Date: 21/10/2021

Q1. Professor C.V Raman surprised his students by suspending freely a tiny light ball in a transparent vacuum chamber by shining a laser beam on it. Which property of EM waves was he exhibiting? Give one more example of this property.

Q2. Why is the orientation of the portable radio with respect to broadcasting station important?

Q3. The charge on a parallel plate capacitor varies as $q = q_0 \cos 2\pi\nu t$. The plates are very large and close together (area = A, separation = d). Neglecting the edge effects, find the displacement current through the capacitor?

Q4. A variable frequency a.c source is connected to a capacitor. How will the displacement current change with decrease in frequency?

Q5. The magnetic field of a beam emerging from a filter facing a floodlight is given by

$$B_0 = 12 \times 10^{-8} \sin (1.20 \times 10^7 z - 3.60 \times 10^{15} t)\ T.$$

What is the average intensity of the beam?

Q6. Pointing vectors S is defined as a vector whose magnitude is equal to the wave intensity and whose direction is along the direction of wave propagation. Mathematically, it is given by $S = \frac{1}{\mu_0} E \times B$. Show the nature of S vs t graph.

Q7. Why does microwave oven heats up a food item containing water molecules most efficiently?

Q8. Show that the magnetic field B at a point in between the plates of a parallel-plate capacitor during charging is $\frac{\varepsilon_0 \mu_r}{2} \frac{dE}{dt}$ (symbols having usual meaning).

Q9. What happens to the intensity of light from a bulb if the distance from the bulb is doubled? As a laser beam travels across the length of a room, its intensity essentially remains constant.

What geometrical characteristic of LASER beam is responsible for the constant intensity which is missing in the case of light from the bulb?

Q10. Even though an electric field E exerts a force qE on a charged particle yet the electric field of an EM wave does not contribute to the radiation pressure (but transfers energy). Explain.

Q11. Electromagnetic waves with wavelength

(i) λ_1 is used in satellite communication.

(ii) λ_2 is used to kill germs in water purifies.

(iii) λ_3 is used to detect leakage of oil in underground pipelines.

(iv) λ_4 is used to improve visibility in runways during fog and mist conditions.

(a) Identify and name the part of electromagnetic spectrum to which these radiations belong.

(b) Arrange these wavelengths in ascending order of their magnitude.

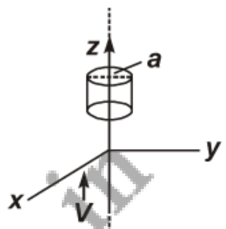
(c) Write one more application of each.

Q12. Show that average value of radiant flux density 'S' over a single period 'T' is given by $S = \frac{1}{2c\mu_0} E_0^2$.

Q13. Show that the radiation pressure exerted by an EM wave of intensity I on a surface kept in vacuum is I/c.

Q14. You are given a 2μF parallel plate capacitor. How would you establish an instantaneous displacement current of 1mA in the space between its plates?

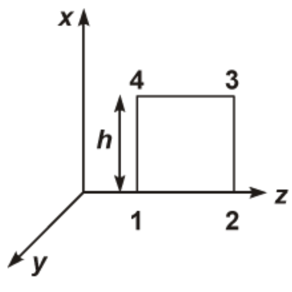
Q15. An infinitely long thin wire carrying a uniform linear static charge density λ is placed along the z-axis (see figure). The wire is set into motion along its length with a uniform velocity $v = v\hat{k}_z$. Calculate the pointing vector $S = \frac{1}{\mu_0}(E \times B)$.



Q16. A plane EM wave travelling in vacuum along z direction is given by $E = E_0 \sin(kz - \omega t)\,\hat{i}$ and $B = B_0 \sin(kz - \omega t)\,\hat{j}$.

(a) Evaluate $\oint E.dl$ over the rectangular loop 1234, shown in figure.

(b) Evaluate $\int B.ds$ over the surface bounded by loop 1234, shown in figure.



(c) Use equation $\oint E.dl = \frac{-d\phi_0}{dt}$ to prove $\frac{E_0}{B_0} = c$.

(d) By using similar process and the equation $\oint B.dl = \mu_0 I + \varepsilon_0 \frac{d\phi_0}{dt}$ prove that $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$.

