Binomial Theorem

Single Correct Answer Type

1. If 7 divides $32^{32^{32}}$, the remainder is

A) 1 B) 0 C) 4 D) 6

Key. C

Sol. $32 = 2^5 \Rightarrow (32)^{32} = (2^5)^{32}$

$= 2^{160} = (3-1)^{160} = 3m+1, m \in N$

$\therefore (32)^{32^{32}} = (32)^{3m+1} = 2^{5(3m+1)}$

$2^{3(5m+1)}\, 2^2 = 4.8^{5m+1}$

$4(7+1)^{5m+1} = 4(7n+1), n \in N = 28n+4$

$\therefore$ When 7 divides $(32)^{32^{32}}$ remainder =4

2. If {x} represents the fractional part of x, then $\left\{\frac{5^{200}}{8}\right\}$ is

A) $\frac{1}{4}$ B) $\frac{1}{8}$ C) $\frac{3}{8}$ D) $\frac{5}{8}$

Key. B

Sol. $\frac{5^{200}}{8} = \frac{(5^2)^{100}}{8} = \frac{(1+24)^{100}}{8}$

$= \frac{1+{}^{100}C_1.24+{}^{100}C_2(24)^2+....+{}^{100}C_{100}(24)^{100}}{8}$

$= \frac{1}{8} + \text{int}\,eger \Rightarrow \left\{\frac{5^{200}}{8}\right\} = \frac{1}{8}$

3. For n>3 ,1.2 ${}^nC_r - 2.3\ {}^nC_{r-1} + ... + (-1)^r (r+1)(r+2)$ is

A) ${}^{n-3}C_r$ B) 2. ${}^{n-3}C_r$ C) ${}^{n+3}C_{r+1}$ D) ${}^{n-2}C_r$

Key. B

Sol. We have $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + ...$

$+{}^nC_{r-1}\ x^{r-1} + {}^nC_r\ x^r + ... + {}^nC_n x^n$ .....(1)

and $(1+x)^{-3} = 1 - {}^3C_1 x + {}^4C_2 x^2 ...... +$

$(-1)^{r-1}\ {}^{r+1}C_{r-1}\ x^{r-1} + (-1)^r$

$^{r+2}C_r\ x^r + ...to\infty$ ....(2)

Multiply (1) and (2) , we get

$(1+x)^{n-3} = \left(^nC_0 + ^nC_1x + ^nC_2\ x^2 + ... + ^nC_n\ x^n\right)\left(1 - ^3C_1x + ^4C_2\ x^2 - .....\infty\right)$

Clearly , the coefficient of x^r from the product in R.H.S is

$1.^nC_r - ^3C_1.^nC_{r-1} + ^4C_2.^nC_{r-2} - ...... + (-1)^r\ ^{r+2}C_r.^nC_0$

$= ^nC_r - 3^nC_{r-1} + \frac{4.3}{2.1}\ ^nC_{r-2} - \frac{5.4}{2.1}\ ^nC_{r-3}$

$+ ...... + (1)^r \frac{(r+2)(r+1)}{2.1}$

$= \frac{1}{2}\left[1.2\ ^nC_r - 2.3\,^nC_{r-1} + 4.3\ ^nC_{r-2} - ...... + (-1)r(r+2)(r+1)\right]$

$\therefore$ Required series = $2\times$ coefficient of x^r in $(1+x)^{n-3} = 2.^{n-3}C_r$

4. The coefficient of x^{50} in the expansion of

$(1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + ... + 1001\ x^{1000}$

A) $1000_{C_{50}}$ B) $1001_{C_{50}}$ C) $1002_{C_{50}}$ D) 2^{1001}

Key. C

Sol. Let, $S = (1+x)^{1000} + 2x(1+x)^{999} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + ... + 1001\ x^{1000}$

$\frac{x}{1+x}S = x(1+x)^{999} + +2x^2(1+x)^{998} + ...$

$+1000_x{}^{1000} + \frac{1001x^{1001}}{1+x}$

Subtract above equations,

$\left(1 - \frac{x}{1+x}\right)S = (1+x)^{1000} + (1+x)^{999} +$

$x^2(1+x)^{998} + ... + x^{1000} - \frac{1001x^{1001}}{1+x}$

$\Rightarrow S = (1+x)^{1001} + x(1+x)^{1000} + x^2(1+x)^{999}$

$+ .... + x^{1000}(1+x) - 100]\ x^{1001}$

$= \frac{(1+x)^{1001}\left[\left(\frac{x}{1+x}\right)^{1001} - 1\right]}{\frac{x}{1+x} - 1} - 1001\ x^{1001}$

[sum of G.P]

$= (1+x)^{1002} - x^{1002} - 1002\ x^{1001}$

$\therefore$ coefficient of x^{50} in S= coefficient of x^{50} in $\left[(1+x)^{1002}-x^{1002}-1002x^{1001}\right]={}^{1002}C_{50}$

5. The coefficient of the term independent of x in the expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$

A) 70 B) 112 C) 105 D) 210

Key. D

Sol. Given expression $=\frac{\left(x^{1/3}\right)^3+(1)^3}{x^{2/3}-x^{1/3}+1}-\frac{(x-1)}{x^{1/2}\left(x^{1/2}-1\right)}$

$=\frac{\left(x^{1/3}+1\right)\left(x^{2/3}-x^{1/3}+1\right)}{\left(x^{2/3}-x^{1/3}+1\right)}-\frac{\left(x^{1/2}+1\right)\left(x^{1/2}-1\right)}{x^{1/2}\left(x^{1/2}-1\right)}$

$=\left(x^{1/3}+1\right)-\left(1+x^{-1/2}\right)=x^{1/3}-x^{-1/2}$

$\Rightarrow\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$

$=\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$

$=\left(x^{1/3}-x^{-1/2}\right)^{10}$

T_{r+1} *in* $\left(x^{1/3}-x^{-1/2}\right)^{10}$ is

${}^{10}C_r\left(x^{1/3}\right)^{10-r}.(-1)^r.\left(x^{-1/2}\right)^r$

$=(-1)^r\ {}^{10}C_r\ x^{\left(\frac{10-r}{3}-\frac{r}{2}\right)}$ which is independent of x

If $\left(\frac{10-r}{3}-\frac{r}{2}\right)=0\Rightarrow r=4$

Hence required coefficient $={}^{10}C_4(-1)^4=210$

6. Let n be an odd natural number greater than 1. Then the number of zeros at the end of the sum 99^n+1 is

(A) 3 (B) 4 (C) 2 (D) None of these

Key. C

Sol. $1+99^n = 1+(100-1)^n = 1+$

$\left\{{}^nC_0 100^n - {}^nC_1.100^{n-1} + ... - {}^nC_n\right\}$

Because n is odd $= 100\left\{{}^nC_0.100^{n-1} - {}^nC_1.100^{n-2} + ... -\right.$

$\left.{}^nC_{n-2}.100 + {}^nC_{n-1}\right\}$

= $100 \times$ integer whose units place is different from 0

$\left[\because {}^nC_{n-1} = n, has\ odd\ digit\ at\ unit\ place\right]$

$\therefore$ There are two zeros at the end of the sum $99^n + 1$

7. If $C_0, C_1, C_2, ...C_n$ are the Binomial coefficients in the expansion $(1+x)^n$. 'n' being even, then $C_0 + (C_0 + C_1) + (C_0 + C_1 + C_2) + ...(C_0 + C_1 + C_2 + ... + C_{n-1}) =$ is equal to

(A) $n2^n$ (B) $n.2^{n-1}$ (C) $n.2^{n-3}$ (D) $n.2^{n-2}$

Key. B

Sol. Sum $= \left\{C_0 + \left(c_1 + c_2 + ... + c_{n-1}\right)\right\} + = \left\{\left(c_0 + c_1\right) + \left(c_0 + c_1 + ... + c_{n-2}\right)\right\} +$

$\left\{\left(c_0 + c_1 + c_2\right) + \left(c_0 + c_1 + ... + c_{n-3}\right)\right\} + ...\ to \left(\dfrac{n}{2}\right)$

Terms = $= \left(c_0 + c_1 + ...c_n\right) \times \dfrac{n}{2} = n.2^{n-1}$

8. If $\displaystyle\sum_{r=0}^{n}\left\{\frac{{}^nC_{r-1}}{{}^nC_r + {}^nC_{r-1}}\right\}^3 = \frac{25}{24}$, then n is equal to

(A) 3 (B) 4 (C) 5 (D) 6

Key. C

Sol. Let $t_r = \dfrac{{}^nC_{r-1}}{{}^nC_r + {}^nC_{r-1}} = \dfrac{{}^nC_{r-1}}{{}^{n+1}C_r} = \dfrac{{}^nC_{r-1}}{\dfrac{n+1}{r}\ {}^nC_{r-1}}$

$\therefore l_r = \dfrac{r}{n+1}$

Now,

$S = \displaystyle\sum_{r=0}^{n}\left\{l_r\right\}^3 \Rightarrow S = \sum_{r=0}^{n}\frac{r^3}{(n+1)^3} = \frac{1}{(n+1)^3}\sum_{r=0}^{n} r^3$

$\Rightarrow S = \dfrac{1}{(n+1)^3}\left\{\dfrac{n(n+1)}{2}\right\}^2 \Rightarrow S = \dfrac{n^2}{4(n+1)}$

Now, $S = \dfrac{25}{24}$ (given) which is only possible for 5

9. If m and n are any two odd positive integers with n < m, then the largest positive integer which divides all numbers of the form, $m^2 - n^2$ can be

(A) 4 (B) 6 (C) 8 (D) 9

Key. C

Sol. Let $m = 2k - 1$ *and* $n = 2p - 1, p < k$

Then $m^2 - n^2 = (m+n)(m-n)$

$= (2k + 2p - 2)(2k - 2p) = 4(k + p - 1)(k - p)$

Further if k and p both even, then k-p is even but k+p-1 is odd

If k and p both odd then k-p is even but k+p-1 is odd. If one is even and other odd then k-p is odd but k+p-1 is even. Thus in every case $(k-p)(k+p-1)$ even

$\therefore m^2 - n^2$ is divisible by $4 \times 2 = 8$. Hence , $m^2 - n^2$ is divisible by 8 or any multiple of 8. The largest integer among the given options is 8

10. The number of terms in $(a_1 + a_2 + a_3 + a_4)^3$ is

(A) 64 (B) 81 (C) 30 (D) 20

Key: D

Hint Any term of $(a_1 + \alpha_2 + a_3 + a_4)^3$ is of the form $a_1^{\alpha} . a_2^{\beta} . a_3^{\gamma} . a_4^{\delta}$.

where $\alpha + \beta + \gamma + \delta = 3$, $\alpha, \beta, \gamma, \delta \in \{0, 1, 2, 3\}$

Thus number of terms is 20.

