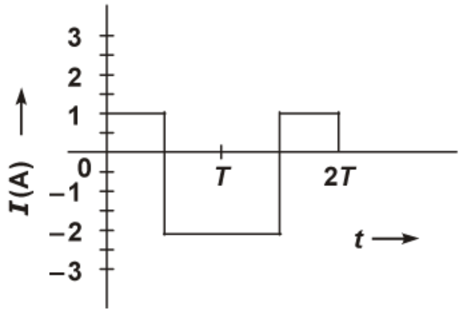
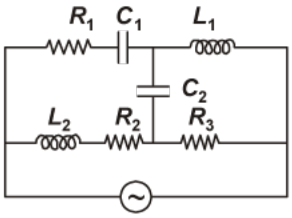
Date: 21/10/2021

Q1. **The alternating current in a circuit is described by the graph shown in figure. Show r.m.s. current in this graph.**



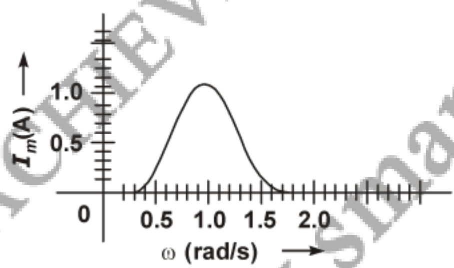
Q2. **If a *LC* circuit is considered analogous to a harmonically oscillating spring block system, which energy of the *LC* circuit would be analogous to potential energy and which one analogous to kinetic energy?**

Q3. **Draw the effective equivalent circuit of the circuit shown in figure, at very high frequencies and find the effective impedance.**

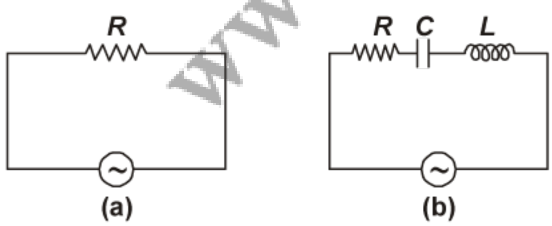


Q4. **How does the sign of the phase angle ϕ, by which the supply voltage leads the current in an *LCR* series circuit, change as the supply frequency is gradually increased from very low to very high values.**

Q5. **In series LCR circuit, the plot of I_{max} vs ω is shown in figure. Find the bandwidth and mark in the figure.**



Q6. **Study the circuits (a) and (b) shown in figure and answer the following questions.**



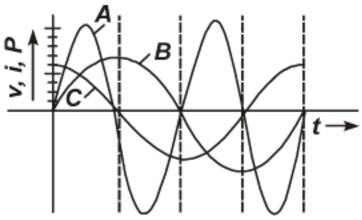
(a) **Under which conditions would the r.m.s. currents in the two circuits be the same?**

(b) **Can the r.m.s. current in circuit (b) be larger than that in (a)?**

Q7. Can the instantaneous power output of an ac source ever be negative? Can the average power output be negative?

Q8. Both alternating current and direct current are measured in amperes. But how is the ampere defined for an alternating current?

Q9. A device 'X' is connected to an a.c source. The variation of voltage, current and power in one complete cycle is shown in figure.



(a) Which curve shows power consumption over a full cycle?

(b) What is the average power consumption over a cycle?

(c) Identify the device 'X'.

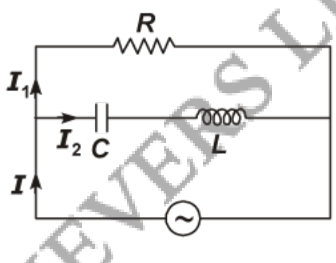
Q10. Explain why the reactance offered by an inductor increases with increasing frequency of an alternating voltage.

Q11. A 60 W load is connected to the secondary of a transformer whose primary draws line voltage. If a current of 0.54 A flows in the load, what is the current in the primary coil? Comment on the type of transformer being used.

Q12. A coil of 0.01 henry inductance and 1 ohm resistance is connected to 200 volt, 50 Hz A.C. supply. Find the impedance of the circuit and time lag between max. alternating voltage and current.

Q13. Explain why the reactance provided by a capacitor to an alternating current decreases with increasing frequency.

Q14. Consider the LCR circuit shown in figure. Find the net current I and the phase of I. Show that $I = \frac{V}{Z}$. Find the impedance Z for this circuit.



