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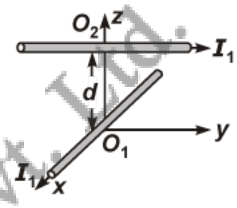
Q1. The magnetic force depends on v which depends on the inertial frame of reference. Does then the magnetic force differ from inertial frame to frame? Is it reasonable that the net acceleration has a different value in different frames of reference?

Q2. Show that a force that does no work must be a velocity dependent force.

Q3. Verify that the cyclotron frequency $\omega = eB/m$ has the correct dimensions of $[T]^{-1}$.

Q4. Describe the motion of a charged particle in a cyclotron if the frequency of the radio frequency (rf) field were doubled.

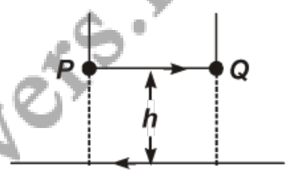
Q5. Two long wires carrying current I_1 and I_2 are arranged as shown in figure. The one carrying current I_1 is along is the x-axis. The other carrying current I_2 is along a line parallel to the y-axis given by $x = 0$ and $z = d$. Find the force exerted at O_2 because of the wire along the x-axis.



Q6. A current carrying loop consists of 3 identical quarter circles of radius R, lying in the positive quadrants of the x-y, y-z and z-x planes with their centres at the origin, joined together. Find the direction and magnitude of B at the origin.

Q7. A charged particle of charge e and mass m is moving in an electric field E and magnetic field B. Construct dimensionless quantities and quantities of dimension $[T]^{-1}$.

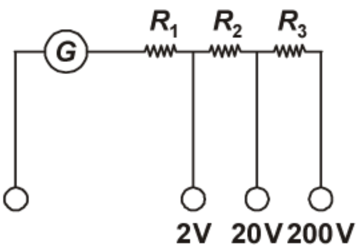
Q8. A long straight wire carrying current of 25 A rests on a table as shown in figure. Another wire PQ of length 1 m, mass 2.5 g carries the same current but in the opposite direction. The wire PQ is free to slide up and down. To what height will PQ rise?



Q9. Do magnetic forces obey Newton's third law. Verify for two current elements $dl_1 = dl\hat{i}$ located at the origin and $dl_2 = dl\hat{j}$ located at (0, R, 0). Both carry current I.

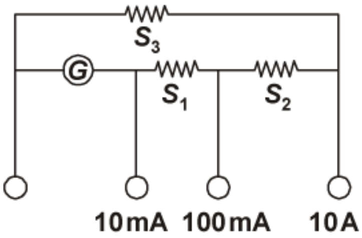
Q10. An electron enters with a velocity $v = v_0\hat{i}$ into a cubical region (faces parallel to coordinate planes) in which there are uniform electric and magnetic fields. The orbit of the electron is found to spiral down inside the cube in plane parallel to the x-y plane. Suggest a configuration of fields E and B that can lead to it.

Q11. A multirange voltmeter can be constructed by using a galvanometer circuit as shown in figure. We want to construct a voltmeter that can measure 2 V, 20 V and 200 V using a galvanometer of resistance 10 Ω and that produces maximum deflection for current of 1 mA. Find R_1, R_2 and R_3 that have to be used.



Q12. An electron and a positron are released from (0, 0, 0) and (0, 0, 1.5 R) respectively, in a uniform magnetic field $B = B_0\hat{i}$, each with an equal momentum of magnitude $p = eBR$. Under what conditions on the direction of momentum will the orbits be nonintersecting circles?

Q13. A multirange current meter can be constructed by using a galvanometer circuit as shown in figure. We want a current meter that can measure 10 mA, 100 mA and 1 A using a galvanometer of resistance 10 Ω and that produces maximum deflection for current of 1 mA. Find S_1, S_2 and S_3 that have to be used



Q14. Consider a circular current-carrying loop of radius R in the x-y plane with centre at origin.

Consider the line integral $\Im(L) = \left| \int_{-L}^{L} B.dl \right|$ taken along z-axis.

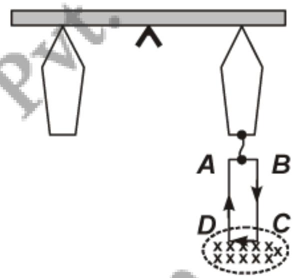
(a) Show that $\Im(L)$ monotonically increases with L.

(b) Use an appropriate Amperian loop to show that $\Im(\infty) = \mu_0 I$, where I is the current in the wire.

(c) Verify directly the above result.

