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Q1. For all $n \geq 1$, prove that $1^2 + 2^2 + 3^2 + 4^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$.

Q2. Prove that the rule of exponents $(ab)^n = a^n b^n$ is valid for all n, (a, $b \geq 0$) by using principle of mathematical induction for every natural number.

Q3. Prove that $1 + 3 + 3^2 + ... + 3^{n-1} = \frac{1}{2}(3^n - 1)$

Q4. Using the principle of mathematical induction prove that

$$\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + ... + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}.$$

Q5. Using the principle of mathematical induction prove that

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + ... + n(n+1) = \frac{1}{3}n(n+1)(n+2)$$

Q6. Using mathematical induction show that

$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + 3 \cdot 4 \cdot 5 + ... + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}.$$

Q7. Using Principle of Mathematical Induction for all $\{n \geq 1, n \in N\}$ prove that $1 + 2 + 3 + 4 + 5 + ... + n = \frac{n(n+1)}{2}$.

Q8. Using Principle of Mathematical Induction for $n \geq 1$ prove that

$$3 + 8 + 13 + 18 + ... + (5n - 2) = \frac{n(5n+1)}{2}.$$

Q9. Using Principle of Mathematical Induction for all $n \geq 1$ prove that $1 + 3 + 5 + 7 + ... = n^2$.

Q10. Using principle of mathematical induction prove that $2 + 3 \cdot 2 + 4 \cdot 2^2 + ... + (n+1)2^{n-1} = n \cdot 2^n$.

Q11. Using principle of mathematical induction prove that

$$2 + 6 + 18 + . . . + 2 \cdot 3^{n-1} = 3^n - 1.$$

Q12. Using Principle of Mathematical Induction prove that $2 + 4 + 6 + 8 + ... + 2n = n(n + 1)$.

Q13. Prove the following by using the principle of mathematical induction for all $n \in N$.

$$1^3 + 2^3 + 3^3 + ... + n^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

Q14. Prove the following by using the principle of mathematical induction for all $n \in N$.

$$1.2 + 2.2^2 + 3.2^2 + ... + n.2^n = (n-1)\,2^{n+1} + 2.$$

Q15. Prove the following by using the principle of mathematical induction for all $n \in N$.

$$\frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + ... + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}.$$

Q16. Prove the following by using the principle of mathematical induction for all $n \in N$.

$$1^2 + 3^2 + 5^2 + ... + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}.$$

Q17. Using principle of mathematical induction prove that

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ... + \frac{1}{2^n} = \left(1 - \frac{1}{2^n}\right).$$

Q18. Using the principle of mathematical induction, prove that

$$a + ar + ar^2 + ... + ar^{n-1} = \frac{a(r^n - 1)}{(r-1)}.$$

Q19. Using principle of mathematical induction prove that

$$(1+1)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\ldots\left(1+\frac{1}{n}\right) = (n+1).$$

Q20. Using principle of mathematical induction prove that

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + ... + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}.$$

Q21. Using principle of mathematical induction prove that

$$1.3 + 3.5 + 5.7 + ... + (2n-1)(2n+1) = \frac{n(4n^2+6n-1)}{3}.$$

Q22. Using principle of mathematical induction prove that

$$1.3 + 2.3^2 + 3.3^3 + ... + n.3^n = \frac{(2n-1).3^{n+1}+3}{4}.$$

Q23. Using principle of mathematical induction prove that

$$a + (a+d) + (a+2d) + ... + [a+(n-1)d] = \frac{n}{2}[2a + (n-1)d].$$

Q24. Using principle of mathematical induction prove that

$$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\left(1+\frac{2n+1}{n^2}\right) = (n+1)^2.$$

Q25. Using principle of mathematical induction prove that

$$1 + \frac{1}{1+2} + \frac{1}{1+2+3} + ... + \frac{1}{1+2+3+...+n} = \frac{2n}{n+1}.$$

Q26. Using principle of mathematical induction prove that

$$1 + 4 + 7 + ... + (3n-2) = \frac{1}{2}n(3n-1)$$

Q27. Using principle of mathematical induction prove that

$$\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + ... + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}.$$

Q28. Using the principle of mathematical induction prove that

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + ... + \frac{1}{n(n+1)} = \frac{n}{n+1}.$$

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S1. **Step I:** Let the given statement be $P(n)$.

$$P(n):\ 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

Step II: For $n = 1$, we have

$$\text{R.H.S.} = \frac{1(1+1)(2\times1+1)}{6} = \frac{1\times2\times3}{6} = 1 = 1^2 = \text{L.H.S.}$$

Therefore, $P(1)$ is true.

Step III: For $n = k$,

Assume that $P(k)$ is true for some positive integer k,

$$\therefore\quad P(k) : 1^2 + 2^2 + 3^2 + 4^2 + \ldots + k^2 = \frac{k(k+1)(2k+1)}{6}$$

Step IV: For $n = (k + 1)$

We shall now prove that $P(k+1) = \dfrac{(k+1)(k+2)\,[2(k+1)+1]}{6}$

Now, we have,

$$\text{L.H.S.} = (1^2 + 2^2 + 3^2 + \ldots + k^2) + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{(k+1)(k+1+1)(2(k+1)+1)}{6} = \text{R.H.S.}$$

Thus, $P(k+1)$ is true, whenever $P(k)$ is true. Hence from the principle of mathematical induction. The statement $P(n)$ is true for all natural number n.

S2. **Step I:** Let $P(n)$ be the given statement

$$P(n): (ab)^n = a^n b^n.$$

Step II: For $n = 1$, $(ab)^1 = a^1 b^1$. So $P(n)$ is true for $n = 1$.

Step III: Let $P(k)$ be true,

$$(ab)^k = a^k b^k$$

Step IV: We shall now prove that $P(k + 1)$ is true, whenever $P(k)$ is true.

For $n = (k + 1)$

$$(ab)^{k+1} = (ab)^k \cdot ab$$

$$= (a^k b^k) \cdot ab$$

$$= (a^k \cdot a^1) \cdot (b^k \cdot b^1)$$
$$= a^{k+1} \cdot b^{k+1}$$

Therefore, $P(k + 1)$ is also true, whenever $P(k)$ is true. Hence by principle of mathematical induction, $P(n)$ is true for all $n \in N$.

S3. **Step I:** Let the given statement be $P(n)$.

$$1 + 3 + 3^2 + ... + 3^{n-1} = \frac{1}{2}(3^n - 1)$$

Step II: For $n = 1$, we have

$$\text{R.H.S.} = \frac{1}{2}(3^1 - 1) = 1 = \text{L.H.S.}$$

Therefore $P(1)$ is true.

Step III: For $n = k$

Assume that $P(k)$ is true for some positive integer k.

$$\therefore \quad P(k) : 1 + 3 + 3^2 + ... + 3^{k-1} = \frac{1}{2}(3^k - 1)$$

Step IV: For $n = k + 1$

We shall now prove that $P(k + 1)$ is also true.

Now we have,

$$\text{L.H.S. } (1 + 3 + 3^2 + ... + 3^{k-1}) + 3^k.$$
$$= \frac{1}{2}(3^k - 1) + 3^k.$$
$$= \frac{3}{2} \cdot 3^k - \frac{1}{2}$$

$$\text{R.H.S. } \frac{1}{2}(3^{k+1} - 1)$$

Hence $P(k + 1)$ is true, whenever $P(k)$ is true.

Hence from the principle of mathematical induction the statement $P(n)$ is true for all natural number n.

S4. **Step I:** Let the given statement be $P(n)$.

$$\therefore \quad P(n): \frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + ... + \frac{1}{(2n+1)(2n+3)} = \frac{n}{3(2n+3)}$$

Step II: For $n = 1$, we have

$$\text{R.H.S.} = \frac{1}{3(2 \times 1 + 3)} = \frac{1}{3.5} = \text{L.H.S.}$$

Therefore $P(1)$ is true.

Step III: For $n = k$

Assume that $P(k)$ is true for some positive integer k.

$$\therefore \quad P(k) = \frac{1}{3.5} + \frac{1}{5.7} + \ldots + \frac{1}{(2k+1)(2k+3)} = \frac{k}{3(2k+3)}$$

Step IV: For $n = k + 1$

We shall now prove that $P(k + 1)$ is also true.

$$P(k+1): \frac{1}{3.5} + \frac{1}{5.7} + \ldots + \frac{1}{(2k+1)(2k+3)} + \frac{1}{\{2(k+1)+1\}\{2(k+1)+3\}}$$

$$= \frac{k+1}{3\{2(k+1)+3\}}$$

$\Rightarrow$ $P(k + 1)$ is true, whenever $P(k)$ is true.

Hence by principle of mathematical induction $P(n)$ is true for all $n \in N$.

S5. Step I: Let the given statement be $P(n)$, then

$$P(n): 1{\cdot}2 + 2{\cdot}3 + 3{\cdot}4 + \ldots + n(n+1) = \frac{1}{3}n(n+1)(n+2)$$

Step II: For $n = 1$

L.H.S. = 2, R.H.S. = $\frac{1}{3} \times 1 \times 2 \times 3 = 2$

L.H.S. = R.H.S.

Hence $P(1)$ is true.

