Date: 20/10/2021

Q1. A proton has spin and magnetic moment just like an electron. Why then its effect is neglected in magnetism of materials?

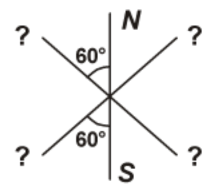
Q2. A permanent magnet in the shape of a thin cylinder of length 10 cm has $M = 10^6$ A/m. Calculate the magnetisation current I_M.

Q3. Explain quantitatively the order of magnitude difference between the diamagnetic susceptibility of N_2 ($\sim 5 \times 10^{-9}$) (at STP) and Cu ($\sim 10^{-5}$).

Q4. A ball of superconducting material is dipped in liquid nitrogen and placed near a bar magnet. (a) In which direction will it move? (b) What will be the direction of it's magnetic moment?

Q5. From molecular view point, discuss the temperature dependence of susceptibility for diamagnetism, paramagnetism and ferromagnetism.

Q6. Three identical bar magnets are riveted together at centre in the same plane as shown in figure. This system is placed at rest in a slowly varying magnetic field. It is found that the system of magnets does not show any motion. The north-south poles of one magnet is shown in the figure. Determine the poles of the remaining two.



Q7. Use (i) the Ampere's law for *H* and (ii) continuity of lines of *B*, to conclude that inside a bar magnet, (a) lines of *H* run from the *N* pole to *S* pole, while (b) lines of *B* must run from the *S*-pole to *N*-pole.

Q8. Verify the Gauss's law for magnetic field of a point dipole of dipole moment *m* at the origin for the surface which is a sphere of radius *R*.

Q9. A bar magnet of magnetic moment *M* and moment of inertia *I* (about centre, perpendicular to length) is cut into two equal pieces, perpendicular to length. Let *T* be the period of oscillations of the original magnet about an axis through the mid-point, perpendicular to length, in a magnetic field *B*. What would be the similar period T^2 for each piece?

Q10. Suppose we want to verify the analogy between electrostatic and magneto static by an explicit experiment. Consider the motion of (i) electric dipole *p* in an electrostatic field *E* and (ii) magnetic dipole *M* in a magnetic field *B*. Write down a set of conditions on *E*, *B*, *p*, *M* so that the two motions are verified to be identical. (Assume identical initial conditions.)

Q11. Verify the Ampere's law for magnetic field of a point dipole of dipole moment $m = m\hat{k}$. Take *C* as the closed curve running clockwise along

(a) the *z*-axis from $z = a > 0$ to $z = R$,

(b) along the quarter circle of radius *R* and centre at the origin, in the first quadrant of *x*-*z* plane,

(c) along the *x*-axis from $x = R$ to $x = a$, and

(d) along the quarter circle of radius *a* and centre at the origin in the first quadrant of *x*-*z* plane.

Q12. What are the dimensions of χ, the magnetic susceptibility? Consider an H-atom. Guess an expression for χ, upto a constant by constructing a quantity of dimensions of χ, out of parameters of the atom: e, m, v, R and μ_0. Here, m is the electronic mass, v is electronic velocity, R is Bohr radius. Estimate the number so obtained and compare with the value of $|\chi| \sim 10^{-5}$ for many solid materials.

Q13. Assume the dipole model for Earth's magnetic field B which is given by B_V = vertical component of magnetic field = $\frac{\mu_0}{4\pi}\frac{2m\cos\theta}{r^3}$, B_H = Horizontal component of magnetic field = $\frac{\mu_0}{4\pi}\frac{\sin\theta m}{r^3}$, $\theta = 90° -$ latitude as measured from magnetic equator.

Find loci of points for which (a) $|B|$ is minimum; (b) dip angle is zero; and (c) dip angle is $\pm 45°$.

Q14. Consider the plane S formed by the dipole axis and the axis of Earth.

Let P be point on the magnetic equator and in S. Let Q be the point of intersection of the geographical and magnetic equators. Obtain the declination and dip angles at P and Q.

Q15. There are two current carrying planar coils made each from identical wires of length L. C_1 is circular (radius R) and C_2 is square (side a). They are so constructed that they have same frequency of oscillation when they are placed in the same uniform B and carry the same current. Find a in terms of R.

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S1. The comparison between the spinning of a proton and an electron can be done comparing their magnetic dipole moment which can be given by

$$M = \frac{eh}{4\pi m} \quad \text{or} \quad M \propto \frac{1}{m} \qquad \left(\because \frac{eh}{4\pi} = \text{constant}\right)$$

$$\therefore \qquad \frac{M_p}{M_e} = \frac{m_e}{m_p}$$

$$= \frac{M_e}{1837\, M_e} \qquad (\because M_p = 1937\, m_e)$$

$$\Rightarrow \qquad \frac{M_p}{M_e} = \frac{1}{1837} << 1$$

$$\Rightarrow \qquad M_p << M_e$$

Thus, effect of magnetic moment of proton is neglected as compared to that of electron.

