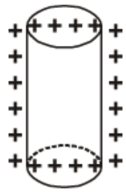
Date: 20/10/2021

Q1. An arbitrary surface encloses a dipole. What is the electric flux through this surface?

Q2. Sketch the electric field lines for a uniformly charged hollow cylinder shown in figure.



Q3. If the total charge enclosed by a surface is zero, does it imply that the electric field everywhere on the surface is zero? Conversely, if the electric field everywhere on a surface is zero, does it imply that net charge inside is zero.

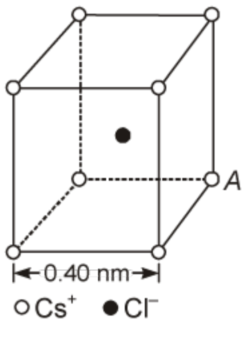
Q4. The dimensions of an atom are of the order of an Angstrom. Thus there must be large electric fields between the protons and electrons. Why, then is the electrostatic field inside a conductor zero?

Q5. A metallic spherical shell has an inner radius R_1 and outer radius R_2. A charge Q is placed at the centre of the spherical cavity. What will be surface charge density on (a) the inner surface, and (b) the outer surface?

Q6. A paisa coin is made up of Al-Mg alloy and weighs 0.75g. It has a square shape and its diagonal measures 17 mm. It is electrically neutral and contains equal amounts of positive and negative charges.

Treating the paisa coins made up of only Al, find the magnitude of equal number of positive and negative charges. What conclusion do you draw from this magnitude?

Q7. In the figure, represents a crystal unit of cesium chloride, CsCl. The cesium atoms, represented by open circles are situated at the corners of a cube of side 0.40 nm, whereas a Cl atom is situated at the centre of the cube. The Cs atoms are deficient in one electron while the Cl atom carries an excess electron.



(a) What is the net electric field on the Cl atom due to eight Cs atoms?

(b) Suppose that the Cs atom at the corner A is missing. What is the net force now on the Cl atom due to seven remaining Cs atoms?

Q8. Two charges q and $-3q$ are placed fixed on x-axis separated by distance 'd'. Where should a third charge $2q$ be placed such that it will not experience any force?

Q9. A paisa coin is made up of Al-Mg alloy and weighs 0.75g. It has a square shape and its diagonal measures 17 mm. It is electrically neutral and contains equal amounts of positive and negative charge of magnitude 34.8 kC. Suppose that these equal charges were concentrated in two point charges separated by

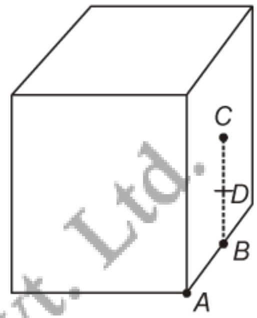
(a) 1 cm $\left(\sim \frac{1}{2} \times \text{ diagonal of the one paisa coin}\right)$,

(b) 100 m (~ length of a long building), and

(c) 10^6 m (radius of the Earth).

Find the force on each such point charge in each of the three cases. What do you conclude from these results?

Q10. What will be the total flux through the faces of the cube (shown in figure) with side of length a if a charge q is placed at



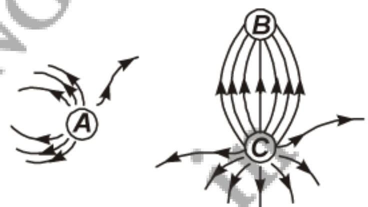
(a) A: a corner of the cube.

(b) B: mid-point of an edge of the cube.

(c) C: centre of a face of the cube.

(d) D: mid-point of B and C.

Q11. The figure shows that, the electric field lines around three point charges *A*, *B* and *C*.



(a) Which charges are positive?

(b) Which charge has the largest magnitude? Why?

(c) In which region or regions of the picture could the electric field be zero? Justify your answer.

(i) near *A*, (ii) near *B*, (iii) near *C*, (iv) nowhere.

