

MISSION NDA MATHEMATICS

COMPLETE TOPICWISE QUESTIONS

LIMITS, CONTINUITY & DIFFERENTIABILITY

- $\lim_{\theta \rightarrow 0} \frac{4\theta(\tan \theta - 2\theta \tan \theta)}{(1 - \cos 2\theta)^2}$ is equal to
 (a) $-\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$
 (c) 1 (d) 2
- $\lim_{x \rightarrow 0} \frac{4^x - 1}{3^x - 1}$ equals
 (a) 1 (b) $\log_4 3$
 (c) $\log 4$ (d) $\log_3 4$
- If α is a repeated root of $ax^2 + bx + c = 0$, then $\lim_{x \rightarrow \alpha} \frac{\sin(ax^2 + bx + c)}{(x - \alpha)^2}$ is
 (a) 0 (b) a
 (c) b (d) c
- The value of $\lim_{x \rightarrow 0} \frac{\sqrt{\frac{1}{2}(1 - \cos 2x)}}{x}$ is
 (a) -1 (b) 0
 (c) 1 (d) none of these
- $\lim_{x \rightarrow -2} \frac{\sin^{-1}(x+2)}{x^2 + 2x}$ is equal to
 (a) $-\frac{1}{2}$ (b) 0
 (c) ∞ (d) none of these
- If $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by

$$f(x) = \begin{cases} \frac{x-2}{x^2 - 3x + 2}, & \text{if } x \in \mathbb{R} - (1, 2) \\ 2, & \text{if } x = 1 \\ 1, & \text{if } x = 2 \end{cases}$$

then $\lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2}$ is equal to

- (a) -1 (b) $-(1/2)$
 (c) 1 (d) does not exist
- The value of $\lim_{x \rightarrow 0} \left\{ \frac{3x - \sin^{-1} x}{4x - \tan^{-1} x} \right\}$ is
 (a) $\frac{3}{4}$ (b) $\frac{4}{3}$
 (c) $\frac{2}{3}$ (d) $\frac{3^2}{4^2}$



8. The value of $\lim_{x \rightarrow 0} \frac{\sin \sqrt[3]{x} \ln(1-5x)}{(\tan^{-1} \sqrt{x})^2 (e^{\sqrt[3]{x}} - 1)}$ is
- (a) $-\frac{5}{3}$ (b) -5
(c) $-\frac{2}{3}$ (d) $-\frac{3}{5}$
9. The value of $\lim_{x \rightarrow 0} \frac{(1-\cos x)^2}{\tan^3 x - \sin^3 x}$ is
- (a) $\frac{1}{12}$ (b) 0
(c) ∞ (d) does not exist
10. The value of $\lim_{x \rightarrow -\infty} x + \sqrt{x^2 + x^2 \sin\left(\frac{1}{x}\right)}$ is
- (a) -2 (b) 0
(c) 2 (d) none of these
11. $\lim_{n \rightarrow \infty} \left\{ \frac{1 + \frac{1}{3} + \frac{1}{9} + \dots \text{ to } (n+1) \text{ terms}}{1 + \frac{1}{2} + \frac{1}{4} + \dots \text{ to } (n+1) \text{ terms}} \right\} =$
- (a) $\frac{3}{4}$ (b) $\frac{4}{3}$
(c) $\frac{3}{2}$ (d) $\frac{2}{3}$
12. $\lim_{n \rightarrow \infty} \left(\frac{1+a+a^2+\dots+a^{n-1}}{1+b+b^2+\dots+b^{n-1}} \right) = 0$, if
- (a) $a = b$ (b) $a < b$
(c) $a > b$ (d) $1 < a < b$
13. $\lim_{n \rightarrow \infty} \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{(2n-1) \cdot (2n+1)}$ is equal to
- (a) $\frac{1}{4}$ (b) $\frac{1}{3}$
(c) $\frac{1}{2}$ (d) none of these
14. $\lim_{x \rightarrow 0} \left(\frac{1^x + 2^x + 3^x + \dots + n^x}{n} \right)^{a/x}$ is equal to
- (a) $(\ln)^a$ (b) $\frac{1}{a \ln}$
(c) $(\ln)^{a/n}$ (d) $\frac{n}{(\ln)^a}$
15. The value of $\lim_{x \rightarrow \frac{\pi}{2}} \left\{ \frac{1^{\cos^2 x} + 2^{\cos^2 x} + \dots + n^{\cos^2 x}}{n} \right\}^{\frac{1}{\cos^2 x}}$ is
- (a) $\frac{1}{(\ln)^n}$ (b) $(\ln)^{\frac{1}{n}}$
(c) $\frac{n^n}{\ln}$ (d) $\left(\frac{\ln}{1-n} \right)^{\frac{n}{n+1}}$