S2. Given, M (intensity of magnetisation) = 10^6 A/m.

l (length) = 10 cm = 10×10^{-2} m = 0.1 m

and I_M = Magnetisation current

We know that $M = \frac{I_M}{l}$

$\Rightarrow \qquad I_M = M \times l$

$= 10^6 \times 0.1 = 105\ A$

Note: Here, M = intensity of magnetisation as its unit is given as A/m.

S3. We know that,

$$\text{Density of nitrogen } \rho_{N_2} = \frac{28\text{ g}}{22.4\text{ L}} = \frac{28\text{ g}}{22400\text{ cc}}$$

Also, $$\text{Density of copper } \rho_{Cu} = \frac{8\text{ g}}{22.4\text{ L}} = \frac{8\text{ g}}{22400\text{ cc}}$$

Now, comparing both densities

$$\frac{\rho_{N_2}}{\rho_{Cu}} = \frac{28}{22400} \times \frac{1}{8} = 1.6 \times 10^{-4}$$

Also given, $$\frac{\chi_{N_2}}{\chi_{Cu}} = \frac{5 \times 10^{-9}}{10^{-5}} = 5 \times 10^{-4}$$

We know that, $\chi = \dfrac{\text{Magnetisation}\,(M)}{\text{Magnetic intensity}\,(H)}$

$$= \frac{\text{Magnetic moment}\,(M)/\text{Volume}\,(V)}{H}$$

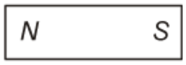
$$= \frac{M}{HV} = \frac{M}{H\,(mass/density)} = \frac{M\rho}{Hm}$$

$$\chi \propto \rho \qquad \left(\because \quad \frac{M}{Hm} = \text{constant}\right)$$

Hence, $\dfrac{\chi_{N_2}}{\chi_{Cu}} = \dfrac{\rho_{N_2}}{\rho_{Cu}} = 1.6 \times 10^{-4}$

Thus, we can say that magnitude difference or major difference between the diamagnetic susceptibility of N_2 and Cu.

S4. When a diamagnetic material is dipped in liquid nitrogen, it again behaves as a diamagnetic material. Thus, superconducting material will again behave as a diamagnetic material. When this diamagnetic material is placed near a bar magnet, it will be feebly magnetised opposite to the direction of magnetising field.



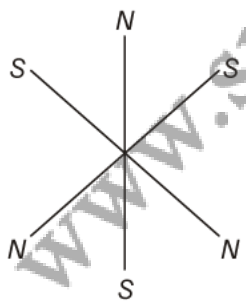
(a) Thus, it will be repelled.

(b) Also its direction of magnetic moment will be opposite to the direction of magnetic field of magnet.

S5. Diamagnetism is due to orbital motion of electrons developing magnetic moments opposite to applied field and hence is not much affected by temperature.

Paramagnetism and ferromagnetism is due to alignments of atomic magnetic moments in the direction of the applied field. As temperature increases, this alignment is disturbed and hence susceptibilities of both decrease as temperature increases.

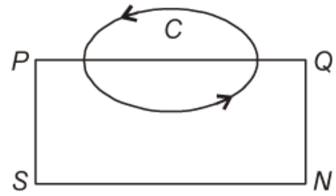
S6. The system will be in stable equilibrium if the net force on the system is zero and net torque on the system is also zero. This is possible only when the poles of the remaining two magnets are as given in the figure.



S7. Consider a magnetic field line of ***B*** through the bar magnet as given in the figure below:

The magnetic field line of *B* through the bar magnet must be a closed loop.

