Date: 22/10/2021

Q1. There are two sources of light, each emitting with a power of 100 W. One emits X-rays of wavelength 1nm and the other visible light at 500 nm. Find the ratio of number of photons of X-rays to the photons of visible light of the given wavelength?

Q2. Do all the electrons that absorb a photon come out as photoelectrons?

Q3. There are materials which absorb photons of shorter wavelength and emit photons of longer wavelength. Can there be stable substances which absorb photons of larger wavelength and emit light of shorter wavelength.

Q4. (a) In the explanation of photo electric effect, we assume one photon of frequency ν collides with an electron and transfers its energy. This leads to the equation for the maximum energy E_{max} of the emitted electron as

$$E_{max} = h\nu - \phi_0,$$

where ϕ_0 is the work function of the metal. If an electron absorbs 2 photons (each of frequency ν) what will be the maximum energy for the emitted electron?

(b) Why is this fact (two photon absorption) not taken into consideration in our discussion of the stopping potential?

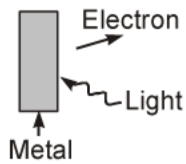
Q5. A proton and an α-particle are accelerated, using the same potential difference. How are the deBroglie wavelengths λ_p and λ_α related to each other?

Q6. Two monochromatic beams *A* and *B* of equal intensity ***I***, hit a screen. The number of photons hitting the screen by beam *A* is twice that by beam *B*. Then what inference can you make about their frequencies?

Q7. Consider a metal exposed to light of wavelength 600 nm. The maximum energy of the electron doubles when light of wavelength 400 nm is used. Find the work function in eV.

Q8. Two particles *A* and *B* of de Broglie wavelengths λ_1 and λ_2 combine to form a particle *C*. The process conserves momentum. Find the de Broglie wavelength of the particle *C*. (The motion is one dimensional).

Q9. Consider (see figure) for photoemission.

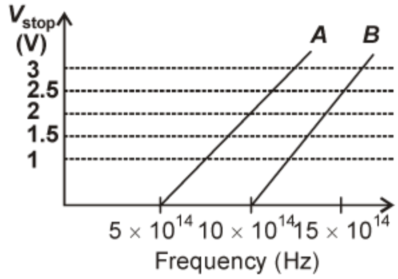


How would you reconcile with momentum-conservation? Note light (photons) have momentum in a different direction than the emitted electrons.

Q10. Assuming an electron is confined to a 1nm wide region, find the uncertainty in momentum using Heisenberg Uncertainty principle. You can assume the uncertainty in position Δx as 1 nm. Assuming $p - \Delta p$, find the energy of the electron in electron volts.

Q11. A neutron beam of energy *E* scatters from atoms on a surface with a spacing $d = 0.1$ nm. The first maximum of intensity in the reflected beam occurs at $\theta = 30°$. What is the kinetic energy *E* of the beam in eV?

Q12. A student performs an experiment on photoelectric effect, using two materials *A* and *B*. A plot of V_{stop} vs ν is given in figure.



(a) Which material *A* or *B* has a higher work function?

(b) Given the electric charge of an electron = 1.6×10^{-19} C, find the value of *h* obtained from the experiment for both *A* and *B*.

Comment on whether it is consistent with Einstein's theory:

Q13. Consider a thin target $(10^{-2})^2$ m^2, 10^{-3} m thickness) of sodium, which produces a photocurrent of 100 μA when a light of intensity 100 W/m^2 (λ = 660 nm) falls on it. Find the probability that a photoelectron is produced when a photons strikes a sodium atom. [Take density of Na = 0.97 kg/m^3].

Q14. Consider a 20 W bulb emitting light of wavelength 5000 Å and shining on a metal surface kept at a distance 2 m. Assume that the metal surface has work function of 2 eV and that each atom on the metal surface can be treated as a circular disk of radius 1.5Å.

(a) Estimate no. of photons emitted by the bulb per second. [Assume no other losses]

(b) Will there be photoelectric emission?

(c) How much time would be required by the atomic disk to receive energy equal to work function (2 eV)?

