AP,GP,HP, Sequences

Single Correct Answer Type

1. The 2008th term of the sequence $1, \underbrace{2, 2, 2}_{3}, \underbrace{3, 3, 3, 3, 3, 3}_{6}, \underbrace{4, 4, 4, 4, 4, 4, 4, 4, 4, 4}_{10}, \ldots$ where n occurs $\frac{n(n+1)}{2}$ times in the sequence, equals

(A) 24 (B) 23
(C) 22 (D) 21

Key. C

Sol.

		No. of terms
Group (1)	1	1
Group (2)	2, 2, 2	3
Group (3)	3, 3, ..., 3	6
Group (4)	4, 4, ..., 4	10
. . . . .		
. . . . .		
Group (r)	r, r, ..., r	$\frac{r^2+r}{2}$

Let 2008th term falls in rth group

$\Rightarrow 1 + 3 + 6 + 10 + \ldots + \frac{(r-1)^2+(r-1)}{2} < 2008 \le 1 + 3 + 6 + \ldots + \frac{r^2+r}{2}$

$\Rightarrow \frac{(r-1)r(r+1)}{6} < 2008 \le \frac{r(r+1)(r+2)}{6}$

$\Rightarrow r^3 - r < 12048 \le (r+1)^3 - (r+1)$ ... (i)

$\Rightarrow$ r will be nearer to cube root of 12048

Note: $22 < \sqrt[3]{12048} < 23$

for r = 22 inequality (i) holds

for r < 22 RHS of (1) is less than 12048

for $r \ge 23$ LHS of (1) is greater than 12048

$\Rightarrow$ r = 22 is the required value $\Rightarrow$ 2008th term is 22

Ans. (C) 22.

1. If $a_k = \frac{1}{k(k+1)}, \text{ for } k = 1, 2, 3 \ldots\ldots n, \text{ then } \left(\sum_{k=1}^{n} a_k\right)^2 =$

1) $\frac{n}{n+1}$ 2) $\frac{n^2}{(n+1)^2}$ 3) $\frac{n^4}{(n+1)^4}$ 4) $\frac{n^6}{(n+1)^6}$

Key. 2

Sol. $\left(\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \ldots\ldots + \frac{1}{n} - \frac{1}{n+n}\right)^2$

$\frac{n^2}{(n+1)^2}$

2. $\sum_{k=1}^{\infty}\frac{1}{k!}\left(\sum_{n=1}^{k}2^{n-1}\right)=$

1)e 2) e^2+e 3) e^2 4) e^2-e

Key. 4

Sol. $\sum_{k=1}^{\infty}\frac{1}{k!}\left(1+2+2^2+.......+2^{k-1}\right)$

$\sum_{k=1}^{\infty}\frac{2^k-1}{k!}=e^2-e$

3. Coefficient of x^{10} in the expansion of $(2+3x)e^{-x}$ is

1) $\frac{-26}{(10)!}$ 2) $\frac{-28}{(10)!}$ 3) $\frac{-30}{(10)!}$ 4) $\frac{-32}{(10)!}$

Key. 2

Sol. $(2+3x)\left(1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}....+\frac{x^{10}}{10!}\right)$

$\frac{2}{10!}-\frac{3}{9!}=\frac{2-30}{10!}=\frac{-28}{10!}$

4. $\frac{1^2}{1!}+\frac{1^2+2^2}{2!}+\frac{1^2+2^2+3^2}{3!}+.....$

1) $\frac{17e}{6}$ B) $\frac{6e}{17}$ C) $\frac{11e}{7}$ D) $\frac{7e}{11}$

Key. 1

Sol. $\sum_{n=1}^{\infty}\frac{1^2+2^2+3^2+....+n^2}{n!}$

$=\frac{1}{6}\left(\sum_{n=1}^{\infty}\frac{2n^3}{n!}+\sum_{n=1}^{\infty}\frac{3n^2}{n!}+\sum_{n=1}^{\infty}\frac{n}{n!}\right)$

$=\frac{1}{6}(2\times5e+3\times2e+e)=\frac{17e}{6}$

5. $\sum_{n=1}^{\infty}\frac{2n^2+n+1}{n!}=$

1) $2e-1$ 2) $2e+1$ 3) $6e-1$ 4) $6e+1$

Key. 3

Sol. $\sum_{n=1}^{\infty}\frac{2n(n-1)+3n+1}{n!}=\sum_{n=1}^{\infty}\frac{2n(n-1)}{n!}+\sum_{n=1}^{\infty}\frac{3n}{n!}+\sum_{n=1}^{\infty}\frac{1}{n!}$

$2e+3e+e-1=6e-1$

6. The sum of the series $1+\frac{1}{4.2!}+\frac{1}{16.4!}+\frac{1}{64.6!}+.......$

1) $\frac{e-1}{\sqrt{e}}$ 2) $\frac{e+1}{\sqrt{e}}$ 3) $\frac{e-1}{\sqrt{e}}$ 4) $\frac{e+1}{2\sqrt{e}}$

Key. 4

Sol. $1+\frac{1}{4.2!}+\frac{1}{16.4!}+\ldots\ldots$

$=\frac{e^{1/2}+e^{-1/2}}{2}=\frac{e+1}{2\sqrt{e}}$

8. If $|x|<1$ *and* $y=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+..........$ *then* $x=$

1) $y+\frac{y^2}{2}+\frac{y^3}{3}+......$ 2) $y-\frac{y^2}{2}+\frac{y^3}{3}-\frac{y^4}{4}+......$

3) $y+\frac{y^2}{2!}+\frac{y^3}{3!}+......$ 4) $y-\frac{y^2}{2!}+\frac{y^3}{3!}-\frac{y^4}{4!}+......$

Key. 3

Sol. $y=x-x^2+\frac{x^3}{3}-\frac{x^4}{4}+....$

$y=\log_e(1+x)\Rightarrow 1+x=e^y$

$\Rightarrow x=y+\frac{y^2}{2!}+\frac{y^3}{3!}+.....$

9. In a sequence of (4n+1) terms, the first (2n+1) terms are in A.P., whose common difference is 2, and the last (2n+1) terms are in G.P whose common ratio is 0.5 if the middle terms of the A.P and G.P are equal then the middle term of the sequence is

A) $n2^{n-1}/2^n-1$ B) $n.2^{n+1}/\ 2^{2n}-1$ C) $n.2^n$ D) $n2^{n+1}/2^n-1$

Key. D

Sol. Let the first term is a , then first (2n+1) terms are $a, a+2, a+4, ...a+2.2n$. Clearly the middle term of the sequence of $4n+1$ term is $(2n+1)^{th}$ term, i.e . $a+4n$ also the middle term of the A.P of $(2n+1)$ term is $(n+1)^{th}$ term i.e., $a+2n$ .Again for the last $(2n+1)$ terms the first term will be $(2n+1)^{th}$ term of the A.P i.e. $a+4n$

$\therefore G.P\ is\ (a+4n),(1+4n)(0.5)^n$

Its middle term is $(a+4n)(0.5)^n$

According to the given condition,

$a+2n=(1+4n)(0.5)^n$

$\therefore\ a=\frac{2n-4n(0.5)^n}{(0.5)^n-1}$

$\therefore$ Required middle term = $a+4n=$

$$\frac{2n-4n(0.5)^n}{(0.5)^n-1}+4n=\frac{2n}{1-\left(\frac{1}{2}\right)^2}=\frac{n.2^{n-1}}{2^n-1}$$

10. The sum of the series $\frac{x}{1-x^2}+\frac{x^2}{1-x^4}+\frac{x^4}{1-x^8}+......$ to infinite terms, if $|x|<1$ is

A) $\frac{x}{1-x}$ B) $\frac{1}{1-x}$ C) $\frac{1+x}{1-x}$ D) 1

Key. A

Sol. The general term of the series is $t_n=\frac{x^{2^{n-1}}}{1-x^{2^n}}$

$$=\frac{1+x^{2^{n-1}}-1}{\left(1+x^{2^{n-1}}\right)\left(1-x^{\;2n-1}\right)}$$

$$\therefore l_n=\frac{1}{1-x^{2n-1}}-\frac{1}{1-x^{2^n}}$$

Now , $S_n=\sum_{n=1}^{n}t_n=\left[\left\{\frac{1}{1-x}-\frac{1}{1-x^2}\right\}\right]$

$$+\left\{\frac{1}{1-x^2}-\frac{1}{1-x^4}\right\}+....$$

$$+\left\{\frac{1}{1-x^{2^{n-1}}}-\frac{1}{1-x^{2^n}}\right\}\Big]=\frac{1}{1-x}-\frac{1}{1-x^{2^n}}$$

$\therefore$ The sum to infinite terms

$$=\lim_{n\to\infty}S_n=\frac{1}{1-x}-1=\frac{x}{1-x}$$

$$\left[\because \lim_{n\to\infty} x^{2^n}=0, as\ |x|<1\right]$$

11. If n arithmetic means are inserted between two sets of numbers a, 2b and 2a, b, where a,b $\in R$. Suppose that m^{th} arithmetic mean between these two sets of numbers is same, then the ratio a:b equals

A) $n-m+1:m$ B) $n-m+1:n$ C) $m:n-m+1$ D) $n:n-m+1$

Key. C

Sol. Let $A_1, A_2.....A_n$ be airthmetric means between a and 2b , then $A_m=a+m\left(\frac{2b-a}{n+1}\right)$

Again , let $B_1, B_2........B_n$ be arithmetic means

Between 2a and b then $B_m = 2a + m\left(\frac{b-2a}{n+1}\right)$

Now , $A_m = B_m \Rightarrow a + m\left(\frac{2b-a}{n+1}\right) = 2a +$

$m\left(\frac{b-2a}{n+1}\right) \Rightarrow m\left(\frac{b+a}{n+1}\right) = a \Rightarrow \frac{a}{b} = \frac{m}{n-m+1}$

12. The ratio of sum of first three terms of a G.P. to the sum of first six terms is $64:91$,the common ratio of G.P. is

1. $\frac{1}{4}$ 2. $\frac{3}{4}$ 3. $\frac{5}{4}$ 4. $\frac{7}{4}$

Key. 2

Sol. Given $\frac{S_3}{S_6} = \frac{64}{91} = \frac{a\left(r^3-1\right)}{a\left(r^6-1\right)}$

$\Rightarrow \frac{\left(r^3-1\right)}{\left(r^3+1\right)\left(r^3-1\right)} = \frac{64}{91}$

$\Rightarrow r^3 = \frac{27}{64} \quad \therefore \quad r = \frac{3}{4}$

13. Sum of the series $3+5+9+17+33+.....$to n terms is

1. $2^{n+1}-n-2$ 2. $2^{n+1}+n-2$ 3. 2^n+n-2 4. $2^{n+1}-n+2$

Key. 2

Sol. $S_n = 3+5+9+17+33+......$

$= (2+1)+\left(2^2+1\right)\left(2^3+1\right)+\left(2^4+1\right)+....$

$= \left(2+2^2+2^3+2^4+......n \text{ terms}\right)+n$

$= 2\left(2^n-1\right)+n = 2^{n+1}+n-2$

$= 2^{n+1}+n-2$

14. If one A.M. A and two G.M.s p and q be inserted between two numbers a and b, then which of the following is hold good

1. $a^3+b^3=2Apq$ 2. $p^3+q^3=2Apq$ 3. $a^3+b^3=2Aab$ 4. None of these.

