Hyperbola

Single Correct Answer Type

1. A line drawn through the point P (-1, 2) meets the hyperbola $xy = c^2$ at the points A and B. (points A and B lie on same side of P) and Q is a point on AB such that PA, PQ and PB are in H.P then locus of Q is

A. $x-2y=2c^2$ B. $2x-y=2c^2$ C. $2x+y+2c^2=0$ D. $x+2y=2c^2$

Key. B
Sol. Locus of Q is $S_1 = 0$

$2x-y=2c^2$

2. If the asymptote of the hyperbola $(x+y+1)^2-(x-y-3)^2=5$ cut each other at A and the coordinate axis at B and C then radius of circle passing through the points A,B,C is

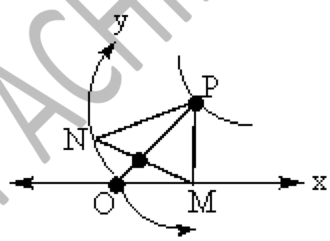
A. 3 B. $\frac{\sqrt{5}}{2}$ C. $\frac{\sqrt{3}}{2}$ D. $\sqrt{3}$

Key. B
Sol. Centre of rectangular hyperbola = (1,-2)
So equation of asymptotes are x = 1, y = -2
So radius of circle $=\frac{\sqrt{5}}{2}$

3. PM and PN are the perpendiculars from any point P on the rectangular hyperbola xy = 8 to the asymptotes. If the locus of the mid point of MN is a conic, then the least distance of (1, 1) to director circle of the conic is

A. $\sqrt{3}$ B. $\sqrt{2}$ C. $2\sqrt{3}$ D. $2\sqrt{5}$

Key. B
Sol. OMPN is rectangle.



$P=(Ct,\frac{c}{t})$

Mid point $=\left(\frac{ct}{2},\frac{c}{2t}\right)=(x,y)$ $\therefore cy=\frac{c^2}{4}\Rightarrow e=\sqrt{2}$

4. A hyperbola passing through origin has 3x – 4y – 1=0 and 4x – 3y – 6 = 0 as its asymptotes. Then the equations of its transverse and conjugate axes are
A) x – y – 5 = 0 and x + y + 1 = 0 B) x – y = 0 and x + y + 5 = 0
C) x + y – 5 = 0 and x – y – 1 = 0 D) x + y – 1 = 0 and x – y – 5 = 0

Key. C
Sol. Transverse and conjugate axes are the bisectors of the angle between asymptotes.

$$\frac{3x-4y-1}{5}=\pm\left(\frac{4x-3y-6}{5}\right) \text{ etc......}$$

5. If the asymptotes of the hyperbola $(x+y+1)^2-(x-y-3)^2=5$ cuts each other at A and the coordinate axes at B and C, then radius of the circle passing through the points A, B, C is

A) 3 B) $\frac{\sqrt{5}}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $\sqrt{3}$

Key. B

Sol. (B) Centre of rectangular hyperbola (1, –2)

So equation of asymptotes are x = 1, y = –2

So radius of circle = $\frac{\sqrt{5}}{2}$

6. If a chord joining P($a\operatorname{Sec}\theta, a\tan\theta$), Q($a\operatorname{Sec}\alpha, a\tan\alpha$) on the hyperbola $x^2-y^2=a^2$ is the normal at P,then $\operatorname{Tan}\alpha=$

A) $\operatorname{Tan}\theta(4\sec^2\theta+1)$ B) $\operatorname{Tan}\theta(4\sec^2\theta-1)$ C) $\operatorname{Tan}\theta(2\operatorname{Sec}^2\theta-1)$ D) $\operatorname{Tan}\theta(1-2\operatorname{Sec}^2\theta)$

Key. B

Sol. Slope of chord joining P and Q = slope of normal at P

$\frac{\operatorname{Tan}\alpha-\operatorname{Tan}\theta}{\sec\alpha-\sec\theta}=-\frac{\operatorname{Tan}\theta}{\sec\theta} \Rightarrow \operatorname{Tan}\alpha-\operatorname{Tan}\theta=-k\operatorname{Tan}\theta$ and $\sec\alpha-\sec\theta=k\sec\theta$

$\therefore (1-k)Tan\theta=Tan\alpha \to 1.\ (1+k)\sec\theta=\sec\alpha \to 2.$

$\left[(1+k)\sec\theta\right]^2-\left[(1-k)\operatorname{Tan}\theta\right]^2 = \sec^2\alpha-\operatorname{Tan}^2\alpha$

$\Rightarrow k=-2\left(\sec^2\theta+\operatorname{Tan}^2\theta\right) = -4\sec^2\theta+2$

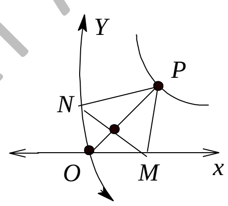
From (1) $Tan\alpha=Tan\theta\ \left(1+4\sec\theta^2-2\right)=\operatorname{Tan}\theta\left(4\sec\theta^2-1\right).$

7. PM and PN are the perpendiculars from any point P on the rectangular hyperbola $xy=c^2$ to the asymptotes. If the locus of the mid point of MN is a conic, then its eccentricity is

A) $\sqrt{3}$ B) $\sqrt{2}$ C) $\frac{1}{\sqrt{3}}$ D) $\frac{1}{\sqrt{2}}$

Key. B

Sol. OMPN is rectangle.



$P=\left(Ct,\frac{c}{t}\right)$

Mid point = $\left(\frac{ct}{2},\frac{c}{2t}\right)=(x,y)$ $\therefore xy=\frac{c^2}{4} \Rightarrow e=\sqrt{2}$

8. A variable straight line of slope 4 intersects the hyperbola xy = 1 at two points. The locus of the point which divides the line segment between these two points in the ratio 1 : 2 is

A) $16x^2+10xy+y^2=2$ B) $16x^2-10xy+y^2=2$

C) $16x^2+10xy+y^2=4$ D) $16x^2-10xy+y^2=4$

Key. A

Sol. Let P(h, k)

y - k = 4(x - h) --- (1)

Let it meets xy = 1 ----(2) at A (x_1, y_1) and B (x_2, y_2)

$$x_1+x_2=\frac{4h-k}{4}, x_1x_2=-\frac{1}{4} \text{Also} \Rightarrow \therefore \frac{2x_1+x_2}{3}=h \Rightarrow x_1=\frac{8h+k}{4}, x_2=\frac{2h+k}{2}$$

$$\Rightarrow 16x^2+10xy+y^2=2$$

9. The length of the transverse axis of the hyperbola $9x^2-16y^2-18x-32y-151=0$ is

1) 8 2) 4 3) 6 4) 2

Key. 1

Sol. Given hyperbola is $\frac{(x-1)^2}{16}-\frac{(y+1)^2}{9}=1$

Length of the transverse axis is 2a=8.

10. The equation of a hyperbola , conjugate to the hyperbola $x^2+3xy+2y^2+2x+3y=0$ is

1) $x^2+3xy+2y^2+2x+3y+1=0$ 2) $x^2+3xy+2y^2+2x+3y+2=0$

3) $x^2+3xy+2y^2+2x+3y+3=0$ 4) $x^2+3xy+2y^2+2x+3y+4=0$

Key. 2

Sol. Let $H=x^2+3xy+2y^2+2x+3y=0$ and C=0 is its conjugate. Then C + H=2A, where A=0 is the combined equation of asymptotes. Equation of asymptotes is $x^2+3xy+2y^2+2x+3y+\lambda=0$, where $\Delta=abc+2fgh-af^2-bg^2-ch^2=0 \Rightarrow \lambda=1$

$$\therefore C=2\left(x^2+3xy+2y^2+2x+3y+1\right)-\left(x^2+2y^2+3xy+2x+3y\right)$$

$\Rightarrow$ equation of conjugate hyperbola is $x^2+3xy+2y^2+2x+3y+2=0$

11. If AB is a double ordinate of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that ΔOAB is an equilateral triangle O being the origin, then the eccentricity of the hyperbola satisfies

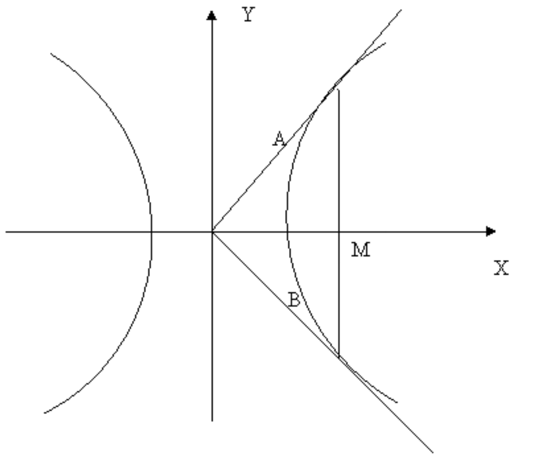
1) $e>\sqrt{3}$ 2) $1<e<\frac{1}{\sqrt{3}}$ 3) $e=\frac{2}{\sqrt{3}}$ 4) $e>\frac{2}{\sqrt{3}}$

Key. 4

Sol. Let the length of the double ordinate be 2ℓ

$\therefore$ AB=2ℓ and AM=BM=ℓ

Clearly ordinate of point A is ℓ .



The abscissa of the point A is given by

$$\frac{x^2}{a^2}-\frac{l^2}{b^2}=1 \Rightarrow x=\frac{a\sqrt{b^2+l^2}}{b}$$

$\therefore$ A is $\left(\frac{a\sqrt{b^2+l^2}}{b}, l\right)$

Since ΔOAB is equilateral triangle, therefore

OA=AB=OB=2l

Also, $OM^2+AM^2=OA^2 \therefore \frac{a\left(b^2+l^2\right)}{b}+l^2=4l^2$

We get $l^2=\frac{a^2b^2}{3b^2-a^2}$

Since $l^2>0 \therefore \frac{a^2b^2}{3b^2-a^2}>0 \Rightarrow 3b^2-a^2>0$

$$\Rightarrow 3a^2\left(e^2-1\right)-a^2>0 \Rightarrow e>\frac{2}{\sqrt{3}}$$

12. If the line $5x+12y-9=0$ is a tangent to the hyperbola $x^2-9y^2=9$, then its point of contact is

1) (-5,4/3) 2) (5,-4/3) 3) $(3,-1/2)$ 4) (5,4/3)

Key. 2

Sol. Common Point

13. Any chord passing through the focus $(ae, 0)$ of the hyperbola $x^2 - y^2 = a^2$ is conjugate to the line

1) $ex - a = 0$ 2) $ae + x = 0$ 3) $ax + e = 0$ 4) $ax - e = 0$

Key. 1

Sol. $S_1 = 0$

14. Number of points from where perpendicular tangents to the curve $\frac{x^2}{16} - \frac{y^2}{25} = 1$ can be drawn, is:

1) 1 2) 2 3) 0 4) 3

Key. 3

Sol. Director circle is set of points from where drawn tangents are perpendicular in this case $x^2 + y^2 = a^2 - b^2$ (equation of director circle)i.e., $x^2 + y^2 = -9$ is not a real circle so there is no points from where tangents are perpendicular.

15. $x^2 - y^2 + 5x + 8y - 4 = 0$ represents

1) Rectangular hyperbola 2)Ellipse

3) Hyperbola with centre at (1,1) 4)Pair of lines

Key. 1

Sol. $\Delta \neq 0,\ x^2 - ab > 0,\ a + b = 0$

16.

1) $\left(2\sqrt{2}, 2\sqrt{2}\right), \left(-2\sqrt{2}, -2\sqrt{2}\right)$ 2) $\left(-3\sqrt{2}, -3\sqrt{2}\right), \left(3\sqrt{2}, 3\sqrt{2}\right)$

3) $\left(2\sqrt{2}, -2\sqrt{2}\right), \left(-2\sqrt{2}, 2\sqrt{2}\right)$ 4) (-2. 2)

Key. 1

Sol. foci of $xy = c^2$ is $\left(\pm c\sqrt{2}, \pm c\sqrt{2}\right)$

17. Which of the following is INCORRECT for the hyperbola $x^2 - 2y^2 - 2x + 8y - 1 = 0$

1) Its eccentricity is $\sqrt{2}$ 2) Length of the transverse axis is $2\sqrt{3}$

3) Length of the conjugate axis is $2\sqrt{6}$ 4) Latus rectum $4\sqrt{3}$

Key. 1

Sol. The equation of the hyperbola is $x^2 - 2y^2 - 2x + 8y - 1 = 0$

Or $(x-1)^2 - 2(y-2)^2 + 6 = 0$

Or $\frac{(x-1)^2}{-6} + \frac{(y-2)^2}{3} = 1;$ or $\frac{(y-2)^2}{3} - \frac{(x-1)^2}{6} = 1 \rightarrow 1$

Or $\frac{Y^2}{3} - \frac{X^2}{6} = 1$, where X = x -1 and Y = y - 2 $\rightarrow 2$

$\therefore$ the centre=(0,0)in the X-Y coordinates.

$\therefore$ the centre=(1,2)in the x-y coordinates .using $\rightarrow 2$

If the transverse axis be of length 2a, then a= $\sqrt{3}$,since in the equation (1) the transverse axis is parallel to the y-axis.

If the conjugate axis is of length 2b,then b = $\sqrt{6}$

But $b^2 = a^2(e^2 - 1)$

$\therefore$ $6 = 3(e^2 - 1), \therefore e^2 = 3$ or $e = \sqrt{3}$

The length of the transverse axis = $2\sqrt{3}$

The length of the conjugate axis = $2\sqrt{6}$

Latus rectum $4\sqrt{3}$

18. If the curve $xy = R^2 - 16$ represents a rectangular hyperbola whose branches lies only in the quadrant in which abscissa and ordinate are opposite in sign but not equal in magnitude, then

1) $|R| < 4$ 2) $|R| \geq 4$ 3) $|R| = 4$ 4) $|R| = 5$

Key. 1

Sol. conceptual

19. If the line ax + by + c=0 is a normal to the curve xy=1,then

1) a > 0, b > 0 2) a < 0,b < 0 3) a < 0,b > 0 4) a=b=1

Key. 3

Sol. Slope of the line $\frac{-a}{b}$ is equal to slope of the normal to the curve.

∴ either a > 0 & b < 0 (or) a < 0 & b > 0.

20. The equation of normal at $\left(at, \frac{a}{t}\right)$ to the hyperbola $xy = a^2$ is

1) $xt^3 - yt + at^4 - a = 0$
2) $xt^3 - yt - at^4 + a = 0$
3) $xt^3 + yt + at^4 - a = 0$
4) $xt^3 + yt - at^4 - a = 0$

Key. 2

Sol. Equation of tangent is $s_1 = 0$ normal is $\perp^r$ to tangent and passing through $\left(at, \frac{a}{t}\right)$ is $xt^3 - yt - at^4 + a = 0$

21. The product of perpendiculars from any point P (θ) on the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ to its asymptotes is equal to:

1) $\frac{6}{5}$
2) $\frac{36}{13}$
3) Depending on θ
4) $\frac{5}{6}$

Key. 2

Sol. The product of perpendiculars from any point P (θ) on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to its asymptotes is equal to $\frac{a^2b^2}{a^2+b^2}$

22. The foot of the perpendicular from the focus to an asymptote of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

1) (ae , be)
2) (a/e,b/e)
3) (e/a,e/b)
4) (be,ae)

Key. 2

Sol. Focus S=(ae,0) Equation of one asymptote is bx-ay=0

Let (h,k) be the foot of the perpendicular from s to bx-ay=0

Then $\frac{h-ae}{b} = \frac{k-0}{-a} = \frac{-abe}{a^2+b^2} \Rightarrow \frac{h-ae}{b} = \frac{-abe}{a^2e^2} \;\&\; \frac{k}{-a} = \frac{-abe}{a^2e^2}$

On simplification, we get h=a/e, k=b/e

Foot of the perpendicular is (a/e,b/e)

23. The area of the triangle formed by the asymptotes and any tangent to the hyperbola $x^2 - y^2 = a^2$

1) $4a^2$
2) $3a^2$
3) $2a^2$
4) a^2

Key. 4

Sol. Equation of any tangent to $x^2 - y^2 = a^2$

i.e. $\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1 \; is \; \frac{x}{a}\sec\theta - \frac{y}{a}\tan\theta = 1 \rightarrow (1)$

$or \; x\sec\theta - y\tan\theta = a$

equation of other two sides of the triangle are

x-y=0..(2) x + y=0(3)

The two asymptotes of the hyperbola $x^2 - y^2 = a^2$

Are x-y=0 and x + y=0)

Solving (1) (2) and (3) in pairs the coordinates of the vertices of the triangle are (0,0)

$\left(\frac{a}{\sec\theta + \tan\theta}, \frac{a}{\sec\theta + \tan\theta}\right)$

And $\left(\frac{a}{\sec\theta - \tan\theta}, \frac{-a}{\sec\theta - \tan\theta}\right) -$

Area of triangle = $\frac{1}{2}\left|\frac{a^2}{\sec^2\theta - \tan^2\theta} + \frac{a^2}{\sec^2\theta - \tan^2\theta}\right|$

$\frac{1}{2}\left(a^2 + a^2\right) \qquad \because \sec^2\theta - \tan^2\theta = 1$

$= a^2$

24. The foot of the normal $3x + 4y = 7$ to the hyperbola $4x^2 - 3y^2 = 1$ is

1) $(1,1)$ 2) $(1,-1)$ 3) $(-1,1)$ 4) $(-1,-1)$

Key. 1

Sol. Since the point $(1,1)$ lies on the normal and hyperbola it is the foot of the normal

25. Tangent at the point $\left(2\sqrt{2},3\right)$ to the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ meet its asymptotes at A and B, then area of the triangle OAB, O being the origin is

1) 6 sq. units 2) 3 sq. units 3) 12 sq. units 4) 2 sq. units

Key. 1

Sol. Since area of the Δ formed by tangent at any point lying on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and its asymptotes is always constant and is equal to ab. Therefore, required area is 2 X 3=6 square units.

