3D-Geometry

Single Correct Answer Type

1. In a three dimensional co - ordinate system P, Q and R are images of a point A(a, b, c) in the x - y the y - z and the z - x planes respectively. If G is the centroid of triangle PQR then area of triangle AOG is (O is the origin)

a) 0 b) $a^2+b^2+c^2$ c) $\frac{2}{3}\left(a^2+b^2+c^2\right)$ d) none of these

Key. A

Sol. Point A is (a, b, c)

$\Rightarrow$Points P, Q, R are (a, b, –c), (-a, b, c) and (a, –b, c) respectively.

$\Rightarrow$centroid of triangle PQR is $\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$

$\Rightarrow G \equiv \left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$

$\Rightarrow$A, O, G are collinear $\Rightarrow$ area of triangle AOG is zero.

2. The four lines drawing from the vertices of any tetrahedron to the centroid of the opposite faces meet in a point whose distance from each vertex is 'k' times the distance from each vertex to the opposite face, where k is

a) $\frac{1}{3}$ b) $\frac{1}{2}$ c) $\frac{3}{4}$ d) $\frac{5}{4}$

Key. C

Sol. Let A $\left(x_1,y_1,z_1\right)$ B $\left(x_2,y_2,z_2\right)$C$\left(x_3,y_3,z_3\right)$ D$\left(x_4,y_4,z_4\right)$ be the vertices of tetrahydron. If E is the centroid of face BCD and G is the centroid of A B C D the AG=$3/4\left(AE\right) \therefore K=3/4$

3. The coordinates of the circumcentre of the triangle formed by the points (3, 2, -5), (-3, 8, -5) (-3, 2, 1) are

a) (-1, 4, -3) b) (1, 4, -3) c) (-1, 4, 3) d) (-1, -4, -3)

Key. A

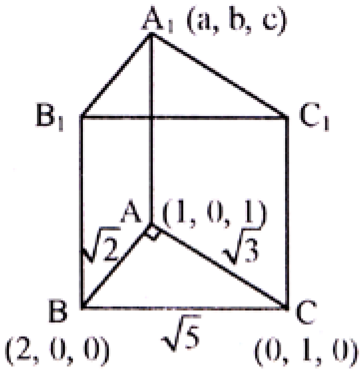
Sol. Triangle formed is an equilateral $\Rightarrow$ Circum centre = centroid = (-1, 4, -3)

4. The volume of a right triangular prism $ABCA_1B_1C_1$ is equal to 3. Than the co-ordinates of the vertex A_1, if the co-ordinates of the base vertices of the prism are A(1, 0, 1), B(2, 0, 0) and C(0, 1, 0)

a) (–2, 2, 2) or (0, –2, 1) b) (2, 2, 2) or (0, –2, 0)
c) (0, 2, 0) or (1, –2, 0) d) (3, –2, 0) or (1, –2, 0)

Key. B

Sol. Volume = Area of base $\times$ height



$3=\frac{1}{2}'\ \sqrt{2}'\ \sqrt{3}'\ h$

h = $\sqrt{6}$

$(A_1A)^2 = h^2 = 6$

$\overrightarrow{A_1A}.\overrightarrow{AB} = 0$

$\overrightarrow{A_1A}.\overrightarrow{AC} = 0$

$\overrightarrow{AA_1}.\overrightarrow{BC} = 0$

solving we get position vector of A_1 are (0, –2, 0) or (2, 2, 2)

5. If $\vec{a},\vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a}+\vec{b}+\vec{c}$ is also a unit vector and θ_1,θ_2 *and* θ_3 are angles between the vectors $\vec{a},\vec{b};\vec{b},\vec{c}$ *and* $\vec{c},\vec{a}$, respectively, then among θ_1, θ_2 *and* θ_3.

a) all are acute angles b) all are right angles

c) at least one is obtuse angle d) None of these

Key. C

Sol. Since $\left|\vec{a}+\vec{b}+\vec{c}\right|=1 \Rightarrow \left(\vec{a}+\vec{b}+\vec{c}\right).\left(\vec{a}+\vec{b}+\vec{c}\right)=1 \Rightarrow \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}=-1$

$\Rightarrow \cos\theta_1+\cos\theta_2+\cos\theta_3=-1$

So, at least one of $\cos\theta_1,\cos\theta_2$ *and* $\cos\theta_3$ must be negative

6. Given that the points $A(3,2,-4), B(5,4,-6)$ *and* $C(9,8,-10)$ are collinear, the ratio in which B divides $\overline{AC}$ is :

1)1 : 2 2)2 : 1 3)3 : 2 4)2 : 3

Key. 1

Sol. $\left(\frac{9m+3n}{m+n},\frac{8m+2n}{m+n},\frac{-10m-4n}{m+n}\right)=(5,4,-6)$

$$\frac{m}{n}=\frac{1}{2}$$

7. If $A(0,1,2), B(2,-1,3)\, and\ C(1,-3,1)$ are the vertices of a triangle, then its circumcentre and orthocenter are situated at a distance of

1)3 units 2)2 units 3)3/2 units 4) $3/\sqrt{2}$ units

Key. 4

Sol. ortho center- $(2,-1,3)$

Circum center- $\left(\frac{1}{2},-1,\frac{3}{2}\right)$

8. Equation of the plane passing through the origin and perpendicular to the planes $x+2y+z=1, 3x-4y+z=5$ is

1) $x+2y-5z=0$ 2) $x-2y-3z=0$ 3) $x-2y+5z=0$ 4) $3x+y-5z=0$

Key. 4

Sol. $\begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 3 & -4 & 1 \end{vmatrix}=0$

$A=3i+j-5k$

$\Rightarrow 3x+y-5z=0$

9. If θ is the angle between $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda}z+4=0$ and is such that $\sin\theta$ =1/3, the value of λ =

1) $-\frac{4}{3}$ 2) $\frac{4}{3}$ 3) $-\frac{3}{5}$ 4) $\frac{5}{3}$

Key. 4

Sol. $Sin\theta=\left|\frac{2-2+2\sqrt{\lambda}}{3\sqrt{5+\lambda}}\right|=\frac{1}{3}$

$\lambda=\frac{5}{3}$

10. The image of the point $(-1,3,4)$ in the plane x-2y=0 is

1) $(15,11,4)$ 2) $\left(-\frac{17}{3},-\frac{19}{3},1\right)$ 3) $\left(\frac{9}{5},-\frac{13}{5},4\right)$ 4) $\left(-\frac{17}{3},-\frac{19}{3},4\right)$

Key. 3

Sol. $\frac{h+1}{1}=\frac{k-3}{-2}=\frac{p-4}{0}=-2\left(\frac{-1-6}{5}\right)$

$(h,k,p)=\left(\frac{9}{5},\frac{-13}{5},4\right)$

11. The plane passing through the points $(-2,-2,2)$ and containing the line joining the points $(1,1,1)$ and $(1,-1,2)$ makes intercepts on the coordinates axes, the sum of whose lengths is

1. 3 2. 4 3. 6 4. 12

Key. 4

Sol. Equation of the plane be $a(x+2)+b(y+2)+c(z-2)=0.$ As it passes through $(1,1,1)$ and $(1,-1,2), \frac{a}{1}=\frac{b}{-3}=\frac{c}{-6}$. Equation of the plane is $\frac{x}{-8}+\frac{y}{8/3}+\frac{z}{8/6}=1$ and the required sum $=12.$

12. An equation of the plane containing the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and the point $(0,7,-7)$ is

1. $x+y+z=0$
2. $x+2y-3z=35$
3. $3x-2y+3z+35=0$
4. $3x-2y-z=21$

Key. 1

Sol. Equation of the plane is $A(x+1)+B(y-3)+C(z+2)=0$ where $3A+2B+1=0$ and $A+B(7-3)+C(-7+2)=0$

13. The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the coordinate axes at A,B,C respectively. D and E are the mid-points of AB and AC respectively. Coordinates of the mid-point of DE are

1. $(a,b/4,c/4)$ 2. $(a/4,b,c/4)$ 3. $(a/4,b/4,c)$ 4. $(a/2,b/4,c/4)$

Key. 4

Sol. $A(a,0,0), B(0,b,0), C(0,0,c), D(a/2,b/2,0), E(a/2,0,c/2)$ so midpoint of DE is $(a/2,b/4,c/4).$

14. The coordinates of a point on the line $x=4y+5, z=3y-6$ at a distance $3\sqrt{26}$ from the point $(5,0,-6)$ are

1. $(17,3,3)$ 2. $(-7,3,-15)$ 3. $(-17,-3,-3)$ 4. $(7,-3,15)$

Key. 1

Sol. Line is $\frac{x-5}{4/\sqrt{26}}=\frac{y}{1/\sqrt{26}}=\frac{z+6}{3/\sqrt{26}}$. A point on this line at a distance $3\sqrt{26}$ from $(5,0,-6)$ is $(5\pm(3\times 4),\pm 3,-6\pm 9)=(17,3,3)$ or $(-7,-3,-15).$

15. The points $(0,7,10),(-1,6,6)$ and $(-4,9,6)$ are the vertices of

1. A right angled isosceles triangle
2. Equilateral triangle
3. An isosceles triangle
4. An obtuse angled triangle

Key. 1

Sol. Length of the sides are 18, 18 and 36.

16. Equation of a plane bisecting the angle between the planes $2x-y+2z+3=0$ and $3x-2y+6z+8=0$ is

1. $5x-y-4z-45=0$
2. $5x-y-4z-3=0$
3. $23x+13y+32z-45=0$
4. $23x-13y+32z+5=0$

Key. 2

Sol. Equations of the planes bisecting the angle between the given planes are

$$\frac{2x-y+2z+3}{\sqrt{2^2+(-1)^2+2^2}}=\pm\frac{3x-2y+6z+8}{\sqrt{3^2+(-2)^2+6^2}}$$

$$\Rightarrow\ 7(2x-y+2z+3)=\pm3(3x-2y+6z+8)$$

$\Rightarrow$ $5x-y-4z-3=0$ taking the $+ve\ sign$, and $23x-13y+32z+45=0$ taking the – ve sign.