Key. 2
Sol. Given $a+b=2A$

And $a, p, q, \quad b \in$ G.P.

$\therefore \quad p^2=aq$ and $q^2=pb$

$\Rightarrow \; p^3=apq$ and $q^3=bpq$

by adding we get

$p^3+q^3=apq+bpq$

$=pq(a+b)=2Apq$

15. If fourth term of a G.P. is 3, the product of the first seven terms is

1. 3^4 2. 3^7 3. 7^4 4. 4^7

Key. 2
Sol. As the number of terms are odd (7) let r, be the common ratio

So terms can be taken as $\frac{a}{r^3}, \frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2, ar^3$

$\therefore$ Product of the term $=a^7$

$=3^7$ as $(t_4=a=3)$

16. The number of divisors of 6912, 52480,32000 are in

1. A.P Only 2. G.P. Only 3. A.P. , G.P.& H.P. 4. None of these

Key. 3
Sol. If n is a + ve number.

$n=P_1^{k_1}.P_2^{k_2}...P_r^{k_r}$

(where $p_1, p_2, p_3, ... p_r$ are prime number) then number of divisors of n are

$=(k_1+1)(k_2+1)....(k_r+1)$

$\therefore$ Number of prime factor of 6912 are $=2^8.3^3$ so no. of divisors $=9\times 4=36$

Prime factors of 52,400 are $=3^8\times 2^3$

$\therefore$ No. of divisors $=9\times 4=36$

Prime factors of 32,000 are $=5^3\times 2^8$

$\therefore$ No. of divisors $=9\times 4=36$

Now each number having same number of divisors *i.e.,* 36,36,36

Each and every term is constant & constant sequence is always in A.P.& G.P. both as common difference is 0 and common ratio is 1.

17. If 1, $\log_{81}(3^x+48)$, $\log_9\left(3^x-\frac{8}{3}\right)$ are in A.P., then the value of x equals

1. 9 2. 6 3. 2 4. 4

Key. 3

Sol. Given 1, $\log_9 2\left(3^x+48\right), \log_9\left(3^x-8/3\right), \in$ A.P.

$$\Rightarrow \log_9 9,\ \frac{1}{2}\log_9\left(3^x+48\right),\ \log_9\left(3^x-8/3\right) \in \text{A.P.}$$

$$\Rightarrow 9,\left(3^x+48\right)^{1/2}, 3^x-8/3 \in \text{G.P.} \quad \text{(By concept)}$$

$$\Rightarrow \log a, \log b, \log c \in \text{ A.P.}$$

$$\therefore a,b,c \in \text{ G.P.} \quad \therefore 3^x+48=9\left(3^x-8/3\right)$$

$$8.3^x=72$$

$$3^x=9,\ 3^x=3^2, \quad x=2.$$

18. If a, b, c are in H.P., then $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in

1. A.P. 2. H.P. 3. G.P. 4. None of these

Key. 2
Sol. Given $a, b, c \in$ H.P.

So, $\frac{1}{a}, \frac{1}{b}, \frac{1}{c} \in$ A.P.

$\frac{a+b+c}{a}, \frac{b+c+a}{b}, \frac{c+a+b}{c} \in$ A.P.

By using concept if $a, b, c \in$ A.P.

Then their reciprocals are in H.P.

19. Between the numbers 2 and 20, 8 means are inserted, then their sum is

1. 88 2. 44 3. 176 4. None of these

Key. 1
Sol. Let $a, A_1, A_2 .... A_g, b \in A.P$

Where $a = 2, b = 20, n = 8$

$\therefore$ sum of the means $= \frac{n}{2}(a+b) = \frac{8}{2}(2+20) = 88$

20. In the expansion of $(1+x)^{70}$,the sum of coefficients of odd powers of x is

1. 0 2. 2^{69} 3. 2^{70} 4. 2^{71}

Key. 2
Sol. Fact. The sum of the coefficients of odd powers in the expansion of $(1+x)^n$ = sum of the coefficients of even powers in $(1+x)^n$

$= 2^{n-1}$

$2^{70-1} = 2^{69}$

21 If the arithmetic mean of two positive numbers $a \& b$ $(a > b)$ is twice their G.M., then

$a:b$ is

1. $6+\sqrt{7}:6-\sqrt{7}$ 2. $2+\sqrt{3}:2-\sqrt{3}$ 3. $5+\sqrt{6}:5-\sqrt{6}$ 4. None of these

Key. 2

Sol. $\frac{a+b}{2}=2\sqrt{ab}$

$a+b-4\sqrt{ab}=0$

$\frac{a}{b}+1-4\sqrt{\frac{a}{b}}=0$ (Dividing by b)

Or $\left(\sqrt{\frac{a}{b}}\right)^2-4\sqrt{\frac{a}{b}}+1=0$

$\therefore \sqrt{\frac{a}{b}}=\frac{4\pm 2\sqrt{3}}{2}=\left(2\pm\sqrt{3}\right)$

$\frac{a}{b}=\frac{2+\sqrt{3}}{2-\sqrt{3}}$

22 The number of terms common between the two series 2 + 5 + 8 + …up to 50 terms and the series 3 + 5 + 7 + 9 + ….up to 60 terms.

1. 24 2. 26 3. 25 4. None of these

Key. 4

Sol. Let m^{th} term of first A.P. be equal to the n^{th} term of the second A.P. then

2 , 5 , 8 ,….50 terms series 1

3 , 5 ,7 , …., 60 terms series 2

Common series 5 , 11, 17 , ….., 119

40^{th} term of series 1 = 59^{th} term of series 2 = 119 = last term of common series

$\Rightarrow a_n=5+(n-1)d \Rightarrow 119+1=6n \Rightarrow n=20.$

$\therefore$ Number of common terms is 20.

23 The sum of the series $1+\frac{9}{4}+\frac{36}{9}+\frac{100}{16}+....$ up to n terms if $n=16$ is

1. 446 2. 746 3. 546 4. 846

Key. 1

Sol. The given series can be written as $1^3+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+...$

$$t_n=\frac{1^3+2^3+3^3+.....n^3}{1+3+5+....+(2n-1)}$$

$$t_n=\frac{n^2(n+1)^2}{4n^2}=\frac{(n+1)^2}{4}$$

$$t_n=\frac{1}{4}(n+1)(n+1)$$

$$=\frac{1}{4}(n^2+2n+1)=\frac{1}{4}\left[\sum_{k=1}^{n}k^2+2\sum_{k=1}^{n}k+n\right]$$

$$\therefore S_n=\frac{1}{4}\left[\frac{n(n+1)(2n+2)}{6}+n(n+1)+n\right]$$

$$\therefore S_{16}=\frac{1}{4}\left[\frac{16.17.33}{6}+16.17+16\right]=\frac{1}{4}[88\times17+16\times8+16]=446$$

24 Sum of n terms of series

$ab+(a+1)(b+1)+(a+2)(b+2)+....+(a+(n-1))(b+(n-1))$

if $ab=\frac{1}{6}$ and $(a+b)=\frac{1}{3}$, is

(A) $\frac{n}{6}(1-2n)^2$ (B) $\frac{n}{6}(1+n-2n^2)$ (C) $\frac{n}{6}(1-2n+2n^2)$ (D) none of these

Key. C

Sol. $s=ab+[ab+(a+b)+1]+[ab+2(a+b)+2^2]+....[ab+(n-1)(a+b)+(n-1)^2]$

$=nab+(a+b)\sum_{r=1}^{n-1}r+\sum_{r=1}^{n-1}r^2$

$= nab+(a+b)\frac{n(n-1)}{2}+\frac{(n-1))(n)(2n-1)}{6}$

$= \frac{n}{6}[1+(n-1)\{1+2n-1\}]$

$= \frac{n}{6}[1+2n(n-1)] = \frac{n}{6}(1-2n+2n^2)$

25 If log (a+c),log(a+b), log(b+c) are in A.P. and a,c,b are in H.P, then the value of a+b is (given a,b,c >0)

(A) 2c (B) 3c (C) 4c (D) 6c

Key. A

$\log(a+c)+\log(b+c) = 2\log(a+b)$

$(a+c)(b+c) = (a+b)^2$

Sol. $\Rightarrow ab + c(a+b)+c^2 = (a+b)^2$ (1)

$also, c = \frac{2ab}{a+b} \Rightarrow 2ab = c(a+b)$

$\Rightarrow 2ab+2c(a+b)+2c^2 = 2(a+b)^2$ ..... (2)

From (1) and(2),

$c(a+b)+2c(a+b)+2c^2 = 2(a+b)^2$

$2(a+b)^2 - 3c(a+b) - 2c^2 = 0$

$\therefore a+b = \frac{3c \pm \sqrt{9c^2+16c^2}}{4} = \frac{3c \pm 5c}{4} = 2c \; or -\frac{c}{2}$

