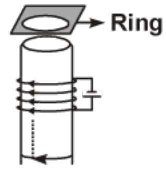
Date: 21/10/2021

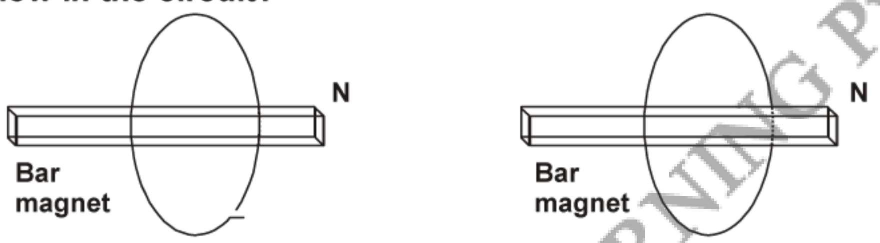
Q1. Consider a metal ring kept on top of a fixed solenoid (say on a cardboard) (shown in figure). The centre of the ring coincides with the axis of the solenoid. If the current is suddenly switched on, the metal ring jumps up. Explain



Q2. A solenoid is connected to a battery so that a steady current flows through it. If an iron core is inserted into the solenoid, will the current increase or decrease? Explain.

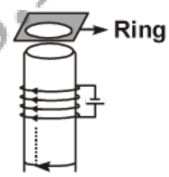
Q3. A wire in the form of a tightly wound solenoid is connected to a D.C. source, and carries a current. If the coil is stretched so that there are gaps between successive elements of the spiral coil, will the current increase or decrease? Explain.

Q4. Consider a magnet surrounded by a wire with an on/off switch S (shown in figure). If the switch is thrown from the off position (open circuit) to the on position (closed circuit), will a current flow in the circuit?



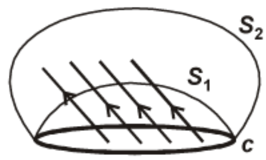
Circuit open   Circuit closed

Q5. Consider a metal ring kept (supported by a cardboard) on top of a fixed solenoid carrying a current I (see figure). The centre of the ring coincides with the axis of the solenoid. If the current in the solenoid is switched off, what will happen to the ring?



Q6. Consider a metallic pipe with an inner radius of 1 cm. If a cylindrical bar magnet of radius 0.8 cm is dropped through the pipe, it takes more time to come down than it takes for a similar unmagnetised cylindrical iron bar dropped through the metallic pipe. Explain.

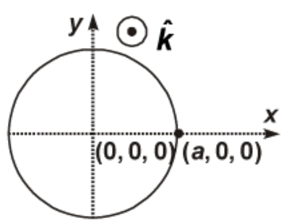
Q7. Consider a closed loop C in a magnetic field (shown in figure). The flux passing through the loop is defined by choosing a surface whose edge coincides with the loop and using the formula $\phi = B_1 \cdot dA_1 + B_2 \cdot dA_2 + ...$ Now if we chose two different surfaces S_1 and S_2 having C as their edge, would we get the same answer for flux. Justify your answer.



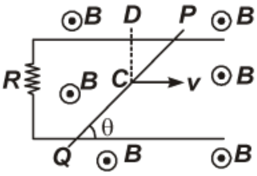
Q8. There are two coils A and B separated by some distance. If a current of 2 A flows through A, a magnetic flux of 10^{-2} Wb passes through B (no current through B). If no current passes through A and a current of 1 A passes through B, what is the flux through A?

