PHYSICS

The following question given below consist of an "Assertion" (A) and "Reason" (R) Type questions. Use the following Key to choose the appropriate answer.

(A) If both (A) and (R) are true, and (R) is the correct explanation of (A).

(B) If both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) If (A) is true but (R) is false.

(D) If (A) is false but (R) is true.

Q. 1 **Assertion:** The frequency of a second pendulum in an elevator moving up with an acceleration half the acceleration due to gravity is 0.612 s^{-1}.

Reason : The frequency of a second pendulum does not depend upon acceleration due to gravity. **[C]**

Q.2 **Assertion :** In SHM let x be the maximum speed, y the frequency of oscillation and z the maximum acceleration then $\frac{xy}{z}$ is a constant quantity.

Reason : This is because $\frac{xy}{z}$ becomes a dimensionless quantity. **[B]**

Q.3 **Assertion :** In simple harmonic motion A is the amplitude of oscillation. If t_1 be the time to reach the particle from mean position to $\frac{A}{\sqrt{2}}$ and t_2 the time to reach from $\frac{A}{\sqrt{2}}$ to A. Then $t_1 = \frac{t_2}{\sqrt{2}}$.

Reason : Equation of motion for the particle starting from mean position is given by $x = \pm A \sin \omega t$ and of the particle starting from extreme position is given by $x = \pm A \cos \omega t$.

[D]

Q.4 **Assertion :** In SHM the total mechanical energy may be negative.

Reason : Potential energy is always negative and if it is greater than kinetic energy total mechanical energy will be negative. **[C]**

Q.5 **Assertion :** In case of circular motion the projection of body on any diameter will not execute SHM if radius is not very small.

Reason : In SHM time period is independent of amplitude. **[D]**

Q.6 **Assertion (A)** : The amplitude of oscillation can never be infinite.

Reason (R) : The energy of oscillator is continuously dissipated. **[A]**

(A) Both 'A' and 'R' are true and 'R' is the correct explanation of 'A'

(B) Both 'A' and 'R' are true but 'R' is not the correct explanation of 'A'.

(C) 'A' is true but 'R' is false

(D) Both are false.

Q.7 **Assertion (A)** : Earth is in periodic motion around the sun.

Reason (R) : The motion of earth around the sun in not S.H.M. **[B]**

(A) Both 'A' and 'R' are true and 'R' is the correct explanation of 'A'

(B) Both 'A' and 'R' are true but 'R' is not the correct explanation of 'A'.

(C) 'A' is true but 'R' is false

(D) Both are false.

Q.8 **Assertion** : A particle of mass 0.1 kg executes SHM under a force F = (–10 x) newton. Speed of particle at mean position is 6 m/s. Then amplitude of oscillations is 0.6 m.

Reason : There is a conservation of mechanical energy. **[A]**

Q.9 **Assertion** : The maximum velocity is SHM is v_m. The average velocity during motion from one extreme position to other extreme position will be $2v_m/\pi$.

Reason : Average velocity is the mean of the maximum and minimum velocity of particle is SHM. **[C]**

Q.10 **Assertion** : Two SHM's along x and y axes with angular frequency ratio $\omega_1 : \omega_2 = 1 : 2$, with same amplitude results in a parabolic path on super-position.

Reason : The x and y displacements are related as $y \propto x^2$. **[A]**

Q.11 **Assertion** : The length of a simple pendulum is increased by 4%. The corresponding decrease in time period will be 2%.

Reason : $t \propto \sqrt{\ell}$. **[B]**

Q.12 **Assertion** : A simple pendulum has time period 2 s in air. If the whole arrangement is placed in a non-viscous liquid of density $\sigma = 1/2$ times the density of bob ρ_1, the time period in the liquid will be $2\sqrt{2}$ s.

Reason : $g_{eff} = \left(1 - \frac{\sigma}{\rho}\right)$ **[A]**

Q.13 In the SHM of a particle

Assertion : (1) the average value of particle velocity over one time period is zero

Reason : (2) the acceleration of particle is maximum at extreme position

(A) (1) and (2) both are wrong

(B) (1) and (2) both are correct

(C) (1) is wrong, (2) is correct

(D) (1) is correct, (2) is wrong **[B]**

Sol. As in SHM ; $v = A\omega \cos \omega t$

and $a = -\omega^2 x$

Q.14 **Assertion :** If in SHM x be the maximum speed, y be the frequency as oscillation and z be the maximum acceleration then $\frac{xy}{z}$ is a constant quantity.

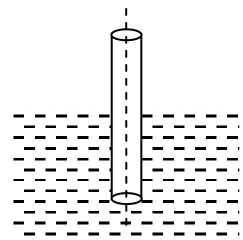
Reason : $\frac{xy}{z}$ is a dimensionless quantity. **[B]**

Sol. $x = \omega A$, $y = \frac{1}{T}$, $z = \omega^2 A$

$$\therefore xy = \frac{\omega A}{T} = \frac{\omega^2 A}{2\pi} = \frac{z}{2\pi}$$

Q.15 A homogeneous cylinder is floating in liquid as shown.

Assertion : The cylinder can perform both translational and rotational simple harmonic oscillations, for small displacement.



Reason : Translatory equilibrium of cylinder in stable and rotational equilibrium is unstable. **[D]**

Sol. The meta-centre is below the centre of gravity.

Q.16 **Assertion :** An oscillatory motion will be SHM only if amplitude of oscillation is very small.

Reason : In simple harmonic motions time period is independent of amplitude. **[D]**

Q.17 **Statement-I:** In SHM let x be the maximum speed, y the frequency of oscillation and z the maximum acceleration then $\frac{xy}{z}$ is a constant quantity.

Statement-II: This is because $\frac{xy}{z}$ becomes a dimensionless quantity. **[B]**

Q.18 **Statement-I :** In simple harmonic motion A is the amplitude of oscillation. If t_1 be the time to reach the particle from mean position to $\frac{A}{\sqrt{2}}$ and t_2 the time to reach from $\frac{A}{\sqrt{2}}$ to A. Then $t_1 = \frac{t_2}{\sqrt{2}}$

Statement-II: Equation of motion for the particle starting from mean position is given by $x = \pm A \sin \omega t$ and of the particle starting from extreme position is given by $x = \pm A \cos \omega t$ **[D]**

Q.19 **Statement-I:** If a man with a wrist watch on his hand falls from the top of a tower, its watch gives correct time during the free fall.

Statement-II: The working of the wrist watch depends on spring action and it has nothing to do with gravity. **[A]**

Q.20 **Statement-I :** The graph of potential energy and kinetic energy of a particle in SHM with respect to position is a parabola.

Statement-II : Potential energy and kinetic energy do not vary linearly with position. **[B]**

Q.1 In SHM match the following :

Column-I	**Column-II**
(A) Acceleration-displacement graph	(P) Parabola
(B) Velocity-acceleration graph	(Q) Straight line
(C) Acceleration-time graph	(R) Circle
(D) Velocity-time graph	(S) None

Sol. **A → Q ; B → S ; C → S ; D → S**

$\alpha = -\,\omega^2$ i.e., a – x graph is straight line passing through origin.

If $v = v_0 \sin \omega t$

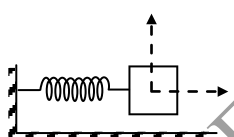
then $a = \frac{dv}{dt} = v_0\, \omega \cos \omega t = v_0\, \omega \sqrt{1-\sin^2 \omega t}$

$= v_0\omega \sqrt{1-\frac{v^2}{v_0^2}}$

$a = \omega\sqrt{v_0^2 - v^2}$

So, a-v graph is neither a straight line nor a parabola. Further, acceleration and velocity time graphs are sine or consine functions.

Q.2 A block of mass m = 2 kg is connected with spring (k = 3200 N/m) and placed on a frictionless horizontal surface as shown in figure. Initially it is compressed by a distance 10 cm. It is then released at t = 0.



Column I	**Column II**
(A) Time for the mass to move by distance first 5 cm	(P) $\frac{\pi}{160}$ sec
(B) Time for the mass to move by next 5 cm distance	(Q) $\frac{\pi}{240}$ sec
(C) Time at which kinetic energy and potential energy becomes equal for the first time	(R) $\frac{\pi}{120}$ sec
(D) Time at which kinetic energy become one-fourth of its maximum value	(S) $\frac{\pi}{40}$ sec

Sol. **A → R; B → Q; C → P ; D → Q**

Time period is given by

$T = 2\pi\sqrt{\frac{m}{k}} = \frac{\pi}{20}$ sec

Displacement of particle is given by (taking mean position as origin)

$x = -A\cos\omega t = -10\cos\frac{\pi t}{20}$

<u>Kinetic energy :</u>

$K = \frac{1}{2}m\omega^2A^2\sin^2\omega t = K_{max}\sin^2\left(\frac{\pi}{20}t\right)$

<u>Potential energy :</u>

$U = \frac{1}{2}m\omega^2A^2\cos^2\omega t = U_{max}\cos^2\omega t$

$= U_{max}\left(\frac{\pi}{20}\right)t$

Now,

$x = -5 \text{ cm} \Rightarrow t = \frac{T}{6}$

$x = 0 \Rightarrow t = \frac{T}{4}$

Time traveling in first 5 cm distance

$= \frac{T}{6} = \frac{\pi}{120}$ sec

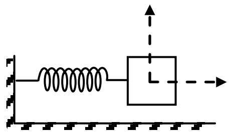
Time traveling in next 5 cm distance

$= \frac{T}{4} - \frac{T}{6} = \frac{\pi}{240}$ sec

$U = K \Rightarrow \omega t = \frac{\pi}{4} \Rightarrow t = \frac{\pi}{160}$ sec

$K = \frac{K_{max}}{4} \Rightarrow \omega t = \frac{\pi}{12} \Rightarrow t = \frac{\pi}{240}$ sec

Q.3 A block of mass m = 2 kg is connected with spring (k = 3200 N/m) and placed on a frictionless horizontal surface as shown in figure. Initially it is compressed by a distance 10 cm. It is then released at t = 0.



Column I	Column II
(A) Time for the mass to move by distance first 5 cm	(P) $\frac{\pi}{160}$ sec
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(C) Time at which kinetic energy and potential energy becomes equal for the first time	(R) $\frac{\pi}{120}$ sec
(D) Time at which kinetic energy become one-fourth of its maximum value	(S) $\frac{\pi}{40}$ sec

Ans. **A → R ; B → Q ; C → P ; D → Q**

Q.4 x-t equation of a particle in SHM is given as : x = 1.0 sin (12πt) in SI units. Potential energy at mean position is zero. Mass of particle is $\frac{1}{4}$ kg. Match the following table :

Table-1	Table-2
(A) Frequency with which kinetic energy oscillates	(P) $\frac{1}{2}$
(B) Speed of particle is maximum at time t =	(Q) $18\pi^2$
(C) Maximum potential energy	(R) 12
(D) Force constant K	(S) $36\pi^2$

Ans. **A → R ; B → P ; C → Q ; D → S**

Q.5 In SHM match the following :

Table-1	Table-2
(A) Acceleration-displacement graph	(P) Parabola
(B) Velocity-acceleration graph	(Q) Straight line
(C) Acceleration-time graph	(R) Circle
(D) Velocity-time graph	(S) None

Ans. **A → Q ; B → S ; C → S ; D → S**

Q.6 **Column** I describes some situations in which a small object moves. **Column** II describes some characteristics of these motions. Match the situations in **Column** I with the characteristics in **Column** II. **[IIT-2007]**

Column – I	Column – II
(A) The object moves on the x–axis under a conservative force in such away that its "speed" and "position satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants.	(P) The object executes a simple harmonic motion.
(B) The object moves on the x–axis in such away that its velocity and its displacement from the origin satisfy v = –kx, where k is a positive constant.	(Q) The object does not change its direction.
(C) The object is attached to one end of a mass–less spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upward with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration.	(R) The kinetic energy of the object keeps on decreasing.
(D) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{GM_e/R_e}$, where M_e is the mass of the earth and R_e is the radius of the earth. Neglect forces from objects other than the earth.	(S) The object can change its direction only once.

Ans. **A→P ; B → Q, R ; C → P ; D → R,Q**

Q.7 **Column I** gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in **Column II**. Match the set of parameters given in **Column I** with the graphs given in **Column II**.