$\therefore a+b = 2c$ $(\because a,b,c>0)$

26 If $a_1, a_2, a_3, ..., a_n$ are in A.P. with s_n as the sum of first 'n' terms $(s_0 = 0)$, then

$\sum_{k=0}^{n} {}^nC_k s_k$ is equal to

(A) $2^{n-2}[na_1+s_n]$ (B) $2^n[a_1+s_n]$ (C) $2[na_1+s_n]$ (D) $2^{n-1}[a_1+s_n]$

Key. A

Sol. $\sum_{k=o}^{n} {}^nc_k s_k = \sum_{k=0}^{n} {}^nc_k \frac{k}{n}[2a+(k-1)d]$

$=[(a_1-\frac{d}{2})\sum_{k=0}^{n} k\,{}^nc_k + \frac{d}{2}\sum_{k=0}^{n} k^2\,{}^nc_k]$

$=\left(a_1-\frac{d}{2}\right)n.2^{n-1}+\frac{d}{2}[n.2^{n-1}+n(n-1)2^{n-2}]$

$= a_1.n.2^{n-1}+dn(n-1)2^{n-3}$

$= n.2^{n-3}[4a_1 + a_n - a_1] = n.2^{n-3}[3a_1 + a_n]$

$= 2^{n-3}[2na_1 + 2n\left(\frac{a_1 + a_n}{2}\right)]$

$= 2^{n-2}[na_1 + s_{n]}.$

27 The positive integral values of n such that $1.2^1 + 2.2^2 + 3.2^3 + 4.2^4 + 5.2^5 + ... + n.2^n = 2^{(n+10)} + 2$ is

(A) 313 (B) 513 (C) 413 (D) 613

Key. B

Sol.

$2^1 + 2^2 + 2^3 + ... + 2^n = 2^{n+1} - 2$

$2^2 + 2^3 + ... 2^n = 2^{n+1} - 2^2$

$2^3 + ... + 2^n = 2^{n+1} - 2^3$

... ...

$+2^n = 2^{n+1} - 2$

$= n\left(2^{n+1}\right) - \left(2^{n+1} - 2\right)$

$= 2^{n+1}(n-1) + 2$

Given that $2^{n+1}(n-1) + 2 = 2^{2+10} + 2$

$\Rightarrow (n-1)2^{n+1} = 2^{n+10}$

$\Rightarrow n - 1 = 2^9$

$\Rightarrow n = 2^9 + 1 = 513$

28 If a,b,c, are in A.P. and p, p' are respectively A.M. and G.M. between a and b while q, q' are respectively AM. And G.M. between b and c, then

(A) $p^2 + q^2 = p'^2 + q'^2$ (B) $pq = p'q'$

(C) $p^2 - q^2 = p'^2 - q'^2$ (D) $p^2 + p'^2 = q^2 + q'^2$

Key. C

Sol. We have $2b = a + c$ and a,p,b,q,c are in A.P

$\Rightarrow p = \frac{a+b}{2}, q = \frac{b+c}{2}$

Again , $p' = \sqrt{ab}$ and $q' = \sqrt{bc}$

$\therefore p^2 - q^2 = \frac{(a+b)^2 - (b+c)^2}{4}$

$= \frac{(a-c)(a+c+2b)}{4} = (a-c)b = p'^2 - q'^2$

29. The arithmetic mean of the nine numbers in the given set {9, 99, 999, ... 999999999} is a 9 digit number N, all whose digits are distinct. The number N does not contain the digit

(A) 0 (B) 2 (C) 5 (D) 9

Key. A

Sol. N = $\frac{1}{9}$ {9, 99, 999, ... 999999999} = 1 + 11 + 111 + ...+111111111

= 1 2 3 4 5 6 7 8 9 (A)

30. The minimum value of the expression $\frac{9x^2 \sin^2 x + 4}{x \sin x}$ for x ∈ (0, π) is

(A) $\frac{16}{3}$ (B) 6 (C) 12 (D) $\frac{8}{3}$

Key. C

Sol. E = 9x sin x + $\frac{4}{x \sin x}$ [note that x sin x > 0 in (0, π)]

E = $\left(3\sqrt{x \sin x} - \frac{2}{\sqrt{x \sin x}}\right)^2$ + 12

⇒ E_{min} = 12 which occurs when 3 x sin x = 2 ⇒x sin x = 2/3]

note that x sin x is continuous at x = 0 and attains the value π/2 which is greater than 2/3 at x = π/2, hence it must take the 2/3 in $(0, \pi/2)$]

31. There is a certain sequence of positive real numbers. Beginning from the third term, each term of the sequence is the sum of all the previous terms. The seventh term is equal to 1000 and the first term is equal to 1. The second term of this sequence is equal to

(A) 246 (B) $\frac{123}{2}$ (C) $\frac{123}{4}$ (D) 124

Key. B

Sol. sequence is $t_1 + t_2 + t_3 + t_4$ + ...

$t_3 = t_1 + t_2$; $t_7 = 1000$

$t_1 = 1$

but $t_7 = t_1 + t_2 + t_3 + t_4 + t_5 + t_6$

$1000 = 2(t_1 + t_2 + t_3 + t_4 + t_5)$

$= 4(t_1 + t_2 + t_3 + t_4)$

$= 8(t_1 + t_2 + t_3)$

$1000 = 16(t_1 + t_2)$

$t_1 + t_2 =$ ⇒ $t_2 =$ – 1 = – 1 =

32. If $(1 + x + x^2)^{25} = a_0 + a_1 x + a_2 x^2 + ... + a_{50} . x^{50}$ then $a_0 + a_2 + a_4 + ... + a_{50}$ is :

(A) even (B) odd & of the form 3n

(C) odd & of the form (3n 1) (D) odd & of the form (3n + 1)

Key. A

Sol. putting x = 1 and 1 and adding

$a_0 + a_2 + ... + a_{50} =$ = ()

=

= = 2 [13 + $^{25}C_2$ + ... + $^{25}C_{25} . 2^{23}$]

 even

33. The sum of the series $(1^2 + 1).1! + (2^2 + 1).2! + (3^2 + 1). 3! + ... + (n^2 + 1). n!$ is :

(A) (n + 1). (n+2)! (B) n.(n+1)! (C) (n + 1). (n+1)! (D) none of these

Key. B

Sol. T_n = [n (n + 1) (n 1)] n! = n . (n+1)! (n 1). n!

Now put n = 1, 2, 3, ... , n and add

34. Find the sum of the infinite series $\frac{1}{9}+\frac{1}{18}+\frac{1}{30}+\frac{1}{45}+\frac{1}{63}+.....$

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{5}$ (D) $\frac{2}{3}$

Key. A

Sol. T_n = =

hence T_n using method of diff; T_n = =

 S_n = = Ans.

35. The sequence a_1, a_2, a_3, ... satisfies $a_1 = 19$, $a_9 = 99$, and for all n 3, a_n is the arithmetic mean of the first n – 1 terms. Then a_2 is equal to

(A) 179 (B) 99 (C) 79 (D) 59

Key. A

Sol. n 3, a_3 = ...(1)

a_4 = = $a_4 = a_3$

a_5 = = = a_4

$a_3 = a_4 = a_5 = ... = a_9 = 99$

put in equation (1)

99 = $a_2 = 179$ Ans.

36. If a, b, c are in G.P. then $\frac{1}{b-a}, \frac{1}{2b}, \frac{1}{b-c}$ are in

(A) A.P. (B) G.P. (C) H.P. (D) none

Key. A

Sol. Let a = x; b = xr; c = xr^2

hence the number are , ,

now, =

\+ = =

hence , , are in A.P.

37. Let $d_1, d_2, \ldots, d_k$ be all the distinct factors of a positive integer n including 1 and n. Suppose

$d_1 + d_2 + \ldots + d_k = 72$, then the value of $\frac{1}{d_1} + \frac{1}{d_2} + \ldots\ldots \frac{1}{d_k}$

(A) $\frac{72}{nk}$

(B) cannot be computed from the given information

(C) $\frac{72}{n}$

(D) None of these

Key. C

Sol. , ,..... are all distinct and each of these represents one of the number d_1, $d_2, \ldots\ldots d_k$.

□ =

□

38. If b is the arithmetic mean between a and x; b is the geometric mean between 'a' and y; 'b' is the harmonic mean between a and z, ($a, b, x, y, z > 0$) then the value of xyz is

(A) a^3 (B) b^3 (C) $\frac{b^3(2a-b)}{2b-a}$ (D) $\frac{b^3(2b-a)}{2a-b}$

Key. D

39. The first term of an infinite geometric series is 2 and its sum be denoted by S. If $|S - 2| < 1/10$ then the true set of the range of common ratio of the series is

(A) $\left(\frac{1}{10}, \frac{1}{5}\right)$ (B) $\left(-\frac{1}{2}, \frac{1}{2}\right) - \{0\}$

(C) $\left(-\frac{1}{19}, \frac{1}{20}\right) - \{0\}$ (D) $\left(-\frac{1}{19}, \frac{1}{21}\right) - \{0\}$

Key. D

40. The number of real values of the parameter $'k'$ for which $\left(\log_{16} x\right)^2 - \log_{16} x + \log_{16} k = 0$ will have unique solution

A) 2 B) 1 C) 4 D) 5

Key. B

Sol. For exactly one solution $4\log_{16} k = 1, k > 0 \Rightarrow k = 2$

41. If $3^{37} = 80\lambda + k$, where $\lambda \in N$, then $'k'$ is

A) 78 B) 3 C) 2 D) 9

Key. B

Sol. $3^{37} = 3^{4\times 9}.3 = 3(81)^9 = 3(80+1)^9 = 3\left({}^9C_0 80^9 + {}^9C_1 80^8 + ... + {}^9C_9\right)$.

