

MISSION NDA MATHEMATICS

COMPLETE TOPICWISE QUESTIONS

COMPLEX NUMBER

1. The equation $z\bar{z} + a\bar{z} + \bar{a}z + b = 0, b \in \mathbb{R}$ represents a circle if
 - (a) $|a|^2 = b$
 - (b) $|a|^2 > b$
 - (c) $|a|^2 < b$
 - (d) none of these
2. If $\frac{c+i}{c-i} = a + ib$, where a, b, c are real, then $a^2 + b^2$ is equal to
 - (a) 7
 - (b) 1
 - (c) c^2
 - (d) $-c^2$
3. $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ will be real, if θ is
 - (a) $2n\pi$
 - (b) $n\pi + (\pi/2)$
 - (c) $n\pi$
 - (d) none of these
4. If $1, \alpha_1, \alpha_2, \dots, \alpha_{3n}$ are the roots of equation $x^{3n+1} - 1 = 0$ and ω, ω^2 are the imaginary cube roots of unity, then $\frac{(\omega^2 - \alpha_1)(\omega^2 - \alpha_2) \dots (\omega^2 - \alpha_{3n})}{(\omega - \alpha_1)(\omega - \alpha_2) \dots (\omega - \alpha_{3n})} = ?$
 - (a) ω
 - (b) $-\omega$
 - (c) 1
 - (d) ω^2
5. If $(1+i)(1+2i)(1+3i) \dots (1+ni) = x + iy$, then the value of $2 \times 5 \times 10 \times \dots \times (1+n^2)$ is
 - (a) $x^2 + y^2$
 - (b) $x^2 - y^2$
 - (c) $\frac{\sqrt{x}}{2} + \frac{\sqrt{y}}{3}$
 - (d) $\frac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}}$
6. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then for any +ve integer n , x is equal to
 - (a) $4n$
 - (b) $4n + 1$
 - (c) $2n$
 - (d) $2n + 1$
7. If $1, \alpha_1, \alpha_2, \dots, \alpha_{n-1}$ are the n roots of unity, then $(1 - \alpha_1)(1 - \alpha_2) \dots (1 - \alpha_{n-1})$ is equal to
 - (a) $n - 1$
 - (b) n
 - (c) -1
 - (d) 1
8. If z is a complex number, then the minimum value of $|z| + |z - 1|$ is
 - (a) 1
 - (b) 0
 - (c) $1/2$
 - (d) none of these
9. If $|z^n \sin \theta_0 + z^{n-1} \sin \theta_1 + z^{n-2} \sin \theta_2 + \dots + z \sin \theta_{n-1} + \sin \theta_n| = 2$, then all the roots of the equation lie
 - (a) inside the circle $|z| = 1/2$
 - (b) outside the circle $|z| = 1/2$
 - (c) on the circle $|z| = 1/2$
 - (d) none of these
10. If the conjugate and the reciprocal of a complex number $z = x + iy$ are equal, then
 - (a) $x + y = 1$
 - (b) $x^2 + y^2 = 1$
 - (c) $x = 1$ and $y = 0$
 - (d) $x = 0$ and $y = 1$



