Date: 9/10/2021

Q1. Find the radian measures corresponding to each of the following degree measures?

(a) $1°$ (b) $1'$

Q2. Find the radian measure corresponding to $40°20'$.

Q3. Represent $10°$ into radian measure.

Q4. If $f(\theta) = 30°$ and $g(\theta) = 30'$, let $h(\theta) = f(\theta) + g(\theta)$. Represent $h(\theta)$ into degree measure.

Q5. Find the degree measure corresponding to $\frac{\pi}{32}$ rad.

Q6. Find the degree measure of $\frac{5\pi}{36}$ Rad.

Q7. If $f(\theta) = 10''$. Represent $f(\theta)$ into minutes.

Q8. If $f(\theta) = 30°$ and $g(\theta) = \frac{2\pi}{3}$ Rad. and $h(\theta) = f(\theta) + g(\theta)$, represent $h(\theta)$ into radian.

Q9. If $f(\theta) = 30°$ and $g(\theta) = \frac{2\pi}{3}$ radius such that $h(\theta) = f(\theta) + g(\theta)$, represent $h(\theta)$ into degree measure.

Q10. Find the angle in radian subtended at the centre of the circle by an arc whose length is 2.2 times the radius.

Q11. If the angular diameter of the moon be $30'$, how far from the eye a coin of diameter 2.2 cm be kept to hide the moon?

Q12. Find the radius of the circle in which a central angle of $60°$ intercepts an arc of length 37.4 cm (Use $\pi = 22/7$).

Q13. The minute hand of a watch is 1.5 cm long. How far does its tip move in 40 minutes? (Use $\pi = 3.14$).

Q14. A wheel makes 360 revolutions in one minute. Through how many radians does it turn in one second?

Q15. Find the radian measures corresponding to the following degree measures:

(i) $240°$, (ii) $520°$.

Q16. Find the degree measures corresponding to the following radian measures (Use $\pi = 22/7$).

(i) $\frac{11}{16}$ (ii) -4

Q17. Find the degree measures corresponding to the following radian measures (Use $\pi = 22/7$).

(i) $\frac{5\pi}{3}$ (ii) $\frac{7\pi}{6}$

Q18. Convert $40°\ 20'$ into radian measure.

Q19. Find the Radian measures corresponding to following degree measures:

(a) $25°$ (b) $-47°30'$

Q20. Find the radian measure corresponding to each of the following degree measures.

(a) $15°$ (b) $-37°30'$

Q21. Find the radian measure of $47°30'$.

Q22. Convert 6 radian into degree measure.

Q23. Find the length of an arc of a circle of radius 3 cm, if the angle subtended at the centre is $30°$. ($\pi = 3.14$).

Q24. If $f(\theta) = 30'$ and $g(\theta) = -30°$. Let $h(\theta) = f(\theta) + g(\theta)$, represent $h(\theta)$ into degree.

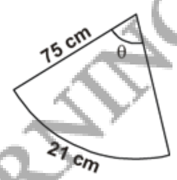
Q25. If a circle of diameter 10 cm and a chord is of length 5 cm, find the length of the minor arc of the chord.

Q26. Find the length of a arc of an unit circle subtending a central angle measuring $\frac{5\pi}{8}$.

Q27. Find the length of an arc of a unit circle if the angle subtended at the centre is $60°$.

Q28. Find the length of an arc of a circle of radius 6 cm if the angle subtended at the centre is $\left(\frac{\pi}{2}\right)$, ($\pi = 3.14$)

Q29. Find angle in radian through which a pendulum swings and its length is 75 cm and tip describes an arc of length 21 cm.



Q30. If a circle of diameter 40 cm, a chord is 20 cm find the length of the minor arc of the chord.

Q31. A train is travelling on a curved of 700 m radius at 14 km/h through what angle will it turn in 1 minute.

Q32. What is the ratio of radii of two circles at the centre of which two arc of same length subtend angles $60°$ and $75°$?

Q33. Find in degrees and radians, the angle between the hour hand and the minute hand of a clock at half past three.

Q34. In a right angled triangle the difference between two acute angles is $\left(\frac{\pi}{6}\right)^c$. Find the angles in degrees.

Q35. Find the radius of a circle in which a central angle of $72°$ intercepts an arc of length 22 cm. $\left(\text{Use } \pi = \frac{22}{7}\right)$.

Q36. The moons distance from the earth is 360000 km and its diameter subtends an angle of $31'$ at the eye of the observer on the earth, find the radius of the moon.

Q37. Find the angle between the minute hand and hours hand of a clock when the time is 7 : 20.

Q38. Find the angle between minute hand and the hours hand of a clock when the time is 10:10.

Q39. A wheel makes 360 revolutions in 1 minute. Through how many radians does it turn in 1 second.

Q40. The angles of a triangle are in A.P. and ratio of number of degrees in the least to the number of radians in the greatest in 60 π, find angles in degrees and radians.

Q41. The angles of a triangle are in A.P. and the greatest angle is double the least. Find all the angles in degrees and radians.

Q42. Express $10'$ into radian measure.

Q43. Find in degrees the angle subtended at the centre of a circle of radius 25 cm by an arc of length 11 cm.

Q44. The minute hand of a watch is 1.4 cm long. How for does it tip move in 45 minute. $\left(\pi = \frac{22}{7}\right)$.

Q45. A wheel makes 180 revolutions in 1 minute through how many radian does it turn in 30 second.

Q46. Convert $1''$ into radian measure.

Q47. Convert $-10°10'$ into radian measure.

Q48. Express $48°37'30''$ into radian.

Q49. Express $10''$ into radian measures.

Q50. The minute hand of a watch is 2.1 cm long. How far does it tip move in 15 minutes? $\left(\text{Use } \pi = \frac{22}{7}\right)$.

Q51. If the arcs of the same lengths in two circles subtend angles 65° and 110° at the centre, find the ratio of their radii.

Q52. A horse is tied to a post by a rope. If the horse moves along a circular path, always keeping the rope tight and describes 88 meters when it traces 72° at the centre, find the length of the rope.

Q53. The difference between the two acute angles of a right triangle is $\left(\frac{\pi}{5}\right)^c$. Find these angles in radians and degrees.

Q54. Find the degree measure of the angle subtended at the centre of a circle of radius 100 cm by an arc of length 22 cm (Use π = 22/7).

Q55. Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ in each of the following: $\tan x = -\frac{4}{3}$, x in quadrant **II**.

Q56. Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ in each of the following: $\sin x = \frac{1}{4}$, x in quadrant **II**.

Q57. Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ in each of the following: $\cos x = -\frac{1}{3}$, x in quadrant **III**.

Q58. If in two circles, arcs of the same length subtend angles 60° and 75° at the centre, find the ratio of their radii.

Q59. Find the angle in radian through which a pendulum swings if its length is 75 cm and the tip describes an arc of length. (i) 10cm, (ii) 15 cm, (iii) 21 cm.

Date: 9/10/2021

S1. (a)

$$180° = \pi \quad \Rightarrow \quad 1° = \left(\frac{\pi}{180}\right)^c$$

Hence, $$1° = \left(\frac{\pi}{180}\right)^c$$

(b) $$1' = \left(\frac{1}{60}\right)^\circ$$

$\therefore$ Radian measure of $1' = \left(\frac{\pi}{180} \times \frac{1}{60}\right)^c$.

S2. $\because$ $$1' = \left(\frac{1}{60}\right)^\circ$$

$\therefore$ $$20' = \left(\frac{20}{60}\right)^\circ = \left(\frac{1}{3}\right)^\circ$$

Now, total degree to convert

$$= \left(40 + \frac{1}{3}\right)^\circ = \left(\frac{121}{3}\right)^\circ$$

$\therefore$ Radian measure = $\left(\frac{\pi}{180}\right) \times$ degree measure.

$\therefore$ Required Radian measure

$$= \frac{\pi}{180} \times \frac{121}{3} = \frac{121\pi}{540}$$

S3. Required radian measure

$$= \left(\frac{\pi}{180}\right) \times \text{degree measure}$$

$$= \left(\frac{\pi}{180} \times 10^\circ\right) \text{radian} = \left(\frac{\pi}{18}\right)^\circ$$

S4. $\because$ $f(\theta) = 30°$ (Degree measure)

$g(\theta) = 30'$

$\because$ $1' = \left(\frac{1}{60}\right)^\circ$ $\quad \therefore \quad 30' = \left(\frac{1}{60} \times 30\right)^\circ$

$$= \left(\frac{1}{2}\right)^\circ$$

$\therefore \quad h(\theta) = f(\theta) + g(\theta) = 30^\circ + \left(\frac{1}{2}\right)^\circ$

$$= \left(\frac{61}{2}\right)^\circ$$

S5. Required degree measure

$$= \left(\frac{180}{\pi}\right) \times \text{radian measure}$$

$$= \frac{180}{\pi} \times \frac{\pi}{32} = \left(5\frac{5}{8}\right)^\circ$$

S6. Required degree measure

$$= \left(\frac{180}{\pi}\right) \times \text{radian measure}$$

$$= \frac{\overset{5}{\cancel{180}}}{\pi} \times \frac{5\pi}{\underset{1}{\cancel{36}}} = 25^\circ$$

S7. $\because \quad 60'' = 1'$

$\therefore \quad 1'' = \left(\frac{1}{60}\right)'$

$\therefore \quad 10'' = \left(\frac{1}{60} \times 10\right)' = \left(\frac{1}{6}\right)'$

S8. $\because \quad f(\theta) = 30^\circ$

$\therefore$ Required radian measure for $f(\theta)$

$$= 30 \times \frac{\pi}{180} = \frac{\pi}{6} \text{ rad.}$$

$\therefore \quad h(\theta) = f(\theta) + g(\theta) = \frac{\pi}{6} + \frac{2\pi}{3}$

$$= \frac{\pi + 4\pi}{6} = \frac{5\pi}{6} \text{ rad.}$$

S9. $f(\theta) = 30^\circ$

$$g(\theta) = \frac{2\pi}{3} \text{ rad} = \frac{2\overset{1}{\cancel{\pi}}}{\cancel{3}} \times \frac{\overset{60}{\cancel{180}}}{\underset{1}{\cancel{\pi}}} = 120^\circ$$

$\therefore \qquad h(\theta) = f(\theta) + g(\theta)$

$= 30° + 120° = 150°$

S10. $\because \qquad \theta = \frac{l}{r}$, θ is in radian

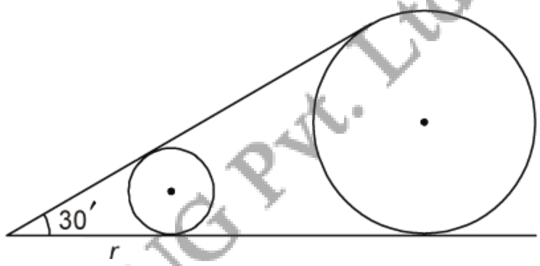
$\because \qquad l = 2.2\,r$

$\therefore \qquad \theta = \frac{2.2r}{r} = 2.2$

Hence, $\qquad \theta = 2.2$ radian.

S11. $\theta = 30' \quad l = 2.2$ cm, $\quad r = ?$ $\qquad 1' = \left(\frac{1}{60}\right)^\circ$

$\theta = 30' = \frac{30}{60} \times \frac{\pi}{180} = \frac{\pi}{360}$ Rad.



$\therefore \qquad \frac{\pi}{360} = \frac{2.2}{r} \quad \Rightarrow \quad r = \frac{22 \times 360 \times 7}{22 \times 10}$

$= 252$ cm

$\therefore \qquad r = 252$ cm

S12. Here $l = 37.4$ cm and $\theta = 60° = \frac{60\pi}{180}$ radian $= \frac{\pi}{3}$.

Hence, $\qquad$ by $r = \frac{l}{\theta}$, we have

$$r = \frac{37.4 \times 3}{\pi} = \frac{37.4 \times 3 \times 7}{22} = 35.7 \text{ cm}.$$

S13. In 60 minutes, the minute hand of a watch completes one revolution. Therefore in 40 minutes, the minute hand turns through $\frac{2}{3}$ of a revolution. Therefore, $\theta = \frac{2}{3} \times 360°$ or $\frac{4\pi}{3}$ radian. Hence, the required distance travelled is given by

$$l = r\theta = 1.5 \times \frac{4\pi}{3} \text{ cm} = 2\pi \text{ cm}$$

$$= 2 \times 3.14 \text{ cm} = 6.28 \text{ cm}.$$

S14. Given that a wheel makes 360 revolutions in one minute.

$\therefore$ In 1 second the wheel makes $\frac{360}{60}$ revolutions.

Now, in each revolution the wheel completes $360° = 2\pi$ radians $\qquad$ [As $180° = \pi$ radians]

$\therefore$ Required radians in one second

$$= \frac{360}{60} \times 2\pi = 12\pi.$$

S15. (i) Here,

$$240° = 240 \times \frac{\pi}{180} = \frac{4\pi}{3} \text{ radians.}$$

(ii)

$$520° = 520 \times \frac{\pi}{180} = \frac{26\pi}{9} \text{ radians.}$$

S16.

$$\pi \text{ radians} = 180°$$

$$\Rightarrow \quad 1 \text{ radian} = \left(\frac{180}{\pi}\right)^{\circ}$$

(i)

$$\frac{11}{16} \text{ radians} = \left(\frac{11}{16} \times \frac{180}{\pi}\right)^{\circ}$$

$$= \left(\frac{11}{16} \times \frac{180 \times 7}{22}\right)^{\circ} = \left(\frac{315}{8}\right)^{\circ} = 39° \, 22' \, 30''.$$

(ii)

$$-4 \text{ radians} = \left(-4 \times \frac{180}{\pi}\right)^{\circ}$$

$$= -\left(\frac{4 \times 180 \times 7}{22}\right)^{\circ} = -\left(\frac{2520}{11}\right)^{\circ} = -229° \, 5' \, 27'' \text{ (nearly).}$$

S17. (i)

$$\frac{5\pi}{3} \text{ radians} = \left(\frac{5\pi}{3} \times \frac{180}{\pi}\right)^{\circ}$$

$$= \left(\frac{5 \times 180}{3}\right)^{\circ} = 300°.$$

(ii)

$$\frac{7\pi}{6} \text{ radians} = \left(\frac{7\pi}{6} \times \frac{180}{\pi}\right)^{\circ}$$

$$= \left(\frac{7 \times 180}{6}\right)^{\circ} = 210°.$$

S18. We know that $180' = \pi$ radian.

Hence,

$$40° \, 20' = 40\frac{1}{3} \text{ degree} = \frac{\pi}{180} \times \frac{121}{3} \text{ radian} = \frac{121\pi}{540} \text{ radian}$$

Therefore,

$$40° \, 20' = \frac{121\pi}{540} \text{ radian.}$$

S19. (a) We know that

$$\text{Radian measure} = \frac{\pi}{180} \times \text{degree measure}$$

$\therefore$ Required radian measure

$$= \frac{\pi}{180} \times 25 = \frac{5\pi}{36}$$

(b) We have

$$= -\left[47^\circ + \frac{30}{60}\right)^\circ\Bigg] \qquad \because \quad 1' = \left(\frac{1}{60}\right)^\circ$$

$$= -\left[47^\circ + \frac{1^\circ}{2}\right) = -\left[\frac{95}{2}\right]^\circ$$

Now required radian measure

$$= \frac{\pi}{180} \times \text{degree measure}$$

$$= -\left(\frac{\pi}{180} \times \frac{95}{2}\right) = \frac{-19\pi}{72}$$

Hence, Radian measure of $(-47^\circ 30')$ is $\frac{-19\pi}{72}$.

S20. (a)

$$\because \qquad 180^\circ = \pi \quad \Rightarrow \quad 1^\circ = \left(\frac{\pi}{180}\right)^c$$

$$\Rightarrow \qquad 15^\circ = \left(\frac{\pi}{180} \times 15\right)^c = \left(\frac{\pi}{12}\right)^c$$

Hence, $15^\circ = \left(\frac{\pi}{12}\right)^c$

(b)

$$180^\circ = \pi^c \quad \Rightarrow \quad 1^\circ = \left(\frac{\pi}{180}\right)^c$$

$$\Rightarrow \qquad -37^\circ 30' = \left\{\frac{\pi}{180} \times \left(\frac{-75}{2}\right)\right\}^c$$

$$\because \qquad \left[-37^\circ 30' = -\left(37\frac{1}{2}\right)^\circ = -\left(\frac{75}{2}\right)^\circ\right]$$

$$= \left(\frac{-5\pi}{24}\right)^c$$

Hence, $(-37^\circ 30') = \left(\frac{-5\pi}{24}\right)^c$.

S21. We know that

$$\text{Radian measure} = \frac{\pi}{180} \times \text{degree measure.}$$

$\therefore$ Required radian measure = ?

We have, $47°30' = \left[47° + \left(\frac{30}{60}\right)^{\circ}\right]$

$\because$ $1' = \left(\frac{1}{60}\right)^{\circ}\Bigg]$

$$= \left[47° + \frac{1°}{2}\right] = \left(\frac{95}{2}\right)^{\circ}$$

Hence, radian measure of 47°30′ = ?

Now required radian measure

$$= \left(\frac{\pi}{180} \times \frac{95}{2}\right) = \frac{19\pi}{72}$$

Hence, radian measure of 47°30′ is $\frac{19\pi}{72}$.

S22. We know that degree measure

$$= \frac{180}{\pi} \times \text{radian measure.}$$

$\therefore$ Required degree measure

$$= \frac{180}{\pi} \times 6$$

$\because$ $\pi = \frac{22}{7}$

$\therefore$ Required degree measure

$$= \frac{1080 \times 7}{22} \text{ degree}$$

$$= 343\frac{7}{11} \text{ degree}$$

$$= 343° + \frac{7}{11} \times 60 \text{ min}$$

$$= 343° + 38\frac{2}{11} \text{ min} \qquad (1' = 60'')$$

$$= 343° + 38' + \frac{2}{11} \text{ min}$$

$$= 343° + 38' + \frac{2}{11} \times 60''$$

$$= 343°38'11'' \text{ (Approx.)}$$

Hence, 6 Radian = 343°38′11″ (Approx.)

S23. Let l be the length of the arc

$$\because \quad \text{Angle } \theta = \frac{l}{r}$$

Where θ is in radian.

$$\therefore \quad r = 3 \text{ cm}$$

$$\theta = 30^\circ = 30 \times \frac{\pi}{180} = \frac{\pi}{6} \text{ rad}$$

On putting the values of r and θ, we get

$$\frac{\pi}{6} = \frac{l}{3} \quad \Rightarrow \quad l = \frac{\pi}{2} = \frac{3.14}{2} = 1.57 \text{ cm}$$

S24. Step-1:

Convert minutes into degree

$$\therefore \quad 1' = \left(\frac{1}{60}\right)^\circ$$

$$\therefore \quad 30' = \left(\frac{1}{60} \times 30\right)^\circ = \left(\frac{1}{2}\right)^\circ$$

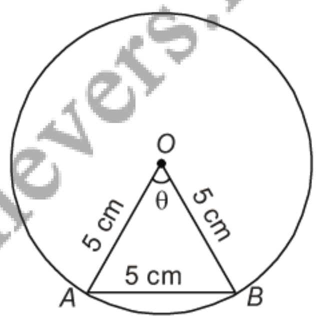
$$\therefore \quad h(\theta) = f(\theta) + g(\theta) = \frac{1^\circ}{2} - 30^\circ = -29\frac{1^\circ}{2}$$

S25. $\because \quad 2r = 10 \text{ cm}$

$\Rightarrow \quad r = 5 \text{ cm}$

where r is radius of the circle.

Thus, ΔOAB is an equilateral triangle.



$$\therefore \quad \theta = 60^\circ = 60 \times \frac{\pi}{180} = \frac{\pi}{3} \text{ rad.}$$

We know that $\theta = \dfrac{\text{Arc . AB}}{\text{radius}}$

$$\therefore \quad \text{Arc } AB = \theta \times r = \frac{\pi}{3} \times 20 = \frac{20\pi}{3} \text{ cm}$$

S26. Let l be the length of the arc.

$$\therefore \quad \text{Angle } \theta = \frac{l}{r}$$

Where θ is in radian.

$$r = 1 \text{ cm}, \quad \theta = \frac{5\pi}{8}$$

$\therefore$ $$\frac{5\pi}{8} = \frac{l}{1} \Rightarrow l = \frac{5\pi}{8} = \frac{5 \times 3.14}{8}$$

$$= \frac{15.70}{8} = 1.9625 \text{ cm}$$

Hence, length of the arc is 1.9625 cm

S27. Since unit circle is a circle having radius 1 unit.

Let l be the length of the arc.

$\therefore$ $$\text{Angle } \theta = \frac{l}{r}$$

Where θ is in radian.

$$r = 1 \text{ cm}$$

$$\theta = 60° = \left(60 \times \frac{\pi}{180}\right) \text{ rad.} = \left(\frac{\pi}{3}\right) \text{ rad.}$$

$\because$ $$\frac{\pi}{3} = \frac{l}{1} \Rightarrow l = \frac{\pi}{3} = 1.04 \text{ cm}$$

S28. Let r be the length of the arc

$\therefore$ $$\text{Angle } \theta = \frac{l}{r}$$

$\therefore$ θ is in radian $\quad \because$ $\theta = \left(\frac{\pi}{2}\right)$

Given, $r = 6$ cm, $\theta = \frac{\pi}{2}$

On putting the values of r and θ, we get

$$\frac{\pi}{2} = \frac{l}{6} \quad , \quad l = 3\pi \quad , \quad l = 9.42 \text{ cm}$$

S29. Given length of Pendulum = 75 cm

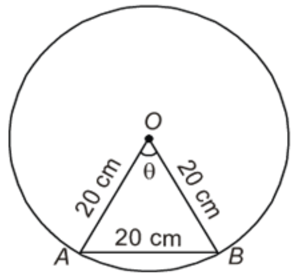
Radius (r) = length of pendulum = 75 cm

Length of arc (l) = 21 cm

Now, $$\theta = \frac{l}{r} = \frac{21}{75} = \frac{7}{25} \text{ rad.}$$

Hence, angle in radian measure is $\frac{7}{25}$ rad.

S30. Let r be the radius of circle



$\because$ Diameter $(2r) = 40$ cm

$\therefore$ $r = 20$ cm

and length of the chord $AB = 20$ cm

Thus, triangle OAB is an equilateral triangle.

$$\therefore \quad \theta = 60^\circ \times \frac{\pi}{180} = \frac{\pi}{3} \text{ rad.}$$

We know that $$\theta = \frac{\text{Arc . AB}}{\text{radius}}$$

$$\therefore \quad \text{Arc } AB = \theta \times r = \frac{\pi}{3} \times 20$$

$$= \frac{20}{3}\pi \text{ cm.}$$

S31. $r = 700$ m, speed = 14 km/hr

l = distance covered in 1 minute

$$= \frac{1400}{60} \text{ m} = \frac{700}{3} \text{ m}$$

$$\therefore \quad \theta = \frac{l}{r} = \frac{700}{3} \times \frac{1}{700} = \frac{1^C}{3}$$

$$\therefore \quad \theta = \frac{1^C}{3}$$

S32. Let the radii be r_1 and r_2.

$$\therefore \quad \theta_1 = 60^\circ = \left(60 \times \frac{\pi}{180}\right) = \frac{\pi}{3} \text{ radians.}$$

$$\theta_2 = 75^\circ = 75 \times \frac{\pi}{180} \text{ radian}$$

$$\therefore \quad \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1}$$

$\because$ (arcs are of same length)

$$\therefore \quad \frac{r_1}{r_2} = \frac{75\pi}{180} \times \frac{3}{\pi}$$

$$\therefore \quad \frac{r_1}{r_2} = \frac{5}{4}.$$

S33. The angle traced by the hour hand in $12h = 360°$

$\therefore$ The angle traced by hour hand in $3h$ 30 min.

$$= \left(\frac{360}{12} \times \frac{7}{2}\right)^{\circ} = 105°$$

$\because$ $\left(3h\ 30\ \text{min} = \frac{7}{2}h\right)$

The angle traced by minute hand in 60 minute = 360°.

$\therefore$ The angle traced by minute hand in 30 min.

$$= \left(\frac{360}{60} \times 30\right)^{\circ} = 180°$$

Hence, the required angle between two hands

$$= 180° - 105° = 75°$$

$$= \frac{5\pi}{12} \text{ rad.}$$

S34. Clearly the sum of two acute angles of a right triangle is 90°.

$\because$ $\left(\frac{\pi}{6}\right)^{c} = 30°$

$\therefore$ Let the two acute angles be $x°$ and $y°$.

$\therefore$ $x + y = 90$ ... (i)

$x - y = 30$ ... (ii)

Solving Eq. (i) and (ii),

$\therefore$ $x = 60°, y = 30°$.

S35. Let the required radius be r, then

$$l = 22 \text{ cm}, \quad \theta = 72 = \left(72 \times \frac{\pi}{180}\right)^{c}$$

$$= \left(\frac{2\pi}{5}\right)^{c}$$

Now, $\theta = \frac{l}{r} \Rightarrow r = \frac{l}{\theta}$

$\therefore$ $r = \left(22 \times \frac{5}{2\pi}\right)$ cm

$$= \left(22 \times \frac{5}{2} \times \frac{7}{22}\right) \text{cm} = \frac{35}{2} \text{ cm} = 17.5 \text{ cm}$$

Hence, radius of the circle is 17.5 cm.

S36.

$$31' = \frac{31}{60} = \frac{\pi}{180} \times \frac{31}{60} \text{ radian}$$

$\therefore$ $$\text{Diameter} = \frac{\pi}{180} \times \frac{31}{60} \times 360000 \text{ km}$$

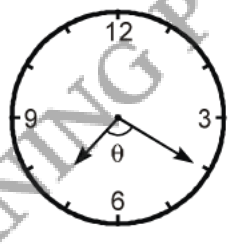
$\therefore$ $$\text{Radius} = \frac{1}{2} \times \frac{\pi}{180} \times \frac{31}{60} \times 360000 \text{ km}$$

$$= \frac{31 \times 300}{18} = \frac{1550}{3}$$

$$= 516.66 \text{ km}$$

$\therefore$ Radius of the moon is 516.66 km.

S37. $\because$ Angle traced by hour hand in 12 hours = 360°.



$\therefore$ Angle traced in 7 hours 20 minutes.

$\Rightarrow$ $\frac{22}{3}$ hours

$$= \left(\frac{360}{12} \times \frac{22}{3}\right)^\circ = 220^\circ$$

Angle traced by minute hand in 60 min. = 360°

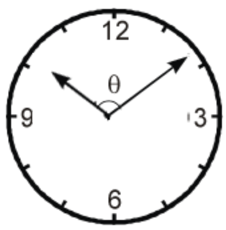
$\therefore$ Angle traced by minute hand in 20 min = $\left(\frac{360}{60} \times 20\right)^\circ = 120^\circ$

Hence, required angle between two hands = (220° – 120°) = 100°

S38. Angle traced by the hour hand in 12 hours = 360°.

$\therefore$ Angle traced by it in $10h$ 10 min.

$$= \left(\frac{360}{12} \times \frac{61}{6}\right)^\circ = 305^\circ$$



$\because$ $$10h \text{ 10 min} = 10 + \frac{10}{60} = 10 + \frac{1}{6} = \frac{61}{6} h$$

Angle traced by minute hand in 10 min = ?

$\because$ Angle traced by minute hand in 60 min

$$= 360^\circ$$

$\therefore$ Angle traced by minute hand in 10 min

$$= \left(\frac{360}{60} \times 10\right)^\circ = 60^\circ$$

$\therefore$ Hence required angle between two hands.

$$= 305^\circ - 60^\circ = 245^\circ.$$

S39. $\because$ Number of revolutions made in 60 seconds

$$= 360$$

Number of revolution made in 1 second

$$= \frac{360}{60} = 6$$

Angle moved in 1 revolution

$$= (2\pi)^C$$

$\therefore$ Angle moved in 6 revolutions

$$= (2\pi \times 6)^C = (12\pi)^C$$

S40. Let the angles of the triangle be $(a - d)^\circ$, a°, $(a + d)^\circ$ then

$$(a - d) + a + (a + d) = 180^\circ \quad \Rightarrow \quad a = 60$$

Thus the angles are $(60 - d)^\circ$, 60°, and $(60 + d)^\circ$.

Number of degrees in the least degree = $(60 - d)^\circ$.

$\therefore$ Number of radians in greatest angle = $\left\{(60 + d) \times \frac{\pi}{180}\right\}$.

$$\Rightarrow \quad \frac{(60 - d)}{(60 - d) \times \frac{\pi}{180}} = \frac{60}{\pi}$$

$$\Rightarrow \quad d = 30$$

$\therefore$ The required angles are 30°, 60°, and 90°.

These angles can be represented in radians as $\left(\frac{\pi}{6}\right)^C$, $\left(\frac{\pi}{3}\right)^C$, $\left(\frac{\pi}{2}\right)^C$ respectively.

S41. $\because$ Let the required angles be $(a - d)^\circ$, a°, $(a + d)^\circ$.

$\because$ Then, the sum of the angles of a triangle is being 180°, we have

$$(a - d) + a + (a + d) = 180^\circ \quad \Rightarrow \quad a = 60^\circ$$

Also, $\quad (a + d) = 2(a - d) \quad \Rightarrow \quad a = 3d$

$$\Rightarrow \quad d = 20$$

Hence, the angles in degrees are 40°, 60°, 80°.

∴ The angles in radians are $\left(\frac{2\pi}{9}\right)^C, \left(\frac{\pi}{3}\right)^C, \left(\frac{4\pi}{9}\right)^C$ respectively.

S42. **Step-1:** Convert minutes into degree

∵ $$1' = \left(\frac{1}{60}\right)^\circ$$

∴ $$10' = \left(\frac{1}{60}\times 10\right)^\circ = \left(\frac{1}{6}\right)^\circ$$

Step-2: Convert degree into radian.

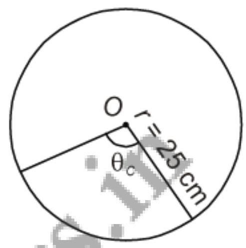
Degree obtained from Step-1 = $\left(\frac{1}{6}\right)^\circ$

∴ Required radian measure

$$= \frac{\pi}{180}\times \text{degree measure} = \left(\frac{\pi}{180}\times\frac{1}{6}\right)\text{radian} = \left(\frac{\pi}{1080}\right)\text{rad.}$$

S43. Here, $r = 25$ cm and $l = 11$ cm

Let the measure of required angle be θ^C.



Then, $$\theta^C = \left(\frac{l}{r}\right)^C = \left(\frac{11}{25}\right)^C$$

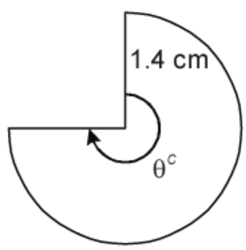
$$= \left(\frac{11}{25}\times\frac{180}{\pi}\right)^\circ \quad (\because \pi^C = 180^\circ)$$

$$= \left(\frac{11}{25}\times\frac{7}{22}\times 180\right)^\circ = \left(\frac{126}{5}\right)^\circ$$

Hence, the required angle is 25°12′.

S44. In 60 minutes the minute hand moves through $(2\pi)^C$.

In 45 minutes, the minute hand moves through



$$\left(\frac{2\pi}{60}\times 45\right)^C = \left(\frac{3\pi}{2}\right)^C$$

∴ $$r = 1.4\text{ cm}, \quad \theta = \left(\frac{3\pi}{2}\right)^C$$

Distance moved by the tip of minute hand in 45 minutes is given by

$$l = r\theta = \left(1.4\times\frac{3\pi}{2}\right) = \left(1.4\times\frac{3}{2}\times\frac{22}{7}\right)$$

$$= 6.6 \text{ cm}$$

S45. $\because$ no. of revolutions made in 60 seconds

$$= 180$$

$\therefore$ no. of revolution made in 1 second

$$= \frac{180}{60} = 3$$

No. of revolution made in 30 seconds

$$= 30 \times 3 = 90$$

$\because$ Angle moved in one revolution

$$= (2\pi)^C$$

$\therefore$ Angle moved in 90 revolutions

$$= (2\pi \times 90)^C = (180\pi)^C$$

S46. Step-1: Convert second into minutes

$$\because \quad 1'' = \left(\frac{1}{60}\right)'$$

Step-2: Convert minutes into degree.

Minute obtained from Step-1 = $\left(\frac{1}{60}\right)'$

$$\because \quad 1' = \left(\frac{1}{60}\right)^\circ$$

$$\therefore \quad \left(\frac{1}{60}\right)' = \left(\frac{1}{60} \times \frac{1}{60}\right)^\circ = \left(\frac{1}{3600}\right)^\circ$$

Step-3: Convert degree into radian.

Degree obtained from Step-2 = $\left(\frac{1}{3600}\right)^\circ$

$$\therefore \quad \text{radian measure} = \frac{\pi}{180} \times \text{degree measure}$$

$$= \frac{\pi}{180} \times \frac{1}{3600} = \frac{\pi}{648000} \text{ rad.}$$

$$= \frac{\pi}{6.48} \times 10^{-5} \text{ rad.}$$

S47. Step-1: Convert minutes into degree

$$\because \quad 1' = \left(\frac{1}{60}\right)^\circ$$

$\therefore \quad 10' = \left(\frac{1}{60} \times 10\right)^\circ = \left(\frac{1}{6}\right)^\circ$

Step-2: Total degree obtained including Step-1.

$$= 10^\circ + \left(\frac{1}{6}\right)^\circ = \left(\frac{61}{6}\right)^\circ$$

Step-3: Assign negative sign as mentioned.

$$= -\left(\frac{61}{6}\right)^\circ$$

Step-4: Convert degree into radian

$\because$ Required radian measure.

$$= \left(\frac{\pi}{180}\right) \times \text{degree measure}$$

$$= -\frac{\pi}{180} \times \frac{61}{6} = \frac{-61\pi}{1080} \text{ rad.}$$

S48. Step-1: Convert seconds into minutes.

Here given degree measures have seconds and minutes. So firstly convert second into minutes.

$\because \quad 1'' = \left(\frac{1}{60}\right)'$

$\therefore \quad 30'' = \left(\frac{30}{60}\right)' = \left(\frac{1}{2}\right)'$

Step-2: Convert minutes into degree.

Minutes obtained from Step-1 = $\left(\frac{1}{2}\right)'$.

$\therefore$ Total minutes to convert

$$= 37' + \left(\frac{1}{2}\right)' = \left(\frac{74+1}{2}\right)' = \left(\frac{75}{2}\right)'$$

$\because \quad 1' = \left(\frac{1}{60}\right)^\circ$

$\because \quad \left(\frac{75}{2}\right)' = \left(\frac{75}{2 \times 60}\right)^\circ = \left(\frac{5}{8}\right)^\circ$

Step-3: Convert degree in radian.

Degree obtained from Step-2 = $\left(\frac{5}{8}\right)^\circ$

$\therefore$ Total degree to convert

$$= 48^\circ + \left(\frac{5}{8}\right)^\circ = \left(\frac{389}{8}\right)^\circ$$

$\because$ Radian measure = $\frac{\pi}{180} \times$ degree measure

$$\because \quad = \left(\frac{389}{8}\right) \times \frac{\pi}{180} = \frac{389\pi}{1440} \text{ radian.}$$

S49. **Step-1:** Convert second into minutes.

Here given degree measure have seconds

$$\because \quad 1'' = \left(\frac{1}{60}\right)'$$

$$\therefore \quad 10'' = \left(\frac{1}{60} \times 10\right)' = \left(\frac{1}{6}\right)'$$

Step-2: Convert minutes into degree.

Minute obtained from Step-1 = $\left(\frac{1}{6}\right)'$

$$\because \quad 1' = \left(\frac{1}{60}\right)^\circ$$

$$\therefore \quad \left(\frac{1}{6}\right)' = \left(\frac{1}{6} \times \frac{1}{60}\right)^\circ = \left(\frac{1}{360}\right)^\circ$$

Step-3: Convert degree into radian.

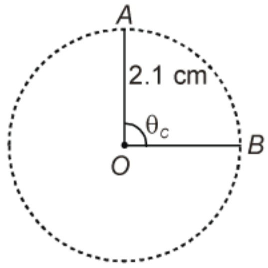
Degree obtained from Step-2 = $\left(\frac{1}{360}\right)^\circ$

$\therefore$ Required radian measure

$$= \frac{\pi}{180} \times \text{degree measure.} = \left(\frac{\pi}{180} \times \frac{1}{360}\right) \text{ radian} = \left(\frac{\pi}{64800}\right)$$

radian.

S50. $\because$ In 60 minutes, the minute hand moves through $(2\pi)^C$.



$\therefore$ In 1 minute, the minute hand moves through = $\left(\frac{2\pi}{60}\right)^C$

$\therefore$ In 15 minute the minute hand movers through

$$= 15 \times \frac{2\pi}{60} = \left(\frac{\pi}{2}\right)^C$$

$\therefore$ $$r = 2.1 \text{ cm}, \qquad \theta = \left(\frac{\pi}{2}\right)^C$$

$\therefore$ Distance moved by a tip of minute hand in 15 minute is given by

$$l = r\theta = \left(2.1 \times \frac{\pi}{2}\right) = 2.1 \times \frac{1}{2} \times \frac{22}{7} = 3.3 \text{ cm}$$

S51. Let r_1 and r_2 be the radii of the two circles. Given that

$$\theta_1 = 65° = \frac{\pi}{180} \times 65 = \frac{13\pi}{36} \text{ radian}$$

and $$\theta_2 = 110° = \frac{\pi}{180} \times 110 = \frac{22\pi}{36} \text{ radian}$$

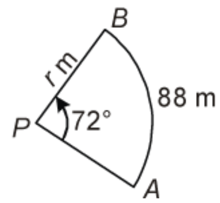
Let l be the length of each of the arc. Then $l = r_1\theta_1 = r_2\theta_2$, which gives

$$\frac{13\pi}{36} \times r_1 = \frac{22\pi}{36} \times r_2, \qquad i.e., \quad \frac{r_1}{r_2} = \frac{22}{13}$$

Hence, $r_1 : r_2 = 22 : 13$.