Q17. Sea water at frequency $\nu = 4 \times 10^8$ Hz has permittivity $\varepsilon = 80\,\varepsilon_0$, permeability $\mu \approx \mu_0$ and resistivity $\rho = 0.25\ \Omega$–m. Imagine a parallel plate capacitor immersed in sea water and driven by an alternating voltage source $V(t) = V_0 \sin(2\pi \nu t)$. What fraction of the conduction current density is the displacement current density?

Q18. A plane *EM* wave travelling along *z* direction is described by $E = E_0 \sin(kz - \omega t)\hat{i}$ and $B = B_0 \sin(kz - \omega t)\hat{j}$. Show that

(a) The average energy density of the wave is given by

$$u_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\frac{B_0^2}{\mu_0}.$$

(b) The time averaged intensity of the wave is given by

$$I_{av} = \frac{1}{2}c\varepsilon_0 E_0^2.$$

Q19. A long straight cable of length *l* is placed symmetrically along *z*-axis and has radius $a (<<l)$. The cable consists of a thin wire and a co-axial conducting tube. An alternating current $I(t) = I_0 \sin(2\pi \nu t)$ flows down the central thin wire and returns along the co-axial conducting tube. The induced electric field at a distance *s* from the wire inside the cable is $E(s, t) = \mu_0 I_0 \nu \cos(2\pi \nu t) \ln\left(\frac{s}{a}\right)\hat{k}$.

(a) Calculate the displacement current density inside the cable.

(b) Integrate the displacement current density across the cross-section of the cable to find the total displacement current I^d.

(c) Compare the conduction current I_0 with the displacement current I_0^d.

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S1. An electromagnetic wave carries energy and momentum like other waves.

Since, it carries momentum, an electromagnetic wave also exerts pressure called radiation pressure. This property of electromagnetic waves helped professor CV Raman surprised his students by suspending freely a tiny light ball in a transparent vacuum chamber by shining a laser beam on it. The tails of the comets are also due to radiation pressure.

S2. As electromagnetic waves are plane polarised, so the receiving antenna should be parallel to electric/magnetic part of the wave.

S3. The displacement current through the capacitor is,

$$I_d = I_c = \frac{dq}{dt} \quad \text{... (i)}$$

Here, $q = q_0 \cos 2\pi\nu t \times 2\pi\nu$

Putting this value in Eq. (i), we get

$$I_d = I_c = -q_0 \sin 2\pi\nu t \times 2\pi\nu$$

$$I_d = I_c = -2\pi\nu q_0 \sin 2\pi\nu t.$$

S4. On decreasing the frequency, reactance $X_c = \frac{1}{\omega C}$ = will increase which will lead to decrease in conduction current. In this case $i_D = i_C$; hence displacement current will decrease.

S5. $i_{av} = \frac{1}{2} c \frac{B_0^2}{\mu_0} = \frac{1}{2} \times \frac{3 \times 10^8 \times (12 \times 10^{-8})^2}{1.26 \times 10^{-6}} = 1.71 \text{ W/m}^2$

S6. Consider and electromagnetic waves, let $\boldsymbol{E}$ be varying along y-axis, $\boldsymbol{B}$ is along z-axis and propagation of wave be along x-axis. Then $\boldsymbol{E} \times \boldsymbol{B}$ will tell the direction of propagation of energy flow in electromagnetic wave, along x-axis.

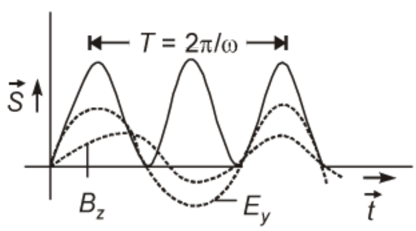
Let $\boldsymbol{E} = E_0 \sin(\omega t - kx)\,\hat{\boldsymbol{i}}$

$\boldsymbol{B} = B_0 \sin(\omega t - kx)\,\hat{\boldsymbol{j}}$

$$\boldsymbol{S} = \frac{1}{\mu_0}(\boldsymbol{E} \times \boldsymbol{B}) = \frac{1}{\mu_0} E_0 B_0 \sin^2(\omega t - kx)\,[\hat{\boldsymbol{i}} \times \hat{\boldsymbol{j}}]$$

$$= \frac{E_0 B_0}{\mu_0} \sin^2(\omega t - kx)\,\hat{\boldsymbol{i}}$$

The variation of $|\boldsymbol{S}|$ with time t will be as given in the figure below:



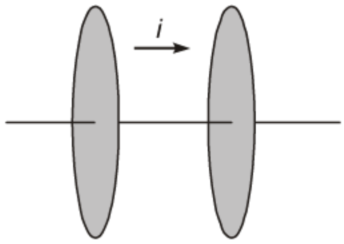
S7. Frequency of the microwave matches the resonant frequency of water molecules.