Hence remainder is 3

42. The sum of first $'n'$ terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+.....$ is

A) 2^{n-1} B) $1-2^{-n}$ C) $2^{-n}-n+1$ D) $2^{-n}+n-1$

Key. D

Sol. $S = \frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+........$ to $'n'$ terms

$S = \left(1-\frac{1}{2}\right)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{8}\right)+.........$ to $'n'$ terms

$$= (1+1+1+......)-\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+......\right) = n-\frac{1}{2}\frac{1-\left(\frac{1}{2}\right)^n}{2-\frac{1}{2}} = n-\left(1-\frac{1}{2^n}\right) = 2^{-n}+n-1$$

43. The sum to n terms of the series

$\frac{1}{\lfloor 1}\left(\frac{1}{2}\right)+\frac{1.3}{\lfloor 2}\left(\frac{1}{2}\right)^2+\frac{1.3.5}{\lfloor 3}\left(\frac{1}{2}\right)^3+.....$ upto n terms is

(A) $\frac{1.3.5.....(2n-1)(2n+1)}{2^n\lfloor n}-1$ (B) $1-\frac{1.3.5.....(2n-1)}{\lfloor n \ \lfloor n}$

(C) $1-\frac{1.3.5.....(2n-3)}{2^{n-1}\lfloor n-1}$ (D) $\frac{1.3.5.....(2n-3)}{2^{n-1}\lfloor n-1}$

Key. A

Sol. $t_{n+1} = \frac{t_n \times (2n+1)}{(n+1)} \times \frac{1}{2}$

$(2n+2)t_{n+1} = (n+1)t_n$

$(2n+3)t_{n+1} - (2n+1)t_n = t_{n+1}$

Put n = 1

$5t_2 - 3t_1 = t_2$

n = 2, $7t_3 - 5t_2 = t_3$

$(2n+1)t_n - (2n-1)t_{n-1} = t_n$

$(2n+1)t_n - 2t_1 = S$

$$S = \frac{1.3.5.....(2n+1)}{\lfloor n \times 2^n} - 1$$

44. The sequence $\{x_1, x_2, ..., x_{50}\}$ has the property that for each k, x_k is k less than the sum of other 49 numbers. The value of $96x_{20}$ is

a) 300 b) 315 c) 1024 d) 0

Key : B

Sol : We have $x_k + k = S - x_k$ where $x_1 + x_2 + ... + x_k = s$

$\Rightarrow$ $2x_k + k = S$

$\Rightarrow$ $2(S) + \frac{50.51}{2} = 50S$

$\Rightarrow$ $48(S) = 25.51$

$\Rightarrow$ $x_{20} = \left(\frac{25.51}{48} - 20\right)\frac{1}{2} = \frac{315}{96}.$

45. If the first and $(2n-1)^{th}$ terms of an A.P; a G.P. and H.P. are equal and their n^{th} terms are p,q and s respectively, then which of the following options is/are correct?

a) $p \geq q \geq s$ b) $p + s = q$ c) $ps = q^2$ d) $p = q = s$

KEY : C

HINT: Let the first term be a and $(2n-1)^{th}$ term be b then

$$p = a + (n-1)d = a + (n-1)\left(\frac{b-a}{2n-2}\right) = \frac{a+b}{2}$$

$$q = a.r^{n-1} = a\left(\frac{b}{a}\right)^{\frac{n-1}{2n-2}} = a\left(\frac{b}{a}\right)^{\frac{1}{2}} = \sqrt{ab}$$

$$\frac{1}{s} = \frac{1}{a} + (n-1)\left(\frac{\frac{1}{b} - \frac{1}{a}}{2n-2}\right) = \frac{\frac{1}{a} + \frac{1}{b}}{2}$$

p, q, r are the A.M, G.M, H.M of a, b.

$\therefore p \geq q \geq r$ and $ps = q^2$

46. If $a_1, a_2, a_3, \ldots a_{4001}$ are terms of an AP such that $\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + ... + \frac{1}{a_{4000}a_{4001}} = 10$ and $a_2 + a_{4000} = 50$ then $|a_1 - a_{4001}|$ is equal to

(A) 20 (B) 30

(C) 40 (D) 10

KEY : B

HINT: $\frac{4000}{a_1a_{4001}} = 10 \Rightarrow a_1a_{4001} = 400$

$a_1 + a_{4001} = 50$

$(a_1 - a_{4001})^2 = (a_1 + a_{4001})^2 - 4a_1 a_{4001}$

$\Rightarrow |a_1 - a_{4001}| = 30$

47. Statement-1 : The series for which the sum to n terms ($n \geq 1$), S_n is given by $S_n = 3n^2 + 4n + 5$ is an arithmetic progression (AP).
Statement-2 : The sum to n terms of an AP having non-zero common difference is a quadratic in n.

KEY : D

HINT: CONCEPTUAL

48. The fourth and fifth term of a sequence $\{t_n\}_{n\geq 1}$ are 4 and 5 respectively and the n[th] term is given as $t_n = 2t_{n-1} - t_{n-2},\ n \geq 3\ (n \in N)$. Then the sum to first 2009 terms is

(A) 2019045 (B) 2013021
(C) 2017036 (D) 2018040

KEY : A

HINT: $t_n = 2\,t_{n-1} - t_{n-2}$

$t_n - t_{n-1} = t_{n-1} - t_{n-2}$

$a_n = t_n - t_{n-1}, n \geq 3$

WE HAVE $a_n = a_{n-1}$

THUS $\{a_n\}$ IS A CONSTANT SEQUENCE

$a_5 = t_5 - t_4 = 1$

NOW $a_4 = t_4 - t_3 \Rightarrow 1 = 4 - t_3 \Rightarrow t_3 = 3$

SIMILARLY $t_2 = 2,\ t_1 = 1$

THUS $\{t_n\}$ IS AN A.P WITH $r = 1$ AND COMMON DIFFERENCE 1

$$\sum_{n=1}^{2009} t_n = \frac{2009 \times 2010}{2} = 2003 \times 1005 = 2019045$$

49. If $x^6 = 2x^3 - 1$ *and* x is not real then $\sum_{r=1}^{50}\left(x^r + x^{2r}\right)^3 =$

A) 100 B) 256 C) 76 D) 94

KEY : D

HINT : $x^3 = 1 \Rightarrow x = \omega, \omega^2$ $\quad x^r + x^{2r} = \begin{cases} 2 & if\ \ r\ \ is\ a\ \ multiple\ \ of\ \ 3 \\ -1 & if\ \ r\ \ is\ \ not\ \ a\ \ multiple\ \ of\ \ 3 \end{cases}$

50. If a, b, c are in A.P. b,c,d are in G.P. and c,d,e are in H.P. then a,c,e are in

(A) AP (B) GP
(C) HP (D) none

KEY : B

HINT : a, b, c are in AP $\Rightarrow a + c = 2b; \quad (1)$

b, c, d are in GP $\Rightarrow c^2 = bd \quad (2)$

c, d, e are in HP $\Rightarrow \frac{2ce}{c+e} = d \quad (3)$

$(1)\times(3) \Rightarrow \frac{(a+c)ce}{c+e} = bd = c^2$

$\therefore (a+c)e = c(c+e)$

$ae = c^2 \Rightarrow a, c, e$ are in G.P

51. If $a_n = \sum_{k=0}^{n} \frac{(\log_e 10)^n}{k!(n-k)!}$ for $n \ge 0$ then $a_0 + a_1 + a_2 + a_3 + \ldots$ upto ∞ equal is

(A) 10 (B) 10^2 (C) 10^3 (D) 10^4

Key : B

Hint : $a_n = \frac{(\log_e 10)^n}{n!} \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} = \frac{(\log_e 10)^n}{n!}\left[2^n\right] = \frac{(2\log_e 10)^n}{n!}$

Thus, $a_0 + a_1 + a_2 + \ldots$ upto infinity is

$= \sum_{n=0}^{\infty} \frac{(2\log_e 10)^n}{n!} = e^{2\log_e 10} = 100$

$\therefore$ (B) is the correct answer.

52. If a_1 is the greatest value of $f(x)$; where $f(x) = \left(\frac{1}{2+[\sin x]}\right)$ (where [.] denotes greatest integer function) and $a_{n+1} = \frac{(-1)^{n+2}}{(n+1)} + a_n$, then $\lim_{n\to\infty}(a_n)$ is

(A) 1 (B) e^2

(C) ln2 (D) ln3

Key: C

Hint: $a_1 = 1$

$\Rightarrow a_2 = 1 - \frac{1}{2}$

$\Rightarrow a_3 = 1 - \frac{1}{2} + \frac{1}{3}$

………..

………..

$a_\infty = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots\ldots\ldots$

= ln2

53. The sequence $\{x_k\}$ is defined by $x_{k+1} = x_k^2 + x_k$ and $x_1 = \frac{1}{2}$. Then $\left[\frac{1}{x_1+1} + \frac{1}{x_2+1} + \ldots + \frac{1}{x_{100}+1}\right]$ (where [.] denotes the greatest integer function) is equal to

(A) 0 (B) 2

(C) 4 (D) 1

Key: D

Hint: $\frac{1}{x_{k+1}}=\frac{1}{x_k(x_k+1)}=\frac{1}{x_k}-\frac{1}{x_k+1}\Rightarrow\frac{1}{x_k+1}=\frac{1}{x_k}-\frac{1}{x_{k-1}}$

$\therefore\frac{1}{x_1+1}=\frac{1}{x_2+1}+...+\frac{1}{x_{100}+1}=\frac{1}{x_1}-\frac{1}{x_{101}}$

As $0<\frac{1}{x_{101}}<1$

$\therefore\left[\frac{1}{x_1+1}+\frac{1}{x_2+1}+...+\frac{1}{x_{100}+1}\right]=1$

54. If $S_n=\sum_{r=1}^{n}t_r=\frac{1}{6}n\left(2n^2+9n+13\right)$, then $\sum_{r=1}^{n}\sqrt{t_r}$ equals

(a) $\frac{1}{2}n(n+1)$ (b) $\frac{1}{2}n(n+2)$ (c) $\frac{1}{2}n(n+3)$ (d) $\frac{1}{2}n(n+5)$

Key: c

Hint: We have $t_n = S_n - S_{n-1}\ \forall\ n\ \geq 2$

$\therefore\quad t_n=\frac{1}{6}\left[2\left(n^3-(n-1)^3\right)+9\left(n^2-(n-1)^2\right)+13(n-n+1)\right]$