11. The value of ${}^{12}C_2 + {}^{13}C_3 + {}^{14}C_4 + ... + {}^{999}C_{989}$ is

(A) ${}^{1000}C_{11} - 12$ (B) ${}^{1000}C_{11} + 12$ (C) ${}^{999}C_{11} - 12$ (D) ${}^{1000}C_{989}$

Key: A

Hint Since ${}^{10}C_0 + {}^{11}C_1 + {}^{12}C_2 + {}^{13}C_3 + ... + {}^{999}C_{989}$

$= {}^{1000}C_{989} = {}^{1000}C_{11}$

$\left(\text{Since , } {}^{10}C_0 = {}^{11}C_0 \text{ and } {}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r\right)$

So, ${}^{12}C_2 + {}^{13}C_3 + {}^{14}C_4 + ... + {}^{999}C_{989} = {}^{1000}C_{11} - 12$

12. The sum $S_n = \sum_{k=0}^{n} (-1)^K . {}^{3n}C_k$, where $n = 1, 2, ...$ is

(A) $(-1)^n . {}^{3n-1}C_{n-1}$ (B) $(-1)^n . {}^{3n-1}C_n$ (C) $(-1)^n . {}^{3n-1}C_{n+1}$ (D) None of these

Key: B

Hind: $S_n = {}^{3n}C_0 - {}^{3n}C_1 + {}^{3n}C_2 + ... + (-1)^n . {}^{3n}C_n$

But ${}^{3n}C_0 = {}^{3n-1}C_0$

$-{}^{3n}C_1 = -{}^{3n-1}C_0 - {}^{3n-1}C_1$

${}^{3n}C_2 = {}^{3n-1}C_1 + {}^{3n-1}C_2$

$-{}^{3n}C_3 = -{}^{3n-1}C_2 - {}^{3n-1}C_3$

....

$(-1)^n.{}^{3n}C_n = (-1)^n.{}^{3n-1}C_{n-1} + (-1)^n.{}^{3n-1}C_n$

On adding we get $S_n = (-1)^n.{}^{3n-1}C_n$

13. The Coefficient of x^9 in $(x-{}^{21}C_0)(x-{}^{21}C_1)(x-{}^{21}C_2)........(x-{}^{21}C_{10})$ *is*

(A) $2^{40}-\frac{1}{2}\,{}^{42}C_{20}$ (B) $2^{39}-\frac{1}{2}\,{}^{42}C_{21}$ (C) $2^{40}-{}^{42}C_{20}$ (D) $2^{39}-\frac{1}{4}\,{}^{42}C_{21}$

Key: D

Hint: Coefficient x^{10} =sum of products of ${}^{20}C_0, {}^{20}C_1 ....{}^{20}C_{10}$. Taking '2' at a time.

14. $\sum_{K=1}^{10}\frac{(-1)^{K-1}}{K}.(10_{C_K}) =$

(A) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+---+\frac{1}{11}$ (B) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+---+\frac{1}{10}$

(C) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+---+\frac{1}{9}$ (D) $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+---+\frac{1}{12}$

Key: B

Hint: Required value is $\frac{10_{C_1}}{1}-\frac{10_{C_2}}{2}+\frac{10_{C_3}}{3}---\frac{10_{C_{10}}}{10}$

To find which, consider $(1-x)^{10} = 10_{C_0} - 10_{C_1}x + 10_{C_2}x^2 -----+10_{C_{10}}x^{10}$

$$\Rightarrow \frac{(1-x)^{10}-1}{x} = -\left[10_{C_1} - 10_{C_2}x + ------+10_{C_{10}}x^9\right]$$

$$\Rightarrow \int_0^1 \frac{1-(1-x)^{10}}{x}dx = \int_0^1\left[10_{C_1} - 10_{C_2}x + ---+10_{C_{10}}x^9\right]dx$$

$$= 10_{C_1} - \frac{10_{C_2}}{2} + \frac{10_{C_3}}{3} ----- \frac{10_{C_{10}}}{10}$$

To find LHS consider $I_n = \int_0^1 \frac{1-(1-x)^n}{x}dx \Rightarrow I_{n+1} - I_n = \int_0^1 (1-x)^n\,dx = \frac{1}{n+1}$

$$\therefore I_{n+1} = \frac{1}{n+1} + I_n$$

$$\therefore I_{10} = \int_0^1 \frac{1-(1-x)^{10}}{x}dx = \frac{1}{10} + I_9 = \frac{1}{10} + \frac{1}{9} + I_8 \approx ---$$

$$= \frac{1}{10} + \frac{1}{9} + \frac{1}{8} + ----+1$$

15. The sum $\sum_{k=1}^{\infty} \frac{6^k}{\left(3^k - 2^k\right)\left(3^{k+1} - 2^{k+1}\right)} =$

(A) 1 (B) 2 (C) 3 (D) 4

Key: B

Hint: $\frac{6^k}{\left(3^k - 2^k\right)\left(3^{k+1} - 2^{k+1}\right)} = \frac{3^k - 2^k}{\left(3^k - 2^k\right)\left(3^{k+1} - 2^{k+1}\right)}$

$$= \frac{3^k}{3^k - 2^k} - \frac{3^{k+1}}{3^{k+1} - 2^{k+1}}$$

$$\sum_{k=1}^{\infty} \frac{3^k}{3^k - 2^k} - \frac{3^{k+1}}{3^{k+1} - 2^{k+1}} = \frac{3}{3-2} - \underset{n\to\infty}{Lt} \frac{3^2}{3^n - 2^n}$$

$$= 3 - 1 = 2$$

16. The value of the expression ${}^{10}C_0 10^9 - {}^{10}C_1 9^9 + {}^{10}C_2 8^9 \ldots\ldots\ldots - {}^{10}C_9$ *is*

A) $\underline{|9}$ B) $\underline{|10}$ C) $9\underline{|10}$ D) 0

Key: D

Hint: Given expression = No.of on to functions from a set of 9 elements to a set of 10 elements = 0

17. $\sum_{r=0}^{n} \frac{n-3r+1}{n-r+1} \frac{{}^nC_r}{2^r}$ is equal to

a) $\frac{1}{2^n}$ b) $\frac{1}{3^n}$ c) $\frac{1}{4^n}$ d) $\frac{1}{2^n} + 1$

Key: A

Hint: $S = \sum_{r=0}^{n} \left(1 - \frac{2r}{n-r+1}\right) \frac{{}^nC_r}{2^r} = \sum_{r=0}^{n} \frac{{}^nC_r}{2^r} - \sum_{r=0}^{n} \frac{{}^nC_{r-1}}{2^{r-1}} = \frac{1}{2^n}$

18. The sum of all the coefficient of those terms in the expansion of $(a + b + c + d)^8$ which contains b but not c

(A) 6305 (B) 6561 (C) 256 (D) 4^8

Key: A

Hint: Sum of the coefficients of the terms not containing c is 3^8 and of the term not containing b and c both is 2^8, so required sum = $3^8 - 2^8$.

19. If $\underline{|15} = 2^{p_1} 3^{p_2} 5^{p_3} 7^{p_4} 11^{p_5} 13^{p_6}$ then $\sum_{r=1}^{6} P_r$ is

(A) 24 (B) 23 (C) 22 (D)21

Key: A

Hint: $15! = 2^{11} \times 3^6 \times 5^3 \times 7^2 \times 11^1 \times 13^1$

$$\therefore \sum_{r=1}^{6} P_r = 11 + 6 + 3 + 2 + 1 + 1 = 24$$

20. The number of the positive integer pairs (x, y) such that $\frac{1}{x} + \frac{1}{y} = \frac{1}{2007}$ where $x < y$ is

(A) 5 (B) 6 (C) 7 (D) 8

Key: C

Hint: $\frac{1}{x}+\frac{1}{y}=\frac{1}{2007}$

$\Rightarrow (x+y)2007 = xy$

$\Rightarrow xy - 2007x - 2007y = 0$

$(x-2007)(y-2007) = 2007^2 = 3^4 \times 223^2$

The number of pairs is equal to the number of divisors of 2007^2 that is $(4+1)\times(2+1)=15$

Since $x < y$, so required number of pairs = 7

21. The greatest integer less than or equal to $\left(\sqrt{2}+1\right)^6$ is

(a) 194 (b) 195 (c) 196 (d) 197

Key: d

Hint: Let $\left(\sqrt{2}+1\right)^6$ = 1 + F, where I is an integer and 0 < F <1.

Let f = $\left(\sqrt{2}-1\right)^6$. We have

$\sqrt{2}-1=\frac{1}{\sqrt{2}+1} \Rightarrow 0<\sqrt{2}-1<1 \Rightarrow$ 0 < f < 1.

Also 1 + F + f = $\left(\sqrt{2}+1\right)^6+\left(\sqrt{2}-1\right)^6$

= $2[{}^6C_0 . 2^3 + {}^6C_2 . 2^2 + {}^6C_4 .2 + {}^6C_6]$

= 2(8 + 60 + 30 + 1) = 198

Hence F + f = 198 – I is an integer. But 0 < F + f < 2.

Therefore F + f = 1, and thus, I = 197.

22. The coefficient of x^6 in the expansion of $\left(1+\frac{x}{\lfloor 1}+\frac{x^2}{\lfloor 2}+\frac{x^3}{\lfloor 3}+\frac{x^4}{\lfloor 4}+\frac{x^5}{\lfloor 5}\right)^2$ is

(A) $\frac{2}{15}$ (B) $\frac{4}{15}$ (C) $\frac{31}{360}$ (D) $\frac{2}{45}$

Key: C

Hint: Coefficient of x^6 is $\frac{1}{\lfloor 5}\frac{1}{\lfloor 1}+\frac{1}{\lfloor 4}\frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}\frac{1}{\lfloor 3}+\frac{1}{\lfloor 2}\frac{1}{\lfloor 4}+\frac{1}{\lfloor 1}\frac{1}{\lfloor 5}$

$=\frac{1}{\lfloor 6}\left[{}^6C_1+{}^6C_2+{}^6C_3+{}^6C_4+{}^6C_5\right]$

$=\frac{1}{\lfloor 6}\left(2^6-2\right)=\frac{31}{360}$

23. Coefficient of x^{2009} in $(1 + x + x^2 + x^3 + x^4)^{1001} (1 - x)^{1002}$ is

(A) 0 (B) $4. {}^{1001}C_{501}$

(C) -2009 (D) none of these

Key: A

Hint: $(1 + x + x^2 + x^3 + x^4)^{1001} (1 - x)^{1002}$

= (1 - x) (1 - x^5)^{1001}, so all the powers of x will be of the 5m or 5m + 1 (m∈I)

So coff. of x^{2009} will be 0

24. If $\left(x^2+2x+4\right)^n=\sum_{r=0}^{2n}a_r x^r$, then $\sum_{r=n}^{2r}\left(\frac{a_{2n-r}}{a_r}\right)$ is

a) $2^{2n+1}-1$ b) $\frac{4^{n+1}-1}{3}$

c) 4^{n-1} d) $\frac{4^{n-1}+1}{2}$

Key : b

Sol : Put x = 2x

$$\left(4x^2+4x+4\right)^n=\sum_{r=0}^{2n}a_r 2^r x^r$$

Put $x=\frac{1}{x}$

$$\left[1+2x+(2x)^2\right]^n=\sum_{r=0}^{2n}a_r\,x^{2n-r}$$

Put 2x = y

$$\left(1+y+y^2\right)^n=\sum_{r=0}^{2n}a_r\frac{y^{2n-r}}{2^{2n-r}}$$

Equating coefficient of $x^r\ \frac{1}{4^n}a_r 2^r=a_{2n-r}2^{-r}$

$$\Rightarrow\frac{a_{2n-r}}{a_r}=\frac{1}{4n}4^r$$

$$\sum_{r=n}^{2n}\frac{a_{2n-r}}{a_n}=\sum_{r=n}^{2n}\frac{1}{4n}4r\ =\left(\frac{4^{n+1}-1}{3}\right)$$

25. Coefficient of x^6 in $\left((1+x)\left(1+x^2\right)^2\left(1+x^3\right)^3.....\left(1+x^n\right)^n\right)$ is

a) 26 b) 28 c) 30 d) 35

Key : b

Sol : The coefficient of x^6 in the given expression = coefficient of x^6 in

$\left(1+{}^6C_1x^6\right)\left(1+{}^5C_1x^5\right)\left(1+{}^4C_1x^4\right)\left(1+{}^3C_1x^3+{}^3C_2x^6\right)\left(1+{}^2C_1x^2+{}^2C_2x^4\right)(1+x)$