S8.

$$B = \frac{\mu_0 2I_D}{4\pi r} = \frac{\mu_0 I_D}{2\pi r} = \frac{\mu_0}{2\pi r}\,\varepsilon_0\,\frac{d\phi_E}{dt}$$

$$= \frac{\mu_0\varepsilon_0}{2\pi r} = \frac{d}{dt}(E\pi r^2)$$

$$= \frac{\mu_0\varepsilon_0 r}{2}\frac{dE}{dt}.$$



S9. As the distance is doubled, the area of spherical region ($4\pi r^2$) will become four times, so the intensity becomes one fourth the initial value $\left(\because\ I \propto \frac{1}{r^2}\right)$ but in case of laser it does not spread considieriably, so its intensity remain same.

Geometrical characteristic of LASER beam which is responsible for the constant intensity are as following:

(a) Unidirection (b) Monochromatic (c) Coherent light (d) Highly collimated

These characteristic are missing in the case of light from the bulb.

S10. Electric field of an *EM* wave is an oscillating field and so is the electric force caused by it on a charged particle. This electric force averaged over an integral number of cycles is zero since its direction changes every half cycle. Hence, electric field is not responsible for radiation pressure.

S11. (a) (i) Microwave is used in satellite communications.

So, λ_1 is the wavelength of microwave.

(ii) Ultraviolet rays are used to kill germs in water purifier. So, λ_2 is the wavelength of UV rays.

(iii) X-rays are used to detect leakage of oil in underground pipelines. So, λ_3 is the wavelength of X-rays.

(iv) Infrared is used to improve visibility on runways during fog and mist conditions. So, it is the wavelength of infrared waves.

(b) Wavelength of X-rays < wavelength of UV < wavelength of infrared < wavelength of microwave.

$$\Rightarrow \quad \lambda_3 < \lambda_2 < \lambda_4 < \lambda_1$$

(c) (i) Microwave is used in radar.

(ii) UV is used in LASIK eye surgery.

(iii) X-ray is used to detect a fracture in bones.

(iv) Infrared is used in optical communication.

S12.

$$S_{av} = c^2\varepsilon_0\,|\mathbf{E}_0 \times \mathbf{B}_0|\,\frac{1}{T}\int_0^T \cos^2(kx - \omega t)\,dt \text{ as } S^2 = c^2\varepsilon_0(\mathbf{E} \times \mathbf{B})$$

$$= c^2\varepsilon_0 E_0 B_0 \frac{1}{\cancel{T}} \times \frac{\cancel{T}}{2}$$

$$= c^2\varepsilon_0 E_0\left(\frac{E_0}{c}\right)\times\frac{1}{2}\left(as\ c=\frac{E_0}{B_0}\right)$$

$$= \frac{1}{2}\varepsilon_0 E_0^2 c$$

$$= \frac{E_0^2}{2\mu_0 c}\ as\left(c=\frac{1}{\sqrt{\mu_0\varepsilon_0}}\right).$$

S13. Pressure

$$P = \frac{\text{Force}}{\text{Area}}=\frac{F}{A}=\frac{1}{A}=\frac{\Delta p}{\Delta t} \qquad \left(F = \frac{\Delta p}{\Delta t}\ \text{rate of change of momentum}\right)$$

$$= \frac{1}{A}\cdot\frac{U}{\Delta t c} \qquad (\Delta pc = \Delta U\ \text{energy imparted by wave in time}\ \Delta t)$$

$$= \frac{I}{c} \qquad \left(\text{intensity}\ I = \frac{U}{A\Delta t}\right)$$

S14.

$$i_D = C\frac{dv}{dt}$$

$$1\times10^{-3} = 2\times10^{-6}\ \frac{dV}{dt}$$

$$\frac{dV}{dt} = \frac{1}{2}\times10^3 = \mathbf{5\times10\ V/s}.$$

Hence, applying a varying potential difference of 5×10^2 V/s would produce a displacement current of desired value.

