Date: 9/10/2021

Q1. If $z = (x + iy)(ix + y)$, represent z in form of $(a + ib)$.

Q2. If $z = \frac{a - ib}{a + ib}$, represent z in terms of $x + iy$.

Q3. If $z = (x + y)(ix + y)$, represent z in terms of $(a + ib)$, where x and y are real numbers.

Q4. If $z = \left[\frac{1 + i}{1 - i}\right]^{4n+1}$, where $n \in N$, represent z in terms of $x + iy$.

Q5. If $z = \frac{1}{i^7}$ represent z in terms of $x + iy$.

Q6. If $z = (1 + i)^4$ represent z in terms of $x + iy$.

Q7. If $z = (1 - i)^4$ represent z in terms of $x + iy$.

Q8. Express $z = \sqrt{3} \times \sqrt{-3}$ in terms of $x + iy$.

Q9. If $z = \sqrt{37} + \sqrt{-19}$ find the real and imaginary part of complex number z.

Q10. If $z = [(\sqrt{3} + i)(\sqrt{3} - i)]^{-3/2}$ express z in terms of $x + iy$.

Q11. If $z = 2i^2 + 6i^3 + 3i^{16} - 6i^{49} + 4i^{25}$, then represent it in terms of $x + iy$.

Q12. Represent $z = \frac{(1 - i)^3}{1 - i^3}$ in terms of $x + iy$.

Q13. If $z = \frac{1 + i}{1 - i} - \frac{1 - i}{1 + i}$ represent z in terms of $x + iy$.

Q14. Find the real and imaginary part of $(x + i)(x - y)$. Where x and y are real numbers.

Q15. Express $(5 - 3i)^3$ in the form $a + ib$.

Q16. Express the following in the form of $a + bi$: $(5i)\left(\frac{1}{8}i\right)$.

Q17. If $z = (a + ib)(ia - b)$, represent z in terms of $(x + iy)$.

Q18. If $z = (x - iy)(ix + y)$, represent z in form of $(a + ib)$.

Q19. If $z = (ia + b)(a - ib)$, represent z in terms of $(x + iy)$.

Q20. Express the following in the form of $a + bi$: $(i)(2i)\left(\frac{1}{8}i\right)^3$.

Q21. Express the complex number given below in the form $a + ib$:

$$(1 - i) - (-i + i6).$$

Q22. Find real θ such that $\frac{3 + 2i \sin\theta}{1 - 2i \sin\theta}$ is purely real.

Q23. Express the following in the form $a + ib$: i^{-35}.

Q24. Express the following in the form $a + ib$: $\frac{5+\sqrt{2}i}{1-\sqrt{2}i}$.

Q25. Express $(-\sqrt{3}+\sqrt{-2})(2\sqrt{3}-i)$ in the form $a + ib$.

Q26. Express the complex number given below in the form $a + ib$:

$$\left(\frac{1}{5}+i\frac{2}{5}\right)-\left(4+\frac{i5}{2}\right).$$

Q27. Express the complex number given below in the form $a + ib$:

$$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(-\frac{4}{3}+i\right).$$

Q28. Express the complex number given below in the form $a + ib$:

$$\left(\frac{1}{3}+3i\right)^3.$$

Q29. Find the real and imaginary part of $z = \frac{(i+1)}{(i-1)}$.

Q30. If $z = \frac{3+4i}{2-4i}$ represent z in terms of $x + iy$.

Q31. If $z = \frac{(1-i)(2-i)(3-i)}{1+i}$, represent z in terms of $x + iy$.

Q32. If $z = \frac{1}{1-\cos\theta+2i\sin\theta}$, represent z in terms of $x + iy$.

Q33. If $z = \frac{(a+ib)(a-ib)}{(1-i)}$, represent z in terms of $(x + iy)$.

Q34. Represent $z = \frac{a+ib}{(a-ib)}$ in term of $(x + iy)$.

Q35. If $z = \frac{(a+ib)^2}{(a-ib)^2}-\frac{(a-ib)^2}{(a+ib)^2}$, represent z in the form of $x + iy$.

Q36. Find real and imaginary part of complex number $\frac{(x+y)}{(1+i)}$.

Q37. If z = square of $\left(\frac{i}{1+i}\right)$ represent z in terms of $x + iy$.

Q38. Let $z = \sqrt{3} \times \sqrt{-5} \times \sqrt{-6} \times \sqrt{-8}$ express z in terms of $x + iy$.

Q39. If $z = \frac{1+2i}{1-(1-i)^2}$ then represent z in terms of $x + iy$.

Q40. If $z = \left[\frac{1}{(1-2i)}+\frac{3}{1+i}\right]\left[\frac{3+4i}{2-4i}\right]$ find the value of x and y such that $z = x + iy$.

Q41. If $z = \frac{3+2i\sin\theta}{1-2i\sin\theta}$, represent z in terms of $x + iy$.

Q42. Represent $\frac{3}{1+i}-\frac{2}{2-i}+\frac{2}{1-i}$ in terms of $x + iy$.

Q43. If $z = \frac{(a+ib)^2}{a-ib}-\frac{(a-ib)^2}{a+ib}$ such that it can be expressible in terms of $x + iy$ then find the value of x and y respectively.

Q44. If $z = \left(-1 - \frac{1}{\sqrt{3}}i\right)^3$, represent z in terms of $x + iy$.

Q45. If $z = \frac{(i+1)(i+2)}{(i-1)(i-2)}$ represent z in terms of $x + iy$.

Q46. Express the complex number given below in the form $a + ib$:

$$\left(-2 - \frac{1}{3}i\right)^3.$$

Q47. Express the following expression in the form of $a + ib$:

$$\frac{(3 + i\sqrt{5})(3 - i\sqrt{5})}{(\sqrt{3} + \sqrt{2}i) - (\sqrt{3} - i\sqrt{2})}.$$

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S1. $\because$ $z = (x + iy)(ix + y)$

$= ix^2 + xy - xy + iy^2 = i(x^2 + y^2)$

$\therefore$ $z = 0 + i(x^2 + y^2)$

S2.

$$z = \frac{(a - ib)}{(a + ib)} \times \frac{(a - ib)}{(a - ib)} = \frac{(a - ib)^2}{a^2 + b^2}$$

$$= \frac{(a^2 - b^2) - i2ab}{a^2 + b^2} = \frac{(a^2 - b^2)}{(a^2 + b^2)} - i\frac{2ab}{(a^2 + b^2)}$$

S3. $z = (x + y)(ix + y)$

$= ix(x + y) + y(x + y)$

$= y(x + y) + i(x + y).x$

$\therefore$ z can be represented in $(x + iy)$ as $y(x + y) + i(x + y).x$

S4. $\because$

$$\frac{(1+i)^2}{(1-i)(1+i)} = \frac{1+i^2+2i}{1-i^2}$$

$$= \frac{2i}{2} = i$$

$\therefore$ $z = [i]^{4n+1} = i^{4n}.i = i$

$z = 0 + i$

S5. $\because$

$$z = \frac{1}{i^7} = \frac{1}{(i^4)\cdot i^3} = \frac{1}{i^3} = -\frac{1}{i}$$

$$z = \frac{i^2}{i} = i$$

$z = 0 + i$

S6. $z = (1 + i)^4$

$= (1 + i)^2 \cdot (1 + i)^2$

$= (1 + i^2 + 2i)(1 + i^2 + 2i)$

$= (2i)(2i) = -4$

$z = -4 + 0i$

S7. $z = (1 - i)^4$

$\Rightarrow$ $z = (1 - i)^2 \cdot (1 - i)^2$

$= (1 + i^2 - 2i)\,(1 + i^2 - 2i)$

$= (-2i) \times (-2i) = 4i^2 = -4$

$z = -4 + 0i$

S8. $z = \sqrt{3} \times \sqrt{-3} = \sqrt{3} \times i\sqrt{3} = 3i$

$\therefore$ $z = 0 + 3i$

S9. $\because$ $\sqrt{-1} = i \quad \therefore \quad \sqrt{-19} = \sqrt{-1 \times 19}$

$= \sqrt{-1} \cdot \sqrt{19} = i\sqrt{19}$

$\therefore$ $z = \sqrt{37} + i\sqrt{19}$

Hence its real part is $\sqrt{37}$ and imaginary part is $\sqrt{19}$.

S10. $\because$ $(\sqrt{3} + i)\,(\sqrt{3} - i) = (\sqrt{3})^2 - (i)^2 = 4$

$\therefore$ $z = (4)^{-3/2} = (\sqrt{4})^{-3} = \frac{1}{8}$

$z = \frac{1}{8} + 0i$

S11. $\because$ $(i)^{4n} = 1, \qquad (n \in \mathrm{I})$

$\therefore$ $z = 2i^2 + 6i^3 + 3(i^4)^4 - 6(i)^{4 \times 12 + 1} + 4(i)^{4 \times 6 + 1}$

$= -2 - 6i + 3 - 6i + 4i = -8i + 1$

$z = 1 - 8i$

S12. Since, $z = \frac{(1-i)^3}{1-i^3} = \frac{(1-i)^3}{(1-i)(1+i-1)}$

$= \frac{(1-i)^2}{i} = \frac{1-1-2i}{i} = -2$

$z = -2 + 0i$

S13. Since, $z = \frac{1+i}{1-i} - \frac{1-i}{1+i} = \frac{(1+i)^2 - (1-i)^2}{2}$

$$= \frac{(1+i+1-i)(1+i-1+i)}{2} = \frac{2(2i)}{2} = 2i$$

$$z = 0 - 2i$$

S14. $(x + i)(x - y) = x^2 - xy + ix - iy$

$$= (x^2 - xy) + i(x - y)$$

Hence it's real part is $x^2 - xy$ and imaginary part is $(x - y)$.

S15. We have, $(5 - 3i)^3 = 5^3 - 3 \times 5^2 \times (3i) + 3 \times 5\,(3i)^2 - (3i)^3$

$$= 125 - 225i - 135 + 27i$$

$$= -10 - 198i.$$

S16. $(5i)\left(\frac{1}{8}i\right) = \frac{5}{8}i^2 = \frac{5}{8}(1) = \frac{5}{8} = \frac{5}{8} + i0.$

S17. $z = (a + ib)(ia - b)$

$\Rightarrow$ $z = ia^2 - ab - ab - ib^2$

$$= -2ab + i(a^2 + b^2)$$

$\therefore$ z can be represented in forms of $(x + iy)$ as

$$z = -2ab + i(a^2 + b^2)$$

S18. $z = (x - iy)(ix - y)$

$$= ix^2 - xy + xy + iy^2 = i(x^2 + y^2)$$

$\therefore$ z can be represented in $(a + ib)$ as

$$z = 0 + i(x^2 + y^2)$$

S19. $\because$ $z = (ia + b)(a - ib)$

$\Rightarrow$ $z = ia^2 + ab + ab - ib^2$

$\Rightarrow$ $= 2ab + i(a^2 - b^2)$

$\therefore$ z can be represented in forms of $(x + iy)$ as

$$z = 2ab + i(a^2 - b^2).$$

S20. $(i)(2i)\left(\frac{1}{8}i\right)^3 = 2\frac{1}{8\ 8\ 8}i^5 = \frac{1}{256}(i^2)^2\, i = \frac{-1}{256}i.$

S21. Let $(1 - i) - (-i + i6) = 1 - i + 1 - i6$

$$= 2 - i - 6i = 2 - 7i.$$

S22. We have, $\frac{3 + 2i\sin\theta}{1 - 2i\sin\theta} = \frac{(3 + 2i\sin\theta)(1 + 2i\sin\theta)}{(1 - 2i\sin\theta)(1 + 2i\sin\theta)}$

$$= \frac{3 + 6i \sin\theta + 2i \sin\theta - 4\sin^2\theta}{1 + 4\sin^2\theta}$$

$$= \frac{3 - 4\sin^2\theta}{1 + 4\sin^2\theta} + \frac{8i \sin\theta}{1 + 4\sin^2\theta}$$

We are given the complex number to the real. Therefore

$$\frac{8\sin\theta}{1 + 4\sin^2\theta} = 0, \qquad i.e., \quad \sin\theta = 0$$

Thus, $\theta = n\pi, \quad n \in Z.$

S23. We have, $i^{-35} = \frac{1}{(i^2)^{17}\, i} = \frac{1}{-i} \times \frac{i}{1} = \frac{i}{-i^2} = i$

S24. We have, $\frac{5 + \sqrt{2}i}{1 - \sqrt{2}i} = \frac{5 + \sqrt{2}i}{1 - \sqrt{2}i} \times \frac{1 + \sqrt{2}i}{1 + \sqrt{2}i} = \frac{5 + 5\sqrt{2}i + \sqrt{2}i - 2}{1 - (\sqrt{2}i)^2}$

$$= \frac{3 + 6\sqrt{2}i}{1 + 2} = \frac{3(1 + 2\sqrt{2}i)}{3} = 1 + 2\sqrt{2}i\,.$$

S25. We have, $(-\sqrt{3} + \sqrt{-2})(2\sqrt{3} - i) = (-\sqrt{3} + \sqrt{2}i)(2\sqrt{3} - i)$

$$= -6 + \sqrt{3}i + 2\sqrt{6}i - \sqrt{2}i^2$$

$$= (-6 + \sqrt{2}) + \sqrt{3}(1 + 2\sqrt{2})\,i\,.$$

S26. We have $\left(\frac{1}{5} + i\,\frac{2}{5}\right) - \left(4 + \frac{i5}{2}\right) = \frac{1}{5} + \frac{2i}{5} - 4 - \frac{5i}{2}$

$$= \frac{1}{5} - 4 + \frac{2i}{5} - \frac{5i}{2}$$

$$= \frac{1 - 20}{5} + \frac{4i - 25i}{10}$$

$$= -\frac{19}{5} - \frac{21i}{10}.$$

S27. We have

$$\left[\left(\frac{1}{3} + i\,\frac{7}{3}\right) + \left(4 + i\,\frac{1}{3}\right)\right] - \left(-\frac{4}{3} + i\right) = \left[\frac{1}{3} + \frac{7i}{3} + 4 + \frac{i}{3}\right] + \frac{4}{3} - i$$

$$= \left(\frac{1}{3} + 4\right) + \frac{7i}{3} + \frac{i}{3} + \frac{4}{3} - i$$

$$= \frac{13}{3} + \frac{8i}{3} + \frac{4}{3} - i = \frac{17}{3} + \frac{5i}{3}\,.$$

S28. We have

$$\left(\frac{1}{3}+3i\right)^3 = \left(\frac{1}{3}\right)^3 + (3i)^3 + 3\times\frac{1}{3}\times 3i\left(\frac{1}{3}+3i\right)$$

$$= \frac{1}{27} + 27i^3 + 3i\left(\frac{1}{3}+3i\right)$$

$$= \frac{1}{27} + 27i^3 + i + 9i^2$$

$$= \frac{1}{27} - 27i + i - 9$$

$$= -\frac{242}{27} - 26i\,.$$

S29. Since,

$$z = \frac{(i+1)}{(i-1)}$$

$$z = \frac{(i+1)(i+1)}{(i-1)(i+1)} = \frac{i^2+2i+1}{i^2-1}$$

$\Rightarrow$ $$z = \frac{-1+2i+1}{-2} \quad \Rightarrow \quad z = -i$$

$$z = -i$$

Hence, it's real part is 0 and imaginary part is –1.

S30. Since,

$$z = \frac{3+4i}{2-4i}$$

$\because$ $$z = \frac{(3+4i)(2+4i)}{(2-4i)(2+4i)} = \frac{6+12i+8i-16}{20}$$

$$= \frac{-10+20i}{20} = \frac{-1}{2}+i$$

Hence, it's real part is $\frac{-1}{2}$ and imaginary part is 1.