S52. Let us denote the post by P and let PA be the length of the rope in the highest position. Suppose that the horse moves along the arc AB so that $\angle APB = 72°$ and arc $AB = 88$m.

Let the length of the rope of PA be r meters.



Then $$\theta = 72° = \left(72 \times \frac{\pi}{180}\right)^C$$

$$= \left(\frac{2\pi}{5}\right)^C \quad \text{and} \quad l = 88 \text{ m}$$

$\therefore$ $$r = \frac{l}{\theta} = \frac{88}{\left(\frac{2\pi}{5}\right)} \text{ m} = 70 \text{ m}$$

Hence the length of the rope is 70 m.

S53. $\because$ Let the acute angles of the given triangle be x^C and y^C. Then,

$$x + y = \frac{\pi}{2} \qquad \text{... (i)}$$

$$x - y = \frac{\pi}{5} \qquad \dots \text{(ii)}$$

Solving Eq. (i) and (ii), we get

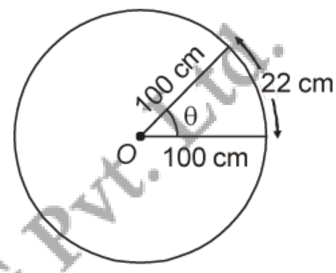
$$x = \frac{7\pi}{20},\ y = \frac{3\pi}{20}$$

Thus required angles are $\left(\frac{7\pi}{20}\right)^C$ and $\left(\frac{3\pi}{20}\right)^C$.

These angles in degrees are $\left(\frac{7\pi}{20} \times \frac{180}{\pi}\right)^\circ$ and $\left(\frac{3\pi}{20} \times \frac{180}{\pi}\right)^\circ$

$\therefore$ These angles in degree are 63° and 27°.

S54. We know that $l = r\theta$



Radius = 100 cm

where, l = Length of arc

r = Radius of circle

θ = Angle subtended at the centre

Here, l = 22 cm

r = 100 cm

$$\therefore \quad \theta = \frac{l}{r} = \frac{22}{100} = 0.22 \text{ radians}$$

$$= 0.22 \times \frac{180^\circ}{\pi} = \left(\frac{0.22 \times 180 \times 7}{22}\right)$$

$$= \left(\frac{22}{100} \times \frac{180 \times 7}{22}\right)^\circ = \left(\frac{126}{10}\right)^\circ = 12.6^\circ.$$

S55. It is given that x lies in IInd quadrant in which $\cos x$ is negative.

$$\therefore \quad \cos x = \frac{1}{\sqrt{1 + \tan^2 x}} = -\frac{1}{\sqrt{1 + 16/9}} = -\frac{3}{5}$$

Now, x lies in IInd quadrant

$$\Rightarrow \quad \frac{\pi}{2} < x < \pi$$

$$\Rightarrow \quad \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

$\Rightarrow$ $\frac{x}{2}$ lies in first quadrant

$\Rightarrow$ $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$ are positive

$$\therefore \quad \cos\frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$$

$$\Rightarrow \quad \cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1-3/5}{2}} = \frac{1}{\sqrt{5}}$$

$$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}}$$

$$\Rightarrow \quad \sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+3/5}{2}} = \frac{2}{\sqrt{5}}$$

and $$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$\Rightarrow \quad \tan\frac{x}{2} = \sqrt{\frac{1-\cos x}{1+\cos x}} = \sqrt{\frac{1+3/5}{1-3/5}} = 2.$$

S56. Since $\frac{\pi}{2} < x < \pi$, cos x is negative

Also, $$\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

Therefore, $\sin\frac{x}{2}$ and $\cos\frac{x}{2}$ both will also be positive

Now, $$\sin^2 x = \frac{1}{4}$$

As $$\cos^2 x = 1 - \sin^2 x$$

$$\Rightarrow \quad \cos^2 x = 1 - \frac{1}{16} = \frac{15}{16}$$

$$\Rightarrow \quad \cos x = \frac{-\sqrt{15}}{4}$$

We have $$2\sin^2\frac{x}{2} = 1 - \cos x$$

$$= 1 + \frac{\sqrt{15}}{4} = \frac{4+\sqrt{15}}{4}$$

$$\Rightarrow \quad \sin^2\frac{x}{2} = \frac{4+\sqrt{15}}{8}$$

$$\sin\frac{x}{2} = \sqrt{\frac{4+\sqrt{15}}{8}} = \frac{\sqrt{4+\sqrt{15}}}{2\sqrt{2}}$$

$$= \frac{\sqrt{2}\sqrt{4+\sqrt{15}}}{4} = \frac{\sqrt{8+2\sqrt{15}}}{4}$$

Again $$2\cos^2\frac{x}{2} = 1 + \cos x$$

$$= 1 - \frac{\sqrt{15}}{4} = \frac{4-\sqrt{15}}{4}$$

$\Rightarrow$ $$\cos^2\frac{x}{2} = \frac{4-\sqrt{15}}{8}$$

$\Rightarrow$ $$\cos\frac{x}{2} = \sqrt{\frac{4-\sqrt{15}}{8}} = \frac{\sqrt{4-\sqrt{15}}}{2\sqrt{2}} = \frac{\sqrt{8-2\sqrt{15}}}{4}$$

$\therefore$ $$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\sqrt{8+2\sqrt{15}}}{\sqrt{8-2\sqrt{15}}}$$

$$= \frac{8+2\sqrt{15}}{2} = 4+\sqrt{15}.$$

S57. It is given that x lies in the III quadrant.

$\therefore$ $$\pi < x < \frac{3\pi}{2}$$

$\Rightarrow$ $$\frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

$\Rightarrow$ $\frac{x}{2}$ lies in IInd quadrant

$\Rightarrow$ $$\cos\frac{x}{2} < 0, \quad \sin\frac{x}{2} > 0 \quad \text{and} \quad \tan\frac{x}{2} < 0$$

$\therefore$ $$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$\Rightarrow$ $$\cos\frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}}$$

$\Rightarrow$ $$\cos\frac{x}{2} = -\sqrt{\frac{1-1/3}{2}} = \frac{1}{\sqrt{3}} \qquad \left[\because \ \cos x = -\frac{1}{3}\right]$$

and $$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} \qquad \left[\because \ \sin\frac{x}{2} > 0\right]$$

$$\Rightarrow \qquad \sin\frac{x}{2} = \sqrt{\frac{1+1/3}{2}} = \sqrt{\frac{2}{3}} \qquad \left[\because \ \cos x = -\frac{1}{3}\right]$$

$$\text{and} \qquad \tan\frac{x}{2} = \frac{\sin x/2}{\cos x/2} = \sqrt{\frac{2}{3}} \times (-\sqrt{3}) = -\sqrt{2}.$$

S58. Let r_1 and r_2 be the radii of the two circles.

Given that $\theta_1 = 60° = \frac{\pi}{180} \times 60 = \frac{\pi}{3}$ radians

and $\theta_2 = 75° = \frac{\pi}{180} \times 75 = \frac{5\pi}{12}$ radians

Let l be length of each of the arcs.

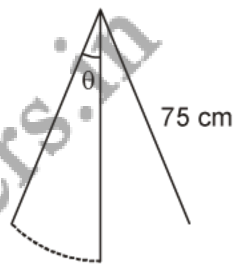
Then $l = r_1\,\theta_1 = r_2\,\theta_2$

Which gives $\frac{\pi}{3} \times r_1 = \frac{5\pi}{12} \times r_2$

i.e., $$\frac{r_1}{r_2} = \frac{\frac{5\pi}{12}}{\frac{\pi}{3}} = \frac{5\pi}{12} \times \frac{3}{\pi} = \frac{5}{4}$$

Hence, $r_1 : r_2 = 5 : 4$.

S59. (i) Let $r = 75$ cm, $l = 10$ cm, $\theta = ?$



$\Rightarrow \quad \theta = \frac{l}{r} = \frac{10}{75} = \frac{2}{15}$ radians

(ii) Given, $r = 75$ cm, $l = 15$ cm

$\Rightarrow \quad \theta = \frac{l}{r} = \frac{15}{75} = \frac{1}{5}$ radians

(iii) Given, $r = 75$ cm, $l = 21$ cm

$\Rightarrow \quad \theta = \frac{l}{r} = \frac{21}{75} = \frac{7}{25}$ radians.

Date: 9/10/2021

Q1. Find the value of sin 210°.

Q2. Find the value of sin 765°.

Q3. Find the value of cos 120°.

Q4. If $\sin\theta + \text{cosec}\,\theta = 2$, then find the value of $\sin^2\theta + \text{cosec}^2\theta$.

Q5. Prove that $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

Q6. If $\cos\theta = -\frac{1}{2}$ lies in third quadrant then find the value of $\sin\theta$.

Q7. Find the value of $\tan\left(-\frac{16\pi}{3}\right)$.

Q8. Find the value of $\cos(-\pi)$.

Q9. Find the value of $\cot\left(\frac{29\pi}{4}\right)$.

Q10. Find the value of $\sec(-6\pi)$.

Q11. Prove that $\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4} = -\frac{1}{2}$.

Q12. Find the value of $\frac{\tan(90^\circ - \theta)\cdot\sec(180^\circ - \theta)\cdot\sin(-\theta)}{\sin(180^\circ + \theta)\cdot\cot(360^\circ - \theta)\cdot\text{cosec}(90^\circ - \theta)}$.

Q13. Prove that $\tan 225^\circ \cdot \cot(405^\circ) + \tan 765^\circ \cdot \cot 675^\circ = 0$.

Q14. Show that: $\cos 130^\circ \cos 40^\circ + \sin 130^\circ \sin 40^\circ = 0$.

Q15. Find the value of $\sin\frac{31\pi}{3}$.

Q16. Find the value of $\cos(-1710^\circ)$.

Q17. Find the value of sin 15°.

Q18. Find the value of $\tan\frac{13\pi}{12}$.

Q19. Find the value of the following trigonometric function: sin 765°.

Q20. Find the value of the following trigonometric function: cosec (– 1410°).

Q21. Find the value of the following trigonometric function: $\tan\frac{19\pi}{3}$.

Q22. Find the value of the following trigonometric function: $\sin\left(-\frac{11\pi}{3}\right)$.

Q23. Find the value of the following trigonometric function: $\cot\left(-\frac{15\pi}{4}\right)$.

Q24. Find the value of cosec (– 1110°).

Q25. Prove that $\tan^2\frac{\pi}{3} + 2\cos^2\frac{\pi}{4} + 3\sec^2\frac{\pi}{6} + 4\cos^2\frac{\pi}{2} = 8$

Q26. Prove that:

$$\frac{\sin A}{\sec A + \tan A - 1} + \frac{\cos A}{\operatorname{cosec} A + \cot A - 1} = 1.$$

Q27. Prove that: $\sin(n+1)x \sin(n+2)x + \cos(n+1)x\cos(n+2)x = \cos x$

Q28. Prove that:

$$\frac{\sin\theta + 1 - \cos\theta}{\cos\theta - 1 + \sin\theta} = \frac{1+\sin\theta}{\cos\theta}.$$

Q29. Show that: $\cos 70° \cos 10° + \sin 70° \sin 10° = \frac{1}{2}$.

Q30. Prove that $\frac{(\tan\theta + \sec\theta - 1)}{(\tan\theta - \sec\theta + 1)} = \frac{1+\sin\theta}{\cos\theta}$.

Q31. If $\sin\theta + \operatorname{cosec}\theta = 2$ then find the value of $\sin^8\theta + \operatorname{cosec}^8\theta$.

Q32. If $\sin\theta = -\frac{1}{2}$ and θ lies in quadrant four, find the values of all other five trigonometric functions.

Q33. If $\tan\theta = 3$ and θ lies in third quadrant, then find the value of $\sin\theta$.

Q34. If $\sin x = \frac{-2\sqrt{6}}{5}$ and x lies in quadrant third then find the values of $\cos x$ and $\cot x$.

Q35. If $\sec\theta = -2$ and $\pi < \theta < \frac{3\pi}{2}$, find the value of other five trigonometric functions.

Q36. Prove that $\frac{\cos A + \sin A - 1}{\cos A - \sin A + 1} = \operatorname{cosec} A + \cot A$.

Q37. If $\tan\theta = a - \frac{1}{4a}$, $a \in R$ then find the value of $\sec\theta - \tan\theta$.

Q38. If $\frac{2\sin\alpha}{1+\sin\alpha+\cos\alpha} = \lambda$ then find the value of $\frac{1+\sin\alpha-\cos\alpha}{1+\sin\alpha}$ in terms of λ.

Q39. Prove that: $\sin^2 A = \cos^2(A-B) + \cos^2 B - 2\cos(A-B)\cos A\cos B$.

Q40. Prove that:

$$\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}.$$

Q41. Prove that:

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = \sec\theta\operatorname{cosec}\theta + 1.$$

Q42. Prove that: $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$.

Q43. If $\sin\theta = \frac{3}{5}$, θ lies in second quadrant, then find the other five trigonometric functions.

Q44. Find all trigonometric ratios, $\sin\theta = -\frac{2\sqrt{6}}{5}$ and θ lies in quadrant IV.

Q45. If $\tan A = \frac{m}{m-1}$ and $\tan B = \frac{1}{2m-1}$ prove that $A - B = \frac{\pi}{4}$.

Q46. Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$, if $\sin x = \frac{1}{4}$, x in **II** quadrant.

Q47. Prove that: tan 6° tan 42° tan 66° tan 78° = 1

Q48. Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$, if $\cos = -\frac{1}{3}$, x in **II** quadrant.

Q49. Find $\sin\frac{x}{2}$, $\cos\frac{x}{2}$ and $\tan\frac{x}{2}$, if $\tan = -\frac{4}{3}$, x in **II** quadrant.

Q50. If $\tan\frac{\theta}{2} = \sqrt{\frac{1-e}{1+e}}\tan\frac{\phi}{2}$, prove that $\cos\phi = \frac{\cos\theta - e}{1 - e\cos\theta}$.

Q51. Prove that: $4(\cos^3 20° + \cos^3 40°) = 3(\cos 20° + \cos 40°)$

Q52. If sec $(\phi + \alpha)$, sec ϕ and sec $(\phi - \alpha)$, are in A.P., prove that $\cos\phi = \pm\sqrt{\left(2\cos^2\frac{1}{2}\alpha\right)}$.

Q53. Show that:

$$\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right)=\frac{1}{16}$$

Q54. Prove that:

(i) $\sin^2 72° - \sin^2 60° = \frac{\sqrt{5}-1}{8}$ (ii) $\sin\frac{\pi}{5}\sin\frac{2\pi}{5}\sin\frac{3\pi}{5}\sin\frac{4\pi}{5} = \frac{5}{16}$

Q55. Find the value of: (i) sin 75°; (ii) tan 15°

Q56. Prove the following identities:

(i) $\frac{\sin^2 A}{\cos^2 A} + \frac{\cos^2 A}{\sin^2 A} = \frac{1}{\sin^2 A\cos^2 A} - 2 = \sec^2 A\,\text{cosec}^2 A - 2$.

(ii) $\sec^4 A\,(1 - \sin^4 A) - 2\tan^2 A = 1$.

Q57. Prove that:

$$\sin 10° \sin 30° \sin 50° \sin 70° = \frac{1}{6}.$$

Q58. Prove that:

$$\sin 20° \sin 40° \sin 80° = \frac{\sqrt{3}}{8}.$$

Q59. Prove that: $\sin(150° + x) + \sin(150° - x) = \cos x$.

Q60. Prove that:

$$\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)=-\sqrt{2}\sin x.$$

Q61. If $\cos x = -\frac{3}{5}$, x lies in the third quadrant, find the values of other five trigonometric functions.

Q62. Find the values of all trigonometric function of the following problems: $\cos x = -\frac{1}{2}$, x lies in third quadrant.

Q63. If $\sin x = \frac{3}{5}$, $\cos y = -\frac{12}{13}$, where x and y both lie in second quadrant, find the value of $\sin(x + y)$.

Q64. If $\cot x = -\frac{5}{12}$, x lies in second quadrant, find the values of other five trigonometric functions.

Q65. Find the values of trigonometric function of the following problems: sin $x = \frac{3}{5}$, x lies in second quadrant.

Q66. Find the values of trigonometric function of the following problems: cot $x = \frac{3}{4}$, x lies in third quadrant.

Q67. Find the values of trigonometric function of the following problems: sec $x = \frac{13}{5}$, x lies in fourth quadrant.

Q68. Find the values of trigonometric function of the following problems: tan $x = \frac{-5}{12}$, x lies in second quadrant.

Q69. Find the value of: (i) sin 75°, (ii) tan 15°.

Q70. Prove that:

(i) $\frac{\cos\theta}{1-\sin\theta} + \frac{1-\sin\theta}{\cos\theta} = 2\sec\theta$ (ii) $\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2 = \frac{1-\cos\theta}{1+\sin\theta}$

Q71. Solve : $4\sin x\cos x + 2\sin x + 2\cos x + 1 = 0$

Q72. Prove that:

$$\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15} = \frac{1}{128}$$

Q73. Prove the following identities:

(i) $\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}} = \operatorname{cosec}\alpha + \cot\alpha$ (ii) $\sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}} = \sec\alpha + \tan\alpha$

Q74. Prove that:

(i) $\frac{\cot A + \tan B}{\cot B + \tan A} = \cot A \cdot \tan B$ (ii) $\tan^2 A - \tan^2 B = \frac{\sin^2 A - \sin^2 B}{\cos^2 A\cos^2 B}$

Q75. Prove that:

$$\frac{(1+\cot A + \tan A)(\sin A - \cos A)}{\sec^3 A - \operatorname{cosec}^3 A} = \sin^2 A \cdot \cos^2 A.$$

Q76. Prove the following identities:

(i) $\frac{\sin\theta + 1 - \cos\theta}{\cos\theta - 1 + \sin\theta} = \frac{1+\sin\theta}{\cos\theta}$ (ii) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$

Q77. If $\tan\beta = \frac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}$; prove that $\tan(\alpha-\beta) = (1-n)\tan\alpha$

Q78. Prove that:

$$\left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\operatorname{cosec}^2\theta - \sin^2\theta}\right)\sin^2\theta\cdot\cos^2\theta = \frac{1-\sin^2\theta\cdot\cos^2\theta}{2+\sin^2\theta\cdot\cos^2\theta}$$

Q79. If $\sin\theta = \frac{4}{5}$, find the value of $\frac{5\cos\theta + 4\operatorname{cosec}\theta + 3\tan\theta}{4\cot\theta + 3\sec\theta + 5\sin\theta}$

Date: 9/10/2021

S1. We know that

$$\sin(2n\pi + \theta) = \sin\theta \qquad (n \in z)$$

$$\therefore \qquad \sin(210°) = \sin\left(2\times\pi + \frac{\pi}{6}\right)$$

$$= \sin\frac{\pi}{6} = \frac{1}{2}.$$

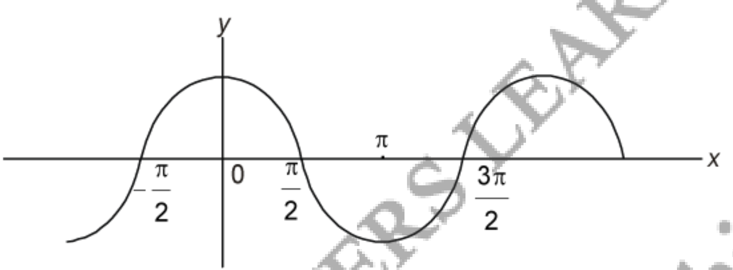
S2.

$$\because \qquad 765° = \left(\frac{\pi}{180}\times 765\right)^{C} = \left(\frac{17\pi}{4}\right)^{C}$$

$$\therefore \qquad \sin 765° = \sin\left(4\pi + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}.$$

S3. $\because \qquad \cos(90° + \theta) = -\sin\theta$

$$\therefore \qquad \cos(120°) = \cos(90 + 30)° = -\sin 30° = -\frac{1}{2}$$



S4.

$$\sin\theta + \frac{1}{\sin\theta} = 2 \qquad \text{... (i)}$$

$\because$ $\sin\theta$ should be at maximum value for eq. (i)

$$\therefore \qquad \theta = 90°$$

$$\therefore \qquad \sin^2\theta + \mathrm{cosec}^2\theta = \sin^2 90° + \mathrm{cosec}^2 90° = 2.$$

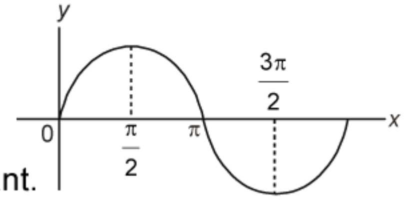
S5. $\because$ We know that

$$(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$$

$$\therefore \qquad (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$$

$$= 2\{(\sin^2\theta) + (\cos^2\theta)\} = 2\times 1 = 2 \qquad [\because (\sin^2\theta + \cos^2\theta = 1)]$$

S6. $\because \quad \sin^2\theta + \cos^2\theta = 1$



$\therefore \quad \sin^2\theta + \dfrac{1}{4} = 1 \quad \Rightarrow \quad \sin\theta = \pm\dfrac{\sqrt{3}}{2}.$

$\because$ From the graph it is clear that $\sin\theta$ is negative in third quadrant.

Hence, $\quad \sin\theta = -\dfrac{\sqrt{3}}{2}.$

S7. $\because \quad \tan\left(-\dfrac{16\pi}{3}\right) = -\tan\dfrac{16\pi}{3}$

$\because \quad \tan(-\theta) = -\tan\theta$

$= -\tan\left(5\pi + \dfrac{\pi}{3}\right) \qquad \because \tan(n\pi + \theta) = \tan\theta$

$= -\tan\dfrac{\pi}{3} = -\sqrt{3}$

S8. $\because \quad \cos\theta = \cos(-\theta)$

$\therefore \quad \cos\pi = \cos(-\pi)$

$\therefore \quad \cos(-\pi) = -1$

S9. $\cot\left(\dfrac{29\pi}{4}\right) = \left(7\pi + \dfrac{\pi}{4}\right) \qquad (\cot(n\pi + \theta)) = \cot\theta.$

$= \cot\dfrac{\pi}{4} = 1$

S10. $\because \quad \sec(-\theta) = +\sec\theta$

$\therefore \quad \sec(-6\pi) = \sec 6\pi$

$= \sec(6\pi + 0) \qquad \because \sec(2n\pi + \theta) = \sec\theta$

$= \sec 0 = 1.$

S11. From L.H.S. $\sin^2\dfrac{\pi}{6} + \cos^2\dfrac{\pi}{3} - \tan^2\dfrac{\pi}{4} \qquad \left\{\because \sin\dfrac{\pi}{6} = \dfrac{1}{2}, \quad \cos\dfrac{\pi}{3} = \dfrac{1}{2}, \ \tan\dfrac{\pi}{4} = 1\right\}$

$= \left(\dfrac{1}{2}\right)^2 + \left(\dfrac{1}{2}\right)^2 - 1^2 = -\dfrac{1}{2}$

Hence, $\quad$ L.H.S. = R.H.S.

S12. $\because \quad \dfrac{\tan(90^\circ - \theta)\cdot\sec(180^\circ - \theta)\cdot\sin(-\theta)}{\sin(180^\circ + \theta)\cdot\cot(360^\circ - \theta)\cdot\operatorname{cosec}(90^\circ - \theta)}$

$= \dfrac{\cot\theta\cdot(-\sec\theta)\cdot(-\sin\theta)}{(-\sin\theta)\cdot(-\cot\theta)\cdot\sec\theta} = 1$

S13. From R.H.S. $\tan 225° \cdot \cot(405°) + \tan 765° \cdot \cot 675°$

$= \tan(180° + 45°) \cdot \cot(360° + 45°) + \tan(24 \times 180° + 45°) \,.\, \cot(3 \times 180° + 135°)$

$= -\tan 45° \cdot \cot 45° + \tan 45° \cdot \tan 45°$

$= -1 + 1 = 0.$ L.H.S = R.H.S Proved

S14. L.H.S. $= \cos 130° \cos 40° + \sin 130° \sin 40°$

$= \cos(130° - 40°)$ $[\because \cos(A - B) = \cos A \cos B + \sin A \sin B]$

$= \cos 90°$

$= 0 =$ RHS. **Proved.**

S15. We know that values of $\sin x$ repeats after an interval of 2π. Therefore

$$\sin\frac{31\pi}{3} = \sin\left(10\pi + \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}.$$

S16. We know that values of $\cos x$ repeats after an interval of 2π or 360°.

Therefore, $\cos(-1710°) = \cos(-90° + 5 \times 360°)$

$= \cos(-90° + 1800°) = \cos 90° = 0°.$

S17. We have, $\sin 15° = \sin(45° - 30°)$

$= \sin 45° \cos 30° - \cos 45° \sin 30°$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}.$$

S18. We hve, $\tan\frac{13\pi}{12} = \tan\left(\pi + \frac{\pi}{12}\right) = \tan\frac{\pi}{12} = \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$$= \frac{\tan\frac{\pi}{4} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{4}\tan\frac{\pi}{6}} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3}.$$

S19. We know that values of $\sin\theta$ repeats after an interval of 2π or 360. Therefore

$\sin 765° = \sin(2 \times 360° + 45°)$

$$= \sin 45° = \frac{1}{\sqrt{2}}.$$

S20. Let $\operatorname{cosec}(-1410°) = -\operatorname{cosec}(1410°)$

$= -\operatorname{cosec}(4 \times 360° - 30°)$

$= -(-\operatorname{cosec} 30°)$

$= \operatorname{cosec} 30° = 2.$

S21. We have,

$$\frac{19\pi}{3} = \left(\frac{19}{3}\times 180\right)^\circ = 1140^\circ$$

$$= (90^\circ \times 12 + 60^\circ)$$

Clearly, it lies in first quadrant.

$$\therefore \quad \tan\frac{19\pi}{3} = \tan(90^\circ \times 12 + 60^\circ)$$

$$= \tan 60^\circ = \sqrt{3}.$$

S22. We have,

$$\frac{11\pi}{3} = \left(\frac{11\times 180}{3}\right)^\circ = 660^\circ$$

$$= 180^\circ \times 4 - 60^\circ$$

$$\therefore \quad \sin\left(\frac{-11\pi}{3}\right) = -\sin\frac{11\pi}{3} \qquad [\because \sin(-\theta) = -\sin\theta]$$

$$= -\sin(180^\circ \times 4 - 60^\circ)$$

$$= -(-\sin 60^\circ) = -\left(\frac{-\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}.$$

S23. We have,

$$\frac{15\pi}{4} = \left(\frac{15\times 180}{4}\right)^\circ = 675^\circ$$

$$= 90^\circ \times 7 + 45^\circ$$

$$\therefore \quad \cot\left(\frac{-15\pi}{4}\right) = -\cot\left(\frac{15\pi}{4}\right) \qquad [\because \cot(-\theta) = -\cot\theta]$$

$$= -\cot(90^\circ \times 7 + 45^\circ)$$

$$= -(-\cot 45^\circ) = -(-1) = 1.$$

S24. $\because \quad \text{cosec}(2n\pi + \theta) = \text{cosec}\,\theta, \quad n \in z$

$\because \quad \text{cosec}(-\theta) = -\text{cosec}\,\theta$

$\therefore \quad \text{cosec}(-1110^\circ) = -\text{cosec}(1110^\circ)$

$$= -\text{cosec}\left(\frac{37\pi}{6}\right)$$

$$= -\text{cosec}\left(6\pi + \frac{\pi}{6}\right). \qquad (\text{cosec}(2n\pi + \theta)) = \text{cosec}\,\theta$$

$$= -\text{cosec}\left(\frac{\pi}{6}\right) \quad = -2$$

S25. From L.H.S.

$$\tan^2\frac{\pi}{3} + 2\cos^2\frac{\pi}{4} + 3\sec^2\frac{\pi}{6} + 4\cos^2\frac{\pi}{2}$$

$$= (\sqrt{3})^2 + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 3\left(\frac{2}{\sqrt{3}}\right)^2 + 4(0)^2$$

$$= 3 + 1 + 3 \times \frac{4}{3} = 4 + 4 = 8$$

Hence, L.H.S. = R.H.S.

S26.

$$\text{L.H.S.} = \frac{\sin A}{\sec A + \tan A - 1} + \frac{\cos A}{\text{cosec } A + \cot A - 1}$$

$$= \frac{\sin A \text{ cosec } A + \sin A \cot A - \sin A + \cos A \sec A + \cos A \tan A - \cos A}{(\sec A + \tan A - 1)(\text{cosec } A + \cot A - 1)}$$

$$= \frac{1 + \cos A - \sin A + 1 + \sin A - \cos A}{\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} - 1\right)\left(\frac{1}{\sin A} + \frac{\cos A}{\sin A} - 1\right)}$$

$$= \frac{1 + \cos A - \sin A + 1 + \sin A - \cos A}{\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A} - 1\right)\left(\frac{1}{\sin A} + \frac{\cos A}{\sin A} - 1\right)}$$

$$= \frac{2}{\left(\frac{1 + \sin A - \cos A}{\cos A}\right)\frac{(1 + \cos A - \sin A)}{\sin A}}$$

$$= \frac{2 \sin A \cos A}{[1 + (\sin A - \cos A)][1 - (\sin A - \cos A)]} = \frac{2 \sin A \cos A}{1 - (\sin A - \cos A)^2}$$

$$= \frac{2 \sin A \cos A}{1 - (\sin^2 A + \cos^2 A - 2 \sin A \cos A)} \quad [\because (a + b)(a - b) = a^2 - b^2]$$

$$= \frac{2 \sin A \cos A}{1 - (1 - 2 \sin A \cos A)} = \frac{2 \sin A \cos A}{1 - 1 + 2 \sin A \cos A} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{2 \sin A \cos A}{2 \sin A \cos A} = 1 = \text{R.H.S.}$$

Proved.

S27. We have, L.H.S = $\sin (n + 1) x \sin (n + 2) x + \cos (n + 1) x \cos (n + 2) x$

$= \cos (n + 2) x \cos (n + 1) x + \sin (n + 2) x \sin (n + 1) x$

Put, $(n + 2) x = A, \quad (n - 1) x = B$

L.H.S. = $\cos A \cos B + \sin A \sin B$

$= \cos (A - B)$

$= \cos [(n + 2) x - (n + 1) x]$

$= \cos x.$

Proved.

S28.

$$\text{L.H.S.} = \sqrt{\frac{1 + \sin\theta}{1 - \sin\theta}} + \sqrt{\frac{1 - \sin\theta}{1 + \sin\theta}} = \sqrt{\frac{1 + \sin\theta}{1 - \sin\theta} \times \frac{1 + \sin\theta}{1 + \sin\theta}} + \sqrt{\frac{1 - \sin\theta}{1 + \sin\theta} \times \frac{1 - \sin\theta}{1 - \sin\theta}}$$

$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} + \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1+\sin\theta}{\cos\theta} + \frac{1-\sin\theta}{\cos\theta} = \frac{1+\sin\theta+1-\sin\theta}{\cos\theta}$$

$$= \frac{2}{\cos\theta} = 2\sec\theta = \text{R.H.S.}$$ **Proved.**

S29. (i) We have, L.H.S = cos 70° cos 10° + sin 70° sin 10°

= cos (70° – 10°) [$\because$ cos (A – B) = cos A cos B + sin A sin B]

= cos 60° = $\frac{1}{2}$ = RHS. **Proved.**

S30. From L.H.S. $\frac{(\tan\theta+\sec\theta-1)}{(\tan\theta-\sec\theta+1)}$

$$= \frac{(\tan\theta+\sec\theta)-(\sec^2\theta-\tan^2\theta)}{(\tan\theta-\sec\theta+1)}$$ [$\because$ $\sec^2\theta - \tan^2\theta = 1$]

$$= \frac{(\tan\theta+\sec\theta)(1+\tan\theta-\sec\theta)}{(1+\tan\theta-\sec\theta)}$$

$$= \tan\theta + \sec\theta = \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}$$

$$= \frac{\sin\theta+1}{\cos\theta} = \frac{1+\sin\theta}{\cos\theta}.$$

S31. $\because$ $\sin\theta + \text{cosec}\,\theta = 2$

$\Rightarrow$ $\sin\theta$ takes its maximum value.

$\Rightarrow$ $\theta = \frac{\pi}{2}$

$\therefore$ $\sin^8\theta + \text{cosec}^8\theta = \left(\sin\frac{\pi}{2}\right)^8 + \left(\text{cosec}\frac{\pi}{2}\right)^8$

= 1 + 1 = 2.

S32. $\because$ In fourth quadrant only $\cos\theta$ and $\sec\theta$ positive.

$\because$ $\sin^2\theta + \cos^2\theta = 1$

$\therefore$ $\left(-\frac{1}{2}\right)^2 + \cos^2\theta = 1$

$\Rightarrow \qquad \cos^2\theta = \frac{3}{4} \quad \Rightarrow \quad \cos\theta = \pm\frac{\sqrt{3}}{2}$

Hence, $\qquad \cos\theta = \frac{\sqrt{3}}{2}$

$\therefore \qquad \sec\theta = \frac{2}{\sqrt{3}}, \qquad \text{cosec}\,\theta = -2$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-1/2}{\sqrt{3}/2} = -\frac{1}{\sqrt{3}}$$

$\therefore \qquad \cot\theta = -\sqrt{3}$.

S33. $\because \qquad \sec^2\theta - \tan^2\theta = 1$

$\therefore \qquad \sec^2\theta = 1 + \tan^2\theta$

$$= 1 + \left(\frac{3}{1}\right)^2 = 10$$

$\therefore \qquad \sec\theta = \pm\sqrt{10}$

Since θ lies in third quadrant hence sec θ is negative.

$\therefore \qquad \sec\theta = -\sqrt{10}$

$\therefore \qquad \cos\theta = \frac{1}{\sec\theta} = -\frac{1}{\sqrt{10}}$

$\because \qquad \sin^2\theta + \cos^2\theta = 1$

$\therefore \qquad \sin^2\theta = 1 - \cos^2\theta$

$\therefore \qquad \sin^2\theta = 1 - \frac{1}{10} = \frac{9}{10}$

$\therefore \qquad \sin\theta = \pm\sqrt{\frac{9}{10}}$

Since, θ lies in third quadrant.

$\therefore \qquad \sin\theta = \frac{-3}{\sqrt{10}}$.

S34. $\because$ cos x is negative in third quadrant.

$\because \qquad \sin^2 x + \cos^2 x = 1$

$\therefore \quad \left(\frac{-2\sqrt{6}}{5}\right)^2 + \cos^2 x = 1$

$\therefore \quad \frac{24}{25} + \cos^2 x = 1$

$\Rightarrow \quad \cos^2 x = 1 - \frac{24}{25} \quad \Rightarrow \quad \cos^2 x = \frac{1}{25}$

$\therefore \quad \cos x = \pm\frac{1}{5}$

$\therefore \quad \cos x = -\frac{1}{5}$

$\therefore \quad \cot x = \frac{\cos x}{\sin x} = \frac{-1/5}{-2\sqrt{6}/5} = \frac{1}{2\sqrt{6}}.$

S35. $\because$ in third quadrant only tan θ, cot θ is positive.

$\because \quad \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \quad 4 - \tan^2\theta = 1 \qquad \tan^2\theta = 3$

$\Rightarrow \quad \tan\theta = \pm\sqrt{3}$

$\therefore \quad \tan\theta = \sqrt{3}$

$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = \sqrt{3} \quad \Rightarrow \quad \sin\theta \times \sec\theta = \sqrt{3}$

$\Rightarrow \quad \sin\theta = \frac{-\sqrt{3}}{2} \qquad (\because \ \sec\theta = -2)$

$\therefore \quad \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\sqrt{3}}$

$\text{cosec}\,\theta = \frac{1}{\sin\theta} = -\frac{2}{\sqrt{3}}$

$\cos\theta = \frac{1}{\sec\theta} = \frac{-1}{2}.$

S36. From L.H.S. $\frac{\cos A + \sin A - 1}{\cos A - \sin A + 1}$

$= \frac{\sin A}{\sin A}\left\{\frac{\cot A + \text{cosec}\, A - 1}{\cot A - \text{cosec}\, A + 1}\right\}$

$$= \left(\frac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1}\right) \times \frac{\cot A + \operatorname{cosec} A}{\cot A + \operatorname{cosec} A}$$

$$= \frac{\cot^2 A + \cot A \cdot \operatorname{cosec} A - \cot A + \cot A \cdot \operatorname{cosec} A + \operatorname{cosec}^2 A - \operatorname{cosec} A}{\cot^2 A - \cot A \cdot \operatorname{cosec} A + \cot A + \operatorname{cosec} A \cdot \cot A - \operatorname{cosec}^2 A + \operatorname{cosec} A}$$

$$= \frac{(\cot A + \operatorname{cosec} A)^2 - (\cot A + \operatorname{cosec} A)}{(-1 + \cot A + \operatorname{cosec} A)}$$

$$= \frac{(\operatorname{cosec} A + \cot A)(\operatorname{cosec} A + \cot A - 1)}{(\cot A + \operatorname{cosec} A - 1)}$$

$= \operatorname{cosec} A + \cot A$ R.H.S.

S37. Let $(\sec\theta - \tan\theta) = \lambda$ ... (i)

$\Rightarrow$ $(\sec\theta + \tan\theta) = \dfrac{1}{\lambda}$ ... (ii) $[(\sec^2\theta - \tan^2\theta) = 1]$

Subtracting Eq. (ii) from Eq. (i), we get

$\Rightarrow$ $2\tan\theta = -\lambda + \dfrac{1}{\lambda}$

$\Rightarrow$ $2\left(a - \dfrac{1}{4a}\right) = -\lambda + \dfrac{1}{\lambda}$

$\Rightarrow$ $\lambda = \left(\dfrac{1}{2a}, -2a\right)$

Hence, $(\sec\theta - \tan\theta) = \left(\dfrac{1}{2a}, -2a\right)$.