Q15. An electrical device draws 2 kW power from A.C. mains (voltage 223 V (r.m.s.) = $\sqrt{50,000}$ V). The current differs (lags) in phase by $\phi\left(\tan\phi = \frac{-3}{4}\right)$ as compared to voltage. Find (a) R, (b) $X_C - X_L$, and (c) I_M. Another device has twice the values for R, X_C and X_L. How are the answers affected?

Q16. 1 MW power is to be delivered from a power station to a town 10 km away. One uses a pair of Cu wires of radius 0.5 cm for this purpose. Calculate the fraction of ohmic losses to power transmitted if

(a) power is transmitted at 220 V. Comment on the feasibility of doing this.

(b) a step-up transformer is used to boost the voltage to 11000 V, power transmitted, then a step-down transformer is used to bring voltage to 220 V. ($\rho_{Cu} = 1.7 \times 10^{-8}$ SI unit)

Q17. For an *LCR* circuit driven at frequency ω, the equation reads $L\dfrac{dI}{dt}+RI+\dfrac{q}{C}=V_i=V_m\sin\omega t$

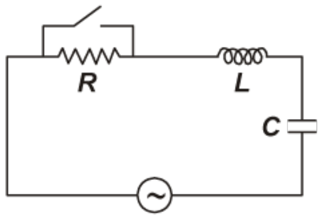
(a) Multiply the equation by I and simplify where possible.

(b) Interpret each term physically.

(c) Cast the equation in the form of a conservation of energy statement.

(d) Integrate the equation over one cycle to find that the phase difference between v and i must be acute.

Q18. In the *LCR* circuit shown in figure, the A.C. driving voltage is $V = V_m \sin \omega t$.



(a) Write down the equation of motion for $q(t)$.

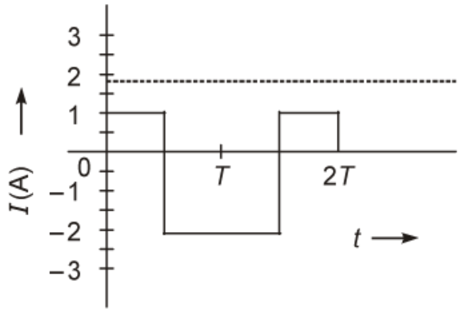
(b) At $t = t_0$, the voltage source stops and R is short circuited. Now write down how much energy is stored in each of L and C.

(c) Describe subsequent motion of charges.

Date: 21/10/2021

S1.

$$I_{r.m.s.} = \sqrt{\frac{1^2 + 2^2}{2}} = \sqrt{\frac{5}{2}} = 1.6\text{A}$$



S2. If we consider a *L*-*C* circuit analogous to a harmonically oscillating spring block system. The electrostatic energy $\frac{1}{2}CV^2$ is analogous to potential energy and energy associated with moving charges (current) that is magnetic energy $\left(\frac{1}{2}LI^2\right)$ is analogous to kinetic energy.

S3. We know that inductive reactance

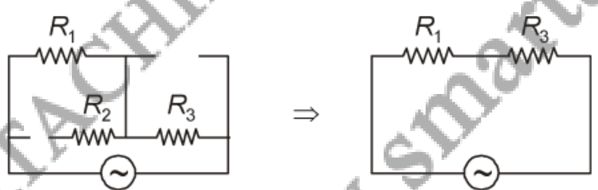
$$X_L = 2\pi fL$$

and capacitive reactance $X_C = \frac{1}{2\pi fC}$

For very high frequencies ($f \to \infty$), $X_L \to \infty$ and $X_C \to 0$

When reactance of a circuit is infinite it will be considered as open circuit. When reactance of a circuit is zero it will be considered as short circuited.

So, C_1, $C_2 \to$ shorted and L_1, $L_2 \to$ opened.



So, effective impedance = $R_{eq} = R_1 + R_3$.

S4. The phase angle (ϕ) by which voltage leads the current in *LCR* series circuit is given by

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{2\pi\nu L - \frac{1}{2\pi\nu C}}{R}$$

$$\tan\phi = 0 \text{ (for } \nu < \nu_0)$$

$$\tan\phi = 0 \text{ (for } \nu > \nu_0)$$

$$\tan\phi = 0 \qquad \left(\text{For} \quad \nu = \nu_0 = \frac{1}{2\pi\sqrt{LC}}\right)$$

S5. Bandwidth corresponds to frequencies at which $I_m = \frac{1}{\sqrt{2}} I_{\max} \approx 0.7\, I_{\max}$.

