y-\left(\frac{x+y}{2}\right)\right]
$$

Q. 32 In hydraulic press radii of connecting pipes $r_{1}$ and $r_{2}$ are in ratio $1: 2$. In order to lift a heavy
mass M on larger piston, the small piston must be pressed through a minimum force $f$ equal to -

(A) Mg
(B) $\mathrm{Mg} / 2$
(C) $\mathrm{Mg} / 4$
(D) $\mathrm{Mg} / 8$
[C]

Sol. According to Pascal's principle
$\frac{\mathrm{f}_{1}}{\mathrm{f}_{2}}=\frac{\mathrm{A}_{1}}{\mathrm{~A}_{2}}=\frac{\mathrm{r}_{1}^{2}}{\mathrm{r}_{2}^{2}}=\frac{1}{4}$

$$
\mathrm{f}_{1}=\frac{1}{4} \mathrm{Mg}
$$

Q. 33 A U-tube of cross section $A$ and 2 A as shown contains liquid of density d. Initially, the liquid in the two arms are held with a level difference h. After being released the levels equalize after some time. The work done by gravity forces on the liquid in the process is -

(A) $\frac{\mathrm{Adgh}^{2}}{3}$
(B) $\frac{\mathrm{Adgh}^{2}}{2}$
(C) $\frac{\mathrm{Adgh}^{2}}{6}$
(D) $\frac{\text { Adgh }^{2}}{8}$

Sol. $\quad \Delta U=m g\left(\frac{h}{2}-\frac{h}{6}\right)=m g \frac{h}{3}$
$=$ Adhg. $\frac{\mathrm{h}}{3}=\frac{\mathrm{Adh}^{2} \mathrm{~g}}{3}$
Q. 34 A given shaped glass tube having uniform area of cross-section is filled with water and is mounted on a rotatable shaft as shown in Fig. If the tube is rotated with a constant angular velocity $\omega$, then -

(A) Water level in A will rise and fall in B
(B) Water level in both A and B will rise
(C) Water level in B will rise and will fall in A
(D) Water level remains same in both A and B

Sol. [B] Extra force $F=\frac{m \omega^{2} r}{2}$ \& extra pressure
Q. 35 A container has a liquid filled upto the height H .

There is a hole at height $H / 4$. The area of hole is ' $a$ '. Density of liquid is $\rho$. Torque due to the efflux coming out about an axis passing through ' A ' and perpendicular to the plane of figure is

(A) $\frac{\rho a g H^{2}}{4}$
(B) $\frac{3 \rho a g H^{2}}{4}$
(C) $\frac{3}{8} \rho a g H^{2}$
(D) $\rho a g H^{2}$
[B]

A container has a hole at a height of 2 m . If the time taken by the efflux to strike the inclined plane perpendicularly is 1 sec . Then the height of liquid level initially is (Take $g=10 \mathrm{~m} / \mathrm{s}^{2}$ ) -

(A) 2 m
(B) 5 m
(C) 7 m
(D) 3 m

Q. 37 Water from a tap emerges vertically downwards with an initial speed of $1.0 \mathrm{~m} / \mathrm{s}$. The crosssectional area of tap is $10^{-4} \mathrm{~m}^{2}$. Assume that the pressure is constant throughout and that the flow is steady, the cross-sectional area of stream 0.15 m below the tap is :
(A) $5.0 \times 10^{-4} \mathrm{~m}^{2}$
(B) $1.0 \times 10^{-4} \mathrm{~m}^{2}$
(C) $5.0 \times 10^{-5} \mathrm{~m}^{2}$
(D) $2.0 \times 10^{-5} \mathrm{~m}^{2}$
[C]
Sol. From conservation of energy

$$
\begin{equation*}
\mathrm{v}_{2}^{2}=\mathrm{v}_{1}^{2}+2 \mathrm{gh} \tag{1}
\end{equation*}
$$