= coefficient of x^6 in $\left(1+6x^6+5x^5+4x^4\right)\left(1+2x^2+3x^3+x^4+6x^5+3x^6\right)(1+x)$

= coefficient of x^6 in $\left(11x^5+17x^6\right)\ (1+x)$ = 28

26. The coefficient of x^2 in $(1+x)(1+2x)(1+4x).....\left(1+2^n x\right)$ is

a) $\frac{\left(2^n+1\right)\left(2^n+2\right)}{3}$

b) $\frac{\left(2^{n+1}-1\right)\left(2^{n+1}-2\right)}{3}$

c) $\frac{\left(2^{n+1}+1\right)\left(2^{n+1}+2\right)}{3}$

d) $\frac{\left(2^{n+1}+1\right)\left(2^{n+1}-2\right)}{5}$

Key : b

Sol : Coefficient of $x^2=\frac{1}{2}\left[\left(1+2+2^2......+2^n\right)^2\right]-\left[1^2+2^2+....\left(2^n\right)^2\right]$

$$=\frac{1}{2}\left[\left(\frac{2^{n+1}-1}{1}\right)^2-\left(\frac{\left(2^2\right)^{n+1}-1}{3}\right)\right]$$

$$=\frac{1}{2}\left[\left(2^{n+1}-1\right)\left[\left(2^{n+1}-1\right)-\frac{2^{n+1}+1}{3}\right]\right]$$

$$=\frac{\left(2^{n+1}-1\right)\left(2^{n+1}-2\right)}{3}$$

27. Let $n\in N$ and $n<\left(5\sqrt{3}+8\right)^4$. Then the greatest value of n is

a) 77040 b) 77042

c) 77041 d) none of these

Key : c

Sol : Suppose $x=\left(5\sqrt{3}+8\right)^4$

= [x] + [x]

$0<5\sqrt{3}-8<1$

$0<\left(5\sqrt{3}-8\right)^4<1$

Let $F = \left(5\sqrt{3}-8\right)^4$

$0 < F < 1$

$[x]+\{x\}+F = \left(5\sqrt{3}+8\right)^4 + \left(5\sqrt{3}-8\right)^4$

$= {}^4C_0\left(5\sqrt{3}\right)^4 + {}^4C_1\left(5\sqrt{3}\right)^3(8)+....+\left({}^4C_0\left(5\sqrt{3}\right)^4 - {}^4C_1\left(5\sqrt{3}\right)^3(8)+...\right)$

$= 2.\left[C_0\left(5\sqrt{3}\right)^4 + C_2(8)^2\left(5\sqrt{3}\right)^2 + C_4(8)^4\right]$

$= 2\left[625(9)+(6)64(25)(3)+(64)(64)\right]$

$= 2[5625+28800+4096] = 77042$

$\{x\}$ + F must be an integer

Also $0 < \{x\}+F < 2$

$\Rightarrow \{x\}+f = 1$

$\Rightarrow [x] = 77042-1 = 77041$

28. If $P_k = \dfrac{1-x^{k+1}}{1-x}$, the number of terms in the product $P_1.P_2....P_n$ is

a) $\dfrac{n(n+1)}{2}$

b) $\dfrac{n^2-n}{2}$

c) $\dfrac{n^2+n-2}{2}$

d) $\dfrac{n^2+n+2}{2}$

Key ; D

Sol : $P_k = \dfrac{1-x^{k+1}}{1-x}$

$$P_1P_2P_3.....P_n = \frac{\left(1-x^2\right)\left(1-x^3\right)\left(1-x^4\right).....\left(1-x^{n+1}\right)}{\left(1-x^n\right)}$$

No. of terms = 1 + max. power of x = $1+\dfrac{n(n+1)}{2} = \dfrac{n^2+n+2}{2}$

29. The value of $a_0+a_2+a_4+....$ is

a) $\frac{2^n-1}{2}$ b) $\frac{2^n+1}{2}$

c) $\frac{(n-1)!}{2}$ d) $\frac{(n+1)!}{2}$

Key: d

Sol : $(1+x)(1+x+x^2)....(1+x+...+x^n)=a_0+a_1x+a_2x^2+a_3x^3+....$

Put, x = 1,

$2\times3\times4\times...(n-1)=a_0+a_1+a_2+a_3+....$

$0=a_0-a_1+a_2-a_3+...$

Put x = -1,

$(n+1)!=2[a_0+a_2+...]\Rightarrow a_0+a_2+....=\frac{(n+1)!}{2}$

30. The coefficient of abc^3de^2 in the expansion of $(a+b+c+d+e)^8$ is equal to

a) 3630 b) 3600

c) 3360 d) none of these

KEY : c

Sol : Coefficient of $abc^3de^2 is \frac{8!}{3!2!}=3360$

31. If each coefficient in the expansion of the expression $x(1+x)^n (n\in N)$ in powers of $'x'$ is divided by the exponent of corresponding power, then the sum of the values thus obtained is equal to

A) $\frac{2^n}{n+1}$ B) $\frac{2^n-1}{n+1}$ C) $\frac{2^n+1}{n+1}$ D) $\frac{2^{n+1}-1}{n+1}$

Key. D

Sol. $\frac{c_0}{1}+\frac{c_1}{2}+\frac{c_2}{3}+.....+\frac{c_{n-1}}{n}+\frac{c_n}{n+1}=\frac{2^{n+1}-1}{n+1}$

32. If $x^{15}-x^{13}+x^{11}-x^9+x^7-x^5+x^3-x=7$ then

(A) x^{16} is 15 (B) x^{16} is less than 15

(C) x^{16} greater than 15 (D) Nothing can be said about x^{16}

Key. C

Sol. $x^{15}-x^{13}+x^{11}-x^9+x^7-x^5+x^3-x$

$$= x(x^8+1)(x^4+1)(x^2-1)$$

$$x^{16}-1=(x^8+1)(x^4+1)(x^2+1)(x^2-1)$$

$$= \frac{7}{x}(x^2+1) = 7\left[\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2+2\right] > 14$$

$$\therefore x^{16} > 15$$

33. The value of the expression $\sum_{0\le i}\sum_{<j\le n}(-1)^{i+j-1}\ {}^nC_i.{}^nC_j =$

1) ${}^{2n-1}C_n$ 2) ${}^{2n}C_n$ 3) ${}^{2n+1}C_n$ 4) none

Key: 1

Hint: Let the required value be S

$$\sum_{i=0}^{n}\sum_{j=0}^{n}(-1)^{i+j-1}\ {}^nC_i{}^nC_j = \sum_{i=0}^{n}(-1)^{2i-1}\left({}^nC_i\right)^2+2S$$

$$0 \qquad = -\sum_{i=0}^{n}\left({}^nC_i\right)^2+2S$$

$$S = {}^{2n-1}C_{n-1} = {}^{2n-1}C_n$$

34. If $A_{(i,j)}$ be the co-efficient of $a^ib^jc^{2010-i-j}$ in the expansion of $(a+b-c)^{2010}$ then

(A) $A_{i,\,i}$ is defined for $i \le 1010$ (B) $A_{i,\,j} = A_{j,\,i}$

(C) $A_{2i,\,3i}$ is defined for $i \le 405$ (D) $A_{0,\,1} = 2000$

Key. B

Sol. Clearly, $A_{i,j} = \frac{2010!}{i!\ j!\left(2010-i-j\right)!}$

$$A_{j,i} = \frac{2010!}{j!i!\left(2010-i-j\right)!}$$

Hence, $A_{i,j} = A_{j,i}$

35. If n > 3 and $a, b \in R$, then the value of $ab - n(a-1)(b-1) + \frac{n(n-1)}{1.2}(a-2)$ $(b-2) - ..... + (-1)^n(a-n)(b-n)$ is equal to

(A) $a^n + b^n$ (B) $\frac{a^n - b^n}{a-b}$

(C) $(ab)^n$ (D) 0

Key. D

Sol. $T_{k+1} = (-1)^k\,.{}^nC_k\,(a-k)(b-k)$

$$= (-1)^{k.n}C_k\left[ab - k(a+b) + k^2\right]$$

Thus, the sum of series in (i)

$$= \sum_{k=0}^{n}(-1)^k.{}^nC_k\left[ab - k(a+b) + k^2\right]$$

$= ab\sum_{k=0}^{n}(-1)^k.{}^nC_k - (a+b)\sum_{k=0}^{n}(-1)^k k.{}^nC_k + \sum_{k=0}^{n}(-1)^k k^2.{}^nC_k$

We know that

$(1+x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + ... + {}^nC_nx^n ...(ii)$

Differentiating both sides w.r.t. x, we get

$n(1+x)^{n-1} = 1.\ {}^nC_1 + 2.{}^nC_2x + 3.\ {}^nC_3x^2 + ... + n.{}^nC_nx^{n-1} ....(iii)$

Multiplying it by x we get

$n.x(1+x)^{n-1} = 1.{}^nC_1x + 2.\ {}^nC_2x^2 + 3.\ {}^nC_3x^3 + ... + n.{}^nC_nx_n$

Differentiating w.r.t. x, we get

$n(1+x)^{n-1} + n(n-1)x(1+x)^{n-2}$

$= 1^2.{}^nC_1 + 2^2.{}^nC_2x + ... + n^2.{}^nC_nx^{n-1}$ ...(iv)

Putting x = –1 in (ii), (iii) and (iv), we get

$\sum_{k=0}^{n}(-1)^k.{}^nC_k = 0$

$\sum_{k=0}^{n}(-1)^k k.{}^nC_k = 0$

$\sum_{k=0}^{n}(-1)^k k.^2\ {}^nC_k = 0$

Thus, the sum of the series is 0

36. The expression $\frac{1}{\sqrt{4x+1}}\left[\left[\frac{1+\sqrt{4x+1}}{2}\right]^7 - \left[\frac{1-\sqrt{4x+1}}{2}\right]^7\right]$ is a polynomial in x of degree

(A) 7 (B) 5 (C) 4 (D) 3

Key. D

Sol. put = y and expand

37. Co-efficient of α^t in the expansion of,

$(\alpha + p)^{m-1} + (\alpha + p)^{m-2}(\alpha + q) + (\alpha + p)^{m-3}(\alpha + q)^2 + ...(\alpha + q)^{m-1}$

where $\alpha \neq -q$ and $p \neq q$ is :

(A) $\frac{{}^mC_t\left(p^t - q^t\right)}{p-q}$ (B) $\frac{{}^mC_t\left(p^{m-t} - q^{m-t}\right)}{p-q}$

(C) $\frac{{}^mC_t\left(p^t + q^t\right)}{p-q}$ (D) $\frac{{}^mC_t\left(p^{m-t} + q^{m-t}\right)}{p-q}$

Key. B

Sol. E = $(\alpha + p)^{m-1}$

$\Rightarrow$ *co-efficient of* α^t = = =

38. Let $\left(5+2\sqrt{6}\right)^n = p + f$ where $n \in N$ and $p \in N$ and $0 < f < 1$ then the value of, $f^2 - f + pf - p$ is :

(A) a natural number (B) a negative integer
(C) a prime number (D) are irrational number

Key. B

Sol. Ans is – 1

39. The coefficient of x^4 of in the expansion $(1+5x+9x^2+......\infty)(1+x^2)^{11}$ is

(A) ${}^{11}C_2 + 4\,{}^{11}C_1 + 3$ (B) ${}^{11}C_2 + 3\,{}^{11}C_1 + 4$
(C) $3\,{}^{11}C_2 + 4\,{}^{11}C_1 + 3$ (D) 171

Key. D

40. The number of distinct terms in the expansion of $\left(x+y^2\right)^{13} + \left(x^2+y\right)^{14}$ is

A) 27 B) 29 C) 28 D) 25

Key. C

Sol. To get common terms in both the expansions

$x^{r_1}\left(y^2\right)^{13-r_1} = \left(x^2\right)^{r_2}\left(y\right)^{14-r_2}$

$r_1 = 2r_2$ & $26 - 2r_1 = 14 - r_2$

$r_1 = 8;\ r_2 = 4$

$\therefore$ Only one term is common.