Let C be the amperian loop. Then,



$$\int_Q^P \boldsymbol{H}.\boldsymbol{dl} = \int_Q^P \frac{\boldsymbol{B}}{\mu_0}.\boldsymbol{dl}$$

We know that the angle between $\boldsymbol{B}$ and $\boldsymbol{dl}$ is less than 90° inside the bar magnet. So, it is positive.

i.e., $$\int_Q^P \boldsymbol{H}.\boldsymbol{dl} = \int_Q^P \frac{\boldsymbol{B}}{\mu_0}.\boldsymbol{dl} > 0$$

Hence, the lines of B must run from South pole (S) to North pole (N) inside the bar magnet. According to Ampere's law,

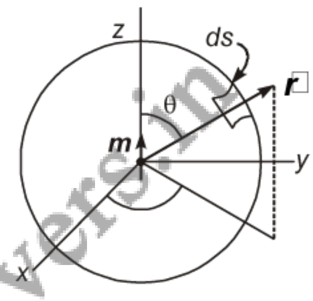
$\therefore$ $$\oint_{PQP} \boldsymbol{H}.\boldsymbol{dl} = 0$$

$\therefore$ $$\oint_{PQP} \boldsymbol{H}.\boldsymbol{dl} = \int_P^Q \boldsymbol{H}.\boldsymbol{dl} + \int_Q^P \boldsymbol{H}.\boldsymbol{dl} = 0$$

As $\int_Q^P \boldsymbol{H}.\boldsymbol{dl} > 0$, so, $\int_P^Q \boldsymbol{H}.\boldsymbol{dl} < 0$ [*i.e.*, negative]

It will be so if angle between $\boldsymbol{H}$ and $\boldsymbol{dl}$ is more than 90°, so that cos θ is negative. It means the line of $\boldsymbol{H}$ must run from N-pole to S-pole inside the bar magnet.

S8.



$$\boldsymbol{B} = \frac{\mu_0}{4\pi}\frac{2\boldsymbol{m}.\hat{\boldsymbol{r}}}{r^3}, \quad m = m\hat{\boldsymbol{k}}$$

$$d\boldsymbol{s} = \hat{\boldsymbol{r}} \ . \ \boldsymbol{r}^2 \sin\theta d\theta$$

$$0 \le \theta \le \pi, \qquad 0 \le \phi \le \pi$$

$$\oint \boldsymbol{B}.ds = \frac{\mu_0 m}{4\pi}\int \frac{2\cos\theta}{r^3} r^2 \sin\theta d\theta$$

$$= 0 \text{ [due to } \theta \text{ integral]}.$$

S9. Given, I = moment of inertia of the bar magnet

m = mass of bar magnet

l = length of magnet about an any passing through its centre and perpendicular to its length

M = magnetic moment of the magnet

B = uniform magnetic field in which magnet is oscillating, we get time period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

Here, $$I = \frac{ml^2}{12}$$

When magnet is cut into two equal pieces, perpendicular to length, then moment of inertia each piece of magnet about an axis perpendicular to length passing through its centre is

$$I' = \frac{m}{2}\frac{(l/2)^2}{12} = \frac{ml^2}{12} \times \frac{1}{8} = \frac{I}{8}$$

Magnetic dipole moment $M' = M/2$

Its time period of oscillation is

$$T' = 2\pi\sqrt{\frac{I'}{M'B}} = 2\pi\sqrt{\frac{I/8}{(M/2)B}} = \frac{2\pi}{2}\sqrt{\frac{I}{MB}}$$

$$T' = \frac{T}{2}.$$

S10. Now, suppose that the angle between $\boldsymbol{M}$ and B is θ.

Torque on magnetic dipole moment $\boldsymbol{M}$ in magnetic field $\boldsymbol{B}$,

$$\tau' = MB \sin\theta$$

Two motions will be identical, if

$$pE \sin\theta = MB \sin\theta$$

$\Rightarrow$ $$pE = MB \quad \text{... (i)}$$

But, $$E = cB$$

$\therefore$ Putting this value in Eq. (i),

$$pcB = MB$$

$\Rightarrow$ $$p = \frac{M}{c}.$$

S11. From P to Q, every point on the z-axis lies at the axial line of magnetic dipole of moment $\boldsymbol{M}$. Magnetic field induction at a point distance z from the magnetic dipole of moment is

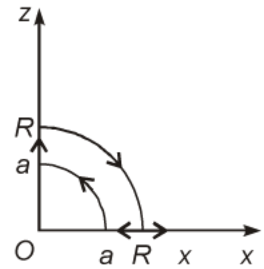
$$|\boldsymbol{B}| = \frac{\mu_0}{4\pi}\frac{2|\boldsymbol{M}|}{z^3} = \frac{\mu_0 M}{2\pi z^3}$$

(a) Along z-axis

$$\boldsymbol{B} = \frac{\mu_0}{4\pi}\frac{2\boldsymbol{m}}{r^3}$$

$$\int_a^R \boldsymbol{B}.d\boldsymbol{l} = \frac{\mu_0}{4\pi}2m\int_a^R \frac{dz}{z^3} = \frac{\mu_0 m}{2\pi}\left(-\frac{1}{2}\right)\left(\frac{1}{a^2} - \frac{1}{R^2}\right).$$