[IIT-2008]

Column I

(A) Potential energy of a simple pendulum

(y-axis) as a function of displacement (x axis)

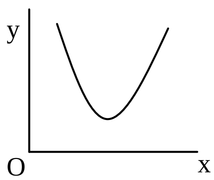
(B) Displacement (y axis) as a function of time (x axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x –direction

(C) Range of a projectile (y axis) as a function of its velocity (x axis) when projected at a fixed angle

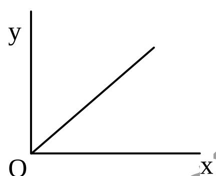
(D) The square of the time period (y axis) of a simple pendulum as a function of its length (x axis)

Column II

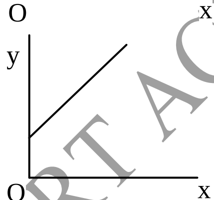
(P)



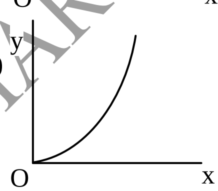
(Q)



(R)



(S)



Ans. **A → P; B → Q, S C → S; D → Q**

Q.8 A body of mass 'm' is acted upon by net force $\vec{F}$. $\vec{r}$ and $\vec{v}$ be their initial position and velocity. All quantity are in S.I. unit . Then match the following.

Column I	**Column II**
(A) $\vec{F} = -4x^2\hat{i}$, $\vec{r} = 0$, $\vec{v} = 2\hat{i}, m = 2$	(P) Motion : S.H.M.
(B) $\vec{F} = (2-4y)\hat{j}$, $\vec{r} = 0$, $\vec{v} = 0$, m = 1	(Q) Motion : Non-periodic
(C) $\vec{F} = -2(x+x^3)\hat{i}$, $\vec{r} = 10^{-2}\hat{i}$, $\vec{v} = 0$, m = 1	(R) Path : Straight line
(D) $\vec{F} = -4(x\hat{i}+y\hat{j})$, $\vec{r} = 2\hat{j}$, $\vec{v} = 6\hat{i}, m = 1$	(S) Time period : π

Sol. **A → Q; B → P,R,S; C → P,R ; D → S**

$\vec{F} = -4x^2\,\hat{i}$ is a non-restoring force, hence motion be non-periodic. As $\vec{F}$ and $\vec{v}$ along same line path will be straight line.

$\vec{F} = (2-4y)\hat{j}$

Mean position : $y = \frac{1}{2}$

Let 'x' be displacement from mean position

∴ F = – 4x ∴ Motion = S.H.M.

$\Rightarrow T = \frac{2\pi}{\omega} = \pi$ sec

$\vec{F} = -2(x+x^3)\hat{i}$

For small displacement = 10^{-2} m

$F \approx -2x$

∴ Motion will be S.H.M. and

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi$ sec

$\vec{F} = -4(x\hat{i} + y\hat{j})$,

$\Rightarrow F_x = -4x\hat{i}$, $F_y = -4y\hat{j}$

⇒ x = 3 sin ωt, y = 2 cos ωt, [where ω = 2]

∴ Motion periodic and time period

$T = \frac{2\pi}{2} = \pi$ sec

Q.9 In SHM match the following columns :

Column I	**Column II**
(a) Displacement and velocity	(p) Phase difference of zero
(b) Displacement and acceleration	(q) Phase difference of $\pi/2$
(c) Velocity and acceleration	(r) Phase difference of π

(A) (a → q), (b → r), (c → q)
(B) (a → r), (b → q), (c → p)
(C) (a → p), (b → q), (c → r)
`
(D) None of these

Sol. **[A]** Suppose x = A sin ωt

then $v = \frac{dx}{dt} = \omega A \cos \omega t$

and $a = \frac{dv}{dt} = -\omega^2 A \sin\omega t$

Q.10 In the equation y = A sin (ωt + π/4) match the following: For $x = \frac{A}{2}$.

Column-I	**Column-II**
(A) Kinetic energy	(P) Half the maximum value
(B) Potential energy	(Q) 3/4 times the maximum value
(C) Acceleration	(R) 1/4 times the maximum value
	(S) cannot say anything

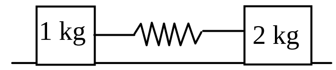
Ans. A → Q; B → S; C → P

Q.11 In $y = A \sin \omega t + A \sin t\left(\omega t + \frac{2\pi}{3}\right)$ match the following table:

Column-I	**Column-II**
(A)Motion	(P) is periodic but not SHM
(B) Amplitude	(Q) is SHM
(C) Initial phase	(R) A
(D) Maximum velocity	(S) $\pi/3$
	(T) ω A/2
	(U) None

Ans. A → Q; B → R; C → S; D → U

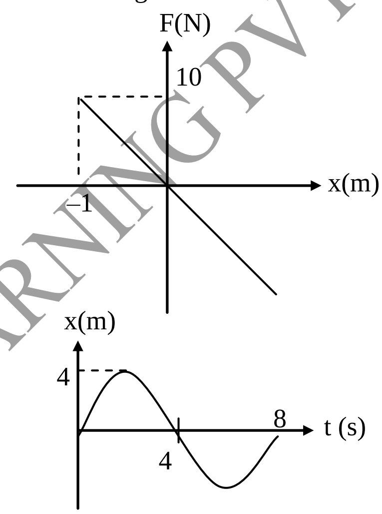
Q.12 In the two block spring system, force constant of spring is K = 6N/m. Spring is stretched by 12 cm and then left. Match the following:



Column-I	**Column-II**
(A) Angular frequency of oscillation	(P) 4.8×10^{-3} SI unit
(B) Maximum kinetic energy of 1 kg	(Q) 3 SI unit
(C) Maximum kinetic energy of 2 kg	(R) 2.4×10^{-3} SI unit
	(S) None

Ans. A → Q; B → S; C → P

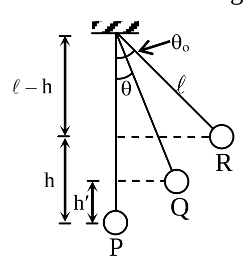
Q.13 F-*x* and x-t graph of a particle in SHM are as shown in figure. Match the following:



Column-I	**Column-II**
(A) Mass of the particle	(P) $\pi/2$ SI unit
(B) Maximum kinetic energy of particle	(Q) $(160/\pi^2)$SI unit
(C) Angular frequency of particle	(R) (8.0×10^{-3})SI unit
	(T) None

Ans. A → Q; B → R; C → T

Q.14 A simple pendulum is vibrating with maximum angle θ_0 If ℓ is length of pendulum and m is mass of bob then match the following for $\theta < \theta_0$.



Column -I	Column-II
(A) Potential Energy	(P) $mg(\cos\theta - \cos\theta_0)$
(B) Kinetic Energy	(Q) $mg\ell(1 - \cos\theta)$
(C) Momentum	(R) $m\sqrt{2g\ell(\cos\theta - \cos\theta_o)}$
(D) Momentum at midpoint	(S) $m\sqrt{2gh}$

Sol. **(A)** $\to$ Q **(B)** $\to$ P
(C) $\to$ R **(D)** $\to$ S

Q. 1 A particle is subjected to two simple harmonic motion along x and y-directions according to equations

$x = 4\sin 100\pi t$ and $y = 3\sin 100\pi t$

Choose the correct statement –

(A) Motion of particle will be on an ellipse

(B) Motion of the particle will be on a straight line

(C) Particle will execute SHM of amplitude 5

(D) Particle will not execute SHM **[B,C]**

Q.2 Which of the following quantities are always negative in a simple harmonic motion ?

(A) $\vec{F}.\vec{a}$ (B) $\vec{v}.\vec{r}$

(C) $\vec{a}.\vec{r}$ (D) $\vec{F}.\vec{r}$ **[C,D]**

Q.3 Which of the following quantities are always zero in a simple harmonic motion ?

(A) $\vec{F}\times\vec{a}$ (B) $\vec{v}\times\vec{r}$

(C) $\vec{a}\times\vec{r}$ (D) $\vec{F}\times\vec{r}$

[A,B,C,D]

Q.4 The speed v of a particle moving along a straight line, when it is at a distance x from a fixed point on the line is $v^2 = 144 - 9x^2$. Select the correct alternative(s) :

(A) The motion of the particle is SHM with time period $T = \frac{2\pi}{3}$ unit

(B) The maximum displacement of the particle from the fixed point is 4 unit

(C) The magnitude of acceleration at a distance 3 units from the fixed point is 27 unit

(D) The motion of the particle is periodic but not simple harmonic

Sol. **[A,B,C]**

$v^2 = 144 - 9x^2$

$2v\frac{dv}{dx} = -18x$

$\therefore v\frac{dv}{dx} = -9x \Rightarrow \omega = 3 \Rightarrow T = \frac{2\pi}{3}$ units

Also, $v^2 = 144 - 9A^2 = 0$

$A = 4$ units

Now, $|a| = \omega^2 x = 9 \times 3 = 27$ units

Q.5 A SHM is given by $y = (\sin\omega t + \cos\omega t)$. Which of the following statement are true-

(A) The amplitude is 1m

(B) The amplitude is $\sqrt{2}$ m

(C) When t = 0, the amplitude is 0 m

(D) When t = 0, the amplitude is 1 m

Sol. **[B,D]** $y = \frac{1}{\sqrt{2}}(\sqrt{2}\sin\omega t + \sqrt{2}\cos\omega t)$

$y = \sqrt{2}\sin(\frac{\pi}{4} + \omega t) = A\sin(\omega t + \frac{\pi}{4})$

$A = \sqrt{2}$, $t = 0$

$y = \sqrt{2}\sin\frac{\pi}{4}$

$y = \sqrt{2} \times \frac{1}{\sqrt{2}} = 1$m

Q.6 For a particle undergoing S.H.M -

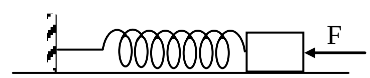
(A) Total mechanical energy must be conserved in a round trip

(B) Total mechanical energy must be conserved throughout the motion

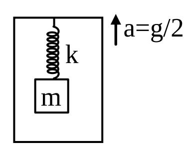
(C) Minimum potential energy must correspond maximum kinetic energy

(D) Minimum kinetic energy may correspond maximum potential energy **[A, D]**

Sol. Consider a situation



Q.7 A spring block system is in a lift moving upwards with acceleration a = g/2. Block has mass m and spring constant is k. Assuming ideal conditions-



(A) Time period of SHM depends on a

(B) Time period of SHM is independent of a

(C) In mean position elongation in spring is $\frac{3mg}{2k}$

(D) In mean position elongation in spring is $\frac{mg}{k}$ **[B,C]**

Sol. Here time period $T = 2\pi\sqrt{\frac{m}{K}}$ independent of g and a and if mean position elongation is x_0 then,

$$Kx_0 = m(g + a) = m\left(g + \frac{g}{2}\right)$$

$$Kx_0 = \frac{3mg}{2}$$

$$x_0 = \frac{3mg}{2k}$$

Q.8 A wire is clamped horizontally between two rigid support and a small mass is hanged from middle of the wire. The mass is displaced a little in vertical direction -

(A) Mass will undergo S.H.M for all value of 'x' (large or small)

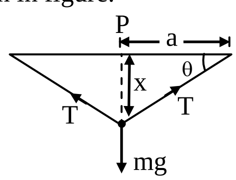
(B) Mass will not undergo S.H.M for any value of 'x' (large or small)

(C) mass will undergo S.H.M. only if value of 'x' is very small

(D) Mass will undergo periodic motion

[B,D]

Sol. Let 'x' be the displacement from mean position 'P' shown in figure.



Restoring force $= 2T \sin\theta$

$= 2.K.x \sin^2\theta$

$= 2.K. \frac{x^3}{a^2}$

Thus restoring force $\alpha\ x^3$.

Hence mass will not undergo S.H.M. but it will undergo periodic motion.

Q.9 The speed v of a particle moving along a straight line, when it is at a distance x from a fixed point on the line is

$$V^2 = 144 - 9x^2$$

Select correct option(s) –

(A) The maximum displacement from point is 4 units

(B) The motion is SHM with time period $T = \frac{2\pi}{3}$ units

(C) The magnitude of acceleration at a distance 3 units from fixed point is 27 units

(D) The motion is periodic but not SHM

[A,B,C]

Sol.