(d) How many photons would atomic disk receive within time duration calculated in (iii) above?

(e) Can you explain how photoelectric effect was observed instantaneously?

Q15. A particle A with a mass m_A is moving with a velocity *v* and hits a particle *B* (mass m_B) at rest (one dimensional motion). Find the change in the de Broglie wavelength of the particle *A*. Treat the collision as elastic.

Q16. Consider an electron in front of metallic surface at a distance *d* (treated as an infinite plane surface). Assume the force of attraction by the plate is given as $\frac{1}{4}\frac{q^2}{4\pi\varepsilon_0 d^2}$

Calculate work in taking the charge to an infinite distance from the plate. Taking *d* = 0.1 nm. find the work done in electron volts. [Such a force law is not valid for *d* < 0.1 nm].

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S1. Suppose wavelength of X-rays is λ_1 and the wavelength of visible light is λ_2.

Given, $P = 100\,\text{W}; \quad \lambda_1 = 1\,\text{nm}; \quad \lambda_2 = 500\,\text{nm}$

Also, n_1 and n_2 represents number of photons of X-rays and visible light emitted from the two sources per sec.

So, $$\frac{E}{t} = P = n_1\frac{hc}{\lambda_1} = n_2\frac{hc}{\lambda_2}$$

$$\Rightarrow \quad \frac{n_1}{\lambda_1} = \frac{n_2}{\lambda_2}$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{1}{500}.$$

S2. No, most electrons get scattered into the metal. Only a few come out of the surface of the metal.

S3. In the first case energy given out is less than the energy supplied. In the second case, the material has to supply the energy as the emitted photon has more energy. This cannot happen for stable substances.

S4. (a) Here, it is given that, an electron absorbs 2 photons each of frequency ν then $\nu' = 2\nu$ where ν is the frequency of emitted electron

Given, $E_{max} = h\nu - \phi_0$.

Now, maximum energy for emitted electrons is

$$E'_{max} = h(2\nu) - \phi_0 = 2h\nu - \phi_0$$

(b) The probability of absorbing 2 photons by the same electron is very low. Hence such emissions will be negligible.

S5. As, $$\lambda = \frac{h}{\sqrt{2mqV}}$$ [V = Potential difference]

$$\therefore \quad \lambda \propto \frac{1}{\sqrt{mq}}$$

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{\sqrt{m_\alpha q_\alpha}}{\sqrt{m_p q_p}} = \frac{\sqrt{4m_p \times 2e}}{\sqrt{m_p \times e}} = \sqrt{8}$$

$$\therefore \quad \lambda_p = \sqrt{8}\,\lambda_\alpha$$

i.e., wavelength of proton is $\sqrt{8}$ times wavelength of α-particle.

S6. Suppose, n_A is the number of photons falling per second of beam A and n_B is the number of photons falling per second of beam B.

Thus, $$n_A = 2n_B$$

Energy of falling photon of beam $A = n_A h \nu_A$

Energy of falling photon of beam $B = n_B h \nu_B$

Now, according to question,

$$\text{intensity of } A = \text{intensity of } B$$

$$n_A h\nu_A = n_B h\nu_B$$

$$\frac{n_A}{n_B} = 2 = \frac{\nu_B}{\nu_A}.$$

The frequency of beam B is twice that of A.

S7. Given, For the first condition,

Wavelength of light $\lambda = 600$ nm

and for the second condition,

Wavelength $\lambda' = 400$ nm

Also, maximum kinetic energy for the second condition is equal to the twice of the kinetic energy in first condition.