[can also be found by applying Bernouilli's theorem between 1 and 2]

From continuity equation
$\mathrm{A}_{1} \mathrm{v}_{1}=\mathrm{A}_{2} \mathrm{v}_{2}$
$\mathrm{v}_{2}=\left(\frac{\mathrm{A}_{1}}{\mathrm{~A}_{2}}\right) \mathrm{v}_{1}$
Substituting value of $\mathrm{v}_{2}$ from Eq. (2) in Eq. (1)
$\frac{\mathrm{A}_{1}^{2}}{\mathrm{~A}_{2}^{2}} \cdot \mathrm{v}_{1}^{2}=\mathrm{v}_{1}^{2}+2 \mathrm{gh}$
or $A_{2}^{2}=\frac{A_{1}^{2} v_{1}^{2}}{v_{1}^{2}+2 g h}$
$\therefore \mathrm{A}_{2}=\frac{\mathrm{A}_{1} \mathrm{v}_{1}}{\sqrt{\mathrm{v}_{1}^{2}+2 \mathrm{gh}}}$


Substituting the given values
$\mathrm{A}_{2}=\frac{\left(10^{-4} \mathrm{~m}^{2}\right)(1.0 \mathrm{~m} / \mathrm{s})}{\sqrt{(1.0 \mathrm{~m} / \mathrm{s})^{2}+2(10)(0.15)}}$
$\mathrm{A}_{2}=5.0 \times 10^{-5} \mathrm{~m}^{2}$
Q. 38 A cylinder of radius $R$ is floating in a liquid as shown. The work done in submerging the cylinder completely in the liquid of density ' $\rho$ ' is -

(A) $\frac{2}{9} \rho \pi R^{2} L^{2} g$
(B) $\frac{8}{18} \rho \pi R^{2} \mathrm{D}^{2} g$
(C) $\frac{1}{3} \rho \pi R^{2} L^{2} g$
(D) $\frac{2}{9} \rho R^{2} L^{2} g$
[A]
Q. 39 A sealed tank of length $\ell$ and height $h$, completely filled with liquid of density $\rho$, moves with horizontal acceleration a. The maximum difference in pressure between any two points in the liquid is -
(A) $\rho \mathrm{gh}$
(B) $\rho g \ell$
(C) $\rho(\mathrm{gh}+\mathrm{a} \ell)$
(D) all points in the liquid are at the same pressure
Sol. $\quad[C] P_{2}-P_{1}=x \rho a ; P_{3}-P_{2}=y \rho g$
$P_{3}-P_{1}=x \rho a+y \rho g \therefore P_{4}-P_{1}=\rho(g h+a \ell)$

Q. 40 A tank is filled with water to a height H. A hole is punched in one of the walls at a depth $h$ below the water surface. Then the distance x from the foot of the wall at which the stream strikes the floor is -
(A) $2 \sqrt{\mathrm{Hh}}$
(B) $\sqrt{2(\mathrm{H}-\mathrm{h}) \mathrm{H}}$
(C) $2 \sqrt{(\mathrm{H}-\mathrm{h}) \mathrm{h}}$
(D) $2 \mathrm{~h}\left(\frac{\mathrm{H}-\mathrm{h}}{\mathrm{H}}\right) \quad[\mathrm{C}]$

Sol. Using Bernoulli's equation, $h \rho g=1 / 2 \rho v^{2}$
$\mathrm{F}=\sqrt{2 \mathrm{gh}}=$ horizontal speed of water just from the hole.
Time of fall $=\sqrt{\frac{2(\mathrm{H}-\mathrm{h})}{\mathrm{g}}}, \mathrm{x}=\mathrm{vt}$

A hemispherical portion of radius $R$ is removed from the bottom of a cylinder of radius R . The volume of the remaining cylinder is V and its mass M. It is suspended by a string in a liquid of density $\rho$ where it stays vertical. The upper surface of the cylinder is at a depth $h$ below the liquid surface. The force on the bottom of the cylinder by the liquid is -