$v^2 = 144 - 9x^2$

$2v\frac{dv}{dx} = -18x$

$\therefore v\frac{dv}{dx} = -9x \Rightarrow \omega = 3 \Rightarrow T = \frac{2\pi}{3}$ units

Also, $v^2 = 144 - 9A^2 = 0$

$A = 4$ units

Now, $|a| = \omega^2 x = 9 \times 3 = 27$ units

Q.10 A body of mass m is suspended from two light springs of force constants k_1 and k_2 separately. The periods of vertical oscillations are T_1 and T_2 respectively. Now the same body is suspended from the same two springs which are first connected in series and then in parallel. The period of vertical oscillations are T_s and T_p respectively –

(A) $T_p < T_1 < T_2 < T_s$ for $k_1 > k_2$

(B) $\frac{1}{T_p^2} = \frac{1}{T_1^2} + \frac{1}{T_2^2}$

(C) $T_s^2 = T_1^2 + T_2^2$

(D) $\sqrt{T_s} = \sqrt{T_1} + \sqrt{T_2}$ **[A,B,C]**

Q.11 A particle of mass 2 kg moving along x-axis has potential energy given by

$U = 16x^2 - 32x$ (in joule), where x is in metre. Its speed when passing through x = 1m is $2ms^{-1}$. Then-

(A) The motion of particle is uniformly accelerated motion

(B) The motion of particle is oscillatory from x = 0.5 m to x = 1.5 m

(C) The motion of particle is simple harmonic

(D) The period of oscillatory motion is $\frac{\pi}{2}$ s

[B,C,D]

Q.12 A particle is oscillating with frequency f. Then- (Assume no damping effects)

(A) Its potential energy varies periodically with frequency 2f

(B) Its kinetic energy varies periodically with frequency 2f

(C) Its total mechanical energy (potential energy + kinetic energy) varies periodically with period 4f

(D) Its total mechanical energy is constant with infinite period **[A,B,D]**

Q.13 A particle of mass m is executing a motion in which the velocity when plotted against displacement follows the curve $\frac{x^2}{a^2}+\frac{v^2}{b^2}=1$, where a and b are positive constants and a < b. If U, K and T denote the average potential energy, average kinetic energy and total energy of the particle respectively, then –

(A) $U = K = \frac{T}{3} = \frac{1}{6}mb^2$

(B) $U \neq K \neq T$

(C) $U = K \neq T$

(D) $U = K = \frac{T}{2} = \frac{1}{4}mb^2$ **[C,D]**

Q.14 A horizontal spring-mass system of mass M executes oscillatory motion of amplitude a_0 and time period T_0. When the mass M is passing through its equilibrium position another mass m is placed on it such that both move together. If a and T be the new amplitude and time period respectively then –

(A) $a = \sqrt{\frac{M}{M+m}}a_0$ (B) $a = \sqrt{\frac{M+m}{M}}a_0$

(C) $T = \sqrt{\frac{M}{M+m}}T_0$ (D) $T = \sqrt{\frac{M+m}{M}}T_0$

[A,D]

Q.15 A simple pendulum of Length ℓ and mass (M) is oscillating in a plane about a vertical line between the angular limits $-\phi$ to $+\phi$. For an angular displacement $\theta(<|\phi|)$, the tension in the string T and velocity of the bob V are related as-

(A) $T\cos\theta = Mg$

(B) $T = Mg\cos\theta + MV^2$

(C) Tangential acceleration is $g \sin\theta$

(D) $T = Mg\cos\theta$ **[B,C]**

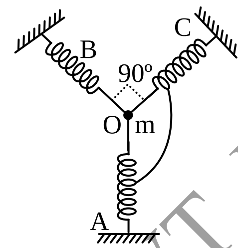
Q.16 A cylindrical log of wood is floating in a large pool of water with its length normal to water's surface. The log has a radius r, mass m, length ℓ and has density σ. If the log is depressed below its equilibrium depth d (but not beneath the surface of water) and then released, it executes harmonic oscillations with time period T. (Assume density of water to be ρ)-

(A) $T = 2\pi\sqrt{\frac{m}{2\pi\rho r\ell g}}$ (B) $T = \frac{2\pi}{r}\sqrt{\frac{m}{\pi\rho g}}$

(C) $d = \frac{m}{2\pi r\rho\ell}$ (D) $d = \frac{m}{\pi r^2\rho}$

[B,D]

Q.17 A particle of mass m is attached to three identical springs A, B and C each of force constant k as shown in figure. If the particle of mass m is pushed slightly against the spring A and released, then the time period of oscillation is –



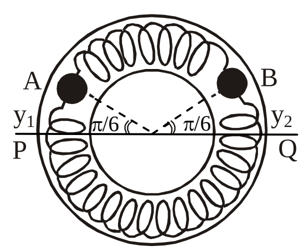
(A) Extension in springs are same

(B) $2\pi\sqrt{\frac{m}{2k}}$

(C) Extension in A is different from B and C

(D) $2\pi\sqrt{\frac{m}{3k}}$ **[B,C]**

Q.18 Two identical balls A and B each of mass 0.1 kg are attached to two identical massless springs. The spring mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in the figure. The pipe is fixed in a horizontal plane. The centres of the balls can move in a circle of radius 0.06 m. Each spring has a natural length of 0.06 π m and force constant 0.1 N/m. Initially, both the balls are displaced by angle $\theta = \pi/6$ radian with respect to the diameter PQ of the circle and released from rest. The frequency of oscillation of the ball B is –



(A) π Hz (B) $\frac{1}{\pi}$ Hz

(C) $K_{eq} = \frac{K}{2}$ (D) $K_{eq} = 2K$

[B,D]

Q.19 A pendulum suspended from the roof of an elevator at rest has a time period T_1; when the elevator moves up with an acceleration a its time period becomes T_2; when the elevator moves down with an acceleration a; its time period becomes T_3 then –

(A) $T_3 > T_2$ and T_1 (B) $T_2 > T_3 > T_1$

(C) $T_1 = \frac{T_2T_3\sqrt{2}}{\sqrt{T_2^2 + T_3^2}}$ (D) $T_1 = \sqrt{T_2^2 + T_3^2}$

[A,D]

Q.20 A linear harmonic oscillator of force constant 2×10^6 N/m and amplitude 0.01 m has a total mechanical energy of 160 J. Its – **[IIT-1989]**

(A) Maximum potential energy is 100 J

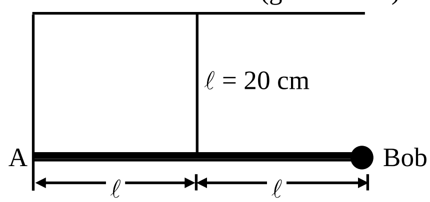
(B) Maximum kinetic energy is 100 J

(C) Maximum potential energy is 160 J

(D) Minimum potential energy is zero

[C,D]

Q.1 A weightless rigid rod with a small iron bob at the end is hinged at point A to the wall so that it can rotate in all directions. The rod is kept in the horizontal position by a vertical inextensible string of length 20 cm, fixed at its mid point. The bob is displaced slightly, perpendicular to the plane of the rod and string. Find period of small oscillations of the system in the form $\frac{\pi x}{10}$ sec. and fill value of x. ($g = 10\ m/s^2$)



Sol. The bob will execute SHM about a stationary axis passing through AB. If its effective length is 'ℓ' then $T = 2\pi\sqrt{\frac{\ell'}{g'}}$

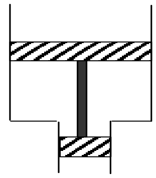
$\ell' = \frac{\ell}{\sin\theta} = \sqrt{2}\ell$

$g' = g\cos\theta = g/\sqrt{2}$

$T = 2\pi\sqrt{\frac{2\ell}{g}} = \frac{2\pi}{5} = \frac{4\pi}{10}$

x = 4.

Q.2 A smooth vertical conducting tube have two different section is open from both ends and equipped with two piston of different areas. Each piston slides in respective tube section. 1 liter of ideal gas at pressure 1.5×10^5 Pa is enclosed between the piston connected with a light rod. The cross section area of upper piston is $10\pi\ cm^2$ greater than lower one. Combined mass of two piston is 1.5 kg. If the piston is displaced slightly. Time period of oscillation will be (in 10^{-1} sec).



[0005]

Sol. Let the piston are displaced by 'x'

Process isothermal

$\therefore\ \Delta P = \frac{P}{V}\Delta V$

$\therefore\ a = \frac{\Delta P(A_1 - A_2)}{m}$

[A_1 : Area of upper piston
m = 1.5 kg]

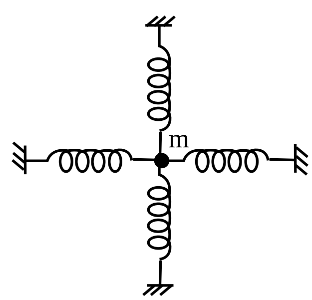
$= \frac{P}{mV}(A_1 - A_2).\Delta V$

$= \frac{P}{mV}(A_1 - A_2)^2 x$

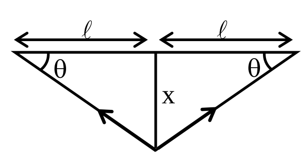
$\Rightarrow \omega = \sqrt{\frac{P}{mV}}\,.\,(A_1 - A_2)$

$\Rightarrow$ T = 0.5 sec

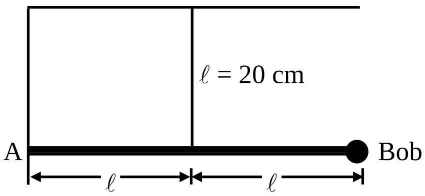
Q.3 As shown in figure a particle of mass m = 100 gm is attached with four identical springs each of length ℓ = 10 cm. Initial tension in each spring is f_0 = 25 N. Neglecting gravity the period of small oscillations of the particle in 10^{-2} sec along a line perpendicular to plane of figure is nearly.



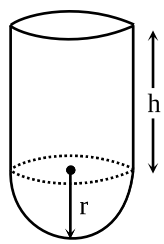
Sol. [6] $F = -4F_0\sin\theta = -\frac{4F_0 x}{\ell}$



Q.4 A weightless rigid rod with a small iron bob at the end is hinged at point A to the wall so that it can rotate in all directions. The rod is kept in the horizontal position by a vertical inextensible string of length 20 cm, fixed at its mid point. The bob is displaced slightly, perpendicular to the plane of the rod and string. Find period of small oscillations of the system in the form $\frac{\pi x}{10}$ sec. and fill value of x. (g = 10 m/s^2)



Q.5 A container consist of hemispherical shell of radius 'r' and cylindrical shell of height 'h' radius of same material and thickness. The maximum value h/r so that container remain stable equilibrium in the position shown (neglect friction) is -



Sol. **[1]**

For stable equilibrium $h \le r \Rightarrow \frac{h}{r} \le 1$

Q.6 A solid uniform sphere of radius r rolls without slipping along the inner surface of a fixed spherical shell of radius R and performs small oscillations and its period is found with an unknown x to be $2\pi\sqrt{\frac{x(R-r)}{5g}}$ then the value of x will be.

Sol. **[7]**

$[P_0 + \rho_2 gh + \rho_1 g(h-20)] = P_0 + \frac{1}{2}\rho_1 V^2$

$\Rightarrow V = \left[\frac{2[\rho_2 gh + \rho_1 g(h-20)]}{\rho_1}\right]^{1/2}$

$= 4$ m/sec

Q. 1 A uniform spring of normal length ℓ has a force constant k. It is cut into two pieces of lengths ℓ_1, and ℓ_2 such that $\ell_1 = n\ell_2$ where n is an integer. Then the value of k_1 (force constant of spring of length ℓ_1) is -

(A) $\frac{kn}{(n+1)}$ (B) $\frac{k(n+1)}{n}$

(C) $\frac{k(n-1)}{n}$ (D) $\frac{kn}{n-1}$

Sol.[B] $k_1\ell_1 = k_2\ell_2 = k(\ell_1 + \ell_2)$

$k_1 = \frac{k(\ell_1+\ell_2)}{\ell_1}$ or $k_1 = \frac{k(n\ell_2+\ell_2)}{n\ell_2}$

or $k_1 = \frac{k(n+1)}{n}$

Q.2 To study the dissipations of energy student Plots a graph between square root of time and amplitude. The graph would be a -

(A) Straight line (B) hyperbola

(C) Parabola (D) Exponential **[B]**

Sol. $a^2 t$ = constant

$a\sqrt{t}$ = constant

so hyperbola.