S15.

$$\mathbf{E} = \frac{\lambda\hat{\mathbf{e}}_s}{2\pi\varepsilon_0 a}\hat{\mathbf{j}}$$

$$\mathbf{B} = \frac{\mu_0 i}{2\pi a}\hat{\mathbf{i}}$$

$$= \frac{\mu_0\lambda v}{2\pi a}\hat{i}$$

$$\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B}) = \frac{1}{\mu_0}\left(\frac{\lambda\hat{\mathbf{j}}_s}{2\pi\varepsilon_0 a}\hat{\mathbf{j}}\times\frac{\mu_0\lambda v}{2\pi a}\hat{\mathbf{i}}\right)$$

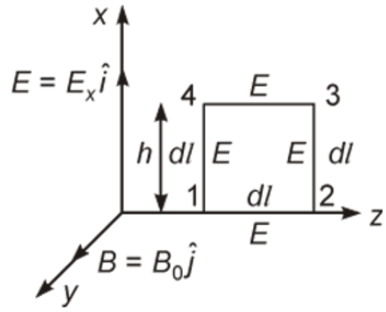
$$= \frac{-\lambda^2 v}{4\pi^2\varepsilon_0 a^2}\hat{\mathbf{k}}.$$

S16. (a)

$$\oint E.dl = \int_1^2 E.dl + \int_2^3 E.dl + \int_3^4 E.dl + \int_4^1 E.dl$$

$$= \int_1^2 E.dl\cos 90° + \int_2^3 E.dl\cos 0° + \int_3^4 E.dl\cos 90° + \int_4^1 E.dl\cos 180°$$

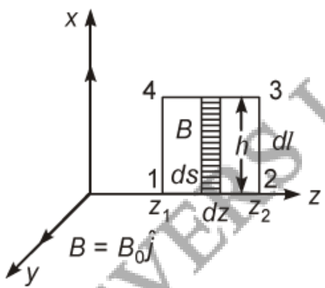
$$= \boldsymbol{E}_0 h[\sin(kz_2 - \omega t) - \sin(kz_1 - \omega t)] \quad \text{... (i)}$$



(b) For evaluating $\int \mathbf{B}.\mathbf{ds}$, let us consider the rectangle 1234 to be made of strips of area $ds = h\,dz$ each.

$$\int \mathbf{B}.\mathbf{ds} = \int B ds\cos 0 = \int B ds = \int_{z_1}^{z_2} B_0 \sin(kz_2 - \omega t) h dz$$

$$= \frac{-B_0 h}{k}[\cos(kz_2 - \omega t) - \cos(kz_1 - \omega t)] \quad \text{... (ii)}$$



(c)

$$\oint \mathbf{E}.\mathbf{d}l = \frac{-d\phi_B}{dt}$$

Using the relations obtained in Equations (i) and (ii) and simplifying, we get

$$E_0 h[\sin(kz_2 - \omega t) - \sin(kz_1 - \omega t)] = \frac{B_0 h}{k}\omega[\sin(kz_2 - \omega t) - \sin(kz_1 - \omega t)]$$

$$E_0 = B_0 \frac{\omega}{k}$$

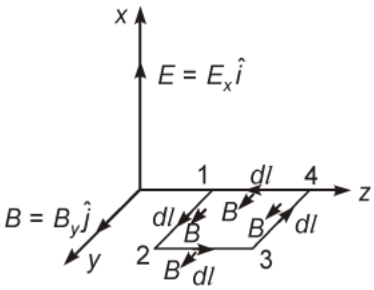
$$\frac{E_0}{B_0} = c$$

(d) For evaluating $\oint \mathbf{E}.\mathbf{d}l$, let us consider the loop 1234 in yz plane as shown in figure.

