CLASS : CC (ADVANCED) **GRAVITATION & ELASTICITY** **M.M. 93** **TEST -14**

[SINGLE CORRECT CHOICE TYPE]

Q.1 A satellite in force free space sweeps stationary interplanetary dust at a rate (dM/dt) = αv, where M is the mass and v is the velocity of satellite and α is a constant. What is the deceleration of the satellite ? **[3]**

(A) $\frac{\alpha v^2}{M}$ (B) $-\alpha v^2 / M$ (C) $-\frac{\alpha v^2}{2M}$ (D) $-\frac{2\alpha v^2}{M}$

Q.2 The escape velocity from the earth is about 11 km/s. The escape velocity from the planet having twice the radius and the same density as the earth is **[3]**

(A) 22 km/s (B) 11 km/s (C) 5.5 km/s (D) 15.5 km/s

Q.3 A substance breaks down by a stress of 10^6 N/m^2. If the density of the material of the wire is 3×10^3 kg/m^2, then the length of the wire of that substance which will break under its own weight when suspended vertically is **[3]**

(A) 3.4 m (B) 30 m (C) 340 m (D) none of these

Q.4 One end of a long metallic wire of length L is tied to ceiling. The other end is tied to mass less spring of spring constant K. A mass m hangs freely from the end of spring. The area of cross-section and the Young modulus of wire are A and Y respectively. It mass is slightly pulled down and released, it will oscillate with a time period T equal to **[3]**

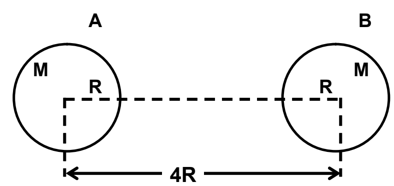
(A) $2\pi \sqrt{m/K}$ (B) $2\pi \sqrt{m(YA+KL)/YAK}$

(C) $2\pi \sqrt{mYA/KL}$ (D) None of these

Q.5 Two spherical massive bodies of uniform density each of mass M and radius R are kept at a distance 4R apart. Then the minimum speed (V_{min}) required to project a particle from the surface of body A such that it will never return to the surface of the same body is **[3]**

(A) $V_{min} = \sqrt{\frac{12GM}{5R}}$ (B) $V_{min} = \sqrt{\frac{2GM}{3R}}$

(C) $V_{min} = \sqrt{\frac{5GM}{7R}}$ (D) $V_{min} = \sqrt{\frac{2GM}{R}}$



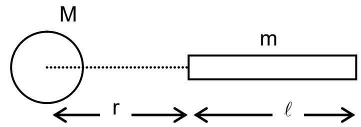
Q.6 Two rods A and B of the same material and length have radii r_1 and r_2, respectively. When they are rigidly fixed at one end and twisted by the same couple applied at the other end, the ratio $\left(\frac{\text{angle of twist at end of A}}{\text{angle of twice at the end of B}}\right)$ is : **[3]**

(A) $\frac{r_1^2}{r_2^2}$ (B) $\frac{r_1^3}{r_2^3}$ (C) $\frac{r_2^4}{r_1^4}$ (D) $\frac{r_1^4}{r_2^4}$

Q.7 There is a solid sphere of radius R and uniform mass density ρ in which there is a spherical cavity of radius (R/4). The centre of spherical cavity is at a distance of $\left(\frac{R}{2}\right)$ from the centre of the solid sphere. Then the gravitational field intensity at the centre of the spherical cavity is **[3]**

(A) $\frac{\pi G R \rho}{6}$ (B) $\frac{2}{3}\rho R \pi G$ (C) $\frac{\pi G R \rho}{3}$ (D) $\frac{\pi G R \rho}{4}$

Q.8 The gravitational force of attraction between a uniform sphere of mass M and a uniform rod of length ℓ and mass m oriented as shown is **[3]**



(A) $\frac{GMm}{r(r+\ell)}$ (B) $\frac{GM}{r^2}$ (C) $\frac{GMm}{r^2}$ (D) $\frac{GMm}{r}$

