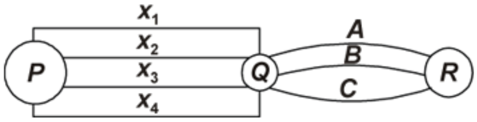
Date: 9/10/2021

**Q1.** In a class there are twenty boys and 16 girls. The teacher wants to select either a boy or a girl to represent the class in a function. In how many ways teacher can made his selection.

**Q2.** In a class there are 20 boys and 16 girls. The teacher wants to select a boy and a girls to represent a class in a function. In how many ways can the teacher made this selection.

**Q3.** The following figure represent route chart containing three station $P$, $Q$, $R$. In how many ways can a person go form station $P$ to station $R$.




**Q4.** There are four books on Physics and five on Chemistry on display in a show room. In how many ways can a person buy a book either of physics or of chemistry?

**Q5.** In an election there are five candidates for President, six for vice-president, seven for secretary and three for the post of Treasurer. There will be how many possible results of this election?

**Q6.** A carpenter has ten patterns of chairs and eight patterns of tables. In how many ways can he make a pair of table and chair?

**Q7.** In a cinema hall there are four entrances and three exit doors. In how many ways can a person enter the hall and then come out?

**Q8.** If there are 6 periods in each working day of a school, in how many ways can one arrange 5 subject such that each subject is allowed at least one period?

**Q9.** Shobhit wants to arrange 4 English, 3 Maths and 2 physics books on a shelf. If the books on the same subject are different. determine the number of all possible arrangements.

**Q10.** From among the 50 teachers in a school one principal and two vice-Principals are to be appointed. In how many ways can this be done?

**Q11.** There are 6 multiple choice questions in an examination. How many sequences of answers are possible if the first three questions have four choices each and remaining three have 3 each?

**Q12.** How many Arithmetic progressions with 10 terms are there whose first term belongs to {2, 3, 4} and common difference to {5, 6, 7, 8}?

**Q13.** In a class of 25 boys and 15 girls the teacher wants to select one boy and one girl. In how many ways this can be done?

**Q14.** In how many ways can 5 women draw water from 5 taps, if no tap can be used more than once?

**Q15.** In Delhi, telephone numbers consist of 6 digits and none of them begins with 0. How many such telephone numbers are possible?

**Q16.** 25 buses are running between Delhi and Noida. In how many ways can a person go from Delhi to Noida and return by a different bus?

**Q17.** In how many ways can 50 voters vote for four candidates contesting the election for the post of Secretary of their association?

**Q18.** How many six digit telephone numbers be made if each number starts with 35 and no digit appears more than once?

**Q19.** There are 3 different rings to be worn in 4 fingers with at most one in each finger. In how many ways can this be done?

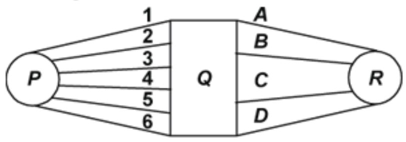
**Q20.** In how many ways can 6 boys and five girls be arranged for a group photograph if the girls are to sit on chairs in a row and the boys are to stand in a row behind them?

**Q21.** In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

**Q22.** Given 4 flags of different colours, how many different signals can be generated, if a signal requires the use of 2 flags one below the other?

**Q23.** In a house there are 3 doors and 4 windows. In how many ways can a thief commit the theft by entering through a window and exiting through a door.

**Q24.** The following figure represents the route chart connecting three stations *P*, *Q* and *R*. In how many ways can a person go form station *P* to Station *R*.




**Q25.** There are *n* locks and *n* matching keys. If all the Locks and keys are to be perfectly matched find the maximum number of trials required to open a lock.

**Q26.** A code word is to consists of single English alphabets followed by two distinct number between 1 to 9. for example N34 is a code word. How many such code words are there? How many them and with an even integer?

**Q27.** A coin is tossed twice and the outcomes are recorded. Find the number of possible outcomes.

**Q28.** A coin is tossed three times. Determine the number of possible outcomes and also list them.

**Q29.** In how many ways can 4 people be seated in a row containing 7 seats.

**Q30.** How many four letter code can be formed using the first 10 letters of the English alphabet, if no letter can be repeated.

**Q31.** Poor Dolly's TV has only 5 channels, all of them quite boring hence it is not surprising that she desires to switch (change) channel after every one minute. Then find the number of ways in which she can change the channels so that she is back to her original channel for the first time after 4 minutes.

**Q32.** How many numbers are there between 100 and 1000 which have exactly one of their digit as 7?

**Q33.** In how many ways can the letters of word ASSASSINATION be arranged so that all the S's are together?

**Q34.** Find the number of different signals that can be made by arranging at least three flags in order on a vertical pole, if 6 different flags are available.

**Q35.** A code word is to consists of two distinct English alphabets followed by two distinct no. between 1 and 9. For example CA23 is a code word. How many such code word are here? And how many them end with an even integer?

**Q36.** How many no. are there between 100 and 1000. Such that at least one of their digit is 7.

Date: 9/10/2021

**S1.** Since teacher has to perform either of the operations.

(i) Selecting a boy among 20 boys and

(ii) Selecting a girl among 16 girls.

The first of these can be performed in 20 ways and the second in 16 ways.

Therefore by fundamental principle of addition either of the two operations can be performed in (20 + 16) = 36 ways.

**S2.** Here the teacher has to perform two operation.

(i) Selecting a boy among 20 boys.

(ii) Selecting a girl among 16 girls.

The first of these can be performed in 20 ways and the second in 16 ways. Therefore by the fundamental principle of multiplication the required no of ways is $20 \times 16 = 320$.

**S3.** Overall no of possible paths for $P$ to $R$ is.

$\{x_1 A, x_1 B, x_1 C\}, \{x_2 A, x_2 B, x_2 C\}, \{x_3 A, x_3 B, x_3 C\}, \{x_4 A, x_4 B, x_4 C\}$

Total no. of ways = $4 \times 3 = 12$.

**S4.** In total there are 4 + 5 = 9 books he has to purchase one book only for which he has 9 options. Therefore he can purchase a book in 9 ways.

**S5.** One President can be elected in 5 ways, one Vice-President can be elected in 6 ways, one Secretary can be elected in 7 ways, one Treasurer can be elected in 3 ways. Therefore, one President, one Vice-President, one Secretary and one treasurer can be elected in $5 \times 6 \times 7 \times 3$ = 630 ways $\Rightarrow$ there are 630 possible results of this election.

**S6.** A chair can be made in 10 ways and a table in 8 ways.

$\Rightarrow$ one chair and one table can be made in $10 \times 8 = 80$ ways.

**S7.** A person can enter the hall in 4 ways and come out in 3 ways.

$\Rightarrow$ the number of ways he can enter and then come out = $4 \times 3 = 12$.

**S8.** Out of 6 periods 5 may be arranged in ${}^6p_5$ ways and remaining one period can be arranged in ${}^5p_1$ ways.

Therefore, the total number of arrangement = ${}^6p_5 \times {}^5p_1 = 6! \times 5 = 3600$

**S9.** In all there are 9 books to be arranged and this can be done in 9! = 362880 ways.

**S10.** The number of options for principal and two Vice principals are 50, 49 and 48 respectively. Therefore, the selection can be made in $50 \times 49 \times 48 = 117600$ ways.

**S11.** The total possible sequences are $4 \times 4 \times 4 \times 3 \times 3 \times 3 = 1728$.

**S12.** There are 3 options for the first term and 4 options for the common difference. Therefore, the required number of AP's is $3 \times 4 = 12$.

**S13.** One boy can be selected in 25 ways and one girl in 15 ways. Therefore, one boy and one girl can be selected in $25 \times 15 = 375$ ways.

**S14.** The number of options for first, second, third fourth and fifth will be 5, 4, 3, 2, 1 respectively.

Therefore, the required possibilities are $5 \times 4 \times 3 \times 2 \times 1 = 120$.

**S15.** In case of telephone numbers, the repetition of digits is allowed and the number does not begin with 0.

Therefore, the total possible numbers will be $9 \times 10 \times 10 \times 10 \times 10 \times 10 = 900000$.

**S16.** He can go in 25 ways and return in 24 ways (because he does not have to come by the same bus).

Therefore, to and fro journey can be performed in $25 \times 24 = 600$ ways.

**S17.** One voter can vote in 4 ways. Therefore 50 voters can vote in $4 \times 4 \times \ldots$ (50 times) $= 4^{50}$ ways.

**S18.** As first two places are served for 3 and 5 therefore, the next 4 places to be filled with remaining 8 digits and this can be done in ${}^8p_4 = 8 \times 7 \times 6 \times 5 = 1680$ ways.

**S19.** For the first ring there are 4 options as one finger cannot have more than one ring the remaining two rings will have 3 and 2 options respectively.

Therefore, the total possibilities are $4 \times 3 \times 2 = 24$

**S20.** 5 girls can sit in 5! ways and behind them 6 boys can stand in 6! ways.

Therefore, the total number of ways $= 5! \times 6! = 120 \times 720 = 86400$.

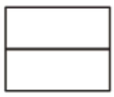
**S21.** Number of ways of choosing 3 boys from 5 boys $= {}^5C_3$

$$= \frac{5!}{3!2!} = \frac{5 \times 4}{2} = 10$$

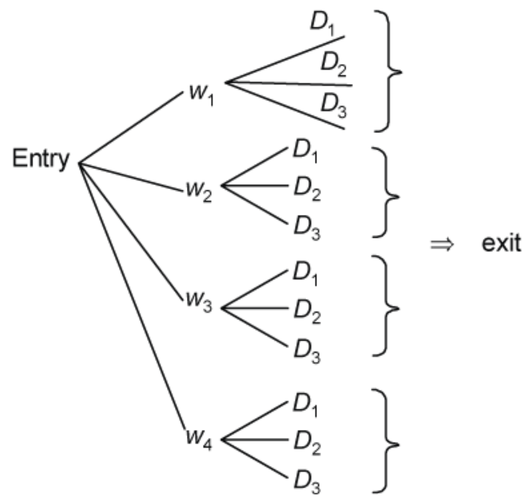
Number of ways of choosing 3 girls from 4 girls $= {}^4C_3$

$$= \frac{4!}{3!1!} = 4$$

By F.P.C., number of ways in which the team can be selected $= 10 \times 4 = 40$.

**S22.** There will be as many signals as there are ways of filling in 2 vacant places  in succession by the 4 flags of different colours. The upper vacent place can be filled in 4 different ways by anyone of the 4 flags, following which, the lower vacant place can be filled in 3 different ways by anyone of the remaining 3 different flags. Hence, by the multiplication principle, the required number of signals $= 4 \times 3 = 12$.

**S23.** Let four windows be $w_1, w_2, w_3, w_4$ and three doors be $D_1, D_2, D_3$ respectively.




Possible ways: $(w_1D_1, w_1D_2, w_1D_3), (w_2D_1, w_2D_2, w_2D_3), (w_3D_1, w_3D_2, w_3D_3), (w_4D_1, w_4D_2, w_4D_3)$

$\therefore$ Total no. of ways = $4 \times 3 = 12$ ways.

**S24.** He can go from $P$ to $Q$ in 6 ways $Q$ to $R$ in 4 ways.

Therefore, from $P$ to $R$ he can go in $6 \times 4 = 24$ ways.

**S25.** The maximum number of trials used for the first key is $n$.

For second key, it will be $n - 1$.

Now, for the $r$th key the maximum number of trials needed is $(n - r + 1)$.

Hence total required no of trials = $n + (n - 1) + 9 .... + 1 = \dfrac{n(n+1)}{2}$.

**S26.** There are in all 26 alphabets, one alphabet can be chosen in 26 ways. Again out of nine digits.

unit place can be filled up in 4 ways, (by 2, 4, 6, 8},

Tenth place can be filled up in 8 ways (since one digit is already used.)

Thus, number of such codes

$$26 \times 4 \times 8 = 832 \text{ codes.}$$

**S27.** Since a coin has two different faces. head ($H$), and Tail. ($T$).

So in two tosses the possible outcomes could be as follows.

(a) Either $H$ in first toss and $T$ in second toss = $HT$

(b) Either $T$ in first toss and $H$ in second toss = $TH$

(c) Either it could be $H$ in first toss and H in second = $HH$

(d) Either $T$ in first or $T$ in second = $TT$.

So all outcomes are $HH$, $HT$, $TH$, $TT$ = 4 outcomes.

**S28.** When a coin is tossed once, there are two possible outcomes namely head and tale. when it is tossed three times the number of outcomes will be

$$2 \times 2 \times 2 = 8$$

*i.e.*, $HHH, HHT, HTH, THH, HTT, THT, TTH, TTT$.

**S29.** First person can be seated in 7 ways. When the first person has taken a seat, the number of seats for the second person is 7 – 1 = 6.

Hence second person can be seated in 6 ways and third person can be seated in 5 ways and fourth person can be 5 ways and third person can be seated in 4 ways.

Hence by the, fundamental principle of counting total number of ways in which four people can be seated in seven seats in a row is equal to

$$= 7 \times 6 \times 5 \times 4 = 840 \text{ ways.}$$

**S30.**

| | | | |
|---|---|---|---|
| IV$^{th}$ place | III$^{rd}$ place | II$^{nd}$ place | I$^{st}$ place |

First place can be filled in 10 ways. Second place can be filled in (10 – 1) = 9 ways.

Third place can be filled in (10 – 2) ways. Fourth place can be filled in (10 – 3) ways.

Hence, total no. of letter code = $10 \times 9 \times 8 \times 7 = 5040$.

**Alternate Method:**

Required no. of ways of selecting 4 letter code out of first 10 letters of English alphabet = ${}^{10}P_4 = 5040$.

**S31.** Let the five channels be $C_1$, $C_2$, $C_3$, $C_4$, $C_5$ at $t = 0$ minute she is watching channel $C_5$.

∴ After first minute she has 4 choices to switch the channel ($C_1$, $C_2$, $C_3$, $C_4$).

After 2$^{nd}$ minute she has 3 choices to switch the channel.

After 3rd minute she has 2 choices to switch the channel.

But after 4th minute she has only one choice to switch. The channels $C_1$

∴ Total no. of ways = $4 \times 3 \times 2 \times 1 = 24$.

**S32.** We have to form 3 digit numbers having exactly one of their digit as 7.

**Case - I:** 3 digit no. with the 7 at the unit place but neither at ten's place nor at the hundredth place.

Hence the number of ways to fill the unit place = 1.

The number of ways to fill ten's place = 9.

and the number of ways to fill the hundredth place = 8.

∴ Number of such numbers = $1 \times 9 \times 8 = 72$

**Case - II:** 3 Digit number with 7 at the ten's place but neither at hundredth place nor at unit place.

The number of ways to fill the ten's place = 1.

The number of ways to fill the units place = 9.

The number of ways to fill the hundredth place = 8.

∴ Number of such numbers = $1 \times 9 \times 8 = 72$

**Case - III:** 3 digit number with 7 at the hundredth place but neither at the unit place nor at the ten's place.

The number of ways to fill the hundredth place = 1.

The number of ways to fill the tenth place = 9.

The number of ways to fill the unit place = 9.

$\therefore$ Number of such numbers = $1 \times 9 \times 9 = 81$

Hence, total no. of required type of numbers = 72 + 72 + 81 = 225

**S33.** There are 13 letters, of which A appears 3 times, S appears 4 times, I appears 2 times, N appears 2 times, and the rest all are different.

For the given condition, all S's have to the together.

We treat them as a single object $\boxed{S\ S\ S\ S}$ for the time being. This single object together with 9 remainig objects will account for 10 objects.

These 10 objects in which there are 3 A's, 2 I's and 2 N's can be rearranged in $\frac{10!}{3!2!2!}$ ways. Corresponding to each of these arrangements, the 4 S's can be arranged in $\frac{4!}{4!}$ ways.

Therefore, by F.P.C., the required number of arrangements

$$\frac{10!}{3!2!2!} \times \frac{4!}{4!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4}{2 \times 2} = 151200.$$

**S34.** There are six different flags

**Case - I:** Number of signals with three flags. First place can be filled in 6 ways, second place can be filled in 5 ways and third place can be filled in 4 ways.

Hence total no. of signals with the help of three flags = $6 \times 5 \times 4 = 120$ ways.

**Case - II:** No. of signals with four flags. First place can be filled in 6 ways second place can be filled in 5 ways and so on.

Hence, total no. of signals using four flags = $6 \times 5 \times 4 \times 3 = 360$ ways.

**Case - III:** No. of signals using five flags.

Similarly, no. of signals using five flags = $6 \times 5 \times 4 \times 3 \times 2 = 720$ ways.

**Case - IV:** No. of signals using six flags.

Similarly, total no. of signals using six flags = $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ ways.

Hence total no. of signals = (120 + 360 + 720 + 720) = 1920 signals.

**Alternative Method:**

No. of signals with 3 flags out of 6 flags can be chosen in $^6P_3$ ways.

$\because$ at least three flags in order on a vertical pole.

Hence required no. of ways.

$$= {}^6P_3 + {}^6P_4 + {}^6P_5 + {}^6P_6$$

$$= \frac{6\,!}{(6-3)\,!} + \frac{6\,!}{(6-4)\,!} + \frac{6\,!}{(6-5)\,!} + 6\,!$$

$$= \frac{6 \times 5 \times 4 \times 3\,!}{3\,!} + \frac{6 \times 5 \times 4 \times 3 \times 2\,!}{2\,!} + \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1!}{1!} + 6\,!$$

$$= 120 + 360 + 720 + 720$$

$= 1920$ signals.

**S35.** There are in all 26 English alphabets.

Hence first alphabet can be selected in 26 ways.

Second alphabet can be selected in 25 ways.

Again out of 9 digits (1 to 9) first digit can be selected in 9 ways and second digit can be selected in 8 ways.

Thus total no. of distinct codes = $26 \times 25 \times 9 \times 8 = 46800$

For the word to end with an even integer.

As above, two distinct alphabet can be selected in $26 \times 25$ ways.

We have in all 1, 2, 3, 4, 5, 6, 7, 8, 9 digits unit place can be filled up in 4 ways (by 2, 4, 6, 8).

Tenth place can be filled up in 8 ways (since one of the digit is already used.)

Thus the number of codes = $26 \times 25 \times 4 \times 8 = 20800$

**S36.** Clearly, a number between 100 and 1000 has three digits.

$\therefore$ Total no. of 3 digits no. having at least one of their digit as 7.

= (total no. of three digit no.) – (total no. of three digit no in which 7 does not appear at all).

Total no. of three digit no.

We have to form three digit no. by using the digit 0, 2, 3, ... 9.

| | | |
|---|---|---|
| OX | | · |

9 ways

Clearly hundredth place can be filled in 9 ways and each of ten's and one's place can be filled in 10 ways.

So, total no. of three digit no. = $9 \times 10 \times 10 = 900$

Total no. of three digit no. in which 7 does not appear at all.

Here, we have to form three digit no. by using digits 0 to 9, except 7.

So, hundredth place can be filled in 8 ways and each of ten's and one's place can be filled in 9 ways.

$\therefore$ Total number of three digit no. in which 7 does not appear at all = $8 \times 9 \times 9$.

Hence, total no. of three digit no. having at least one of their digit as 7

$$9 \times 10 \times 10 - 8 \times 9 \times 9 = 252$$

Date: 9/10/2021

**Q1.** In how many ways can 7 person stand in a queue?

**Q2.** 8 Athletics are participating in a race. In how many ways can the first three athletics win the prizes.

**Q3.** Eleven animals of a circus have to be placed in 11 cages (one in each cage). If 4 of the cages are too small for 6 of the animals, then find the number of ways of caging all the animals.

**Q4.** If $A = \{x/x$ is a prime no. and $x < 30\}$. Find the number of different rational numbers whose numerator and denominator belong to $A$.

**Q5.** If ${}^9P_r = 72$, find the value of $r$.

**Q6.** From a committee of 8 persons, in how many ways can we choose a chairman and a vice chairman assuming one person cannot hold more than one position?

**Q7.** How many words, with or without meaning can be formed using all the letters of the word EQUATION using each letter exactly once?

**Q8.** If ${}^{10}p_r = 5040$, find the value of $r$.

**Q9.** Prove that ${}^np_n = 2.{}^np_{n-2}$.

**Q10.** If the best and worst papers never appear together. Find the how many ways six examination paper can be arranged.

**Q11.** Find $n$, if ${}^{2n}p_3 = 100.{}^np_2$.

**Q12.** Find $n$, if ${}^np_4 = 20.{}^np_2$.

**Q13.** There are two identical white balls, 3 identical red balls, and 4 green balls of different shades. Find the number of ways in which they can be arranged in a row so that at least one ball is separated from the balls of the same colour.

**Q14.** Find the number of ways in which 5 girls and 5 boys can be arranged in a row if boys and girls are alternate.

**Q15.** Find the number of ways in which 5 girls and 5 boys can be arranged in a row if no two boys are together.

**Q16.** There are six periods in each working day of a school. Find the number of ways in which 5 subjects can be arranged if each subject is allotted at least one period and no period remains vacant.