Step III: Let $P(k)$ be true then

$$P(k): 1{\cdot}2 + 2{\cdot}3 + 3{\cdot}4 + \ldots.. + k(k+1) = \frac{1}{3}k(k+1)(k+2)$$

Now,

$$P(k+1) = 1{\cdot}2 + 2{\cdot}3 + 3{\cdot}4 + \ldots.. + k(k+1) + (k+1)(k+2)$$

$$= \frac{1}{3}(k+1)\,(k+2)\,(k+3)$$

L.H.S. $1{\cdot}2 + 2{\cdot}3 + 3{\cdot}4 + \ldots.. + k(k+1) + (k+1)(k+2)$

$$= \frac{1}{3}k(k+1)\,(k+2) + (k+1)\,(k+2)$$

$$= \frac{1}{3}(k+1)\,(k+2)\,(k+3) = \text{R.H.S.}$$

$\Rightarrow$ $P(k+1)$ is true, whenever $P(k)$ is true.

Hence by the principle of mathematical induction $P(n)$ is true for all $n \in N$.

S6. **Step I:** Denote the given statement by $(P(n))$

$$P(n): 1\cdot2\cdot3 + 2\cdot3\cdot4 + 3\cdot4\cdot5 + ... + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}.$$

Step II: Put $n = 1$ and show that $P(1)$ is true statement

For $n = 1$

L.H.S. = $1\cdot2\cdot3$ and

$$\text{R.H.S.} = \frac{1(1+1)(1+2)(1+3)}{4} = 1\cdot2\cdot3$$

Therefore, $P(1)$ is true.

Step III: Suppose that statement is true for $n = k$.

Let it is true for $n = k$, then ;

$$P(k): 1\cdot2\cdot3 + 2\cdot3\cdot4 + 3\cdot4\cdot5 + ... + k(k+1)(k+2)$$
$$= \frac{k(k+1)(k+2)(k+3)}{4}$$

Step IV: $P(k+1)$: $1\cdot2\cdot3 + 2\cdot3\cdot4 + ... + k(k+1)(k+2) + (k+1)(k+2)(k+3)$

$$= \frac{(k+1)(k+2)(k+3)(k+4)}{4}$$

$$\text{L.H.S.} = [1\cdot2\cdot3 + 2\cdot3\cdot4 + ... + k(k+1)(k+2)] + (k+1)(k+2)(k+3)$$
$$= \left[\frac{k(k+1)(k+2)(k+3)}{4}\right] + (k+1)(k+2)(k+3)$$
$$= \frac{k(k+1)(k+2)(k+3) + 4(k+1)(k+2)(k+3)}{4}$$

Now taking $(k+1)(k+2)(k+3)$ common in numerator part, we get

$$\text{L.H.S.} = \frac{(k+1)(k+2)(k+3)(k+4)}{4}$$
$$= \frac{(k+1)[(k+1)+1][(k+1)+2][(k+1)+3]}{4} = \text{R.H.S.}$$

Therefore $P(k+1)$ is true whenever $P(k)$ is true. Hence from principle of mathematical induction, the statement is true for all natural number n.

S7. **Step I:** Let the given statement be $P(n)$:

$$\therefore \quad P(n): 1 + 2 + 3 + 4 + 5 + ... + n = \frac{n(n+1)}{2}.$$

Step II: For $n = 1$, we have

$$\text{R.H.S.} = \frac{1(1+1)}{2} = 1 = \text{L.H.S.}$$

Therefore $P(1)$ is true.

Step III: Assume that $P(k)$ is true for some positive integer k.

$$\therefore \quad P(k) = 1 + 2 + 3 + 4 + 5 + \ldots = \frac{k(k+1)}{2}$$

Step IV: For $n = k + 1$

We shall now prove that $P(k + 1)$ is also true.

$$\therefore \quad P(k + 1) = 1 + 2 + 3 + \ldots k + (k + 1)$$

Now, we have,

$$\text{L.H.S.} = 1 + 2 + 3 + \ldots k + (k + 1)$$

$$= \frac{(k)(k+1)}{2} + (k+1) = \frac{k(k+1) + 2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2} = \frac{(k+1)(k+2)}{2}$$

Thus $P(k + 1)$ is true, whenever $P(k)$ is true.

Hence, from the principle of mathematical induction the statement $P(n)$ is true for all natural number n.

S8. Step I: Let the given statement be $P(n)$.

$$\therefore \quad P(n) = 3 + 8 + 13 + 18 + \ldots + (5n - 2) = \frac{n(5n+1)}{2}.$$

Step II: For $n = 1$, we have

$$\text{R.H.S.} = \frac{1(5 \times 1 + 1)}{2} = 3 = \text{L.H.S.}$$

Therefore, $P(1)$ is true.

Step III: For $n = k$

Assume that $P(k)$ is true for some positive integer k.

$$\therefore \quad P(k) = 3 + 8 + 13 + \ldots + (5k - 2) = \frac{k(5k+1)}{2}$$

Step IV: For $n = (k + 1)$

We shall now prove that $P(k + 1)$ is also true.

$$\therefore \quad P(k + 1) : 3 + 8 + 13 + (5k - 2) + (5k + 3) = \frac{(1+k)(5(k+1)+1)}{2}.$$

From L.H.S. = 3 + 8 + 13 + ... + (5k – 2) + (5k + 3)

$$= \frac{k(5k+1)}{2} + (5k+3) = \frac{(1+k)[(5(k+1)+1]}{2} = \text{R.H.S.}$$

Thus, P(k + 1) is true, whenever, $P(k)$ is true.

Hence, from the principle of mathematical induction the statement $P(n)$ is true for all natural number n.

S9. **Step I:** Let the given statement be $P(n)$.

$\therefore$ $P(n) = 1 + 3 + 5 + 7 + \dots = n^2$

Step II: For $n = 1$, we have

R.H.S. = $1^2 = 1$ = L.H.S.

Therefore, $P(1)$ is true.

Step III: For $n = k$

Assume that $P(k)$ is true for some positive integer k.

$\therefore$ $P(k)$:

$\Rightarrow$ $1 + 3 + 5 + \dots + (2k - 1) = k^2$

Step IV: We shall now prove that $P(k + 1)$ is also true.

$\therefore$ $P(k + 1): 1 + 3 + 5 + 7 + \dots + (2k - 1) + (2k + 1) = (k + 1)^2$

Now we have

L.H.S. = $1 + 3 + 5 + 7 + \dots + (2k - 1) + (2k + 1)$

$= k^2 + (2k + 1) = (k + 1)^2$ = R.H.S.

Thus $P(k + 1)$ is true, whenever $P(k)$ is true.

Hence from the principle of mathematical induction, the statement $P(n)$ is true for all natural number n.

S10. $P(n)$: $2 + 3 \cdot 2 + 4 \cdot 2^2 + \dots + (n + 1)2^{n-1} = n \cdot 2^n$

We verify for $n = 1$

$P(1)$: $2 = 1 \cdot 2^1 \Rightarrow 2 = 2$

$\Rightarrow$ $P(1)$ is true.

We assume $P(k)$ to be true.

$P(k)$: $2 + 3 \cdot 2 + \dots + (k + 1)\, 2^{k-1} = k \cdot 2^k$

We will prove that $P(k + 1)$ is true.

$P(k + 1)$: $2 + 3 \cdot 2 + 4 \cdot 2^2 + \dots + (k + 1)2^{k-1} + (k + 2)2^k = (k + 1)2^{k+1}$

Proof of $P(k + 1)$:

L.H.S. = $[2 + 3 \cdot 2 + 4 \cdot 2^2 + \dots + (k + 1)2^{k-1} + (k + 2)2^k]$

$= k \cdot 2^k + (k + 2)2^k$ [From $P(k)$]

$$= (2k + 2)2^k = 2(k + 1)2^k = (k + 1)2^{k+1}$$

$\Rightarrow$ $P(k + 1)$ is true. Hence, by Principle of Mathematical Induction, $P(n)$ is true for all n.

S11. Let the given statement be $P(n)$.

$\therefore$ $P(n)$: $2 + 6 + 18 + \ldots + 2 \cdot 3^{n-1} = 3^n - 1$

For $n = k$: L.H.S. = 2, R.H.S. = $3^1 - 1 = 2$

L.H.S. = R.H.S.

Hence, $P(1)$ is true.

Let for $n = k$, $P(k)$ be true.

$P(k)$: $2 + 6 + 18 + \ldots 2 \cdot 3^{n-1}$

$$= (3^n - 1)$$

Now we shall prove that for $n = (k + 1)$, $P(k + 1)$ is true.

$P(k + 1)$: $2 + 6 + 18 + \ldots + 2 \cdot 3^{k-1} + 2 \cdot 3^k$

$$= 3^k - 1 + 2 \cdot 3^k$$

$$= 3^{k+1} - 1 = \text{R.H.S.}$$

Hence, $P(k + 1)$ is true, whenever $P(k)$ is true.

Hence, $P(n)$ is true for all $n \in N$.

S12. **Step I:** Let the given statement be $P(n)$.

$\therefore$ $P(n)$: $2 + 4 + 6 + 8 + \ldots + 2n = n(n + 1)$.

Step II: For $n = 1$, we have

R.H.S. = $1(1 + 1) = 2$ = L.H.S.

Therefore, $P(1)$ is true.

Step III: For $n = k$,

Assume that $P(k)$ is true for some positive integer k.

$$P(k) = 2 + 4 + 6 + 8 + \ldots + 2k = k(k + 1)$$

Step IV: For $n = (k + 1)$

We shall now prove that $P(k + 1)$ is also true.