Q.9 One cm^3 of water is taken from the surface to the bottom of a lake 200 m deep. If the bulk modulus of water is 2.2×10^4 atmosphere, the density 10^3 kg/m^3 and atmospheric pressure is 10^5 N/m^2, then the change in volume is [Take g = 9.8 m/s^2] **[3]**
(A) 8.9×10^{-8} cm^3 (B) 8.9×10^{-7} cm^3 (C) 8.9×10^{-4} cm^3 (D) 8.9×10^{-2} cm^3

Q.10 In a double star system one of mass m_1 and other of mass m_2 with a separation d, rotate about their common centre of mass. Then the ratio of the areal velocity of star of mass m_1 to that of star of mass m_2 about their common C.M is **[3]**

(A) $\frac{m_1}{m_2}$ (B) $\frac{m_2}{m_1}$ (C) $\frac{m_1^2}{m_2^2}$ (D) $\frac{m_2^2}{m_1^2}$

Q.11 A geostationary satellite is revolving at a height 6R above the earth's surface (R being radius of earth). The period of revolution for a satellite revolving at a height 2.5 R above earth's surface will be (in hours) **[3]**

(A) 24 (B) $12\sqrt{2}$ (C) 12 (D) $6\sqrt{2}$

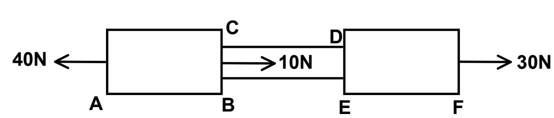
Q.12 A wire of length L and cross-section A is made of material of Young's modulus Y. It is stretched by an amount x the work done by external agent is **[3]**

(A) $\frac{YxA}{2L}$ (B) $\frac{Yx^2A}{L}$ (C) $\frac{Yx^2A}{2L}$ (D) $\frac{2Yx^2A}{L}$

Q.13 A rocket of initial mass 2m (rocket plus fuel) takes off in a straight line from a station in outer space. If the velocity of ejection of gas is u (relative to the rocket), the net K.E. of the rocket when its mass is m is (approximately) : **[3]**

(A) $\frac{1}{2}mu^2$ (B) $\frac{1}{4}mu^2$ (C) mu^2 (D) $\frac{1}{3}mu^2$

Q.14 Find the stress in CD. **[3]**
Area of CD = 2 m^2



(A) 15 N/m^2 (B) 5 N/m^2
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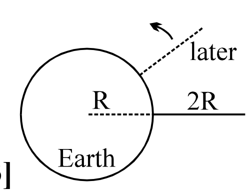
Q.15 Consider a very long stick of length 2R, which extends from just above the surface of the earth, to a radius 3R. If initial conditions have been set up so that this stick moves in a circular orbit while always pointing radially. What is the period of this orbit. g represents acceleration due to gravity on surface of earth. **[3]**



(A) $6\pi\sqrt{\frac{R}{2g}}$ (B) $2\pi\sqrt{6}\sqrt{\frac{R}{g}}$ (C) $\pi\sqrt{6}\sqrt{\frac{R}{g}}$ (D) none of these

[MULTIPLE CORRECT CHOICE TYPE]

Q.16 Earth orbiting satellite will escape if **[4]**

(A) Its speed is increased by 41 %

(B) Its speed in the orbit is made $\sqrt{1.5}$ times of its value

(C) Its KE is doubled

(D) It moves in a spiraling path moving in the orbit

Q.17 When a steel wire fixed at one end is pulled by a constant force F at its other end, its length in equilibrium increases by *l*. Which of the following statements is not correct? **[4]**

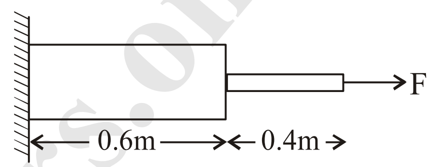
(A) Work done by the external force is F*l*.

(B) Some heat is produced in the wire in the process.

(C) The elastic potential energy of the wire is F*l*/2.

(D) The heat produced is equal to half of the elastic potential energy of the wire.