i.e., $$K'_{\max} = 2K_{\max}$$

Here, $$K'_{\max} = \frac{hc}{\lambda} - \phi$$

$$\Rightarrow \quad 2K_{\max} = \frac{hc}{\lambda'} - \phi$$

$$\left(\frac{1230}{600} - \phi\right) = \frac{1}{2}\left(\frac{1230}{400} - \phi\right)$$

$$\phi = \frac{1230}{1200} = 1.02 \text{ eV}.$$

S8. $$p_c = |p_A| + |p_B|$$

$$\Rightarrow \quad \frac{h}{\lambda_C} = \frac{h}{\lambda_A} + \frac{h}{\lambda_B} \qquad \left[\because \ \lambda = \frac{h}{mv} = \frac{h}{p} \ \Rightarrow \ p = \frac{h}{\lambda}\right]$$

$$\Rightarrow \quad \frac{h}{\lambda_C} = \frac{h\lambda_B + h\lambda_A}{\lambda_A \lambda_B}$$

$$\Rightarrow \quad \frac{\lambda_C}{h} = \frac{\lambda_A \lambda_B}{h\lambda_B + h\lambda_A}$$

$\Rightarrow$ $$\lambda_c = \frac{\lambda_A \lambda_B}{\lambda_A + \lambda_B}$$

Case I: Suppose both p_A and p_B are positive,

then $$\lambda_c = \frac{\lambda_A \lambda_B}{\lambda_A + \lambda_B}$$

Case II: When both p_A and p_B are negative,

then $$\lambda_c = \frac{\lambda_A \lambda_B}{\lambda_A + \lambda_B}$$

Case III: When $p_A > 0, \ p_B < 0$

then $$\lambda_c = \frac{\lambda_A \lambda_B}{\lambda_A - \lambda_B}$$

Case IV: When $p_A < 0, \ p_B > 0$

then $$\lambda_c = \frac{\lambda_A \lambda_B}{\lambda_A - \lambda_B}.$$

S9. The momentum is transferred to the metal. At the microscopic level, atoms absorb the photon and its momentum is transferred mainly to the nucleus and electrons. The excited electron is emitted. Conservation of momentum needs to be accounted for the momentum transferred to the nucleus and electrons.

S10. Here, Δx = 1 nm = 10^{-9}, Δp = ?

As $\Delta x \, \Delta p = h$

$\therefore$ $$\Delta p = \frac{h}{\Delta x} = \frac{h}{2\pi \Delta x}$$

$\Rightarrow$ $$= \frac{6.62 \times 10^{-34} \text{ Js}}{2 \times (22/7)(10^{-9}) \text{ m}}$$

$$= 1.05 \times 10^{-25} \text{ kg m/s}$$

$$E = \frac{p^2}{2m} = \frac{(1.05 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31}}$$

$$= \frac{1.05^2}{18.2} \times 10^{-19} \text{ J} = \frac{1.05^2}{18.2 \times 1.6} \text{ eV}$$

$$= 3.8 \times 10^{-2} \text{ eV}.$$

S11. Given, d = 0.1 nm

$$\theta = 30° \quad \Rightarrow \quad n = 1$$

$$2d \sin \theta = \text{Path difference}$$

Now, according Bragg's law:

$$2d \sin \theta = n\lambda \qquad \text{[Condition for constructive interference]}$$

$$\Rightarrow \qquad 2 \times 0.1 \times \sin 30 = 1\lambda$$

$$p = \frac{h}{10^{-10}} = \frac{6.6 \times 10^{-34}}{10^{-10}} = 6.6 \times 10^{-21}\,\text{kgm/s}$$

$$E = \frac{(6.6 \times 10^{-24})^2}{2 \times (1.7 \times 10^{-27})} \times 1.6 \times 10^{-19} = \frac{6.6^2}{2 \times 1.7} \times 1.6 \times 10^{-2}\,\text{eV}$$

$$= 20.5 \times 10^{-2}\,\text{eV} = 0.21\,\text{eV}.$$

S12. (a) Stopping potential = 0 at a higher frequency for *B*. Hence it has a higher work function.

(b)

$$\text{Slope} = \frac{2}{(10-5) \times 10^{14}} \quad \text{for } A.$$

$$= \frac{2.5}{(15-10) \times 10^{14}} \quad \text{for } B.$$

$$h = \frac{1.6 \times 10^{-19}}{5} \times 2 \times 10^{-14} = 6.04 \times 10^{-34}\,\text{Js for } A$$

$$= \frac{1.6 \times 10^{-19} \times 2.5 \times 10^{-14}}{5} = 8 \times 10^{-34}\,\text{Js for B}.$$

Since *h* works out differently, experiment is not consistent with the theory.

