PHYSICS - XII | Nuclear Energy NCERT

Date: 22/10/2021

Q1. Why do all the electrons emitted during β-decay not have the same energy?

Q2. Define critical mass.

Q3. Which process generate energy in a nuclear reactor?

Q4. How much energy is released during the fission of ${}_{92}U^{235}$ per nucleus?

Q5. What are moderators?

Q6. What is critical size?

Q7. What is meant by the Q-value of a nuclear reaction?

Q8. Why chain reaction can not occur in natural uranium?

Q9. Nuclear fusion reactions are also known as thermonuclear reactions. Why?

Q10. If the total number of neutrons and protons in a nuclear reaction is conserved, how then is the energy absorbed or evolved in the reaction?

Q11. Why nuclear fusion is not possible in a laboratory?

Q12. Suppose, we think of fission of a ${}^{56}_{26}\text{Fe}$ nucleus into two equal fragments, ${}^{28}_{13}\text{Al}$. Is the fission energetically possible? Argue by working out Q of the process. Given

$$m({}^{56}_{26}\text{Fe}) = 55.93494 \text{ u} \quad \text{and} \quad m({}^{28}_{13}\text{Al}) = 27.98191 \text{ u}$$

Q13. Draw a curve to show the variation of binding energy per nucleon with mass number. Show the position of H^2 and U^{238} on this curve.

Q14. If 200 MeV energy is released in the fission of a single nucleus of ${}_{92}U^{235}$, how many fissions must occur per second to produce a power of 1 kW?

Q15. A heavy nucleus X of mass number 240 and binding energy per nucleon 7.6 MeV splits into two fragments Y and Z of mass number 110 and 130. The binding energy of nucleons in Y and Z is 8.5 MeV per nucleon. Calculate the energy Q released per fission in MeV.

Q16. In a nuclear chain reaction, 2 g of ${}_{92}U^{235}$ undergoes fission. If the energy released per fission is 200 MeV, what is the total energy released?

Q17. Consider the following reaction: ${}_{1}H^{2} + {}_{1}H^{2} \longrightarrow {}_{2}He^{4} + Q$

If mass of the deuterium atom = 2.0141 a.m.u. and mass of helium atom = 4. 0024 a.m.u., find the energy released in the reaction.

Q18. The nucleus ${}^{23}_{10}\text{Ne}$ decays by β^- emission. Write down the β decay equation and determine the maximum kinetic energy of the electrons emitted. Given that:

$$m({}^{23}_{10}\text{Ne}) = 22.994466 \text{ u}$$

$$m({}^{23}_{11}\text{Na}) = 22.989770 \text{ u}.$$

Q19. Find the Q-value and the kinetic energy of the emitted α-particle in the α-decay of (a) ${}^{226}_{88}\text{Ra}$ and (b) ${}^{220}_{86}\text{Rn}$.

Given, $m({}^{226}_{88}\text{Ra}) = 226.02540\text{ u}$, $m({}^{222}_{86}\text{Rn}) = 222.01750\text{ u}$,
$m({}^{220}_{86}\text{Rn}) = 220.01137\text{ u}$, $m({}^{216}_{84}\text{Po}) = 216.00189\text{ u}$.

Q20. The fission properties of ${}^{239}_{94}\text{Pu}$ are very similar to those of ${}^{235}_{92}\text{U}$.

The average energy released per fission is 180 MeV. How much energy, in MeV, is released if all the atoms in 1 kg of pure ${}^{239}_{94}\text{Pu}$ undergo fission?

Q21. The Q value of a nuclear reaction $A + b \rightarrow C + d$ is defined by

$$Q = [m_A + m_b - m_C - m_d]\, c^2$$

where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

(a) ${}^{1}_{1}\text{H} + {}^{3}_{1}\text{H} \longrightarrow {}^{2}_{1}\text{H} + {}^{2}_{1}\text{H}$ (b) ${}^{12}_{6}\text{C} + {}^{12}_{6}\text{C} \longrightarrow {}^{20}_{10}\text{Ne} + {}^{4}_{2}\text{He}$

Atomic masses are given to be

$m({}^{2}_{1}\text{H}) = 2.014102\text{ u}$; $m({}^{3}_{1}\text{H}) = 3.016049\text{ u}$; $m({}^{12}_{6}\text{C}) = 12.000000\text{ u}$; $m({}^{20}_{10}\text{Ne}) = 19.992439\text{ u}$

Q22. A 1000 MW fission reactor consumes half of its fuel in 5.00 y. How much ${}^{235}_{92}\text{U}$ did it contain initially? Assume that the reactor operates 80% of the time, that all the energy generated arises from the fission of ${}^{235}_{92}\text{U}$ and that this nuclide is consumed only by the fission process.

Q23. Suppose India had a target of producing by 2020 AD, 200,000 MW of electric power, ten percent of which was to be obtained from nuclear power plants. Suppose we are given that, on an average, the efficiency of utilization (*i.e.*, conversion to electric energy) of thermal energy produced in a reactor was 25%. How much amount of fissionable uranium would our country need per year by 2020? Take the heat energy per fission of ${}^{235}\text{U}$ to be about 200 MeV.

Q24. Calculate and compare the energy released by (a) fusion of 1.0 kg of hydrogen deep within Sun and (b) the fission of 1.0 kg of ${}^{235}\text{U}$ in a fission reactor.

Q25. Consider the D-T reaction (deuterium-tritium fusion)

$${}^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \longrightarrow {}^{4}_{2}\text{He} + n$$

(a) Calculate the energy released in MeV in this reaction from the data:

$m({}^{2}_{1}\text{H}) = 2.014102\text{ u}$ $m({}^{3}_{1}\text{H}) = 3.016049\text{ u}$

(b) Consider the radius of both deuterium and tritium to be approximately 2.0 fm. What is the kinetic energy needed to overcome the coulomb repulsion between the two nuclei? To what temperature must the gas be heated to initiate the reaction? (Hint: Kinetic energy required for one fusion event = average thermal kinetic energy available with the interacting particles = $2(3kT/2)$; k = Boltzman's constant, T = absolute temperature.)

Q26. Consider the fission of ${}^{238}_{92}\text{U}$ by fast neutrons. In one fission event, no neutrons are emitted and the final end products, after the beta decay of the primary fragments, are ${}^{140}_{58}\text{Ce}$ and ${}^{99}_{44}\text{Ru}$. Calculate Q for this fission process. The relevant atomic and particle masses are:

$m({}^{238}_{92}\text{U}) = 238.05079\text{ u}$; $m({}^{140}_{58}\text{Ce}) = 139.90543\text{ u}$; $m({}^{99}_{44}\text{Ru}) = 98.90594\text{ u}$

Q27. How long can an electric lamp of 100 W be kept glowing by fusion of 2.0 kg of deuterium? Take the fusion reaction as

$$^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \longrightarrow {}^{3}_{1}\text{He} + n + 3.27 \text{ MeV}$$

Q28. Draw the curve showing variation of binding energy nucleon with mass number of different nuclei. Briefly state, how nuclear fusion and nuclear fission can be explained on the basic of this graph?