26. Eccentricity of hyperbola $\frac{x^2}{k}+\frac{y^2}{k}=1(k<0)$ is :

1) $\sqrt{1+k}$ 2) $\sqrt{1-k}$ 3) $\sqrt{1+\frac{1}{k^2}}$ 4) $\sqrt{1-\frac{1}{k}}$

Key. 4

Sol. Given equation can be rewritten as $\frac{y^2}{k^2}-\frac{x^2}{(-k)}=1(-k>0)$

$e^2=1+\frac{(-k)}{k^2}=1-\frac{1}{k}\Rightarrow e=\sqrt{1-\frac{1}{k}}$

27. If the circle $x^2+y^2=a^2$ intersect the hyperbola $xy=c^2$ in four points $P(x_1,y_1),Q(x_2,y_2),R(x_3,y_3),S(x_4,y_4)$ then which of the following does not hold

1) $x_1+x_2+x_3+x_4=0$ 2) $x_1x_2x_3x_4=y_1y_2y_3y_4=c^4$

3) $y_1+y_2+y_3+y_4=0$ 4) $x_1+y_2+x_3+y_4=0$

Key. 4

Sol. $x^2+\frac{c^4}{x^2}=a^2$ $\Rightarrow \mathbf{x^4-a^2x^2+c^4=0}$, 4th option does not hold

28. If a normal to the hyperbola x y = c^2 at $\left(ct_1,\frac{c}{t_1}\right)$ meets the curve again at $\left(ct_2,\frac{c}{t_2}\right)$, then:

1) $t_1t_2=-1$ 2) $t_2=-t_1-\frac{2}{t_1}$ 3) $t_2^3t_1=-1$ 4) $t_1^3t_2=-1$

Key. 4

Sol. Equation of normal at $\left(ct_1,\frac{c}{t_1}\right)$ is

$t_1^3x-t_1y-ct_1^4+c=0$

It passes through $\left(ct_2,\frac{c}{t_2}\right)$

Ie., $t_1^3.ct_2-t_1.\frac{c}{t_2}-ct_1^4+c=0$

$\Rightarrow(t_1-t_2)(t_1^3t_2+1)=0$

$\Rightarrow t_1^3t_2=-1$

29. The equation of the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola xy=c^2 is

1) $\frac{x}{x_1+x_2}+\frac{y}{y_1+y_2}=1$ 2) $\frac{x}{x_1-x_2}+\frac{y}{y_1-y_2}=1$ 3) $\frac{y}{x_1+x_2}+\frac{x}{y_1+y_2}=1$ 4) $\frac{x}{y_1-y_2}+\frac{y}{x_1-x_2}=1$

Key. 1

Sol. Mid point of the chord is $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

The equation of the chord in terms of its mid-point is $S_1 = S_{11}$

30. A rectangular hyperbola whose centre is C is cut by any circle of radius r in four points P,Q,R and S. Then $CP^2+CQ^2+CR^2+CS^2 =$

1) r^2 2) $2r^2$ 3) $3r^2$ 4) $4r^2$

Key. 4

Sol. $CP = CQ = CR = CS = r$

31. The product of focal distances of the point (4,3) on the hyperbola $x^2 - y^2 = 7$ is

1) 25 2) 12 3) 9 4) 16

Key. 1

Sol. $e = \sqrt{2},\ sp.s'p = (ex_1 + a)(ex_1 - a) = 25$

32. Let $y = 4x^2$ & $\frac{x^2}{a^2}-\frac{y^2}{16}=1$ intersect iff

1) $|a| \le \frac{1}{\sqrt{2}}$ 2) $a > \frac{1}{\sqrt{2}}$ 3) $a > -\frac{1}{\sqrt{2}}$ 4) $a > \sqrt{2}$

Key. 1

Sol. $y = 4x^2$ & $\frac{1}{4}y = x^2$

Using $\frac{1}{4a^2}y - \frac{y^2}{16} = 1$

$\Rightarrow 4y - a^2y^2 = 16a^2$

$\Rightarrow a^2y^2 - 4y + 16a^2 = 0$

$\Rightarrow D \geq 0$ for intersection of two curves

$\Rightarrow 16 - 4a^2\left(16a^2\right) \geq 0$

$\Rightarrow 1 - 4a^4 \geq 0$

$\Rightarrow \left(2a^2\right) \leq 1$

$\Rightarrow \left|\sqrt{2}a\right| \leq 1 \Rightarrow -\frac{1}{\sqrt{2}} \leq a \leq \frac{1}{\sqrt{2}}$

33. If angle between the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is 45°, then value of eccentricity e is

1) $\sqrt{4 \pm 2\sqrt{2}}$ 2) $\sqrt{4 + 2\sqrt{2}}$ 3) $\sqrt{4 - 2\sqrt{2}}$ 4) $\sqrt{4 - 3\sqrt{2}}$

Key. 3

Sol. $2\tan^{-1}\frac{b}{a} = 45^\circ \Rightarrow \frac{b}{a} = \tan 22^o = \frac{a^2\left(e^2 - 1\right)}{a^2} = \left(\sqrt{2} - 1\right)^2$

$\Rightarrow e^2 - 1 = 3 - 2\sqrt{2} \Rightarrow e = \sqrt{4 - 2\sqrt{2}}$.

34. A hyperbola, having the transverse axis of length $2\sin\theta$, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is

1) $x^2 \operatorname{cosec}^2\theta - y^2 \sec^2\theta = 1$ 2) $x^2 \sec^2\theta - y^2 \operatorname{cosec}^2\theta = 1$

3) $x^2 \sin^2\theta - y^2 \cos^2\theta = 1$ 4) $x^2 \cos^2\theta - y^2 \sin^2\theta = 1$

Key. 1

Sol. Equation of the ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$. Its eccentricity is $e = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

Coordinates of foci are $(\pm 1, 0)$.

Let the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $a = \sin\theta$

Also, $ae_1 = 1 \Rightarrow$ $e_1 = \operatorname{cosec}\theta$

$\therefore b^2 = a^2\left(e_1^2 - 1\right) = 1 - \sin^2\theta = \cos^2\theta$

Equation of the hyperbola is thus $\frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$

35. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinates axes, then

1) Equation of ellipse is $x^2 + 2y^2 = 1$
2) the foci of ellipse are $(\pm 1, 0)$
3) equation of ellipse are $x^2 + 2y^2 = 4$
4) the foci of ellipse are $(\pm\sqrt{2}, 0)$

Key. 2

Sol. If two concentric conics intersect orthogonally then they must be confocal, so ellipse and hyperbola will be confocal

$\Rightarrow (\pm ae, 0) \equiv (\pm 1, 0)$

[foci of hyperbola are $(\pm 1, 0)$]

36. Let $P(6,3)$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x axis at $(9,0)$, then the eccentricity of the hyperbola is:

1) $\sqrt{\frac{5}{2}}$ 2) $\sqrt{\frac{3}{2}}$ 3) $\sqrt{2}$ 4) $\sqrt{3}$

Key. 2

Sol. Normal at (6,3) is

$\frac{a^2 x}{6} + \frac{b^2 y}{3} = a^2 + b^2$,

$\Rightarrow \frac{9a^2}{6} = a^2 + b^2 \Rightarrow \frac{3}{2} = 1 + \frac{b^2}{a^2}$

$\therefore \frac{b^2}{a^2} = \frac{1}{2} \Rightarrow e^2 - 1 = \frac{1}{2} \Rightarrow e = \sqrt{\frac{3}{2}}$

37. For hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant with change in 'α'

1) abscissae of vertices
2) abscissae of foci
3) Eccentricity
4) directrix

Key. 2

Sol. Hyperbola is $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$

Coordinates of vertices are $(\pm\cos\alpha, 0)$, eccentricity of the hyperbola is $e = \sqrt{1 + \frac{\sin^2\alpha}{\cos^2\alpha}} = |\sec\alpha|$

$\therefore$ Coordinates of foci are thus $(\pm1, 0)$, which are independent of α.

Directrix is $x = \pm\cos^2\alpha$

38. Equation of a common tangent to the curves $y^2 = 8x$ and $xy = -1$ is

(a) $3y = 9x + 2$ (b) $y = 2x + 1$ (c) $2y = x + 8$ (d) $y = x + 2$

Key. D

Sol. $y^2 = 8k, xy = -1$

Let $P\left(t, \frac{-1}{t}\right)$ be any point on xy = - 1

Equation of the tangent to $xy = -1$ at $P\left(t, \frac{-1}{t}\right)$ is

$$\frac{xy_1 + yx_1}{2} = -1$$

$$\frac{-x}{t} + yt = -2$$

$$y = \frac{x}{t^2} + \left(\frac{-2}{t}\right) \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

If (1) is tangent to the parabola $y^2 = 8x$ then

$$\frac{-2}{t} = \frac{2}{1/t^2} \Rightarrow t^3 = -1$$

$$t = -1$$

$\therefore$ Common tangent is y = x+2

39. If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that OPQ is an equilateral triangle, O being the centre of the hyperbola. Then the eccentricity *e* of the hyperbola, satisfies

(a) $1 < e < 2/\sqrt{3}$ (b) $e = 2/\sqrt{3}$ (c) $e = \sqrt{3}/2$ (d) $e > 2/\sqrt{3}$

Key. D

Sol. If OPQ is equilateral triangle then OP makes 30^0 with x-axis.

$\left(\frac{\sqrt{3}r}{2}, \frac{r}{2}\right)$ ties on hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$\Rightarrow r^2 = \frac{16a^2b^2}{12b^2 - 4a^2} > 0$$

$$\Rightarrow 12b^2 - 4a^2 > 0 \Rightarrow \frac{b^2}{a^2} > \frac{4}{12}$$

$$e^2 - 1 > \frac{1}{3}$$

$$e^2 > \frac{4}{3} \Rightarrow e > \frac{2}{\sqrt{3}}$$

40. The locus of a point, from where tangents to the rectangular hyperbola $x^2 - y^2 = a^2$ contain an angle of 45^0, is

(A) $\left(x^2+y^2\right)+a^2\left(x^2-y^2\right)=4a^2$ (B) $2\left(x^2+y^2\right)+4a^2\left(x^2-y^2\right)=4a^2$

(C) $\left(x^2+y^2\right)^2+4a^2\left(x^2-y^2\right)=4a^4$ (D) $\left(x^2+y^2\right)^2+a^2\left(x^2-y^2\right)=a^4$

Key. C

Sol. Equation of tangent to the hyperbola : $y=mx\pm\sqrt{m^2a^2-a^2}$

$\Rightarrow$ Let $P\left(x_1,y_1\right)$ be locus

$\Rightarrow y-mx=\pm\sqrt{m^2a^2-a^2}$

S.B.S

$\Rightarrow m^2\left(x_1^2-a^2\right)-2y_1x_1m+y_1^2+a^2=0$

$m_1+m_2=\dfrac{2x_1y_1}{x_1^2-a^2}$; $m_1m_2=\dfrac{y_1^2+a^2}{x_1^2-a^2}$

$\tan 45^0=\left|\dfrac{m_1-m_2}{1+m_1m_2}\right|$

$\Rightarrow\left(1+m_1m_2\right)^2=\left(m_1-m_2\right)^2=\left(m_1+m_2\right)^2-4m_1m_2$

$\Rightarrow\left(1+\dfrac{y_1+a^2}{x_1^2-a^2}\right)=\left(\dfrac{2x_1y_1}{x_1^2-a^2}\right)-4\left(\dfrac{y_1^2+a^2}{x_1^2-a^2}\right)$

41. If a circle cuts the rectangular hyperbola xy=1 in 4 points $\left(x_r,y_r\right)$ where r =1,2,3,4. Then ortho centre of triangle with vertices at $\left(x_r,y_r\right)$ where r=1,2,3 is

1. $\left(x_4,y_4\right)$ 2. $\left(-x_4,-y_4\right)$ 3. $\left(-x_4,+y_4\right)$ 4. $\left(+x_4,-y_4\right)$

Key. 2

Sol. $xy=1$ cuts the circle in 4-points then $x_1x_2x_3x_4=1,\ y_1y_2y_3y_4=1$

Ortho centre of triangle with vertices $\left(x_1,y_1\right)\left(x_2,y_2\right)\left(x_3,y_3\right)$

Ie $\left(\dfrac{-1}{x_1x_2x_3},-\left(y_1y_2y_3\right)^{-1}\right)$

$-\left(-x_4,-y_4\right)$

42. A hyperbola passing through origin has 3x – 4y – 1=0 and 4x – 3y – 6 = 0 as its asymptotes. Then the equations of its transverse and conjugate axes are

A) x – y – 5 = 0 and x + y + 1 = 0 B) x – y = 0 and x + y + 5 = 0

C) x + y – 5 = 0 and x – y – 1 = 0 D) x + y – 1 = 0 and x – y – 5 = 0

Key. C

Sol. Transverse and conjugate axes are the bisectors of the angle between asymptotes.

$\dfrac{3x-4y-1}{5}=\pm\left(\dfrac{4x-3y-6}{5}\right)$ etc......

43. If the asymptotes of the hyperbola $\left(x+y+1\right)^2-\left(x-y-3\right)^2=5$ cuts each other at A and the coordinate axes at B and C, then radius of the circle passing through the points A, B, C is

A) 3 B) $\frac{\sqrt{5}}{2}$ C) $\frac{\sqrt{3}}{2}$ D) $\sqrt{3}$

Key. B

Sol. Centre of rectangular hyperbola (1, –2)

So equation of asymptotes are x = 1, y = –2

So radius of circle = $\frac{\sqrt{5}}{2}$

44. If a chord joining P($a\sec\theta, a\tan\theta$), Q($a\sec\alpha, a\tan\alpha$) on the hyperbola $x^2 - y^2 = a^2$ is the normal at P,then $\tan\alpha =$

A) $\tan\theta(4\sec^2\theta+1)$ B) $\tan\theta(4\sec^2\theta-1)$ C) $\tan\theta(2\sec^2\theta-1)$ D) $\tan\theta(1-2\sec^2\theta)$

Key. B

Sol. Slope of chord joining P and Q = slope of normal at P

$\frac{\tan\alpha-\tan\theta}{\sec\alpha-\sec\theta} = -\frac{\tan\theta}{\sec\theta} \Rightarrow \tan\alpha-\tan\theta = -k\tan\theta$ and $\sec\alpha-\sec\theta = k\sec\theta$

$\therefore (1-k)\tan\theta = \tan\alpha \to 1.\ (1+k)\sec\theta = \sec\alpha \to 2.$

$\left[(1+k)\sec\theta\right]^2 - \left[(1-k)\tan\theta\right]^2 = \sec^2\alpha - \tan^2\alpha$

$\Rightarrow k = -2\left(\sec^2\theta+\tan^2\theta\right) = -4\sec^2\theta+2$

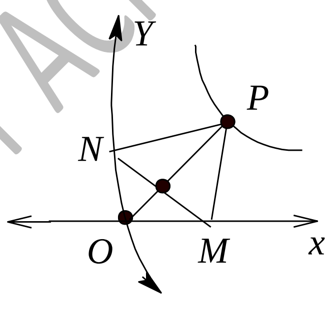
From (1) $\tan\alpha = \tan\theta\ \left(1+4\sec\theta^2-2\right) = \tan\theta\left(4\sec\theta^2-1\right).$

45. PM and PN are the perpendiculars from any point P on the rectangular hyperbola $xy = c^2$ to the asymptotes. If the locus of the mid point of MN is a conic, then its eccentricity is

A) $\sqrt{3}$ B) $\sqrt{2}$ C) $\frac{1}{\sqrt{3}}$ D) $\frac{1}{\sqrt{2}}$

Key. B

Sol. OMPN is rectangle.