Q.3 The pendulum suspended from the ceiling of a train has a period T when the train is at rest. When the train is accelerating with an uniform acceleration, the period of oscillation will -

(A) increase (B) decrease

(C) remain unaffected (D) become infinite

[B]

Sol. Comparing with $y = 2\pi\sqrt{\frac{\ell}{g}}$;

$T' = 2\pi\sqrt{\frac{1}{\sqrt{g^2+a^2}}}$

clearly, T' < T

Q.4 For definite length of wire, if the weight used for applying tension is immersed in water, then frequency will -

(A) become less (B) become more

(C) remain equal (D) become zero **[A]**

Sol. For stretched string

$n \propto \sqrt{T} \propto \sqrt{M.g}$

When weight is dipped in water due to buoyancy force, tension decreases and hence frequency decreases

Q. 5 The amplitude and the time period in SHM are 0.8 cm and 0.2 s respectively. If the initial phase is $\frac{\pi}{2}$ radian, then the equation representing SHM is-

(A) y = 0.8 cos 10πt (B) y = 0.8 sin πt

(C) y = 3 × 0.8 sin πt (D) y = 0.8 sin 10πt

[A]

Sol. $y = a \sin(\omega t + \phi_0)$

$= 0.8 \sin\left[\frac{2\pi}{0.2}t + \frac{\pi}{2}\right]$

$= 0.8 \cos 10\,\pi t$

Q.6 The time period of a mass suspended from a spring is 5s. The spring is cut into four equal parts and same mass is now suspended from one of its parts. The period is now -

(A) 5s (B) 2.5 s

(C) 1.25 s (D) $\frac{5}{16}$ s **[B]**

Sol. $T = 2\pi\sqrt{\frac{m}{k}}$

Q.7 A particle executes SHM along a straight line so that its period is 12s. The time it takes in traversing a distance equal to half its amplitude from its equilibrium position is -

(A) 6s (B) 4s

(C) 2s (D) 1s **[D]**

Sol. E = a sin ωt

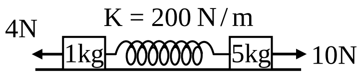
$\frac{a}{2} = a \sin \omega t$

or $\frac{1}{2} = \sin \omega t$

or $\sin\frac{\pi}{6} = \sin\frac{2\pi}{12}t$

t = 1s

Q.8 The maximum elongation in the spring is -



(A) 2 cm (B) 3 cm (C) 4 cm (D) 5 cm

Sol.[D] $x = \frac{2\times10\times1}{200\times6} + \frac{2\times4\times5}{200\times6} = \frac{60}{200\times6} = \frac{1}{20}$ m = 5cm

Q.9 A particle executes SHM along a straight line so that its period is 12 s. The time it takes in traversing a distance equal to half its amplitude from its equilibrium position is-

(A) 6 s (B) 4 s

(C) 2 s (D) 1s **[D]**

Sol. $E = a \sin \omega t \Rightarrow \frac{a}{2} = a\sin\omega t$ or $\frac{1}{2} = \sin\omega t$

or $\sin\frac{\pi}{6} = \sin\frac{2\pi}{12}t \Rightarrow t = 1s$

Q.10 The length of simple pendulum executing SHM is increased by 21%. The percentage increase in the time period of the pendulum is -

(A) 10% (B) 11% (C) 21% (D) 42%

Sol. **[A]**

$\frac{T'}{T} = \sqrt{\frac{121}{100}}$, $T' = \frac{11}{10}T$

$\left(\frac{T'}{T} - 1\right) \times 100\% = \left(\frac{11}{10} - 1\right) \times 100\%$

= 10%

Q.11 A particle executes SHM of amplitude 5cm and period 3s. The velocity of the particle at a distance 4 cm from the mean position-

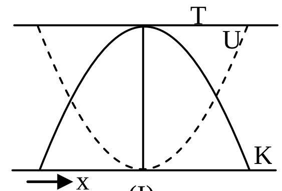
(A) 8 cm/s (B) 12 cm/s

(C) 4 cm/s (D) 6 cm/s

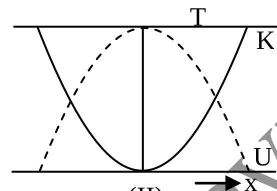
Sol.[D] $v = \omega\sqrt{a^2 - x^2} = \frac{2\pi}{T}\sqrt{a^2 - x^2} = \frac{2\times3}{3}\sqrt{25-16}$

= 6 cm/s

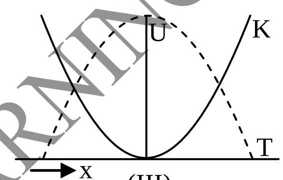
Q.12 A particle is executing S.H.M. along a straight line. The graph showing the variation of kinetic, potential and total energy K, U and T respectively with displacement is



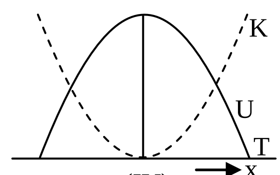
(I)



(II)



(III)

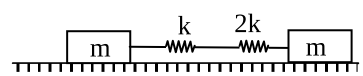


(IV)

(A) I (B) II

(C) III (D) IV **[A]**

Q.13 A system is shown in the figure. The time period for small oscillations of the two blocks will be -



(A) $2\pi\sqrt{\frac{3m}{k}}$ (B) $2\pi\sqrt{\frac{3m}{4k}}$

(C) $2\pi\sqrt{\frac{3m}{8k}}$ (D) $2\pi\sqrt{\frac{3m}{2k}}$

Sol. **[B]**

Both the spring are in series

$K_{eq} = \frac{k(2k)}{k+2k} = \frac{2k}{3}$

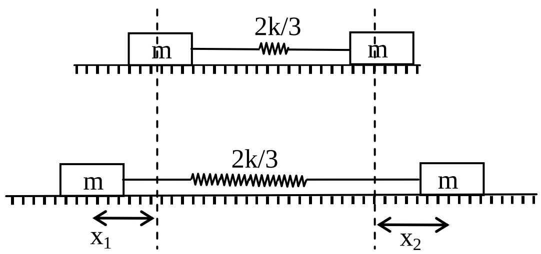
Time period $T = 2\pi\sqrt{\frac{\mu}{K_{eq}}}$

where $\mu = \frac{m_1.m_2}{m_1 + m_2}$

Here $\mu = \frac{m}{2}$

$\therefore T = 2\pi\sqrt{\frac{m}{2}.\frac{3}{2k}} = 2\pi\sqrt{\frac{3m}{4k}}$

Alternative method :



$\therefore mx_1 = mx_2 \Rightarrow x_1 = x_2$

force equation for first block;

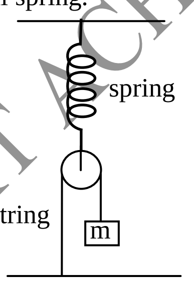
$\frac{2k}{3}(x_1+x_2) = m\frac{d^2x_1}{dt^2}$

Put $x_1 = x_2 \Rightarrow \frac{d^2x_1}{dt^2} + \frac{4k}{3m}x_1 = 0$

$\Rightarrow \omega^2 = \frac{4k}{3m}$

$\therefore T = 2\pi\sqrt{\frac{3m}{4k}}$

Q. 14 Find the period of low amplitude vertical vibrations of the system shown The mass of the block is m. The pulley hangs from the ceiling on a spring with a force constant k. The block hangs from an ideal spring.



(A) $2\pi\sqrt{\frac{m}{k}}$ (B) $6\pi\sqrt{\frac{m}{k}}$

(C) $4\pi\sqrt{\frac{m}{k}}$ (D) $8\pi\sqrt{\frac{m}{k}}$

Sol. **[C]**

When block is given displacement x spring will stretch by $\frac{x}{2}$.

$\therefore$ spring force $= \frac{kx}{2}$

Tension in the string $= \frac{1}{2} \times$ spring force

$= \frac{kx}{4}$

$T = 2\pi\sqrt{\frac{m}{\frac{k}{4}}} = 4\pi\sqrt{\frac{m}{k}}$

Q. 15 A spring has a force constant k and mass m . The spring hangs vertically and a block of unknown mass is attached to its bottom end: It is known that the mass of the block is much greater than that of the spring. The hanging block stretches the spring the twice its relaxed length. How long (t) would it take for a low amplitude transverse pulse to travel the length of the spring stretched by the hanging block ?

(A) $\sqrt{\frac{2m}{k}}$ (B) $\sqrt{\frac{m}{k}}$

(C) $\sqrt{\frac{m}{2k}}$ (D) $\sqrt{\frac{2m}{3k}}$

Sol. **[A]**

Since mass of spring is small compared to the mass m, the tension force is approximately constant

$\therefore T = mg$

$kx = mg$ $x = \frac{mg}{k}$

When x-acceleration of the spring

x = 2L given

Where L – Relaxed length.

New length = 2L

$= \frac{2mg}{k}$

$\therefore$ mass per unit length

$\mu = \frac{mk}{2mg} = \frac{k}{2g}$

Speed of wave $V = \sqrt{\frac{T}{\mu}}$

$= \sqrt{\frac{mg \times 2mg}{mk}}$

$$V = g\sqrt{\frac{2m}{k}}$$

$$\text{time} = \frac{2L}{V}$$

$$= \frac{2mg}{k}\,\frac{\sqrt{k}}{\sqrt{2m}}$$

$$\text{time} = \sqrt{\frac{2m}{k}}$$

Q. 16 A particle of mass m is acted upon by a force $F = t^2 - kx$. Initially the particle is at rest at the origin. Then –

(A) Its displacement will be in simple harmonic

(B) Its velocity will be in simple harmonic

(C) Its acceleration will be in simple harmonic

(D) Particle will move with constant velocity

Sol. **[C]**

Conceptual

Q.17 Two particles A and B execute simple harmonic motion with periods of T and $\frac{5T}{4}$ respectively . They start simultaneously from mean position. The phase difference between them when A completes one oscillation will be -

(A) 0 (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$ (D) $\frac{2\pi}{5}$

Sol. **[D]**

$$\Delta\phi = (\omega_1 - \omega_2)t = \left(\frac{2\pi}{T} - \frac{2\pi}{5T/4}\right)T = \frac{2\pi}{5}$$

Q. 18 A simple harmonic oscillator has amplitude 'A' angular frequency ω and mass m. Then average kinetic energy in one time period is -

(A) $\frac{1}{2}m\omega^2A^2$ (B) $\frac{1}{4}m\omega^2A^2$

(C) $m\omega^2A^2$ (D) zero

Sol. **[B]**

$$K_{av} = \frac{1}{T}\int_6^T \frac{1}{2}m\omega^2A^2\cos^2(\omega t + \phi)$$

Q.19 A student says that he had applied a force $F = -k\sqrt{x}$ on a particle and the particle moved in simple harmonic motion. He refuses to tell whether k is a constant or not. Assume that he has worked only with positive x and no other force acted on the particle.

(A) As x increases k increases

(B) As x increases k decreases

(C) As x increases k remains constant

(D) The motion cannot be simple harmonic

[A]

Q.20 When the displacement is half of the amplitude. The ratio of potential energy to the total energy is -

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$

(C) 1 (D) $\frac{1}{8}$

Sol. **[B]** P.E. = $\frac{1}{2}m\omega^2x^2$ & T.E. = $\frac{1}{2}m\omega^2A^2$

So ratio at x = A/2 $\Rightarrow \frac{P.E.}{T.E.} = \frac{1}{4}$

Q.21 The distance moved by a particle in simple harmonic motion in one time period is –

(A) A (B) 2A

(C) 4A (D) zero **[C]**

Q.22 The time period of a particle in simple harmonic motion is equal to the smallest time between the particle acquiring a particular velocity $\vec{v}$. The value of v is –

(A) v_{max}

(B) 0

(C) between 0 and v_{max}

(D) between 0 and $-v_{max}$ **[D]**

Q.23 Which of the following quantities are always positive in a simple harmonic motion ?

(A) $\vec{F}.\vec{a}$ (B) $\vec{v}.\vec{r}$

(C) $\vec{a}.\vec{r}$ (D) $\vec{F}.\vec{r}$ **[A]**

Q.24 A particle moves such that its acceleration is given by

$$a = -\beta(x - 2)$$

Here : β is a positive constant and x is the position from origin. Time period of oscillations is -

(A) $2\pi\sqrt{\beta}$ (B) $2\pi\sqrt{\frac{1}{\beta}}$

(C) $2\pi\sqrt{\beta+2}$ (D) $2\pi\sqrt{\frac{1}{\beta+2}}$

Sol. **[B]**

$a = -\beta(x - 2)$

as $a = -\omega^2(x - x_0)$

$\therefore \omega^2 = \beta \Rightarrow T = 2\pi\sqrt{\frac{1}{\beta}}$

Q.25 The displacement of two identical particles executing SHM are represented by equations

$x_1 = 4\sin\left(10t + \frac{\pi}{6}\right)$ and $x_2 = 5\cos\omega t$

For what value of ω energy of both the particles is same ?

