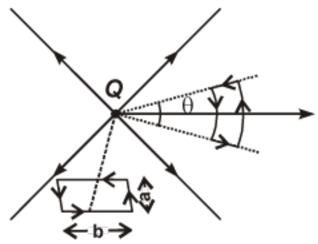
Date: 20/10/2021

Q1. Can the potential function have a maximum or minimum in free space?

Q2. A test charge q is made to move in the electric field of a point charge Q along two different closed paths (shown in figure). First path has sections along and perpendicular to lines of electric field. Second path is a rectangular loop of the same area as the first loop. How does the work done compare in the two cases?



Q3. Consider two conducting spheres of radii R_1 and R_2 with $R_1 > R_2$. If the two are at the same potential, the larger sphere has more charge than the smaller sphere. State whether the charge density of the smaller sphere is more or less than that of the larger one.

Q4. Do free electrons travel to region of higher potential or lower potential?

Q5. Can there be a potential difference between two adjacent conductors carrying the same charge?

Q6. Prove that a closed equipotential surface with no charge within itself must enclose an equipotential volume.

Q7. Prove that, if an insulated, uncharged conductor is placed near a charged conductor and no other conductors are present, the uncharged body must be intermediate in potential between that of the charged body and that of infinity.

Q8. Calculate potential energy of a point charge $-q$ placed along the axis due to a charge $+Q$ uniformly distributed along a ring of radius R. Sketch P.E. as a function of axial distance z from the centre of the ring. Looking at graph, can you see what would happen if $-q$ is displaced slightly from the centre of the ring (along the axis)?

Q9. Calculate potential on the axis of a ring due to charge Q uniformly distributed along the ring of radius R.

Q10. A capacitor has some dielectric between its plates, and the capacitor is connected to a D.C. source. The battery is now disconnected and then the dielectric is removed. State whether the capacitance, the energy stored in it, electric field, charge stored and the voltage will increase, decrease or remain constant.

Q11. Two point charges of magnitude $+q$ and $-q$ are placed at $(-d/2, 0,0)$ and $(d/2, 0,0)$, respectively. Find the equation of the equipoential surface where the potential is zero.

Q12. Find the equation of the equipotentials for an infinite cylinder of radius r_0, carrying charge of linear density λ.

Q13. A parallel plate capacitor is filled by a dielectric whose relative permittivity varies with the applied voltage (U) as $\varepsilon = \alpha U$ where $\alpha = 2V^{-1}$. A similar capacitor with no dielectric is charged to $U_0 = 78V$. It is then connected to the uncharged capacitor with the dielectric. Find the final voltage on the capacitors.

Q14. **(a)** In a quark model of elementary particles, a neutron is made of one up quarks [charge (2/3) e] and two down quarks [charges – (1/3) e]. Assume that they have a triangle configuration with side length of the order of 10^{-15} m. Calculate electrostatic potential energy of neutron and compare it with its mass 939 MeV.

(b) Repeat above exercise for a proton which is made of two up and one down quark.

Q15. Two charges $-q$ each are separated by distance $2d$. A third charge $+q$ is kept at mid point O. Find potential energy of $+q$ as a function of small distance x from O due to $-q$ charges. Sketch P.E. v/s x and convince yourself that the charge at O is in an unstable equilibrium.

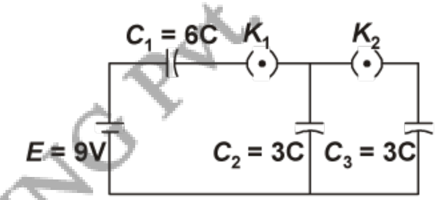
Q16. Two charges q_1 and q_2 are placed at (0, 0, d) and (0, 0, $-d$) respectively. Find locus of points where the potential a zero.

Q17. Calculate potential on the axis of a disc of radius R due to a charge Q uniformly distributed on its surface.

Q18. A capacitor is made of two circular plates of radius R each, separated by a distance $d \ll R$. The capacitor is connected to a constant voltage. A thin conducting disc of radius $r \ll R$ and thickness $t \ll r$ is placed at a centre of the bottom plate. Find the minimum voltage required to lift the disc if the mass of the disc is m.

Q19. In the circuit shown in figure, initially K_1 is closed and K_2 is open. What are the charges on each capacitors.