41. If $A_{(i,j)}$ be the co-efficient of $a^i b^j c^{2010-i-j}$ in the expansion of $(a+b-c)^{2010}$ then

(A) $A_{i,i}$ is defined for $i \le 1010$ (B) $A_{i,j} = A_{j,i}$
(C) $A_{2i,3i}$ is defined for $i \le 405$ (D) $A_{0,1} = 2000$

Key. A

Sol. Clearly, $A_{i,j} = \dfrac{2010!}{i!\,j!\left(2010-i-j\right)!}$

$A_{j,i} = \dfrac{2010!}{j!i!\left(2010-i-j\right)!}$

Hence, $A_{i,j} = A_{j,i}$

42. The value of $\dfrac{{}^{11}C_0}{1} + \dfrac{{}^{11}C_1}{2} + \dfrac{{}^{11}C_2}{3} + ... + \dfrac{{}^{11}C_{11}}{12}$ will be

A) $\frac{1}{12}\left(2^{12}-1\right)$ B) $\frac{1}{12}\left(2^{11}-1\right)$ C) $\frac{1}{12}\left(2^{11}+1\right)$ D) None of these

Key. A

Sol. Using $^{n}C_{k}=\frac{n}{k}\cdot{}^{n-1}C_{k-1}$

For $0\le k\le 11$

$$\frac{^{11}C_{k}}{k+1}=\frac{^{12}C_{k+1}}{12}$$

So, given expression is

$$\Rightarrow \frac{1}{12}\sum_{k=0}^{11}{}^{12}C_{k+1}\Rightarrow \frac{1}{12}\left[\sum_{k=0}^{11}{}^{12}C_{k}-{}^{12}C_{0}\right]\Rightarrow \frac{1}{12}\left(2^{12}-1\right)$$

43. $\sum_{m=1}^{n}\left(\sum_{k=1}^{m}\left(\sum_{p=k}^{m}{}^{n}C_{m}\cdot{}^{m}C_{p}\cdot{}^{p}C_{k}\right)\right)=$

a) $3^{n}-2^{n}$ b) $4^{n}-3^{n}$ c) $3^{n}+2^{n}$ d) $4^{n}-1$

Key: B

Hint: $\sum_{m=1}^{n}{}^{n}C_{m}\left(\sum_{k=1}^{m}\left(\sum_{p=k}^{m}\frac{m!}{p!(m-p)!}\cdot\frac{p!}{k!(p-k)!}\right)\right)$

$$=\sum_{m=1}^{n}{}^{n}C_{m}\left(\sum_{k=1}^{m}\left(\sum_{p=k}^{m}{}^{m-k}C_{p-k}\right)\frac{m!}{k!(m-k)!}\right)$$

$$=\sum_{m=1}^{n}{}^{n}C_{m}\left(\sum_{k=1}^{m}2^{m-k}\cdot{}^{m}C_{k}\right)$$

$$=\sum_{m=1}^{n}{}^{n}C_{m}\left((1+2)^{m}-2^{m}\right)=\sum_{m=1}^{n}\left({}^{n}C_{m}3^{m}-{}^{n}C_{m}2^{m}\right)$$

$$=(1+3)^{n}-1-(1+2)^{n}+1=4^{n}-3^{n}$$

44. The value of $2000_{C_2}+2000_{C_5}+2000_{C_8}+......+2000_{C_{2000}}=?$

a) $\frac{2^{1999}-1}{3}$ b) $\frac{2^{1999}+1}{3}$ c) $\frac{2^{2000}+1}{3}$ d) $\frac{2^{2000}-1}{3}$

Key: D

Hint $(1+x)^{n}=n_{C_0}+n_{C_1}x+n_{C_2}x^{2}+......+n_{C_n}x^{n}$

Put x = 1, w, w^{2} and add

$$\Rightarrow C_{2}+C_{5}+C_{8}+......=\frac{1}{3}\left\{2^{n}+(-1)^{n}\left(w^{2n+1}+w^{n+2}\right)\right\}$$

$$=\frac{1}{3}\left\{2^{2000}+(-1)^{2000}\left(w^{4001}+w^{2002}\right)\right\}$$

$= \dfrac{2^{2000}-1}{3}$

45. $3^{10}\,{}^{20}C_0\,{}^{20}C_{10} - 3^9\,{}^{20}C_1\,{}^{19}C_9 + ... + {}^{20}C_{10}\,{}^{10}C_0$ equals (${}^nC_r$ denote coefficient of x^r in $(1 + x)^n$.)

(A) ${}^{10}C_{10}.3^{10}$ (B) ${}^{20}C_{10}.3^{10}$

(C) ${}^{20}C_{10}.2^{10}$ (D) ${}^{20}C_{10}\,{}^{10}C_8$

Key. C

Sol. Coefficient of x^{10} in $\left[{}^{20}C_0(3+x)^{20} - {}^{20}C_1(3+x)^{19} + ... + {}^{20}C_{10}(3+x)^{10}\right]$

$= x^{10}$ in $(3 + x - 1)^{20}$

$= x^{10}$ in $(2 + x)^{20}$

$= 2^{10}\,{}^{20}C_{10}$

46. Number of ways, 3 persons having 6 one rupee coins, 7 one rupee coins, 8 one rupee coins respectively donate 10 one rupee coin collectively is ……….

a) 29 b) 83 c) 44 d) 47

Key. D

Sol. Coeff of $x^{10}\left(1+x+x^2+-----+x^6\right)\left(1+x+--+x^7\right)\left(1+x+---+x^8\right)$ is

47. $\sum_{r=1}^{n}\sum_{p=0}^{r-1} {}^nC_r.\,{}^rC_p.2^p$ is equal to

a) $4^n - 3^n + 1$ b) $4^n - 3^n - 1$

c) $4^n - 3^n + 2$ d) $4^n - 3^n$

Key. D

Sol. $\sum_{r=1}^{n} nC_r\left((1+2)^r - 2^r\right) = \sum_{r=1}^{n} nC_r 2^r - \sum_{r=1}^{n} nC_r 2^r$

$=\left(4^n - 1\right) - \left(3^n - 1\right) = 4^n - 3^n$

48. The value of $\binom{50}{6} - \binom{5}{1}\binom{40}{6} + \binom{5}{2}\binom{30}{6} - \binom{5}{3}\binom{20}{6} + \binom{5}{4}\binom{10}{6}$ where $\binom{n}{r}$ denotes ${}^nC_r$, is

(A) 15625 (B) 0

(C) 1000000 (D) 2250000

Key. D

Sol. ${}^{50}C_6 - {}^5C_1\,{}^{40}C_6 + {}^5C_2\,{}^{30}C_6 - {}^5C_3\,{}^{20}C_6 + {}^5C_4\,{}^{10}C_6$ = coefficient of x^6 in $[{}^5C_0(1 + x)^{50} - {}^5C_1(1 + x)^{40} + {}^5C_2(1 + x)^{30} - {}^5C_3(1 + x)^{20} + {}^5C_4(1 + x)^{10} - {}^5C_5(1 + x)^0]$ = coefficient x^6 in $[(1 + x)^{10} - 1]^5$
= coefficient of x^6 in $({}^{10}C_1\,x + {}^{10}C_2\,x^2 + ...)^5 = {}^5C_1({}^{10}C_2)({}^{10}C_1)^4 = 2250000$.

49. The value of the expression ${}^{10}C_0 10^9 - {}^{10}C_1 9^9 + {}^{10}C_2 8^9 .... - {}^{10}C_9$ *is*

a) $\lfloor 9$ b) $\lfloor 10$ c) $9\lfloor 10$ d) 0

Key. D

Sol. Given expression = No.of on to functions from a set of 9 elements to a set of 10 elements = 0

50. The term independent of x and y in the expansion of

$[(\sqrt{x}+\frac{1}{\sqrt{x}})^2+(\sqrt{y}+\frac{1}{\sqrt{y}})^2+(\sqrt{xy}+\frac{1}{\sqrt{xy}})^2-4]^n$ is

a) $(\sum_{r=0}^{n} {}^nc_r)^2$ b) $\sum_{r=0}^{n}({}^nc_r)^2$ c) $(\sum_{r=0}^{n} {}^nc_r)^3$ d) $\sum_{r=0}^{n}({}^nc_r)^3$

Key. D

Sol. The given expression can be written as $(1+x)^n.(1+y)^n.(1+\frac{1}{xy})^n$. The constant term is clearly $C_0^3+C_1^3+-----+C_n^3$ where $c_r={}^nc_r$.

51. The sum of the rational terms in the expansion of $\left(\sqrt{2}+\sqrt[5]{3}\right)^{10}$ is :

a) 41 b) 42 c) 39 d) 45

Key. A

Sol. $T_{r+1}=10C_r\frac{10-r}{2^2}.3^{\frac{r}{5}}$

This is rational , if $\frac{10-r}{2}$ and $\frac{r}{5}$ are integers.

$\therefore$ There are only two rational terms

Namely $10C_0\left(\sqrt{2}\right)^{10}\left(3^{\frac{1}{5}}\right)^0$ and $10C_{10}\left(\sqrt{2}\right)^0\left(3^{\frac{1}{5}}\right)^{10}$

$\therefore$ sum = 32 + 9 = 41

52. The values of 'r' such that $(100)C_r\left(\frac{1}{5^8}\right)^{100-r}\left(\frac{1}{2^6}\right)^r$ is rational is :

a) 84 b) 85 c) 86 d) 42

Key. A

Sol. Direct verification in sufficient.

53. If $\left(1+x\right)^n=a_0+a_1x+a_2x^2+.....+a_nx^n$ in which a_{n-3},a_{n-2},a_{n-1} are in AP, then

a) a_1,a_2,a_3 are in GP b) a_1,a_2,a_3 are in HP c) $n=7$ d) $n=14$

Key. C

Sol. $a_{n-3}=a_3,a_{n-2}=a_2,a_{n-1}=a_1\left(\because nC_r=nC_{n-r}\right)$

$\therefore$ (A) is correct.

a_1,a_2,a_3 are in AP $\Rightarrow n,\frac{n(n-1)}{2},\frac{n(n-1)(n-2)}{6}$ are in AP.

$$\frac{n+\frac{n(n-1)(n-2)}{6}}{2}=\frac{n(n-1)}{2}$$

$$6n+n^3-3n^2+2n=6n^2-6n$$

$n^3-9n^2+14n=0$, $(n-7)(n-2)=0$

$\therefore$ n = 7.