(A) 16 unit (B) 6 unit
(C) 4 unit (D) 8 unit

Sol. **[D]**

$E_1 = E_2$

$\therefore \frac{1}{2} m_1 \omega_1^2 A_1^2 = \frac{1}{2} m_2 \omega_2^2 A_2^2$

but $m_1 = m_2$

$\therefore \omega_1^2 \times 16 = \omega_2^2 \times 25$

$\therefore 100 \times 16 = \omega^2 \times 25$

$\omega = 8$ units

Q.26 A simple pendulum 4 m long swings with an amplitude of 0.2 m. What is its acceleration at the ends of its path ? ($g = 10\ m/s^2$)

(A) zero (B) $10\ m/s^2$
(C) $0.5\ m/s^2$ (D) $2.5\ m/s^2$

Sol. **[C]**

$\omega = \sqrt{\frac{g}{L}}$ $\therefore a_{max} = \omega^2 A = \frac{g}{L} \times A = 0.5\ m/s^2$

Q.27 A particle of mass 5×10^{-5} kg is placed at the lowest point of a smooth parabola having the equation $x^2 = 40y$ (x, y in cm). If it is displaced slightly and it moves such that it is constrained to move along the parabola, the angular frequency of oscillation will be, approximately -

(A) $1\ s^{-1}$ (B) $7\ s^{-1}$
(C) $5\ s^{-1}$ (D) None of these

Sol. **[D]**

Restoring force = $F_R = -mg\sin\theta$ where

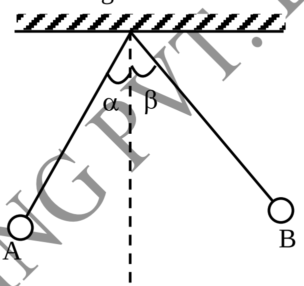
$\tan\theta = \frac{dy}{dx} = \frac{x}{20}$

$\therefore F_R = \frac{-mgx}{20}$

$\therefore F_R = -m\omega^2 x$

$\therefore \omega = \sqrt{\frac{g}{20}}$

Q.28 Two identical simple pendulums A and B are fixed at same point. They are displaced by very small angles α and β ($\beta > \alpha$) and released from rest. Find the time after which B reaches its initial position for the first time. Collisions are elastic and length of strings is ℓ.



(A) $\pi\sqrt{\frac{\ell}{g}}$ (B) $2\pi\sqrt{\frac{\ell}{g}}$

(C) $\frac{\pi\beta}{\alpha}\sqrt{\frac{\ell}{g}}$ (D) $\frac{2\pi\beta}{\alpha}\sqrt{\frac{\ell}{g}}$

Sol. **[B]**

Time period of both A and B $T = 2\pi\sqrt{\frac{\ell}{g}}$

After first collision, B acquires amplitude of A and after second collision it acquires its own amplitude in this process time taken is

$= \frac{T}{4} + \frac{T}{4} + \frac{T}{4} + \frac{T}{4} = T = 2\pi\sqrt{\frac{\ell}{g}}$

Q. 29 If a simple harmonic motion is represented by $\frac{d^2x}{dt^2} + \alpha x = 0$, its time period is -

(A) $\frac{2\pi}{\alpha}$ (B) $\frac{2\pi}{\sqrt{\alpha}}$

(C) $2\pi\alpha$ (D) $2\pi\sqrt{\alpha}$ **[B]**

Q. 30 If the displacement (x) and velocity (v) of a particle executing simple harmonic motion are related through the expression $4v^2 = 25 - x^2$, then its time period is -

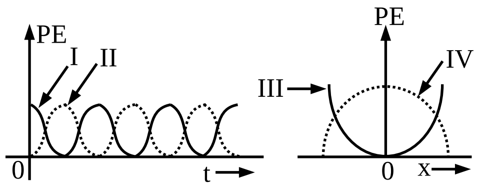
(A) π (B) 2π
(C) 4π (D) 6π **[C]**

Q. 31 A simple pendulum has a period T. It is taken inside a lift moving up with uniform acceleration of g/3. Now its time period will be

(A) $\frac{\sqrt{2}}{3}T$ (B) $\frac{\sqrt{3}}{2}T$

(C) $\frac{2T}{\sqrt{3}}$ (D) $\frac{3T}{\sqrt{2}}$ **[B]**

Q. 32 For a particle executing simple harmonic motion, the displacement x is given by x = A cos ωt. Identify the graph which represents the variation of potential energy (PE) as a function of time t and displacement x -



(A) I, III (B) II, IV

(C) II, III (D) I, IV **[A]**

Q. 33 A body of mass 1 kg is executing simple harmonic motion. Its displacement x (in cm) at time t (in second) is given by,

$$x = 6\sin\left(100t + \frac{\pi}{4}\right)$$

The maximum kinetic energy of the body is -

(A) 6 J (B) 18 J

(C) 24 J (D) 36 J **[B]**

Q. 34 A uniform spring has an unstreched length ℓ and a force constant k. The spring is cut into two parts of unstreched length ℓ_1 and ℓ_2 such that $\ell_1 = \eta\ell_2$ where η is an integer. The corresponding force constants k_1 and k_2 are :

(A) kη and k(η + 1)

(B) $\frac{k(\eta+1)}{\eta}$ and k(η – 1)

(C) $\frac{k(\eta-1)}{\eta}$ and k(η+1)

(D) $\frac{k(\eta+1)}{\eta}$ and k(η+1) **[D]**

Sol. $\ell_1 = \eta\,\ell_2 \Rightarrow \ell_1 : \ell_2 = \eta : 1$

$\Rightarrow \ell_1 = \frac{\eta}{\eta+1}\ell$ & $\ell_2 = \frac{1}{(\eta+1)}\ell$

so $k_1 = \frac{\eta+1}{\eta}k$, $k_2 = (\eta + 1)k$

Q. 35 A particle is vibrating in simple harmonic motion with an amplitude of 4 cm. At what displacement from the equilibrium position is its energy half potential and half kinetic ?

(A) 10 cm (B) $\sqrt{2}$ cm

(C) 2 cm (D) $2\sqrt{2}$ cm

Sol.

[D] The total energy E of a particle vibrating SHM is given by

$E = \frac{1}{2}m\omega^2a^2$ .....(1)

The kinetic energy K is given by

$K = \frac{1}{2}m\omega^2(a^2 - y^2)$

where y = displacements of the particle

but $K = \frac{E}{2} = \frac{1}{2}\left[\frac{1}{2}m\omega^2a^2\right]$

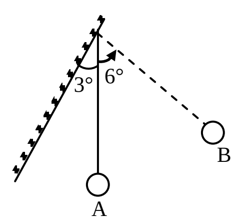
$\therefore \frac{1}{2}\left[\frac{1}{2}m\omega^2a^2\right] = \frac{1}{2}m\omega^2(a^2 - y^2)$

or $\frac{a^2}{2} = a^2 - y^2$ or $y^2 = \frac{a^2}{2}$ $\therefore y = \frac{a}{\sqrt{2}}$

Hence the kinetic energy is half of the total energy when displacement of the particle is $a/\sqrt{2}$. Given that a = 4cm.

$\therefore y = 4/\sqrt{2} = 2\sqrt{2}$.

Q.36 A pendulum of length 10 cm is hanged by wall making an angle 3° with vertical. It is swinged to position B. Time period of pendulum will be



(A) π/5 sec

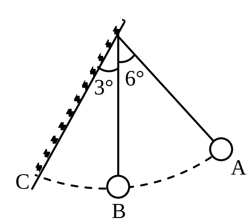
(B) $\frac{2\pi}{15}$ sec

(C) π/6 sec

(D) Subsequent motion will not be periodic

[B]

Sol.



Time taken by pendulum in going from A to B

$= \dfrac{T}{4}$ where $T = 2\pi\sqrt{\dfrac{\ell}{g}}$

Time taken by pendulum in going from B to C

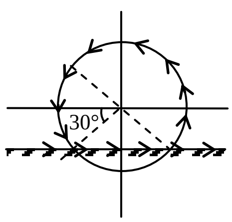
$= \dfrac{T}{12}$

$\therefore$ Time period of pendulum

$= 2\left(\dfrac{T}{4} + \dfrac{T}{12}\right)$

$= \dfrac{2T}{3} = \dfrac{2}{3} \cdot \dfrac{\pi}{5} = \dfrac{2\pi}{15}$ sec

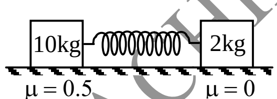
Altier :



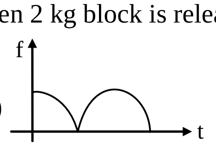
$T' = \dfrac{240}{360} \cdot T$

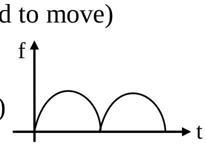
$= \dfrac{2}{3}T$

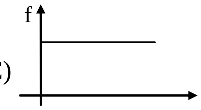
Q.37 Two blocks of mass 10 kg and 2 kg are connected by an ideal spring of spring constant 1000 N/m and the system is placed on a horizontal surface as shown.



The coefficient of friction between 10 kg block and surface is 0.5 but friction is assumed to be absent between 2 kg and surface. Initially blocks are at rest and spring is unstretched then 2 kg block is displaced by 1 cm to elongate the spring then released. Then the graph representing magnitude of frictional force on 10 kg block and time t is : (Time t is measured from that instant when 2 kg block is released to move)

(A) 

(B) 

(C) 

(D) None of these **[A]**

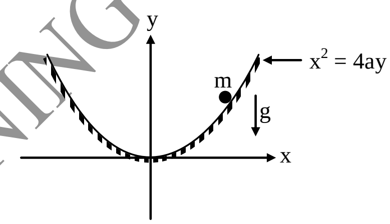
Sol. $f_s = kx$

(where f_s is frictional force on 20 kg block and xis instantaneous elongation or compression in spring)

$f_s = k\,(A \cos \omega t)$

$\therefore \quad |f_s| = kA\,|\cos \omega t|$

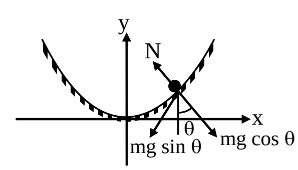
Q.38 A particle of mass m is allowed to oscillate near the minimum point of a vertical parabolic path having the equation $x^2 = 4ay$, then the angular frequency of small oscillations of particle is –



(A) $\sqrt{ga}$ (B) $\sqrt{2ga}$

(C) $\sqrt{\dfrac{g}{a}}$ (D) $\sqrt{\dfrac{g}{2a}}$ **[D]**

Sol.



$ma = -\,mg \sin\theta$

$a = -\,g \sin\theta$ or $a = -\,g \tan\theta$ … (1)

(as θ is small)

Now,

$x^2 = 4ay$

$\therefore \quad \dfrac{dy}{dx} = \dfrac{x}{2a}$

$\therefore \quad a = -\,g\dfrac{x}{2a}$

$-\,\omega^2 x = -\,\dfrac{gx}{2a}$

$\omega = \sqrt{\dfrac{g}{2a}}$

Q.39 If the same weight is suspended from three springs having lengths 1 : 3 : 5, the period of oscillations shall be in the ratio of -

(A) 1 : 3 : 5 (B) $1 : \sqrt{3} : \sqrt{5}$

(C) 15 : 5 : 3 (D) $1 : \frac{1}{\sqrt{3}} : \frac{1}{\sqrt{15}}$ **[B]**

Sol. $T \propto \frac{1}{\sqrt{K}}$ and $K \propto \frac{1}{\ell}$

$\therefore T \propto \sqrt{\ell}$

Q.40 The displacement of two identical particles executing SHM are represented by equations

$x_1 = 4 \sin\left(10t + \frac{\pi}{6}\right)$ and $x_2 = 5 \cos \omega t$

For what value of ω energy of both are particles is same ?

(A) 16 unit (B) 6 unit

(C) 4 unit (D) 8 unit **[D]**

Sol. $E = \frac{1}{2} mA^2\omega^2$ i.e., $E \propto (A\omega)^2$

or $(A_1\omega_1)^2 = (A_2\omega_2)^2$

or $A_1\omega_1 = A_2\omega_2$

or $4 \times 10 = 5 \times \omega$

or $\omega = 8$ unit

Q.41 A simple pendulum 4 m long swings with an amplitude of 0.2 m. What is its acceleration at the ends of its path ? ($g = 10$ m/s^2)

(A) zero (B) 10 m/s^2

(C) 0.5 m/s^2 (D) 2.5 m/s^2 **[C]**

Sol.

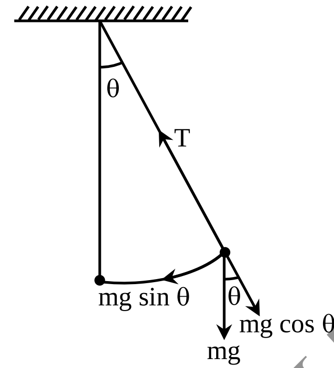


Fig.