$P = \left(Ct, \frac{c}{t}\right)$

Mid point = $\left(\frac{ct}{2}, \frac{c}{2t}\right) = (x, y)$ $\therefore xy = \frac{c^2}{4} \Rightarrow e = \sqrt{2}$

46. A variable straight line of slope 4 intersects the hyperbola xy = 1 at two points. The locus of the point which divides the line segment between these two points in the ratio 1 : 2 is

A) $16x^2 + 10xy + y^2 = 2$ B) $16x^2 - 10xy + y^2 = 2$

C) $16x^2+10xy+y^2=4$ D) $16x^2-10xy+y^2=4$

Key. A

Sol. Let P(h, k)

y - k = 4(x - h) --- (1)

Let it meets xy = 1 ----(2) at A (x_1, y_1) and B (x_2, y_2)

$$x_1+x_2=\frac{4h-k}{4}, x_1x_2=-\frac{1}{4} \text{Also} \Rightarrow \therefore \frac{2x_1+x_2}{3}=h \Rightarrow x_1=\frac{8h+k}{4}, x_2=\frac{2h+k}{2}$$

$$\Rightarrow 16x^2+10xy+y^2=2$$

47. From a point *P* on the hyperbola $\frac{x^2}{16}-\frac{y^2}{4}=1$ straight lines are drawn parallel to the asymptotes of the hyperbola. Then the area of parallelogram formed by the asymptotes and the two lines through P is

A) dependent on coordinates of P B) 4 C) 6 D) $8\sqrt{2}$

Key. B

Sol. Area of parallelogram is $\frac{ab}{2}=\frac{4\times 2}{2}=4$

48. The eccentricity of the conic defined by $\left|\sqrt{(x-1)^2+(y-2)^2}-\sqrt{(x-5)^2+(y-5)^2}\right|=3$

A) 5/2 B) 5/3 C) $\sqrt{2}$ D) $\sqrt{11}/3$

Key. B

Sol. Hyperbola for which (1, 2) and (5, 5) are foci and length of transverse axis 3.

$2ae=5$ and $2a=3$ $\therefore e=5/3$

49. The asymptotes of a hyperbola are $3x-4y+2=0$ and $5x+12y-4=0$. If the hyperbola passes through the point (1, 2) then slope of transverse axis of the hyperbola is

A) 6 B) –7/2 C) –8 D) 1/8

Key. C

Sol. Axes of hyperbola are bisectors of angles between asymptotes.

50. If P is a point on the rectangular hyperbola $x^2 - y^2 = a^2$, C being the center and S, S' are two foci, then $SP.S'P$ =

a) 2 b) $(CP)^2$ c) $(CS)^2$ d) $(SS')^2$

Key. B

Sol. Let P = (a sec θ,a tan θ), S_1 S^1 = $\left(\pm a\sqrt{2},0\right)$

SP= $a\left(\sqrt{2}\sec\theta-1\right)$, S^1 P = $a\left(\sqrt{2}\sec\theta+1\right)$

SP-S^1P = a^2 ($\sec^2\theta$ + $\tan^2\theta$) = CP^2

51. An equation of common tangent to the parabola $y^2 = 8x$ and the hyperbola $3x^2 - y^2 = 3$ is

a) 2x – y + 1 = 0 b) x – y + 2 = 0 c) x + y + 2 = 0 d) 2x + y – 1 = 0

Key. A

Sol. Let m be the slope of the common tangent

$$\therefore \frac{2}{m} = \sqrt{m^2 - 3} \Rightarrow m = \pm 2$$

Equation of common tangents are y = 2x+1 or y = -2x-1

52. Let P(θ), Q(φ) be two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ satisfying θ + φ = π/2. If (h, k) be the point of intersection of normals at P and Q, then k is equal to

a) $\frac{a^2+b^2}{a}$ b) $-\frac{a^2+b^2}{a}$ c) $-\frac{a^2+b^2}{b}$ d) $\frac{a^2+b^2}{b}$

Key. C

Sol. Solving the normals at θ, φ and using $\theta + \varphi = \frac{\pi}{2}$

53. A chord of the hyperbola $x^2 - 2y^2 = 1$ is bisected at the point (–1, 1). Then the area of the triangle formed by the chord and the coordinate axes is

a) 1 b) 2 c) 1/2 d) 1/4

Key. D

Sol. Equation of the chord as $S_1 = S_{11}$ = Req Area $\frac{1}{4}$

54. The angle of intersection between the curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2 - k^2} - \frac{y^2}{k^2 - b^2} = 1, (a > k > b > 0)$ is

a) $\tan^{-1}\left(\frac{b}{a}\right)$ b) $\tan^{-1}\left(\frac{b}{ka}\right)$

c) $\tan^{-1}\left(\frac{a}{kb}\right)$ d) None of these

Key. D

Sol. Confocal ellipse and hyperbola cut at right angles

55. Let A is the number of tangents drawn from a point on the asymptote of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (except origin) to the hyperbola itself. B is the number of normals which can be drawn from centre of $xy = c^2$ to the $xy = c^2$. C is the maximum number of normals which can be drawn from a point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. D is the number of tangent common to both branches of a hyperbola. Then number of normals which can be drawn from the point (ABD, BC) to $y^2 - 48y - 4x + 616 = 0$ is (If A =3, B = 5, C = 4 then ABC = 354)

a) 1 b) 0 c) 2 d) 3

Key. D

Sol. A – 1, B = 2, C = 4, D = 0

From (120, 24) we can draw 3 normals to

$(y - 24)^2 = 4(x - 10)$ since (x – 10) > 2

56. If the normal at P(8, 2) on the curve xy = 16 meets the curve again at Q. Then angle subtended by PQ at the origin is

a) $\tan^{-1}\left(\frac{15}{4}\right)$ b) $\tan^{-1}\left(\frac{4}{15}\right)$

c) $\tan^{-1}\left(\frac{261}{55}\right)$ d) $\tan^{-1}\left(\frac{55}{261}\right)$

Key. A

Sol. If a normal cuts the hyperbola at point $\left(t,\frac{1}{t}\right)$ meets the curve again at $\left(ct^1,\frac{C}{t^1}\right)$ then $t^3t^1=-1$

57. A triangle is inscribed in the curve $xy=c^2$ and two of its sides are parallel to $y+m_1x=0$ and $y+m_2x=0$. Then the third side touches the hyperbola

a) $4m_1m_2xy=c^2\left(m_1+m_2\right)^2$ b) $m_1m_2xy=c^2\left(m_1+m_2\right)$

c) $2m_1m_2xy=c^2\left(m_1+m_2\right)^2$ d) $4m_1m_2xy=c^2\left(m_1-m_2\right)^2$

Key. A

Sol. $m(AC)=\frac{-1}{t_1t_3}=-m_1, m(BC)=-m_2=\frac{-1}{t_2t_3}, m_1m_2=\frac{1}{t_3^2.t_1t_2}$

$m_1+m_2=\frac{1}{t_3}\left(\frac{t_2+t_1}{t_1t_2}\right)$ Compare chord $Ab=x+yt_1t_2=c\left(t_1+t_2\right)$ with $\frac{x}{t}+y+=2k$

58. Let a hyperbola passes through the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$. The transverse and conjugate axes of this hyperbola coincide with the major and minor axes of the given ellipse, also the product of eccentricities of given ellipse and hyperbola is 1, then

a) The equation of hyperbola is $\frac{x^2}{16}-\frac{y^2}{9}=1$ b) The equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{25}=1$

c) Focus of hyperbola is (5, 0) d) vertex of hyperbola is $\left(5\sqrt{3},0\right)$

Key. C

Sol. Conceptual

59. Consider a branch of the hyperbola $x^2-2y^2-2\sqrt{2x}-4\sqrt{2}y-6=0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, Then area of triangle ABC is

a) $\sqrt{\frac{3}{2}}+1$ b) $1-\sqrt{\frac{2}{3}}$

c) $1+\sqrt{\frac{2}{3}}$ d) $\sqrt{\frac{3}{2}}-1$

Key. D

Sol. Area $=\frac{1}{2}a\left(e-1\right)\times\frac{b^2}{a}=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}}=\sqrt{\frac{3}{2}}-1$

60. If the equation to the hyperbola is $3x^2-5xy-2y^2+5x+11y-8=0$ then equation to the conjugate hyperbola is

a) $3x^2-5xy-2y^2+5x+11y-16=0$

b) $3x^2-5xy-2y^2+5x+11y-12=0$

c) $3x^2-5xy-2y^2+5x+11y-4=0$

d) $3x^2-5xy-2y^2+5x+11y-20=0$

Key. A

Sol. $3x^2 - 5xy - 2y^2 + 5x + 11y + c = 0$ be the equation to the pair of asymptotes then c = -12. And hence equation to the conjugate hyperbola is $3x^2 - 5xy - 2y^2 + 5x + 11y - 16 = 0$

61. Locus of the mid points of the chords of the hyperbola $x^2-y^2=a^2$ that touch the parabola $y^2=4ax$ is

(A) $x^2(x-a)=y^3$

(B) $y^2(x-a)=x^3$

(C) $x^3(x-a)=y^2$

(D) $y^3(x-a)=x^2$

Key. B

Sol. Let the mid point = (*h, k*)

$\therefore$ equation of the chord $xh-yk=h^2-k^2$

$yk=xh+\left(k^2-h^2\right)$

$y=\frac{xh}{k}+\frac{\left(k^2-h^2\right)}{k}$

$\frac{k^2-h^2}{k}=\frac{ak}{h}$

$\Rightarrow k^2h-h^3=ak^2 \quad \Rightarrow k^2(h-a)=h^3 \quad \therefore x^3=y^2(x-a)$

62. If e is the eccentricity of of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and θ is the angle between the asymptotes. The $\cos\theta/2$ is equal to

(A) $\sqrt{e}$

(B) $\frac{e}{1+e}$

(C) $\frac{1}{\sqrt{e}}$

(D) $\frac{1}{e}$

Key. D

Sol. $\tan\frac{\theta}{2}=\frac{b}{a}$

$$\cos\frac{\theta}{2}=\frac{a}{\sqrt{a^2+b^2}}=\frac{1}{\sqrt{1+\frac{b^2}{a^2}}}=\frac{1}{e}.$$

63. Locus of the midpoints of the chords of the hyperbola $x^2-y^2=a^2$ that touch the parabola $y^2=4ax$ is

A) $x^2(x-a)=y^3$ B) $y^2(x-a)=x^3$ C) $x^3(x-a)y^2$ D) $y^3(x-a)x^2$

Key. B

Sol. let the mid point (h,k) equation of the chord is $xh - yk = h^2 - k^2$

$$y = \frac{xh}{k} + \frac{(k^2 - h^2)}{k};\ \frac{(k^2 - h^2)}{k} = \frac{ak}{h} \Rightarrow k^2(h-a) = h^3 \Rightarrow x^3 = y^3(x-a)$$

64. Consider a branch of the hyperbola $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is

(A) $1 - \sqrt{\frac{2}{3}}$ (B) $\sqrt{\frac{3}{2}} - 1$

(C) $1 + \sqrt{\frac{2}{3}}$ (D) $\sqrt{\frac{3}{2}} + 1$

Key. B

Sol. $x^2 - 2\sqrt{2}x - 2(y^2 + 2\sqrt{2}y) = 6$

$\Rightarrow (x - \sqrt{2})^2 - 2(y + \sqrt{2})^2 - 2 + 4 = 6$

$\Rightarrow (x - \sqrt{2})^2 - 2(y + \sqrt{2})^2 = 4$

$\Rightarrow \frac{(x-\sqrt{2})^2}{4} - \frac{(y+\sqrt{2})^2}{2} = 1$

$b^2 = a^2(e^2 - 1)$

$\Rightarrow 2 = 4(e^2 - 1)$

$\Rightarrow e^2 - 1 = 1/2$

$e = \sqrt{3}/2$

area = $\frac{1}{2}(ae - a) \times b^2/a$

$= (e - 1) = \left(\sqrt{\frac{3}{2}} - 1\right)$

65. The equations of the transverse and conjugate axes of a hyperbola respectively are $x + 2y - 3 = 0$ and $2x - y + 4 = 0$ and their respective lengths are $\sqrt{2}$ and $\frac{2}{\sqrt{3}}$. The equation of the hyperbola is

(A) $\frac{2}{5}(x + 2y - 3)^2 - \frac{3}{5}(2x - y + 4)^2 = 1$ (B) $\frac{2}{5}(2x - y + 4)^2 - \frac{3}{5}(x + 2y - 3)^2 = 1$

(C) $2(2x - y + 4)^2 - 3(x + 2y - 3)^2 = 1$ (D) $2(x + 2y - 3)^2 - 3(2x - y + 4)^2 = 1$

Key. B

Sol. The equation of the hyperbola is

$$\frac{\left(\frac{|2x-y+4|}{\sqrt{5}}\right)^2}{\left(\frac{\sqrt{2}}{2}\right)^2} - \frac{\left(\frac{|x+2y-3|}{\sqrt{5}}\right)^2}{\left(\frac{2}{\sqrt{3}}.\frac{1}{2}\right)^2} = 1$$

66. If $P(\theta_1)$ and $Q(\theta_2)$ are the extremities of any focal chord of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,$ then $\cos^2\frac{\theta_1+\theta_2}{2} = \lambda \cos^2\frac{\theta_1-\theta_2}{2}$, where λ is equal to

(A) $\frac{a^2+b^2}{a^2}$ (B) $\frac{a^2+b^2}{b^2}$

(C) $\frac{a^2+b^2}{ab}$ (D) $\frac{a^2+b^2}{2ab}$

Key. A

Sol. Equation of any chord joining the points $P(\theta_1)$ and $Q(\theta_2)$ is,

$\frac{x}{a}\cos\left(\frac{\theta_1-\theta_2}{2}\right)-\frac{y}{b}$

$\sin\left(\frac{\theta_1+\theta_2}{2}\right)=\cos\left(\frac{\theta_1+\theta_2}{2}\right)$. If it passes through (ae , 0), then

$\Rightarrow\ e^2\cos^2\left(\frac{\theta_1-\theta_2}{2}\right)=\cos^2\left(\frac{\theta_1+\theta_2}{2}\right)$

$\Rightarrow\ \lambda = e^2 = 1+\frac{b^2}{a^2}=\frac{a^2+b^2}{a^2}$

67. If the normal at the points P_i (x_i, y_i), i = 1 to 4 on the hyperbola $xy = c^2$ are concurrent at the point Q(h, k), then $\frac{(x_1+x_2+x_3+x_4)(y_1+y_2+y_3+y_4)}{x_1x_2x_3x_4}$ is equal to

(A) $\frac{hk}{c^4}$ (B) $\frac{h^2k^2}{c^6}$

(C) $\frac{\sqrt{|hk|}}{c^3}$ (D) $-\frac{hk}{c^4}$

Key. D

Sol. Equation of normal at any point $P(ct, \frac{c}{t})$ on xy

$= c^2$, is $xt^3 - yt - ct^4 + c = 0$

If it passes through Q (h, k), then

$ct^4 - ht^3 + kt - c = 0$

If it's roots are t_1, t_2, t_3 and t_4, then

$t_1 + t_2 + t_3 + t_4 = h/c$

$\Rightarrow ct_1 + ct_2 + ct_3 + ct_4 = h \ \Rightarrow\ \Sigma x_i = h,\ \Sigma t_1 t_2 t_3 = -\frac{k}{c},\ t_1 t_2 t_3 t_4 = -1$

$\Rightarrow\ (ct_1)(ct_2)(ct_3)(ct_4) = -c^4\ \Rightarrow\ \Sigma\frac{c}{t_i}=k \Rightarrow \Sigma y_i = k$ and $x_1x_2x_3x_4$

$= -c^4 \Rightarrow \frac{\Sigma x_i \Sigma y_i}{x_1x_2x_3x_4}=-\frac{hk}{c^4}$

68. A tangent to the hyperbola $y = \frac{x+9}{x+5}$ passing through the origin is

(A) x + 25y = 0 (B) 5x + y = 0

(C) 5x – y = 0 (D) x – 25y = 0

Key. A

Sol. $y=\frac{x+9}{x+5}=1+\frac{4}{x+5}$

$\frac{dy}{dx}$ at $(x_1, y_1)=\frac{-4}{(x_1+5)^2}$

Equation of tangent

$$y - y_1 = \frac{-4}{(x_1+5)^2}(x - x_1)$$

$$y - 1 - \frac{4}{x_1+5} = \frac{-4}{(x_1+5)^2}\cdot(x - x_1)$$

Since it passes through (0, 0)

$(x_1 + 5)^2 + 4(x_1 + 5) + 4x_1 = 0$

$x_1 = -15$ or $x_1 = -3$. So equation are

$x + 25y = 0$ or, $x + y = 0$.