(A) Mg
(B) $\mathrm{Mg}-\mathrm{V} \rho \mathrm{g}$
(C) $\mathrm{Mg}+\pi \mathrm{R}^{2} \mathrm{~h} \rho g$
(D) $\rho g\left(V+\pi R^{2} h\right)$
[D]
Q. 42 A liquid having area of free surface ' A ' has an orifice at a depth ' h ' with an area 'a' below liquid surface, then the velocity V of flow through the orifice is -
(A) $\sqrt{2 \mathrm{gh}}$
(B) $\sqrt{2 g h} \sqrt{\frac{A^{2}}{A^{2}-a^{2}}}$
$\begin{array}{ll}\text { (C) } \sqrt{2 g h} \sqrt{\frac{\mathrm{~A}}{\mathrm{~A}-\mathrm{a}}} & \text { (D) None of these }\end{array}$
Sol. [B]
$P+\frac{1}{2} \rho V^{2}+\rho g h=P+\frac{1}{2} \rho v^{2}+0$ $\qquad$
also $\mathrm{AV}=\mathrm{av}$
Q. 43 Water is flowing in streamline motion through a tube of varying cross-section, as shown. The pressure p at points along the axis of the tube is represented by.

(A) $\begin{aligned} & \text { P } \\ & \text { P } \\ & \\ & \longrightarrow x\end{aligned}$


(D) P
[C] $1 ?$
Q. 44 A cube of density $0.5 \mathrm{~g} / \mathrm{cm}^{3}$ is placed in vessel and a liquid of density $1 \mathrm{~g} / \mathrm{cm}^{3}$ is gradually filled in the vessel at a constant rate then the graph representing variation of normal reaction of vessel on cube and time is -
(A)

(B)


(D)


Sol. Here force of buoyancy will act on cube when the liquid level rises above it and it will rise up.
Q. 45 A tank is filled with water to a height H. A hole is made in one of the walls at a depth D below the water surface. The distance x from the foot
of the wall at which the stream of water coming out of the tank strikes the ground is given by
(A) $x=2[D(H-D)]^{1 / 2}$
(B) $\mathrm{x}=2(\mathrm{gD})^{1 / 2}$
(C) $x=2[D(H+D)]^{1 / 2}$
(D) None of these
Q. 46 When the system shown in the following figure is in equilibrium and if the areas of crosssection of the smaller piston and bigger piston are 'a' and '10a' then

(A) $\mathrm{m}=\mathrm{M}$
(B) $\mathrm{M}=10 \mathrm{~m}$
(C) $\mathrm{m}=10 \mathrm{M}$
(D) $\mathrm{M}=100 \mathrm{~m}$
[B]
Q. 47 A small hole is made at the bottom of a symmetrical jar as shown in figure. A liquid is filled into the jar upto a certain height. The rate of descension of liquid is independent of the level of the liquid in the jar. Then the surface of the jar is a surface of revolution of the curve -

(A) $y=k x^{4}$
(B) $\mathrm{y}=\mathrm{kx} \mathrm{x}^{2}$
(C) $y=k x^{3}$
(D) $y=k x^{5}$

## Sol. [A]

Let $y$ be the height of liquid at some instant.
Then $-\frac{\mathrm{dy}}{\mathrm{dt}}=$ constant (given)
From equation of continuity,
$\left(\pi x^{2}\right)\left(-\frac{d y}{d t}\right)=a \sqrt{2 g y} \quad(a=$ area of hole $)$
Here, $\pi\left(\frac{-\mathrm{dy}}{\mathrm{dt}}\right)$, a and g are constants
Hence, squaring the equation, we get $y=k x^{4}$.
Q. 48


The volume of brick is 2.197 litres. The submerged brick is balanced by a 2.54 kg mass on the beam scale. The weight of the brick is -
(A) 46 N
(B) 50 N
(C) 56 N
(D) 72 N