$T = mg \cos\theta$

$\therefore F_{net} = mg \sin\theta$

and acceleration = $g \sin\theta = g \tan\theta$ ($\sin\theta = \tan\theta$)

$= (10)\frac{(0.2)}{4} = 0.5$ m/s^2

Q.42 A clock with an Iron Pendulum keeps correct time at 20°C. How much will it lose or gain if temperature changes to 40°C? [Given cubical expansion of iron = 36×10^{-6} °C^{-1}]

(A) 10.368 sec gain (B) 10.368 sec loss

(C) 5.184 sec gain (D) 5.184 sec loss

[B]

Sol. $T = 2\pi\sqrt{\frac{L}{g}}$

and $T' = 2\pi\sqrt{\frac{L'}{g}}$

or $\frac{T'}{T} = \sqrt{\frac{L'}{L}}$

$L' = L(1 \times \alpha\,\Delta t)$

$\therefore \alpha = \frac{\gamma}{3} = \frac{36 \times 10^{-6}}{3}$

$= L(1 + 12 \times 10^{-6} \times 20) = 12 \times 10^{-6}$ °C^{-1}

$L' = L(1.00024)$

$\frac{T'}{T} = \sqrt{\frac{1.00024\,L}{L}}$

or $\frac{T'}{2} = 1.00012$ $(\because T = 2 \text{ sec})$

$T' = 2.00024$

Loss in time per day

$= \frac{(2.00024 - 2) \times 24 \times 60 \times 60}{2}$ sec.

= 10.368 sec. Loss/day

Q.43 A particle of mass 'm' kept at origin is subjected to a force $\vec{F} = (pt - qx)\hat{i}$ where 't' : time elapsed and x : x-coordinate of position of particle. Particle starts its motion at t = 0 with zero initial velocity. If p and q are positive constants, then -

(A) Acceleration of the particle will continuously keep on increasing with time

(B) Particle will execute S.H.M.

(C) Force on particle will have no upper limit

(D) The acceleration of particle varies sinusoidally with time

Sol. **[D]**

F = pt – qx

$\Rightarrow a = \frac{p}{m}t - \frac{q}{m}x$

$\Rightarrow \frac{d^2a}{dt^2} = -\left(\frac{q}{m}\right)a$

Q.44 A particle executes SHM of amplitude 5cm and period 3s. The velocity of the particle at a distance 4 cm from the mean position -

(A) 8 cm/s (B) 12 cm/s

(C) 4 cm/s (D) 6 cm/s

Sol. **[D]**

$v = \omega\sqrt{a^2 - x^2}$

$= \frac{2\pi}{T}\sqrt{a^2 - x^2} = \frac{2\times 3}{3}\sqrt{25 - 16} = 6$ cm/s

Q.45 The displacement y of a particle executing periodic motion is given by

$y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000\, t)$

this expression may be considered to be a result of the superposition of ……independent harmonic motions -

(A) Two (B) Three

(C) Four (D) Five

Sol. **[B]** $y = 4\cos^2\left(\frac{t}{2}\right)\sin(1000\, t)$

$= 2\{1 + \cos t\}\sin(1000\, t)$

$y = 2\sin(1000\, t) + 2\cos t \sin 1000\, t$

$= 2\sin(1000\, t) + \sin(999\, t) + \sin(1001\, t)$

⇒ it is the super of three

Q.46 A particle undergoes simple harmonic motion having time-period T. The time taken 3/8th oscillation is-

(A) (3/8)T (B) (5/8) T

(C) (5/12)T (D) (7/12)T **[C]**

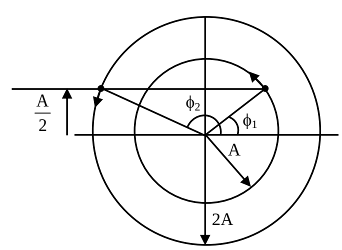
Q.47 Two particles are in SHM with same angular frequency and amplitudes A and 2A respectively along same straight line with same mean position. They cross each other at position $\frac{A}{2}$ distance from mean position in opposite direction. The phase difference between them is

(A) $\frac{5\pi}{6} - \sin^{-1}\left(\frac{1}{4}\right)$ (B) $\frac{\pi}{6} - \sin^{-1}\left(\frac{1}{4}\right)$

(C) $\frac{5\pi}{6} - \cos^{-1}\left(\frac{1}{4}\right)$ (D) $\frac{\pi}{6} - \cos^{-1}\left(\frac{1}{4}\right)$

[A]

Sol.



$\sin\phi_1 = \frac{A/2}{A} = \frac{1}{2}$

$\phi_1 = \frac{\pi}{6}$

$\sin(\pi - \phi_2) = \frac{A/2}{2A} = \frac{1}{4}$

$\phi_2 = \pi - \sin^{-1}\left(\frac{1}{4}\right)$

Phase difference

$\phi_2 - \phi_1 = \frac{5\pi}{6} - \sin^{-1}\left(\frac{1}{4}\right)$

Q.48 The velocities of a particle in SHM at displacements x_1 and x_2 from mean position are v_1 and v_2 respectively. Its amplitude will be -

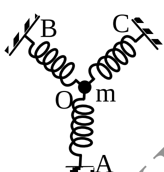
(A) $\left(\frac{x_2^2v_1^2 - x_1^2v_2^2}{v_1^2 - v_2^2}\right)^{1/2}$

(B) $\left(\frac{x_2^2v_2^2 - x_1^2v_1^2}{v_1^2 - v_2^2}\right)^{1/2}$

(C) $\left(\frac{x_2^2v_1^2 - x_1^2v_2^2}{v_2^2 - v_1^2}\right)^{1/2}$

(D) $\left(\frac{x_2^2v_2^2 - x_1^2v_1^2}{v_2^2 - v_1^2}\right)^{1/2}$ **[A]**

Q.49 A particle of mass m is attached to three identical springs A, B and C each of force constant k as shown in figure. If the particle of mass m is pushed slightly against the spring A and released, then the time period of oscillation is -



(A) $2\pi\sqrt{\frac{2m}{k}}$ (B) $2\pi\sqrt{\frac{m}{2k}}$

(C) $2\pi\sqrt{\frac{m}{k}}$ (D) $2\pi\sqrt{\frac{m}{3k}}$ **[B]**

Sol. $T = 2\pi\sqrt{\frac{M}{K(1+2\cos^2\theta)}}$

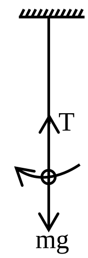
$= 2\pi\sqrt{\frac{M}{K(1+2\cos^2 45)}} = 2\pi\sqrt{\frac{M}{2K}}$

Q.50 A simple pendulum with angular frequency ω oscillates simple harmonically. The tension in the string at lowest point is T. The total acceleration of the bob at its lowest position is -

(A) $\frac{T}{m} - g$ (B) zero

(C) $g - \frac{T}{m}$ (D) $\frac{T}{m} + g$ **[A]**

Sol.



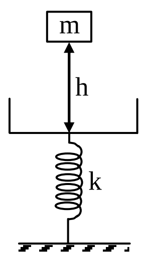
$T - mg = m\omega^2 R$

$\frac{T}{m} - g = \omega^2 R = a$

Q.1 The displacement equation for a particle executing simple harmonic motion is $y = 0.2 \sin 50\pi (t + 0.01)$ meter, where y is the displacement at the instant t. Calculate the amplitude, time-period, maximum velocity and the displacement at the start of motion.

Ans. **[0.2 m, 0.04 sec, 31.4 m/s, 0.2 m]**

Q.2 A body of mass m falls from height h on to the pan of a spring of force constant k. Having struck the pan the body starts making vertical S.H.M. Neglect the mass of pan and spring, calculate the amplitude of the oscillation.



Ans. **[** $x = \frac{mg}{k} \pm \frac{mg}{k}\sqrt{1+\frac{2kh}{mg}}$ **]**

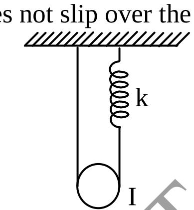
Q.3 A pan with a set of weights is attached to a spring. The period of vertical oscillations is 0.4s. After additional weights are placed on the pan, the period of vertical oscillations becomes 0.5s. By how much does the spring stretch owing to the additional weights ?

Ans. **[**2.25×10^{-2} m**]**

Q.4 A pan with set of weights is attached to a light spring. The period of vertical oscillations is 0.5 s. When some additional weights are put in the pan, the period of the vertical oscillations increases by 0.1 s. Find the extension causes by the extra weight.

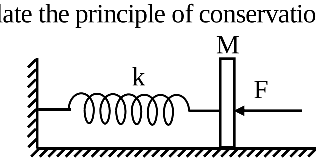
Ans. **[**2.7 cm**]**

Q.5 The pulley shown in figure has a moment of inertia I about its axis and mass m. Find the time period of vertical oscillation of its centre of mass. The spring has spring constant k and the string does not slip over the pulley



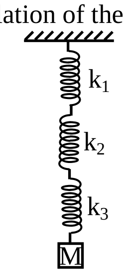
Ans. $T = 2\pi\sqrt{\frac{\left(\frac{I}{r^2}+m\right)}{4k}}$

Q.6 In figure k = 100 N/m, M = 1 kg and F = 10 N (a) Find the compression of the spring in the equilibrium position. (b) A sharp blow by some external agent imparts a speed of 2 m/s to the block towards left. Find the sum of the potential energy of the spring and the kinetic energy of the block at this instant. (c) Find the time period of the resulting simple harmonic motion. (d) Find the amplitude. (e) Write the potential energy of the spring when the block is at the left extreme. (f) Write the potential energy of the spring when the block is at the right extreme. The answers of (b), (e) and (f) are different. Explain why this does not violate the principle of conservation of energy.



Ans. (a) 10 cm (b) 2.5 J (c) $\pi/5$ sec. (d) 20 cm (e) 4.5 J (f) 0.5 J

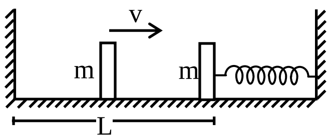
Q.7 Find the elastic potential energy stored in each spring shown in figure, when the block is in equilibrium. Also find the time period of vertical oscillation of the block.



Ans. $\frac{M^2g^2}{2k_1}, \frac{M^2g^2}{2k_2}$ and $\frac{M^2g^2}{2k_3}$,

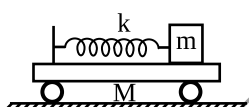
$T = 2\pi\sqrt{M\left(\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{k_3}\right)}$

Q.8 The left block in figure moves at a speed v towards the right block placed in equilibrium. All collisions to take place are elastic and the surfaces are frictionless. Show that the motions of the two blocks are periodic. Find the time period of these periodic motions. Neglect the widths of the blocks.



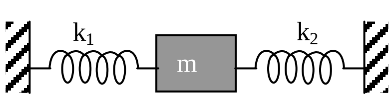
Ans. $\left(\pi\sqrt{\dfrac{m}{k}}+\dfrac{2L}{v}\right)$

Q.9 All the surfaces shown in figure are frictionless. The mass of the car is M, that of the block is m and the spring has spring constant k. Initially, the car and the block are at rest and the spring is stretched through a length x_0 when the system is released. (a) Find the amplitudes of the simple harmonic motion of the block and of the car as seen from the road. (b) Find the time period(s) of the two simple harmonic motions.



Ans. (a) $\dfrac{Mx_0}{M+m}$, $\dfrac{mx_0}{M+m}$ (b) $2\pi\sqrt{\dfrac{mM}{k(M+m)}}$

Q.10 A body of mass m oscillates, tied to two springs. The force constants of the springs are k_1 and k_2, and their natural length is L_0. Write the equation of motion for the body and solve it for the following two cases :



(a) At the state of equilibrium, the springs are both at their natural length.

(b) At the state of equilibrium, the springs are pressed, so that each is of the length L, where $L < L_0$.

Sol. We denote the displacement of the mass from equilibrium by x. The forces exerted by spring 1 and spring 2 are $-k_1x$ and $-k_2x$, respectively. The equation of motion is, therefore:

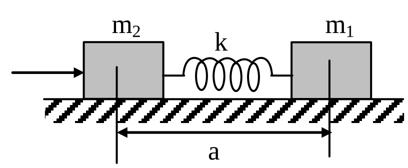
$m\ddot{x} = -k_1x - k_2x = -(k_1 + k_2)x$ …..(i)

Solving this equation, we find :

$x(t) = A \sin(\omega_0 t + \phi)$ …..(ii)

where $\omega_0 = \sqrt{\dfrac{k_1+k_2}{m}}$. ϕ and A are determined by the initial conditions, i.e., the values of x and $\dot{x}$ at t = 0. This solution now holds for both of the cases. Note that as far as the forces exerted on the mass are concerned, there is no difference between the two sections.