69. The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

(A) $2x - \sqrt{5}y - 20 = 0$ (B) $2x - \sqrt{5}y + 4 = 0$

(C) $3x - 4y + 8 = 0$ (D) $4x - 3y + 4 = 0$

Key. B

Sol. Equation of tangent at point P(θ)

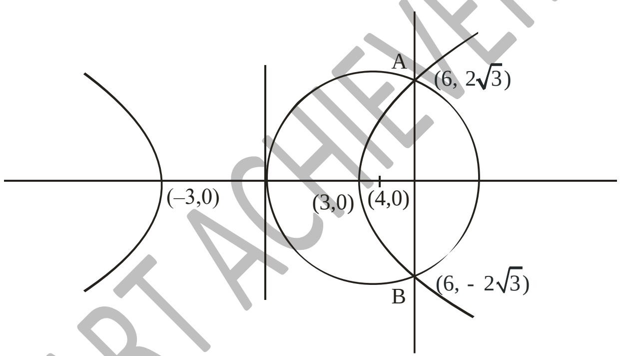
$$\frac{x\sec\theta}{3} - \frac{y\tan\theta}{2} - 1 = 0 \quad \text{.....(i)}$$

since eq. (i) will be a tangent to the circle

$$\therefore \frac{\frac{4\sec\theta}{3} - 1}{\sqrt{\frac{\sec^2\theta}{9} + \frac{\tan^2\theta}{4}}} = 4$$

by solving it we will get

$$2x - \sqrt{5}y + 4 = 0$$



70. There is a point P on the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ such that its distance to the right directrix is the average of its distance to the two foci. Let the x-coordinate of P be $\frac{m}{n}$ with m and n being integers, (n > 0) having no common factor except 1. Then n – m equals

(A) 59 (B) 69 (C) -59 (D) -69

Key. B

Sol. It turns out that P has to be on the left branch. x-coordinate is found to be $-64/5$

71. The reflection of the hyperbola $xy=1$ in the line $y=2x$ is the curve $12x^2+r\,xy+s\,y^2+t=0$ then the value of $'r'$ is

a) – 7 b) 25 c) – 175 d) 90

Key. A

Sol. The reflection of (α,β) in the line $y=2x$ is

$$(\alpha_1,\beta_1)=\left(\frac{4\beta-3\alpha}{5},\frac{4\alpha+3\beta}{5}\right)=\ \alpha_1\beta_1=1$$

$$\Rightarrow 12\alpha^2-7\alpha\beta-12\beta^2+25=0$$

72. Chords of the parabola $y^2=4x$ touch the hyperbola $x^2-y^2=1$. The locus of the point of intersection of the tangents drawn to the parabola at the extremities of such chords is

a) a circle b) a parabola

c) an ellipse d) a rectangular hyperbola

Key. C

Sol. The chord of contact $y\,y_o=2\left(x+x_o\right)$ of the point $P\left(x_o,y_o\right)$ w.r.t the parabola is tangent to the hyperbola $x^2-y^2=1\ iff\ \ 2x_0^2+y_0^2=4$. Locus of P is the ellipse $2x^2+y^2=4$

73. A chord of the hyperbola $x^2 – 2y^2 = 1$ is bisected at the point (–1, 1). Then the area of the triangle formed by the chord and the coordinate axes is

a) 1 b) 2 c) $\frac{1}{2}$ d) 1/4

Key. D

Sol. Equation of the chord as $S_1 = S_{11}$ = Req Area $\frac{1}{4}$

74. A pair of tangents with inclinations α,β are drawn from an external point P to the parabola $y^2=16x$. If the point P varies in such a way that $\tan^2\alpha+\tan^2\beta=4$ then the locus of P is a conic whose eccentricity is

A) $\frac{\sqrt{5}}{2}$ B) $\sqrt{5}$ C) 1 D) $\frac{\sqrt{3}}{2}$

Key. B

Sol. Let $m_1=\tan\alpha, m_2=\tan\beta$, Let $P=(h,k)$

m_1, m_2 are the roots of $K=mh+\frac{4}{m}\Rightarrow hm^2-Km+4=0$

$$m_1+m_2=\frac{K}{h};\quad m_1m_2=\frac{4}{h}$$

$$m_1^2+m_2^2=\frac{K^2}{h^2}-\frac{8}{h}=4$$

Locus of P is $y^2-8x=4x^2\Rightarrow y^2=4(x+1)^2-4\Rightarrow\frac{(x+1)^2}{1}-\frac{y^2}{4}=1$

75. From a point *P* on the hyperbola $\frac{x^2}{16}-\frac{y^2}{4}=1$ straight lines are drawn parallel to the asymptotes of the hyperbola. Then the area of parallelogram formed by the asymptotes and the two lines through P is

A) dependent on coordinates of P B) 4 C) 6 D) $8\sqrt{2}$

Key. B

Sol. Area of parallelogram is $\frac{ab}{2}=\frac{4\times 2}{2}=4$

76. The asymptotes of a hyperbola are $3x-4y+2=0$ and $5x+12y-4=0$. If the hyperbola passes through the point (1, 2) then slope of transverse axis of the hyperbola is

A) 6 B) –7/2 C) –8 D) 1/8

Key. C

Sol. Axes of hyperbola are bisectors of angles between asymptotes.

77. Locus of the midpoints of the chords of the hyperbola $x^2-y^2=a^2$ that touch the parabola $y^2=4ax$ is

A) $x^2(x-a)=y^3$ B) $y^2(x-a)=x^3$ C) $x^3(x-a)y^2$ D) $y^3(x-a)x^2$

Key. B

Sol. let the mid point (h,k) equation of the chord is $xh-yk=h^2-k^2$

$$y=\frac{xh}{k}+\frac{(k^2-h^2)}{k};\ \frac{(k^2-h^2)}{k}=\frac{ak}{h}\Rightarrow k^2(h-a)=h^3\Rightarrow x^3=y^2(x-a)$$

78. Two distinct tangents can be drawn from the point $(\alpha,2)$ to different branches of the hyperbola $\frac{x^2}{9}-\frac{y^2}{16}=1,$ if 'α' belongs to

A) $\left(\frac{-3}{2},\frac{5}{2}\right)$ B) $\left(\frac{-5}{2},\frac{5}{2}\right)$ C) $\left(\frac{-7}{2},\frac{7}{2}\right)$ D) $\left(\frac{-3}{2},\frac{3}{2}\right)$

Key. D

Sol. The point on the line $y=2$ that should lie between the asymptotes where the curve do not exist.Equation of asymptotes are $4x=\pm 3y$. The point of intersection of $y=2$ with asymptotes are $x=\pm\frac{3}{2}$

$\therefore \frac{-3}{2}<\alpha<\frac{3}{2}$

79. A hyperbola passing through origin has $3x-4y-1=0$ and $4x-3y-6=0$ as its asymptotes. Then the equation of its transverse axis is

A) $x-y-5=0$ B) $x+y+1=0$

C) $x+y-5=0$ D) $x-y-1=0$

Key. A

Sol. Asymptotes are equally inclined to the axes of hyperbola. Find the bisector of the asymptotes which bisects the angle containing the origin.

80. A hyperbola has centre $'C'$ and one focus at $P(6,8)$. If its two directrices are $3x+4y+10=0$ and $3x+4y-10=0$ then $CP=$

A) 14 B) 8 C) 10 D) 6

Key. A

Sol. $\frac{2a}{e}=4 \Rightarrow a=2e,\ P$ is nearest to $3x+4y-10=0$

$\Rightarrow ae-\frac{a}{e}=8 \Rightarrow e=\sqrt{5},\ a=2\sqrt{5}$

$CP=ae=10$

81. If a variable tangent to the circle $x^2+y^2=1$ intersects the ellipse $x^2+2y^2=4$ at points P and Q, then the locus of the point of intersection of tangents to the ellipse at P and Q is a conic whose

a) eccentricity is $\frac{\sqrt{3}}{2}$ b) eccentricity is $\frac{\sqrt{5}}{2}$

c) latus -rectum is of length 2 units d) foci are $\left(\pm 2\sqrt{5},0\right)$

Key: A,C

Hint: A tangent to the circle $x^2+y^2=1\ is\ x\cos\theta+y\sin\theta=1. R\left(x_o,y_o\right)$ is the point of intersection of the tangents to the ellipse at P and Q $\Leftrightarrow x\cos\theta+y\sin\theta=1$ and $x_o x+2y_o y=4$ represent the same line

$\Leftrightarrow x_o=4\cos\theta\, and\ y_o=2\sin\theta$

$\Leftrightarrow \frac{x_0^2}{16}+\frac{y_0^2}{4}=1.$ Hence, locus of P is the ellipse $\frac{x^2}{16}+\frac{y^2}{4}=1$

82. A variable straight line with slope $m(m\neq 0)$ intersects the hyperbola xy=1 at two distinct points .Then the locus of the point which divides the line segment between these two points in the ratio 1:2 is

(A) An ellipse (B) A hyperbola (C) A circle (D) A parabola

Key: B

Hint: Let the points of intersection be $\left(t_1,\frac{1}{t_1}\right)\left(t_2,\frac{1}{t_2}\right)$.given $m=-\frac{1}{t_1t_2}\ or\ t_1t_2=-\frac{1}{m}$

also by section formula ,

solving for t_1,t_2 and eliminating them gives $2m^2x^2+5mxy+2y^2=m$ which is always a hyperbola as

$\frac{25m^2}{4}-4m^2=\frac{9m^2}{4}>0,\forall m\neq 0$

83. A tangent to the parabola x^2 = 4ay meets the hyperbola $x^2-y^2=a^2$ at two points P and Q, then midpoint of P and Q lies on the curve

a) $y^3=x\left(y-a\right)$ b) $y^3=x^2\left(y-a\right)$

c) $y^2=x^2\left(y-a\right)$ d) $y^2=x^3\left(a-y\right)$

Key: B

Hint: Equation of tangent to parabola $y = mx - am^2 ....(1)$ equation of chord of hyperbola whose midpoint is (h, k) is $hx - ky = h^2 - k^2 ......(2)$ form (1) and (2)

$$\frac{m}{h} = \frac{1}{k} = \frac{am^2}{h^2 - k^2} \Rightarrow k^3 = h^2(k - a)$$

84. The equation of a tangent to the hyperbola $3x^2 - y^2 = 3$, parallel to the line $y = 2x + 4$ is

(A) y = 2x + 3 (B) y = 2x + 1

(C) y = 2x + 4 (D) y = 2x + 2

Key. B

Sol. $3x^2 - y^2 = 3, \frac{x^2}{1} - \frac{y^2}{3} = 1$

Equation of tangent in terms of slope.

$y = mx \pm \sqrt{(m^2 - 3)}$

Here, m = 2,

then $y = 2x \pm 1$

85. A circle cuts the X-axis and Y-axis such that intercept on X-axis is a constant a and intercept on Y-axis is a constant b. Then eccentricity of locus of centre of circle is

1. 1 2. $\frac{1}{2}$ 3. $\sqrt{2}$ 4. $\frac{1}{\sqrt{2}}$

Key. 3

Sol. Locus of centre of circle is a rectangular hyperbola hence its eccentricity is $\sqrt{2}$

86. If a circle cuts the rectangular hyperbola xy=1 in 4 points (x_r, y_r) where r =1,2,3,4. Then ortho centre of triangle with vertices at (x_r, y_r) where r=1,2,3 is

1. (x_4, y_4) 2. $(-x_4, -y_4)$

3. $(-x_4, +y_4)$ 4. $(+x_4, -y_4)$

Key. 2

Sol. $xy = 1$ cuts the circle in 4-points then $x_1x_2x_3x_4 = 1,\ y_1y_2y_3y_4 = 1$

Ortho centre of triangle with vertices $(x_1, y_1)(x_2, y_2)(x_3, y_3)$

Ie $\left(\frac{-1}{x_1x_2x_3}, -(y_1y_2y_3)^{-1}\right)$

$-(-x_4, -y_4)$

87. The locus of a point, from where tangents to the rectangular hyperbola $x^2 - y^2 = a^2$ contain an angle of 45^0, is

(A) $(x^2 + y^2) + a^2(x^2 - y^2) = 4a^2$ (B) $2(x^2 + y^2) + 4a^2(x^2 - y^2) = 4a^2$

(C) $(x^2 + y^2)^2 + 4a^2(x^2 - y^2) = 4a^4$ (D) $(x^2 + y^2)^2 + a^2(x^2 - y^2) = a^4$

Key. C

Sol. Equation of tangent to the hyperbola : $y = mx \pm \sqrt{m^2a^2 - a^2}$

$\Rightarrow$ Let $P(x_1, y_1)$ be locus

$\Rightarrow y - mx = \pm\sqrt{m^2a^2 - a^2}$

S.B.S

$\Rightarrow m^2(x_1^2 - a^2) - 2y_1x_1m + y_1^2 + a^2 = 0$

$m_1 + m_2 = \dfrac{2x_1y_1}{x_1^2 - a^2}$; $m_1m_2 = \dfrac{y_1^2 + a^2}{x_1^2 - a^2}$

$\tan 45^0 = \left|\dfrac{m_1 - m_2}{1 + m_1m_2}\right|$

$\Rightarrow (1 + m_1m_2)^2 = (m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1m_2$

$\Rightarrow \left(1 + \dfrac{y_1 + a^2}{x_1^2 - a^2}\right) = \left(\dfrac{2x_1y_1}{x_1^2 - a^2}\right) - 4\left(\dfrac{y_1^2 + a^2}{x_1^2 - a^2}\right)$

88. If PQ is a double ordinate of the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ such that OPQ is an equilateral triangle, O being the centre of the hyperbola. Then the eccentricity *e* of the hyperbola, satisfies

(a) $1 < e < 2/\sqrt{3}$ (b) $e = 2/\sqrt{3}$ (c) $e = \sqrt{3}/2$ (d) $e > 2/\sqrt{3}$

Key. D

Sol. If OPQ is equilateral triangle then OP makes 30^0 with x-axis.

$\left(\dfrac{\sqrt{3}r}{2}, \dfrac{r}{2}\right)$ ties on hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

$\Rightarrow r^2 = \dfrac{16a^2b^2}{12b^2 - 4a^2} > 0$

$\Rightarrow 12b^2 - 4a^2 > 0 \Rightarrow \dfrac{b^2}{a^2} > \dfrac{4}{12}$

$e^2 - 1 > \dfrac{1}{3}$

$e^2 > \dfrac{4}{3} \Rightarrow e > \dfrac{2}{\sqrt{3}}$

89. Consider a hyperbola xy=4 and a line 2x+y=4. Let the given line intersect the x-axis at R. If a line through 'R' intersects the hyperbola at S and T. The minimum value of $RS \times RT$ is

A) 24 B) 16 C) 8 D) 4

Key. C

Sol. S, T = $(2 + r\cos\theta, 0 + r\sin\theta)$

$r^2\cos\theta\sin\theta + 2\sin\theta - 4 = 0$

RS.RT = $\dfrac{4}{\sin\theta\cos\theta} = \dfrac{8}{\sin 2\theta} \geq 8$

90. The normal at 'P' on a hyperbola of eccentricity 'e' intersects its transverse and conjugate axes at L and M respectively. If the locus of the mid point of LM is a hyperbola then its eccentricity is

A) $\dfrac{e+1}{e-1}$ B) $\dfrac{e}{\sqrt{e^2 - 1}}$ C) e D) $\dfrac{2e}{\sqrt{e^2 - 1}}$

Key. B

Sol. Normal : $ax\cos\theta + by\cot\theta = a^2 + b^2$

$$L = \left(\frac{a^2+b^2}{a}\sec\theta, 0\right),\ M = \left(0, \frac{a^2+b^2}{b}\tan\theta\right)$$

Locus is $\dfrac{x^2}{\frac{a^2e^2}{4}} - \dfrac{y^2}{\frac{a^2e^2}{4b^2}} = 1$

$$e_1 = \frac{e}{\sqrt{e^2-1}}$$

91. Consider a branch of the hyperbola $x^2 - 2y^2 - 2\sqrt{2x} - 4\sqrt{2}y - 6 = 0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, Then area of triangle ABC is

a) $\sqrt{\frac{3}{2}}+1$ b) $1-\sqrt{\frac{2}{3}}$ c) $1+\sqrt{\frac{2}{3}}$ d) $\sqrt{\frac{3}{2}}-1$

Key. D

Sol. Area $= \frac{1}{2}a(e-1)\times\frac{b^2}{a} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}} = \sqrt{\frac{3}{2}}-1$

92. If the equation to the hyperbola is $3x^2 - 5xy - 2y^2 + 5x + 11y - 8 = 0$ then equation to the conjugate hyperbola is

a) $3x^2 - 5xy - 2y^2 + 5x + 11y - 16 = 0$

b) $3x^2 - 5xy - 2y^2 + 5x + 11y - 12 = 0$

c) $3x^2 - 5xy - 2y^2 + 5x + 11y - 4 = 0$

d) $3x^2 - 5xy - 2y^2 + 5x + 11y - 20 = 0$

Key. A

Sol. $3x^2 - 5xy - 2y^2 + 5x + 11y + c = 0$ be the equation to the pair of asymptotes then c = -12. And hence equation to the conjugate hyperbola is $3x^2 - 5xy - 2y^2 + 5x + 11y - 16 = 0$

93. A tangent to the circle $x^2 + y^2 = 4$ intersects the hyperbola $x^2 - 2y^2 = 2$ at P and Q. If locus of mid–point of PQ is $(x^2 - 2y^2)^2 = \lambda(x^2 + 4y^2)$; then λ equals

(A) 2 (B) 4

(C) 6 (D) 8

Key. B

Sol. Equation of chord of hyperbola $\frac{x^2}{2} - \frac{y^2}{1} = 1,$ whose mid–point is (h, k) is

$$\frac{hx}{2} - ky = \frac{h^2}{2} - \frac{k^2}{1}$$

It is tangent to the circle $x^2 + y^2 = 4$, then $\left|\dfrac{\frac{h^2}{2}-k^2}{\sqrt{\frac{h^2}{4}+k^2}}\right| = 2$

$\Rightarrow \left(\frac{h^2}{2}-k^2\right)^2 = 4\left(\frac{h^2}{4}+k^2\right) \Rightarrow (x^2-2y^2)^2 = 4(x^2+4y^2) \Rightarrow \lambda = 4.$

94. Length of latusrectum of the conic satisfying the differential equation $xdy + ydx = 0$ and passing through the point (2, 8) is

A) $4\sqrt{2}$ B) 8 C) $8\sqrt{2}$ D) 16

Key. C

Sol. $\frac{dy}{y}+\frac{dx}{x}=0 \Rightarrow xy=16$

$\therefore\ y=-x$ is conjugate axis centre is (0, 0).