$\Rightarrow$ $$z = \frac{-1}{2}+i$$

S31. Since,

$$z = \frac{(1-i)(2-i)(3-i)}{1+i}$$

$$\frac{(1-i)(1-i)}{(1+i)(1-i)} = \frac{1+i^2-2i}{2} = -i$$

$$z = -i(2-i)(3-i)$$
$$= (-2i+1)(3-i) = -6i - 2 + 3 - i$$
$$z = 1 - 7i.$$

S32. Since,

$$z = \frac{1}{1-\cos\theta+2i\sin\theta} \times \frac{1-\cos\theta-2i\sin\theta}{(1-\cos\theta-2i\sin\theta)}$$

$$= \frac{1-\cos\theta-2i\sin\theta}{(1-\cos\theta)^2-(2i\sin\theta)^2}$$

$$= \frac{1-\cos\theta-2i\sin\theta}{1+\cos^2\theta-2\cos\theta+4\sin^2\theta} = \frac{1-\cos\theta-2i\sin\theta}{2-2\cos\theta+3\sin^2\theta}$$

$$z = \frac{1-\cos\theta}{2+3\sin^2\theta-2\cos\theta} - \frac{2i\sin\theta}{2-2\cos\theta+3\sin^2\theta}.$$

S33.

$$z = \frac{(a+ib)(a-ib)}{(1-i)}$$

$\Rightarrow$ $$z = \frac{(a^2+b^2)(1+i)}{(1-i)\times(1+i)}$$

$$= \frac{(a^2+b^2)+i(a^2+b^2)}{2}$$

$\Rightarrow$ $$= \left(\frac{a^2+b^2}{2}\right) + i\frac{(a^2+b^2)}{2}$$

$\therefore$ z in form of $(x + iy)$ can be represented as

$$z = \left(\frac{a^2+b^2}{2}\right) + i\left(\frac{a^2+b^2}{2}\right).$$

S34.

$$z = \frac{(a+ib)}{(a-ib)} \times \frac{(a+ib)}{(a+ib)}$$

$\Rightarrow$ $$z = \frac{(a^2-b^2)+2iba}{a^2+b^2}$$

$\Rightarrow$ $$z = \frac{(a^2-b^2)}{(a^2+b^2)} + i\frac{2ab}{(a^2+b^2)}$$

S35.

$$z = \frac{(a+ib)^2}{(a-ib)^2} - \frac{(a-ib)^2}{(a+ib)^2}$$

$$= \frac{(a+ib)^4-(a-ib)^4}{(a-ib)^2(a+ib)^2} = \frac{((a+ib)^2)^2-((a-ib)^2)^2}{(a^2+b^2)^2}$$

$$= \frac{1}{(a^2+b^2)^2}\left\{\left\{(a+ib)^2+(a-ib)^2\right\}-\left\{(a+ib)^2-(a-ib)^2\right\}\right\}$$

$$= \frac{1}{(a^2+b^2)^2}\left\{2(a^2-b^2)-(2a\times 2ib)\right\}$$

$$= \frac{2(a^2-b^2)}{a^2+b^2} - i\frac{4ab}{a^2+b^2}$$

S36. $\because$ $\frac{x+y}{1+i} = \frac{(x+y)(1-i)}{(1+i)(1-i)}$

$$= \frac{x-ix+y-iy}{1-i^2} = \frac{(x+y)-i(x+y)}{2}$$

$$= \left(\frac{x+y}{2}\right) - i\left(\frac{x+y}{2}\right)$$

Hence, $\text{Re}(z) = \frac{x+y}{2}, \quad im(z) = \frac{-(x+y)}{2}$

S37. $z = \left(\frac{i}{1+i}\right)^2 = \frac{i^2}{(1+i)^2}$

$$= \frac{-1}{1-1+2i} = \frac{-1}{2i}$$

$\therefore$ $z = 0 - \frac{1}{2i}$

S38. $z = \sqrt{3} \times \sqrt{-5} \times \sqrt{-6} \times \sqrt{-8}$

$$= \sqrt{3} \times i\sqrt{5} \times i\sqrt{6} \times i\sqrt{8}$$

$$= i^3 \cdot \sqrt{3\times 5\times 6\times 8}$$

$$= -i\sqrt{3^2\times 5\times 2^4} = -i\times 12\sqrt{5}$$

$$= -i \cdot 12\sqrt{5}$$

$\therefore$ $z = 0 - i\cdot 12\sqrt{5}$

S39. Since, $z = \frac{1+2i}{1-(1-i)^2} = \frac{1+2i}{(1-1+i)(1+1-i)}$

$$= \frac{1+2i}{i(2-i)} = \frac{(1+2i)(1+2i)}{(1-2i)(1+2i)}$$

$$= \frac{1+4i^2+4i}{1+4} = \frac{-3+4i}{5} = \frac{-3}{5}+\frac{4}{5}i$$

$$z = \frac{-3}{5}+\frac{4}{5}i$$

S40. $\because$ $$\frac{1}{1-2i}+\frac{3}{1+i} = \frac{1+i+3(1-2i)}{(1-2i)(1+i)}$$

$$= \frac{1+i+3-6i}{1+i-2i+2} = \frac{4-5i}{3-i}$$

$$z = \frac{(4-5i)}{(3-i)}\times\frac{(3+4i)}{(2-4i)} = \frac{(2-4i)(4-5i)-(3-i)(3+4i)}{(3-i)(2-4i)}$$

$$= \frac{8-10i-16i+20-9-12i+3i-4}{6-12i-2i+4} = \frac{15-35i}{10-14i}$$

$$= \frac{(15-35i)(10+14i)}{(10-14i)(10+14i)} = \frac{150+210i-350i+490}{100+196}$$

$$= \frac{640-140i}{296} = \frac{640}{296}-\frac{140i}{296}$$

Hence, $x = \frac{640}{296}$, $y = \frac{-140}{296}$.

S41. $$z = \frac{(3+2i\sin\theta)(1+2i\sin\theta)}{(1-2i\sin\theta)(1+2i\sin\theta)}$$

$$= \frac{3+6i\sin\theta+2i\sin\theta-4\sin^2\theta}{1+4\sin^2\theta}$$

$$= \frac{3-4\sin^2\theta+8i\sin\theta}{1+4\sin^2\theta} = \frac{3-4\sin^2\theta}{1+4\sin^2\theta}+\frac{8i\sin\theta}{1+4\sin^2\theta}$$

$$z = \frac{3-4\sin^2\theta}{1+4\sin^2\theta}+\frac{8i\sin\theta}{1+4\sin^2\theta} = \frac{(3-4\sin^2\theta)}{1+4\sin^2\theta}+\frac{8i\sin\theta}{1+4\sin^2\theta}.$$

S42. $$\frac{3}{1+i}-\frac{2}{2-i}+\frac{2}{1-i} = \frac{3}{1+i}+\frac{2}{1-i}-\frac{2}{2-i}$$

$$= \frac{3(1-i)+2(1+i)}{2}-\frac{2}{2-i}$$

$$= \frac{3-3i+2+2i}{2} - \frac{2}{2-i}$$

$$= \frac{5-i}{2} - \frac{2}{2-i} = \frac{(5-i)(2-i)-4}{2(2-i)}$$

$$= \frac{10-5i-2i+1-4}{4-2i} = \frac{(6-7i)(4+2i)}{(4-2i)(4+2i)}$$

$$= \frac{24+12i-28i+14}{20} = \frac{38-16i}{20}$$

$$= \frac{19}{10} - \frac{4}{5}i$$

$$z = \frac{19}{10} - \frac{4}{5}i.$$

S43.

$$z = \frac{(a+ib)^2}{a-ib} - \frac{(a-ib)^2}{a+ib}$$

$$= \frac{(a+ib-a+ib)\{(a+ib)^2+(a-ib)^2-(a+ib)(a+ib)\}}{a^2+b^2}$$

$$= \frac{2ib\{2(a^2+b^2)-a^2-b^2\}}{a^2+b^2}$$

$$(x+y)^2 + (x-y)^2 = 2(x^2+y^2)$$

$$= \frac{2ib\{a^2+b^2\}}{a^2+b^2} = 2ib$$

$$z = 0 + i \cdot 2b$$

Hence, $x = 0$, $y = 2b$

S44. Since,

$$z = \left(-1 - \frac{1}{\sqrt{3}}i\right)^3$$

$$= (-1)^3 \left[1 + \frac{1}{\sqrt{3}}i\right]^3$$

$$= -\left\{1 + \left(\frac{1}{\sqrt{3}}i\right)^3 + \frac{3}{\sqrt{3}}i\left(1 + \frac{1}{\sqrt{3}}i\right)\right\}$$

$$= -\left\{1 + \frac{1}{3\sqrt{3}}i + \sqrt{3}i\left(1 + \frac{1}{\sqrt{3}}i\right)\right\}$$

$$= -\left\{1 - \frac{1}{3\sqrt{3}} + \sqrt{3}i - 1\right\}$$

$$= -\left\{\sqrt{3}i - \frac{1}{3\sqrt{3}}i\right\} = \frac{1}{3\sqrt{3}}i - \sqrt{3}i$$

$$z = 0 + i\left(\frac{1}{3\sqrt{3}} - \sqrt{3}\right).$$

S45.

$$\frac{i+1}{i-1} = \frac{(i+1)^2}{i^2-(1)^2} = \frac{i^2+1+2i}{-1-1}$$

$$= \frac{2i}{-2} = -i$$

and
$$\frac{i+2}{i-2} = \frac{(i+2)^2}{i^2-4} = \frac{i^2+4+4i}{-5}$$

$$= \frac{3+4i}{-5}$$

$\therefore$
$$z = \frac{-i(3+4i)}{-5} = \frac{3i+4i^2}{5}$$

$$= \frac{3i+4}{5} = \frac{4}{5} + \frac{3i}{5}$$

$\therefore$
$$z = \frac{4}{5} + \frac{3i}{5}.$$

S46. We have
$$\left(-2-\frac{1}{3}i\right)^3 = (-2)^3 + \left(-\frac{1}{3}i\right)^3 + 3\times(-2)\times\left(-\frac{1}{3}\right)i\left(-2-\frac{1}{3}i\right)$$

$$= -8 - \frac{1}{27}i^3 + 2i\left(-2-\frac{1}{3}i\right)$$

$$= -8 + \frac{1}{27}i - 4i - \frac{2}{3}i^2$$

$$= -8 + \frac{1}{27}i - 4i + \frac{2}{3}$$

$$= -\frac{22}{3} - \frac{107}{27}i.$$

S47. We have
$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})} = \frac{(3)^2-(i\sqrt{5})^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+i\sqrt{2}}$$

$$= \frac{9-i^2\times 5}{2\sqrt{2}\,i} = \frac{9-(-1)5}{2\sqrt{2}\,i}$$

$$= \frac{9+5}{2\sqrt{2}\,i} = \frac{14}{2\sqrt{2}\,i}$$

$$= \frac{7}{\sqrt{2}\,i} \times \frac{i}{i} = \frac{7i}{-\sqrt{2}}$$

$$= -\frac{7}{\sqrt{2}}\,i = \frac{-7\sqrt{2}}{2}\,i$$

Date: 9/10/2021

Q1. Find the conjugate of complex number $x + iy$ where x and y are real.

Q2. Find the conjugate of complex number $1 + i$.

Q3. Find the conjugate of complex number $z = -1 + i$.

Q4. Find the amp (z) and $\bar{z}$ of complex number $z = (1 + i)^2$.

Q5. If $z = (2 + 5i)^2$, find $\bar{z}$ show that $z\bar{z}$ is purely real.

Q6. Find the amp (z) if $z = (1 - i)^2$.

Q7. If $z = (1 + i)^2$ find the amp $(z + \bar{z})$.

Q8. Find the amp of $z = 1 + \sqrt{3}\,i$.

Q9. Find the amp of $z = (1 + i)$.

Q10. Find the amp of $z = (1 + i)(1 - i)$.

Q11. Prove that the sum of complex number and it's conjugate is real.

Q12. Prove that the product of a complex number and it's conjugate is always real.

Q13. Let $z_1 = 2 - i$, $z_2 = 2 - i$, Find : $\text{Im}\left(\frac{1}{z_1\bar{z}_2}\right)$.

Q14. If $z = \frac{1-i}{1+i}$ find $\bar{z}$.

Q15. If $z = (1 + i)^2$, find the amplitude of $(z - \bar{z})$.

Q16. Find the amp (z), if $z = \left(\frac{1+i}{1-i}\right)$.

Q17. If $z = \cos\theta + i\sin\theta$, find the amp ($z$), where $\theta = \frac{\pi}{4}$.

Q18. If $z = (1 - i)^4$ then find the conjugate and amplitude of z.

Q19. If $z_1 = 1 + \sqrt{3}\,i$, then prove that amp (z_1) + amp $(z_2) = \frac{\pi}{4}$.

Q20. Find the real number x and y if $(x - iy)$ is conjugate of $-6 - 24i$.

Q21. If $z_1 = x + iy$ and $z_2 = \cos\theta + i\sin\theta$, then prove that $x^2 + y^2 = 1$ subject to condition that z_1 and z_2 are conjugate of each other.

Q22. If $z_1 = \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ and $z_2 = (x + iy)$ such that z_1 and z_2 are conjugate of each other then find the amp (z_2).

Q23. Find the real number x and y such that $(x + iy)(1 + i)$ is the conjugate of $(2 - 3i)$.

Q24. Find the modulus and argument of the complex numbers: $\frac{1}{1+i}$.

Q25. Find the real number x and y such that $(x - iy)(1 + i)$ is conjugate of $2 + 3i$.

Q26. Find the conjugate of $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$.

Q27. Find the modulus and the arguments of the complex numbers given as: $z = -1 - i\sqrt{3}$.

Q28. Find the modulus and the arguments of the complex numbers given as: $z = -\sqrt{3} + i$.

Q29. Let $z_1 = 2 - i$, $z_2 = -2 + i$, Find : $\text{Re}\left(\frac{z_1 z_2}{z_1}\right)$.

Q30. Find the real numbers x and y if $(x + iy)(3 + 5i)$ is the conjugate of $-6 - 24i$.

Q31. Find the real number x and y such that $-3 + ix^2y$ and $x^2 + y + 4i$ are conjugate of each other.

Date: 9/10/2021

S1. Conjugate of complex number $(x + iy)$ is given by $x - iy$.

S2. If $z = 1 + i$, $\bar{z} = 1 - i$ where $\bar{z}$ is conjugate of z.

S3. If $z = -1 + \text{i}$, then $\bar{z} = -1 - \text{i}$, where $\bar{z}$ is conjugate of z.

S4.
$$z = (1 + i)^2 = 1 + i^2 + 2i = 2i$$
$\because$ $z = 2i$, $\bar{z} = -2i$

$\therefore$ $\text{amp}(z) = \tan^{-1} 2$

S5.
$$z = (2 + 5i)^2 = 4 - 25 + 20i$$
$$= -21 + 20i$$
$\therefore$ $\bar{z} = -21 - 20i$

$\therefore$ $z\cdot\bar{z} = (-21 + 20i)(-21 - 20i)$
$$= 441 + 400 = 840$$
Hence, $z\cdot\bar{z}$ is purely real.

S6.
$$(1 - i)^2 = 1 + i^2 - 2i = -2i$$
$\therefore$ Since amp (z) is given by $\theta = \tan^{-1}\dfrac{y}{x}$

$\therefore$ $\text{amp}(z) = \tan^{-1}(-2)$

S7.
$$z = (z + \bar{z}) = 1 + i^2 + 2i = 2i$$
$\therefore$ $\bar{z} = -2\text{i}$

$\therefore$ $z + \bar{z} = 0$

$\therefore$ $\text{amp}(z + \bar{z}) = \tan^{-1} 0 = 0$

S8. $\because$ If $z = x + iy$ then amp (z) is given by $z = \tan^{-1}\dfrac{y}{x}$

Since, $z = 1 + \sqrt{3}\,i$

$\therefore$ $\text{amp}(z) = \tan^{-1}\sqrt{3} = \dfrac{\pi}{3}$.

S9. Since, $z = 1 + i$

$\therefore$ $\text{amp}(z) = \tan^{-1}(1) = \dfrac{\pi}{4}$

S10. $z = (1 + i)(1 - i)$

$= 1 - i^2 = 2$

$z = 2 + 0i$

$\therefore$ $\text{amp}(z) = \tan^{-1}\left(\dfrac{0}{2}\right) = \tan^{-1}(0) = 0$

S11. Let $z = x + iy$ be a complex number such that $x, y \in R$ then

$\bar{z} = x - iy$

$\therefore$ $z + \bar{z} = (x + iy)(x - iy) = 2x = \text{Real}$ Proved.