Q12. There is another useful system of units, besides the SI/mks A system, called the cgs (centimeter-gram-second) system. In this system Coloumb's law is given by

$$F = \frac{Qq}{r^2}\hat{r}$$

where the distance r is measured in cm (= 10^{-2} m), F in dynes (= 10^{-5} N) and the charges in electrostatic units (es units), where 1 es unit of charge $\frac{1}{[3]} \times 10^{-9}$ C

The number [3] actually arises from the speed of light in vaccum which is now taken to be exactly given by $c = 2.99792458 \times 10^8$ m/s. An approximate value of c then is $c = [3] \times 10^8$ m/s.

(a) Show that the coloumb law in cgs units yields

1 esu of charge = 1 $(\text{dyne})^{1/2}$ cm.

Obtain the dimensions of units of charge in terms of mass M, length L and time T. Show that it is given in terms of fractional powers of M and L.

(b) Write 1 esu of charge = xC, where x is a dimensionless number. Show that this gives

$$\frac{1}{4\pi\varepsilon_0} = \frac{10^{-9}}{x^2}\frac{\text{N.m}^2}{\text{C}^2}$$

With $$x = \frac{1}{[3]} \times 10^9,$$

we have $$\frac{1}{4\pi\varepsilon_0} = [3]^2 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$$

or, $$\frac{1}{4\pi\varepsilon_0} = (2.99792458)^2 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \text{ (exactly).}$$

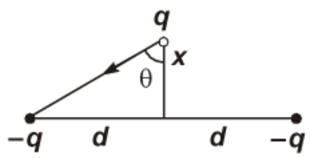
Q13. Total charge $-Q$ is uniformly spread along length of a ring of radius R. A small test charge $+q$ of mass m is kept at the centre of the ring and is given a gentle push along the axis of the ring.

(a) Show that the particle executes a simple harmonic oscillation.

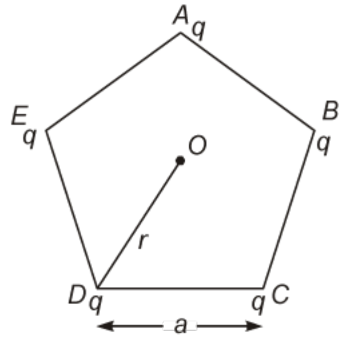
(b) Obtain its time period.

Q14. Two charges $-q$ each are fixed separated by distance $2d$. A third charge q of mass m placed at the mid-point is displaced slightly by x ($x \ll d$) perpendicular to the line joining the two fixed charged as shown in figure. Show that q will perform simple harmonic oscillation of time period.

$$T = \left[\frac{8\pi^3\varepsilon_0 md^3}{q^2}\right]^{1/2}$$



Q15. Five charges, q each are placed at the corners of a regular pentagon of side 'a' (as shown in the figure).



(a) (i) What will be the electric field at O, the centre of the pentagon?

(ii) What will be the electric field at O if the charge from one of the corners (say A) is removed?

(iii) What will be the electric field at O if the charge q at A is replaced by $-q$?

(b) How would your answer to (a) be affected if pentagon is replaced by n-sided regular polygon with charge q at each of its corners?

Q16. Consider a sphere of radius R with charge density distributed as

$$\rho(r) = kr \quad \text{for} \quad r \leq R$$
$$= 0 \quad \text{for} \quad r > R.$$

(a) Find the electric field at all points r.

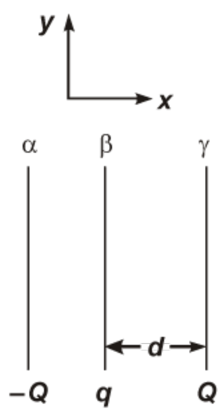
(b) Suppose the total charge on the sphere is $2e$ where e is the electron charge. Where can two protons be embedded such that the force on each of them is zero. Assume that the introduction of the proton does not alter the negative charge distribution.

Q17. In 1959 Lyttleton and Bondi suggested that the expansion of the Universe could be explained if matter carried a net charge. Suppose that the Universe is made up of hydrogen atoms with a number density N, which is maintained a constant. Let the charge on the proton be: $e_p = -(1 + y)e$ where e is the electronic charge.