S38.

$$\frac{1+\sin\alpha - \cos\alpha}{1+\sin\alpha} = \frac{(1+\sin\alpha)^2 - \cos^2\alpha}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}$$

$$= \frac{2\sin\alpha + 2\sin^2\alpha}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}$$

$$= \frac{2\sin\alpha}{1+\sin\alpha+\cos\alpha} = \lambda$$

Hence, $\dfrac{1+\sin\alpha - \cos\alpha}{1+\sin\alpha} = \lambda$.

S39. R.H.S. $= \cos^2(A-B) + \cos^2 B - 2\cos(A-B)\cos A\cos B$

$= \cos^2 B + \cos^2(A-B) - 2\cos(A-B)\cos A\cos B$

$= \cos^2 B + \cos(A-B)\{\cos(A-B) - 2\cos A\cos B\}$

$= \cos^2 B + \cos(A-B)\{\cos A\cos B + \sin A\sin B - 2\cos A\cos B\}$

$$= \cos^2 B + \cos(A - B)\{\sin A \sin B - \cos A \cos B\}$$
$$= \cos^2 B + \cos(A - B)(\cos A \cos B - \sin A \sin B)$$
$$= \cos^2 B - \cos(A - B)\cos(A + B) = \cos^2 B - (\cos^2 A - \sin^2 B)$$
$$= \cos^2 B + \sin^2 B - \cos^2 A = 1 - \cos^2 A = \sin^2 A = \text{LHS.}$$ **Proved.**

S40. We have, $$\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$$

$$\Rightarrow \quad \frac{1}{\operatorname{cosec}\theta - \cot\theta} + \frac{1}{\operatorname{cosec}\theta + \cot\theta} = \frac{1}{\sin\theta} + \frac{1}{\sin\theta}$$

$$\Rightarrow \quad \frac{1}{\operatorname{cosec}\theta - \cot\theta} + \frac{1}{\operatorname{cosec}\theta + \cot\theta} = \frac{2}{\sin\theta}$$

Now, $$\text{L.H.S.} = \frac{1}{\operatorname{cosec}\theta - \cot\theta} + \frac{1}{\operatorname{cosec}\theta + \cot\theta} = \frac{\operatorname{cosec}\theta + \cot\theta + \operatorname{cosec}\theta - \cot\theta}{(\operatorname{cosec}\theta - \cot\theta)(\operatorname{cosec}\theta + \cot\theta)}$$

$$= \frac{2\operatorname{cosec}\theta}{(\operatorname{cosec}^2\theta - \cot^2\theta)} \qquad \left[\because \operatorname{cosec}^2\theta - \cot^2\theta = 1\right]$$

$$= \frac{2\operatorname{cosec}\theta}{1} = \frac{2}{\sin\theta} = \text{R.H.S.} \qquad \left[\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right]$$ **Proved.**

S41.

$$\text{L.H.S.} = \frac{\tan\theta}{1 - \cot\theta} + \frac{\cot\theta}{1 - \tan\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{1 - \frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1 - \frac{\sin\theta}{\cos\theta}} = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta - \cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta - \sin\theta}{\cos\theta}}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta - \cos\theta)} = \frac{\sin^2\theta}{\cos\theta(\sin\theta - \cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta - \cos\theta)}$$

$$= \frac{\sin^3\theta - \cos^3\theta}{(\sin\theta - \cos\theta)\sin\theta\cos\theta} = \frac{(\sin\theta - \cos\theta)(\sin^2\theta + \sin\theta\cos\theta + \cos^2\theta)}{(\sin\theta - \cos\theta)\sin\theta\cos\theta}$$
$$[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$$

$$= \frac{1 + \sin\theta\cos\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta \cdot \cos\theta} + 1 = \sec\theta \operatorname{coses}\theta + 1 = \text{R.H.S.}$$ **Proved.**

Alternative Method:

$$\text{L.H.S.} = \frac{\tan\theta}{1 - \frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1 - \tan\theta} = \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{\tan\theta(\tan\theta - 1)}$$

$$= \frac{\tan^3\theta - 1}{\tan\theta(\tan\theta - 1)} = \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{\tan\theta(\tan\theta - 1)}$$

$$= \frac{\sec^2\theta + \tan\theta}{\tan\theta} = \frac{\sec^2\theta}{\tan\theta} + \frac{\tan\theta}{\tan\theta}$$

$= \sec\theta \operatorname{cosec}\theta + 1$ = RHS. **Proved.**

S42. (i) L.H.S. $= \sin^6\theta + \cos^6\theta = (\sin^2)^3 + (\cos^2\theta)^3$ $[\because\ a^3 + b^3 = (a+b)(a^2 - ab + b^2)]$

$= (\sin^2\theta + \cos^2\theta) \times [(\sin^2\theta)^2 - \sin^2\theta\cos^2\theta + (\cos^2\theta)^2]$

$= (\sin^2\theta)^2 + (\cos^2\theta)^2 - \sin^2\theta\cos^2\theta$ $[\therefore\ \sin^2\theta + \cos^2\theta = 1]$

$= (\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - 3\sin^2\theta\cos^2\theta$ $[\therefore\ \sin^2\theta + \cos^2\theta = 1]$

$= (\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta = 1 - 3\sin^2\theta\cos^2\theta$ = R.H.S. **Proved.**

Alternative Method:

L.H.S. $= \sin^6\theta + \cos^6\theta = (\sin^2\theta)^3 + (\cos^2\theta)^3$

$= (\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$ $[\because\ a^3 + b^3 = (a+b)^3 - 3ab(a+b)]$

$= 1 - 3\sin^2\theta\cos^2\theta$ = RHS. **Proved.**

S43. We have, $\sin\theta = \frac{3}{5}$, θ lies in second quadrant,

We know that, $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow$ $\cos\theta = \pm\sqrt{1 - \sin^2\theta}$

In second quadrant, $\cos\theta$ is negative, therefore

$$\cos\theta = -\sqrt{1 - \sin^2\theta} = -\sqrt{1 - \left(\frac{3}{5}\right)^2} = -\sqrt{\frac{25 - 9}{25}} = -\frac{4}{5} \quad \left[\because\ \sin\theta = \frac{3}{5}\right]$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{3}{5}}{-\frac{4}{5}} = -\frac{3}{4}$$

Also, the reciprocal relations are:

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{3/5} = \frac{5}{3}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{-4/5} = -\frac{5}{4}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\frac{3}{4}} = -\frac{4}{3}$$

Hence, the other five trigonometric functions are

$\operatorname{cosec}\theta = \frac{5}{3}$, $\cos\theta = -\frac{4}{5}$, $\sec\theta = -\frac{5}{4}$, $\tan\theta = -\frac{3}{4}$, $\cot\theta = -\frac{4}{3}$ **Ans.**

S44. We have, $\sin\theta = -\frac{2\sqrt{6}}{5}$

We know that $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \quad \cos\theta = \pm\sqrt{1-\sin^2\theta} = \pm\sqrt{1-\left(\frac{-2\sqrt{6}}{5}\right)^2} = \pm\sqrt{\frac{25-24}{25}} = \pm\sqrt{\frac{1}{25}} = \pm\frac{1}{5}$$

In the fourth quadrant, cos θ is positive, therefore $\cos\theta = \frac{1}{5}$

and $$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\frac{2\sqrt{6}}{5}}{\frac{1}{5}} = -2\sqrt{6}$$

The other trigonometric ratios are as follows:

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{-\frac{2\sqrt{6}}{5}} = -\frac{5}{2\sqrt{6}}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-2\sqrt{6}} = -\frac{1}{2\sqrt{6}}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{1}{5}} = 5$$ **Ans.**

S45. We have, $\tan A = \frac{m}{m-1}$ and $\tan B = \frac{1}{2m-1}$

Now, $$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B} \quad \text{... (i)}$$

Putting the values of tan A and tan B in (i), we get

$$\tan(A-B) = \frac{\frac{m}{m-1} - \frac{1}{2m-1}}{1+\left(\frac{m}{m-1}\right)\left(\frac{1}{2m-1}\right)}$$

$$= \frac{2m^2-m-m+1}{(m-1)(2m-1)} \times \frac{(m-1)(2m-1)}{2m^2-3m+1+m} = \frac{2m^2-2m+1}{2m^2-2m+1} = 1$$

$$\Rightarrow \quad \tan(A-B) = \tan\frac{\pi}{4} \quad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$$\Rightarrow \quad A-B = \frac{\pi}{4}.$$ **Proved.**

S46. We have, $\sin x = \dfrac{1}{4}, \quad \dfrac{\pi}{2} < x < \pi \quad \Rightarrow \quad \dfrac{\pi}{4} < \dfrac{x}{2} < \dfrac{\pi}{2}$

i.e., $\dfrac{x}{2}$ lies in the first quadrant, so that all *t*-ratios of $\dfrac{x}{2}$ are positive.

Also, $\cos^2 x = 1 - \sin^2 x = 1 - \dfrac{1}{16} = \dfrac{15}{16}$ and $\cos x$ is negative in the second quadrant.

$$\Rightarrow \qquad \cos x = -\frac{\sqrt{15}}{4}$$

$$\therefore \qquad \sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1+\frac{\sqrt{15}}{4}}{2}} = \sqrt{\frac{4+\sqrt{15}}{8}} \qquad \left[\because \cos x = \frac{-\sqrt{15}}{4}\right]$$

$$= \sqrt{\frac{8+2\sqrt{15}}{16}} = \frac{\sqrt{5}+\sqrt{3}}{4}$$

$$[\because (\sqrt{5}+\sqrt{3})^2 = 5+3+2\sqrt{5}\times\sqrt{3} = 8+2\sqrt{15}]$$

$$\cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1-\frac{\sqrt{15}}{4}}{2}} = \sqrt{\frac{4-\sqrt{15}}{8}} = \sqrt{\frac{8-2\sqrt{5}}{16}} = \frac{\sqrt{5}-\sqrt{3}}{4}$$

$$[\because (\sqrt{5}-\sqrt{3})^2 = 5+3-2\sqrt{5}\times\sqrt{3} = 8-2\sqrt{15}]$$

$$\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{(\sqrt{5}+\sqrt{3})^2}{5-3} = \frac{8+2\sqrt{15}}{2} = 4+\sqrt{15}$$ **Ans.**

S47. L.H.S. = $\tan 6^\circ \tan 42^\circ \tan 66^\circ \tan 78^\circ$

$$= \frac{\sin 6^\circ \sin 42^\circ \sin 66^\circ \sin 78^\circ}{\cos 6^\circ \cos 42^\circ \cos 66^\circ \cos 78^\circ} = \frac{(2\sin 66^\circ \sin 6^\circ)(2\sin 78^\circ \sin 42^\circ)}{(2\cos 66^\circ \cos 6^\circ)(2\cos 78^\circ \cos 42^\circ)}$$

$$= \frac{(\cos 60^\circ - \cos 72^\circ)(\cos 36^\circ - \cos 120^\circ)}{(\cos 60^\circ + \cos 72^\circ)(\cos 36^\circ + \cos 120^\circ)}$$

$$= \frac{(\cos 60^\circ - \sin 18^\circ)(\cos 36^\circ + \sin 30^\circ)}{(\cos 60^\circ + \sin 18^\circ)(\cos 36^\circ - \sin 30^\circ)} = \frac{\left(\frac{1}{2}-\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}+\frac{1}{2}\right)}{\left(\frac{1}{2}+\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}-\frac{1}{2}\right)}$$

$$= \frac{(3-\sqrt{5})(3+\sqrt{5})}{(\sqrt{5}+1)(\sqrt{5}-1)} = \frac{9-5}{5-1} = 1 = \text{RHS.}$$ **Proved.**

S48. We have,

$$\cos x = -\frac{1}{3}, \quad \pi < x < \frac{3\pi}{2} \quad \Rightarrow \quad \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

i.e., $\frac{x}{2}$ lies in the first quadrant, so that sin $\frac{x}{2}$ > 0, cos $\frac{x}{2}$ < 0 and tan $\frac{x}{2}$ < 0.

Now,
$$\sin \frac{x}{2} = +\sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1-\left(-\frac{1}{3}\right)}{2}} = \sqrt{\frac{2}{3}} = \frac{\sqrt{6}}{3}$$

$$\cos \frac{x}{2} = -\sqrt{\frac{1+\cos x}{2}} = -\sqrt{\frac{1-\frac{1}{3}}{2}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\sqrt{\frac{2}{3}}}{-\frac{1}{\sqrt{3}}} = -\sqrt{2}$$ **Ans.**

S49. We have,

$$\tan x = -\frac{4}{3}, \quad \frac{\pi}{2} < x < \pi \quad \Rightarrow \quad \frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$$

i.e., $\frac{x}{2}$ lies in the first quadrant, so that all *t*-ratios of $\frac{x}{2}$ are positive.

Also,
$$\cos^2 x = \frac{1}{\sec^2 x} = \frac{1}{1+\tan^2 x} = \frac{1}{1+\left(\frac{16}{9}\right)} = \frac{9}{25}$$

cos x is negative in the second quadrant.

$\Rightarrow$
$$\cos x = -\frac{3}{5}$$

$\therefore$
$$\sin \frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1-\left(-\frac{3}{5}\right)}{2}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\cos \frac{x}{2} = \sqrt{\frac{1+\cos x}{2}} = \sqrt{\frac{1-\frac{3}{5}}{2}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\tan \frac{x}{2} = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \frac{\frac{2}{\sqrt{5}}}{\frac{1}{\sqrt{5}}} = 2$$ **Ans.**

S50.

We have, $\cos\phi = \dfrac{1-\tan^2\dfrac{\phi}{2}}{1+\tan^2\dfrac{\phi}{2}} = \dfrac{1-\dfrac{1+e}{1-e}\tan^2\dfrac{\theta}{2}}{1+\dfrac{1+e}{1-e}\tan^2\dfrac{\theta}{2}}$ $\left[\because \tan\dfrac{\phi}{2} = \sqrt{\dfrac{1+e}{1-e}}\tan\dfrac{\theta}{2}\right]$

$$= \frac{1-e-\tan^2\dfrac{\theta}{2}-e\tan^2\dfrac{\theta}{2}}{1-e+\tan^2\dfrac{\theta}{2}+e\tan^2\dfrac{\theta}{2}}$$

$\Rightarrow$ $\cos\phi = \dfrac{1-\tan^2\dfrac{\theta}{2}-e\left[1+\tan^2\dfrac{\theta}{2}\right]}{1+\tan^2\dfrac{\theta}{2}-e\left[1-\tan^2\dfrac{\theta}{2}\right]} = \dfrac{\cos^2\dfrac{\theta}{2}-\sin^2\dfrac{\theta}{2}-e\left[\cos^2\dfrac{\theta}{2}+\sin^2\dfrac{\theta}{2}\right]}{\cos^2\dfrac{\theta}{2}+\sin^2\dfrac{\theta}{2}-e\left[\cos^2\dfrac{\theta}{2}-\sin^2\dfrac{\theta}{2}\right]}$

$= \dfrac{\cos\theta-e}{1-e\cos\theta}$ $\left[\begin{array}{l}\because \cos 2A = \cos^2 A - \sin^2 A \\ \Rightarrow \cos A = \cos^2\dfrac{A}{2}-\sin^2\dfrac{A}{2}\end{array}\right]$

= R.H.S. **Proved.**

S51. Let us consider, $\cos 60° = \cos 3\,(20°)$

$\Rightarrow$ $\cos 60° = 4\cos^3 20° - 3\cos 20°$ $[\because \cos 3\theta = 4\cos^3\theta - 3\cos\theta]$

$\Rightarrow$ $\dfrac{1}{2} = 4\cos^3 20° - 3\cos 20°$

$\Rightarrow$ $4\cos^3 20° = \dfrac{1}{2} + 3\cos 20°$ ... (i)

Also, $\cos 120° = \cos(3\times 40°)$

$\Rightarrow$ $\cos 120° = 4\cos^3 40° - 3\cos 40°$

$\Rightarrow$ $-\dfrac{1}{2} = 4\cos^3 40° - 3\cos 40°$

$\left[\because \cos 120° = (\cos 180° - 60°) = -\cos 60° = -\dfrac{1}{2}\right]$

$\Rightarrow$ $4\cos^3 40° = 3\cos 40° - \dfrac{1}{2}$ ... (ii)

Now, L.H.S. = $4(\cos^3 20° + \cos^3 40°)$ ... (iii)

Putting values of $4\cos^3 20°$ and $4\cos^3 40°$ from (i) and (ii) respectively in (3), we get

$$= \left(\frac{1}{2} + 3\cos 20^3 + 3\cos 40° - \frac{1}{2}\right)$$

= $3(\cos 20° + \cos 40°)$ = R.H.S. **Proved.**

S52. Let, sec $(\phi + \alpha)$, sec ϕ and sec $(\phi - \alpha)$, are in A.P.

$$\Rightarrow \quad 2 \sec \phi = \sec(\phi + \alpha) + \sec(\phi - \alpha)$$

$$= \frac{1}{\cos(\phi + \alpha)} + \frac{1}{\cos(\phi - \alpha)}$$

$$= \frac{\cos(\phi - \alpha) + \cos(\phi + \alpha)}{\cos(\phi + \alpha)\cos(\phi - \alpha)}$$

$$\Rightarrow \quad \frac{2}{\cos\phi} = \frac{\cos(\phi - \alpha) + \cos(\phi + \alpha)}{\cos^2\phi - \sin^2\alpha} \quad [\cos(A - B)\cos(A + B) = \cos^2 A - \sin^2 B]$$

$$\Rightarrow \quad \frac{2}{\cos\phi} = \frac{2\cos\phi\sin\alpha}{\cos^2\phi - \sin^2\alpha} \quad \Rightarrow \quad \cos^2\phi\cos\alpha = \cos^2\phi - \sin^2\alpha$$

$$\Rightarrow \quad \cos^2\phi(\cos\alpha - 1) = -\sin^2\alpha \quad \Rightarrow \quad \cos^2\phi(1 - \cos\alpha) = \sin^2\alpha$$

$$\Rightarrow \quad \cos^2\phi(1 - \cos\alpha) = 1 - \cos^2\alpha \quad \Rightarrow \quad \cos^2\phi = 1 + \cos\alpha$$

$$\Rightarrow \quad \cos^2\phi = 1 + 2\cos^2\frac{\alpha}{2} - 1 \quad \Rightarrow \quad \cos\phi = \pm\sqrt{2\cos^2\frac{\alpha}{2}}$$

S53.

$$\text{L.H.S.} = \left(1 + \cos\frac{\pi}{10}\right)\left(1 + \cos\frac{3\pi}{10}\right)\left(1 + \cos\frac{7\pi}{10}\right)\left(1 + \cos\frac{9\pi}{10}\right)$$

$$= (1 + \cos 18°)(1 + \cos 54°)(1 + \cos 126°)(1 + \cos 162°)$$

$$= (1 + \cos 18°)(1 + \cos 54°)(1 + \cos(180° - 54°))(1 + \cos(180° - 18°))$$

$$= (1 - \cos 18°)(1 + \cos 54°)(1 - \cos 54°)(1 - \cos 18°)$$

$$= (1 - \cos^2 18°)(1 - \cos^2 54°) = \sin^2 18° \sin^2 54° = \sin^2 18° [\sin(90° - 36°)]^2$$

$$= \sin^2 18° \cos^2 36°$$

$$= \left(\frac{\sqrt{5} - 1}{4}\right)^2 \left(\frac{\sqrt{5} + 1}{4}\right)^2$$

$$= \left(\frac{\sqrt{5} - 1}{4} \times \frac{\sqrt{5} + 1}{4}\right)^2 = \left(\frac{5 - 1}{16}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16} = \text{RHS.}$$

Proved.

S54. (i)

$$\text{L.H.S.} = \sin^2 72° - \sin^2 60°$$

$$= \{\sin(90° - 18°)\}^2 - (\sin 60°)^2 = (\cos 18°)^2 - (\sin^2 60°)$$

$$= \left(\frac{\sqrt{10 + 2\sqrt{5}}}{4}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 \quad \left[\because \cos 18° = \frac{\sqrt{10 + 2\sqrt{5}}}{4}\right]$$

$$= \frac{10+2\sqrt{5}}{16} - \frac{3}{4} = \frac{10+2\sqrt{5}-12}{16} = \frac{2\sqrt{5}-2}{16} = \frac{\sqrt{5}-1}{8} = \text{R.H.S.}$$ **Proved.**

(ii) $$\text{L.H.S.} = \sin\frac{\pi}{5}\sin\frac{2\pi}{5}\sin\frac{3\pi}{5}\sin\frac{4\pi}{5} = \sin\frac{\pi}{5}\sin\frac{2\pi}{5}\sin\left(\pi-\frac{2\pi}{5}\right)\sin\left(\pi-\frac{\pi}{5}\right)$$

$$= \sin\frac{\pi}{5}\sin\frac{2\pi}{5}\cdot\sin\frac{2\pi}{5}\cdot\sin\frac{\pi}{5} = \left(\sin\frac{\pi}{5}\cdot\sin\frac{2\pi}{5}\right)^2$$

$$= (\sin 36°\ \sin 72°)^2 \qquad \left[\because\ \frac{\pi}{5} = 36° \text{ etc.}\right]$$

$$= (\sin 36°\ .\ \cos 18°)^2 \qquad \left[\begin{aligned}\because\ \sin 72° &= \sin(90°-18°)\\ &= \cos 18°\end{aligned}\right]$$

$$= \left(\frac{\sqrt{10-2\sqrt{5}}}{4}\cdot\frac{\sqrt{10+2\sqrt{5}}}{4}\right)^2$$

$$= \frac{(10-2\sqrt{5})(10+2\sqrt{5})}{16\times16} = \frac{100-20}{16\times16} = \frac{80}{16\times16} = \frac{5}{16} = \text{R.H.S.}$$ **Proved.**

S55. (i) We have, $\sin 75° = \sin(45° + 30°)$

$$= \sin 45° \cos 30° + \cos 45° \sin 30°$$

$[\sin(A+B) = \sin A\cos B + \cos A \sin B]$

$$= \frac{1}{2}\times\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}\cdot\sqrt{3}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}.$$ **Ans.**

(ii) $\tan 15° = \tan(45° - 30°)$

$$= \frac{\tan 45° - \tan 30°}{1+\tan 45° \tan 30°} \qquad \left[\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}\right]$$

$$= \frac{1-\frac{1}{\sqrt{3}}}{1+1\times\frac{1}{\sqrt{3}}} \qquad \left[\begin{aligned}\because\ \tan 45° &= 1\\ \tan 45° &= \frac{1}{\sqrt{3}}\end{aligned}\right]$$

$$= \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}-1)^2}{3-1} = \frac{3+1-2\sqrt{3}}{2} = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}.$$ **Ans.**

S56. (i) $$\frac{\sin^2 A}{\cos^2 A}+\frac{\cos^2 A}{\sin^2 A}=\frac{\sin^2 A+\cos^2 A}{\sin^2 A\cos^2 A}$$

$$= \frac{(\sin^2 A)^2+(\cos^2 A)^2+2\sin^2 A\cos^2 A-2\sin^2 A\cdot\cos^2 A}{\sin^2 A\cos^2 A}$$

$$= \frac{(\sin^2 A+\cos^2 A)^2-2\sin^2 A\cos^2 A}{\sin^2 A\cos^2 A}=\frac{1-2\sin^2 A\cos^2 A}{\sin^2 A\cos^2 A}$$

$$= \frac{1}{\sin^2 A\cos^2 A}-\frac{2\sin^2 A\cos^2 A}{\sin^2 A\cos^2 A}$$

$= \sec^2 A\ \text{cosec}^2 A - 2 =$ R.H.S. **Proved.**

(ii) L.H.S. $= \sec^4 A\,(1-\sin^4 A) - 2\tan^2 A$

$$= \sec^4 A - \frac{\sin^4 A}{\cos^4 A} - 2\tan^2 A$$

$= \sec^4 A - \tan^4 A - 2\tan^2 A$

$= (1+\tan^2 A)^2 - \tan^4 A - 2\tan^2 A$

$= 1 + 2\tan^2 A + \tan^4 A - \tan^4 A - 2\tan^2 A = 1 =$ R.H.S. **Proved.**

S57. L.H.S. $= \sin 30°\,(\sin 10°\sin 50°)\sin 70° = \frac{1}{2}\cdot\frac{1}{2}[2\sin 10°\sin 50°]\sin 70°$

$= \frac{1}{4}[\cos(50°-10°)-\cos(50°+10°)]\sin 70°$

$[\because\ 2\sin A\sin B = \cos(A-B)-\cos(A+B)]$

$= \frac{1}{4}(\cos 40° - \cos 60°)\sin 70° = \frac{1}{4}[\sin 70°\cos 40° - \sin 70°\cos 60°]$

$= \frac{1}{4}[\sin(70°+40°)+\sin(70°-40°)-\sin 70°]$

$[\because\ 2\sin A\sin B = \sin(A+B)+\sin(A+B)]$

$= \frac{1}{4}[\sin 110° + \sin 30° - \sin 70°] = \frac{1}{4}[\sin(180°-70°)+\sin 30° - \sin 70°]$

$= \frac{1}{8}\left[\sin 70° + \frac{1}{2} - \sin 70°\right] = \frac{1}{8}\times\frac{1}{2} = \frac{1}{16}$ = R.H.S. **Proved.**

S58. We have,

L.H.S. $= \sin 20°\sin 40°\sin 80°$

$= \frac{1}{2}[2\sin 20°\sin 40°]\cdot\sin 80°$

$$= \frac{1}{2}\ [\cos(20° - 40°) - \cos(20° + 40°)] \sin 80°$$

$[\because\ 2 \cos A \cos B = \cos(A - B) - \cos(A + B)]$

$$= \frac{1}{2}\ [\cos - (-20°) - \cos 60°] \sin 80° = \frac{1}{2}\left(\cos 20° - \frac{1}{2}\right) \sin 80°$$

$$= \frac{1}{4}\ (2 \cos 20° \sin 80° - \sin 80°) = \frac{1}{4}\ [\sin 100° - \sin(-60°) - \sin 80°]$$

$$= \frac{1}{4}\ [\sin(180° - 80°) + \sin 60° - \sin 80°] = \frac{1}{4}\ [\sin 80° + \sin 60° - \sin 80°]$$

$[\because\ \sin(180 - \theta) = \sin\theta]$

$$= \frac{1}{4} \times \sin 60° = \frac{1}{4} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8} = \text{R.H.S.}$$ **Proved.**

S59. We have,

$$\text{L.H.S.} = \sin(150° + x) + \sin(150° - x)$$

$$= 2 \sin\left(\frac{150° + x + 150° - x}{2}\right) - \cos\left(\frac{150° + x - 150° + x}{2}\right)$$

$$= 2 \sin 150° \cdot \cos x \qquad \left[\because\ \sin C + \sin D = 2 \sin\frac{C + D}{2} \cos\frac{C - D}{2}\right]$$

$$= 2 \sin(180° - 30°) \cdot \cos x = 2 \sin 30° \cdot \cos x \qquad [\because\ \sin(180° - \theta) = \sin\theta]$$

$$= 2 \times \frac{1}{2} \cdot \cos x = \cos x \qquad \left[\because\ \sin 30° = \frac{1}{2}\right]$$

$$\text{L.H.S.} = \cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right)$$

S60.

$$\text{L.H.S.} = \cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right)$$

$$= 2 \sin\left(\frac{\frac{3\pi}{4} + x + \frac{3\pi}{4} - x}{2}\right) \sin\left(\frac{\frac{3\pi}{4} - x - \frac{3\pi}{4} - x}{2}\right)$$

$$\left[\because\ \cos C - \cos D = 2 \sin\frac{C + D}{2} \sin\frac{D - C}{2}\right]$$

$$= 2 \sin\left(\frac{3\pi}{4}\right) \cdot \sin(-x) = -2 \sin\left(\pi - \frac{\pi}{4}\right) \cdot \sin x \qquad [\because\ \sin(-\theta) = -\sin\theta]$$

$$= -2 \times \frac{\pi}{4} \cdot \sin x \qquad [\because\ \sin(\pi - \theta) = \sin\theta]$$

$$= -2 \times \frac{1}{\sqrt{2}} \sin x = -\sqrt{2} \sin x = \text{R.H.S.}$$ **Proved.**

S61. Since, $\cos x = -\dfrac{3}{5}$, we have $\sec x = -\dfrac{5}{3}$

Now, $\sin^2 x + \cos^2 x = 1$, *i.e.*, $\sin^2 x = 1 - \cos^2 x$

or $$\sin^2 x = 1 - \frac{9}{25} = \frac{16}{25}$$

Hence, $$\sin x = \pm \frac{4}{5}$$

Since x lies in third quadrant, $\sin x$ is negative. Therefore

$$\sin x = -\frac{4}{5}$$

which also gives $$\text{cosec } x = -\frac{5}{4}$$

Further, we have $$\tan x = \frac{\sin x}{\cos x} = \frac{4}{3} \quad \text{and} \quad \cot x = \frac{\cos x}{\sin x} = \frac{3}{4}.$$

S62. Given: x lies in third quadrant

and $$\cos x = -\frac{1}{2}$$

$\Rightarrow$ $$\sec x = -2$$

Now, $\sin^2 x + \cos^2 x = 1$ or $\sin^2 x = 1 - \cos^2 x$

or $$\sin^2 x = 1 - \left(\frac{-1}{2}\right)^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

$$\sin x = \pm \frac{\sqrt{3}}{2}$$

Since, x lies in third quadrant, $\sin x$ is negative

or $$\sin x = -\frac{\sqrt{3}}{2}$$

which also gives $$\text{cosec } x = -\frac{2}{\sqrt{3}}$$

Further, we have $$\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}$$

$\therefore$ $$\cot x = \frac{1}{\tan x} = \frac{1}{\sqrt{3}}.$$

S63. We know that, $\sin(x + y) = \sin x \cos y + \cos x \sin y$ ... (i)

Now, $\cos^2 x = 1 - \sin^2 x = 1 - \frac{9}{25} = \frac{16}{25}$

Therefore, $\cos x = \pm\frac{4}{5}$

Since x lies in second quadrant, $\cos x$ is negative.

Hence, $\cos x = -\frac{4}{5}$

Now, $\sin^2 y = 1 - \cos^2 y = 1 - \frac{144}{169} = \frac{25}{169}$

i.e., $\sin y = \pm\frac{5}{13}$

Since y lies in second quadrant, hence $\sin y$ is positive. Therefore, $\sin y = \frac{5}{13}$. Substituting the values of $\sin x$, $\sin y$, $\cos x$ and $\cos y$ in Eq. (i), we get

$$\sin(x + y) = \frac{3}{5}\times\left(-\frac{12}{13}\right) + \left(-\frac{4}{5}\right)\times\frac{5}{13}$$

$$= -\frac{36}{65} - \frac{20}{65} = -\frac{56}{65}.$$

S64. Since, $\cot x = -\frac{5}{12}$, we have $\tan x = -\frac{12}{5}$

Now, $\sec^2 x = 1 + \tan^2 x = 1 + \frac{144}{25} = \frac{169}{25}$

Hence, $\sec x = \pm\frac{13}{5}$

Since x lies in second quadrant, $\sec x$ will be negative. Therefore

$$\sec x = -\frac{13}{5}$$

which also gives $\cos x = -\frac{5}{13}$

Further, we have $\sin x = \tan x \cos x = \left(-\frac{12}{5}\right)\times\left(-\frac{5}{13}\right) = \frac{12}{13}$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{13}{12}.$$

S65. Given: x lies in second quadrant

and $\sin x = \frac{3}{5}$

$\Rightarrow$ $\operatorname{cosec} x = \frac{5}{3}$

Now, $\sin^2 x + \cos^2 x = 1$ or $\cos^2 x = 1 - \sin^2 x$

or $$\cos^2 x = 1 - \left(\frac{3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{16}{25}$$

$\Rightarrow$ $$\cos x = \pm\frac{4}{5}$$

Since, x lies in second quadrant

$\therefore$ $$\cos x = -\frac{4}{5}$$

Which also gives $$\sec x = -\frac{5}{4}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{3}{5}}{-\frac{4}{5}} = -\frac{3}{4}$$

So, $$\cot x = \frac{1}{-\frac{3}{4}} = -\frac{4}{3}.$$

S66. Given: x lies in third quadrant

and $$\cot x = \frac{3}{4}$$

$\Rightarrow$ $$\tan x = \frac{4}{3}$$

Now, $1 + \tan^2 x = \sec^2 x$

or $$\sec^2 x = 1 + \left(\frac{4}{3}\right)^2 = 1 + \frac{16}{9} = \frac{25}{9}$$

$$\sec x = \pm\frac{5}{3}$$

Since, x lies in third quadrant,

We get $$\sec x = -\frac{5}{3}$$

Which also gives $$\cos x = -\frac{3}{5}$$

Further, we have $$\tan x = \frac{\sin x}{\cos x}$$

$\Rightarrow$ $$\sin x = \tan x \cdot \cos x$$

or $\sin x = \frac{4}{3} \times \left(-\frac{3}{5}\right) = -\frac{4}{5}$

Which also gives $\operatorname{cosec} x = \frac{1}{\sin x} = -\frac{5}{4}$.

S67. Given: *x* lies in fourth quadrant

$$\sec x = \frac{13}{5}$$

$\Rightarrow$ $\cos x = \frac{5}{13}$

Now, $\sin^2 x + \cos^2 x = 1$ or $\sin^2 x = 1 - \cos^2 x$

or $\sin^2 x = 1 - \frac{25}{169} = \frac{144}{169}$

$$\sin x = \pm\frac{12}{13}$$

Since, *x* lies in fourth quadrant,

$\therefore$ $\sin x = -\frac{12}{13}$

which also gives $\operatorname{cosec} x = -\frac{13}{12}$

Further, we have $\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{12}{13}}{\frac{5}{13}} = -\frac{12}{5}$

Which gives $\cot x = -\frac{5}{12}$.