$$\oint \mathbf{E}.\mathbf{d}l = \int_1^2 \mathbf{E}.\mathbf{d}l + \int_2^3 \mathbf{E}.\mathbf{d}l + \int_3^4 \mathbf{E}.\mathbf{d}l + \int_4^1 \mathbf{E}.\mathbf{d}l$$

$$= \int_1^2 E.dl \cos 0^\circ + \int_2^3 E.dl \cos 90^\circ + \int_3^4 E.dl \cos 180^\circ + \int_4^1 E.dl \cos 90^\circ$$

$$= B_0 h[\sin(kz_1 - \omega t) - \sin(kz_2 - \omega t)] \quad \text{... (iii)}$$



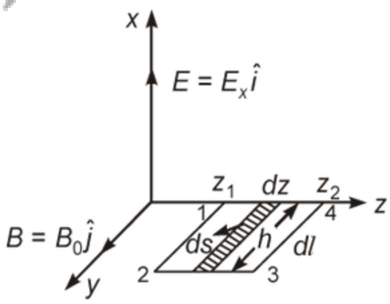
Now to evaluate $\phi_E = \int \mathbf{E}.\mathbf{ds}$, let us consider the rectangle 1234 to be made of strips of area hdz each.

$$\phi_E = \int \mathbf{E}.\mathbf{ds} = \int Eds \cos 0 = \int Eds = \int_{z_1}^{z_2} E_0 \sin(kz_1 - \omega t) hdz$$

$$= \frac{-E_0 h}{k}[\cos(kz_2 - \omega t) - \cos(kz_1 - \omega t)]$$

$$\therefore \quad \frac{d\phi_E}{dt} = \frac{E_0 h \omega}{k}[\sin(kz_1 - \omega t) - \sin(kz_2 - \omega t)] \quad \text{... (iv)}$$

In $\oint \mathbf{B}.\mathbf{d}l = \mu_0 \left(I + \varepsilon_0 \frac{d\phi_E}{dt} \right)$,



I = conduction current = 0 in vacuum.

$$\therefore \quad \oint \mathbf{B}.\mathbf{d}l = \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

Using relations obtained in Equations (iii) and (iv) and simplifying, we get

$$B_0 = E_0 \frac{\omega}{k} \cdot \mu_0 \varepsilon_0$$

$$\frac{E_0}{B_0}\frac{\omega}{k} = \frac{1}{\mu_0 \varepsilon_0}. \quad \text{But } E_0/B_0 = c, \text{ and } \omega = ck$$

or $$c.c = \frac{1}{\mu_0\varepsilon_0}. \quad \text{Therefore, } c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}.$$

S17. Let the distance between the plates be d. Then the electric field $E = \frac{V_0}{d}\sin(2\pi\nu t)$. The conduction current density is given by the Ohm's law = E.

$$\Rightarrow \quad J^c = \frac{1}{\rho}\frac{V_0}{d}\sin(2\pi\nu t)$$

$$= \frac{V_0}{\rho d}\sin(2\pi\nu t)$$

$$= J_0^c \sin 2\pi\nu t$$

where, $$J_0^c = \frac{V_0}{\rho d}$$

The displacement current density is given as

$$J^d = \varepsilon\frac{\partial E}{dt} = \varepsilon\frac{\partial}{dt}\left\{\frac{V_0}{d}\sin(2\pi\nu t)\right\}$$

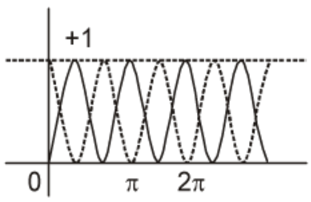
$$= \frac{\varepsilon 2\pi\nu V_0}{d}\cos(2\pi\nu t)$$

$$= J_0^d\cos(2\pi\nu t), \text{ where } J_0^d = \frac{2\pi\nu\varepsilon V_0}{d}$$

$$J_0^d \Big/ J_0^d = \frac{2\pi\nu\varepsilon V_0}{d}\cdot\frac{\rho d}{V_0} = 2\pi\nu\varepsilon\rho = 2\pi\times 80\,\varepsilon_0\nu\times 0.25 = 4\pi\varepsilon_0\nu\times 10$$

$$= \frac{10\nu}{9\times 10^9} = \frac{4}{9}.$$

S18. (a) E - field contribution is $$u_E = \frac{1}{2}\varepsilon_0 E^2$$



B - field contribution is $$u_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

Total energy density $$u = u_E + u_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0} \quad \text{... (i)}$$

The values of E^2 and B^2 vary from point to point and from moment to moment. Hence, the effective values of E^2 and B^2 are their time averages.