16. $\lim_{x \rightarrow \infty} x^2 \sqrt{\left(1 - \cos \frac{1}{x}\right) \sqrt{\left(1 - \cos \frac{1}{x}\right) \sqrt{\left(1 - \cos \frac{1}{x}\right) \sqrt{\dots \infty}}}$ is equal to

(a) 0 (b) 1/2
(c) 1 (d) 2/3

17. Let $f(x) = \begin{cases} \frac{(x^3 + x^2 - 16x)}{(x-2)^2} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$. If $f(x)$ is continuous for all x , then $k =$

(a) 7 (b) 2
(c) 0 (d) -1

18. The function $f(x) = \sin^{-1}(\cos x)$ is

(a) discontinuous at $x = 0$ (b) continuous at $x = 0$
(c) differentiable at $x = 0$ (d) none of these

19. The function $f(x) = \begin{cases} x + 2, & 1 \leq x \leq 2 \\ 4, & x = 2 \\ 3x - 2, & x > 2 \end{cases}$ is continuous at

(a) $x = 2$ only (b) $x \leq 1$
(c) $x \geq 1$ (d) none of these

20. Let the function $f(x) = \frac{x}{1+e^{1/x}}$, if $x \neq 0$; $f(x) = 0$. If $x = 0$, the derivative from the right $f'(0^+)$ and the derivative from the left $f'(0^-)$ are

(a) 0 and 1 (b) -1 and 1
(c) 1 and -1 (d) 1 and 0

21. The function $f(x) = 1 + |\sin x|$ is

(a) continuous nowhere
(b) continuous everywhere and not differentiable at $x = 0$
(c) differentiable nowhere
(d) differentiable at $x = 0$

22. Let $f(x) = \begin{cases} 0, & x < 0 \\ x^2, & x \geq 0 \end{cases}$.

Then for all x ,

- (a) f'' is differentiable
(b) f is differentiable
(c) f' is not continuous
(d) f is not continuous



23. The function $f(x) = \begin{cases} \frac{1}{4x^2-1}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous
- (a) everywhere except at $x = 0$ and $x = \infty$ (b) nowhere
(c) everywhere (d) everywhere except at $x = 0$
24. At point $x = 1$, the function $f(x) = \begin{cases} x^3 - 1; & 1 < x < \infty \\ x - 1; & -\infty < x \leq 1 \end{cases}$ is
- (a) continuous and differentiable
(b) continuous but not differentiable
(c) discontinuous and differentiable
(d) discontinuous and not differentiable
25. A function is defined as $f(x) = \begin{cases} e^x; & x \leq 0 \\ |x - 1|; & x > 0 \end{cases}$, then $f(x)$ is
- (a) continuous at $x = 0, 1$ (b) differentiable at $x = 0$
(c) differentiable at $x = 1$ (d) none of these
26. For the function $f(x) = \begin{cases} \frac{1-x}{|x-1|}, & x < 1 \\ 1, & x = 1 \\ x^2, & x > 1 \end{cases}$ which of the following is true?
- (a) It is continuous at all points
(b) It is continuous at all points except at $x = 1$
(c) It is differentiable at all points
(d) none of these
27. If $f(x) = \begin{cases} \frac{1-\cos x}{x^2}, & x < 0 \\ \frac{1}{2}e^x, & x \geq 0 \end{cases}$, then at $x = 0$, f is
- (a) continuous (b) not continuous
(c) differentiable (d) none of these
28. The function $f(x) = \begin{cases} |2x - 3| \cdot [x], & x \geq 1 \\ \sin\left(\frac{\pi x}{2}\right), & x < 1 \end{cases}$ (where $[x]$ denotes the greatest integer $\leq x$) is
- (a) continuous at $x = 2$ (b) differentiable at $x = 1$
(c) continuous but not differentiable at $x = 1$ (d) none of these
29. Let $f(x) = \begin{cases} \frac{x^4 - 5x^2 + 4}{(x-1)(x-2)}, & x \neq 1, 2 \\ 6, & x = 1 \\ 12, & x = 2 \end{cases}$
- then $f(x)$ is continuous on the set
- (a) $\mathbb{R}$ (b) $\mathbb{R} - \{1\}$
(c) $\mathbb{R} - \{2\}$ (d) $\mathbb{R} - \{1, 2\}$