Q.18 Two bars of steel ($Y = 2 \times 10^{11}$ N/m^2) are joined together as shown. The area of cross section of the left bar is 15 cm^2 and the area of right bar is unknown. The extension in both bars is same. **[4]**



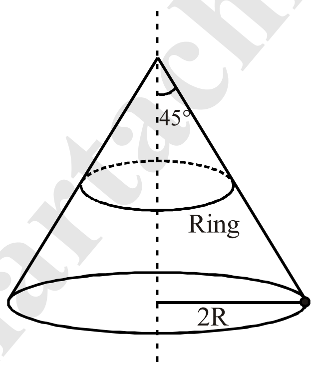
(A) The area of right bar is 10 cm^2

(B) The stresses in left and right bar are in ratio 3 : 2

(C) The decrease in thickness of bar is more for the left

(D) The decrease in thickness of bar is more for right bar

Q.19 A ring of mass m, radius R, cross-sectional area a and Young's modulus Y is kept on a smooth cone of radius 2R and semi-vertical angle 45°, as shown in the figure. Assume that the extension in the ring is small. **[4]**



(A) The tension in the ring will be same throughout.

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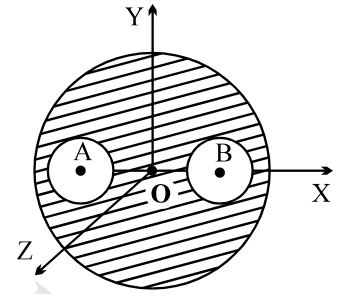
(D) Elastic potential energy stored in the ring will be $\frac{m^2g^2R}{8\pi Ya}$

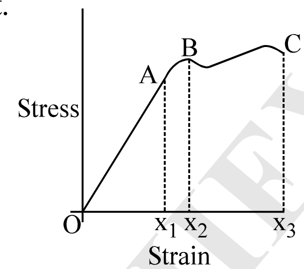
Q.20 A solid sphere of uniform density and radius 4 units is located with its centre at the origin O of coordinates. Two spheres of equal radii 1 unit, with their centres at A (– 2, 0, 0) and B (2, 0, 0) respectively are taken out of the solid leaving behind spherical cavities as shown **[4]** in figure.



(A) the gravitational field due to this object at the origin is zero
(B) the gravitational field at the point B (2, 0, 0) is zero
(C) the gravitational potential is the same at all points of the circle $y^2 + z^2 = 36$
(D) the gravitational potential is the same at all points of the circle $y^2 + z^2 = 4$

[MATCH THE COLUMN TYPE]

Q.21 A metal wire is pulled by equal forces at the ends. The stress-strain (x) relationship of the wire is given. Point B of graph is the elastic limit. **[8]**



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[SUBJECTIVE TYPE]

Q.22 A metal disc of radius R, density ρ and thickness t is rotating with angular velocity ω in gravity free space. Find the tensile stress in the disc at any distance r from its centre. Hence find the total elastic strain energy in the entire disc. The young's modulus of the disc's material is Y. **[5]**

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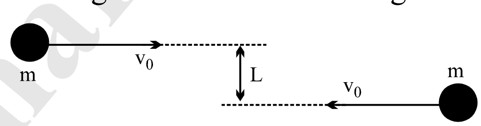

Q.24 A rocket starts vertically upwards with speed v_0 from the earth surface. Find its speed v at a height h, where R is the radius of the earth. Hence deduce the maximum height reached by a rocket fired with speed equal to 90% of escape velocity. **[5]**

Q.25 A cylindrical wire of radius 1 mm and length 1 m is subjected to a tension of 100 N. Find the change in radius of the wire. Poisson's ratio for the material = 0.25, Young's modulus = 2×10^{10} N/m^2. **[5]**

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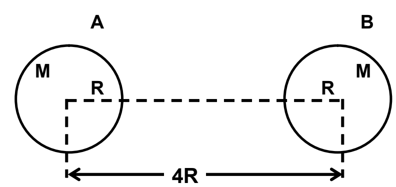

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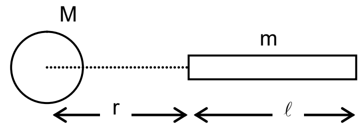