Vertices are (4, 4), $(-4,-4)$. $e=\sqrt{2}$

Length of transverse axis = $8\sqrt{2}=2a$

L.R = $2a(e^2-1)$

95. From a point *P* on the hyperbola $\frac{x^2}{16}-\frac{y^2}{4}=1$ straight lines are drawn parallel to the asymptotes of the hyperbola. Then the area of parallelogram formed by the asymptotes and the two lines through P is

A) dependent on coordinates of P B) 4 C) 6 D) $8\sqrt{2}$

Key. B

Sol. Area of parallelogram is $\frac{ab}{2}=\frac{4\times 2}{2}=4$

96. The eccentricity of the conic defined by $\left|\sqrt{(x-1)^2+(y-2)^2}-\sqrt{(x-5)^2+(y-5)^2}\right|=3$

A) 5/2 B) 5/3 C) $\sqrt{2}$ D) $\sqrt{11}/3$

Key. B

Sol. Hyperbola for which (1, 2) and (5, 5) are foci and length of transverse axis 3.

$2ae=5$ and $2a=3$ $\quad \therefore e=5/3$

97. The asymptotes of a hyperbola are $3x-4y+2=0$ and $5x+12y-4=0$. If the hyperbola passes through the point (1, 2) then slope of transverse axis of the hyperbola is

A) 6 B) –7/2 C) –8 D) 1/8

Key. C

Sol. Axes of hyperbola are bisectors of angles between asymptotes.

98. A triangle is inscribed in the curve $xy=c^2$ and two of its sides are parallel to $y+m_1x=0$ and $y+m_2x=0$. Then the third side touches the hyperbola

a) $4m_1m_2xy=c^2(m_1+m_2)^2$ b) $m_1m_2xy=c^2(m_1+m_2)$

c) $2m_1m_2xy=c^2(m_1+m_2)^2$ d) $4m_1m_2xy=c^2(m_1-m_2)^2$

Key. A

Sol. $m(AC)=\frac{-1}{t_1t_3}=-m_1, m(BC)=-m_2=\frac{-1}{t_2t_3}, m_1m_2=\frac{1}{t_3^2.t_1t_2}$

$m_1 + m_2 = \frac{1}{t_3}\left(\frac{t_2 + t_1}{t_1 t_2}\right)$ Compare chord $Ab = x + yt_1 t_2 = c(t_1 + t_2)$ with $\frac{x}{t} + y + = 2k$

99. Let a hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. The transverse and conjugate axes of this hyperbola coincide with the major and minor axes of the given ellipse, also the product of eccentricities of given ellipse and hyperbola is 1, then

a) The equation of hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = 1$ b) The equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{25} = 1$

c) Focus of hyperbola is (5, 0) d) vertex of hyperbola is $\left(5\sqrt{3}, 0\right)$

Key. C

Sol. Conceptual

100. The length of the transverse axis of the hyperbola $9x^2 - 16y^2 - 18x - 32y - 151 = 0$ is

1) 8 2) 4 3) 6 4) 2

Key. 1

Sol. Given hyperbola is $\frac{(x-1)^2}{16} - \frac{(y+1)^2}{9} = 1$

Length of the transverse axis is 2a=8.

101. The equation of a hyperbola , conjugate to the hyperbola $x^2 + 3xy + 2y^2 + 2x + 3y = 0$ is

1) $x^2 + 3xy + 2y^2 + 2x + 3y + 1 = 0$ 2) $x^2 + 3xy + 2y^2 + 2x + 3y + 2 = 0$

3) $x^2 + 3xy + 2y^2 + 2x + 3y + 3 = 0$ 4) $x^2 + 3xy + 2y^2 + 2x + 3y + 4 = 0$

Key. 2

Sol. Let $H = x^2 + 3xy + 2y^2 + 2x + 3y = 0$ and C=0 is its conjugate. Then C + H=2A, where A=0 is the combined equation of asymptotes. Equation of asymptotes is $x^2 + 3xy + 2y^2 + 2x + 3y + \lambda = 0$, where $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0 \Rightarrow \lambda = 1$

$\therefore C = 2\left(x^2 + 3xy + 2y^2 + 2x + 3y + 1\right) - \left(x^2 + 2y^2 + 3xy + 2x + 3y\right)$

$\Rightarrow$ equation of conjugate hyperbola is $x^2 + 3xy + 2y^2 + 2x + 3y + 2 = 0$

102. If AB is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that ΔOAB is an equilateral triangle O being the origin, then the eccentricity of the hyperbola satisfies

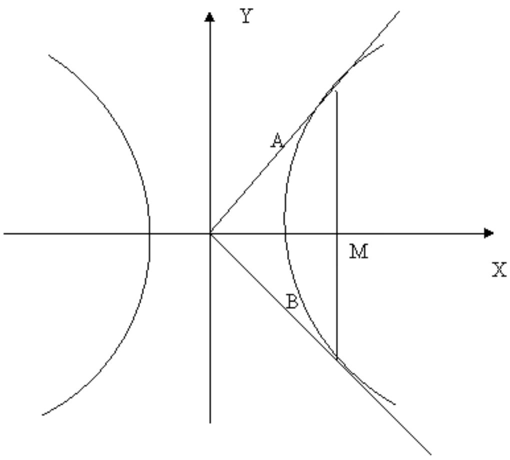
1) $e > \sqrt{3}$ 2) $1 < e < \frac{1}{\sqrt{3}}$ 3) $e = \frac{2}{\sqrt{3}}$ 4) $e > \frac{2}{\sqrt{3}}$

Key. 4

Sol. Let the length of the double ordinate be 2ℓ

$\therefore$ AB=2ℓ and AM=BM=ℓ

Clearly ordinate of point A is ℓ.



The abscissa of the point A is given by

$$\frac{x^2}{a^2} - \frac{l^2}{b^2} = 1 \Rightarrow x = \frac{a\sqrt{b^2 + l^2}}{b}$$

$\therefore$ A is $\left(\frac{a\sqrt{b^2 + l^2}}{b}, l\right)$

Since ΔOAB is equilateral triangle, therefore

OA=AB=OB=$2l$

Also, $OM^2 + AM^2 = OA^2 \therefore \frac{a\left(b^2 + l^2\right)}{b} + l^2 = 4l^2$

We get $l^2 = \frac{a^2b^2}{3b^2 - a^2}$

Since $l^2 > 0 \therefore \frac{a^2b^2}{3b^2 - a^2} > 0 \Rightarrow 3b^2 - a^2 > 0$

$$\Rightarrow 3a^2\left(e^2 - 1\right) - a^2 > 0 \Rightarrow e > \frac{2}{\sqrt{3}}$$

103. If the line $5x+12y-9=0$ is a tangent to the hyperbola $x^2-9y^2=9$, then its point of contact is

1) (-5,4/3) 2) (5,-4/3) 3) $(3,-1/2)$ 4) (5,4/3)

Key. 2

Sol. Common Point

104. Any chord passing through the focus $(ae,0)$ of the hyperbola $x^2-y^2=a^2$ is conjugate to the line

1) $ex-a=0$ 2) $ae+x=0$ 3) $ax+e=0$ 4) $ax-e=0$

Key. 1

Sol. $S_1=0$

105. Number of points from where perpendicular tangents to the curve $\frac{x^2}{16}-\frac{y^2}{25}=1$ can be drawn, is:

1) 1 2) 2 3) 0 4) 3

Key. 3

Sol. Director circle is set of points from where drawn tangents are perpendicular in this case $x^2+y^2=a^2-b^2$ (equation of director circle)i.e., $x^2+y^2=-9$ is not a real circle so there is no points from where tangents are perpendicular.

106. $x^2-y^2+5x+8y-4=0$ represents

1) Rectangular hyperbola 2)Ellipse

3) Hyperbola with centre at (1,1) 4)Pair of lines

Key. 1

Sol. $\Delta \neq 0,\ x^2-ab>0,\ a+b=0$

107. Coordinates if foci of the hyperbola xy=4 are

1) $(2\sqrt{2},2\sqrt{2}),(-2\sqrt{2},-2\sqrt{2})$ 2) $(-3\sqrt{2},-3\sqrt{2}),(3\sqrt{2},3\sqrt{2})$

3) $(2\sqrt{2},-2\sqrt{2}),(-2\sqrt{2},2\sqrt{2})$ 4) (-2. 2)

Key. 1

Sol. foci of $xy=c^2$ is $(\pm c\sqrt{2},\pm c\sqrt{2})$

108. Which of the following is INCORRECT for the hyperbola $x^2-2y^2-2x+8y-1=0$

1) Its eccentricity is $\sqrt{2}$ 2) Length of the transverse axis is $2\sqrt{3}$

3) Length of the conjugate axis is $2\sqrt{6}$ 4) Latus rectum $4\sqrt{3}$

Key. 1

Sol. The equation of the hyperbola is $x^2-2y^2-2x+8y-1=0$

Or $(x-1)^2-2(y-2)^2+6=0$

Or $\frac{(x-1)^2}{-6}+\frac{(y-2)^2}{3}=1;$ or $\frac{(y-2)^2}{3}-\frac{(x-1)^2}{6}=1\rightarrow 1$

Or $\frac{Y^2}{3}-\frac{X^2}{6}=1$, where X = x -1 and Y = y - 2 $\rightarrow 2$

$\therefore$ the centre=(0,0)in the X-Y coordinates.

$\therefore$ the centre=(1,2)in the x-y coordinates .using $\rightarrow 2$

If the transverse axis be of length 2a, then a= $\sqrt{3}$,since in the equation (1) the transverse axis is parallel to the y-axis.

If the conjugate axis is of length 2b,then b = $\sqrt{6}$

But $b^2=a^2\left(e^2-1\right)$

$\therefore$ $6=3\left(e^2-1\right),\therefore e^2=3$ or $e=\sqrt{3}$

The length of the transverse axis = $2\sqrt{3}$

The length of the conjugate axis = $2\sqrt{6}$

Latus rectum $4\sqrt{3}$

109. If the curve $xy=R^2-16$ represents a rectangular hyperbola whose branches lies only in the quadrant in which abscissa and ordinate are opposite in sign but not equal in magnitude, then

1) $|R|<4$ 2) $|R|\geq 4$ 3) $|R|=4$ 4) $|R|=5$

Key. 1

Sol. Conceptual

110. Assertion: The pair of asymptotes of $\frac{x^2}{10}-\frac{y^2}{4}=1$ and the pair of asymptotes of $\frac{x^2}{10}-\frac{y^2}{4}=-1$ coincide.

Reason : A hyperbola and its conjugate hyperbola possess the same pair of asymptotes

1) Both A and R are true and R is the correct explanation of A

2) Both A and R are true but R is not correct explanation of A

3) A is true R is false

4) A is false R is true

Key. 1

Sol. Conceptual

111. If the line ax + by + c=0 is a normal to the curve xy=1,then

1) a > 0, b > 0 2) a < 0,b < 0 3) a < 0,b > 0 4) a=b=1

Key. 3

Sol. Slope of the line $\frac{-a}{b}$ is equal to slope of the normal to the curve.

$\therefore$ either a > 0 & b < 0 (or) a < 0 & b > 0.

112. The equation of normal at $\left(at, \frac{a}{t}\right)$ to the hyperbola $xy = a^2$ is

1) $xt^3 - yt + at^4 - a = 0$ 2) $xt^3 - yt - at^4 + a = 0$

3) $xt^3 + yt + at^4 - a = 0$ 4) $xt^3 + yt - at^4 - a = 0$

Key. 2

Sol. Equation of tangent is $s_1 = 0$ normal is $\perp^r$ to tangent and passing through

$\left(at, \frac{a}{t}\right)$ is $xt^3 - yt - at^4 + a = 0$

113. The product of perpendiculars from any point P (θ) on the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ to its asymptotes is equal to:

1) $\frac{6}{5}$ 2) $\frac{36}{13}$ 3) Depending on θ 4) $\frac{5}{6}$

Key. 2

Sol. The product of perpendiculars from any point P (θ) on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to its asymptotes is equal to $\frac{a^2b^2}{a^2+b^2}$

114. The foot of the perpendicular from the focus to an asymptote of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

1) (ae , be) 2) (a/e,b/e) 3) (e/a,e/b) 4) (be,ae)

Key. 2

Sol. Focus S=(ae,0) Equation of one asymptote is bx-ay=0

Let (h,k) be the foot of the perpendicular from s to bx-ay=0

Then $\frac{h-ae}{b} = \frac{k-0}{-a} = \frac{-abe}{a^2+b^2} \Rightarrow \frac{h-ae}{b} = \frac{-abe}{a^2e^2} \& \frac{k}{-a} = \frac{-abe}{a^2e^2}$

On simplification, we get h=a/e, k=b/e

Foot of the perpendicular is (a/e,b/e)

115. The area of the triangle formed by the asymptotes and any tangent to the hyperbola $x^2 - y^2 = a^2$

1) $4a^2$ 2) $3a^2$ 3) $2a^2$ 4) a^2

Key. 4

Sol. Equation of any tangent to $x^2 - y^2 = a^2$

i.e. $\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1 \; is \; \frac{x}{a}\sec\theta - \frac{y}{a}\tan\theta = 1 \rightarrow (1)$

$or \; x\sec\theta - y\tan\theta = a$

equation of other two sides of the triangle are

x-y=0..(2) x + y=0(3)

The two asymptotes of the hyperbola $x^2 - y^2 = a^2$

Are x-y=0 and x + y=0)

Solving (1) (2) and (3) in pairs the coordinates of the vertices of the triangle are (0,0)

$\left(\frac{a}{\sec\theta + \tan\theta}, \frac{a}{\sec\theta + \tan\theta}\right)$

And $\left(\frac{a}{\sec\theta - \tan\theta}, \frac{-a}{\sec\theta - \tan\theta}\right)$ –

Area of triangle = $\frac{1}{2}\left|\frac{a^2}{\sec^2\theta - \tan^2\theta} + \frac{a^2}{\sec^2\theta - \tan^2\theta}\right|$

$\frac{1}{2}(a^2 + a^2) \qquad \because \sec^2\theta - \tan^2\theta = 1$

$= a^2$

116. The foot of the normal $3x + 4y = 7$ to the hyperbola $4x^2 - 3y^2 = 1$ is

1) $(1,1)$ 2) $(1,-1)$ 3) $(-1,1)$ 4) $(-1,-1)$

Key. 1

Sol. Since the point $(1,1)$ lies on the normal and hyperbola it is the foot of the normal

117. Tangent at the point $(2\sqrt{2},3)$ to the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$ meet its asymptotes at A and B, then area of the

triangle OAB, O being the origin is

1) 6 sq. units 2) 3 sq. units 3) 12 sq. units 4) 2 sq. units

Key. 1

Sol. Since area of the Δ formed by tangent at any point lying on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and its asymptotes is always constant and is equal to ab. Therefore, required area is 2 X 3=6 square units.