30. The set of all points, where the function $f(x) = \frac{x}{1+|x|}$ is differentiable, is

- (a) $(-\infty, \infty)$ (b) $(0, \infty)$
(c) $(-\infty, 0) \cup (0, \infty)$ (d) none of these

31. The value of $\lim_{n \rightarrow \infty} \frac{1+2^4+3^4+\dots+n^4}{n^5} - \lim_{n \rightarrow \infty} \frac{1+2^3+3^3+\dots+n^3}{n^5}$ is

- (a) 0 (b) $1/4$
(c) $1/5$ (d) $1/30$

32. The function $f(x) = \frac{1+\sin x - \cos x}{1-\sin x - \cos x} x \neq 0$ is continuous at $x = 0$ if $f(0)$ is

- (a) 1 (b) -1
(c) $\sqrt{2}$ (d) $-\sqrt{2}$

33. The value of $\lim_{x \rightarrow 1} \frac{x+x^2+\dots+x^n-n}{x-1}$ is

- (a) n (b) $\frac{n+1}{2}$
(c) $\frac{n(n+1)}{2}$ (d) $\frac{n(n-1)}{2}$

34. If the function $f(x) = \begin{cases} (1 + |\sin x|)^{\frac{a}{\sin x}}, & -\frac{\pi}{6} < x < 0 \\ b, & x = 0 \\ e^{\frac{\tan 2x}{\tan 3x}}, & 0 < x < \frac{\pi}{6} \end{cases}$ is continuous at $x = 0$, then

- (a) $a = \log_e b, a = \frac{2}{3}$ (b) $b = \log_e a, a = \frac{2}{3}$
(c) $a = \log_e b, b = 2$ (d) none of these

35. If $f(x) = \sin |x| - e^{|x|}$ then at $x = 0$, $f(x)$ is

- (a) continuous but not differentiable
(b) neither continuous nor differentiable
(c) both continuous and differentiable
(d) none of these

36. The value of $\lim_{x \rightarrow 1} \frac{x^n + x^{n-1} + x^{n-2} + \dots + x^2 + x - n}{x-1}$ is

- (a) $\frac{n(n+1)}{2}$ (b) 0
(c) 1 (d) n

37. If $f(x) = \begin{cases} \frac{x \log \cos x}{\log(1+x^2)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

then

- (a) $f(x)$ is not continuous at $x = 0$
(b) $f(x)$ is continuous at $x = 0$
(c) $f(x)$ is continuous at $x = 0$ but not differentiable at $x = 0$
(d) none of these