S68. Given: *x* lies in second quadrant

$$\tan x = \frac{-5}{12}$$

$\therefore$ $\cot x = \frac{-12}{5}$

Also, $\sec^2 x = 1 + \tan^2 x$

$\Rightarrow$ $\sec^2 x = 1 + \frac{25}{144} = \frac{169}{144}$

$\Rightarrow$ $\sec x = \pm\frac{13}{12}$

As x lies in second quadrant, so sec x is negative

$\Rightarrow$ $$\sec x = \frac{-13}{12}$$

$\therefore$ $$\cos x = \frac{-12}{13}$$

Now as, $$\tan x = \frac{\sin x}{\cos x}$$

$\Rightarrow$ $$\sin x = \cos x \cdot \tan x$$

$$\sin x = \frac{-12}{13} \times \left(\frac{-5}{12}\right) = \frac{5}{13}$$

$\therefore$ $$\operatorname{cosec} x = \frac{13}{5}.$$

S69. (i) We have
$$\sin 75° = \sin(45° + 30°)$$
$$= \sin 45° \cos 30° + \cos 45° \sin 30°$$
$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$
$$= \frac{(\sqrt{3}+1)\sqrt{2}}{4} = \frac{1}{4}(\sqrt{2}+\sqrt{6})$$

(ii)
$$\tan 15° = \tan(45° - 30°)$$
$$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \cdot \tan 30°}$$
$$= \frac{1 - \frac{1}{\sqrt{3}}}{1 + (1)\left(\frac{1}{\sqrt{3}}\right)} = \frac{\frac{\sqrt{3}-1}{\sqrt{3}}}{\frac{\sqrt{3}+1}{\sqrt{3}}}$$
$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$
$$= \frac{(\sqrt{3})^2 - 2\sqrt{3} + (1)^2}{3-1}$$
$$= \frac{3 - 2\sqrt{3} + 1}{2} = \frac{4 - 2\sqrt{3}}{2}$$
$$= \frac{2(2-\sqrt{3})}{2} = 2 - \sqrt{3}.$$

S70. (i)
$$\text{L.H.S.} = \frac{\cos\theta}{1-\sin\theta} + \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta + 1 - 2\sin\theta + \sin^2\theta}{\cos\theta\,(1-\sin\theta)} = \frac{\cos^2\theta + \sin^2\theta + 1 - 2\sin\theta}{\cos\theta\,(1-\sin\theta)}$$

$$= \frac{1 + 1 - 2\sin\theta}{\cos\theta\,(1-\sin\theta)} = \frac{2\,(1-\sin\theta)}{\cos\theta\,(1-\sin\theta)} = 2\sec\theta = \text{R.H.S.}$$ **Proved.**

(ii) $$\text{L.H.S.} = \left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2 = \frac{(1-\sin\theta)^2 + \cos^2\theta - 2\cos\theta\,(1+\sin\theta)}{(1+\sin\theta)^2 + \cos^2\theta + 2\cos\theta\,(1+\sin\theta)}$$

$$= \frac{1 + 2\sin\theta + \sin^2\theta + \cos^2\theta - 2\cos\theta\,(1+\sin\theta)}{1 + 2\sin\theta + \sin^2\theta + \cos^2\theta + 2\cos\theta\,(1+\sin\theta)}$$

$$= \frac{2\sin\theta + 2 - 2\cos\theta\,(1+\sin\theta)}{2\sin\theta + 2 + 2\cos\theta\,(1+\sin\theta)} = \frac{(1+\sin\theta) - \cos\theta\,(1+\sin\theta)}{(1+\sin\theta) + \cos\theta\,(1+\sin\theta)}$$

$$= \frac{(1+\sin\theta)(1-\cos\theta)}{(1+\sin\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\sin\theta} = \text{R.H.S.}$$ **Proved.**

S71. We have,

$$4\sin x\cos x + 2\sin x + 2\cos x + 1 = 0$$

$\Rightarrow$ $2\sin x\,(2\cos x + 1) + 1\,(2\cos x + 1) = 0$

$\Rightarrow$ $(2\sin x + 1)\,(2\cos x + 1) = 0$

$\Rightarrow$ $2\sin x + 1 = 0$ or $2\cos x = 0$

$\Rightarrow$ $\sin x = -\frac{1}{2}$ or $\cos x = -\frac{1}{2}$

Now, $\sin x = -\frac{1}{2} \Rightarrow \sin x = \sin\left(-\frac{\pi}{6}\right) \Rightarrow x = -\frac{\pi}{6}$

The general solution of this is

$$x = n\pi + (-1)^n\left(-\frac{\pi}{6}\right) = n\pi + (-1)^{n+1}\left(\frac{\pi}{6}\right)$$

$\Rightarrow$ $$x = \pi\left[n + \frac{(-1)^{n+1}}{6}\right] \quad \text{... (i)}$$

And $\cos x = -\frac{1}{2} \Rightarrow \cos x = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3} \Rightarrow x = \frac{2\pi}{3}$

The general solution of this is

$$x = 2n\pi \pm \frac{2\pi}{3} \quad \textit{i.e.,} \quad x = 2\pi\left(n \pm \frac{1}{3}\right) \quad \text{... (ii)}$$

From Eq. (i) and (ii), we have

$\pi\left[n + \frac{(-1)^{n+1}}{6}\right]$ and $2\pi\left(n \pm \frac{1}{3}\right)$ are the required solutions. **Ans.**

S72.

$$\text{L.H.S.} = \cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{5\pi}{15}\cos\frac{6\pi}{15}\cos\frac{7\pi}{15}$$

$$= \cos\frac{\pi}{15}\cos\frac{2\pi}{15}\left(\frac{\sqrt{5}+1}{4}\right)\cos\frac{4\pi}{15}\left(\frac{1}{2}\right)\cos\left(\frac{\pi}{2}-\frac{\pi}{10}\right)\cos\frac{7\pi}{15}$$

$$\left[\because\ \frac{3\pi}{15} = 36^\circ,\quad \frac{5\pi}{15} = 60^\circ\right]$$

$$= \frac{1}{2}\left(\frac{\sqrt{5}+1}{4}\right)\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\sin\frac{\pi}{10}\cos\frac{7\pi}{15}$$

$$= \frac{1}{2}\left(\frac{\sqrt{5}+1}{4}\right)\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\left(\frac{\sqrt{5}-1}{4}\right)\cos\frac{7\pi}{15}$$

$$= \frac{1}{2}\left(\frac{5-1}{16}\right)\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\times\cos\frac{4\pi}{15}\cos\frac{7\pi}{15}$$

$$= \frac{1}{8}\left(\frac{1}{2}\right)\left(2\cos\frac{\pi}{15}\cdot\cos\frac{4\pi}{15}\right)\frac{1}{2}\left(2\cos\frac{2\pi}{15}\cos\frac{7\pi}{15}\right)$$

$$= \frac{1}{32}\left[\left(\cos\frac{5\pi}{15}+\cos\frac{3\pi}{15}\right)\left(\cos\frac{9\pi}{15}+\cos\frac{5\pi}{15}\right)\right]$$

$$= \frac{1}{32}\left(\frac{1}{2}+\frac{\sqrt{5}+1}{4}\right)\left(\cos\left(\frac{\pi}{2}+\frac{\pi}{10}\right)+\frac{1}{2}\right) = \frac{1}{32}\left(\frac{1}{2}+\frac{\sqrt{5}+1}{4}\right)\left(-\sin\frac{\pi}{10}+\frac{1}{2}\right)$$

$$= \frac{1}{32}\left(\frac{1}{2}+\frac{\sqrt{5}+1}{4}\right)\left(-\frac{\sqrt{5}+1}{4}+\frac{1}{2}\right) = \frac{1}{512}[2+\sqrt{5}+1][-\sqrt{5}+1+2]$$

$$= \frac{1}{512}(3+\sqrt{5})(3-\sqrt{5}) = \frac{1}{512}(9-5) = \frac{1}{128} = \text{RHS.}$$ **Proved.**

S73. (i)

$$\text{L.H.S.} = \sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}} = \sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}\times\frac{1+\cos\alpha}{1+\cos\alpha}} = \sqrt{\frac{(1+\cos\alpha)^2}{1-\cos^2\alpha}}$$

$$= \sqrt{\frac{(1+\cos\alpha)^2}{\sin^2\alpha}} = \frac{1+\cos\alpha}{\sin\alpha} = \frac{1}{\sin\alpha}+\frac{\cos\alpha}{\sin\alpha}$$

$$= \text{cosec}\,\alpha + \cot\alpha = \text{R.H.S.}$$ **Proved.**

(ii) $$\text{L.H.S.} = \sqrt{\frac{1+\sin\alpha}{1-\sin\alpha}} = \sqrt{\frac{(1+\sin\alpha)(1+\sin\alpha)}{(1-\sin\alpha)(1+\sin\alpha)}} = \sqrt{\frac{(1+\sin\alpha)^2}{1-\sin^2\alpha}}$$

$$= \sqrt{\frac{(1+\sin\alpha)^2}{\cos^2\alpha}} = \frac{1+\sin\alpha}{\cos\alpha} = \frac{1}{\cos\alpha} + \frac{\sin\alpha}{\cos\alpha}$$

$= \sec\alpha + \tan\alpha =$ R.H.S. **Proved.**

S74.

(i) $$\text{L.H.S.} = \frac{\cot A + \tan B}{\cot B + \tan A} = \frac{\dfrac{\cos A}{\sin A} + \dfrac{\sin B}{\cos B}}{\dfrac{\cos B}{\sin B} + \dfrac{\sin A}{\cos A}} = \frac{\dfrac{\cos A \cos B + \sin A \sin B}{\sin A \cos B}}{\dfrac{\cos A \cos B + \sin A \sin B}{\sin B \cos A}}$$

$$= \frac{\cos A \cos B + \sin A \sin B}{\sin A \cos B} \times \frac{\sin B \cos A}{\cos A \cos B + \sin A \sin B}$$

$$= \frac{\sin B \cos A}{\sin A \cos B} = \left(\frac{\cos A}{\sin B}\right)\left(\frac{\sin B}{\cos B}\right) = \cot A \tan B = \text{R.H.S.}$$ **Proved.**

(ii) $$\text{L.H.S.} = \tan^2 A - \tan^2 B = \frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}$$

$$= \frac{\sin^2 A \cos^2 B - \cos^2 A \sin^2 B}{\cos^2 A \cos^2 B}$$

$$= \frac{\sin^2 A\,(1-\sin^2 B) - (1-\sin^2 A)\sin^2 B}{\cos^2 A \cos^2 B}$$

$$= \frac{\sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 A}{\cos^2 A \cos^2 B}$$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A \cos^2 B} = \text{R.H.S.}$$ **Proved.**

S75. $$\text{L.H.S.} = \frac{(1+\cot A + \tan A)(\sin A - \cos A)}{\sec^3 A - \operatorname{cosec}^3 A}$$

$$= \frac{\left(1+\dfrac{\cos A}{\sin A} + \dfrac{\sin A}{\cos A}\right)(\sin A - \cos A)}{(\sec A - \operatorname{cosec} A)(\sec^2 A + \sec A \operatorname{cosec} A + \operatorname{cosec}^2 A)}$$

$[\because\ a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$

$$= \frac{\dfrac{(\sin A \cos A + \sin^2 A + \sin^2 A)(\sin A - \cos A)}{\sin A \cos A}}{(\sec A - \operatorname{cosec} A)\left(\dfrac{1}{\cos^2 A} + \dfrac{1}{\cos A \sin A} + \dfrac{1}{\sin^2 A}\right)}$$

$$= \frac{(\sin A \cos A + 1)\left(\frac{\sin A}{\sin A \cos A} - \frac{\cos A}{\sin A \cos A}\right)}{(\sec A - \operatorname{cosec} A)\left(\frac{\sin^2 A + \sin^2 A \cos A + \cos^2 A}{\sin^2 A \cos^2 A}\right)}$$

$$= \frac{(\sin A \cos A + 1)(\sec A - \operatorname{cosec} A)}{(\sec A - \operatorname{cosec} A)(1 + \sin A \cos A)} \times \sin^2 A \cos^2 A \qquad [\sin^2\theta + \cos^2\theta = 1]$$

$= \sin^2 A \cos^2 A$ = R.H.S. **Proved.**

S76. L.H.S. $= \frac{\sin\theta + 1 - \cos\theta}{\cos\theta - 1 + \sin\theta}$

Dividing numerator and denominator by $\cos\theta$, we get

$$= \frac{\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} - \frac{\cos\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta} - \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}} = \frac{\tan\theta + \sec\theta - 1}{1 - \sec\theta + \tan\theta}$$

Using formula $\sec^2\theta - \tan^2\theta = 1$, we get

$$= \frac{\tan\theta + \sec\theta - (\sec^2\theta - \tan^2\theta)}{1 - \sec\theta + \tan\theta}$$

$$= \frac{(\tan\theta + \sec\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{1 - \sec\theta + \tan\theta}$$

$$= \frac{(\tan\theta + \sec\theta)[1 - (\sec\theta - \tan\theta)]}{1 - \sec\theta + \tan\theta}$$

$$= \frac{(\tan\theta + \sec\theta)(1 - \sec\theta + \tan\theta)}{1 - \sec\theta + \tan\theta} = \tan\theta + \sec\theta$$

$= \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} = \frac{\sin\theta + 1}{\cos\theta} = \frac{1 + \sin\theta}{\cos\theta}$ = R.H.S. **Proved.**

S77. L.H.S. $= \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}$ ... (i)

Putting $\tan\beta = \frac{n \sin\alpha \cos\alpha}{1 - n\sin^2\alpha}$ in (i), we get

$$\text{L.H.S.} = \frac{\frac{\sin\alpha}{\cos\alpha} - \frac{n\sin\alpha\cos\alpha}{1 - n\sin^2\alpha}}{1 + \frac{\sin\alpha}{\cos\alpha}\cdot\frac{n\sin\alpha\cos\alpha}{1 - n\sin^2\alpha}} \qquad \left[\because\ \tan\alpha = \frac{\sin\alpha}{\cos\alpha}\right]$$

$$= \frac{\sin\alpha\,(1 - n\sin^2\alpha) - n\sin\alpha\cos^2\alpha}{\cos\alpha\,(1 - n\sin^2\alpha) + n\sin^2\alpha\cos\alpha}$$

$$= \frac{\sin\alpha - n\sin^3\alpha - n\sin\alpha\cos^2\alpha}{\cos\alpha - n\sin^2\alpha\cos\alpha + n\sin^2\alpha\cos\alpha}$$

$$= \frac{\sin\alpha - n\sin\alpha(\sin^2\alpha + \cos^2\alpha)}{\cos\alpha} = \frac{\sin\alpha - n\sin\alpha}{\cos\alpha} \qquad [\because\ \sin^2\alpha + \cos^2\alpha = 1]$$

$$= \frac{\sin\alpha\,(1-n)}{\cos\alpha} = (1-n)\tan\alpha \ = \text{RHS.}$$ **Proved.**

S78. $$\text{LHS.} = \left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{\text{cosec}^2\theta - \sin^2\theta}\right)\sin^2\theta\cdot\cos^2\theta$$

$$= \left[\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\frac{1}{\sin^2\theta} - \sin^2\theta}\right]\sin^2\theta\cos^2\theta$$

$$= \left[\frac{\cos^2\theta}{1-\cos^2\theta} + \frac{\sin^2\theta}{1-\sin^2\theta}\right]\sin^2\theta\cos^2\theta$$

$$= \left[\frac{\cos^2\theta}{(1+\cos^2\theta)(1-\cos^2\theta)} + \frac{\sin^2\theta}{(1-\sin^2\theta)(1+\sin^2\theta)}\right]\sin^2\theta\cos^2\theta$$

$$[\because\ a^2 - b^2 = (a-b)(a+b)]$$

$$= \left[\frac{\cos^2\theta}{(1+\cos^2\theta)\sin^2\theta} + \frac{\sin^2\theta}{\cos^2\theta\,(1+\sin^2\theta)}\right]\sin^2\theta\cos^2\theta$$

$$= \frac{\cos^4\theta}{1+\cos^2\theta} + \frac{\sin^4\theta}{1+\sin^2\theta} = \frac{[\cos^4\theta(1+\sin^2\theta) + \sin^4\theta(1+\cos^2\theta)]}{(1+\cos^2\theta)(1+\sin^2\theta)}$$

$$= \frac{\cos^4\theta + \sin^2\theta\cos^4\theta + \sin^4\theta + \sin^4\theta\cos^2\theta}{(1+\cos^2\theta)(1+\sin^2\theta)}$$

$$= \frac{\sin^4\theta + \cos^4\theta + \sin^2\theta\cos^2\theta\,(\cos^2\theta + \sin^2\theta)}{(1+\cos^2\theta)(1+\sin^2\theta)}$$

$$= \frac{(\sin^2\theta)^2 + (\cos^2\theta)^2 + 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta}{(1+\cos^2\theta)(1+\sin^2\theta)}$$

$$= \frac{(\sin^2\theta + \cos^2\theta)^2 - \sin^2\theta + \cos^2\theta}{1 + \sin^2\theta + \cos^2\theta + \sin^2\theta + \cos^2\theta} = \frac{1 - \sin^2\theta\cos^2\theta}{1 + 1 + \sin^2\theta\cos^2\theta}$$

$$= \frac{1 - \sin^2\theta\cos^2\theta}{2 + \sin^2\theta\cos^2\theta} \ = \text{R.H.S.}$$ **Proved.**

S79. We have, $\sin\theta = \frac{4}{5}$

We know that, $\sin^2\theta + \cos^2\theta = 1$

$$\Rightarrow \quad \cos\theta = \pm\sqrt{1-\sin^2\theta} = \pm\sqrt{1-\left(\frac{4}{5}\right)^2} = \pm\sqrt{\frac{25-16}{25}} = \pm\sqrt{\frac{9}{25}} = \pm\frac{3}{5}$$

Since, $\sin\theta$ is positive, θ lies either in I quadrant or in II quadrant.

Case 1: θ is in I quadrant.

$$\therefore \quad \cos\theta = \frac{3}{5}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}, \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{3}{4}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{5}{3}, \qquad \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{5}{4}$$

$$\therefore \quad \frac{5\cos\theta + 4\operatorname{cosec}\theta + 3\tan\theta}{4\cot\theta + 3\sec\theta + 5\sin\theta} = \frac{5\left(\frac{3}{5}\right)+4\left(\frac{5}{4}\right)+3\left(\frac{4}{3}\right)}{4\left(\frac{3}{4}\right)+3\left(\frac{5}{3}\right)+5\left(\frac{4}{5}\right)} = \frac{3+5+4}{3+5+4} = \frac{12}{12} = 1$$

Case 2: θ is in II quadrant.

$$\therefore \quad \cos\theta = -\frac{3}{5}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{4}{5}}{-\frac{3}{5}} = -\frac{4}{3}, \qquad \cot\theta = \frac{1}{\tan\theta} = -\frac{3}{4}$$

$$\sec\theta = \frac{1}{\cos\theta} = -\frac{5}{3}, \qquad \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{5}{4}$$

$$\therefore \quad \frac{5\cos\theta + 4\operatorname{cosec}\theta + 3\tan\theta}{4\cot\theta + 3\sec\theta + 5\sin\theta} = \frac{5\left(-\frac{3}{5}\right)+4\left(\frac{5}{4}\right)+3\left(-\frac{4}{3}\right)}{4\left(-\frac{3}{4}\right)+3\left(-\frac{5}{3}\right)+5\left(\frac{4}{5}\right)} = \frac{-3+5-4}{-3-5+4} = \frac{-2}{-4} = \frac{1}{2}$$

Ans.

Date: 9/10/2021

Q1. Prove that $\cos 2x = (2\cos^2 x - 1)$.

Q2. Prove that $\sin 2x = 2\sin x\cos x$.

Q3. Represent $\tan 2x$ in terms of $\tan x$.

Q4. Find the value of $2\sin 15^\circ \cdot \cos 75^\circ$.

Q5. Prove that $\dfrac{\tan x + \tan y}{\tan x - \tan y} = \dfrac{\sin(x+y)}{\sin(x-y)}$.

Q6. Prove that $\tan\left(\dfrac{\pi}{4} + x\right) = \dfrac{1+\tan x}{1-\tan x}$.

Q7. Prove that $\dfrac{\tan(A+B)}{\cot(A-B)} = \dfrac{\sin^2 A - \sin^2 B}{\cos^2 A - \sin^2 B}$.

Q8. Prove that $\dfrac{\sin 2x}{1-\cos 2x} = \cot x$.

Q9. Prove that $\dfrac{\sin A + \sin 3A}{\cos A + \cos 3A} = \tan 2A$.

Q10. Prove that $\dfrac{1-\cos 2x}{1+\cos 2x} = \tan^2 x$.

Q11. Prove that $\dfrac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ} = \tan 56^\circ$.

Q12. Prove that $\tan\left(\dfrac{\pi}{4} - x\right) = \dfrac{1-\tan x}{1+\tan x}$.

Q13. Prove that: $3\sin\dfrac{\pi}{6}\sec\dfrac{\pi}{3} - 4\sin\dfrac{5\pi}{6}\cot\dfrac{\pi}{4} = 1$.

Q14. Show that $\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$.

Q15. Prove that: $\cos\left(\dfrac{\pi}{4} + x\right) + \cos\left(\dfrac{\pi}{4} - x\right) = \sqrt{2}\cos x$.

Q16. Prove that $\dfrac{\cos 7x + \cos 5x}{\sin 7x - \sin 5x} = \cot x$.

Q17. Prove that: $\dfrac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$.

Q18. Prove that: $2\sin^2\dfrac{\pi}{6} + \operatorname{cosec}^2\dfrac{7\pi}{6}\cos^2\dfrac{\pi}{3} = \dfrac{3}{2}$.

Q19. Prove that: $\cot^2\dfrac{\pi}{6} + \operatorname{cosec}\dfrac{5\pi}{6} + 3\tan^2\dfrac{\pi}{6} = 6$.

Q20. Prove that: $2\sin^2\dfrac{3\pi}{4} + 2\cos^2\dfrac{\pi}{4} + 2\sec^2\dfrac{\pi}{3} = 10$.

Q21. Represent $\cos 2x$ in terms of $\tan x$.

Q22. Find the value of $\cos 3x$ in terms of $\cos x$.

Q23. Represent $\sin 3x$ in terms of $\sin x$.

Q24. Represent tan $3x$ in terms of tan x.

Q25. Prove that $\sin A \cdot \sin(60 - A) \cdot \sin(60 + A) = \frac{1}{4}\sin 3A$.

Q26. Represent cos $4x$ in terms of cos x and sin x.

Q27. Prove that $\cos\alpha \cdot \cos(60 - \alpha) \cdot \cos(60 + \alpha) = \frac{1}{4}\cos 3\alpha$.

Q28. Prove that $\cos A + \cos(120 - A) + \cos(120 + A) = 0$

Q29. Prove that $\sqrt{2 + \sqrt{2(1 + \cos 4\theta)}} = 2\cos\theta$.

Q30. If $f(\theta) = \tan\frac{\pi}{16} + 2\tan\frac{\pi}{8} + 4$, represent $f(\theta)$ in terms of $\cot\theta$.

Q31. Prove that $\frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \cot\left(\frac{\alpha + \beta}{2}\right)$.

Q32. Prove that $\sin\left(\frac{\pi}{4} + A\right) \cdot \sin\left(\frac{\pi}{4} - A\right) = \frac{1}{2}\cos 2A$.

Q33. If $\tan A = \frac{3}{4}$ and $\cos B = \frac{9}{41}$ where $\pi < A < \frac{3\pi}{2}$ and $0 < B < \frac{\pi}{2}$, find the value of $\tan(A + B)$.

Q34. Show that $\cos\left(\frac{\pi}{4} - \theta\right) \cdot \cos\left(\frac{\pi}{4} - \phi\right) - \sin\left(\frac{\pi}{4} - \theta\right) \cdot \sin\left(\frac{\pi}{4} - \phi\right) = \sin(\theta + \phi)$

Q35. Represent sin $2x$ in terms of tan x.

Q36. Prove that $\cos^2\left(\frac{\pi}{8}\right) + \cos^2\left(\frac{3\pi}{8}\right) + \cos^2\left(\frac{5\pi}{8}\right) + \cos^2\left(\frac{7\pi}{8}\right) = 2$

Q37. If $2\tan\alpha = 3\tan\beta$, prove that $\tan(\alpha - \beta) = \frac{\sin 2\beta}{5 - \cos 2\beta}$.

Q38. If $\sin x + \sin y = a$, $\cos x + \cos y = b$, show that: $\cos(x - y) = \frac{1}{2}(a^2 + b^2 - 2)$.

Q39. If $\tan\theta + \tan\phi = a$ and $\cot\theta + \cot\phi = b$, prove that $\cot(\theta + \phi) = \frac{1}{a} - \frac{1}{b}$.

Q40. Find the values of: $\sin 22\frac{1}{2}^\circ$.

Q41. Find the values of: $\cos 22\frac{1}{2}^\circ$.

Q42. Prove that: $\frac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta} = \tan\frac{\theta}{2}$.

Q43. Prove that: $2\tan 2x = \frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x}$

Q44. If $2\cos\theta = x + \frac{1}{x}$, prove that $2\cos 3\theta = x^3 + \frac{1}{x^3}$.

Q45. Solve the following equations:

$$\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$$

Q46. Prove that $\frac{\sin x - \sin y}{\cos x + \cos y} = \tan\left(\frac{x - y}{2}\right)$

Q47. If $\cos\alpha + \cos\beta = \sin\alpha + \sin\beta = 0$, then find the value of $\cos 2\alpha + \cos 2\beta$.

Q48. Prove that $\dfrac{\cos 4x \cdot \sin 3x - \cos 2x \cdot \sin x}{\sin 4x \cdot \sin x + \cos 6x \cdot \cos x} = \tan 2x$.

Q49. Prove that $\cos 5x = 16 \cos^5 x - 20 \cos^3 x + 5 \cos x$.

Q50. Prove that $\dfrac{2\sin(\alpha - \gamma) \cdot \cos(\gamma) - \sin(\alpha - 2\gamma)}{2\sin(\beta - \gamma) \cdot \cos(\gamma) - \sin(\beta - 2\gamma)} = \dfrac{\sin \alpha}{\sin \beta}$.

Q51. An angle α is divided into two parts such that the ratio of the tangents of the two parts is equal to k and difference of two parts is equal to x, show that $\sin x = \dfrac{k-1}{k+1} \cdot \sin \alpha$.

Q52. Let $a = \cos A + \cos \beta - \cos (A + \beta)$ and $b = 4 \sin \dfrac{A}{2} \cdot \sin \dfrac{\beta}{2} \cdot \cos \left(\dfrac{A+\beta}{2}\right)$, then find the value of $(a - b)$.

Q53. Prove that:

$$\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x .$$

Q54. Find the values of: $\sin 7\dfrac{1^\circ}{2}$.

Q55. Show that: $\cot 7\dfrac{1^\circ}{2} = \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$.

Q56. Prove that: $\cos 6x = 32 \cos^6 x - 48 \cos^4 x + 18 \cos^2 x - 1$

Q57. Prove that: $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 8\theta}}} = 2 \cos \theta$.

Q58. Prove that: $\cot A + \cot (60^\circ + A) + \cot (120^\circ + A) = 3 \cot 3A$.

Q59. If $13\alpha = \pi$, then prove that,

$$\cos \alpha \cos 2\alpha \cos 3\alpha \cos 4\alpha \cos 5\alpha \cos 6\alpha = \frac{1}{64}$$

Q60. Prove that:

$$\cos^3 \left(x - \frac{2\pi}{3}\right) + \cos^3 x + \cos^3 \left(x + \frac{2\pi}{3}\right) = \frac{3}{4} \cos 3x$$

Q61. Prove the following:

$$\cos \left(\frac{3\pi}{2} + x\right) \cos (2\pi + x) \left[\cot \left(\frac{3\pi}{2} - x\right) + \cot (2\pi + x)\right] = 1$$

Q62. Solve the following:

(i) $\tan \theta + \tan 2\theta + \tan \theta \tan 2\theta = 1$ (ii) $\tan \theta + \tan 2\theta + \tan 3\theta = \tan \theta \tan 2\theta \tan 3\theta$

Q63. Prove that:

$$(\cos \alpha + \cos \beta)^2 + (\sin \alpha + \sin \beta)^2 = 4 \cos^2 \frac{\alpha - \beta}{2}$$

Q64. Prove that:

$$\left(\frac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\frac{\sin A + \sin B}{\cos A - \cos B}\right)^n = \begin{cases} 2 \cot^n \left(\dfrac{A - B}{2}\right), & \text{if } n \text{ is even} \\ 0, & \text{if } n \text{ is odd} \end{cases}.$$

Q65. Prove that:

$$\frac{\cos A}{1 - \sin A} = \tan \left(45^\circ + \frac{A}{2}\right).$$

Q66. Prove that $\cos 2x \cos \frac{x}{2} - \cos 3x \cos \frac{9x}{2} = \sin 5x \sin \frac{5x}{2}$.

Q67. Prove that:

$$\frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \left(\frac{1 + \tan x}{1 - \tan x}\right)^2.$$

Q68. If $\tan x = \frac{3}{4}$, $\pi < x < \frac{3\pi}{2}$, find the value of $\sin \frac{x}{2}$, $\cos \frac{x}{2}$ and $\tan \frac{x}{2}$.

Q69. Find the value of $\tan \frac{\pi}{8}$.

Q70. Prove that: $\cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \cos^2\left(x - \frac{\pi}{3}\right) = \frac{3}{2}$.

Q71. Prove that: $\dfrac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)} = \cot^2 x$.

Q72. Prove that:

$$\cos\left(\frac{3\pi}{2} + x\right)\cos(2\pi + x)\left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x)\right] = 1.$$

Q73. Prove that: $\sin(n+1)x \sin(n+2)x + \cos(n+1)x \cos(n+2)x = \cos x$.

Q74. Prove that:

$$\cos\left(\frac{3\pi}{4} + x\right) - \cos\left(\frac{3\pi}{4} - x\right) = -\sqrt{2}\sin x.$$

Q75. Prove that: $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2 \frac{x - y}{2}$.

Q76. Prove that: $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2 \frac{x + y}{2}$.

Q77. Prove that: $(\sin 3x + \sin x)\sin x + (\cos 3x - \cos x)\cos x = 0$.

Q78. Prove that: $2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13} = 0$.

Q79. Prove that: $\dfrac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x$.

Q80. Prove that: $\dfrac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$.

Q81. Prove that: $\dfrac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\dfrac{\sin 2x}{\cos 10x}$.

Q82. Prove that: $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x$.

Q83. Prove that: $\cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$.

Q84. Prove that: $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$.

Q85. Find the value of (a) $\sin^2 22°30'$, (b) $\cos^2 22°30'$, (c) $\tan^2 22°30'$

Q86. Show that: $\tan 142\frac{1°}{2} = \sqrt{2} - \sqrt{3} + \sqrt{4} - \sqrt{6}$.

Q87. Prove that:

(i) $\dfrac{\cos 3\theta + 2\cos 5\theta + \cos 7\theta}{\cos\theta + 2\cos 3\theta + \cos 5\theta} = \cos 2\theta - \sin 2\theta \tan 3\theta$

(ii) $\dfrac{\cos\theta + \cos 2\theta + \cos 3\theta + \cos 4\theta}{\sin\theta + \sin 2\theta + \sin 3\theta + \sin 4\theta} = \cot \dfrac{5}{2}\theta$

Q88. Prove that:

$$\cos\alpha + \cos\beta + \cos\gamma + \cos(\alpha+\beta+\gamma) = 4\cos\frac{\alpha+\beta}{2}\cos\frac{\beta+\gamma}{2}\cos\frac{\gamma+\alpha}{2}.$$

Q89. Prove that:

$$\sin\alpha + \sin\beta + \sin\gamma + \sin(\alpha+\beta+\gamma) = 4\sin\frac{\alpha+\beta}{2}\sin\frac{\beta+\gamma}{2}\sin\frac{\gamma+\alpha}{2}.$$

Q90. If $\sin\alpha + \sin\beta = a$ and $\cos\alpha + \cos\beta = b$, find the values of

(i) $\tan\dfrac{\alpha+\beta}{2}$ (ii) $\tan\dfrac{\alpha-\beta}{2}$

Q91. Prove that:

(i) $\sin^2 6x - \sin^2 4x = \sin 10x \sin 2x$ (ii) $\cos^2 2x - \cos^2 6x = \sin 8x \sin 4x$

Q92. If $x\cos\theta = y\cos\left(\theta + \dfrac{2\pi}{3}\right) = z\cos\left(\theta + \dfrac{4\pi}{3}\right)$, then show that $xy + yz + zx = 0$.

Q93. If $\sin\theta = n\sin(\theta + 2\alpha)$, show that $\tan(\theta + \alpha) = \dfrac{1+n}{1-n}\tan\alpha$.

Q94. Prove that: $\dfrac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)} = \tan 6x$.

Q95. Prove that: $\sin x + \sin 3x + \sin 5x + \sin 7x = 4\cos x\cos 2x\sin 4x$.

Q96. Prove that: $\tan 4x = \dfrac{4\tan x(1-\tan^2 x)}{1 - 6\tan^2 x + \tan^4 x}$.

Q97. Prove that: $\cot x\cot 2x\cot 3x - \cot 3x\cot x = 1$.

Q98. Prove that: $\dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$.

Q99. Prove that: $\cot 4x(\sin 5x + \sin 3x) = \cot x(\sin 5x - \sin 3x)$.

Q100 Prove that, $\cos\theta + \cos(120° + \theta) + \cos(\theta - 120°) = 0$, and hence, deduce that $\cos^3\theta + \cos^3(120° + \theta) + \cos^3(\theta - 120°) = \dfrac{3}{4}\cos 3\theta$.

Q101 Prove that:

$$\cos\theta\cos 2\theta\cos 4\theta \ldots \cos 2^{n-1}\theta = \frac{\sin(2^n\theta)}{2^n(\sin\theta)}.$$

Q102 Prove that:

(i) $\cos 4x = 1 - 8\sin^2 x \cdot \cos^2 x$ (ii) $\tan 4\theta = \dfrac{4\tan\theta(1-\tan^2\theta)}{1 - 6\tan^2\theta + \tan^4\theta}$

Q103 Prove that: $\sin 3x + \sin 2x - \sin x = 4\sin x\cos\dfrac{x}{2}\cos\dfrac{3x}{2}$.

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S1. $\because$ $\cos(A + B) = \cos A \cdot \cos B - \sin A \sin B$

$\therefore$ $\cos 2x = \cos(x + x)$

$= \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x)$

$= 2\cos^2 x - 1.$

S2. $\because$ $\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\therefore$ $\sin 2x = \sin(x + x)$

$= \sin x \cos x + \cos x \sin x$

$= 2 \sin x \cos x.$

S3. $\because$ $\tan 2x = \tan(x + x)$

$$= \frac{\tan x + \tan x}{1 - \tan x \cdot \tan x}$$

$$\Rightarrow \quad \tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

S4. Given $2\sin 15° \cdot \cos 75°$. $[\because 2\sin A\cos B = \sin(A + B) + \sin(A - B)$

$\Rightarrow$ $2\sin 15° \cdot \cos 75° = \sin(15° + 75°) + \sin(15° - 75°)$

$\Rightarrow$ $= \sin 90° + \sin(-60°)$

$\Rightarrow$ $= \sin 90° - \sin 60°$ $(\sin(-\theta) = -\sin\theta)$

$$= 1 - \frac{\sqrt{3}}{2} = \frac{2 - \sqrt{3}}{2}$$

S5. From R.H.S.

$$\frac{\sin(x + y)}{\sin(x - y)} = \frac{\sin x \cos y + \cos x \sin y}{\sin x \cos y - \cos x \sin y}$$

Dividing numerator and denomenator by $\cos x \cdot \cos y$, we get

$$= \frac{\tan x + \tan y}{\tan x - \tan y} \quad \text{L.H.S.}$$

S6. From L.H.S.

$$\tan\left(\frac{\pi}{4} + x\right) = \frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4} \cdot \tan x}$$

$$= \frac{1+\tan x}{1-\tan x}$$

Hence, L.H.S. = R.H.S.

S7. From L.H.S.

$$\frac{\tan(A+B)}{\cot(A-B)} = \frac{\sin(A+B)\cdot\sin(A-B)}{\cos(A+B)\cdot\cos(A-B)}$$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 A - \sin^2 B}$$

Hence, L.H.S. = R.H.S.

S8. From L.H.S.

$$\frac{\sin 2x}{1-\cos 2x} = \frac{2\sin x\cos x}{2\sin^2 x}$$

$$= \cot x$$

Hence, L.H.S. = R.H.S.

S9. From L.H.S.

$$\frac{\sin A + \sin 3A}{\cos A + \cos 3A} = \frac{2\sin 2A\cdot\cos A}{2\cos 2A\cdot\cos A}$$

$$= \tan 2A \qquad \left[\frac{\sin A + \sin 3A + \sin 5A + \ldots\ldots + \sin(2n-1)A}{\cos A + \cos 3A + \cos 5A + \ldots\ldots + \cos(2n-1)A} = \tan nA\right]$$

Hence, L.H.S. = R.H.S.

S10. From L.H.S.

$$\frac{1-\cos 2x}{1+\cos 2x} = \frac{2\sin^2 x}{2\cos^2 x} = \tan^2 x \qquad [\cos 2x = 1 - 2\sin^2 x]$$

Hence, L.H.S. = R.H.S.

S11. From L.H.S. $\frac{\cos 11^\circ + \sin 11^\circ}{\cos 11^\circ - \sin 11^\circ}$, Dividing numerator and denominator both by cos 11°.

$$= \frac{1+\tan 11^\circ}{1-\tan 11^\circ} = \frac{\tan 45^\circ + \tan 11^\circ}{1-\tan 45^\circ\cdot\tan 11^\circ} \qquad \left[\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\cdot\tan B}\right]$$

$$= \tan(45^\circ + 11^\circ) = \tan 56^\circ$$

S12. From L.H.S.

$$\tan\left(\frac{\pi}{4} - x\right) = \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\cdot\tan x}$$

$$= \frac{1-\tan x}{1+\tan x}$$

Hence, L.H.S. = R.H.S.

S13. We have L.H.S. $= 3\sin\frac{\pi}{6}\sec\frac{\pi}{3} - 4\sin\frac{5\pi}{6}\cot\frac{\pi}{4}$

$$= 3\times\frac{1}{2}\times 2 - 4\sin\left(\pi-\frac{\pi}{6}\right)\times 1 = 3 - 4\sin\frac{\pi}{6}$$

$$= 3 - 4\times\frac{1}{2} = 1 \text{ R.H.S.}$$

S14. We know that $3x = 2x + x$

Therefore, $\tan 3x = \tan(2x + x)$

or $\tan 3x = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$

or $\tan 3x - \tan 3x \tan 2x \tan x = \tan 2x + \tan x$

or $\tan 3x - \tan 2x - \tan x = \tan 3x \tan 2x \tan x$

or $\tan 3x \tan 2x \tan x = \tan 3x - \tan 2x - \tan x$.

S15. L.H.S. $= \cos\left(\frac{\pi}{4}+x\right) + \cos\left(\frac{\pi}{4}-x\right)$

$$= 2\cos\left(\frac{\frac{\pi}{4}+x+\frac{\pi}{4}-x}{2}\right)\cos\left(\frac{\frac{\pi}{4}+x-\left(\frac{\pi}{4}-x\right)}{2}\right)$$

$$= 2\cos\frac{\pi}{4}\cos x = 2\times\frac{1}{\sqrt{2}}\cos x = \sqrt{2}\cos x = \text{R.H.S.}$$

S16. L.H.S. $= \frac{2\cos\frac{7x+5x}{2}\cos\frac{7x-5x}{2}}{2\cos\frac{7x+5x}{2}\sin\frac{7x-5x}{2}} = \frac{\cos x}{\sin x} = \cot x$ = R.H.S.