$P(k + 1)$: $(2 + 4 + 6 + \ldots + 2k) + (2k + 2) = (k + 1)(k + 2)$

Now we have

$$\text{L.H.S.} = (2 + 4 + 6 + \ldots + 2k) + 2(k + 1)$$

$$= k(k + 1) + (2k + 2) = (k + 1)(k + 2)$$

Thus, $P(k + 1)$ is true, whenever, $P(k)$ is true.

Hence, from the principle of mathematical induction the statement $P(n)$ is true for all natural number n.

S13. Let $P(n)$ be the given statement *i.e.*,

i.e., $P(n)$: $$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4} \quad \ldots \text{(i)}$$

For $n = 1$, L.H.S. $= 1^3 = 1$

and R.H.S. $= \frac{1^2(1+1)^2}{4} = \frac{1 \times 4}{4} = 1$

L.H.S. = R.H.S. *i.e.*, $P(1)$ is true.

Let us suppose that $P(k)$ is true

$\therefore$ Putting $n = k$ in Eq. (i), we have

$P(k)$: $$1^3 + 2^3 + 3^3 + \ldots + k^3 = \frac{k^2(k+1)^2}{4} \quad \ldots \text{(ii)}$$

Changing k to $(k + 1)$ in last term k^3 of L.H.S. of Eq. (ii), adding $(k + 1)^3$ to both sides

$$1^3 + 2^3 + 3^3 + \ldots + k^3 + (k+1)^3 = \frac{k^2(k+1)^2}{4} + (k+1)^3$$

$$= (k+1)^2 \left[\frac{k^2 + 4(k+1)}{4}\right]$$

$$= \frac{(k+1)^2(k^2 + 4k + 4)}{4}$$

$$= \frac{(k+1)^2(k+2)^2}{4}$$

$$= \frac{(k+1)^2(\overline{k+1}+1)^2}{4}$$

$P(n)$ is true for $n = k + 1$, *i.e.*, $P(k + 1)$ is true.

By principle of mathematical induction $P(n)$ is true for all natural numbers n.

S14. Let $P(n)$ be the given statement *i.e.*,

i.e., $P(n)$: $1.2 + 2.2^2 + 3.2^2 + \ldots + n\,.\,2^n = (n - 1)\,2^{n+1} + 2$

Putting $n = 1$, L.H.S. $= 1\,.\,2 = 2$

R.H.S. $= 0 + 2 = 2$

$\therefore$ $P(n)$ is true for $n = 1$

Assume that $P(n)$ is true for $n = k$, *i.e.*, $P(k)$ is true *i.e.*,

Adding it to both sides

L.H.S. $= 1.2 + 2.2^2 + 3.2^2 + \ldots + k\,.\,2^{k+1} + (k + 1)\,2^{k+1}$

R.H.S. $= (k - 1)\,2^{k+1} + 2 + (k + 1)\,.\,2^{k+1}$

$= 2^{k+1}\,.\,[k - 1 + k + 1] + 2$

$= 2k\,.\,2^{k+1} + 2$

This proves $P(n)$ is true for n = $k + 1$.

Thus $P(k + 1)$ is true whenever $P(n)$ is true.

By principle of mathematical induction $P(k)$ is true for all values of $n \in N$.

S15. Let $P(n)$ be the given statement *i.e.*,

$$P(n): \quad \frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \ldots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$$

Putting $n = 1$ L.H.S. $= \frac{1}{1.2.3} = \frac{1}{6}$

$$\text{R.H.S.} = \frac{1.(1+3)}{4(1+1)(1+2)} = \frac{4}{4.2.3} = \frac{1}{6}$$

$\therefore$ $P(n)$ is true for $n = 1$

Assuming $P(n)$ is true for $n = k$, *i.e.*, $P(k)$ is supposed to be true *i.e.*,

$$P(k): \quad \frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \ldots + \frac{1}{k(k+1)(k+2)} = \frac{k(k+3)}{4(k+1)(k+2)}$$

$$k^{\text{th}} \text{ term} = \frac{1}{k(k+1)(k+2)}$$

$$\therefore \quad (k+1)^{\text{th}} \text{ term} = \frac{1}{(k+1)(k+2)(k+3)}$$

Adding this term to both sides.

$$\text{L.H.S.} = \frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \ldots + \frac{1}{k(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)}$$

$$\text{R.H.S.} = \frac{k(k+3)}{4(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)}$$

$$= \frac{1}{(k+1)(k+2)}\left[\frac{k(k+3)}{4} + \frac{1}{k+3}\right]$$

$$= \frac{1}{(k+1)(k+2)} \times \frac{k(k+3)^2 + 4}{4(k+3)}$$

$$= \frac{1}{4(k+1)(k+2)(k+3)}[k(k^2 + 6k + 9) + 4]$$

$$= \frac{k^3 + 6k^2 + 9k + 4}{4(k+1)(k+2)(k+3)}$$

$$= \frac{(k+1)(k^2 + 5k + 4)}{4(k+1)(k+2)(k+3)}$$

$$= \frac{(k+1)(k+4)}{4(k+2)(k+3)}$$

$$= \frac{(\overline{k+1})(\overline{k+1}+3)}{4(k+1+1)(k+1+2)}$$

This shows $P(n)$ is true for $n = k$, *i.e.*, P (k + 1) is true whenever $P(k)$ is true.

Hence, by principle of mathematical induction $P(n)$ is true for all values of $n \in N$.

S16. Let the given statement be $P(n)$ *i.e.*,

$$P(n): \quad 1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

For $n = 1$ L.H.S. $= 1^2 = 1$

$$\text{R.H.S.} = \frac{1.(2-1)(2+1)}{3} = \frac{1.1.3}{3} = 1$$

$\therefore$ $P(n)$ is true for $n = 1$

Suppose $P(k)$ is true for $n = k$, *i.e.*,

$$1^2 + 3^2 + 5^2 + \dots + (2k-1)^2 = \frac{k(2k-1)(2k+1)}{3}$$

$$k^{th} \text{ term} = (2k-1)^2$$

$$\therefore \quad (k+1)^{th} \text{ term} = (2(k+1)-1)^2 = (2k+1)^2$$

Adding $(2k+1)^2$ to both sides.

$$\text{L.H.S.} = 1^2 + 3^2 + 5^2 + \dots + (2k-1)^2 + (2k+1)^2$$

$$\text{R.H.S.} = \frac{k(2k-1)(2k+1)}{3} + (2k+1)^2$$

$$= (2k+1)\left[\frac{k(2k-1)}{3} + (2k+1)\right]$$

$$= (2k+1)\left[\frac{k(2k-1)+3(2k+1)}{3}\right]$$

$$= (2k+1)\left(\frac{2k^2+5k+3}{3}\right)$$

$$= \frac{(2k+1)(k+1)(2k+3)}{3}$$

$$= \frac{(\overline{k+1})[2(\overline{k+1})-1][2(\overline{k+1})+1]}{3}$$

Thus, $P(k+1)$ is true for $n = k$, *i.e.*,

$P(k+1)$ is true whenever $P(k)$ is true.

By principle of mathematical induction $P(n)$ is true for all values of $n \in N$.

S17. **Step I:** Let the given statement be $P(n)$

$$P(n): \quad \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} = \left(1 - \frac{1}{2^n}\right)$$

Step II: For $n = 1$

R.H.S. $= \left(1 - \frac{1}{2^1}\right) = \frac{1}{2}$ = L.H.S.

Thus $P(1)$ is true.

Step III: For $n = k$

Assume that $P(k)$ is true for some positive integer k.

$$\therefore \quad P(k) : \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^k} = \left(1 - \frac{1}{2^k}\right)$$

Step IV: For $n = (k + 1)$

We shall now prove that $P(k + 1)$ is also true.

$$P(k+1) : \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots + \frac{1}{2^k} + \frac{1}{2^{k+1}}$$

$$= \left(1 - \frac{1}{2^k}\right) + \frac{1}{2^{k+1}}$$

$$= \left(1 - \frac{1}{2^{k+1}}\right) = \text{R.H.S.}$$

Thus $P(k + 1)$ is true when $P(k)$ is true.

Hence, $P(n)$ is true for all $n \in N$.

S18. Let the given statement be $P(n)$.

Then,

$$P(n) : a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(r^n - 1)}{(r - 1)}.$$

When, $n = 1$

$$\text{L.H.S.} = a \text{ and R.H.S.} = \frac{a(r^1 - 1)}{(r - 1)} = a$$

$\therefore$ L.H.S. = R.H.S.

Hence $P(1)$ is true.

Let $P(k)$ be true, then

$$P(k): a + ar + ar^2 + \ldots + ar^{k-1} = \frac{a(r^k - 1)}{(r - 1)}$$

Now,

$$(a + ar + \ldots + ar^{k-1}) + ar^k = \frac{a(r^k - 1)}{(r-1)} + ar^k = \frac{a(r^{k+1} - 1)}{(r-1)}$$

$$\therefore \quad P(k+1): \quad a + ar + ar^2 + \ldots + ar^{k-1} + ar^k = \frac{a(r^{k+1} - 1)}{(r-1)}$$

$\Rightarrow$ $P(k + 1)$ is true whenever $P(k)$ is true.

Hence, by principle of mathematical induction, $P(n)$ is true for all $n \in N$.