Then K_1 was opened and K_2 was closed (order is important), What will be the charge on each capacitor now? [$C = 1\ \mu$F]



Q20. Two metal spheres, one of radius R and the other of radius $2R$, both have same surface charge density σ. They are brought in contact and separated. What will be new surface charge densities on them?

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S1. No. The absence of atmosphere around conductor prevents the phenomenon of electric discharge or potential leakage and hence, potential function do not have a maximum or minimum in free space.

S2. As electric field is conservative, work done will be zero in both the cases.

S3. Since, the two spheres are at the same potential, therefore

$$\frac{kq_1}{R_1} = \frac{kq_2}{R_2} \quad \Rightarrow \quad \frac{kq_1R_1}{4\pi R_1^2} = \frac{kq_2R_2}{4\pi R_2^2}$$

or $$\sigma_1 R_1 = \sigma_2 R_2 \quad \Rightarrow \quad \frac{\sigma_1}{\sigma_2} = \frac{R_1}{R_2}$$

$$R_2 > R_1$$

This imply that $\sigma_1 > \sigma_2$.

The charge density of the smaller sphere is more than that of the larger one.

S4. The free electrons experiences electrostatic force in a direction opposite to the direction of electric field being is of negative charge. The electric field always directed from higher potential to lower potential.

Therefore, electrostatic force and hence direction of travel of electrons is from lower potential to region of higher potential.

S5. Yes, if the sizes are different.

S6. Let's assume contradicting statement that the potential is not same inside the closed equipotential surface. Let the potential just inside the surface is different to that of the surface causing in a potential gradient $\left(\frac{dV}{dr}\right)$. Consequently electric field comes into existence, which is given by as $E = -\frac{dV}{dr}$.

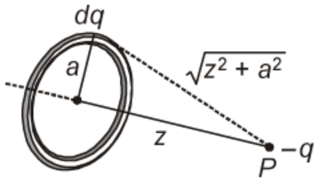
Consequently field lines pointing inwards or outwards from the surface. These lines cannot be again on the surface, as the surface is equipotential. It is possible only when the other end of the field lines are originated from the charges inside.

This contradict the original assumption. Hence, the entire volume inside must be equipotential.

S7. Let us take point P to be at a distance x from the centre of the ring, as shown in figure. The charge element dq is at a distance x from point P. Therefore, V can be written as

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{z^2 + a^2}}$$

where, $k = \dfrac{1}{4\pi\varepsilon_0}$, since each element dq is at the same distance from point P, so we have net potential



$$V = \frac{k_e}{\sqrt{z^2 + a^2}} \int dq = \frac{k_e Q}{\sqrt{z^2 + a^2}}$$

Considering $-q$ charge at P, the potential energy is given by

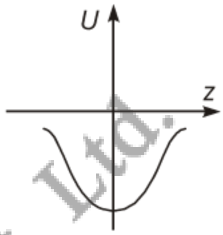
$$U = W = q \times \text{potential difference}$$

$$U = \frac{k_e Q(q)}{\sqrt{z^2 + a^2}}$$

or

$$U = \frac{1}{4\pi\varepsilon_0} \frac{-Qq}{\sqrt{z^2 + a^2}}$$

$$= \frac{1}{4\pi\varepsilon_0 a} \frac{-Qq}{\sqrt{1 + \left(\frac{z}{a}\right)^2}}$$



This is the required expression.

The variation of potential energy with z is shown in the figure. the charge $-q$ displaced would perform oscillations.

Nothing can be concluded just by looking at the graph.

S8. Let us take point P to be at a distance x from the centre of the ring, as shown in figure. The charge element dq is at a distance x from point P. Therefore, V can be written as

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{z^2 + R^2}}$$

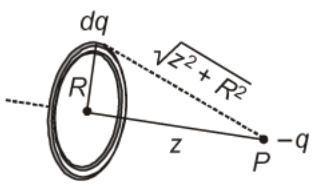
where, $k = \dfrac{1}{4\pi\varepsilon_0}$, since each element dq is at the same distance from point P, so we have net potential

$$V = \frac{k_e}{\sqrt{z^2 + R^2}} \int dq = \frac{k_e Q}{\sqrt{z^2 + R^2}}$$

Considering $-q$ charge at P, the potential energy is given by

$$U = W - q \times \text{potential difference}$$

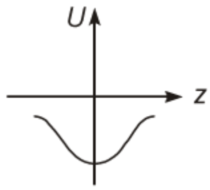
$$U = \frac{k_e Q(-q)}{\sqrt{z^2 + R^2}}$$



or

$$U = \frac{1}{4\pi\varepsilon_0}\frac{-Qq}{\sqrt{z^2+R^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{-Qq}{\sqrt{1+\left(\frac{z}{R}\right)^2}}$$



This is the required expression.