17. If the perpendicular distance of a point P other than the origin from the plane $x+y+z=p$ is equal to the distance of the plane from the origin, then the coordinates of P are

1. $(p,2p,0)$
2. $(0,2p,-p)$
3. $(2p,p,-p)$
4. $(2p,-p,2p)$

Key. 3

Sol. The perpendicular distance of the origin $(0,0,0)$ from the plane $x+y+z=p$ is

$$\left|\frac{-p}{\sqrt{1+1+1}}\right|=\frac{|p|}{\sqrt{3}}.$$

If the coordinates of P are (x,y,z), then we must have

$$\left|\frac{x+y+z-p}{\sqrt{3}}\right|=\frac{|p|}{\sqrt{3}}$$

$$\Rightarrow\ |x+y+z-p|=|p|$$

Which is satisfied by (c)

18. If p_1, p_2, p_3 denote the distances of the plane $2x-3y+4z+2=0$ from the planes $2x-3y+4z+6=0, 4x-6y+8z+3=0$ and $2x-3y+4z-6=0$ respectively, then

1. $p_1+8p_2-p_3=0$
2. $p_3^2=16p_2^2$
3. $8p_2^2=p_1^2$
4. $p_1+2p_2+3p_3=\sqrt{29}$

Key. 1 or 4

Sol. Since the planes are all parallel planes, $p_1 = \frac{|2-6|}{\sqrt{2^2+3^2+4^2}} = \frac{4}{\sqrt{4+9+16}} = \frac{4}{\sqrt{29}}$

Equation of the plane $4x-6y+8z+3=0$ can be written as $2x-3y+4z+3/2=0$

So $p_2 = \frac{|2-3/2|}{\sqrt{2^2+3^2+4^2}} = \frac{1}{2\sqrt{29}}$ and $p_3 = \frac{|2+6|}{\sqrt{2^2+3^2+4^2}} = \frac{8}{\sqrt{29}}$

$\Rightarrow \quad p_1 + 8p_2 - p_3 = 0$

19. The radius of the circle in which the sphere $x^2+y^2+z^2+2x-2y-4z-19=0$ is cut by the plane $x+2y+2z+7=0$ is

1. 2 2. 3 3. 4 4. 1

Key. 2

Sol. Centre of the sphere is $(-1,1,2)$ and its radius is $\sqrt{1+1+4+19}=5.$

Length of the perpendicular from the centre on the plane is $\left|\frac{-1+2+4+7}{\sqrt{1+4+4}}\right| = 4$

Radius of the required circle is $\sqrt{5^2-4^2}=3.$

20. The shortest distance from the plane $12x+4y+3z=327$ to the sphere $x^2+y^2+z^2+4x-2y-6z=155$ is

1. $11\frac{3}{4}$ 2. 13 3. 39 4. 26

Key. 2

Sol. The centre of the sphere is $(-2,1,3)$ and its radius is $\sqrt{4+1+9+155}=13$

Length of the perpendicular from the centre of the sphere on the plane is

$$\left|\frac{-24+4+9-327}{\sqrt{144+16+9}}\right| = \frac{338}{13} = 26$$

So the plane is outside the sphere and the required distance is equal to $26-13=13.$

21. An equation of the plane passing through the line of intersection of the planes $x+y+z=6$ and $2x+3y+4z+5=0$ and the point $(1,1,1)$ is

1. $2x+3y+4z=9$ 2. $x+y+z=3$ 3. $x+2y+3z=6$ 4. $20x+23y+26z=69$

Key. 4

Sol. Equation of any plane through the line of intersection of the given planes is

$2x+3y+4z+5+\lambda(x+y+z-6)=0$

It passes through $(1,1,1)$ if $(2+3+4+5)+\lambda(1+1+1-6)=0 \Rightarrow \lambda=14/3$ and the required equation is therefore, $20x+23y+26z=69.$

22. The volume of the tetrahedron included between the plane $3x+4y-5z-60=0$ and the coordinate planes is

1. 60 2. 600 3. 720 4. None of these

Key. 2

Sol. Equation of the given plane can be written as $\frac{x}{20}+\frac{y}{15}+\frac{z}{-12}=1$

Which meets the coordinates axes in points $A(20,0,0), B(0,15,0)$ and $C(0,0,-12)$ and the coordinates of the origin are $(0,0,0).$

$\therefore$ the volume of the tetrahedron $OABC$ is

$$\frac{1}{6}\begin{vmatrix} 0 & 0 & 0 & 1 \\ 20 & 0 & 0 & 1 \\ 0 & 15 & 0 & 1 \\ 0 & 0 & -12 & 1 \end{vmatrix} = \left|\frac{1}{6}\times 20\times 15\times(-12)\right| = 600.$$

23. Two lines $x=ay+b, z=cy+d$ and $x=a^1y+b^1, z=c^1y+d^1$ will be perpendicular, if and only if

1. $aa^1+bb^1+cc^1=0$ 2. $(a+a^1)(b+b^1)(c+c^1)=0$

3. $aa^1+cc^1+1=0$ 4. $aa^1+bb^1+cc^1+1=0$

Key. 3

Sol. Lines can be written as $\frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c}$ and $\frac{x-b^1}{a^1}=\frac{y}{1}=\frac{z-d^1}{c^1}$ which will be perpendicular if and only if $aa^1+1+cc^1=0$

24. A tetrahedron has vertices at $O(0,0,0), A(1,2,1), B(2,1,3)$ and $C(-1,1,2).$ Then the angle between the faces OAB and ABC will be

1. $\cos^{-1}(17/31)$ 2. 30^0 3. 90^0 4. $\cos^{-1}(19/35)$

Key. 4

Sol. Let the equation of the face OAB be $ax+by+cz=0$ where

$a+2b+c=0$ and $2a+b+3c=0 \Rightarrow \frac{a}{5}=\frac{b}{-1}=\frac{c}{-3}$

25. If the angle θ between the lines $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda}z+4=0$ is such that $\sin\theta=1/3$, then the value of λ is

1. $3/4$ 2. $-4/3$ 3. $5/3$ 4. $-3/5$

Key. 3

Sol. Since the line makes an angle θ with the plane in makes an angle $\pi/2-\theta$ with normal to the plane

$$\therefore \cos(\frac{\pi}{2}-\theta)=\frac{2(1)+(-1)(2)+(\sqrt{\lambda})(2)}{\sqrt{1+4+4}\times\sqrt{4+1+\lambda}}$$

$$\Rightarrow \frac{1}{3}=\frac{2\sqrt{\lambda}}{3\sqrt{\lambda+5}} \Rightarrow \lambda+5=4\lambda$$

$$\Rightarrow \lambda=5/3$$

26. The ratio in which the yz plane divides the segment joining the points $(-2,4,7)$ and $(3,-5,8)$ is

1. $2:3$ 2. $3:2$ 3. $4:5$ 4. $-7:8$

Key. 1

Sol. Let yz plane divide the segment joining $(-2,4,7)$ and $(3,-5,8)$ in the ration $\lambda:1$. Then $\Rightarrow \frac{3\lambda-2}{\lambda+1}=0 \Rightarrow \lambda=\frac{2}{3}$ and the required ratio is $2:3$.

27. The coordinates of the point equidistant from the points $(a,0,0),(0,a,0),(0,0,a)$ and $(0,0,0)$ are

1. $(a/3,a/3,a/3)$ 2. $(a/2,a/2,a/2)$ 3. (a,a,a) 4. $(2a,2a,2a)$

Key. 2

Sol. Let the coordinates of the required point be (x,y,z) then

$x^2+y^2+z^2=(x-a)^2+y^2+z^2=x^2+(y-a)^2+z^2=x^2+y^2+(z-a)^2$

$\Rightarrow x=a/2=y=z.$ Hence the required point is $(a/2,a/2,a/2)$.

28. Algebraic sum of the intercepts made by the plane $x+3y-4z+6=0$ on the axes is

1. $-13/2$ 2. $19/2$ 3. $-22/3$ 4. $26/3$

Key. 1

Sol. Equation of the plane can be written as $\frac{x}{-6}+\frac{y}{-2}+\frac{z}{3/2}=1$

So the intercepts on the coordinates axes are $-6,-2,3/2$ and the required sum is

$-6-2+3/2=-13/2.$

29. If a plane meets the co-ordinate axes in A,B,C such that the centroid of the triangle ABC is the point $(1,r,r^2)$, then equation of the plane is

1. $x+ry+r^2z=3r^2$ 2. $r^2x+ry+z=3r^2$ 3. $x+ry+r^2z=3$ 4. $r^2x+ry+z=3$

Key. 2

Sol. Let an equation of the required plane be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

This meets the coordinates axes in $A(a,0,0), B(0,b,0)$ and $C(0,0,c)$.

So that the coordinates of the centroid of the triangle ABC are

$(a/3,b/3,c/3)=(1,r,r^2)\,(given)\Rightarrow a=3,b=3r,3r^2$ and the required equation of the plane is

$\frac{x}{3}+\frac{y}{3r}+\frac{z}{3r^2}=1$ or $r^2x+ry+z=3r^2$.

30. An equation of the plane passing through the point $(1,-1,2)$ and parallel to the plane $3x+4y-5z=0$ is

1. $3x+4y-5z+11=0$ 2. $3x+4y-5z=11$ 3. $6x+8y-10z=1$ 4. $3x+4y-5z=2$

Key. 1

Sol. Equation of any plane parallel to the plane $3x+4y-5z=0$ is $3x+4y-5z=K$

If it passes through $(1,-1,2)$, then $3-4-5(2)=K\Rightarrow K=-11$

So the required equation is 3x+4y-5z+11=0.

31. Equations of a line passing through $(2,-1,1)$ and parallel to the line whose equations are $\frac{x-3}{2}=\frac{y+1}{7}=\frac{z-2}{-3}$,is

1. $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-1}{2}$
2. $\frac{x-2}{2}=\frac{y+1}{7}=\frac{z-1}{-3}$
3. $\frac{x-2}{2}=\frac{y-7}{-1}=\frac{z+3}{1}$
4. $\frac{x-3}{2}=\frac{y+1}{-1}=\frac{z-2}{1}$

Key. 2

Sol. The required line passes through $(2,-1,1)$ and its direction cosines are proportional to $2,7,-3$ so its equation is $\frac{x-2}{2}=\frac{y+1}{7}=\frac{z-1}{-3}$

32. The ratio in which the plane $2x-1=0$ divides the line joining $(-2,4,7)$ and $(3,-5,8)$ is

1. $2:3$
2. $4:5$
3. $7:8$
4. $1:1$

Key. 4

Sol. Let the required ratio be $k:1$, then the coordinates of the point which divides the join of the points $(-2,4,7)$ and $(3,-5,8)$ in this ratio are given by $\left(\frac{3k-2}{k+1},\frac{-5k+4}{k+1},\frac{8k+7}{k+1}\right)$

As this point lies on the plane $2x-1=0$.

$\Rightarrow \frac{3k-2}{k+1}=\frac{1}{2}\Rightarrow k=1$ and thus the required ratio as $1:1$.

33. If l_1, m_1, n_1 and l_2, m_2, n_2, are d.c.'s of $\overrightarrow{OA}$, $\overrightarrow{OB}$ such that $\angle AOB=\theta$ where 'O' is the origin, then the d.c.'s of the internal bisector of the angle $\angle AOB$ are

(A) $\frac{l_1+l_2}{2\sin\theta/2},\frac{m_1+m_2}{2\sin\theta/2},\frac{n_1+n_2}{2\sin\theta/2}$

(B) $\frac{l_1+l_2}{2\cos\theta/2},\frac{m_1+m_2}{2\cos\theta/2},\frac{n_1+n_2}{2\cos\theta/2}$

(C) $\frac{l_1-l_2}{2\sin\theta/2},\frac{m_1-m_2}{2\sin\theta/2},\frac{n_1-n_2}{2\sin\theta/2}$

(D) $\frac{l_1-l_2}{2\cos\theta/2},\frac{m_1-m_2}{2\cos\theta/2},\frac{n_1-n_2}{2\cos\theta/2}$

Key. B

Sol. Let OA and OB be two lines with d.c's l_1, m_1, n_1 and l_2, m_2, n_2. Let OA = OB = 1. Then, the coordinates of A and B are (l_1, m_1, n_1) and (l_2, m_2, n_2), respectively. Let OC be the bisector of $\angle AOB$. Then, C is the mid point of AB and so its coordinates are $\left(\frac{l_1+l_2}{2},\frac{m_1+m_2}{2},\frac{n_1+n_2}{2}\right)$.