S12. Let $z = x + iy \quad (x, y \in R)$

$\bar{z} = x - iy$

$\therefore$ $z \cdot \bar{z} = (x + iy)(x - iy)$

$= x^2 + y^2 = \text{Real}$ Proved.

S13. Let $\dfrac{1}{z_1 \bar{z}_2} = \dfrac{1}{(2 - i)(\overline{2 - i})}$

$= \dfrac{1}{(2 - i)(2 + i)}$

$= \dfrac{1}{4 + 1} = \dfrac{1}{5} = \dfrac{1}{5} + 0i$

Hence, $\text{Im}\left(\dfrac{1}{z_1 \bar{z}_2}\right) = 0$

Hence the result.

S14. $\because$ $\dfrac{(1 - i)^2}{(1 + i)(i - i)} = \dfrac{1 + i^2 - 2i}{1 - i^2}$

$= \dfrac{-2i}{2} = -i$

$\therefore$ $\bar{z} = +i$, $\bar{z}$ is conjugate of z.

S15. $\because$ $z = (1 + i)^2 = 1 + i^2 + 2i$

$= 2i$

$\bar{z} = -2i$

$\therefore$ $(z - \bar{z}) = 4i$

$\therefore$ $\text{amp}(z - \bar{z}) = \tan^{-1}\left(\dfrac{y}{x}\right) = \tan^{-1}(4)$

S16. $\because$ $$z = \frac{(1+i)(1+i)}{(1-i)(1+i)} = \frac{1+i^2+2i}{1-i^2}$$
$$= i$$

$\therefore$ If $z = x + iy$, amp (z) is given by $\tan^{-1}\frac{y}{x}$.

$\because$ $$z = 0 + 1i$$

$\therefore$ $$\theta = \tan^{-1}\infty = \frac{\pi}{2}$$

Hence, amp $(z) = \frac{\pi}{2}$

S17. $$z = \cos\theta + i\sin\theta,$$

$\therefore$ $$z = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$$

$\therefore$ $$\text{amp }(z) = \left(\tan^{-1}\frac{\left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{1}{\sqrt{2}}\right)}\right) = \tan^{-1}(1) = \frac{\pi}{4}$$

Hence, amp (z) $= \frac{\pi}{4}$

S18. $\because$ $$z = (1-i)^4 = (1-i)^2(1-i)^2$$
$$= (-2i)^2\cdot(-2i)^2 = 4i^2\cdot 4i^2 = 16$$

Hence, $z = 16 + 0i$

Conjugate of $z = 16 - 0i$

$\therefore$ amp $(z) = \tan^{-1} 0 = 0$

S19. $$z_1 = 1 + \sqrt{3}\,i$$

$\therefore$ $$\text{amp }(z_1) = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

Similarly, $$\text{amp }(z_2) = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$$

$\therefore$ amp (z_1) + amp $(z_2) = \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{4}$ Proved

S20. Let $z_1 = x - iy$, $z_2 = -6 - 24i$.

If z_1 and z_2 are conjugate of each other then $z_1 \cdot z_2$ = Purely real and real part of z_1 = real part of z_2.

$\therefore$ img. part of z_1 + img. part of $z_2 = 0$.

$\Rightarrow \qquad z_1 = x - iy$

$z_2 = -6 - 24i$

$\therefore \qquad x = -6$

$-y - 24 = 0 \quad \Rightarrow \quad y = -24$

Hence, $x = -6, \quad y = -24$

S21. $\because \qquad z_1 = x + iy, \qquad z_2 = \cos\theta + i\sin\theta$

$\therefore \qquad x = \cos\theta, \quad y = -\sin\theta$

$\Rightarrow \qquad x^2 + y^2 = \cos^2\theta + (-\sin\theta)^2$

$= \cos^2\theta + \sin^2\theta = 1$

Hence, $\qquad x^2 + y^2 = 1$ **Proved**.

S22. Since $\qquad z_1 = \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$z_2 = (x + iy)$

$\therefore \qquad x = \cos\frac{\pi}{3}, \quad y = -\sin\frac{\pi}{3}$

$x = \frac{1}{2}, \quad y = \frac{-\sqrt{3}}{2}$

$\frac{y}{x} = \frac{-\sqrt{3}}{2} \times \frac{2}{1} = -\sqrt{3}$

$\therefore \qquad \tan^{-1}\frac{y}{x} = \tan^{-1}(-\sqrt{3}) = \frac{-\pi}{3}$

$\therefore \qquad \text{amp } (z_2) = \frac{-\pi}{3}.$

S23. Let $\qquad z_1 = (x + iy)(1 + i),$

$z_2 = (2 - 3i)$

$z_1 = x + xi + iy - y = (x - y) + i(x + y)$

$z_2 = 2 - 3i$

$\Rightarrow \qquad x - y = 2 \qquad \dots \text{(i)}$

$x + y = 3 \qquad \dots \text{(ii)}$

$2x = 5 \quad \Rightarrow \quad x = 5/2$

$y = \frac{1}{2}$

Hence, $x = \frac{5}{2}, \quad y = \frac{1}{2}.$

S24. We have, $\frac{1}{1+i} = \frac{1-i}{(1+i)(1-i)} = \frac{1-i}{1+1} = \frac{1}{2} - \frac{i}{2}$

Let $\frac{1}{2} = r\cos\theta, \quad -\frac{1}{2} = r\sin\theta$

Further as we get $r = \frac{1}{\sqrt{2}}, \quad \cos\theta = \frac{1}{\sqrt{2}}, \quad \sin\theta = \frac{-1}{\sqrt{2}}$

Therefore, $\theta = \frac{-\pi}{4}$

Hence, the modulus of $\frac{1}{1+i}$ is $\frac{1}{\sqrt{2}}$, argument is $\frac{-\pi}{4}$.

S25. Let $z_1 = (x - iy)(1 + i)$

$= x + ix - iy + y = (x + y) + i(x - y)$

$z_2 = 2 + 3i$

$\therefore \quad x + y = 2 \quad \text{... (i)}$

$x - y = -3 \quad \text{... (ii)}$

$2x = -1 \quad \Rightarrow \quad x = -1/2$

Hence, $y = \frac{5}{2}$

Hence, $x = \frac{1}{2}, \quad y = \frac{5}{2}$.

S26. We have, $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)} = \frac{6+9i-4i+6}{2-i+4i+2} = \frac{12+5i}{4+3i} \times \frac{4-3i}{4-3i}$

$= \frac{48-36i+20i+15}{16+9} = \frac{63-16i}{25} = \frac{63}{25} - \frac{16}{25}i$

Therefore, conjugate of $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$ is $\frac{63}{25} + \frac{16}{25}i$.

S27. Let $z = -1 - i\sqrt{3}$

Therefore, $x = r\cos\theta = -1$

and $y = r\sin\theta = -\sqrt{3}$

Now squaring and adding

$$r^2\cos^2\theta + r^2\sin^2\theta = 1 + 3 = 4$$

$$r^2(\cos^2\theta + r\sin q) = 4$$

$$r^2 = 4 \quad \Rightarrow \quad r = 2$$

Now $\tan\theta = \dfrac{r\sin\theta}{r\cos\theta} = \dfrac{-\sqrt{3}}{-1} = \sqrt{3}$

$\Rightarrow$ $\theta = \dfrac{-2\pi}{3}$

Hence, modulus $r = 2$ and argument $\theta = \dfrac{-2\pi}{3}$.

S28. Let $z = -\sqrt{3} + i = x + iy$

Therefore, $x = -\sqrt{3}, \quad y = 1$

Now, $r = \sqrt{x^2+y^2} = \sqrt{(-\sqrt{3})^2+1}$

$= \sqrt{3+1} = 2$

Now, $\tan\theta = \dfrac{y}{x} = \dfrac{1}{-\sqrt{3}}$

$\Rightarrow$ $\theta = \pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6}$

Hence, modulus $r = 2$ and argument $\theta = \dfrac{5\pi}{6}$.

S29. Let $\dfrac{z_1z_2}{z_1} = \dfrac{(2-i)(-2+i)}{(2-i)}$

$= \dfrac{(2-i)(-2+i)}{2+i} = \dfrac{-(2-i)(2-i)}{2+i}$

$= \dfrac{-(2-i)^2}{2+i} = \dfrac{-(4+i^2-4i)}{2+i}$

$= \dfrac{-(4-1-4i)}{2+i}$

$= \dfrac{-(3-4i)}{2+i} \times \dfrac{(2-i)}{2-i}$

$= \dfrac{-(3-4i)(2-i)}{4+1}$

$= \dfrac{-(6-8i-3i-4)}{5}$

$= \dfrac{-2+11i}{5} = \dfrac{-2}{5} + \dfrac{11i}{5}$

Now, $\text{Re}\left(\dfrac{z_1z_2}{z_1}\right) = -\dfrac{2}{5}$.

S30. Let $(x + iy)(3 + 5i) = \overline{-6 - 24i}$

or $x(3 + 5i) - iy(3 + 5i) = -6 - 24i$

or $3x + 5ix - 3iy - 5yi^2 = -6 - 24i$

$3x + 5y + i(5x - 3y) = -6 - 24i$

$$\left.\begin{aligned} 3x + 5y &= -6 \\ 5x - 3y &= 24 \end{aligned}\right\}$$

$$\left.\begin{aligned} 9x + 15y &= -18 \\ 25x - 15y &= 120 \end{aligned}\right\}$$

$34x = 102$ (on addition)

$x = 3$

Hence, $3 \times 3 + 5y = -6$

$9 + 5y = -6$

$\Rightarrow$ $y = -3$

Hence, $x = 3, \quad y = -3.$

S31. $z_1 = -3 + ix^2y$

$z_2 = x^2 + y + 4i$

$\Rightarrow$ $x^2 + y = -3$ ... (i)

$x^2y + 4 = 0$ ... (ii)

Multiply Eq. (i) by y, we get

$x^2y + y^2 = -3y$

$\Rightarrow$ $-4 + y^2 = -3y$

$\Rightarrow$ $y^2 + 3y - 4 = 0$

$\Rightarrow$ $(y + 4)(y - 1) = 0 \Rightarrow y = 1, -4$

Case I: When $y = 1$

$x^2y + 4 = 0 \Rightarrow x^2 + 4 = 0$

$\Rightarrow$ $x = \pm 2i$

Case II: When $y = -4$

$-4x^2 + 4 = 0 \Rightarrow x^2 = 2 \Rightarrow x = \pm 1$

Hence, $x = 1, \quad y = 4$ or $x = -1$ and $y = -4.$

Date: 9/10/2021

Q1. If $z = 2 + 3i$, then find the additive inverse of z.

Q2. Find the additive inverse of complex number $z = \frac{1+i}{1-i}$.

Q3. Find the additive inverse of $z = -1 + i$.

Q4. Find the multiplicative inverse of $z = i$.

Q5. Find the multiplicative inverse of $z = (1 - i)^2$.

Q6. Find the multiplicative inverse of $z = -i \sin\theta$.

Q7. If $z = 1 + 2i$, prove that $z^2 - 2z + 5 = 0$.

Q8. If $z = 1 + i$, prove that $z^2 - 2z + 2 = 0$.

Q9. If $\sqrt{1-ib} = x - iy$ prove that $\sqrt{1+ib} = x + iy$.

Q10. If $1 + 4\sqrt{3}\, i = (x + iy)^2$ prove that $x^2 - y^2 = 1$ and $xy = 2\sqrt{3}$.

Q11. Find the multiplicative inverse of $2 - 3i$.

Q12. Find the multiplicative inverse of the complex number given as: $4 - 3i$.

Q13. Find the multiplicative inverse of the complex number given as: $-i$.

Q14. Find the multiplicative inverse of $z = \sin\theta + i\cos\theta$.

Q15. Find the multiplicative inverse of $z = (1 - i)^3$.

Q16. Find the multiplicative inverse of $z = (1 + i)^3$.

Q17. If $z = 1 + 3i$, prove that $z^3 - 3z^2 + 3z + 27i = 1$.

Q18. Find the multiplicative inverse of $z = 3 + 2i$.

Q19. If $3 + 5\sqrt{2}\, i = (x + iy)^2$ prove that $(x - y)(x + y) = 3$ and $xy = \frac{5}{\sqrt{2}}$.

Q20. If $x + iy = \frac{a+ib}{a-ib}$, prove that $x^2 + y^2 = 1$.

Q21. Find the multiplicative inverse of $z = (1 + i)^2$.

Q22. Find the multiplicative inverse of $z = \left(\frac{1+i}{1-i}\right)$.

Q23. Find the multiplicative inverse of $(6 + 5i)^2$.

Q24. Find the multiplicative inverse of $z = 1 + i^3$.

Q25. Find the multiplicative inverse of $z = \sec\theta + \tan\theta$.

Q26. Find the multiplicative inverse of $z = (\cos\theta + i\sin\theta)$.

Q27. If $x = 1 + i$, prove that $x^3 - 3x^2 + 3x + i = 1$.

Q28. If $a + ib = \frac{k+i}{k-i}$ where k is real, prove that $a^2 + b^2 = 1$.

Q29. Find the multiplicative inverse of $z = \frac{1+2i}{(2+i)}$ and write it in the form $(a + ib)$.

Q30. If $\frac{a+ib}{c+id} = x + iy$ show that $x^2 + y^2 = \frac{a^2+b^2}{c^2+d^2}$.

Q31. If $a + ib = \frac{k+i}{k-i}$ where k is real, prove that $\frac{b}{a} = \frac{2k}{k^2-1}$.

Q32. If $x + iy = \sqrt{\frac{a-ib}{c+id}}$, prove that $(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$.

Q33. If $p + iq = \frac{(a-i)^2}{(2a-i)}$, show that $p^2 + q^2 = \frac{(a^2-1)^2}{(4a^2+1)}$.

Q34. Find the number of non-zero integral solutions of the equation $|1 - i|^x = 2^x$.

Q35. If : $a + ib = \frac{(x+i)^2}{2x^2+1}$, prove that : $a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$.

Q36. If : $z_1 = 2 - i$, $z_2 = 1 + i$, find $\left|\frac{z_1 + z_2 + 1}{z_1 - z_2 + i}\right|$.

Q37. Find the multiplicative inverse of the complex numbers given as: $\sqrt{5} + 3i$.

Q38. If $x + iy = (a - ib)^3$ then prove that $\frac{x}{a} + \frac{y}{b} = -2(a^2 + b^2)$.

Q39. If $(x + iy) = (a + ib)^3$, show that $\frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2)$.

Date: 9/10/2021

S1. Since – (*a* + *ib*) is the additive inverse of *a* + *ib*.

Hence, additive inverse of $z = 2 + 3i$ is $-2-3i$

S2. Given that

$$z = \frac{1+i}{1-i}$$

$$z = \frac{(1+i)^2}{(1-i)(1+i)} = \frac{1+i^2+2i}{1-i^2}$$

$$= \frac{2i}{2} = i$$

Hence, additive inverse of *z* is $-i$.

S3. Additive inverse of $z = -1 + i$ is $1 - i$.

S4. Let (*a* + *ib*) be the multiplicative inverse of $z = i$.

$\therefore \quad (a+ib)\cdot i = 1$

$\Rightarrow \quad a + ib = \frac{1}{i \times i} \times i \quad \Rightarrow \quad a + ib = -i$

Hence, multiplicative inverse of $z = i$ is $z = -i$.

S5. Let (*a* + *ib*) be multiplicative inverse of *z*, then

Hence, $(a + ib)(1-i)^2 = 1$

$\Rightarrow \quad (a+ib) = \frac{1}{(1-i)^2} = -\frac{(1-i)^2}{2i} \times \frac{1}{(1-i)^2}$

$\Rightarrow \quad (a+ib) = \frac{-1}{2i} = \frac{+1}{2}i$

$$a + ib = \frac{1}{2}i.$$

S6. Let (*a* + *ib*) be multiplicative inverse of *z*.