S19. Let the given statement be P(n)

$$\therefore \quad P(n): \quad (1+1)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\ldots\left(1+\frac{1}{n}\right) = (n+1)$$

For $n = 1$

L.H.S. = 2, R.H.S. = 2

$\therefore$ L.H.S. = R.H.S.

Thus, $P(1)$ is true.

For $n = k$, $P(k)$ be true for some positive integer k.

$$P(k): \quad (1+1)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\ldots\left(1+\frac{1}{k}\right) = (k+1)$$

Now, for $n = (k + 1)$

We prove that $P(k + 1)$ is also true.

$$P(k+1): \quad (1+1)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\ldots\left(1+\frac{1}{k}\right)\ldots\left(1+\frac{1}{k+1}\right)$$

$$= (k+1)\left(1+\frac{1}{k+1}\right)$$

$$= (k+2) = \text{R.H.S.}$$

$\therefore$ $P(k + 1)$ is true whenever $P(k)$ is true.

Hence, $P(n)$ is true for all $n \in N$.

S20. $\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \ldots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$

We verify for $n = 1$

$$P(1): \quad \frac{1}{1.4} = \frac{1}{3+1} \quad \Rightarrow \quad \frac{1}{4} = \frac{1}{4}$$

$\Rightarrow$ $P(1)$ is true.

We assume $P(k)$ to be true.

$P(k)$: $\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+...+\frac{1}{(3k-2)(3k+1)}=\frac{k}{3k+1}$

We will prove that $P(k + 1)$ is true.

$P(k + 1)$: $\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+...+\frac{1}{(3k-2)(3k+1)}+\frac{1}{(3k+1)(3k+4)}$

$$= \frac{k+1}{3k+4}$$

Proof of $P(k + 1)$:

$$\text{L.H.S.} = \left[\frac{1}{1\cdot 4}+\frac{1}{4\cdot 7}+\frac{1}{7\cdot 10}+...+\frac{1}{(3k-2)(3k+1)}\right]+\frac{1}{(3k+1)(3k+4)}$$

$$= \frac{k}{3k+1}+\frac{1}{(3k+1)(3k+4)} \quad \text{[From } P(k)\text{]}$$

$$= \frac{k(3k+4)+1}{(3k+1)(3k+4)} = \frac{3k^2+4k+1}{(3k+1)(3k+4)}$$

$$= \frac{(k+1)(3k+1)}{(3k+1)(3k+4)} = \frac{k+1}{3k+4}$$

⇒ $P(k + 1)$ is true. Hence, by the Principle of Mathematical Induction, $P(n)$ is true for all n.

S21. $P(n)$:

$$1.3 + 3.5 + 5.7 + ... + (2n - 1)(2n + 1) = \frac{n(4n^2+6n-1)}{3}$$

$P(1)$: $1.3 = \frac{(4+6-1)}{3}$

⇒ $P(1)$ is true.

We assume $P(k)$ to be true.

P(k): $1.3 + 3.5 + 5.7 + ... + (2n - 1)(2n + 1) = \frac{k(4k^2+6k-1)}{3}$

We will prove that $P(k + 1)$ is true.

$P(k + 1)$: $1.3 + 3.5 + ... + (2k - 1)(2k + 1) + (2k + 1)(2k + 3)$

$$= (k+1)\ \frac{[4(k+1)^2+6k+5]}{3}$$

Proof of $P(k + 1)$:

$$\text{L.H.S.} = [1.3 + 3.5 + ... + (2k - 1)(2k + 1)] + (2k + 1)(2k + 3)$$

$$= \frac{k(4k^2+6k-1)}{3} + (2k + 1)(2k + 3)$$

$$= \frac{1}{3}[4k^3 + 18k^2 + 23k + 9]$$

$$= \frac{(k+1)}{3}\ [4k^2 + 14k + 9]$$

$$= \frac{(k+1)[4(k+1)^2 + 6k + 5]}{3}$$

$\Rightarrow$ $P(k + 1)$ is true.

Hence, by the Principle of Mathematical Induction, $P(n)$ is true for all n.

S22. $P(n)$:

$$1.3 + 2.3^2 + 3.3^3 + ... + n.3^n = \frac{(2n-1).3^{n+1} + 3}{4}$$

$P(1)$: $\quad 1.3 = \frac{(2-1)3^2 + 3}{4} = 3$

$\Rightarrow$ $P(1)$ is true.

We assume $P(k)$ to be true.

$P(k)$: $\quad 1.3 + 2.3^2 + 3.3^3 + ... + k{\cdot}3^k = \frac{(2k-1)3^{k+1} + 3}{4}$

We will prove that $P(k + 1)$ is true.

$P(k + 1)$: $\quad 1.3 + 2.3^2 + 3.3^3 + ... + k{\cdot}3^k + (k + 1)3^{k+1} = \frac{(2k+1)3^{k+2} + 3}{4}$

Proof of $P(k + 1)$:

$$\text{L.H.S.} = [1.3 + 2.3^2 + ... + k{\cdot}3^k] + (k + 1)3^{k+1}$$

$$= \frac{(2k-1)3^{k+2} + 3}{4} + (k + 1)3^{k+1} \qquad \text{[From } P(k)\text{]}$$

$$= \frac{(2k-1)3^{k+1} + 3 + 4(k+1)3^{k+1}}{4}$$

$$= \frac{3^{k+1}(2k - 1 + 4k + 4) + 3}{4}$$

$$= \frac{3^{k+1}(6k+3) + 3}{4} = \frac{(2k+1).3^{k+2} + 3}{4}$$

$\Rightarrow$ $P(k + 1)$ is true.

Hence, by the Principle of Mathematical Induction, $P(n)$ is true for all n.

S23. $P(n)$:

$$a + (a + d) + (a + 2d) + ... + [a + (n - 1)d] = \frac{n}{2}[2a + (n - 1)d]$$

We verify for $n = 1$

$P(1)$: $a = \frac{1}{2}[2a + (1-1)d]$

$\Rightarrow$ $a = a$ $\Rightarrow$ $P(1)$ is true.

We assume $P(k)$ to be true.

$P(k)$: $a + (a+d) + (a+2d) + ... + [a + (k-1)d] = \frac{k}{2}[2a + (k-1)d]$

We will prove that $P(k+1)$ is true.

$P(k+1)$: $a + (a+d) + (a+2d) + ... + [a + (k-1)d] + [a + kd] = \frac{k+1}{2}[2a + kd]$

Proof of $P(k+1)$:

L.H.S. $[a + (a+d) + (a+2d) + ... + [a + (k-1)d] + [a + kd]$

$$= \frac{k}{2}[2a + (k-1)d] + [a + kd] \quad \text{[From } P(k)\text{]}$$

$$= \frac{k+1}{2}[2a + kd]$$

$\Rightarrow$ $P(k+1)$ is true. Hence, by the Principle of Mathematical Induction, $P(n)$ is true for all n.

S24. $P(n)$: $\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)...\left(1+\frac{2n+1}{n^2}\right) = (n+1)^2$

$P(1)$: $\left(1+\frac{3}{1}\right) = (1+1)^2$ $\Rightarrow$ $4 = 4$

$\Rightarrow$ $P(1)$ is true.

We assume $P(k)$ to be true.

$$P(k): \left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)...\left(1+\frac{2k+1}{k^2}\right) = (k+1)^2$$

We assume $P(k+1)$ to be true.

$$P(k+1): \left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)...\left(1+\frac{2k+1}{k^2}\right)\left(1+\frac{2k+3}{(k+1)^2}\right) = (k+2)^2$$

Proof of $P(k+1)$:

$$\text{L.H.S.} = \left[\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)...\left(1+\frac{2k+1}{k^2}\right)\right]\left(1+\frac{2k+3}{(k+1)^2}\right)$$

$$= (k+1)^2\left(\frac{(k+1)^2 + 2k + 3}{(k+1)^2}\right) = (k+2)^2$$

$\Rightarrow$ $P(k + 1)$ is true.

Hence, by the Principle of Mathematical Induction $P(n)$ is true for all n.

S25. $P(n)$: $1+\frac{1}{1+2}+\frac{1}{1+2+3}+...+\frac{1}{1+2+3+...+n} = \frac{2n}{n+1}$

$P(1)$: $1 = \frac{2(1)}{1+1} = 1 \Rightarrow P(1)$ is true.

We assume $P(k)$ to be true.

$P(k)$: $1+\frac{1}{1+2}+\frac{1}{1+2+3}+...+\frac{1}{1+2+...+k} = \frac{2k}{k+1}$

We will prove that $P(k + 1)$ is true.

$P(k + 1)$: $1+\frac{1}{1+2}+\frac{1}{1+2+3}+...+\frac{1}{1+2+...+k}+\frac{1}{1+2+...+(k+1)} = \frac{2(k+1)}{k+2}$

Proof of $P(k + 1)$:

$$\text{L.H.S.} = \left[1+\frac{1}{1+2}+\frac{1}{1+2+3}+...+\frac{1}{1+2+...+k}\right]+\frac{1}{1+2+...+(k+1)}$$

$$= \frac{2k}{k+1}+\frac{1\times 2}{(k+1)(k+2)}$$

$$= \frac{2}{k+1}\left[k+\frac{1}{k+2}\right]=\frac{2}{k+1}\frac{(k+1)^2}{k+2} = \frac{2(k+1)}{k+2}$$

$\Rightarrow$ $P(k + 1)$ is true.

Hence, by the Principle of Mathematical Induction $P(n)$ is true for all n.