**Q17.** How many different signals can be given using any number of flags from five flags of different colours?

**Q18.** If ${}^9p_5 + 5 \cdot {}^9p_4 = {}^{10}p_r$ then find the value of $r$.

**Q19.** Find $n$ if, (a) ${}^np_4 : {}^{n-1}p_3 = 9 : 1$; (b) ${}^np_4 : {}^np_5 = 1 : 2$.

**Q20.** Find $n$ if, (a) $30 . {}^{n}p_6 = {}^{n+2}p_7$; (b) ${}^{n}p_5 : {}^{n-1}p_4 = 6 : 1$.

**Q21.** A shelf contains 20 books of which 4 are single volume and the other form sets of 8, 5 and 3 volumes respectively. Find the number of ways in which the books may be arranged on the shelf so that

(a) volume of each set will not be separated.

(b) volume of each set remains in their due order.

**Q22.** Find the number of ways in which 6 boys and 6 girls can be seated in a row so that

(a) all the girls sit together (b) all the girls are never together.

**Q23.** Prove that if $r \le s \le n$, then ${}^{n}p_s$ is divisible by ${}^{n}p_r$.

**Q24.** If ${}^{2n+1}p_{n-1} : {}^{2n-1}p_n = 3 : 5$, then find the value of $n$.

**Q25.** How many 4-digit numbers are there with no digit repeated?

**Q26.** It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangement are possible?

**Q27.** Find r if : ${}^{5}P_r = {}^{6}P_{r-1}$.

**Q28.** How many 3-digit even numbers can be made using the digits 1, 2, 3, 4, 5, 6, 7 if no digit is repeated?

**Q29.** Find $r$, if $5 \cdot {}^{4}P_r = 6 \cdot {}^{5}P_{r-1}$.

**Q30.** In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together?

**Q31.** In how many ways can 4 red, 3 yellow and 2 green discs be arranged in a row if the discs of the same colour are indistinguishable?

**Q32.** Find the value of n such that

(i) ${}^{n}P_5 = 42 \cdot {}^{n}P_3$, $n > 4$ (ii) $\dfrac{{}^{n}P_4}{{}^{n-1}P_4} = \dfrac{5}{3}$, $n > 4$

**Q33.** Prove that,

(a) ${}^{n}p_r = \dfrac{n!}{(n-r)!}$ (b) ${}^{n}p_n = {}^{n}p_{n-1}$

**Q34.** Find $r$, if (a) ${}^{20}p_r = 13\ {}^{20}p_{r-1}$; (b) ${}^{5}p_r = 2\ {}^{6}p_{r-1}$.

**Q35.** Find $r$, if (a) ${}^{10}p_r = 2 . {}^{9}p_r$, (b) $4 . {}^{6}p_r = {}^{6}p_{r+1}$.

**Q36.** Find $n$ if, (a) ${}^{n-1}p_3 : {}^{n+1}p_3 = 5 : 12$; (b) ${}^{2n-1}p_n : {}^{2n+1}p_{n-1} = 22 : 7$.

**Q37.** Find $n$ if, (a) $16 \cdot {}^{n}p_3 = 13 \cdot {}^{n+1}p_3$; (b) ${}^{n}p_5 = 20 . {}^{n}p_3$.

**Q38.** Prove that,

(a) ${}^{n}p_r = n \cdot {}^{n-1}p_{r-1}$ (b) ${}^{n}p_r = {}^{n-1}p_r + r \cdot {}^{n-1}p_{r-1}$

Date: 9/10/2021

**S1.** The number of ways in which 7 person can stand in a queue is same as filling 7 places with 7 persons.

Hence, the number of permutations of 7 objects taken all at a time is

$$^{7}P_{7} = 7! = 5040.$$

**S2.** Basically, it is equivalent to filling 3 places (prizes) with 8 persons.

The number of permutations of 8 objects taken three at a time is $^{8}P_{3} = 8 \times 7 \times 6 = 336$.

**S3.** Let the 6 animals be placed in 7 of larger cages. This can be done in $^{7}P_{6}$ ways. In each of these ways, one larger cage is left vacant. The remaining 5 animals can be placed in the remaining five cages in 5! ways.

Hence, by the fundamental theorem of counting the required number of ways is $^{7}P_{6} \times 5! = 604800$.

**S4.** Let $A = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$.

A rational number is made by taking any two numbers in any order.

Hence, the required number of rational numbers is $(^{10}P_{2} + 1)$ (including 1).

**S5.** Since, $^{9}p_{r} = 72$

$\Rightarrow$ $^{9}p_{r} = 9 \times 8$

$\Rightarrow$ $^{9}p_{r} = {}^{9}P_{2}$

$\Rightarrow$ $r = 2.$

**S6.** No. of persons = 8.

Number of ways of choosing chairman and vice-chairman is equal to the number of ways of arranging 8 persons taking 2 at a time (first place for chairman and second for vice-chairman).

$$^{8}P_{2} = 8 \times 7 = 56.$$

**S7.** There are 8 different letters in the word EQUATION. Number of permutations of 8 objects taken al at a time is $^{8}P_{8}$ which is given by

$$^{8}P_{8} = 8!$$

$$= 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 40320.$$

**S8.** Since, $^{10}p_{r} = 5040$

$\Rightarrow$ $^{10}p_{r} = 10 \times 504$

$\Rightarrow$ $^{10}p_{r} = 10 \times 9 \times 8 \times 7$

$\Rightarrow$ $^{10}p_{r} = {}^{10}p_{4}$

$\Rightarrow$ $r = 4$

**S9.** R.H.S.

$$2 \cdot {}^n p_{n-2} = 2 \frac{n!}{[n-(n-2)]!} = 2 \cdot \frac{n!}{2!} = n!$$

L.H.S. $${}^n p_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$$

Hence, L.H.S. = R.H.S.

**S10.** If the best and worst paper appear always together, the number of ways is $5! \times 2$.

Therefore required number of ways is as follows.

$\Rightarrow$ total no of ways without any restrictions – number of ways when best and worst paper are together = $6! - 5! \times 2 = 480$.

**S11.** Since, $${}^{2n}p_3 = 100.{}^n p_2$$

$$\Rightarrow \quad \frac{(2n)!}{(2n-3)!} = 100 \cdot \frac{n!}{(n-2)!}$$

$$\Rightarrow \quad (2n)(2n-1)(2n-2) = 100 \cdot n(n-1)$$

$$\Rightarrow \quad 4(2n-1) = 100$$

$$\Rightarrow \quad 2n-1 = 25$$

$$\Rightarrow \quad n = 13.$$

**S12.** Since, $${}^n p_4 = 20.{}^n p_2$$

$$\frac{n!}{(n-4)!} = 20 \cdot \frac{n!}{(n-2)!}$$

$$\Rightarrow \quad \frac{1}{(n-4)!} = 20 \cdot \frac{1}{(n-2)(n-3)(n-4)!}$$

$$\Rightarrow \quad (n-2)(n-3) = 20$$

$$\Rightarrow \quad n^2 - 5n - 14 = 0$$

$$\Rightarrow \quad (n-7)(n+2) = 0$$

$$\Rightarrow \quad n = 7, -2 \qquad \text{As } n \geq 4$$

$$n = 7.$$

**S13.** Total no of arrangement without any restrictions = $\frac{9!}{3! \cdot 2!}$.

Now number of ways when balls of the same colour are together = $3! \times 4!$.

Now required number of ways = total no. of arrangements – number of ways when the balls of the same colour are together.

$$= \frac{9!}{2! \cdot 3!} - 3! \cdot 4! = 6(7! - 4!).$$

**S14.** First five girls can be arranged in 5 ! ways.

X G X G X G X G X G

or

G X G X G X G X G X

Now, if girls and boys are alternate then boys can occupy places with "X" mark in the diagram.

Hence total no of arrangement is $5! \times 5! + 5! \times 5! = 2 \times 5! \times 5!$

**S15.** Since, in this question, there is no condition for arranging the girls. Now 5 girls can be arrnaged in 5! ways.

X G X G X G X G X G X

When girls are arrnaged, six gaps are generated as shown above with X.

Therefore, 5 boys can be arranged in these six gap in ${}^6p_5$ ways.

Hence, total no. of arrangement is $5! \times {}^6p_5$.

**S16.** Let the five subjects are *a*, *b*, *c*, *d*, *e*.

Since number of subjects are less than the number of periods any one of the five subject occur twice. If subject a occur twice (*a*, *a*, *b*, *c*, *d*, *e*).

Then six subjects can be arranged in $\frac{6!}{2!}$ ways.

Similarly, number of ways when subject *b*, *c*, *d*, *e* occur twice.

Hence, total no. of ways are

$$5 \times \frac{6!}{2!} = 1800$$

**S17.** The signals can be made by using one or more flags at a time.

The total number of signals when *r* flags are used at a time from five flags is equal to number of arrangements of 5, taking r at a time $\Rightarrow {}^5p_r$.

Since *r* can take values 1, 2, 3, 4, 5.

Hence, by the fundamental principal of addition, the total no. of signals is

$$= {}^5p_1 + {}^5p_2 + {}^5p_3 + {}^5p_4 + {}^5p_5$$

$$= 5 + (5 \times 4) + (5 \times 4 \times 3) + (5 \times 4 \times 3 \times 2) + (5\,!)$$

$$= 5 + 20 + 60 + 120 + 120 = 325$$

**S18.** Since,

$${}^{10}p_r = {}^9p_5 + 5 \cdot {}^9p_4$$

$$= \frac{9!}{(9-5)!} + 5 \times \frac{9!}{(9-4)!}$$

$$= \frac{9!}{4!} + 5 \times \frac{9!}{5!}$$

$$= \frac{9!}{4!} + \frac{9!}{4!}$$

$$= 2 \times \frac{9!}{4!} = \frac{5 \times 2 \times 9!}{5 \times 4!}$$

$$= \frac{10 \times 9!}{5!} = \frac{10!}{5!} = {}^{10}p_5$$

Hence, $r = 5$.

**S19.** (a) ${}^{n}p_4 : {}^{n-1}p_3 = 9 : 1$

$$\Rightarrow \quad \frac{n!}{(n-4)!} \times \frac{(n-4)!}{(n-1)!} = 9$$

$$\Rightarrow \quad n = 9$$

(b) ${}^{n}p_4 : {}^{n}p_5 = 1 : 2$

$$\Rightarrow \quad \frac{n!}{(n-4)!} \times \frac{(n-5)!}{n!} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{1}{n-4} = \frac{1}{2}$$

$$\Rightarrow \quad n = 6.$$

**S20.** (a) $30 . {}^{n}p_6 = {}^{n+2}p_7$

$$\Rightarrow \quad 30 \cdot \frac{n!}{(n-6)!} = \frac{(n+2)!}{(n-5)!}$$

$$\Rightarrow \quad 30 \cdot \frac{n!}{(n-6)!} = \frac{(n+2)(n+1)!(n!)}{(n-5)(n-6)!}$$

$$\Rightarrow \quad 30 = \frac{(n+2)(n+1)}{(n-5)}$$

$$\Rightarrow \quad 30(n-5) = n^2 + 3n + 2$$

$$\Rightarrow \quad n^2 - 27n + 152 = 0$$

$$\Rightarrow \quad (n-19)(n-8) = 0 \quad \Rightarrow \quad n = 8, 19$$

(b) ${}^{n}p_5 : {}^{n-1}p_4 = 6 : 1$

$$\Rightarrow \quad \frac{n!}{(n-5)!} \times \frac{(n-5)!}{(n-1)!} = 6$$

$$\Rightarrow \quad n = 6.$$

**S21.** (a) Considering each set as a single unit, permutation of 7 units is 7!.

Permutation of the books of the set of 8 volumes among themselves is 8!.

Respective permutations of books of the set 5 volumes is 5 ! and that of books of 3 volumes is 3 !

Hence, by the product rule, total number of permutation is $7! \cdot 8! \cdot 5! \cdot 3!$.

(b) Since the books in a set of books containing any number of volumes can be arranged in due order in 2 ways.

Hence, total no. of permutations is $7! \times 2 \times 2 \times 2 = 8 \times 7! = 8!$.

**S22.** (a) $b_1, b_2, b_3, b_4, b_5, b_6$

$g_1, g_2, g_3, g_4, g_5, g_6$

Considering boys and girls as two units the number of permutation is

$$^7P_7 \times 6! = 7!.\ 6!$$

(b) The total arrangement where all girls are not together is equal to total arrangement without restriction – arrangement when all girls are together.

$$= 12! - 7! \cdot 6!.$$

**S23.** Let $s = r + k$ where $0 \le k \le (s - r)$

$$^np_s = \frac{n!}{(n-s)!}$$

$$\because \quad \frac{n!}{(n-s)!} = n(n-1)(n-2) \ldots (n-(s-1))$$

$$= n(n-1)(n-2) \ldots (n-(r-1))(n-r)(n-(r+1)) \ldots (n-(r+k-1)).$$

$$= [(n(n-1)(n-2) \ldots (n-r-1)] \cdot \{((n-r)(n-(r+1) \ldots (n-(n-(r+k-1)\}$$

$$= {}^np_r \cdot \text{integer}$$

Hence, $^np_s$ is divisible by $^np_r$.

**S24.**

$$^{2n+1}p_{n-1} : {}^{2n-1}p_n = 3 : 5$$

$$\Rightarrow \quad \frac{^{2n+1}p_{n-1}}{^{2n-1}p_n} = \frac{3}{5} \quad \Rightarrow \quad \frac{(2n+1)!}{(n+2)!} \times \frac{(n-1)!}{(2n-1)!} = \frac{3}{5}$$

$$\Rightarrow \quad \frac{(2n+1)(2n)}{(n+2)(n+1)(n)} = \frac{3}{5}$$

$$\Rightarrow \quad 10(2n+1) = 3(n+2)(n+1)$$

$$\Rightarrow \quad 3n^2 - 11n - 4 = 0$$

$$\Rightarrow \quad (n-4)(3n+1) = 0 \quad \Rightarrow \quad n = 4.$$

**S25.** To determine 4-digit numbers using the 10 digits *i.e.*, 0, 1, 2, 3, ..., 9. We first have to count the arrangement of 10 digits taken 4 at a time. This number would be ${}^{10}P_4$. But, this permutation will include arrangements where 0 is at the 1000's place. For example, 0125, 0532, ... etc., are such numbers which are actualy 3-digit numbers and hence the number of such numbers has to be subtracted from ${}^{10}P_4$ to get the required number. To get the number of such numbers, we fix 0 at the 1000's place and rearrange the remaining 9 digits taking 3 at a time. This numbers is ${}^{9}P_3$.

So, the requireed number

$$ {}^{10}P_4 - {}^{9}P_3 = \frac{10!}{6!} - \frac{9!}{6!} $$

$$ = (10 \times 9 \times 8 \times 7) - (9 \times 8 \times 7) $$

$$ = 5040 - 504 $$

$$ = 4536. $$

**S26.** Let 5 men be $M_1$, $M_2$, $M_3$, $M_4$ and $M_5$.

Since 4 women should occupy even places, this will be so, if the women are make to sit at places marked with 'X' as given below.

$$ M_1 \times M_2 \times M_3 \times M_4 \times M_5 $$

There are 4 places, to which 4 women can be seated upon. They can sit in ${}^4P_4 = 4! = 24$ ways.

Also, 5 men can sit among themselves in

$$ {}^5P_5 = 120 $$

The required number of ways in which men and women can be seated under given condition is $24 \times 120 = 2880$.

**S27.** We have $\quad {}^5P_r = {}^6P_{r-1}$

or $\quad \dfrac{5!}{(5-r)!} = \dfrac{6!}{[6-(r-1)!]}$

or $\quad \dfrac{5!}{(5-r)!} = \dfrac{6 \times 5!}{[6-r+1]!}$

or $\quad \dfrac{5!}{(5-r)!} = \dfrac{6 \times 5!}{(7-r)!}$

or $\quad \dfrac{5!}{(5-r)!} = \dfrac{6 \times 5!}{(7-r)(6-r)(5-r)!}$

or $\quad (7-r)(6-r) = 6$

or $\quad r^2 - 13r + 36 = 0$

or $\quad r = 4, 9$

or $\quad r = 4$

[$r = 9 \Rightarrow {}^5P_9$ which is meaningless]. Hence, $r = 4$.

**S28.** A number is even if the digit at its unit's place is one out of 2, 4, 6 from the digits given. Therefore, the unit place can be filled in $^3P_1$ ways (any one of 2, 4, 6). Now we have to choose 2 digits out of the remaining 5 digits for filling hundred's and thousand's place. This can be done in $^5P_2$ ways (any 2 out of 5) after filling the unit's place.

Required number of 3-digit even numbers

$$^3P_1 \times {}^5P_2 = 3 \times 5 \times 4 = 60.$$

**S29.** We have $5 \cdot {}^4P_r = 6 \cdot {}^5P_{r-1}$

or $5 \times \frac{4!}{(4-r)!} = \frac{6 \times 5!}{(5-r+1)(5-r)(5-r-1)!}$

or $\frac{5!}{(4-r)!} = 6 \times \frac{5!}{(6-r)(5-r)(4-r)!}$

or $(6-r)(5-r) = 6$

or $r^2 - 11r + 24 = 0$

or $r^2 - 8r - 3r + 24 = 0$

or $(r-8)(r-3) = 0$

or $r = 8$ or $r = 3$.

Hence, $r = 8, 3$.

**S30.** Let us first seat the 5 girls. This can be done in 5! ways. For each such arrangement, the three boys can be seated only at the cross marked places.

$$\times G \times G \times G \times G \times G \times.$$

There are 6 cross marked places and the three boys can be seated in $^6P_3$ ways. Hence, by multiplication principle, the total number of ways

$$= 5! \times {}^6P_3 = 5! \times \frac{6!}{3!}$$

$$= 4 \times 5 \times 2 \times 3 \times 4 \times 5 \times 6 = 14400.$$

**S31.** Total number of discs are 4 + 3 + 2 = 9. Out of 9 discs, 4 are of the first kind (red), 3 are of the second kind (yellow) and 2 are of the third kind (green).

Therefore, the number of arrangements $\frac{9!}{4!\,3!\,2!} = 1260$.

**S32.** (i) Given that $^nP_5 = 42\ {}^nP_3$

or $n(n-1)(n-2)(n-3)(n-4) = 42\, n(n-1)(n-2)$

Since $n > 4$ So, $n(n-1)(n-2) \neq 0$

Therefore, by dividing both sides by $n(n-1)(n-2)$, we get

$(n-3)(n-4) = 42$

or $n^2 - 7n - 30 = 0$

or $n^2 - 10n + 3n - 30 = 0$

or $(n-10)(n+3) = 0$

or $n - 10 = 0$ or $n + 3 = 0$

or $n = 10$ or $n = -3$

As $n$ cannot be negative, so $n = 10$.

(ii) Given that $\frac{{}^{n}P_4}{{}^{n-1}P_4} = \frac{5}{3}$

Therefore, $3n(n-1)(n-2)(n-3) = 5(n-1)(n-2)(n-3)(n-4)$

or $3n = 5(n-4)$ [as $(n-1)(n-2)(n-3) \neq 0, n > 4$]

or $n = 10$.

**S33.** (a) ${}^{n}p_r$ represents the number of ways of arranging $r$ objects from given $n$ objects in a row which is same as filling up $r$ places from given $n$ objects. For, first place there are $n$ options for second place there are $n-1$ options and similarly for $r^{th}$ place there are $n-r+1$ options.

By applying Fundamental Principle of Counting, we get

$$ {}^{n}p_r = n \cdot (n-1) \cdot (n-2) \ldots (n-r+1)$$

$$= \frac{[n(n-1)(n-2)\ldots(n-r+1)][(n-r)(n-r-1)\ldots 1]}{[(n-r)(n-r-1)\ldots 1]}$$

$$= \frac{n!}{(n-r)!}$$

(b) $${}^{n}p_n = \frac{n!}{(n-n)!} = \frac{n!}{0!}$$

and $${}^{n}p_{n-1} = \frac{n!}{[n-(n-1)]!} = \frac{n!}{1!} = n!$$

$\Rightarrow$ $${}^{n}p_n = {}^{n}p_{n-1}.$$

**S34.** (a) ${}^{20}p_r = 13\ {}^{20}p_{r-1}$

$\Rightarrow$ $$\frac{20!}{(20-r)!} = 13 \cdot \frac{20!}{(21-r)!}$$

$\Rightarrow$ $21 - r = 13$

$\Rightarrow$ $r = 8$

(b) ${}^{5}p_r = 2\ {}^{6}p_{r-1}$

$\Rightarrow$ $$\frac{5!}{(5-r)!} = 2 \cdot \frac{6!}{(7-r)!}$$

$\Rightarrow$ $$\frac{5!}{(5-r)!} = \frac{2 \cdot 6 \cdot 5!}{(7-r)(6-r)(5-r)!}$$

$\Rightarrow$ $(7-r)(6-r) = 12$

$\Rightarrow$ $r^2 - 13r + 42 = 12$

$\Rightarrow \quad r^2 - 13r + 30 = 0$

$\Rightarrow \quad (r - 3)(r - 10) = 0$

$\Rightarrow \quad r = 3, 10 \quad \text{As} \quad r \le 5.$

Hence, $r = 3$.