(d) Suppose we replace the circular coil by a square coil of sides R carrying the same current I. What can you say about $\Im(L)$ and $\Im(\infty)$?

Q15. A 100 turn rectangular coil *ABCD* (in *XY*-plane) is hung from one arm of a balance (shown in figure). A mass 500 g is added to the other arm to balance the weight of the coil. A current 4.9 A passes through the coil and a constant magnetic field of 0.2 *T* acting inward (in *xz*-plane) is switched on such that only arm *CD* of length 1 cm lies in the field. How much additional mass '*m*' must be added to regain the balance?



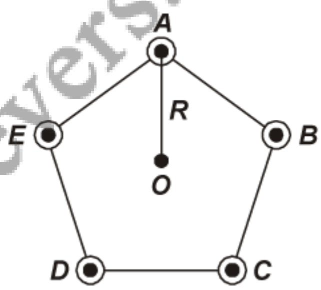
Q16. Five long wires *A*, *B*, *C*, *D* and *E*, each carrying current I are arranged to form edges of a pentagonal prism as shown in figure.

Each carries current out of the plane of paper.

(a) What will be magnetic induction at a point on the axis *O*? Axis is at a distance *R* from each wire.

(b) What will be the field if current in one of the wires (say *A*) is switched off?

(c) What if current in one of the wire (say) *A* is reversed?



Q17. A uniform conducting wire of length $12a$ and resistance R is wound up as a current carrying coil in the shape of (a) an equilateral triangle of side a; (b) a square of sides a and, (c) a regular hexagon of sides a. The coil is connected to a voltage source V_0. Find the magnetic moment of the coils in each case.

Q18. A rectangular conducting loop consists of two wires on two opposite sides of length l joined together by rods of length d. The wires are each of the same material but with cross-sections differing by a factor of 2. The thicker wire has a resistance R and the rods are of low resistance, which in turn are connected to a constant voltage source V_0. The loop is placed in uniform a magnetic field B at 45° to its plane. Find τ, the torque exerted by the magnetic field on the loop about an axis through the centres of rods.

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S1. Yes, the magnetic force is frame dependent. Net acceleration arising from this is however frame independent (non-relativistic physics) for inertial frames.

S2.
$$dW = \boldsymbol{F} . d\boldsymbol{l} = 0$$
$$\Rightarrow \quad \boldsymbol{F} . \boldsymbol{v}dt = 0$$
$$\Rightarrow \quad \boldsymbol{F} . \boldsymbol{v} = 0$$

F must be velocity dependent which implies that angle between ***F*** and ***v*** is 90°. If ***v*** changes (direction) then (directions) ***F*** should also change so that above condition is satisfied.

S3. For a charge particle moving perpendicular to the magnetic field:

$$\frac{mv^2}{R} = qvB$$ [∵ Centripetal force is produced by the magnetic force]

$$\therefore \quad \frac{qB}{m} = \frac{v}{R} = \omega$$

$$\therefore \quad [\omega] = \left[\frac{qB}{m}\right] = \left[\frac{v}{R}\right] = [T]^{-1}.$$

S4. Here, the condition of magnetic resonance is violated.

When the frequency of the radio frequency (*rf*) field were doubled, the time period of the radio frequency (*rf*) field were halved. Therefore, the duration in which particle completes half revolution inside the dees, radio frequency completes the cycle.

Hence, particle will accelerate and decelerate alternatively. So, the radius of path in the dees will remain same.

S5. At O_2, the magnetic field due to I_1 is along the *y*-axis. The second wire is along the *y*-axis and hence the force is zero.

S6. For the current carrying loop quarter circles of radius *R*, lying in the positive quadrants of the *x*-*y* plane

$$B_1 = \frac{\mu_0}{4\pi}\frac{I(\pi/2)}{R}\hat{k} = \frac{\mu_0}{4}\frac{I}{2R}\hat{k}$$

For the current carrying loop quarter circles of radius *R*, lying in the positive quadrants of the *y*-*z* plane and hence the magnetic field is along positive direction of *x*-axis.

$$B_2 = \frac{\mu_0}{4}\frac{I}{2R}\hat{i}$$

For the current carrying loop quarter circles of radius R, lying in the positive quadrants of the z-x plane

$$B_3 = \frac{\mu_0}{4}\frac{I}{2R}\hat{j}$$

Current carrying loop consists of 3 identical quarter circles of radius R, lying in the positive quadrants of the x-y, y-y and z-z planes with their centres at the origin, joined together is equal to the vector sum of magnetic field due to each quarter and given by

$$B = \frac{1}{4\pi}\frac{\mu_0 I}{2R}(\hat{i}+\hat{j}+\hat{k}).$$

S7. No dimensionless quantity can be constructed using given quantities.