54. The coefficient of $a^8b^6c^4$ in the expansion of $\left(a+b+c\right)^{18}$ is :

a) $18C_4\times14C_6$ b) $18C_{10}\times10C_8$

c) $18C_8 \times 12C_8$ d) $18C_{14} \times 14C_7$

Key. A

Sol. $(a+b+c)^{18} = \left[a+(b+c)\right]^{18}$

$= 18C_0 a^{18} + 18C_1 a^{17}(b+0^{71} + 18C_2 a^{16}(b+c)^2 + ... + 18C_{10} a^8 (b+c)^{10} + ...$

$\therefore$ coefficient of $a^8 b^6 c^4 = 18C_{10} \times 10C_6$

55. In the expansion of $(1 + x)(1 + x + x^2) ..... (1 + x + x^2 + ... + x^{2n})$, the sum of coefficients is

(A) 1 (B) (2n)!

(C) (2n)! + 1 (D) (2n + 1)!

Key. D

Sol. Let $(1 + x)(1 + x + x^2) ..... (1 + x + ... + x^{2n}) = a_0 + a_1x + a_2x^2 + ...$

Put x = 1, we get

$a_0 + a_1 + a_2 + ... = 2.3.4 ..... (2n + 1) = (2n + 1)!$

56. If x = $\left(\sqrt{3}+\sqrt{2}\right)^6$, then the largest integer not exceeding x is

(A) 729 (B) 971

(C) 969 (D) 970

Key. C

Sol. x = $(5 + 2\sqrt{6})^3$

Let x = k + f $k \in N$, 0 < f < 1

Consider: $5 - 2\sqrt{6} = \dfrac{1}{5+2\sqrt{6}} \Rightarrow 0 < 5 - 2\sqrt{6} < 1 \Rightarrow 0 < (5 - 2\sqrt{6})^n < 1$

Let F = $(5 - 2\sqrt{6})^n$, 0 < F < 1

$k + f + F = (5 + 2\sqrt{6})^3 + (5 - 2\sqrt{6})^3 = 2[5^3 + {}^3C_2\, 5.\, (2\sqrt{6})^2]$

= 2[125 + 360] = 970.

$\Rightarrow$ f + F is a integer, but 0 < f + F < 2

$\Rightarrow$ f + F = 1

$\Rightarrow$ k = 969.

57. The coefficient of x^{53} in the expansion of $\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} 2^m$ is

A) ${}^{100}C_{47}$ B) ${}^{100}C_{53}$ C) $-{}^{100}C_{53}$ D) $-{}^{100}C_{100}$

Key. C

Sol. $(x-3)^{100} + {}^{100}C_1 (x-3)^{99} 2^1 + ... + {}^{100}C_{100} 2^{100}$

$= \{(x-3)+2\}^{100} = (x-1)^{100} = (1-x)^{100} = {}^{100}C_{53}(-1)^{53} = -{}^{100}C_{53}$

58. If 7 divides $32^{32^{32}}$, the reminder is

A) 1 B) 0 C) 4 D) 81

Key. C

Sol. $32^{32^{32}} = 3m+1, m \in I^+$

$32^{32^{32}} = (2^5)^{3m+1} = 2^{15m+5} = 28n+5$

59. The number of irrational terms in the expansion of $\left(5^{\frac{1}{3}}+2^{\frac{1}{4}}\right)^{100}$ is

A) 94 B) 92 C) 93 D) 91

Key. B

Sol. $\left(5^{\frac{1}{3}}+2^{\frac{1}{4}}\right)^{100}=\left(2^{\frac{1}{4}}+5^{\frac{1}{3}}\right)^{100}$

$T_{r+1}={}^{100}C_r\left(2^{\frac{1}{4}}\right)^{100-r}.\left(5^{\frac{1}{3}}\right)^{r}={}^{100}C_r\,2^{25-\frac{r}{4}}.5^{\frac{r}{3}}$

For rational terms, 'r' should be divisible by 12.

$\therefore$ No. of rational terms = 9

$\therefore$ No. of irrational terms = 101-9=92

60. The greatest integer less than or equal to $(\sqrt{3}+1)^6$ is

A) 416 B) 414 C) 417 D) 415

Key. D

Sol. $(\sqrt{3}+1)^6+(\sqrt{3}-1)^6=2\left[{}^6C_0(\sqrt{3})^6+{}^6C_2(\sqrt{3})^4+{}^6C_4(\sqrt{3})^2+{}^6C_6\right]=416$

Let $(\sqrt{3}+1)^6=I+F$ where I is integral part and F is fractional part

Let $(\sqrt{3}-1)^6=G$

$0<F<1; 0<G<1 \Rightarrow 0<F+G<2 \Rightarrow F+G=1$

$I+F+G=416 \Rightarrow I+1=416 \Rightarrow I=415$

61. $2\,{}^{10}C_0+\frac{2^2}{2}\,{}^{10}C_1+\frac{2^3}{3}\,{}^{10}C_2+......+\frac{2^{11}}{11}\,{}^{10}C_{10}=$

A) $2^{11}-1/11$ B) $3^{11}-1/11$ C) $2^{11}-2/11$ D) $4^{11}-1/11$

Key. B

Sol. Conceptual

62. If the fourth term in the expansion of $\left(2+\frac{3}{8}x\right)^{10}$ has the maximum numerical value then the range of x contains.

(a) $-\frac{64}{21}<x<-2$ (b) $1<x<\frac{64}{21}$ (c) $-2<x<\frac{64}{21}$ (d) $-\frac{64}{21}<x<2$

Key. A

Sol. Conceptual

63. If n is an odd natural number then $\sum_{r=0}^{n}\frac{(-1)^r}{n_{c_r}}$ is equal to

(a) 0 (b) 1 / n (c) $\frac{n}{2^n}$ (d) $n.2^n$

Key. A

Sol. Conceptual

64. If $(1+x)^n=\sum_{r=0}^{n}n_{c_r}x^r$ then $C_0^{\,2}+\frac{C_1^{\,2}}{2}+\frac{C_2^{\,2}}{3}+.......+\frac{C_n^{\,2}}{n+1}=$

(a) $\frac{\lfloor 2n}{(\lfloor n)^2}$ (b) $\frac{\lfloor 2n+1}{(\lfloor n+1)^2}$ (c) $\frac{\lfloor 2n-1}{(\lfloor n-1)^2}$ (d) $\frac{\lfloor n}{(\lfloor n-1)^2}$

Key. B

Sol. Conceptual

65. In the expansion of $\left(\sqrt[3]{\frac{a}{b}}+\sqrt[3]{\frac{b}{\sqrt{a}}}\right)^{21}$ the term containing same powers of a and b is

(a) 11th (b) 13th (c) 12th (d) 6th

Key. B

Sol. $T_{r+1}=21_{C_r}a^{\frac{42-3r}{6}}b^{\frac{2r-21}{3}}$

$42-3r=4r-42\Rightarrow r=12$

66. The coefficient of x^4 of the expansion $(1+5x+9x^2+.......\infty)(1+x^2)^{11}$ is

(a) $11_{C_2}+4.11_{C_1}+3$ (b) $11_{C_2}+3.11_{C_1}+4$ (c) $3.11_{C_2}+4.11_{C_1}+3$ (d) 171

Key. D

Sol. Co-efficient of x^4

= $(1+5x+9x^2+13x^3+17x^4+......)(1+11x^2+11_{C_2}x^4+.....)$

$=11_{C_2}+99+17$

67. The coefficient of x^{n-2} in the polynomial $(x-1)(x-2)(x-3).....(x-n)$ is

(a) $\frac{n(n^2+2)(3n+1)}{24}$ (b) $\frac{n(n^2-1)(3n+2)}{24}$ (c) $\frac{n(n^2+1)(3n+4)}{24}$ (d) None

Key. B

Sol. $T_2=(x-\alpha_1)(x-\alpha_2)(x-\alpha_3)......(x-\alpha_n)$

$=x^n-\sum\alpha_i x^{n-1}+\alpha_1\alpha_2 x^{n-2}+..............$

68. If $1,\propto_1,\propto_2,\propto_3,\propto_4$ are the fifth roots of unity then $\sum_{i=1}^{4}\frac{1}{2-\propto_i}=$

(a) $\frac{51}{31}$ (b) $\frac{49}{31}$ (c) $\frac{25}{32}$ (d) $\frac{25}{16}$

Key. B

Sol. we know that $z^5-1=(z-1)(z-\alpha_1)(z-\alpha_2)(z-\alpha_3)(z-\alpha_4)$

Take log on both sides, diff.w.r.t. z and put z = 2.

69 If a_1 and a_2 be the coefficients of x^n in the expansion of $(1+x)^{2n}$ and $(1+x)^{2n-1}$ respectively, then

1. $a_2=2a_1$ **2.** $a_1=2a_2$ **3.** $a_1=a_2$ **4.** None of these

Key. **2**

Sol. Consider T_{r+1} in $(1+x)^{2n}$ $\therefore$ $T_{r+1}={}^{2n}C_r x^r$

$a_1=$ Coefficient of $x^n={}^{2n}C_n$

$$= \frac{2n!}{n!n!} = \frac{2n(2n-1)!}{n(n-1)!(n)!} = \frac{2(2n-1)!}{(n-1)!(n!)}$$

Again coefficient of T_{r+1} in $(1+x)^{2n-1}$ is ${}^{2n-1}C_r$

$a^2 =$ Coefficient of x^n in $(1+x)^{2n-1}$

$$= {}^{2n-1}C_n = \frac{(2n-1)!}{n!(n-1)!}$$

$$a_2 = \frac{1}{2}\frac{2(2n-1)!}{(n-1)!n!} = \frac{1}{2}a_1$$

$\therefore\ 2a_2 = a_1$

70. If $C_0, C_1, C_2 .....$ are binomial coefficients in the expansion $\sum_{r=0}^{n} C_r x^r$, then value of the expression (series)

$$\frac{2C_0}{1} + \frac{3C_1}{2} + \frac{4C_2}{3} + \frac{5C_3}{4} + ... + \text{ is}$$

1. $\frac{2^n+1}{n+1}$ **2.** $\frac{2^n-1}{n+1}$ **3.** $\frac{2^n(n+3)-1}{n+1}$ **4.** $\frac{2^n(n+2)-1}{n+1}$

Key. **3**

Sol. Given

$$(1+x)^n = C_0 + C_1x + c_2x^2 + ... + C_nx^n$$

Integrating both sides with respect to x, we get

$$\frac{(1+x)^{n+1}}{n+1} = \frac{C_0x}{1} + \frac{C_1x^2}{2} + \frac{C_2x^3}{3} + ... + \frac{C_nx^{n+1}}{n+1} + k$$

Putting $x = 0$,

We get $k = \frac{1}{n+1}$

$$\therefore\quad \frac{(1+x)^{n+1}-1}{n+1}$$

$$= \frac{C_0 x}{1} + \frac{C_1 x^2}{2} + \frac{C_2 x^3}{3} + ... + \frac{C_n x^{n+1}}{n+1}$$

Multiplying with x both sides

$$\frac{x(1+x)^{n+1} - x}{n+1} = \frac{C_0 x^2}{1} + \frac{C_1 x^3}{2} + \frac{C_2 x^4}{3} + ... + \frac{C_n x^{n+1}}{n+1}$$

Differentiating with respect to x

$$\frac{(n+1)x(1+x)^n + (1+x)^{n+1} - 1}{n+1}$$

$$= \frac{2C_0 x}{1} + \frac{3C_1 x^2}{2} + \frac{4C_2 x^3}{3} + ... + \frac{(n+2)C_n x^{n+1}}{n+1}$$

Now putting $x = 1$ both sides, we get

$$\frac{2^{n+1} + (n+1)2^n - 1}{n+1}$$

$$= \frac{2C_0}{1} + \frac{3C_1}{2} + \frac{4C_2}{3} + ... + \frac{(n+2)C_n}{n+1}$$

$$\frac{2^n(n+3) - 1}{n+1} = \frac{2C_0}{1} + \frac{3C_1}{2} + \frac{4C_2}{3} + ... + \frac{(n+2)C_n}{n+1}$$

