CLASS : CC (Advanced) **VECTORS & 3-D** **TEST-10**

M.M.: 68 **Time: 60 Min**

PART-A

[SINGLE CORRECT CHOICE TYPE]

Q.1 to Q.10 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct. **[10 × 3 = 30]**

Q.1 A normal line with positive direction cosines to the plane P makes equal angle with coordinate axes. The distance of the point A(1, 2, 3) from the line $\frac{x-1}{1}=\frac{y+2}{1}=\frac{z-3}{2}$ measured parallel to the plane P is equal to

(A) $\sqrt{14}$ (B) $\sqrt{30}$ (C) $\sqrt{35}$ (D) 7

Q.2 $\vec{a}_1, \vec{a}_2, \vec{a}_3$ are non zero and non-coplanar vectors such that

$$(x+y-3)\vec{a}_1+(2x-y+2)\vec{a}_2+(2x+y+\lambda)\vec{a}_3=\vec{0}$$

holds for some x and y, then λ is

(A) $\frac{7}{3}$ (B) 2 (C) $\frac{-10}{3}$ (D) $\frac{5}{3}$

Q.3 Equation of line in the plane $\Pi : 2x-y+z-4=0$ which is perpendicular to the line l whose equation is $\frac{x-2}{1}=\frac{y-2}{-1}=\frac{z-3}{-2}$ and which passes through the point of intersection of l and Π, is

(A) $\frac{x-2}{3}=\frac{y-1}{5}=\frac{z-1}{-1}$ (B) $\frac{x-1}{3}=\frac{y-3}{5}=\frac{z-5}{-1}$

(C) $\frac{x+2}{2}=\frac{y+1}{-1}=\frac{z+1}{1}$ (D) $\frac{x-2}{2}=\frac{y-1}{-1}=\frac{z-1}{1}$

Q.4 Equation of plane which passes through the point of intersection of lines $\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and at greatest distance from the point (0, 0, 0) is

(A) $4x + 3y + 5z = 25$ (B) $4x + 3y + 5z = 50$

(C) $3x + 4y + 5z = 49$ (D) $x + 7y - 5z = 2$

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Q.5 If $\vec{V}_1 = \hat{i} + \hat{j} + \hat{k}$ and $\vec{V}_2 = a\hat{i} + b\hat{j} + c\hat{k}$ where $a, b, c \in \{-2, -1, 0, 1, 2\}$, then number of possible non-zero vectors $\vec{V}_2$ perpendicular to $\vec{V}_1$ is

(A) 10 (B) 13 (C) 15 (D) 18

Q.6 Centroid of the tetrahedron OABC, where $A \equiv (a, 2, 3)$, $B \equiv (1, b, 2)$, $C \equiv (2, 1, c)$ and O is the origin is (1, 2, 3) then the value of $(a^2 + b^2 + c^2)$ is equal to

(A) 75 (B) 80 (C) 121 (D) 125

Q.7 The direction ratios of a normal to the plane through (1, 0, 0), (0, 1, 0) which makes an angle of $\sec^{-1}\sqrt{2}$ with the plane $x + y = 3$ is

(A) $(1, \sqrt{2}, 1)$ (B) $(1, 1, \sqrt{2})$ (C) (1, 1, 2) (D) $(\sqrt{2}, 1, 1)$

Q.8 If the angles between the vectors $\vec{v}_1$ and $\vec{v}_2$, $\vec{v}_2$ and $\vec{v}_3$, $\vec{v}_3$ and $\vec{v}_1$ be $\frac{\pi}{3}, \frac{\pi}{4}$ and $\frac{\pi}{6}$ respectively, then the angle the vector $\vec{v}_1$ makes with the plane containing $\vec{v}_2$ and $\vec{v}_3$ is

(A) $\sin^{-1}\sqrt{1-\sqrt{\frac{2}{3}}}$ (B) $\sin^{-1}\sqrt{\sqrt{\frac{3}{2}}-1}$ (C) $\cos^{-1}\sqrt{1-\sqrt{\frac{2}{3}}}$ (D) $\cos^{-1}\sqrt{\sqrt{\frac{3}{2}}-1}$

Q.9 Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c} = |\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a}\times\vec{b}$ and $\vec{c}$ is 30º, then $\left|(\vec{a}\times\vec{b})\times\vec{c}\right|$ equals

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) 3 (D) $\frac{3\sqrt{3}}{2}$

Q.10 The shortest distance between the lines $2x + y + z - 1 = 0 = 3x + y + 2z - 2$ and $x = y = z$, is

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) $\frac{3}{\sqrt{2}}$ (D) $\frac{\sqrt{3}}{2}$

[PARAGRAPH TYPE]

Q.11 to Q.14 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct. **[4 × 3 = 12]**

Paragraph for question nos. 11 & 12

Let $\hat{x}, \hat{y}$ and $\hat{z}$ be unit vectors such that

$\hat{x}+\hat{y}+\hat{z}=\vec{a}$, $\hat{x}\times(\hat{y}\times\hat{z})=\vec{b}$, $(\hat{x}\times\hat{y})\times\hat{z}=\vec{c}$, $\vec{a}.\hat{x}=\frac{3}{2}$, $\vec{a}.\hat{y}=\frac{7}{4}$ and $|\vec{a}|=2$.