For a charge particle moving perpendicular to the magnetic field, the magnetic Lorentz forces provides necessary centripetal force for revolution.

$$\frac{mv^2}{R} = qvB$$

On simplifying the terms, we have

$$\therefore \qquad \frac{qB}{m} = \frac{v}{R} = \omega$$

Finding the dimensional formula of angular frequency

$$[\omega] = \left[\frac{qB}{m}\right] = \left[\frac{v}{R}\right] = [T]^{-1}$$

This is the required expression.

S8. $F = BIl \text{ Sin } \theta = BIl$

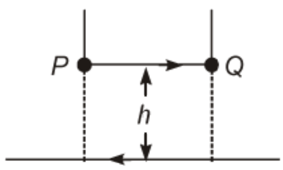
$$B = \frac{\mu_0 I^2}{2\pi h}$$

$$F = mg = \frac{\mu_0 I^2 l}{2\pi h} \qquad [\because \text{ in equilibrium}]$$

$$h = \frac{\mu_0 I^2 l}{2\pi mg} = \frac{4\pi \times 10^{-7} \times 250 \times 25 \times 1}{2\pi \times 2.5 \times 10^{-3} \times 9.8}$$

$$= 51 \times 10^{-4}$$

$$h = 0.51 \text{ cm.}$$



S9. In Biot-Savart's law, magnetic field $\boldsymbol{B}$ is parallel (II) to $\hat{\boldsymbol{i}}\,\boldsymbol{dl} \times \boldsymbol{r}$ and idl have its direction along the direction of flow of current.

Here, for the direction of magnetic field, At $\boldsymbol{dl}_2$, located at (0, R, 0) due to wire d_1 is given by $B \parallel \hat{\boldsymbol{i}}\,\boldsymbol{dl} \times \boldsymbol{r}$ or $\hat{\boldsymbol{i}} \times \hat{\boldsymbol{j}}$ (because point (0, R, 0) lies on y-axis), but $\hat{\boldsymbol{i}} \times \hat{\boldsymbol{j}} = \hat{\boldsymbol{k}}$.

So, the direction of magnetic field at d_2 is along z-direction.

The direction of magnetic force exerted at d_2 because of the first wire along the x-axis.

$$F = \hat{i}\ (I \times B) \quad i.e., \quad F \parallel (i \times k) \text{ or along } -\hat{j} \text{ direction.}$$

Therefore, force due to $d\boldsymbol{l}_1$ or $d\boldsymbol{l}_2$ is non-zero.

Now, for the direction of magnetic field, At d_1, located at (0, 0, 0) due to wire d_2 is given by $B \parallel \hat{i}\,d\boldsymbol{l} \times \boldsymbol{r}$ or $\hat{i} \times -\hat{j}$ (because origin lies on y-direction w.r.t. point (0, R, 0), but $\hat{j} \times -\hat{j} = 0$.

So, the magnetic field at d_1 does not exist.

Force due to $d\boldsymbol{l}_2$ on $d\boldsymbol{l}_1$ is zero.

So, magnetic forces do not obey Newton's third law.

S10. Considering magnetic field $\boldsymbol{B} = B_0\hat{\boldsymbol{k}}$, and electron enters with a velocity $\boldsymbol{v} = v_0\hat{\boldsymbol{i}}$ into a cubical region (faces parallel to coordinate planes).

The force on electron, using magnetic Lorentz force, is given by

$$\boldsymbol{F} = -e\,(v_0\hat{\boldsymbol{i}} \times B_0\hat{\boldsymbol{k}}) = ev_0B_0\hat{\boldsymbol{j}}$$

which revolves the electron in x-y plane.

The electric force $\boldsymbol{F} = -\boldsymbol{e}\boldsymbol{B}_0\hat{\boldsymbol{k}}$ accelerates e along z-axis which in turn increases the radius of circular path and hence particle traversed on spiral path.