(a) Find the critical value of y such that expansion may start.

(b) Show that the velocity of expansion is proportional to the distance from the centre.

Q18. Two fixed, identical conducting plates (α & β), each of surface area S are charged to $-Q$ and q, respectively, where $Q > q > 0$. A third identical plate (γ), free to move is located on the other side of the plate with charge q at a distance d (as shown in the figure). The third plate is released and collides with the plate β. Assume the collision is elastic and the time of collision is sufficient to redistribute charge amongst β & γ.



(a) Find the electric field acting on the plate γ before collision.

(b) Find the charges on β and γ after the collision.

(c) Find the velocity of the plate γ after the collision and at a distance d from the plate β.

Date: 20/10/2021

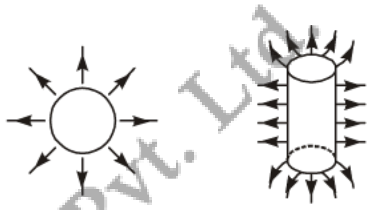
S1. From Gauss's Law, the electric flux through an enclosed surface is given by $\oint_s E.dS = \frac{q}{\varepsilon_0}$.

Here, q is the net charge inside that enclosed surface.

Now, the net charge on a dipole is given by $-q + q = 0$

$\therefore$ Electric flux through a surface enclosing a dipole = $\frac{-q+q}{\varepsilon_0} = \frac{0}{\varepsilon_0} = 0$.

S2. Thus, the electric field lines will start from positive charges and move towards infinity as given in the figure below:



S3. Gauss' law also implies that when the surface is so chosen that there are some charges inside and some outside.

The flux in such situation is given by $\oint E.dS = \frac{q}{\varepsilon_0}$.

In such situations, the electric field in the L.H.S. is due to all the charges both inside and outside the surface. The term q on the right side of the equation given by Gauss' low represent only the total charge inside the surface.

Thus, despite being total charge enclosed by a surface zero, it doesn't imply that the electric field everywhere on the surface is zero, the field may be normal to the surface.

Also, conversely if the electric field everywhere on a surface us zero, it doesn't imply that net charge inside it is zero.

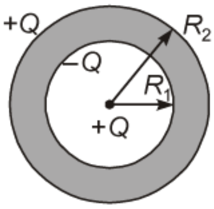
i.e., Putting $E = 0$ in $\oint E.dS = \frac{q}{\varepsilon_0}$

we get, $q = 0$.

S4. The protons and electrons are bound into a atom with distinct and independent existence and neutral in charge. Electrostatic fields are caused by the presence of excess charges.

But there can be no excess charge on the inter surface of an isolated conductor. So, the electrostatic fields inside a conductor is zero despite the fact that the dimensions of an atom are of the order of an Angstrom.

S5. Here, the charge placed at the centre of the spherical cavity is positively charged. So, the charge created at the inner surface of the sphere, due to induction will be $-Q$ and due to this charge created at outer surface of the sphere is $+Q$.



(a) Now, surface charge density on the inner surface = $\dfrac{-Q}{4\pi R_1^2}$ and

(b) Surface charge density on the inner surface = $\dfrac{Q}{4\pi R_2^2}$.

S6. 1 Molar mass M of Al has $N_A = 6.023 \times 10^{23}$ atoms.

$\therefore$ m = mass of Al paisa coin has $N = N_A \dfrac{m}{M}$ atoms

Now, $Z_{Al} = 13$, $M_{Al} = 26.9815\,g$

Hence, $N = 6.02 \times 10^{23}$ atoms/mol $\times \dfrac{0.75}{26.9815\text{ g/mol}}$

$= 1.6733 \times 10^{22}$ atoms

$\therefore$ q = +ve charge in paisa = NZe

$= (1.67 \times 10^{22})(13)(1.60 \times 10^{-19}\,C)$

$= 3.48 \times 10^{4}\,C.$

$q = 34.8$ kC of ±ve charge.

This is an enormous amount of charge. Thus we see that ordinary neutral matter contains enormous amount of ± charges.

S7. (a) Zero, from symmetry.