Q9. A magnetic field in a certain region is given by $B = B_0 \cos(\omega t)\hat{k}$ and a coil of radius a with resistance R is placed in the x-y plane with its centre at the origin in the magnetic field (see figure). Find the magnitude and the direction of the current at (a, 0, 0) at

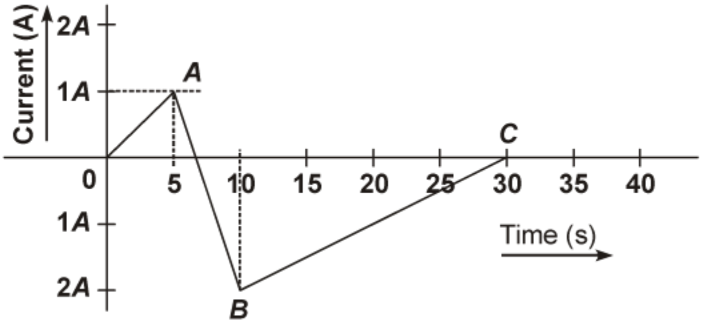
$$t = \pi/2\omega, \quad t = \pi/\omega \quad \text{and} \quad t = 3\pi/2\omega.$$



Q10. Find the current in the wire for the configuration shown in figure. Wire PQ has negligible resistance. B, the magnetic field is coming out of the paper. θ is a fixed angle made by PQ travelling smoothly over two conducting parallel wires seperated by a distance d.

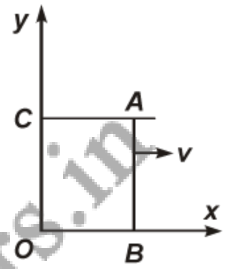


Q11. A (current vs time) graph of the current passing through a solenoid is shown in figure. For which time is the back electromotive force (u) a maximum. If the back e.m.f. at $t = 3$s is e, find the back e.m.f. at $t = 7$s, 15s and 40s. OA, AB and BC are straight line segments.



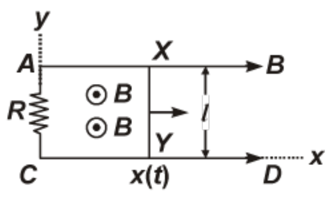
Q12. A magnetic field B is confined to a region $r \leq a$ and points out of the paper (the z-axis), $r = 0$ being the centre of the circular region. A charged ring (charge = Q) of radius b, $b > a$ and mass m lies in the x-y plane with its centre at the origin. The ring is free to rotate and is at rest. The magnetic field is brought to zero in time Δt. Find the angular velocity ω of the ring after the field vanishes.

Q13. A magnetic field $B = B_0 \sin(\omega t)\ \hat{k}$ covers a large region where a wire AB slides smoothly over two parallel conductors separated by a distance d (see figure). The wires are in the x-y plane. The wire AB (of length d) has resistance R and the parallel wires have negligible resistance. If AB is moving with velocity v, what is the current in the circuit. What is the force needed to keep the wire moving at constant velocity?



Q14. A metallic ring of mass m and radius l (ring being horizontal) is falling under gravity in a region having a magnetic field. If z is the vertical direction, the z-component of magnetic field is $B_z = B_0 (1 + \lambda z)$. If R is the resistance of the ring and if the ring falls with a velocity v, find the energy lost in the resistance. If the ring has reached a constant velocity, use the conservation of energy to determine v in terms of m, B, λ and acceleration due to gravity g.

Q15. A conducting wire XY of mass m and negligible resistance slides smoothly on two parallel conducting wires as shown in figure. The closed circuit has a resistance R due to AC. AB and CD are perfect conductors. There is a magnetic field $B = B(t)\hat{k}$.



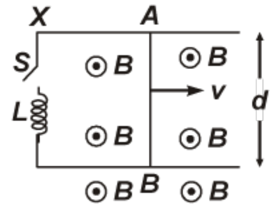
(a) Write down equation for the acceleration of the wire XY.

(b) If B is independent of time, obtain $v(t)$, assuming $v(0) = u_0$.

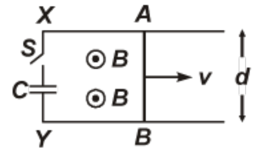
(c) For (b), show that the decrease in kinetic energy of XY equals the heat lost in R.