Hence,

$$(a+ib)(-i\sin\theta) = 1$$

$\Rightarrow$ $(a + ib) = \dfrac{-1}{i} \times \dfrac{1}{\sin\theta} = i \operatorname{cosec}\theta$

Hence, $a + ib = i \operatorname{cosec}\theta$

S7. $z = 1 + 2i$

$\Rightarrow$ $(z - 1) = 2i$ ... (i)

Squaring both sides Eq (i), we get

$(z - 1)^2 = (2i)^2 \Rightarrow z^2 - 2z + 1 = -4$

$\Rightarrow$ $z^2 - 2z + 5 = 0$ Proved.

S8. $z = 1 + i$

$\Rightarrow$ $\{z - 1\} = i$ ... (i)

Squaring both sides Eq (i), we get

$z^2 - 2z + 1 = -1 \Rightarrow z^2 - 2z + 2 = 0$ Proved

S9. $\because$ $\sqrt{1 - ib} = x - iy$

Replacing $i \to -i$

$\therefore$ $\sqrt{1 + ib} = x + iy$ Proved

S10. $\because$ $1 + 4\sqrt{3}\, i = (x + iy)^2$

$\Rightarrow$ $x^2 - y^2 + 2xy \cdot i = 1 + 4\sqrt{3}\, i$

$\Rightarrow$ $x^2 - y^2 = 1, \quad 2xy = 4\sqrt{3}$

$\Rightarrow$ $xy = 2\sqrt{3}$ Proved

S11. Let $z = 2 - 3i$

Then $\bar{z} = 2 + 3i$ and $|z|^2 = 2^2 + (-3)^2 = 13$

Therefore, the multiplicative inverse of $2 - 3i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{2 + 3i}{13} = \frac{2}{13} + \frac{3}{13}i$$

S12. Let $z = 4 - 3i$

Now, Multiplicative inverse $= z^{-1}$

$$= \frac{1}{z} = \frac{1}{4 - 3i} \times \frac{4 + 3i}{4 + 3i} = \frac{4 + 3i}{(4)^2 - (3i)^2}$$

$$= \frac{4+3i}{16-9i^2} = \frac{4+3i}{16-9(-1)}$$

$$= \frac{4+3i}{25} = \frac{4}{25} + \frac{3}{25}i.$$

S13. Let $z = -i$

Now, Multiplicative inverse = z^{-1}

$$= \frac{1}{z} = \frac{1}{-i} = \frac{1}{-i} \times \frac{i}{i}$$

$$= \frac{-i}{i^2} = \frac{-i}{(-1)} = i.$$

S14. Let $a + ib$ be multiplicative inverse of z.

$\therefore (a + ib)(\sin\theta + i\cos\theta) = 1$

$$\Rightarrow \quad a + ib = \frac{1}{(\sin\theta + i\cos\theta)} \times \frac{(\sin\theta - i\cos\theta)}{(\sin\theta - i\cos\theta)}$$

$$= \frac{\sin\theta - i\cos\theta}{\sin^2\theta + \cos^2\theta} = \sin\theta - i\cos\theta$$

Hence, $a + ib = \sin\theta - i\cos\theta$

S15. Let $(a + ib)$ be multiplicative inverse of z.

Then, $(a + ib)\cdot(1 - i)^3 = 1$

$$\Rightarrow \quad (a + ib) = \frac{1}{(1-i)^3} = -\frac{(1-i)^2}{2i} \times \frac{1}{(1-i)^3}$$

$$\Rightarrow \quad (a + ib) = \frac{1-1}{(2i)(1-i)} = \frac{1}{2i(1-1)}$$

Hence, $a + ib = -\frac{1}{4} + \frac{1}{4}i$

S16. Let $(a + ib)$ be multiplicative inverse of z.

Hence, $(a + ib)(1 + i)^3 = 1$

$$\Rightarrow \quad (a + ib) = \frac{1}{(1+i)^3}$$

$$= \frac{(1+i)^2}{2i} \times \frac{1}{(1+i)^3}$$

$$= \frac{1}{2i(1+i)} = \frac{1}{2i-2} = \frac{1}{2} \times \frac{1 \times (i+1)}{(i-1)(i+1)}$$

$$= \frac{1}{2} \times \frac{(i+1)}{-2} = \frac{-1}{4}i - \frac{1}{4}$$

$\therefore$ $$a + ib = \frac{-1}{4} - \frac{1}{4}i$$

S17. $z = 1 + 3i \Rightarrow (z - 1) = 3i$ ... (i)

Cubing both sides Eq (i), we get

$$(z - 1)^3 = (3i)^3$$

$\Rightarrow$ $z^3 - 1 - 3z(z - 1) = -27i$

$\Rightarrow$ $z^3 - 3z^2 + 3z - 1 = -27i$

$\Rightarrow$ $z^3 - 3z^2 + 3z + 27i = 1$ **Proved.**

S18. Let $a + ib$ be the multiplicative inverse of $(3 + 2i)$.

$\therefore$ $(a + ib)(3 + 2i) = 1$

$\Rightarrow$ $$(a + ib) = \frac{1}{(3+2i)} \times \frac{(3-2i)}{(3-2i)} = \frac{3-2i}{13}$$

$\Rightarrow$ $$a + ib = \frac{3}{13} - \frac{2}{13}i$$

Hence required multiplicative inverse is $\frac{3}{13} - \frac{2}{13}i$.

S19. $(x + iy)^2 = 3 + 5\sqrt{2}\,i$

$\Rightarrow$ $x^2 - y^2 + 2ixy = 3 + 5\sqrt{2}\,i$

$\Rightarrow$ $x^2 - y^2 = 3, \quad 2xy = 5\sqrt{2}$

$\Rightarrow$ $(x - y)(x + y) = 3, \quad xy = \frac{5}{\sqrt{2}}$. **Proved.**

S20. $\because$ $$x + iy = \frac{a+ib}{a-ib}$$ ... (i)

$$x - iy = \frac{a-ib}{a+ib}$$ ... (ii)

Since $x^2 + y^2 = (x - iy)(x + iy)$

$\therefore \quad x^2 + y^2 = \frac{(a+ib)}{(a-ib)} \times \frac{(a-ib)}{(a+ib)} = 1$

$\therefore \quad x^2 + y^2 = 1$ Proved.

S21. Let $(a + ib)$ be multiplicative inverse of z.

Hence, $(a + ib)\,(1 + i)^2 = 1$

$$\Rightarrow \quad (a + ib) = \frac{1}{(1+i)^2}$$

$$= \frac{(1+i)^2}{2i} \times \frac{1}{(1+i)^2}$$

$$a + ib = \frac{1}{2i} = -\frac{1}{2}i$$

$$\therefore \quad a + ib = -\frac{1}{2}i\,.$$

S22.

Let $(a + ib)$ be multiplicative inverse of $\left(\frac{\mathbf{1}+\boldsymbol{i}}{\mathbf{1}-\boldsymbol{i}}\right)$.

$$\therefore \quad (a + ib)\left(\frac{1+i}{1-i}\right) = 1$$

$$\Rightarrow \quad (a + ib) = \frac{(1-i)^2}{(1+i)(1-i)}$$

$$\Rightarrow \quad a + ib = \frac{1+i^2-2i}{2}$$

$$\Rightarrow \quad a + ib = -\mathrm{i}$$

Hence multiplicative inverse of z = $-i$.

S23. Let $(a + ib)$ be multiplicative inverse of $(6 + 5i)^2$

$$\therefore \quad (a + ib)\,(6 + 5i)^2 = 1$$

$$\Rightarrow \quad (a + ib) = \frac{1}{(6+5i)^2} \Rightarrow \frac{1}{36-25+60i}$$

$$\Rightarrow \quad a + ib = \frac{1}{(11+60i)} \times \frac{(11-60i)}{(11-60i)}$$

$\Rightarrow \quad a + ib = \dfrac{11 - 60i}{121 + 3600} = \dfrac{11}{3721} - \dfrac{60i}{3721}$

$\Rightarrow \quad a + ib = \dfrac{11}{3721} - \dfrac{60i}{3721}.$

S24. Let (a + ib) be multiplicative inverse of z.

$\therefore \quad (a + ib)(1 + i^3) = 1$

$\Rightarrow \quad (a + ib) = \dfrac{1}{1 + i^3}$

$\Rightarrow \quad (a + ib) = \dfrac{1}{(1 - i)} \times \dfrac{(1 + i)}{(1 + i)} = \dfrac{(1 + i)}{2}$

$\Rightarrow \quad a + ib = \dfrac{1}{2} + \dfrac{1}{2}i$

Hence $\quad a + ib = \dfrac{1}{2} + \dfrac{1}{2}i$

S25. Let (a + ib) be multiplicative inverse of z. Then

$(a + ib)(\sec\theta + \tan\theta) = 1$

$\Rightarrow \quad (a + ib) = \dfrac{1}{(\sec\theta + \tan\theta)} \times \dfrac{(\sec\theta - \tan\theta)}{(\sec\theta - \tan\theta)}$

$\Rightarrow \quad (a + ib) = \dfrac{\sec\theta - \tan\theta}{\sec^2 - \tan^2\theta} = \sec\theta - \tan\theta \qquad (\because \sec^2\theta - \tan^2\theta = 1)$

Hence $\quad a + ib = \sec\theta - \tan\theta$

S26. Let (a + ib) be multiplicative inverse of z = $(\cos\theta + i\sin\theta)$

$\therefore (a + ib)(\cos\theta + i\sin\theta) = 1$

$\Rightarrow \quad (a + ib) = \dfrac{1}{(\cos\theta + i\sin\theta)} \times \dfrac{(\cos\theta - i\sin\theta)}{(\cos\theta - i\sin\theta)}$

$\Rightarrow \quad a + ib = \dfrac{\cos\theta - i\sin\theta}{1}$

$\Rightarrow \quad a + ib = \cos\theta - i\sin\theta$

Hence $\quad a + ib = \cos\theta - i\sin\theta$

S27. Since, $\quad x = 1 + i$

$\Rightarrow \quad (x - 1) = i \qquad \ldots$ (i)

Cubing both sides Eq. (i), we get

$(x - 1)^3 = i^3$

$\Rightarrow \quad x^3 - 1 - 3x(x - 1) = -i$

$\Rightarrow \quad x^3 - 3x^2 + 3x - 1 + i = 0$

$\Rightarrow \quad x^3 - 3x^2 + 3x + i = 1$ **Proved.**

S28. Since,

$$a + ib = \frac{k+i}{k-i}$$

$$\Rightarrow \quad a - ib = \frac{k-i}{k+i}$$

$$\Rightarrow \quad a^2 + b^2 = (a + ib)(a - ib)$$

$$\Rightarrow \quad a^2 + b^2 = 1$$

S29. Let $(a + ib)$ be multiplicative inverse, then

$$(a+ib)\frac{(1+2i)}{(2+i)} = 1$$

$$\Rightarrow \quad (a + ib) = \frac{(2+i)(1-2i)}{(1+2i)(1-2i)}$$

$$= \frac{2-4i+i+2}{1+4} = \frac{4-3i}{5} = \frac{4}{5} - \frac{3}{5}i$$

$$\Rightarrow \quad a + ib = \frac{4}{5} - \frac{3}{5}i$$

S30.

$$x + iy = \frac{a+ib}{c+id}$$

$$\therefore \quad x - iy = \frac{a-ib}{c-id}$$

$$\therefore \quad x^2 + y^2 = (x + iy)(x - iy)$$

$$\therefore \quad x^2 + y^2 = \frac{(a+ib)}{(c+id)}\frac{(a-ib)}{(c-id)}$$

$$\Rightarrow \quad x^2 + y^2 = \frac{a^2+b^2}{c^2+d^2}$$ **Proved**.

S31. Since,

$$(a + ib) = \frac{(k+i)^2}{(k-i)(k+i)}$$

$$\Rightarrow \quad a + ib = \frac{k^2-1+2ki}{k^2-1}$$

$\Rightarrow \quad a + ib = 1 + \frac{2k}{k^2 - 1} i$

$\Rightarrow \quad a = 1,\ b = \frac{2k}{k^2 - 1}$

Hence $\quad \frac{b}{a} = \frac{2k}{k^2 - 1}$ **Proved.**

S32. Since, $\quad x + iy = \sqrt{\frac{a - ib}{c + id}}$

$x - iy = \sqrt{\frac{a + ib}{c - id}}$

$\because \quad x^2 + y^2 = (x + iy)(x - iy)$

$\Rightarrow \quad x^2 + y^2 = \sqrt{\frac{(a - ib)(a + ib)}{(c + id)(c - id)}}$

$\Rightarrow \quad x^2 + y^2 = \sqrt{\frac{a^2 + b^2}{c^2 + d^2}}$

$\Rightarrow \quad (x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$ **Proved.**

S33. Since, $\quad (p + iq) = \frac{(a - i)^2}{(2a - i)}$

$\therefore \quad (p - iq) = \frac{(a + i)^2}{(2a + i)}$

$\therefore \quad p^2 + q^2 = (p + iq)(p - iq)$

$\therefore \quad p^2 + q^2 = \frac{(a - i)^2}{(2a - i)} \times \frac{(a + i)^2}{(2a + i)}$

$= \frac{(a^2 - 1)^2}{4a^2 + 1}$

$\Rightarrow \quad p^2 + q^2 = \frac{(a^2 - 1)^2}{4a^2 + 1}$ **Proved.**

S34. Let

$$|1-i|^x = 2^x$$
$$(\sqrt{1^2+(-1^2)})^x = 2^x$$
$$(\sqrt{2})^x = 2^x$$
$$2^{x/2} = 2^x$$

Hence, $\frac{x}{2} = x \Rightarrow 1 = 2$ which is false.

Hence no value of x satisfies the value of x.

S35. Let

$$a + ib = \frac{(x+i)^2}{2x^2+1}$$
$$|a + ib| = \left|\frac{|x+i|^2}{2x^2+1}\right|$$
$$\sqrt{a^2+b^2} = \frac{|(\sqrt{(x^2+1)})^2|}{(2x^2+1)}$$
$$a^2 + b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$$

Hence, the result.

S36. Let

$$z_1 = 2 - i, \quad z_2 = 1 + i$$
$$z_1 + z_2 + 1 = 2 - i + 1 + i + 1 = 4$$
$$z_1 - z_2 + i = 2 - i - 1 - i + i$$
$$= 1 - i$$

Now,
$$\left|\frac{z_1+z_2+1}{z_1-z_2+i}\right| = \left|\frac{4}{1-i}\right|$$
$$= \frac{|4|}{|1-i|} = \frac{4}{\sqrt{1+1}} = 2\sqrt{2}.$$

S37. Let

$$z = \sqrt{5} + 3i$$

Now, Multiplicative inverse $= z^{-1}$

$$= \frac{1}{z} = \frac{1}{\sqrt{5}+3i}$$
$$= \frac{1}{\sqrt{5}+3i} \times \frac{\sqrt{5}-3i}{\sqrt{5}-3i}$$
$$= \frac{\sqrt{5}-3i}{(\sqrt{5})^2-(3i)^2} = \frac{\sqrt{5}-3i}{5-9i^2}$$
$$= \frac{\sqrt{5}-3i}{5-9(-1)} = \frac{\sqrt{5}}{14} - \frac{3i}{14}.$$

S38. Since, $x + iy = (a - ib)^3$

$$= a^3 - (ib)^3 - 3iab(a - ib)$$

$$= a^3 + b^3 i - 3a^2bi + 3i^2ab^2$$

$$= a^3 + ib^3 - i\,3a^2b - 3ab^2$$

$$= (a^3 - 3ab^2) + i(b^3 - 3a^2b)$$

$\Rightarrow$ $x = a^3 - 3ab^2; \quad y = b^3 - 3a^2b$

$$\frac{x}{a} = a^2 - 3b^2, \quad \frac{y}{b} = b^2 - 3a^2$$

$\Rightarrow$ $\frac{x}{a} + \frac{y}{b} = -2a^2 - 2b^2$

$\Rightarrow$ $\frac{x}{a} + \frac{y}{b} = -2(a^2 + b^2).$

S39. Since, $(x + iy) = (a + ib)^3$

$\Rightarrow$ $(x + iy) = a^3 + (ib)^3 + 3ia^2b + 3i^2ab^2$

$\Rightarrow$ $x + iy = a^3 - 3ab^2 + i(3a^2b - b^3)$

$\Rightarrow$ $x = a^3 - 3ab^2, \quad y = 3a^2b - b^3$

$\Rightarrow$ $\frac{x}{a} = a^2 - 3b^2, \quad \frac{y}{b} = 3a^2 - b^2$

$\Rightarrow$ $\frac{x}{a} + \frac{y}{b} = a^2 - 3b^2 + 3a^2 - b^2$

$\Rightarrow$ $\frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2)$ **Proved.**

Date: 9/10/2021

Q1. Find the square roots of $\left(\frac{(2+3i)}{(5-4i)}+\frac{(2-3i)}{(5+4i)}\right)$.