S17. We have L.H.S. $= \frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \frac{\sin 5x + \sin x - 2\sin 3x}{\cos 5x - \cos x}$

$$= \frac{2\sin 3x\cos 2x - 2\sin 3x}{-2\sin 3x\sin 2x} = -\frac{\sin 3x(\cos 2x - 1)}{\sin 3x\sin 2x}$$

$$= \frac{1-\cos 2x}{\sin 2x} = \frac{2\sin^2 x}{2\sin x\cos x} = \tan x = \text{R.H.S.}$$

S18. Here, L.H.S. $= 2\sin^2\frac{\pi}{6} + \operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$$= 2\sin^2\frac{\pi}{6} + \operatorname{cosec}^2\left(\pi+\frac{\pi}{6}\right)\cos^2\frac{\pi}{3}$$

$$= 2\left(\frac{1}{2}\right)^2 + \operatorname{cosec}^2\frac{\pi}{6}\left(\frac{1}{2}\right)^2$$

$$= \frac{1}{2} + (2)^2\left(\frac{1}{2}\right)^2$$

$$= \frac{1}{2} + 1 = \frac{3}{2} = \text{R.H.S.}$$

S19. Here,

$$\text{L.H.S.} = \cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6}$$

$$= \cot^2\frac{\pi}{6} + \operatorname{cosec}\left(\pi - \frac{\pi}{6}\right) + 3\tan^2\frac{\pi}{6}$$

$$= (\sqrt{3})^2 + \operatorname{cosec}\frac{\pi}{6} + 3\left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 3 + 2 + 3\left(\frac{1}{3}\right) = 3 + 2 + 1 = 6 = \text{R.H.S.}$$

S20. Here,

$$\text{L.H.S.} = 2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3}$$

$$= 2\sin^2\left(\pi - \frac{\pi}{4}\right) + 2\left(\frac{1}{\sqrt{2}}\right)^2 + 2(2)^2$$

$$= 2\sin^2\frac{\pi}{4} + 1 + 8$$

$$= 2\left(\frac{1}{\sqrt{2}}\right)^2 + 9 = 1 + 9 = 10 = \text{R.H.S.}$$

S21. $\because$

$$\cos 2x = \frac{\cos^2 x - \sin^2 x}{1}$$

$$= \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} \qquad \because \cos^2 x + \sin^2 x = 1$$

Dividing numerator and denominator by $\cos^2 x$, we get

$$\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}.$$

S22. $\because$

$$\cos 3x = \cos(2x + x)$$

$$= \cos 2x \cdot \cos x - \sin 2x \cdot \sin x \qquad [\because \sin 2x = 2\sin x\cos x]$$

$$= (2\cos^2 x - 1)\cdot\cos x - 2\sin^2 x\cos x$$

$$= 2\cos^3 x - \cos x - 2(1 - \cos^2 x)\cdot\cos x$$

$$= 4\cos^3 x - 3\cos x$$

S23. $\because$ $\sin 3x = \sin(2x + x)$

$= \sin 2x \cdot \cos x + \cos 2x \cdot \sin x$ $\quad [\because \sin 2x = 2 \sin x \cos x]$

$= 2 \sin x \cos x \cdot \cos x + (1 - 2 \sin^2 x) \cdot \sin x$

$= 2 \sin x(1 - \sin^2 x) + \sin x - 2 \sin^3 x$

$= 3 \sin x - 4 \sin^3 x$

S24. $\because$ $\tan 3x = \tan(2x + x)$

$$= \frac{\tan 2x + \tan x}{1 - \tan 2x \cdot \tan x} = \frac{\frac{2 \tan x}{1 - \tan^2 x} + \tan x}{1 - \frac{2 \tan x}{1 - \tan^2 x} \cdot \tan x}$$

$\Rightarrow$ $$\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$$

S25. From L.H.S. $\sin A \sin (60 - A) \cdot \sin (60 + A)$ $\quad [\because \{\sin (A + B) \cdot \sin (A - B) = \sin^2 A - \sin^2 B\}]$

$$= \sin A \left(\frac{3}{4} - \sin^2 A\right) \qquad \because \sin 60° = \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} \sin A (3 - 4 \sin^2 A)$$

$$= \frac{1}{4} (3 \sin A - 4 \sin^3 A) = \frac{1}{4} \cdot \sin 3A$$ Proved

S26. $\because$ $\cos 2x = 1 - 2 \sin^2 x$

$\therefore$ $\cos 4x = 1 - 2 \sin^2 2x$

$= 1 - 2(\sin 2x)^2 = 1 - 2(2 \sin x \cos x)^2$

$= 1 - 8 \sin^2 x \cos^2 x.$

S27. From L.H.S. $\cos \alpha \cdot \cos (60 - \alpha) \cdot \cos (60 + \alpha)$

$= \cos \alpha [\cos^2 \alpha - \sin^2 60]$

$$= \cos \alpha \left[\cos^2 \alpha - \frac{3}{4}\right]$$

$$= \frac{1}{4} [4 \cos^3 \alpha - 3 \cos \alpha] = \frac{1}{4} \cos 3\alpha$$

Hence, L.H.S. = R.H.S.

S28. From L.H.S. $\cos A + \cos(120 - A) + \cos(120 + A) = 0$

$$= \cos A + 2\cos\left(\frac{240}{2}\right)^{\circ} \cos(-A)$$

$$= \cos A + 2\cos 120^{\circ} \cdot \cos A$$

$$= \cos A + 2 \cdot \left(-\frac{1}{2}\cos A\right)$$

$$= 0$$

Hence, L.H.S. = R.H.S.

S29. From L.H.S.

$$\sqrt{2+\sqrt{2(1+\cos 4\theta)}} = \sqrt{2+2\cos 2\theta}$$

$$= \sqrt{2(1+\cos 2\theta)} = \sqrt{4\cos^2\theta} = 2\cos\theta$$

Hence, L.H.S. = R.H.S.

S30. We know that $\tan\theta = \cot\theta - 2\cot 2\theta$

Hence, $\tan\frac{\pi}{16} = \left(\cot\frac{\pi}{16} - 2\cot\frac{\pi}{8}\right)$ and $\tan\frac{\pi}{8} = \left(\cot\frac{\pi}{8} - 2\cot\frac{\pi}{4}\right)$

Putting all these values, we get

$$= \left(\cot\frac{\pi}{16} - 2\cot\frac{\pi}{8}\right) + 2\left(\cot\frac{\pi}{8} - 2\cot\frac{\pi}{4}\right) + 4$$

$$= \cot\frac{\pi}{16}.$$

S31. From L.H.S.

$$\frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \frac{2\cos\left(\frac{\alpha+\beta}{2}\right)\cdot\sin\left(\frac{\alpha-\beta}{2}\right)}{2\sin\left(\frac{\alpha+\beta}{2}\right)\cdot\sin\left(\frac{\alpha-\beta}{2}\right)}$$

$$= \cot\left(\frac{\alpha+\beta}{2}\right)$$

Hence, L.H.S. = R.H.S.

S32. From L.H.S.

$$\sin\left(\frac{\pi}{4}+A\right)\cdot\sin\left(\frac{\pi}{4}-A\right) = \sin^2\frac{\pi}{4} - \sin^2 A$$

$$= \frac{1}{2} - \sin^2 A = \frac{1}{2}(1 - 2\sin^2 A)$$

$$= \frac{1}{2}\cos 2A$$

Hence, L.H.S. = R.H.S.

S33. $\because$ $$\tan A = \frac{3}{4}, \cos B = \frac{9}{41}$$

$A \in$ 3rd quadrant, $B \in$ 1st quadrant

$\therefore$ $$\tan B = \frac{40}{9}.$$

$\therefore$ $$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$= \frac{\frac{3}{4} + \frac{40}{9}}{1 - \frac{3}{4} \times \frac{40}{9}} = \frac{-187}{84}.$$

S34. From L.H.S.

$$\cos\left(\frac{\pi}{4} - \theta\right) \cdot \cos\left(\frac{\pi}{4} - \phi\right) - \sin\left(\frac{\pi}{4} - \theta\right) \cdot \sin\left(\frac{\pi}{4} - \phi\right)$$

$$= \cos\left[\left(\frac{\pi}{4} - \theta\right) + \left(\frac{\pi}{4} - \phi\right)\right]$$

$\because$ $$\cos (x + y) = \cos x \cos y - \sin x \sin y$$

$$= \cos\left[\frac{\pi}{2} - (\theta + \phi)\right] \qquad \because \cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$= \sin(\theta + \phi)$$

Hence, L.H.S. = R.H.S.

S35. $\because$ $$\sin 2x = \frac{2 \sin x \cos x}{1}$$

$$= \frac{2 \sin x \cos x}{\sin^2 x + \cos^2 x} \qquad \{\sin^2 x + \cos^2 x = 1\}$$

$$= \frac{2 \cos x \sin x}{\cos^2 x \left(\frac{\sin^2 x}{\cos^2 x} + 1\right)}$$

$$= \frac{2 \tan x}{1 + \tan^2 x}.$$

S36. $\cos^2\left(\frac{\pi}{8}\right)+\cos^2\left(\frac{3\pi}{8}\right)+\cos^2\left(\frac{5\pi}{8}\right)+\cos^2\left(\frac{7\pi}{8}\right)$

$$= \frac{1}{2}\left[2\cos^2\frac{\pi}{8}+2\cos^2\frac{3\pi}{8}+2\cos^2\frac{5\pi}{8}+2\cos^2\frac{7\pi}{8}\right]$$

$$= \frac{1}{2}\left[1+\cos\frac{\pi}{4}+1+\cos\frac{3\pi}{4}+1+\cos\frac{5\pi}{4}+1+\cos\frac{7\pi}{4}\right]$$

= 2 Proved

S37. From L.H.S. $\tan(\alpha-\beta) = \frac{\tan\alpha-\tan\beta}{1+\tan\alpha\cdot\tan\beta}$

$$= \frac{\frac{3}{2}\tan\beta-\tan\beta}{1+\frac{3}{2}\tan^2\beta} = \frac{2\tan\beta}{4+6\tan^2\beta}$$

$$= \frac{2\tan\beta}{5(1+\tan^2\beta)-(1-\tan^2\beta)} = \frac{\frac{2\tan\beta}{1+\tan^2\beta}}{\left(5-\frac{1-\tan^2\beta}{1+\tan^2\beta}\right)}$$

$$= \frac{\sin 2\beta}{5-\cos 2\beta}$$

R.H.S.

S38. We have, $a = \sin x + \sin y$ ... (i)

and $b = \cos x + \cos y$ ... (ii)

Squaring (i) and (ii) and adding, we get

$$a^2 + b^2 = (\sin^2 x + \sin^2 y) + 2\sin x \sin y + (\cos^2 x + \cos^2 y) + 2\cos x\cos y$$

$\Rightarrow$ $a^2 + b^2 = 1 + 1 + 2(\cos x\cos y + \sin x \sin y)$

$\Rightarrow$ $a^2 + b^2 = 2 + 2\cos(x-y)$

$\Rightarrow$ $\cos(x-y) = \frac{1}{2}(a^2+b^2-2)$ **Proved.**

S39. We have, $a = \tan\theta + \tan\phi$ ... (i)

and $b = \cot\theta + \cot\phi$ ...(ii)

We know that, $\cot(\theta+\phi) = \frac{1}{\tan(\theta+\phi)} = \frac{1}{\frac{\tan\theta+\tan\phi}{1-\tan\theta\cdot\tan\phi}} = \frac{1-\tan\theta\tan\phi}{\tan\theta+\tan\phi}$

$$= \frac{1}{\tan\theta + \tan\phi} - \frac{\tan\theta \cdot \tan\phi}{\tan\theta + \tan\phi} = \frac{1}{a} - \frac{\frac{1}{\cot\theta} \cdot \frac{1}{\cot\phi}}{\frac{1}{\cot\theta} + \frac{1}{\cot\phi}} = \frac{1}{a} - \frac{1}{\cot\theta + \cot\phi}$$

$$= \frac{1}{a} - \frac{1}{b}.$$ [Using (i) and (ii)] **Proved.**

S40. We have, $$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}} \quad \text{... (i)}$$

Putting, $A = 45°$ in Eq. (i), we get

$$\sin 22\frac{1°}{2} = \sqrt{\frac{1-\cos 45°}{2}} \qquad \left[\because \ \sin 22\frac{1°}{2} \text{ is} + \text{ve}\right]$$

$$\Rightarrow \quad \sin 22\frac{1°}{2} = \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$$ **Ans.**

S41. We have, $$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}} \quad \text{... (i)}$$

Putting, $A = 45°$ in Eq. (i), we get

$$\therefore \quad \cos 22\frac{1°}{2} = \sqrt{\frac{1+\cos 45°}{2}} \qquad \left[\because \ \cos 22\frac{1°}{2} \text{ is} + \text{ve}\right]$$

$$\Rightarrow \quad \cos 22\frac{1°}{2} = \sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$$ **Ans.**

S42. We have $$\text{L.H.S.} = \frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta} = \frac{(1-\cos\theta)+\sin\theta}{(1+\cos\theta)+\sin\theta}$$

$$= \frac{2\sin^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \frac{\sin^2\frac{\theta}{2} + \sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2} + \sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$= \frac{\sin\frac{\theta}{2}\left[\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right]}{\cos\frac{\theta}{2}\left[\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right]} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \tan\frac{\theta}{2}$$ **Proved.**

S43. $$\text{R.H.S.} = \frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = \frac{(\cos x + \sin x)^2 - (\cos x - \sin x)^2}{\cos^2 x - \sin^2 x}$$

$$= \frac{(\cos^2 x + \sin^2 x) + 2\cos x \sin x - (\cos^2 x + \sin^2 x - 2\cos x \sin x)}{\cos 2x}$$

$$= \frac{1 + \sin 2x - 1 + \sin 2x}{\cos 2x} = \frac{2\sin 2x}{\cos 2x} = 2\tan 2x = \text{R.H.S.}$$ **Proved.**

S44. We have, $2\cos\theta = x + \frac{1}{x}$

$\therefore$ $2\cos 3\theta = 2(4\cos^3\theta - 3\cos\theta) = 8\cos^3\theta - 6\cos\theta = (2\cos\theta)^3 - 3(2\cos\theta)$

$$= \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$$

$$= x^3 + \frac{1}{x^3} + 3x \cdot \frac{1}{x}\left(x + \frac{1}{x}\right) - 3\left(x + \frac{1}{x}\right) = x^3 + \frac{1}{x^3}$$ **Proved.**

S45. We have, $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$

$\Rightarrow$ $\tan\theta + \tan 2\theta = \sqrt{3}\,(1 - \tan\theta\tan 2\theta)$

$\Rightarrow$ $\frac{\tan\theta + \tan 2\theta}{1 - \tan\theta + \tan 2\theta} = \sqrt{3}$ $\Rightarrow$ $\tan(\theta + 2\theta) = \sqrt{3}$

$\Rightarrow$ $\tan 3\theta = \sqrt{3}$ $\Rightarrow$ $\tan 3\theta = \tan\frac{\pi}{3}$

$\Rightarrow$ $3\theta = n\pi + \frac{\pi}{3},\quad n \in Z$

$\Rightarrow$ $\theta = \frac{n\pi}{3} + \frac{\pi}{9},\quad n \in Z$ **Ans.**

S46.

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cdot\cos\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cdot\cos\left(\frac{A-B}{2}\right)$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\cdot\sin\frac{A-B}{2}$$

$\therefore$ $\frac{\sin x - \sin y}{\cos x + \cos y}$

$$= \frac{2\cos\left(\frac{x+y}{2}\right)\cdot\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cdot\cos\left(\frac{x-y}{2}\right)}$$

$$= \tan\left(\frac{x-y}{2}\right) \quad \textbf{Proved}.$$

S47. Given, $\cos\alpha + \cos\beta = \sin\alpha + \sin\beta = 0$

On squaring on both side, we get

$$(\cos\alpha + \cos\beta)^2 = 0 \qquad \text{... (i)}$$

$$(\sin\alpha + \sin\beta)^2 = 0 \qquad \text{... (ii)}$$

On subtracting Eq. (ii) from Eq. (i), we get

$$(\cos\alpha + \cos\beta)^2 - (\sin\alpha + \sin\beta)^2 = 0$$

$\Rightarrow$ $(\cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta) - (\sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta) = 0$

$\Rightarrow$ $(\cos^2\alpha - \sin^2\alpha) + (\cos^2\beta - \sin^2\beta) + 2[\cos\alpha\cos\beta - \sin\alpha\sin\beta] = 0$

$\Rightarrow$ $\cos 2\alpha + \cos 2\beta + 2\cos(\alpha + \beta) = 0$

$\Rightarrow$ $\cos 2\alpha + \cos 2\beta = -2\cos(\alpha + \beta)$

S48. From L.H.S. $\frac{\cos 4x \cdot \sin 3x - \cos 2x \cdot \sin x}{\sin 4x \cdot \sin x + \cos 6x \cdot \cos x}$

$$= \frac{\{\sin(4x+3x) - \sin(4x-3x)\} - \{(\sin(2x+x) - \sin(2x-x)\}}{\{\cos(4x-x) - \cos(4x+x)\} + \{\cos(6x+x) + \cos(6x-x)\}}$$

$$= \frac{(\sin 7x - \sin x) - (\sin(3x) - \sin x)}{(\cos 3x - \cos 5x) + (\cos 7x + \cos 5x)}$$

$$= \frac{(\sin 7x - \sin 3x)}{(\cos 7x + \cos 3x)}$$

$$= \frac{2\cos\left(\frac{7x+3x}{2}\right)\cdot\sin\left(\frac{7x-3x}{2}\right)}{2\cos\left(\frac{7x+3x}{2}\right)\cdot\cos\left(\frac{7x-3x}{2}\right)} = \frac{\sin 2x}{\cos 2x} = \tan 2x \quad \text{R.H.S.}$$

S49. From L.H.S.

$\cos 5x = \cos(3x + 2x)$

$= \cos 3x \cdot \cos 2x - \sin 3x \cdot \sin 2x$

$= (4\cos^3 x - 3\cos x)(2\cos^2 x - 1) - (3\sin x - 4\sin^3 x)(2\sin x\cos x)$

$= (8\cos^5 x - 10\cos^3 x + 3\cos x) - 6\sin^2 x \cdot \cos x + 8\sin^4 x \cdot \cos x$

$$= (8\cos^5 x - 10\cos^3 x + 3\cos x) - 6(1-\cos^2 x)\cdot\cos x + 8(1-\cos^2 x)^2\cos x$$

$$= 16\cos^5 x - 20\cos^3 x + 5\cos x$$

Hence, $\cos 5x = 16\cos^5 x - 20\cos^3 x + 5\cos x$

S50. From L.H.S. $\dfrac{2\sin(\alpha-\gamma)\cdot\cos(\gamma)-\sin(\alpha-2\gamma)}{2\sin(\beta-\gamma)\cdot\cos(\gamma)-\sin(\beta-2\gamma)}$

$$= \frac{[\sin(\alpha-\gamma+\gamma)+\sin(\alpha-\gamma-\gamma)-\sin(\alpha-2\gamma)]}{[\sin(\beta-\gamma+\gamma)+\sin(\beta-\gamma-\gamma)-\sin(\beta-2\gamma)]} \quad [\because 2\sin A\cdot\cos B = \sin(A+B)+\sin(A-B)]$$

$$= \frac{\sin\alpha+\sin(\alpha-2\gamma)-\sin(\alpha-2\gamma)}{\sin\beta+\sin(\beta-2\gamma)-\sin(\beta+2\gamma)}$$

$$= \frac{\sin\alpha}{\sin\beta} \text{ R.H.S.}$$

S51. Let to parts be A and β.

$\therefore \quad A+\beta=\alpha \quad \text{and} \quad A-\beta = x,$

$\because \quad \tan A = k\tan\beta.$

$\therefore \quad \dfrac{\tan A}{\tan\beta} = \dfrac{k}{1} \Rightarrow \dfrac{\tan A-\tan\beta}{\tan A+\tan\beta} = \dfrac{k-1}{k+1}$

$\Rightarrow \quad \dfrac{\sin(A-\beta)}{\sin(A+\beta)} = \left(\dfrac{k-1}{k+1}\right)$

$\Rightarrow \quad \dfrac{\sin x}{\sin\alpha} = \left(\dfrac{k-1}{k+1}\right)$

$\Rightarrow \quad \sin x = \dfrac{k-1}{k+1}\cdot\sin\alpha$

S52. $\because \quad a = \cos A + \cos\beta - \cos(A+\beta)$

$$= 2\cos\frac{A+\beta}{2}\cdot\cos\left(\frac{A-\beta}{2}\right) - 2\cos^2\left(\frac{A+\beta}{2}\right)+1$$

$$= 2\cos\frac{A+\beta}{2}\left[\cos\left(\frac{A-\beta}{2}\right)-\cos\left(\frac{A+\beta}{2}\right)\right]+1$$

$$= 2\cos\left(\frac{A+\beta}{2}\right)\cdot 2\sin\frac{A}{2}\cdot\sin\frac{\beta}{2}+1$$

$$= (b+1)$$

$$a = b+1 \Rightarrow (a-b) = 1.$$

S53.

$$\text{L.H.S.} = \frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \frac{\cos 4x + \cos 2x + \cos 3x}{\sin 4x + \sin 2x + \sin 3x}$$

$$= \frac{2\cos\frac{4x+2x}{2}\cdot\cos\frac{4x-2x}{2} + \cos 3x}{2\sin\frac{4x+2x}{2}\cdot\cos\frac{4x-2x}{2} + \sin 3x}$$

$$= \frac{2\cos 3x\cos x + \cos 3x}{2\sin 3x\cos x + \sin 3x} = \frac{\cos 3x(2\cos x + 1)}{\sin 3x(2\cos x + 1)}$$

$$= \frac{\cos 3x}{\sin 3x} = \cot 3x = \text{RHS.}$$ **Proved.**

S54. We have, $\cos 15° = \cos(45° - 30°) = \cos 45°\cos 30° + \sin 45°\sin 30°$

$\Rightarrow$ $$\cos 15° = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\cdot\frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Putting, $A = 15°$ in $\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$, we get

$$\sin 7\frac{1°}{2} = \sqrt{\frac{1-\cos 15°}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}+1}{2\sqrt{2}}}{2}} \qquad \left[\because\ \sin 7\frac{1°}{2} \text{ is} + \text{ve}\right]$$

$$= \sqrt{\frac{2\sqrt{2}-\sqrt{3}-1}{4\sqrt{2}}} = \sqrt{\frac{4-\sqrt{6}-\sqrt{2}}{8}} \qquad [\text{Multiplying num. and den. by } \sqrt{2}]$$

$$= \frac{\sqrt{4-\sqrt{6}-\sqrt{2}}}{2\sqrt{2}}$$ **Ans.**

S55.

$$\text{L.H.S.} = \cot 7\frac{1°}{2} = \frac{\cot 7\frac{1°}{2}}{\sin 7\frac{1°}{2}} = \frac{2\cos^2 7\frac{1°}{2}}{2\sin 7\frac{1°}{2}\cos 7\frac{1°}{2}} = \frac{1+\cos 15°}{\sin 15°}$$

$$= \frac{1+\cos(45°-30°)}{\sin(45°-30°)} = \frac{1+[\cos 45°\cos 30° + \sin 45°\sin 30°]}{\sin 45°\cos 30° - \cos 45°\sin 30°}$$

$$= \frac{1+\left[\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\times\frac{1}{2}\right]}{\frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\times\frac{1}{2}} = \frac{1+\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{2\sqrt{2}+\sqrt{3}+1}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{2\sqrt{6}+2\sqrt{2}+3+\sqrt{3}+\sqrt{3}+1}{3-1} = \frac{2\sqrt{6}+2\sqrt{2}+2\sqrt{3}+4}{2}$$

$= \sqrt{6} + \sqrt{2} + \sqrt{3} + 2 = \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$ = R.H.S. **Proved.**

S56. $\cos 6x = \cos 3(2x) = 5\cos^3 2x - 3\cos 2x$

Putting $\cos 2x = 2\cos^2 x - 1$

$[\because \cos 3A = \cos(2A + A) = \cos 2A \cos A - \sin 2A \sin A]$

$= (2\cos^2 A - 1)\cos A - 2\cos A \sin^2 A$

$= 2\cos^3 A - \cos A - 2\cos A(1 - \cos^2 A)$

$= 4\cos^3 A \cos A$

L.H.S. $\cos 6x = 4(2\cos^2 x - 1)^3 - 3(2\cos^2 x - 1)$

$= 4(8\cos^6 x - 12\cos^4 x + 6\cos^2 x - 1) - 6\cos^2 x + 3$

$= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$ = R.H.S. **Proved.**

S57.

$$\text{LHS.} = \sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 8\theta}}} = \sqrt{2 + \sqrt{2 + \sqrt{2(1 + \cos 8\theta)}}}$$

$$= \sqrt{2 + \sqrt{2 + \sqrt{2(2\cos^2 4\theta)}}} \qquad [\because 1 + \cos 2\theta = 2\cos^2\theta]$$

$$= \sqrt{2 + \sqrt{2 + 2\cos 4\theta}} = \sqrt{2 + \sqrt{2(1 + \cos 4\theta)}}$$

$$= \sqrt{2 + \sqrt{2(2\cos^2 2\theta)}} = \sqrt{2 + 2\cos 2\theta}$$

$$= \sqrt{2(1 + \cos 2\theta)} = \sqrt{2.2\cos^2\theta} = \sqrt{4\cos^2\theta}$$

$= 2\cos\theta$ = R.H.S. **Proved.**

S58.

$$\text{L.H.S.} = \cot A + \frac{1 - \tan 60^\circ \tan A}{\tan 60^\circ + \tan A} + \frac{1 - \tan 120^\circ \tan A}{\tan 120^\circ + \tan A}$$

$$= \cot A + \frac{1 - \sqrt{3}\tan A}{\sqrt{3} + \tan A} + \frac{1 + \sqrt{3}\tan A}{-\sqrt{3} + \tan A}$$

$$= \cot A + \frac{(1 - \sqrt{3}\tan A)(-\sqrt{3} + \tan A) + (1 + \sqrt{3}\tan A)(\sqrt{3} + \tan A)}{(\sqrt{3} + \tan A)(-\sqrt{3} + \tan A)}$$

$$= \cot A + \frac{-\sqrt{3} + \tan A + 3\tan A - \sqrt{3}\tan^2 A + \sqrt{3} + \tan A + 3\tan A + \sqrt{3}\tan^2 A}{\tan^2 A - 3}$$

$$= \cot A + \frac{8\tan A}{\tan^2 A - 3} = \cot A + \frac{\frac{8}{\cot A}}{\frac{1}{\cot^2 A} - 3} = \cot A + \frac{8\tan A}{1 - 3\cot^2 A}$$

$$= \frac{\cot A\,(1 - 3\cot^3 A) + 8\cot A}{1 - 3\cot^2 A} = \frac{9\cot A - 3\cot^3 A}{1 - 3\cot^2 A}$$

$$= 3\left[\frac{3\cot A - \cot^3 A}{1 - 3\cot^2 A}\right] = 3\cot 3A.$$

S59. Let, $x = \cos\alpha\cos 2\alpha\cos 3\alpha\cos 4\alpha\cos 5\alpha\cos 6\alpha$

Multiplying both sides by $\sin\alpha$, we get

$$x\sin\alpha = (\sin\alpha\cos\alpha)\cos 2\alpha\cos 3\alpha\cos 4\alpha\cos 5\alpha\cos 6\alpha$$

$$= \frac{1}{2}(\sin 2\alpha\cos 2\alpha)\cos 2\alpha\cos 3\alpha\cos 4\alpha\cos 5\alpha\cos 6\alpha;\ \text{etc.}$$

$$= \frac{1}{16}\sin(13\alpha - 3\alpha)\cos 3\alpha\cos 6\alpha = \frac{1}{16}\sin(\pi - 3\alpha)\cos 3\alpha\cos 6\alpha$$

$$= \frac{1}{16}(\sin 3\alpha\cos 3\alpha)\cos 6\alpha = \frac{1}{16}\sin 6\alpha\cos 6\alpha$$

$$= \frac{1}{64}\sin 12\alpha = \frac{1}{64}\sin(13\alpha - \alpha) = \frac{1}{64}\sin(\pi - \alpha) = \frac{1}{64}\sin\alpha \Rightarrow x = \frac{1}{64}$$

S60. L.H.S. $= \cos^3\left(x - \frac{2\pi}{3}\right) + \cos^3 x + \cos^3\left(x + \frac{2\pi}{3}\right)$ $\left[\begin{array}{l}\cos 3A = 4\cos^3 A - 3\cos A \\ \cos^3 A = \frac{1}{4}\cos 3A + \frac{3}{4}\cos A\end{array}\right]$

$$= \left[\frac{1}{4}\cos 3\left(x - \frac{2\pi}{3}\right) + \frac{3}{4}\cos\left(x - \frac{2\pi}{3}\right)\right] + \frac{1}{4}\left[\cos 3x + \frac{3}{4}\cos x\right] + \left[\frac{1}{4}\cos 3\left(x + \frac{2\pi}{3}\right) + \frac{3}{4}\cos\left(x + \frac{2\pi}{3}\right)\right]$$

$$= \frac{3}{4}\left[\cos\left(x - \frac{2\pi}{3}\right) + \cos x + \cos\left(x + \frac{2\pi}{3}\right)\right] + \frac{1}{4}[\cos(3x - 2\pi) + \cos 3x + \cos(3x + 2\pi)]$$

$$= \frac{3}{4}\left[\cos x + 2\cos x\cos\frac{2\pi}{3}\right] + \frac{1}{4}[\cos 3x + 2\cos 3x\cos 2\pi]$$

$$= \frac{3}{4}\cos x\left[1 + 2\cos\frac{2\pi}{3}\right] + \frac{1}{4}\cos 3x[1 + 2\cos 2\pi]$$

$$= \frac{3}{4}\cos x\left[1 + 2\times\frac{-1}{2}\right] + \frac{1}{4}\cos 3x[3]$$

$= 0 + \frac{3}{4}\cos 3x = \frac{3}{4}\cos 3x$ = R.H.S.

S61. L.H.S. = $\cos\left(\frac{3\pi}{2} + x\right)\cos(2\pi + x)\left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x)\right]$

Now, $\cos\left(\frac{3\pi}{2} + \pi\right) = \sin x$, $\cos(2\pi + x) = \cos x$

$\cot\left(\frac{3\pi}{2} - \pi\right) = \tan x$, $\cot(2\pi + x) = \cot x$

L.H.S. = $\sin x \cos x\,[(\tan x) + \cot x]$

$= \sin x \cos x\left[\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right] = \sin x \cos x\left[\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}\right]$

$= (\sin x \cos x)\frac{1}{\sin x \cos x}$ $[\sin^2 x + \cos^2 x = 1]$

= 1. **Proved.**

S62. (i) we have,

$\tan\theta + \tan 2\theta + \tan\theta\tan 2\theta = 1$

$\tan\theta + \tan 2\theta = 1 - \tan\theta\tan 2\theta$

$\Rightarrow \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\cdot\tan 2\theta} = 1$

$\Rightarrow \tan 3\theta = 1 \Rightarrow \tan 3\theta = \tan\frac{\pi}{4}$

$\Rightarrow 3\theta = n\pi + \frac{\pi}{4}, \quad n \in Z$

$\Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{12}, \quad n \in Z$ **Ans.**

(ii) We have,

$\tan\theta + \tan 2\theta + \tan 3\theta = \tan\theta\tan 2\theta\tan 3\theta$

$\Rightarrow \tan\theta + \tan 2\theta = -\tan 3\theta + \tan\theta\tan 2\theta\tan 3\theta$

$\Rightarrow \tan\theta + \tan 2\theta = -\tan 3\theta(1 - \tan\theta\tan 2\theta)$

$\Rightarrow \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\cdot\tan 2\theta} = -\tan 3\theta$

$\Rightarrow \tan(\theta + 2\theta) = -\tan 3\theta$

$\Rightarrow \tan 3\theta = -\tan 3\theta$

$\Rightarrow 2\tan 3\theta = 0 \quad = \quad \tan 3\theta = 0$

$$\Rightarrow \qquad 3\theta = n\pi, \quad n \in Z \quad \Rightarrow \quad \theta = \frac{n\pi}{3}, \quad n \in Z$$ **Ans.**

S63. L.H.S. = $(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2$

$$= \left(2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}\right)^2 + \left(2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}\right)^2$$

$$= 4\cos^2\left(\frac{\alpha+\beta}{2}\right)\cos^2\left(\frac{\alpha-\beta}{2}\right) + 4\sin^2\left(\frac{\alpha+\beta}{2}\right)\cos^2\left(\frac{\alpha-\beta}{2}\right)$$

$$= 4\cos^2\frac{\alpha-\beta}{2}\left(\cos^2\frac{\alpha+\beta}{2} + \sin^2\frac{\alpha+\beta}{2}\right)$$

$$= 4\cos^2\frac{\alpha+\beta}{2} = \text{R.H.S.}$$ $[\because \sin^2\theta + \cos^2\theta = 1]$ **Proved.**

S64. L.H.S. = $\left(\dfrac{\cos A + \cos B}{\sin A - \sin B}\right)^n + \left(\dfrac{\sin A + \sin B}{\cos A - \cos B}\right)^n$

$$= \left(\frac{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}}{2\sin\frac{A-B}{2}\cos\frac{A+B}{2}}\right)^n + \left(\frac{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}{-2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}\right)^n$$

$$= \left[\cot\left(\frac{A-B}{2}\right)\right]^n + \left[-\cot\left(\frac{A-B}{2}\right)\right]^n = \cot^n\left(\frac{A-B}{2}\right) + (-1)^n\cot^n\left(\frac{A-B}{2}\right)$$

$$= \cot^n\left(\frac{A-B}{2}\right)\{1 + (-1)^n\} = \begin{cases} 2\cot^n\left(\frac{A-B}{2}\right), & \text{if } n \text{ is even} \\ 0, & \text{if } n \text{ is odd} \end{cases}$$ **Proved.**

S65. L.H.S. = $\dfrac{\cos A}{1 - \sin A} = \dfrac{\cos^2\frac{A}{2} - \sin^2\frac{A}{2}}{\cos^2\frac{A}{2} + \sin^2\frac{A}{2} - 2\sin\frac{A}{2}\cos\frac{A}{2}}$

$$= \frac{\left(\cos\frac{A}{2} - \sin\frac{A}{2}\right)\left(\cos\frac{A}{2} + \sin\frac{A}{2}\right)}{\left(\cos\frac{A}{2} - \sin\frac{A}{2}\right)^2}$$ $[\because \cos^2\theta + \sin^2\theta = 1]$

$$= \frac{\left(\cos\frac{A}{2} + \sin\frac{A}{2}\right)}{\left(\cos\frac{A}{2} - \sin\frac{A}{2}\right)} = \frac{1 + \tan\frac{A}{2}}{1 - \tan\frac{A}{2}}$$ $\left[\text{Dividing num. and den. by } \cos\frac{A}{2}\right]$

$$= \frac{\tan 45^\circ + \tan \frac{A}{2}}{1 - \tan 45^\circ \tan \frac{A}{2}} = \tan\left(45^\circ + \frac{A}{2}\right) = \text{R.H.S.}$$ **Proved.**

S66. We have $\text{L.H.S.} = \frac{1}{2}\left[2\cos 2x \cos\frac{x}{2} - 2\cos\frac{9x}{2}\cos 3x\right]$

$$= \frac{1}{2}\left[\cos\left(2x + \frac{x}{2}\right) + \cos\left(2x - \frac{x}{2}\right) - \cos\left(\frac{9x}{2} + 3x\right) - \cos\left(\frac{9x}{2} - 3x\right)\right]$$

$$= \frac{1}{2}\left[\cos\frac{5x}{2} + \cos\frac{3x}{2} - \cos\frac{15x}{2} - \cos\frac{3x}{2}\right] = \frac{1}{2}\left[\cos\frac{5x}{2} - \cos\frac{15x}{2}\right]$$

$$= \frac{1}{2}\left[-2\sin\left\{\frac{\frac{5x}{2} + \frac{15x}{2}}{2}\right\}\sin\left\{\frac{\frac{5x}{2} - \frac{15x}{2}}{2}\right\}\right]$$

$$= -\sin 5x \sin\left(-\frac{5x}{2}\right) = \sin 5x \sin\frac{5x}{2} = \text{R.H.S.}$$

S67.

$$\text{LHS.} = \frac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)} = \frac{\frac{1 + \tan x}{1 - \tan x}}{\left(\frac{1 - \tan x}{1 + \tan x}\right)}$$

$$= \left(\frac{1 + \tan x}{1 - \tan x}\right)^2 = \text{R.H.S.}$$

S68. Since, $\pi < x < \frac{3\pi}{2}$, cos x is negative.

Also, $\frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$

Therefore, sin $\frac{x}{2}$ is positive and cos $\frac{x}{2}$ is negative.

Now, $\sec^2 x = 1 + \tan^2 x = 1 + \frac{9}{16} = \frac{25}{16}$

Therefore, $\cos^2 x = \frac{16}{25}$ or $x = -\frac{4}{5}$

Now, $2\sin^2\frac{x}{2} = 1 - \cos x = 1 + \frac{4}{5} = \frac{9}{5}$

Therefore, $\sin^2\frac{x}{2} = \frac{9}{10}$ or $\sin\frac{x}{2} = \frac{3}{\sqrt{10}}$

Again, $2\cos^2\frac{x}{2} = 1 + \cos x = 1 - \frac{4}{5} = \frac{1}{5}$

Therefore, $\cos^2\frac{x}{2} = \frac{1}{10}$ or $\cos\frac{x}{2} = -\frac{1}{\sqrt{10}}$

Hence, $\tan\frac{x}{2} = \frac{\sin\frac{x}{2}}{\cos\frac{x}{2}} = \frac{3}{\sqrt{10}} \times \left(\frac{-\sqrt{10}}{1}\right) = -3.$

S69. Let, $x = \frac{\pi}{8}.$

Then $2x = \frac{\pi}{4}$

Now, $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$ or $\tan\frac{\pi}{4} = \frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$

Let $y = \tan\frac{\pi}{8}.$

Then $1 = \frac{2y}{1-y^2}$

or $y^2 + 2y - 1 = 0$

Therefore, $y = \frac{-2 \pm 2\sqrt{2}}{2} = -1 \pm \sqrt{2}$

Since $\frac{\pi}{8}$ lies in the first quadrant, $y = \tan\frac{\pi}{8}$ is positive. Hence,

$$\tan\frac{\pi}{8} = \sqrt{2} - 1.$$

S70.