**S35.** (a) ${}^{10}p_r = 2\ {}^{9}p_r$

$$\Rightarrow \quad \frac{10!}{(10-r)!} = 2\cdot\frac{9!}{(9-r)!}$$

$$\Rightarrow \quad \frac{10\cdot 9!}{(10-r)\cdot(9-r)!} = \frac{2\cdot 9!}{(9-r)!}$$

$$\Rightarrow \quad \frac{10}{10-r} = 2$$

$$\Rightarrow \quad 20 - 2r = 10 \quad \Rightarrow \quad r = 5$$

(b) $4\cdot{}^{6}p_r = {}^{6}p_{r+1}$

$$\Rightarrow \quad 4\cdot\frac{6!}{(6-r)!} = \frac{6!}{(5-r)!}$$

$$\Rightarrow \quad \frac{4}{(6-r)\cdot(5-r)!} = \frac{1}{(5-r)!}$$

$$\Rightarrow \quad \frac{4}{6-r} = 1$$

$$\Rightarrow \quad r = 2.$$

**S36.** (a) ${}^{n-1}p_3 : {}^{n+1}p_3 = 5 : 12$

$$\Rightarrow \quad \frac{(n-1)!}{(n-4)!}\times\frac{(n-2)!}{(n+1)!} = \frac{5}{12}$$

$$\Rightarrow \quad \frac{(n-1)!\cdot(n-2)\cdot(n-3)\cdot(n-4)!}{(n-4)!\cdot(n+1)\cdot n(n-1)!} = \frac{5}{12}$$

$$\Rightarrow \quad \frac{(n-2)(n-3)}{n(n+1)} = \frac{5}{12}$$

$$\Rightarrow \quad 12\,[n^2 - 5n + 6] = 5[n^2 + n]$$

$$\Rightarrow \quad 7n^2 - 65n + 72 = 0$$

$$\Rightarrow \quad 7n^2 - 56n - 9n + 72 = 0$$

$$\Rightarrow \quad (7n - 9)(n - 8) = 0$$

$$\Rightarrow \quad n = 8, \ \frac{9}{7} \quad \text{As} \quad n \in N$$

$$\Rightarrow \quad n = 8.$$

(b) $^{2n-1}p_n : {}^{2n+1}p_{n-1} = 22 : 7$

$$\Rightarrow \quad \frac{(2n-1)!}{(n-1)!} \times \frac{(n+2)!}{(2n+1)!} = \frac{22}{7}$$

$$\Rightarrow \quad \frac{(2n-1)!\cdot(n+2)\cdot(n+1)\cdot n(n-1)!}{(n-1)!\cdot(2n+1)(2n)(2n-1)!} = \frac{22}{7}$$

$$\Rightarrow \quad \frac{(n+2)(n+1)}{2(2n+1)} = \frac{22}{7}$$

$$\Rightarrow \quad 7(n+2)(n+1) = 44(2n+1)$$

$$\Rightarrow \quad 7(n^2+3n+2) = 44(2n+1)$$

$$\Rightarrow \quad 7n^2 - 67n - 30 = 0$$

$$\Rightarrow \quad 7n^2 - 70n + 3n - 30 = 0$$

$$\Rightarrow \quad 7n(n-10) + 3(n-10) = 0$$

$$\Rightarrow \quad n = 10,\ -\frac{3}{7}. \qquad \text{As } n \in N$$

$$\Rightarrow \quad n = 10$$

**S37.** (a) $16 \cdot {}^{n}p_3 = 13 \cdot {}^{n+1}p_3$

$$\Rightarrow \quad 16 \cdot \frac{n!}{(n-3)!} = 13 \cdot \frac{(n+1)!}{(n-2)!}$$

$$\Rightarrow \quad 16\, n(n-1)(n-2) = 13(n+1)(n)(n-1)$$

$$\Rightarrow \quad 16(n-2) = 13(n+1)$$

$$\Rightarrow \quad 3n = 45$$

$$\Rightarrow \quad n = 13$$

(b) ${}^{n}p_5 = 20\, {}^{n}p_3$

$$\Rightarrow \quad \frac{n!}{(n-5)!} = 20 \cdot \frac{n!}{(n-3)!}$$

$$\Rightarrow \quad \frac{1}{(n-5)!} = 20 \cdot \frac{1}{(n-3)(n-4)(n-5)!}$$

$$\Rightarrow \quad (n-3)(n-4) = 20$$

$$\Rightarrow \quad n^3 - 7n - 8 = 0$$

$$\Rightarrow \quad (n-8)(n+1) = 0$$

$$\Rightarrow \quad n = -1, 8 \qquad \text{As, } \quad n \geq 5$$

$$\Rightarrow \quad n = 8.$$

**S38.** (a) R.H.S.

$$n \cdot {}^{n-1}p_{r-1} = n \cdot \frac{(n-1)!}{[(n-1)-(r-1)]!}$$

$$= \frac{n!}{(n-r)!} = {}^{n}p_{r}$$

(b) R.H.S.

$${}^{n-1}p_{r} + r \cdot {}^{n-1}p_{r-1} = \frac{(n-1)!}{(n-1-r)!} + r \times \frac{(n-1)!}{[(n-1)-(r-1)]!}$$

$$= (n-1)!\left[\frac{1}{(n-r-1)!} + \frac{r}{(n-r)!}\right]$$

$$= (n-1)!\left[\frac{n-r+r}{(n-r)!}\right]$$

$$= \frac{n(n-1)!}{(n-r)!} = \frac{n!}{(n-r)!} = {}^{n}p_{r}.$$

Date: 10/10/2021

**Q1.** How many three letter words can be made using the letter of the word "ORIENTAL"? (Repetition not allowed.)

**Q2.** How many four letter words, with or without meaning can be formed out of letters of the word "LOGARTHIMS" if repetition of letter is not allowed.

**Q3.** How many words with or without meaning, can be formed by using the letter of the word "TRIANGLE". (Repetition not allowed.)

**Q4.** In how many ways letters of the word SANDEEP be arranged such that each word starts with *E* and ends with *E*. (Repetition not allowed.)

**Q5.** In how many ways letter of the word "MANMOHAN" can be arranged such that each word ends with *N*. (Repetition not allowed.)

**Q6.** In how many ways can the word "PENCIL" be arranged so that *N* is always next to *E* ?

**Q7.** In how many arrangements of the word "GOLDEN" will the vowels never occur together ?

**Q8.** If the different permutations of the word "EXAMINATION" are arranged as in a dictionary, how many words are there before the first word starting with *E*?

**Q9.** Find the number of 4 letter words, with or without meaning, which can be formed out of the letters of the word ROSE, where the repetition of the letters is not alllowed.

**Q10.** How many 4-letter code can be formed using the first 10 letters of the English alphabet, if no letter can be repeated?

**Q11.** Find the number of permutations of the letters of the word ALLAHABAD.

**Q12.** How many 5-digit telephone numbers can be constructed using the digits 0 to 9 if each number starts with 67 and no digit appears more than once?

**Q13.** A coin is tossed 3 times and the outcomes are recorded. How many possible outcomes are there?

**Q14.** Given 5 flags of different colours, how many different signals can be generated if each signal requires the use of 2 flags, one below the other?

**Q15.** In how many ways each letter of the word "NEELAM" be arranged so that each word ends with *L* and also find total no. of words. (Repetition not allowed.)

**Q16.** In how many ways the letters of the word 'BORDEN' be arranged so that the each word begins with *B* and also find total words of 6 letters. (Repetition not allowed.)

**Q17.** In how many ways the letter of the word "BORDEN" be arranged so that
(a) all vowels together. (b) all vowels never together.

**Q18.** In the how many ways the letter of the word "BORDEN" be arranged so that
(a) Each word begin with *B* and end with *N*. (b) Each word end with *N*.

**Q19.** The letter of the word "TUESDAY" arranged in a line and each arrangement ending with *S*. How many differents arrangements are possible? How many of them will start with letter *D*? (Repetition not allowed.)

**Q20.** In how many ways the letter of the word "CHITRA" be arranged so that the vowels occupy only even places. (Repetition not allowed.)

**Q21.** The letter of the word "BHABHA" be arranged in a line. How many arrangement is possible and how many words begins with *B*. (Repetition not allowed.)

**Q22.** In how many ways 5 boys and 3 girls should be arranged in a row so that
(a) all girls together (b) all girls never together (c) no two girls together

**Q23.** In how many ways letter of the word "NEELAM" be arranged so that
(a) all vowels together. (b) all vowels never together.

**Q24.** In how many ways can letter of the word "EDUCATION" be arranged so that
(a) all vowels together. (b) all vowels never together.

**Q25.** How many arrangements can be made with the letters of the word "MATHEMATICS" ? In how many of them are the vowels together ?

**Q26.** How many words can be formed by using the letters of the word "ORIENTAL" so that the vowels always occupy the odd places?

**Q27.** In how many ways letter of the word "TEACHERHAND" be arranged so that
(a) all vowels together. (b) all vowels never together.

**Q28.** How many words can be formed by using all the letters of the word "ALLAHABAD" ? How many of these words will not contain both L together ?

**Q29.** Find the number of arrangements of the letters of the word INDEPENDENCE. In how many of these arrangements,
(i) do the words start with P (ii) do all the vowels always occur together
(iii) do the vowels never occur together (iv) do the words begin with I and end in P?

**Q30.** Find the number of arrangement of the letter of the word "COMBINATION" such that all vowels are never together.

**Q31.** Find the number of arrangement of the letter of the word "ARRANGEMENT" such that
(a) do all the vowels occur together. (b) do all the vowels never occur together.

**Q32.** How many words with or without meaning can be made from the letters of the word MONDAY, assuming that no letter is repeated, if: (i) 4 letters are used at a time? (ii) all letters are used at a time? (iii) all letters are used but first letters is a vowel?

**Q33.** In how many ways the letter of the word "BHABHA" be arranged so that
(a) each word begins with *B* and ends with *A*. (b) all vowels together.
(c) all vowels never together.

| | | | | | |
|---|---|---|---|---|---|
| *B* | | | | | *A* |

**Q34.** In how many ways the letter of the word "BORDEN" be arranged so that
(a) relative order of consonant and vowels remains unchanged.
(b) order of vowels does not change.

**Q35.** Find the number of different 8 letter arrangements that can be made from the letter of the word "DAUGHTER" so that all vowels never occur together.

**Q36.** In how many ways can be letters of the word "PERMUTATIONS" be arranged if the
(a) words starts with P and ends with S.
(b) vowels are all together.

**Q37.** In how many of the distinct permutations of the letters in "MISSISSIPPI" do the four I's not come together.

**Q38.** Find the number of words which can be made using all the letters of the word 'AGAIN'. If these words are written as in a dictionary, what will be the $50^{th}$ word?

**Q39.** If "a" denotes the number of permutations of ($x$ + 2) things taken all at a time, $b$ the number of permutation of $x$ things taken 11 at a time and $c$ the number of permutation of ($x$ – 11) things taken all at a time such that $a = 182bc$. Find the value of $x$.

**Q40.** Find the number of arrangement of the letters of the word "ILLUSTRATIVE"
(a) do all the vowels occur together. (b) do all the vowels never occur together.

Date: 10/10/2021

**S1.** Total no. of letters in the given word $n = 8$

No. of letters to be used in forming a word, $r = 3$

Hence, required no. of words = $^8p_3$

$$= \frac{8!}{(8-3)!} = \frac{8!}{5!} = 8 \times 7 \times 6 = 336.$$

**S2.** LOGARTHIMS

Total no. of different words = 10

$\therefore$ Total no. of four letters = $^{10}p_4$

$$= \frac{10!}{(10-4)!} = \frac{10 \times 9 \times 8 \times 7 \times 6!}{6!}$$

$$= 720 \times 7 = 5040$$

**S3.** TRIANGLE

Total no. of different letters = 8

$\therefore$ Total no. of words = 8!.

**S4.**

| E | | | | | | E |
|---|---|---|---|---|---|---|
| 1 ways | | | 5! | | | 1 ways |

SANDEEP
2E, 1A, 1S, 1N, 1P, 1D

$\therefore$ Total no. of words = 5! = 120

**S5.** MANMOHAN

2N, 2M, 2A, 1H, 1O

| | | | | | | | N |
|---|---|---|---|---|---|---|---|

$\frac{7!}{2!\cdot 2!}$    1 ways

$\therefore$ Total no. of words = $\frac{7!}{2!\cdot 2!} = \frac{5040}{4}$ = 1260 words.

**S6.** We consider *EN* as one letter.

Now, 5 letters can be arranged in 5! = 120 ways.

**S7.** Total arrangements = 6!

When vowels are together, the number of arrangement = $5! \times 2$

Therefore, when the vowels are not together, the number of arrangements

$$= 6! - 5! \times 2 = 5\,!\,(6 - 2) = 480.$$

**S8.** Starting with $E$, the number of words with remaining letters

$$= \frac{10\,!}{2\,! \times 2\,!} = 907200 \qquad [\because \text{ there are 2 } I\text{'s and 2 } N\text{'s}]$$

According to the dictionary, the next word will begin with $E$.

**S9.** There are as many words as there are ways of filling in 4 vacant places □□□□ by the 4 letters, keeping in mind that the repetition is not allowed. The first place can be filled in 4 different ways by anyone of the 4 letters R, O, S, E following which, the second place can be filled in by anyone of the remaining 3 letters in 3 different ways, following which the third place can be filled in 2 different ways, following which, the fourth place can be filled in 1 way. Thus, the number of ways in which the 4 places can be filled, by the multiplication principle, is $4 \times 3 \times 2 \times 1 = 24$. Hence, the required number of words is 24.

**S10.** No. of ways of filling first letter = 10

No. of ways of filling second letter = 9

No. of ways of filling third letter = 8

No. of ways of filling fourth letter = 7

By F.P.C., total possibilities = $10 \times 9 \times 8 \times 7 = 5040$.

**S11.** Here, there are 9 objects (letters) of which there are 4 A's, 2 L's and rest are all different.

Therefore, the required number of arrangements

$$\frac{9!}{4!2!} = \frac{5 \times 6 \times 7 \times 8 \times 9}{2} = 7560.$$

**S12.** Out of 10 digits we are left with 8 digits to form the telephone numbers because starting two digits of the telephone number are fixed.

No. of ways of filling unit's place = 8

No. of ways of filling ten's place = 7

No. of ways of filling hundred's place = 6

No. of ways of filling thousand's place = 1

No. of ways of filling ten thousand's place = 1

By F.P.C., total possibilities = $8 \times 7 \times 6 \times 1 \times 1 = 336$.

**S13.** We will denote by H, the result, the coin turns up 'Head' and by T, the result, the coin turns up 'Tail'.

No. of ways in which the coin can turn up in the first toss = 2 (H or T).

No. of ways in which the coin can turn up in the second toss = 2 (H or T).

No. of ways in which the coin can turn up in the third toss = 2 (H or T).

By F.P.C., total no. of outcomes = $2 \times 2 \times 2 = 8$.

**S14.** There will be as many signals as there are ways of filling in 2 vacant places $\boxed{\frac{\quad}{\quad}}$ in succession by the 5 flags of different colours. The upper vacant place can be filled in 5 different ways by any one of the 5 flags. Following which, the lower vacent place can be filled in 4 different ways be any one of the remaining 4 different flags. Hence, by multiplication principle, the required number of signals = $5 \times 4 = 20$.

**S15.** NEELAM

Vowel = 2E, A

$$\therefore \qquad \text{Total words} = \frac{6!}{2!} = \frac{720}{2} = 360$$

| | | | | | |
|---|---|---|---|---|---|
| · | · | · | · | · | L |

$\frac{5!}{2!}$ 1 ways

Total no. of words ending with $L$.

$$= \frac{5!}{2!} = 60.$$

**S16.**

| | | | | | |
|---|---|---|---|---|---|
| · | · | · | · | · | · |

${}^6p_6$

Total 6 letter word = ${}^6p_6$.

Now word beginning with $B$ can be represented as

| | | | | | |
|---|---|---|---|---|---|
| $B$ | | | | | |

1 ways ${}^5p_5$

Total no. of ways = $1 \times {}^5p_5 = 5! = 120$ ways.

**S17.** (a) All vowels together = ${}^5p_5 \times 2! = 240$

Vowels = $O$, $E$

Consonant = $B$, $R$, $D$, $N$

Two vowels can be arranged in 2! ways and after that treat them as single unit (string method).

(b) All vowels never together = total – together

$$= {}^6p_6 - {}^5p_5 \times 2! = 480$$

**S18.** (a)

| | | | | | |
|---|---|---|---|---|---|
| *B* | · | · | · | · | *N* |

1 ways $\quad {}^4p_4 \quad$ 1 ways

Total words = $1 \times {}^4p_4 \times 1 = 4\,!$

(b)

| | | | | | |
|---|---|---|---|---|---|
| · | · | · | · | · | *N* |

${}^5p_5 \quad$ 1 ways

Total words = $1 \times {}^5p_5 \times 1 = {}^5p_5$

**S19.** TUESDAY $\Rightarrow$ 7 letters

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | *S* |

${}^6p_6 \quad$ 1 ways

Total no. of ways $= {}^6p_6 \times 1 = 6!$

Words begin with *D*.

| | | | | | | |
|---|---|---|---|---|---|---|
| D | | | | | | |

1 ways $\quad {}^6p_6$

Total no. of words starting with letter

$$D = 1 \times {}^6P_6 = 6!.$$

**S20.**

| | | | | | |
|---|---|---|---|---|---|
| | ✓ | | ✓ | | ✓ |

In "CHITRA" there are two different vowels = I, A

There are two vowels *I* and *A* and 3 even places 2, 4, 6. These two vowels can be arranged in three even places in 3! ways. The remaining 4 letters can be arranged in 3 places (1, 3, 5) in 3!. ways.

Hence, by the fundamental principle of counting the total no of arrangements = 3! × 3! = 36.

**S21.**

| | | | | | |
|---|---|---|---|---|---|
| | | | | | |

BHABHA
2B, 2H, 2A

$$\text{Total words} = \frac{6\,!}{2\,!\cdot 2\,!\cdot 2\,!} = \frac{720}{8} = 90$$

Words begin with *B* are

| | | | | | |
|---|---|---|---|---|---|
| B | | | | | |

$$\text{Total ways} = \frac{5\,!}{2\,!\cdot 2\,!} = 20 \text{ ways.}$$

**S22.** (a) All girls together = ${}^6p_6 \times 3!$

(b) All girls never together = ${}^8p_8 - 6! \cdot 3! = 36000$

(c) $X\ B_1\ X\ B_2\ X\ B_3\ X\ B_4\ X\ B_5\ X$

Total ways = ${}^6p_3 \times 5!$

**S23.** NEELAM

No. of vowels = 3(2E, 1A)

Total no. of 6 letter words = $\dfrac{6!}{2!} = 360$

If we consider these three vowels as one letter, then the number of different words

$= (6 - 3) + 1 = 4$

Hence total no. of 6 letter words in which vowels are always together

$$= \frac{3!}{2!} \times 4! = 3 \times 24 = 72$$

Hence, total no. of words in which vowels are never together

$= 360 - 72 = 288.$

**S24.** (a) EDUCATION

No. of vowels = 5($A$, $E$, $I$, $O$, $U$)

Total no. of 8 letter words = 8!

If we consider these five vowels as one letter then no. of different words = (8 – 5) + 1 = 4

Hence, total no. of 8 letter words in which vowels are always together

$= 5! \times 4! = 2880$

Hence, total no. of ways in which vowels are never together.

$$= 8! - (5! \times 4!) = 8 \times 7 \times 6 \times 5! - 5! \times 4!$$

$$= 5!\,(8 \times 7 \times 6 - 24)$$

$$= 5!\,(312) = 37440.$$

**S25.** There are 11 letters in the word "MATHEMATICS", out of these letters M, A and T are appearing twice, therefore, the required number of arrangements.

$$= \frac{11!}{2!\cdot 2!\cdot 2!} = 4989600$$

Now, there are 4 vowels ($A$, $A$, $E$, $I$) considering them as one unit there are 8 letters in the given word with M and T appearing twice.

Therefore, such arrangements are $\dfrac{8!}{2!\cdot 2!}$ and 4 vowels can be arranged in $\dfrac{4!}{2!}$ ways. Therefore, the number of arrangements when the vowels are together.

$$= \frac{8!}{2!\cdot 2!} \cdot \frac{4!}{2!} = 120960$$

**S26.** In the given 8 letter word, there are 4 vowels ($A$, $E$, $I$, $O$).

The odd places are 1, 3, 5, 7.