Q29. A neutron is absorbed by a ${}_{3}\text{Li}^{6}$ nucleus with subsequent emission of an alpha particle. Write the corresponding nuclear reaction. Calculate the energy release in this reaction.

Given that $m({}_{3}\text{Li}^{6}) = 6.015125$ a.m.u.; $m({}_{1}\text{H}^{3}) = 3.016049$ a.m.u.;

$m({}_{2}\text{He}^{4}) = 4.002604$ a.m.u.; $m_n = 1.008665$ a.m.u.

Q30. Calculate the binding energy per nucleon of ${}_{26}\text{Fe}^{56}$ [mass of ${}_{26}\text{Fe}^{56} = 55.934939$ a.m.u., mass of proton = 1.007825 a.m.u., mass of neutron = 1.008665 a.m.u].

Q31. Answer the following questions:

(a) Are the equations of nuclear reactions (such as those given in Section 13.7) 'balanced' in the sense a chemical equation (*e.g.*, $2H_2 + O_2 \rightarrow 2H_2O$) is? If not, in what sense are they balanced on both sides?

(b) If both the number of protons and the number of neutrons are conserved in each nuclear reaction, in what way is mass converted into energy (or vice-versa) in a nuclear reaction?

(c) A general impression exists that mass-energy interconversion takes place only in nuclear reaction and never in chemical reaction. This is strictly speaking, incorrect. Explain.

Q32. Calculate binding energy per nucleon fo ${}_{83}\text{Bi}^{209}$. Given:

$m({}_{83}\text{Bi}^{209}) = 208.980388$ a.m.u

m(neutron) = 1.008665 a.m.u

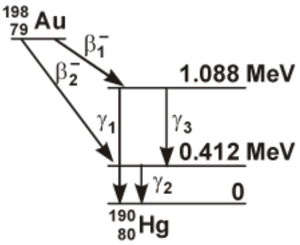
m(proton) = 1.007825 a.m.u

Q33. The fission of one nucleus of ${}_{92}\text{U}^{235}$ releases 200 MeV energy. How many fissions should occur per second for producing a power of 1 MW ($1 \text{ eV} = 1.6 \times 10^{-19}$ J).

Q34. Obtain the maximum kinetic energy of β-particles, and the radiation frequencies of γ decays in the decay scheme shown in figure. You are given that

$m({}^{198}\text{Au}) = 197.968233$ u

$m({}^{198}\text{Hg}) = 197.966760$ u



Q35. Describe the process of release of energy in a nuclear reactor. Draw a labeled diagram of a nuclear reactor and write the function of each part.

Q36. Distinguish between the phenomena of nuclear fusion and fission. Explain, using the graph for the *B.E.*/*A* versus mass number (*A*).

Q37. What is nuclear reaction? Give one example of it. Explain the phenomenon of nuclear fission and nuclear fusion.

Q38. **Draw a graph showing the variation of binding energy per nucleon mass number for different nuclei. Explain, with the help of this graph, the release of energy by the process of nuclear fusion and nuclear fission.**

Q39. **(a)** **Why is the mass of a nucleus always less than the sum of the masses of its constituents, neutrons and protons?**

If the total number of neutrons and protons in a nuclear reaction is conserved, then how is the energy absorbed or evolved in the reaction? Explain.

(b) **The fission properties of $^{239}_{94}\mathrm{Pu}$ are very similar to those of $^{235}_{92}\mathrm{U}$. The average energy released per fission is 180 Mev. How much energy in MeV, is released if all the atoms is 1 kg of pure $^{239}_{94}\mathrm{Pu}$ unedergo fission?**

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S1. According to Pauli, it is due to existence of an uncharged particle called antineutrino ($\overline{\nu}$) which is emitted along with the β-particle. The energy Q is shared by the β^--particle and antineutrino. When antineutrino carries maximum energy,, the energy of the β^--particle is minimum and vice-versa.

S2. For a self-sustained nuclear reaction, it is essential that the fissionable material used has a minimum mass called the critical mass.

S3. The controlled nuclear fission process produces energy inside a nuclear reactor.

S4. 200 MeV (approx.).

S5. Certain substances like paraffin, graphite, heavy water (or deuterium), etc. which are rich in hydrogen are used to slow down the neutrons for a fission to proceed in a controlled manner. Such substances are called ***moderators***.

S6. In order to have a sustained chain reaction in a sample of U^{235}, it is required that the number of neutrons lost due to leakage and absorption should be much smaller than the number of greater than a certain critical value, called critical size.

S7. The Q-value of a nuclear reaction gives the total energy change of the reaction.

S8. In natural uranium, the percentage of U^{235} (0.714%) is much smaller than that of U^{238} (99.28%). U^{238} is fissionable only with fast neutrons with energy higher than 2 MeV. Almost all the neutrons colliding with U^{238} get slowed down and, as a result, further fission of U^{238} is not possible. That is why, chain reaction can not occur in natural uranium.

S9. To carry out nuclear fusion, the two light nuclei have to be brought very close to each other against the electrostatic repulsion. For this, energy is made available by raising them to very high temperature ($\approx 10^7$ K). Due to this reason, nuclear fusion is called thermonuclear reaction.

S10. The total rest mass of protons and neutrons is same on the two sides of the nuclear reaction as their number is conserved. But the total binding energy of the nuclei on the two sides need not be equal. The difference in these energies appears as the energy released or absorbed (Q-value of the reaction)

S11. To carry out nuclear fusion, the two light nuclei have to be raised to very high temperature ($\approx 10^7$ K). Such a high temperature cannot be created in the laboratory.

S12. The fission of $^{56}_{26}\text{Fe}$ can be given as:

$$^{56}_{13}\text{Fe} \longrightarrow {}^{28}_{13}\text{Al}$$

It is given that:

$$\text{Atomic mass of } m\,({}^{56}_{26}\text{Fe}) = 55.93494 \text{ u}$$

$$\text{Atomic mass of } m\,({}^{28}_{13}\text{Al}) = 27.98191 \text{ u}$$

The Q-value of this nuclear reaction is given as:

$$Q = \Delta\, \text{cm}^2$$

$$Q = [m\,({}^{56}_{26}\text{Fe}) - 2m\,({}^{28}_{13}\text{Al})]\, c^2$$

$$= [55.93494 - 2 \times 27.98191]\, c^2$$

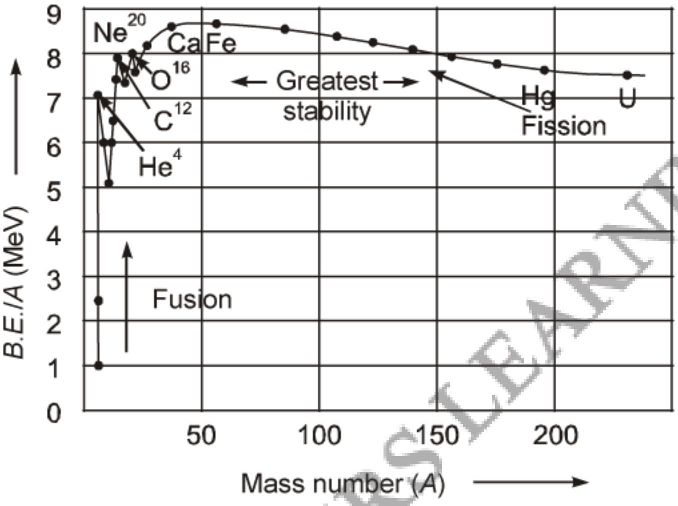
$$= (-0.02888\, c^2)\, \text{u}$$

But $\quad 1 \text{ u} = 931.5 \text{ MeV}/c^2$

$\therefore \quad Q = -0.02888 \times 931.5 = 26.902 \text{ MeV}$

The Q-value of the fission is negative. Therefore, the fission is not possible energetically. For an energetically-possible fission reaction, the Q-value must be positive.