Q16. A long solenoid 'S' has 'n' turns per meter, with diameter 'a'. At the centre of this coil we place a smaller coil of 'N' turns and diameter 'b' (where $b < a$). If the current in the solenoid increases linearly, with time, what is the induced emf appearing in the smaller coil. Plot graph showing nature of variation in e.m.f., if current varies as a function of $mt^2 + C$.

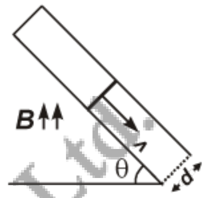
Q17. Find the current in the sliding rod *AB* (resistance = *R*) for the arrangement shown in figure. *B* is constant and is out of the paper. Parallel wires have no resistance. *v* is constant. Switch *S* is closed at time $t = 0$.



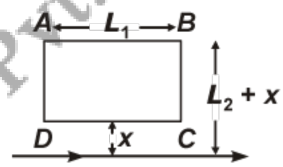
Q18. Find the current in the sliding rod *AB* (resistance = *R*) for the arrangement shown in figure. *B* is constant and is out of the paper. Parallel wires have no resistance. v is constant. Switch *S* is closed at time $t = 0$.



Q19. A rod of mass *m* and resistance *R* slides smoothly over two parallel perfectly conducting wires kept sloping at an angle ¸ with respect to the horizontal (see figure). The circuit is closed through a perfect conductor at the top. There is a constant magnetic field B along the vertical direction. If the rod is initially at rest, find the velocity of the rod as a function of time.

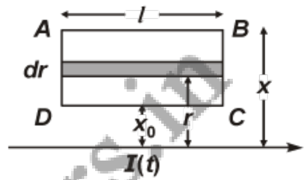


Q20. A rectangular loop of wire *ABCD* is kept close to an infinitely long wire carrying a current $I(t) = I_0(1 - t/T)$ for $0 \le t \le T$ and $I(0) = 0$ for $t > T$ (see figure). Find the total charge passing through a given point in the loop, in time *T*. The resistance of the loop is *R*.



Q21. Consider an infinitely long wire carrying a current $I(t)$, with

$\frac{dI}{dt} = \lambda =$ constant. Find the current produced in the rectangular loop of wire *ABCD* if its resistance is *R* (see figure).



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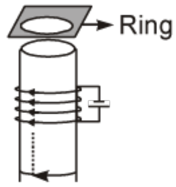
S1. No flux was passing through the metal ring initially. When the current is switched on, flux passes through the ring. According to Lenz's law this increase will be resisted and this can happen if the ring moves away from the solenoid. One can analyse this in more detail (see figure). If the current in the solenoid is as shown, the flux (downward) increases and this will cause a counterclockwise current (as seen form the top in the ring). As the flow of current is in the opposite direction to that in the solenoid, they will repel each other and the ring will move upward.



S2. The current will decrease. As the iron core is inserted in the solenoid, the magnetic field increases and the flux increases. Lenz's law implies that induced e.m.f. should resist this increase, which can be achieved by a decrease in current.

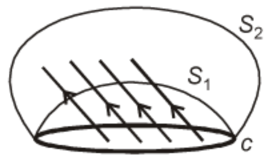
S3. The current will increase. As the wires are pulled apart the flux will leak through the gaps. Lenz's law demands that induced e.m.f. resist this decrease, which can be done by an increase in current.

S4. No part of the wire is moving and so motional e.m.f. is zero. Since, motional e.m.f. is Bvl and V is 0. The magnet is stationary and hence the magnetic field does not change with time. This means no electromotive force is produced and hence no current will flow in the circuit.

S5. When the current in the solenoid decreases a current flows in the same direction in the metal ring as in the solenoid. Thus there will be a downward force. This means the ring will remain on the cardboard. The upward reaction of the cardboard on the ring will increase.

