CLASS : CC (Advanced) **Determinant & Matrices** **TEST-12**

Note : In this worksheet symbols used P^T, adj. (P), det.(P) or $|P|$ and $T_r(P)$ denotes transpose, adjoint, value of determinant and trace of matrix P respectively.

M.M.: 68 **Time: 60 Min**

PART-A
[SINGLE CORRECT CHOICE TYPE]

Q.1 to Q.8 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct. **[8 × 3 = 24]**

Q.1 Let $\Delta_r = \begin{vmatrix} r & r-2 & a \\ 2r & 2r-2 & b \\ 3r & 3r-2 & c \end{vmatrix}$ where a, b, c are distinct real numbers. If $\sum_{r=1}^{n} \Delta_r = \lambda \begin{vmatrix} 1 & 1 & a \\ 2 & 1 & b \\ 3 & 1 & c \end{vmatrix}$, then λ is equal to

(A) $\frac{n(n+1)}{2}$ (B) $n(n+1)$ (C) $\frac{-n(n+1)}{2}$ (D) $-n(n+1)$

Q.2 Let $f(x) = \begin{vmatrix} e^x-1 & e^{3x}-1 & x^3-1 \\ x & 4x(e^x-1) & x^4-x \\ x^3+2 & e^{2x}-1 & 1 \end{vmatrix}$ then the value of $\frac{d}{dx}\left(f\left(f(x)\right)\right)$ at $x = 0$, is

(A) 0 (B) – 1 (C) – 2 (D) 1

Q.3 Consider, $f(x) = \frac{3x-2}{x+1}$. If [a, b) is the range of $y = f(\{x\})$ and $A = [a_{ij}]_{2\times 2}$ is a matrix, where $a_{ij} \in \{a, b, a^2, b^2\}$, $1 \le i, j \le 2$ (all elements of matrix A are distinct) then least absolute value of det. (A) is
[**Note:** {y} denotes fractional part function of y and det. (P) denotes determinant of matrix P.]

(A) 1 (B) 2 (C) 3 (D) 4

Q.4 If M is a 3 × 3 matrix such that $M^2 = \mathbf{O}$, then det. $\left((I+M)^{50} - 50M\right)$ where I is an identity matrix of order 3, is equal to

(A) 3 (B) 50 (C) 2 (D) 1

Q.5 If $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ where ${a_i}^2 + {b_i}^2 + {c_i}^2 = 1$, where i = 1, 2, 3 and $a_i \cdot a_j + b_i \cdot b_j + c_i \cdot c_j = 0$, $i \ne j$

given that $|A| = 1$ then the value of of $\begin{vmatrix} a_1-1 & b_1 & c_1 \\ a_2 & b_2-1 & c_2 \\ a_3 & b_3 & c_3-1 \end{vmatrix}$ is equal to

(A) 0 (B) – 1 (C) 1 (D) 2

Q.6 Let a square matrix A of order 3 be the zero of the polynomial $f(x) = x^3 - 5x^2 + 7x - 6$. If $l = \text{Tr}(A)$ and $m = \det.(A)$ then $(l + m)$ equals

(A) 5 (B) – 2 (C) 11 (D) 18

Q.7 If diagonal elements (α, β, γ) of a non-singular diagonal matrix of order 3 are the roots of the equation $x^3 - 9x^2 + kx - 27 = 0,\ k \in R$ and $\alpha, \beta, \gamma > 0$ then number of such matrices is

(A) 1 (B) 3 (C) 6 (D) 27

Q.8 If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where a, b, c, d are the roots of the equation $x^4 - 13x^3 + px^2 + qx - 64 = 0$ whose three roots are positive and one is negative then minimum positive value of det. (A) is

(A) 8 (B) 16 (C) 27 (D) 64

[PARAGRAPH TYPE]

Q.9 to Q.13 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct. **[5 × 3 = 15]**

Paragraph for question nos. 9 & 10

If A is a square matrix of order less than 4 such that $|A - A^T| \neq 0$ and $B = \text{adj } A$.

Q.9 If $\det.(A) = 3$ then $\text{Tr}(\text{adj.}(AB))$ is equal to

(A) 6 (B) 9 (C) 27 (D) 81

Q.10 The matrix $\text{adj}\left(B^2A^{-1}B^{-1}A\right)$ is equal to

(A) A (B) B (C) | A | A (D) | B | B

Paragraph for question nos. 11 to 13

Let A and B are inverse matrices of each other of order 3 such that $A(B + A) = 2I$.