S13.



S14. Here, $P = 1 \text{ kW} = 10^3 \text{ W} = 10^3 \text{ J s}^{-1}$

Energy released in fission of one U^{235} nucleus,

$$Q = 200 \text{ MeV} = 200 \times 1.6 \times 10^{-13} \text{ J}$$

Therefore, number of fission that should occur per second,

$$n = \frac{P}{Q} = \frac{10^3}{200 \times 1.6 \times 10^{-13}} = \mathbf{3.125 \times 10^{13}\ s^{-1}}.$$

S15. Binding energy of the nucleus X

$$= 7.6 \times 240 = 1{,}824 \text{ MeV}$$

Binding energy of the nuclei X and Y

$$= 8.5 \times (110 + 130) = 8.5 \times 240 = 2{,}040 \text{ Mev}$$

The energy released per fission,

$$Q = 2{,}040 - 1{,}824 = \mathbf{216\ MeV}.$$

S16. Number of U^{235} nuclei in 2 g,

$$n = \frac{6.02 \times 10^{23} \times 2}{235}.$$

Therefore, energy released by fission of 2 g of U^{235},

$$E = n \times 200 = \frac{6.02 \times 10^{23} \times 2}{235} \times 200 \text{ MeV}$$

$$= \frac{6.02 \times 10^{23} \times 2 \times 200 \times 1.6 \times 10^{-13}}{235} = \mathbf{1.64 \times 10^{11}\ J}.$$

S17.

$$Q = [2m\,({}_1\text{H}^2) - \text{m}\,({}_2\text{He}^4))] \times 931$$
$$= (2 \times 2.0141 - 4.0024) \times 931$$
$$= 0.0258 \times 931 = \mathbf{24.02\ MeV}.$$

S18. In β^- emission, the number of protons increases by 1, and one electron and an antineutrino are emitted from the parent nucleus.

β^- emission of the nucleus ${}^{23}_{10}\text{Ne}$ is given as:

$${}^{23}_{10}\text{Ne} \longrightarrow {}^{23}_{11}\text{Na} + e^- + \overline{\nu} + Q$$

It is given that:

Atomic mass of $m\,({}^{23}_{10}\text{Ne}) = 22.994466\,\text{u}$

Atomic mass of $m\,({}^{23}_{11}\text{Na}) = 22.989770\,\text{u}$

Mass of an electron, $m_e = 0.000548\,\text{u}$

Q-value of the given reaction is given as:

$$Q = [m\,({}^{23}_{10}\text{Ne}) - [m\,({}^{23}_{11}\text{Na}) + m_e]]\,c^2$$

There are 10 electrons in ${}^{23}_{10}\text{Ne}$ and 11 electrons in ${}^{23}_{11}\text{Na}$. Hence, the mass of the electron is cancelled in the Q-value equation.

$\therefore$ $$Q = [22.994466 - 22.989770]$$
$$= (0.004696\,c^2)\,\text{u}$$

But $$1\ \text{u} = 931.5\ \text{MeV}/c^2$$

$\therefore$ $$Q = 0.004696 \times 931.5 = 4.374\ \text{MeV}$$

The daughter nucleus is too heavy as compared to e^- and $\overline{\nu}$. Hence, it carries negligible energy. The kinetic energy of the antineutrino is nearly zero. Hence, the maximum kinetic energy of the emitted electrons is almost equal to the Q-value, *i.e.*, 4.374 MeV.

S19. (a) Alpha particle decay of ${}^{226}_{88}$Ra emits a helium nucleus. As a result, its mass number reduces to (226 – 4) 222 and its atomic number reduces to (88 – 2) 86. This is shown in the following nuclear reaction.

$$ {}^{226}_{88}\text{Ra} \longrightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\text{He} $$

Q-value of

emitted α-particle = (Sum of initial mass – Sum of final mass) c^2

Where, c = Speed of light

It is given that: $m({}^{226}_{88}\text{Ra}) = 226.02540\,\text{u}$

$m({}^{222}_{86}\text{Rn}) = 222.01750\,\text{u}$

$m({}^{4}_{2}\text{He}) = 4.002603\,\text{u}$

$$ Q\text{-value} = [226.02540 - (222.01750 + 4.002603)]\ \text{u}\ c^2 $$
$$ = 0.005297\ \text{u}\ c^2 $$

But $1\ \text{u} = 931.5\ \text{MeV}/c^2$

$\therefore$ $Q = 0.005297 \times 931.5 \approx 4.94\ \text{MeV}$

$$ \text{Kinetic energy of the } \alpha\text{-particle} = \left(\frac{\text{Mass number after decay}}{\text{Mass number before decay}}\right) \times Q $$

$$ = \frac{222}{226} \times 4.94 = 4.85\ \text{MeV} $$

(b) Alpha particle decay of (${}^{220}_{86}$Rn) is shown by the following nuclear reaction.

$$ {}^{220}_{86}\text{Rn} \longrightarrow {}^{216}_{84}\text{Po} + {}^{4}_{2}\text{He} $$

It is given that: Mass of (${}^{220}_{86}$Rn) = 220.01137 u

Mass of (${}^{216}_{84}$Po) = 216.00189 u

$$ Q\text{-value} = [220.01137 - (216.00189 + 4.002603)] \times 931.5 $$
$$ \approx 641\ \text{MeV} $$

$$ \text{Kinetic energy of the } \alpha\text{-particle} = \left(\frac{220 - 4}{220}\right) \times 6.41 $$

$$ = 6.29\ \text{MeV}. $$

S20. Average energy released per fission of ${}^{239}_{94}$Pu, $E_{av} = 180\,\text{MeV}$

Amount of pure ${}_{94}\text{Pu}^{239}$, $m = 1\,\text{kg} = 1000\,\text{g}$

N_A = Avogadro number = 6.023×10^{23}

Mass number of ${}^{239}_{94}$Pu = 239 g

1 mole of ${}_{94}\text{Pu}^{239}$ contains N_A atoms.