118. Eccentricity of hyperbola $\frac{x^2}{k}+\frac{y^2}{k}=1(k<0)$ is :

1) $\sqrt{1+k}$ 2) $\sqrt{1-k}$ 3) $\sqrt{1+\frac{1}{k^2}}$ 4) $\sqrt{1-\frac{1}{k}}$

Key. 4

Sol. Given equation can be rewritten as $\frac{y^2}{k^2}-\frac{x^2}{(-k)}=1(-k>0)$

$e^2=1+\frac{(-k)}{k^2}=1-\frac{1}{k}\Rightarrow e=\sqrt{1-\frac{1}{k}}$

119. If the circle $x^2+y^2=a^2$ intersect the hyperbola $xy=c^2$ in four points $P(x_1,y_1),Q(x_2,y_2),R(x_3,y_3),S(x_4,y_4)$ then which of the following does not hold

1) $x_1+x_2+x_3+x_4=0$ 2) $x_1x_2x_3x_4=y_1y_2y_3y_4=c^4$

3) $y_1+y_2+y_3+y_4=0$ 4) $x_1+y_2+x_3+y_4=0$

Key. 4

Sol. $x^2+\frac{c^4}{x^2}=a^2$ $\Rightarrow \mathbf{x^4-a^2x^2+c^4=0}$, 4th option does not hold

120. If a normal to the hyperbola x y = c² at $\left(ct_1,\frac{c}{t_1}\right)$ meets the curve again at $\left(ct_2,\frac{c}{t_2}\right)$, then:

1) $t_1t_2=-1$ 2) $t_2=-t_1-\frac{2}{t_1}$ 3) $t_2^3t_1=-1$ 4) $t_1^3t_2=-1$

Key. 4

Sol. Equation of normal at $\left(ct_1,\frac{c}{t_1}\right)$ is

$t_1^3x-t_1y-ct_1^4+c=0$

It passes through $\left(ct_2,\frac{c}{t_2}\right)$

Ie., $t_1^3.ct_2 - t_1.\frac{c}{t_2} - ct_1^4 + c = 0$

$\Rightarrow (t_1 - t_2)(t_1^3 t_2 + 1) = 0$

$\Rightarrow t_1^3 t_2 = -1$

121. The equation of the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola xy=c^2 is

1) $\frac{x}{x_1+x_2}+\frac{y}{y_1+y_2}=1$ 2) $\frac{x}{x_1-x_2}+\frac{y}{y_1-y_2}=1$ 3) $\frac{y}{x_1+x_2}+\frac{x}{y_1+y_2}=1$ 4) $\frac{x}{y_1-y_2}+\frac{y}{x_1-x_2}=1$

Key. 1

Sol. Mid point of the chord is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

The equation of the chord in terms of its mid-point is $s_1 = s_{11}$

122. A rectangular hyperbola whose centre is C is cut by any circle of radius r in four points P, Q, R and S. Then $CP^2 + CQ^2 + CR^2 + CS^2 =$

1) r^2 2) $2r^2$ 3) $3r^2$ 4) $4r^2$

Key. 4

Sol. $CP = CQ = CR = CS = r$

123. The product of focal distances of the point (4,3) on the hyperbola $x^2 - y^2 = 7$ is

1) 25 2) 12 3) 9 4) 16

Key. 1

Sol. $e = \sqrt{2},\ sp.s'p = (ex_1 + a)(ex_1 - a) = 25$

124. Let $y = 4x^2$ & $\frac{x^2}{a^2} - \frac{y^2}{16} = 1$ intersect iff

1) $|a| \le \frac{1}{\sqrt{2}}$ 2) $a > \frac{1}{\sqrt{2}}$ 3) $a > -\frac{1}{\sqrt{2}}$ 4) $a > \sqrt{2}$

Key. 1

Sol. $y = 4x^2$ & $\frac{1}{4}y = x^2$

Using $\frac{1}{4a^2}y - \frac{y^2}{16} = 1$

$\Rightarrow 4y - a^2y^2 = 16a^2$

$\Rightarrow a^2y^2 - 4y + 16a^2 = 0$

$\Rightarrow D \ge 0$ for intersection of two curves

$\Rightarrow 16 - 4a^2\left(16a^2\right) \ge 0$

$\Rightarrow 1 - 4a^4 \ge 0$

$\Rightarrow \left(2a^2\right) \le 1$

$\Rightarrow \left|\sqrt{2}a\right| \le 1 \Rightarrow -\frac{1}{\sqrt{2}} \le a \le \frac{1}{\sqrt{2}}$

125. If angle between the asymptotes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is 45°, then value of eccentricity e is

1) $\sqrt{4 \pm 2\sqrt{2}}$ 2) $\sqrt{4 + 2\sqrt{2}}$ 3) $\sqrt{4 - 2\sqrt{2}}$ 4) $\sqrt{4 - 3\sqrt{2}}$

Key. 3

Sol. $2\tan^{-1}\frac{b}{a} = 45° \Rightarrow \frac{b}{a} = \tan 22^o = \frac{a^2\left(e^2 - 1\right)}{a^2} = \left(\sqrt{2} - 1\right)^2$

$\Rightarrow e^2 - 1 = 3 - 2\sqrt{2} \Rightarrow e = \sqrt{4 - 2\sqrt{2}}$.

126. A hyperbola, having the transverse axis of length $2\sin\theta$, is confocal with the ellipse $3x^2 + 4y^2 = 12$. Then its equation is

1) $x^2 \operatorname{cosec}^2\theta - y^2 \sec^2\theta = 1$ 2) $x^2 \sec^2\theta - y^2 \operatorname{cosec}^2\theta = 1$

3) $x^2 \sin^2\theta - y^2 \cos^2\theta = 1$ 4) $x^2 \cos^2\theta - y^2 \sin^2\theta = 1$

Key. 1

Sol. Equation of the ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$. Its eccentricity is $e = \sqrt{1 - \frac{3}{4}} = \frac{1}{2}$

Coordinates of foci are $(\pm 1, 0)$.

Let the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $a = \sin\theta$

Also, $ae_1 = 1 \Rightarrow$ $e_1 = \operatorname{cosec}\theta$

$\therefore b^2 = a^2\left(e_1^2 - 1\right) = 1 - \sin^2\theta = \cos^2\theta$

Equation of the hyperbola is thus $\frac{x^2}{\sin^2\theta} - \frac{y^2}{\cos^2\theta} = 1$

127. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinates axes, then

1) Equation of ellipse is $x^2 + 2y^2 = 1$
2) the foci of ellipse are $(\pm 1, 0)$
3) equation of ellipse are $x^2 + 2y^2 = 4$
4) the foci of ellipse are $(\pm\sqrt{2}, 0)$

Key. 2

Sol. If two concentric conics intersect orthogonally then they must be confocal, so ellipse and hyperbola will be confocal

$\Rightarrow (\pm ae, 0) \equiv (\pm 1, 0)$

[foci of hyperbola are $(\pm 1, 0)$]

128. Let $P(6,3)$ be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x axis at $(9,0)$, then the eccentricity of the hyperbola is:

1) $\sqrt{\frac{5}{2}}$
2) $\sqrt{\frac{3}{2}}$
3) $\sqrt{2}$
4) $\sqrt{3}$

Key. 2

Sol. Normal at (6,3) is

$$\frac{a^2x}{6} + \frac{b^2y}{3} = a^2 + b^2,$$

$$\Rightarrow \frac{9a^2}{6} = a^2 + b^2 \Rightarrow \frac{3}{2} = 1 + \frac{b^2}{a^2}$$

$$\therefore \quad \frac{b^2}{a^2} = \frac{1}{2} \Rightarrow e^2 - 1 = \frac{1}{2} \Rightarrow e = \sqrt{\frac{3}{2}}$$

129. For hyperbola $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$, which of the following remains constant with change in 'α'

1) abscissae of vertices
2) abscissae of foci
3) Eccentricity
4) directrix

Key. 2

Sol. Hyperbola is $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$

Coordinates of vertices are $(\pm\cos\alpha, 0)$, eccentricity of the hyperbola is $e = \sqrt{1 + \frac{\sin^2\alpha}{\cos^2\alpha}} = |\sec\alpha|$

∴ Coordinates of foci are thus $(\pm 1, 0)$, which are independent of α.

Directrix is $x = \pm\cos^2\alpha$

130. A ray emanating from the point (5, 0) is incident on the hyperbola $9x^2 - 16y^2 = 144$ at the point P with abscissa 8, then the equation of the reflected ray after first reflection is (P lies in first quadrant)

A) $\sqrt{3}x - y + 7 = 0$ B) $3\sqrt{3}x - 13y + 15\sqrt{3} = 0$

C) $\sqrt{3}x + y - 14 = 0$ D) $3\sqrt{3}x + 13y - 15\sqrt{3} = 0$

Key. B

Sol. foci = $(\pm 5, 0)$

$\therefore$ Equation of reflected ray after first reflection passes through $P, S^1; P = (8, 3\sqrt{3}), S^1 = (-5, 0)$

131. If PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that OPQ is an equilateral triangle, O being the centre of the hyperbola then eccentricity e of the hyperbola satisfies

A) $1 < e < \frac{2}{\sqrt{3}}$ B) $e = \frac{2}{\sqrt{3}}$ C) $e = \frac{\sqrt{3}}{2}$ D) $e > \frac{2}{\sqrt{3}}$

Key. D

Sol. Let $PQ = 2\ell$

$\ell^2 = \frac{a^2b^2}{3b^2 - a^2} > 0 \Rightarrow e > \frac{2}{\sqrt{3}}$

132. $P(a\sec\theta, b\tan\theta)$ and $Q(a\sec\phi, b\tan\phi)$ are the ends of a focal chord of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $\tan\frac{\theta}{2}\tan\frac{\phi}{2}$ is

A) $\frac{e-1}{e+1}$ B) $\frac{1-e}{1+e}$ C) $\frac{1+e}{1-e}$ D) $\frac{e+1}{e-1}$

Key. B or C

Sol. Conceptual Question

133. If a variable straight line $x\cos\alpha + y\sin\alpha = P$, which is a chord of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 (b > a)$, subtend a right angle at the centre of the hyperbola, then it always touches a fixed circle whose radius is

A) $\frac{ab}{\sqrt{b-2a}}$ B) $\frac{a}{\sqrt{a-b}}$ C) $\frac{ab}{\sqrt{b^2 - a^2}}$ D) $\frac{ab}{b\sqrt{b+a}}$

Key. C

Sol. Making homogeneous equation of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with the help of $x\cos\alpha + y\sin\alpha = P$

$(\ )x^2 + (\ )y^2 = 0$

$\frac{1}{a^2} - \frac{1}{b^2} = \frac{1}{P^2} \Rightarrow P = \frac{ab}{\sqrt{b^2 - a^2}}$

P is the length of perpendicular drawn from (0, 0) to $x\cos\alpha + y\sin\alpha = P$

Radius $= P = \dfrac{ab}{\sqrt{b^2-a^2}}$

134. The normal at P to a hyperbola of eccentricity e, intersects its transverse and conjugate axes at L and M respectively. If locus of the mid point of LM is a hyperbola, then eccentricity of the hyperbola is

A) $\dfrac{e+1}{e-1}$ B) $\dfrac{e}{\sqrt{e^2-1}}$ C) e D) $\dfrac{2e}{\sqrt{e^2-1}}$

Key. B

Sol. $N_P : ax\cos\theta + by\cot\theta = a^2+b^2$

$L\left(\dfrac{a^2+b^2}{a}\sec\theta, 0\right)$

$M\left(0, \dfrac{a^2+b^2}{b}\tan\theta\right)$

Locus is $\dfrac{x^2}{\left(\dfrac{a^2+b^2}{2a}\right)^2} - \dfrac{y^2}{\left(\dfrac{a^2+b^2}{2b}\right)^2} = 1 \Rightarrow e_1 = \dfrac{e}{\sqrt{e^2-1}}$

135. If e is the eccentricity of the hyperbola $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ and θ is the angle between the asymptotes, then $\cos\dfrac{\theta}{2}$ is equal to

A) $\dfrac{1-e}{e}$ B) $\dfrac{2}{e}-e$ C) $\dfrac{1}{e}$ D) $\dfrac{2}{e}$

Key. C

Sol. $\theta = 2\tan^{-1}\dfrac{b}{a} \Rightarrow \tan\dfrac{\theta}{2} = \dfrac{b}{a}$

$\cos\dfrac{\theta}{2} = \dfrac{a}{\sqrt{a^2+b^2}} = \dfrac{1}{\sqrt{1+\dfrac{b^2}{a^2}}} = \dfrac{1}{e}$

136. Area of triangle formed by the lines $x-y=0, x+y=0$ and any tangent to the hyperbola $x^2-y^2=a^2$ is

A) $|a|$ B) $\dfrac{1}{2}|a|$ C) a^2 D) $\dfrac{1}{2}a^2$

Key. C

Sol. Any tangent to $x^2-y^2=a^2$ is $x\sec\phi - y\tan\phi = a$

Area $=|a|$

137. The locus of the point of intersection of the line $\sqrt{3}x - y - 4\sqrt{3}K = 0$ and $\sqrt{3}Kx + Ky - 4\sqrt{3} = 0$ is a hyperbola of eccentricity is

A) 1 B) 2 C) 2.5 D) $\sqrt{3}$

Key. B

Sol. $K = \dfrac{\sqrt{3}x-y}{4\sqrt{3}} = \dfrac{4\sqrt{3}}{\sqrt{3}x-y}$

$\Rightarrow 3x^2 - y^2 = 48 \Rightarrow \dfrac{x^2}{16} - \dfrac{y^2}{48} = 1$

$48 = 16(e^2 - 1) \Rightarrow e = 2$

138. The locus of the middle points of chords of hyperbola $3x^2 - 2y^2 + 4x - 6y = 0$ parallel to $y = 2x$ is

A) $3x - 4y = 4$ B) $3y - 4x + 4 = 0$ C) $4x - 4y = 3$ D) $3x - 4y = 2$

Key. A

Sol. Let locus be $P(h,k)$, $T = S_1$

$3hx - 2ky + 2(x+h) - 3(k+y) = 3h^2 - 2k^2 + 4h - 6k$

Slope $= \dfrac{3h+2}{2k+3} = 2 \Rightarrow 3x - 4y = 4$

139. From a point P(1, 2) pair of tangent's are drawn to a hyperbola 'H' in which one tangent to each arm of hyperbola. Equation of asymptotes of hyperbola H are $\sqrt{3}x - y + 5 = 0$ & $\sqrt{3}x + y - 1 = 0$ then eccentricity of 'H' is

A) 2 B) $\dfrac{2}{\sqrt{3}}$ C) $\sqrt{2}$ D) $\sqrt{3}$

Key. B

Sol. Since $c_1c_2\left(a_1a_2 + b_1b_2\right) < 0$

$\therefore$ origin lies in acute angle

P(1, 2) lies in obtuse angle

Acute angle between the asymptotes is $\dfrac{\pi}{3}$

$\therefore$ $e = \sec\dfrac{\theta}{2} = \sec\dfrac{\pi}{6} = \dfrac{2}{\sqrt{3}}$

140. If a variable line has its intercepts on the co-ordinates axes e, e', where $\dfrac{e}{2}, \dfrac{e'}{2}$ are the eccentricities of a hyperbola and its conjugate hyperbola, then the line always touches the circle $x^2 + y^2 = r^2$, where r =

A) 1 B) 2 C) 3 D) can not be decided

Key. B

Sol. Since $\dfrac{e}{2}$ and $\dfrac{e'}{2}$ are eccentricities of a hyperbola and its conjugate

$\therefore$ $\dfrac{4}{e^2} + \dfrac{4}{e'^2} = 1$

i.e. $4 = \dfrac{e^2 e'^2}{e'^2 + e'^2}$

line passing through the points (e, 0) and $\left(0, e'\right)$ $e'x + ey - ee' = 0$

it is tangent to the circle $x^2 + y^2 = r^2$

$\therefore$ $\dfrac{ee'}{\sqrt{e^2 + e'^2}} = r$

$\therefore$ $r^2 = \dfrac{e^2 e'^2}{e^2 + e'^2} = 4$

$\therefore$ r = 2

141. If angle between asymptote's of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is 120° and product of perpendiculars drawn from foci upon its any tangent is 9, then locus of point of intersection of perpendicular tangents of the hyperbola can be –

A) $x^2+y^2=6$ B) $x^2+y^2=9$ C) $x^2+y^2=3$ D) $x^2+y^2=18$

Key. D

Sol. $b^2=9$

$\frac{b}{a}=\tan 30°=\frac{1}{\sqrt{3}}$

$\therefore$ $a^2=3b^2=27$

$\therefore$ Required locus is director circle of the hyperbola & which is $x^2+y^2=27-9,\ x^2+y^2=18$

If $\frac{b}{a}=\tan 60°$ is taken then

$$a^2=\frac{b^2}{3}=\frac{9}{3}=3.$$

$\therefore$ Required locus is $x^2+y^2=3-9=-6$ which is not possible.