(b) Removing a +ve Cs ion is equivalent to adding singly charged –ve Cs ion at that location.

Net force then is

$$F = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

where r = distance between the Cl ion and a Cs ion.

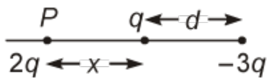
$$= \sqrt{(0.20)^2 + (0.20)^2 + (0.20)^2} \times 10^{-9} = \sqrt{3(0.20)^2} \times 10^{-9}$$

$$= 0.346 \times 10^{-9}\,m$$

Hence, $$F = \frac{(8.99 \times 10^9)(1.6 \times 10^{-19})^2}{(0.346 \times 10^{-9})^2} = 1.92 \times 10^{-9}\,N$$

Ans.: 1.92×10^{-9} N, directed from A to Cl^-.

S8. At P: on $2q$, Force due to q is to the left and that due to $-3q$ is to the right. For net force to be zero.



$$\therefore \quad \frac{2q^2}{4\pi\varepsilon_0 x^2} = \frac{6q^2}{4\pi\varepsilon_0 (d+x)^2}$$

$$\therefore \quad (d + x)^2 = 3x^2$$

$$\therefore \quad 2x^2 - 2dx - d^2 = 0$$

$$x = \frac{d}{2} \pm \frac{\sqrt{3}d}{2}$$

(–ve sign would be between q and $-3q$ and hence is unacceptable.)

$$x = \frac{d}{2} + \frac{\sqrt{3}d}{2} = \frac{d}{2}(1+\sqrt{3}) \text{ to the left of } q.$$

S9. Given, $q = \pm 34.8 \text{ kC} = \pm 3.48 \times 10^4 \text{ C}$,

$r_1 = 1 \text{ cm} = 10^{-2} \text{ m}$, $r_2 = 100 \text{ m}$, $r_3 = 10^6 \text{ m}$

(a) $F_1 = \frac{|q|^2}{4\pi\varepsilon_0 r_1^2} = \left(8.99 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}\right) \frac{(3,48 \times 10^4 \text{ C})}{10^{-4} \text{ m}^2} = 1.1 \times 10^{23} \text{ N}$

(b) $\frac{F_2}{F_1} = \frac{r_1^2}{r_2^2} = \frac{(10^{-2}\text{m})^2}{(10^2 \text{ m})^2} = 10^{-8} \quad \Rightarrow \quad F_2 = F_1 \times 10^{-8} = 1.1 \times 10^{15} \text{ N}$

(c) $\frac{F_3}{F_1} = \frac{r_1^2}{r_2^2} = \frac{(10^{-2}\text{ m})^2}{(10^6 \text{ m})^2} = 10^{-8} \quad \Rightarrow \quad F_3 = F_1 \times 10^{-16} = 1.1 \times 10^{7} \text{ N}$

Conclusion: When separated as point charges these charges exert an enormous force. It is not easy to disturb electrical neutrality.

S10. (a) There are eight corners in a cube so, total charge for the cube is $\frac{q}{8}$.

Thus, electric flux at $A = \frac{q}{8\varepsilon_0}$.

(b) When the charge q is place at B, middle point of an edge of the cube, it is being shared equally by 4 cubes. Therefore, total flux through the faces of the given cube = $q/4\varepsilon_0$.

(c) When the charge q is place at C, the centre of a face of the cube, it is being shared equally by 2 cubes. Therefore, total flux through the faces of the given cube = $q/2\varepsilon_0$.

(d) Similarly, when the charge q is place at Q, the mid-point of B and C, it is being shared equally by 2 cubes. Therefore, total flux through the faces of the given cube = $q/2\varepsilon_0$.

S11. (a) Charges A and C are positive since lines of force emanate from them.

(b) Charge C has the largest magnitude since maximum number of field lines are associated with it.

(c) (i) near A. There is no neutral point between a positive and a negative charge. A neutral point may exist between two like charges. From the figure we see that a neutral point exists between charges A and C. Also between two like charges the neutral point is closer to the charge with smaller magnitude. Thus, electric field is zero near charge A.