71. In the expansion of $(1+x)^{70}$,the sum of coefficients of odd powers of x is

1. 0 **2.** 2^{69} **3.** 2^{70} **4.** 2^{71}

Key. **2**

Sol. Fact. The sum of the coefficients of odd powers in the expansion of $(1+x)^n$ = sum of the coefficients of even powers in $(1+x)^n$

$$= 2^{n-1}$$

$$2^{70-1} = 2^{69}$$

72. Number of irrational terms in the expansion of $\left(\sqrt[5]{2}+\sqrt[10]{3}\right)^{60}$ are

1. 54 **2.** 61 **3.** 30 **4.** 31

Key. **1**

Sol. Given $\left(\sqrt[5]{2}+\sqrt[10]{3}\right)^{60}$ = $\left(2^{\frac{1}{5}}+3^{\frac{1}{10}}\right)^{60}$

Now L.C.M. of 5 and 10 is 10

$\therefore$ Number of rational terms let us writes $T_{r+1}={}^{60}C_r\left(2^{\frac{1}{5}}\right)^{60-r}\left(3^{\frac{1}{10}}\right)^{r}$

$={}^{60}C_r\,2^{12-\frac{r}{5}}3^{\frac{r}{10}}$

As $0\le r\le 60$

$\therefore$ $r=0,10,20,30,40,50,60$

$\therefore$ Number of rational terms is 7

$\therefore$ Number of irrational terms equals to

Total number of terms - Number of rational terms

$=61-7=54$

73. If $C_0,C_1,C_2,....,C_n$ are Binomial Coefficients, such that $S_n=\sum_{r=0}^{n}\frac{1}{{}^nC_r}$ and $t_n=\sum_{r=0}^{n}\frac{r}{{}^nC_r}$ then $\frac{t_n}{S_n}$ equals

1. $\frac{n}{2}$ **2.** $\frac{n(n+1)}{2}$ **3.** $\frac{n+1}{2}$ **4.** None of these

Key. **1**

Sol. Given $t_n = \sum_{r=o}^{n} \frac{r}{C_r} = \sum_{r=0}^{n} \frac{n-(n-r)}{C_{n-r}}$ $\left(\because {}^n C_r = {}^n C_{n-r}\right)$

$$= \sum_{r=0}^{n} \frac{n}{C_{n-r}} - \sum_{r=0}^{n} \frac{n-r}{C_{n-r}}$$

$$= nS_n - \left[\frac{n}{C_n} + \frac{n-1}{C_{n-1}} + ... + \frac{1}{C_1} + 0\right]$$

$$t_n = nS_n - \sum_{r=0}^{n} \frac{r}{C_r}$$

$$t_n = nS_n - t_n$$

$$2t_n = nS_n$$

$$\therefore \frac{t_n}{S_n} = \frac{n}{2}$$

74. If $(1+x)^n = \sum_{r=0}^{n} C_r x^r$, then the value of $3C_1 + 7C_2 + 11C_3 + ... + (4n-1)C_n$ is

1. $(4n-1)2^n$ **2.** $(2n-1)2^n$ **3.** $(2n-1)2^n + 1$ **4.** $(4n-1)2^n - 1$

Key. **3**

Sol. Let $S = 3C_1 + 7C_2 + 11C_3 + ... + (4n-1)C_n$

Let us write

$$T_r = (4r-1)C_r$$

$$T_r = 4rC_r - C_r$$

$= 4r \frac{n}{r} C_{r-1} - C_r$ using ${}^n C_r = \frac{n}{r} . {}^{n-1}C_{r-1}$ \

$$\therefore \sum_{r=1}^{n} T_r = 4n \sum_{r=1}^{n} C_{r-1} - \sum_{r=1}^{n} C_r$$

$$= 4\ n . 2^{n-1} \left(2^n - 1\right)$$

$$= 2n.2^n - 2^n + 1$$

$S = 2^n (2n-1) + 1$ By using

$(1+x)^{n-1} = C_0 + C_1x + C_2x^2 + ... + C_{n-1}x^{n-1}$

$2^{n-1} = C_0 + C_1 + C_2 + ... + C_{n-1}$

75. Middle term in the expansion of $\left(1-3x+3x^2-x^3\right)^{2n}$ is

1. $\frac{(6n)!x^n}{(3n)!(3n)!}$ **2.** $\frac{(6n)!.x^{3n}}{(3n)!}$ **3.** $\frac{(6n)!}{(3n)!(3n)!}(-x)^{3n}$ **4.** None of these

Key. 3

Sol. $\left(1-3x+3x^2-x^3\right)^{2n} = (1-x)^{6n}$

Concept: Index $= 6n$ which is even so most middle term

is $\left(\frac{6n}{2}+1\right)^{th}$ $i.e.,(3n+1)^{th}$ term is middle term,

$T_{3n+1} = {}^{6n}C_{3n}(-x)^{3n} = \frac{6n!}{3n!3n!}(-x)^{3n}$

76. Let n be an odd natural number greater than 1. Then the number of zeros at the end of the sum $99^n + 1$ is

(A) 3 (B) 4 (C) 2 (D) None of these

Key. C

Sol. $1+99^n = 1+(100-1)^n = 1+$
$\left\{{}^nC_0 100^n - {}^nC_1.100^{n-1} + ... - {}^nC_n\right\}$
Because n is odd $= 100\left\{{}^nC_0.100^{n-1} - {}^nC_1.100^{n-2} + ... -\right.$
$\left.{}^nC_{n-2}.100 + {}^nC_{n-1}\right\}$
= 100 × integer whose units place is different from 0
$\left[\because {}^nC_{n-1} = n, has\ odd\ digit\ at\ unit\ place\right]$
∴ There are two zeros at the end of the sum $99^n + 1$

77. If $C_0, C_1, C_2, ...C_n$ are the Binomial coefficients in the expansion $(1+x)^n$. 'n' being even, then $C_0 + (C_0+C_1) + (C_0+C_1+C_2) + ...(C_0+C_1+C_2+....+C_{n-1}) =$ is equal to

(A) $n2^n$ (B) $n.2^{n-1}$ (C) $n.2^{n-3}$ (D) $n.2^{n-2}$

Key. B

Sol. Sum $= \{C_0 + (c_1+c_2+......+c_{n-1})\} + = \{(c_0+c_1)+(c_0+c_1+.......+c_{n-2})\} +$

$\{(c_0+c_1+c_2)+(c_0+c_1+......+c_{n-3})\}+.......\ to \left(\frac{n}{2}\right)$

Terms = $=(c_0+c_1+......c_n)\times\frac{n}{2}=n.2^{n-1}$

78. The positive integral values of n such that $1.2^1+2.2^2+3.2^3+4.2^4+5.2^5+....+n.2^n=2^{(n+10)}+2$ is

(A) 313 (B) 513 (C) 413 (D) 613

Key. B

Sol.

$2^1+2^2+2^3+...+2^n=2^{n+1}-2$

$2^2+2^3+...2^n=2^{n+1}-2^2$

$2^3+...+2^n=2^{n+1}-2^3$

... ...

$+2^n=2^{n+1}-2$

$=n(2^{n+1})-(2^{n+1}-2)$

$=2^{n+1}(n-1)+2$

Given that $2^{n+1}(n-1)+2=2^{2+10}+2$

$\Rightarrow(n-1)2^{n+1}=2^{n+10}$

$\Rightarrow n-1=2^9$

$\Rightarrow n=2^9+1=513$

79. If $\sum_{r=0}^{n}\left\{\frac{{}^nC_{r-1}}{{}^nC_r+{}^nC_{r-1}}\right\}^3=\frac{25}{24}$, then n is equal to

(A) 3 (B) 4 (C) 5 (D) 6

Key. C

Sol. Let $t_r=\frac{{}^nC_{r-1}}{{}^nC_r+{}^nC_{r-1}}=\frac{{}^nC_{r-1}}{{}^{n+1}C_r}=\frac{{}^nC_{r-1}}{\frac{n+1}{r}\ {}^nC_{r-1}}$

$\therefore l_r=\frac{r}{n+1}$

Now,

$S=\sum_{r=0}^{n}\{l_r\}^3\Rightarrow S=\sum_{r=0}^{n}\frac{r^3}{(n+1)^3}=\frac{1}{(n+1)^3}\sum_{r=0}^{n}r^3$

$$\Rightarrow S=\frac{1}{(n+1)^3}\left\{\frac{n(n+1)}{2}\right\}^2 \Rightarrow S=\frac{n^2}{4(n+1)}$$

Now, $S=\frac{25}{24}$ (given) which is only possible for 5

80. If m and n are any two odd positive integers with n < m, then the largest positive integer which divides all numbers of the form, m^2-n^2 can be

(A) 4 (B) 6 (C) 8 (D) 9

Key. C

Sol. Let $m=2k-1$ *and* $n=2p-1, p<k$

Then $m^2-n^2=(m+n)(m-n)$

$=(2k+2p-2)(2k-2p)=4(k+p-1)(k-p)$

Further if k and p both even, then k-p is even but k+p-1 is odd

If k and p both odd then k-p is even but k+p-1 is odd. If one is even and other odd then k-p is odd but k+p-1 is even. Thus in every case $(k-p)(k+p-1)$ even

$\therefore m^2-n^2$ is divisible by $4\times2=8$. Hence , m^2-n^2 is divisible by 8 or any multiple of 8. The largest integer among the given options is 8

81. If $C_0, C_1, C_2, ..., C_n$ denote the binomial coefficients in the expansion of $(1+x)^n$, then $\sum_{r=0}^{n}(-1)^r\, {}^nC_r\,\frac{1+r\log_e 10}{(1+\log_e 10^n)^r}$ is equal to

A) 0 B) 1 C) 2 D) 3

Key. A

Sol. Let $\log_e 10 = x$

$$=\sum_{r=0}^{n}(-1)^r\; {}^nC_r\,\frac{1+rx}{(1+nx)^r}$$

$$=\left(1-\frac{1}{1+nx}\right)^n-\frac{nx}{1+nx}\left(1-\frac{1}{1+nx}\right)^{n-1}=0$$

82. In the expansion of $\left(\sqrt[3]{\frac{a}{b}}+\sqrt[3]{\frac{b}{\sqrt{a}}}\right)^{21}$ the term containing same powers of a and b is

A) 11[th] B) 13[th] C) 12[th] D) 6[th]

Key. B

Sol. $t_{r+1}={}^{21}C_r\left(\frac{a}{b}\right)^{\frac{21-r}{3}}\frac{b^{r/3}}{a^{r/6}}={}^{21}C_r\; a^{\frac{42-3r}{6}}\; b^{\frac{2r-21}{3}}$

$\therefore$ 42 – 3r = 4r – 42 i.e. r = 12

$\therefore$ 13[th] term contains same powers of a and b

83. The coefficient of x^4 of in the expansion $\left(1+5x+9x^2+....\right)\left(1+x^2\right)^{11}$ is

A) ${}^{11}C_2+4\;{}^{11}C_1+3$ B) ${}^{11}C_2+3\;{}^{11}C_1+4$ C) $3\,{}^{11}C_2+4\;{}^{11}C_1+3$ D) 171

Key. D

Sol. Coefficient of x^4 is $\left(1+5x+9x^2+...\right)\left(1+x^2\right)^{11}$

$=\left(1+5x+9x^2+...\right)\left(1+11x^2+{}^{11}C_2\left(x^2\right)^2+...\right)$