We have L.H.S. $= \frac{1+\cos 2x}{2} + \frac{1+\cos\left(2x+\frac{2\pi}{3}\right)}{2} + \frac{1+\cos\left(2x-\frac{2\pi}{3}\right)}{2}$

$$= \frac{1}{2}\left[3 + \cos 2x + \cos\left(2x+\frac{2\pi}{3}\right) + \cos\left(2x-\frac{2\pi}{3}\right)\right]$$

$$= \frac{1}{2}\left[3 + \cos 2x + 2\cos 2x\cos\frac{2\pi}{3}\right]$$

$$= \frac{1}{2}\left[3 + \cos 2x + 2\cos 2x\cos\left(\pi - \frac{\pi}{3}\right)\right]$$

$$= \frac{1}{2}\left[3 + \cos 2x - 2\cos 2x \cos\frac{\pi}{3}\right]$$

$$= \frac{1}{2}\left[3 + \cos 2x - \cos 2x\right] = \frac{3}{2} = \text{R.H.S.}$$

S71.

$$\text{LHS.} = \frac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)}$$

$$= \frac{(-\cos x)\cos x}{\sin x\,(-\sin x)} = \frac{-\cos^2 x}{-\sin^2 x}$$

$$= \cot^2 x = \text{R.H.S.}$$

$\therefore\ \cos(\pi + \theta) = -\cos\theta$

$\cos(-\theta) = \cos\theta$

$\sin(\pi - \theta) = \sin\theta$

$\cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta.$

S72.

$$\text{L.H.S.} = \cos\left(\frac{3\pi}{2} + x\right)\cos(2\pi + x)\left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x)\right]$$

$$= (\sin x)(\cos x)(\tan x + \cot x)$$

$$= \sin x \cos x\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$$

$$= \sin x \cos x\left(\frac{\sin^2 x + \cos^2 x}{\cos x \sin x}\right)$$

$$= 1 = \text{R.H.S.}$$

S73. Here,

$$\text{L.H.S.} = \sin(n + 1)x \sin(n + 2)x + \cos(n + 1)x \cos(n + 2)x$$

Write

$$A = (n + 1)x$$

$$B = (n + 2)x$$

$$\text{L.H.S.} = \sin A \sin B + \cos A \cos B$$

$$= \cos(A - B)$$

$$= \cos(n + 1)x - (n + 2)x$$

$$= \cos(x - 2x)$$

$$= \cos(-x) \qquad [\because\ \cos(-\theta) = \cos\theta\ \forall\theta \in R]$$

$$= \cos x = \text{R.H.S.}$$

S74. L.H.S. = $\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)$

Let $\frac{3\pi}{4}+x = A$ and $\frac{3\pi}{4}-x = B$

$\therefore$ L.H.S. = $\cos A - \cos B$

$$= -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

Now $A + B = \frac{3\pi}{4}+x+\frac{3\pi}{4}-x=\frac{3\pi}{2}$

$$A - B = \frac{3\pi}{4}+x-\left(\frac{3\pi}{4}-x\right)$$

$$= \frac{3\pi}{4}+x-\frac{3\pi}{4}+x=2x$$

$\therefore$ L.H.S. = $-2\sin\frac{3\pi}{4}\sin\frac{2x}{2}$

$$= -2\sin\left(\pi-\frac{\pi}{4}\right)\sin x$$

$$= -2\sin\frac{\pi}{4}\sin x$$

$$= -2\left(\frac{1}{\sqrt{2}}\right)\sin x$$

$$= -\sqrt{2}\sin x = \text{R.H.S.}$$

S75. L.H.S. = $(\cos x - \cos y)^2 + (\sin x - \sin y)^2$

$$= \left(-2\sin\frac{x+y}{2}\sin\frac{x-y}{2}\right)^2+\left(2\cos\frac{x+y}{2}\sin\frac{x-y}{2}\right)^2$$

$$= 4\sin^2\frac{x+y}{2}\sin^2\frac{x-y}{2}+4\cos^2\frac{x+y}{2}\sin^2\frac{x-y}{2}$$

$$= 4\sin^2\frac{x-y}{2}\left[\sin^2\frac{x+y}{2}+\cos^2\frac{x+y}{2}\right]$$

$$= 4\sin^2\frac{x-y}{2} = \text{R.H.S.}$$

S76. L.H.S. = $(\cos x + \cos y)^2 + (\sin x - \sin y)^2$

$$= \left(2\cos\frac{x+y}{2}\cos\frac{x-y}{2}\right)^2+\left(2\cos\frac{x+y}{2}\sin\frac{x-y}{2}\right)^2$$

$$= 4\cos^2\frac{x+y}{2}\cos^2\frac{x-y}{2} + 4\cos^2\frac{x+y}{2}\sin^2\frac{x-y}{2}$$

$$= 4\cos^2\frac{x+y}{2}\left[\cos^2\frac{x-y}{2} + \sin^2\frac{x-y}{2}\right]$$

$$= 4\cos^2\frac{x+y}{2} = \text{R.H.S.}$$

S77. L.H.S. = $(\sin 3x + \sin x)\sin x + (\cos 3x - \cos x)\cos x$

$$= \left(2\sin\frac{3x+x}{2}\cos\frac{3x-x}{2}\right)\sin x - \left(2\sin\frac{3x+x}{2}\sin\frac{3x-x}{2}\right)\cos x$$

$$= \left[\sin\theta + \sin\phi = 2\sin\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2} \quad \text{and} \quad \cos\theta - \cos\phi = -2\sin\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}\right]$$

$= (2\sin 2x\cos x)\sin x - (2\sin 2x\sin x)\cos x$

$= 2\sin 2x\cos x\,[\sin x - \sin x] = 0 = \text{R.H.S.}$

S78.

$$\text{L.H.S.} = 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= \cos\left(\frac{\pi}{13} + \frac{9\pi}{13}\right) + \cos\left(\frac{\pi}{13} - \frac{9\pi}{13}\right) + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$[2\cos\theta\cos\phi = \cos(\theta+\phi) + \cos(\theta-\phi)]$

$$= \cos\frac{10\pi}{13} + \cos\left(\frac{-8\pi}{13}\right) + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$$

$$= \left(\cos\frac{10\pi}{13} + \cos\frac{3\pi}{13}\right) + \left(\cos\frac{8\pi}{13} + \cos\frac{5\pi}{13}\right)$$

$$= \left(2\cos\frac{10\pi+3\pi}{26}\cos\frac{10\pi-3\pi}{26}\right) + \left(2\cos\frac{8\pi+5\pi}{26}\cos\frac{8\pi-5\pi}{26}\right)$$

$$= 2\cos\frac{\pi}{2}\cos\frac{7\pi}{26} + 2\cos\frac{\pi}{2}\cos\frac{3\pi}{26}$$

$= 0 + 0 = 0 = \text{R.H.S.}$ $\left[\because \cos\frac{\pi}{2} = 0\right]$

S79.

$$\text{L.H.S.} = \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = \frac{2\cos\frac{x+3x}{2}\sin\frac{x-3x}{2}}{-\cos 2x}$$

$$= \frac{2\cos 2x\sin(-x)}{-\cos 2x} = \frac{-2\cos 2x\sin x}{-\cos 2x}$$

$= 2\sin x = \text{R.H.S.}$

S80.

$$\text{L.H.S.} = \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$$

$$= \frac{2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}$$

$$= \frac{\sin 4x}{\cos 4x} = \tan 4x = \text{R.H.S.}$$

S81. $\text{L.H.S.} = \frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

$$= \frac{-2\sin\left(\frac{9x+5x}{2}\right)\sin\left(\frac{9x-5x}{2}\right)}{2\sin\left(\frac{17x-3x}{2}\right)\cos\left(\frac{17x+3x}{2}\right)}$$

$$= \frac{-2\sin 7x \sin 2x}{2\sin 7x \cos 10x} = \frac{-\sin 2x}{\cos 10x} = \text{R.H.S.}$$

S82. $\text{L.H.S.} = \sin 2x + 2\sin 4x + \sin 6x$

$= (\sin 2x + \sin 6x) + 2\sin 4x$

$$= 2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right) + 2\sin 4x$$

$$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$= 2\sin(4x)\cos(-2x) + 2\sin 4x$

$= 2\sin 4x(\cos 2x + 1)$ $[\because \cos(-x) = \cos x \ \forall x \in R]$

$= 2\sin 4x(2\cos^2 x)$

$= 4\cos^2 x \sin 4x = \text{R.H.S.}$

S83. $\text{L.H.S.} = \cos^2 2x - \cos^2 6x$

$= (\cos 2x + \cos 6x)(\cos 2x - \cos 6x)$

$$= \left[\left(2\cos\frac{2x+6x}{2}\cos\frac{2x-6x}{2}\right)\left(-2\sin\frac{2x+6x}{2}\sin\frac{2x-6x}{2}\right)\right]$$

$= [(2\cos 4x\cos(-2x))(-2\sin 4x\sin(-2x))]$

$= (2\cos 4x\cos 2x)(2\sin 4x\sin 2x)$ $[\because \cos(-\theta) = \theta, \sin(-\theta) = -\sin\theta]$

$= (2\sin 4x\cos 4x)(2\sin 2x\cos 2x)$

$= \sin 8x\sin 4x = \text{R.H.S.}$ $[\because \sin 2A = 2\sin A\cos A]$

S84. Here $\text{L.H.S.} = \sin^2 6x - \sin^2 4x$

$= (\sin 6x + \sin 4x)(\sin 6x - \sin 4x)$ $[\because a^2 - b^2 = (a+b)(a-b)]$

$$= \left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right] \times \left[2\cos\left(\frac{6x+4x}{2}\right)\sin\left(\frac{6x-4x}{2}\right)\right]$$

$= (2 \sin 5x \cos x)\,(2 \cos 5x \sin x)$

$= (2 \sin 5x \cos 5x)\,(2 \sin x \cos x)$

$= \sin 10x \,.\, \sin 2x =$ R.H.S. $\qquad [\because\ \sin 2A = 2 \sin A \cos A]$

S85. (a)

$$\sin^2 \theta = \left(\frac{1-\cos 2\theta}{2}\right)$$

$$\Rightarrow \quad \sin^2(22°30') = \left(\frac{1-\cos 45°}{2}\right)$$

$$\Rightarrow \quad \sin^2(22°30') = \frac{\sqrt{2}-1}{2\sqrt{2}}$$

(b)

$$\cos^2 \theta = \left(\frac{1+\cos 2\theta}{2}\right)$$

$$\Rightarrow \quad \cos^2(22°30') = \left(\frac{1+\cos 45°}{2}\right) = \frac{\sqrt{2}+1}{2\sqrt{2}}$$

$$\therefore \quad \cos^2(22°30') = \frac{\sqrt{2}+1}{2\sqrt{2}}$$

(c)

$$\tan^2(22°30') = \frac{\sin^2(22°30')}{\cos^2(22°30')} = \frac{\sqrt{2}-1}{2\sqrt{2}} \times \frac{2\sqrt{2}}{\sqrt{2}+1}$$

$$\Rightarrow \quad \tan^2(22°30') = (\sqrt{2}-1)^2$$

S86.

$$\text{L.H.S.} = \tan 142\frac{1°}{2} = \tan\left(180° - 37\frac{1°}{2}\right) = -\tan 37\frac{1°}{2}$$

$$= -\tan\left(45° - 7\frac{1°}{2}\right) = -\frac{\tan 45° - 7\frac{1°}{2}}{1+\tan 45° \tan 7\frac{1°}{2}} = -\frac{1-\tan 7\frac{1°}{2}}{1+\tan 7\frac{1°}{2}} \quad [\because\ \tan 45° = 1]$$

$$= -\frac{\cos 7\frac{1°}{2} - \sin 7\frac{1°}{2}}{\cos 7\frac{1°}{2} + \sin 7\frac{1°}{2}} = -\frac{\left(\cos 7\frac{1°}{2} - \sin 7\frac{1°}{2}\right)^2}{\cos^2 7\frac{1°}{2} - \sin^2 7\frac{1°}{2}}$$

$$= -\frac{\left(\cos^2 7\frac{1^\circ}{2} + \sin^2 7\frac{1^\circ}{2}\right) - 2\sin 7\frac{1^\circ}{2}\cdot\cos 7\frac{1^\circ}{2}}{\cos 7\frac{1^\circ}{2} + \cos 2\left(7\frac{1^\circ}{2}\right)}$$

$$= -\frac{1-\sin 15^\circ}{\cos 15^\circ} = -\frac{1-\frac{\sqrt{3}-1}{2\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}}}$$

$$\left[\text{As shown above } \sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}},\quad \cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}\right]$$

$$= -\frac{2\sqrt{2}-\sqrt{3}+1}{\sqrt{3}+1}\times\frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{-2\sqrt{6}+2\sqrt{2}+3-\sqrt{3}-\sqrt{3}+1}{3-1} = \frac{-2\sqrt{6}-2\sqrt{3}+2\sqrt{2}+4}{2}$$

$$= -\sqrt{6}-\sqrt{3}+\sqrt{2}+2 = \sqrt{2}-\sqrt{3}+\sqrt{4}-\sqrt{6} = \text{R.H.S.}$$ **Proved.**

S87. (i)

$$\text{L.H.S.} = \frac{(\cos 3\theta + \cos 7\theta) + 2\cos 5\theta}{(\cos\theta + \cos 5\theta) + 2\cos 3\theta} = \frac{2\cos 5\theta\cos 2\theta + 2\cos 5\theta}{2\cos 3\theta\cos 2\theta + 2\cos 3\theta}$$

$$= \frac{2\cos 5\theta(\cos 2\theta + 1)}{2\cos 3\theta(\cos 2\theta + 1)} = \frac{\cos 5\theta}{\cos 3\theta} = \frac{\cos(2\theta + 3\theta)}{\cos 3\theta}$$

$$= \frac{\cos 2\theta\cos 3\theta - \sin 2\theta\sin 3\theta}{\cos 3\theta} = \frac{\cos 2\theta\cos 3\theta}{\cos 3\theta} - \frac{\sin 2\theta\,.\,\sin 3\theta}{\cos 3\theta}$$

$= \cos 2\theta - \sin 2\theta\ .\ \tan 3\theta =$ RHS. **Proved.**

(ii)

$$\text{L.H.S.} = \frac{(\cos\theta + \cos 4\theta) + (\cos 2\theta + \cos 3\theta)}{(\sin\theta + \sin 4\theta) + (\sin 2\theta + \sin 3\theta)}$$

$$= \frac{2\cos\frac{5\theta}{2}\cos\left(-\frac{3\theta}{2}\right) + 2\cos\frac{5\theta}{2}\cos\left(-\frac{\theta}{2}\right)}{2\sin\frac{5\theta}{2}\cos\left(-\frac{3\theta}{2}\right) + 2\sin\frac{5\theta}{2}\cos\left(-\frac{\theta}{2}\right)}$$

$$= \frac{2\cos\frac{5\theta}{2}\left(\cos\frac{3\theta}{2} + \cos\frac{\theta}{2}\right)}{2\sin\frac{5\theta}{2}\left(\cos\frac{3\theta}{2} + \cos\frac{\theta}{2}\right)} = \frac{\cos\frac{5\theta}{2}}{\sin\frac{5\theta}{2}} = \cot\frac{5\theta}{2} = \text{R.H.S.}$$ **Proved.**

S88. L.H.S. = $\cos\alpha + \cos\beta + \cos\gamma + \cos(\alpha + \beta + \gamma)$

$= (\cos\alpha + \cos\beta) + [\cos\gamma + \cos(\alpha + \beta + \gamma)]$

$$= 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)+2\cos\left(\frac{\alpha+\beta+\gamma+\gamma}{2}\right)\cdot\cos\left(\frac{\alpha+\beta+\gamma-\gamma}{2}\right)$$

$$= 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)+2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\beta+2\gamma}{2}\right)$$

$$= 2\cos\left(\frac{\alpha+\beta}{2}\right)\left\{\cos\left(\frac{\alpha-\beta}{2}\right)+\cos\left(\frac{\alpha+\beta+2\gamma}{2}\right)\right\}$$

$$= 2\cos\left(\frac{\alpha+\beta}{2}\right)\left\{2\cos\left(\frac{\frac{\alpha-\beta}{2}+\frac{\alpha+\beta+2\gamma}{2}}{2}\right)\cos\left(\frac{\frac{\alpha+\beta+2\gamma}{2}-\frac{\alpha-\beta}{2}}{2}\right)\right\}$$

$$= 2\cos\left(\frac{\alpha+\beta}{2}\right)\left\{2\cos\left(\frac{\alpha+\gamma}{2}\right)\cos\left(\frac{\beta+\gamma}{2}\right)\right\}$$

$$= 4\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\beta+\gamma}{2}\right)\cos\left(\frac{\gamma+\alpha}{2}\right) = \text{R.H.S.}$$ **Proved.**

S89. L.H.S. $= \sin\alpha + \sin\beta + \sin\gamma + \sin(\alpha+\beta+\gamma)$

$= (\sin\alpha + \sin\beta) + [\sin\gamma + \sin(\alpha+\beta+\gamma)]$

$$= 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}+2\cos\frac{\gamma+\alpha+\beta+\gamma}{2}\sin\frac{\gamma-\alpha-\beta-\gamma}{2}$$

$$= 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}+2\cos\frac{\alpha+\beta+2\gamma}{2}\sin\left(\frac{-\alpha-\beta}{2}\right)$$

$$= 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}-2\cos\frac{\alpha+\beta+2\gamma}{2}\sin\left(\frac{\alpha+\beta}{2}\right) \quad [\because \sin(-\theta) = -\sin\theta]$$

$$= 2\sin\frac{\alpha+\beta}{2}\left\{\cos\frac{\alpha-\beta}{2}-\cos\frac{\alpha+\beta+2\gamma}{2}\right\}$$

$$= 2\sin\left(\frac{\alpha+\beta}{2}\right)\left\{2\sin\left(\frac{\frac{\alpha-\beta}{2}+\frac{\alpha+\beta+2\gamma}{2}}{2}\right)\sin\left(\frac{\frac{\alpha+\beta+2\gamma}{2}-\frac{\alpha-\beta}{2}}{2}\right)\right\}$$

$$= 2\sin\frac{\alpha+\beta}{2}\left\{2\sin\left(\frac{\alpha-\beta+\alpha+\beta+2\gamma}{4}\right)\cdot\sin\left(\frac{\alpha+\beta+2\gamma-\alpha+\beta}{4}\right)\right\}$$

$$= 2\sin\frac{\alpha+\beta}{2}\left(2\sin\frac{\alpha+\gamma}{2}\sin\frac{\beta+\gamma}{2}\right)$$

$$= 4\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha+\gamma}{2}\right)\sin\left(\frac{\beta+\gamma}{2}\right) = \text{R.H.S.}$$ **Proved.**

S90. $\sin\alpha + \sin\beta = a$ ... (i)

$\cos\alpha + \cos\beta = b$ ... (ii)

$$2 \sin \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2} = a \quad \text{... (iii)}$$

$$2 \cos \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2} = b \quad \text{... (iv)}$$

(i) Dividing (iii) by (iv), we get

$$\boxed{\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{a}{b}}$$

(ii) On squaring (iii) and (iv), then adding, we get

$$4 \sin^2 \frac{(\alpha+\beta)}{2} \cos^2 \frac{(\alpha-\beta)}{2} + 4 \cos^2 \frac{(\alpha+\beta)}{2} \cos^2 \frac{(\alpha-\beta)}{2} = a^2 + b^2$$

$$4 \cos^2 \frac{\alpha-\beta}{2}\left[\sin^2 \frac{(\alpha-\beta)}{2} + \cos^2 \frac{(\alpha+\beta)}{2}\right] = a^2 + b^2$$

$$4 \cos^2 \frac{(\alpha-\beta)}{2} = a^2 + b^2$$

$$\Rightarrow \quad \cos^2 \frac{(\alpha-\beta)}{2} = \frac{a^2+b^2}{4} \quad \text{... (v)}$$

Now, $\sin^2 \frac{(\alpha-\beta)}{2} = 1 - \cos^2 \frac{(\alpha-\beta)}{2} = 1 - \frac{a^2+b^2}{4} = \frac{4-a^2-b^2}{4}$ ... (vi)

On dividing (v) by (vi), we get

$$\tan^2 \frac{(\alpha-\beta)}{2} = \frac{4-a^2-b^2}{4} \times \frac{4}{a^2+b^2}$$

$$\Rightarrow \quad \tan \frac{\alpha-\beta}{2} = \pm\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}.$$

S91. (i) L.H.S. = $\sin^2 6x - \sin^2 4x$

We apply the formula

$$\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$$

Proof: $\sin(A+B)\sin(A-B) = (\sin A \cos B + \cos A \sin B)(\sin A \cos B - \cos A \sin B)$

$= \sin^2 A \cos^2 B - \cos^2 A \sin^2 B$

$= \sin^2 A (1 - \sin^2 B) - (1 - \sin^2 A) \sin^2 B$

$= \sin^2 A - \sin^2 A \sin^2 B - \sin^2 B + \sin^2 A \sin^2 B$

$= \sin^2 A - \sin^2 B$

$\therefore$ L.H.S. = $\sin^2 6x - \sin^2 4x = \sin(6x+4x)\sin(6x-4x)$

$= \sin 10x \sin 2x$ = R.H.S. Proved.

(ii) L.H.S. = $\cos^2 2x - \cos^2 6x = 1 - \sin^2 2x - (1 - \sin^2 6x)$

$$= \sin^2 6x - \sin^2 2x = \sin(6x + 2x)\sin(6x - 2x)$$

$$= \sin 8x \sin 4x = \text{R.H.S.}$$ **Proved.**

S92. Let $x\cos\theta = y\cos\left(\theta + \frac{2\pi}{3}\right) = z\cos\left(\theta + \frac{4\pi}{3}\right) = k$ ... (i)

$$\Rightarrow \quad \frac{1}{x} = \frac{\cos\theta}{k}, \quad \frac{1}{y} = \frac{\cos\left(\theta + \frac{2\pi}{3}\right)}{k}, \quad \frac{1}{z} = \frac{\cos\left(\theta + \frac{4\pi}{3}\right)}{k}$$

Now, $\text{L.H.S.} = xy + yz + zx = \frac{xyz}{z} + \frac{xyz}{x} + \frac{xyz}{y} = xyz\left(\frac{1}{z} + \frac{1}{x} + \frac{1}{y}\right)$

$$= xyz\left[\frac{\cos\left(\theta + \frac{4\pi}{3}\right)}{k} + \frac{\cos\theta}{k} + \frac{\cos\left(\theta + \frac{2\pi}{3}\right)}{k}\right]$$ [Using Eq. (i)

$$= \frac{xyz}{k}\left[\cos\left(\theta + \frac{4\pi}{3}\right) + \cos\left(\theta + \frac{2\pi}{3}\right) + \cos\theta\right]$$

$$\left[\because \cos(C + D) = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\right]$$

$$= \frac{xyz}{k}\left[2\cos\frac{2\theta + 2\pi}{2}\cos\frac{2\pi}{6} + \cos\theta\right]$$

$$= \frac{xyz}{k}\left[2\cos(\pi + \theta)\cos\frac{\pi}{3} + \cos\theta\right] = \frac{xyz}{k}\left[-2\cos\theta\cdot\left(\frac{1}{2}\right) + \cos\theta\right]$$

$$= \frac{xyz}{k}[-\cos\theta + \cos\theta] = \frac{xyz}{k}(0) = 0 = \text{R.H.S.}$$ **Proved.**

$\Rightarrow \quad xy + yz + zx = 0.$

S93.

$$\sin\theta = n\sin(\theta + 2\alpha)\ \frac{\sin(\theta + 2\alpha)}{\sin\theta} = \frac{1}{n}$$

Applying componendo dividendo, we get

$$\frac{\sin(\theta + 2\alpha) + \sin\theta}{\sin(\theta + 2\alpha) - \sin\theta} = \frac{1+n}{1-n}$$

$$\frac{2\sin\frac{\theta + 2\alpha + \theta}{2}\cos\frac{\theta + 2\alpha - \theta}{2}}{2\sin\frac{\theta + 2\alpha - \theta}{2}\cos\frac{\theta + 2\alpha + \theta}{2}} = \frac{1+n}{1-n}$$

$$\left[\begin{array}{l}\because \sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2} \\ \& \sin C - \sin D = 2\sin\frac{C-D}{2}\cos\frac{C+D}{2}\end{array}\right]$$

$$\frac{2\sin(\theta+\alpha)\cos\alpha}{2\sin\alpha\cos(\theta+\alpha)} = \frac{1+n}{1-n} \quad \Rightarrow \quad \frac{\sin(\theta+\alpha)}{\cos(\theta+\alpha)}\cdot\frac{\cos\alpha}{\sin\alpha} = \frac{1+n}{1-n}$$

$$\frac{\tan(\theta+\alpha)}{\tan\alpha} = \frac{1+n}{1-n} \qquad \left[\because \ \tan\theta = \frac{\sin\alpha}{\cos\theta}\right]$$

$$\tan(\theta+\alpha) = \frac{1+n}{1-n}\tan\alpha.$$

S94.

$$\text{L.H.S.} = \frac{(\sin 7x + \sin 5x) + (\sin 9x + \sin 3x)}{(\cos 7x + \cos 5x) + (\cos 9x + \cos 3x)}$$

$$= \frac{2\sin\frac{7x+5x}{2}\cos\frac{7x-5x}{2} + 2\sin\frac{9x+3x}{2}\cos\frac{9x-3x}{2}}{2\cos\frac{7x+5x}{2}\cos\frac{7x-5x}{2} + 2\cos\frac{9x+3x}{2}\cos\frac{9x-3x}{2}}$$

$$= \frac{2\sin 6x\cos x + 2\sin 6x\cos 3x}{2\cos 6x\cos x + 2\cos 6x\cos 3x}$$

$$= \frac{2\sin 6x(\cos x + \cos 3x)}{2\cos 6x(\cos x + \cos 3x)}$$

$$= \frac{\sin 6x}{\cos 6x} = \tan 6x = \text{R.H.S.}$$

S95.

$$\text{L.H.S.} = \sin x + \sin 3x + \sin 5x + \sin 7x$$

$$= (\sin x + \sin 7x) + (\sin 3x + \sin 5x)$$

$$= \left(2\sin\frac{x+7x}{2}\cos\frac{x-7x}{2}\right) + \left(2\sin\frac{3x+5x}{2}\cos\frac{3x-5x}{2}\right)$$

$$= 2\sin 4x\cos(-3x) + 2\sin 4x\cos(-x)$$

$$= 2\sin 4x\cos 3x + 2\sin 4x\cos x$$

$$= 2\sin 4x[\cos 3x + \cos x]$$

$$= 2\sin 4x\left[2\cos\frac{3x+x}{2}\cos\frac{3x-x}{2}\right]$$

$$= 2\sin 4x[2\cos 2x\cos x]$$

$$= 4\cos x\cos 2x\sin 4x = \text{R.H.S.}$$

S96.

$$\text{L.H.S.} = \tan 4x = \frac{2\tan 2x}{1-\tan^2 2x}$$

$$= \frac{2\left[\frac{2\tan x}{1-\tan^2 x}\right]}{1-\left[\frac{2\tan x}{1-\tan^2 x}\right]^2} \qquad \left[\because \ \tan 2x = \frac{2\tan x}{1-\tan^2 x}\right]$$

$$= \frac{\dfrac{4\tan x}{1-\tan^2 x}}{1-\dfrac{4\tan^2 x}{(1-\tan^2 x)^2}} = \frac{\dfrac{4\tan x}{1-\tan^2 x}}{\dfrac{(1-\tan^2 x)^2 - 4\tan^2 x}{(1-\tan^2 x)^2}}$$

$$= \frac{4\tan x}{(1-\tan^2 x)} \times \frac{(1-\tan^2 x)^2}{(1-\tan^2 x)^2 - 4\tan^2 x}$$

$$= \frac{4\tan x\,(1-\tan^2 x)}{1-2\tan^2 x+\tan^4 x-4\tan^2 x}$$

$$= \frac{4\tan x\,(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x} = \text{R.H.S.}$$

S97. $\cot 3x = \cot(2x + x) = \dfrac{\cot 2x \cot x - 1}{\cot 2x + \cot x}$

$= \cot 3x\,(\cot 2x + \cot x) = \cot x \cot 2x - 1$

$= \cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1 = \text{R.H.S.}$

S98. $\text{L.H.S.} = \dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$

$$= \frac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$$

$$= \frac{2\cos\dfrac{4x+2x}{2}\cos\dfrac{4x-2x}{2} + \cos 3x}{2\sin\dfrac{4x+2x}{2}\cos\dfrac{4x-2x}{2} + \sin 3x}$$

$$= \frac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$$

$$= \frac{\cos 3x\,(2\cos x + 1)}{\sin 3x\,(2\cos x + 1)}$$

$$= \frac{\cos 3x}{\sin 3x} = \cot 3x = \text{R.H.S.}$$

S99. $\text{L.H.S.} = \cot 4x\,(\sin 5x + \sin 3x)$

$$= \frac{\cos 4x}{\sin 4x}(\sin 5x + \sin 3x)$$

$$= \frac{\cos 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$$

$$= \frac{\cos 4x}{\sin 4x}(2\sin 4x \cos x)$$

$$= \frac{\cos 4x}{2 \sin 2x \cos 2x} (2 \sin 4x \cos x)$$

$$= \frac{\cos 4x}{4 \sin x \cos x \cos 2x} (4 \sin 2x \cos 2x \cos x)$$

$$= \frac{\cos 4x \cdot (2 \sin x \cos x)}{\sin x}$$

$$= (2 \cos 4x \sin x)(\cot x)$$

$$= \cot x (\sin 5x - \sin 3x) = \text{R.H.S.}$$

S100. L.H.S. $= \cos \theta + \cos (120° + \theta) + \cos (\theta - 120°)$

$$= \cos \theta + 2 \cos \theta \cos (240°)$$

$$= \cos \theta + 2 \cos \theta \cos (180° + 60°)$$

$$= \cos \theta + 2 \cos \theta \left(\frac{-1}{2}\right)$$

$$= \cos \theta - \cos \theta = 0 = \text{R.H.S.}$$ **Proved.**

Now, let $a = \cos \theta$, $b = \cos (120° + \theta)$, $c = \cos (\theta - 120°)$

Here, $a + b + c = 0$

$\Rightarrow$ $a^3 + b^3 + c^3 = 3abc$

$\Rightarrow$ $\cos^3 \theta + \cos^3(120° + \theta) + \cos^3(\theta - 120°) = 3 \cos \theta \cos \theta (120° + \theta) \cos (\theta - 120°)$

$$= \frac{3}{2} \cos \theta [\cos 2\theta + \cos 240°]$$

$$= \frac{3}{2} \cos \theta \left[\cos 2\theta - \frac{1}{2}\right]$$

$$= \frac{3}{2} \cos \theta \left[2 \cos^2 \theta - 1 - \frac{1}{2}\right]$$

$$= \frac{3}{4} \cos \theta (4 \cos^2 \theta - 3)$$

$$= \frac{3}{4} (4 \cos^3 \theta - 3 \cos \theta) = \frac{3}{4} \cos 3\theta$$ **Proved.**

S101. Here, we observe that each angle in L.H.S. is double of the preceding angle.

L.H.S. $= \cos \theta \cos 2\theta \cos 4\theta \ldots \cos 2^{n-1} \theta$

$$= \frac{1}{2 \sin \theta} (2 \sin \theta . \cos \theta) \cos 2\theta . \cos 4\theta \ldots \cos 2^{n-1} \theta$$

$$= \frac{1}{2^2 \sin \theta} (2 \sin 2\theta \cos 2\theta) (\cos 4\theta \ldots \cos 2^{n-1} \theta)$$

$$= \frac{1}{2^3 \sin\theta} (2 \sin 4\theta \,.\, \cos 4\theta) [\cos 8\theta \cos 16\theta \ldots \cos 2^{n-1} \theta]$$

$$= \frac{1}{2^4 \sin\theta} (2 \sin 8\theta \,.\, \cos 8\theta) [\cos 16\theta \ldots \cos 2^{n-1} \theta]$$

- -

- -

$$= \frac{1}{2^n \sin\theta} . [2 \sin 2^{n-1}\theta \cos 2^{n-1}\theta] = \frac{\sin(2^n\theta)}{2^n \sin\theta} = \text{R.H.S.}$$ **Proved.**

S102.(i) L.H.S. $= \cos 4x = \cos 2(2x)$

$= 2[2\cos^2 x - 1]^2 - 1$

$= 2[4\cos^4 x - 4\cos^2 x + 1] - 1$

$= 8\cos^4 x - 8\cos^2 x + 2 - 1$

$= 1 - 8\cos^2 x(1 - \cos^2 x) = 1 - 8\cos^2 x \sin^2 x =$ R.H.S. **Proved.**

(ii) L.H.S. $= \tan 4x = \tan 2(2x)$

$$= \frac{2\tan 2x}{1-\tan^2 x} = \frac{2\,\frac{2\tan x}{1-\tan^2 x}}{1-\left(\frac{2\tan x}{1-\tan^2 x}\right)^2} \qquad \left[\because\ \tan^2 A = \frac{2\tan A}{1-\tan^2 A}\right]$$

$$= \frac{\frac{4\tan x}{1-\tan^2 x}}{\frac{(1-\tan^2 x)^2 - 4\tan^2 x}{(1-\tan^2 x)^2}}$$

$$= \frac{4\tan x}{1-\tan^2 x} \times \frac{(1-\tan^2 x)^2}{(1-2\tan^2 x + \tan^4 x)^2 - 4\tan^2 x}$$

$$= \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x + \tan^4 x} = \text{R.H.S.}$$ **Proved.**

S103. L.H.S. $= \sin 3x + \sin 2x - \sin x$

$= (\sin 3x - \sin x) + \sin 2x$

$$= 2\cos\frac{3x+x}{2}\sin\frac{3x-x}{2} + \sin 2x$$

$= 2\cos 2x \sin x + 2\sin x \cos x$

$= 2\sin x[\cos 2x + \cos x]$

$$= 2\sin x\left[2\cos\frac{2x+x}{2}\cos\frac{2x-x}{2}\right]$$

$$= 2 \sin x \left[2 \cos \frac{3x}{2} \cos \frac{x}{2} \right]$$

$$= 4 \sin x \cos \frac{x}{2} \cos \frac{3x}{2} = \text{R.H.S.}$$

Date: 9/10/2021

Q1. Find the principle solution of $\sin x = \frac{1}{2}$.

Q2. Find general solution of $\sin 2x = 0$.

Q3. Find general solution of $\sin\left(x - \frac{\pi}{4}\right) = 0$.

Q4. Find the general solution of $\tan 5x = 0$.

Q5. Find the general solution of $\cos\left(x + \frac{\pi}{10}\right) = 0$.

Q6. Find the general solution of $\tan\left\{2x + \frac{\pi}{3}\right\} = 0$.

Q7. Find the general solution of $\tan\left(4x + \frac{\pi}{6}\right) = 0$.

Q8. If $\sin x + \sin^2 x = 1$ then find the value of $\cos^2 x + \cos^4 x$.

Q9. If $\cos x + \cos^2 x = 1$ then find the value of $\sin^2 x + \sin^4 x$.

Q10. If $0 \le \theta \le \pi$, find θ for which, $\sin\theta = \cos\theta$.

Q11. If $0 \le \theta \le \pi$, find θ for which, $\sin\theta = \sin(-\theta)$.

Q12. Find the principal solutions of the equation $\sin x = \frac{\sqrt{3}}{2}$.

Q13. Find the solution of $\sin x = -\frac{\sqrt{3}}{2}$.

Q14. Solve $\cos x = \frac{1}{2}$.

Q15. Solve $\tan 2x = -\cot\left(x + \frac{\pi}{3}\right)$.

Q16. Find the principle solution of $\tan x = \sqrt{3}$.

Q17. Find the principle solution of the equation $\operatorname{cosec} x = -2$

Q18. If $a \sec\alpha - c\tan\alpha = d$ and $b\sec\alpha + d\tan\alpha = c$ then prove that $a^2 + b^2 = c^2 + d^2$.

Q19. If $\cos^4\alpha - \sin^4\alpha = a$, then find the value of $\frac{1-a}{1+a}$.

Q20. Solve for x, $\sin x \cdot \tan x - 1 = \tan x - \sin x$.

Q21. If $3\sin\theta + 4\cos\theta = 5$, then find the value of $4\sin\theta - 3\cos\theta$.

Q22. If $\sin\theta = \sin\alpha$ prove that $\theta = n\pi + (-1)^n\alpha$, where $n \in I$.

Q23. Find x from the following equation $\operatorname{cosec}(90° + A) + x\cos A \cdot \cot(90° + A) = \sin(90° + A)$.

Q24. If $\sin\alpha + \sin\beta = a$ and $\cos\alpha - \cos\beta = b$, then find the value of $\tan\left(\frac{\alpha-\beta}{2}\right)$.

Q25. Let $P = a\cos\theta - b\sin\theta$, then prove that $-\sqrt{a^2+b^2} \le P \le \sqrt{a^2+b^2}$.

Q26. Find the value of $2\tan\frac{\pi}{10} + 3\sec\frac{\pi}{10} - 4\cos\frac{\pi}{10}$.

Q27. If $\tan^2\theta + \sec\theta = 5$ then find the value of $\sec\theta$.

Q28. If $\sec A + \tan A = 3$ then find the value of $\sec A$.

Q29. If $x = 1 + \tan\theta$, $y = 2 + \cot\theta$, then prove that $xy + 1 = 2x + y$.

Q30. Find the general solution of $\sin mx + \sin nx = 0$.

Q31. If α and β are complementary angles and $\sin\alpha = \frac{3}{5}$, then find the value of $\sin\alpha\cos\beta - \cos\alpha\sin\beta$.

Q32. If $\cos 20° - \sin 20° = P$, then find the value of $\cos 40°$.

Q33. Let $a\sin x = b\cos x = \frac{2c\tan x}{1-\tan^2 x}$ and $a^2 - b^2 = k\cdot c^2(a^2+b^2)$, then find the value of k.

Q34. Solve for x, $\tan x + \tan 2x + \tan x\cdot\tan 2x = 1$

Q35. Find the general solution for equation $\sin x + \sin 2x + \sin 3x = 0$.

Q36. Solve, $\cot^2 x + 3\operatorname{cosec} x + 3 = 0$

Q37. Prove that if $a\cos\theta + b\sin\theta = c \Rightarrow \theta = 2n\pi + \alpha \pm \beta$, where $\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}$, $\cos\beta = \frac{c}{\sqrt{a^2+b^2}}$ and $\sin\alpha = \frac{b}{\sqrt{a^2+b^2}}$.

Q38. If $n\tan\frac{\theta}{2} = m$, prove that $m\sin\theta + n\cos\theta = n$.

Q39. If $(A + B) = 45°$, then prove that $(1 + \tan A)(1 + \tan B) = 2$.

Q40. Prove that $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$

Q41. If $\cos^2 x + \cos x = 1$, then find the value of $\sin^2 x(2 - \cos^2 x)$.

Q42. Find the set of values of $\lambda \in R$, such that $\tan^2\theta + \sec\theta = \lambda$ holds for some real θ.

Q43. Solve for x, $\sec x - \tan x = \sqrt{3}$.

Q44. Find the principle solution of $\tan x = \frac{-1}{\sqrt{3}}$.

Q45. If $\tan x = \frac{b}{a}$, then find the value of $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$.

Q46. If $\tan^2\alpha = 1 - P^2$ then find the value of $\sec\alpha + \tan^3\alpha\cdot\operatorname{cosec}\alpha$ in terms of P.

Q47. Find the general solution of $\cos 3x = \sin 2x$.

Q48. If $a\sec\theta + b\tan\theta = c$ then find the value of $(a\tan\theta + b\sec\theta)^2$.

Q49. Prove that, $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x\sin 4x$.