S6. For the magnet, eddy currents are produced in the metallic pipe. These currents will oppose the motion of the magnet. Therefore magnet's downward acceleration will be less than the acceleration due to gravity g. On the other hand, an unmagnetised iron bar will not produce eddy currents and will fall with an acceleration g. Thus the magnet will take more time.

S7. One gets the same answer for flux. Flux can be through of as the number of magnetic field lines passing through the surface (we draw $dN = B\Delta A$ lines in an area $\Delta A \perp$ to $\boldsymbol{B}$), As lines of $\boldsymbol{B}$ cannot end or start in space (they form closed loops) number of lines passing through surface S_1 must be the same as the number of lines passing through the surface S_2.



S8. Applying the mutual inductance of coil A with respect to coil B

$$M_{21} = \frac{N_2\phi_2}{I_1}$$

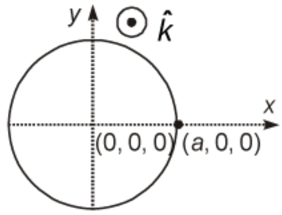
Therefore, we have

$$\text{Mutual inductance} = \frac{10^{-2}}{2} = 5\,\text{mH}$$

Again applying this formula for other case

$$N_1\phi_1 = M_{12}I_2 = 5\,\text{mH} \times 1\,\text{A} = 5\,\text{mWb}.$$

S9. Flux through the ring



$$\phi = B_0 (\pi a^2) \cos \omega t$$
$$e = B (\pi a^2) \omega \sin \omega t$$
$$I = B (\pi a^2) \omega \sin \omega t / R$$

Current at $t = \dfrac{\pi}{2\omega}$; $I = \dfrac{B(\pi a^2)\omega}{R}$ along $\hat{\mathbf{j}}$

Because $\sin \omega t = \sin\left(\omega \dfrac{\pi}{2\omega}\right) = \sin \dfrac{\pi}{2} = 1$

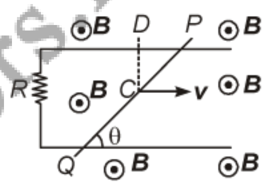
$t = \dfrac{\pi}{\omega}$; $I = \dfrac{B(\pi a^2)\omega}{R}$ along $\hat{\mathbf{j}}$

$\sin \omega t = \sin\left(\omega \dfrac{\pi}{\omega}\right) = \sin \pi = 0$

$t = \dfrac{3}{2}\dfrac{\pi}{\omega}$; $I = \dfrac{B(\pi a^2)\omega}{R}$ along $-\hat{\mathbf{j}}$

$\sin \omega t = \sin\left(\omega \dfrac{3\pi}{2\omega}\right) = \sin \dfrac{3\pi}{2} = -1.$

S10. Motional electric field E along the dotted line CD ($\perp$ to both $\boldsymbol{v}$ and $\boldsymbol{B}$ and along $\boldsymbol{v} \times \boldsymbol{B}$) = vB



E.M.F. along PQ = (length PQ) $\times$ (Field along PQ)

$$= \frac{d}{\cos \theta} \times uB \cos \theta = dvB.$$

Therefore, $I = \dfrac{dvB}{R}$ and is independent of q.

S11. Maximum rate of change of current is in AB. So maximum back e.m.f. will be obtained between $5s < t < 10s$.

If $u = L\ 1/5 \left(\text{for } t = 3s, \dfrac{dI}{dt} = 1/5\right)$ (L is a constant)

For $5s < t < 10s$ $u_1 = -L\dfrac{3}{5} = -\dfrac{3}{5}L = -3\,e$

Thus at $t = 7s$, $u_1 = -3\,e.$

For $10s < t < 30s$ $u_2 = L\dfrac{2}{20} = \dfrac{L}{10} = \dfrac{1}{2}e$

For $t > 30s$ $u_2 = 0.$

S12. Since, the magnetic field is brought to zero in time Δt, the magnetic flux linked with the ring also reduces from maximum to zero. This, in turn, induces an e.m.f. in ring by the phenomenon of EMI. The induces e.m.f. causes the electric field E generation around the ring.