$=\left(1+5x+9x^2+13x^3+17x^4+...\right)\left(1+11x^2+{}^{11}C_2x^4+...\right)$

Coefficient of x^4 is ${}^{11}C_2+9+11+17=55+99+17=171$

84. If $\left(1+x+x^2\right)^{25}=a_0+a_1x+a_2x^2+...+a_{50}x^{50}$, then $a_0+a_2+a_4+...+a_{50}$ is

A) even B) odd and of the form 3n

C) odd and of the form (3n – 1) D) odd and of the form (3n + 1)

Key. A

Sol. putting x = 1 and – 1 and adding

$a_0+a_2+...+a_{50}=\dfrac{3^{25}+1}{2}=\dfrac{(1+2)^{25}+1}{2}$

$=\dfrac{{}^{25}C_0+{}^{25}C_1 2+{}^{25}C_2.2^2+...+{}^{25}C_{25}.2^{25}+1}{2}$

$=\dfrac{2\left[1+{}^{25}C_1+{}^{25}C_2.2+...+{}^{25}C_{25}.2^{24}\right]}{2}=2\left[13+{}^{25}C_2+...+{}^{25}C_{25}.2^{23}\right]$ is an even integer

85. The co-efficient of x^{n-2} in the polynomial (x – 1) (x – 2) (x – 3) … (x – n) is

A) $\dfrac{n\left(n^2+2\right)(3n+1)}{24}$ B) $\dfrac{n\left(n^2-1\right)(3n+2)}{24}$

C) $\dfrac{n\left(n^2+1\right)(3n+4)}{24}$ D) none of these

Key. B

Sol. $E=\left(x-\alpha_1\right)\left(x-\alpha_2\right)\left(x-\alpha_3\right)...\left(x-\alpha_n\right)$ wher $\alpha_1=1,\ \alpha_2=2$ etc

$=x^n-\left(\sum\alpha_1\right)x^{n-1}+\left(\sum\alpha_1\alpha_2\right)x^{n-2}+...$

Hence co-efficient of x^{n-2} = sum of all the products of the first ‘n’ natural numbers taken two at a time $=\dfrac{(1+2+3+...+n)^2-\left(1^2+2^2+...+n^2\right)}{2}=\dfrac{n\left(n^2-1\right)(3n+2)}{24}$

86. The remainder when 27^{40} is divided by 12 is

A) 3 B) 7 C) 9 D) 11

Key. C

Sol. $27^{40}=3^{120}$

$3^{119}=(4-1)^{119}={}^{119}C_0 4^{119}-{}^{119}C_1 4^{118}+{}^{119}C_2 4^{117}-{}^{119}C_3 4^{116}+...+{}^{119}C_{118}4-1$

$\therefore\quad 3^{119}=4k-1$

$\therefore\quad 3^{120}=12k-3=12(k-1)+9$

$\therefore$ The required remainder is 9

87. If $\sum_{r=0}^{2n}a_r(x-1)^r=\sum_{r=0}^{2n}b_r(x-2)^r$ and $b_r=(-1)^{r-n}$ for all $r\ge n$, then $a_n=$

A) ${}^{2n+1}C_{n-1}$ B) ${}^{3n}C_n$ C) ${}^{2n+1}C_n$ D) 0

Key. C

Sol. Let x – 1 = t, then

$\sum_{r=0}^{2n} a_r t^r = \sum_{r=0}^{2n} b_r (t-1)^r$

$\therefore \quad a_n =$ coefficient of t^n in $\sum_{r=0}^{2n} b_r (t-1)^r$

= coefficient of t^n b in $\left(b_0 + b_1(t-1) + ... + b_n(t-1)^n + b_{n+1}(t-1)^{n+1} + ... + b_{2n}(t-1)^{2n}\right)$

$= b_n \, {}^nC_0 + b_{n+1} \, {}^{n+1}C_1(-1)^1 + b_{n+2} \, {}^{n+2}C_2(-1)^2 + ... + b_{2n} \, {}^{2n}C_n(-1)^n$

$= (-1)^{n-n} . {}^nC_0 + (-1)^{n+1-n+1}, {}^{n+1}C_1 + ... + (-1)^{2n-n+n} \, {}^{2n}C_n$

$= {}^nC_0 + {}^{n+1}C_1 + {}^{n+2}C_2 + ... + {}^{2n}C_2 + ... + {}^{2n}C_n = {}^{2n}C_n = {}^{2n+1}C_{n+1}$

$= {}^{2n+1}C_n$

88. In the expansion of $\left(7^{\frac{1}{3}} + 11^{\frac{1}{9}}\right)^{6561}$ total number of terms free from radical signs is

A) 729 B) 730 C) 731 D) none of these

Key. B

Sol. $T_{r+1} = {}^{6561}C_r \, 7^{\frac{6561-r}{3}} \, 11^{r/9}$

The term is free from radical sign, if r is multiple of 9 and 6561 – r is a multiply of 3

i.e. r = 0, 9, 18, 27, … 6561. These are 730 in number,

89. The last two digits of the number $(23)^{14}$ are

A) 01 B) 03 C) 09 D) None of these

Key. C

Sol. $(23)^{14} = (529)^7 = (530-1)^7$

$= {}^7C_0(530)^7 - {}^7C_1(530)^6 + ... - 7C_5(530)^2 + {}^7C_6 530 - 1$

$= {}^7C_0(530)^7 - {}^7C_1(530)^6 + ... + 3710 - 1$

= 100 m + 3709

$\therefore$ last two digits are 09

90. $\sum\sum_{0 \le i < j \le n} (C_i + C_j)^2 =$ ____

a) $(n-1).{}^{2n}C_n + 2^{2n}$ b) ${}^{2n}C_n + 2^{2n}$ c) ${}^{2n}C_n - (n+1)2^n$ d) None

Key. A

Sol. $\sum\sum(C_i + C_j)^2 = \sum\sum C_i^2 + C_j^2 + 2C_iC_j = n\left(C_0^2 + C_1^2 + ... + C_n^2\right) + 2\sum\sum C_iC_j$ is

$$= n.{}^{2n}C_n + 2\left(\frac{2^{2n} - {}^{2n}C_n}{2}\right)$$

Since $(C_0 + C_1 + ... + C_n)^2 = C_0^2 + C_1^2 + ... + C_n^2 + 2\sum\sum C_iC_j$

$2^{2n} = {}^{2n}C_n + 2\sum\sum C_iC_j$

91. The value of $\frac{1}{\lfloor 1 \lfloor 15} + \frac{1}{\lfloor 3 \lfloor 13} + \frac{1}{\lfloor 5 \lfloor 11} + \frac{1}{\lfloor 7 \lfloor 9}$ is

a) $\frac{2^{14}}{\lfloor 15}$ b) $\frac{2^{15}}{\lfloor 16}$ c) $\frac{2^{10}}{\lfloor 15}$ d) $\frac{2^{13}}{\lfloor 15}$

Key. C

Sol. Multiply and divide by 16!

92. The term independent of x and y in the expansion of

$$\left[\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2+\left(\sqrt{y}+\frac{1}{\sqrt{y}}\right)^2+\left(\sqrt{xy}+\frac{1}{\sqrt{xy}}\right)^2-4\right]^n \text{ is}$$

a) $\left(\sum_{r=0}^{n} {}^nC_r\right)^2$ b) $\sum_{r=0}^{n}\left({}^nC_r\right)^2$ c) $\left(\sum_{r=0}^{n} {}^nC_r\right)^3$ d) $\sum_{r=0}^{n}\left({}^nC_r\right)^3$

Key. D

Sol. It can be simplified as $(1+x)^n(1+y)^n\left(1+\frac{1}{xy}\right)^n$

The constant term is $C_0^3+C_1^3+.....+C_n^3$

93. If $C_r={}^nC_r$, then $(C_0-C_2+C_4-C_6+...)^2+(C_1-C_3+C_5-C_7+...)^2$ is

a) 2^{2n} b) 2^n c) 2^{n^2} d) $2^{\frac{n+1}{2}}$

Key. B

Sol. Put $x=i$ in the expansion of $(1+x)^n$ we get

$(C_0-C_2+C_4-C_6.....)+i(C_1-C_3+C_5....)=2^{n/2}\left[\cos n\pi/4+i\sin n\pi/4\right]$ Take modulus both sides and square it

94. The coefficient of x^3 in $\left(2x-\frac{3}{x^2}\right)^9$ is

a) 41472 b) $2^8.3^5$ c) $2^8.3^4$ d) 44172

Key. A

Sol. r = 2 , coefficient $={}^9C_2\,(2)^7(-3)^2=(2)^9(3)^4$

95. If the coefficient of x in $\left(x^2+\frac{k}{x}\right)^5$ is 270, then the value of k is

a) 2 b) 3 c) 4 d) 5

Key. B

Sol. r = 3 , ${}^5C_3\,k^3=270$,k = 3

96. In the expansion of $\left(2+\frac{x}{3}\right)^n$, coefficient of x^7 and x^8 are equal. Then the value of n is

a) 49 b) 50 c) 55 d) 56

Key. C

Sol. ${}^nC_7\frac{2^{n-7}}{3^7}={}^nC_8\frac{2^{n-8}}{3^8}$, n = 55.

97. Value of the numerically greatest term in the expansion of $\left(1+\frac{1}{\sqrt{3}}\right)^{20}$ is

a) ${}^{20}C_7 \frac{1}{27}$ b) ${}^{20}C_8 \frac{1}{27}$
c) ${}^{20}C_8 \frac{1}{27\sqrt{3}}$ d) ${}^{20}C_7 \frac{1}{27\sqrt{3}}$

Key. D

Sol. $\frac{21}{\sqrt{3}+1}=7g.....,T_{7+1}={}^{20}C_7 \frac{1}{27\sqrt{3}}.$

98. The coefficients of 9th, 10th and 11th terms in the expansion of $(1+x)^n$ are in A.P. then the Value of n is

a) 23 b) 7 c) 4 d) 21

Key. A

Sol. r = 9, $n^2-(4\times 9+1)n+4\times 9^2-2=0$, n = 14 or 23.

99. The number of terms in the expansion of $(p+q+r+s)^n$ is

a) $\frac{n(n+1)}{2}$ b) $\frac{(n+1)(n+2)(n+3)}{6}$
c) $\frac{n(n+1)(2n+1)}{6}$ d) $\frac{(n+1)^2n^2}{4}$

Key. B

Sol. Standard formulae.

100. The number of irrational terms in the expansion of $\left(\sqrt{3}+\sqrt[4]{5}\right)^{124}$ is

a) 125 b) 32 c) 93 d) 34

Key. C

Sol. No of rational terms are $\left[\frac{124}{4}\right]+1$ = 32,

101. If a_1,a_2,a_3,a_4 are any 4 consecutive binomial coefficients of the expansion of $(1+x)^n$

Respectively, then $\frac{a_1}{a_1+a_2}, \frac{a_2}{a_2+a_3}, \frac{a_3}{a_3+a_4}$ are in

a) A.G.P b) G.P c) H.P d) A.P

Key. D

Sol. $\frac{1}{1+3}, \frac{3}{3+3}, \frac{3}{3+1}=\frac{1}{4},\frac{3}{6},\frac{3}{4}=\frac{3}{12},\frac{6}{12},\frac{9}{12}$, A.P.