$=\frac{1}{6}\left[6n^2-6n+2+9(2n-1)+13\right]$

$=\frac{1}{6}\left(6n^2+12n+6\right)=(n+1)^2$

$\therefore\quad\sum_{r=1}^{n}\sqrt{t_r}=\sum_{r=1}^{n}(r+1)=\frac{1}{2}(n+1)(n+2)-1=\frac{1}{2}n(n+3)$

55. $\{a_n\}$ and $\{b_n\}$ be two sequences given by $a_n=(x)^{1/2^n}+(y)^{1/2^n}$ and $b_n=(x)^{1/2^n}-(y)^{1/2^n}$ for all $n\in N$, then $a_1\,a_2\,a_3\,\ldots\,a_n$ is equal to

(A) $x-y$ (B) $\frac{x+y}{b_n}$

(C) $\frac{x-y}{b_n}$ (D) $\frac{xy}{b_n}$

Key : c

Sol : $a_1\,a_2\,\ldots.\,A_n = b_n\ \frac{a_1a_2...a_n}{b_n}$

$= a_n\,b_n\ \frac{(a_1a_2...a_{n-1})}{b_n}$

$=\left(x^{\frac{1}{2^{n-1}}}-y^{\frac{1}{2^{n-1}}}\right)\frac{(a_1\,a_2...a_{n-1})}{b_n}=a_{n-1}\ b_{n-1}\ \frac{(a_1a_2...a_{n-2})}{b_n}$

$=\frac{a_1b_1}{b_n}=\frac{(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})}{b_n}=\frac{x-y}{b_n}$

56. The sum of the series $\frac{9}{5^2.2.1}+\frac{13}{5^3.3.2}+\frac{17}{5^4.4.3}+---$ upto infinity

(A) 1 (B) $\frac{9}{5}$

(C) $\frac{1}{5}$ (D) $\frac{2}{5}$

Key. C

Sol. $Tr=\frac{4r+1}{5^r r(r-1)}, r\ge 2$

$$\frac{5r-(r-1)}{5^r r(r-1)}=\frac{1}{5^{r-1}(r-1)}-\frac{1}{5^r r}$$

$$\sum_{r=2}^{\infty}T_r=\left(\left(\frac{1}{5^1.1}-\frac{1}{5^2.2}\right)+\left(\frac{1}{5^2.2}-\frac{1}{5^3.3}\right)+\left(\frac{1}{5^3.3}-\frac{1}{5^4.4}\right)+---\infty\right)$$

$$=\frac{1}{5}$$

57. If a, b, c, d are distinct integers in AP such that $d=a^2+b^2+c^2$ then a + b + c + d is

(A) 0 (B) 1

(C) 2 (D) None

Key. C

Sol. $d=a^2+b^2+c^2 \Rightarrow a+3t=(a+t)^2+a^2+(a+2t)^2$

$$5t^2+3(2a-1)t+3a^2-a=0$$

$$D\ge 0 \Rightarrow 24a^2+16a-9\le 0$$

$$\Rightarrow -\frac{1}{3}-\frac{\sqrt{70}}{2}<a<-\frac{1}{3}+\frac{\sqrt{70}}{2}$$

$$\Rightarrow a=-1,0$$

$$a=0, t=0,\frac{3}{5}$$

$$a=-1, t=1,\frac{4}{5}$$

$$\Rightarrow t=1$$

$$a+b+c+d=2$$

58. If $b+c, c+a, a+b$ are in H.P then show that a^2, b^2, c^2 are in

(a) A.P (b) G.P (c) H.P (d) A.G.P

Key. A

Sol. $\frac{1}{c+a}-\frac{1}{b+c}=\frac{1}{a+b}-\frac{1}{c+a}$

59. Sum of first n terms of a sequence is given by $3\,S_n = T_n^{\,2} + 3T_n - 2,\ (T_n > 0)$ where T_n is the nth term of sequence, then the value of T_2^2 is

A) $2-\sqrt{2}$ B) $2+\sqrt{2}$ C) $2+3\sqrt{2}$ D) $3+2\sqrt{2}$

Key. C

Sol. $S_1 = \dfrac{T_1^{\,2}+3T_1-2}{3} = T_1 \Rightarrow T_1^{\,2} = 2$

$S_2 - S_1 = \dfrac{T_2^{\,2}-T_1^2+3(T_2-T_1)}{3} = T_2$

$T_2^2 - T_1^2 + 3(T_2 - T_1) = 3T_2$

$\Rightarrow T_2^2 = T_1^2 + 3T_1 = 2 + 3\sqrt{2}$

60. If a, b, c are three distinct numbers such that a, b, c are in A.P. and b – a, c – b, a are in G.P., then a : b : c are in

(A) 2 : 3 : 4 (B) 3 : 4 : 5
(C) 1 : 3 : 5 (D) 1 : 2 : 3

Key. D

Sol. a, b, c are in A.P. $\Rightarrow 2b = a + c$ …..(1)

(b – a), (c – b), a are in G.P. $\Rightarrow (c-b)^2 = a(b-a)$

$\Rightarrow c - a = a(b - a)$ ….(2)

from (1) and (2)

$$\frac{a}{1} = \frac{b}{2} = \frac{c}{3}$$

61. The sum to n terms of the series

$\dfrac{1}{\lfloor 1}\left(\dfrac{1}{2}\right) + \dfrac{1.3}{\lfloor 2}\left(\dfrac{1}{2}\right)^2 + \dfrac{1.3.5}{\lfloor 3}\left(\dfrac{1}{2}\right)^3 + ...$ upto n terms is

(A) $\dfrac{1.3.5.....(2n-1)(2n+1)}{2^n \lfloor n} - 1$ (B) $1 - \dfrac{1.3.5.....(2n-1)}{\lfloor n\ \lfloor n}$

(C) $1 - \dfrac{1.3.5.....(2n-3)}{2^{n-1}\lfloor n-1}$ (D) $\dfrac{1.3.5.....(2n-3)}{2^{n-1}\lfloor n-1}$

Key. A

Sol. $t_{n+1} = \dfrac{t_n \times (2n+1)}{(n+1)} \times \dfrac{1}{2}$

$(2n+2)t_{n+1} = (n+1)t_n$

$(2n+3)t_{n+1} - (2n+1)t_n = t_{n+1}$

Put n = 1

$5t_2 - 3t_1 = t_2$

n = 2, $7t_3 - 5t_2 = t_3$

$(2n+1)t_n-(2n-1)t_{n-1}=t_n$

$(2n+1)t_n-2t_1=S$

$S=\frac{1.3.5.....(2n+1)}{\lfloor n\times 2^n}-1$

62. The sum to n terms of the series

$\frac{1}{\lfloor 1}\left(\frac{1}{2}\right)+\frac{1.3}{\lfloor 2}\left(\frac{1}{2}\right)^2+\frac{1.3.5}{\lfloor 3}\left(\frac{1}{2}\right)^3+.....$ upto n terms is

(A) $\frac{1.3.5.....(2n-1)(2n+1)}{2^n\lfloor n}-1$ (B) $1-\frac{1.3.5.....(2n-1)}{\lfloor n\ \lfloor n}$

(C) $1-\frac{1.3.5.....(2n-3)}{2^{n-1}\lfloor n-1}$ (D) $\frac{1.3.5.....(2n-3)}{2^{n-1}\lfloor n-1}$

Key. A

Sol. $t_{n+1}=\frac{t_n\times(2n+1)}{(n+1)}\times\frac{1}{2}$

$(2n+2)t_{n+1}=(n+1)t_n$

$(2n+3)t_{n+1}-(2n+1)t_n=t_{n+1}$

Put n = 1

$5t_2-3t_1=t_2$

n = 2, $7t_3-5t_2=t_3$

$(2n+1)t_n-(2n-1)t_{n-1}=t_n$

$(2n+1)t_n-2t_1=S$

$S=\frac{1.3.5.....(2n+1)}{\lfloor n\times 2^n}-1$

63. If the ratio of the sum to 'n' terms of two A.P's is (5n+3):(3n+4), then the ratio of their 17th terms is

a) 172:99 b)168:103 c)175:99 d)171:103

Key. B

Sol. $\frac{\frac{n}{2}\left[2a_1+(n-1)d_1\right]}{\frac{n}{2}\left[2a_1+(n-1)d_2\right]}=\frac{5n+3}{3n+4}\Rightarrow\frac{a_1+\left(\frac{n-1}{2}\right)d_1}{a_1+\left(\frac{n-1}{2}\right)d_2}=\frac{5n+3}{3n+4}$ put $\frac{n-1}{2}=16$

64. If x,y,z are in G..P and $a^x=b^y=c^z$, then

A) $log_b a=\log_a c$ b) $log_a^b=\log_a c$ C) $log_b a=\log_c b$ D) None

Key. C

Sol. $a^n=b^y=c^z=k.,\ y^2=xz\Rightarrow\left(\text{lo}\,l_b k\right)^2=\log_a k\ ,=\log_a k\Rightarrow\left(\log b\right)^2=\log a.\log c$

65. If the pth, qth, r th terms of an A.P are in G.P, then common ratio of G..P is

a) $\frac{pr}{q^2}$ b) $\frac{r}{p}$ c) $\frac{q+r}{p+q}$ d) $\frac{q-r}{p-q}$

Key. D

Sol.
$a+(p-1)d=k$ $Find\frac{②-①}{③-②}$
$a+(q-1)d=kr$
$a+(r-1)d=kr^2$

66. If H_1, H_2 ...., H_{20} be 20 harmonic means between 2 and 3, then $\frac{H_1+2}{H_1-2}+\frac{H_{20}+3}{H_{20}-3}=$

a) 20 b) 21 c) 40 d) 38

Key. C

Sol. $H_1=\frac{63}{31}, H_{20}=\frac{126}{43}$

67. If $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+$ ....to $\infty=\frac{\pi^2}{6}$, then $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+$...................=

a) $\frac{\pi^2}{8}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{3}$ d) $\frac{\pi^2}{2}$

Key. A

Sol. $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.........=\frac{\pi^2}{6}-\frac{1}{2^2}\left(\frac{\pi^2}{6}\right)=\frac{\pi^2}{8}$

68. The 20th term of 2,9,20,35,54, ...is

a) 819 b) 820 c) 1009 d) 1010

Key. A

Sol. $t_n=2+\left(7+11+15......(n-1)terms\right)$

69. If x > 1, y > 1, z > 1 and x, y, z are in G.P. then $(\ln x^2)^{-1}$, $(\ln xy)^{-1}$, $(\ln xz)^{-1}$ are in
(A) A.P. (B) G.P.
(C) H.P. (D) none of these

Key. C

Sol. x > 1, y > 1, z > 1
x, y, z $\to$ G.P. $\Rightarrow$ *l*nx, *l*ny, *l*nz are in A.P. 2*l*nx, *l*nxy, *l*nxz are in A.P.
$(l\text{n}x^2)^{-1}$, $(\text{in}xy)^{-1}$, $(l\text{n}xz)^{-1}$ are in H.P.