The induced e.m.f. = Electric field $E \times (2\pi b)$ (Because $V = E \times d$) ... (i)

By Faraday's law

$$\text{e.m.f.} = \frac{B.\pi a^2}{\Delta t} \quad \text{... (ii)}$$

From Eqs. (i) and (ii), we have

$$2\pi bE = \text{e.m.f.} = \frac{B.\pi a^2}{\Delta t}$$

Since, the charged ring experienced a electric force = QE

This force try to rotate the coil, and the torque is given by

$$\text{Torque} = b \times \text{Force}$$

$$QEb = Q\left[\frac{B\pi a^2}{2\pi b \Delta t}\right] b$$

$$= Q\frac{Ba^2}{2\Delta t}.$$

If ΔL is the change in angular momentum

$$\Delta L = \text{Torque} \times \Delta t = Q\frac{Ba^2}{2}$$

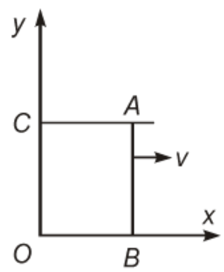
Initial angular momentum = 0

$$\text{Final angular momentum} = mb^2\omega = \frac{QBa^2}{2}$$

$$\omega = \frac{QBa^2}{2mb^2}.$$

On rearranging the terms, we have the required expression of angular speed.

S13. Let us assume that the parallel wires at are $y = 0$ and $y = d$. At $t = 0$, AB has $x = 0$ and moves with a velocity $v\hat{i}$.



At time t, wire is at $x(t) = vt$.

$$\text{Motional e.m.f.} = (B_0 \sin \omega t)\, vd\,(-\hat{j})$$

E.m.f due to change in field (along $OBAC$)

$$= -B_0\omega \cos \omega t\, x(t)\, d$$

$$\text{Total e.m.f} = -B_0\omega\, [\omega x \cos(\omega t) + v \sin(\omega t)]$$

Along $OBAC$,

$$\text{Current (clockwise)} = \frac{B_0 d}{R}(\omega x \cos \omega t + v \sin \omega t)$$

$$\text{Force needed along } \hat{\boldsymbol{i}} = \frac{B_0 d}{R}(\omega x \cos \omega t + v \sin \omega t) \times d \times B_0 \sin \omega t$$

$$= \frac{B_0^2 d^2}{R}(\omega x \cos \omega t + v \sin \omega t) \sin \omega t.$$

S14. $\frac{d\phi}{dt}$ = rate of change in flux = $(\pi/2) B_0 l \frac{dz}{dt} = IR$.

$$I = \frac{\pi l^2 B_0 \lambda}{R} v$$

Energy lost/second = $I^2 R = \frac{(\pi l^2 \lambda)^2 B_0^2 v^2}{R}$

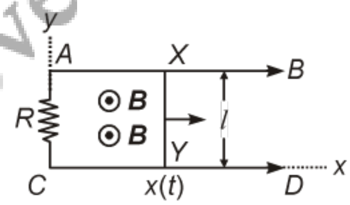
This must come from rate of change in P.E. = $mg\frac{dz}{dt} = mgv$

(as kinetic energy is constant for v = constant)

Thus, $$mgv = \frac{(\pi l^2 \lambda B_0)^2 v^2}{R}$$

Or, $$v = \frac{mgR}{(\pi l^2 \lambda B_0)^2}.$$

S15. (a) Let the wire be at x = $x(t)$ at time t.



$$\text{Flux} = B(t)\, l\, x(t)$$

$$E = -\frac{d\phi}{dt} = -\frac{dB(t)}{dt} l\, x(t) - B(t) l . v(t)$$

(second term due to motional e.m.f.)