Q50. If $\sin\theta + \cos\theta = 0$ and θ lies in the fourth quadrant, find $\sin\theta$ and $\cos\theta$.

Q51. Solve for x, $\sqrt{3}\cos x - \sin x = 1$.

Q52. If $x = \sec\theta - \tan\theta$ and $y = \text{cosec}\,\theta + \cot\theta$, then prove that $xy + x - y + 1 = 0$.

Q53. Find the principal and general solution of the following equation: $\cos 4x = \cos 2x$.

Q54. Find the principal and general solutions of the following equation: $\cot x = -\sqrt{3}$.

Q55. Find the principal and general solution of the following equation: $\sec x = 2$.

Q56. Solve $2\cos^2 x + 3\sin x = 0$.

Q57. Solve $\sin 2x - \sin 4x + \sin 6x = 0$,

Q58. Find the principal solutions of the equation $\tan x = -\frac{1}{\sqrt{3}}$.

Q59. Find the general solution of the following equation:

$$\cos 3x + \cos x - \cos 2x = 0.$$

Q60. Find the general solution of the following equation:

$$\sin 2x + \cos x = 0.$$

Q61. Find the general solution of the following equation:

$$\sec^2 2x = 1 - \tan 2x.$$

Q62. Find the general solution of the following equation:

$$\sin x + \sin 3x + \sin 5x = 0.$$

Q63. Find the value of $\sin\frac{\pi}{14}\cdot\sin\frac{3\pi}{14}\cdot\sin\frac{5\pi}{14}\cdot\sin\frac{7\pi}{14}\cdot\sin\frac{9\pi}{14}\cdot\sin\frac{11\pi}{14}\cdot\sin\frac{13\pi}{14}$.

Q64. Find the general solution of each equation.

(a) $\sqrt{3}\cot x + 1$ (b) $\text{cosec}\,x + \sqrt{2}$

Q65. If $\sin\theta = \frac{3}{5}$, $\tan\phi = \frac{3}{5}$ and $\frac{\pi}{2} < \theta < \pi < \phi < \frac{3\pi}{2}$, find the value of $8\tan\theta - \sqrt{5}\sec\phi$.

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S1. The solution of a trigonometric equation for which the value of unknown angle say x lies between 0 to 2π is called its principle solution.

$$\therefore \quad \sin x = \frac{1}{2} \Rightarrow \sin x = \sin\frac{\pi}{6}$$

$$\therefore \quad x = \frac{\pi}{6}, \text{ which lies between 0 to } 2\pi.$$

S2. $\because \quad \sin 2x = 0 \Rightarrow 2x = n\pi$

$$\Rightarrow \quad x = n\frac{\pi}{2}, \quad n \in I$$

S3. $\because$ $\sin x$ is zero at integral multiple of π.

$$\Rightarrow \quad x - \frac{\pi}{4} = n\pi \Rightarrow x = n\pi + \frac{\pi}{4},\ n \in I.$$

S4. $\because$ $\tan x$ is zero for integral multiple of π.

$$\therefore \quad \tan 5x = \tan n\pi$$

$$\Rightarrow \quad 5x = n\pi \Rightarrow x = \frac{n\pi}{5}, n \in I$$

S5. $\because$ $\cos x$ is zero at odd multiple of $\frac{\pi}{2}$.

$$\therefore \quad \cos\left(x + \frac{\pi}{10}\right) = \cos(2n+1)\frac{\pi}{2}, \quad n \in I$$

$$\Rightarrow \quad x + \frac{\pi}{10} = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow \quad x = \left\{(2n+1)\frac{\pi}{2} - \frac{\pi}{10}\right\}$$

S6.

$$\tan\left(2x + \frac{\pi}{3}\right) = \tan(n\pi)$$

$$\Rightarrow \quad 2x + \frac{\pi}{3} = n\pi \Rightarrow x = \frac{1}{2}\left(n\pi - \frac{\pi}{3}\right).$$

S7.

$$\tan\left(4x + \frac{\pi}{6}\right) = 0 \Rightarrow \tan\left(4x + \frac{\pi}{6}\right) = \tan n\pi$$

$$4x + \frac{\pi}{6} = n\pi$$

$$\Rightarrow \quad x = \frac{1}{4}\left(n\pi - \frac{\pi}{6}\right).$$

S8. $\because \quad \sin x + \sin^2 x = 1$

$$\Rightarrow \quad \sin x = 1 - \sin^2 x \quad \Rightarrow \quad \sin x = \cos^2 x$$

$$\Rightarrow \quad \cos^2 x + \cos^4 x = \sin x + (\sin x)^2$$

$$= \sin x + \sin^2 x = 1.$$

S9. $\because \quad \cos x + \cos^2 x = 1 \quad \Rightarrow \quad \cos x = 1 - \cos^2 x$

$$\Rightarrow \quad \cos x = \sin^2 x$$

$$\therefore \quad \sin^2 x + \sin^4 x = \cos x + \cos^2 x = 1.$$

S10. (i) We know that, $\sin\theta$ and $\cos\theta$ have the same sign, if θ lies in I quadrant of III quadrant. Since, $0 \le \theta \le \pi$, they will be equal in I quadrant.

$$\sin\theta = \cos\theta \quad \Rightarrow \quad \frac{\sin\theta}{\cos\theta} = 1 \quad \Rightarrow \quad \tan\theta = 1$$

$$\tan\theta = \tan 45° \quad \Rightarrow \quad \theta = 45° \qquad [\tan 45° = 1]$$

S11. We have, $\quad \sin = \sin\theta\,(-\theta)$

$$\Rightarrow \quad \sin\theta = -\sin\theta \quad \Rightarrow \quad 2\sin\theta = 0 \quad \Rightarrow \quad \sin\theta = 0$$

$$\Rightarrow \quad \theta = 0$$

$$\therefore \quad \theta = 0, \quad \pi \qquad [\text{SInce, } 0 \le \theta \le \pi]$$ **Ans.**

S12. We know that, $\quad \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

and $\quad \sin\frac{2\pi}{3} = \sin\left(\pi - \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Therefore, principal solutions are

$$x = \frac{\pi}{3} \quad \text{and} \quad \frac{2\pi}{3}.$$

S13. We have, $\quad \sin x = -\frac{\sqrt{3}}{2} = -\sin\frac{\pi}{3} = \sin\left(\pi + \frac{\pi}{3}\right) = \sin\frac{4\pi}{3}$

Hence, $\quad \sin x = \sin\frac{4\pi}{3}$, which gives

$$x = n\pi + (-1)^n \frac{4\pi}{3}, \quad \text{where} \quad n \in Z.$$

S14. We have, $\cos x = \frac{1}{2} = \cos\frac{\pi}{3}$

Therefore $x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$.

S15. We have, $\tan 2x = -\cot\left(x + \frac{\pi}{3}\right) = \tan\left(\frac{\pi}{2} + x + \frac{\pi}{3}\right)$

or $\tan 2x = \tan\left(x + \frac{5\pi}{6}\right)$

Therefore, $2x = n\pi + x + \frac{5\pi}{6}$, where $n \in Z$

or $x = n\pi + \frac{5\pi}{6}$ where $n \in Z$.

S16. Here value of tan x is positive so principle solutions lie in I and III quadrants.

$\therefore$ $\tan x = \sqrt{3}$

$\Rightarrow$ $\tan\left(\frac{\pi}{3}\right) = \sqrt{3} \Rightarrow x = \frac{\pi}{3}$

which lie in 1st quadrant and $\tan\left(\pi + \frac{\pi}{3}\right) = \tan\frac{\pi}{3} = \sqrt{3}$ which lies in 3rd quadrant.

$\therefore$ $\tan\frac{4\pi}{3} = \sqrt{3} \Rightarrow x = \frac{4\pi}{3}$

Hence, $x = \frac{\pi}{3}$ and $\frac{4\pi}{3}$ are principle solution.

S17. We have, $\operatorname{cosec} x = -2$

We know that

$$\operatorname{cosec}\left(\pi + \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2$$

Also, $\operatorname{cosec}\left(2\pi - \frac{\pi}{6}\right) = -\operatorname{cosec}\frac{\pi}{6} = -2$

Thus, $\left(\pi + \frac{\pi}{6}\right)$ and $\left(2\pi - \frac{\pi}{6}\right)$ are principle solutions.

Hence, principle solution of $\operatorname{cosec} x = -2$ are $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$.

S18. $a \sec\alpha - c\tan\alpha = d$

$\Rightarrow$ $a = d\cos\alpha + c\sin\alpha$ ... (i)

$b \sec \alpha + d \tan \alpha = c$

$\Rightarrow \quad b = c \cos \alpha - d \sin \alpha$ ... (ii)

$\Rightarrow \quad a^2 + b^2 = c^2 + d^2.$

S19. $\because \quad a = \cos^4 \alpha - \sin^4 \alpha$

$a = (\cos^2 \alpha + \sin^2 \alpha)(\cos^2 \alpha - \sin^2 \alpha)$

$\Rightarrow \quad 1 - a = 2 \sin^2 \alpha$

Similarly,

$1 + a = 2 \cos^2 \alpha$

$\therefore \quad \frac{1-a}{1+a} = \frac{2\sin^2 \alpha}{2\cos^2 \alpha} = \tan^2 \alpha.$

S20. Given that $\sin x \cdot \tan x - 1 = \tan x - \sin x$

$\because \quad \sin x \tan x + \sin x - 1 = \tan x$

$\Rightarrow \quad \sin x(\tan x + 1) - (\tan x + 1) = 0$

$\Rightarrow \quad (\tan x + 1)(\sin x - 1) = 0$

$\Rightarrow \quad x = \left(m\pi - \frac{\pi}{4}\right)$ or $x = n\pi + (-1)^n \cdot \frac{\pi}{2}$, $(m, n \in z)$.

S21. Given that $3 \sin \theta + 4 \cos \theta = 5$

$\because \quad (3 \sin \theta + 4 \cos \theta)^2 + (4 \sin \theta - 3 \cos \theta)^2 = 16 + 9$

$\Rightarrow \quad 25 + (4 \sin \theta - 3 \cos \theta)^2 = 25$

$\Rightarrow \quad 4 \sin \theta - 3 \cos \theta = 0.$

S22. $\because \quad \sin \theta = \sin \alpha \quad \Rightarrow \quad \sin \theta - \sin \alpha = 0$

$\Rightarrow \quad 2\cos\left(\frac{\theta+\alpha}{2}\right) \times \sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\Rightarrow \quad \cos\left(\frac{\theta+\alpha}{2}\right) = 0 \quad \text{or} \quad \sin\left(\frac{\theta-\alpha}{2}\right) = 0$

$\Rightarrow \quad \frac{\theta+\alpha}{2} = (2n+1)\frac{\pi}{2}, \ \frac{\theta-\alpha}{2} = n\pi$

$\Rightarrow \quad (\theta + \alpha) = (2n + 1)\pi, \ \theta - \alpha = 2n\pi$

$\Rightarrow \quad \theta = [(2n + 1)\pi - \alpha]$ or $\theta = (2n\pi + \alpha)$

$\Rightarrow \quad \theta = [(\text{an odd multiple of } \pi) - \alpha]$

or $\quad \theta = [(\text{an even multiple of } \pi) + \alpha]$

$\Rightarrow \quad \theta = n\pi + (-1)^n\alpha$, where $n \in I$.

S23. cosec (90° + A) + x cos A · cot (90° + A) = sin (90° + A)

$\Rightarrow$ $\sec A + x\cos A(-\tan A) = \cos A$

$\Rightarrow$ $-x\cos A\cdot\tan A = \dfrac{\cos^2 A - 1}{\cos A}$

$\Rightarrow$ $-x\sin A = \dfrac{-\sin^2 A}{\cos A}$

$\Rightarrow$ $x = \tan A$

S24. $\because$ $\sin\alpha + \sin\beta = a$

$\Rightarrow$ $2\sin\dfrac{\alpha+\beta}{2}\cdot\cos\dfrac{\alpha-\beta}{2} = a$ ... (i)

Similarly, $-2\sin\dfrac{\alpha+\beta}{2}\cdot\sin\dfrac{\alpha-\beta}{2} = b$ ... (ii)

Dividing (ii) by (i), we get

$$\tan\frac{\alpha-\beta}{2} = \frac{-b}{a}.$$

S25.

$$P = \sqrt{a^2+b^2}\left\{\frac{a}{\sqrt{a^2+b^2}}\cos\theta - \frac{b}{\sqrt{a^2+b^2}}\cdot\sin\theta\right\}$$

$$= \sqrt{a^2+b^2}\cdot\cos(\theta+\alpha),\quad \text{where } \cos\alpha = \frac{a}{\sqrt{a^2+b^2}}$$

But $-1 \le \cos(\theta+\alpha) \le 1$

$\therefore$ $-\sqrt{a^2+b^2} \le P \le \sqrt{a^2+b^2}$

S26. $\because$ $2\tan\dfrac{\pi}{10} + 3\sec\dfrac{\pi}{10} - 4\cos\dfrac{\pi}{10}$

$$= \frac{2\sin 18^\circ + 3 - 4\cos^2 18^\circ}{\cos 18^\circ}$$

$$= \frac{2\sin 18^\circ + 3 - 2(1+\cos 36^\circ)}{\cos 18^\circ}$$

$$= \frac{2\times\dfrac{\sqrt{5}-1}{4} + 3 - 2 - 2\times\left(\dfrac{\sqrt{5}+1}{4}\right)}{\cos 18^\circ} = 0$$

S27. $\tan^2\theta + \sec\theta = 5$

$\Rightarrow$ $\sec\theta = 5 - \tan^2\theta$

$\Rightarrow$ $\sec\theta = 6 - (1 + \tan^2\theta)$

$\Rightarrow \quad \sec\theta = 6 - \sec^2\theta \quad \Rightarrow \quad \sec^2\theta + \sec\theta - 6 = 0$

$\Rightarrow \quad \sec^2\theta + 3\sec\theta - 2\sec\theta - 6 = 0$

$\Rightarrow \quad \sec\theta(\sec\theta + 3) - 2(\sec\theta + 3) = 0$

$\Rightarrow \quad \sec\theta = 2, -3$

S28. $\because \quad \sec A + \tan A = 3 \quad \ldots \text{(i)}$

$\sec A - \tan A = \dfrac{1}{3} \quad \ldots \text{(ii)} \qquad [\sec^2 A - \tan^2 A = 1]$

Adding Eq. (i) and (ii), we get

$\Rightarrow \quad 2\sec A = 3 + \dfrac{1}{3} \quad \text{or} \quad 2\sec A = \dfrac{10}{3}$

$\sec A = \dfrac{10}{6} \quad \Rightarrow \quad \sec A = \dfrac{5}{3}$

S29. $x = 1 + \tan\theta$

$\Rightarrow \quad x - 1 = \tan\theta \quad \ldots \text{(i)}$

$y = 2 + \cot\theta$

$\Rightarrow \quad y - 2 = \cot\theta \quad \ldots \text{(ii)}$

Multiplying Eq. (i) and (ii), simultaneously

$(x-1)(y-2) = \tan\theta \cdot \cot\theta$

$\Rightarrow \quad (x-1)(y-2) = 1 \quad \Rightarrow \quad xy + 1 = 2x + y,$ **Proved**.

S30. $\because \quad \sin mx + \sin nx = 0$

$\Rightarrow \quad 2\sin\dfrac{(m+n)x}{2} \cdot \cos\dfrac{(m-n)x}{2} = 0$

or $\quad \dfrac{\sin(m+n)x}{2} = 0 \quad$ or $\quad \dfrac{\cos(m-n)x}{2} = 0$

$\Rightarrow \quad \dfrac{(m+n)x}{2} = p\pi \quad$ or $\quad \dfrac{(m-n)x}{2} = (2q+1)\dfrac{\pi}{2}, \quad$ where $p, q \in I$.

$\Rightarrow \quad x = \dfrac{2p\pi}{m+n}, \quad$ or $\quad x = \dfrac{(2q+1)\pi}{m-n}, \quad$ where $p, q \in I$.

S31. $\because \quad \alpha + \beta = 90^\circ \quad \Rightarrow \quad \alpha = (90^\circ - \beta)$

Taking sin on both sides, we get,

$\sin\alpha = \sin(90^\circ - \beta) = \cos\beta$

$\Rightarrow \quad \cos\beta = \dfrac{3}{5}, \quad \sin\beta = \dfrac{4}{5}, \cos\alpha = \dfrac{4}{5}$

Putting all these values in given expression, we get

$$\sin\alpha\cos\beta - \cos\alpha\sin\beta = \frac{3}{5}\cdot\frac{3}{5} - \frac{4}{5}\cdot\frac{4}{5} = -\frac{7}{25}.$$

S32. $(\cos 20° + \sin 20°)^2 + (\cos 20° - \sin 20°)^2 = 2$

$\because \quad (a+b)^2 + (a-b)^2 = 2(a^2+b^2)$

$\therefore \quad \cos 20° + \sin 20° = \sqrt{2-P^2}$

$\therefore \quad \cos 40° = (\cos 20° - \sin 20°)(\cos 20° + \sin 20°)$

$\therefore \quad \cos 40° = P.\sqrt{2-P^2}.$

S33. Let $a\sin x = b\cos x = c\tan 2x = l$

$\Rightarrow \quad \sin x = \frac{l}{a}, \cos x = \frac{l}{b}, \tan 2x = \frac{l}{c}$

$\Rightarrow \quad (a^2 - b^2) = 4c^2(a^2+b^2)$

$\Rightarrow \quad k = 4.$

S34. $\tan x + \tan 2x + \tan x \cdot \tan 2x = 1$

$\Rightarrow \quad \tan x + \tan 2x = 1 - \tan x \cdot \tan 2x$

$\Rightarrow \quad \frac{\tan x + \tan 2x}{1 - \tan x \cdot \tan 2x} = 1$

$\Rightarrow \quad \tan(x+2x) = 1 \quad \Rightarrow \quad \tan 3x = \tan\frac{\pi}{4}$

$\Rightarrow \quad 3x = n\pi + \frac{\pi}{4} \quad \Rightarrow \quad x = \frac{1}{3}\left(n\pi + \frac{\pi}{4}\right), n \in z.$

S35. $\because \quad \sin x + \sin 2x + \sin 3x = 0$

$\Rightarrow \quad (\sin 3x + \sin x) + \sin 2x = 0 \qquad \left\{\because \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cdot\cos\left(\frac{C-D}{2}\right)\right\}$

$\Rightarrow \quad 2\sin 2x \cdot \cos x + \sin 2x = 0$

$\Rightarrow \quad \sin 2x(2\cos x + 1) = 0$

$\Rightarrow \quad \sin 2x = 0 \text{ or } \cos x = \frac{-1}{2}$

$x = \frac{m\pi}{2}, \quad \text{or} \quad x = 2n\pi \pm \frac{2\pi}{3}, \quad m, n \in z$

S36. $\because \quad \cot^2 x + 3\operatorname{cosec} x + 3 = 0$

$\Rightarrow \quad \operatorname{cosec}^2 x - 1 + 3\operatorname{cosec} x + 3 = 0$

$\Rightarrow \quad \operatorname{cosec}^2 x + 3\operatorname{cosec} x + 2 = 0$

$\Rightarrow \quad \operatorname{cosec} x = -1 \text{ or } \operatorname{cosec} x = -2$

$$x = n\pi + (-1)^{n+1} \cdot \frac{\pi}{6}, \quad \text{or} \quad x = n\pi + (-1)^n \cdot \frac{\pi}{2} \quad n \in z.$$

S37. The given Eq. is $a \cos \theta + b \sin \theta = c$ . ... (i)

Dividing (i) throughout by $\sqrt{a^2 + b^2}$, we get

$$\frac{a}{\sqrt{a^2+b^2}} \cdot \cos\theta + \frac{b}{\sqrt{a^2+b^2}} \cdot \sin\theta = \frac{c}{\sqrt{a^2+b^2}}$$

$\Rightarrow \quad \cos\theta \cos\alpha + \sin\theta \sin\alpha = \cos\beta$

where $\cos\alpha = \frac{a}{\sqrt{a^2+b^2}}$, $\sin\alpha = \frac{b}{\sqrt{a^2+b^2}}$ and $\cos\beta = \frac{c}{\sqrt{a^2+b^2}}$

$\Rightarrow \quad \cos(\theta - \alpha) = \cos\beta$

$\Rightarrow \quad (\theta - \alpha) = 2n\pi \pm \beta$

$\Rightarrow \quad \theta = (2n\pi + \alpha \pm \beta).$

S38. $\because \quad m \sin\theta + n \cos\theta$

$$= m\left[\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right] + n\left[\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right]$$

$$= m\left[\frac{\frac{2m}{n}}{1+\frac{m^2}{n^2}}\right] + n\left[\frac{1-\frac{m^2}{n^2}}{1+\frac{m^2}{n^2}}\right]$$

$$= \frac{2m^2n}{m^2+n^2} + \frac{n(n^2-m^2)}{m^2+n^2}$$

$$= n$$

S39. $\because \quad (A + B) = 45°$

$\Rightarrow \quad \tan(A + B) = \tan 45°$

$\Rightarrow \quad \frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$

$\Rightarrow \quad \tan A + \tan B = 1 - \tan A \tan B$

$\Rightarrow \quad \tan A + \tan A \tan B + \tan B + 1 = 2$

$\Rightarrow \quad \tan A(1 + \tan B) + 1\,(1 + \tan B) = 2$

$\Rightarrow \quad (1 + \tan A)\,(1 + \tan B) = 2$ **Proved**.

S40. $\because$ $(\tan\alpha - \cot\alpha) + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = 0$

L.H.S. $= \left(\dfrac{\sin^2\alpha - \cos^2\alpha}{\sin\alpha\cdot\cos\alpha}\right) + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha$

$= -2\cot 2\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha$

$\Rightarrow$ $= 2[\tan 2\alpha - \cot 2\alpha] + 4\tan 4\alpha + 8\cot 8\alpha$

$\Rightarrow$ $= 4[\tan 4\alpha - \cot 4\alpha] + 8\cot 8\alpha = 0$

$\Rightarrow$ $= 8[\tan 8\alpha - \tan 8\alpha]$

$= 0$

Hence, $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha = \cot\alpha$

S41. $\because$ $\cos^2 x + \cos x = 1$

$\cos^2 x + \cos x - 1 = 0$ or $\cos x = \dfrac{-1\pm\sqrt{5}}{2}$ $\left\{\because \cos x \neq \left(\dfrac{-1-\sqrt{5}}{2}\right) < -1\right\}$

$\therefore$ $\cos^2 x = \left(\dfrac{\sqrt{5}-1}{2}\right)^2 = \dfrac{3-\sqrt{5}}{2}$

$\therefore$ $\sin^2 x\,(2-\cos^2 x) = (1-\cos^2 x)(2-\cos^2 x)$

$= \left(1-\left(\dfrac{3-\sqrt{5}}{2}\right)\right)\left(2-\dfrac{3-\sqrt{5}}{2}\right) = \left(\dfrac{\sqrt{5}-1}{2}\right)\left(\dfrac{\sqrt{5}+1}{2}\right) = 1$

S42. $\because$ $\tan^2\theta + \sec\theta = \lambda$

$\sec^2\theta + \sec\theta - (\lambda+1) = 0$ $[\sec^2\theta - 1 = \tan^2\theta]$

$\sec\theta = \dfrac{-1\pm\sqrt{1+4(\lambda+1)}}{2} = \dfrac{-1\pm\sqrt{4\lambda+5}}{2}$

For real $\sec\theta$, $4\lambda + 5 \geq 0$ or $\lambda \geq \dfrac{-5}{4}$

Also, $\sec\theta \geq 1$ or $\sec\theta \leq -1$

$\therefore$ $\dfrac{-1\pm\sqrt{4\lambda+5}}{2} \leq -1$

$\Rightarrow$ $\lambda \in [-1, \infty)$.

S43. $\therefore$ $\sec x - \tan x = \sqrt{3}$

$\Rightarrow$ $\dfrac{1}{\cos x} - \dfrac{\sin x}{\cos x} = \sqrt{3}$ $\Rightarrow$ $(1-\sin x) = \sqrt{3}\cos x$.

$$\Rightarrow \qquad \sqrt{3}\ \cos x + \sin x = 1 \qquad \dots \text{(i)}$$

Dividing both sides by $\sqrt{(\sqrt{3})^2 + 1^2}$, we get

$$\frac{\sqrt{3}}{2}\ \cos x + \frac{1}{2}\sin x = \frac{1}{2}$$

$$\Rightarrow \quad \cos x \cdot \cos \frac{\pi}{6} + \sin x \cdot \sin \frac{\pi}{6} = \frac{1}{2}$$

$$\Rightarrow \quad \cos \left(x - \frac{\pi}{6}\right) = \cos \frac{\pi}{3}$$

$$\Rightarrow \quad \left(x - \frac{\pi}{6}\right) = 2n\pi \pm \frac{\pi}{3}$$

$$\Rightarrow \quad x = \left(2n\pi + \frac{\pi}{2}\right) \quad \text{or} \quad \left(2n\pi - \frac{\pi}{6}\right), \quad \text{where } n \in I.$$

S44. $\because$ tan x is negative in II and IV quadrants.

$$\because \quad \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

Thus,

$$\tan \left(\pi - \frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}} \quad \Rightarrow \quad \tan \frac{5\pi}{6} = -\frac{1}{\sqrt{3}}$$

$\therefore$ $x = \frac{5\pi}{6}$, which lies in II quadrant.

Also, $\tan \left(2\pi - \frac{\pi}{6}\right) = -\tan \frac{\pi}{6}$

$\Rightarrow$ $x = \frac{11\pi}{6}$, which lies in IV quadrant.

Hence, principle solutions are

$$x = \frac{5\pi}{6} \text{ and } x = \frac{11\pi}{6}.$$

S45. $\because$ $\tan x = \frac{b}{a}$

$$\therefore \quad \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} = \frac{a+b+a-b}{\sqrt{a-b}\sqrt{a+b}}$$

$$= \frac{2a}{\sqrt{a^2 - b^2}} = \frac{2a}{\sqrt{a^2 - (a^2 \tan^2 x)}}$$

$$= \frac{2a}{a\sqrt{1-\tan^2 x}} = \frac{2}{\sqrt{1-\frac{\sin^2 x}{\cos^2 x}}}$$

$$= \frac{2\cos x}{\sqrt{\cos^2 x - \sin^2 x}} \qquad [\because \cos^2 x - \sin^2 x = \cos 2x]$$

$$= \frac{2\cos x}{\sqrt{\cos 2x}}.$$

S46. $\because \quad \tan^2\alpha = 1 - P^2$

Now,

$$\sec\alpha + \tan^3\alpha \cdot \text{cosec}\,\alpha = \sec\alpha\left[1 + \frac{\sin^3\alpha}{\cos^3\alpha}\cdot\frac{\cos\alpha}{\sin\alpha}\right]$$

$$= \sec\alpha[1 + \tan^2\alpha] = \sec^3\alpha$$

$\because \qquad \sec^2\alpha = 1 + \tan^2\alpha$

$\therefore \qquad \sec^2\alpha = 1 + 1 - P^2 = 2 - P^2$

$\Rightarrow \qquad \sec^3\alpha = (\sec^2\alpha)^{3/2} = (2 - P^2)^{3/2}$

Hence required value is $(2 - P^2)^{3/2}$

S47.

$$\cos 3x = \sin 2x \quad \Rightarrow \quad \cos 3x = \cos\left(\frac{\pi}{2} - 2x\right)$$

$$3x = 2n\pi \pm \left(\frac{\pi}{2} - 2x\right)$$

For positive sign

$$3x = 2n\pi + \frac{\pi}{2} - 2x \quad \Rightarrow x = (4n+1)\frac{\pi}{10}, \quad n \in z$$

For negative sign

$$3x = 2n\pi - \frac{\pi}{2} + 2x \quad \Rightarrow \quad x = (4n-1)\frac{\pi}{10}, \qquad n \in z$$

S48. $\qquad (a\sec\theta + b\tan\theta)^2 = c^2 \qquad \ldots$ (i)

Let $\qquad (a\tan\theta + b\sec\theta)^2 = k^2 \qquad \ldots$ (ii)

Subtracting Eq (ii) from (i) , we get

$$(a\sec\theta + b\tan\theta)^2 - (a\tan\theta + b\sec\theta)^2 = c^2 - k^2$$

$$(a\sec\theta + b\tan\theta - a\tan\theta - b\sec\theta)\cdot(a\sec\theta + b\tan\theta + a\tan\theta + b\sec\theta) = c^2 - k^2$$

$$(a-b)\cdot(\sec\theta - \tan\theta)\cdot(a+b)\cdot(\sec\theta + \tan\theta) = c^2 - k^2$$

$$a^2 - b^2 = c^2 - k^2$$

$$k^2 = -a^2 + b^2 + c^2$$

S49. L.H.S. = sin 2x + 2 sin 4x + sin 6x

= (sin 6x + sin 2x) + 2 sin 4x

$$= 2\sin\frac{6x+2x}{2}\cos\frac{6x-2x}{2}+2\sin 4x$$

= 2 sin 4x cos 2x + 2 sin 4x

= 2 sin 4x [cos 2x + 1] [$\because$ cos 2x + 1 = 2 $\cos^2 x$ – 1 + 1 = 2 $\cos^2 x$]

= 2 sin 4x 2 $\cos^2 x$

= 4 sin 4x $\cos^2 x$ = R.H.S. **Proved.**

S50. Here, $\sin\theta + \cos\theta = 0$ ... (i)

Squaring both sides, we get

$$(\sin\theta + \cos\theta)^2 = 0$$

$\Rightarrow$ $\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 0$ [$\because$ $(a+b)^2 = a^2 + b^2 + 2ab$]

$\Rightarrow$ $1 + 2\sin\theta\cdot\cos\theta = 0$ [$\because$ $\sin^2\theta + \cos^2\theta = 1$]

$\Rightarrow$ $\sin\theta\ \cos\theta = \dfrac{-1}{2}$ ... (ii)

From Eq. (i), $\sin\theta = -\cos\theta$

$\therefore$ Eq. (ii) becomes $-\cos\theta\cdot\cos\theta = \dfrac{-1}{2}$

$$\cos^2\theta = \frac{1}{2} \quad = \quad \cos\theta = \pm\frac{1}{\sqrt{2}}$$

But in the IV quadrant, $\cos\theta$ is +ve

$\therefore$ $\cos\theta = +\dfrac{1}{\sqrt{2}}$ and $\sin\theta = \dfrac{-1}{\sqrt{2}}$.

S51. $\sqrt{3}\cos x - \sin x = 1$ ... (i)

Dividing both sides of Eq. (i) by $\sqrt{(\sqrt{3})^2 + (-1)^2}$, we get

$$\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = \frac{1}{2}$$

$\Rightarrow$ $\cos x\cdot\cos\dfrac{\pi}{6} - \sin x\cdot\sin\dfrac{\pi}{6} = \dfrac{1}{2}$

$\Rightarrow$ $\cos\left(x+\dfrac{\pi}{6}\right) = \cos\dfrac{\pi}{3}$

$\Rightarrow$ $x + \dfrac{\pi}{6} = 2n\pi \pm \dfrac{\pi}{3}$

$\Rightarrow \quad x = \left(2n\pi + \frac{\pi}{6}\right)$ or $x = \left(2n\pi - \frac{\pi}{2}\right), \quad n \in I.$

S52. $\because \quad x = \sec\theta - \tan\theta = \frac{1-\sin\theta}{\cos\theta}$

$y = \text{cosec}\,\theta + \cot\theta = \frac{1+\cos\theta}{\sin\theta}$

$\therefore \quad xy + x - y + 1 = 0$

$= \frac{1-\sin\theta}{\cos\theta} \times \frac{1+\cos\theta}{\sin\theta} + \frac{1-\sin\theta}{\cos\theta} - \frac{1+\cos\theta}{\sin\theta} + 1$

$= \frac{1-(\sin^2\theta + \cos^2\theta)}{\sin\theta \cdot \cos\theta} = \frac{1-1}{\sin\theta \cdot \cos\theta} = 0.$

S53. We have $\cos 4x = \cos 2x$

$\Rightarrow \quad 4x = 2n\pi \pm 2x$

Taking +ve sign, we get

$4x = 2n\pi + 2x$

$\Rightarrow \quad 4x - 2x = 2n\pi$

$\Rightarrow \quad 6x = 2n\pi$

$x = n\pi$

$\Rightarrow \quad x = n\pi, \quad n \in Z$

Taking –ve sign, we get

$4x = 2n\pi - 2x$

$\Rightarrow \quad 4x + 2x = 2n\pi$

$\Rightarrow \quad 6x = 2n\pi$

$x = \frac{n\pi}{3}, \quad n \in Z$

$\therefore$ General solution is $x = \frac{n\pi}{3}$ or $x = n\pi, \quad n \in Z.$

S54. We have $\cot x = -\sqrt{3}$

$\Rightarrow \quad \frac{1}{\tan x} = -\sqrt{3}$

$\Rightarrow \quad \tan x = -\frac{1}{\sqrt{3}}$

We know that $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

Thus, $\tan\left(\pi - \frac{\pi}{6}\right) = -\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}}$

$\tan\left(2\pi - \frac{\pi}{6}\right) = -\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}}$

Thus, $\tan\frac{5\pi}{6} = \tan\frac{11\pi}{6} = -\frac{1}{\sqrt{3}}$

Principal solutions are $\frac{5\pi}{6}$ and $\frac{11\pi}{6}$

Now, $\tan x = -\frac{1}{\sqrt{3}}$

$\Rightarrow$ $\tan x = \tan\frac{5\pi}{6}$

Which gives $x = n\pi + \frac{5\pi}{6}, \quad n \in Z$

General solution is $x = n\pi + \frac{5\pi}{6}, \quad n \in Z.$

S55. We have $\sec x = 2$

$\Rightarrow$ $\frac{1}{\cos x} = 2$

$\Rightarrow$ $\cos x = \frac{1}{2}$

We know that $\cos\frac{\pi}{3} = \frac{1}{2}$

and $\cos\left(2\pi - \frac{\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$

Thus, $\cos\frac{\pi}{3} = \cos\frac{5\pi}{3} = \frac{1}{2}$

Principal solutions are $\frac{\pi}{3}$ and $\frac{5\pi}{3}$

Now, $\cos x = \frac{1}{2}$

$\Rightarrow$ $\cos x = \cos\frac{\pi}{3}$

Which gives $x = 2n\pi \pm \frac{\pi}{3}, \quad n \in Z$

General solution is $x = 2n\pi \pm \frac{\pi}{3}, \quad n \in Z.$

S56. The equation can be written as

$$2(1 - \sin^2 x) + 3 \sin x = 0$$

or $$2 \sin^2 x - 3 \sin x - 2 = 0$$

or $$(2 \sin x + 1)(\sin x - 2) = 0$$

Hence, $\sin x = \frac{1}{-2}$ or $\sin x = 2$

But $\sin x = 2$ is not possible $[\because -1 \le \sin x \le 1]$

Therefore, $\sin x = -\frac{1}{2} = \sin \frac{7\pi}{6}$

Hence, the solution is given by $x = n\pi + (-1)^n \frac{7\pi}{6}$, where $n \in Z$.

S57. The equation can be written as

$$\sin 6x + \sin 2x - \sin 4x = 0$$

or $$2 \sin 4x \cos 2x - \sin 4x = 0$$

or $$\sin 4x (2 \cos 2x - 1) = 0$$

Therefore, $\sin 4x = 0$ or $\cos 2x = \frac{1}{2}$

i.e., $\sin 4x = 0$ or $\cos 2x = \cos \frac{\pi}{3}$

Hence, $4x = n\pi$ or $2x = 2n\pi \pm \frac{\pi}{3}$, where $n \in Z$

i.e., $x = \frac{n\pi}{4}$ or $x = n\pi \pm \frac{\pi}{6}$, where $n \in Z$.

S58. We know that, $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$

Thus, $\tan\left(\pi - \frac{\pi}{6}\right) = -\tan \frac{\pi}{6} = -\frac{1}{\sqrt{3}}$

and $\tan\left(2\pi - \frac{\pi}{6}\right) = -\tan \frac{\pi}{3} = -\frac{1}{\sqrt{3}}$

Therefore, principal solutions are $\frac{5\pi}{6}$ and $\frac{11\pi}{6}$.

S59. Let $\cos 3x + \cos x - \cos 2x = 0$

$$\Rightarrow \left(2 \cos \frac{3x + x}{2} \cos \frac{3x - x}{2}\right) - \cos 2x = 0$$

$\Rightarrow \quad (2 \cos 2x \cos x) - \cos 2x = 0$

$\Rightarrow \quad \cos 2x\,(2 \cos x - 1) = 0$

$\Rightarrow \quad \cos 2x = 0$

or $\quad 2 \cos x - 1 = 0$

i.e., $\quad \cos 2x = 0$

or $\quad \cos x = \frac{1}{2}$

i.e., $\quad \cos 2x = 0$

or $\quad \cos 2x = \cos \frac{\pi}{2}$

Hence, $\quad 2x = (2n+1)\frac{\pi}{2}$

or $\quad x = 2n\pi \pm \frac{\pi}{3}, \quad$ where $\quad n \in Z$

i.e., $\quad x = (2n+1)\frac{\pi}{4}$

or $\quad x = 2n\pi \pm \frac{\pi}{3}, \quad$ where $\quad n \in Z$

Which is the required general solution.

S60. Let $\quad \sin 2x + \cos x = 0$

$\Rightarrow \quad 2 \cos x \sin x + \cos x = 0$

$\Rightarrow \quad \cos x\,(2 \sin x + 1) = 0$

$\Rightarrow \quad \cos x = 0$

or $\quad 2 \sin x + 1 = 0$

$\Rightarrow \quad \cos x = 0$

or $\quad \sin x = -\frac{1}{2}$

$\Rightarrow \quad \cos x = 0$

or $\quad \sin x = \sin\left(\pi + \frac{\pi}{6}\right)$

$\Rightarrow \quad \cos x = 0$

or $\sin x = \sin \frac{7\pi}{6}$

$\Rightarrow$ $x = (2n+1)\frac{\pi}{2}$

or $x = n\pi + (-1)^n \frac{7\pi}{6}$, where $n \in Z$

Hence, general solution is $x = (2n+1)\frac{\pi}{2}$

or $x = n\pi + (-1)^n \frac{7\pi}{6}$, where $n \in Z$.