102. If the sum of the coefficients in the expansion $(x - 2y+3z)^n$ is 128, then the greatest coefficient in $(1+x)^n$ is

a) 25 b) 35 c) 45 d) 31

Key. B

Sol. Put $x = y = z = 1$ then $2^n = 128$, n=7, $7_{C_3} = \frac{7.6.5}{1.2.3} = 35$

103. If n = 2009, then N = $2009^n - 1982^n - 1972^n + 1945^n$ is divisible by

a) 658 b) 1977 c) 1988 d) 2009

Key. B

Sol. Since n is odd $x^n + y^n$ has divisor x+y.

104. If $C_0, C_1 \ldots C_{10}$ are the binomial coefficient in the expansion of $(1+x)^{10}$, then

$2C_0 + \frac{2^2}{2} C_1 + \frac{2^3}{3}. C_2 + \ldots + \frac{2^{11}}{11}. C_{10}$

a) $\frac{2^{11}}{11}$ b) $\frac{2^{11}-1}{11}$ c) $\frac{3^{11}}{11}$ d) $\frac{3^{11}-1}{11}$

Key. D

Sol. $\int_0^2 (1+x)^{10} dx = 10_{C_0} 2 + \frac{2^2 10_{C_1}}{2} + \ldots\ldots\ldots \frac{2^{11} 10_{C_{10}}}{11} = \frac{3^{11}-1}{11}$

105. If $(1+px+x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then $\sum_{r=0}^{2n} (2r+1) a_r =$

a) $(p+2)^n$ b) $(2p+1)(p+2)^2$

c) $(2n+1)(p+2)^n$ d) $(p+2)^{n+1}$

Key. C

Sol. $(2\sum_{r=0}^{2n} ra_r + \sum_{r=0}^{2n} a_r = 2n(2+p)^{n-1}(P+2) + (2+P)^n$

$= (2n+1)(P+2)^n$

106. Let $(x^3 + \alpha x^2 + 2x - 5)^{19} (x^2 + \beta x - 41)^8 (x^4 - x^3 + x - 7)^6 =$

$x^{97} + 391x^{96} + a_{95}x^{95} + a_{94}x^{94} + \ldots + a_1 x + a_0$ be an identity, where

$\alpha, \beta, a_{95}, a_{94}, \ldots, a_1, a_0$ are integers. If $\alpha + \beta < 10$, then the smallest possible value of α is

a) 7 b) 8 c) 31 d) 23

Key. C

Sol. It will be an identity even if we replace x by $\frac{1}{y}$ and considering numerator alone.

Differentiating on both sides with respect to y, at y =0 we get $19\alpha + 8\beta = 397$, $\alpha + \beta = 10 - k$ where k is positive integer. Put $\beta = 10 - \alpha - k$ in first equation we get $11\alpha - 8k = 317$

$\therefore \alpha = 31$

107. The number of different terms in the expansion of

$(1+x)^{2009} + (1+x^2)^{2008} + (1+x^3)^{2007}$ is

a) 3683 b) 4017 c) 4018 d) 4352

Key. B

Sol. $(1+x)^{2009}$ has 2010 terms in total. $(1+x^2)^{2008}$ has a constant, even power of x starting from 2 to 4016 but already even powers of x from 2 to 2008 were enumerated in $(1+x)^{2009}$. The remaining terms containing even powers of x are from 2010 to 4016. They are 1004 in number. In $(1+x^3)^{2007}$ has a constant, multiples of 3 as powers of x. Even multiples of 3 from 6 to 4014 were already enumerated in above expansions. The remaining even multiples of 3 from 4020 to 6018 which are 334 in number. Odd multiples of 3 as powers of x from 3 to 2007 were enumerated in above expansions and the remaining from 2013 to 6021 are to be enumerated. They are 669 in number.

$\therefore$ the number of terms in the expansion = 2010 + 1004 + 669 + 334 = 4017.

108. The term independent of x and y in the expansion of

$[(\sqrt{x}+\frac{1}{\sqrt{x}})^2+(\sqrt{y}+\frac{1}{\sqrt{y}})^2+(\sqrt{xy}+\frac{1}{\sqrt{xy}})^2-4]^n$ is

A) $(\sum_{r=0}^{n} {}^nc_r)^2$ B) $\sum_{r=0}^{n}({}^nc_r)^2$ C) $(\sum_{r=0}^{n} {}^nc_r)^3$ D) $\sum_{r=0}^{n}({}^nc_r)^3$

Key. D

Sol. The given expression can be written as $(1+x)^n.(1+y)^n.(1+\frac{1}{xy})^n$. The constant term is clearly $C_0^3+C_1^3+----+C_n^3$ where $c_r={}^nc_r$.

109. The sum of the rational terms in the expansion of $\left(\sqrt{2}+\sqrt[5]{3}\right)^{10}$ is :

A) 41 B) 42 C) 39 D) 45

Key. A

Sol. $T_{r+1}=10C_r\,\frac{10-r}{2^2}.3^{\frac{r}{5}}$

This is rational , if $\frac{10-r}{2}$ and $\frac{r}{5}$ are integers.

$\therefore$ There are only two rational terms

Namely $10C_0\left(\sqrt{2}\right)^{10}\left(3^{\frac{1}{5}}\right)^0$ and $10C_{10}\left(\sqrt{2}\right)^0\left(3^{\frac{1}{5}}\right)^{10}$

$\therefore$ sum = 32 + 9 = 41

110. If $(1+x)^n=a_0+a_1x+a_2x^2+.....+a_nx^n$ in which a_{n-3},a_{n-2},a_{n-1} are in AP, then

A) a_1,a_2,a_3 are in GP B) a_1,a_2,a_3 are in HP

C) $n=7$ D) $n=14$

Key. C

Sol. $a_{n-3}=a_3,a_{n-2}=a_2,a_{n-1}=a_1\left(\because nC_r=nC_{n-r}\right)$

$\therefore$ (A) is correct.

$a_{,},a_2,a_3$ are in AP $\Rightarrow n,\frac{n(n-1)}{2},\frac{n(n-1)(n-2)}{6}$ are in AP.

$$\frac{n+\frac{n(n-1)(n-2)}{6}}{2}=\frac{n(n-1)}{2}$$

$6n+n^3-3n^2+2n=6n^2-6n$

$n^3-9n^2+14n=0$, $(n-7)(n-2)=0$

$\therefore$ n = 7.

111. If x = $\left(2+\sqrt{3}\right)^n$, then the value of $x-x^2+x[x]$ where [.] denotes the greatest integer function, is equal to

A) 1 B) 2 C) 2^{2n} D) 2^n

Key. A

Sol. $x-x^2+x[x]=x-x(x-[x])=x(1-\{x\})$

Now $x+x_1$ = even integer where $x_1=\left(2-\sqrt{3}\right)^n$ clearly $x_1 \in (0, 1)\ \forall\ n \in N$.

$\therefore \{x\}+x_1$ = integer

$\Rightarrow \{x\}+x_1=1$

112. In the binomial expansion of $\left(\sqrt{x}+\frac{1}{2\sqrt[4]{x}}\right)^n$, $n \in N$ the coefficients of first, second and third terms form an A.P. The number of rational terms in the expansion is (Assume that x is a rational number and $\sqrt{x}$, $\sqrt[4]{x}$ are irrational)

A) 1 B) 2 C) 3 D) 4

Key. C

Sol. $\left(\sqrt{x}+\frac{1}{2\sqrt[4]{x}}\right)^n=\sum_{r=0}^{n}{}^nC_r\left(\sqrt{x}\right)^{n-r}.\left(\frac{1}{2\sqrt[4]{x}}\right)^r=\sum_{r=0}^{n}{}^nC_r.\frac{1}{2^r}x^{\frac{2n-3r}{4}}$

1, $\frac{n}{2}$, $\frac{n(n-1)}{8}$ are in A.P. $\Rightarrow n=8$

113. $\left(\lfloor\underline{1}+2\lfloor\underline{2}+3\lfloor\underline{3}+.......+2009\lfloor\underline{2009}\right)+1$

A) $2\lfloor\underline{2010}$ B) $\lfloor\underline{2010}$ C) $\lfloor\underline{2011}$ D) $2\lfloor\underline{2011}$

Key. B

Sol. $\lfloor\underline{1}+2\lfloor\underline{2}+3\lfloor\underline{3}+..........+n\lfloor\underline{n}=\lfloor\underline{n+1}-1$

114. The sum of coefficients of the terms of degree 'm' in the expansion of $(1+x)^n(1+y)^n(1+z)^n$ is

(A) $\left({}^nC_r\right)^3$ (B) $3\left({}^nC_r\right)$ (C) ${}^nC_{3r}$ (D) ${}^{3n}C_m$

Key. D

Sol. Putting y = z = x we get $(1+x)^{3n}$ coeff $x^m={}^{3n}C_m$

115. If C_r denotes ${}^nC_r$ then the value of $\frac{C_0}{2}-\frac{C_1}{3}+\frac{C_2}{4}-\frac{C_3}{5}+------(-1)^n\frac{C_n}{n+2}=$

(A) $\frac{1}{n+1}$ (B) $\frac{1}{n+2}$ (C) $\frac{1}{n(n+1)}$ (D) $\frac{1}{(n+1)(n+2)}$

Key. D

Sol. Req sum = $\int_0^1 x(1-x)^n\,dx=\frac{1}{(n+1)(n+2)}$

116. If the 4[th] term in the expansion of $\left(2+\frac{3x}{8}\right)^{10}$ has the maximum numerical value, then 'x' lies in the interval

(A) $\left(\frac{-64}{21},-2\right)\cup\left(2,\frac{64}{4}\right)$ (B) $\left(\frac{-60}{23},-2\right)\cup\left(2,\frac{64}{23}\right)$

(C) $\left(\frac{-64}{21},2\right)$ (D) $\left(-2,\frac{-64}{21}\right)$

Key. A or B

Sol. $\left|\frac{t_3}{t_4}\right|<1 \ \ and \left|\frac{t_5}{t_4}\right|<1$

i.e., $\left|\frac{2}{x}\right|<1;\ \left|\frac{21}{64}x\right|<1$

$\therefore x\in\left(\frac{-64}{21},-2\right)\cup\left(2,\frac{64}{21}\right)$

117. The Value of $\sum_{r=0}^{15} {}^{15}C_r\left(r-\frac{15}{2}\right)^2$ is

A) $2^{10}.15$ B) $2^{12}.15$ C) $2^{13}.15$ D) $2^{15}.15$

Key. C

Sol. $\sum_{r=0}^{15} {}^{15}C_r.r^2-15\sum_{r=0}^{15} r.{}^{15}C_r+\frac{225}{4}\times 2^{15}$

$\sum_{r=0}^{15} r^2.{}^{15}C_r=15\sum_{r=1}^{15}[r-1+1]\times{}^{14}C_{r-1}=15.2^{14}+15.14.2^{13}=2^{15}.60$

$\therefore$ Required Sum $=\frac{225}{4}\times 2^{15}+2^{15}.60-225.2^{14}$

118. The Value of $\frac{1}{\lfloor 1\lfloor 15}+\frac{1}{\lfloor 3\lfloor 13}+\frac{1}{\lfloor 5\lfloor 11}+\frac{1}{\lfloor 7\lfloor 9}$ is

A) $\frac{2^{14}}{\lfloor 15}$ B) $\frac{2^{15}}{\lfloor 16}$ C) $\frac{2^{10}}{\lfloor 15}$ D) $\frac{2^{13}}{\lfloor 15}$

Key. C

Sol. Let S be the required Sum. Then we have $2S\times\angle 16={}^{16}C_1+{}^{16}C_3+{}^{16}C_5+{}^{16}C_7+\ldots+{}^{16}C_{15}$