$\therefore$ mg of ${}_{94}\text{Pu}^{239}$ contains $\left(\dfrac{N_A}{\text{Mass number}} \times m\right)$ atoms

$$= \frac{6.023 \times 10^{23}}{239} \times 1000 = 2.52 \times 10^{24} \text{ atoms}$$

$\therefore$ Total energy released during the fission of 1 kg of ${}^{239}_{94}\text{Pu}$ is calculated as:

$$E = E_{\text{av}} \times 2.52 \times 10^{24}$$

$$= 180 \times 2.52 \times 10^{24} = 4.536 \times 10^{26} \text{ MeV}$$

Hence, 4.536×10^{26} MeV is released if all the atoms in 1 kg of pure ${}_{94}\text{Pu}^{239}$ undergo fission.

S21. (a) The given nuclear reaction is:

$$_1\text{H}^1 + {}^3_1\text{H} \longrightarrow {}^2_1\text{H} + {}^2_1\text{H}$$

It is given that:

Atomic mass $m({}^1_1\text{H}) = 1.007825\,\text{u}$

Atomic mass $m({}^3_1\text{H}) = 3.016049\,\text{u}$

Atomic mass $m({}^2_1\text{H}) = 2.014102\,\text{u}$

According to the question, the Q-value of the reaction can be written as:

$$Q = \Delta\, cm^2$$

$$Q = [m({}^1_1\text{H}) - m({}^3_1\text{H}) - 2m({}^2_1\text{H})]\, c^2$$

$$= [1.007825 + 3.016049 - 2 \times 2.014102]\, c^2$$

$$Q = (-0.00433\, c^2)\,\text{u}$$

But $\quad 1\,\text{u} = 931.5\ \text{MeV}/c^2$

$\therefore \quad Q = -0.00433 \times 931.5 = -4.0334\ \text{MeV}$

The negative Q-value of the reaction shows that the reaction is endothermic.

(b) The given nuclear reaction is:

$${}^{12}_{6}\text{C} + {}^{12}_{6}\text{C} \longrightarrow {}^{20}_{10}\text{Ne} + {}^{4}_{2}\text{He}$$

It is given that:

Atomic mass of $m({}^{12}_{6}\text{C}) = 12.0\,\text{u}$

Atomic mass of $m({}^{20}_{10}\text{Ne}) = 19.992439\,\text{u}$

Atomic mass of $m({}^{4}_{2}\text{He}) = 4.002603\,\text{u}$

The Q-value of this reaction is given as:

$$Q = [2m({}^{12}_{6}\text{C}) - m({}^{20}_{10}\text{Ne}) - m({}^{4}_{2}\text{He})]\, c^2$$

$$= [2 \times 12.0 - 19.992439 - 4.002603]\, c^2$$

$$= (0.004958\, c^2)\, \text{u}$$

$$= 0.004958 \times 931.5 = 4.618377 \text{ MeV}$$

The positive Q-value of the reaction shows that the reaction is exothermic.

S22. Half life of the fuel of the fission reactor, $t_{1/2} = 5$ years

$$= 5 \times 365 \times 24 \times 60 \times 60 \text{ s}$$

We know that in the fission of 1 g of $^{235}_{92}\text{U}$ nucleus, the energy released is equal to 200 MeV.

1 mole, *i.e.*, 235 g of $^{235}_{92}\text{U}$ contains 6.023×10^{23} atoms.

$\therefore$ 1 g $^{235}_{92}\text{U}$ contains $\dfrac{6.023 \times 10^{23}}{235}$ atoms

The total energy generated per gram of $^{235}_{92}\text{U}$ is calculated as:

$$E = \frac{6.023 \times 10^{23}}{235} \times 200 \text{ MeV/g}$$

$$= \frac{200 \times 6.023 \times 10^{23} \times 1.6 \times 10^{-19} \times 10^{6}}{235} = 8.20 \times 10^{10} \text{ J/g}$$

The reactor operates only 80% of the time.

Hence, the amount of $^{235}_{92}\text{U}$ consumed in 5 years by the 1000 MW fission reactor is calculated as:

$$= \frac{5 \times 80 \times 60 \times 60 \times 365 \times 24 \times 1000 \times 10^{6}}{100 \times 8.20 \times 10^{10}} \text{ g} \approx 1538 \text{ kg}$$

$\therefore$ Initial amount of $^{235}_{92}\text{U} = 2 \times 1538 = 3076$ kg

S23. Amount of electric power to be generated, $P = 2 \times 10^5$ MW

10% of this amount has to be obtained from nuclear power plants.

$\therefore$ Amount of nuclear power, $P_1 = \dfrac{10}{100} \times 2 \times 10^5$

$$= 2 \times 10^4 \text{ MW}$$

$$= 2 \times 10^4 \times 10^6 \text{ J/s}$$

$$= 2 \times 10^{10} \times 60 \times 60 \times 24 \times 365 \text{ J/y}$$

Heat energy released per fission of a ^{235}U nucleus, E = 200 MeV

Efficiency of a reactor = 25%

Hence, the amount of energy converted into the electrical energy per fission is calculated as:

$$\frac{25}{100} \times 200 = 50 \text{ MeV}$$

$$= 50 \times 1.6 \times 10^{-19} \times 10^{6} = 8 \times 10^{-12}\ \text{J}$$

Number of atoms required for fission per year:

$$\frac{2 \times 10^{10} \times 60 \times 60 \times 24 \times 365}{8 \times 10^{-12}} = 78840 \times 10^{24}\ \text{atoms}$$

1 mole, *i.e.*, 235 g of U^{235} contains 6.023×10^{23} atoms.

$\therefore$ Mass of 6.023×10^{23} atoms of $U^{235} = 235\ \text{g} = 235 \times 10^{-3}\ \text{kg}$

$\therefore$ Mass of 78840×10^{24} atoms of U^{235}

$$\frac{235 \times 10^{-3}}{6.023 \times 10^{23}} \times 78840 \times 10^{24} = 3.076 \times 10^{4}\ \text{kg}$$

Hence, the mass of uranium needed per year is 3.076×10^{4} kg.

S24. (a) Amount of hydrogen, $m = 1\ \text{kg} = 1000\ \text{g}$

1 mole, *i.e.*, 1 g of hydrogen ($^{1}_{1}H$) contains 6.023×10^{23} atoms.

$\therefore$ 1000 g of $^{1}_{1}H$ contains $6.023 \times 10^{23} \times 1000$ atoms.

Within the sun, four $^{1}_{1}H$ nuclei combine and form one $^{4}_{2}He$ nucleus. In this process 26 MeV of energy is released.

Hence, the energy released from the fusion of 1 kg $^{1}_{1}H$ is:

$$E_1 = \frac{6.023 \times 10^{23} \times 26 \times 10^{3}}{4} = 39.1495 \times 10^{26}\ \text{MeV}$$

(b) Amount of $^{235}_{92}U = 1\ \text{kg} = 1000\ \text{g}$

1 mole, i.e., 235 g of $^{235}_{92}U$ contains 6.023×10^{23} atoms.