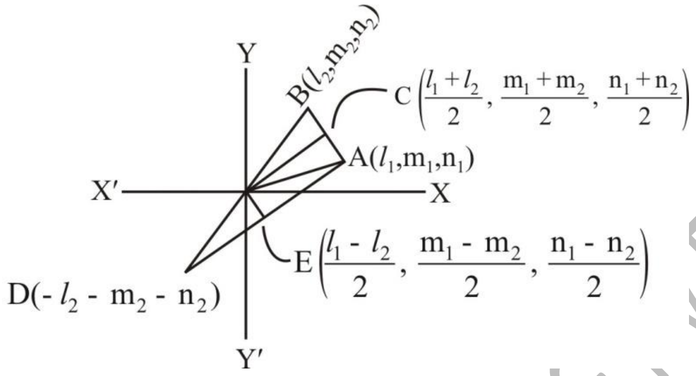
$\therefore$ d.r's of OC are $\frac{l_1+l_2}{2},\frac{m_1+m_2}{2},\frac{n_1+n_2}{2}$

We have, $OC=\sqrt{\left(\frac{l_1+l_2}{2}\right)^2+\left(\frac{m_1+m_2}{2}\right)^2+\left(\frac{n_1+n_2}{2}\right)^2}$

$$= \frac{1}{2}\sqrt{\left(l_1^2 + m_1^2 + n_1^2\right) + \left(l_2^2 + m_2^2 + n_2^2\right) + 2\left(l_1 l_2 + m_1 m_2 + n_1 n_2\right)}$$

$$= \frac{1}{2}\sqrt{2 + 2\cos\theta} \qquad \left[\because \cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2\right]$$

$$= \frac{1}{2}\sqrt{2\left(1 + \cos\theta\right)} = \cos\left(\frac{\theta}{2}\right)$$



$\therefore$ d.c's of OC are $\frac{l_1 + l_2}{2(OC)}, \frac{m_1 + m_2}{2(OC)}, \frac{n_1 + n_2}{2(OC)}$

34. A line is drawn from the point $P(1,1,1)$ and perpendicular to a line with direction ratios $(1,1,1)$ to intersect the plane $x + 2y + 3z = 4$ at Q. The locus of point Q is

A) $\frac{x}{1} = \frac{y-5}{-2} = \frac{z+2}{1}$ B) $\frac{x}{-2} = \frac{y-5}{1} = \frac{z+2}{1}$

C) $x = y = z$ D) $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$

Key. A

Sol. Locus of 'Q' is the line of intersection of the plane $x + 2y + 3z = 4$ and $1(x-1) + 1(y-1) + 1(z-1) = 0 \Rightarrow$ then the line is $\frac{x}{1} = \frac{y-5}{-2} = \frac{z+2}{1}$

35. A line is drawn from the point P(1, 1, 1) and perpendicular to a line with direction ratios $(1,1,1)$ to intersect the plane $x + 2y + 3z = 4$ at Q. The locus of point Q is

A) $\frac{x}{1} = \frac{y-5}{-2} = \frac{z+2}{1}$ B) $\frac{x}{-2} = \frac{y-5}{1} = \frac{z+2}{1}$ C) $x = y = z$ D) $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$

Key: A

Hint: Locus of Q is the line of intersection of the plane $x + 2y + 3z = 4$ and $1(x-1) + 1(y-1) + 1(z-1) = 0 \Rightarrow$ then line is $\frac{x}{1} = \frac{y-5}{-2} = \frac{z+2}{1}$

36. If a line with direction ratios 2 : 2: 1 intersects the line $\frac{x-7}{3}=\frac{y-5}{2}=\frac{z-3}{1}$ and $\frac{x-1}{2}=\frac{y+1}{4}=\frac{z+1}{3}$ at A and B then AB=.

a) $\sqrt{2}$ b) 2 c) $\sqrt{3}$ d) 3

Key:

Hint $A(7+3\alpha,5+2\alpha,3+\alpha),B(1+2\beta,-1+4\beta,-1+3\beta)$

Dr's of AB are 2:2:1

$$\frac{6+3\alpha-2\beta}{2}=\frac{3+\alpha-2\beta}{1}=\frac{4+\alpha-3\beta}{1}$$

$\alpha=-2,\beta=1$

A(1,1,1)B(3,3,2)

AB = 3

37. A, B, C are the points on x, y and z axes respectively in a three dimensional co-ordinate system with O as origin. Suppose the area of triangles OAB, OBC and OCA are 4, 12 and 6 respectively, then the area of the triangle ABC equals

(A) 16 (B) 14 (C) 28 (D) 32

Key: B

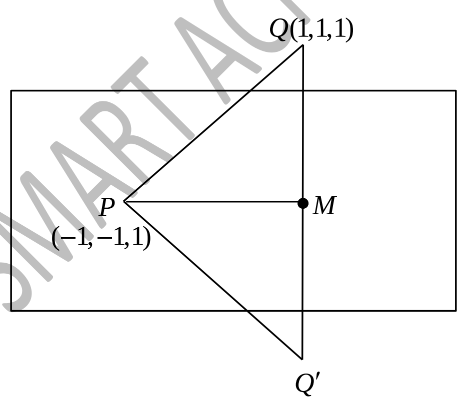
Hint $[ABC]=\sqrt{[OAB]^2+[OBC]^2+[OCA]^2}$

where $[ABC]$ = area of triangle ABC

38. The area of the figure formed by the points $(-1,-1,1);(1,1,1)$ and their mirror images on the plane $3x+2y+4z+1=0$ is

(a) $\frac{5\sqrt{33}}{29}$ (b) $\frac{4\sqrt{33}}{29}$ (c) $\frac{20\sqrt{33}}{27}$ (d) $\frac{20\sqrt{33}}{29}$

Key. D



Sol.

Req. area = ΔPQQ^1

$=2\Delta PQM$

$=2\cdot\frac{1}{2}\cdot QM\cdot PM$

39. If a plane passes through the point $(1,1,1)$ and is perpendicular to the line $\frac{x-1}{3}=\frac{y-1}{0}=\frac{z-1}{4}$ then its perpendicular distance from the origin is

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{7}{5}$ (D) 1

Key: C

Hint: The d.r of the normal to the plane is 3, 0, 4 . The equation of the plane is $3x+0y+4z+d=0$ since it passes through $(1,1,1)$ so; $d=-7$

Now distance of the plane $3x+4z-7=0$ from $(0,0,0)$ is $\frac{7}{\sqrt{3^2+4^2}}=\frac{7}{5}$ unit

40. Three straight lines mutually perpendicular to each other meet in a point P and one of them intersects the x-axis and another intersects the y-axis, while the third line passes through a fixed point (0, 0, c) on the z-axis. Then the locus of P is

A) $x^2+y^2+z^2-2cx=0$ B) $x^2+y^2+z^2-2cy=0$

C) $x^2+y^2+z^2-2cz=0$ D) $x^2+y^2+z^2-2c(x+y+z)=0$

Key: C

Hint: Let L_1, L_2, L_3 be the mutually perpendicular lines and $P(x_0, y_0, z_0)$ be their point of concurrence. If L_1 cuts the x-axis at A(a, 0, 0), L_2 meets the y-axis at B(0, b, 0) and C(0, 0, c) $\in L_3$, then $L_1 11(x_0-a,\ y_0,\ z_0)$, $L_2 11(x_0,\ y_0-b,\ z_0)$ and $L_3 11(x_0,\ y_0,\ z_0-c)$. Hence

$$\left.\begin{array}{l} x_0(x_0-a)+y_0(y_0-b)+z_0^2=0 \\ x_0^2+(y_0-b)y_0+z_0(z_0-c)=0 \end{array}\right\}$$

$$x_0(x_0-a)+y_0^2+z_0(z_0-c)=0$$

Eliminating a and b from the equations, we get

$$x_0^2+y_0^2+z_0^2-2cz_0=0$$

41. The centroid of the triangle formed by (0, 0, 0) and the point of intersection of $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-1}{1}$ with $x=0$ *and* $y=0$ is

(a) $(1,1,1)$ (b) $\left(\frac{1}{6},\frac{-1}{3},\frac{1}{6}\right)$ (c) $\left(\frac{-1}{6},\frac{1}{3},\frac{-1}{6}\right)$ (d) $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

Key. B

Sol. Any point on the given line $(K+1, 2K+1, K+1)$

but $x=0 \quad \Rightarrow A(0,-1,0)$

$y=0 \quad \Rightarrow B(\frac{1}{2},0,\frac{1}{2});0(0,0,0)$

42. The plane $x-y-z=4$ is rotated through 90^o about its line of intersection with the plane $x+y+2z=4$ and equation in new position is $Ax+By+Cz+D=0$ where A,B,C are least positive integers and $D<0$ then

(a) $D = -10$ (b) $ABC = -20$

(c) $A + B + C + D = 0$ (d) $A + B + C = 10$

Key. D

Sol. Given planes are $x - y - z = 4$ ---------- (1) and $x + y + 2z = 4$ ---------- (2)

Since required plane passes through the line of intersection (1) & (2)

$\Rightarrow$ Its equation is $(x - y - z - 4) + \alpha(x + y + 2z - 4) = 0$

$\Rightarrow (1+\alpha)x + (\alpha - 1)y + (2\alpha - 1)z - (4\alpha + 4) = 0$ ---------- (3)

Since (1) & (3) are perpendicular

$\Rightarrow 1(1+\alpha) - 1(\alpha - 1) - 1(2\alpha - 1) = 0$

$1 + \alpha - \alpha + 1 - 2\alpha + 1 = 0 \qquad \Rightarrow \alpha = 3/2$

$\Rightarrow$ Its equations is $(x - y - z - 4) + \frac{3}{2}(x + y + 2z - 4) = 0$

$5x + y + 4z - 20 = 0$

43. Three lines $y - z - 1 = 0 = x;\ z + x + 1 = 0 = y;\ x - z - 1 = 0 = y$ intersect the xy plane at A, B, C then orthocenter of triangle ABC is

(a) $(0,1,0)$ (b) $(-1,0,0)$ (c) $(0,0,0)$ (d) $(1,1,1)$

Key. A

Sol. Intersection of $y - z - 1 = 0 = x$ with xy plane gives $A(0,1,0)$ similarly $B(-1,0,0)$, $C(1,0,0)$

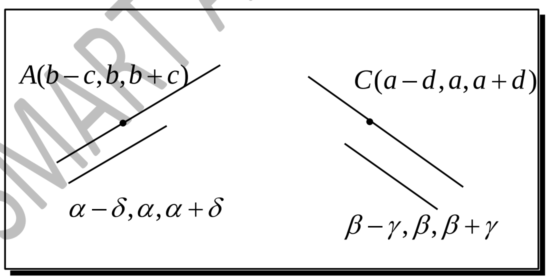
$\therefore$ orthocentre is $(0,1,0)$

44. The lines $\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta}; \frac{x-b+c}{\beta-r} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+r}$ are coplanar and the equation of the plane in which they lie is

(a) $x + y + z = 0$ (b) $x - y + z = 0$ (c) $x - 2y + z = 0$ (d) $x + y - 2z = 0$

Key. C

Sol.