38. The value of $\lim_{x \rightarrow \infty} [\sqrt{x + \sqrt{x + \sqrt{x}}} - \sqrt{x}]$ is
 (a) $\frac{1}{2}$ (b) 1
 (c) 0 (d) none of these
39. The set of all points where $f(x) = \sqrt[3]{x^2|x|} - |x| - 1$ is not differentiable is
 (a) $\{0\}$ (b) $\{-0, 0, 1\}$
 (c) $\{0, 1\}$ (d) none of these
40. $\lim_{x \rightarrow 0} \frac{x\sqrt{y^2 - (y-x)^2}}{(\sqrt{8xy - 4x^2} + \sqrt{8xy})^3}$ equals
 (a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2}$
 (c) $\frac{1}{4}$ (d) none of these
41. Let $f(x) = (x + |x|)|x|$. Then for all x ,
 (a) f is continuous (b) f' is differentiable for some x
 (c) f' is continuous (d) f'' is continuous
42. The value of $\lim_{x \rightarrow 0} \frac{\ln(1+2h) - 2\ln(1+h)}{h^2}$ is
 (a) 1 (b) -1
 (c) 0 (d) none of these
43. If $f(x) = \begin{cases} ax^2 + b, & b \neq 0, c \leq 1 \\ bx^2 + ax + c, & x > 1 \end{cases}$ then $f(x)$ is continuous and differentiable at $x = 1$ if
 (a) $c = 0, a = 2b$ (b) $a = b, c \in \mathbb{R}$
 (c) $a = b, c = 0$ (d) $a = b, c \neq 0$
44. $\lim_{n \rightarrow \infty} \frac{1 - (2\sin x)^{2n}}{1 + (2\sin x)^{2n}}$ is not continuous at $x =$
 (a) $\pi/6$ (b) $\pi/4$
 (c) $\pi/2$ (d) none of these
45. Let $f(x) = \sin \frac{1}{x}, x \neq 0$. Then $f(x)$ can be continuous at $x = 0$
 (a) if $f(0) = 1$ (b) if $f(0) = 0$
 (c) if $f(0) = -1$ (d) for no definite value of $f(0)$
46. The set of all points where the function $f(x) = \frac{x}{1+|x|}$ is differentiable is
 (a) $(-\infty, \infty)$ (b) $(\infty, 0)$
 (c) $(-\infty, 0) \cup (0, \infty)$ (d) none of these
47. If $f(x) = [x]$, where $[]$ is the greatest integer function, and $g(x) = x(1 - x^2)(4 - x^2)$, then $g[f(x)]$ is



- (a) discontinuous at all integers
(c) continuous at $x = 0, \pm 1, \pm 2$

- (b) continuous at all integers
(d) discontinuous at $x = 0, \pm 1, \pm 2$

48. Let $f(x) = [\cos x + \sin x], 0 < x < 2\pi$, where $[x]$ denotes the greatest integer less than or equal to x . The number of points of discontinuity of $f(x)$ is

- (a) 6
(c) 4
- (b) 5
(d) 3

49. If $[x]$ denotes the greatest integer $\leq x$, then $f(x) = \frac{\tan(\pi[x-\pi])}{1+[x]^2}$ is

- (a) discontinuous at some x
(b) continuous at all x , but the derivative $f'(x)$ does not exist for some x
(c) $f'(x)$ exists for all x but $f''(x)$ does not exist
(d) $f'(x)$ exists for all x

50. The number of points where $f(x) = \frac{1}{\log|x^2-3x+3|}$ is discontinuous is

- (a) 2
(c) 6
- (b) 4
(d) 0

51. Let $f(x) = \sin x$ if x is rational and $f(x) = 1 - 2\cos x$ if x is irrational, then

- (a) $f(x)$ is nowhere continuous
(b) $f(x)$ is continuous at one point only
(c) $f(x)$ is injective
(d) $f(x)$ is continuous at infinite number of points

52. The value of k for which the function

$$f(x) = \begin{cases} \sin \frac{1}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

is continuous is

- (a) 8
(c) -1
- (b) 1
(d) none of these

53. If $f(x) = \frac{x}{1+e^{1/x}}, x \neq 0$ and $f(0) = 0$, then at $x = 0$

- (a) f is continuous
(b) f is discontinuous
(c) f is left continuous only
(d) f is right continuous only



ANSWER KEY

1. (b)	2. (d)	3. (a)	4. (d)	5. (a)	6. (a)	7. (c)	8. (b)	9. (d)	10. (d)
11. (a)	12. (d)	13. (c)	14. (c)	15. (b)	16. (b)	17. (a)	18. (b)	19. (c)	20. (a)
21. (b)	22. (b)	23. (d)	24. (b)	25. (a)	26. (a)	27. (a)	28. (c)	29. (a)	30. (a)
31. (c)	32. (b)	33. (c)	34. (a)	35. (c)	36. (a)	37. (b)	38. (a)	39. (d)	40. (d)
41. (c)	42. (b)	43. (a)	44. (a)	45. (d)	46. (a)	47. (c)	48. (c)	49. (d)	50. (d)
51. (a)	52. (d)	53. (a)							