Q.11 Two masses, m_1 and m_2, are tied to the ends of a spring whose force constant is k, and whose natural length is a. This system is placed horizontally on a perfectly smooth table, as shown in figure. At t = 0, m_1 is bumped and receives a linear momentum of $\vec{p}_1 = p_0\hat{x}$, where p_0 is a constant.



(a) Write the equations of motion for m_1 and m_2. What is the velocity of the center of mass ?

(b) Prove that the harmonic oscillation equation of the system is

$\mu(\ddot{x}_2 - \ddot{x}_1) = -k(x_2 - x_1)$ where $\mu = \dfrac{m_1m_2}{m_1+m_2}$

(c) What is the oscillation amplitude of $(x_2 - x_1)$?

Sol. The motion of the system of the two bodies can be conveniently described by using the center of mass frame of reference. The center of mass moves in a straight line with constant velocity, due to the conservation of linear momentum. In the center of mass frame the two bodies perform simple harmonic oscillations. Denoting the position of the masses m_1 and m_2 by x_1 and x_2, respectively, we can express the distance between the masses as $x_2 - x_1$. The change in the length of the spring is then $x = x_2 - x_1 - a$.

(a) The forces applied by the spring on the two systems are :

$m_1 \ddot{x}_1 = kx$ .....(i)

$m_2 \ddot{x}_2 = -kx$ …..(ii)

The signs are used according to the position of the mass relative to the spring. Multiplying Eq.(i) by m_2, Eq.(ii) by m_1, and subtracting Eq.(i) from Eq.(ii) we have

$m_1m_2(\ddot{x}_2 - \ddot{x}_1) = -k(m_1 + m_2)x$ …..(iii)

Since $x = x_2 - x_1 - a$, we have $\dot{x} = \dot{x}_2 - \dot{x}_1$ and $\ddot{x} = \ddot{x}_2 - \ddot{x}_1$. Therefore,

$m_1m_2\ddot{x} = -k(m_1 + m_2)x$ …..(iv)

and the solution to this equation is :

$x(t) = A\cos(\omega_0 t + \phi)$ …..(v)

where $\omega_0 = \sqrt{\dfrac{m_1 + m_2}{m_1m_2}k}$. Notice that x does not denote the position of any of the masses. It denotes the difference between the distance between the masses and the initial state, so that $x = (x_2 - x_1) - a$. The velocity of the center of mass is given by:

$\vec{v}_{cm} = \dfrac{p_0\hat{x}}{m_1 + m_2}$ …..(vi)

(b) The equation specified in the problem is easily derived from Eq.(iii), which we found in the first section. The constant μ is called the "reduced mass" of the system and is defined as

$\dfrac{1}{\mu} \equiv \dfrac{1}{m_1} + \dfrac{1}{m_2}$ …..(vii)

or in a different form, $\mu = \dfrac{m_1m_2}{m_1 + m_2}$ …..(viii)

(c) Taking energy into consideration, we have

$E_k = \dfrac{1}{2}m_1\left(\dfrac{p_0}{m_1}\right)^2 = \dfrac{1}{2}\dfrac{p_0^2}{m_1}$ …..(ix)

where E_k is the initial kinetic energy. The kinetic energy of the center of mass is :

$E_{k(cm)} = \dfrac{1}{2}\dfrac{p_0^2}{m_1 + m_2}$ …..(x)

and therefore, the total kinetic energy in the center of mass frame becomes:

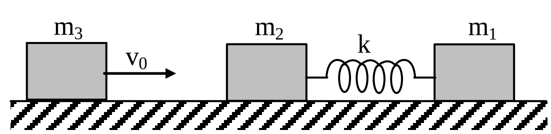
$E'_{k(cm)} = E_k - E_{k(cm)} = \dfrac{m_2}{2m_1(m_1 + m_2)}p_0^2$ …..(xi)

The kinetic energy is proportional to the square of amplitude,

$E'_k = \dfrac{1}{2}kA^2$

Therefore, $A = \sqrt{\dfrac{m_2p_0^2}{km_1(m_1 + m_2)}}$ …..(xii)

Q.12 Two masses m_1 and m_2 are attached to the two ends of a spring of force constant k. The system lies horizontally on a perfectly smooth surface. A third mass m_3, is thrown with a velocity of v_0, horizontally onto the plane to hit mass m_2. The two masses m_2 and m_3 stick together at the moment of collision. The sticking process occurs almost immediately, so that the length of the spring does not change. It is given that $m_1 = 2m$, $m_2 = m$ and $m_3 = \dfrac{1}{2}m$.



(a) What is the velocity of the center of mass before and after the collision ?

(b) How much kinetic energy is lost during the collision ?

(c) What are the velocities of the masses m_1 and $m_2 + m_3$ immediately after the collision in c.m. frame ?

(d) What is the maximum that the spring shrinks ?

Sol. **(a)** There are no external forces acting on the system, therefore, its total linear momentum is conserved. This means that $\vec{v}_{cm}$ is conserved throughout the collision process or

$$\vec{v}_{cm}^{before} = \frac{\sum_i m_i \vec{v}_i}{\sum_i m_i} = \frac{m_3 v_0}{m_1 + m_2 + m_3}\hat{x}$$

$$= \vec{v}_{cm}^{after} \qquad .....(i)$$

Since this problem is unidimensional, we omit the vector notation from now on.

(b) Let us write the expressions for the kinetic energy :

$$E_{before} = E_k^{(m_3)} = \frac{1}{2} m_3 v_0^2 \qquad .....(ii)$$

$$E_{after} = E_k^{(m_2+m_3)} = \frac{1}{2}(m_2 + m_3)\ v_{2+3}^2 \qquad .....(iii)$$

Notice that since the time that elapses when masses m_2 and m_3 stick together is short, mass m_1 stays at rest during the process. We calculate v_{2+3} using the law of conservation of linear momentum.

$$m_3 v_0 = (m_2 + m_3) v_{2+3} \rightsquigarrow v_{2+3} = \frac{m_3}{m_2 + m_3} v_0 \qquad .....(iv)$$

Substituting the value of v_{2+3} into E_{after}, we find

$$E_{after} = \frac{1}{2}\frac{m_3^2}{m_2 + m_3} v_0^2 \qquad .....(v)$$

Therefore, the loss of kinetic energy, ΔE, is

$$\Delta E = E_{before} - E_{after} = \frac{1}{2}\left(m_3 - \frac{m_3^2}{m_2 + m_3}\right) v_0^2 =$$

$$\frac{1}{2}\frac{m_2 m_3}{m_2 + m_3} v_0^2 \qquad .....(vi)$$

(c) Velocities in the center of mass frame are determined by :

$$v' = v - v_{cm} \qquad .....(vii)$$

where v is the velocity in the laboratory frame.Hence,

$$v'_{2+3} = v_{2+3} - v_{cm} = \frac{m_1 m_3}{(m_2 + m_3)(m_1 + m_2 + m_3)} \qquad .....(viii)$$

and since $v_1 = 0$, we have

$$v'_1 = v_1 - v_{cm} = -\frac{m_3}{m_2 + m_3} v_0 \qquad .....(ix)$$

(d) We use the law of conservation of energy

$$E_{after} = \frac{1}{2}(m_2 + m_3)v^2_{2+3} \qquad .....(x)$$

$$E'_{after} = \frac{1}{2}(m_1 + m_2 + m_3)v^2_{cm} + \frac{1}{2}k(\Delta x)^2_{max} \qquad .....(xi)$$

To determine the maximal shrinking of the spring. E'_{after} is the energy in the state of maximal shrinking. Note that in the state of maximal shrinking, the three masses move at the same velocity as the center of mass. Using the equality $E_{after} = E'_{after}$, and substituting v_{cm} and v_{2+3}, we obtain :

$$(\Delta x)_{max} = m_3 v_0 \sqrt{\frac{m_1}{k(m_1 + m_3)(m_1 + m_2 + m_3)}} \qquad .....(xii)$$

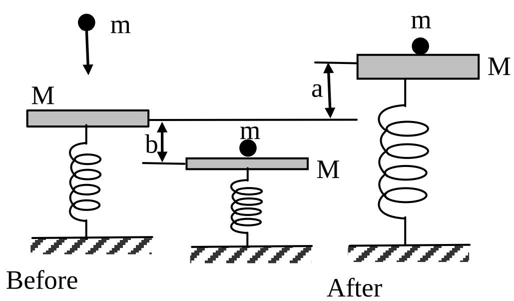
Q.13 **(a)** A mass m with kinetic energy E_k collides with another mass M, initially at rest, and sticks to it at the moment of contact. What is the total kinetic energy immediately after the collision ?

(b) The mass M is used as the weighting surface of a spring-scale whose spring is ideal (a massless spring). A body of known mass m is released from a certain height from where it falls to hit M. The two masses M and m stick together at the moment they touch, and move together from then on. The oscillations they perform reach to height a above the original level of the scale, and depth b below it (see Fig.)

(i) Find the constant of force of the spring.

(ii) Find the oscillation frequency.

(iii) What is the height above the initial level from which the mass m was released ?



Sol. (i) The initial kinetic energy is given by $E_k = \frac{1}{2m}p^2$, and the final value of the kinetic energy is given by $E'_k = \frac{1}{2(m+M)}p'^2$. Using linear-momentum conservation ($p' = p$), we have

$$E'_k = \frac{m}{m+M}E_k \qquad .....(i)$$

(ii)(a) Since the point of equilibrium changes, we know that $a \neq b$. (it moves down from the original level due to the extra mass m). Furthermore, using $a - (-b) = a + b = 2A$, where A is the amplitude of oscillations, and $a - y = A$, where y is the height of the new point of equilibrium relative to the original one ($y < 0$), we find that the point of equilibrium is $\frac{b-a}{2}$ below the original level of the scale. Applying the equilibrium of forces, we can write :

$$k|y| = k\left(\frac{b-a}{2}\right) = mg \qquad .....(ii)$$

And, therefore, $k = \frac{2mg}{b-a}$.

(b) The oscillation frequency is $\nu = \frac{\omega}{2\pi}$, where ω is defined as :

$$\omega = \sqrt{\frac{k}{m+M}} = \sqrt{\frac{2mg}{(m+M)(b-a)}} \qquad .....(iii)$$

(c) The potential energy in harmonic motion is known to be $\frac{1}{2}kx^2$. Therefore, the law of conservation of energy yields

$$E\,(t=0) = E'_k + \frac{1}{2}ky^2 = E'_k + \frac{k}{2}\left(\frac{b-a}{2}\right)^2 =$$

$$E'' = \frac{k}{2}\left(\frac{b+a}{2}\right)^2 \qquad .....(iv)$$

Where E″ denotes the total energy when the mass stops; i.e., when the amplitude is maximal and the kinetic energy vanishes. Using the result of the first section ($E_k = mgh$ and the relation between E_k and E'_k) along with Eq.(iv), we obtain :

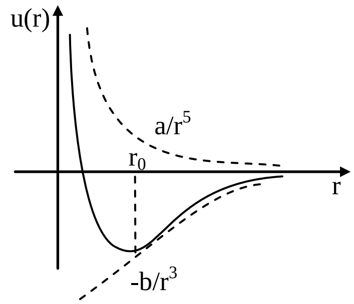
$$E'_k = mgh\frac{m}{m+M} = \frac{k}{2}\left(\frac{b+a}{2}\right)^2 - \frac{k}{2}\left(\frac{b-a}{2}\right)^2 = \frac{k}{2}ab \qquad .....(v)$$

Plugging in the value of k, we arrive at :

$$h = \frac{m+M}{m}\frac{ab}{b-a} \qquad .....(iv)$$

Q.14 Assume that the potential energy of a bi-atomic molecule is given by a two term expression, composed of a binding term $\frac{b}{r^3}$ and short-range repulsion term $\frac{a}{r^5}$, where r is the distance between atoms,

$$U(r) = \frac{a}{r^5} - \frac{b}{r^3}$$



(a) Find the equilibrium point and the force constant between the two atoms.

(b) What is the oscillation frequency of the molecule if the mass of each atom equals m?