Q2. Find the value of $\sqrt{3+4i}$.

Q3. Find the value of $\sqrt{4+3i}$.

Q4. Find the value of $\sqrt{5+12i}$.

Q5. Find the value of $\sqrt{8+15i}$.

Q6. Find the value of $\sqrt{15+8i}$.

Q7. Find the square root of $x^2+\frac{1}{x^2}-\frac{4}{i}\left(x-\frac{1}{x}\right)-6$.

Q8. Find the value of $\sqrt{3-4i}$.

Q9. Find the value of $\sqrt{1+i}$.

Q10. Find the value of $\sqrt{4+3\sqrt{-20}}+\sqrt{4-3\sqrt{-20}}$.

Q11. Find the value of $\sqrt{-7+24i}$.

Q12. Find the value of $\sqrt{12-5i}$.

Q13. Find the value of $\sqrt{1+2i}$.

Q14. Find the value of $\sqrt{8-5i}$.

Date: 9/10/2021

S1. We have,

$$\frac{2+3i}{5-4i}+\frac{2-3i}{5+4i} = \frac{(2+3i)(5+4i)+(2-3i)(5-4i)}{5^2-(4i)^2}$$

$$= \frac{(-2+23i)+(-2-23i)}{41}=\frac{-4}{41}$$

Hence required square root is $\pm\frac{2}{\sqrt{41}}$

S2. Let, $\sqrt{3+4i} = a + ib$

$\Rightarrow$ squaring both sides, we get

$$a^2 - b^2 = 3, \quad 2ab = 4$$

$\therefore \quad (a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$\Rightarrow \quad a^2 + b^2 = 5 \quad$ ... (i)

$a^2 - b^2 = 3 \quad$ ... (ii)

After solving these two Eqn, we get

$$2a^2 = 8 \Rightarrow a = \pm 2$$

$$2b^2 = 2 \Rightarrow b = \pm 1$$

Hence $a + ib = \pm 2 \pm i = \pm (2 + i)$

S3. Let, $\sqrt{4+3i} = a - ib$

$\Rightarrow \quad 4 - 3i = a^2 - b^2 - i \cdot 2ab$

$\Rightarrow \quad a^2 - b^2 = 4$ and $2ab = 3$

$\because \quad (a^2 + b^2)^2 = (a^2 - b^2)^2 + 4a^2b^2$

$= 16 + 9 = 25$

$\Rightarrow \quad a^2 + b^2 = 5 \quad$ ... (i)

$a^2 - b^2 = 4 \quad$ ... (ii)

$\Rightarrow$ After solving Eq (i) and (ii), we get

$$a = \pm\frac{3}{\sqrt{2}}, \quad b = \pm\frac{1}{\sqrt{2}}$$

$\therefore \quad \sqrt{4+3i} = \pm\frac{1}{\sqrt{2}}(3 - i).$

S4. Let, $\sqrt{5+12i} = a + ib$

Squaring both side, we get

$$a^2 - b^2 = 5 \quad \text{and} \quad 2ab = 12$$

$$\because \quad (a^2 + b^2)^2 = (a^2 - b^2)^2 + 4a^2b^2$$

$$= (5)^2 + (12)^2 = 169$$

$$\Rightarrow \quad a^2 + b^2 = 13 \quad \text{... (i)}$$

$$a^2 - b^2 = 5 \quad \text{... (ii)}$$

After solving Eq (i) and (ii), we get

$$\Rightarrow \quad a = \pm 3, \quad b = \pm 2. \qquad \sqrt{5+12i} = \pm(3+2i)$$

S5. Let, $\sqrt{8+15i} = a - ib$

$$\Rightarrow \quad a^2 - b^2 = 8, \quad 2ab = 15$$

$$(a^2 + b^2)^2 = (a^2 - b^2)^2 + 4a^2b^2$$

$$\Rightarrow \quad a^2 + b^2 = 17 \quad \text{... (i)}$$

$$a^2 - b^2 = 8 \quad \text{... (ii)}$$

After solving Eq (i) and (ii), we get

$$\Rightarrow \quad a = \pm\frac{5}{\sqrt{2}}, \quad b = \pm\frac{3}{\sqrt{2}}$$

$$\therefore \quad a - ib = \pm\frac{1}{\sqrt{2}}(5 - 3i).$$

S6. Let, $\sqrt{15+8i} = a + ib$

$$\Rightarrow \quad 15 + 8i = a^2 - b^2 + 2iab$$

$$\Rightarrow \quad a^2 - b^2 = 15, \quad 2ab = 8$$

$$\because \quad (a^2 + b^2)^2 = (a^2 - b^2)^2 + 4a^2b^2$$

$$\Rightarrow \quad a^2 + b^2 = 17 \quad \text{... (i)}$$

$$a^2 - b^2 = 15 \quad \text{... (ii)}$$

Solving Eq (i) and (ii), we get

$$a = \pm 4, \quad b = \pm 1$$

$$\therefore \quad a + ib = \pm(4 + i).$$

S7.

$$x^2 + \frac{1}{x^2} - \frac{4}{i}\left(x - \frac{1}{x}\right) - 6 = x^2 + \frac{1}{x^2} - 2 \cdot x \cdot \frac{1}{x} - \frac{4i}{i^2}\left(x - \frac{1}{x}\right) - 6 + 2 \cdot x \cdot \frac{1}{x}$$

$$= \left(x - \frac{1}{x}\right)^2 + 4i\left(x - \frac{1}{x}\right) - 4$$

$$= \left(x - \frac{1}{x}\right)^2 + 4i\left(x - \frac{1}{x}\right) + 4i^2$$

$$= \left(x - \frac{1}{x}\right)^2 + 2\left(x - \frac{1}{x}\right)\cdot 2i + (2i)^2$$

$$= \left(x - \frac{1}{x} + 2i\right)^2$$

Hence, the required square roots are equal to $\pm\left(x - \frac{1}{x} + 2i\right)$.

S8. Let, $\sqrt{3-4i} = x + iy$

$\Rightarrow \quad 3 - 4i = (x + iy)^2$

$\Rightarrow \quad 3 - 4i = x^2 - y^2 + i(2xy)$

$\Rightarrow \quad x^2 - y^2 = 3 \quad \text{and} \quad 2xy = -4$

$\Rightarrow \quad xy = -2$

Now, $(x^2 + y^2)^2 = (x^2 - y^2) + 4x^2y^2$

$\Rightarrow \quad x^2 + y^2 = 5$

$\Rightarrow \quad x = \pm 2, \quad y = \pm 1$

$\because$ xy is negative, which indicates that x and y are of opposite signs.

$\therefore$ when $x = 2, \quad y = -1, \quad$ and

$x = -2, \quad y = 1$

$\therefore \quad \sqrt{3-4i} = 2 - i \quad \text{or} \quad -2 + i = \pm(2 - i).$

S9. Let, $\sqrt{1+i} = x + iy$

$\Rightarrow \quad (x^2 - y^2) + 2ixy = 1 + i$

$\Rightarrow \quad x^2 - y^2 = 1$ ... (i)

$2xy = 1$ ... (ii)

$$xy = \frac{1}{2}$$

$\because \quad (x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$

$= 1 + 1$

$(x^2 + y^2)^2 = 2 \quad \Rightarrow \quad x^2 + y^2 = \sqrt{2}$ ... (iii)

Adding Eq (i) and (ii), we get

$$2x^2 = 1 + \sqrt{2}$$

$\Rightarrow \qquad x^2 = \frac{1}{2}(1 + \sqrt{2})$

$\Rightarrow \qquad x = \pm\frac{1}{\sqrt{2}}(\sqrt{\sqrt{2}+1})$.

Subtracting Eq (i) from Eq (iii), we get

$$2y^2 = \sqrt{2} - 1 \quad \Rightarrow \quad y = \pm\frac{1}{\sqrt{2}}(\sqrt{\sqrt{2}-1}).$$

$\therefore \qquad \sqrt{1+i} = \pm\frac{1}{\sqrt{2}}\left(\sqrt{\sqrt{2}+1} + \sqrt{\sqrt{2}-1}\, i\right)$

S10. $\because \qquad \sqrt{-20} = 2\sqrt{5}\, i$

$\therefore \sqrt{4+3\sqrt{-20}} + \sqrt{4-3\sqrt{-20}} = \sqrt{4+6\sqrt{5}\, i} + \sqrt{4-6\sqrt{5}\, i}$

Let $\qquad \sqrt{4+6\sqrt{5}\, i} = x + iy$

$\Rightarrow \qquad 4 + 6\sqrt{5}\, i = (x^2 - y^2) + i(2xy)$

$\Rightarrow \qquad x^2 - y^2 = 4, \quad 2xy = 6\sqrt{5}$

$\because \qquad (x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$

$\Rightarrow \qquad (x^2 + y^2)^2 = 196$

$\Rightarrow \qquad x^2 + y^2 = 14 \qquad \ldots \text{(i)}$

$x^2 - y^2 = 4 \qquad \ldots \text{(ii)}$

After solving Eq (i) and (ii), we get

$$x = \pm 3 \quad \text{and} \quad y = \pm\sqrt{5}$$

$\therefore \qquad \sqrt{4+6\sqrt{5}\, i} = \pm(3 + \sqrt{5}\, i)$

Similarly, $\qquad \sqrt{4-6\sqrt{5}\, i} = \pm(3 - \sqrt{5}\, i)$

Hence,

$$\sqrt{4+6\sqrt{5}\, i} + \sqrt{4-6\sqrt{5}\, i} = \pm 6.$$

S11. Let, $\qquad \sqrt{-7+24i} = a + ib$

Squaring both sides, we get

$\Rightarrow \qquad -7 + 24i = a^2 - b^2 + 2iab$

$\Rightarrow \qquad a^2 - b^2 = -7 \quad \text{and} \quad 2ab = 24$

Since $\qquad (a^2 + b^2)^2 = (a^2 - b^2)^2 + 4a^2b^2$

$\qquad = (-7)^2 + (24)^2 = 625$

$\Rightarrow \quad a^2 + b^2 = 25 \quad \ldots \text{(i)}$

$a^2 - b^2 = -7 \quad \ldots \text{(ii)}$

After solving Eq (i) and (ii), we get

$$a = \pm 3, \quad b = \pm 4$$

Hence, $\quad a + ib = \pm (3 + 4i)$

S12. Let, $\quad \sqrt{12 - 5i} = a - ib$

Squaring both sides, we get

$$a^2 - b^2 - 2iab = 12 - 5i$$

$\Rightarrow \quad a^2 - b^2 = 12 \quad \text{and} \quad 2ab = 5$

Since, $\quad (a^2 + b^2)^2 = (a^2 - b^2)^2 + 4a^2b^2$

$\Rightarrow \quad (a^2 + b^2)^2 = 169$

$\Rightarrow \quad a^2 + b^2 = 13 \quad \ldots \text{(i)}$

$a^2 - b^2 = 12 \quad \ldots \text{(ii)}$

After solving Eq (i) and (ii), we get

$$a = \pm \frac{5}{\sqrt{2}}, \quad b = \pm \frac{1}{\sqrt{2}}$$

$\therefore \quad a - ib = \pm \frac{5}{\sqrt{2}} \mp \frac{1}{\sqrt{2}} i$

$$= \pm \frac{1}{\sqrt{2}} (5 - i).$$

S13. Let, $\quad \sqrt{1 + 2i} = a + ib$

Squaring both sides, we get

$$a^2 - b^2 = 1$$

$$2ab = 2$$

$\because \quad (a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$$= 1 + 4 = 5$$

$$(a^2 + b^2)^2 = 5$$

$\Rightarrow \quad (a^2 + b^2) = \sqrt{5} \quad \ldots \text{(i)}$

$a^2 - b^2 = 1 \quad \ldots \text{(ii)}$

$\Rightarrow \quad 2a^2 = \sqrt{5} + 1 \quad \Rightarrow \quad a^2 = \frac{1}{2}(\sqrt{5} + 1)$

$\Rightarrow \quad a = \pm\sqrt{\frac{1}{2}(\sqrt{5} + 1)}$

Similarly $$b = \pm\sqrt{\frac{1}{2}(\sqrt{5}-1)}$$

$\therefore$ $$\sqrt{1+2i} = \pm\frac{1}{\sqrt{2}}(\sqrt{5}+1+(\sqrt{5}+1)i).$$

S14. Let, $$\sqrt{8-5i} = a - ib$$

$\Rightarrow$ $$a^2 - b^2 - 2iab = 8 - 5i$$

$\Rightarrow$ $$a^2 - b^2 = 8$$

$$2ab = 5$$

$\because$ $$(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$$

$$= 8^2 + 25 = 64 + 25$$

$\Rightarrow$ $$(a^2 + b^2)^2 = 99 \quad \Rightarrow \quad a^2 + b^2 = \sqrt{99} \qquad \text{... (i)}$$

$$a^2 - b^2 = 8 \qquad \text{... (ii)}$$

Adding Eq (i) and (ii), we get

$$2a^2 = 8 + \sqrt{99} \quad \Rightarrow \quad a = \pm\frac{1}{\sqrt{2}}(8+\sqrt{99})^{\frac{1}{2}}$$

Similarly, $$b = \pm\frac{1}{\sqrt{2}}(\sqrt{99}-8)^{\frac{1}{2}}$$

$\therefore$ $$a + ib = \pm\frac{1}{\sqrt{2}}\left((8+\sqrt{99})^{\frac{1}{2}} + i(\sqrt{99}-8)^{\frac{1}{2}}\right).$$

Date: 9/10/2021

Q1. Find the modulus of $\frac{1+i}{1-i}-\frac{1-i}{1+i}$.

Q2. Find the modulus of $\frac{1+i}{1-i}+\frac{1-i}{1+i}$.

Q3. Find the modulus of $z = 3 + 4i$.

Q4. If $(a_1 + ib_1)(a_2 + ib_2) \ldots (a_n + ib_n) = X + iY$, then find the value of $(a_1^2 + b_1^2)(a_2^2 + b_2^2) \ldots (a_n^2 + b_n^2)$.

Q5. For any two complex number z_1 and z_2 prove that $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$.

Q6. Find non zero integral solutions of $|1 - i|^x = 2^x$.

Q7. For any two complex number z_1 and z_2 prove that $\overline{z_1 - z_2} = \bar{z}_1 - \bar{z}_2$.

Q8. For any two complex number z_1 and z_2 prove that $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$.

Q9. For any two complex number z_1 and z_2 prove that $\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$.

Q10. For any two complex number z_1 and z_2 prove that $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$.

Q11. If z is any complex number prove that $|z|^2 = |z^2|$.

Q12. For any two complex numbers z_1 and z_2 prove that $\overline{(z_1 + z_2)} = \bar{z}_1 + \bar{z}_2$.

Q13. If $(1 + i)^{100} = 2^{49} \cdot (x + iy)$, then find the value of $(x^2 + y^2)$.

Q14. If $(1 + i)(1 + 2i)(1 + 3i) \ldots (1 + ni) = a + ib$ prove that $2 \cdot 5 \cdot 10 \ldots (1 + n^2) = a^2 + b^2$.

Q15. Find the modulus and argument of the complex number $\frac{1+2i}{1-3i}$.

Q16. If α and β are different complex numbers with $|\beta| = 1$, find the the value of $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$.