Therefore, the vowels can be arranged in 4 ! ways.

Now, the remaining 4 letters can be arranged in 4 ! ways.

Therefore, the total number of arrangements will be $4! \times 4! = 576$.

**S27.** TEACHERHAND

Vowel = 4 (2$E$, 2$A$)

$$\text{Total no. of 11 letter words} = \frac{11!}{2!\cdot 2!\cdot 2!} = 4989600$$

Consonants, $IT$ , $IC$ , 2$H$, 1$D$, 1$N$, 1$R$.

If we consider these 4 vowels as one letter, then the number of different letters

$$= (11 - 4) + 1 = 8$$

Hence, total no. of 11 letter words in which vowels are always together.

$$= \frac{4!}{2!\cdot 2!} \times \frac{8!}{2!} = 120960$$

Total no. of ways in which vowels are never together

$$= \left(\frac{11!}{2!\cdot 2!\cdot 2!} - 120960\right) = 4868640$$

**S28.** In the word ALLAHABAD, A is appearing 4 times and $L$ two times.

$\Rightarrow$ The number of words = $\frac{9!}{4!\cdot 2!}$ = 7560 when both $L$ are together we consider $LL$ as one letter.

Therefore, the number of arrangements with both $L$ together.

$$\frac{8!}{4!} = 1680$$

Hence the number of arrangements when both L are not together.

$$7560 - 1680 = 5880$$

**S29.** There are 12 letters, of which N appears 3 times, E appears 4 times and D appears 2 times and the rest are all different. Therefore

$$\text{The required number of arrangements} = \frac{12!}{3!\,4!\,2!} = 1663200.$$

(i) Let us fix P at the extreme left position, we, then, count the arrangments of the remaining 11 letters. Therefore, the required number of words starting with P are

$$= \frac{11!}{3!\,4!\,2!} = 138600$$

(ii) There are 5 vowels in the given word, which are 4 Es and 1 I. Since, they have to always occur together, we treat them as a sigle object EEEEI for the time being. This single object together with 7 remaining objects will account for 8 objects. These 8 objects, in which there are 3 Ns and 2 Ds, can be rearranged in $\frac{8!}{3!\,2!}$ ways. Corresponding to each of these arrangements, the 5 vowels E, E, E, E and I can be rearranged in $\frac{5!}{4!}$ ways. Therefore, by multiplication principle the required number of arrangements

$$= \frac{8!}{3!\,2!} \times \frac{5!}{4!} = 16800$$

(iii) The required number of arrangements = the total number of arrangements (without any restriction) – the number of arrangements where all the vowels occur together.

$$= 1663200 - 16800 = 1646400.$$

(iv) Let us fix I and P at the extreme ends (I at the left end and P at the right end). We are left with 10 letters.

Hence, the required number of arrangements

$$= \frac{10!}{3!\,2!\,4!} = 12600.$$

**S30.** COMBINATION

{1C, 2O, 1M, 1B, 2N, 1A, 1T, 2I}

Vowels $\Rightarrow$ {2O, 1A, 2I}

Total no. of arrangements without restriction = $\frac{11!}{2!\cdot 2!\cdot 2!}$.

If we treat all vowels as single object then total no. of letters = 11 – 5 + 1 = 7

$\therefore$ Total no. of ways in which all vowels together = $\frac{7!}{2!} \times \frac{5!}{2!\cdot 2!}$ = 75600.

Hence the case in which all vowels are never together = total – together

$$= \frac{11!}{2!\cdot 2!\cdot 2!} - \frac{7!\times 5!}{2!\cdot 2!\cdot 2!} = 4914000.$$

**S31.** ARRANGEMENT

{2A, 2R, 2N, 2E, 1G, 1M, 1T}

Total no. of words without restriction = $\frac{11!}{2!\cdot 2!\cdot 2!\cdot 2!}$

$\therefore$ Vowels are (2A, 2E)

If we take vowels as one letter then totol no. of letter = 11 – 4 + 1 = 8.

$\therefore$ Total no. of permutations when all vowels together

$$= \frac{8!}{2!\cdot 2!} \times \frac{4!}{2!\cdot 2!} = 60480 \; .$$

$\therefore$ Total no. of permutations when all vowels never together

$$= \frac{11!}{2!\cdot 2!\cdot 2!\cdot 2!} - \frac{4!}{2!\cdot 2!} \times \frac{8!}{2!\cdot 2!} = 2434320.$$

**S32.** Number of letters in the word MONDAY is 6.

(i) We have to form words using 4 letters of the word MONDAY. It means we want to find the number of permutations of 6 objects taken 4 at a time, *i.e.*, ${}^6P_4$ which is given by

$${}^6P_4 = \frac{6!}{(6-4)!} = 360.$$

(ii) Number of permutations of 6 objects taken all at a time is ${}^6P_6$ which is given by

$${}^6P_6 = 6! = 720.$$

(iii) The first letter in each permutation is to be either 'O' or 'A'. Thus, we have 2 choices for the first letter. After selecting the first letter, we can select the remaining 5 without restrictions and consequently in ${}^5P_5$ or 5! ways or 120 ways.

The required number of words

$$2 \times 120 = 240.$$

**S33.** (a) 1 ways $\quad \frac{4!}{2!} \quad$ 1 ways

$$\text{Total ways} = \frac{4!}{2!} = 12$$

(b) $$\text{all vowels together} = \frac{5!}{2!\cdot 2!} \times \frac{2!}{2!} = 30$$

(c) $$\text{all vowels never together} = \left(\frac{6!}{2!\cdot 2!\cdot 2!} - \frac{5!}{2!\cdot 2!}\right)$$

$$= 90 - 30 = 60.$$

**S34.** BORDEN

(a)

| | | | | | |
|---|---|---|---|---|---|
| *B* | *O* | *R* | *D* | *E* | *N* |
| *C* | *V* | *C* | *C* | *V* | *C* |

Required no. of ways = ${}^4p_4 \times {}^2p_2 = 4! \times 2!$.

(b) order of vowels does not change (first $O$ comes after that $E$).

| | | | | | |
|---|---|---|---|---|---|
| $O$ | | | | | |

Total no. of ways = $\dfrac{6\,!}{2\,!}$ = 360.

Treat all vowels as identical.

**Alternate method:**

Order of vowels does not change.

| | | | | | |
|---|---|---|---|---|---|
| $O$ | .. | .. | .. | .. | .. |

$B$, $R$, $D$, $N$ can be arranged in 4! ways.

Total ways of arranging $O$, $E$.

$$= 5 + 4 + 3 + 2 + 1 = 15$$

$\therefore$ Total ways = $15 \times 4\,! = 360$.

**S35.** DAUGHTER

No. of vowels = 3 ($A$, $E$, $U$)

Total no. of 8 letter words = $\dfrac{8\,!}{(8-8)\,!}$ = 8! (0! = 1)

If we consider these three vowels as one letter, then the number of different words

$$= (8 - 3) + 1 = 5 + 1 = 6$$

Hence, total no. of 8 letter words in which vowels always together

$$= {}^3p_3 \times {}^6p_6 = 3! \times 6!$$

If $p$ is the number of words in which vowels are never together

$$p = 8! - 3! \cdot 6!$$

$$= 6!\,(8 \times 7 - 6) = 6! \times 50$$

$$= 720 \times 50 = 36000.$$

**S36.** (a) PERMUTATIONS (1P, 1E, 1R, 1M, 1U, 2T, 1A, 1I, 1O, 1N)

| | | | | | |
|---|---|---|---|---|---|
| P | | | | | S |
| 1 ways | | $\dfrac{{}^9P_4}{2!}$ | | | 1 ways |

Total all such words = $\dfrac{{}^9p_4}{2!} = 1512$

(b) There are 12 letters in the word "PERMUTATIONS" but there are five vowels. Since they have to always occur together. We treat them as single object (EUAIO) for the time being.

The single object together with 7 remaining objects will count for 8 objects.

These 8 objects can be arranged in 8! ways, but there are 2T.

So, The no. of arrangements = $\frac{{}^{8}p_{8}}{2}$

Corresponding to each of these arrangements the five vowels can be arranged in ${}^{5}p_{5}$ ways. Therefore, by multiplication principle the required no. of arrangements

$$= \frac{{}^{8}p_{8}}{2} \times {}^{5}p_{5} = \frac{8!}{2} \times 5! = 2419200$$

**S37.**

| Letter | $M$ | $I$ | $S$ | $P$ | Total |
|---|---|---|---|---|---|
| No. | 1 | 4 | 4 | 2 | 11 |

Total no. of permutations with no restriction

$$= \frac{11!}{1! \cdot 4! \cdot 4! \cdot 2!}$$

If we take 4 'I's as one letter then, we have

| Letter | $M$ | $IIII$ | $S$ | $P$ | Total |
|---|---|---|---|---|---|
| No. | 1 | 1 | 4 | 2 | 8 |

Total no. of permutations when four 'I's together

$$= \frac{8!}{1! \cdot 1! \cdot 4! \cdot 2!}$$

∴ Total no. of permutations when 4 'I' are not together.

$$= \frac{11!}{4! \cdot 4! \cdot 2!} - \frac{8!}{4! \cdot 2!} = 34650 - 840 = 33810.$$

**S38.**

| Letters | A | G | I | N | Total |
|---|---|---|---|---|---|
| No. | 2 | 1 | 1 | 1 | 5 |

We start with A, remaining letters GAIN can be arranged in 4! = 24 ways

⇒ 24 words can be formed when A is first letters.

Now we from the word beginning with G remaining letters AAIN can be arranged in $\frac{4!}{2!}$ = 12 ways ⇒ There are 12 words beginning with G. Now we from the words beginning with I, remaining letters AAGN can be arranged in $\frac{4!}{2!}$ = 12 ways.

⇒ There are 12 words beginning with I. So for we have to formed 24 + 12 + 12 = 48 words the 49th word is NAAGI and 50th word is NAAIG.

**S39.** $a = {}^{x+2}p_{x+2} = (x+2)!$, $b = {}^{x}p_{11}$, $c = {}^{x-11}p_{x-11} = (x-11)!$

Also $a = 182\,bc$

$\Rightarrow$ $(x+2)! = 182 \cdot \dfrac{x!}{(x-11)!}(x-11)!$

$\Rightarrow$ $(x+2)(x+1)\cdot x! = x! \cdot 182$

$\Rightarrow$ $x^2 + 2x + x + 2 = 182$

$\Rightarrow$ $x^2 + 3x - 180 = 0$

$\Rightarrow$ $(x-12)(x+15) = 0$

$\Rightarrow$ $x = 12, -15$, but $x \neq -15$

Hence $x = 12$.

**S40.** ILLUSTRATIVE

{2I, 2L, 2T, 1U, 1V, 1E, 1S, 1R, 1A}

| Letter | *I* | *L* | *T* | *U* | *V* | *E* | *S* | *R* | *A* | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| No. | 2 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 12 |

Total no. of permutations with no restriction

$$= \frac{12!}{2!\cdot 2!\cdot 2!}$$

There are five vowels (2I, 1U, 1A, 1E).

If we take all vowels as one letter.

| Letter | *IIUAE* | *L* | *T* | *V* | *S* | *R* | Total |
|---|---|---|---|---|---|---|---|
| No. | 1 | 2 | 2 | 1 | 1 | 1 | 8 |

Total no. of permutations when all vowels are together

$$= \frac{5!}{2!} \times \frac{8!}{2!\cdot 2!} = 604800$$

$\therefore$ Total no. of arrangement when all vowels never together $= \dfrac{12!}{2!\cdot 2!\cdot 2!} - 604800 = 59270400$

Date: 10/10/2021

**Q1.** How many three digit numbers are there such that 5 is at unit's place?

**Q2.** How many numbers are there between 100 and 1000 such that at least one of the digits is 6 ?

**Q3.** How many 4 digit numbers can be formed with the digits 1, 2, 3, 4, 5, 6 when the repetition of the digits is allowed?

**Q4.** How many 2-digit even numbers can be formed from the digits 1, 2, 2, 4, 5 if the digits can be repeated?

**Q5.** How many 4-digit numbers can be formed by using the digits 1 to 9, if repetition of digits is not allowed?

**Q6.** How many 3-digit numbers can be formed from the digits 1, 2, 3, 4 and 5, assuming that:
(i) repetition of the digits is a llowed? (ii) repetition of the digitis is not allowed?

**Q7.** How many 3-digit even numbers can be formed from the digits 1, 2, 3, 4, 5, 6 if the digits can be repeated?

**Q8.** How many 3-digit numbers can be formed by using the digits 1 to 9 if no digits is repeated?

**Q9.** How many numbers of 4 digits can be formed with the help of digits 1, 2, 3, 4, 5, 6 and also find how many all such even numbers are possible (Repetitions not allowed)?

**Q10.** How many numbers of 4 digit can be formed with the help of digits 1 , 1 , 1 , 2 , 2 , 3 (Repetitions not allowed)?

**Q11.** How many numbers of 4 digit can be formed with the help of digits 1 , 1 , 1 , 2 , 2 , 3 (Repetitions not allowed) such that all are even?

**Q12.** How many numbers of 4 digit can be formed with the help of digits 1, 2, 3, 4, 5, 6 such that all such numbers are divisible by 4 (Repetitions not allowed)?

**Q13.** How many numbers of 6 digit can be formed with the help of digits 1, 1, 1, 2, 2, 3 (Repetitions not allowed)?

**Q14.** How many numbers of 6 digits can be formed with the help of digits 1, 1, 1, 2, 2, 3 such that all are even numbers (Repetitions not allowed)?

**Q15.** How many numbers of 4 digits can be formed with the help of digits 0, 1, 2, 3, 4, 5 such that all are even numbers (Repetitions not allowed)?

**Q16.** How many numbers of 4 digits can be formed with the help of digits 0, 1, 2, 3, 4, 5 (Repetitions not allowed)?

**Q17.** How many numbers can be formed with the digits 1, 2, 3, 4, 3, 2, 1 so that odd digits always occupy the odd places?

Q18. How many three digit odd numbers can be formed by using the digits 1, 2, 3, 4, 5, 6 when
(a) the repetition of digits is not allowed? (b) the repetition of digits is allowed?

Q19. How many 4-digit odd numbers can be formed with the help of the digits 1, 2, 3, 4, 5 if
(a) no digit is repeated? (b) digits are repeated?

Q20. How many numbers of 6 digit can be formed with the digits 1, 2, 3, 4, 5, 6 such that (Repetition not allowed)
(a) digits are divisible by 4. (b) digits are divisible by 5.

Q21. How many numbers of 4 digits can be formed with the help of digits 1,2,3,4,5,6 such that all such numbers are greater than 3400 (Repetition not allowed)?

Q22. How many numbers lying between 100 and 1000 can be formed with the digits 0, 1, 2, 3, 4, 5, if the repetition of the digits is not allowed?

Q23. How many numbers of greater than 1000000 can be formed by using the digits 1, 2, 0, 2, 4, 2, 4?

Q24. Find the number of 4-digit numbers that can be formed using the digits 1, 2, 3, 4, 5 if no digits is repeated. How many of these will be even?

Q25. How many 6-digit numbers can be formed from the digits 0, 1, 3, 5, 7 and 9 which are divisible by 10 and no digit is repeated?

Q26. How many odd numbers less than 10,000 can be formed using the digits 0, 2, 3, 5, allowing repetition of digits?

Q27. How many number of 6 digit can be formed with the digits 1, 2, 3, 4, 5, 6 such that (Repetition not allowed)
(a) all are even (b) all are odd

Q28. How many numbers of 6 digit can be formed with the digits 1, 2, 3, 4, 5, 6 such that (Repetition not allowed)
(a) total even no. occupy at odd places. (b) How many no. greater than 540000.

Q29. How many numbers of 6 digit can be formed with the digits 0, 1, 2, 3, 4, 5 such that (Repetition not allowed) all are even numbers?

Q30. How many even no. of 4 digits can be formed with the help of digits 0, 1, 1, 1, 2, 2 (Repetition not allowed)?

Q31. How many numbers of 4 digits can be formed with the help of digits 0, 1, 1, 1, 2, 2. (Repetition not allowed)?

Q32. How many numbers of 6 digits can be formed with the help of 0, 1, 1, 1, 2, 2 such that all are even numbers (Repetition not allowed)?

Q33. How many numbers of 6 digits can be formed with the help of digits 0, 1, 1, 1, 2, 2 (Repetition not allowed)?

Q34. How many numbers of 4 digits can be formed with the help of digits 0, 1, 2, 3, 4, 5 such that all such numbers are divisible by 3 (Repetition not allowed)?

Q35. How many numbers of 4 digits can be formed with the help of digits 1,2,3,4,5,6 such that all such digits are divisible by 6 (Repetition not allowed)?

**Q36.** How many numbers of 4 digits can be formed with the help of digits 1,2,3,4,5,6 such that all such digits are divisible by 3 (Repetition not allowed)?

**Q37.** How many numbers of 6 digit can be formed with the help of digits 0, 1, 2, 3, 4, 5 such that (Repetition not allowed) all are divisible by 5?

**Q38.** How many numbers of 6 digit can be formed with the digits 0, 1, 2, 3, 4, 5 such that (Repetition not allowed) they are divisible by 4?

Date: 10/10/2021

**S1.** The number of options for unit's place = 1 [$\because$ only 5 can come]

The number of options for ten's place = 10 [0 to 9, any digit can come]

The number of options for hundred's place = 9 [$\because$ 0 can not come]

Therefore, the total possible numbers are $9 \times 10 \times 1 = 90$

**S2.** Total three digit numbers are $9 \times 10 \times 10 = 900$

Now, three digit numbers without 6 at any of the places are $8 \times 9 \times 9 = 648$

Therefore, the number with at least one 6 will be $900 - 648 = 252$

**S3.** As there is no restriction and repetition is allowed, each of the four digits can be filled in 6 ways.

$\Rightarrow$ The total number of ways = $6^4 = 1296$.

**S4.** There will be as many ways as there are ways of filling 2 vacent places $\square\square$ in succssion by the five given digits. Here, in this case, we start filling in unit's place, because the options for this place are 2 and 4 only and this can be done in 2 ways, following which the ten's place can be filled by any of the 5 digits in 5 different ways as the digits can be repeated. Therefore, by the multiplication principle, the required number of two digits even numbers is $2 \times 5$, *i.e.*, 10.

**S5.** Here, order matters for example 1234 and 1324 are two different numbers. Therefore, there will be as many 4-digit numbers as there are permutations of 9 different digits taken 4 at a time.

Therefore, the required 4-digit nubers = $^9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = 9 \times 8 \times 7 \times 6 = 3024$.

**S6.** (i) No. of ways of filling unit's place = 5

No. of ways of filling ten's place = 5

No. of ways of filling hundred's place = 5

By F.P.C., total possibilities = $5 \times 5 \times 5 = 125$.

(ii) No. of ways of filling unit's place = 5

No. of ways of filling ten's place = 4

No. of ways of filling hundred's place = 3

By F.P.C., total possibilities = $5 \times 4 \times 3 = 60$.

**S7.** A number is even if the digit at its unit's place is one out of 2, 4, 6 from the digits given

No. of ways of filling unit's place = 3

No. of ways of filling ten's place = 6

No. of ways of filling hundred's place = 6

By F.P.C., total posibilities = $3 \times 6 \times 6 = 108$.