Q.11 The value of $\hat{y}.\hat{z}$ is equal to

(A) $\frac{-1}{4}$ (B) $\frac{3}{4}$ (C) 0 (D) $\frac{1}{2}$

Q.12 The vector $\hat{x}$ in terms of $\vec{a}, \vec{b}$ and $\vec{c}$ is equal to

(A) $\frac{2}{3}(3\vec{a}+4\vec{b}+8\vec{c})$ (B) $\frac{1}{3}(3\vec{a}+4\vec{b}+8\vec{c})$ (C) $\frac{2}{3}(3\vec{a}+2\vec{b}+4\vec{c})$ (D) $\frac{1}{3}(3\vec{a}+2\vec{b}+4\vec{c})$

Paragraph for question nos. 13& 14

Consider, three vectors $\vec{v}_1=\sin\theta\hat{i}+\cos\theta\hat{j}+(a-3)\hat{k}$

$\vec{v}_2=(\sin\theta+\cos\theta)\hat{i}+(\cos\theta-\sin\theta)\hat{j}+(b-4)\hat{k}$

$\vec{v}_3=\cos\theta\hat{i}-\sin\theta\hat{j}+(c-5)\hat{k}$

Q.13 If $\vec{v}_1+\vec{v}_2+\vec{v}_3=\lambda\hat{i}$ where $\theta\in[0,2\pi]$ and $a, b, c\in N$ then number of quadruplets (a, b, c, θ) are

(A) 55 (B) 91 (C) 110 (D) 182

Q.14 If $\vec{v}_1, \vec{v}_2, \vec{v}_3$ represent sides of a triangle PQR in xy-plane and corresponding to triangle PQR there exist one more triangle ABC whose sides are denoted by a, b, c then $\frac{Ar(\Delta ABC)}{Ar(\Delta PQR)}$ is equal to

(A) 6 (B) 9 (C) 12 (D) 24

[MULTIPLE CORRECT CHOICE TYPE]

Q.15 to Q.18 has four choices (A), (B), (C), (D) out of which **ONE OR MORE** may be correct. **[4 × 4 = 16]**

Q.15 Consider the lines $L_1: \vec{r}=\hat{i}-3\hat{k}+\lambda(2\hat{i}-\hat{j}+\hat{k})$, $L_2: \vec{r}=4\hat{i}-3\hat{j}-3\hat{k}+\mu(\hat{i}+\hat{j}+2\hat{k})$, $(\lambda, \mu\in R)$ and the planes $\sigma_1: \vec{r}\cdot(7\hat{i}+\hat{j}+2\hat{k})-3=0$, $\sigma_2: \vec{r}\cdot(3\hat{i}+5\hat{j}-6\hat{k})=4$.

Let $ax-3y+bz=c$ be the equation of plane passing through the point of intersection of lines L_1 and L_2, and perpendicular to the planes σ_1 and σ_2. Then

(A) $a+b=-3$ (B) $a+b=-1$ (C) $b+c=12$ (D) $b+c=11$

Q.16 Let A(1, 1, 1), B(2, 3, 5), C(–1, 0, 2) be three points, then equation of a plane parallel to the plane ABC which is at a distance 2 from it, is

(A) $2x - 3y + z + 2\sqrt{14} = 0$ (B) $2x - 3y + z - 2\sqrt{14} = 0$

(C) $3x - 3y + z + 2 = 0$ (D) $2x - 3y + z - 2 = 0$

Q.17 Let $L_1 : \frac{x-1}{2} = \frac{y-2}{-1} = \frac{z-3}{3}$, $L_2 : \frac{x-1}{-1} = \frac{y-2}{3} = \frac{3(z-3)}{5}$ and $L_3 : \frac{x-1}{-32} = \frac{y-2}{-19} = \frac{z-3}{15}$ be three lines. A plane intersecting these lines at A, B and C respectively such that PA = 2, PB = 3 and PC = 6 where $P \equiv (1, 2, 3)$. If V is the volume of the tetrahedron PABC and d is the perpendicular distance of the plane from the point P then-

(A) V = 18 cubic units (B) V = 6 cubic units

(C) $d = \frac{6}{\sqrt{14}}$ units (D) d = 7 units

Q.18 If $\vec{a}, \vec{b}$ are non-zero and non-collinear vectors, then $\vec{v} = \frac{\vec{a}}{|\vec{a}|} + \frac{\vec{b}}{|\vec{b}|}$ is

(A) a unit vector (B) in the plane of $\vec{a}$ and $\vec{b}$.

(C) equally inclined to $\vec{a}$ and $\vec{b}$ (D) perpendicular to $\vec{a} \times \vec{b}$

PART-D
[INTEGER TYPE]

Q.1 & Q.2 are "Integer Type" questions. (The answer to each of the questions **are upto <u>4 digits</u>**) **[2 × 5 = 10]**

Q.1 If vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are coplanar such that $\overrightarrow{OA}\,\hat{}\,\overrightarrow{OB} = \overrightarrow{OB}\,\hat{}\,\overrightarrow{OC} = 30^\circ$ and area of parallelograms OADB, OBEC and ΔOAC are 6, 10 and 4 square units respectively. If ODFE is a parallelogram and area of quadrilateral BDFE is S, then find the value of $\left[\frac{S}{3}\right]$.

[**Note :** [k] denotes greatest integer less than or equal to k.]

Q.2 Line L meets lines $L_1 : \frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $L_2 : \frac{x-2}{2} = \frac{y-1}{4} = \frac{z-4}{5}$ orthogonally at points P and Q. $(PQ)^2$ is D. DRs of line L are (a, b, c) {a, b, c ∈ I}, then least value of $\frac{5D}{3} + |a| + |b| + |c|$.

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ANSWER KEY
PART-A

Q.1	A	Q.2	C	Q.3	B	Q.4	B	Q.5	D
Q.6	A	Q.7	B	Q.8	B	Q.9	B	Q.10	A
Q.11	C	Q.12	B	Q.13	C	Q.14	C	Q.15	BD
Q.16	AB	Q.17	BC	Q.18	BCD				

PART-D

Q.1	5	Q.2	6