$$I = \frac{1}{R} E$$

$$\text{Force} = \frac{l B(t)}{R}\left[-\frac{dB}{dt} l\, x(t) - B(t) l . v(t)\right]\hat{\boldsymbol{i}}$$

$$m\frac{d^2x}{dt^2} = -\frac{l^2 B}{R}\frac{dB}{dt} x(t) - \frac{l^2 B^2}{R}\frac{dx}{dt} \qquad \left[\because \text{Force} = m \times a = m \times \frac{d^2x}{dt^2}\right]$$

(b) $$\frac{dB}{dt} = 0, \quad \frac{d^2x}{dt^2} + \frac{l^2 B^2}{mR}\frac{dx}{dt} = 0$$

$$\frac{dv}{dt} + \frac{l^2B^2}{mR} v = 0$$

$$v = A \exp\left(\frac{-l^2B^2t}{mR}\right)$$

At $t = 0$, $v = u$

$$v(t) = u \exp(-l^2B^2t/mR).$$

(c) $$I^2R = \frac{B^2l^2c^2(t)}{R^2} \times R = \frac{B^2l^2}{R} u^2 \exp(-2l^2B^2t/mR)$$

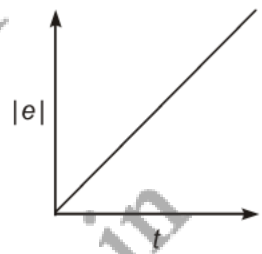
$$\text{Power lost} = \int_0^t I^2R^2\,dt = \frac{B^2l^2}{R} u^2 \frac{mR}{2l^2B^2}[1 - e^{-(l^2B^2t/mR)}]$$

$$= \frac{m}{2}u^2 - \frac{m}{2}v^2(t)$$

= decrease in kinetic energy.

S16. Magnetic field due to a solenoid S, $B = \mu_0 nI$

Magnetic flux in smaller coil $\phi = NBA$ where $A = \pi b^2$



So $$e = -\frac{d\phi}{dt} = \frac{-d}{dt}(NBA)$$

$$= -N\pi b^2 \frac{d(B)}{dt} = -N\pi b^2 \frac{d}{dt}(\mu_t)$$

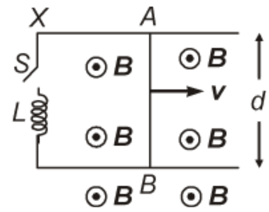
$$= -N\pi b^2 \mu_0 n \frac{dI}{dt}$$

$$= -Nn\pi\mu_0 b^2 \frac{d}{dt}(mt^2 + C) = -\mu_0 Nn\pi b^2 2mt$$

$$e = -\mu_0 Nn\pi b^2 2mt.$$

Negative sign signifies opposite nature of induced e.m.f. The magnitude of e.m.f. varies with time as shown in the figure.

S17. The conductor of length d moves with speed v, perpendicular to magnetic field B as shown in figure. This produces motional e.m.f. across two ends of rod, which is given by = vBd. Since, switch D is closed at time $t = 0$. Current start growing in inductor by the potential difference due to motional e.m.f.



$$-L\frac{dI}{dt} + vBd = IR$$

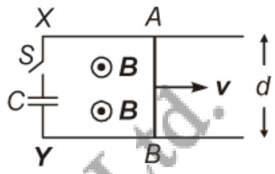
$$L\frac{dI}{dt} + IR = vBd$$

$$I = \frac{vBd}{R} + Ae^{-Rt/2}$$

At $\quad t = 0 \qquad I = 0 \quad \Rightarrow \quad A = -\frac{vBd}{R}$

$$I = \frac{vBd}{R}(1 - e^{-Rt/L}).$$

S18. The conductor of length d moves with speed v, perpendicular to magnetic field B as shown in figure. This produces motional e.m.f. across two ends of rod. which is given by vBd. Since, switch S is closed at time $t = 0$, capacitor is charged by this potential difference. Let $Q(t)$ is charge on the capacitor (note current flows from A to B)