142. 'C' be a curve which is locus of point of intersection of lines x = 2 + m and my = 4 – m. A circle $s\equiv(x-2)^2+(y+1)^2=25$ intersects the curve C at four points P,Q,R and S. If O is centre of the curve 'C' then $OP^2+OQ^2+OR^2+OS^2$ is

A) 50 B) 100 C) 25 D) 25/2

Key. B

Sol. x – 2 = m

$y+1=\frac{4}{m}$

$\therefore$ (x – 2) (y + 1) = 4

$\Rightarrow$ XY = 4, where X = x – 2, Y = y + 1

$S\equiv(x-2)^2+(y+1)^2=25$

$\Rightarrow$ $X^2+Y^2=25$

Curve 'C' & circle S both are concentric

$\therefore$ $OP^2+OQ^2+OR^2+OS^2=4r^2=4.25=100$

143. The combined equation of the asymptotes of the hyperbola $2x^2+5xy+2y^2+4x+5y=0$ is

A) $2x^2+5xy+2y^2+4x+5y+2=0$ B) $2x^2+5xy+2y^2+4x+5y-2=0$

C) $2x^2+5xy+2y^2=0$ D) none of these

Key. A

Sol. Let the equation of asymptotes be

$2x^2+5xy+2y^2+4x+5y+\lambda=0$ …(1)

This equation represents a pair of straight lines therefore

$abc+2fgh-af^2-bg^2-ch^2=0$

$\therefore$ $4\lambda+25-\frac{25}{2}-8-\lambda\frac{25}{4}=0$ $\Rightarrow$ $-\frac{9\lambda}{4}+\frac{9}{2}=0$

$\Rightarrow$ $\lambda=2$

Putting the value of λ in (i), we get $2x^2+5xy+2y^2+4x+5y+2=0$ this is the equation of the asymptotes.

144. If $\alpha+\beta=3\pi$ then the chord joining the points α and β for the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ passes through

A) focus
B) centre
C) one of the end points of the transverse axis
D) one of the end points of the conjugates axis

Key. B

Sol. (i) Equation of chord joining α and β is

$$\frac{x}{a}\cos\left(\frac{\alpha-\beta}{2}\right)-\frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right)=\cos\left(\frac{\alpha+\beta}{2}\right)$$

$\therefore$ $\alpha+\beta=3\pi$

$$\frac{x}{a}\cos\left(\frac{\alpha-\beta}{2}\right)=\frac{y}{b}=0$$

If passes through the centre (0, 0)

145. For a given non-zero value of m each of the lines $\frac{x}{a}-\frac{y}{b}=m$ and $\frac{x}{a}+\frac{y}{b}=m$ meets the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at a point. Sum of the ordinates of these points, is

A) $\frac{a\left(1+m^2\right)}{m}$
B) $\frac{b\left(1-m^2\right)}{m}$
C) 0
D) $\frac{a+b}{2m}$

Key. C

Sol. Ordinate of the point of intersection of the line $\frac{x}{a}-\frac{y}{b}=m$ and the hyperbola is given by

$\left(\frac{x}{a}-\frac{y}{b}\right)\left(\frac{x}{a}-\frac{y}{b}+\frac{2y}{b}\right)=1$ i.e. $m\left(m+\frac{2y}{b}\right)=1$ i.e. $y=\frac{b\left(1-m^2\right)}{2m}$

Similarly ordinate of the point of intersection of the line $\frac{x}{a}+\frac{y}{b}=m$ and the hyperbola is given by

$y=\frac{b\left(m^2-1\right)}{2m}$ $\therefore$ Sum of the ordinates is 0.

146. The equation of the transverse axis of the hyperbola $(x-3)^2+(y+1)^2=(4x+3y)^2$ is

A) x + 3y = 0
B) 4x + 3y = 9
C) 3x – 4y = 13
D) 4x + 3y = 0

Key. C

Sol. $(x-3)^2+(y+1)^2=(4x+3y)^2$

$$(x-3)^2+(y+1)^2=25\left(\frac{4x+3y}{5}\right)^2$$

PS = 5PM

$\therefore$ directrix is 4x + 3y = 0 and focus (3, - 1)

So transverse axis has slope $=\frac{3}{4}$ and equation of transverse axis $y+1=\frac{3}{4}(x-3)$

$\Rightarrow$ 3x – 4y = 13

147. For which of the hyperbola we can have more than one pair of perpendicular tangents?

A) $\frac{x^2}{4}-\frac{y^2}{9}=1$
B) $\frac{x^2}{4}-\frac{y^2}{9}=-1$
C) $x^2-y^2=4$
D) xy = 4

Key. B

Sol. Locus of point of intersection of perpendicular tangents is director circle for $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ equation of director circle is $x^2+y^2=a^2-b^2$ which is real if a > b

$\Rightarrow$ B is correct answer.

148. From point (2, 2) tangents are drawn to the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ then point of contact lie in

A) I & II quadrants B) I & IV quadrants C) I & III quadrants D) III & IV quadrants

Key. D

Sol. Equation of Asymplote are 4y – 3x = 0 and 4y + 3x = 0
Since point (2, 2) lies above the asymptotes 4y – 3x = 0,
Hence point of constant of pair of tangent are in III & IV quadrant

149. The equation to the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola $xy=c^2$ is

A) $\frac{x}{x_1+x_2}+\frac{y}{y_1+y_2}=1$ B) $\frac{x}{x_1-x_2}+\frac{y}{y_1-y_2}=1$

C) $\frac{x}{y_1+y_2}+\frac{y}{x_1+x_2}=1$ D) $\frac{x}{y_1-y_2}+\frac{y}{x_1-x_2}=1$

Key. A

Sol. Mid point is $M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$

$\therefore$ equation of the chord to the hyperbola $xy=c^2$ whose midpoint is M, is $\frac{x}{\frac{x_1+x_2}{2}}=\frac{y}{\frac{y_1+y_2}{2}}=2$

$\Rightarrow \quad \frac{x}{x_1+x_2}+\frac{y}{y_1+y_2}=1$

150. The locus of the foot of the perpendicular from the centre of the hyperbola $xy=c^2$ on a variable tangent is

A) $(x^2-y^2)^2=4c^2xy$ B) $(x^2+y^2)^2=2c^2xy$ C) $(x^2+y^2)=4x^2xy$ D) $(x^2+y^2)^2=4c^2xy$

Key. D

Sol. Equation of tangent at $P, \frac{x}{t}+ty=2c$.

or $x+t^2y=2ct$ …(i)

slope of tangent $=-\frac{1}{t^2}$

$\therefore$ equation of CM is $y=t^2\ x$ …(ii)

Squaring (i), $(x+t^2y)^2=4c^2t^2$

Using (ii), we get $\left(x+\frac{y^2}{x}\right)^2=4c^2+\frac{y}{x}\Rightarrow(x^2+y^2)=4c^2xy$

151. If $P(x_1,y_1), Q(x_2, y_2), R(x_3, y_3)$ & $S(x_4, y_4)$ are 4 concyclic points on the rectangular hyperbola $xy=c^2$, the co-ordinates of the orthocenter of the triangle PQR are

A) $(x_4,-y_4)$ B) (x_4,y_4) C) $(-x_4,-y_4)$ D) $(-x_4,y_4)$

Key. C

Sol. Let P,Q,R,S are $\left(ct, \frac{c}{t}\right)$

Where t is t_1, t_2, t_3, t_4 respectively let equation of circle is $x^2 + y^2 = r^2$

$\left(ct \frac{c}{t}\right)$ satisfy this equation

$\therefore \quad c^2t^2 + \frac{c^2}{t^2} - r^2 = 0$

$c^2t^4 - r^2t^2 + c^2 = 0$

Its roots are t_1, t_2, t_3, t_4

$t_1, t_2, t_3, t_4 = 1$ …(i)

Coordinates of orthocenter of ΔPQR are $\left(\frac{-c}{t_1t_2t_3}, -ct_1t_2t_3\right)$

$\Rightarrow \quad \left(-ct_4, -\frac{c}{t_4}\right)$ (using (i))

$\Rightarrow \quad (-x_4, -y_4)$

152. If the curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, (a > b) and $x^2 - y^2 = c^2$ cut at right angles then

A) $(x_4, -y_4)$ B) (x_4, y_4) C) $(-x_4, -y_4)$ D) $(-x_4, y_4)$

Key. C

Sol. Let P on the ellipse is $(a\cos\theta, b\sin\theta)$

Slope of tangent at P on the ellipse $m_1 = -\frac{b\cos\theta}{a\sin\theta}$

Slope of tangent at P on the hyperbola $x^2 - y^2 = c^2$, is

$m_2 = \frac{a\cos\theta}{b\sin\theta}$

Since these curves are intersecting at right angle

$\therefore \quad m_1m_2 = -1$

$-\frac{b}{a} \times \frac{\cos\theta}{\sin\theta} \times \frac{a}{b}\frac{\cos\theta}{\sin\theta} = -1 \Rightarrow \quad \tan^2\theta = 1$

$P(a\cos\theta, b\sin\theta)$ also lies on hyperbola

$\therefore \quad a^2\cos^2\theta - b^2\sin^2\theta = c^2$

$a^2 - b^2\tan^2\theta = c^2 + c^2\tan^2\theta$

$\Rightarrow \quad a^2 - b^2 = c^2 + c^2 \quad [\because \tan^2\theta = 1]$

$a^2 - b^2 = 2c^2$

153. If radii of director circles of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{(b')^2} = 1$ are 2r and r respectively and e_e and e_h be the eccentricities of the ellipse and the hyperbola respectively then

A) $2e_n^2 - e_e^2 = 6$ B) $e_e^2 - 4e_n^2 = 6$ C) $4e_n^2 - e_e^2 = 6$ D) none of these

Key. C

Sol. Equation of director circles of ellipse and hyperbola are respectively.

$x^2 + y^2 = a^2 + b^2$

and $x^2+y^2=a^2-b^2$

$a^2+b^2=4r^2$ …(1)

$a^2-b^2=r^2$ …(2)

So $2a^2=5r^2$

$a^2=\frac{5r^2}{2}$

$b^2=4r^2-\frac{5r^2}{2}$

$b^2=\frac{3r^2}{2}$

$e_n^2=1-\frac{b^2}{a^2}$

$\Rightarrow \quad e_n^2=1-\frac{3r^2}{2}\times\frac{2}{5r^2}=1-\frac{3}{5}=\frac{2}{5}$

$e_n^2=1+\frac{b^2}{a^2}$

$\Rightarrow \quad e_n^2=1+\frac{3}{5}=\frac{8}{5}$

So $\quad 4e_n^2-e_n^2=4\times\frac{8}{5}-\frac{2}{5}=\frac{30}{5}=6$

154. If the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{b^2}=1$ & the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide then the value of b^2 is

A) 4 B) 9 C) 16 D) none

Key. C

Sol. For ellipse $a^2=16$ $\Rightarrow$ $e=\sqrt{1-\frac{b^2}{a^2}}=\frac{\sqrt{25-b^2}}{5}$

$\Rightarrow \quad \text{focii}-(\pm ae,\ 0)=\left(\pm\sqrt{25-b^2},0\right)$

For hyperbola, $e=\sqrt{1+\frac{b^2}{a^2}}=\frac{5}{4}$

$\therefore \quad \text{focii}=(\pm ae,0)=(\pm 3,0)$

$\therefore \quad \sqrt{25-b^2}=3\Rightarrow b^2=16$

155. The tangent at any point $P(x_1,y_1)$ on the hyperbola $xy=c^2$ meets the co-ordinate axes at points Q & R. The circumcentre of ΔOQR has co-ordinates.

A) (0, 0) B) (x_1,y_1) C) $\left(\frac{x_1}{2},\frac{y_1}{2}\right)$ D) $\left(\frac{2x_1}{3},\frac{2y_1}{3}\right)$

Key. B

Sol. Tangent at $P(x_1,y_1)$ on $xy=c^2$ is

$\frac{x}{x_1}+\frac{y}{y_1}=2$

$\therefore \quad Q=(2x_1,0),\ R=(0,2y_1)$

Now OQR is a right Δ and QR is the hypotenuse.

$\therefore$ circumcentre = mid pt, of $QR = (x_1, y_1)$

156. The locus of the mid points of the chords passing through a fixed point (α, β) of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

A) a circle with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ B) an ellipse with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

C) a hyperbola with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ D) straight line passing through $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

Key. C

Sol. Let (h, k) be the mid point

$\therefore \quad T = S_1 \quad \Rightarrow \quad \frac{xh}{a^2} - \frac{yk}{b^2} = \frac{h^2}{a^2} - \frac{k^2}{b^2}$ …(i)

(1) passes through (α, β) so putting (α, β) in it

$$\Rightarrow \quad \frac{\alpha x}{a^2} - \frac{\beta y}{b^2} = \frac{x^2}{a^2} - \frac{y^2}{b^2} \Rightarrow \left(\frac{x^2}{a^2} - \frac{\alpha x}{a^2}\right) - \left(\frac{y^2}{b^2} - \frac{\beta y}{b^2}\right) = 0$$

$$\Rightarrow \quad \frac{\left(x - \frac{\alpha}{2}\right)^2}{a^2} - \frac{\left(y - \frac{\beta}{2}\right)^2}{b^2} + \frac{\alpha^2}{4a^2} - \frac{\beta^2}{4b^2} = 0$$

Which is a hyperbola with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

157. If two conics $a_1x^2 + 2h_1xy + b_1y^2 = c_1$ and $a_2x^2 + 2h_2xy + b_2y^2 = c_2$ intersect in four concyclic points, then

A) $(a_1 - b_1)h_2 = (a_2 - b_2)h_1$ B) $(a_1 - b_1)h_1 = (a_2 - b_2)h_2$

C) $(a_1 + b_1)h_2 = (a_2 + b_2)h_1$ D) $(a_1 + b_1)h_1 = (a_2 + b_2)h_2$

Key. A

Sol. On removing xy terms by multiplying $a_1x^2 + 2h_1xy + b_1^2 = C_1$ by h_2 and $a_2x^2 + 2h_2xy + b_2y^2 = C_2$ by h_1 and subtracting we have

$(a_1h_2 - a_2h_1)x^2 + (b_1h_2 - b_2h_1)y^2 = C_1h_2 - C_2h_1$

Now this will represent a circle if coefficient of x^2 = coefficient of y^2

i.e. $a_1h_2 - a_2h_1 = b_1h_2 - b_2h_1$

i.e. $(a_1 - b_1)h_2 = (a_2 - b_2)h_2$

158. The transverse axis of a hyperbola is of length 2a and a vertex divides the segment of the axis between the centre and the corresponding focus in the ratio 2: 1, the equation of the hyperbola is

A) $4x^2 - 5y^2 = 4a^2$ B) $4x^2 - 5y^2 = 5a^2$ C) $5x^2 - 4y^2 = 4a^2$ D) $5x^2 - 4y^2 = 5a^2$

Key. D

Sol.

Clearly $\frac{2ae}{3} = a \quad \Rightarrow \quad e = \frac{3}{2}$

$\therefore \quad S = \left(\frac{3a}{2}, 0\right)$

Directrix is $x = \frac{2a}{3}$

$\therefore$ equation of hyperbola will be $\left(x-\frac{3a}{2}\right)^2+y^2=\frac{9}{4}\left(x-\frac{2a}{3}\right)^2$

Which reduces to $5x^2-4y^2=5a^2$

Hyperbola

Multiple Correct Answer Type

1. If the normal at P to the rectangular hyperbola meets the axes in G and g and C is centre of the hyperbola, then

A. PG = PC B. Pg = PC C. PG = Pg D. Gg = 2PC

Key. A,B,C,D

Sol. Let $P(x_1, y_1)$ pt on $x^2 - y^2 = 4$

Normal is $x_1 y + x y_1 = 2x_1 y_1$

$\Rightarrow G = (2x_1, 0) g(0, 2y_1)$

$PG = \sqrt{(2x_1 - x_1)^2 + y_1^2} = PC$

$Pg = \sqrt{x_1^2 + (2y_1 - y_1)^2} = PC$

$Gg = \sqrt{(2x_1)^2 + (2y_1^2)} = 2\sqrt{x_1^2 + y_1^2} = 2PC$

2. For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, let n be the number of points on the plane through which perpendicular tangents are drawn

A. If n = 1 then $e = \sqrt{2}$ B. If n = 0 then $e < \sqrt{2}$
C. If n > 1 then $0 < e < \sqrt{2}$ D. None of these

Key. A,C

Sol. Lous of point of intersection of tangents is bisector circle $x^2 + y^2 = a^2 - b^2$

If $a^2 > b^2$ there are infinite points an circle $\Rightarrow e^2 < 2$

n > 1 $\Rightarrow e^2 < \sqrt{2}$

If $a^2 < b^2$ there doe not exist any point on the plane $e^2 > 2 \Rightarrow e > \sqrt{2}$

If $a^2 = b^2$ there exists exactly one point (centre of hyperbola) $\Rightarrow e = \sqrt{2}$

3. A rectangular hyperbola of latus rectum 4 units passes through (0, 0) and has (2, 0) as its one focus. The equation of locus of the other focus is

A) $x^2 + y^2 = 36$ B) $x^2 + y^2 = 4$ C) $x^2 - y^2 = 4$ D) $x^2 + y^2 = 9$

Key. A

Sol. The difference between the focal distances is a constant for a hyperbola. For a rectangular hyperbola latusrectum = transverse axis.