Q17. If $|z_1| = |z_2| = \ldots |z_n| = 1$, prove that $|z_1 + z_2 + \ldots + z_n| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\ldots+\frac{1}{z_n}\right|$.

Q18. If z_1, z_2, z_3 are unimodulo complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}\right|$, then prove that $|z_1 + z_2 + z_3| = 1$.

Q19. If z_1 and z_2 are complex numbers such that $\frac{2z_1}{3z_2}$ is purely imaginary number then find the value of $\left|\frac{z_1-z_2}{z_1+z_2}\right|$.

Date: 9/10/2021

S1.
$$\frac{1+i}{1-i}-\frac{1-i}{1+i} = \frac{(1+i)^2-(1-i)^2}{(1-i)(1+i)}$$
$$= \frac{4i}{2} = 2i$$
$\therefore$ $$|z| = \sqrt{0+4} = 2$$

S2. Let,
$$z = \frac{1+i}{1-i}+\frac{1-i}{1+i}$$
$$= \frac{(1+i)^2+(1-i)^2}{2} = \frac{2(1+i^2)}{2} = 0$$
$\therefore$ $$|z| = 0$$

S3. If $z = x + iy$, then modulus of z is given by
$$|z| = \sqrt{x^2+y^2}$$
$\therefore$ $$|z| = \sqrt{3^2+4^2} = \sqrt{25}$$
$$|z| = 5$$

S4. $\because$ $$(a_1 + ib_1)(a_2 + ib_2) \dots (a_n + ib_n) = X + iY$$
Taking modulus on both sides, we get
$$(a_1^2 + b_1^2)(a_2^2 + b_2^2) \dots (a_n^2 + b_n^2) = X^2 + Y^2$$

S5. Let $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$
$$z_1 \cdot z_2 = (a_1 + ib_1)(a_2 + ib_2)$$
$\therefore$ $$z_1 \cdot z_2 = (a_1a_2 - b_1b_2) + i(a_1b_2 + a_2b_1)$$
$\therefore$ $$|z_1 \cdot z_2| = \sqrt{(a_1a_2-b_1b_2)^2+(a_1b_2+a_2b_1)^2}$$
$$= \sqrt{(a_1^2+b_1^2)(a_2^2+b_2^2)} = |z_1|\cdot|z_2|$$
$$|z_1 \cdot z_2| = |z_1|\cdot|z_2|.$$

S6. $\because$ $$|1-i|^x = (\sqrt{2})^x$$
$\therefore$ $$(\sqrt{2})^x = 2^x \quad \Rightarrow \quad = 2^{\frac{1}{2}x} = 2^x$$
Equating exponents on both sides we get

$\Rightarrow \qquad x = \frac{1}{2}$ (not possible)

Hence there is no integral solution.

S7. Let $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$

$$\therefore \qquad z_1 - z_2 = (a_1 + ib_1) - (a_2 + ib_2)$$

$$= (a_1 - a_2) + i(b_1 - b_2)$$

$$\therefore \qquad \overline{(z_1 - z_2)} = (a_1 - a_2) - i(b_1 - b_2) \qquad \text{... (i)}$$

$$\bar{z}_1 - \bar{z}_2 = (a_1 - ib_2) - (a_2 - ib_2)$$

$$= (a_1 - a_2) - i(b_2 - b_2) \qquad \text{... (ii)}$$

From Eq. (i) and (ii), we get

$$\overline{(z_1 - z_2)} = \bar{z}_1 - \bar{z}_2.$$

S8. Let $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$

$$\therefore \qquad \frac{z_1}{z_2} = \frac{(a_1 + ib_1)}{(a_2 + ib_2)} \times \frac{(a_2 - ib_2)}{(a_2 - ib_2)}$$

$$\therefore \qquad \frac{z_1}{z_2} = \frac{(a_2a_2 + b_1b_2) + i(b_1a_1 - b_2a_1)}{a_2^2 + b_2^2}$$

$$\therefore \qquad \left(\overline{\frac{z_1}{z_2}}\right) = \frac{a_1a_2 + b_1b_2}{a_2^2 + b_2^2} - \frac{i(b_1a_2 - b_2a_1)}{(a_2^2 + b_2^2)} \qquad \text{... (i)}$$

$$\frac{\bar{z}_1}{\bar{z}_2} = \frac{(a_1 - ib_1)}{(a_2 - ib_2)}$$

$$= \frac{(a_1 - ib_1)}{(a_2 - ib_2)} \times \frac{(a_2 + ib_2)}{(a_2 + ib_2)}$$

$$= \frac{a_1a_2 + b_1b_2}{a_2^2 + b_2^2} - \frac{i\,(b_1a_2 - b_2a_1)}{(a_2^2 + b_2^2)} \qquad \text{... (ii)}$$

From Eq. (i) and (ii), we get

$$\left(\overline{\frac{z_1}{z_2}}\right) = \frac{\bar{z}_1}{\bar{z}_2}.$$

S9. Let $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$

$$\therefore \qquad z_1 \cdot z_2 = (a_1 + ib_1)\,(a_2 + ib_2)$$

$$\therefore \qquad z_1 \cdot z_2 = (a_1a_2 - b_1b_2) + i\,(a_1b_2 + b_1a_2)$$

$\therefore \qquad \overline{z_1 \cdot z_2} = (a_1a_2 - b_1b_2) + i(a_1b_2 + b_1a_2)$ ... (i)

$$\bar{z}_1 \cdot \bar{z}_2 = (a_1 - ib_1)(a_2 - ib_2)$$

$$= (a_1a_2 - b_1b_2) - i(a_1b_2 - a_2b_1) \quad \text{... (ii)}$$

From Eq. (i) and (ii), we get

$$\overline{z_1 \cdot z_2} = \bar{z}_1 \cdot \bar{z}_2 .$$

S10. Let $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$

$$\therefore \qquad \frac{z_1}{z_2} = \frac{(a_1 + ib_1)}{(a_2 + ib_2)} \times \frac{(a_2 - ib_2)}{(a_2 - ib_2)}$$

$$\therefore \qquad \frac{z_1}{z_2} = \frac{(a_2a_2 + b_1b_2)}{a_2^2 + b_2^2} - \frac{i(a_2b_1 - a_1b_2)}{a_2^2 + b_2^2}$$

$$\therefore \qquad \left|\frac{z_1}{z_2}\right| = \sqrt{\left(\frac{a_1a_2 + b_1b_2}{a_2^2 + b_2^2}\right)^2 + \left(\frac{a_2b_1 - a_1b_2}{a_2^2 + b_2^2}\right)^2}$$

$$= \sqrt{\frac{a_1^2 + b_1^2}{a_2^2 + b_2^2}} = \frac{\sqrt{a_1^2 + b_1^2}}{\sqrt{a_2^2 + b_2^2}} = \frac{|z_1|}{|z_2|}.$$

S11. Let $z = x + iy$, $\quad x, y \in R$

$$|z| = \sqrt{x^2 + y^2}$$

$$|z|^2 = x^2 + y^2$$

$$\Rightarrow \qquad |z|^2 = x^2 + y^2 \quad \text{... (i)}$$

$$\because \qquad z = x + iy$$

$$z^2 = (x + iy)^2 = x^2 - y^2 + 2ixy$$

$$\therefore \qquad |z^2| = \sqrt{(x^2 - y^2) + (2xy)^2} = \sqrt{(x^2 + y^2)^2}$$

$$\therefore \qquad |z^2| = x^2 + y^2 \quad \text{... (ii)}$$

From Eq. (i) and (ii), we get

$$|z|^2 = |z^2|.$$

S12. Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$$\bar{z}_1 = x_1 - iy_1, \quad \bar{z}_2 = x_2 - iy_2$$

$$\therefore \qquad z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$$

$$\Rightarrow \qquad \overline{(z_1 + z_2)} = (x_1 + x_2) - i(y_1 + y_2) \quad \text{... (i)}$$

$$\bar{z}_1 + \bar{z}_2 = x_1 - iy_1 + x_2 - iy_2$$

$$= (x_1 + x_2) + i(y_1 + y_2) \quad \dots \text{(ii)}$$

From Eq. (i) and (ii), we get $\therefore$

$$\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2.$$

S13. $\because$ $(1 + i)^2 = 2i$

$$(1 + i)^{100} = (2i)^{50} = 2^{50} \cdot i^{50}$$
$$= -2^{50}$$

$\therefore$ $(1 + i)^{100} = 2^{49} \cdot (x + iy)$

$\therefore$ $-2^{50} = 2^{49} \cdot (x + iy)$

$\Rightarrow$ $x + iy = -2$

Taking modulus on both sides, we get

$$|x^2 + y^2| = 2.$$

S14. $(1 + i)(1 + 2i)(1 + 3i) \dots (1 + ni) = a + ib$

Taking modulus on both sides, we get

$$|(1 + i)(1 + 2i)(1 + 3i) \dots (1 + ni)| = |a + ib|$$

$\because$ $|z_1 \cdot z_2 \dots z_n| = |z_1| \cdot |z_1| \dots |z_n|$

$\Rightarrow$ $|1 + i| \cdot |1 + 2i| \dots |1 + ni| = \sqrt{a^2 + b^2}$

Squaring both sides, we get

$$2 \cdot 5 \cdot 10 \dots (1 + n^2) = a^2 + b^2.$$

S15. Let $z = \dfrac{1+2i}{1-3i} = \dfrac{1+2i}{1-3i} \times \dfrac{1+3i}{1+3i}$

$$= \frac{1+3i+2i+6i^2}{1+9}$$

$$= \frac{1+5i-6}{10} = \frac{-5+5i}{10}$$

$$= -\frac{1}{2} + \frac{1}{2}i$$

Now, $|z| = \sqrt{\left(-\dfrac{1}{2}\right)^2 + \left(\dfrac{1}{2}\right)^2}$

$$= \sqrt{\frac{1}{4} + \frac{1}{4}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\text{Argument } \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta = \tan^{-1}\left(\frac{\frac{1}{2}}{-\frac{1}{2}}\right) = \tan^{-1}(-1)$$

$$= \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

Hence, modulus = $\frac{\sqrt{2}}{2}$ and argument = $\frac{3\pi}{4}$.

S16.

$$\therefore \quad \left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|^2 = \left(\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right)\left(\overline{\frac{\beta-\alpha}{1-\alpha\beta}}\right)$$

$$= \left(\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right)\left(\frac{\bar{\beta}-\bar{\alpha}}{1-\bar{\alpha}\beta}\right)$$

$$= \frac{(\beta-\alpha)(\bar{\beta}-\bar{\alpha})}{(1-\bar{\alpha}\beta)(1-\bar{\alpha}\beta)}$$

$$= \frac{|\beta|^2 - \alpha\bar{\beta} - \bar{\beta}\alpha + |\alpha|^2}{1-\bar{\alpha}\beta - \alpha\beta + |\alpha|^2|\beta|^2}$$

$$= \frac{1-\alpha\bar{\beta}-\bar{\alpha}\beta+|\alpha|^2}{1-\alpha\bar{\beta}-\bar{\alpha}\beta+|\alpha|^2} \qquad \because \ |\beta| = 1$$

$$= 1$$

Hence $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = 1$

S17.

$$|z_1| = 1 \Rightarrow z_1 \cdot \bar{z}_1 = 1$$

$$\bar{z}_1 = \frac{1}{z_1}, \quad \bar{z}_2 = \frac{1}{z_2}, \quad \bar{z}_3 = \frac{1}{z_3}$$

$$\therefore \quad \left|\frac{1}{z_1} + \frac{1}{z_2} + \ldots + \frac{1}{z_n}\right| = |\bar{z}_1 + \bar{z}_2 + \ldots + \bar{z}_n|$$

$$= |\overline{z_1 + z_2 + \ldots + z_n}|$$

$$= |z_1 + z_2 + \ldots + z_n|.$$

S18.

$$|z_1| = |z_2| = |z_3| = 1$$

$$\Rightarrow \quad z_1\bar{z}_1 = z_2\bar{z}_2 = z_3\bar{z}_3 = 1$$

$\Rightarrow \qquad \bar{z}_1 = \frac{1}{z_1},\ \bar{z}_2 = \frac{1}{z_2}, \bar{z}_3 = \frac{1}{z_3}.$

$\therefore \qquad \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = |\bar{z}_1 + \bar{z}_2 + \bar{z}_3|$

$= \left|\overline{z_1 + z_2 + z_3}\right| = 1$

$\Rightarrow \qquad = |z_1 + z_2 + z_3| = 1.$

S19. $\because \quad \frac{2z_1}{3z_2}$ is purely imaginary.

Let, $\frac{2z_1}{3z_2} = xi$, where $x \in R$

$\therefore \qquad \frac{z_1}{z_2} = \frac{3}{2}\, xi \qquad$ ... (i)

Now, $\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = \left|\frac{\frac{z_1}{z_2} - 1}{\frac{z_1}{z_2} + 1}\right| = \left|\frac{\frac{3x}{2}i - 1}{\frac{3x}{2}i + 1}\right|$

$= \left|\frac{3x\,i - 2}{3x\,i + 2}\right| = 1 \qquad$ Using eq. (i)

Hence $\left|\frac{z_1 - z_2}{z_1 + z_2}\right| = 1.$

Date: 9/10/2021

Q1. Represent 1 in polar form.

Q2. If $z = 1 + i$, represent z in polar form.

Q3. If $z = (1 - i)$, represent z in polar form.

Q4. If $z = -1 + i$, represent z in polar form.

Q5. If $z = (-1 - i)$, represent z in the form of $r(\cos\theta + i\sin\theta)$.

Q6. Represent i in polar form.

Q7. Express $(\sqrt{3} + i)$ in polar form.

Q8. If $z = (3 + 4i)$, represent z in polar form.

Q9. If $z = (1 + \sqrt{3}\,i)$, represent z in polar form.

Q10. If $z = x + iy$, where x and y are real numbers, express z in polar form.

Q11. Convert the complex number $\frac{-16}{1 + i\sqrt{3}}$ into polar form.

Q12. Convert the complex number $z = \frac{i - 1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$ in the polar form.

Q13. Convert the following in the polar form: $\frac{1 + 3i}{1 - 2i}$.

Q14. Convert the following in the polar form: $\frac{1 + 7i}{(2 - i)^2}$.

Q15. Convert the complex number i into polar form.

Q16. Convert the complex number -3 into polar form.