It is shown in the Fig.

$$\Delta\omega = 1.2 - 0.8 = 0.4 \text{ rad/s}$$

S6. Let, $(I_{\text{r.m.s.}})\, a$ = r.m.s. current in circuit (*a*)

$(I_{\text{r.m.s.}})\, b$ = r.m.s. current in circuit (*b*)

$$(I_{\text{r.m.s.}})\, a = \frac{V_{\text{r.m.s.}}}{R} = \frac{V}{R}$$

$$(I_{\text{r.m.s.}})\, a = \frac{V_{\text{r.m.s.}}}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

(a) When $(I_{\text{r.m.s.}})\, a = (I_{\text{r.m.s.}})\, b$

$$R = \sqrt{R^2 + (X_L - X_C)^2}$$

$\Rightarrow$ $X_L = X_C$, resonance condition

(b) As $Z \geq R$

$$\Rightarrow \quad \frac{(I_{\text{r.m.s.}})a}{(I_{\text{r.m.s.}})b} = \frac{\sqrt{R^2 + (X_L - X_C)^2}}{R}$$

$$\Rightarrow \quad = \frac{Z}{R} \geq 1$$

No, the r.m.s. current in circuit (b), cannot be larger than that in (a).

S7. Let the applied e.m.f.

$$E = E_0 \sin(\omega t)$$

and current developed is

$$I = I_0 \sin(\omega t \pm \phi)$$

Instantaneous power output of the A.C. source

$$P = EI = (E_0 \sin \omega t) \qquad [I_0 \sin(\omega t \pm \phi)]$$

$$= E_0 I_0 \sin \omega t \cdot \sin(\omega t + \phi)$$

$$= \frac{E_0 I_0}{2}[\cos\phi - \cos(2\omega t + \phi)] \quad \text{... (i)}$$

Average power $$P_{av} = \frac{V_0}{\sqrt{2}}\frac{I_0}{\sqrt{2}}\cos\phi$$

$$= V_{r.m.s.} I_{r.m.s.} \cos\phi \quad \text{... (ii)}$$

where ϕ is the phase difference.

Clearly, from Eq. (i)

when $\cos\phi < \cos(2\omega t + \phi)$

$P < 0$

Yes, the instantaneous power output of an A.C. source can be negative

From Eq. (ii) $P_{av} > 0$

Because $\cos\phi = \frac{R}{Z} > 0$

No, the average power output of an A.C. source cannot be negative.

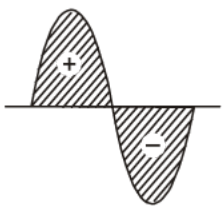
S8. An A.C. current changes direction with the source frequency and the attractive force would average to zero. Thus, the A.C. ampere must be defined in terms of some property that is independent of the direction of current. Joule's heating effect is such property and hence it is used to define r.m.s. value of A.C.

S9. (a) We know that Power = $P = VI$

that is curve of power will be having maximum amplitude, equals to multiplication of amplitudes of voltage (V) and current (I) curve, So, the curve will be represented by A.

(b) As shown by shaded area in the diagram, the full cycle of the graph consists of one positive and one negative symmetrical area.

Hence, average power over a cycle is zero.



(c) As the average power is zero, hence the device may be inductor (L) or capacitor (C) or the series combination of L and C.

S10. An inductor opposes flow of current through it by developing a back e.m.f. according to Lenz's law. The induced voltage has a polarity so as to maintain the current at its present value. If the current is decreasing, the polarity of the induced e.m.f. will be so as to increase the current and vice versa. Since the induced e.m.f. is proportional to the rate of change of current, it will provide greater reactance to the flow of current if the rate of change is faster, *i.e.*, if the frequency is higher. The reactance of an inductor, therefore, is proportional to the frequency, being given by ωL.

S11. $$P_L = 60\,\text{W}, \quad I_L = 0.54\,\text{A}$$

$$V_L = \frac{60}{0.54} = 110\,\text{V}. \quad \text{... (i)}$$

Voltage in the secondary (E_s) is less than voltage in the primary (E_p).

Hence, the transformer is step down transformer.