S26. $P(n)$: $1 + 4 + 7 + ... + (3n - 2) = \frac{1}{2}n(3n - 1)$

We verify for $n = 1$.

$P(1)$: $1 = \frac{1}{2}\cdot 1\cdot(3-1)$

$\Rightarrow$ $1 = 1 \Rightarrow P(1)$ is true.

We assume $P(k)$ to be true.

$P(k)$: $1 + 4 + 7 + ... + (3k - 2) = \frac{1}{2}k(3k - 1)$

We will prove that $P(k + 1)$ is true.

$P(k+1)$: $1 + 4 + 7 + ... + (3k-2) + (3k+1) = \frac{1}{2}(k+1)(3k+2)$

Proof of $P(k+1)$:

L.H.S. $[1 + 4 + 7 + ... + (3k-2)] + (3k+1)$

$$= \frac{1}{2}k(3k-1) + (3k+1) \qquad \text{[From } P(k)\text{]}$$

$$= \frac{1}{2}[k(3k-1) + 2(3k+1)]$$

$$= \frac{1}{2}[3k^2 + 5k + 2]$$

$$= \frac{1}{2}(k+1)(3k+2)$$

$\Rightarrow$ $P(k+1)$ is true. Hence, by Principle of Mathematical Induction, $P(n)$ is true for all n.

S27. $P(n)$: $\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + ... + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$

$P(1)$: $\frac{1}{2.5} = \frac{1}{6+4} \Rightarrow \frac{1}{10} = \frac{1}{10}$

$\Rightarrow$ $P(1)$ is true.

We assume $P(k)$ to be true.

$P(k)$: $\frac{1}{2.5} + \frac{1}{5.8} + \frac{1}{8.11} + ... + \frac{1}{(3k-1)(3k+2)} = \frac{k}{6k+4}$

We will prove that $P(k+1)$ is true.

$P(k+1)$: $\frac{1}{2.5} + \frac{1}{5.8} + ... + \frac{1}{(3k-1)(3k+2)} + \frac{1}{(3k+2)(3k+5)} = \frac{k+1}{6k+10}$

Proof of $P(k+1)$:

$$\text{L.H.S.} = \left[\frac{1}{2.5} + \frac{1}{5.8} + ... + \frac{1}{(3k-1)(3k+2)}\right] + \frac{1}{(3k+2)(3k+5)}$$

$$= \frac{k}{6k+4} + \frac{1}{(3k+2)(3k+5)}$$

$$= \frac{1}{(3k+2)}\left[\frac{k}{2} + \frac{1}{3k+5}\right]$$

$$= \frac{3k^2 + 5k + 2}{2(3k+2)(3k+5)} = \frac{(3k+2)(k+1)}{2(3k+2)(3k+5)}$$

$$= \frac{k+1}{6k+10}$$

$\Rightarrow$ $P(k + 1)$ is true.

Hence, by Principle of Mathematical Induction, $P(n)$ is true for all n.

S28. **Step I:** Let the given statement be $P(n)$

$$\therefore \quad P(n) : \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \ldots + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

Step II: For $n = 1$

$$\text{R.H.S.} = \frac{1}{1+1} = \frac{1}{2} = \frac{1}{1\cdot 2} = \text{L.H.S.}$$

Thus $P(1)$ is true.

Step III: For $n = k$,

Assume that $P(k)$ is true for some positive integer k.

$$\therefore \quad P(k) = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \ldots + \frac{1}{k(k+1)} = \frac{k}{k+1}$$

Step IV: Now, we shall prove that $P(k + 1)$ is also true.

$$P(k+1) = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \ldots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)}$$

$$= \frac{k}{(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{1}{(k+1)}\left(k + \frac{1}{k+2}\right)$$

$$= \frac{k+1}{k+2} = \text{R.H.S.}$$

Hence, $P(k + 1)$ is true, whenever $P(k)$ is true hence $P(k)$ is true for all $n \in N$.

Date: 9/10/2021

Q1. Using the principle of mathematical induction prove that $(x^n - y^n)$ is divisible by $(x - y)$ for all $n \in N$.

Q2. Using the principle of mathematical induction prove that $(10^{2n-1} + 1)$ is divisible by 11 for all $n \in N$.

Q3. Using the principle of mathematical induction, prove that $(2 \cdot 7^n + 3 \cdot 5^n - 5)$ is divisible by 24 for all $n \in N$.

Q4. Prove by mathematical induction that $n(n + 1)(2n + 1)$ is multiple of 6 for all $n \in N$.

Q5. By using Principle of Mathematical Induction for every integer n, prove that $7^n - 3^n$ is divisible by 4.

Q6. For every positive integer n, prove that $5^n - 3^n$ is divisible by 2 using principle of mathematical induction.

Q7. Using principle of mathematical induction prove that $(2^{3n} - 1)$ is divisible by 7.

Q8. Prove that $n(n + 1)(n + 5)$ is multiple of 3 using principle of mathematical induction.

Q9. By using principle of mathematical induction prove that $n^3 + 3n^2 + 5n + 3$ is divisible by 3.

Q10. Using the principle of mathematical induction prove that $n^2 + n$ is divisible by 2.

Q11. Using the principle of mathematical induction prove that $5^n - 5$ is divisible by 4 for all $n \in N$.

Q12. Using the principle of mathematical induction prove that $12^n + 25^{n-1}$ is divisible by 13.

Q13. Using the principle of mathematical induction prove that $5^{2n} - 1$ is divisible by 24.

Q14. Using the principle of mathematical induction prove that $4^n + 15n - 1$ is divisible by 9

Q15. Using the principle of mathematical induction prove that $p^{n+1} + (p + 1)^{2n-1}$ is divisible by $p^2 + p + 1$ where p is a natural number.

Q16. Using the principle of mathematical induction prove that $n^3 + (n + 1)^3 + (n + 2)^3$ is divisible by 9.

Q17. Using the principle of mathematical induction prove that $11^{n+2} + 12^{2n+1}$ is divisible by 133.

Q18. Using the principle of mathematical induction prove that $5^{2n+2} - 24n - 25$ is divisible by 576.

Q19. Using the principle of mathematical induction prove that $(x^n - y^n)$ is divisible by $(x + y)$ when n is even.

Q20. Using the principle of mathematical induction that $n(n^2 - 1)$ is divisible by 24 when n is odd.

Date: 9/10/2021

S1. Let the given statement be $P(n)$, then

$P(n)$: $(x^n - y^n)$ is divisible by $(x - y)$.

When, $n = 1$, the given statement becomes $(x^1 - y^1)$ which is clearly true.

$\therefore$ $P(1)$ is true.

Let $P(k)$ be true than

$P(k)$: $(x^k - y^k)$ is divisible by $(x - y)$.

Now, $$(x^{k+1} - y^{k+1}) = x^{k+1} - x^k \cdot y + x^k \cdot y - y^{k+1}$$
$$= x^k(x - y) + y(x^k - y^k)$$

$\Rightarrow$ $P(k + 1)$: $(x^{k+1} - y^{k+1})$ is divisible by $(x - y)$.

$\Rightarrow$ $P(k + 1)$ is true, whenever $P(k)$ is true.

Hence, given statement is true for all $n \in N$.

S2. Let $P(n)$: $(10^{2n-1} + 1)$ is divisible by 11.

For $n = 1$, the given expression becomes as $\{10^{(2\times 1-1)} + 1\} = 11$ which is divisible by 11.

So, $P(1)$ is true.

Let $P(k)$ be true.

$P(k)$: $(10^{2k-1} + 1)$ is divisible by 11.

$\Rightarrow$ $$\{10^{2(k+1)-1} + 1\} = \{10^{2k+1} + 1\}$$
$$= \{10^2 \cdot 10^{2k-1} + 1\} = 100 ´ \{10^{2k-1} + 1\} - 99$$
$$= \{100 \times 11\, m\} - 99$$
$$= 11 \times (100\, m - 9) \text{ which is divisible by 11.}$$

$\Rightarrow$ $P(k + 1)$: $\{10^{2(k+1)-1} + 1\}$ is divisible by 11.

$\Rightarrow$ $P(k + 1)$ is true whenever $P(k)$ is true.

Thus, $P(1)$ is true and $P(k + 1)$ is true, whenever $P(k)$ is true.

Hence, by the principle of mathematical induction, $P(n)$ is true for all $n \in N$.

S3. Let $P(n)$: $(2 \cdot 7^n + 3 \cdot 5^n - 5)$ is divisible by 24.

For $n = 1$, $P(1) = (2 \cdot 7^1 + 3 \cdot 5^1 - 5) = 24$ which is clearly divisible by 24.

So, the given statement is true for $n = 1$

$P(1)$ is true.

Let $P(k)$ be true.

$P(k)$: $(2 \cdot 7^k + 3 \cdot 5^k - 5)$ is divisible by 24.

$\Rightarrow (2 \cdot 7^k + 3 \cdot 5^k - 5) = 24\,m$ for $m \in N$.

Now, $(2 \cdot 7^{k+1} + 3 \cdot 5^{k+1} - 5) = 2 \cdot 7^k (6 + 1) + 3 \cdot 5^k (4 + 1) - 5$

$= 12 \cdot (7^k - 5^k) + (2 \cdot 7^k + 3 \cdot 5^k - 5)$

Since, 7^k and 5^k are odd numbers. Therefore $(7^k + 5^k)$ will be even and $12\,(7^k + 5^k)$ will be divisible by 24, the second term will be devisible by 24 (from $P(k)$)

$\Rightarrow P(k + 1)$: $(2 \cdot 7^{k+1} + 3 \cdot 5^{k+1} - 5)$ is divisible by 24.