Date: 9/10/2021

S1. Let $1 = r[\cos\theta + i\sin\theta]$

$\Rightarrow \quad r\cos\theta = 1, \quad r\sin\theta = 0$

Squaring and adding, we get

$$r^2 = 1 \Rightarrow r = 1$$

$\Rightarrow \quad \sin\theta = 0 \Rightarrow \theta = 0$

$\therefore$ 1 can be represented in polar form as

$$1 = (\cos 0 + i\sin 0).$$

S2. Let $1 + i = r(\cos\theta + i\sin\theta)$

$\Rightarrow \quad r\cos\theta = 1, \quad r\sin\theta = 1$

On squaring and adding, we get

$$r^2(\cos^2\theta + \sin^2\theta) = 1^2 + 1^2$$

$\Rightarrow \quad r = \sqrt{2}$

$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{2}}, \quad \sin\theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \theta = \frac{\pi}{4}$

$\therefore \quad (1 + i) = \frac{1}{\sqrt{2}}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

S3. $\because \quad z = (1 - i)$

Let $\quad (1 - i) = r(\cos\theta + i\sin\theta)$

$\Rightarrow \quad r\cos\theta = 1, \quad r\sin\theta = -1$

Squaring and adding both sides, we get

$$r^2(\cos^2\theta + \sin^2\theta) = 1 + 1$$

$\Rightarrow \quad r = \sqrt{2}$

$\therefore \quad \cos\theta = \frac{1}{\sqrt{2}}, \quad \sin\theta = -\frac{1}{\sqrt{2}}$

$\therefore \qquad (1 - i) = \sqrt{2} \cdot \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$

$= \sqrt{2} \cdot \left\{ \cos \left(\frac{-\pi}{4} \right) + i \sin \left(\frac{-\pi}{4} \right) \right\}$

S4. Let $\qquad -1 + i = r(\cos \theta + i \sin \theta)$

$\Rightarrow \qquad r \cos \theta = -1, \quad r \sin \theta = 1$

Squaring and adding both sides, we get

$r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1$

$\Rightarrow \qquad r = \sqrt{2}$

$\cos \theta = \frac{-1}{\sqrt{2}} \quad \Rightarrow \quad \sin \theta = \frac{1}{\sqrt{2}}$

$\Rightarrow \qquad \theta = \frac{3\pi}{4}$

$\therefore \qquad (-1 + i) = \sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$

S5. Let $\quad -1 - i = r(\cos \theta + i \sin \theta)$

$\Rightarrow \qquad r \cos \theta = -1$

$r \sin \theta = -1$

$\Rightarrow \qquad r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1$

$\Rightarrow \qquad r^2 = 2 \quad \Rightarrow \quad r = \sqrt{2}$

$\therefore \qquad \cos \theta = \frac{-1}{\sqrt{2}}, \quad \sin \theta = \frac{-1}{\sqrt{2}}$

$\theta = \frac{-3\pi}{4}$

$\therefore \qquad -1 - i = \sqrt{2} \left[\cos \left(\frac{-3\pi}{4} \right) + i \sin \left(\frac{-3\pi}{4} \right) \right]$

S6. Let $\quad i = r[\cos \theta + i \sin \theta]$

$\Rightarrow \quad r \cos \theta = 0 \quad$ and $\quad r \sin \theta = 1$

On squaring and adding

$r^2[\cos^2 \theta + \sin^2 \theta] = 0 + 1$

$\Rightarrow \qquad r^2 = 1 \quad \Rightarrow \quad r = 1$

Now, $\qquad \cos \theta = \frac{0}{r} \quad$ and $\quad \sin \theta = \frac{1}{r} = 1$

$\Rightarrow \qquad \theta = \frac{\pi}{2}$

Therefore, $i = \left[\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right]$

S7. Let $\sqrt{3} + i = r[\cos\theta + i\sin\theta]$

$\Rightarrow \quad r\cos\theta = \sqrt{3} \quad \text{and} \quad r\sin\theta = 1$

On squaring and adding

$$r^2[\cos^2\theta + \sin^2\theta] = 3 + 1$$

$\Rightarrow \quad r^2 = 4 \Rightarrow r = \sqrt{4} \Rightarrow r = 2$

Now, $\cos\theta = \frac{\sqrt{3}}{r} = \frac{\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{r} = \frac{1}{2}$

$\Rightarrow \quad \sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

Therefore, $\sqrt{3} + i = 2\left[\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right]$

S8. Let $(3 + 4i) = r[\cos\theta + i\sin\theta]$

$\Rightarrow \quad r\cos\theta = 3$ and $r\sin\theta = 4$

On squaring and adding, we get

$$r^2[\cos^2\theta + \sin^2\theta] = 9 + 16$$

$\Rightarrow \quad r^2 = 25 \Rightarrow r = 5$

Now, $\sin\theta = \frac{4}{r} = \frac{4}{5}$

and $\cos\theta = \frac{3}{r} = \frac{3}{5}$

$\therefore \quad \tan\theta = \frac{4/5}{3/5} \Rightarrow \theta = \tan^{-1}\frac{4}{3}$

Therefore, $(3 + 4i) = 5\left[\cos\left(\tan^{-1}\frac{4}{3}\right) + i\sin\left(\tan^{-1}\frac{4}{3}\right)\right]$

S9. Let $(1 + \sqrt{3}\,i) = r[\cos\theta + i\sin\theta]$

$\Rightarrow \quad r\cos\theta = 1 \quad \text{and} \quad r\sin\theta = \sqrt{3}$

On squaring and adding, we get

$$r^2[\cos^2\theta + \sin^2\theta] = 1 + 3$$

$\Rightarrow \quad r^2 = 4 \Rightarrow r = 2$

Now, $\cos\theta = \dfrac{1}{r} = \dfrac{1}{2}$ and $\sin\theta = \dfrac{\sqrt{3}}{r} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow$ $\theta = \dfrac{\pi}{3}$

Therefore $(1 + \sqrt{3}\,i) = 2\left[\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right]$

S10. Let $x + iy = r(\cos\theta + i\sin\theta)$

$\Rightarrow$ $r\cos\theta = x,\quad r\sin\theta = y$

Squaring and adding both, we get

$$r^2(\cos^2\theta + \sin^2\theta) = x^2 + y^2$$

$\Rightarrow$ $r^2 = x^2 + y^2 \Rightarrow r = \sqrt{x^2 + y^2}$

$\therefore$ $r\cos\theta = x \Rightarrow \sqrt{x^2 + y^2}\cdot\cos\theta = \text{x}$

$\Rightarrow$ $\cos\theta = \dfrac{x}{\sqrt{x^2 + y^2}} \Rightarrow \theta = \cos^{-1}\dfrac{x}{\sqrt{x^2 + y^2}}$

$\therefore$ $(x + iy)$ can be represented in r $(\cos\theta + i\sin\theta)$ as

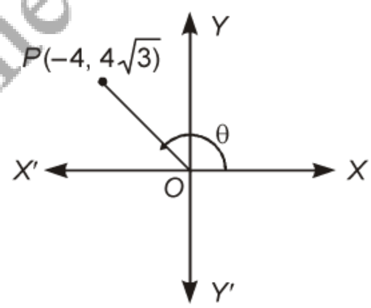
$$\sqrt{x^2 + y^2}\left\{\cos\left(\frac{\cos^{-1}x}{\sqrt{x^2 + y^2}}\right) + i\sin\left(\frac{\cos^{-1}x}{\sqrt{x^2 + y^2}}\right)\right\}.$$

S11. The given complex number

$$\frac{-16}{1 + i\sqrt{3}} = \frac{-16}{1 + i\sqrt{3}} \times \frac{1 - i\sqrt{3}}{1 - i\sqrt{3}}$$

$$= \frac{-16(1 - i\sqrt{3})}{1 - (i\sqrt{3})^2} = \frac{-16(1 - i\sqrt{3})}{1 + 3}$$

$$= -4(1 - i\sqrt{3}) = -4 + i4\sqrt{3}$$



Let $-4 = r\cos\theta,\quad 4\sqrt{3} = r\sin\theta$

By squaring and adding, we get

$$16 + 48 = r^2(\cos^2\theta + \sin^2\theta)$$

which gives $r^2 = 64$, *i.e.*, $r = 8$

Hence $\cos\theta = -\dfrac{1}{2},\quad \sin\theta = \dfrac{\sqrt{3}}{2}$

$$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Thus, the required polar form is $8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$.

S12. We have,

$$z = \frac{i-1}{\frac{1}{2}+\frac{\sqrt{3}}{2}i} = \frac{2(i-1)}{1+\sqrt{3}i} \times \frac{1-\sqrt{3}i}{1-\sqrt{3}i}$$

$$= \frac{2(i+\sqrt{3}-1+\sqrt{3}i)}{1+3} = \frac{\sqrt{3}-1}{2} + \frac{\sqrt{3}+1}{2}i$$

Now, put $\frac{\sqrt{3}-1}{2} = r\cos\theta, \quad \frac{\sqrt{3}+1}{2} = r\sin\theta$

Squaring and adding, we obtain

$$r^2 = \left(\frac{\sqrt{3}-1}{2}\right)^2 + \left(\frac{\sqrt{3}+1}{2}\right)^2 = \frac{2((\sqrt{3})^2+1)}{4} = \frac{2\times 4}{4} = 2$$

Hence, $r = \sqrt{2}$ which gives

$$\cos\theta = \frac{\sqrt{3}-1}{2\sqrt{2}}, \quad \sin\theta = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Therefore, $\theta = \frac{\pi}{4} + \frac{\pi}{6} = \frac{5\pi}{12}$

Hence, the polar form is $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$.

S13. Let $\frac{1+3i}{1-2i} = \frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i}$

$$= \frac{1+2i+3i+6i^2}{1-(2i)^2}$$

$$= \frac{1+5i-6}{1-4i^2}$$

$$= \frac{-5+5i}{1+4} = -1+i$$

Let $-1 + i = r(\cos\theta + i\sin\theta)$

Then $r\cos\theta = -1$

and $r\sin\theta = 1$

Squaring and adding, we get

$$r^2 (\cos^2 \theta + \sin^2 \theta) = (-1)^2 + 1^2 = 2$$

or $$r^2 = 2 \Rightarrow r = \sqrt{2}$$

Now, $$\cos \theta = \frac{-1}{\sqrt{2}} \quad \text{and} \quad \sin \theta = \frac{1}{\sqrt{2}}$$

or $$\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

Thus, $$\frac{1+3i}{1-2i} = \sqrt{2}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right).$$

S14. Let $$\frac{1+7i}{(2-i)^2} = \frac{1+7i}{4+i^2-4i}$$

$$= \frac{1+7i}{4-1-4i}$$

$$= \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i}$$

$$= \frac{3+4i+21i+28i^2}{9-16i^2}$$

$$= \frac{-25+25i}{25} = -1+i$$

Let $$-1 + i = r(\cos \theta + i \sin \theta)$$

Then $$r \cos \theta = -1$$

and $$r \sin \theta = 1$$

Squaring and adding, we get

$$r^2 (\cos^2 \theta + \sin^2 \theta) = (-1)^2 + 1^2 = 2$$

or $$r^2 = 2 \Rightarrow r = \sqrt{2}$$

or $$\cos \theta = \frac{-1}{\sqrt{2}} \quad \text{and} \quad \sin \theta = \frac{1}{\sqrt{2}}$$

or $$\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$$

Thus, $$-1 + i = \sqrt{2}\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right).$$

S15. Let $$i = r(\cos \theta + i \sin \theta)$$

Therefore, $$r \cos \theta = 0$$

and $$r \sin \theta = 1$$

$$r^2 \cos^2\theta + r^2 \sin^2\theta = 0 + (1)^2 = 1$$

$$r^2(\cos^2\theta + \sin^2\theta) = 1$$

$$r^2 = 1 \quad \Rightarrow \quad r = 1$$

Now, $\cos\theta = 0, \quad \sin\theta = 1$

or $\theta = \frac{\pi}{2}$

Hence, $i = \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$$= \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}.$$

S16. Let $-3 = r(\cos\theta + i\sin\theta)$

Therefore, $r\cos\theta = -3$

and $r\sin\theta = 0$

Squaring and adding, we get

$$r^2(\cos^2\theta + \sin^2\theta) = (-3)^2 + (0)^2 = 9$$

$$r^2 = 9$$

$\Rightarrow$ $r = 3$

Here, $\cos\theta = -1$

and $\sin\theta = 0$

$$\theta = -\pi$$

Hence, the polar form is $-3 = 3[\cos(-\pi) + i\sin(-\pi)]$.

Date: 9/10/2021

Q1. Solve $x^2 + 1 = 0$.

Q2. Solve $x^2 + 2 = 0$.

Q3. Solve: $x^2 + 3 = 0$.

Q4. If α, β are the roots of eq. $x^2 + x + 1 = 0$, then find the value of α and β.

Q5. Find the roots of $x^2 - 7ix - 12 = 0$.

Q6. Solve the equation $x^2 - x + 2 = 0$.

Q7. Solve $x^2 - 5ix + 6 = 0$.

Q8. Solve $x^2 - 9ix + 10 = 0$.

Q9. Solve $ix^2 - x + 12i = 0$

Q10. Solve the eq. $x^2 - x + 1 + i = 0$.

Q11. Solve $x^2 - (2 + i)x = 1 - 7i$.

Q12. Solve: $2x^2 + x + 1 = 0$.

Q13. Solve: $x^2 + 3x + 9 = 0$.

Q14. Solve: $x^2 + 3x + 5 = 0$.

Q15. Solve: $\sqrt{2}\,x^2 + x + \sqrt{2} = 0$.

Q16. Solve: $\sqrt{3}\,x^2 + \sqrt{2}\,x + 3\sqrt{3} = 0$.

Q17. Solve: $x^2 + x + \frac{1}{\sqrt{2}} = 0$.

Q18. Solve: $3x^2 - 4x + \frac{20}{3} = 0$.

Q19. Solve: $x^2 - 2x + \frac{3}{2} = 0$.

Q20. Solve: $27x^2 - 10x + 1 = 0$.

Q21. Solve: $21x^2 - 28x + 10 = 0$.

Date: 9/10/2021

S1. We have

$$x^2 + 1 = 0 \Rightarrow x^2 = -1$$

$$\Rightarrow \quad x^2 = i^2 \Rightarrow (x - i)(x + i) = 0$$

$$\Rightarrow \quad x = \pm i$$

S2.

$$x^2 + 2 = 0 \Rightarrow x^2 - (\sqrt{2}\, i)^2 = 0$$

$$\Rightarrow (x - (\sqrt{2}\, i)(x + (\sqrt{2}\, i) = 0$$

$$\Rightarrow \quad x = \pm\sqrt{2}\, i$$

S3. Here, $x^2 + 3 = 0$

$$\Rightarrow \quad x^2 = -3$$

$$\Rightarrow \quad x = \pm\sqrt{-3} = \pm\sqrt{3}\, i.$$

S4. Since, $x^2 + x + 1 = 0$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$\Rightarrow \quad x = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$$

$$\Rightarrow \quad \alpha = \frac{-1 + i\sqrt{3}}{2}, \quad \beta = \frac{-1 - i\sqrt{3}}{2}$$

S5. Since, $x^2 - 7ix - 12 = 0$

$$\Rightarrow \quad x^2 - 3ix - 4ix - 12 = 0$$

$$\Rightarrow \quad x(x - 3i) - 4i(x - 3i) = 0$$

$$\Rightarrow \quad (x - 3i)(x - 4i) = 0$$

Hence, $x = 3i, 4i$

S6.

$$x^2 - x + 2 = 0$$

Let x_1 and x_2 be roots of this eq.

Here, $a = 1, \quad b = -1, \quad c = 2$

$\therefore \quad \langle x_1,\ x_2\rangle = \dfrac{+1 \pm \sqrt{1-4\times 2}}{2}$

$= \dfrac{1 \pm \sqrt{-7}}{2}$

$\therefore \quad \langle x_1,\ x_2\rangle = \left\langle \dfrac{1+\sqrt{-7}}{2}, \dfrac{1-\sqrt{-7}}{2} \right\rangle = \left\langle \dfrac{1+\mathrm{i}\sqrt{7}}{2}, \dfrac{1-\mathrm{i}\sqrt{7}}{2} \right\rangle$

S7. Since, $x^2 - 5ix + 6 = 0$

$\Rightarrow \quad x^2 - 2ix - 3ix + 6 = 0$

$\Rightarrow \quad x(x - 2i) - 3i(x - 2i) = 0$

$\Rightarrow \quad (x - 3i)(x - 2i) = 0$

$\Rightarrow \quad x = 3i, 2i.$

S8. Since, $x^2 - 9ix + 10 = 0$

$\Rightarrow \quad x^2 - 10ix + ix + 10 = 0$

$\Rightarrow \quad x(x - 10i) + i(x - 10i) = 0$

$\Rightarrow \quad (x + i)(x - 10i) = 0$

$\Rightarrow \quad x = 10i, -i.$

S9. The given Eq. is $ix^2 - x + 12i = 0$.

$a = i, \quad b = -1, \quad c = 12i$

$\therefore \quad x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4i(12i)}}{2i}$

$= \dfrac{1 \pm \sqrt{1+48}}{2i} = \dfrac{1 \pm 7}{2i}$

$\therefore \quad x_1 = \dfrac{1}{2i} + \dfrac{7}{2i}, \quad x_2 = \dfrac{1}{2i} - \dfrac{7}{2i}$

After simplifying, we get

Hence, $(x_1, x_2) = (4i, -3i)$.

S10. Let x_1 and x_2 be roots of this equation.