11. The value of the sum $\sum_{n=1}^{13} (i^n + i^{n+1})$, where $i = \sqrt{-1}$, is
 (a) i (b) $i - 1$
 (c) $-i$ (d) 0
12. If $1, \omega$ and ω^2 are the three cube roots of unity and α, β and γ are the cube roots of p ($p < 0$), then for any x, y and z the expression $\frac{x\alpha + y\beta + z\gamma}{x\beta + y\gamma + z\alpha}$ is equal to
 (a) 1 (b) ω
 (c) ω^2 (d) none of these
13. The smallest positive integral value of n for which $\left(\frac{1-i}{1+i}\right)^n$ is purely imaginary with positive imaginary part, is
 (a) 1 (b) 3
 (c) 5 (d) none of these
14. If $\left|\frac{z-2}{z+2}\right| = n$, where n is a constant and $n \neq 1$, then the locus of z is
 (a) a straight line (b) a circle
 (c) a parabola (d) an ellipse
15. Let $\alpha = z_1 + z_2, \beta = \omega z_1 + \omega^2 z_2$ and $\gamma = \omega^2 z_1 + \omega z_2$, where ω is a complex cube root of unity, then
 (a) $\alpha^2 + \beta^2 + \gamma^2 = 8z_1 z_2$ (b) $\alpha\beta + \beta\gamma + \gamma\alpha = 3z_1 z_2$
 (c) $\alpha^3 + \beta^3 + \gamma^3 = 3(z_1^3 + z_2^3)$ (d) $\alpha\beta\gamma = 2(z_1^3 + z_2^3)$
16. If z_1, z_2 and z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$, then the value of $|z_1 + z_2 + z_3|$ is
 (a) equal to 1 (b) less than 1
 (c) equal to 3 (d) greater than 3
17. If $(1 + x + x^2)^n = a_0 + a_1 x + a_2 x^2 + \dots + a_{2n} x^{2n}$, then $a_0 + a_3 + a_6 + \dots$ is equal to
 (a) 3^n (b) 3^{n-1}
 (c) 3^{n-2} (d) none of these
18. If $z = x + iy$, then $|3z - 1| = |3(z - 2)|$ represents
 (a) x -axis (b) y -axis
 (c) circle (d) $x = c$
19. For a complex number z , the minimum value of $|z| + |z - 2|$ is
 (a) 1 (b) 2
 (c) 3 (d) none of these
20. If $|z_1| = |z_2| = \dots = |z_n| = 1$, then the value of $|z_1 + z_2 + z_3 + \dots + z_n|$ is
 (a) 1 (b) $|z_1| + |z_2| + \dots + |z_n|$
 (c) $\left|\frac{1}{z_1} + \frac{1}{z_2} + \dots + \frac{1}{z_n}\right|$ (d) none of these
21. If z_1 and z_2 are the roots of the equation $z^2 + az + b = 0$, then prove that the origin, z_1 and z_2 form an equilateral triangle if and only if
 (a) $a^2 = 3b$ (b) $b^2 = 3a$
 (c) $a^2 + 3b = 0$ (d) $b^2 + 3a = 0$
22. The value of z satisfying the equation $\log z + \log z^2 + \dots + \log z^n = 0$ is
 (a) $\cos\left[\frac{4m\pi}{n(n+1)}\right] + i\sin\left[\frac{4m\pi}{n(n+1)}\right], m = 0, 1, 2, \dots$



- (b) $\cos \left[\frac{4m\pi}{n(n+1)} \right] - i \sin \left[\frac{4m\pi}{n(n+1)} \right], m = 0, 1, 2, \dots$
 (c) $\sin \left(\frac{4m\pi}{n} \right) + i \cos \left(\frac{4m\pi}{n} \right), m = 0, 1, 2, \dots$
 (d) 0
23. If $a = \cos \alpha + i \sin \alpha, b = \cos \beta + i \sin \beta, c = \cos \gamma + i \sin \gamma$ and $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = 1$, then $\cos(\alpha - \beta) + \cos(\beta - \gamma) + \cos(\gamma - \alpha)$ is equal to
 (a) $3/2$ (b) $-(3/2)$
 (c) 0 (d) 1
24. The greatest value of the moduli of the complex number z satisfying the equation $\left| z - \frac{4}{z} \right| = 2$ is
 (a) $\sqrt{5}$ (b) $\sqrt{5} - 1$
 (c) $\sqrt{5} + 1$ (d) none of these
25. If $|z| = 1$ and $z \neq \pm 1$, then all the values of $\frac{z}{1-z^2}$ lie on
 (a) a line not passing through the origin (b) $|z| = \sqrt{2}$
 (c) the x -axis (d) the y -axis
26. If z is a complex number, then $|3z - 1| = 3|z - 2|$ represents
 (a) the y -axis (b) a circle
 (c) the x -axis (d) a line parallel to the y -axis
27. If a, b, c are integers not all equal and $\omega \neq 1$ is a cube root of unity, then min value of $|a + b\omega + c\omega^2|$ is
 (a) 0 (b) 1
 (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$
28. The values of x and y satisfying the equation $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$ are
 (a) $x = -1, y = 3$ (b) $x = 3, y = -1$
 (c) $x = 0, y = 1$ (d) $x = 1, y = 0$
29. The locus of the complex number z in an Argand plane satisfying the equation $\arg(z + i) - \arg(z - i) = (\pi/2)$ is
 (a) the boundary of a circle (b) the interior of a circle
 (c) the exterior of a circle (d) none of these
30. Let $3 - i$ and $2 + i$ be affixes of two points A and B in the Argand plane and P represent the complex number $z = x + iy$. Then the locus of P if $|z - 3 + i| = |z - 2 - i|$ is
 (a) the circle on AB as diameter (b) the line AB
 (c) the perpendicular bisector of AB (d) none of these
31. The complex numbers $\sin x + i \cos 2x$ and $\cos x - i \sin 2x$ are conjugate to each other for
 (a) $x = n\pi$ (b) $x = \left(n + \frac{1}{2}\right)\pi$
 (c) $x = 0$ (d) no value of x
32. If the equation $(p + iq)x^2 + (m + in)x + r = 0$ has real roots, where p, q, m, n and r are real ($r \neq 0$), then $pn^2 + rq^2 - mnq$ is equal to
 (a) 1 (b) 0
 (c) -1 (d) none of these