$\therefore$ 1000 g of $^{235}_{92}U$ contains $\dfrac{6.023 \times 10^{23} \times 1000}{235}$ atoms

It is known that the amount of energy released in the fission of one atom of $^{235}_{92}U$ is 200 MeV.

Hence, energy released from the fission of 1 kg of $^{235}_{92}U$ is:

$$E_1 = \frac{6 \times 10^{23} \times 1000 \times 200}{235} = 5.106 \times 10^{26}\ \text{MeV}$$

$$E_1 = \frac{39.1495 \times 10^{23}}{5.106 \times 10^{26}} = 7.67 \approx 8$$

Therefore, the energy released in the fusion of 1 kg of hydrogen is nearly 8 times the energy released in the fission of 1 kg of uranium.

S25. (a) Take the *D*-*T* nuclear reaction:

$$^{2}_{1}H + {}^{3}_{1}H \longrightarrow {}^{4}_{2}He + n$$

It is given that:

Mass of $^{2}_{1}H$, $m_1 = 2.014102\ \text{u}$

Mass of ${}_{1}^{3}\text{H}$, $m_2 = 3.016049\,\text{u}$

Mass of ${}_{2}^{4}\text{He}$, $m_3 = 4.002603\,\text{u}$

Mass of ${}_{0}^{1}n$, $m_4 = 1.008665\,\text{u}$

Q-value of the given *D*-*T* reaction is:

$$Q = [m_1 + m_2 - m_3 - m_4]\,c^2$$

$$= [2.014102 + 3.016049 - 4.002603 - 1.008665]\,c^2$$

$$= [0.018883\,c^2]\,\text{u}$$

But $1\,\text{u} = 931.5\ \text{MeV}/c^2$

$\therefore$ $Q = 0.018883 \times 931.5 = 17.59\ \text{MeV}$

(b) Radius of deuterium and tritium,

$$r \approx 2.0\ \text{fm} = 2 \times 10^{-15}\,\text{m}$$

Distance between the two nuclei at the moment when they touch each other,

$$d = r + r = 4 \times 10^{-15}\,\text{m}$$

Charge on the deuterium nucleus = *e*

Charge on the tritium nucleus = *e*

Hence, the repulsive potential energy between the two nuclei is given as:

$$V = \frac{e^2}{4\pi\varepsilon_0(d)}$$

Where, ε_0 = Permittivity of free space

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\,\text{N m}^2\,\text{C}^{-2}$$

$\therefore$ $$V = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})}{4 \times 10^{-15}} = 5.76 \times 10^{-14}\,\text{J}$$

$$= \frac{5.76 \times 10^{-14}}{1.6 \times 10^{-19}} = 3.6 \times 10^5\ \text{eV} = 360\ \text{keV}$$

Hence, 5.76×10^{-14} J or 360 keV of kinetic energy (K.E.) is needed to overcome the Coulomb repulsion between the two nuclei.

However, it is given that:

$$\text{K.E.} = 2 \times \frac{3}{2}kT$$

Where, k = Boltzmann constant = $1.38 \times 10^{-23}\,\text{m}^2\,\text{kg s}^{-2}\,\text{K}^{-1}$

T = Temperature required for triggering the reaction

$\therefore$ $$T = \frac{\text{K.E.}}{3K}$$

$$= \frac{5.76 \times 10^{-14}}{3 \times 1.38 \times 10^{-23}} = 1.39 \times 10^{9}\ \text{K}$$

Hence, the gas must be heated to a temperature of 1.39×10^9 K to initiate the reaction.

S26. In the fission of $^{238}_{92}\text{U}$, 10 β^- particles decay from the parent nucleus. The nuclear reaction can be written as:

$$^{238}_{92}\text{U} + {}^{1}_{0}n \longrightarrow {}^{140}_{58}\text{Ce} + {}^{99}_{44}\text{Ru} + 10\,{}^{0}_{-1}e$$

It is given that:

Mass of a nucleus $^{238}_{92}\text{U}$, $m_1 = 238.05079\,\text{u}$

Mass of a nucleus $^{140}_{58}\text{Ce}$, $m_2 = 139.90543\,\text{u}$

Mass of a nucleus $^{99}_{44}\text{Ru}$, $m_3 = 98.90594\,\text{u}$

Mass of a neutron $^{1}_{0}n$, $m_4 = 1.008665\,\text{u}$

Q-value of the above equation,

$$Q = [m'(^{238}_{92}\text{U}) + m(^{1}_{0}n) - m'(^{140}_{58}\text{Ce}) - m'(^{99}_{44}\text{Ru}) - 10\,m_e]\,c^2$$

Where, m' = Represents the corresponding atomic masses of the nuclei

$$m'(^{238}_{92}\text{U}) = m_1 - 92\,m_e$$
$$m'(^{140}_{58}\text{Ce}) = m_2 - 58\,m_e$$
$$m'(^{99}_{44}\text{Ru}) = m_3 - 44\,m_e$$
$$m(^{1}_{0}n) = m_4$$

$$Q = [m_1 - 92\,m_e + m_4 - m_2 + 58\,m_e - m_3 + 44\,m_e - 10\,m_e]\,c^2$$
$$= [m_1 + m_4 - m_2 - m_3]\,c^2$$
$$= 238.05079 + 1.008665 - 139.90543 - 98.90594]\,c^2$$
$$= [0.247995\,c^2]\,\text{u}$$

But $1\,\text{u} = 931.5\ \text{MeV}/c^2$

$\therefore$ $Q = 0.247995 \times 931.5 = 231.007$ MeV

Hence, the Q-value of the fission process is 231.007 MeV.

S27. The given fusion reaction is:

$$^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \longrightarrow {}^{3}_{2}\text{He} + n + 3.27\ \text{MeV}$$

Amount of deuterium, $m = 2$ kg

1 mole, *i.e.*, 2 g of deuterium contains 6.023×10^{23} atoms.

$\therefore$ 2.0 kg of deuterium contains $= \dfrac{6.023 \times 10^{23}}{2} \times 2000 = 6.023 \times 10^{26}$ atoms.

It can be inferred from the given reaction that when two atoms of deuterium fuse, 3.27 MeV energy is released.