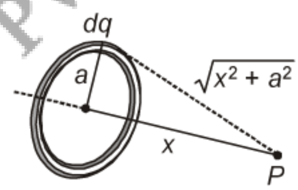
The variation of potential energy with z is shown in the figure. The charge $-q$ displaced would perform oscillations.

Nathing can be concluded just by looking at the graph

S9. Let us take point P to be at a distance x from the centre of the ring, as shown in figure. The charge element dq is at a distance x from point P. Therefore, V can be written as

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{x^2+a^2}}$$

where, $k = \frac{1}{4\pi\varepsilon_0}$, since each element dq is at the same distance from point P, so we have net potential



$$V = \frac{k_e}{\sqrt{x^2+a^2}}\int dq = \frac{k_e Q}{\sqrt{x^2+a^2}}$$

The net electric potential $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x^2+a^2}}$.

S10. The capacitance of the parallel plate capacitor, filled with dielectric medium of dielectric constant K is given by

$$C = \frac{K\varepsilon_0 A}{d}, \text{ where signs are as usual.}$$

The capacitance of the parallel plate capacitor decreases with the removal of dielectric medium as for air or vacuum $K = 1$.

After disconnection from battery charge stored will remain the same due to conservation of charge.

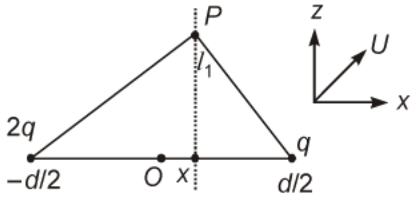
The energy stored in an isolated charge capacitor = $q^2/2C$; as q is constant, energy stored $\propto 1/C$ and C decreases with the removal of dielectric medium, therefore energy stored increases. Since q is constant and $V = q/C$ and C decreases which in turn increases V and therefore E increases as $E = V/d$.

Note: One of the very imortant questions with the competitive point of view.

S11. Let the plane be at a distance x from the origin. The potential at the point P is

$$\frac{1}{4\pi\varepsilon_0}\frac{q}{[(x+d/2)^2+h^2]^{1/2}} - \frac{1}{4\pi\varepsilon_0}\frac{q}{[(x-d/2)^2+h^2]^{1/2}}$$

If this is to be zero.



$$\frac{1}{[(x+d/2)^2+h^2]^{1/2}} = \frac{1}{[(x-d/2)^2+h^2]^{1/2}}$$

Or, $$(x - d/2)^2 + h^2 = (x + d/2)^2 + h^2$$

$\Rightarrow$ $$x^2 - dx + d^2/4 = x^2 + dx + d^2/4$$

Or, $$2dx = 0$$

$$x = 0$$

The equation is that of a plane $x = 0$.

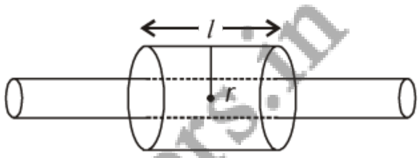
S12. To find the potential at distance r from the line consider the electric field. We note that from symmetry the field lines must be radially outward. Draw a cylindrical Gaussian surface of radius r and length l. Then

$$\oint \mathbf{E}.d\mathbf{S} = \frac{1}{\varepsilon_0}\lambda l$$ [$q = \lambda l$ since λ is linear charge density]

Or $$E_r 2\pi r l = \frac{1}{\varepsilon_0}\lambda l$$

$\Rightarrow$ $$E_r = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Hence, if r_0 is the radius,



$$V(r) - V(r) = -\int_{r_0}^{r} \mathbf{E}.dr = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_0}{r}$$

For a given V,

$$\ln\frac{r}{r_0} = -\frac{2\pi\varepsilon_0}{\lambda}[V(r) - V(r_0)]$$

$\Rightarrow$ $$r = r_0 e^{-2\pi\varepsilon_0 V(r_0)/\lambda} . e^{+2\pi\varepsilon_0 V(r)/\lambda}$$