33. If $|z| = 1$ and $\omega = \frac{z-1}{z+1}$ (where $z \neq -1$), then the value of $\operatorname{Re}(\omega)$ is
- (a) 0 (b) $-\frac{1}{|z+1|^2}$
 (c) $\frac{\sqrt{2}}{|z+1|^2}$ (d) $\left|\frac{z}{z+1}\right| \cdot \frac{1}{|z+1|^2}$
34. If $z = \frac{1+7i}{(2-i)^2}$, then the polar form of z is
- (a) $\sqrt{2} \left[\cos \frac{\pi}{4} + i \sin \frac{3\pi}{4} \right]$ (b) $\sqrt{2} \left[\cos \frac{3\pi}{4} - i \sin \frac{3\pi}{4} \right]$
 (c) $\sqrt{2} \left[\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right]$ (d) none of these
35. The points representing the complex number z , for which $|z + 3i|^2 - |z - 3|^2 = 6$, lie on the line given by
- (a) $x + y = 1$ (b) $x + y = -1$
 (c) $x + y = 2$ (d) $x + y = 0$
36. If $\tan^{-1}(\alpha + i\beta) = x + iy$, then x is equal to
- (a) $\frac{1}{2} \tan^{-1} \left(\frac{2\alpha}{1-\alpha^2-\beta^2} \right)$ (b) $\frac{1}{2} \tan^{-1} \left(\frac{2\alpha}{1+\alpha^2+\beta^2} \right)$
 (c) $\tan^{-1} \left(\frac{2\alpha}{1-\alpha^2-\beta^2} \right)$ (d) none of these
37. The value of $-2i \sum_{p=1}^6 \left(\sin \frac{2p\pi}{7} + i \cos \frac{2p\pi}{7} \right)$ is
- (a) $2i$ (b) $-2i$
 (c) 2 (d) 1
38. If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, then the value of $(x^2 + y^2)^2$ is
- (a) $\frac{a^2+b^2}{c^2+d^2}$ (b) $\frac{c^2+d^2}{a^2+b^2}$
 (c) $\frac{a^2-b^2}{c^2-d^2}$ (d) $\sqrt{\frac{a^2+b^2}{c^2+d^2}}$
39. If for the complex numbers z_1 and z_2 , $|z_1 + z_2| = |z_1 - z_2|$, then $\arg z_1 \sim \arg z_2$ is equal to
- (a) π (b) $\pi/2$
 (c) $\pi/4$ (d) none of these
40. If n is a positive integer greater than unity and z is a complex number satisfying the equation $z^n = (z + 1)^n$, then
- (a) $\operatorname{Re}(z) < 0$ (b) $\operatorname{Re}(z) > 0$
 (c) $\operatorname{Re}(z) = 0$ (d) none of these
41. The locus of z , if $\arg \left(\frac{z-1}{z+1} \right) = \frac{\pi}{2}$, is
- (a) a circle (b) a semicircle
 (c) a straight line (d) none of these
42. Suppose z_1, z_2, z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 2$. If $z_1 = (1 + i\sqrt{3})$ and z_1, z_2, z_3 are in the clockwise sense, then
- (a) $z_2 = 1 - i\sqrt{3}, z_3 = -2$ (b) $z_2 = 2, z_3 = 1 - i\sqrt{3}$
 (c) $z_2 = -1 + i\sqrt{3}, z_3 = -2$ (d) none of these