S12. (a)

$$F = \frac{kQq}{r^2} = 1\,\text{dyne} = \frac{[1\,\text{esu of charge}]^2}{[1\,\text{cm}]^2}$$

Or,

$$1 \text{ esu of charge} = 1\ (\text{dyne})^{1/2}\,(\text{cm})$$

Hence, $[1 \text{ esu of charge}] = [F]^{1/2} L = [MLT^{-2}]^{1/2} L = M^{1/2} L^{3/2} T^{-1}$

$[1 \text{ esu of charge}] = M^{1/2} L^{3/2} T^{-1}$

Thus charge in cgs unit is expressed as fractional powers (1/2) of M and (3/2) of L.

(b) Consider the coloumb force on two charges, each of magnitude 1 esu of charge separated by a distance of 1 cm:

The force is then 1 dyne = 10^{-5} N.

This situation is equivalent to two charges of magnitude xC separated by 10^{-2} m.

This gives:

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{x^2}{10^{-4}}$$

which should be 1 dyne = 10^{-5} N. Thus

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{x^2}{10^{-4}} = 10^{-5} \quad \Rightarrow \quad \frac{1}{4\pi\varepsilon_0} = \frac{10^{-9}}{x^2} \frac{\text{Nm}^2}{\text{C}^2}$$

With $x = \frac{1}{[3] \times 10^9}$, this yields

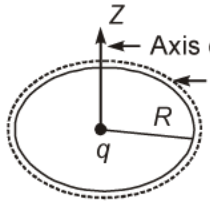
$$\frac{1}{4\pi\varepsilon_0} = 10^{-9} \times [3]^2 \times 10^{18} = [3]^2 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$$

With $[3] \to 2.99792458$, we get

$$\frac{1}{4\pi\varepsilon_0} = 8.98755 \ldots. \times 10^9 \frac{\text{Nm}^2}{\text{C}^2} \text{ exactly.}$$

S13. (a) Slight push on q along the axis of the ring gives rise to the situation shown in Fig (b). A and B are two points on the ring at the end of a diameter.

Force on q due to line elements $\frac{-Q}{2\pi R}$ at A and B is



(a)

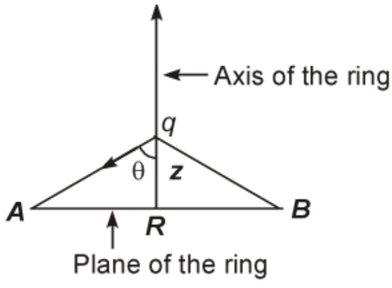
$$F_{A+B} = 2 \cdot \frac{-Q}{2\pi R} \cdot q \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{1}{r^2} \cdot \cos\theta$$

$$= \frac{-Qq}{\pi R \cdot 4\pi\varepsilon_0} \cdot \frac{1}{(z^2 + R^2)} \cdot \frac{z}{(z^2 + R^2)^{1/2}}$$

Total force due to ring on $q = (F_{A+B})(\pi R)$

$$= \frac{-Qq}{4\pi\varepsilon_0} \frac{z}{(z^2 + R^2)^{3/2}}$$

$$\Rightarrow \quad \frac{-Qq}{4\pi\varepsilon_0} \quad \text{for} \quad z \ll R$$



(b)

Thus, the force is proportional to negative of displacement. Motion under such forces is harmonic. Since acceleration is directly proportional to displacement.

(b) From (a)

$$m\frac{d^2z}{dt^2} = \frac{Qqz}{4\pi\varepsilon_0 R^3} \quad \text{or} \quad \frac{d^2z}{dt^2} = -\frac{Qq}{4\pi\varepsilon_0 mR^3}z$$

That is, $$\omega^2 = \frac{Qq}{4\pi\varepsilon_0 mR^3}.$$

Hence, $$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 mR}{Qq}}.$$

S14. Let us elaborate the figure first.