Here, $a = 1, \quad b = -1, \quad c = 1 + i$

$\therefore \quad \langle x_1,\ x_2\rangle = \dfrac{1 \pm \sqrt{1-4(1+i)}}{2}$

$\therefore \quad \langle x_1,\ x_2\rangle = \dfrac{1 \pm \sqrt{-3-4i}}{2}$

$$\langle x_1, x_2\rangle = \left\langle \frac{1+\sqrt{-3-4i}}{2}, \frac{1-\sqrt{-3-4i}}{2} \right\rangle.$$

S11. $x^2 - (2 + i)x + (7i - 1) = 0$

Here, $a = 1$, $b = -(2 + i)$, $c = (7i - 1)$

$$\therefore \quad \langle x_1, x_2\rangle = \frac{(2+i) \pm \sqrt{(2+i)^2 - 4(7i-1)}}{2}$$

$$= \frac{(2+i) \pm \sqrt{4 - 1 + 4i - 28i + 4}}{2}$$

$$= \frac{(2+i) \pm \sqrt{7 - 24i}}{2}$$

$$\langle x_1, x_2\rangle = \left\langle \frac{(2+i) + \sqrt{7-24i}}{2}, \frac{(2+i) - \sqrt{7-24i}}{2} \right\rangle.$$

S12. Let $2x^2 + x + 1 = 0$

$a = 2$, $b = 1$, $c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4 \times 2 \times 1}}{2 \times 2}$$

$$x = \frac{-1 \pm \sqrt{-7}}{4} = \frac{-1 \pm \sqrt{7}\,i}{4}.$$

S13. Let $x^2 + 3x + 9 = 0$

$a = 1$, $b = 3$, $c = 9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4 \times 1 \times 9}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{9 - 36}}{2} = \frac{-3 \pm \sqrt{-27}}{2}$$

$$x = \frac{-3 \pm \sqrt{27}\,i}{2}$$

$$x = \frac{-3 \pm 3\sqrt{3}\,i}{2}.$$

S14. Let $x^2 + 3x + 5 = 0$

$a = 1, \quad b = 3, \quad c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4 \times 1 \times 5}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{9 - 20}}{2} = \frac{-3 \pm \sqrt{-11}}{2}$$

$$x = \frac{-3 \pm \sqrt{11}i}{2}.$$

S15. Let $\sqrt{2}\,x^2 + x + \sqrt{2} = 0$

$a = \sqrt{2}, \quad b = 1, \quad c = \sqrt{2}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4 \times \sqrt{2} \times \sqrt{2}}}{2\sqrt{2}}$$

$$x = \frac{-1 \pm \sqrt{1 - 8}}{2\sqrt{2}} = \frac{-1 \pm \sqrt{-7}}{2\sqrt{2}} = \frac{-1 \pm \sqrt{7}\,i}{2\sqrt{2}}.$$

S16. Let $\sqrt{3}\,x^2 + \sqrt{2}\,x + 3\sqrt{3} = 0$

$a = \sqrt{3}, \quad b = -\sqrt{2}, \quad c = 3\sqrt{3}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-\sqrt{2}) \pm \sqrt{(-\sqrt{2})^2 - 4 \times \sqrt{3} \times 3\sqrt{3}}}{2\sqrt{3}}$$

$$x = \frac{\sqrt{2} \pm \sqrt{2 - 36}}{2\sqrt{3}} = \frac{\sqrt{2} \pm \sqrt{-34}}{2\sqrt{3}} = \frac{\sqrt{2} \pm \sqrt{34}\,i}{2\sqrt{3}}.$$

S17. Let $x^2 + x + \frac{1}{\sqrt{2}} = 0$

$a = 1, \quad b = 1, \quad c = \frac{1}{\sqrt{2}}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4 \times 1 \times \frac{1}{\sqrt{2}}}}{2 \times 1}$$

$$x = \frac{-1 \pm \sqrt{1 - 2\sqrt{2}}}{2}$$

$$= \frac{-1 \pm \sqrt{-(2\sqrt{2} - 1)}}{2}$$

$$= \frac{-1 \pm i\sqrt{(2\sqrt{2} - 1)}}{2}.$$

S18. Let $3x^2 - 4x + \frac{20}{3} = 0$

or $9x^2 - 12x + 20 = 0$

$a = 9, \quad b = -12, \quad c = 20$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \times 9 \times 20}}{2 \times 9}$$

$$x = \frac{12 \pm \sqrt{144 - 720}}{18}$$

$$x = \frac{12 \pm \sqrt{-576}}{18}$$

$$x = \frac{12 \pm 24i}{18}$$

$$x = \frac{2}{3} + \frac{4}{3}i.$$

S19. Let $x^2 - 2x + \frac{3}{2} = 0$

or $2x^2 - 4x + 3 = 0$

$a = 2, \quad b = -4, \quad c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 2 \times 3}}{2 \times 2}$$

$$x = \frac{4 \pm \sqrt{16 - 24}}{4}$$

$$x = \frac{4 \pm \sqrt{-8}}{4}$$

$$x = \frac{4 \pm 2\sqrt{2}\, i}{4}$$

Hence, $$x = 1 \pm \frac{\sqrt{2}}{2}\, i .$$

S20. Let $27x^2 - 10x + 1 = 0$

$a = 27, \quad b = -10, \quad c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4 \times 27 \times 1}}{2 \times 27}$$

$$x = \frac{10 \pm \sqrt{100 - 108}}{54} = \frac{10 \pm \sqrt{-8}}{54}$$

$$x = \frac{10 \pm 2\sqrt{2}\, i}{54}$$

Hence, $$x = \frac{5}{27} \pm \frac{\sqrt{2}}{27}\, i .$$

S21. Let $21x^2 - 28x + 10 = 0$

$a = 21, \quad b = -28, \quad c = 10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-28) \pm \sqrt{(-28)^2 - 4 \times 21 \times 10}}{2 \times 21}$$

$$x = \frac{28 \pm \sqrt{784 - 840}}{42} = \frac{28 \pm \sqrt{-56}}{42}$$

$$x = \frac{28 \pm 2\sqrt{14}\, i}{42}$$

Hence, $$x = \frac{2}{3} \pm \frac{\sqrt{14}}{21}\, i .$$

Date: 9/10/2021

Q1. Prove that $1 + i^2 + i^4 + i^6 = 0$.

Q2. Prove that $i^{-1} - i^{-2} + i^{-3} - i^{-4} = 0$

Q3. Prove that $i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0$, $n \in N$.

Q4. Prove that $(1+i)^4\left(1+\frac{1}{i}\right)^4 = 16$.

Q5. If $x + iy = (2 + 3i)^2$, find the value of x and y.

Q6. Find the smallest positive integer m for which $\left[\frac{1+i}{1-i}\right]^m = 1$.

Q7. Represent the point $(1 - i)$ in complex plane.

Q8. Find the value of x and y such that $2 + (x + iy) = (5 - i)$.

Q9. Find the value of x and y such that $3 + (x + iy) = (2 + i)^2$.

Q10. Find the value of x and y such that $(x + iy) = (1 + i)(1 - i)$.

Q11. Prove that: $3(1 - 2i) - (-4 - 5i) + (-8 + 3i) = 2i - 1$.

Q12. Prove that: $i^{107} + i^{112} + i^{117} + i^{122} = 0$.

Q13. Find the value of x and y such that $3x + (3x + 2)i = 12 + yi$.

Q14. Find the value of x and y such that $i(x + iy) = (2 + 3i)(3 + 2i)$.

Q15. If $4x + i(3x - y) = 3 + i(-6)$, where x and y are real numbers, then find the values of x and y.

Q16. Express the complex number given in the form $a + ib$: $(5i)\left(-\frac{3}{5}i\right)$.

Q17. Express the complex number given in the form $a + ib$: $i^9 + i^{19}$.

Q18. Express the complex number given in the form $a + ib$: i^{-39}.

Q19. Express the complex number given in the form $a + ib$: $3(7 + i7) + i(7 + i7)$.

Q20. Prove that $(x + 1 + i)(x + 1 - i)(x - 1 + i)(x - 1 - i) = x^4 + 4$.

Q21. Solve for x and y, $3x + (2x - y)i = 6 - 3i$.

Q22. If $z = 1 + 2i$, represent z on the complex plane.

Q23. Find the value of x and y such that $5x + 3iy + 7 = 3 + 4i$

Q24. Find the value of x and y such that $(1 + i)(x + iy) = (3 + i)^2$.

Q25. Find the value of x and y such that $(x + iy)(2 - 3i) = 2 + 3i$.

Date: 9/10/2021

S1. $\because \quad i^4 = 1, \quad i^6 = i^4 \cdot i^2 = -1, \quad i^2 = -1$

$\therefore \quad 1 + i^2 + i^4 + i^6 = 1 - 1 + 1 - 1 = 0$ **Proved**.

S2. From L.H.S.

$$i^{-1} - i^{-2} + i^{-3} - i^{-4} = \frac{1}{i} - \frac{1}{i^2} + \frac{1}{i^3} - \frac{1}{i^4} = \frac{1}{i} + 1 - \frac{1}{i} - 1 = 0 \qquad \text{Proved}$$

S3. Given, $i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0$

$$i^n + i^n \cdot i + i^n \cdot i^2 + i^n \cdot i^3 = i^n + i^n \cdot i - i^n - i \cdot i^n = 0$$

Hence,

$i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0$ **Proved**.

S4. From L.H.S.

$$(1+i)^4\left(1+\frac{1}{i}\right)^4 = (1+i)^4\,(1-i)^4 = \{(1+i)(1-i)\}^4 = 2^4 = 16$$

L.H.S. = R.H.S. Proved

S5. Since, $x + iy = 4 - 9 + 12i$

$\Rightarrow \quad x + iy = -5 + 12i$

Equating real and imaginary coefficients, we get

$$x = -5, \quad y = 12$$

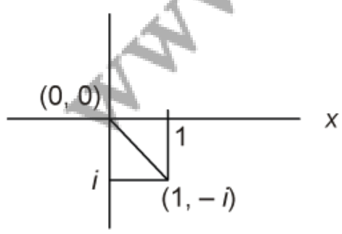
S6.

$$\frac{(1+i)}{(1-i)} = \frac{(1+i)^2}{(1-i)(1+i)} = \frac{2i}{2} = \text{i}$$

$\Rightarrow \quad i^m = 1 \quad \Rightarrow \quad m = 4k \; (K \in I)$

Hence smallest positive integer should be 4.

S7.



S8. Given, $2 + (x + iy) = (5 - i)$

$\Rightarrow (x + iy) = 3 - i$

Equating real and imaginary coefficients, we get

$$x = 3, \quad y = -1$$

S9. Given, $3 + (x + iy) = (2 + i)^2$

$\Rightarrow 3 + (x + iy) = 4 - 1 + 4i$

$\Rightarrow x + iy = 3 + 4i - 3$

$\Rightarrow x + iy = 4i$

Equating real and imaginary coefficients, we get

$$x = 0, \quad y = 4$$

S10. Given, $(x + iy) = (1 + i)(1 - i)$

$\Rightarrow (x + iy) = 1 - i^2$

$\Rightarrow x + iy = 2$

Equating real and imaginary coefficient, we get

$$x = 2, \quad y = 0$$

S11. From L.H.S.

$3(1 - 2i) - (-4 - 5i) + (-8 + 3i) = 3 - 6i + 4 + 5i - 8 + 3i$

$= -1 + 2i$

$= 2i - 1$ Proved

S12. Since, we have that $i^{4k} = 1$, $K \in \mathrm{I}$.

$\therefore \quad i^{107} + i^{112} + i^{117} + i^{122} = i^{(4 \times 26 + 3)} + i^{4 \times 28} + i^{4 \times 29 + 1} + i^{4 \times 30 + 2}$

$= i^3 + 1 + i + i^2$

$= -i + 1 + i - 1 = 0$ Proved

S13. Given, $(3x - 12) + (3x - y + 2)i = 0$

Hence, $3x - 12 = 0$ and $3x - y + 2 = 0$

$\Rightarrow x = 4, \quad y = 14.$

S14. Given, $i(x + iy) = 6 + 4i + 9i - 6$

$\Rightarrow i(x + iy) = 13i$

$\Rightarrow x + iy = 13$

Equating real and imaginary coefficients, we get

$$x = 13, \quad y = 0$$

S15. We have

$$4x + i(3x - y) = 3 + i(-6) \quad \dots \text{(i)}$$

Equating the real and the imaginary parts of Eq. (i), we get

$$4x = 3, \quad 3x - y = -6,$$

which, on solving simultaneously, give $x = \frac{3}{4}$ and $y = \frac{33}{4}$.

S16. We have $(5i)\left(-\frac{3}{5}i\right) = -3i^2 = -3(-1) = 3.$

$\therefore \quad a + ib = 3 + 0i.$

S17. Let $i^9 + i^{19} = (i^2)^4 \cdot i + (i^2)^9 \cdot i$

$= (-1)^4 i + (-1)^9 i$

$= i - i = 0.$

$\therefore \quad a + ib = 0 + 0i.$

S18. Let $(i)^{-39} = (i^2)^{-19} i^{-1}$

$= (-1)^{-19} i^{-1}$

$= \frac{1}{(-1)^{19}} \times \frac{1}{i} = -\frac{1}{i} \times \frac{i}{i}$

$= \frac{-i}{(i^2)} = \frac{-i}{-1} = i.$

$\therefore \quad a + ib = 0 + i.$

S19. Let $3(7 + i7) + i(7 + i7) = 21 + 21i + 7i + 7i^2$

$= 21 + 28i + 7(-1)$

$= 14 + 28i.$

$\therefore \quad a + ib = 14 + 28i.$

S20. $(x + 1 + i)(x + 1 - i)(x - 1 + i)(x - 1 - i)$

$= (x + 1 + i)(x - (1 + i))(x + (1 - i))(x - (1 - i))$

$= (x^2 - (1 + i)^2)(x^2 + (1 - i)^2)$

$= (x^2 - 2i)(x^2 + 2i) = x^4 + 4$ **Proved**.

S21. Since, $3x + (2x - y)i = 6 - 3i$

$\Rightarrow \quad 3x + 2xi - yi - 6 + 3i = 0$

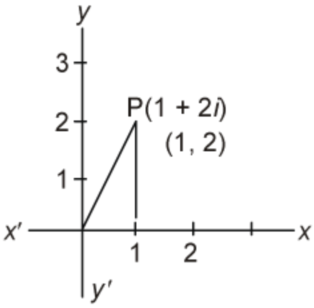
$\Rightarrow \quad (3x - 6) + i(2x - y + 3) = 0$

$\Rightarrow$ $3x - 6 = 0 \quad \Rightarrow \quad x = 2$

$2x - y + 3 = 0 \quad \Rightarrow \quad y = 7$

Hence, $x = 2, \quad y = 7$

S22.



Here, $z = 1 + 2i$, this represents the point $P(1, 2)$.

S23. Given, $5x + 3iy + 7 = 3 + 4i$

$\Rightarrow$ $(5x + 4) + i(3y - 4) = 0$

Hence, $5x + 4 = 0, \quad 3y - 4 = 0$

$\Rightarrow$ $x = \frac{-4}{5}, \quad y = \frac{4}{3}$

S24. $(1 + i)(x + iy) = (3 + i)^2$

$\Rightarrow$ $x + iy = \frac{(9 - 1 + 6i)(1 - i)}{(1 + i)(1 - i)}$

$\Rightarrow$ $x + iy = \frac{(8 + 6i)(1 - i)}{2}$

$\Rightarrow$ $x + iy = (4 + 3i)(1 - i)$

$\Rightarrow$ $x + iy = 4 - 4i + 3i + 3$

$\Rightarrow$ $x + iy = 7 - i$

Equating real and imaginary coefficients, we get

$x = 7, \quad y = -1.$

S25. $(x + iy)(2 - 3i) = 2 + 3i$

$\Rightarrow$ $(x + iy) = \frac{(2 + 3i)^2}{(2 - 3i)(2 + 3i)}$

$\Rightarrow$ $(x + iy) = \frac{4 - 9 + 12i}{4 + 9}$

$\Rightarrow$ $x + iy = \frac{-5 + 12i}{13}$

$\Rightarrow$ $x + iy = \frac{-5}{13} + \frac{12}{13}i$

Equating real and imaginary coefficients, we get

$$x = \frac{-5}{13}, \quad y = \frac{12}{13}.$$