Date: 28/9/2021

Q1. "The product of two consecutive positive intergers is divisible by 2". Is this statement true or false? Give reasons.

Q2. Write whether the square of any positive integer can be of the form $3m + 2$, where m is a natural number. Justify your answer.

Q3. A positive integer is of the form $3q + 1$, q being a natural number, Can you write its square in any form other than $3m + 1$, *i.e.*, $3m + 2$ for some integer m? Justify your answer.

Q4. Explain why $3 \times 5 \times 7 \div 7$ is a composite number.

Q5. Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons.

Q6. Without actually performing the long division, find if $\frac{987}{10500}$ will have terminating or non-terminating (repeating) decimal expansion. Give reasons for your answer.

Q7. On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm, respectively. What is the minimum distance each should walk, so that each can cover the same distance in complete steps?

Q8. Write whether every positive integer can be of the form $4q + 2$, where q is an integer. Justify your answer.

Q9. "The product of three consecutive positive integers is divisible by 6". Is this statement true or false? Justify your answer.

Q10. Prove that, if x and y both odd positive integers, then $x^2 + y^2$ is even but not divisible by 4.

Q11. If n is an odd integer, then show that $n^2 - 1$ is divisible by 8.

Q12. Show that the square of any odd integer is of the form $4m + 1$, for some integer m.

Q13. Show that the square of any positive integer cannot be of the form $6m + 2$ or $6m + 5$ for any integer m.

Q14. Show that the square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$ for any integer q.

Q15. Show that cube of any positive integer is of the form $4m$, $4m + 1$ or $4m + 3$ for some integer m.

Q16. A rational number in its decimal expansion is 327.7081. What can you say about the prime factors of q, when this number is expressed in the form $\frac{p}{q}$? Give reasons.

Q17. The numbers 525 and 3000 are both divisible only by 3, 5, 15, 25 and 75. What is HCF (525, 3000)? Justify your answer.

Q18. Show that the cube of a positive integer of the form $6q + r$, q is an integer and $r = 0, 1, 2, 3, 4, 5$ is also of the form $6m + r$.

Q19. For any positive integer n, prove that $n^3 - n$ is divisible by 6.

Q20. Prove that one of any three consecutive positive integers must be divisible by 3.

Q21. Prove that $\sqrt{3} + \sqrt{5}$ is irrational.

Q22. Write the denominator of rational number $\frac{257}{5000}$ in the form $2^m \times 5^n$, where m, n are non-negative integers. Hence, write its decimal expansion, without actual division.

Q23. Use Euclid's division algorithm to find the HCF of 441, 567 and 693.

Q24. Using Euclid's division algorithm, find the largest number that divides 1251, 9377 and 15628 leaving remainders 1, 2 and 3, respectively.

Q25. Show that 12^n cannot end with the digit 0 or 5 for any natural number n.

Q26. Prove that $\sqrt{p} + \sqrt{q}$ is irrational, where p and q are primes.

Q27. Prove that one and only one out of n, $(n + 2)$ and $(n + 4)$ is divisible by 3, where n is any positive integer.

Q28. Show that one and only one out of n, $n + 4$, $n + 8$, $n + 12$ and $n + 16$ is divisible by 5, where n is any positive integer.

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S1. True.

S2. Square of any positive integer is in the form of $3m$ and $3m + 1$.

S3. No, square of a positive integer is of the form $3q + 1$ is always in the form $3m + 1$ for some integer m.

S4. It is the product of prime factors 2 and 7. i.e., it has more than two factors. Hence, it is composite number.

S5. No, because HCF is always a factor of LCM but here 18 is not a factor of 380.

S6. Yes, after simplification denominator has factor $5^3 . 2^2$ and which is of the type $2^m . 5^n$. So, this is terminating decimal.

S7. Minimum distance each should walk 2520 cm. So that, each can cover the same distance in complete steps.

S8. No, every positive integer must be in the form 4q, $4q + 1$, $4q + 2$ or $4q + 3$.

S9. True.

S10. Proved.

S11. Proved.

S12. Proved.

S13. Proved.

S14. Proved.

S15. Proved.

S16. 327.7081 is terminating decimal number. So, it represents a rational number and also its denominator must have the form $2^m \times 5^n$.

$$327.7081 = \frac{3277081}{10000} = \frac{p}{q}$$

$$q = 10^4 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$$

$$= 2^4 \times 5^4 = (2 \times 5)^4$$

Hence, the prime factors of q is 2 and 5.

S17. The HCF (525, 3000) = 75.

S18. Proved.

S19. Proved.

S20. Proved.

S21. Proved.

S22. Denominator, $5000 = 2^3 \times 5^4$

$$\text{Decimal expansion} = \frac{257}{5000} = \frac{514}{10000} = \frac{514}{10^4} = 0.0514.$$

S23. HCF (843, 567, 441) = 63.

S24. 625 is the largest number which divides 1251, 9377 and 15628 leaving remainder 1, 2 and 3 respectively.

S25. Proved.

S26. Proved.

S27. Proved.

S28. Proved.