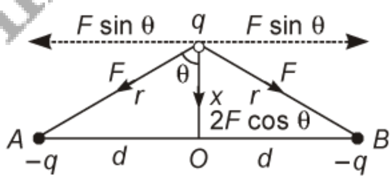
Given, two charge $-q$ at A and B

$$AB = AO + OB = 2d$$

x = small distance perpendicular to O.

i.e., $x < d$ mass of charge q is. So, force of attraction at P towards. A and B are each $F = \frac{q(q)}{4\pi\varepsilon_0 r^2}$, where $AP = BP = r$.

Horizontal components of these forces F_n are cancel out. Vertical components along PO add. If $\angle APO = \theta$, the net force on q along PO is $F' = 2F\cos\theta$



$$= \frac{2q^2}{4\pi\varepsilon_0 r^2}\left(\frac{x}{r}\right)$$

$$= \frac{2q^2 x}{4\pi\varepsilon_0 (d^2 + r^2)^{3/2}}$$

When $x \ll d$, $$F' = \frac{2q^2 x}{4\pi\varepsilon_0 d^3} = Kx$$

where, $$K = \frac{2q^2}{4\pi\varepsilon_0 d^3}$$

$$\Rightarrow \quad F \propto x$$

i.e., force on charge q is proportional to its displacement from the centre O and it is directed towards O.

Hence, motion of charge q would be smile harmonic, where

$$\omega = \sqrt{\frac{K}{m}}$$

and $$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}$$

$$= 2\pi\sqrt{\frac{m\,4\pi\varepsilon_0 d^3}{2q^2}} = \left[\frac{8\pi^2\varepsilon_0 n d^3}{q^2}\right]^{1/2}.$$

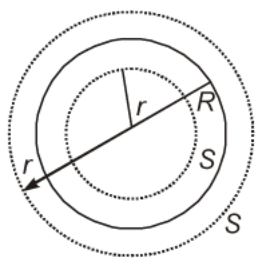
S15. (a) (i) The point O is equidistant from all the charges at the end point of pentagon. Thus, due to symmetry, the forces due to all the charges are cancelled out. As a result electric field at O is zero.

(ii) When charge q is removed a negative charge will develop at A giving electric field $E = \frac{q \times 1}{4\pi\varepsilon_0 r^2}$ along OA.

(iii) If charge q at A is replaced by $-q$, then two negative charges $-2q$ will develop there. Thus, the value of electric field $E = \frac{2q}{4\pi\varepsilon_0 r^2}$ along OA.

(b) When pentagon is replaced by n sided regular polygon with charge q at each of its corners, the electric field at O would continue to be zero as symmetricity of the charges is due to the regularity of the polygon. It doesn't depend on the number of sides or the number of charges.

S16. (a) Electric field is radial in case of sphere. For points $r < R$, consider a spherical Gaussian surfaces. Then on the surface



$$\oint \mathbf{E}_r \cdot d\mathbf{S} = \frac{1}{\varepsilon_0}\int_V \rho\, dv$$

$$4\pi r^2 E_r = \frac{1}{\varepsilon_0} 4\pi k \int_0^r r'^3 dr' = \frac{1}{\varepsilon_0}\frac{4\pi k}{4} r^4$$

$$\therefore \quad E_r = \frac{1}{4\varepsilon_0} k r^2$$

$$\mathbf{E}(r) = \frac{1}{4\varepsilon_0} k r^2 \hat{\mathbf{r}}$$

For points $r > R$, consider a spherical Gaussian surfaces' of radius

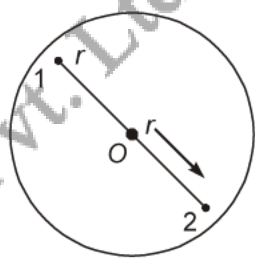
$$\oint \boldsymbol{E}_r \cdot d\boldsymbol{S} = \frac{1}{\varepsilon_0} \int_V \rho dv$$

$$4\pi r^2 E_r = \frac{4\pi k}{\varepsilon_0} \int_0^R r^3 dr = \frac{4\pi k}{\varepsilon_0} \frac{R^4}{4}$$

$$\therefore \quad E_r = \frac{k}{4\varepsilon_0} \frac{R^4}{4}$$

$$\mathbf{E}(r) = (k/4\varepsilon_0)\ (R^4/r^2)\ \hat{\mathbf{r}}$$

(b) The two protons must be on the opposite sides of the centre along a diameter. Suppose the protons are at a distance r from the centre.