$\Rightarrow P(k + 1)$ is true, whenever $P(k)$ is true.

Thus $P(1)$ is true and $P(k + 1)$ is true, whenever $P(k)$ is true.

Hence, by the principle of mathematical induction $P(n)$ is true for all $n \in N$.

S4. **Step I:** Let $P(n)$: $n(n + 1)(2n + 1)$ is multiple of 6.

Step II: For n = 1, we have

$P(1)$: $1(1 + 1)(2 + 1) = 6$ which is a multiple of 6. So $P(1)$ is true.

Step III: For $n = k$, assume that $P(k)$ is true.

$P(k)$: $k(k + 1)(2k + 1) = 6\lambda$ for some integer λ.

Step IV: For $n = k + 1$, we have to show that $P(k + 1)$ is true.

$P(k + 1) = (k + 1)(k + 1 + 1)[2(k + 1) + 1]$ is a multiple of 6.

Now,

$(k + 1)(k + 1 + 1)[2(k + 1) + 1]$

$= k(k + 1)(2k + 1) + 6(k + 1)^2$

$= 6[\lambda + (k + 1)^2]$ which is multiple of 6.

So, $P(k + 1)$ is true, whenever $P(k)$ is true.

Hence, by principle of mathematical induction $P(n)$ is true for all $n \in N$.

S5. **Step I:** Consider the given statement as $P(n)$.

Let $P(n)$: $7^n - 3^n$ is divisible by 4.

Step II: For $n = 1$, show that $P(1)$ is true statement.

For $n = 1$, we have,

$P(1) = 7^1 - 3^1 = 4$, which is divisible by 4.

Thus $P(n)$ is true for $n = 1$.

Step III: Suppose that result is true.

For $n = k$

Let $P(k)$ be true for $k \in N$.

$P(k)$: $7^k - 3^k$ is divisible by 4.

So, we can write

$$7^k - 3^k = 4d, \text{ when } d \in N.$$

Show that the result is true for $n = (k + 1)$

Now, $P(k + 1) = (7^{k+1} - 3^{k+1})$

$= 7^{k+1} - 7.3^k + 7.3^k - 3^{k+1} \quad = 7\,(7^k - 3^k) + 3^k\,(7 - 3)$

$= 4(7d + 3^k) \quad d \in N$, which is divisible by 4.

Thus, $7^{k+1} - 3^{k+1}$ is divisible by 4, so, $P(k + 1)$ is true, whenever $P(k)$ is true.

Hence, by the principle of mathematical induction the statement is true for every positive integer $n \in N$.

S6. **Step I:** Consider the given statement as $P(n)$.

Let $P(n)$: $5^n - 3^n$ is divisible by 2.

Step II: Put $n = 1$ and show that $P(1)$ is true statement.

L.H.S. $= 5^1 - 3^1 = 2$, which is divisible by 2.

Thus $P(n)$ is true for $n = 1$.

Step III: Suppose that the result is true for $n = k$

Let $P(k)$ be true for some natural number k.

$P(k)$: $5^k - 3^k$ is divisible by 2.

$\therefore \quad 5^k - 3^k = 2d \quad$ where $d \in N$.

Step IV: Show that result is true for $n = (k + 1)$

Now, $P(k + 1) = 5^{k+1} - 3^{k+1}$

$= 5^{k+1} - 5 \cdot 3^k + 5 \cdot 3^k - 3^{k+1}$

$= 5(5^k - 3^k) + 3^k(5 - 3) = 5 \cdot 2d + 3k \cdot 2 = 2(5d + 3k)$

Which is divisible by 2.

Thus, $5^{k+1} - 3^{k+1}$ is divisible by 2. So, $P(k + 1)$ is true, whenever $P(k)$ is true.

Hence, by principle of mathematical induction given statement is true for every positive integer $n \in N$.

S7. Consider the given statement as $P(n)$.

$\therefore$ $P(n)$: $2^{3n} - 1$ is divisible by 7.

Step I: For, $n = 12$

L.H.S. = $2^3 - 1 = 7$

Which is divisible by 7.

Hence, $P(1)$ is true statement.

Now, let the given statement is being true to $n = k$.

$\therefore$ $P(k)$: $2^{3k}-1$ is divisible by 7.

$\therefore$ $2^{3k} - 1 = 7d$, where $d \in N$.

Now, we shall prove that given statement is true for $n = (k + 1)$.

$$P(k + 1) = 2^{3(k+1)} - 1$$
$$= 2^{3k} \cdot 2^3 - 1 = 2^{3k} \cdot 2^3 - 2^3 + 2^3 - 1 = 2(2^{3k} - 1) + 7.$$

which is clearly divisible by 7.

Hence, $P(k + 1)$ is true, whenever $P(k)$ is true. Hence, given statement is true for all $n \in N$.

S8. **Step I:** Let the given statement be $P(n)$.

$\therefore$ $P(n)$: $n(n + 1)(n + 5)$ is multiple of 3.

Step II: For n = 1

$$P(1) = 1(1 + 1)(1 + 5) = 12 = 3 \times 4.$$

Which is a multiple of 3. So, $P(n)$ is true for $n = 1$

Step III: Let it is true for $n = k$.

$$P(k) = k(k + 1)(k + 5)$$
$$= k^3 + 6k^2 + 5k = 3\lambda \qquad (\lambda \in N)$$

Step IV: For $n = (k + 1)$, we have

$$(k + 1)(k + 1 + 1)(k + 1 + 5)$$
$$= k^3 + 9k^2 + 20k + 12$$
$$= (3\lambda - 6k^2 - 5k) + 9k^2 + 20k + 12$$
$$= 3\lambda + 3k^2 + 15k + 12 = 3(\lambda + k^2 + 5k + 4)$$

which is a multiple of 3.

Therefore,

$P(k + 1)$ is true, whenever $P(k)$ is true.

Hence, by principle of mathematical induction the statement is true for all natural number n.

S9. **Step I:** Let the given statement be $P(n)$.

$\therefore$ $P(n)$: $n^3 + 3n^2 + 5n + 3$ is divisible by 3.

Step II: For $n = 1$

$P(1) = 1 + 3 + 5 + 3 = 12 = 3 \times 4$

which is multiple of 3, so $P(n)$ is true for $n = 1$.

Step III: Let it is true for $n = k$.

Then $P(k) = k^3 + 3k^2 + 5k + 3 = 3\lambda$ $(\lambda \in N)$

Step IV: For $n = (k + 1)$

$$= (k + 1)^3 + 3(k + 1)^2 + 5(k + 1) + 3$$
$$= (k^3 + 1 + 3k^2 + 3k + 3k^2 + 6k + 3 + 5k + 5 + 3)$$
$$= 3(\lambda + 3k + 3 + k^2)$$

which is multiple of 3.

Therefore, $P(k + 1)$ is true whenever $P(k)$ is true.

Hence by principle of mathematical induction the statement is true for all natural number n.

S10. $P(n)$: $n^2 + n$ is divisible by 2.

We verify for $n = 1$.

$P(1)$: $1^2 + 1$ is divisible by 2

$1^2 + 1 = 2$, which is divisible by 2.

We assume $P(k)$ to be true.

$P(k)$: $k^2 + k$ is divisible by 2

We will prove that $P(k + 1)$ is true.

$P(k + 1)$: $(k + 1)^2 + (k + 1)$ is divisible by 2.

Proof of $P(k + 1)$:

$$(k + 1)^2 + (k + 1) = (k + 2)(k + 1)$$
$$= k(k + 1) + 2(k + 1)$$

The first term is divisible by 2 [from $P(k)$] and the second is also divisible by 2.

$\Rightarrow$ $(k + 1)^2 + (k + 1)$ is divisible by 2.

$\Rightarrow$ $P(k + 1)$ is true. Hence, by the Principle of Mathematical Induction, $P(n)$ is true for all n.

S11. Let $P(n)$ be statement $5^n - 5$ is divisible by 4.

Now, $P(1) = 5^1 - 5$ is divisible by 4.

$\Rightarrow$ $P(1)$ is true.

{0 is divisible by 4, which is true}

Let $P(k)$ be true.

$$(5^k - 5) = 4\lambda, \text{ for some integer } \lambda.$$

$\Rightarrow$ $5k = 4\lambda + 5$

$P(k + 1)$: $5^{k+1} - 5$ is divisible by 4.

Consider $(5^{k+1} - 5) = 5 \cdot 5^k - 5$

$= 5(4\lambda + 5) - 5 = 4(5\lambda + 5)$, which is divisible by 4

$\Rightarrow$ $P(k + 1)$ is true.

Hence, by the principle of mathematical induction $P(n)$ is true for all $n \in N$.

S12. $P(n)$: $12^n + 25^{n-1}$ is divisible by 13.

We verify for $n = 1$.

$P(1)$: $12^1 + 25^{1-1}$ is divisible by 13

$12^1 + 25^{1-1} = 12 + 1 = 13$, which is divisible by 13

$\Rightarrow$ $P(1)$ is true.

We assume $P(k)$ to be true.

$P(k)$: $12^k + 25^{k-1}$ is divisible by 13

We will prove that $P(k + 1)$ is true.