Sol. [A]

$\mathrm{T}+\mathrm{F}_{\mathrm{B}}-\mathrm{W}=0$
$\mathrm{W}=\mathrm{T}+\mathrm{F}_{\mathrm{B}}=2.54 \times 9.8+10^{3} \times 2.197 \times 10^{-3} \times$ 10
$\mathrm{W}=46.43 \mathrm{~N}$
Q. 49 Water is flowing through a channel that is 12 m wide with a speed of $0.75 \mathrm{~m} / \mathrm{s}$. The water then flows into four identical channels that have a width of 4.0 m . The depth of the water does not change as it flows into the four channels. What is speed of the water in one of the smaller channels?

(A) $0.56 \mathrm{~m} / \mathrm{s}$
(B) $2.3 \mathrm{~m} / \mathrm{s}$
(C) $0.25 \mathrm{~m} / \mathrm{s}$
(D) $0.75 \mathrm{~m} / \mathrm{s}$
$\mathrm{Av}=\mathrm{A}_{1} \mathrm{~V}_{1}+\mathrm{A}_{2} \mathrm{v}_{2}+\mathrm{A}_{3} \mathrm{v}_{3}+\mathrm{A}_{4} \mathrm{~V}_{4}$
$\mathrm{Av}=4\left(\mathrm{~A}_{1} \mathrm{v}_{1}\right)$
$12 \times 0.75=4[4 \times \mathrm{v}]$
$\mathrm{v}=\frac{3}{4} \times 0.75$
$=0.75 \times 0.75$
$\mathrm{v}=0.56 \mathrm{~m} / \mathrm{s}$
Q. 50 The figure shows two fish tank, each having ends of width 1 foot. Tank A is 3 feet long while tank B is 6 feet long. Both tanks are filled with 1 foot of water.

$\mathrm{S}_{\mathrm{A}}=$ the <magnitude of the force of the water on the end of tank $A$
$S_{B}=$ the magnitude of the force of the water on
the end of tank B
$\mathrm{B}_{\mathrm{A}}=$ the magnitude of the force of the water on the bottom of tank A
$B_{B}=$ the magnitude of the force of the water on the bottom of tank B Using the notation given above, Which one of the following sets of equations below is correct for this situation?
(A) $\mathrm{S}_{\mathrm{A}}=\mathrm{S}_{\mathrm{B}}$ and $\mathrm{B}_{\mathrm{A}}=\mathrm{B}_{\mathrm{B}}$
(B) $\mathrm{S}_{\mathrm{A}}=2 \mathrm{~S}_{\mathrm{B}}$ and $\mathrm{B}_{\mathrm{A}}=\mathrm{B}_{\mathrm{B}}$
(C) $2 \mathrm{~S}_{\mathrm{A}}=\mathrm{S}_{\mathrm{B}}$ and $2 \mathrm{~B}_{\mathrm{A}}=\mathrm{B}_{\mathrm{B}}$
(D) $\mathrm{S}_{\mathrm{A}}=\mathrm{S}_{\mathrm{B}}$ and $2 \mathrm{~B}_{\mathrm{A}}=\mathrm{B}_{\mathrm{B}}$

Sol. [D]

$S_{A}=(\rho g h) A$
$\mathrm{S}_{\mathrm{B}}=(\rho \mathrm{gh})\left(1 \mathrm{ft}^{2}\right)$
$\mathrm{S}_{\mathrm{A}}=(\rho \mathrm{gh})\left(1 \mathrm{ft}^{2}\right) \quad \mathrm{B}_{\mathrm{B}}=(\rho \mathrm{gh})\left(6 \mathrm{ft}^{2}\right)$
$B_{A}=\rho g h\left(3 \mathrm{ft}^{2}\right)$
$S_{A}=S_{B} ; B_{B}=2 B_{A}$

## Sol. [A]