45. The reflection of the point P(1, 0, 0) in the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ is

(a) (3, –4, –2) (b) (5, –8, –4) (c) (1, –1, –10) (d) (2, –3, 8)

Key: b

Hint: Coordinates of any point Q on the given line are

(2r + 1, –3r – 1, 8r – 10) for some $r \in R$

So the direction ratios of PQ are 2r, –3r – 1, 8r – 10

Now PQ is perpendicular to the given line

if 2(2r) – 3 (–3r – 1) + 8(8r – 10) = 0

$\Rightarrow$ 77r – 77 = 0 $\Rightarrow$ r = 1

and the coordinates of Q, the foot of the perpendicular from P on the line are (3, –4, –2).

Let R(a, b, c) be the reflection of P in the given lines when Q is the mid-point of PR

$\Rightarrow \frac{a+1}{2}=3, \frac{b}{2}=-4, \frac{c}{2}=-2$

$\Rightarrow$ a = 5, b = –8, c = –4

and the coordinates of the required point are $(5,-8,-4)$.

46. Reflection of plane 2x + 3y + 4z + 1 = 0 in plane x + 2y + 3z – 2 = 0 is

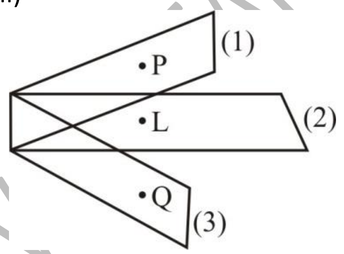
(A) 6x – 19y + 32z = 47 (B) 6x + 19y + 32z = 47

(C) 6x + 19y + 16z = 47 (D) 3x + 19y + 16z = 47

Key. B

Sol. 2x + 3y + 4z + 1 = 0 .....(i)

x + 2y + 3z – 2 = 0 .....(ii)



(iii) is reflection of plane

reflection of ax + by + cz + d = 0 in $a'x+b'y+c'z+d'=0$

$=(aa'+bb'+cc')(a'x+b'y+c'z+d')$

$=(a'^2+b'^2+c'^2)(ax+by+cz+d)$

$2(2+6+12)(x+2y+3z-2)=(1^2+2^2+3^2)(2x+3y+4z+1)$

$4(x+2y+32-2)=14(2x+3y+4z+1)$

$12x+38y+64z=94$

$\Rightarrow 6x+19y+32z=47$

47. The reciprocal of the distance between two points, one on each of the lines $\frac{x-2}{3}=\frac{y-4}{2}=\frac{z-5}{5}$ and $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$

(A) cannot be less than 9 (B) having minimum value $5\sqrt{3}$

(C) cannot be greater than 78 (D) cannot be $2\sqrt{19}$

Key. D

Sol. The shortest distance (SD) = $\dfrac{\left|\begin{vmatrix} 2-1 & 4-2 & 5-3 \\ 2 & 3 & 4 \\ 3 & 2 & 5 \end{vmatrix}\right|}{\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 2 & 5 \end{vmatrix}\right|} = \dfrac{1}{\sqrt{78}}$

So, $\dfrac{1}{SD} = \sqrt{78}$

48. Equation of the plane containing the straight line $\dfrac{x}{2} = \dfrac{y}{3} = \dfrac{z}{4}$ and perpendicular to the plane containing the straight lines $\dfrac{x}{3} = \dfrac{y}{4} = \dfrac{z}{2}$ and $\dfrac{x}{4} = \dfrac{y}{2} = \dfrac{z}{3}$ is

(A) $x + 2y - 2z = 0$
(B) $3x + 2y - 2z = 0$
(C) $x - 2y + z = 0$
(D) $5x + 2y - 4z = 0$

Key. C

Sol. Vector along the required plane is $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix} = 8\hat{i} - \hat{j} - 10\hat{k}$

So, normal vector $(\vec{n})$ to the plane is $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & -1 & -10 \\ 2 & 3 & 4 \end{vmatrix} = 26\hat{i} - 52\hat{j} + 26\hat{k}$.

So, equation of the plane is $\vec{r}.\vec{n} = 0 \Rightarrow x - 2y + z = 0$.

49. The distance between the plane $x - 2y + z - 6 = 0$ and the plane containing the sets of points $(1 + 2\lambda, 2 + 3\lambda, 3 + 4\lambda)$ and $(2 + 3\mu, 3 + 4\mu, 4 + 5\mu)$, where λ, μ are parameters, is

(A) $\sqrt{3/2}$
(B) $\sqrt{6}$
(C) $\sqrt{12}$
(D) $2\sqrt{6}$

Key. B

Sol. Normal vector : $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i} + 2\hat{j} - \hat{k}$

equation of plane: $-1(x - 1) + 2(y - 2) - 1(z - 3) = 0$
$\Rightarrow x - 2y + z = 0$

So, required distance = $\dfrac{|6|}{\sqrt{1+4+1}} = \sqrt{6}$

50. If the point $(0, \lambda, 1)$ lies within the triangular prism formed by the planes $x = 0$, $2y - z + 2 = 0$ and $2y + 3z - 6 = 0$ then the set of values of λ is

(A) $(-2, 2)$
(B) $\left(-\dfrac{1}{2}, \dfrac{3}{2}\right)$

(C) $\left(-4,-\frac{4}{3}\right)$ (D) (0, 4)

Key. B

Sol. The planes are 2y + z = 0, 5x – 12y = 13 and 3x + 4z = 10

Solving we get z = $\frac{11}{2}$

51. Number of lattice point (x, y, z all being integers) inside the tetrahedron (not on the surface) having vertices (0, 0, 0), (21, 0, 0), (0, 21, 0), (0, 0, 21) is

(A) 1140 (B) 4000

(C) 2024 (D) none of these

Key. A

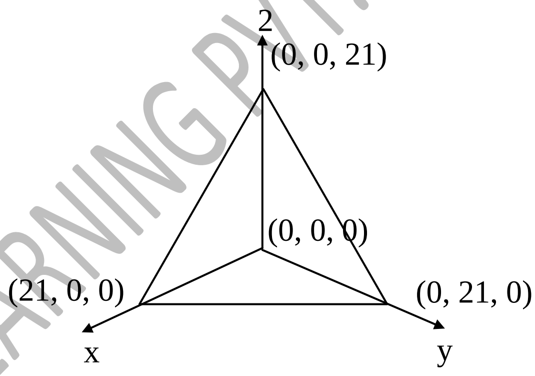
Sol.

Tetrahedron is bounded by $x \geq 0$, $y \geq 0$, $z \geq 0$ and

x + y + z = 21

Total no. of lattice point in side the tetrahedron is

= 1140



52. The equations of hypotenuse of a right angled isosceles triangle are $\frac{x+6}{5}=\frac{y+10}{3}=\frac{z+14}{8}$ and the centroid of the triangle is $\left(\frac{20}{3},-1,\frac{16}{3}\right)$. If (α,β,γ) is the circumcentre of the triangle then $\gamma =$

A) 6 B) -4 C) 5 D) 3

Key. A

Sol. Let $\bar{a}=5i+3j+8k$ (vector parallel to given line)

$G=\left(\frac{20}{3},-1,\frac{16}{3}\right), P=(5\lambda-6,3\lambda-10,8\lambda-14)$

P is the circumcentre $\overrightarrow{GP}.\bar{a}=0$.

53. The distance of the point of intersection of lines $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$ and $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ from $(7,-4,7)$ is

A) 6 B) $\sqrt{24}$ C) $\sqrt{14}$ D) 5

Key. C

Sol. Point of intersection = $(5,-7,6)$

54. Let ABCD be a tetrahedron in which position vectors of A, B, C & D are $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+\hat{j}+2\hat{k}$, $3\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+3\hat{j}+2\hat{k}$. If ABC be the base of tetrahedron then height of tetrahedron is

A) $\sqrt{\frac{3}{2}}$ B) $\sqrt{\frac{3}{5}}$ C) $\frac{2\sqrt{2}}{\sqrt{3}}$ D) $\frac{1}{\sqrt{3}}$

Key. C

Sol. $\overrightarrow{AB}\times\overrightarrow{AC}=-\hat{i}+2\hat{j}+\hat{k}$

Height $=\frac{\left|\overrightarrow{AD}.(\overrightarrow{AB}\times\overrightarrow{AC})\right|}{\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|}=\frac{2\sqrt{3}}{\sqrt{3}}$

55. The plane passing through the point whose position vector is $i+j-k$ and parallel to the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x-1}{-1}=\frac{y+1}{-2}=\frac{z-1}{1}$ has l,m,n as direction cosines of its normal then $|l+m+n|=$

A) $1/\sqrt{3}$ B) $1/\sqrt{2}$ C) $1/\sqrt{5}$ D) $1/\sqrt{6}$

Key. C

Sol. $a+2b+3c=0$

$-a-2b+c=0$

$\Rightarrow a:b:c=2:-1:0$

56. If a line with direction ratios 2 : 2 : 1 intersects the lines $\frac{x-7}{3}=\frac{y-5}{2}=\frac{z-3}{1}$ and $\frac{x-1}{2}=\frac{y+1}{4}=\frac{z+1}{3}$ at A and B then AB=

A) $\sqrt{2}$ B) 2 C) $\sqrt{3}$ D) 3

Key. D

Sol. Let $A(7+3\alpha,5+2\alpha,3+\alpha),\ B(1+2\beta,-1+4\beta,-1+3\beta)$

D.R's of AB are in 2 : 2 : 1

$\therefore\frac{6+3\alpha-2\beta}{2}=\frac{3+\alpha-2\beta}{1}=\frac{4+\alpha-3\beta}{1}$

$\therefore\alpha=-2,\beta=1,\ A(1,1,1),\ B(3,3,2)$

57. The two lines whose direction cosines are connected by the relations $al+bm+cn=0$ and $ul^2+vm^2+wn^2=0$ are perpendicular then

(a) $a^2(v-w)+b^2(w-u)+c^2(u-v)=0$

(b) $\frac{a^2}{u}+\frac{b^2}{v}+\frac{c^2}{w}=0$

(c) $a(v^2+w^2)+b(w^2+u^2)+c(u^2+v^2)=0$

(d) $a^2(v+w)+b^2(w+u)+c^2(u+v)=0$

Key. D

Sol. Given relations are

$al+bm+cn=0$ ---------- (1)