70. If $S_n=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+.....+\frac{1}{n}$ and n > 2, then S_n would always be

(A) more than $n(n+1)^{\frac{1}{n}}-n$ (B) less than $n(n+1)^{1/n}-n$

(C) equal to $n(n+1)^{\frac{1}{n}}-n$ (D) greater than or equal to $\frac{n(n+1)^{\frac{1}{n}}}{(n+5)}$

Key. A

Sol. $$\frac{(1+1)+\left(1+\frac{1}{2}\right)+\left(1+\frac{1}{3}\right)+\left(1+\frac{1}{4}\right)+.....+\left(1+\frac{1}{n}\right)}{n} > \left(2\times\frac{3}{2}\times\frac{4}{3}\times\frac{5}{4}\times.....\times\frac{n+1}{n}\right)^{1/n}$$

$$\Rightarrow \frac{n+S_n}{n} > (n+1)^{1/n} \Rightarrow S_n > n(n+1)^{1/n} - n$$

71. If a,b,c are in AP, then the sum of the coefficients of $\left\{1+\left(ax^2-2bx+c\right)^2\right\}^{1973}$ is

a) -2 b) -1 c) 0 d) 1

Key. D

Sol. ∵ a, b, c are in A.P.

$\Rightarrow 2b = a+c$

$\Rightarrow a-2b+c=0$

Putting x=1

Required sum = $\left(1+a-2b+c\right)^{1973} = \left(1+0\right)^{1973} = 1$

72. $\{a_n\}$ and $\{b_n\}$ be two sequences given by $a_n = (x)^{1/2^n} + (y)^{1/2^n}$ and $b_n = (x)^{1/2^n} - (y)^{1/2^n}$ for all $n \in N$, then $a_1 a_2 a_3 \ldots a_n$ is equal to

(A) $x - y$ (B) $\frac{x+y}{b_n}$

(C) $\frac{x-y}{b_n}$ (D) $\frac{xy}{b_n}$

Key. C

Sol. $a_1 a_2 \ldots a_n = b_n \frac{a_1 a_2 \ldots a_n}{b_n}$

$= a_n b_n \frac{(a_1 a_2 \ldots a_{n-1})}{b_n}$

$= \left(x^{\frac{1}{2^{n-1}}} - y^{\frac{1}{2^{n-1}}}\right)\frac{(a_1 a_2 \ldots a_{n-1})}{b_n} = a_{n-1}\ b_{n-1}\ \frac{(a_1 a_2 \ldots a_{n-2})}{b_n}$

$= \frac{a_1 b_1}{b_n} = \frac{(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})}{b_n} = \frac{x-y}{b_n}$

73. $\frac{1}{2} - \frac{1}{2.2^2} + \frac{1}{3.2^3} - \frac{1}{4.2^4} + ... =$

1) $\frac{1}{4}$ 2) $\log_e\left(\frac{3}{4}\right)$ 3) $\log_e\left(\frac{3}{2}\right)$ 4) $\log_e\left(\frac{2}{3}\right)$

Key. 3

Sol. $\log_e\left(1+\frac{1}{2}\right) = \log_e\frac{3}{2}$

74. The ratio of sum of first three terms of a G.P. to the sum of first six terms is $64:91$,the common ratio of G.P. is

1. $\frac{1}{4}$ 2. $\frac{3}{4}$ 3. $\frac{5}{4}$ 4. $\frac{7}{4}$

Key. 2

Sol. Given $\frac{S_3}{S_6} = \frac{64}{91} = \frac{a(r^3-1)}{a(r^6-1)}$

$\Rightarrow \frac{(r^3-1)}{(r^3+1)(r^3-1)} = \frac{64}{91}$

$\Rightarrow r^3 = \frac{27}{64} \quad \therefore \quad r = \frac{3}{4}$

75. Sum of the series $3+5+9+17+33+.....$to n terms is

1. $2^{n+1}-n-2$ 2. $2^{n+1}+n-2$ 3. 2^n+n-2 4. $2^{n+1}-n+2$

Key. 2

Sol. $S_n = 3+5+9+17+33+......$

$= (2+1)+(2^2+1)(2^3+1)+(2^4+1)+....$

$= (2+2^2+2^3+2^4+......n \text{ terms})+n$

$= 2(2^n-1)+n = 2^{n+1}+n-2$

$= 2^{n+1}+n-2$

76. If one A.M. A and two G.M.s p and q be inserted between two numbers a and b, then which of the following is hold good

1. $a^3+b^3=2Apq$ 2. $p^3+q^3=2Apq$ 3. $a^3+b^3=2Aab$ 4. None of these.

Key. 2

Sol. Given $a+b=2A$

And $a, p, q, \quad b \in$ G.P.

$\therefore \quad p^2 = aq$ and $q^2 = pb$

$\Rightarrow p^3 = apq$ and $q^3 = bpq$

by adding we get

$p^3 + q^3 = apq + bpq$

$= pq(a+b) = 2Apq$

77. If fourth term of a G.P. is 3, the product of the first seven terms is

1. 3^4 2. 3^7 3. 7^4 4. 4^7

Key. 2

Sol. As the number of terms are odd (7) let r, be the common ratio

So terms can be taken as $\frac{a}{r^3}, \frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2, ar^3$

$\therefore$ Product of the term $= a^7$

$= 3^7$ as $(t_4 = a = 3)$

78. The number of divisors of 6912, 52480,32000 are in

1. A.P Only 2. G.P. Only 3. A.P. , G.P.& H.P. 4. None of these

Key. 3

Sol. If n is a $+$ ve number.

$n = P_1^{k_1}.P_2^{k_2}...P_r^{k_r}$

(where $p_1, p_2, p_3, ...p_r$ are prime number) then number of divisors of n are

$= (k_1+1)(k_2+1)....(k_r+1)$

$\therefore$ Number of prime factor of 6912 are $= 2^8.3^3$ so no. of divisors $= 9 \times 4 = 36$

Prime factors of 52,400 are $= 3^8 \times 2^3$

$\therefore$ No. of divisors $= 9 \times 4 = 36$

Prime factors of 32,000 are $= 5^3 \times 2^8$

$\therefore$ No. of divisors $= 9 \times 4 = 36$

Now each number having same number of divisors *i.e.*, 36,36,36

Each and every term is constant & constant sequence is always in A.P.& G.P. both as common difference is 0 and common ratio is 1.

79. If 1, $\log_{81}(3^x + 48)$, $\log_9\left(3^x - \frac{8}{3}\right)$ are in A.P., then the value of x equals

1. 9 2. 6 3. 2 4. 4

Key. 3

Sol. Given 1, $\log_9 2\left(3^x + 48\right), \log_9\left(3^x - 8/3\right), \in$ A.P.

$$\Rightarrow \log_9 9,\ \frac{1}{2}\log_9\left(3^x + 48\right),\ \log_9\left(3^x - 8/3\right) \in \text{A.P.}$$

$$\Rightarrow 9, \left(3^x + 48\right)^{1/2}, 3^x - 8/3 \in \text{G.P.} \quad \text{(By concept)}$$

$$\Rightarrow \log a, \log b, \log c \in \text{ A.P.}$$

$$\therefore\ a, b, c \in \text{ G.P.} \quad \therefore\ 3^x + 48 = 9\left(3^x - 8/3\right)$$

$$8.3^x = 72$$

$$3^x = 9,\ 3^x = 3^2, \quad x = 2.$$

80. If a, b, c are in H.P., then $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in

1. A.P. 2. H.P. 3. G.P. 4. None of these

Key. 2

Sol. Given $a, b, c \in$ H.P.

So, $\frac{1}{a}, \frac{1}{b}, \frac{1}{c} \in$ A.P.

$$\frac{a+b+c}{a}, \frac{b+c+a}{b}, \frac{c+a+b}{c} \in \text{ A.P.}$$

By using concept if $a, b, c \in$ A.P.

Then their reciprocals are in H.P.

81. Between the numbers 2 and 20, 8 means are inserted, then their sum is

1. 88 2. 44 3. 176 4. None of these

Key. 1
Sol. Let $a, A_1, A_2....A_g, b \in A.P$

Where $a = 2, b = 20, n = 8$

$\therefore$ sum of the means $= \frac{n}{2}(a+b) = \frac{8}{2}(2+20) = 88$

82 If the arithmetic mean of two positive numbers $a \& b$ $(a > b)$ is twice their G.M., then $a : b$ is

1. $6+\sqrt{7} : 6-\sqrt{7}$ 2. $2+\sqrt{3} : 2-\sqrt{3}$ 3. $5+\sqrt{6} : 5-\sqrt{6}$ 4. None of these

Key. 2
Sol. $\frac{a+b}{2} = 2\sqrt{ab}$

$a + b - 4\sqrt{ab} = 0$

$\frac{a}{b} + 1 - 4\sqrt{\frac{a}{b}} = 0$ (Dividing by b)

Or $\left(\sqrt{\frac{a}{b}}\right)^2 - 4\sqrt{\frac{a}{b}} + 1 = 0$

$\therefore \sqrt{\frac{a}{b}} = \frac{4 \pm 2\sqrt{3}}{2} = \left(2 \pm \sqrt{3}\right)$

$\frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$

83 The number of terms common between the two series 2 + 5 + 8 + …up to 50 terms and the series 3 + 5 + 7 + 9 + ….up to 60 terms.