$P(k + 1)$: $12^{k+1} + 25^k$ is divisible by 13

Proof of $P(k + 1)$:

$$12^{k+1} + 25^k = 12^k(13 - 1) + 25^{k-1}(26 - 1)$$
$$= 13(12^k + 2 \cdot 25^{k-1}) - (12^k + 25^{k-1})$$

The first term is divisible by 13 and because of $P(k)$ the second is also divisible by 13.

$\Rightarrow$ $12^{k+1} + 25^k$ is divisible by 13

$\Rightarrow$ $P(k + 1)$ is true. Hence, by the Principle of Mathematical Induction, $P(n)$ is true for all n.

S13. $P(n)$: $5^{2n} - 1$ is divisible by 24

We verify for $n = 1$.

$P(1)$: $5^2 - 1$ is divisible by 24

$5^2 - 1 = 24$, which is divisible by 24.

$\Rightarrow$ $P(1)$ is true.

We assume $P(k)$ to be true.

$P(k)$: $5^{2k} - 1$ is divisible by 24.

We will prove that $P(k + 1)$ is true.

$P(k + 1)$: $5^{2(k+1)-1}$ is divisible by 24.

Proof of $P(k + 1)$:

$$5^{2(k+1)} - 1 = 5^{2k}(24 + 1) - 1$$
$$= 24(5^{2k}) + 5^{2k} - 1$$

The first term is divisible by 24 [from $P(k)$] and the second is also divisible by 24 [from $P(k)$].

$\Rightarrow$ $5^{2(k+1)} - 1$ is divisible by 24.

$\Rightarrow$ $P(k + 1)$ is true. Hence, by the Principle of Mathematical Induction, $P(n)$ is true for all n.

S14. $P(n)$: $4^n + 15n - 1$ is divisible by 9

We verify for $n = 1$.

$P(1)$: $4^1 + 15(1) - 1$ is divisible by 9

$4^1 + 15(1) - 1 = 18$, which is divisible by 9.

$\Rightarrow$ $P(1)$ is true.

We assume $P(k)$ to be true.

$P(k)$: $4^k + 15k - 1$ is divisible by 9

We will prove that $P(k + 1)$ is true.

$P(k + 1)$: $4^{k+1} + 15(k + 1) - 1$ is divisible by 9

Proof of $P(k + 1)$:

$$4^{k+1} + 15(k + 1) - 1 = 4^k \cdot 4 + 15k + 14$$
$$= 4^k(9 - 5) + (90k - 75k) + (9 + 5) = 9[4^k + 10k + 1] - 5[4^k + 15k - 1]$$

The first term is divisible by 9 and because of $P(k)$, the second term is also divisible by 9.

$\Rightarrow$ $4^{k+1} + 15k + 14$ is divisible by 9

$\Rightarrow$ $P(k + 1)$ is true. Hence, by the Principle of Mathematical Induction, $P(n)$ is true for all n.

S15. $P(n)$: $p^{n+1} + (p + 1)^{2n-1}$ is divisible by $p^2 + p + 1$

We verify for $n = 1$.

$P(1)$: $p^2 + (p + 1)^{2-1}$ is divisible by $p^2 + p + 1$

$p^2 + (p + 1)^{2-1} = p^2 + p + 1$, which is divisible by $p^2 + p + 1$

$\Rightarrow$ $P(1)$ is true.

We assume $P(k)$ to be true.

$P(k)$: $p^{k+1} + (p + 1)^{2k-1}$ is divisible by $p^2 + p + 1$.

We will prove that $P(k + 1)$ is true.

$P(k + 1)$: $p^{k+2} + (p + 1)^{2k+1}$ is divisible by $p^2 + p + 1$

Proof of $P(k + 1)$:

$$p^{k+2} + (p + 1)^{2k+1} = p \cdot p^{k+1} + (p + 1)^2 (p + 1)^{2k-1}$$
$$= p \cdot p^{k+1} + [(p^2 + 2p + 1)] (p + 1)^{2k-1}$$

$$= p[p^{k+1} + (p+1)^{2k-1}] + (p^2 + p + 1)(p+1)^{2k-1}$$

Because of $P(k)$ the first term is divisible by $p^2 + p + 1$ and the second is also divisible by $p^2 + p + 1$.

$\Rightarrow$ $p^{k+2} + (p+1)^{2k+1}$ is divisible by $p^2 + p + 1$.

$\Rightarrow$ $P(k+1)$ is true. Hence, by the Principle of Mathematical Induction, $P(n)$ is true for all n.

S16. $P(n)$: $n^3 + (n+1)^3 + (n+2)^3$ is divisible by 9.

We verify for $n = 1$.

$P(1)$: $1^3 + (1+1)^3 + (1+2)^3$ is divisible by 9.

, which is divisible by 9.

$\Rightarrow$ $P(1)$ is true.

We assume $P(k)$ to be true.

$P(k)$: $k^3 + (k+1)^3 + (k+2)^3$ is divisible by 9

We will prove that $P(k+1)$ is true.

$P(k+1)$: $(k+1)^3 + (k+2)^3 + (k+3)^3$ is divisible by 9

Proof of $P(k+1)$: $(k+1)^3 + (k+2)^3 + (k+3)^3$

$$= (k+1)^3 + (k+2)^3 + k^3 + 9k^2 + 27k + 27$$

$$= [k^3 + (k+1)^3 + (k+2)^3] + 9[k^2 + 3k + 3]$$

The first term is divisible by 9 [from $P(k)$ and the second is also divisible by 9]

$\Rightarrow$ $(k+1)^3 + (k+2)^3] + (k+3)^3$ is divisible by 9.

$\Rightarrow$ $P(k+1)$ is true. Hence, by the Principle of Mathematical Induction, $P(n)$ is true for all n.

S17. $P(n)$: $11^{n+2} + 12^{2n+1}$ is divisible by 133

We verify for $n = 1$.

$P(1)$: $11^{1+2} + 12^{2+1}$ is divisible by 133.

$11^3 + 12^3 = (11 + 12)[11^2 + 12^2 - 11 \cdot 12]$, which is divisible by 133.

$\Rightarrow$ $P(1)$ is true.

We assume $P(k)$ to be true.

$P(k)$: $11^{k+2} + 12^{2k+1}$ is divisible by 133

We will prove that $P(k+1)$ is true.

$P(k+1)$: $11^{k+3} + 12^{2k+3}$ is divisible by 133

Proof of $P(k+1)$:

$$11^{k+3} + 12^{2k+3} = 11 \cdot 11^{k+2} + 12^2 \cdot 12^{2k+1}$$

$$= 11 \cdot 11^{k+2} + (11 + 133)12^{2k+1}$$

$$= 11[11^{k+2} + 12^{2k+1}] + 133 \cdot 12^{2k+1}$$

The first term is divisible by 133 [from $P(k)$] and the second is also divisible by 133.

$\Rightarrow$ $11^{k+3} + 12^{2k+3}$ is divisible by 133.

$\Rightarrow$ $P(k + 1)$ is true. Hence, by the Principle of Mathematical Induction, $P(n)$ is true for all n.

S18. $P(n)$: $5^{2n+2} - 24n - 25$ is divisible by 576.

We verify for $n = 1$.

$P(1)$: $5^{2+2} - 24(1) - 25$ is divisible by 576

$5^4 - 24 - 25 = 625 - 49 = 576$, which is divisible by 576.

$\Rightarrow$ $P(1)$ is true.

We assume $P(k)$ to be true.

$P(k)$: $5^{2k+2} - 24k - 25$ is divisible by 576

We will prove that $P(k + 1)$ is true.

$P(k + 1)$: $5^{2k+4} - 24k - 49$ is divisible by 576

Proof of $P(k + 1)$:

$$5^{2k+4} - 24k - 25 = 25 \cdot 5^{2k+2} - 600k + 576k - 625 + 576$$
$$= 25[5^{2k+2} - 24k - 25] + 576(k + 1)$$

Because of $P(k)$, the first term is divisible by 576 and the second is also divisible by 576.

$\Rightarrow$ $5^{2k+4} - 24k - 49$ is divisible by 576.

$\Rightarrow$ $P(k + 1)$ is true. Hence, by the Principle of Mathematical Induction, $P(n)$ is true for all n.

S19. $P(n)$: $x^n - y^n$ is divisible by $x + y$

We verify for $n = 2$

$P(x)$: $x^2 - y^2$ is divisible by $x + y$

$x^2 - y^2 = (x - y)(x + y)$, which is divisible by $x + y$

$\Rightarrow$ $P(2)$ is true.

We assume $P(k)$ to be true.

$P(k)$: $x^k - y^k$ is divisible by $x + y$.

We will prove that $P(k + 2)$ is true.

$P(k + 2)$: $x^{k+2} - y^{k+2}$ is divisible by $x + y$.

Proof of $P(k + 2)$:

$$x^{k+2} - y^{k+2} = x^{k+2} - x^2y^k + x^2y^k - y^{k+2}$$
$$= x^2(x^k - y^k) + y^k(x^2 - y^2)$$
$$= x^2(x^k - y^k) + y^k(x - y)(x + y)$$

Because of $P(k)$ the first term is divisible by $x + y$, the second term is also divisible by $x + y$.

$\Rightarrow$ $x^{k+2} - y^{k+2}$ is divisible by $x + y$.

$\Rightarrow$ $P(k + 1)$ is true. Hence, by the Principle of Mathematical Induction, $P(n)$ is true for all even values of n.