$$I = \frac{vBd}{R} - \frac{Q}{RC}$$

$\Rightarrow$
$$\frac{Q}{RC} + \frac{dQ}{dt} = \frac{vBd}{R}$$

$\therefore$
$$Q = vBdC + e^{-t/RC}$$

$\Rightarrow$
$$Q = vBdC[1 - e^{-t/RC}]$$

(At time $t = 0$, $Q = 0 = A = -vBdc$). Differentiating, we get

$$I = \frac{vBd}{R}e^{-t/RC}.$$

S19. Here,
$$m\frac{d^2x}{dt^2} = mg\sin\theta - \frac{B\cos\theta d}{mR}\left(\frac{dx}{dt}\right) \times (Bd)\cos\theta$$

$$\frac{dv}{dt} = g\sin\theta - \frac{B^2d^2}{mR}(\cos\theta)^2 v$$

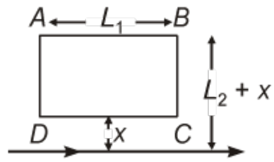
$$\frac{dv}{dt} + \frac{B^2d^2}{mR}(\cos\theta)^2 v = g\sin\theta$$

$$v = \frac{g\sin\theta}{\left(\frac{B^2d^2\cos^2\theta}{mR}\right)} + A\exp\left(-\frac{B^2d^2}{mR}(\cos^2\theta)t\right)$$

(A is a constant to be determine by initial conditions)

$$= \frac{mgR\sin\theta}{B^2d^2\cos^2\theta}\left(1 - \exp\left(-\frac{B^2d^2}{mR}(\cos^2\theta)t\right)\right).$$

S20. If $I(t)$ is the current in the loop.



$$I(t) = \frac{1}{R}\frac{d\phi}{dt}$$

If Q is the charge that passed in time t,

$$I(t) = \frac{dQ}{dt} \quad \text{or} \quad \frac{dQ}{dt} = \frac{1}{R}\frac{d\phi}{dt}$$

Integrating $$Q(t_1) - Q(t_2) = \frac{1}{R}[\phi(t_1) - \phi(t_2)]$$

$$\phi(t_1) = L_1 \frac{\mu_0}{2\pi}\int_x^{L_2+x}\frac{dx'}{x'} I(t_1)$$

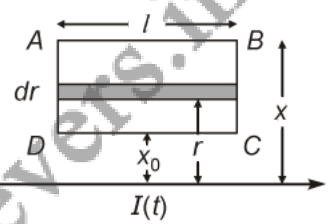
$$= \frac{\mu_0 L_1}{2\pi} I(t_1) \ln\frac{L_2 + x}{x}$$

The magnitude of charge is

$$Q = \frac{\mu_0 L_1}{2\pi}\ln\frac{L_2 + x}{x}[I_0 - 0]$$

$$= \frac{\mu_0 L_1 I_1}{2\pi}\ln\left(\frac{L_2 + x}{x}\right).$$

S21. Let us consider a strip of length l and width dr at a distance r from infinite long current carrying wire. The magnetic field at strip due to current carrying wire is given by



$$\text{Field } B(r) = \frac{\mu_0 I}{2\pi r} \text{ (out of paper).}$$

Total flux through the loop is

$$\text{Flux} = \frac{\mu_0 I}{2\pi} l \int_{x_0}^{x}\frac{dr}{r} = \frac{\mu_0 I}{2\pi} \ln\frac{x}{x_0} \quad \text{... (i)}$$

The e.m.f. induced can be obtained by differentiating the eq. (i) w.r.t. t and then applying Ohm's law

$$\frac{\varepsilon}{R} = I$$

We have, $$\text{Induced current} = \frac{1}{R}\frac{d\phi}{dt} = \frac{\varepsilon}{R} = \frac{\mu_0 I}{2\pi}\frac{\lambda}{R}\ln\frac{x}{x_0}. \qquad \left(\because \frac{dI}{dt} = \lambda\right)$$