S11.

$$i_G(G + R_1) = 2 \text{ for } 2\,\text{V range}$$

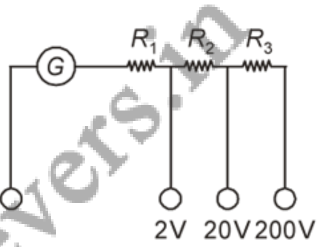
$$i_G(G + R_1 + R_2) = 20 \text{ for } 20\,\text{V range}$$

and $$i_G(G + R_1 + R_2 + R_3) = 200 \text{ for } 200\,\text{V range}$$

Gives $$R_1 = 1990\,\Omega$$

$$R_2 = 18\,\text{k}\Omega$$

and $$R_3 = 180\,\text{k}\Omega.$$



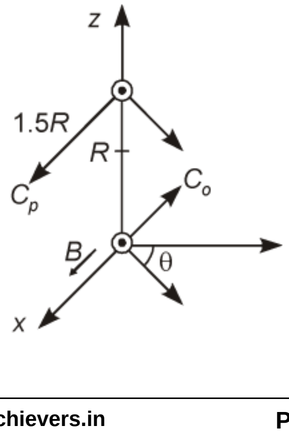
S12. As $\boldsymbol{B}$ is along the x-axis, for a circular orbit the momenta of the two particles are in the y-z plane. Let $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ be the momentum of the electron and positron, respectively. Both of them define a circle of radius R. They shall define circles of opposite sense. Let $\boldsymbol{p}_1$ make an angle θ with the y-axis $\boldsymbol{p}_2$ must make the same angle. The centres of the respective circles must be perpendicular to the momenta and at a distance R. Let the center of the electron be at C_e and of the positron at C_p.

The coordinates of C_e is

$$C_e \equiv (0, -R\sin\theta, R\cos\theta)$$

The coordinates of C_p is

$$C_p \equiv \left(0, -R\sin\theta, \frac{3}{2}R - R\cos\theta\right)$$



The circles of the two shall not overlap if the distance between the two centers are greater than $2R$.

Let d be the distance between C_p and C_e.

Then, $$d^2 = (2R\sin\theta)^2 + \left(\frac{3}{2}R - 2R\cos\theta\right)^2$$

$$= 4R^2\sin^2\theta + \frac{9^2}{4}R - 6R^2\cos\theta + 4R^2\cos^2\theta$$

$$= 4R^2 + \frac{9}{4}R^2 - 6R^2\cos\theta$$

Since d has to be greater than $2R$

$$d^2 > 4R^2$$

$$\Rightarrow \quad 4R^2 + \frac{9}{4}R^2 - 6R^2\cos\theta > 4R^2$$

$$\Rightarrow \quad \frac{9}{4} > 6\cos\theta$$

Or, $$\cos\theta = \frac{3}{8}.$$

S13. $$i_G \cdot G = (i_1 - i_G)(S_1 + S_2 + S_3) \quad \text{for} \quad i_1 = 10\,\text{mA}$$

$$i_G(G + S_1) = (i_2 - i_G)(S_2 + S_3) \quad \text{for} \quad i_2 = 100\,\text{mA}$$

and $$i_G(G + S_1 + S_2) = (i_3 - i_G)(S_3) \quad \text{for} \quad i_3 = 1\,\text{A}$$

gives $$S_1 = 1\,\Omega, \quad S_2 = 0.1\,\Omega \quad \text{and} \quad S_3 = 0.01\,\Omega.$$

S14. (a) $B(z)$ points in the same direction on z-axis and hence $J(L)$ is a monotonically increasing function of L.

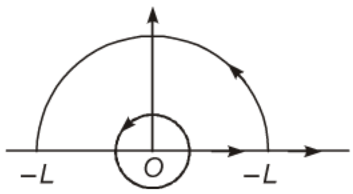
(b) $J(L)$ + Contribution from large distance on contour $C = \mu_0 I$.

$\therefore$ as $L \to \infty$.

Contribution from large distance $\to 0$ (as $B \to 1/r^3$)

$$J(0) - \mu_0 I.$$

(c) $$B_z = \frac{\mu_0 I R^2}{2(z^2 + R^2)^{3/2}}$$



$$\int_{-\infty}^{\infty} B_z\,dz = \int_{-\infty}^{\infty} \frac{\mu_0 I R^2}{2(z^2 + R^2)^{3/2}}\,dz$$

Put $z = R\tan\theta$ $\quad dz = R\sec^2\theta\,d\theta$

$$\therefore \quad \int_{-\infty}^{\infty} B_z\,dz = \frac{\mu_0 I}{2}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta = \mu_0 I.$$

(d) $$B(z)_{\text{square}} < B(z)_{\text{circular coil}}$$

$\therefore$ $$\Im(L)_{\text{square}} < \Im(L)_{\text{circular coil}}$$

But by using arguments as in (b)

$$\Im(\infty)_{\text{square}} < \Im(\infty)_{\text{circular coil}}.$$

S15. When the field is off $\sum\tau = 0$

$$Mgl = W_{\text{coil}}\, l$$

$$500\, gl = W_{\text{coil}}\, l$$

$$W_{\text{coil}} = 500 \times 9.8\ \text{N}$$

When the magnetic field is switched on

$$Mgl + mgl = W_{\text{coil}}\, l + IBL \sin 90°\, l$$

$$mgl = BIL\, l$$

$$m = \frac{BIL}{g} = \frac{0.2 \times 4.9 \times 1 \times 10^{-2}}{0.2} = 10^{-3}\ \text{kg}.$$

$$= 1\ \text{g}.$$

S16. (a) Suppose the five wires A, B, C, D and E be perpendicular to the plane of paper at locations as shown in figure.