**S8.** There will be as many 3-digit numbers as there are permutations of 9 different things taken 3 at a time. Therefore, the reqired 3-digit numbers.

$$^9P_3 = \frac{9!}{(9-3)!} = \frac{9!}{6!} = 9 \times 8 \times 7 = 504.$$

**S9.** $\therefore$ Total no. = $6 \times 5 \times 4 \times 3 = {}^6p_4 = 360$

| | | | |
|---|---|---|---|
| | | | 2, 4, 6 |

$^5p_3$    3 ways

Total no. of even numbers = $3 \times {}^5p_3$

$= 3 \times 5 \times 4 \times 3 = 180.$

**S10.** Digits are 1, 1, 1, 2, 2, 3

| | | | |
|---|---|---|---|
| | | | |

$\therefore$ Total 4 digit no. = $\dfrac{^6p_4}{1! \cdot 2! \cdot 3!}$

$$= \frac{6 \times 5 \times 4 \times 3}{2 \times 3 \times 2} = \frac{360}{12} = 30$$

**S11.** Digits are

| | | | |
|---|---|---|---|
| | | | 2 |

$\dfrac{^5p_3}{3!}$    1 way

$\therefore$ Total even no. of 4 digits = $\dfrac{^5p_3}{3!} \times 1$

$$= \frac{5 \times 4 \times 3}{6} = 10$$

**S12.** Digits are 1, 2, 3, 4, 5, 6

No. divisible by 4 are {12, 16, 24, 32, 36, 52, 56, 64}

| | | | |
|---|---|---|---|
| | | · | · |

$^4p_2$    8 ways

$\therefore$ Total ways = $^4p_2 \times 8 = 96.$

**S13.** No. are 1, 1, 1, 2, 2, 3.

| | | | | | |
|---|---|---|---|---|---|
| · | · | · | · | · | · |

Total no. of 6 digit = $\dfrac{6!}{3! \cdot 2! \cdot 1!}$

$$= \frac{720}{12} = 60$$

**S14.** Digits are 1, 1, 1, 2, 2, 3

| | | | | | |
|---|---|---|---|---|---|
| | | | | | 2 |

$\frac{5!}{3! \times 1!}$     1 way

$\therefore$ Total no. of even no. $= \frac{5!}{3! \times 1!} \times 1 = 20$

**S15.** **Case I:** When zero is not at unit place.

| | | | |
|---|---|---|---|
| 0X | | | 2, 4, 0 |
| 4 ways | $^4p_2$ | | 3 ways |

$\Rightarrow 4 \times {^4p_2} \times 3$

**Case II:** When zero is at unit place.

| | | | |
|---|---|---|---|
| | | | 0 |
| $^5p_3$ | | | 1 way |

$\Rightarrow$ $^5p_3 \times 1 = {^5p_3}$

$\therefore$ Total no. of even numbers $= {^5p_3} + 4 \times {^4p_2} \times 3$

$= 20 + 4 \times 12 \times 3 = 164.$

**S16.** Digits are 0, 1, 2, 3, 4, 5.

| | | | |
|---|---|---|---|
| OX | | | |
| 5 ways | $^5p_3$ | | |

$\therefore$ Total ways $= 5 \times {^5p_3} = 5 \times 60 = 300$

**Alternative method:**

The number of options for thousand's place = 5 [$\because$ 0 cannot come]

The number of options for hundred's place = 5 (because one digit has come at thousand's place).

The number of options for ten's place = 4 (because two digits have come at thousand's and hundred's places)

The numbers of options for units place = 3 (because three digits have occupied thousand's, hundred's and ten's places).

Hence, the total numbers are $5 \times 5 \times 4 \times 3 = 300$.

**S17.** The given digits are 1, 2, 3, 4, 3, 2, 1 out of these 1, 3, 3, 1 are odd digits the odd digits occupy the odd place ($1^{st}$, $3^{rd}$, $5^{th}$, $7^{th}$ place).

$\therefore$ 4 odd places can be filled with 4 odd digits

$$= \frac{4\,!}{2\,!\cdot 2\,!} = 6 \text{ ways}$$

The three even places to be filled with the even digits 2, 4, 2

These places can be filled $= \dfrac{3\,!}{2\,!\cdot 1!} = 3$ ways

Hence the required no. of numbers = $6 \times 3 = 18$.

**S18.** (a) For the number to be odd, there are 3 options (i.e., 1, 3, 5) for the unit place.

The ten's place can be occupied in 5 ways. [$\because$ one digit has occupied the unit place]

The hundred's place can be occupied in 4 ways. [$\because$ two digits have gone]

$\Rightarrow$ the required number of numbers = $3 \times 5 \times 4 = 60$.

(b) For the number to be odd, there are 3 options (i.e., 1, 3, 5) for the unit's place. As the repetition is allowed, there are 6 options each to ten's and hundred's places.

$\Rightarrow$ the required number of numbers = $3 \times 6 \times 6 = 108$.

**S19.** (a) There are 3 options (1, 3, 5) for unit's place and as the repetition of the digits is not allowed, the options for other places are 4, 3, 2. Therefore, the total possible 4 digit odd numbers (without repetition) are $3 \times 4 \times 3 \times 2 = 72$

(b) There are 3 options (1, 3, 5) for unit place and as the repetition of the digits is allowed, the options for the other places are 5, 5, 5.

Therefore, the total possible 4 digits odd numbers are $5 \times 5 \times 5 \times 3 = 375$

**S20.** (a) Divisibility by 4.

$\Rightarrow$ Last two places should be divisible by 4.

| | | | | | |
|---|---|---|---|---|---|
| | | | | · | · |

${}^4p_4$ 8 ways

No. are $\Rightarrow$ 12, 16, 24, 32, 36, 52, 56, 64.

$\therefore$ Total no. of 6 digits which are divisible by 4 = $4! \times 8 = 24 \times 8 = 192$

(b) Digits are divisible by 5.

| | | | | | |
|---|---|---|---|---|---|
| | | | | | 5 |

${}^5p_5$ 1 way

Total no. of 6 digits divisible by 5 = ${}^5p_5 \times 1 = 5\,! = 120$

**S21.** **Case - I:** when 3 is at thousand's place

| | | | |
|---|---|---|---|
| 3 | 4, 5, 6 | | |
| 1 way | 3 ways | ${}^4p_2$ | |

$\Rightarrow 1 \times 3 \times {}^4p_2$

**Case - II:** when 3 is not at thousand place.

| 4, 5, 6 | | | |
|---|---|---|---|
| 3 ways | $^5p_3$ | | |

$$\Rightarrow 3 \times {^5p_3}$$

$$\text{Total ways} = 1 \times 3 \times {^4p_2} + 3 \times {^5p_3}$$
$$= 3 \times 12 + 3 \times 20 = 96$$

**S22.** Every number between 100 and 1000 is a 3-digit number. We, first, have to count the permutations of 6 digits taken 3 at a time. This number would be $^6P_3$. But, these permutations will include those also where 0 is at the 100's place. For example, 092, 042, ... etc., are such numbers which are actually 2-digit numbers and hence the number of such numbers has to be subtracted from $^6P_3$ to get the required number. To get the number of such numbers, we fix 0 at the 100's place and rearrange the remaining 5 digits taking 2 at a time. This number is $^5P_2$. So,

$$\text{The required number} \quad {^6P_3} - {^5P_2} = \frac{6!}{3!} - \frac{5!}{3!}$$
$$= 4 \times 5 \times 6 - 4 \times 5 = 100.$$

**S23.** Since, 1000000 is a 7-digit number and the number of digits to be used is also 7. Therefore, the numbers to be counted will be 7-digit only. Also, the numbers have to be greater than 1000000, so they can begin either with 1, 2 or 4.

The number of numbers beginning with $1 = \frac{6!}{3!\,2!} = \frac{4 \times 5 \times 6}{2} = 60$, as when 1 is fixed at the extreme left position, the remaining digits to be rearranged will be 0, 2, 2, 2, 4, 4, in which there are 3, 2*s* and 2, 4*s*.

$$\text{Total numbers begining with } 2 = \frac{6!}{2!\,2!} = \frac{3 \times 4 \times 5 \times 6}{2} = 180.$$

and $\quad$ Total numbers begining with $4 = \frac{6!}{3!} = 4 \times 5 \times 6 = 120$.

Therefore, the required number of numbers = 60 + 180 + 120 = 360.

**Alternative Method:**

The number of 7-digit arrangements, clearly, $\frac{7!}{3!\,2!} = 420$. But this will include those numbers also, which have 0 at the extreme left position. The number of such arrangements $\frac{6!}{3!\,2!}$ (by fixing 0 at the extreme left position) = 60.

Therefore, the required number of numbers = 420 – 60 = 360.

**S24.** There are 5-digits. Therefore, there will be as many 4-digit numbers as there are permutations of 5 different digits taken 4 at a time. Therefore, the required 4-digit numbers.

$$^5P_4 = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

A number is even if the digit at its unit's place is one out of 2, 4.

Number of four digit numbers with 2 at unit's place = $^4P_3$. $1 = 4 \times 3 \times 2 \times 1 = 24$.

Number of four digit numbers with 4 at unit's place = $^4P_3 . 1 = 4 \times 3 \times 2 \times 1 = 24$.

Required number of 4-digit even numbers = 24 + 24 = 48.

**S25.** (i) The given digits are 0, 1, 3, 5, 7, 9. The number of arrangements of 6-digits by using given digits = $^6P_6 = 6! = 720$.

These arrangements also include numbers with 0 at the million's place. The number of numbers with 0 at the million's place

$$= 1 . {}^5P_5 = 120.$$

Required number of numbers

$$= 720 - 120 = 600$$

(ii) The required numbers are 6-digit numbers with 0 at the unit's place.

Number of 6-digit numbers with 0 at the unit's place = $^5P_5 . 1 = 120$.

**S26.** As the number is less than 10,000 there are following possibilities.

(a) **One digit numbers:** It can be either 3 or 5. Therefore, there are two one digit odd numbers.

(b) **Two digit numbers:** There are two options (3, 5) for unit's place and three (2, 3, 5) options for ten's place.

Therefore, there are $2 \times 3 = 6$ two digit odd numbers.

(c) **Three digit numbers:** The number of options for unit's place = 2 (3, 5).

The number of options for ten's place = 4 (0, 2, 3, 5)

The number of options for hundred's place = 3 (2, 3, 5)

Therefore, there are $2 \times 4 \times 3 = 24$

Three digit odd numbers.

(d) **Four digit numbers:** The number of options for unit's place = 2 (3, 5).

The number of options for ten's place = 4 (0, 2, 3, 5)

The number of options for hundred's place = 4 (0, 2, 3, 5)

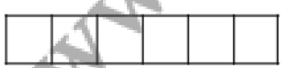
The number of options for thousand place = 3 (2, 3, 5)

Therefore, there are $2 \times 4 \times 4 \times 3 = 96$

4 digit odd numbers.

Hence, there are in all 2 + 6 + 24 + 96 = 128.

**S27.**



$^6p_6$

Total possible no. of six digit = $^6p_6$

**Case - I:** All are even

| | | | | | |
|---|---|---|---|---|---|
| | | | | | 2, 4, 6 |
| | | $^5p_5$ | | | 3 ways |

$\therefore$ Total no. of even digit = $3 \times {}^5p_5$

**Case - II:** All are odd.

$= \text{total digit} - \text{even digits}$

$= {}^6p_6 - 3 \times {}^5p_5 = 720 - 3 \times 120$

$= 720 - 360 = 360$

**Alternate method:**

**Case - II:** All are odd.

| | | | | | |
|---|---|---|---|---|---|
| | | | | | 1, 3, 5 |
| | | $^5p_5$ | | | 3 ways |

$\therefore$ Total no. of odd digits = $3 \times {}^5p_5 = 360$

**S28.** (a) Total even no occupy odd places.

$\therefore$ Total ways = $3\,! \cdot 3\,!$

(b) **Case - I:** when digit starts with 5.

| | | | | | |
|---|---|---|---|---|---|
| 5 | 4, 6 | | | | |
| 1 way | 2 ways | $^4p_4$ | | | |

$\therefore$ Total ways for case - I = $1 \times 2 \times {}^4p_4$

**Case - II:** when digit starts with 6.

| | | | | | |
|---|---|---|---|---|---|
| 6 | | | | | |
| 1 way | | $^5p_5$ | | | |

Total ways of case - II = ${}^5p_5 \times 1 = {}^5p_5$

$\therefore$ Total no. of digits greater than 540000.

$= 2 \times {}^4p_4 + {}^5p_5 = 2 \times 24 + 120$

$= 48 + 120 = 168$

**S29.** Digits are 0, 1, 2, 3, 4, 5

| | | | | | |
|---|---|---|---|---|---|
| 0 X | | | | | |
| 5 ways | | 5 ! | | | |

$\therefore$ Total numbers = $5 \times 5\,!$

**Case - I:** 0 is at unit place.

| | | | | | |
|---|---|---|---|---|---|
| | | | | | 0 |
| | 5! | | | | 1 ways |

Total ways for case - I = $1 \times 5!$

**Case - II:** 0 is not at unit place.

| | | | | |
|---|---|---|---|---|
| 0 X | | | | 2, 4 |
| 4 ways | | 4! | | 2 ways |

$\therefore$ Total ways for case - II = $4! \times 2 \times 4$

$\therefore$ Total no. of even digits = $1 \times 5! + 4! \times 2 \times 4$

$= 120 + 24 \times 2 \times 4 = 120 + 192 = 312$

**S30. Case - I:** when zero is at unit place

| | | | |
|---|---|---|---|
| | | | 0 |
| $\frac{^5p_3}{3!\cdot 2!}$ | | | 1 way |

**Case - II:** when zero is not at unit place.

**Case - II.a:** when 1 is at left most place

| | | | |
|---|---|---|---|
| 1 | | | 2 |
| 1 way | | $\frac{^4p_2}{2!}$ | 1 ways |

**Case - II.b:** when 2 is at left most place.

| | | | |
|---|---|---|---|
| 2 | | | 2 |
| 1 way | | $\frac{^4p_2}{3!}$ | 1 way |

$\therefore$ Total even no. of 4 digits = $\frac{^5p_3}{3!\cdot 2!} + \frac{^4p_2}{2!} + \frac{^4p_2}{3!}$

$= 5 + 6 + 4 = 15$

**S31.** Digits are 0, 1, 1, 1, 2, 2

**Case - I:** when 1 is at left most place.

| | | | |
|---|---|---|---|
| 1 | | | |
| 1 way | | $\frac{^5p_3}{2!\cdot 2!}$ | |

**Case - II:** when 2 is at left most place.

| 2 | | | |
|---|---|---|---|

1 way $\qquad \dfrac{^5p_3}{3!\cdot 1!}$

$\therefore$ Total 4 digit no. $= \dfrac{^5p_3}{2!\cdot 2!} + \dfrac{^5p_3}{3!}$

$= 15 + 20 = 35$

**S32.** Digits are 0, 1, 1, 1, 2, 2

**Case - I:** when zero is at unit place.

| · | · | · | · | · | 0 |
|---|---|---|---|---|---|

$\dfrac{^5p_5}{3!\cdot 2!}$ $\qquad$ 1 way

$\Rightarrow \dfrac{^5p_5}{3!\cdot 2!} \times 1$

**Case - II:** when zero is not at unit place.

**Case - II.a** when 1 is at left most place.

| 1 | | | | | 2 |
|---|---|---|---|---|---|

1 way $\qquad \dfrac{^4p_4}{2!} \times 1 \qquad$ 1 way

**Case - II.b** when 2 is at left most place.

| 2 | | | | | 2 |
|---|---|---|---|---|---|

1 way $\qquad \dfrac{^4p_4}{3!} \qquad$ 1 way

$\therefore$ Total no. of even no. $= \dfrac{^4p_4}{3!} + \dfrac{^4p_4}{2!} + \dfrac{^5p_5}{3!\cdot 2!}$

$= \dfrac{4!}{3!} + \dfrac{4!}{2!} + \dfrac{120}{12} = 18$

**S33.** Digits are 0, 1, 1, 1, 2, 2.

**Case - I:** when 1 is at left most place.

| 1 | | | | | |
|---|---|---|---|---|---|

1 way $\qquad \dfrac{^5p_5}{2!\cdot 2!}$

$$\Rightarrow \frac{{}^5p_5}{2!\cdot 2!}$$

**Case - II:** when 2 is at left most place.

| 2 | | | | | |
|---|---|---|---|---|---|

1 way $\qquad \frac{{}^5p_5}{3!}$

$$\Rightarrow \frac{{}^5p_5}{3!}$$

$\therefore$ Total 6 digit no. $= \frac{{}^5p_5}{2!\cdot 2!} + \frac{{}^5p_5}{3!}$

$$= \frac{60}{2} + 40 = 70$$

**S34.** Digits are 0, 1, 2, 3, 4, 5

Max. sum = 0 + 1 + 2 + 3 + 4 + 5 = 15

But when max. sum occurs we should have to take atleast 5 digits.

**Case - I:** when max. sum is 12.

0, 3, 4, 5

1, 2, 4, 5 $\Rightarrow$ 4 !

0, 3, 4, 5 $\Rightarrow$

| 0X | | | |
|---|---|---|---|

3 ways $\quad {}^3p_3$

0, 3, 4, 5 $\Rightarrow 3 \times {}^3p_3$

**Case - II:** when max. sum is 9.

0, 2, 3, 4 $\Rightarrow$

| 0X | | | |
|---|---|---|---|

3 ways $\quad {}^3p_3$

0, 1, 3, 5 $\Rightarrow$

| 0X | | | |
|---|---|---|---|

3 ways $\quad {}^3p_3$

$\Rightarrow$ 0, 2, 3, 4 $\Rightarrow 3 \times {}^3p_3$

0, 1, 3, 5 $\Rightarrow 3 \times {}^3p_3$

**Case - III:** when max. sum is six.

0, 1, 2, 3 $\Rightarrow$

| 0X | | | |
|---|---|---|---|

3 ways $\quad {}^3p_3$

$0, 1, 2, 3 \Rightarrow 3 \times {}^3p_3$

**Case - IV:** when max. sum is 3, X possible (since after removing 2 digits max. sum should be reduced upto 6 only).

$\therefore$ Total ways $= 4\,! + 3 \times {}^3p_3 + 3 \times {}^3p_3 + 3 \times {}^3p_3 + 3 \times {}^3p_3$

$= 24 + 72 = 96$

**S35.** Since a number is divisible by 6 if it is divisible by 2 and 3 simultaneously.

It is a similar question as that a 4 digit no. is divisible by 3 as well as it is a even number.

**Case - I:** for 3, 4, 5, 6

| | | | |
|---|---|---|---|
| | | | 4, 6 |

${}^3p_3$ 2 ways

$\Rightarrow {}^3p_3 \times 2$

**Case - II:** for 1, 2, 4, 5

| | | | |
|---|---|---|---|
| | | | 2, 4 |

${}^3p_3$ 2 ways

$\Rightarrow {}^3p_3 \times 2$

**Case - III:** for 1, 3, 5, 6

| | | | |
|---|---|---|---|
| | | | 6 |

${}^3p_3$ 1 way

$\Rightarrow {}^3p_3 \times 1$

**Case - IV:** for 2, 3, 4, 6

| | | | |
|---|---|---|---|
| | | | 2,4,6 |

${}^3p_3$ 3 ways

$\Rightarrow {}^3p_3 \times 3$

**Case - V:** for 1, 2, 3, 6

| | | | |
|---|---|---|---|
| | | | 2, 6 |

3 ! 2 ways

$\Rightarrow 3\,! \times 2$

$\therefore$ Total no. divisible by $6 = {}^3p_3 \times 2 + {}^3p_3 \times 1 + {}^3p_3 \times 2 + {}^3p_3 \times 3 + {}^3p_3 \times 2 = {}^3p_3 \times 10$

$= 10 \times 6 = 60.$

**S36.** 1, 2, 3, 4, 5, 6

Max. sum of digits $= 1 + 2 + 3 + 4 + 5 + 6 = 21$

But when we take max. such then digit should be of 6 digit no.

**Case - I:** when max. sum is 18.

$3, 4, 5, 6 \Rightarrow 4!$

**Case - II:** when max. sum is 15.

$1, 3, 5, 6 \Rightarrow 4!$

$2, 3, 4, 6 \Rightarrow 4!$

**Case - III:** when max. sum is 12.

$1, 2, 3, 6 \Rightarrow 4!$

$1, 2, 4, 5 \Rightarrow 4!$

**Case - IV:** when max. sum is 9. X possible (since after removing 2 digits, max. sum should be reduced upto 11 only).

$\therefore$ Total ways = 4! + 4! + 4! + 4! + 4!