$ul^2+vm^2+wn^2=0$ ---------- (2)

Eliminating 'n' between the given relations we get $ul^2+vm^2+w\left(\frac{al+bm}{-c}\right)^2=0$

$c^2ul^2+c^2vm^2+wa^2l^2+wb^2m^2+2abwlm=0$

$\left(c^2u+wa^2\right)\frac{l^2}{m^2}+2abw\frac{l}{m}+\left(b^2w+c^2v\right)=0\rightarrow 1$

The above is quadratic equation in $\frac{l}{m}$, whose roots are $\frac{l_1}{m_1},\frac{l_2}{m_2}$

$\frac{l_1l_2}{m_1m_2}=\frac{b^2w+c^2v}{c^2u+wa^2}$

$\frac{l_1l_2}{b^2w+c^2v}=\frac{m_1m_2}{c^2u+wa^2}=\frac{n_1n_2}{a^2v+b^2u}$

If the lines are perpendicular, then $l_1l_2+m_1m_2+n_1n_2=0$

$b^2w+c^2v+c^2u+wa^2+a^2v+b^2u=0$

$a^2(v+w)+b^2(u+w)+c^2(u+v)=0$

58. $f(x)$ be a polynomial in x satisfying the condition $f(x)f\left(\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)$ and $f(2)=9$. Then the direction cosines of the ray joining the origin and point $\left(f(0),f(1),f(-1)\right)$ are given by

a) $\left(\frac{1}{\sqrt{5}},\frac{2}{\sqrt{5}},0\right)$ b) $(1,2,0)$ c) $(0,1,-1)$ d) $\left(\frac{1}{\sqrt{5}},\frac{2}{\sqrt{5}},\frac{1}{\sqrt{5}}\right)$

Key. A

Sol. $f(x)=x^n+1$. $f(2)=9$ imply $f(x)=x^3+1$ and $f(0)=1$ $f(1)=2,f(-1)=0$,

Dc's of ray joining $(0,0,0)$ & $(1,2,0)$ is $\left(\frac{1}{\sqrt{5}},\frac{2}{\sqrt{5}},0\right)$.

59. The plane $x-y-z=4$ is rotated through 90^o about its line of intersection with the plane $x+y+2z=4$ and equation in new position is $Ax+By+Cz+D=0$ where A,B,C are least positive integers and $D<0$ then

a) $D=-10$ b) $ABC=-20$ c) $A+B+C+D=0$ d) $A+B+C=10$

Key. D

Sol. Given planes are $x-y-z=4$ ---------- (1) and $x+y+2z=4$ ---------- (2)

Since required plane passes through the line of intersection (1) & (2)

$\Rightarrow$ Its equation is $(x-y-z-4)+\alpha(x+y+2z-4)=0$

$\Rightarrow(1+\alpha)x+(\alpha-1)y+(2\alpha-1)z-(4\alpha+4)=0$ ---------- (3)

Since (1) & (3) are perpendicular

$\Rightarrow 1(1+\alpha)-1(\alpha-1)-1(2\alpha-1)=0\Rightarrow 1+\alpha-\alpha+1-2\alpha+1=0$ $\Rightarrow\alpha=3/2$

$\Rightarrow$ Its equation is $(x-y-z-4)+\frac{3}{2}(x+y+2z-4)=0\Rightarrow 5x+y+4z-20=0$

60. The equation of motion of a point in space is $x=2t,y=-4t,z=4t$. where it is measured in hour and the coordinates of moving point in kilometers.The distance of the point from the starting point O(0,0,0) in 10 hours is

a) 20 km b) 40 km c) 55 km d) 60 km

Key. D

Sol. Eliminating ‘t’ from the equation we get the equation of the path, $\frac{x}{2}=\frac{y}{-4}=\frac{z}{4}$

Thus the path represents a straight line through the origin. For t= 10 h, we have x= 20, y=-40,z=40 and $|\vec{r}|=|O\vec{M}|=\sqrt{(x^2+y^2+z^2)}=\sqrt{400+1600+1600}$ =60 km

61. A mirror and a source of light are situated at the origin O and a point on OX respectively. A ray of light from the source strikes the mirror and is reflected. If the DRs of normal to the plane of mirror are 1,-1,1, then DCs for the reflected ray are

a) $\frac{1}{3},\frac{2}{3},\frac{2}{3}$ b) $-\frac{1}{3},-\frac{2}{3},\frac{2}{3}$ c) $-\frac{1}{3},-\frac{2}{3},-\frac{2}{3}$ d) $-\frac{1}{3},\frac{2}{3},\frac{2}{3}$

Key. B

Sol. DCs of the reflected ray are $-\frac{1}{3},-\frac{2}{3},\frac{2}{3}$

62. Through a point P(a, b, c) a plane is drawn at right angles to OP to meet the co-ordinate axes in A, B and C. If OP = p, then the area of ΔABC is

(A) $\frac{p^2ab}{c^2}$ (B) $\frac{p^3c}{3ab}$ (C) $\frac{p^2c^2}{2ab}$ (D) $\frac{p^5}{2abc}$

Key. D

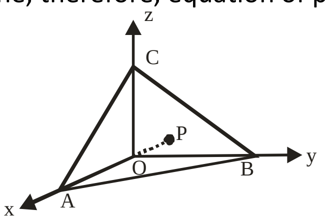
Sol. Here $OP=\sqrt{h^2+k^2+l^2}=p$

$\therefore$ DRs of OP are:

$\frac{h}{\sqrt{h^2+k^2+l^2}},\frac{k}{\sqrt{h^2+k^2+l^2}},\frac{l}{\sqrt{h^2+k^2+l^2}}$

or $\frac{h}{p},\frac{k}{p},\frac{l}{p}$

Since OP is normal to the plane, therefore, equation of plane is



$\frac{h}{p}x+\frac{k}{p}y+\frac{l}{p}z=p$ or $hx+ky+lz=p^2$

$\therefore A\left(\frac{p^2}{h},0,0\right),B\left(0,\frac{p^2}{k},0\right),C\left(0,0,\frac{p^2}{l}\right)$

Now, Area of $\Delta ABC, \Delta=\sqrt{A_{xy}^2+A_{yz}^2+A_{zx}^2}$

Where, A_{xy}^2 is area of projection of ΔABC on xy plane = area of ΔAOB

Now, $A_{xy}=\frac{1}{2}\begin{vmatrix} p^2/h & 0 & 1 \\ 0 & p^2/k & 1 \\ 0 & 0 & 1 \end{vmatrix}=\frac{p^4}{2|hk|}$

Similarly, $A_{yz} = \frac{p^4}{2|kl|}$ and $A_{zx} = \frac{p^4}{2|lh|}$

$$\therefore \Delta^2 = A_{xy}^2 + A_{yz}^2 + A_{zx}^2, \Delta = \frac{p^5}{2hkl}$$

63. If $l_i^2 + m_i^2 + n_i^2 = 1 \forall i \in \{1,2,3\}$ and $l_i l_j + m_i m_j + n_i n_j = 0 \; \forall i, j \in \{1,2,3\} \; (i \neq j)$

$\Delta = \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}$ then

(A) $|\Delta| = 3$ (B) $|\Delta| = 2$ (C) $|\Delta| = 1$ (D) $\Delta = 0$

Key. C

Sol. We have,

$$\Delta^2 = \Delta\Delta = \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}$$

$$= \begin{vmatrix} l_1^2 + m_1^2 + n_1^2 & l_1 l_2 + m_1 m_2 + n_1 n_2 & l_1 l_3 + m_1 m_3 + n_1 n_3 \\ l_1 l_2 + m_1 m_2 + n_1 n_2 & l_2^2 + m_2^2 + n_2^2 & l_2 l_3 + m_2 m_3 + n_1 n_3 \\ l_1 l_3 + m_1 m_3 + n_1 n_3 & l_2 l_3 + m_2 m_3 + n_2 n_3 & l_3^2 + m_3^2 + n_3^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \Rightarrow \Delta = \pm 1 \Rightarrow |\Delta| = 1$$

64. Equation of the straight line in the plane $\vec{r}.\vec{n} = d$ which is parallel to $\vec{r} = \vec{a} + \lambda\vec{b}$ and passes through the foot of perpendicular drawn from the point $P(\vec{a})$ to the plane $\vec{r}.\vec{n} = d$. (where $\vec{n}.\vec{b} = 0$) is

A) $\vec{r} = \vec{a} + \left(\frac{d - \vec{a}.\vec{n}}{n^2}\right)\vec{n} + \lambda\vec{b}$ B) $\vec{r} = \vec{a} + \left(\frac{d - \vec{a}.\vec{n}}{n}\right)\vec{n} + \lambda\vec{b}$

C) $\vec{r} = \vec{a} + \left(\frac{\vec{a}.\vec{n} - d}{n^2}\right)\vec{n} + \lambda\vec{b}$ D) $\vec{r} = \vec{a} + \left(\frac{\vec{a}.\vec{n} - d}{n}\right)\vec{n} + \lambda\vec{b}$

Key. A

Sol. Foot perpendicular from point $A(\vec{a})$ on the plane $\vec{r}.\vec{n} = d$ is $\vec{a} + \frac{(d - \vec{a}.\vec{n})}{|\vec{n}|^2}\vec{n}$

$\therefore$ Equation of line parallel to $\vec{r} = \vec{a} + \lambda\vec{b}$ in the plane $\vec{r}.\vec{n} = d$ is given by

$$\vec{r} = \vec{a} + \frac{(d - \vec{a}.\vec{n})}{|\vec{n}|^2}\vec{n} + \lambda\vec{b}$$

65. If the foot of the perpendicular from the origin to a plane is P(a, b, c), the equation of the plane is

A) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$ B) ax + by + cz = 3

C) ax + by + cz = $a^2+b^2+c^2$ D) ax + bx + cz = a + b + c

Key. C

Sol. Direction ratios of OP are $\langle a,b,c\rangle$

$\therefore$ equation of the plane is

e(x – a) + b(y – b) + c(z – c) = 0

i.e. xa + yb + zc $=a^2+b^2+c^2$

66. Equation of line in the plane $\pi=2x-y+z-4=0$ which is perpendicular to the line *l* whose equation is $\frac{x-2}{1}=\frac{y-2}{-1}=\frac{z-3}{-2}$ and which passes through the point of intersection of *l* and π is

A) $\frac{x-2}{3}=\frac{y-1}{5}=\frac{z-1}{-1}$ B) $\frac{x}{3}=\frac{y-3}{5}=\frac{z-5}{-1}$

C) $\frac{x+2}{2}=\frac{y+1}{-1}=\frac{z+1}{1}$ D) $\frac{x+2}{2}=\frac{y-1}{-1}=\frac{z-1}{1}$