Since, the transformation ratio

$$r = \frac{V_s}{V_p} = \frac{I_p}{I_s}$$

Substituting the values. $\frac{110\text{ V}}{220\text{ V}} = \frac{I_p}{0.54\,A}$

On solving $I_p = 0.27\text{ A}$

S12. $X_L = \omega L = 2\pi f L$ [X_L = Inductive reactance]

$= 3.14\,\Omega$

$$Z = \sqrt{R^2 + L^2}$$

$$= \sqrt{(3.14)^2 + (1)^2} = \sqrt{10.86}\ ; \quad 3.3\,\Omega$$

$$\tan\phi = \frac{\omega L}{R} = 3.14$$

$$\phi = \tan^{-1}(3.14); \quad 72°; \quad \frac{72\times\pi}{180}\text{rad}$$

Timelag $\Delta t = \frac{\phi}{\omega} = \frac{72\times\pi}{180\times 2\pi\times 50} = \frac{1}{250}\text{ s}.$

S13. A capacitor does not allow flow of direct current through it as the resistance across the gap is infinite. When an alternating voltage is applied across the capacitor plates, the plates are alternately charged and discharged. The current through the capacitor is a result of this changing voltage (or charge). Thus, a capacitor will pass more current through it if the voltage is changing at a faster rate, *i.e.*, if the frequency of supply is higher. This implies that the reactance offered by a capacitor is less with increasing frequency; it is given by $1/\omega C$.

S14. In the given figure I is the total current from the source. It is divided into two parts I_1 through R and I_2 through series combination of C and L.

So, we can write $I = I_1 + I_2$

As $V_m \sin\omega t = RI_1$

$$I_1 = \frac{V_m \sin\omega t}{R} \quad \text{... (i)}$$

If q_2 is charge on the capacitor at any time t, then for series combination of C and L.

Applying KVL in the circuit as shown in figure

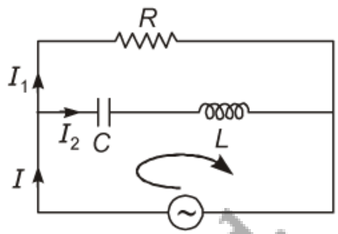
$$\frac{q_2}{C} + \frac{LdI}{dt} - V_m \sin \omega t = 0$$

$$\Rightarrow \quad \frac{q_2}{C} + L\frac{d_2 q_2}{dt^2} = V_m \sin \omega t \quad \text{... (ii)}$$

Let $$q_2 = q_m \sin(\omega t + \phi) \quad \text{... (iii)}$$

$$\frac{dq_2}{dt} = q_m \omega \cos(\omega t + \phi)$$

$$\frac{d_2 q_2}{dt^2} = q_m \omega^2 \sin(\omega t + \phi)$$



Now, putting these values in Eq. (ii), we geet

$$q_m \left(\frac{1}{C} - L\omega^2\right) \sin(\omega t + \phi) = V_m \sin \omega t$$

$$q_m = \frac{V_m}{\frac{1}{C} - L\omega^2}; \quad \phi = 0; \quad \frac{1}{C} - \omega^2 L > 0$$

$$V_R = \frac{V_m}{L\omega^2 - \frac{1}{C}}, \quad \phi = 0; \quad \pi\omega^2 L - \frac{1}{C} > 0$$

$$I_2 = \frac{dq_2}{dt} = \omega q_m \cos(\omega t + \phi)$$

I_1 and I_2 are out of phase. Let us assume $\frac{1}{C} - \omega^2 L > 0$

$$i_1 + i_2 = \frac{V_m \sin \omega t}{R} + \frac{V_m}{L\omega - \frac{1}{c\omega}} \cos \omega t$$

Now, $$A \sin \omega t + B \cos \omega t = C \sin(\omega t + \phi)$$

$$C \cos \phi = A, \quad C \sin \phi = B; \quad C = \sqrt{A^2 + B^2}$$

Therefore, $$I_1 + I_2 = \left[\frac{v_m^2}{R^2} + \frac{v_m^2}{[\omega t - 1/\omega C]^2}\right]^{\frac{1}{2}} \sin(\omega t + \phi)$$

$$\phi = \tan^{-1}\left(\frac{R}{X_L - X_C}\right)$$

$$\frac{1}{Z} = \left\{\frac{1}{R^2} + \frac{1}{(L\omega - 1/\omega C)^2}\right\}^{1/2}.$$