$= 5 \times 4! = 5 \times 24 = 120$

**S37.** **Case - I** when 5 is at unit place.

| 0X | | | | | 5 |
|---|---|---|---|---|---|
| 4 ways | | $^4p_4$ | | | 1 way |

Total no. of ways for case - I = $4 \times {^4p_4} \times 1$

$= 4! \times 4$

**Case - II:** when 0 is at unit place.

| | | | | | 0 |
|---|---|---|---|---|---|
| | | $^5p_5$ | | | 1 way |

Total no. of ways for case - II = $5! \times 1$

$\therefore$ Total no. of 6 digits divisible by 5

$= 5! \times 1 + 4! \times 4$

$= 120 + 24 \times 4 = 120 + 96 = 216$

**S38.** No. divisible by 4 are 04, 12, 20, 24, 32, 40, 52

$\because$ 04 can be treated as single digit number

**Case - I:**

| | | | | · | · |
|---|---|---|---|---|---|
| | 4 ! | | | 3 ways (04, 40, 20) | |

Total ways for case - I = $4! \times 3$

**Case - II:**

| 0X | | | | · | · |
|---|---|---|---|---|---|
| 3 ways | $^3p_3$ | | | 4 ways (12, 24, 32, 52) | |

Total ways for case - II = $3 \times 3! \times 4$

$\therefore$ Total no. of 6 digits = $4! \times 3 + 3 \times 3! \times 4$

$= 72 + 72 = 144$

Date: 10/10/2021

**Q1.** If ${}^nC_9 = {}^nC_8$, find ${}^nC_{17}$.

**Q2.** If ${}^nC_8 = {}^nC_6$ then find the value of ${}^nC_2$.

**Q3.** If ${}^nC_8 = {}^nC_2$, find ${}^nC_2$.

**Q4.** Prove that ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$.

**Q5.** Prove that $\dfrac{{}^nC_r}{{}^nC_{r-1}} = \dfrac{n-r+1}{r}$.

**Q6.** Prove that if ${}^nC_p = {}^nC_q$ then $p = q$ or $p + q = n$

**Q7.** Verify $2 \times {}^7C_4 = {}^8C_4$.

**Q8.** Prove that ${}^nC_r = \dfrac{n!}{r!(n-r)!}$.

**Q9.** Prove that ${}^nC_r + 2.{}^nC_{r-1} + {}^nC_{r-2} = {}^{n+2}C_r$.

**Q10.** Find $n$ and $r$, if ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 3 : 4 : 5$.

**Q11.** If ${}^nC_{10} = {}^nC_{12}$ determine $n$ and hence find ${}^nC_5$.

**Q12.** Determine $n$ if ${}^{2n}C_3 : {}^nC_2 = 12 : 1$.

**Q13.** If ${}^nC_r = 84$, ${}^nC_{r-1} = 36$ and ${}^nC_{r+1} = 126$, then find the value of $n$.

**Q14.** Find $n$ and $r$, if ${}^nC_r : {}^nC_{r+1} : {}^nC_{r+2} = 1 : 2 : 3$

**Q15.** Find $n$ and $r$, if ${}^np_r = {}^nP_{r+1}$, ${}^nC_r = {}^nC_{r-1}$

**Q16.** Prove that the product of $k$ consecutive positive integer is divisible by $k!$.

**Q17.** Prove that ${}^{2n}C_n = \dfrac{2^n \cdot [1 \cdot 3 \cdot 5 \ldots (2n-1)]}{n!}$

**Q18.** Prove that $n \cdot {}^{n-1}C_{r-1} = (n - r + 1).{}^nC_{r-1}$.

**Q19.** A committee of 3 persons is to be constituted from a group of 2 men and 3 women. In how many ways can this be done? How many of these committees would consist of 1 man and 2 women?

**Q20.** Find $r$, if (a) ${}^{18}C_r = {}^{18}C_{r+2}$; (b) ${}^{15}C_r : {}^{15}C_{r-1} = 11 : 5$.

**Q21.** Find $r$, if (a) ${}^{15}C_{3r} = {}^{15}C_{r+3}$; (b) ${}^8C_r - {}^7C_3 = {}^7C_2$.

**Q22.** Find $n$, if (a) ${}^nC_7 = {}^nC_5$; (b) ${}^nC_{10} = {}^nC_{15}$.

**Q23.** What is the number of ways of choosing 4 cards from a pack of 52 playing cards? In how many of these

(i) four cards are of the same suit,
(ii) four cards belong to four different suits,
(iii) are face cards,
(iv) two are red cards and two are black cards,
(v) cards are of the same colour?

**Q24.** Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls of each colour.

**Q25.** A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has (i) no girl? (ii) at least one boy and one girl? (iii) at least 3 girls?

**Q26.** How many words, with or without meaning, each of 3 vowels and 2 consonants can be formed from the letters of the word INVOLUTE?

**Q27.** Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

**Q28.** In how many ways one can select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

**Q29.** A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

**Q30.** Find $n$ and $r$, if ${}^{n+1}C_{r+1} : {}^{n}C_{r} = 11 : 6$, ${}^{n}C_{r} : {}^{n-1}C_{r-1} = 2 : 1$

**Q31.** Find $n$, if (a) ${}^{2n}C_3 : {}^{n}C_3 = 11 : 1$; (b) ${}^{n}C_6 : {}^{n-3}C_3 = 33 : 4$.

**Q32.** If ${}^{15}C_{3r} : {}^{15}C_{r+1} = 11 : 3$ then find the value of $r$.

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**S1.** We have $^nC_9 = {}^nC_8$

*i.e.*, $$\frac{n!}{9!(n-9)!} = \frac{n!}{(n-8)!8!}$$

or $$\frac{1}{9} = \frac{1}{n-8} \quad \text{or} \quad n - 8 = 9 \quad \text{or} \quad n = 17$$

Therefore $^nC_{17} = {}^{17}C_{17} = 1.$

**S2.** If $^nC_x = {}^nC_y$ and $x \neq y$ then $x + y = n$

Hence, $n = 8 + 6 = 14$

Now, $$^nC_2 = {}^{14}C_2 = \frac{14 \times 13}{2} = 91.$$

**S3.** Given, $^nC_8 = {}^nC_2$

So, $8 + 2 = n$

or $n = 10$ $\quad [{}^nC_a = {}^nC_b \Rightarrow a + b = n]$

Thus, $$^nC_2 = {}^{10}C_2 = \frac{10!}{2!(10-2)!}$$

$$= \frac{10!}{2!8!} = \frac{10 \times 9 \times 8!}{2!8!}$$

$$= \frac{10 \times 9}{2} = 45.$$

**S4.** $$^nC_r + {}^nC_{r-1} = \frac{n!}{r!(n-r)!} + \frac{n!}{(r-1)!(n-r+1)!}$$

$$= \frac{(n-r+1)n! + r\,n!}{r!(n-r+1)!}$$

$$= \frac{(n+1)n!}{r!(n-r+1)!} = \frac{(n+1)!}{r!(n+1-r)!}.$$

$$= {}^{n+1}C_r.$$

**S5.** $$\frac{^nC_r}{^nC_{r-1}} = \frac{n!}{r!(n-r)!} \cdot \frac{(n-r+1)!(r-1)!}{n!}$$

$$= \frac{(n-r+1)(n-r)!(r-1)!}{r\cdot(r-1)!(n-r)!}$$

$$= \frac{n-r+1}{r}.$$

**S6.** $${}^nC_p = {}^nC_q$$

$\Rightarrow$ $${}^nC_p = {}^nC_{n-q} \qquad [\because \ {}^nC_r = {}^nC_{n-r}]$$

$\Rightarrow$ $$p = q \quad \text{or} \quad p = n - q$$

$\Rightarrow$ $$p = q \quad \text{or} \quad p + q = n$$

**S7.** From L.H.S.

$$2 \times {}^7C_4 = 2 \times \frac{7!}{4!(7-4)!} = 2 \times \frac{7!}{4!\cdot 3!}$$

$$= 2 \times 7 \times 5 = 70$$

From R.H.S.

$${}^8C_4 = \frac{8!}{4!(8-4)!} = \frac{8!}{4!\cdot 4!}$$

$$= 2 \times 7 \times 5 = 70$$

Hence, L.H.S. = R.H.S. **Proved.**

**S8.** ${}^np_r$ = Number of ways of arranging $r$ objects from given $n$ objects.

${}^nC_r$ = Number of ways of selecting $r$ objects from given $n$ objects.

To find ${}^np_r$, we first select $r$ objects from given $n$ objects which can be done in ${}^nC_r$ ways, having selected we can arrange $r$ objects $r$ ! ways.

$\Rightarrow$ $${}^np_r = {}^nC_r \cdot r!$$

$\Rightarrow$ $$\frac{n!}{(n-r)!} = {}^nC_r \cdot r!$$

$\Rightarrow$ $${}^nC_r = \frac{n!}{r!(n-r)!}$$

**S9.** $${}^nC_r + 2.{}^nC_{r-1} + {}^nC_{r-2} = [{}^nC_r + {}^nC_{r-1}] + [{}^nC_{r-1} + {}^nC_{r-2}]$$

$$= {}^{n+1}C_r + {}^{n+1}C_{r-1}$$

$$= {}^{(n+1)+1}C_r = {}^{n+2}C_r.$$

**S10.** $$\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{4}{3} \Rightarrow \frac{n-r+1}{r} = \frac{4}{3}$$

$\Rightarrow$ $$3n - 7r + 3 = 0 \qquad \dots \text{(i)}$$

$$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{5}{4} \Rightarrow \frac{n-r}{r+1} = \frac{5}{4}$$

$\Rightarrow$ $$4n - 9r - 5 = 0 \qquad \dots \text{(ii)}$$

Solving Eq. (i) and (ii), we get

$$n = 62, \quad r = 27.$$

**S11.** $\because$ $\quad {}^nC_x = {}^nC_y \;\Rightarrow\; n = x + y$

$${}^nC_{10} = {}^nC_{12} \;\Rightarrow\; n = 10 + 12 = 22$$

$\therefore$ $\quad {}^nC_5 = {}^{22}C_5 = \dfrac{22\,!}{5\,!\,(22-5)\,!}$

$$= \frac{22\,!}{5\,!\times 17\,!} = 11 \times 21 \times 19 \times 6 = 26334.$$

**S12.** ${}^{2n}C_3 : {}^nC_2 = 12 : 1$

$$\Rightarrow \quad \frac{(2n)\,!}{3\,!\cdot(2n-3)\,!} : \frac{n\,!}{2\,!\,(n-2)\,!} = \frac{12}{1}$$

$$\Rightarrow \frac{2n(2n-1)(2n-2)}{3} \times \frac{1}{n(n-1)} = \frac{12}{1}$$

$$\Rightarrow \quad \frac{4(2n-1)}{3} = \frac{12}{1}$$

$$\Rightarrow \quad (2n-1) = 12 \times \frac{3}{4} \;\Rightarrow\; (2n-1) = 9$$

$$\Rightarrow \quad 2n = 9 + 1 \;\Rightarrow\; 2n = 10$$

$$\Rightarrow \quad n = 5.$$

**S13.** ${}^nC_r = 84$, ${}^nC_{r-1} = 36$ and ${}^nC_{r+1} = 126$

$$\Rightarrow \quad \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{84}{36} \;\Rightarrow\; \frac{n-r+1}{r} = \frac{84}{36}$$

$$\frac{n-r+1}{r} = \frac{84}{36} \qquad \ldots \text{(i)}$$

$$\frac{{}^nC_{r+1}}{{}^nC_r} = \frac{126}{84} \;\Rightarrow\; \frac{n-r}{r+1} = \frac{126}{84}$$

$$\frac{n-r}{r+1} = \frac{126}{84} \qquad \ldots \text{(ii)}$$

After solving Eq. (i) and (ii), we get

$$r = 3, \quad n = 9$$

**S14.** $\quad \dfrac{{}^nC_{r+1}}{{}^nC_r} = \dfrac{2}{1} \;\Rightarrow\; \dfrac{n-r}{r+1} = 2$

$\Rightarrow \qquad n - 3r - 2 = 0 \qquad \ldots \text{(i)}$

$$\frac{{}^nC_{r+2}}{{}^nC_{r+1}} = \frac{3}{2} \quad \Rightarrow \quad \frac{n-r-1}{r+2} = \frac{3}{2}$$

$\Rightarrow \qquad 2n - 5r - 8 = 0 \qquad \ldots \text{(ii)}$

Solving Eq. (i) and (ii), we get

$$n = 14, \quad r = 4.$$

**S15.**

$${}^np_r = {}^nP_{r+1}$$

$$\Rightarrow \quad \frac{n!}{(n-r)!} = \frac{n!}{(n-r-1)!}$$

$$\frac{1}{(n-r)(n-r-1)!} = \frac{1}{(n-r-1)!}$$

$\Rightarrow \qquad n - r = 1 \qquad \ldots \text{(i)}$

$${}^nC_r = {}^nC_{r-1} \quad \Rightarrow \quad \frac{{}^nC_r}{{}^nC_{r-1}} = 1$$

$$\Rightarrow \quad \frac{(n-r+1)}{r} = 1$$

$\Rightarrow \qquad n - 2r + 1 = 0 \qquad \ldots \text{(ii)}$

Solving Eq. (i) and (ii), we get

$$n = 3, \quad r = 2.$$

**S16.** Let the numbers be $n, n-1, n-2, \ldots n-k+1$.

Let $x = n(n-1)(n-2) \ldots (n-k+1)$

$$x = \frac{[n(n-1)(n-2)\ldots(n-k+1)]\,[(n-k)(n-k-1)\ldots 2\cdot 1]}{(n-k)(n-k-1)\ldots 2\cdot 1}$$

$$= \frac{n!}{(n-k)!}$$

$$\Rightarrow \quad \frac{x}{k!} = \frac{n!}{k!(n-k)!} \quad \Rightarrow \quad \frac{x}{k!} = {}^nC_k$$

Here, R.H.S. gives us the number of ways of selecting $k$ objects from given $n$ objects.

$\Rightarrow$ R.H.S. is a natural number

$\Rightarrow$ L.H.S. must be a natural number

$\Rightarrow$ $x$ is divisible by $k!$.

**S17.**

$${}^{2n}C_n = \frac{(2n)!}{n!(2n-n)!}$$

$$= \frac{[(2n)(2n-1)(2n-2)\ldots 2\cdot 1]}{n!\,n!}$$

$$= \frac{[(2n)(2n-2)(2n-4)\dots 4\cdot 2][(2n-1)\dots 3\cdot 1]}{n!\,n!}$$

$$= \frac{2^n[n(n-1)\dots 1][1\cdot 3\dots (2n-1)]}{n!\,n!}$$

$$= \frac{2^n[1.3\dots(2n-1)]}{n!}.$$

**S18.**

$$n\cdot {}^{n-1}C_{r-1} = \frac{n\cdot(n-1)!}{(r-1)!\cdot[(n-1)-(r-1)]!}$$

$$= \frac{n!}{(r-1)!\,(n-r)!} \quad \dots \text{(i)}$$

$$(n-r+1)\cdot {}^{n}C_{r-1} = (n-r+1)\cdot\frac{n!}{(r-1)!\,[(n-(r-1)]!}$$

$$= \frac{(n-r+1)\cdot n!}{(r-1)!\,(n-r+1)\cdot(n-r)!}$$

$$= \frac{n!}{(r-1)!\,(n-r)!}. \quad \dots \text{(ii)}$$

From Eq. (i) and (ii), we get the result.

**S19.** Here, order does not matter. Therefore, we need to count combinations. There will be as many committees as there are combinations of 5 different persons taken 3 at a time. Hence, the required number of ways = ${}^5C_3 = \frac{5!}{3!\,2!} = \frac{4\times 5}{2} = 10$.

Now, 1 man can be selected from 2 men in ${}^2C_1$ ways and 2 women can be selected from 3 women in ${}^3C_2$ ways. Therefore, the required number of committees = ${}^2C_1 \times {}^3C_2 = \frac{2!}{1!\,1!}\times\frac{3!}{2!\,1!} = 6$.

**S20.** (a)

$${}^{18}C_r = {}^{18}C_{r+2}$$

$$\Rightarrow \quad r + r + 2 = 18$$

$$\Rightarrow \quad r = 8$$

(b)

$$\frac{{}^{15}C_r}{{}^{15}C_{r-1}} = \frac{11}{5}$$

$$\Rightarrow \quad \frac{15!}{r!(15-r)!}\times\frac{(r-1)!(16-r)!}{15!} = \frac{11}{5}$$

$$\frac{(16-r)\cdot(15-r)!\,(r-1)!}{r(r-1)!\,(15-r)!} = \frac{11}{5}$$

$$\frac{16-r}{r} = \frac{11}{5}$$

$$\Rightarrow \quad 80 - 5r = 11r \quad\Rightarrow\quad r = 5.$$

**S21.** (a) $^{15}C_{3r} = {^{15}C_{r+3}}$

$\Rightarrow$ $3r = r + 3$ or $3r + r + 3 = 15$

$\Rightarrow$ $r = \frac{3}{2}$ or $r = 3$, As $r \in N$

$r = 3$

(b) $^{8}C_r - {^7C_3} = {^7C_2}$

$\Rightarrow$ $^{8}C_r = {^7C_3} + {^7C_2}$

$\Rightarrow$ $^{8}C_r = {^8C_3}$

$\Rightarrow$ $r = 3$ or $r + 3 = 8$

$\Rightarrow$ $r = 3$ or $r = 5$.

**S22.** (a) $^nC_7 = {^nC_5}$

$\Rightarrow$ $$\frac{n!}{7!(n-7)!} = \frac{n!}{5!(n-5)!}$$

$\Rightarrow$ $$\frac{1}{7 \cdot 6 \cdot 5! \cdot (n-7)!} = \frac{1}{5!(n-5)(n-6)(n-7)1}$$

$\Rightarrow$ $(n-5)(n-6) = 42$

$\Rightarrow$ $n^2 - 11n - 12 = 0$

$\Rightarrow$ $(n-12)(n+1) = 0$

$\Rightarrow$ $n = 12$, $n = -1$ but $n \in N$

$\Rightarrow$ $n = 12$

[Also, we can say that $^nC_7 = {^nC_5} \Rightarrow n = 7 + 5 = 12$ ]

(b) $^nC_{10} = {^nC_{15}}$

$\Rightarrow$ $n = 10 + 15 = 25$

**S23.** There will be as many ways of choosing 4 cards from 52 cards as there are combinations of 52 different things, taken 4 at a time. Therefore

The required number of ways = $^{52}C_4 = \frac{52!}{4!\,48!} = \frac{49 \times 50 \times 51 \times 52}{2 \times 3 \times 4} = 270725$.

(i) There are four suits: diamond, club, spade, heart and there are 13 cards of each sit. Therefore, there are $^{13}C_4$ ways of choosing 4 diamonds. Similarly, there are $^{13}C_4$ ways of choosing 4 clubs, $^{13}C_4$ ways of choosing 4 spades and $^{13}C_4$ ways choosing 4 hearts. Therefore

The required number of ways = $^{13}C_4 + {^{13}C_4} + {^{13}C_4} + {^{13}C_4} = 4 \times \frac{13!}{4!\,9!} = 2860$.

(ii) There are 13 cards in each suit.

Therefore, there are $^{13}C_1$ ways of choosing 1 card from 13 cards of diamonds, $^{13}C_1$ ways of choosing 1 card from 13 cards of hearts, $^{13}C_1$ ways of choosing 1 card from 13 cards of clubs, $^{13}C_1$ ways of choosing 1 card from 13 cards of spads. Hence, by multiplication principle, the required number of ways

$$= {}^{13}C_1 \cdot {}^{13}C_1 \cdot {}^{13}C_1 \cdot {}^{13}C_1 = 13^4.$$

(iii) There are 12 face cards and 4 are to be selected out of these 12 cards. This can be done in $^{12}C_4$ ways. Therefore, the required number of ways $\frac{12!}{4!\,8!} = 495$.

(iv) There are 26 red cards and 26 black careds. Therefore, the required number of ways

$$= {}^{26}C_2 + {}^{26}C_2 = 105625.$$

(v) 4 red cards can be selected out of 26 red cards in $^{26}C_4$ ways, 4 black cards can be selected out of 26 black cards in $^{26}C_4$ ways.

Therefore, the required number of ways

$$= {}^{26}C_4 + {}^{26}C_4 = 29900.$$

**S24.** 3 red balls can be selected out of 6 red balls in $^6C_3$ ways.

Similarly, 3 white balls can be selected out of 5 white balls in $^5C_3$ ways and 3 blue balls can be selected out of 5 blue balls in $^5C_3$ ways.

By F.P.C., the number of ways in which 9 balls can be selected is $^6C_3 \times {}^5C_3 \times {}^5C_3$

$$= \frac{6!}{3!3!} \times \frac{5!}{3!2!} \times \frac{5!}{3!2!}$$

$$= \frac{6 \times 5 \times 4}{3 \times 2} \times \frac{5 \times 4}{2} \times \frac{5 \times 4}{2}$$

$$= 20 \times 10 \times 10 = 2000.$$

**S25.** (i) Since, the team will not include any girl, therefore, only boys are to be selected. 5 boys out of 7 boys can be selected in $^7C_5$ ways. Therefore, the required number of ways

$$= {}^7C_5 = \frac{7!}{5!\,2!} = \frac{6 \times 7}{2} = 21$$

(ii) Since, at least one boy and one girl are to be there in every team. Therefore, the team can consist of

(a) 1 boy and 4 girls (b) 2 boys and 3 girls

(c) 3 boys and 2 girls (d) 4 boys and 1 girl

1 boy and 4 girls can be selected in $^7C_1 \times {}^4C_4$ ways.

2 boys and 3 girls can be selected in $^7C_2 \times {}^4C_3$ ways.

3 boys and 2 girls can be selected in $^7C_3 \times {}^4C_2$ ways.

4 boys and 1 girl can be selected in $^7C_4 \times {}^4C_1$ ways.

Therefore, the required number of ways

$= {}^7C_1 \times {}^4C_4 + {}^7C_2 \times {}^4C_3 + {}^7C_3 \times {}^4C_2 + {}^7C_4 \times {}^4C_1$

$= 7 + 84 + 210 + 130 = 431.$

(iii) Since, the team has to consist of at least 3 girls, the team can consist of (a) 3 girls and 2 boys or (b) 4 girls and 1 boy.

Note that the team cannot have all 5 girls, because, the group has only 4 girls.

3 girls and 2 boys can be selected in ${}^4C_3 \times {}^7C_2$ ways.