∴ Total energy per nucleus released in the fusion reaction:

$$E = \frac{3.27}{2} \times 6.023 \times 10^{26} \text{ MeV}$$

$$= \frac{3.27}{2} \times 6.023 \times 10^{26} \times 1.6 \times 10^{-19} \times 10^{6}$$

$$= 1.576 \times 10^{14} \text{ J}$$

Power of the electric lamp, $P = 100 \text{ W} = 100 \text{ J/s}$

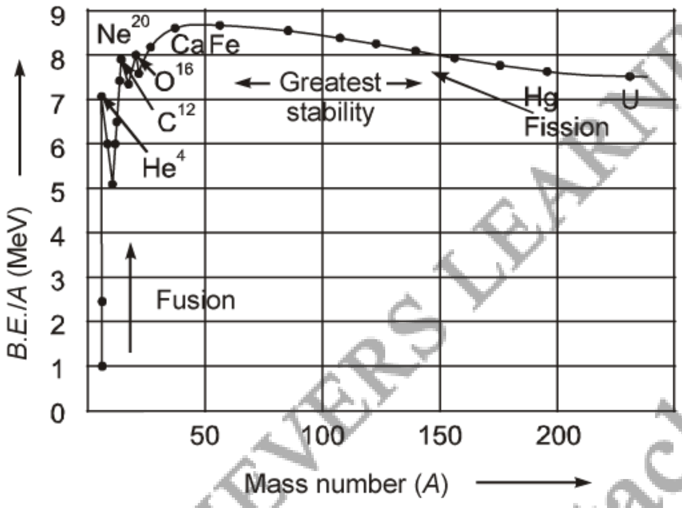
Hence, the energy consumed by the lamp per second = 100 J

The total time for which the electric lamp will glow is calculated as:

$$\frac{1.576 \times 10^{14}}{100} \text{ s}$$

$$\frac{1.576 \times 10^{14}}{100 \times 60 \times 60 \times 24 \times 365} \approx 4.9 \times 10^{4} \text{ years.}$$

S28. The binding energy curve per nucleon is shown below.



Explaination of release of energy in nuclear fission and fusion. The curve reveals that binding energy per nucleon is smaller for heavier nuclei than the middle level nuclei. This shows that heavier nuclei. In nuclear fission, binding energy per nucleon of reactants (heavier nuclei) changes from nearly 7.6 MeV to 8.4 MeV (for nuclei of middle level mass).

Higher value of the binding energy of the nuclear product results in the liberation of energy during the phenomena of nuclear fission.

In nuclear fusion, binding energy per nucleon of lighter nuclei into heavier one changes from low value and release of energy take place in fusion e.g., two ${}_1H^2$ (Be ≃ 1.5 MeV/nucleon) combine to form ${}_2He^4$ (Binding energy per nucleon ≃ 7 MeV/nuclei) and therefore the energy is liberated during nuclear fusion.

S29. The nuclear reaction may be expressed as

$$_0n^1 + {_3}\text{Li}^6 \longrightarrow {_1}\text{H}^3 + {_2}\text{He}^4 + \text{Q},$$

where Q is energy released.

If $m_N({_3}\text{Li}^6)$, $m_N({_1}\text{H}^3)$ and $m_N({_2}\text{He}^4)$ represent nuclear mass, then

$$Q = [m_n + m_N({_3}\text{Li}^6) - m_N({_1}\text{H}^3) - m_N({_2}\text{He}^4)] \times 931.5$$

$$= [m_N + \{m({_3}\text{Li}^6) - 3\,m_e\} - \{m({_1}\text{H}^3) - m_e\} - \{m({_2}\text{He}^4) - 2\,m_e\}] \times 931.5$$

$$= [m_n + m({_3}\text{Li}^6) - m({_1}\text{H}^3) - m({_2}\text{He}^4)] \times 931.5$$

$$= [1.008665 + 6.015125 - 3.016049 - 4.002604] \times 931.5$$

$$= 0.005138 \times 931.5 = \mathbf{4.786\ MeV}.$$

S30. In ${_{26}}\text{Fe}^{56}$ nucleus, No. of protons = 26

Number of neutrons = 56 – 26 = 30

$\therefore$ Mass defect, $\Delta m = \{[26 \times 1.007825 + 30 \times 1.008665] - [55.934939]\}$ a.m.u

$= \{[26.203450 + 30.259950] - [55.934939]\}$ a.m.u.

$= (56.463400 - 55.934939)$ a.m.u

$= 0.528461$ a.m.u

1 a.m.u of mass releases 931 MeV

$\therefore$ Binding energy of ${_{26}}\text{Fe}^{56} = 0.528461 \times 931$ MeV

$= 491.997$ MeV

Binding energy of nucleon $= \dfrac{491.997}{56}$ MeV = 8.786 MeV.

S31. (a) A chemical equation is balanced in the sense that the number of atoms of each element is the same on both sides of the equation. A chemical reaction merely alters the original combinations of atoms. In a nuclear reaction, elements may be transmuted. Thus, the number of atoms of each element is not necessarily conserved in a nuclear reaction. However, the number of protons and the number of neutrons are both separately conserved in a nuclear reaction. [Actually, even this is not strictly true in the realm of very high energies – what is strictly conserved is the total charge and total 'baryon number'. We need not pursue this matter here.] In nuclear reactions (e.g., Eq. 13.26), the number of protons and the number of neutrons are the same on the two sides of the equation.

(b) We know that the binding energy of a nucleus gives a negative contribution to the mass of the nucleus (mass defect). Now, since proton number and neutron number are conserved in a nuclear reaction, the total rest mass of neutrons and protons is the same on either side of a reaction. But the total binding energy of nuclei on the left side need not be the same as that on the right hand side. The difference in these binding energies appears as energy released or absorbed in a nuclear reaction. Since binding energy contributes to mass, we say that the difference in the total mass of nuclei on the two sides get converted into energy or vice-versa. It is in these sense that a nuclear reaction is an example of massenergy interconversion.

(c) From the point of view of mass-energy interconversion, a chemical reaction is similar to a nuclear reaction *in principle.* The energy released or absorbed in a chemical reaction can be traced to the difference in chemical (not nuclear) binding energies of atoms and molecules on the two sides of a reaction. Since, strictly speaking, chemical binding energy also gives a negative contribution (mass defect) to the total mass of an atom or molecule, we can equally well say that the difference in the total mass of atoms or molecules, on the two sides of the chemical reaction gets converted into energy or vice-versa. However, the mass defects involved in a chemical reaction are almost a million times smaller than those in a nuclear reaction.This is the reason for the general impression, (which is *incorrect*) that mass-energy interconversion does not take place in a chemical reaction.

S32. Number of protons in ${}_{83}Bi^{209}$ nucleus = 83

Number of neutrons in ${}_{83}Bi^{209}$ nucleus = 209 – 83 = 126

$$\therefore \quad \text{Mass of protons} = 83 \times 1.007825 \text{ a.m.u} = 83.649475 \text{ a.m.u}$$

$$\text{Mass of nutrons} = 126 \times 1.008665 \text{ a.m.u} = 127.091790$$

$$\therefore \quad \text{Mass defect,} \quad \Delta m = 83.649475 + 127.091790 - 208.980388 \text{ a.m.u}$$

$$= 1.760877 \text{ a.m.u}$$

$$\text{Binding energy of } {}_{83}Bi^{209} \text{ neucleus} = \Delta m \times 931 \text{ MeV}$$

$$= 1.760877 \times 931 \text{ MeV}$$

$$= 1639.38 \text{ MeV}$$

$$\frac{\text{B.E.}}{\text{Nucleon}} = \frac{1639.38}{209} = 7.84 \text{ MeV.}$$

S33.

$$\text{Energy in one fission} = 200 \text{ MeV}$$

$$= 200 \times 10^6 \text{ eV}$$

$$= 200 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

$$\text{Energy to be produced} = 1 \text{ MW} = 10^6 \text{ W} = 10^6 \text{ J/s}$$

$\therefore$ Required number of fission per second

$$= \frac{10^6}{200 \times 10^6 \times 1.6 \times 10^{-19}} \text{ sec}^{-1}$$

$$= \frac{10^{19}}{200 \times 1.6} = \mathbf{3.125 \times 10^{16}\,sec^{-1}}.$$

S34. It can be observed from the given γ-decay diagram that γ_1 decays from the 1.088 MeV energy level to the 0 MeV energy level.