1. 24 2. 26 3. 25 4. None of these

Key. 4

Sol. Let m^{th} term of first A.P. be equal to the n^{th} term of the second A.P. then

2 , 5 , 8 ,….50 terms series 1

3 , 5 ,7 , …., 60 terms series 2

Common series 5 , 11, 17 , ….., 119

40^{th} term of series 1 = 59^{th} term of series 2 = 119 = last term of common series

$\Rightarrow a_n = 5 + (n-1)d \Rightarrow 119 + 1 = 6n \Rightarrow n = 20.$

$\therefore$ Number of common terms is 20.

84. If a, b, c are three positive numbers, then the minimum value of the expression $\dfrac{ab(a+b)+bc(b+c)+ca(c+a)}{bca}$

1. 3 2. 4 3. 6 4. 1

Key. 3

Sol. Given expression equal to

$$\frac{(a+b)}{c}+\frac{(b+c)}{a}+\frac{(c+a)}{b}$$

Or $\dfrac{a}{c}+\dfrac{b}{c}+\dfrac{b}{a}+\dfrac{c}{a}+\dfrac{c}{b}+\dfrac{a}{b}$

Using A.M. $\geq$ G.M. $\dfrac{\frac{a}{c}+\frac{b}{c}+\frac{b}{a}+\frac{c}{a}+\frac{c}{b}+\frac{a}{b}}{6} \geq \sqrt{\frac{a}{c}\frac{b}{c}\frac{b}{a}\frac{c}{a}\frac{c}{b}\frac{a}{b}}$

Or $\dfrac{a+b}{c}+\dfrac{b+c}{a}+\dfrac{c+a}{b} \geq 6$

85. Sum of n terms of series $ab+(a+1)(b+1)+(a+2)(b+2)+....+(a+(n-1))(b+(n-1))$ if $ab=\dfrac{1}{6}\,and\,(a+b)=\dfrac{1}{3}$, is

(A) $\dfrac{n}{6}(1-2n)^2$ (B) $\dfrac{n}{6}(1+n-2n^2)$ (C) $\dfrac{n}{6}(1-2n+2n^2)$ (D) none of these

Key. C

Sol. $s = ab + [ab + (a+b) + 1] + [ab + 2(a+b) + 2^2] + ...[ab + (n-1)(a+b) + (n-1)^2]$

$= nab + (a+b)\sum_{r=1}^{n-1} r + \sum_{r=1}^{n-1} r^2$

$= nab + (a+b)\dfrac{n(n-1)}{2} + \dfrac{(n-1))(n)(2n-1)}{6}$

$= \dfrac{n}{6}[1 + (n-1)\{1 + 2n - 1\}]$

$= \dfrac{n}{6}[1 + 2n(n-1)] = \dfrac{n}{6}(1 - 2n + 2n^2)$

86. If log (a+c),log(a+b), log(b+c) are in A.P. and a,c,b are in H.P, then the value of a+b is (given a,b,c >0)

(A) 2c (B) 3c (C) 4c (D) 6c

Key. A

$\log(a+c) + \log(b+c) = 2\log(a+b)$

$(a+c)(b+c) = (a+b)^2$

Sol. $\Rightarrow ab + c(a+b) + c^2 = (a+b)^2$ (1)

$also, c = \dfrac{2ab}{a+b} \Rightarrow 2ab = c(a+b)$

$\Rightarrow 2ab + 2c(a+b) + 2c^2 = 2(a+b)^2$ ..... (2)

From (1) and(2),

$c(a+b) + 2c(a+b) + 2c^2 = 2(a+b)^2$

$2(a+b)^2 - 3c(a+b) - 2c^2 = 0$

$\therefore a+b = \dfrac{3c \pm \sqrt{9c^2 + 16c^2}}{4} = \dfrac{3c \pm 5c}{4} = 2c \; or - \dfrac{c}{2}$

$\therefore a+b = 2c$ $(\because a,b,c > 0)$

87. If $a_1, a_2, a_3, ..., a_n$ are in A.P. with s_n as the sum of first 'n' terms $(s_0 = 0)$, then $\sum_{k=0}^{n} {}^nC_k s_k$ is equal to

(A) $2^{n-2}\left[na_1 + s_n\right]$ (B) $2^n\left[a_1 + s_n\right]$ (C) $2\left[na_1 + s_n\right]$ (D) $2^{n-1}\left[a_1 + s_n\right]$

Key. A

Sol. $\sum_{k=o}^{n} {}^nc_k s_k = \sum_{k=0}^{n} {}^nc_k \dfrac{k}{n}[2a + (k-1)d]$

$= [(a_1 - \dfrac{d}{2})\sum_{k=0}^{n} k\,{}^nc_k + \dfrac{d}{2}\sum_{k=0}^{n} k^2\,{}^nc_k]$

$= \left(a_1 - \frac{d}{2}\right)n.2^{n-1} + \frac{d}{2}[n.2^{n-1} + n(n-1)2^{n-2}]$

$= a_1.n.2^{n-1} + dn(n-1)2^{n-3}$

$= n.2^{n-3}[4a_1 + a_n - a_1] = n.2^{n-3}[3a_1 + a_n]$

$= 2^{n-3}[2na_1 + 2n\left(\frac{a_1 + a_n}{2}\right)]$

$= 2^{n-2}[na_1 + s_{n]}.$

88. If a,b,c, are in A.P. and p, p' are respectively A.M. and G.M. between a and b while q, q' are respectively AM. And G.M. between b and c, then

(A) $p^2 + q^2 = p'^2 + q'^2$ (B) $pq = p'q'$

(C) $p^2 - q^2 = p'^2 - q'^2$ (D) $p^2 + p'^2 = q^2 + q'^2$

Key. C

Sol. We have $2b = a + c$ and a,p,b,q,c are in A.P

$\Rightarrow p = \frac{a+b}{2}, q = \frac{b+c}{2}$

Again , $p' = \sqrt{ab}$ *and* $q' = \sqrt{bc}$

$\therefore p^2 - q^2 = \frac{(a+b)^2 - (b+c)^2}{4}$

$= \frac{(a-c)(a+c+2b)}{4} = (a-c)b = p'^2 - q'^2$

89. Through the centroid of an equilateral triangle a line parallel to the base is drawn. On this line, an arbitrary point p is taken inside the triangle. Let h denote the distance of p from the base of the triangle. Let h_1 and h_2 be the distance of p from the other two sides of the triangle, then

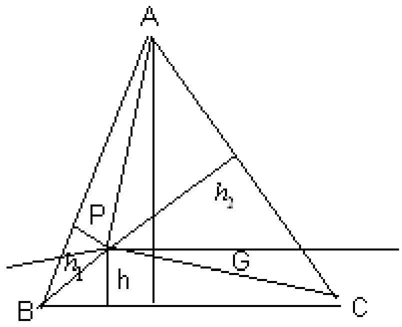
(A) h is the H.M. of h_1, h_2 (B) h is the G.M. of h_1, h_2

(C) h is the A.M, of h_1, h_2 (D) None of these

Key. C

Sol. $\Delta ABC = \Delta PBC + \Delta PAC + \Delta PAB$

$\frac{1}{2}.a.3h = \frac{1}{2}a.h + \frac{1}{2}a\ h_1 + \frac{1}{2}a.h_2$



$$h_1 + h_2 = 2h \Rightarrow h = \frac{h_1 + h_2}{2}$$

90. a,b,c are positive integers forming an increasing G.P. whose common ratio is a natural number, b – a is cube of a natural number and $\log_6 a + \log_6 b + \log_6 c = 6$, then a + b + c =

A) 100 B) 111 C) 122 D) 189

Key. D

Sol. $\log_6(abc) = 6 \Rightarrow (abc) = 6^6$

Let $a = \frac{b}{r}$ and c = br

$\Rightarrow$ b = 36 and $a = \frac{36}{r}$ $\Rightarrow$ r = 2, 3, 4, 6, 9, 12, 18

Also b – a = 36 $\left(1 - \frac{1}{r}\right)$ is a perfect cube. $\therefore$ r = 4

$\Rightarrow$ a + b + c = 36 + 9 + 144=189

91. If S,P and R are the sum, product and sum of the reciprocals of n terms of an increasing G.P. and $S^n = R^n . P^k$, then k is equal to

A) 1 B) 2 C) 3 D) none of these

Key. B

Sol. $S = \frac{a(1-r^n)}{1-r}, P = a^n . r^{\frac{n(n-1)}{2}}$

$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} ....$ to n terms $= \frac{1-r^n}{a(1-r)r^{n-1}}$

$S^n = R^n P^k \quad \Rightarrow \quad \left(\frac{S}{R}\right)^n = P^k$

$\Rightarrow \quad (a^2 r^{n-1})^n = P^k$

$\Rightarrow \quad P^2 = p^k \quad \Rightarrow \quad k = 2$

92. Sum of first hundred numbers common to the two A.P.'s 12, 15, 18, … and 17, 21, 25 …..

A) 56100 B) 65100 C) 61500 D) none of these

Key. C

Sol. AP I = 12, 15, 18, … (common difference $d_1 = 3$)

AP II = 17, 21, 25… (common difference $d_2 = 4$)

First term of the series of common numbers = 21

Here a = 21, common difference of the series of common numbers = L.C.M of d_1 and $d_2 = 12$

$\therefore$ Required sum of first hundred terms

$= \frac{100}{2}\left[2\times 21+(100-1)12\right]=100\left[21+594\right]=61500$

93. If 11 A.M. s are inserted between 28 and 10, then number of integral A.M's is

A) 5 B) 6 C) 7 D) 8

Key. A

Sol. Since $A_1, A_2, A_3, ..., A_{11}$ be 11 A.M. s between 28 and 10.

$\therefore$ 28, $A_1, A_2, ..., A_{11}$, 10 are in A.P.

Let 'd' be the common difference of A.P.