Key. B

Sol. Let direction ratios of the line by $\langle a,b,c\rangle$, then

2a – b + c = 0

a – b – 2c = 0

i.e. $\frac{a}{3}=\frac{b}{5}=\frac{c}{-1}$

$\therefore$ direction ratios of the line are $\langle 3,5,-1\rangle$

Any point on the line is $(2+\lambda,2-\lambda,3-2\lambda)$. It lies on the plane π if

$2(2+\lambda)-(2-\lambda)+(3-2\lambda)=4$

i.e. $4+2\lambda-2+\lambda+3-2\lambda=4$

i.e. $\lambda=-1$

$\therefore$ The point of intersection of the line and the plane is (1, 3, 5)

$\therefore$ equation of the required line is $\frac{x-1}{3}=\frac{y-3}{5}=\frac{z-5}{-1}$

67. Equation of plane which passes through the point of intersection of lines

$\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ at greatest distance from the point (0, 0, 0) is

A) 4x + 3y + 5z = 25 B) 4x + 3y + 5z = 50

C) 3x + 4y + 5z = 49 D) x + 7y – 5z = 2

Key. B

Sol. Let a point $(3\lambda+1,\lambda+2,2\lambda+3)$ of the first line also lies on the second line

Then $\frac{3\lambda+1-3}{1}=\frac{\lambda+2-1}{2}=\frac{2\lambda+3-2}{3}\Rightarrow\lambda=1$

Hence the point of intersection P of the two lines is (4, 3, 5)

Equation of plane perpendicular to OP where O is (0, 0, 0) and passing through P is

4x + 3y + 5z = 50

68. Let $A\left(\vec{a}\right)$ and $B\left(\vec{b}\right)$ be points on two skew lines $\vec{r}=\vec{a}+\lambda\vec{p}$ and $\vec{r}=\vec{b}+u\vec{q}$ and the shortest distance between the skew lines is 1, where $\vec{p}$ and $\vec{q}$ are unit vectors forming adjacent sides of a parallelogram enclosing an area of $\frac{1}{2}$ units. If an angle between AB and the line of shortest distance is 60°, then AB =

A) $\frac{1}{2}$ B) 2 C) 1 D) $\lambda \in R-\{0\}$

Key. B

Sol. $1=\left|\left(\vec{b}-\vec{a}\right).\frac{\left(\vec{p}\times\vec{q}\right)}{\left|\vec{p}\times\vec{q}\right|}\right| \Rightarrow \left|\vec{b}-\vec{a}\right|\cos 60°=\frac{1}{2}AB \Rightarrow AB=2$

69. If plane 2x + 3y + 6z + k = 0 is tangent to the sphere $x^2+y^2+z^2+2x-2y+2z-6=0$, then a value of k is

A) 26 B) 16 C) – 26 D) none of these

Key. A

Sol. Centre and radius of the sphere are (- 1, 1, - 1) and 3 respectively.

Distance of (- 1, 1, - 1) from the plane is $\left|\frac{-2+3-6+k}{\sqrt{4+9+36}}\right|$

Since the plane is tangent to the sphere

$\therefore \left|\frac{k-5}{7}\right|=3$ is |k – 5| = 21

$\therefore$ k = - 16, 26

70. If $P_1:\vec{r}.\vec{n}_1-d_1=0,\ P_2:\vec{r}.\vec{n}_2-d_2=0$ and $P_3:\vec{r}.\vec{n}_3-\vec{d}_3=0$ are three planes and $\vec{n}_1.\vec{n}_2$ and $\vec{n}_3$ are three non-coplanar vectors then, the three lines $P_1=0, P_2=0; P_2=0, P_3=0$ and $P_3=0, P_1=0$ are

A) parallel lines B) coplanar lines C) coincident lines D) concurrent lines

Key. D

Sol. $P_1=P_2=0, P_2=P_3=0$ and $P_3=P_1=0$ are lines of intersection of the three planes P_1, P_2 and P_3. As $\vec{n}_1, \vec{n}_2$ and $\vec{n}_3$ are non-coplanar, planes $P_1, P_2\ P_3$ will intersect at unique point. So the given lives will pass through a fixed point.

71. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors equally inclined to each other at an angle α. Then the angle between $\vec{a}$ and plane of $\vec{b}$ and $\vec{c}$ is

A) $\theta=\cos^{-1}\left(\frac{\cos\alpha}{\cos\frac{\alpha}{2}}\right)$ B) $\theta=\sin^{-1}\left(\frac{\cos\alpha}{\cos\frac{\alpha}{2}}\right)$

C) $\theta=\cos^{-1}\left(\frac{\sin\frac{\alpha}{2}}{\sin\alpha}\right)$ D) $\theta=\sin^{-1}\left(\frac{\sin\frac{\alpha}{2}}{\sin\alpha}\right)$

Key. A

Sol. Let θ be the required angle then θ will be the angle between $\vec{a}$ and $\vec{b}+\vec{c}$ ($\vec{b}+\vec{c}$ lies along the angular bisector of $\vec{a}$ and $\vec{b}$)

$$\cos\theta = \frac{\vec{a}.\left(\vec{b}+\vec{c}\right)}{\left|\vec{a}\right|\left|\vec{b}+\vec{c}\right|}$$

$$= \frac{2\cos\alpha}{\sqrt{2+2\cos\alpha}} = \frac{\cos\alpha}{\cos\frac{\alpha}{2}}$$

$$\theta = \cos^{-1}\left(\frac{\cos\alpha}{\cos\alpha/2}\right)$$

72. The reflection of the point P(1, 0, 0) in the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ is

A) (3, - 4, - 2) B) (5, - 8, - 4) C) (1, - 1, - 10) D) (2, - 3, 8)

Key. B

Sol. Let reflection of P(1, 0, 0) in the line

$\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ be (α, β, γ)

Then $\left(\frac{\alpha+1}{2}, \frac{\beta}{2}, \frac{\gamma}{2}\right)$ lies on the line

and $(\alpha-1)\hat{i}+\beta\hat{j}+\gamma\hat{k}$ is perpendicular to $2\hat{i}-2\hat{j}+8\hat{k}$

$$\therefore \qquad \frac{\frac{\alpha+1}{2}}{2} = \frac{\frac{\beta}{2}+1}{-3} = \frac{\frac{\gamma}{2}+10}{8} = \lambda$$

And $\quad 2(\alpha-1)-3(\beta)+\gamma(8)=0$

$\Rightarrow \quad \alpha=5, \beta=-8, \gamma=-4$

73. Let A(1, 1, 1), B (2, 3, 5), C (- 1, 0, 2) be three points, then equation of a plane parallel to the plane ABC which is at a distance 2 is

A) 2x – 3y + z + $2\sqrt{14}=0$ B) 2x – 3y + z $-\sqrt{14}=0$

C) 2x – 3y + z + 2 = 0 D) 2x – 3y + z – 2 = 0

Key. A

Sol. A(1, 1, 1), B(2,3,5), C (- 1, 0, 2) directions ratios of AB are < 1, 2, 4> direction ratios of AC are < - 2, - 1, 1 >

$\therefore$ direction ratios of normal to plane ABC are < 2, - 3, 1 >

$\therefore$ Equation of the plane ABC is 2x – 3y + z = 0

Let the equation of the required plane be 2x – 3y + z = k, then $\left|\frac{k}{\sqrt{4+9+1}}\right|=2$

$$k=\pm 2\sqrt{14}$$

$\therefore$ Equation of the required plane is $2x-3y+z+2\sqrt{14}=0$

74. The points A(2 – x, 2, 2), B(2, 2 – y, 2), C(2, 2, 2 – z) and D(1, 1, 1) are coplanar, then locus of P(x, y, z) is

A) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$ B) x + y + z = 1

C) $\frac{1}{1-x}+\frac{1}{1-y}+\frac{1}{1-z}=1$ D) None of these

Key. A

Sol. Here $\overrightarrow{AB}=x\hat{i}-y\hat{j}$

$\overrightarrow{AC}=x\hat{i}-z\hat{k}$

$\overrightarrow{AD}=(x-1)\hat{i}-\hat{j}-\hat{k}$

As these vectors are coplanar $\Rightarrow \begin{vmatrix} x & -y & 0 \\ x & 0 & -z \\ x-1 & -1 & -1 \end{vmatrix}=0 \Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$

75. The equation of plane through (1, 2, 3) and at the maximum distance from origin is

A) x + 2y + 3z = 14 B) x + y + z = 6

C) x + 2y + 3z + 14 = 0 D) 3x

Key. A

Sol. Direction rations of normal to the plane is (1, 2, 3)

$\Rightarrow$ Equation of plane (x – 1)1 + (y – 2).2 + (z – 3). 3 = 0

$\Rightarrow$ x + 2y + 3z = 14

76. If P(α, β, γ) be a vertex of an equilateral triangle PQR where vertex Q and R are (-1,0,1) and (1, 0, -1) respectively then P will lie on the plane

a) x + y + z + 6 = 0 b) 2x + 4y + 3z + 10 = 0

c) x – y + z + 12 = 0 d) $x+y+z+3\sqrt{2}=0$

Ans. **d**

$QR=2\sqrt{2}$ = OP = 6

77. The length of the perpendicular from (1, 0, 2) on the line $\frac{x+1}{3}=\frac{y-2}{-2}=\frac{z+1}{-1}$ is

a) $\frac{3\sqrt{6}}{2}$ b) $\frac{6\sqrt{3}}{5}$ c) $3\sqrt{2}$ d) $2\sqrt{3}$

Ans. **a**

$$PM=\sqrt{\left(1-\frac{1}{2}\right)^2+(0-1)^2\left(2+\frac{3}{2}\right)^2}=\frac{3\sqrt{6}}{2}$$

78. In triangle OAB, B = (3, 4). If $H\equiv(1,4)$ be the orthocenter of the triangle, then the coordinates of A are (where O is the origin)

a) $\left(0,\frac{15}{4}\right)$ b) $\left(0,\frac{17}{4}\right)$ c) $\left(0,\frac{21}{4}\right)$ d) $\left(0,\frac{19}{4}\right)$

Ans. **d**

Sol. Let $A=(h,k)$, slope of $AH=\frac{k-4}{h-1}$, slope of OB = $\frac{4}{3}$

$\Rightarrow \dfrac{4(k-4)}{3(h-1)} = -1$

$\Rightarrow 4k + 3h = 19$

Slope of OA = $\dfrac{k}{h}$, slope of BH = 0 As $OA \perp BH$

$\therefore h = 0,$ put in (1)

$k = \dfrac{19}{4}$

79. In an acute angles triangle ABC, AA_1, AA_2 are the median and altitude respectively. Then A_1A_2 is equal to

a) $\dfrac{|a^2 - c^2|}{2b}$ b) $\dfrac{|a^2 - b^2|}{2c}$ c) $\dfrac{|b^2 - c^2|}{2a}$ d) none of these

Ans. c

Sol. $A_2C = AB\cos B = c\cos B = \dfrac{a^2 + c^2 - b^2}{2a}$

Also $A_1B = \dfrac{a}{2}$ and $A_2A_1 = BA_1 - BA_2 = \left|\dfrac{a}{2} - \dfrac{a^2 + c^2 - b^2}{2a}\right|$

= $\left|\dfrac{b^2 - c^2}{2a}\right|$

80. If a chord of the circle $x^2 + y^2 - 4x - 2y - c = 0$ is trisected at the points $\left(\dfrac{1}{3}, \dfrac{1}{3}\right)$ and $\left(\dfrac{8}{3}, \dfrac{8}{3}\right)$, then c =

a) 10 b) 20 c) 30 d) 40

Ans. b

Sol. Cut A : $\left(\dfrac{1}{3}, \dfrac{1}{3}\right)$ and $B\left(\dfrac{8}{3}, \dfrac{8}{3}\right)$. Also C(2, 1).