S13. Given,

$$A = (10^{-2})^2\,\text{m}^2 = 10^{-2} \times 10^{-2}\,\text{m}^2$$

$$= 10^{-4}\,\text{m}^2$$

$$d = 10^{-3}\,\text{m}$$

$$i = 100 \times 10^{-6}\,\text{A} = 10^{-4}\,\text{A}$$

Intensity, $I = 100\,\text{W/m}^2$

$\Rightarrow \qquad \lambda = 660\text{ nm} = 660 \times 10^{-9}\,\text{m}$

$$\rho_{Na} = 0.97\,\text{kg/m}^3$$

Avogadro's number = 6×10^{23} atoms

Volume of sodium target = $A \times d$

$= 10^{-4} \times 10^{-3}$

$\Rightarrow \qquad = 10^{-7}\,\text{m}^3$

We know that 6×10^{23} atoms of Na weights = 23 kg

So, volume of 6×10^{6} Na atoms = $\dfrac{23}{0.97}$ m^3

Volume occupied of 1 Na atom = $\dfrac{23}{0.97 \times 6 \times 10^{26}}\,\text{m}^3 = 3.95 \times 10^{-26}\,\text{m}^3$

No. of sodium atoms in the target = $\dfrac{10^{-7}}{3.95 \times 10^{-26}} = 2.53 \times 10^{18}$

Let n be the number of photons falling per second on the target.

Energy of each photon = hc/λ

Total energy falling per second on target = $\dfrac{nhc}{\lambda} = IA$

$$n = \frac{IA\lambda}{hc}$$

$$= \frac{100 \times 10^{-4} \times (660 \times 10^{-9})}{(6.62 \times 10^{-34}) \times (3 \times 10^{8})} = 3.3 \times 10^{16}$$

Let P be the probability of emission per atom per photon.

The number of photoelectrons emitted per second

$$N = P \times n \times ({}^{n}\text{Na})$$

$$= P \times (3.3 \times 10^{16}) \times (2.53 \times 10^{18})$$

Now, according to question,

$$i = 100\,\mu\text{A} = 100 \times 10^{-6} = 10^{-4}\,\text{A}.$$

Current, $i = Ne$

$\therefore$ $10^{-4} = P \times (3.3 \times 10^{16}) \times (2.53 \times 10^{18}) \times (1.6 \times 10^{-19})$

$\Rightarrow$ $P = \dfrac{10^{-4}}{(3.3 \times 10^{16}) \times (2.53 \times 10^{18}) \times (1.6 \times 10^{-19})}$

$= 7.84 \times 10^{-21}$

Thus, the probability of emission by a single photon on a single atom is very much less than 1. It is due to this reason, the absorption of two photons by an atom is negligible.

S14. Given, $P = 20\,\text{W}, \quad \lambda = 5000\,\text{Å} = 5000 \times 10^{-10}\,\text{m}$

$d = 2\,\text{m}, \quad \phi_0 = 2\,\text{eV}, \quad r = 1.5\,\text{Å} = 15 \times 10^{-10}\,\text{m}$

(a) Number of photon emitted by bulb per second is

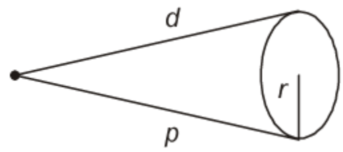
$$n' = \frac{p}{hc/\lambda} = \frac{p\lambda}{hc}$$

$$= \frac{20 \times (5000 \times 10^{-10})}{(6.62 \times 10^{-34}) \times (3 \times 10^{8})}$$

$\Rightarrow$ $= 5 \times 10^{19}\,\text{s}^{-1}$.