(b) Along the quarter circle of radius R



$$B_0 = \frac{\mu_0}{4\pi}\frac{-\boldsymbol{m}.\hat{\theta}}{R^3} = \frac{-\mu_0}{4\pi}\frac{m}{R^3}(-\sin\theta)$$

$$\boldsymbol{B}\,.\,d\boldsymbol{l} = \frac{\mu_0 m}{4\pi R^2}\sin\theta d\theta$$

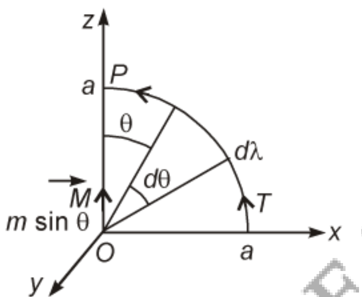
$$\int_0^{\frac{\pi}{2}}\vec{\boldsymbol{B}}.\overrightarrow{d\boldsymbol{l}} = \frac{\mu_0 m}{4\pi R^2}.$$

(c) Along x-axis

$$\boldsymbol{B} = \frac{\mu_0}{4\pi}\left(\frac{-\boldsymbol{m}}{x^3}\right)$$

$$\int\boldsymbol{B}.d\boldsymbol{l} = 0.$$

(d) Along the quarter circle TP of radius a. Consider the figure given below:



m case (ii), we get line integral of $\boldsymbol{B}$ along the quarter circle TP of radius a is circular TP

$$\int\boldsymbol{B}.d\boldsymbol{l} = \int_{\pi/2}^{0}\frac{\mu_0}{4\pi}\frac{\boldsymbol{M}\sin\theta}{a^3}\,ad\theta,$$

$$= \frac{\mu_0}{4\pi}\frac{M}{a^2}\int_{\pi/2}^{0}\sin\theta\,d\theta = \frac{\mu_0}{4\pi}\frac{M}{a^2}[-\cos\theta]_{\pi/2}^{0}$$

$$= \frac{-\mu_0}{4\pi}\frac{M}{a^2}$$

$$\oint_{PQST}\boldsymbol{B}.d\boldsymbol{l} = \int_P^Q\boldsymbol{B}.d\boldsymbol{l} + \int_Q^S\boldsymbol{B}.d\boldsymbol{l} + \int_S^T\boldsymbol{B}.d\boldsymbol{l} + \int_T^P\boldsymbol{B}.d\boldsymbol{l}$$

$$= \frac{\mu_0 M}{4}\left[\frac{1}{a^2} - \frac{1}{R^2}\right] + \frac{\mu_0}{4\pi}\frac{M}{R^2} + 0 + \left(-\frac{\mu_0}{4\pi}\frac{M}{a^2}\right) = 0.$$

S12. As $\boldsymbol{I}$ (intensity or magnetisation) and H both have same units and dimensions, hence, χ has no dimensions. Here, in this question, χ is to be related with e, m, v, R and μ_0. We know that dimensions of μ_0 = $[\text{ML}\theta^{-2}]$

From Biot-Savart's law

$$dB = \frac{\mu_0}{4\pi}\frac{Idl \sin\theta}{r^2}$$

$\Rightarrow$ $$\mu_0 = \frac{4\pi r^2 dB}{Idl \sin\theta} = \frac{4\pi r^2}{Idl \sin\theta} \times \frac{t}{qv \sin\theta} \qquad \left[\because \; dB = \frac{F}{qv \sin\theta}\right]$$

$\therefore$ Dimensions of $\mu_0 = \dfrac{L^2 \times (MLT^{-2})}{(QT^{-1})(L) \times 1 \times (Q)(LT^{-1}) \times (1)} = [MLQ^{-2}]$

where Q is the dimension of charge.

As χ is dimensionless, it should have no involvement of charge Q in its dimensional formula. It will be so if μ_0 and e together should have the value $\mu_0 e^2$, as e has the dimensions of charge.

Let $$\chi = \mu_0 e^2 m^a v^b R^c \qquad \text{... (i)}$$

where, a, b, c are the power of m, v and R respectively, such that relation (i) is satisfied.