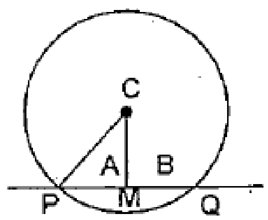
Then equation of AB is y = x, and length AB = $\dfrac{7\sqrt{2}}{3}$

If PQ be the chord, then

Length $PQ = 7\sqrt{2}$

Now $CP^2 = PM^2 + CM^2$

$\Rightarrow 4 + 1 + c = \left(\dfrac{7\sqrt{2}}{2}\right)^2 + \left(\dfrac{1}{\sqrt{2}}\right)^2 = 25 \Rightarrow c = 20$



81. From a point on hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$ tangents are drawn to circle $x^2 + y^2 = 9$ then locus of midpoint of chord of contact

a) $9(9x^2 - 4y^2) = 4(x^2 + y^2)^2$ b) $9(4x^2 - 9y^2) = 4(x^2 + y^2)^2$

c) $5(9x^2 - 4y^2) = 4(x^2 + y^2)^2$ d) $9(9x^2 - 5y^2) = 4(x^2 - y^2)^2$

Ans. b

Sol. Equation of chord of contact is $3x\sec\theta + 2y\tan\theta = 9$ - (1)

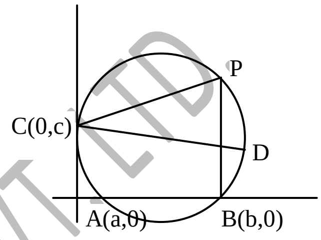
Let midpoint of chord of contact be (h, k) then hx + ky = $h^2 + k^2$ - (2)

(1) and (2) are identical

$$\sec\theta = \frac{9h}{3\left(h^2+k^2\right)}, \tan\theta = \frac{9k}{2\left(h^2+k^2\right)}$$

Then $\sec^2\theta - \tan^2\theta = 1$

82. In figure shown two points A and B are given on x-axis and third point C on y-axis. Then locus of P such that four A, B, P and C lie on a circle



a) $\left(x - \frac{a+b}{2}\right)^2 + \left(y - \frac{c^2+ab}{2c}\right)^2 = \frac{c^4 + a^2b^2 + b^2c^2 + c^2a^2}{4c^2}$

b) $\left(x + \frac{a+b}{2}\right)^2 + \left(y - \frac{c^2+ab}{2c}\right)^2 = \frac{c^4 + a^2b^2 + b^2c^2 + c^2a^2}{4c^2}$

c) $\left(x - \frac{a+b}{2}\right)^2 + \left(y + \frac{c^2+ab}{2c}\right)^2 = \frac{c^4 + a^2b^2 + b^2c^2 + c^2a^2}{4c^2}$

d) none of these

Ans. a

Sol. Let equation of circle be $(x-\alpha)^2 + (y-\beta)^2 = r^2$

Since it passes through A, B, C

$r^2 = (a-\alpha)^2 + \beta^2$

$r^2 = (b-\alpha)^2 + \beta^2$ on solving get equation

$r^2 = \alpha^2 + (c-\beta)^2$

83. Let A be the fixed point (0, 4) and B be a moving point (2t, 0), M be the midpoint of AB and let the perpendicular bisector of AB meet the y-axis at R. The locus of the midpoint of MR is

a) $x^2 = -(y-2)$ b) $x^2 + (y-2)^2 = 1/4$ c) $x^2 + 1/4 = (y-2)^2$ d) none of these

Ans. a

Sol. M(t, 2) $\Rightarrow$ equation of MR is $y - 2 = \frac{t}{2}(x-t)$

$\Rightarrow R \equiv \left(0, 2 - t^2/2\right)$, let midpoint be (h, k)

$\Rightarrow h = t/2,\ k = 2 - t^2/4$

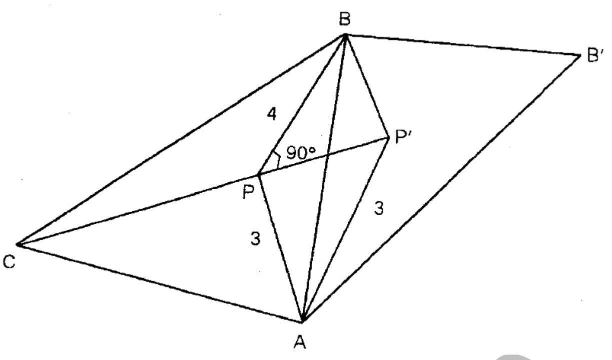
84. If P be a point inside an equilateral ΔABC such that PA = 3, PB = 4 and PC = 5, then the side length of the equilateral ΔABC is

a) $\sqrt{25 - 12\sqrt{3}}$ b) 13 c) $\sqrt{25 + 12\sqrt{3}}$ d) 17

Ans. c

Sol.

Rotate the triangle in clockwise direction through an angle 60^0. Let the points A, B, C and P will be A, B', B and P' respectively after the rotation. We have $PA = P'A = 3$ and $\lfloor PAP' = 60^0 \Rightarrow PP' = 3$. Also $CP = BP' = 5$. So $\Delta BPP'$ is right angle triangle which $\lfloor BPP'$ = 90^0. Now apply cosine rule in ΔBPA because $\lfloor BPA = 90^0 + 60^0 = 150^0$, PA = 3 and BP = 4, we can get AB.



85. Consider $A \equiv (3,4)$, $B \equiv (7,13)$. If P be a point on the line y = x such that PA + PB is minimum, then the coordinate of P are

a) $\left(\frac{13}{7}, \frac{13}{7}\right)$ b) $\left(\frac{23}{7}, \frac{23}{7}\right)$ c) $\left(\frac{31}{7}, \frac{31}{7}\right)$ d) $\left(\frac{33}{7}, \frac{33}{7}\right)$

Ans. c

Sol. Let A, be the reflection of A in $y = x \Rightarrow A_1 \equiv (4,3)$

Now PA + PB = A_1P + PB, which is minimum when A_1, P, B are collinear

Equation of A_1B is $(y-3) = \frac{13-3}{7-4}(x-4) \Rightarrow 3y = 10x - 31$ and y = x gives $P \equiv \left(\frac{31}{7}, \frac{31}{7}\right)$

86. In triangle ABC, equation of the side BC is x – y = 0. Circumcentre and orthocenter of the triangle are (2, 3) and (5, 8) respectively. Equation of the circumcircle of the triangle is

a) $x^2 + y^2 - 4x + 6y - 27 = 0$ b) $x^2 + y^2 - 4x - 6y - 27 = 0$

c) $x^2 + y^2 + 4x + 6y - 27 = 0$ d) $x^2 + y^2 + 4x - 6y - 27 = 0$

Ans. b

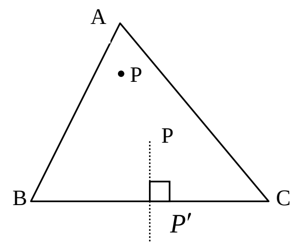
Sol.

Reflection P in BC will lie on BC

$\therefore$ Equation of circumcircle is

$(x-2)^2 + (y-3)^2 = (8-2)^2 + (5-3)^2$ or

$x^2 + y^2 - 4x - 6y - 27 = 0$



87. The locus of the midpoints of the chords of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ that pass through the origin is

a) $x^2 + y^2 + 2gx + 2fy = 0$ b) $x^2 + y^2 + gx + fy + c = 0$

c) $x^2 + y^2 + gx + fy = 0$ d) $2(x^2 + y^2 + gx + |y| + c = 0$

Ans. c

Sol. $T = S_1 \Rightarrow xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

It passes through (0, 0)

$\therefore x_1^2 + y_1^2 + gx_1 + fy_1 = 0$

$\therefore$ Locus is $x^2 + y^2 + gx + fy = 0$

88. Circles C_1 and C_2 having centres G_1 and G_2 respectively intersect each other at the points A and B, secants L_1 and L_2 are drawn to the circles C_1 and C_2 to intersect them in the points A_1, B_1 and A_2, B_2 respectively. If the secants L_1 and L_2 intersect each other at a point P in the exterior

region of circles C_1 and C_2 and $PA_1 \times PB_1 = PA_2 \times PB_2$ then which of the following statement is false

a) points P, A and B are collinear

b) line joining G_1 and G_2 is perpendicular to line joining P and A

c) $PA_1 \times PB_1 = PA \times PB$

d) $PA = PA_1$

Ans. **d**

Sol. Line joining PAB will be the radical axis of the two circles so a, b and c are correct

89. Distance between centres of circles which pass through A(a, a) and B(2a, 2a) and touch the y-axis is

a) 4a b) $2\sqrt{2}a$ c) $4\sqrt{2}\,a$ d) $\sqrt{2}\,a$

Ans. **c**

Sol. Let $(\alpha, 3a-\alpha)$, $(\beta, 3a-\beta)$ be the centres of the circle

$\Rightarrow \alpha, \beta$ are the roots of equation $(x-a)^2 + (2a-x)^2 = x^2$

$$\Rightarrow \alpha + \beta = 6a,\ \alpha\beta = 5a^2$$

$$\Rightarrow |\alpha - \beta| = 4a$$

$$\Rightarrow C_1C_2 = 4a\sqrt{2}$$

90. The locus of the centre of a circle which cuts orthogonally the parabola $y^2 = 4x$ at (1, 2) will pass through points

a) (3, 4) b) (4, 3) c) (5, 3) d) (2, 4)

Ans. **a**

Sol. Tangent to parabola $y^2 = 4x$ at (1, 2) will be the locus

i.e $y \cdot 2 = 2(x+1)$

y = x + 1

91. Let AB be any chord of the circle $x^2 + y^2 - 4x - 4y + 4 = 0$ which subtends an angle of 90^0 at the point (2, 3), then the locus of the midpoint of AB is circle whose centre is

a) (1, 5) b) $\left(1, \frac{5}{2}\right)$ c) $\left(1, \frac{3}{2}\right)$ d) $\left(2, \frac{5}{2}\right)$

Ans. **d**

Sol. Let midpoint of AB is M(h, k)

AB subtends 90^0 at (2, 3)

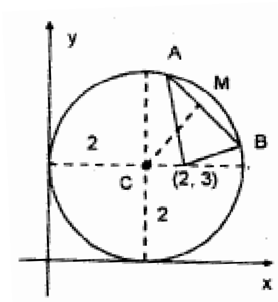
$$\Rightarrow AM = MB$$

$$\Rightarrow \sqrt{(h-2)^2 + (k-3)^2}$$

Also, $CM^2 + MB^2 = CB^2$

$$\Rightarrow (h-2)^2 + (k-2)^2 + (h-2)^2 + (k-3)^2 = 4$$

$$\Rightarrow x^2 + y^2 - 4x - 5y + \frac{17}{2} = 0$$



92. If line y = 2x + c neither cuts the circle $(x-2)^2 + (y-3)^2 = 4$ nor the ellipse $x^2 + 6y^2 = 6$, then the range of c is

a) [-5, 5] b) $(-\infty, -5) \cup (5, \infty)$ c) (-4, 4) d) none of these

Ans. **b**

Sol. Since the given line does not meet the given ellipse and circle.