S15. Given, power drawn = P = 2kW = 2000 W

$$\tan\phi = -\frac{3}{4},\ I_M = I_0 = ?,\ R = ?,\ X_C - X_L$$

$$V_{\text{r.m.s.}} = V = 223\,\text{V}$$

(a) Power $P = \dfrac{V^2}{Z}$ [Z = Impedence of the circuit]

$$Z = \frac{V^2}{P}$$

$$= \frac{50,000}{2000} = 25\,\Omega$$

$$Z^2 = R^2 + (X_C - X_L)^2 = 625$$

$$\tan\phi = \frac{X_C - X_L}{R} = -\frac{3}{4}$$

$$625 = R^2 + \left(-\frac{3}{4}R\right)^2 = \frac{25}{16}$$

$$R^2 = 400 \quad \Rightarrow \quad R = 20\,\Omega$$

(b) $X_C - X_L = -15\,\Omega$

(c) $I = \dfrac{V}{Z} = \dfrac{223}{25} = 9\,\text{A}.$

$$I_M = \sqrt{2} \times 9 = 12.6\,\text{A}.$$

If R, X_C, X_L are all doubled, $\tan\phi$ does not change.

Z is doubled, current is halfed.

Power drawn is halfed.

S16. (a) Resistance of Cu wires, R

$$= \rho\frac{l}{A} = \frac{1.7 \times 10^{-8} \times 20000}{\pi \times \left(\frac{1}{2}\right)^2 \times 10^{-4}} = 4\Omega$$

I at 220 V : $VI = 10^6$ W; $I = \dfrac{10^6}{220} = 0.45 \times 10^4$ A

RI^2 = Power loss

$= 4 \times (0.45)^2 \times 10^8$ W

$> 10^6$ W.

This method cannot be used for transmission

(b) $$V'I' = 10^6 \text{ W} = 11000 I'$$

$$I' = \frac{1}{1.1} \times 10^2$$

$$RI'^2 = \frac{1}{1.21} \times 4 \times 10^4 = 3.3 \times 10^4 \text{ W}$$

$$\text{Fraction of power loss} = \frac{3.3 \times 10^4}{10^6} = 3.3\% .$$

S17. Consider the *LCR* circuit. Applying *KVL* for the loop, we can write

$$\Rightarrow \quad LI\frac{dI}{dt} + \frac{q}{C} + IR = V_m \sin \omega t \quad \text{... (i)}$$

Multiplying both sides by I, we get

$$LI\frac{dI}{dt} + \frac{q}{C}I + I^2R = (V_m I) \sin \omega t = V_m \sin \omega t \quad \text{... (ii)}$$

where, $\frac{q}{C}I = \frac{d}{dt}\left(\frac{q^2}{2C}\right)$ = Rate of change of energy stored in an inductor.

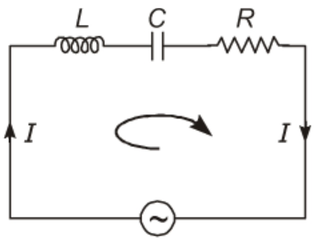
RI^2 = Joule heating loss

$LI\frac{dI}{dt} + \frac{d}{dt}\left(\frac{1}{2}LI^2\right)$ = Rate of change of energy stored in the capacitor.

V_I = rate at which driving force pours in energy. It goes into (i) ohmic loss and (ii) increase of stored energy.

Hence, Eq. (ii) is in the form of conservation of energy statement. Integrating both sides of Eq. (ii) with respect to time over one full cycle ($0 \to T$) we may write

$$\int_0^T \frac{d}{dt}\left(\frac{1}{2}LI^2 + \frac{q^2}{2C}\right)dt + \int_0^T RI^2 dt = \int_0^T VI\,dt$$



$$\Rightarrow \quad 0 + (+\text{ve}) = \int_0^T VI\,dt$$

$\Rightarrow \int_0^T VI\,dt > 0$ if phase difference between V and I is constant and acute angle.

S18. (a) $$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V_m \sin \omega t$$

Let, $$q = q_m \sin(\omega t + \phi) = -q_m \cos(\omega t + \phi)$$

$$I = I_m \sin(\omega t + \phi) = q_m \omega \sin(\omega t + \phi)$$

$$I_m = \frac{V_m}{Z} = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}; \quad \phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

(b)

$$U_L = \frac{1}{2}LI^2 = \frac{1}{2}L\left[\frac{V_m}{\sqrt{R^2 + (X_C - X_L)_0^2}}\right]\sin^2(\omega t_0 + \phi)$$

$$U_C = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2C}\left[\frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]\frac{1}{\omega^2}\cos^2(\omega t_0 + \phi)$$

(c) Left to itself, it is an *LC* oscillator. The capacitor will go on discharging and all energy will go to *L* and back and forth.