Dimensional equation of (i) is

$$[M^0L^0T^0Q^0] = [MLQ^{-2}] \times [Q^2]\,[M^a] \times (LT^{-1})^b \times [L]^c$$
$$= [M^{1+a} + L^{1+b+c}T^{-b}Q^0]$$

Equating the powers of M, L and T, we get

$$0 = 1 + a \;\Rightarrow\; a = -1, \quad 0 = 1 + b + c$$
$$0 = -b \;\Rightarrow\; b = 0, \quad 0 = 1 + 0 + c \;\text{ or }\; c = -1$$

Putting values in Eq. (i), we get

$$c = \mu_0 e^2 m^{-1} v^2 R^{-1} = \frac{\mu_0 e^2}{mR}$$

Here, $\mu_0 = 4\pi \times 10^{-7}$ Tm A^{-1}

$e = 1.6 \times 10^{-19}$ C

$m = 9.1 \times 10^{-31}$ kg, $R = 10^{-10}$ m

$$\chi = \frac{(4\pi \times 10^{-7}) \times (1.6 \times 10^{-19})^2}{(9.1 \times 10^{-31}) \times 10^{-10}} \approx 10^{-4}$$

$\therefore$ $$\frac{\chi}{\chi_{\text{(given solid)}}} = \frac{10^{-4}}{10^{-5}} = 10.$$

S13. (a) $$|\boldsymbol{B}| = \frac{\mu_0}{4\pi}\frac{m}{R^3}(4\cos^2\theta + \sin^2\theta)^{1/2}$$

$$\frac{|\boldsymbol{B}|^2}{\left(\dfrac{\mu_0}{4\pi R^3}\right)^2 m^2} = 3\cos^2\theta + 1, \quad \text{minimum at } \theta = \frac{\pi}{2}.$$

$|\boldsymbol{B}|$ is minimum at magnetic equator.

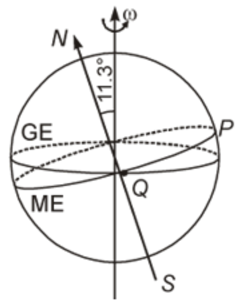
(b) tan (dip angle) = $\dfrac{B_V}{B_H}$ = 2 cot at $\theta = \dfrac{\pi}{2}$ dip angle vanishes. Magnetic equator is again the locus.

(c) Dip angle is ± 45° when $\left|\frac{B_V}{B_H}\right| = 1$

$2 \cot \theta = 1$

$\theta = \tan^{-1} 2$ is the locus.

S14. Refer to the adjacent figure.



(a) P is in S (needle will point both north)

Declination = 0

P is also on magnetic equator.

$\therefore$ dip = 0

(b) Q is on magnetic equator.

$\therefore$ dip = 0

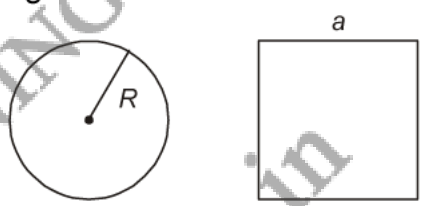
but declination = 11.3°.

S15. C_1 = circular coil of radius R, length L, number of turns per unit length

$$n_1 = \frac{L}{2\pi R}$$

C_2 = square of side a and perimeter L, number of turns per unit length

$$n_2 = \frac{L}{4a}$$



Magnetic moment of C_1

$\Rightarrow$ $$m_1 = n_1 I A_1$$

Magnetic moment of C_2

$\Rightarrow$ $$m_2 = n_2 I A_2$$

$$m_1 = \frac{L \cdot I \cdot \pi R^2}{2\pi R}$$

$$m_2 = \frac{L}{4a} \cdot I \cdot a^2$$

$$m_1 = \frac{LIR}{2} \quad \text{... (i)}$$

$$m_2 = \frac{LIa}{4} \quad \text{... (ii)}$$

Moment of inertia of $C_1 = I_1 = \frac{MR^2}{2}$ ... (iiii)

Moment of inertia of $C_2 = I_2 = \frac{Ma^2}{12}$ ... (iv)

Frequency of $C_1 = f_1 = 2\pi\sqrt{\frac{I_1}{m_1 B}}$

Frequency of $C_2 = f_2 = 2\pi\sqrt{\frac{I_2}{m_2 B}}$

According to question $f_1 = f_2$

$$2\pi\sqrt{\frac{I_1}{m_1 B}} = 2\pi\sqrt{\frac{I_2}{m_2 B}}$$

$$\frac{I_1}{m_1} = \frac{I_2}{m_2} \quad \text{or} \quad \frac{m_2}{m_1} = \frac{I_2}{I_1}$$

Putting the values by Eqs. ((i), (ii), (iii) and (iv)

$$\frac{LIa \cdot 2}{4 \times LIR} = \frac{Ma^2 \cdot 2}{12 \cdot MR^2}$$

$$\frac{a}{2R} = \frac{a^2}{6R^2}$$

$$3R = a$$

Thus, the value of a is $3R$.