S61. Let $\sec^2 2x = 1 - \tan 2x$

$\Rightarrow$ $1 + \tan^2 2x = 1 - \tan 2x$

$\Rightarrow$ $\tan^2 2x + \tan 2x = 0$

$\Rightarrow$ $\tan 2x (\tan 2x + 1) = 0$

$\Rightarrow$ $\tan 2x = 0$

or $\tan 2x = -1$

$\Rightarrow$ $\tan 2x = 0$

or $\tan 2x = \tan\left(\pi - \frac{\pi}{4}\right)$

$\Rightarrow$ $\tan 2x = 0$

or $\tan 2x = \tan \frac{3\pi}{4}$

$\Rightarrow$ $2x = n\pi$

or $2x = n\pi + \frac{3\pi}{4}$, where $n \in Z$

$\Rightarrow$ $x = \frac{n\pi}{2}$

or $x = \frac{n\pi}{2} + \frac{3\pi}{8}$, where $n \in Z$

Hence, general solution is $x = \frac{n\pi}{2}$

or $x = \frac{n\pi}{2} + \frac{3\pi}{8}$, where $n \in Z$.

S62. Let $\sin x + \sin 3x + \sin 5x = 0$

$\Rightarrow$ $(\sin + \sin 5x) + \sin 3x = 0$

$\Rightarrow$ $\left(2 \sin \frac{x+5x}{2} \cos \frac{x-5x}{2}\right) + \sin 3x = 0$

$\Rightarrow \quad 2 \sin 3x \cos(-2x) + \sin 3x = 0$

$\Rightarrow \quad 2 \sin 3x \cos 2x + \sin 3x = 0$

$\Rightarrow \quad \sin 3x (2 \cos 2x + 1) = 0$

$\Rightarrow \quad \sin 3x = 0$

or $\quad \cos 2x = -\frac{1}{2}$

$\Rightarrow \quad \sin 3x = 0$

or $\quad \cos 2x = \cos\left(\pi - \frac{\pi}{3}\right)$

$\Rightarrow \quad \sin 3x = 0$

or $\quad \cos 2x = \cos\frac{2\pi}{3}$

$\Rightarrow \quad 3x = n\pi$

or $\quad 2x = 2n\pi \pm \frac{2\pi}{3}$

$\Rightarrow \quad x = \frac{n\pi}{3}$

or $\quad x = n\pi \pm \frac{\pi}{3}$

Hence, general solution is $\quad x = \frac{n\pi}{3}$

or $\quad x = n\pi \pm \frac{\pi}{3}$, where $n \in Z$.

S63. The given expression can be written as

$$\left(\sin\frac{\pi}{14}\cdot\sin\frac{3\pi}{14}\cdot\sin\frac{5\pi}{14}\right)^2\cdot\sin\frac{7\pi}{14} \qquad \left[\sin\frac{\pi}{14}=\sin\frac{13\pi}{14}\right],\left[\sin\frac{3\pi}{14}=\sin\frac{11\pi}{14}\right],\left[\sin\frac{5\pi}{14}=\sin\frac{9\pi}{14}\right]$$

Clearly,

$$\sin\frac{\pi}{14} = \cos\left(\frac{\pi}{2}-\frac{\pi}{14}\right) = \cos\frac{3\pi}{7}$$

$$= -\cos\left(\pi-\frac{3\pi}{7}\right) = -\cos\frac{4\pi}{7}$$

$\therefore \quad \sin\frac{3\pi}{14} = \cos\frac{2\pi}{7},\ \sin\frac{5\pi}{14} = \cos\frac{\pi}{7}$

$\therefore$ Required value

$$= \left(\cos\frac{\pi}{7}\cdot\cos\frac{2\pi}{7}\right)^2\left(-\cos\frac{4\pi}{7}\right)^2\cdot 1$$

$$= \left\{\frac{2\sin\frac{\pi}{7}\cdot\cos\frac{\pi}{7}\cdot\cos\frac{2\pi}{7}\cdot\cos\frac{4\pi}{7}}{2\sin\frac{8\pi}{7}}\right\}^2$$

$$= ..... = \left(\frac{\sin\frac{8\pi}{7}}{8\sin\frac{8\pi}{7}}\right)^2 = \frac{1}{8^2}$$

$\therefore$ Required value for above expression is $\frac{1}{8^2}$.

S64. (a) $\sqrt{3}\cot x + 1 = 0$

$$\Rightarrow \quad \cot x = \frac{-1}{\sqrt{3}} \quad \Rightarrow \quad \tan x = -\sqrt{3} = -\tan\frac{\pi}{3}$$

$$= \tan\left(\pi - \frac{\pi}{3}\right) = \tan\frac{2\pi}{3}$$

$$\Rightarrow \quad \tan x = \tan\frac{2\pi}{3}$$

$$\Rightarrow \quad x = \left(n\pi + \frac{2\pi}{3}\right), \quad \text{where } n \in I.$$

Hence, the general solution is

$$x = \left(n\pi + \frac{2\pi}{3}\right), \quad \text{where } n \in I.$$

(b) $\operatorname{cosec} x + \sqrt{2} = 0$

$$\Rightarrow \quad \sin x = \frac{-1}{\sqrt{2}} = -\sin\frac{\pi}{4} = \sin\left(\pi + \frac{\pi}{4}\right) = \sin\frac{5\pi}{4}$$

$$\Rightarrow \quad \sin x = \sin\frac{5\pi}{4}$$

$$\Rightarrow \quad x = \left\{n\pi + (-1)^n\cdot\frac{\pi}{4}\right\},\ n \in I.$$

S65.

$$\sin\theta = \frac{3}{5} \quad \text{and} \quad \frac{\pi}{2} < \theta < \pi \qquad [\textit{i.e.}, \text{IInd quadrant}]$$

$$\sin^2\theta - \cos^2\theta = 1$$

$$\therefore \quad \cos\theta = \pm\sqrt{1-\sin^2\theta} = \pm\sqrt{1-\left(\frac{3}{5}\right)^2} = \pm\sqrt{1-\frac{9}{25}} = \pm\sqrt{\frac{16}{25}} = \pm\frac{4}{5}$$

In the IInd quadrant, $\cos\theta$ is –ve

$$\therefore \quad \cos\theta = \frac{-4}{5} \quad \text{and} \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3}{5}\times\frac{5}{-4} = \frac{-3}{4}$$

Again $$\tan\phi = \frac{1}{2}; \quad \pi < \phi < \frac{3\pi}{2} \qquad [\textit{i.e.}, \text{IIIrd quadrant}]$$

$$\sec^2\phi - \tan^2\phi = 1$$

$$\Rightarrow \quad \sec\phi = \pm\sqrt{1+\tan^2\phi} = \pm\sqrt{1+\left(\frac{1}{2}\right)^2} = \pm\sqrt{1+\frac{1}{4}} = \pm\frac{\sqrt{5}}{2}$$

In the IIIrd quadrant, $\sec\phi$ is –ve

$$\therefore \quad \sec\phi = -\frac{\sqrt{5}}{2}$$

Now, $$8\tan\theta - \sqrt{5}\sec\phi = 8\times\left(\frac{-3}{4}\right) - \sqrt{5}\left(\frac{-\sqrt{5}}{2}\right) = -6 + \frac{5}{2} = \frac{-12+5}{2} = \frac{-7}{2}.$$

Date: 9/10/2021

Q1. In a quadrilateral $ABCD$, prove that $\sin(A + B) + \sin(C + D) = 0$.

Q2. In a triangle ABC, prove that $\sin(B + C) = \sin A$.

Q3. In a cyclic quadrilateral $ABCD$ prove that $\sin A = \sin C$.

Q4. In a $\Delta\, ABC$, if $\frac{\cos A}{a} = \frac{\cos B}{b}$, show that triangle is isosceles.

Q5. In a $\Delta\, ABC$, if $\sin^2 A + \sin^2 B = \sin^2 C$ show that the triangle is right angled.

Q6. If $A + B + C = \pi$, then prove that $\tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$.

Q7. If $\tan(A + B) = x$ and $\tan(A - B) = y$ show that

$$\tan 2A = \frac{x + y}{1 - xy}$$

Q8. If $\cos(A + B) \cdot \sin(C + D) = \cos(A - B) \cdot \sin(C - D)$, then show that $\cot A \cot B \cot C = \cot D$.

Q9. In a $\Delta\, ABC$ prove that $\sin \frac{C - A}{2} = \frac{c - a}{b} \cdot \cos \frac{B}{2}$.

Q10. In a $\Delta\, ABC$, if $\cos A = \frac{\sin B}{2\sin C}$, show that the triangle is isosceles.

Q11. If $\cos\theta = \frac{1}{2}\left(a + \frac{1}{a}\right)$, then show that $\cos 3\theta = \frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$.

Q12. If $ABCD$ is a cyclic quadrilateral then find the value of $\cos(180° + A) + \cos(180° - B) + \cos(180° - C) - \sin(90° - D)$.

Q13. If $\sin(\theta + \alpha) = \cos(\theta + \alpha)$, then prove that,

$$\tan\theta = \frac{1 - \tan\alpha}{1 + \tan\alpha}.$$

Q14. In a $\Delta\, ABC$, prove that $a(b\cos C - c\cos B) = b^2 - c^2$.

Q15. In a $\Delta\, ABC$, prove that $(bc\cos A + ca\cos B + ab\cos C) = \frac{a^2 + b^2 + c^2}{2}$.

Q16. In a $\Delta\, ABC$, prove that

$$\frac{\tan B}{\tan C} = \frac{a^2 + b^2 - c^2}{a^2 - b^2 + c^2}.$$

Q17. If $\alpha, \beta, \gamma, \delta$ be four angles of a cyclic quadrilateral then find the value of $\cos\alpha + \cos\beta + \cos\gamma + \cos\delta$.

Q18. In a $\Delta\, ABC$, if $a\cos A = b\cos B$, show that the triangle is either isosceles or right angled.

Q19. Two tree A and B are on the same side of a river from a point C in the river, the distance of trees A and B are 220 m and 300 m respectively. If $\angle C = 45°$, find the distance between the trees. (use $\sqrt{2}$ = 1.44)

Q20. In a Δ *ABC* prove that

$$\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \left(\frac{1}{a^2} - \frac{1}{b^2}\right).$$

Q21. If the angles of a triangle are in ratio 1 : 2 : 3, prove that its corresponding sides are in ratio 1 : $\sqrt{3}$: 2.

Q22. In a triangle *ABC*, if $\angle C = 90°$, then prove that tan *A*, tan *B* are roots of the equation.

$$abx^2 - c^2x + ab = 0$$

Q23. For any triangle *ABC*, prove that

$$\frac{\sin(B-C)}{\sin(B+C)} = \frac{b^2 - c^2}{a^2}$$

Q24. In a Δ *ABC*, prove that

$$\frac{b-c}{b+c} = \frac{\tan\frac{1}{2}(B-C)}{\tan\frac{1}{2}(B+C)}.$$

Q25. The angles of a triangle ABC are in A.P. and if it is being given that

$b : c = \sqrt{3} : \sqrt{2}$, find $\angle A$, $\angle B$, and $\angle C$.

Q26. If $x = \alpha, \beta$, satisfies both the equation $\cos^2 x + a\cos x + b = 0$ and $\sin^2 x + p\sin x + q = 0$, then find the relation between *a*, *b*, *p* and *q*.

Q27. If cosec $A = 4P + \frac{1}{16P}$, then find the value of cosec *A* + cot *A*.

Q28. In any triangle *ABC*, show that

$$\frac{a-b}{a+b} = \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)}$$

Q29. In any triangle *ABC*, show that

$$a\cos\left(\frac{B-C}{2}\right) = (b+c)\sin\frac{A}{2}$$

Q30. With usual notations, if in a triangle *ABC*

$\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$, then prove that $\frac{\cos A}{7} = \frac{\cos B}{19} = \frac{\cos C}{25}$

Q31. If $A + B + C = \pi$, prove that:

$$\frac{\cos A}{\sin B \sin C} + \frac{\cos B}{\sin C \sin A} + \frac{\cos C}{\sin A \sin B} = 2.$$

Q32. Solve the triangle in which $a = (\sqrt{3}+1)$, $b = (\sqrt{3}-1)$ and $\angle C = 60°$.

Q33. In a Δ *ABC* prove that $(b-c)\cot\frac{A}{2} + (c-a)\cot\frac{B}{2} + (a-b)\cot\frac{C}{2} = 0.$

Q34. In a ΔABC, if $a = 18$, $b = 24$, $c = 30$, find sin *A*, sin *B*, sin *C*.

Q35. In any triangle *ABC*, show that

$$\frac{b^2 - c^2}{a^2}\sin 2A + \frac{c^2 - a^2}{b^2}\sin 2B + \frac{a^2 - b^2}{c^2}\sin 2C = 0.$$

Date: 9/10/2021

S1. $A + B + C + D = 2\pi$

$\therefore \quad A + B = 2\pi - (C + D)$

$\Rightarrow \quad \sin(A + B) = \sin(2\pi - (C + D))$

$\Rightarrow \quad \sin(A + B) = -\sin(C + D)$

$\Rightarrow \quad \sin(A + B) + \sin(C + D) = 0$

S2. $\because \quad A + B + C = \pi$

$\Rightarrow \quad B + C = (\pi - A)$

$\Rightarrow \quad \sin(B + C) = \sin(\pi - A)$

$\Rightarrow \quad \sin(B + C) = \sin A$

S3. $\because$ In a cyclic quadrilateral opposite angles are supplementary.

$\therefore \quad A + C = 180, \; B + D = 180$

$A = (180 - C)$

$\Rightarrow \quad \sin A = \sin(180 - C)$

$\Rightarrow \quad \sin A = \sin C$

S4. $$\frac{\cos A}{k \sin A} = \frac{\cos B}{k \sin B}$$

$\Rightarrow \quad \sin A \cos B - \cos A \cdot \sin B = 0$

$\Rightarrow \quad \sin(A - B) = 0$

$\Rightarrow \quad A = B$

Hence, triangle is isosceles.

S5. $\because \quad \sin^2 A + \sin^2 B = \sin^2 C$

$$\Rightarrow \quad \frac{a^2}{k^2} + \frac{b^2}{k^2} = \frac{c^2}{k^2}$$

$\Rightarrow \quad a^2 + b^2 = c^2$

Hence, triangle is right angled.

S6. $\because \quad A + B + C = \pi$

$\Rightarrow \quad A + B = (\pi - C) \quad \ldots \text{(i)}$

Operating tan in both sides in Eq. (i), we get

$\tan(A + B) = \tan(\pi - C)$

$$\Rightarrow \quad \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} = -\tan C$$

$$\Rightarrow \quad \tan A + \tan B + \tan C = \tan A \cdot \tan B \cdot \tan C$$

S7. From L.H.S. $\tan 2A = \tan \{(A + B) + (A - B)\}$

$$= \frac{\tan(A+B) + \tan(A-B)}{1 - \tan(A+B) \cdot \tan(A-B)}$$

$$= \frac{x + y}{1 - xy}$$

Hence, L.H.S. = R.H.S.

S8. $\because \quad \cos(A + B) \cdot \sin(C + D) = \cos(A - B) \cdot \sin(C - D)$

$$\frac{\cos(A + B)}{\cos(A - B)} = \frac{\sin(C - D)}{\sin(C + D)}$$ Applying componendo and dividendo.

$$\frac{\cos(A+B) + \cos(A-B)}{\cos(A+B) - \cos(A-B)} = \frac{\sin(C+D) + \sin(C-D)}{\sin(C+D) - \sin(C-D)}$$

$$\Rightarrow \quad \frac{2\cos A \cdot \cos B}{-2\sin A \sin B} = \frac{2\sin C \cdot \cos D}{-2\cos C \cdot \sin D}$$

$$\Rightarrow \quad \cot A \cot B = \tan C \cot D$$

$\Rightarrow \quad \cot A \cot B \cdot \cot C = \cot D$ Proved

S9. From R.H.S.

$$\frac{c - a}{b} \cdot \cos\frac{B}{2} = \frac{k(\sin C - \sin A)}{k \sin B} \cdot \cos\frac{B}{2}$$

$$= \frac{2\cos\frac{C+A}{2} \cdot \sin\frac{C-A}{2}}{2\sin\frac{B}{2} \cdot \cos\frac{B}{2}} \cdot \cos\frac{B}{2} = \sin\frac{C-A}{2}$$

Hence, R.H.S. = L.H.S.

S10. $\because \quad \cos A = \frac{\sin B}{2\sin C}$

$$\Rightarrow \quad 2\cos A \sin C = \sin B$$

$$\Rightarrow \quad \sin(A + C) - \sin(A - C) = \sin B$$

$$\Rightarrow \quad \sin(A - C) = 0 \quad \Rightarrow \quad A - C = 0$$

$$\Rightarrow \quad A = C$$

Hence, triangle is isosceles.

S11. Given that $\cos\theta = \frac{1}{2}\left(a + \frac{1}{a}\right)$

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$= \frac{1}{2}[8\cos^3\theta - 6\cos\theta]$$

$$= \frac{1}{2}\left[\left(a+\frac{1}{a}\right)^3 - 3\left(a+\frac{1}{a}\right)\right] = \frac{1}{2}\left(a^3 + \frac{1}{a^3}\right).$$

S12. Since sum of opposite angles in a cyclic quadrilateral are supplementary.

$\angle A + \angle C = 180°$ [$\cos A = -\cos C$]

$\therefore$ $\angle B + \angle D = 180°$ [$\cos B = -\cos D$]

$$\cos(180° + A) + \cos(180° - B) + \cos(180° - C) - \sin(90° - D)$$

$$= -\cos A - \cos B - \cos C - \cos D$$

$$= -(\cos A + \cos B + \cos C + \cos D) = 0.$$

S13. $\because$ $\sin(\theta + \alpha) = \cos(\theta + \alpha)$

$\Rightarrow$ $\tan(\theta + \alpha) = 1$

$\Rightarrow$ $\theta + \alpha = \frac{\pi}{4}$

$\Rightarrow$ $\theta = \frac{\pi}{4} - \alpha \Rightarrow \tan\theta = \tan\left(\frac{\pi}{4} - \alpha\right)$

$\Rightarrow$ $\tan\theta = \frac{1 - \tan\alpha}{1 + \tan\alpha}$ Proved.

S14. From L.H.S.

$a(b\cos C - c\cos B)$

$$= ab\cos C - ac\cos B$$

$$= ab\cdot\left(\frac{a^2 + b^2 - c^2}{2ab}\right) - ac\left(\frac{a^2 + c^2 - b^2}{2ac}\right) \qquad \left[\because \cos C = \frac{a^2 + b^2 - c^2}{2ab},\ \cos B = \frac{a^2 + c^2 - b^2}{2ac}\right]$$

$$= \frac{a^2 + b^2 - c^2}{2} - \frac{a^2 + c^2 - b^2}{2}$$

$$= b^2 - c^2 = \text{R.H.S.}$$

S15. $bc\cos A + ca\cos B + ab\cos C$

$$= \frac{bc(b^2 + c^2 - a^2)}{2bc} + \frac{ca(c^2 + a^2 - b^2)}{2ca} + \frac{ab(a^2 + b^2 - c^2)}{2ab}$$

$$= \frac{b^2 + c^2 - a^2}{2} + \frac{c^2 + a^2 - b^2}{2} + \frac{a^2 + b^2 - c^2}{2}$$

$$= \frac{a^2 + b^2 + c^2}{2}$$

S16. From L.H.S.

$$\frac{\tan B}{\tan C} = \frac{\sin B}{\cos B} \times \frac{\cos C}{\sin C} = \frac{k\text{b}}{\cos B} \times \frac{\cos C}{kc}$$

$$= \frac{kb}{\frac{a^2 + c^2 - b^2}{2ac}} \times \frac{\frac{a^2 + b^2 - c^2}{2ab}}{kc} = \frac{a^2 + b^2 - c^2}{a^2 - b^2 + c^2}.$$

S17. $\because$ In a cyclic quadrilateral opposite angles are supplementary.

$\therefore$ $\alpha + \gamma = 180°, \quad \beta + \delta = 180°$

$\Rightarrow$ $\alpha = (180° - \gamma)$

$\therefore$ $\cos \alpha = -\cos \gamma$

$\Rightarrow$ $\cos \alpha + \cos \gamma = 0$ ... (i)

Similarly,

$\cos \beta + \cos \delta = 0$ ... (ii)

Adding Eq. (i) and (ii), we get

$\cos \alpha + \cos \beta + \cos \gamma + \cos \delta = 0$

S18. By the sine rule, we have

$$\frac{a}{\sin A} = \frac{b}{\sin B} = k(\text{say})$$

$\Rightarrow$ $a = k \sin A$ and $b = k \sin B$

$\therefore$ $a \cos A = b \cos B$

$\Rightarrow$ $k \sin A \cos A = k \sin B \cos B$

$\Rightarrow$ $\frac{1}{2}(\sin 2A) = \frac{1}{2} \sin 2B$

$\Rightarrow$ $\sin 2A = \sin 2B$

$\Rightarrow$ $\sin 2A - \sin 2B = 0$

$\Rightarrow$ $2 \cos(A + B) = 0 \quad \text{or} \quad \sin(A - B) = 0$

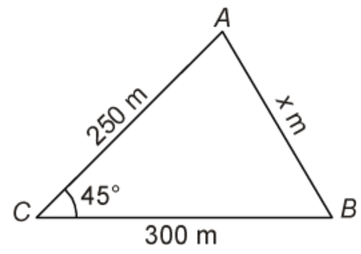
$\Rightarrow$ $A + B = \frac{\pi}{2} \quad \text{or} \quad A - B = 0$

$\Rightarrow \qquad \angle C = \dfrac{\pi}{2} \quad \text{or} \quad A = B$

Hence, triangle *ABC* is right angled or isosceles.

S19. Let *A* and *B* be the trees and *C* be a point in the river. Then $CA = 250\ m$, $CB = 300\ m$ and $\angle ACB = 45°$. Let $AB = x$ meters.

Applying cosine formula on triangle *ABC*, we get



$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\Rightarrow \qquad \cos 45° = \frac{(300)^2 + (250)^2 - x^2}{2 \times 300 \times 250}$$

$$\Rightarrow \qquad \frac{152500 - x^2}{150000} = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \qquad x^2 = 44500$$

$$\Rightarrow \qquad x = 210.95$$

S20. From L.H.S.

$$\frac{\cos 2A}{a^2} - \frac{\cos 2B}{b^2} = \left(\frac{1}{a^2} - \frac{1}{b^2}\right) - \frac{2\sin^2 A}{a^2} + \frac{2\sin^2 B}{b^2} \qquad [\cos^2\theta = 1 - 2\sin^2\theta]$$

$$= \left(\frac{1}{a^2} - \frac{1}{b^2}\right) - \frac{2\sin^2 A}{k^2 \sin^2 A} + \frac{2\sin^2 B}{k^2 \sin^2 B}$$

$$= \left(\frac{1}{a^2} - \frac{1}{b^2}\right) - \frac{2}{k^2} + \frac{2}{k^2} = \left(\frac{1}{a^2} - \frac{1}{b^2}\right).$$

S21. Let $A = x°$, $\angle B = (2x)°$, $\angle C = (3x)°$

Then,

$$A + B + C = 180° \Rightarrow x + 2x + 3x = 180°$$

$$\Rightarrow \qquad x = 30°$$

$\therefore \quad \angle A = 30°, \angle B = 60°$ and $\angle C = 90°$

$\therefore \quad$ Ratio of sides = sin A: sin B : sin C.

$$= \sin 30° : \sin 60° : \sin 90°$$

$$= \frac{1}{2} : \frac{\sqrt{3}}{2} : 1 = 1 : \sqrt{3} : 2.$$

S22. $\angle C = 90°$

$\therefore \quad a^2 + b^2 = c^2 \quad \ldots \text{(i)}$

$$\tan A = \frac{a}{b}, \tan B = \frac{b}{a}$$

$\therefore$ Required eq. is

$x^2 - (\tan A + \tan B)x + \tan A \cdot \tan B = 0$

$$\Rightarrow \quad x^2 - \left(\frac{a}{b} + \frac{b}{a}\right)x + \left(\frac{a}{b} \cdot \frac{b}{a}\right) = 0$$

From Eq. (i), we get

$\Rightarrow \quad abx^2 - c^2x + ab = 0.$

S23. We know that,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$$\Rightarrow \quad a = k \sin A, \quad b = k \sin B, \quad c = k \sin C$$

On putting the values of a, b and c on R.H.S., we get

$$\text{R.H.S.} = \frac{b^2 - c^2}{a^2} = \frac{k^2 \sin^2 B - k^2 \sin^2 C}{k^2 \sin^2 A}$$

$$= \frac{\sin^2 B - \sin^2 C}{\sin^2 A} = \frac{\sin (B + C) \sin (B - C)}{\sin^2 A} = \frac{\sin (\pi + A) \sin (B - C)}{\sin^2 A}$$

$$= \frac{\sin A \sin (B - C)}{\sin^2 A} = \frac{\sin (B - C)}{\sin A} = \frac{\sin (B - C)}{\sin (B + C)} = \text{LHS.}$$ **Prove.**

S24. By the sine rule, we have

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k(\text{say})$$

$\Rightarrow \quad a = k \sin A$, $b = k \sin B$ and $c = k \sin C$.

$\therefore$ From L.H.S.

$$\frac{b - c}{b + c} = \frac{k \sin B - k \sin C}{k \sin B + k \sin C} = \frac{\sin B - \sin C}{\sin B + \sin C}$$

$$= \frac{\tan \frac{1}{2}(B - C)}{\tan \frac{1}{2}(B + C)} = \text{R.H.S.}$$

Hence, $$\frac{b-c}{b+c} = \frac{\tan\frac{1}{2}(B-C)}{\tan\frac{1}{2}(B+C)}.$$

S25. $\because$ $\angle A$, $\angle B$, and $\angle C$ are in A.P.

$\Rightarrow$ $$2\angle B = \angle A + \angle C$$

$$3\angle B = \angle A + \angle B + \angle C = 180°$$

$\Rightarrow$ $$\angle B = 60°$$

Now, $$\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{b}{c} = \frac{\sin B}{\sin C}$$

$\Rightarrow$ $$\frac{\sqrt{3}}{2} = \frac{\sin 60°}{\sin C} \Rightarrow \angle C = 45°$$

$\therefore$ $$\angle A = 75°$$

S26. $\because$ $\cos\alpha + \cos\beta = -a$ [Since given equations are of symmetrical form]

$$\cos\alpha \cdot \cos\beta = b$$

$$\sin\alpha + \sin\beta = -p, \quad \sin\alpha \cdot \sin\beta = q.$$

$\therefore$ $$a^2 + p^2 = 2 + 2\cos(\alpha - \beta)$$

and $$b + q = \cos(\alpha - \beta)$$

$\Rightarrow$ $$a^2 + p^2 = 2 + 2(b + q).$$

S27. $\because$ $\text{cosec}^2 A - \cot^2 A = 1$

Let $\text{cosec}\, A + \cot A = \lambda$ ... (i)

$\therefore$ $\text{cosec}\, A - \cot A = \frac{1}{\lambda}$ ... (ii)

$$(a-b)(a+b) = a^2 - b^2$$

Adding Eq. (i) and (ii), we get

$$2\,\text{cosec}\, A = \lambda + \frac{1}{\lambda} \quad \text{or} \quad 2.\left(4P + \frac{1}{16P}\right) = \lambda + \frac{1}{\lambda}$$

$\Rightarrow$ $$\lambda = \frac{1}{2P} \quad \text{or} \quad (-2P)$$

Hence, $\text{cosec}\, A + \cot A = \frac{1}{2P}$ or $-2P$

S28. We know that,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$\Rightarrow \qquad a = k\sin A, \quad b = k\sin B, \quad c = k\sin C$ ... (i)

On putting the values of a and b from Eq. (i) on, L.H.S., we get

$$\text{L.H.S.} = \frac{a-b}{a+b} = \frac{k\sin A - k\sin B}{k\sin A + k\sin B} = \frac{\sin A - \sin B}{\sin A + \sin B} = \frac{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{2\sin\frac{A+B}{2}\cos\frac{A-B}{2}}$$

$$= \cot\left(\frac{A+B}{2}\right)\tan\left(\frac{A-B}{2}\right) = \frac{\tan\left(\frac{A-B}{2}\right)}{\tan\left(\frac{A+B}{2}\right)} = \text{LHS.}$$ **Prove.**

S29. We know that,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$$

$\Rightarrow \qquad a = k\sin A, \quad b = k\sin B, \quad c = k\sin C$ ... (i)

On putting the values of a, b and c from Eq. (i) in $\frac{b+c}{a}$, L.H.S., we get

Now, $$\frac{b+c}{a} = \frac{k\sin B + k\sin C}{k\sin A}$$

$$= \frac{\sin B + \sin C}{\sin A} = \frac{2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)}{\sin A}$$

$$= \frac{2\sin\left(\frac{180^\circ - A}{2}\right)\cos\left(\frac{B-C}{2}\right)}{\sin A} = \frac{2\cos\frac{A}{2}\cos\left(\frac{B-C}{2}\right)}{2\sin\frac{A}{2}\cos\frac{A}{2}}$$

$$= \frac{\cos\left(\frac{B-C}{2}\right)}{\sin\frac{A}{2}}$$

$\Rightarrow \qquad a\cos\left(\frac{B-C}{2}\right) = (b+c)\sin\frac{A}{2}$ **Proved.**

S30. We know that,

$$\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13} = k \quad \text{(say)}$$

Then, $b + c = 11k, \quad c + a = 12k, \quad a + b = 13k$

Adding these, we get

$$(b + c) + (c + a) + (a + b) = 11k + 12k + 13k$$

$\Rightarrow$ $2(a + b + c) = 36k \Rightarrow a + b + c = 18k$

Now, $b + c = 11k$ and $a + b + c = 18k \Rightarrow a = 7k$

$c + a = 12k$ and $a + b + c = 18k \Rightarrow b = 6k$

$a + b = 13k$ and $a + b + c = 18k \Rightarrow c = 5k$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{36k^2 + 25k^2 - 49k^2}{60k^2} = \frac{12}{60} = \frac{1}{5}$$

$$\cos B = \frac{c^2 + a^2 - b^2}{2ac} = \frac{25k^2 + 49k^2 - 36k^2}{70k^2} = \frac{38}{70} = \frac{19}{35}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{49k^2 + 36k^2 - 25k^2}{84k^2} = \frac{60}{84} = \frac{5}{7}$$

Now, $\cos A : \cos B : \cos C = \frac{1}{5} : \frac{19}{35} : \frac{5}{7} = 7 : 19 : 25$

$\Rightarrow$ $\frac{\cos A}{7} = \frac{\cos B}{19} = \frac{\cos C}{25}$. **Proved.**

S31. We have, $A + B + C = \pi \Rightarrow A = \pi - (B + C)$ ... (i)

Now, $\frac{\cos A}{\sin B \sin C} = \frac{\cos(\pi - (B + C))}{\sin B \sin C}$ [Using Eq. (i)]

$= \frac{-\cos(B + C)}{\sin B \sin C}$ [$\because \cos(\pi - \theta) = -\cos\theta$]

$= \frac{-[\cos B \cos C - \sin B \sin C]}{\sin B \sin C}$

$\Rightarrow$ $\frac{\cos A}{\sin B \sin C} = 1 - \cot B \cot C$ ... (ii)

Similarly, $\frac{\cos B}{\sin C \sin A} = 1 - \cot A \cot C$ ... (iii)

and $\frac{\cos C}{\sin A \sin B} = 1 - \cot A \cot B$ ... (iv)

Adding (ii), (iii) and (iv), we get

$$\frac{\cos A}{\sin B \sin C} + \frac{\cos B}{\sin C \sin A} + \frac{\cos C}{\sin A \sin B} = 3 - (\cot B \cot C + \cot C \cot A + \cot A \cot B) \quad \text{... (v)}$$

But $$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

$\Rightarrow$ $$\cot(\pi - C) = \frac{\cot A \cot B - 1}{\cot B + \cot A} \qquad [\because\ A + B + C = \pi]$$

$\Rightarrow$ $$-\cot C\,[\cot B + \cot A] = \cot A \cot B - 1$$

$\Rightarrow$ $$-(\cot B \cot C + \cot C \cot A) - \cot A \cot B = -1$$

$\Rightarrow$ $$\cot B \cot C + \cot C \cot A + \cot A \cot B = 1 \qquad \dots \text{(vi)}$$

Putting the value of $\cot B \cot C + \cot A \cot C + \cot A \cot B = 1$ from (vi) in (v), we get

$$\frac{\cos A}{\sin B \sin C} + \frac{\cos B}{\sin C \sin A} + \frac{\cos C}{\sin A \sin B} = 3 - 1 = 2$$ **Proved.**

S32. $\because$ $$\tan\frac{A-B}{2} = \frac{a-c}{a+b}\cot\frac{C}{2}$$

$\Rightarrow$ $$\tan\frac{A-B}{2} = \frac{2}{2\sqrt{3}}\cot 30^\circ$$

$\Rightarrow$ $$\frac{A-B}{2} = 45^\circ \quad \Rightarrow \quad A - B = 90^\circ$$

Also, $$A + B + C = 180^\circ \quad \Rightarrow \quad A + B = 120^\circ$$

$\therefore$ $$\angle A = 105^\circ, \quad \angle B = 15^\circ$$

Also, $$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$\Rightarrow$ $$\cos 60^\circ = \frac{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2 - c^2}{2\cdot(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{8 - c^2}{4}$$

$\Rightarrow$ $$\frac{8 - c^2}{4} = \frac{1}{2} \quad \Rightarrow \quad c = \sqrt{6}$$

Hence, $c = \sqrt{6}$ cm, $\angle A = 105^\circ$, $\angle B = 15^\circ$

S33. By the sine rule we have

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k(\text{say})$$

$\Rightarrow$ $a = k \sin A$, $b = k \sin B$ and $c = k \sin C$

$\therefore$ $$(b - c)\cot\frac{A}{2} = k(\sin B - \sin C)\,.\,\cot\frac{A}{2}$$

$$= 2k\cos\frac{B+C}{2}\cdot\sin\frac{B-C}{2}\cdot\frac{\cos\frac{A}{2}}{\sin\frac{A}{2}}$$

$$= 2k \sin\frac{A}{2}\sin\frac{B-C}{2}\cdot\frac{\cos\frac{A}{2}}{\sin\frac{A}{2}} \qquad \left[\cos\frac{B+C}{2} = \cos\left(90^\circ - \frac{A}{2}\right)\right]$$

$$= 2k \sin\frac{B-C}{2}\cdot\cos\frac{A}{2}$$

$$= 2k \sin\frac{B-C}{2}\cdot\sin\frac{B+C}{2} = k(\cos C - \cos B)$$

Similarly, we have

$$(c-a)\cot\frac{B}{2} = k(\cos A - \cos C)$$

$$(a-b)\cot\frac{C}{2} = k(\cos B - \cos A)$$

$\therefore \quad (b-c)\cot\frac{A}{2} + (c-a)\cot\frac{B}{2} + (a-b)\cot\frac{C}{2}$

$$= k[(\cos C - \cos B) + (\cos A - \cos C) + (\cos B - \cos A)] = 0.$$

S34. We have,

$$a = 18, \quad b = 24, \quad c = 30$$

We know that,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k \text{ (say)}$$

$$\Rightarrow \quad \frac{\sin A}{18} = \frac{\sin B}{24} = \frac{\sin C}{30} = k$$

$\Rightarrow \quad \sin A = 18k, \quad \sin B = 24k, \quad \sin C = 20k$ ... (i)

Here, $(30)^2 = (24)^2 + (18)^2$.

So it is a right angled triangle.

But, $\angle C = 90°$ as it is oposite to the biggest side c.

$\therefore \quad \sin C = \sin 90 = 1$

Also, $30k = 1 \Rightarrow k = \frac{1}{30}$ [Using Eq. (i)]

$$\sin A = 18k = 18 \times \frac{1}{30} = \frac{18}{30} = \frac{3}{5}$$

$$\sin B = 24k = 24 \times \frac{1}{30} = \frac{24}{30} = \frac{4}{5}$$

and $\sin C = 1$.

S35. We have,

$$\frac{b^2-c^2}{a^2}\sin 2A = \frac{b^2-c^2}{a^2} 2\sin A\cos A = \frac{b^2-c^2}{a^2}\cdot(2ka)\left(\frac{b^2+c^2-a^2}{2bc}\right)$$

$$= k\left(\frac{b^2-c^2}{a}\right)\left(\frac{b^2+c^2-a^2}{bc}\right)$$

$$= \frac{k}{abc}[(b^2-c^2)(b^2+c^2)-a^2(b^2-c^2)]$$

$$\Rightarrow \quad \frac{b^2-c^2}{a^2}\sin 2A = \frac{k}{abc}(b^4-c^4-a^2b^2+c^2a^2) \quad \text{... (i)}$$

Similarly, $$\frac{c^2-a^2}{b^2}\sin 2B = \frac{k}{abc}(c^4-a^4-b^2c^2+a^2b^2) \quad \text{... (ii)}$$

and $$\frac{a^2-b^2}{c^2}\sin 2C = \frac{k}{abc}(a^4-b^4-c^2a^2+b^2c^2) \quad \text{... (iii)}$$

Adding Eq. (i), (ii) and (iii), we get

$$\frac{b^2-c^2}{a^2}\sin 2A + \frac{c^2-a^2}{b^2}\sin 2B + \frac{a^2-b^2}{c^2}\sin 2C$$

$$= \frac{k}{abc}(b^4-c^4-a^2b^2+c^2a^2+c^4-a^4-b^2c^2+a^2b^2+a^4-b^4-c^2a^2+b^2c^2)$$

$$= \frac{k}{abc} = 0$$ **Proved.**