$$(E^2)_{av} = E_0^2[\sin^2(kz-\omega t)]_{av}$$

$$(B^2)_{av} = (B^2)_{av} = B_0^2[\sin^2(kz-\omega t)]_{av}$$

The graph of $\sin^2\theta$ and $\cos^2\theta$ are identical in shape but shifted by $\pi/2$, so the average values of $\sin^2\theta$ and $\text{Cos}^2\theta$ are also equal over any integral multiple of π.

and also $\sin^2\theta + \cos^2\theta = 1$

So by symmetry the average of $\sin^2\theta$ = average of $\cos^2\theta = \dfrac{1}{2}$

$$\therefore \qquad (E^2)_{av} = \frac{1}{2}E_0^2 \quad \text{and} \quad (B^2)_{av} = \frac{1}{2}B_0^2$$

Substituting in Equation (i),

$$u = \frac{1}{4}\varepsilon_0 E^2 + \frac{1}{4}\frac{B_0^2}{\mu} \qquad \text{... (ii)}$$

We know, $\dfrac{E_0}{B_0} = c$ and $c = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$

$$\therefore \qquad \frac{1}{4}\frac{B_0^2}{\mu_0} = \frac{E_0^2/c^2}{4\mu_0} = \frac{E_0^2}{4\mu_0}\mu_0\varepsilon_0 = \frac{1}{4}\varepsilon_0 E_0^2.$$

Therefore, $u_{av} = \dfrac{1}{4}\varepsilon_0 E_0^2 + \dfrac{1}{4}\varepsilon_0 E_0^2 = \dfrac{1}{2}\varepsilon_0 E_0^2$

and $I_{av} = u_{av}c = \dfrac{1}{2}\varepsilon_0 E_0^2.$

S19. (a) Displacement curing density can be found from the relation be

$$\mathbf{J}_D = \varepsilon_0 \frac{d\mathbf{E}}{dt}$$

$$= \varepsilon_0\mu_0 I_0 \frac{\partial}{\partial t}\cos(2\pi\nu t)\ln\left(\frac{s}{a}\right)\hat{\mathbf{k}}$$

$$= \frac{1}{c^2} I_0 2\pi\nu^2(-\sin(2\pi\nu t)\ln\left(\frac{s}{a}\right)\hat{\mathbf{k}}$$

$$= \left(\frac{\nu}{c}\right)^2 2\pi I_0 \sin(2\pi\nu t)\ln\left(\frac{a}{s}\right)\hat{k}$$

$$= \frac{2\pi}{\lambda^2} I_0 \ln\left(\frac{a}{s}\right) \sin(2\pi\nu t)\, \hat{\mathbf{k}}$$

(b) $$I^d = \int J_D\, s ds d\theta$$

$$= \frac{2\pi}{\lambda^2} I_0 2\pi \int_{s=0}^{a} \ln\left(\frac{a}{s}\right) \cdot sds \sin(2\pi\nu t)$$

$$= \left(\frac{2\pi}{\lambda}\right)^2 I_0 \int_{s=0}^{a} \frac{1}{2} ds^2 \ln\left(\frac{a}{s}\right) \cdot \sin(2\pi\nu t)$$

$$= \frac{a^2}{4}\left(\frac{2\pi}{\lambda}\right)^2 I_0 \int_{s=0}^{a} d\left(\frac{s}{a}\right)^2 \ln\left(\frac{a}{s}\right)^2 \cdot \sin(2\pi\nu t)$$

$$= -\frac{a^2}{4}\left(\frac{2\pi}{\lambda}\right)^2 I_0 \int_0^1 \ln\xi d\xi \cdot \sin(2\pi\nu t)$$

$$= +\left(\frac{a}{4}\right)^2 \left(\frac{2\pi}{\lambda}\right)^2 I_0 \sin 2\pi\nu t \qquad (\because \text{ The integral has value } -1)$$

(c) The displacement current

$$I_d = \left(\frac{a}{2} \cdot \frac{2\pi}{\lambda}\right)^2 I_0 \sin 2\pi\nu t = I_0 \sin 2\pi\nu t$$

$$\frac{I_d}{I_0} = \left(\frac{a\pi}{\lambda}\right)^2.$$