(b) Energy of the incident photon = $\frac{hc}{\lambda} = \frac{(6.62 \times 10^{-34}) \times (3 \times 10^8)}{5000 \times 10^{-10} \times 1.6 \times 10^{-19}}$

$= 2.48 \text{ eV}$



As this energy is greater than 2 eV (*i.e.*, work function of metal surface), hence photoelectric emission takes place.

(c) Let Δt be the time spent in getting the energy ϕ = (work function of metal).

Consider the figure,

$$\frac{P}{4\pi d^2} \times \pi r^2 \Delta t = \phi_0$$

$$\Rightarrow \qquad \Delta t = \frac{4\phi_0 d^2}{Pr^2}$$

$$= \frac{4 \times (2 \times 1.6 \times 10^{-19}) \times 2^2}{20 \times (1.5 \times 10^{-10})^2} \approx 28.4 \text{ s}$$

(d) Number of photons received by atomic disc in time Δt is

$$N = \frac{n' \times \pi r^2}{4\pi d^2} \times \Delta t$$

$$\Rightarrow \qquad = \frac{n' r^2 \Delta t}{4d^2}$$

$$= \frac{(5 \times 10^{19}) \times (1.5 \times 10^{-10})^2 \times 28.4}{4 \times (2)^2} \approx 2$$

(e) As time of emission of electrons is 11.04 s.

Hence, the photoelectric emission is not instantaneous in this problem.

In photoelectric emission, there is an collision between incident photon and free electron of the metal surface, which lasts for very very short interval of time ($\approx 10^{-9}$ s), hence we say photoelectric emission is instantaneous.

S15. As collision is elastic, hence laws of conservation of momentum and kinetic energy are obeyed.

According to law of conservation of momentum,

$$m_A v = m_A v_1 + m_B v_2$$

According to law of conservation of kinetic energy,

$$\frac{1}{2} m_A v^2 = \frac{1}{2} m_A v_1^2 + \frac{1}{2} m_B v_2^2$$

$$\therefore \qquad \frac{1}{2} m_A (v - v_1)(v_A - v_1) = \frac{1}{2} m_B v_B^2$$

$$\therefore \qquad v + v_1 = v_2 \quad \text{or} \quad v = v_2 - v_1$$

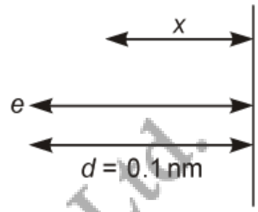
$$\therefore \qquad v_1 = \left(\frac{m_A - m_B}{m_A + m_B}\right)v \quad \text{and} \quad v_2 = \left(\frac{2m_A}{m_A + m_B}\right)v$$

$$\therefore \qquad \lambda_{\text{initial}} = \frac{h}{m_A v}$$

$$\lambda_{\text{initial}} = \frac{h}{m_A v} = \left|\frac{h(m_A + m_B)}{m_A(m_A - m_B)v}\right|$$

$$\therefore \qquad \Delta\lambda = \frac{h}{m_A v}\left[\left|\frac{m_A + m_B}{m_A - m_B}\right| - 1\right].$$

S16. According to question, consider the figure given below



From figure, d = 0.1 nm = 10^{-10} m,

Let the electron be at distance x from metallic surface. Then force of attraction on it is

$$F_x = \frac{q^2}{4 \times 4\pi\varepsilon_0 d^2}$$

Work done by external agency in taking the electron from distance d to infinity is

$$W = \int_d^\infty F_x\, dx = \int_d^\infty \frac{q^2 dx}{4 \times 4\pi\varepsilon_0} \frac{1}{x^2}$$

$$= \frac{q^2}{4 \times 4\pi\varepsilon_0}\left[\frac{1}{d}\right]$$

With d = 0.1nm, $$\text{Energy} = \frac{(1.6 \times 10^{-19})^2 \times 9 \times 10^9}{4 \times 10^{-10}} \text{ J}$$

$$= \frac{(1.6 \times 10^{-19})^2 \times (9 \times 10^9)}{(4 \times 10^{-10}) \times (1.6 \times 10^{-19})} \text{ eV} = 3.6 \text{ eV}.$$