Hence, the energy corresponding to γ_1-decay is given as:

$$E_1 = 1.088 - 0 = 1.088\text{ MeV}$$
$$h\nu_1 = 1.088 \times 1.6 \times 10^{-19} \times 10^6\text{ J}$$

Where, h = Planck's constant = 6.6×10^{-34} Js

ν_1 = Frequency of radiation radiated by γ_1-decay

$$\therefore \quad \nu_1 = \frac{E_1}{h}$$
$$= \frac{1.088 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}} = 2.637 \times 10^{20}\text{ Hz}$$

It can be observed from the given γ-decay diagram that γ_2 decays from the 0.412 MeV energy level to the 0 MeV energy level.

Hence, the energy corresponding to γ_2-decay is given as:

$$E_2 = 0.412 - 0 = 0.412\text{ MeV}$$
$$h\nu_2 = 0.412 \times 1.6 \times 10^{-19} \times 10^6\text{ J}$$

Where, ν_2 = Frequency of radiation radiated by γ_2-decay

$$\therefore \quad \nu_2 = \frac{E_2}{h}$$
$$= \frac{0.412 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}} = 9.998 \times 10^{19}\text{ Hz}$$

It can be observed from the given γ-decay diagram that γ_3 decays from the 1.088 MeV energy level to the 0.412 MeV energy level.

Hence, the energy corresponding to γ_3-decay is given as:

$$E_3 = 1.088 - 0.412 = 0.676\text{ MeV}$$
$$h\nu_3 = 0.676 \times 10^{-19} \times 10^6\text{ J}$$

Where, ν_3 = Frequency of radiation radiated by $\times_3$-decay

$$\therefore \quad \nu_2 = \frac{E_3}{h}$$
$$= \frac{0.676 \times 1.6 \times 10^{-19} \times 10^6}{6.6 \times 10^{-34}} = 1.639 \times 10^{20}\text{ Hz}$$

$$\text{Mass of } m(^{198}_{78}\text{Au}) = 197.968233 \text{ u}$$

$$\text{Mass of } m(^{198}_{80}\text{Hg}) = 197.966760 \text{ u}$$

$$1 \text{ u} = 931.5 \text{ MeV}/c^2$$

Energy of the highest level is given as:

$$E = [m(^{198}_{78}\text{Au}) - m(^{198}_{80}\text{Hg})]$$

$$= 197.968233 - 197.966760 = 0.001473 \text{ u}$$

$$= 0.001473 \times 931.5 = 1.3720995 \text{ MeV}$$

β_1 decays from the 1.3720995 MeV level to the 1.088 MeV level

$\therefore$ Maximum kinetic energy of the β_1 particle = 1.3720995 – 1.088 = 0.2840995 MeV

β_2 decays from the 1.3720995 MeV level to the 0.412 MeV level

$\therefore$ Maximum kinetic energy of the β_2 particle = 1.3720995 – 0.412 = 0.9600995 MeV

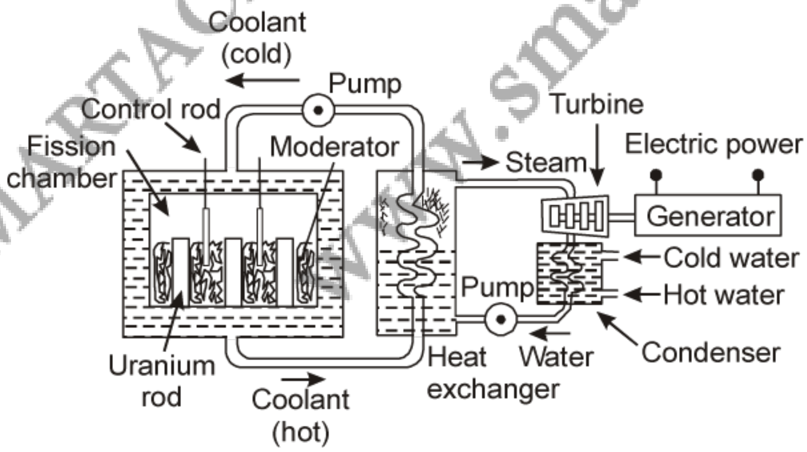
S35. A nuclear reactor is a device used to generate electric energy besides other applications. It works on the principle of controlled chain reaction of nuclear fission process.

A labeled diagram of a nuclear reactor is shown in Fig. 3.1. the main components of a nuclear reactor are:

(a) **Nuclear fuel:** It is a fissionable material *e.g.*, U^{235}, Th^{232}, Pu^{239}, etc. The fuel is sealed in aluminum cylinders. This cylinders are arranged in fission chamber inside reactor. When slow neutrons bombard on the fuel, fission starts.

(b) **Moderator:** The neutrons released by the fission of uranium have high energies of the order of 2 MeV. However, for a controlled chain reaction, slowly moving neutrons are required. Moderators are the substances (rich in protons) used to slow down the fast moving neutrons. Heavy water, graphite, deuterium, paraffin, etc. are some of the commonly used moderators.

When fast moving neutrons collide with the protons of moderator, their energies are interchanged and thus the neutrons are slowed down. The slowed down neutrons are called *thermal neutrons*.



(c) **Control rods:** To control the chain reaction, rods of neutron absorbing material like boron or cadmium are inserted into the reactor core. As a result of it, the desired number of neutrons are absorbed and only limited number of neutrons are left to produce further fission. The depth of control rod inside the reactor control the number of neutrons absorbed.

(d) **Coolant:** A large amount of heat is produced due to nuclear fission inside the reactor. The material which is used to remove the heat produce and transfer it from the core of the nuclear reactor to the surrounding is called coolant. At ordinary temperatures, liquid sodium is used as a coolant. Air at high pressure is used as coolant.

The coolant releases the heat energy to the water in a heat exchanger. Thus, superheated steam is produced which drives the turbines of generator.

(e) **Shielding:** Whole nuclear reactor is enclosed in thick concrete walls called protective shield so that nuclear radiations do not produce harmful effects on the people in the surrounding area.