Also the number of terms = 13.

$10 = T_{13} = T_1 + 12d = 28 + 12d$

$\therefore \quad d = \frac{10-28}{12} = -\frac{18}{12} = -\frac{3}{2}$

$\therefore$ Number of integral A.M's is 5.

94. If a,b,c are in HP, then $\frac{1}{b-a}+\frac{1}{b-c}$ is equal to

A) $\frac{2}{b}$ B) $\frac{2}{a+c}$ C) $\frac{1}{a+c}$ D) none of these

Key. A

Sol. Q a,b,c are in H.P.

Q $b = \frac{2ac}{(a+c)}$

$\therefore \quad \frac{1}{b-a}+\frac{1}{b-c}$

$\Rightarrow \quad \frac{1}{\frac{2ac}{(a+c)}-a}+\frac{1}{\frac{2ac}{(a+c)}-c}$

$\Rightarrow \quad (a+c)\left\{\frac{1}{a(c-a)}+\frac{1}{c(a-c)}\right\} \quad \Rightarrow \quad \frac{(a+c)}{(a-c)}\left\{-\frac{1}{a}+\frac{1}{c}\right\}$

$\Rightarrow \quad \frac{(c+a)(a-c)}{ac(a-c)} \quad \Rightarrow \quad \frac{(a+c)}{ac}=\frac{2}{b}$

95. Let a_n be the n^{th} term of an A.P. If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the A.P. is

A) $\alpha-\beta$ B) $\beta-\alpha$ C) $\frac{\alpha-\beta}{2}$ D) none of these

Key. D

Sol. $a_1 + a_3 + a_5 + ... + a_{199} = \beta$

$a_2 + a_4 + a_6 + ... + a_{200} = \alpha$

$a_2 - a_1 + a_4 - a_3 + a_6 - a_5 ... a_{200} - a_{199} = \alpha - \beta$

$d + d + d \ldots\ldots\ldots d = \alpha - \beta$

$d = \dfrac{\alpha - \beta}{100}$

96. If a,b,c,d are in G.P., then $\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)$ equals to

A) a b + b c + c d B) (a b + b c + c d)^2 C) (a b + b c + c d)^4 D) none of these

Key. B

Sol. a,b,c,d are in G.P., let they are a, ar, ar^2, ar^3

$\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)$

$= a^2 \times a^2\left[1+r^2+r^4\right]\left[r^2+r^4+r^6\right]$

$= a^4r^2\left[1+r^2+r^4\right]^2$

$= \left[a^2r\left[1+r^2+r^4\right]\right]^2$

$= (ab+bc+cd)^2$

97. If a_1, a_2, a_3, a_4, a_5 are in H.P., then $a_1a_2 + a_2a_3 + a_3a_4 + a_4a_5$ is equal to

A) $2a_1a_5$ B) $3a_1a_5$ C) $4a_1a_5$ D) – 4

Key. C

Sol. a_1, a_2, a_3, a_4, a_5 are in H.P.

$\Rightarrow \quad a_2 = \dfrac{2a_1a_2}{a_1+a_3} \Rightarrow 2a_1a_3 = a_2a_1 + a_3a_2$

$a_4 = \dfrac{2a_3a_5}{a_3+a_5} \Rightarrow 2a_3a_5 = a_3a_4 + a_5a_4$

$\Rightarrow \quad a_1a_2 + a_2a_3 + a_3a_4 + a_4a_5 = 2a_1a_3 + 2a_3a_5 \qquad \ldots(i)$

$a_3 = \dfrac{2(a_1a_5)}{a_1+a_5} \Rightarrow a_1a_3 + a_5a_3 = 2a_1a_5 \qquad \ldots(ii)$

using (i) & (ii)

$a_1a_2 + a_2a_3 + a_3a_4 + a_4a_5 = 2(2a_1a_5) = 4a_1a_5$

98. If the sum to infinity of the series, $1 + 4x + 7x^2 + 10x^3 + \ldots\ldots..$, is $\dfrac{35}{16}$, where |x| < 1, then 'x' equals to

A) 19/7 B) 1/5 C) 1/4 D) none of these

Key. B

Sol. $S = 1 + 4x + 7x^2 + 10x^3 + \ldots\ldots\ldots\ldots$

$x.S = x + 4x^2 + 7x^3 + \ldots\ldots\ldots\ldots.$

Subtract

$S(1-x) = 1 + 3x + 3x^2 + 3x^3 + \ldots\ldots\ldots..$

$S(1-x) = 1 + 3x\left(\dfrac{1}{1-x}\right) \qquad |x| < 1$

$S = \dfrac{1+2x}{(1-x)^2}$

Given $\frac{1+2x}{(1-x)^2}=\frac{35}{16}$

$\Rightarrow$ $16+32x=35+35x^2-70x$ $\Rightarrow$ $35x^2-102x+19=0$

$\Rightarrow$ $35x^2-7x-95x+19=0$ $\Rightarrow$ $7x(5x-1)-19(5x-1)=0$

$\Rightarrow$ (5x – 1) (7x – 19) = 0 $\Rightarrow$ $x=\frac{1}{5},\frac{19}{7}$

But |x| < 1 $\therefore$ $x=\frac{1}{5}$

99. If a,b,c and d are four positive real numbers such that abcd = 1, the minimum value of (1 + a) (1 + b) (1 + c) (1 + d) is

A) 4 B) 1 C) 16 D) 18

Key. C

Sol. $1+a\ge 2\sqrt{a}$ $\{AM\ge GM\}$

$1+b\ge 2\sqrt{b}$

$1+c\ge 2\sqrt{c}$

$1+d\ge 2\sqrt{d}$

$\therefore$ (1 + a) (1 + b) (1 + c) (1 + d) $\ge 16\sqrt{abcd}=16$

$\therefore$ min. value = 16 (for a = b = c = d = 1)

100. If the length of sides of a right triangle are in A.P., then the sines of the acute angle are

A) $\frac{3}{5},\frac{4}{5}$ B) $\sqrt{\frac{2}{3}},\sqrt{\frac{1}{3}}$

C) $\sqrt{\frac{\sqrt{5}-1}{2}},\sqrt{\frac{\sqrt{5}+1}{2}}$ D) $\sqrt{\frac{\sqrt{3}-1}{2}},\sqrt{\frac{\sqrt{3}+1}{2}}$

Key. A

Sol. Let the sides be a – d, a, a + d

Where a > d > 0

We have

$(a+d)^2=(a-d)^2+a^2$

$\Rightarrow$ $d=\frac{a}{4}$ we have $\sin\theta=\frac{a}{a+d}$ $\Rightarrow$ $\cos\theta=\frac{3}{5},$ $\sin\theta=\frac{4}{5}$

101. If $a_1,a_2,....a_n$ n distinct odd natural numbers not divisible by any prime greater than 5, then $\frac{1}{a_1}+\frac{1}{a_2}+....\frac{1}{a_n}$ is less than

A) $\frac{15}{8}$ B) $\frac{16}{8}$ C) $\frac{8}{15}$ D) $\frac{15}{4}$

Key. A

Sol. Since each a_1 is an odd number not divisible by a prime greater than 5, a_1 can be written as $a_1=3^r5^8$ where r, s are non-negative integers.

thus for all $n\in N$

$$\frac{1}{a_1}+\frac{1}{a_2}+...+\frac{1}{a_n}<\left(1+\frac{1}{3}+\frac{1}{3^2}+...+\right)\left(1+\frac{1}{5}+\frac{1}{5^2}+...+\right)=\frac{15}{8}$$

102. If the m^{th} term of the sequence defined by $t_n = \frac{\sqrt{n}}{n+2008}$ is the greatest term then $m =$

A) 2006 B) 2007 C) 2008 D) 2009

Key. C

Sol. Consider the function $f(x) = \frac{\sqrt{x}}{x+2008}, x \geq 1$

$$f^1(x) = \frac{(x+2008)\times\frac{1}{2\sqrt{x}} - \sqrt{x}}{(x+2008)^2}$$

$$= \frac{x+2008-2x}{2\sqrt{x}\,(x+2008)^2} = \frac{2008-x}{2\sqrt{x}\,(x+2008)^2}$$

$$f^1(x) = 0 \Rightarrow x = 2008$$

$$x \in (2008-\delta, 2008),\ f^1(x) > 0;\ x \in (2008, 2008+\delta),\ f^1(x) < 0$$

103. If a_1, a_2, a_3, ..., a_9 are in H.P. and $a_4 = 5$, $a_5 = 4$ then $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} =$

A) 31/15 B) 41/18 C) 50/21 D) 61/27

Key. C

Sol. Let $a_i = \frac{1}{a+(i-1)d}$, $i = 1,2,3,....,9$

$$a_4 = \frac{1}{a+3d} = 5 \Rightarrow a+3d = \frac{1}{5}$$

$$a_5 = \frac{1}{a+4d} = 4 \Rightarrow a+4d = \frac{1}{4}$$

$$\therefore a = d = \frac{1}{20} \Rightarrow a_i = \frac{20}{i}$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = (20)^3 \begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \end{vmatrix} = \frac{50}{21}$$

104. If $\log_{ax} x$, $\log_{bx} x$ $\log_{cx} x$ are in H.P. where a, b, c, x belong to (1, ∞), then a, b, c are in

(A) A.P. (B) G.P.

(C) H.P. (D) A.G.P.

Key. B

Sol. Since $\log_{ax}x$, $\log_{bx}x$, $\log_{cx}x$ are in H.P.

$\therefore$ $\log_x ax, \log_x bx, \log_x cx$ are in A.P.

$\Rightarrow 1+\log_x a, 1+\log_x b, 1+\log_x c$ are in A.P.

$\Rightarrow \frac{\log a}{\log x}, \frac{\log b}{\log x}, \frac{\log c}{\log x}$ are in A.P.

$\Rightarrow \log a, \log b, \log c$ are in A.P.

$\Rightarrow 2\log b = \log a + \log c = \log ac$

$\Rightarrow \log b^2 = \log ac \Rightarrow b^2 = ac$

$\Rightarrow$ a, b, c are in G.P.

$\therefore (b)$ holds.