$c^2 > 6 \times 2^2 + 1$ [From $c^2 > a^2m^2 + b^2$]

and $c^2 > 4(1 + 4)$ [From $c^2 > a^2(1 + m^2)$]

$\Rightarrow c^2 > 25$

$\therefore c \in (-\infty, -5) \cup (5.\infty)$

93. If the eccentricity of the hyperbola $x^2 \cos ec^2\theta - y^2 \sec^2\theta = 5$ is 5 times the eccentricity of the ellipse $x^2 \cos ec^2\theta + y^2 \sec^2\theta = 25$, then θ =

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\cot^{-1}\left(\frac{\pm 2}{\sqrt{3}}\right)$ d) $\tan^{-1}\left(\frac{4}{5}\right)$

Ans. c

Sol. $\frac{x^2}{5\sin^2\theta} - \frac{y^2}{5\cos^2\theta} = 1, \ \frac{x^2}{25\sin^2\theta} + \frac{y^2}{25\cos^2\theta} = 1$

$e_H^2 = 1 + \cot^2\theta$

$e_e^2 = 1 - \cot^2\theta$

$1 + \cot^2\theta = 5(1 - \cot^2\theta)$

$6\cot^2\theta = 4$ = $\cot^2\theta = \frac{2}{3}$

$\cot\theta = \pm\sqrt{\frac{2}{3}}$

$\theta = \cot^{-1}\left(\frac{+2}{\sqrt{3}}\right)$

94. Area enclosed by ellipse $x^2 + \sin^4\alpha\, y^2 = \sin^2\alpha$, $\alpha \in \left(0, \frac{\pi}{2}\right)$ is

a) 2π b) π c) 1 d) none of these

Ans. **b**

Sol. Area = πab – π sinα cosecα = π.

95. Find the eccentricity of the conic formed by the locus of the point of intersection of the lines $\sqrt{3}x - y - 4\sqrt{3}k = 0$ and $\sqrt{3}kx + ky - 4\sqrt{3} = 0$

a) 4 b) 1/4 c) 2 d) 1/2

Ans. c

Sol. $\sqrt{3}x - y - 4\sqrt{3}k = 0, \ \sqrt{3}kx + ky - 4\sqrt{3} = 0$

In order to find the locus of point of intersection

We have to eliminate k

$\therefore \frac{\sqrt{3}x - y}{4\sqrt{3}} = k$ put this k in another

$(\sqrt{3}x - y)(\sqrt{3}x + y) = (4\sqrt{3})^2$

or $3x^2 - y^2 = 48$

$\frac{x^2}{16} - \frac{y^2}{48} = 1 \quad \therefore a^2 = 16, b^2 = 48$

Clearly locus form a hyperbola.

$b^2 = a^2(e^2 - 1)$

$48 = 10(e^2 - 1)$

$\therefore$ e = 2

96. If a pair of variable straight lines $x^2+4y^2+\alpha xy=0$ (where α is a real parameter) cut the ellipse $x^2+4y^2=4$ at two points A and B, then locus of point of intersection of tangents at A and B is

a) $x^2-4y^2+8xy=0$ b) $(2x-y)(2x+y)=0$

c) $x^2-4y^2+4xy=0$ d) $(x-2y)(x+2y)=0$

Ans. **d**

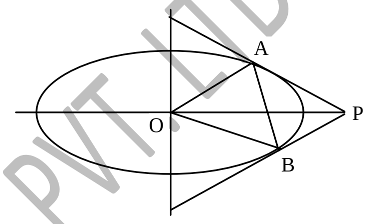
Sol. Let the point of intersection of tangents at A and B be P(h, k) then equation of AB is

$$\frac{xh}{4}+\frac{yk}{1}=1 \qquad - \qquad (1)$$

Homogenizing the ellipse with (1)

$$\frac{x^2}{4}+\frac{y^2}{1}=\left(\frac{xh}{4}+\frac{yk}{1}\right)^2$$

$$\Rightarrow x^2\left(\frac{h^2-4}{16}\right)+y^2\left(k^2-1\right)+\frac{2hk}{4}xy=0 \qquad - \qquad (2)$$



Given, equation of OA and OB is

$x^2+4y^2+\alpha xy=0$ - (3)

(2) and (3) are same

$\Rightarrow$ (h – 2k) (h + 2k) = 0

Therefore locus is (x – 2y) (x + 2y) = 0

97. A man starts from point P(-3, 4) and reaches the point Q(0, 1) touching x-axis at R, such that PR + RQ is minimum, then the coordinates of point R is

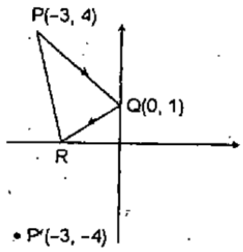
a) $\left(-\frac{3}{5},0\right)$ b) (1, 0) c) (- 1, 0) d) $\left(\frac{3}{5},0\right)$

Ans. **a**

Sol. Let $P'(-3,-4)$ be the image of P with respect to x-axis PR + RQ minimum

$\Rightarrow P'R+RQ$ is minimum

$\Rightarrow P'RQ$ should be collinear



98. Let A, B and C are any three points on the ellipse $36x^2+\frac{y^2}{192}=1$, then the maximum area of the triangle ABC is

a) 1 b) 2 c) 3 d) 4

Ans. **c**

Sol. Area of the triangle inscribed in the ellipse is maximum in difference of the eccentric angles of the point A, B, C is $\frac{2\pi}{3}$

So maximum area of the inscribed triangle is $\frac{3\sqrt{3}}{4}\cdot\frac{1}{6}\cdot 8\sqrt{3}=3\,sq.units$

99. If $P(x_1,y_1), Q(x_2,y_2), R(x_3,y_3)$ and $S(x_4,y_4)$ are four concyclic points on the rectangular hyperbola $xy=c^2$, then the coordinates of the orthocenter of ΔPQR are

a) $(x_4,-y_4)$ b) (x_4,y_4) c) $(-x_4,-y_4)$ d) $(-x_4,y_4)$

Ans. c

Sol. Orthocentre and (x_4, y_4) are the images of each other with respect to the origin.

100. If two of the lines given by $3x^3 + 3x^2y - 3xy^2 + dy^3 = 0$ are at right angled, then the slope of the third line is

a) – 1 b) 1 c) 3 d) – 3

Ans. a

Sol. Let the lines be $y = m_1x$, $y = m_2x$, $y = m_3x$

$$\therefore m_1m_2m_3 = -\frac{3}{d}$$

Let $m_1m_2 = -1$ (two of the lines are perpendicular)

$$\therefore m_3 = \frac{3}{d}$$

$y = \frac{3}{d}x$ satisfying given equation

$$\Rightarrow d\left(\frac{3}{d}\right)^3 - 3\left(\frac{3}{d}\right)^2 + 3\left(\frac{3}{d}\right) + 3 = 0$$

$$\Rightarrow d = -3$$

$\therefore$ The given equation $x^3 + x^2y - xy^2 - y^3 = 0$

$$\Rightarrow (x+y)\left(x^2 - y^2\right) = 0$$

$\therefore$ slopes of other 2 lines are 1, - 1

101. If the angle between tangents drawn to $x^2 + y^2 - 6x - 8y + 9 = 0$ at the points where it is cut by the line $y = 3x + k$ is $\frac{\pi}{2}$, then

a) $k = -5 \pm 2\sqrt{5}$ b) $k = -5 \pm 3\sqrt{5}$ c) $k = 2\sqrt{5} + \sqrt{2}$ d) none of these

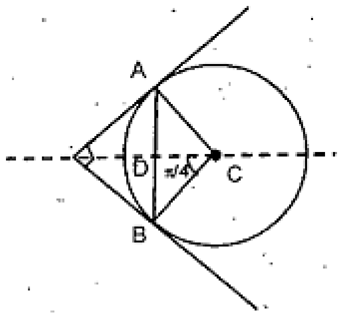
Ans. a

Sol. $CD = CB\cos\frac{\pi}{4} = \sqrt{2}$

$$\sqrt{2} = \frac{|4-9-k|}{\sqrt{1^2+3^2}} = \frac{|-5-k|}{\sqrt{10}}$$

$$20 = (5+k)^2$$

$$\Rightarrow k = -5 \pm 2\sqrt{5}$$



102. If directions of two sides of a triangle are fixed and length of third side is constant and is sliding between these sides, then locus of the orthocenter of the triangle is

a) circle b) ellipse c) straight line d) hyperbola

Ans. a

Sol. Let fixed directions be OA and OB inclined at a constant angle α and AB = c.

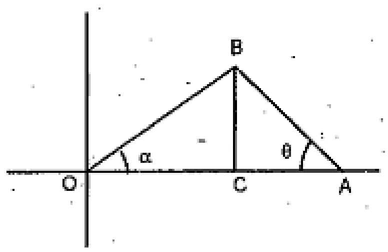
Let $\angle BAO = \theta$ then BC = $c\sin\theta$ and AC = $c\cos\theta$.

$$\therefore OC = c\sin\theta \cdot \cot\alpha$$

Equation of the line passing through A and perpendicular to OB is $y = -\cot\theta(x - c\sin\theta\cot\theta - c\cos\theta)$ and equation of BC is $x = c\sin\theta\cdot\cot\alpha$

$\therefore$ orthocenter is $(c\sin\theta\cdot\cot\theta,\ c\cos\theta\cdot\cot\alpha)$

$\Rightarrow$ Required locus is $x^2 + y^2 = c^2\cot^2\alpha$, which is the



equation of a circle.

103. The number of triangles having two vertices are (1, 2) and (6, 2) and incentre (4,6) is

a) 2 b) 1 c) infinite d) 0

Ans. **d**

Sol. Equation of BC is y = 2, which is parallel to x-axis

$\therefore \tan\frac{B}{2}=\frac{4}{3}\Rightarrow B>\frac{\pi}{2}$ and $\tan\frac{C}{2}=2\Rightarrow C>\frac{\pi}{2}$

In a triangle two angles cannot be greater than 90^0 and hence there is no such triangle.