Thus, magnetic field induction due to five wires will be represented by various sides of a closed pentagon in one order, lying in the plane of paper, So, its value is zero.

(b) Since, the vector sum of magnetic field produced by each wire at O is equal to 0. Therefore, magnetic induction produced by one current carrying wire is equal in magnitude of resultant of four wires and opposite in direction.

Therefore, the field if current in one of the wires (say A) is switched off is $\frac{\mu_0}{2\pi}\frac{i}{R}$ perpendicular to AO towards left.

(c) If current in wire A is reversed, then total magnetic field induction at O = Magnetic field induction due to wire A + magnetic field induction due to wires B, C, D and E.

$$= \frac{\mu_0}{4\pi R}\frac{2I}{R}$$

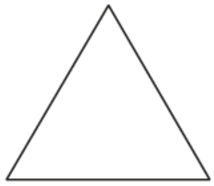
(acting perpendicular to AO towards left) + $\frac{\mu_0}{4\pi\varepsilon_0}\frac{2I}{R}$ (acting perpendicular AO towards left)

$$= \frac{\mu_0}{\pi}\frac{i}{R}$$ acting perpendicular AO towards left.

S17. We know that magnetic moment of the coils $m = nIA$.

Since, the same wire is used in three cases with same potentials, therefore, same current flows in three cases.

(a) For an equilateral triangle of side a.



$n = 3$

$n = 4$ as the total wire of length $= 12a$

Magnetic moment of the coils $m = nIA = 4I\left(\frac{\sqrt{3}}{4}a^2\right)$

$\therefore \quad m = Ia^2\sqrt{3}$

(b) For a square of sides a,

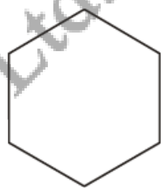
$n = 4$

$n = 3$ as the total wire of length $= 12a$

Magnetic moment of the coils $m = nIA = 3Ia^2$

(c) For a regular hexagon of sides a,

$n = 2$ as the total wire of length $= 12a$



$n = 2$

Magnetic moment of the coils $m = nIA = 2I\left(\frac{6\sqrt{3}}{4}a^2\right)$

$$m = 3\sqrt{3}a^2 I$$

m is in a geometric series.

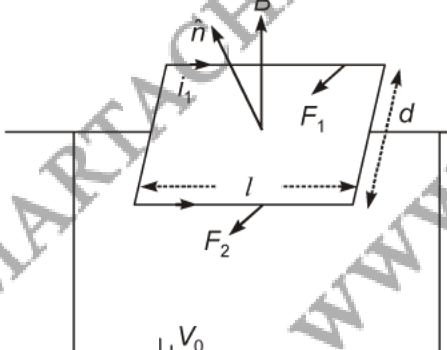
S18. The thicker wire has a resistance R, then the other wire has a resistance $2R$ as the wires are of the same material but with cross-sections differing by a factor 2.

Now, the force and hence, torque on first wire is given by

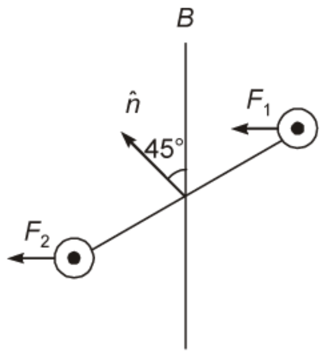
$$F_1 = i_1 lB = \frac{V_0}{R}lB, \quad \tau_1 = \frac{d}{2\sqrt{2}}F_1 = \frac{V_0 ldB}{2\sqrt{2}R}$$

Similarly, the force hence torque on other wire is given by

$$F_2 = i_2 lB = \frac{V_0}{2R}lB, \quad \tau_2 = \frac{d}{2\sqrt{2}}F_2 = \frac{V_0 ldB}{4\sqrt{2}R}$$



Front view



Side view

Net torque $\tau = \tau_1 - \tau_2$

$$\tau = \frac{1}{4\sqrt{2}}\frac{V_0 AB}{R}$$