Now, $$4\pi \int_0^R k r'^3 dr = 2e$$

$$\therefore \quad \frac{4\pi k}{4} R^4 = 2e$$

$$\therefore \quad k = \frac{2e}{\pi R^4}$$

Consider the forces on proton 1. The attractive force due to the charge distribution is

$$-e\mathbf{E}_r = \frac{e}{4\varepsilon_0} kr^2 \hat{\mathbf{r}} = -\frac{2e^2}{4\pi\varepsilon_0} \frac{r^2}{R^4} \hat{\mathbf{r}}$$

The repulsive force is $\left(\frac{e^2}{4\pi\varepsilon_0 4r^2} - \frac{2e^2}{4\pi\varepsilon_0} \frac{r^2}{R^4} \right) \hat{\mathbf{r}}$

Net force is $\frac{e^2}{4\pi\varepsilon_0} \frac{r^2}{(2r)^2} \hat{\mathbf{r}}$

This is zero such that

$$\frac{e^2}{16\pi\varepsilon_0 r^2} = \frac{2e^2}{4\pi\varepsilon_0} \frac{r^2}{R^4}$$

Or, $$r^4 = \frac{4R^2}{32} = \frac{R^4}{8}$$

$$\Rightarrow \quad r = \frac{R}{(8)^{1/4}}$$

Thus, the protons must be at a distance $r = \frac{R}{\sqrt[4]{8}}$ from the centre.

S17. (a) Let the Universe have a radius R. Assume that the hydrogen atoms are uniformly distributed. The charge on each hydrogen atom is

$$e_H = -(1 + y)\, e + e = -ye = |ye|$$

The mass of each hydrogen atom is ~ mp (mass of proton). Expansion starts if the Coulumb repulsion on a hydrogen atom, at R, is larger than the gravitational attraction.

Let the Electric Field at R be **E**. Then

$$4\pi R^2 E = \frac{4}{3\varepsilon_0}\pi R^3 N|ye| \quad \text{(Gauss's law)}$$

$$\mathbf{E}(R) = \frac{1}{3}\frac{N|ye|}{\varepsilon_0} R\hat{\mathbf{r}}$$

Let the gravitational field at R be G_R. Then

$$-4\pi R^2 G_R = 4\pi G\, m_p \frac{4}{3}\pi R^3 N$$

$$G_R = -\frac{4}{3}\pi G\, m_p NR$$

$$\mathbf{G}_R(\mathbf{R}) = -\frac{4}{3}\pi G\, m_p NR\hat{\mathbf{r}}$$

Thus the Coulombic force on a hydrogen atom at R is

$$ye\mathbf{E}(R) = \frac{1}{3}\frac{Ny^2e^2}{\varepsilon_0} R\hat{\mathbf{r}}$$

The gravitational force on this atom is

$$m_p \mathbf{G}_R(R) = -\frac{4\pi}{3} GN m_p^2 NR\hat{\mathbf{r}}$$

The net force on the atom is

$$\mathbf{F} = \left(\frac{1}{3}\frac{Ny^2e^2}{\varepsilon_0} R - \frac{4\pi}{3} GNm_p^2 R\right)\hat{\mathbf{r}}$$

The critical value is when

$$\frac{1}{3}\frac{Ny_c^2e^2}{\varepsilon_0} R = \frac{4\pi}{3} GNm_p^2 R$$

$$\Rightarrow \qquad y_c^2 = 4\pi\varepsilon_0 G\frac{m_p^2}{e^2}$$

$$= \frac{7\times10^{-11}\times1.8^2\times10^6\times81\times10^{-62}}{9\times10^9\times1.6^2\times10^{-28}} = 63\times10^{-38}$$