43. If $8iz^3 + 12z^2 - 18z + 27i = 0$, then
 (a) $|z| = \frac{3}{2}$
 (b) $|z| = \frac{2}{3}$
 (c) $|z| = 1$
 (d) $|z| = \frac{3}{4}$
44. If p, q, r are position integers and ω is an imaginary cube root of unity and $f(x) = x^{3p} + x^{3q+1} + x^{3r+2}$, then $f(\omega)$ is equal to
 (a) ω
 (b) $-\omega^2$
 (c) 0
 (d) none of these
45. If $\arg\left(\frac{z-1}{z+1}\right) = \frac{\pi}{4}$, then the locus of z is
 (a) a straight line
 (b) a circle
 (c) an ellipse
 (d) a hyperbola
46. Let z be a complex number and a be a real parameter such that $z^2 + ax + a^2 = 0$, then
 (a) the locus of z is a pair of straight lines
 (b) the locus of z is a circle
 (c) $\arg(z) = \pm \frac{5\pi}{3}$
 (d) $|z| = -2|a|$
47. If z is a point on the tangent at point z_1 of the circle $|z| = 1$ and z_2 and z_3 are any two other points on the circle, then $\arg\left(\frac{z-z_1}{z_2-z_1}\right)$ is equal to
 (a) $\arg\left(\frac{z_1}{z_2-z_1}\right)$
 (b) $\frac{1}{2}\arg\left(\frac{z_1}{z_2}\right)$
 (c) $\arg(z_1 z_2)$
 (d) none of these
48. A complex number z is such that $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{3}$. The points representing this complex number will lie on
 (a) an ellipse
 (b) a parabola
 (c) a circle
 (d) a straight line
49. The smallest positive integer n for which $(1+i)^{2n} = (1+i)^{2n}$ is
 (a) 4
 (b) 8
 (c) 2
 (d) 12
50. Let A, B and C , respectively, represent the complex numbers z_1, z_2, z_3 on the complex plane. If the circumcentre of the triangle ABC lies at the origin, then the orthocentre is represented by the complex number
 (a) $z_1 + z_2 - z_3$
 (b) $z_2 + z_3 - z_1$
 (c) $z_3 + z_1 - z_2$
 (d) $z_1 + z_2 + z_3$
51. If $a \neq 1$ is any n th root of unity, then $S = 1 + 3a + 5a^2 + \dots$ up to n terms, is equal to
 (a) $\frac{2n}{1-a}$
 (b) $-\frac{2n}{1-a}$
 (c) $\frac{n}{1-a}$
 (d) $-\frac{n}{1-a}$
52. If $\frac{3}{2+\cos\theta+i\sin\theta} = a + ib$, then $(a-2)^2 + b^2$ is equal to
 (a) 0
 (b) 1
 (c) -1
 (d) 2
53. Complex numbers z_1, z_2 and z_3 are associated with vertices A, B and C of a triangle. D, E and F are the midpoints of sides AB, BC and CA respectively. If $I(z_4)$ and $I'(z_5)$ are the incentres of triangles ABC and DEF respectively and $G(z_6)$ is the centroid the triangle



ABC , then

(a) $2z_4 + z_5 = 3z_6$

(c) $z_4 + z_5 = 2z_6$

(b) $2z_5 + z_4 = 3z_6$

(d) none of these

54. If $3^{49}(x + iy) = \left(\frac{3}{2} + i\frac{\sqrt{3}}{2}\right)^{100}$ and $x = ky$, then k is

(a) $-\frac{1}{3}$

(c) $-\sqrt{3}$

(b) $\sqrt{3}$

(d) $-\frac{1}{\sqrt{3}}$

55. If z_1, z_2, z_3, z_4 are the four complex numbers represented by the vertices of a quadrilateral taken in order such that $z_1 - z_4 = z_2 - z_3$ and $\arg\left(\frac{z_4 - z_1}{z_2 - z_1}\right) = \frac{\pi}{2}$, then the quadrilateral is

(a) a square

(c) a rectangle

(b) a rhombus

(d) none of these

56. Let $A_0A_1A_2A_3A_4A_5$ be a regular hexagon inscribed in a circle of unit radius. Then the product of the lengths of the line segments A_0A_1, A_0A_2 and A_0A_4 is

(a) $\frac{3}{4}$

(c) 3

(b) $3\sqrt{3}$

(d) $\frac{3\sqrt{3}}{2}$

57. If $x = a + b, y = a\alpha + b\beta$ and $z = a\beta + b\alpha$, where α and β are complex cube roots of unity, then xyz is equal to

(a) $a^2 + b^2$

(c) a^3b^3

(b) $a^3 + b^3$

(d) $a^3 - b^3$

58. If $1, \alpha, \alpha^2, \dots, \alpha^{n-1}$ are the n th roots of unity, then $\sum_{k=1}^{n-1} \frac{1}{2 - \alpha^k}$ is equal to