Sol. **(a)** The equilibrium and stability point of a physical system is obtained where the potential is minimal; namely, when $\frac{\partial U}{\partial r} = 0$, or $\vec{F} = -\nabla U = 0$. Therefore, we differentiate U with respect to r,

$$\frac{dU}{dr} = -\frac{5a}{r^6} + \frac{3b}{r^4} = 0 \qquad \text{.....(i)}$$

The equation clearly has the solution :

$$r_0 = \sqrt{\frac{5a}{3b}} \qquad \text{.....(ii)}$$

Let us write U(r) as a Taylor Series around $r = r_0$. This technique is correct for any potential function which obeys a few smoothness conditions. Thus,

$$U(r) = U(r_0) + \left.\frac{dU(r)}{dr}\right|_{r=r_0} r + \frac{1}{2}\left.\frac{d^2U(r)}{dr^2}\right|_{r=r_0} r^2 + ... \qquad \text{.....(iii)}$$

Since r_0 is an equilibrium point, we have $\left.\frac{dU(r)}{dr}\right|_{r=r_0} = 0$. Therefore, using a second order expansion of the series, we clearly see that it

behaves like the potential of a spring with a force constant k, where :

$$k = \left.\frac{d^2U(r)}{dr^2}\right|_{r=r_0} \qquad \text{.....(iv)}$$

$U(r_0)$ is an unimportant constant since the potential is always defined by an arbitrary constant since the potential is always defined by an arbitrary constant. In our case,

$$k = \left.\frac{d^2U(r)}{dr^2}\right|_{r=r_0} = \left[\frac{1}{r^5}\left(\frac{30a}{r^2} - 12b\right)\right]_{r=r_0} = 6b\left(\frac{3b}{5a}\right)^{\frac{5}{2}} \qquad \text{.....(v)}$$

(b) The frequency of a system of N masses performing harmonic oscillations is generally

$$v = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}} \qquad \text{.....(vi)}$$

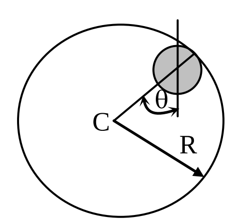
where k is the "spring's" force constant and μ is the reduced mass of the system defined as :

$$\frac{1}{\mu} = \sum_{i=1}^{N}\frac{1}{m_i} \qquad \text{.....(vii)}$$

There are two equal masses; therefore, $\mu = \frac{mm}{m+m} = \frac{m}{2}$. Substituting the constant k found in the previous section, we obtain :

$$v = \frac{1}{2\pi}\sqrt{\frac{12b}{m}\left(\frac{3b}{5a}\right)^{\frac{5}{2}}} \qquad \text{.....(viii)}$$

Q.15 A cylindrical ring of mass m and radius R rolls without sliding, on a fixed cylinder of radius r (see Fig.). The gravitational acceleration is g.



(a) What is the path of the center of the ring C, based on geometrical considerations ?

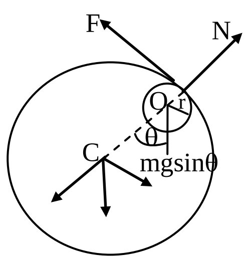
(b) Find the forces exerted on the ring, write the equations of motion, and find the magnitude and direction of the friction force exerted on the ring, recalling that there is no slipping.

(c) If the ring is released very close to the point of equilibrium (θ_0 is very small), calculate θ as function of time.

Sol. **(a)** The path of C is clearly a circle. A circle is defined as the collection of all points whose distance from a given fixed point is constant. In the case defined in figure. We take the fixed point to be O, and the constant

$|\overline{CO}| = |R - r|$ .....(i)

(b) Consider the force diagram in Figure. The equation of tangent force is



$mg \sin\theta - f = ma$ .....(ii)

where f is the friction force and a is the tangential acceleration. Note that $f \neq \mu N$. The equation of the torques is given by :

$fR = I_0\alpha$ .....(iii)

where α is the angular acceleration and I_0 is the moment of inertia of the ring. Since we have $\alpha = \frac{a}{R}$ and $I_0 = mR^2$, we can combine all the above relations to obtain :

$f = \frac{mg}{2}\sin\theta$ .....(iv)

(c) The force equation on the θ axis is given by

$F_\theta = -mg \sin\theta + \frac{mg}{2}\sin\theta \approx -\frac{mg}{2}\theta$ .....(v)

Where the signs are taken relative to the positive θ direction as defined in figure. The length of the arc traveled by the center of the ring, denoted by x, is clearly :

$\theta = \frac{x}{R - r}$ .....(vi)

Therefore, $F_x = -\frac{mg}{2(R-r)}x = -\text{"k"}\,x$ .....(vii)

Where k is an imaginary "spring force constant". Hence,

$\theta = \theta_0 \cos\omega t$ .....(viii)

Where $\omega = \sqrt{\frac{\text{"k"}}{m}} = \sqrt{\frac{g}{2(R-r)}}$ .....(ix)

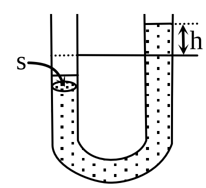
Q.16 Consider a U shaped pipe with a cross sectional area S (see fig.). The pipe contains a liquid of volume V and density ρ. Denote by h the instantaneous height relative to the equilibrium position.

(a) Derive the equation of motion.

(b) Find the solution of the equation of motion corresponding to the initial conditions, $h(t = 0) = h_0$, $\dot{h}(t = 0) = 0$.

(c) Suppose that, due to viscosity effects, one has an effective damping coefficient b. Write the equation of motion including the damping.

(d) An external force pressure $P = P_0\cos\omega_0 t$ is applied at one end of the pipe. Calculate the steady state motion of h.



Sol. (a) The kinetic energy of the fluid is given by :

$E_k = \frac{1}{2}m\dot{h}^2 = \frac{1}{2}\rho V\dot{h}^2$ .....(i)

The potential energy (relative to the equilibrium position) is :

$E_p = 2\rho gSh\frac{h}{2} = \rho gSh^2$ .....(ii)

When no friction forces exist, the total energy is conserved :

$$\frac{d}{dt}(E_p + E_k) = 0 \qquad .....(iii)$$

Therefore,

$$\rho V \ddot{h} + 2\rho g S h = 0 \qquad .....(iv)$$

Thus, we arrive at the equation of motion :

$$\ddot{h} + \omega_n^2 h = 0 \qquad .....(v)$$

Where $\omega_n = \sqrt{\frac{2gS}{V}}$.

(b) The most general solution of the equation of motion is clearly :

$$h(t) = A\cos(\omega_n t + \phi) \qquad .....(vi)$$

Substituting the initial conditions yields $A = h_0$ and $\phi = 0$. Thus,

$$h(t) = h_0 \cos\omega_n t \qquad .....(vii)$$

(c) The equation of motion with damping is :

$$\rho v \ddot{h} + b\dot{h} + 2\rho g S h = 0 \qquad .....(viii)$$

or, $$\ddot{h} + 2\varsigma\omega_n \dot{h} + \omega_n^2 h = 0 \qquad .....(ix)$$

where , $$\begin{cases} \omega_n = \sqrt{\frac{2gS}{V}} \\ \varsigma = \frac{1}{2}\sqrt{\frac{b^2}{2\rho^2 gSV}} \end{cases} \qquad .....(x)$$

(d) The external force is given by :

$$F = PS = P_0 S \cos\omega_0 t \qquad .....(xi)$$

The equation of forced motion becomes:

$$\ddot{h} + 2\varsigma\omega_n \dot{h} + \omega_n^2 h = \alpha\cos\omega_0 t \qquad .....(xii)$$

Where $\alpha = \frac{P_0 S}{\rho V}$.

At steady state, only the particular solution of the equation of motion remains.

$$h = A\cos(\omega_0 t + \phi) \qquad .....(xiii)$$

It can be shown from here that :

$$A[(\omega_n^2 - \omega_0^2)\cos(\omega_0 t + \phi) - 2\varsigma\omega_n\omega_0\sin(\omega_0 t + \phi)] = \alpha\cos\omega_0 t$$

Which may be written as :

$$A\sqrt{(\omega_n^2 - \omega_0^2)^2 + 4\varsigma^2\omega_n^2\omega_0^2}\cos(\omega_0 t + \phi - \phi_0) = \alpha\cos\omega_0 t \qquad .....(xiv)$$

With $\phi_0 = \tan^{-1}\left(\frac{-2\varsigma\omega_n\omega_0}{\omega_n^2 - \omega_0^2}\right)$

Thus, we obtain :

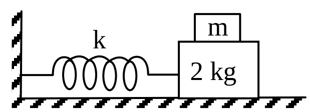
$$\begin{cases} A = \frac{\alpha}{\sqrt{(\omega_n^2 - \omega_0^2)^2 + 4\varsigma^2\omega_n^2\omega_0^2}} \\ \phi = \phi_0 = \tan^{-1}\left(\frac{-2\varsigma\omega_n\omega_0}{\omega_n^2 - \omega_0^2}\right) \end{cases}$$

Q.17 When displaced and released, the 2 kg mass in **figure** oscillates on the frictionless horizontal surface with period $\pi/6$ seconds.

(a) How large a force is necessary to displace the mass 2 cm from equilibrium ?

(b) If a small mass is placed on the 2 kg block and the coefficient of static friction between the small mass and the 2kg block is 0.1, what is the maximum amplitude of oscillation before the small mass slips ?

(Assume the period is unaffected by adding the small mass).



Sol. Let k be the spring constant. The equation of the motion of the mass is

$$2\ddot{x} + kx = 0$$

or $$\ddot{x} + \omega^2 x = 0$$

where **x** is the displacement of the block from its equilibrium position and $\omega^2 = \frac{k}{2}$. The general solution is

$x = A\cos(\omega t + \phi)$.

The period of oscillation is

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2}{k}} = \frac{\pi}{6}$,

Giving $\omega = 12\ s^{-1}$, $k = 288\ Nm^{-1}$. If $x = x_0$ at t = 0, then $\phi = 0$, $A = x_0$ and the solution is

$x = x_0\cos(12t)$.

(a) The force needed is

$f = kx = 288 \times 2 \times 10^{-2} =$ **5.76N**. **Ans.**

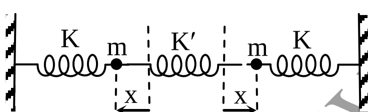
(b) If the small mass moves together with the 2kg block, it has the same acceleration a the latter, i.e. $\ddot{x} = -144x_0\cos(12t)$. Let its mass be **m**. **When it starts to slip, the maximum horizontal force on it just exceeds the static friction :**

$0.1 \times mg = 144\ mx_0$,

Giving, $x_0 = \frac{0.98}{144} = 6.8 \times 10^{-3}$m.

If x_0 exceeds this value m will slip. Hence it gives the maximum amplitude for no slipping.

Q.18 For the system of 2 identical masses and massless springs shown in **figure**. Calculate the period of oscillation if the masses are released from the initial symmetrical configuration shown.



Sol. Due to symmetry, the oscillations of the two masses are the same. Consider one of them and write down the equation of motion

$m\ddot{x} = -Kx - K'(x + x) = -(K + 2K')x$,

where x is the displacement from the respective equilibrium position. Then the angular frequency of oscillation is

$\omega = \sqrt{\frac{K+2K'}{m}}$

And the period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{K+2K'}}$$

Note that generally speaking, there are two modes of linear vibration for this system corresponding to two normal modes. But the symmetrical initial condition determines that only one mode is excited.

Q.19 Human legs are such that a person of normal size finds it comfortable to walk at a natural, swinging pace of about one step per second, the uncomfortable to force a pace susbstantially faster or slower. Neglecting the effect of the knee joint, use the simplest model you can to estimate the frequency which determines this pace, and to find what characteristic of the leg it depends on .

Sol. Consider the human leg to be a uniform pole of length ℓ. In the simplest model, the swinging frequency of the leg should be equal to the characteristic frequency of the pole when it swings about its end as a fixed point. The motion is that of a compound pendulum described by

$$\frac{1}{3}m\ell\ddot{\theta} = -\frac{1}{2}mg\ell\sin\theta.$$

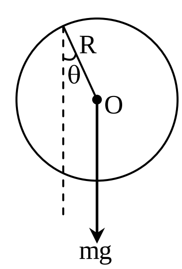
Or $\ddot{\theta} + \frac{3g}{2\ell}\theta = 0$

For θ small. Then the frequency of swing is $\nu = \frac{1}{2\pi}\sqrt{\frac{3g}{2\ell}}$. If the take $\ell \approx 0.4$ m, $\nu \approx 1s^{-1}$.

Q.20 A uniform hoop of mass M and radius R hangs in a vertical plane supported by a knife edge at one point on the inside circumference. Calculate the natural frequency of small oscillation.

Sol. The moment of inertia of the hoop about the supporting knife edge is

$I = MR^2 + MR^2 = 2MR^2$



Refering to figure, we have the equation of motion

$I\ddot{\theta} = -MgR\sin\theta$

Or $\quad I\ddot{\theta} = -MgR\theta$

For small oscillation. Hence the frequency is

$$\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{MgR}{I}} = \frac{1}{2\pi}\sqrt{\frac{g}{2R}}$$