S20. $P(n)$: $n(n^2 - 1)$ is divisible by 24

We verify for $n = 1$.

$P(1)$: $1(1^2 - 1)$ is divisible by 24

$1(1^2 - 1) = 0$, which is divisible by 24

$\Rightarrow$ $P(1)$ is true.

We assume $P(k)$ to be true.

$P(k)$: $k(k^2 - 1)$ is divisible by 24

We will prove that $P(k + 2)$ is true.

$P(k + 2)$: $(k + 2)\,[(k + 2)^2 - 1]$ is divisible by 24

Proof of $P(k + 2)$:

$$(k + 2)\,[(k + 2)^2 - 1] = (k + 2)\,[k^2 + 4k + 3]$$
$$= k^3 + 6k^2 + 11k + 6$$
$$= k(k^2 - 1) + 6(k + 1)^2$$

As k is odd, $(k + 1)$ will be even.

$6(k + 1)^2$ will be divisible by 24 and because of $P(k)$ first term will be divisible by 24.

$\Rightarrow$ $(k + 2)\,[(k + 2)^2 - 1]$ is divisible by 24.

$\Rightarrow$ $P(k + 2)$ is true. Hence $P(n)$ is true for all values of n.

Date: 9/10/2021

Q1. Using the principle of mathematical induction prove that $3^n > 2^n$.

Q2. Using the principle of mathematical induction prove that $(n^2 + n)$ is even for all $n \in N$.

Q3. Using the principle of mathematical induction prove that $(2n + 7) < (n + 3)^2$ for $n \in N$.

Q4. Using the principle of mathematical induction prove that $n < 2^n$ for all $n \in N$.

Q5. Using the principle of mathematical induction, prove that $(1^2 + 2^2 + 3^2 + ... + n^2) > \frac{n^3}{3}$, $n \in N$.

Q6. By mathematical induction prove that $(1 + x)^n \geq (1 + nx)$ for all natural number n, where $x > -1$.

Q7. Using the principle of mathematical induction prove that $1 + 2 + 3 + ... + n < \frac{1}{8}(2n + 1)^2$.

Date: 9/10/2021

S1. $P(n)$: $3^n > 2^n$

We verify for $n = 1$.

$P(1)$: $3^1 > 2^1 \Rightarrow 3 > 2 \Rightarrow P(1)$ is true.

We assume $P(k)$ to be true.

$P(k)$: $3^k > 2^k$

We will prove that $P(k + 1)$ is true.

$P(k + 1)$: $3^{k+1} > 2^{k+1}$

Proof of $P(k + 1)$:

We have $3^k > 2^k$ [From $P(k)$] and $3 > 2$

On multiplication, we get

$$3^k \cdot 3 > 2^k \cdot 2$$

$\Rightarrow$ $3^{k+1} > 2^{k+1}$ $P(k + 1)$ is true

Hence, by the Principle of Mathematical Induction, $P(n)$ is true for all n.

S2. Let $P(n)$: $(n^2 + n)$ is even.

For $n = 1$,

$P(1) = 1^2 + 1 = 2$, which is even.

So, $P(1)$ is true.

Let $P(k)$: $(k^2 + k)$ is even.

$\Rightarrow$ $(k^2 + k) = 2m$, $(m \in N)$

$$P(k + 1) = (k + 1)^2 + (k + 1)$$
$$= (k^2 + k) + 2(k + 1)$$
$$= 2[m + (k + 1)] \text{ which is even}$$

Hence, by the principle of mathematical induction, $P(n)$ is true for all $n \in N$.

S3. Let $P(n)$: $(2n + 7) < (n + 3)^2$

When $n = 1$

L.H.S. $= 2 \times 1 + 7 = 9$, R.H.S. $= (1 + 3)^2 = 16$

Clearly, 9 < 16

Thus, $P(1)$ is true.

Let $P(k)$ be true,

$$P(k) : (2k + 7) < (k + 3)^2$$

Now, $2(k + 1) + 7 = (2k + 7) + 2 < (k + 3)^2 + 2$

$\therefore \quad P(k + 1): 2(k + 1) + 7 < (k + 3)^2 + 2$

$\Rightarrow \quad P(k + 1)$ is true, whenever $P(k)$ is true.

Thus, $P(1)$ is true, and $P(k + 1)$ is true, whenever $P(k)$ is true.

Hence, by the principle of mathematical induction $P(n)$ is true for all $n \in N$.

S4. Let $P(n)$, $n < 2^n$

When $n = 1$, L.H.S. = 1, R.H.S. = $2^1 = 2$

Clearly $1 < 2$

$\therefore \quad P(1)$ is true.

Let $P(k)$ be true, then

$$P(k) : k < 2^k$$

Now, $k < 2^k \quad \Rightarrow \quad 2k < 2^{k+1}$

$\Rightarrow \quad (k + k) < 2^{k+1}$

$\Rightarrow \quad (k + 1) \le (k + k) < 2^{k+1}$

$\Rightarrow \quad (k + 1) < 2^{k+1}$

$\therefore \quad P(k + 1) : (k + 1) < 2^{k+1}$

$\Rightarrow \quad P(k + 1)$ is true, whenever $P(k)$ is true.

Thus $P(1)$ is true and $P(k + 1)$ is true whenever $P(k)$ is true.

Hence, by the principle of mathematical induction $P(n)$ is true for all $n \in N$.

S5. Let $P(n)$:

$$(1^2 + 2^2 + 3^2 + \dots + n^2) > \frac{n^3}{3}$$

When $n = 1$

$$\text{L.H.S.} = 1^2 = 1, \quad \text{R.H.S.} = \frac{1^3}{3} = \frac{1}{3}$$

Since, $1 > \dfrac{1}{3}$, $P(1)$ is true.

Let $P(k)$ be true, then

$$P(k) : (1^2 + 2^2 + \dots + k^2) > \frac{k^3}{3}$$

Now, $P(k + 1)$:

$$1^2 + 2^2 + \dots + k^2 + (k + 1)^2$$

$$= [1^2 + 2^2 + \dots + k^2] + (k + 1)^2 > \frac{k^3}{3} + (k + 1)^3$$

$$= \frac{1}{3} > [\frac{k^3}{3} + (k + 1)^2] > \frac{1}{3}(k + 1)^3$$

$\therefore$ $P(k + 1)$:

$$1^2 + 2^2 + \dots + k^2 + (k + 1)^2 > \frac{1}{3}(k + 1)^3.$$

$\Rightarrow$ $P(k + 1)$ is true, whenever $P(k)$ is true.

Thus $P(1)$ is true and $P(k + 1)$ is true whenever $P(k)$ is true.

Hence, by the principle of mathematical induction $P(n)$ is true for all $n \in N$.

S6. **Step I:** Let $P(n)$ the given statement then

$P(n)$: $(1 + x)^n \geq (1 + nx)$ for $x > -1$

Step II: For $n = 1$, we have

$(1 + x) \geq (1 + x)$ for $x > -1$

Thus $P(n)$ is true when $n = 1$

Step III: For $n = k$, assume that $P(k)$ is true.

$P(k)$: $(1 + x)^k \geq (1 + kx)$, $x > -1$ is true

Step IV: For $n = k + 1$, we have to show that $P(k + 1)$ is true for $x > -1$ whenever $P(k)$ is true.

Consider the identity

$$(1 + x)^{k+1} = (1 + x)^k \cdot (1 + x)$$

Given that $x > -1$ so $(1 + x) > 0$

Therefore by using, $(1 + x)^k \geq (1 + kx)$, we get

$$(1 + x)^{k+1} \geq (1 + kx)(1 + x)$$

Here k is natural number and $x^2 \geq 0$ so that $kx^2 \geq 0$

Therefore, $(1 + x + kx + kx^2) \geq (1 + x + kx)$

$\therefore$ $(1 + x)^{k+1} \geq (1 + x + kx)$

or $(1 + x)^{k+1} > [1 + (1 + k)x]$

Thus, $P(k + 1)$ is true whenever $P(k)$ is true.

Hence, by the principle of mathematical induction, $P(n)$ is true for all $n \in N$.

S7. $P(n)$: $1 + 2 + 3 + ... + n < \frac{1}{8}(2n + 1)^2$

We verify for $n = 1$.

$P(1)$: $1 < \frac{1}{8}(2 + 1)^2 \Rightarrow 1 < \frac{9}{8}$

$\Rightarrow$ $P(1)$ is true.

We assume $P(k)$ to be true.

$P(k)$: $1 + 2 + 3 + ... + k < \frac{1}{8}(2k + 1)^2$

We will prove that $P(k + 1)$ is true.

$P(k + 1)$: $1 + 2 + 3 + ... + k + (k + 1) < \frac{1}{8}[2k + 3]^2$

Proof of $P(k + 1)$:

We have, $1 + 2 + 3 + ... + k < \frac{1}{8}(2k + 1)^2$

$\Rightarrow$ $1 + 2 + 3 + ... + k + (k + 1) < \frac{1}{8}(2k + 1)^2 + (k + 1)$

$$< \frac{1}{8}[(4k + 1)^2 + 8(k + 1)]$$

$$< \frac{1}{8}[4k^2 + 12k + 9)]$$

$$< \frac{1}{8}(2k + 3)^2$$

$\Rightarrow$ $P(k + 1)$ is true. Hence, by the Principle of Mathematical Induction, $P(n)$ is true for all n.