The equipotential surfaces are cylinders of radius

$$r = r_0 e^{-2\pi\varepsilon_0[V(r) - V(r_0)]/\lambda}.$$

S13. Let the final voltage be U: If C is the capacitance of the capacitor without the dielectric, then the charge on the capacitor is

$$Q_1 = CU$$

The capacitor with the dielectric has a capacitance εC. Hence the charge on the capacitor is

$$Q_2 = \varepsilon U = \alpha CU^2$$ [ε = Dielectric constant of the medium]

The initial charge on the capacitor that was charged is

$$Q_0 = CU_0$$

From the conservation of charges,

$$Q_0 = Q_1 + Q_2$$

Or, $$CU_0 = CU + \alpha CU^2$$

$\Rightarrow$ $$\alpha U^2 + U - u_0 = 0$$

$\therefore$ $$U = \frac{-1 \pm \sqrt{1 + 4\alpha U_0}}{2\alpha}$$

$$= \frac{-1 \pm \sqrt{1 + 624}}{4}$$

$$= \frac{-1 \pm \sqrt{625}}{4} \text{ volts}$$

As U is positive

$$U = \frac{\sqrt{625} - 1}{4} = \frac{24}{4} = 6\text{V}.$$

S14.

$$U = \frac{1}{4\pi\varepsilon_0}\left\{\frac{q_d q_d}{r} - \frac{q_u q_d}{r} - \frac{q_u q_d}{r}\right\}$$

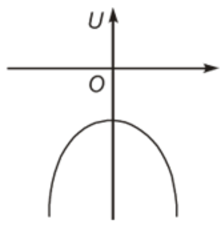
$$= \frac{9 \times 10^9}{10^{-15}} \ \{(1.6 \times 10^{-19}(1/3)^2 - (2/3)(1/3) - (2/3)(1/3)\}$$

$$= 2.304 \times 10^{-13}\left\{\frac{1}{9} - \frac{4}{9}\right\} = -7.68 \times 10^{-14} \text{J}$$

$$= 4.8 \times 10^5 \text{ eV} = 0.48 \text{ MeV} = 5.11 \times 10^{-4}\,(m_n c^2)$$

S15.

$$U = \frac{1}{4\pi\varepsilon_0}\left\{\frac{-q^2}{(d - x)} + \frac{-q^2}{(d - x)}\right\}$$



$$U = \frac{-q^2}{4\pi\varepsilon_0}\frac{2d}{(d^2 - x^2)}$$

$$\frac{dU}{dx} = \frac{-q^2 . 2d}{4\pi\varepsilon_0} \cdot \frac{2x}{(d^2 - x^2)^2}$$

$$U_0 = \frac{2q^2}{4\pi\varepsilon_0 d}$$

$$\frac{dU}{dx} = 0 \quad \text{at} \quad x = 0$$

$x = 0$ is an equilibrium point.

$$\frac{d^2U}{dx^2} = \left(\frac{-2dq^2}{4\pi\varepsilon_0}\right)\left[\frac{2}{(d^2-x^2)^2} - \frac{8x^2}{(d^2-x^2)^3}\right]$$

$$= \left(\frac{-2dq^2}{4\pi\varepsilon_0}\right)\frac{1}{(d^2-x^2)^3}[2(d^2-x^2)^2 - 8x^2]$$

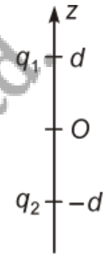
At $x = 0$

$$\frac{d^2U}{dx^2} = \left(\frac{-2dq^2}{4\pi\varepsilon_0}\right)\left(\frac{1}{d^6}\right)(2d^2), \quad \text{which is} < 0.$$

Hence, unstable equilibrium.

S16.

$$\frac{q_1}{\sqrt{x^2+y^2+(z-d)^2}} + \frac{q_2}{\sqrt{x^2+y^2+(z+d)^2}} = 0$$



$\therefore$

$$\frac{q_1}{\sqrt{x^2+y^2+(z-d)^2}} = \frac{-q_2}{\sqrt{x^2+y^2+(z+d)^2}}$$

Thus, to have total potential zero, q_1 and q_2 must have opposite signs. Squaring and simplifying, we get.

$$x^2+y^2+z^2+\left[\frac{(q_1/q_2)^2+1}{(q_1/q_2)^2-1}\right](2zd)+d^2 = 0$$

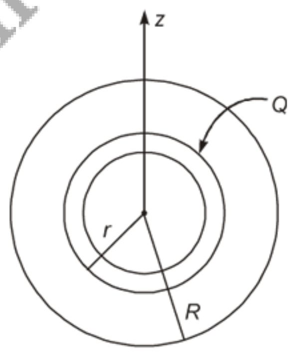
This is the equation of a sphere with centre at $\left(0, 0, -2d\left[\frac{q_1^2+q_1^2}{q_1^2+q_1^2}\right]\right)$.