$\therefore \quad y_C \; 8\times10^{-19}\times10^{-18}.$

Because of the net force, the hydrogen atom experiences an acceleration such that

$$m_p \frac{d^2R}{dt^2} = \left(\frac{1}{3}\frac{Ny^2e^2}{e_0}R - \frac{4p}{3}GNm_p^2R\right) \qquad \left[a = \frac{d^2R}{dt^2}\right]$$

Or, $$\frac{d^2R}{dt^2} = a^2R \quad \text{where} \quad \alpha^2 = \frac{1}{m_p}\left(\frac{1}{3}\frac{Ny^2e^2}{e_0} - \frac{4p}{3}GNm_p^2\right)$$

This has a solution $R = Ae^{at} + Be^{-at}$

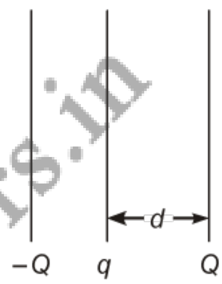
As we are seeking an expansion, $B = 0$.

$\therefore \quad R = Ae^{at}$

$\Rightarrow \quad \dot{R} = \alpha Ae^{at} = \alpha R.$

Thus, the velocity is proportional to the distance from the centre.

S18. (a) The electric field at γ due to plate α is $-\frac{Q}{S2\varepsilon_0}\hat{\mathbf{x}}$

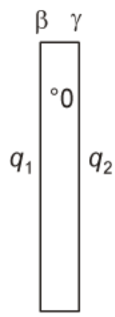


The electric field at γ due to plate β is $\frac{q}{S2\varepsilon_0}\hat{\mathbf{x}}$

Hence, the net electric field is

$$\mathbf{E}_1 = \frac{(Q-q)}{2\varepsilon_0 S}(-\hat{\mathbf{x}})$$

(b) During the collision plates β & γ are together and hence must be at one potential. Suppose the charge on β is q_1 and on γ is q_2. Consider a point O. The electric field here must be zero.



Electric field at 0 due to $\alpha = -\frac{Q}{2\varepsilon_0 S}\hat{\mathbf{x}}$

Electric field at 0 due to $\beta = -\frac{q_1}{2\varepsilon_0 S}\hat{\mathbf{x}}$

Electric Field at 0 due to $\gamma = -\frac{q_2}{2\varepsilon_0 S}\hat{\mathbf{x}}$

$$\therefore \quad \frac{-(Q+q)}{2\varepsilon_0 S} + \frac{q_1}{2\varepsilon_0 S} = 0$$

$$\Rightarrow \quad q_1 - q_2 = Q$$

Further, $q_1 + q_2 = Q + q$

$\Rightarrow$ $q_1 = Q + q/2$ and $q_2 = q/2$

Thus the charge on b and g are $Q + q/2$ and $q/2$, respectively.

(c) Let the velocity be v at the distance d after the collision. If m is the mass of the plate γ, then the gain in K.E. over the round trip must be equal to the work done by the electric field. After the collision, the electric field at γ is

$$\mathbf{E}_2 = -\frac{Q}{2\varepsilon_0 S}\hat{\mathbf{x}} + \frac{(Q+q/2)}{2\varepsilon_0 S}\hat{\mathbf{x}} = \frac{q/2}{2\varepsilon_0 S}\hat{\mathbf{x}}$$

The work done when the plate γ is released till the collision is $F_1 d$ where F_1 is the force on plate γ.

The work done after the collision till it reaches d is $F_2 d$ where F_2 is the force on plate γ.

$$F_1 = E_1 Q = \frac{(Q-q)Q}{2\varepsilon_0 S}$$

and $$F_2 = E_2 q/2 = \frac{(q/2)^2}{2\varepsilon_0 S}$$

$\therefore$ Total work done is force $\times$ displacement in the direction of force.

$$\frac{1}{2\varepsilon_0 S}[(Q-q)Q + (q/2)^2]d = \frac{1}{2\varepsilon_0 S}(Q-q/2)^2 d$$

$$\Rightarrow \quad (1/2)mv^2 = \frac{d}{2\varepsilon_0 S}(Q-q/2)^2$$

$$\therefore \quad v = (Q-q/2)\left(\frac{d}{m\varepsilon_0 S}\right)^{1/2}.$$