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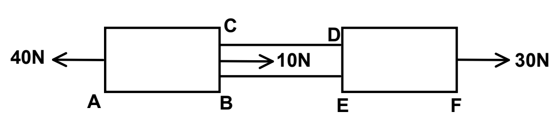

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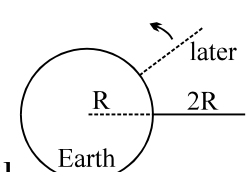

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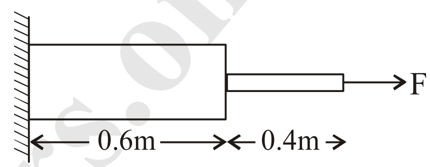

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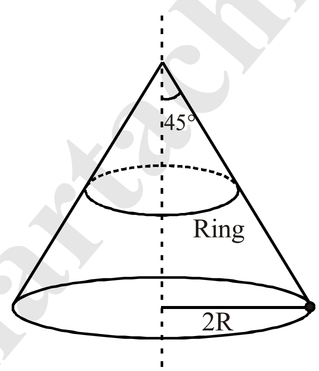

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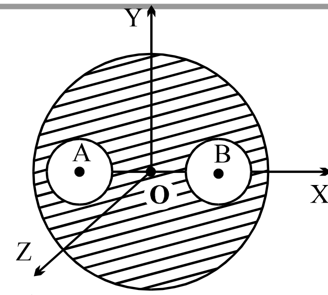

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[MATCH THE COLUMN TYPE]

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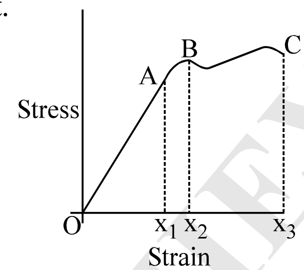


Column I	**Column II**
(A) $x < x_1$	(P) There is permanent deformation
(B) $x_1 < x < x_2$	(Q) Hook's law is valid
(C) $x_2 < x < x_3$	(R) There is no permanent deformation
(D) $x > x_3$	(S) The wire breaks

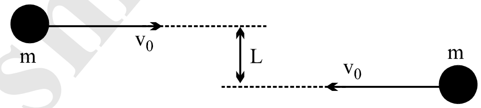
[Ans. (A) Q, R; (B) R; (C) P; (D) P, S]

[SUBJECTIVE TYPE]

Q.22 A metal disc of radius R, density ρ and thickness t is rotating with angular velocity ω in gravity free space. Find the tensile stress in the disc at any distance r from its centre. Hence find the total elastic strain energy in the entire disc. The young's modulus of the disc's material is Y. **[5]**

[Ans. $E = \dfrac{t\pi\rho^2\omega^4 r^6}{6Y}$]

Q.23 In the figure shown, the two tiny but heavy bodies having masses 'm' are initially very far away from each other. At the initial moment, they are imparted velocities $v_0 = \sqrt{\dfrac{Gm}{L}}$ as shown. Find their closest approach distance. The only force acting on them is their mutual gravitation attraction. **[5]**



$$r = \left(\frac{\sqrt{5}-1}{2}\right)L$$

Q.24 A rocket starts vertically upwards with speed v_0 from the earth surface. Find its speed v at a height h, where R is the radius of the earth. Hence deduce the maximum height reached by a rocket fired with speed equal to 90% of escape velocity. **[5]**

[Ans. $v_0^2 - v^2 = \dfrac{(2gh)}{\left(1+\dfrac{h}{r}\right)}$, $\dfrac{81}{19}R$]

Q.25 A cylindrical wire of radius 1 mm and length 1 m is subjected to a tension of 100 N. Find the change in radius of the wire. Poisson's ratio for the material = 0.25, Young's modulus = 2×10^{10} N/m^2. **[5]**

[Ans. $\Delta \mathbf{r} = \dfrac{\mathbf{1.25}}{\boldsymbol{\pi}} \times \mathbf{10^{-6}}$ **m**]