4 girls and 1 boy can be selected in ${}^4C_4 \times {}^7C_1$ ways.

Therefore, the required number of ways

$$= {}^4C_3 \times {}^7C_2 + {}^4C_4 \times {}^7C_1 = 84 + 7 = 91.$$

**S26.** In the word INVOLUTE, there are 4 vowels, namely I, O, E, U and 4 consonants namely N, V, L and T.

The number of ways of selecting 3 vowels out of 4 = ${}^4C_3 = 4$.

The number of ways of selecting 2 consonants out of 4 = ${}^4C_2 = 6$.

Therefore, the number of combinations of 3 vowels and 2 consonants is $4 \times 6 = 24$.

Now, each of these 24 combinations has 5 letters which can be arranged among themselves in 5! ways. Therefore, the required number of different words is $24 \times 5! = 2880$.

**S27.** In a pack of 52 cards there are 4 aces and 48 other cards. We have to choose 1 ace and 4 other cards. This can be done in ${}^4C_1 \times {}^{48}C_4$ ways. Therefore,

The required number of ways $= {}^4C_1 \times {}^{48}C_4 = \frac{4!}{1!3!} \times \frac{48!}{4!44!}$

$$= \frac{4 \times 48 \times 47 \times 46 \times 45}{4 \times 3 \times 2} = 778320.$$

**S28.** No. of players = 17

No. of bowlers = 5

No. of players who are not bowlers = 17 – 5 = 12

To select a cricket team of 11 players we have to choose 7 players who are not bowlers out of 12 players in ${}^{12}C_7$ ways and 4 bowlers can be chosen out of 5 bowlers in ${}^5C_4$ ways.

Total number of ways $= {}^{12}C_7 \times {}^5C_4 = \frac{12!}{7!5!} \times \frac{5!}{4!1!}$

$$= \frac{12 \times 11 \times 10 \times 9 \times 8 \times 5}{5 \times 4 \times 3 \times 2} = 3960.$$

**S29.** Two black balls can be selected out of 5 black balls in $^5C_2$ ways.

Similarly, 3 red balls can be selected out of 6 red balls in $^6C_3$ ways.

Therefore, the number of ways in which 2 black balls and 3 red balls can drawn is $^5C_2 \times {}^6C_3$. Now,

$$^5C_2 \times {}^6C_3 = \frac{5!}{2!3!} \times \frac{6!}{3!3!} = 10 \times 20 = 200$$

Thus, out of 5 black and 6 red balls, 2 black and 3 red balls can be selected in 200 ways.

**S30.**

$$\frac{^{n+1}C_{r+1}}{^nC_r} = \frac{11}{6}$$

$$\Rightarrow \quad \frac{(n+1)!}{(r+1)!(n-2)!} \times \frac{r!(n-r)!}{n!} = \frac{11}{6}$$

$$\Rightarrow \quad \frac{n+1}{r+1} = \frac{11}{6}$$

$$\Rightarrow \quad 6n - 11r - 5 = 0 \qquad \dots (i)$$

$$\frac{^nC_r}{^{n-1}C_{r-1}} = \frac{2}{1}$$

$$\Rightarrow \quad \frac{n!}{r!(n-r)!} \times \frac{(r-1)!(n-r)!}{(n-1)!} = 2$$

$$\Rightarrow \quad \frac{n}{r} = 2 \quad \Rightarrow \quad n = 2r \qquad \dots (ii)$$

Solving Eq. (i) and (ii), we get

$$n = 10, \quad r = 5.$$

**S31.** (a) Here,

$$\frac{^{2n}C_3}{^nC_3} = \frac{11}{1}$$

$$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n!} = \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \cdot \frac{(n-3)!}{n(n-1)\cdot(n-2)(n-3)!} = 11$$

$$\Rightarrow \quad \frac{4(2n-1)}{n-2} = 11$$

$$\Rightarrow \quad 11n - 22 = 8n - 4$$

$$\Rightarrow \quad 3n = 18$$

$$\Rightarrow \quad n = 6$$

(b)

$$\frac{^nC_6}{^{n-3}C_3} = \frac{33}{4}$$

$$\Rightarrow \quad \frac{n!}{6!(n-6)!} \times \frac{3!(n-6)!}{(n-3)!} = \frac{33}{4}$$

$$\frac{n(n-1)(n-2)(n-3)!\,3!}{6 \cdot 5 \cdot 4 \cdot 3!(n-3)!} = \frac{33}{4}$$

$$n(n-1)(n-2) = 990$$

$$= 11 \cdot 10 \cdot 9$$

$$\Rightarrow \quad n = 11$$

**S32.** ${}^{15}C_{3r} : {}^{15}C_{r+1} = 11 : 3$

Clearly $r$ can be 0, 1, 2, 3, 4, 5 but possibilities of $r = 0$ or 5 are clearly ruled out (as ${}^{15}C_0 = {}^{15}C_{15} = 1$)

$\therefore \quad {}^{15}C_{3r} : {}^{15}C_{r+1} = 11 : 3$

For $\quad r = 2$

$${}^{15}C_{3r} = {}^{15}C_6 = \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$${}^{15}C_{r+1} = {}^{15}C_3 = \frac{15 \times 14 \times 13}{6}$$

$\therefore \quad {}^{15}C_{3r} : {}^{15}C_{r+1} \neq 11 : 3$

For $\quad r = 3$

$${}^{15}C_{3r} = {}^{15}C_9 = \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10}{6 \times 5 \times 4 \times 3 \times 2 \times 1}$$

$${}^{15}C_{r+1} = {}^{15}C_4 = {}^{15}C_{11} = \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1}$$

$\therefore \quad {}^{15}C_{3r} : {}^{15}C_{r+1} = 11 : 3$

For $\quad r = 4$

$${}^{15}C_{3r} = {}^{15}C_{12} = {}^{15}C_3 = \frac{15 \times 14 \times 13}{3 \times 2 \times 1}$$

$${}^{15}C_{r+1} = {}^{15}C_5 = {}^{15}C_{10} = \frac{15 \times 14 \times 13 \times 12 \times 11 \times 10}{5 \times 4 \times 3 \times 2 \times 1}$$

$\therefore \quad {}^{15}C_{3r} : {}^{15}C_{r+1} = 1 : 66$

Thus, $\quad r = 3.$

Date: 10/10/2021

**Q1.** For the post of 5 clerks, there are 25 applicants, 2 posts are reserved for SC candidates and remaining for others, there are 7 SC candidates among the applicants. In how many ways can the selection be made?

**Q2.** A question paper has two parts *A* and *B* each containing 10 questions. If a student has to choose 8 from part *A* and 4 from *B*, in how many ways can he choose the questions?

**Q3.** A bag contains 4 red 3 white and 2 blue balls, three balls are drawn at random, determine the number of ways of selecting balls of different colours.

**Q4.** A man has seven friends. In how many ways can he invite one or more of them to a party?

**Q5.** From 7 consonants and 4 vowels, how many different words can be formed consisting of 3 consonants and 2 vowels?

**Q6.** To fill 12 vacancies, there are 25 candidates of which 5 are from schedule castes. If 3 of the vacancies are reserved for schedule caste candidates while the rest are open to all. Find the number of ways in which the selection can be made.

**Q7.** In how many ways a committee of 4 members be selected from 5 men and 4 women?

**Q8.** In an examination, a candidate has to pass in each of the 5 subjects. In how many ways can he fail?

**Q9.** Find the number of ways in which 5 identical balls can be distributed among 10 identical boxes if not more than one ball can go into a box?

**Q10.** How many chords can be drawn through 21 points on a circle?

**Q11.** There are 10 points on a plane of which no three points are collinear if lines are formed joining these points. Find the maximum points of intersection of these lines.

**Q12.** Out of 5 men and 2 women, a committee of 3 is to be formed. In how many ways can it be formed if at least one woman is to be included?

**Q13.** In how many of the permutations of $n$ things taken $r$ at a time will three given things occur?

**Q14.** In a network of railways, a small island has 15 stations. Find the number of different types of tickets to be printed for each class if every station must have tickets for other stations.

**Q15.** 28 games played in a football tournament with each team playing once against each other How many teams were there?

**Q16.** Find the maximum number of points of intersection of 6 circles.

**Q17.** A father with 8 children wants to go to zoo as often as he can without taking the same three children more than once. How often will he go and how often will each child go?

**Q18.** A cricket team of 11 players is to be selected from 16 players including 5 bowlers and 2 wicket keepers. In how many ways can a team be selected so as to consist of exactly 3 bowlers and one wicket keeper?

**Q19.** In how many ways can 10 different books on English and 5 similar books on Hindi be placed in a row on a shelf so that two books on Hindi are not together?

**Q20.** In how many ways a group of 11 boys can be divided into two groups of 6 and 5 boys each?

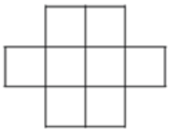
**Q21.** A polygon has 35 diagonals. Find the number of its sides.

**Q22.** How many diagonals are there in a polygon of $n$ sides?

**Q23.** $m$ men and $n$ women are to be seated in a row so that no two women sit together. If $m > n$, then show that the number of ways in which they can be seated is

$$\frac{m!\,(m+1)!}{(m-n+1)!}$$

**Q24.** Six $X$'s have to be placed in the squares of the figure given below such that each row contains at least one $X$. In how many different ways can this be done?



**Q25.** Find the number of ways of dividing 15 things into groups of 8, 4 and 3 respectively.

**Q26.** A box contains two white balls, three black balls, and four red balls. In how many ways can three balls be drawn from the box if at least one black ball to be included in the draw?

**Q27.** A committee of 5 is to be selected from among 6 boys and 5 girls. Determine the number of ways of selections if the committee is to consist of at least one boy and one girl.

**Q28.** From a class of 10 boys and 6 girls, 10 students are to be selected for a competition, at least including 4 boys and 4 girls. The 2 girls who won the prizes last year should be included. In how many ways the selection can be made?

**Q29.** There are two round tables one with $m$ seats and the other with $n$ seats around it. In how many ways can $(m + n)$ guests be seated at them?

**Q30.** There are 15 points in a plane, of which 6 are collinear. How many (a) straight lines (b) triangles can be formed by joining them?

**Q31.** There are $n$ points on a circle, find the number of

(a) lines which can be drawn. (b) triangles which can be formed.

**Q32.** A committee of 5 persons is to be formed out of 6 men and 4 women. In how many ways can this be done, if

(a) at least 2 women are included? (b) at most 2 women are included?

**Q33.** In how many ways can 11 players be chosen out of 15 if

(a) there is no restriction? (b) a particular player is always chosen?

(c) a particular player is never chosen?

**Q34.** A candidate is required to attempt 6 out of 10 questions which are divided into groups each containing 5 questions, and he is not permitted to attempt more than 4 questions from each group. How many ways can he make up his choice?

**Q35.** There are how many rectangles in a chessboard? Of these how many are squares?

**Q36.** Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold five balls. In how many different ways can we place the balls, so that no box remains empty?

**Q37.** A candidate is required to answer 7 questions out of 12 questions which are divided into two groups of 6 questions each. He is not permitted to attempt more than 5 questions from either group. In how many ways can he choose 7 questions?

**Q38.** The English alphabet has 5 vowels and 21 consonants. How many words with two different vowels and 2 different consonants can be formed from the alphabet?

**Q39.** How many words, with or without meaning, each of 2 vowels and 3 consonants can be formed from the letters of the words DAUGHTER?

**Q40.** In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 sepcific courses are compulsory for every student?

**Q41.** From a class of 25 students, 10 are to be chosen for an excursion party. There are 3 students who decide that either all of them will join or none of them will join. In how manv ways can the excursion party be chosen?

**Q42.** Determine the number of 5 card combinations out of a deck of 52 cards, if each selection of 5 cards has exactly one king.

**Q43.** In an examination, a question paper consists of 12 questions divided into two parts *i.e.*, Part I and Part II containing 5 and 7 questions, respectively. A student is required to attempt 8 questions in all, selecting at least 3 from each part. In how many way can a student select the questions?

**Q44.** In how many ways words with or without meaning can be formed with the letter of the word EQUATION so that the vowels and 2 consonants occur together?

**Q45.** A man has 7 relatives, 4 of them are ladies and 3 gentlemen, his wife has 7 relatives and 3 of them are ladies and 4 gentlemen. In how many ways can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of husband's relatives and 3 of wife's relatives?

**Q46.** A student is allowed to select at most $n$ books from a collection of $(2n + 1)$ books. If the total number of ways he can select a book is 63, find the value of $n$.

**Q47.** A question paper contains 12 questions divided into 3 parts, part $A$ contains 6 questions while parts $B$ and $C$ contain 3 questions each. A candidate is required to attempt 6 questions selecting at least two questions from A and at least one form each of parts $B$ and $C$. In how many ways can the candidate select 6 questions?

**Q48.** The sides $AB$, $BC$, $CA$ of a triangle $ABC$ have 3, 4 and 5 interior points respectively on them, find the number of triangles that can be constructed using these points as vertices.

**Q49.** A Committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of : (i) exactly 3 girls? (ii) at least 3 girls? (iii) at most 3 girls?

Date: 10/10/2021

**S1.** 2 SC candidates to be selected from 7 and this can be done in $^7C_2$ ways. The remaining 3 posts are to be filled from the remaining 18 candidates and this can be done in $^{18}C_3$ ways.

$\therefore$ The total ways = $^7C_2 \times {}^{18}C_3 = 21 \times 816 = 17136$.

**S2.** The number of ways of choosing 8 questions from part $A$ = $^{10}C_8$ and the number of ways of choosing 4 questions from part $B$ = $^{10}C_4$.

Therefore, the total number of ways = $^{10}C_8 \times {}^{10}C_4 = 45 \times 210 = 9450$.

**S3.** One red ball can be selected in $^4C_1$ ways, one white ball can be selected in $^3C_1$ ways, one blue ball can be selected in $^2C_1$ ways.

$\Rightarrow$ The total number of ways of choosing balls = $^4C_1 \times {}^3C_1 \times {}^2C_1 = 4 \times 3 \times 2 = 24$.

**S4.** He can invite one, two, ..., seven friends in $^7C_1$, $^7C_2$, ... $^7C_7$ ways respectively. Therefore, the total ways to invite

$$^7C_1 + {}^7C_2 + \dots + {}^7C_7 = 7 + 21 + 35 + 35 + 21 + 7 + 1 = 127.$$

**S5.** The consonants and the vowels can be selected in $^7C_3$ and $^4C_2$ ways respectively. Having selected, these 5 letters can be arranged in 5! ways. Therefore, the total number of words formed.

$$^7C_3 \times {}^7C_2 \times 5\,! = 35 \times 6 \times 120 = 25200.$$

**S6.** 3 SC candidates (out of 5) can be selected in $^5C_3$ ways and remaining 9 candidates (out of remaining 22) can be selected in $^{22}C_9$ ways.

Therefore, the total ways are

$$^5C_3 \times {}^{22}C_9 = 10({}^{22}C_9) = 4974200$$

**S7.** It can be said that out of 9 we have to select 4 persons and this can be done in $^9C_4$ ways.

Now, $$^9C_4 = \frac{9\,!}{4\,!\,5\,!} = 126$$

**S8.** A candidate can fail by failing in 1 or 2 or 3 or 4 or 5 subjects out of 5 in each case and the total number of ways in which this can happen

$$= {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5 = 5 + 10 + 10 + 5 + 1 = 31$$

**S9.** Out of 10 boxes we have to choose 5 boxes, because balls and boxes are identical, and this can be done in

$$^{10}C_5 = \frac{10\,!}{5\,!\cdot 5\,!} = 252 \text{ ways.}$$

**S10.** A chord can be drawn by joining any two points on a circle. Therefore, the number of chords that can be drawn out of 21 points is $^{21}C_2$, which is given by

$$^{21}C_2 = \frac{21!}{2!19!} = \frac{21 \times 20 \times 19!}{2! \times 19!} = 210.$$

Thus, the number of chords is 210.

**S11.** Two points are required to form a line. Then the number of lines is equal to the number of ways two points are selected.

$$= {}^{10}C_2 = 45.$$

Now, two lines intersect at one points. Hence, no. of points of intersection of lines is ${}^{45}C_2 = 990$.

**S12.** There are two possible cases

The committee is consisting of one woman and two men and number of ways of doing this is ${}^2C_1 \times {}^5C_2 = 2 \times 10 = 20$.

The committee is consisting of two women and one man and the number of ways of doing this is ${}^2C_2 \times {}^5C_1 = 1 \times 5 = 5$

$\Rightarrow$ The total number of ways of forming the committee = 20 + 5 = 25.

**S13.** According to given condition, we have to select $(r-3)$ things from remaining $(n-3)$ things and permute these $r$ things.

So, the number of permutation is

$$({}^{n-3}C_{r-3}) \cdot r! = \frac{(n-3)!\,r!}{(r-3)!\,(n-r)!}.$$

**S14.** For each pair of stations two different types of tickets are required.

Now the number of selections of 2 stations from 15 stations = ${}^{15}C_2$.

$\therefore$ Required number of types of tickets

$$= 2 \times {}^{15}C_2 = 2 \times \frac{15!}{2! \cdot 13!}$$

$$= 15 \times 14 = 210.$$

**S15.** Let the number of teams be $n$. Then no. of matches to be played is ${}^nC_2 = 28$

$$\Rightarrow \quad \frac{n(n-1)}{2} = 28 \quad \Rightarrow \quad n^2 - n - 56 = 0$$

$$\Rightarrow \quad (n-8)(n+7) = 0 \quad \Rightarrow \quad n = 8, \quad \text{As } n \neq -7$$

Hence, there are 8 teams in the match.

**S16.** Two circles intersect maximum at two distinct points.

Now, two circle can be selected in ${}^6C_2$ ways.

Again, each selection of two circle gives two points of intersection. Therefore, the total no. of points of intersection is ${}^6C_2 \times 2 = 30$.

**S17.** Out of 8 the father has to take 3 children at a time and this can be done in ${}^8C_3 = 56$ ways. And one child will go as many times as the number of ways of choosing 2 children from remaining 7 and this can be done in ${}^7C_2 = 21$ ways. Therefore, the father goes 56 times and each child will go 21 times.

**S18.** 3 bowlers out of 5 can be selected in ${}^5C_3$ ways. One wicketkeeper out of 2 can be selected in ${}^2C_1$ ways.

Now, 7 more players to be selected from remaining 9 players and this can be done in ${}^9C_7$ ways.

Therefore, the total ways of selection

$${}^5C_3 \times {}^2C_1 \times {}^9C_7 = 10 \times 2 \times 36 = 720.$$

**S19.** $xExExE \ . \ . \ . \ xEx$

There are 10 books on English and as two Hindi books are not to be kept together we can place them at the place them at the places marked with $x$. There are 11 places for Hindi books ($\because$ there are 11, $x$ marks)

Therefore, the total ways to arrange = $10\,! \times {}^{11}C_5 = 462 \cdot 10!. = 1656505600$

**S20.** The number of ways of choosing 6 out of 11 is same as the number of ways of choosing 5 out of 11 which is same as the number of ways of dividing into 6 and 5 and the number of ways of doing this

$${}^{11}C_6 = \frac{11!}{6\,!\cdot 5\,!} = 462.$$

**S21.** The number of sides be $n$.

$$\Rightarrow \quad {}^nC_2 - n = 35$$

$$\Rightarrow \quad \frac{n(n-1)}{2} - n = 35$$

$$\Rightarrow \quad n(n-3) = 70$$

$$\Rightarrow \quad n^2 - 3n - 70 = 0$$

$$\Rightarrow \quad (n-10)(n+7) = 0$$

$$\Rightarrow \quad n = 10, -7 \quad \text{but } n \in N$$

$$\Rightarrow \quad n = 10.$$

**S22.** The number of lines obtained by joining two of $n$ points

$$= {}^nC_2 = \frac{n(n-1)}{2}$$

of these $n$ are the sides of polygon.

Therefore, the number of diagonals

$$= \frac{n(n-1)}{2} - n = n\left[\frac{n-1}{2} - 1\right] = \frac{n(n-3)}{2}.$$

**S23.** $m$ men can be arranged in $m!$ ways. As no two women are to be together we have $m + 1$ places for women. Therefore, the women can be seated in ${}^{m+1}p_n$ ways. Hence the total number of ways of seating men and women

$$= m! \times {}^{m+1}p_n = \frac{m!(m+1)!}{(m-n+1)!}.$$

**S24.** In all there are 8 squares in which 6$X$'s have to be placed which can be done in ${}^8C_6 = 28$ ways.

But this includes the possibilities of top and bottom horizontal rows not having $X$. Therefore, the required number of ways = 28 – 2 = 26

**S25.** First, we select 8 out of 15 and this can be done ${}^{15}C_8$ ways. Now, from the remaining 7 we select 4, this can be done in ${}^7C_4$ ways and finally from remaining 3 we have to select 3, this can be done in ${}^3C_3$ ways. Therefore, the total number of ways of forming the group

$$= {}^{15}C_8 \times {}^7C_4 \times {}^3C_3$$

$$= \frac{15!}{8!\cdot 7!} \times \frac{7!}{4!\cdot 3!} \times \frac{3!}{3!\cdot 0!} = \frac{15!}{8!\cdot 4!\cdot 3!} = 225225.$$

**S26.** We present the solution in the following tabular form

| Black | Possibilities of choosing the balls White or Red | Number of ways of selections |
|---|---|---|
| 1 | 2 | ${}^3C_1 \times {}^6C_2 = 45$ |
| 2 | 1 | ${}^3C_2 \times {}^6C_1 = 18$ |
| 3 | 0 | ${}^3C_3 \times {}^6C_0 = 1$ |

Therefore, the total number of ways = 45 + 18 + 1 = 64.