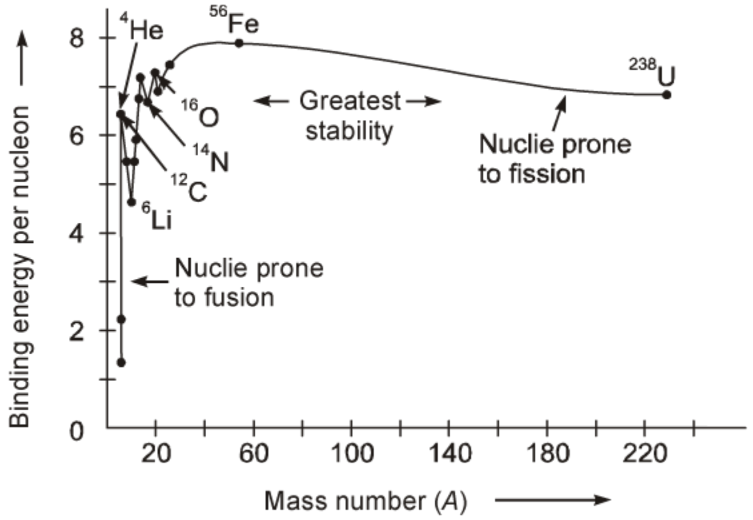
Working: In the beginning, some neutrons are produced by the action of α-particles on beryllium. These neutrons are slowed down and are used to initiate fission of U^{235} nuclei. Fast neutrons liberated are slowed down by the moderator. These thermal neutrons causes fission of more U^{235} nuclei and controlled chain reaction build up with help of control rods.

To stop chain reaction control rods are inserted deep inside the core.

S36.

Nuclear fission	*Nuclear fusion*
1. In nuclear fission, a heavy nucleus splits into two smaller nuclei of nearly same masses.	1. In nuclear fusion, two or more lighter nuclei fuse together to form a behavior nucleus.
2. It does not require extremely high temperature.	2. It take place at extremely high temperatures ($\approx 10^7$ K).
3. Comparatively lower amount of energy is released.	3. Extraordinary huge amount of energy is released in this process.
4. This process is carried out ot generate electric power.	4. This process has not been successfully carried out anywhere in the world till now, in a controlled manner.
5. Fission products are generally radioactive.	5. Fusion products are non-radioactive and do not pose any hazard for life.
6. The sources of fissionable materials (*e.g.*, uranium, thorium, etc.) are limited and exhaustible.	6. The sources of fusion reactions (e.g., hydrogen, etc.) are present in large quantity and inexhaustible.

7. This process is used in making atom bomb.	7. This process is used in hydrogen bomb.



Binding energy per nucleon as a function of mass number (*A*)

S37. A nuclear reaction, in which the projectile used to carry out the fission reaction (such as neutron in U^{235} fission reaction) gets multiplied as more and more such fission reactions take place, is called nuclear chain reaction.

$$_{7}N^{14} + {}_{2}He^{4} \longrightarrow {}_{8}O^{17} + {}_{1}H^{1}$$

Nuclear fission: Nuclear fission is the phenomenon of splitting of a heavy nucleus (usually of mass number greater than 230) into two (or more) lighter nuclei. For example, nuclear fission of $_{92}U^{235}$ when it is hit by a neutron is represented as

$$_{92}U^{235} + {}_{0}n^{1} \longrightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_{0}n^{1} + Q$$

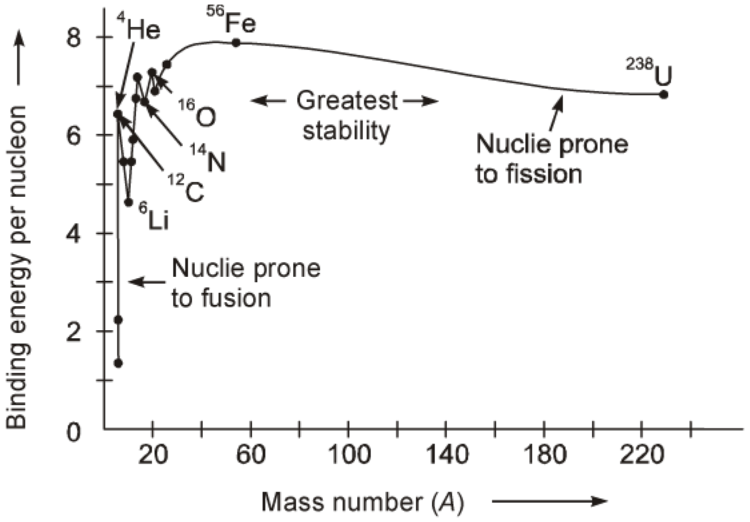
Where Q is the energy released in the process. In the process, certain mass disappears *i.e.*, sum of the masses of final products is found to be slightly less than the sum of the masses of the reactant components. This difference in masses is called mass defect (Δm). The mass defect appears in the form energy in accordance with Einstein mass-energy relation, $E = \Delta m.c^2$.

Nuclear fusion: Nuclear fusion is the phenomenon in which two or more lighter nuclei fuse to form a single heavy nucleus. The nuclear fusion takes place under the conditions of very high temperature ($\approx 10^7$ K) and pressure.

$$_{1}H^{2} + {}_{1}H^{2} \longrightarrow {}_{2}He^{4} + Q$$

where Q is the energy released in the process. The mass of the product nucleus is slightly less than the sum of the masses of the lighter nuclei fusion together. This difference in masses (called mass defect Δm) results in the release of tremendous amount of energy, in accordance with Einstein's mass energy relation, $E = \Delta m.c^2$.

S38. The binding energy curve per nucleon is shown below:



Binding energy per nucleon as a function of mass number (*A*)

Explanation of release of energy in nuclear fission and fusion: The curve reveals that binding energy per nucleon is smaller for heavier nuclei than the middle level nuclei. This shows that heavier nuclei are less stable than middle level nuclei. In nuclear fission, binding energy per nucleon of reactants (heavier nuclei) changes from nearly 7.6 MeV to 8.4 MeV (for nuclei of middle level mass).

Higher value of the binding energy of the nuclear product results in the liberation of energy during the phenomena of nuclear fission.

In nuclear fission, binding energy per nucleon of lighter nuclei into heavier one changes from low value of binding energy per nucleon to high value and release of energy takes place in fusion *e.g.*, two $_1H^2$ (Be $\simeq$ 1.5 MeV/nucleon) combine to form $_2He^4$ (binding energy per nucleon $\simeq$ 7 MeV/nuclei) and therefore the energy is liberated during nuclear fusion.

S39. (a) The difference in mass of sum of masses of nucleons and mass of nucleus is known as mass defect which converts into energy as per Einstein's mass energy relationship, $E = mc^2$.

This energy is known as binding energy which is used to hold the nucleons together inspite of repulsive Coulombian force between positively charged protons. Otherwise nucleons will no longer be stable. The number of neutrons and protons is conserved but it does not mean that the parent nuclei and product nuclei are same. The difference in the mass of parent and product nuclei, converts into energy.

(b) 239 gms of Pu contain 6.023×10^{23} atoms

1000 gm of Pu contain $\dfrac{6.023 \times 10^{23}}{239} \times 1000 = 2.52 \times 10^{24}$ atoms

Fission of each atom releases 180 MeV.

$\therefore$ Energy released due to fission of 1 kg $^{239}_{94}Pu$ or 2.52×10^{24} atoms

$= 2.52 \times 10^{24} \times 180$ MeV

$=$ **4.536×10^{26} MeV**.