Note: If $q_1 = -q_2 \Rightarrow$ Then $z = 0$, which is a plane through mid-point.

S17.

$$\sigma = \frac{\text{Charge}}{\text{Area}} = \frac{Q}{\pi R^2}$$

$$dU = \frac{1}{4\pi\varepsilon_0}\frac{\sigma.2\pi r dr}{\sqrt{r^2+z^2}}$$



$\therefore$

$$U = \frac{\pi\sigma}{4\pi\varepsilon_0}\int_0^R \frac{2rdr}{\sqrt{r^2+z^2}}$$

$$= \frac{\pi\sigma}{4\pi\varepsilon_0}\int_0^R \frac{2rdr}{\sqrt{r^2+z^2}}$$

$$= \frac{2Q}{4\pi\varepsilon_0 R^2}[\sqrt{R^2+z^2} - z]$$

S18. When the disc is in touch with the bottom plate, the entire plate is a equipotential. A change q' is transferred to the disc. The electric field on the disc is

$$= \frac{V}{d}$$

$$\therefore \qquad q' = -\varepsilon_0 \frac{V}{d} \pi r^2$$

The force acting on the disc is

$$-\frac{V}{d} \times q' = \varepsilon_0 \frac{V^2}{d^2} \pi r^2$$

If the disc is to be lifted, then

$$\varepsilon_0 \frac{V^2}{d^2} \pi r^2 = mg$$

$$\Rightarrow \qquad V = \sqrt{\frac{mgd^2}{\pi \varepsilon_0 r^2}}.$$

S19. Initially: $V \propto \frac{1}{C}$ and $V_1 + V_2 = E$ [E = e.m.f. of the battery]

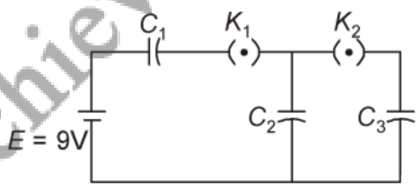
$\Rightarrow$ $V_1 = 3\,V$ and $V_2 = 6\,\text{V}$ [V_1 = potential across C_1 and V_2 = potential across C_2]

$\therefore$ $Q_1 = C_l V_l = 6\text{C} \times 3 = 18\,\mu\text{C}$

$Q_2 = 9\,\mu\text{C}$ and $Q_3 = 0$

Later: $Q_2 = Q_2' + Q_3$

with $C_2 V + C_3 V = Q_2$



$\Rightarrow$ $V = \frac{Q_2}{C_2 + C_3} = (3/2)\,V$

$Q_2' = (9/2)\,\mu\text{C}$ and $Q_3' = (9/2)\,\mu\text{C}$.

S20. Before contact:

$Q_1 = \sigma \,.\, 4\pi R^2$

$Q_2 = \sigma \,.\, 4\pi\,(2R^2) = 4\,(\sigma \,.\, 4\pi R^2) = 4Q_1$

After contact:

$Q_1' + Q_2' = Q_1 + Q_2 = 5Q_1$ [$\because$ Charge remains conserved]

$= 5\,(\sigma \,.\, 4\pi R^2)$

They will be at equal potentials:

$$\frac{Q_1'}{R} = \frac{Q_2'}{2R}$$

$\therefore$ $$Q_2' = 2Q'$$

$\therefore$ $$3Q_1' = 5(\sigma . 4\pi R^2)$$

$\therefore$ $$Q_1' = \frac{5}{3}(\sigma . 4\pi R^2) \quad \text{and} \quad Q_2' = \frac{10}{3}(\sigma . 4\pi R^2)$$

$\therefore$ $$\sigma_1 = 5/3\sigma \quad \text{and} \quad \therefore \quad \sigma_2 = \frac{5}{6}\sigma .$$