**S27.** The possibilities are

| Number of boys | Number of girls | Number of ways of selections |
|---|---|---|
| 1 | 4 | ${}^6C_1 \times {}^5C_4 = 30$ |
| 2 | 3 | ${}^6C_2 \times {}^5C_3 = 150$ |
| 3 | 2 | ${}^6C_3 \times {}^5C_2 = 200$ |
| 4 | 1 | ${}^6C_4 \times {}^5C_1 = 75$ |

The total number of selection = 30 + 150 + 200 + 75 = 455.

**S28.** As 2 particular girls have to be included, we have to choose 8 students out of 10 boys and 4 girls (choosing at least 4 boys and atleast 2 girls).Now the possibilities are

| Number of boys | Number of girls | Number of ways of selections |
|---|---|---|
| 4 | 4 | ${}^{10}C_4 \times {}^4C_4 = 210$ |
| 5 | 3 | ${}^{10}C_5 \times {}^4C_3 = 1008$ |
| 6 | 2 | ${}^{10}C_6 \times {}^4C_2 = 1260$ |

Therefore, the total number of ways = 210 + 1008 + 1260 = 2478.

**S29.** **Step - I:** We divide $(m + n)$ guests into two groups of $m$ and $n$ guests and this can be done in

$$ {}^{m+n}C_m = {}^{m+n}C_n = \frac{(m+n)!}{m!\cdot n!} \text{ ways.} $$

**Step - II:** $m$ guests can be seated at the round table with $m$ seats $(m – 1)!$ ways.

And $n$ guests can be seated at the round table with $n$ seats in $(n – 1)!$ ways. Therefore, the total number of ways of seating the guests $\frac{(m+n)!}{m!\cdot n!} \times (m-1)! \times (n-1)! = \frac{(m+n)!}{m\cdot n}$.

**S30.** (a) Number of straight lines (when no two points are collinear) = ${}^{15}C_2 = 105$. But, 6 points are collinear which means ${}^6C_2 = 15$ lines will not be there

$\Rightarrow$ The number of lines are 105 – 15 + 1 = 91.

(b) Number of triangles = ${}^{15}C_3 = 455$. But, 6 points are collinear which means ${}^6C_3 = 20$ triangles will not be there

$\Rightarrow$ The number of triangles = 455 – 20 = 435.

**S31.** (a) Two points determine a line

$\Rightarrow$ The number of lines will be the number of ways of choosing two points out of $n$.

$\Rightarrow$ The number of line = ${}^nC_2 = \frac{n(n-1)}{2}$.

(b) Three points determine a triangle

$\Rightarrow$ The number of triangles = ${}^nC_3 = \frac{n(n-1)(n-2)}{6}$.

**S32.** (a) The possibilities are (2 women + 3 men), (3 women + 2 men) and (4 women + 1 man) and the number of ways of doing this

$$ = ({}^4C_2 \times {}^6C_3) + ({}^4C_3 \times {}^6C_2) + ({}^4C_4 \times {}^6C_1) $$
$$ = (6 \times 20) + (4 \times 15) + (1 \times 6) = 120 + 60 + 6 = 186. $$

(b) The possibilities are (No woman + 5 men), (1 woman + 4 men) and (2 women + 3 men) and the number of ways of doing this

$$ = ({}^4C_0 \times {}^6C_5) + ({}^4C_1 \times {}^6C_4) + ({}^4C_2 \times {}^6C_3) $$
$$ = (1 \times 6) + (4 \times 15) + (6 \times 20) = 6 + 60 + 120 = 186. $$

**S33.** (a) The number of ways of choosing 11 players out of 15

$$^{15}C_{11} = \frac{15!}{4!\cdot 11!} = 1365.$$

(b) When a particular player is always selected then 10 more will be selected from 14 players and this can be done in

$$^{14}C_{10} = \frac{14!}{10!\cdot 4!} = 1001 \text{ ways.}$$

(c) When a particular player is never selected than 11 players will be selected from 14 players and this can be done in

$$^{14}C_{11} = \frac{14!}{11!\cdot 3!} = 364 \text{ ways.}$$

**S34.** We present the solution in the following tabular form.

| Possibilities of choosing questions from Group I | Group II | Number of ways of selections |
|---|---|---|
| 4 | 2 | $^5C_4 \times {}^5C_2 = 50$ |
| 3 | 3 | $^5C_3 \times {}^5C_3 = 100$ |
| 2 | 4 | $^5C_2 \times {}^5C_4 = 50$ |

Therefore, the total number of ways = 50 + 100 + 50 = 200.

**S35.** In a chess board, there are 9 vertical and 9 horizontal lines we know that a rectangle is formed with 2 vertical and 2 horizontal lines. Two vertical and 2 horizontal lines can be selected in $^9C_2 \times {}^9C_2 = 1296$ ways.

$\Rightarrow$ The number of rectangles = 1296

Now, the number of squares, formed by two adjacent vertical and two adjacent horizontal lines $= 8^2 = 64$.

The number of squares, formed by two vertical lines (lines 1st and 3rd, 2nd and 4th, ...) and two horizontal lines in same order $= 7^2$. Similarly using first and fourth lines, second and fifth lines, etc. we get $6^2 = 36$ squares and so on. Total number of squares $= 8^2 + 7^2 + 6^2 + 5^2 + 4^2 + 3^2 + 2^2 + 1^2 = 204$.

**S36.** We first put one ball in each box in order to ensure that no box remains empty and this can be done in $5 \times 4 \times 3 = 60$ ways.

Now, we are left with 2 more balls to be placed, for which we have two possibilities

(a) One ball in each of the 2 boxes which can be done in $3 \times 2 = 6$ ways.

(b) Both balls in any of the 3 boxes which can be done in 3 ways.

Therefore, we have 6 + 3 = 9 ways of putting 2 balls in 3 boxes.

Hence, the required ways $= 60 \times 9 = 540$

**S37.** We present the solution in the following tabular form

| Possibilities of choosing questions from Group I | Group II | Number of ways of selections |
|---|---|---|
| 5 | 2 | ${}^6C_5 \times {}^6C_2 = 90$ |
| 4 | 3 | ${}^6C_4 \times {}^6C_3 = 300$ |
| 3 | 4 | ${}^6C_3 \times {}^6C_4 = 300$ |
| 2 | 5 | ${}^6C_2 \times {}^6C_5 = 90$ |

Therefore, the total number of ways

$$= 90 + 300 + 300 + 90 = 780.$$

**S38.** Two different vowels can be chosen out of 5 vowels in ${}^5C_2$ ways and two different consonants can be chosen out of 21 consonants in ${}^{21}C_2$ ways. The two vowels and two consonants together within themselves can be arranged in 4! ways.

$$\text{Required number of words} = {}^5C_2 \times {}^{21}C_2 \times 4!$$

$$= \frac{5!}{2!3!} \times \frac{21!}{2!19!} \times 4!$$

$$= \frac{5 \times 4 \times 21 \times 20 \times 4 \times 3 \times 2}{2 \times 2} = 50400.$$

**S39.** DAUGHTER has 3 vowels and 5 consonants.

2 vowels can be selected out of 3 in ${}^3C_2 = {}^3C_1$

$$= \frac{3!}{1!2!} = 3 \text{ ways}$$

3 consonants can be selected out of 5 in ${}^5C_3 = {}^5C_2$

$$= \frac{5!}{2!3!} = 10 \text{ ways}$$

Therefore, the number of combinations of 2 vowels and 3 consonants is $3 \times 10 = 30$.

Now, each of the 30 combinations consists of 5 letters which can be arranged among themselves in ${}^5P_5$ ways. Therefore, the required number of different words is $30 \times 5! = 3600$.

**S40.** Total no. of courses available = 9.

No. of courses which are not compulsory

$$9 - 2 = 7$$

As two specific courses are compulsory, a student can choose them in ${}^2C_2$ ways. Further remaining three courses can be chosen from 7 non-compulsory courses in ${}^7C_3$ ways.

Total number of ways

$$^7C_3 \times {}^2C_2 = \frac{7!}{3!4!} \times 1 = \frac{7 \times 6 \times 5}{3 \times 2} = 35.$$

**S41.** No. of students = 25

No. of students to be selected = 10

For three particular students to join, we are only to select 10 – 3 = 7 students out of 25 – 3 = 22 students and this can be done in

$$^{22}C_7 = \frac{22 \times 21 \times 20 \times 19 \times 18 \times 17 \times 16}{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7}$$

$$= 170544 \text{ ways.}$$

For three particular students not to join, we are to select 10 students out of 25 – 3 = 22 students and this can be done in

$$^{22}C_{10} = \frac{22 \times 21 \times 20 \times 19 \times 18 \times 17 \times 16 \times 15 \times 14 \times 13}{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1} = 646646$$

Total number of ways = 170544 + 646646 = 817190.

**S42.** In a pack of 52 cards there are 4 kings and 48 other cards. We have to choose 1 king and 4 other cards. This can be done in $^4C_1 \times {}^{48}C_4$ ways. Therefore,

The required number of ways = $^4C_1 \times {}^{48}C_4$

$$= \frac{4!}{1!3!} \times \frac{48!}{4!44!}$$

$$= \frac{4 \times 48 \times 47 \times 46 \times 45}{4 \times 3 \times 2} = 778320.$$

**S43.** The candidate has to select 8 questions in all of which at least 3 questions are from each part. Following are the choices.

| Choice | Part I | Part II |
|---|---|---|
| (i) | 3 | 5 |
| (ii) | 4 | 4 |
| (iii) | 5 | 3 |

If the candidate follows choce (i), then the number of ways in which he can select the questions is $^5C_3 \times {}^7C_5$. If the candidate follows choice (ii) then the number of ways will $^5C_4 \times {}^7C_4$ and for choice (iii) the number of ways will be $^5C_5 \times {}^7C_3$.

The total number of ways of selecting the questions (by the student) as required

$$= ({}^5C_3 \times {}^7C_5) + ({}^5C_4 \times {}^7C_4) + ({}^5C_5 \times {}^7C_3)$$

$$= (10 \times 21) + (5 \times 35) + (1 \times 35)$$

$$= 210 + 175 + 35 = 420.$$

**S44.** There are 5 vowels and 3 consonants in the word 'EQUATION'. Three vowels out of 5 and 2 consonants out of 3 can be chosen in $^5C_3 \times {}^3C_2$ ways. So, there are $^5C_3 \times {}^3C_2$ groups each containing 3 consonants and two vowels. Now, each group contains 5 letters which are to be arranged in such a way that 2 consonants occur together. Considering 2 consonants as one letter, we have 4 letters which can be arranged in 4! ways. But two consonants can be put together in 2! ways. Therefore, 5 letters in each group can be arranged in $4! \times 2!$ ways.

Hence, the required number of words

$$= ({}^5C_3 \times {}^3C_2) \times 4! \times 2! = 1440.$$

**S45.** We present the solution in the following tabular form.

| Possibilies of choosing guest from number of ways of selections | | | |
|---|---|---|---|
| Husband's relatives | | Wife's relatives | |
| Ladies | Gents | Ladies | Gents |
| 3 | | | 3 |
| $^4C_3 \times {}^4C_3 = 16$ | | | |
| 2 | 1 | 1 | 2 |
| $^4C_2 \times {}^3C_1 \times {}^3C_1 \times {}^4C_2 = 324$ | | | |
| 1 | 2 | 2 | 1 |
| $^4C_1 \times {}^3C_2 \times {}^3C_2 \times {}^4C_1 = 144$ | | | |
| | 3 | 3 | |
| $^3C_3 \times {}^3C_3 = 1$ | | | |

Hence, the total way = 16 + 324 + 144 + 1 = 485.

**S46.** Since he is allowed to select at most $n$ books out of $(2n + 1)$ books. He can select one or two three or ..., $n$ books which can be done in $^{2n+1}C_1, {}^{2n+1}C_2, \ldots {}^{2n+1}C_n$ ways respectively.

Hence, the total ways of selecting at most n books

$$= {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n = 63 \qquad \text{[Given]}$$

We know that

$$= {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_{2n+1} = 2^{2n+1}$$

Also $$= {}^{2n+1}C_0 = {}^{2n+1}C_{2n+1} = 1, \; {}^{2n+1}C_1 = {}^{2n+1}C_{2n} \text{ etc.}$$

$\Rightarrow \quad [{}^{2n+1}C_0 + {}^{2n+1}C_{2n+1}] + 2[{}^{2n+1}C_1 + {}^{2n+1}C_2 + \ldots + {}^{2n+1}C_n] = 2^{2n+1}$

$\Rightarrow \quad 1 + 1 + 2(63) = 2^{n+1}$

$\Rightarrow \quad 128 = 2^{2n+1}$

$\Rightarrow \quad 2^7 = 2^{2n+1} \quad \Rightarrow \quad 2n + 1 = 7 \quad \Rightarrow \quad n = 3$

**S47.** We present the solution in the following tabular form

| Possibilities of choosing questions from | | | Number of ways of selections |
|---|---|---|---|
| Part A | Part B | Part C | |
| 4 | 1 | 1 | ${}^6C_4 \times {}^3C_1 \times {}^3C_1 = 135$ |
| 3 | 2 | 1 | ${}^6C_3 \times {}^3C_2 \times {}^3C_1 = 180$ |
| 3 | 1 | 2 | ${}^6C_3 \times {}^3C_1 \times {}^3C_2 = 180$ |
| 2 | 3 | 1 | ${}^6C_2 \times {}^3C_3 \times {}^3C_1 = 45$ |
| 2 | 1 | 3 | ${}^6C_2 \times {}^3C_1 \times {}^3C_3 = 45$ |
| 2 | 2 | 2 | ${}^6C_2 \times {}^3C_2 \times {}^3C_2 = 135$ |

Therefore, the total ways are 135 + 180 + 180 + 45 + 45 + 135 = 720.

**S48.** In all there are

$$3 + 4 + 5 = 12 \text{ points}$$

We know that 3 (not collinear) points form a triangle.

Therefore, the total number of triangles = ${}^{12}C_3 = 220$

These include the number of triangles formed by 3 points on $AB = {}^3C_3 = 1$.

The number of triangles formed by 4 points on $BC = {}^4C_3 = 4$

The number of triangles formed by 5 points on CA = ${}^5C_3 = 10$

Hence, the required number of triangles = 220 – (1 + 4 + 10) = 205

**S49.** Here, No. of boys = 9 and No. of girls = 4

(i) Since exactly 3 girls are to be selected, we have to choose 3 girls and 4 boys.

$$\text{No of committees} = {}^4C_3 \times {}^9C_4 = {}^4C_1 \times {}^9C_5$$

$$= 4 \times \frac{9!}{5!4!} = 504$$

(ii) Since at least 3 girls are to be selected, the different combinations are:

(a) 3 girls and 4 boys (b) 4 girls and 3 boys

No. of committees of the type (a)

$$= {}^4C_3 \times {}^9C_4 = {}^4C_1 \times {}^9C_5$$

$$= 4 \times \frac{9!}{5!4!} = 504$$

No. of committees of the type (b)

$$= {}^4C_4 \times {}^9C_3 = 1 \times {}^9C_6$$

$$= \frac{9!}{6!3!} = 84$$

Total number of ways of selection

$$= 504 + 84 = 588$$

(iii) Since at most 3 girls are to be selected, the different groupings are.

(a) 1 girl and 6 boys (b) 2 girls and 5 boys (b) 3 girls and 4 boys

No. of committees of the type (a)

$$= {}^4C_1 \times {}^9C_6 = 4 \times 84 = 336$$

No. of committees of the type (b)

$$= {}^4C_2 \times {}^9C_5 = 756$$

No. of committees of the type (c)

$$= {}^4C_3 \times {}^9C_4 = 504$$

Total number of ways = 336 + 756 + 504 = 1596.

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**Q1.** Find the number of ways in which 8 different beads can be arranged to form a necklace.

**Q2.** In how many ways can 7 person sit round a table so that all shall not have the same neighbours in any two arrangements.

**Q3.** In how many ways can 7 students be arranged in a circle?

**Q4.** Find the number of ways in which 10 different diamonds can be arranged to make a necklace.

**Q5.** Find the number of ways in which six persons can be seated at a round table, so that all shall not have the same neighbours in any two arrangements.

**Q6.** Five boys and five girls sit alternatively around a round table, in how many ways can this be done?

**Q7.** If 20 persons were invited for a party, in how many ways can they and the host be seated at a circular table? In how many of these ways will two particular persons be seated on either of the host?

**Q8.** In how many ways can a party of 4 men and 4 women be seated at a circular table so that no two women are adjacent?

**Q9.** 3 boys and 3 girls are to be seated around a table in a circle. Among them, the boy $X$ does not want any girl neighbour and the girl $Y$ does not want any boy neighbour. How many such arrangement are possible?

**Q10.** In how many ways can 6 persons be arranged in a: (a) a line; (b) a circle.

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**S1.** First fixing the position of one bead, the remaining beads can be arranged in 7! ways, but there is no distinction between the clockwise and anticlockwise arrangements.

So the required no. of arrangements = $\frac{1}{2} \times 7! = 2520$.

**S2.** 7 person can sit at round table in 6 ! ways.

But in clockwise and anticlockwise arrangement, each person will have the same neighbours.

So the required number of ways = $\frac{1}{2} \times 6! = 360$

**S3.** $\because$ The number of ways in which $n$ objects can be arranged in circle is $(n - 1)!$.

The number of ways in which 7 students can be arranged in a circle is $(7 - 1)! = 6!$.

**S4.** Since diamonds do not have natural order of left and right so clockwise and anticlockwise arrangements are taken as identical.

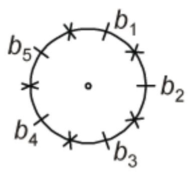
Therefore the number of arrangements of 10 different diamonds to make a necklace is

$$\frac{1}{2} \times 9! = 181440.$$

**S5.** In this case, anticlockwise and clockwise arrangements are same.

Hence, the number of ways of arrangements is $\frac{1}{2} \times 5! = 60$.

**S6.** Five boys can be arranged in a circle in 4! ways.




After that girls can be arranged in the five gaps shown as '$X$' in 5! ways.

Hence, total no. of ways is $4! \times 5! = 2880$

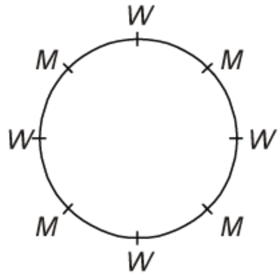
**S7.** Clearly there are 21 persons to be seated round a circular table let us fix the seat of one person say the host, the remaining 20 persons can be arranged in 20! ways.

Hence, the number of ways in which these 21 person can be seated round a circular table = 20!.

If two particular person can be seated on either side of the host in two ways and for each ways of their taking seats, the remaining 18 persons can be seated at circular table in 18! ways.

$\therefore$ The number of ways of seating 21 persons at circular table with two particular persons on either side of the host = $(2 \times 18!)$.

**S8.** The 4 men can be seated at the circular table such that there is a vacant seat between every pair of man.

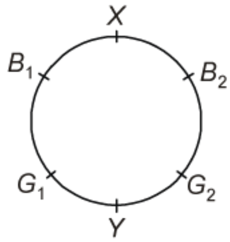



The number of ways in which 4 men can be seated at circular table in $3! = 6$

Now 4 vacant seats can be occupied by 4 women in $4! = 24$ ways.

$\therefore$ Thus the required number of ways $= 6 \times 24 = 144$.

**S9.** Fix the position of $X$ and $Y$ as shown in figure.




Now, the boys $B_1$, $B_2$, and girl $G_1$, $G_2$ may have their neighbour $X$ and $Y$ respectively.

The boys $B_1$, $B_2$ may be arranged among themselves in 2! ways and girl $G_1$, $G_2$ may be arranged among themselves in 2! ways.

Hence, the required no. of arrangements $= 2 \times 2 = 4$.

**S10.** (a) 6 persons can be arranged in a line in 6! ways.

$\Rightarrow$ 720 ways. and

(b) 6 persons can be arranged in a circle in $(6 - 1)! = 5!$ ways

$= 120$ ways.