Q.11 The value of $\det.\left((2\,\text{adj.}A)^2 + 3(\text{adj. }B)^2\right)$ is equal to

(A) 7 (B) 13 (C) 49 (D) 343

Q.12 If $P = (A^{-1} + B^{-1})\left(\text{adj.}(2A) + \text{adj.}(2B)\right)$ then absolute value of trace of P, is

(A) 4 (B) 8 (C) 16 (D) 48

Q.13 If $M(x) = \begin{bmatrix} \left|\left(\dfrac{x^4+1}{x^2}\right)^{1/3} \text{adj.}A\right| & 3x & -1 \\ 3x & 3 & x^2 \\ -1 & x^2 & \left|\left(\dfrac{3}{x^2}\right)^{1/3} \text{adj.}B\right| \end{bmatrix}$

then minimum integral vlaue of trace of M(x) is

(A) 1 (B) 5 (C) 7 (D) 11

[REASONING TYPE]

Q.14 & Q.15 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct. **[2 × 3 = 6]**

Q.14 Consider two matrices $P = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$ and $Q = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Statement-1: $P^{-1} + Q^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Statement-2: If A and B are two invertible matrices then $(A+B)^{-1} = A^{-1} + B^{-1}$

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

Q.15 **Statement-1 :** The system of equations $x + 3y + 4z = 5$, $2x + y + 3z = 3$ and $4x + 7y + 11z = 13$ has infinitely many solutions

Statement-2 : For the system of equations $a_i x + b_i y + c_i z = d_i$, $i = 1, 2, 3$ if $\Delta = \Delta_x = \Delta_y = \Delta_z = 0$ then the given system of equations has infinitely many solutions

(where $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $\Delta_x, \Delta_y\ \Delta_z$ are obtained by replacing columns 1, 2 and 3 in Δ by column vector $\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$ respectively).

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

[MULTIPLE CORRECT CHOICE TYPE]

Q.16 & Q.17 has four choices (A), (B), (C), (D) out of which **ONE OR MORE** may be correct. **[2 × 4 = 8]**

Q.16 Let $A = [a_{ij}]$ be a matrix of order 3 where $a_{ij} = \begin{cases} (j^i + 2j - ij)x, & \text{if } i < j, x \in R \\ 1, & \text{if } i > j \\ 0, & \text{if } i = j \end{cases}$

If $f(x) = \det.(A)$ then identify the correct statement(s) -

(A) minimum value of $f(x) = \frac{-1}{4}$.

(B) $\int_{-1}^{1} f(x)\,dx = 24$

(C) If equation $|f(x)| = k$ has four distinct solutions then $k \in \left(0, \frac{1}{4}\right)$.

(D) Area enclosed by the curve $y = f(x)$ and x-axis is $\frac{1}{36}$ sq. units.

Q.17 Consider matrices $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $C = \begin{bmatrix} 14 \\ 12 \\ 2 \end{bmatrix}$.

If the solution of system of equation $AX = C$ is point $L(x = x_1, y = y_1, z = z_1)$ and L' is the reflection of L in the plane $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 9$, then

(A) sum of coordinates of L is 6.
(B) sum of coordinates of L is 12.
(C) sum of coordinates of L' is 6.
(D) sum of coordinates of L' is 12.

PART-D

[INTEGER TYPE]

Q.1 to Q.3 are "Integer Type" questions. (The answer to each of the questions **are upto <u>4 digits</u>**) **[3 × 5 = 15]**

Q.1 Let p, q, r be three real numbers satisfying $[p\ q\ r] \begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = [0\ 0\ 0]$.

If the point M(p, q, r) lies on the plane $2x + y + z = 1$, then find the value of $(7p + q + r)$.

Q.2 Consider two matrices $A = \begin{bmatrix} x_1 & x_2 & x_3 \\ 0 & x_2 & x_1 \\ 0 & 0 & x_3 \end{bmatrix}$ and $B = \begin{bmatrix} y_1 & 0 & 0 \\ y_3 & y_2 & 0 \\ y_2 & y_1 & y_3 \end{bmatrix}$,

where each of $x_i, y_j \in \{-1, 0, 1\}\ \forall\, i, j = 1, 2, 3$, then if N is the number of possible ordered pair of matrices A and B for which $\det A = \det B$. Find the value of $\frac{N}{131}$.

Q.3 Let A be a non-singular matrix of order 3 satisfying the equation $(A^3 - 4A^2)(A^2 + 4A + 16I) = \mathbf{0}$.

If $N = |\text{adj.}(\text{adj.} 2A)|$ then find the number of digits in the number N.

CLASS : CC (Advanced) **Determinant & Matrices** **TEST-12**

ANSWER KEY
PART-A

Q.1	D	Q.2	A	Q.3	B	Q.4	D	Q.5	A
Q.6	C	Q.7	A	Q.8	B	Q.9	A	Q.10	A
Q.11	D	Q.12	D	Q.13	C	Q.14	C	Q.15	C
Q.16	ABCD	Q.17	AD						

PART-D

Q.1	6	Q.2	3	Q.3	11