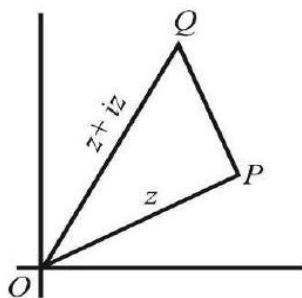
(a) $(n-2)2^n$

(c) $\frac{(n-2)2^{n-1}}{2^n - 1}$

(b) $\frac{(n-2)2^{n-1} + 1}{2^n - 1}$

(d) none of these

59. In Fig. 2.46 if O, P and Q represent the origin and the complex numbers z and $z + iz$, respectively, then the value of $\angle OPQ$ is



(a) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$

(b) $\frac{\pi}{3}$

(d) $\frac{2\pi}{3}$

60. $e^{2\pi i \cot^{-1} p} \cdot \left(\frac{pi+1}{pi-1}\right)^m$ is equal to

(a) 0

(c) -1

(b) 1

(d) none of these



61. If $a = \cos \theta + i \sin \theta$, then $\frac{1+a}{1-a}$ is equal to

- (a) $\cot \frac{\theta}{2}$
(c) $i \cot \frac{\theta}{2}$

- (b) $\cot \theta$
(d) $i \tan \frac{\theta}{2}$

62. If $\frac{(a+ib)^2}{a-ib} - \frac{(a-ib)^2}{a+ib} = x + iy$, then x is equal to

- (a) $\frac{-2b^3}{(a^2+b^2)^2}$
(c) 0

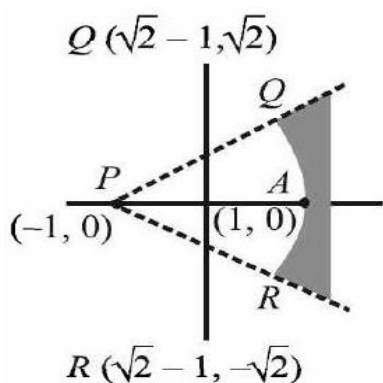
- (b) $\frac{6a^2b}{(a^2+b^2)^2}$
(d) none of these

63. If α, β, γ are the cube roots of $p < 0$, then for any x, y, z , $\frac{\alpha^2 x^2 + \beta^2 y^2 + \gamma^2 z^2}{\beta^2 x^2 + \gamma^2 y^2 + \alpha^2 z^2}$ is

- (a) 1
(c) $\frac{\beta}{\alpha}$

- (b) $\frac{\alpha}{\gamma}$
(d) $\frac{\gamma}{\beta}$

64. PQ and PR are two infinite rays and QAR is an arc. The point lying in the shaded region excluding the boundary, satisfies.



- (a) $|z - 1| > 2; |\arg(z - 1)| < \frac{\pi}{4}$
(c) $|z - 1| > 2; |\arg(z + 1)| < \frac{\pi}{4}$

- (b) $|z - 1| > 2; |\arg(z - 1)| < \frac{\pi}{2}$
(d) $|z - 1| > 2; |\arg(z + 1)| < \frac{\pi}{2}$

65. If $x + iy = \frac{3}{2 + \cos \theta + i \sin \theta}$, then $x^2 + y^2$ is equal to

- (a) $3x - 4$
(c) $4x + 3$

- (b) $4x - 3$
(d) none of these



ANSWER KEY

1. (b)	2. (b)	3. (c)	4. (c)	5. (a)	6. (a)	7. (b)	8. (a)	9. (b)	10. (b)
11. (b)	12. (c)	13. (b)	14. (b)	15. (c)	16. (a)	17. (b)	18. (d)	19. (b)	20. (c)
21. (a)	22. (a)	23. (d)	24. (c)	25. (d)	26. (d)	27. (b)	28. (b)	29. (a)	30. (c)
31. (d)	32. (b)	33. (a)	34. (c)	35. (a)	36. (a)	37. (a)	38. (a)	39. (b)	40. (a)
41. (b)	42. (a)	43. (a)	44. (c)	45. (b)	46. (a)	47. (b)	48. (c)	49. (c)	50. (d)
51. (b)	52. (b)	53. (b)	54. (d)	55. (c)	56. (c)	57. (b)	58. (b)	59. (c)	60. (b)
61. (c)	62. (c)	63. (d)	64. (c)	65. (b)					