S(2, 0) S$'$(h, k) P(0, 0) $|S'p - Sp| = 4$ $\left|\sqrt{h^2 + k^2} - 2\right| = 4 \Rightarrow \sqrt{h^2 + x^2} = 6 \Rightarrow h^2 + k^2 = 36$

Locus of (h, k) is $x^2 + y^2 = 36$

4. A rectangular hyperbola of latus rectum 4 units passes through (0, 0) and has (2, 0) as its one focus. The equation of locus of the other focus is

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Key. A

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S(2, 0) S$'$(h, k) P(0, 0) $|S'p - Sp| = 4$ $\left|\sqrt{h^2+k^2} - 2\right| = 4 \Rightarrow \sqrt{h^2+x^2} = 6 \Rightarrow h^2+k^2 = 36$

Locus of (h, k) is $x^2 + y^2 = 36$

5. The equation of tangent to the hyperbola $5x^2 - y^2 = 5$ passing through the point (2, 8) is(are)

a) $3x - y + 2 = 0$ b) $3x + y - 14 = 0$

c) $23x - 3y - 22 = 0$ d) $3x - 23y + 178 = 0$

Key. A,C

Sol. Let m_1, m_2 be the slopes of tangets

$$\therefore m_1 + m_2 = \frac{2x_1y_1}{x_1^2 - a^2} = \frac{32}{3}, m_1m_2 = \frac{y_1^2 + b^2}{x_1^2 - a^2} = 23$$

$$\therefore m_1 = 3, m_2 = \frac{23}{3}$$

Tangents are 3x-y+2=0 ; 23x-3y-22 = 0

6. If the circle $x^2 + y^2 = a^2$ intersect the hyperbola $xy = c^2$ in four points $P(x_1, y_1), Q(x_2, y_2), R(x_3, y_3)$ and $S(x_4, y_4)$ then

a) $x_1 + x_2 + x_3 + x_4 = 0$ b) $y_1 + y_2 + y_3 + y_4 = 0$

c) $x_1x_2x_3x_4 = c^4$ d) $y_1y_2y_3y_4 = c^4$

Key. A,B,C,D

Sol. Take point on $xy = c^2$ as $\left(t, \frac{c}{t}\right)$

7. If two tangents can be drawn the different branches of hyperbola $\frac{x^2}{1} - \frac{y^2}{4} = 1$ from (α, α^2) then

A) $\alpha \in (-2, 0)$ B) $\alpha \in (0, 2)$ C) $\alpha \in (-\infty, -2)$ D) $\alpha \in (2, \infty)$

Key. C,D

Sol. (cd) (α, α^2) lies on $y = x^2$ (α, α^2) must lie between the asymptotes of hyperbola $\frac{x^2}{1} - \frac{y^2}{4} = 1$ in I and II quadrant. Asymptotes are $y = \pm 2x; \Rightarrow 2\alpha < \alpha^2 \Rightarrow \alpha < 0$ or $\alpha > 2$. And $-2\alpha < \alpha^2 \Rightarrow \alpha < -2$ or $\alpha > 0 \Rightarrow \alpha \in (-\infty, -2) \cup (2, \infty)$

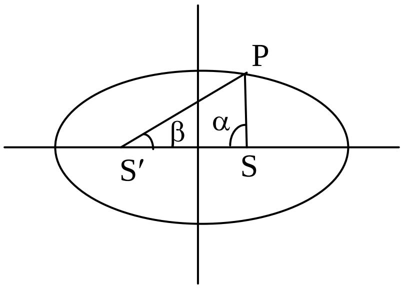
8. The coordinates of the foci of the hyperbola $9x^2 - 16y^2 + 18x + 32y - 151 = 0$ are

(A) (4, 1) (B) (–4, 1)

(C) (6, -1) (D) (–6, 1)

Key. A,D

Sol. $\dfrac{PS}{\sin\beta}=\dfrac{PS'}{\sin\alpha}=\dfrac{2ae}{\sin(\pi-(\alpha+\beta)}$



or, $\dfrac{2a}{\sin\alpha+\sin\beta}=\dfrac{2ae}{\sin(\alpha+\beta)}$

or, $\dfrac{1}{e}=\dfrac{2\sin\dfrac{\alpha+\beta}{2}.\cos\dfrac{\alpha-\beta}{2}}{2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha+\beta}{2}}$

$\therefore\ \dfrac{1-e}{1+e}=\tan\dfrac{\alpha}{2}\tan\dfrac{\beta}{2}$

9. A straight line touches the rectangular hyperbola $9x^2 - 9y^2 = 8$ and the parabola $y^2 = 32x$. An equation of the line is

(A) $9x + 3y - 8 = 0$ (B) $9x - 3y + 8 = 0$

(C) $9x + 3y + 8 = 0$ (D) $9x - 3y - 8 = 0$

Key. B,C

Sol. $y^2 = 32x$

Let equation of tangent $y = mx + \dfrac{8}{m}$

$\dfrac{64}{m^2}=\dfrac{8}{9}m^2-\dfrac{8}{9}$

$m = \pm 3$, $y = \pm 3x \pm 8/3$.

10. If the circle $x^2 + y^2 = a^2$ cuts a rectangular hyperbola $xy = c^2$ in

A $\left(ct_1,\dfrac{c}{t_1}\right)$, B $\left(ct_2,\dfrac{c}{t_2}\right)$, C $\left(ct_3,\dfrac{c}{t_3}\right)$ and D $\left(ct_4,\dfrac{c}{t_4}\right)$, then

(A) $t_1t_2t_3t_4 = 1$ (B) $t_1 + t_2 + t_3 + t_4 = 0$

(C) $\dfrac{1}{t_1}+\dfrac{1}{t_2}+\dfrac{1}{t_3}+\dfrac{1}{t_4} = 0$ (D) $\sum\dfrac{1}{t_1t_2}=0$

Key. A,B,C

Sol. Any point on $xy = c^2$ is $\left(ct,\dfrac{c}{t}\right)$. As it lies on the given circle, we get

$c^2t^2 + \dfrac{c^2}{t^2}=a^2 \Rightarrow c^2t^4 - a^2t^2 + c^2 = 0$

Thus $t_1t_2t_3t_4 = 1$, $t_1 + t_2 + t_3 + t_4 = 0$, $\Sigma t_1t_2 = - -\dfrac{a^2}{c^2}$, $\Sigma t_1t_2t_3 = 0$

Thus, (a), (b), (c) are true.

11. The equation $16x^2 - 3y^2 - 32x - 12y - 44 = 0$ represents a hyperbola with:

(A) length of the transverse axis = $2\sqrt{3}$ (B) length of the conjugate axis = 8

(C) centre at (1, -2) (D) eccentricity = $\sqrt{19}$

Key. A,B,C

Sol. Conceptual

12. If two tangents can be drawn to the different branches of hyperbola $\frac{x^2}{1}-\frac{y^2}{4}=1$ from the point (α, α^2), then

(A) $\alpha \in (-2, 0)$ (B) $\alpha \in (0, 2)$

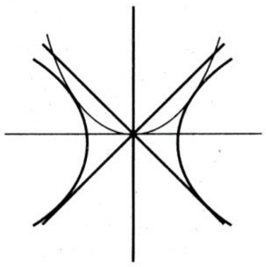
(C) $\alpha \in (-\infty, -2)$ (D) $\alpha \in (2, \infty)$

Key. C,D

Sol. (α, α^2) lie on the parabola $y = x^2$

(α,α^2) must lie between the asymptotes of hyperbola $\frac{x^2}{1}-\frac{y^2}{4}=1$ in 1st and 2nd quadrant

$\therefore$ Asymptotes are $y = \pm 2x$



$\therefore$ $2\alpha < \alpha^2$

$\Rightarrow$ $\alpha < 0$ or $\alpha > 2$

and $-2\alpha < \alpha^2$

$\alpha < -2$ or $\alpha > 0$

$\therefore$ $\alpha \in (-\infty, -2)$ or $\alpha \in (2, \infty)$

13. If two tangents can be drawn the different branches of hyperbola $x^2/1 - y^2/4 = 1$ from (α,α^2) then

A) $\alpha \in (-2,0)$ B) $\alpha \in (0,2)$ C) $\alpha \in (-\infty,-2)$ D) $\alpha \in (2,\infty)$

Key. C,D

Sol. (α,α^2) lies on $y=x^2$ (α,α^2) must lie between the asymptotes of hyperbola $x^2/1 - y^2/4 = 1$ in I and II quadrant. Asymptotes are $y=\pm 2x ; \Rightarrow 2\alpha<\alpha^2 \Rightarrow \alpha<0$ or $\alpha>2$. And $-2\alpha<\alpha^2 \Rightarrow \alpha<-2$ or $\alpha>0 \Rightarrow \alpha \in(-\infty,-2)\cup(2,\infty)$

14. If P(α, β), the point of intersection of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{a^2(1-e^2)}=1$ and the hyperbola

$\frac{x^2}{a^2} - \frac{y^2}{a^2(E^2-1)} = \frac{1}{4}$, is equidistant from the foci of the two curves (all lying in the right of y-axis) then

(A) $2\alpha = a(2e + E)$

(B) $a - e\alpha = E\alpha - a/2$

(C) $E = \frac{\sqrt{e^2+24} - 3e}{2}$

(D) $E = \frac{\sqrt{e^2+12} - 3e}{2}$

Key: B,C

Hint: $S_1P = S_2P \Rightarrow a - e\alpha = E\alpha - \left(\frac{a}{2}\right)$. Also, $\alpha = \frac{ae + \frac{a}{2}E}{2}$

Eliminating α we get $E^2 + 3eE + (2e^2 - 6) = 0 \Rightarrow E = \frac{\sqrt{e^2+24} - 3e}{2}$.

15. A hyperbola having the transverse axis of length $\frac{1}{2}$ unit is confocal with the ellipse $3x^2 + 4y^2 = 12$, then

(A) Equation of the hyperbola is $\frac{x^2}{15} - \frac{y^2}{1} = \frac{1}{16}$

(B) Eccentricity of the hyperbola is 4

(C) Distance between the directrices of the hyperbola is $\frac{1}{8}$ units

(D) Length of latus rectum of the hyperbola is $\frac{15}{2}$ units

Key: B, C, D

Hint: Ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$

Here $\frac{3}{4} = 1 - e^2 \Rightarrow e = \frac{1}{2}$

Foci are $(\pm 1, 0)$

Now the hyperbola is having same focus i.e. $(\pm 1, 0)$. Let e_1 be the eccentricity of hyperbola

$2ae_1 = 2$

But $2a = \frac{1}{2}$ So, $e_1 = 4$

$b^2 = a^2\left(e_1^2 - 1\right) = \frac{1}{16}(16-1) = \frac{15}{16}$

So, the equation of the hyperbola is

$\frac{x^2}{\frac{1}{16}} - \frac{y^2}{\frac{15}{16}} = 1 \Rightarrow \frac{x^2}{1} - \frac{y^2}{15} = \frac{1}{16}$

Its distance between the directrices $= \frac{2a}{e_1} = \frac{1}{2 \times 4} = \frac{1}{8}$ units

Length of latus-rectum $= \frac{2b^2}{a}$

$= \frac{2\times 15\times 4}{16\times 1} = \frac{15}{2}$ units

16. If two tangents can be drawn to the different branches of hyperbola $\frac{x^2}{1} - \frac{y^2}{4} = 1$ from the point (α, α^2), then

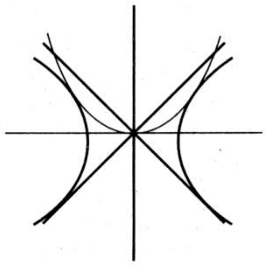
(A) $\alpha \in (-2, 0)$ (B) $\alpha \in (0, 2)$ (C) $\alpha \in (-\infty, -2)$ (D) $\alpha \in (2, \infty)$

Key: C, D

Hint: (α, α^2) lie on the parabola $y = x^2$

(α,α^2) must lie between the asymptotes of hyperbola $\frac{x^2}{1} - \frac{y^2}{4} = 1$ in 1st and 2nd quadrant

$\therefore$ Asymptotes are $y = \pm 2x$



$\therefore$ $2\alpha < \alpha^2$

$\Rightarrow$ $\alpha < 0$ or $\alpha > 2$

and $-2\alpha < \alpha^2$

$\alpha < -2$ or $\alpha > 0$

$\therefore$ $\alpha \in (-\infty, -2)$ or $\alpha \in (2, \infty)$

17. Equations of common tangents to the two hyperbolas $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ (a > b)

A) $y = x + \sqrt{a^2 - b^2}$ B) $y = x - \sqrt{a^2 - b^2}$ C) $y = -x + \sqrt{a^2 - b^2}$ D) $y = -x - \sqrt{a^2 - b^2}$

Key: A,B,C,D

18. A straight line touches the rectangular hyperbola $9x^2 - 9y^2 = 8$ and the parabola $y^2 = 32x$. the equation of the line is

(A) $9x + 3y - 8 = 0$ (B) $9x - 3y + 8 = 0$

(C) $9x + 3y + 8 = 0$ (D) $9x - 3y - 8 = 0$

Key : B, C

Sol : Equation of tangent in terms of slope of $y^2 = 32x$ is

$y = mx + \frac{8}{m}$ ....(i)

which is also tangent of $9x^2 - 9y^2 = 8$

i.e. $x^2 - y^2 = \frac{8}{9}$

then $\left(\frac{8}{m}\right)^2 = \frac{8}{9}m^2 - \frac{8}{9}$

$\Rightarrow \frac{8}{m^2} = \frac{m^2}{9} - \frac{1}{9}$

$\Rightarrow 72 = m^4 - m^2$

$\Rightarrow m^4 - m^2 - 72 = 0$

$\Rightarrow (m^2 - 9)(m^2 + 8) = 0$

$\therefore m^2 = 9, m^2 + 8 \neq 0$

$\therefore m = \pm 3$

from eq (i), $y = \pm 3x + \frac{8}{3}$

$\Rightarrow 3y = \pm 9x \pm 8$

or $\pm 9x - 3y \pm 8 = 0$

i.e. $9x - 3y + 8 = 0, 9x - 3y - 8 = 0$

$-9x - 3y + 8 = 0, -9x - 3y - 8 = 0$

or $9x - 3y + 8 = 0, 9x - 3y - 8 = 0$

$9x + 3y - 8 = 0,$

and $9x + 3y + 8 = 0$

19. If two tangents can be drawn to the different branches of hyperbola $\frac{x^2}{1} - \frac{y^2}{4} = 1$ from the point (α, α^2), then

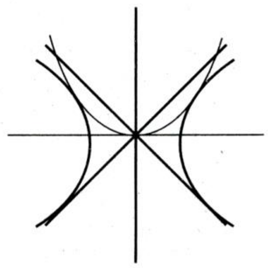
(A) $\alpha \in (-2, 0)$ (B) $\alpha \in (0, 2)$
(C) $\alpha \in (-\infty, -2)$ (D) $\alpha \in (2, \infty)$

Key. C,D

Sol. (α, α^2) lie on the parabola $y = x^2$

(α, α^2) must lie between the asymptotes of hyperbola $\frac{x^2}{1} - \frac{y^2}{4} = 1$ in 1st and 2nd quadrant

$\therefore$ Asymptotes are $y = \pm 2x$



$\therefore 2\alpha < \alpha^2$

$\Rightarrow \alpha < 0$ or $\alpha > 2$

and $-2\alpha < \alpha^2$

$\alpha < -2$ or $\alpha > 0$

$\therefore \alpha \in (-\infty, -2)$ or $\alpha \in (2, \infty)$

20. If the circle $x^2 + y^2 = a^2$ intersect the hyperbola $xy = c^2$ in four points $P(x_1, y_1), Q(x_2, y_2), R(x_3, y_3)$ and $S(x_4, y_4)$ then

a) $x_1 + x_2 + x_3 + x_4 = 0$ b) $y_1 + y_2 + y_3 + y_4 = 0$

c) $x_1 x_2 x_3 x_4 = c^4$ d) $y_1 y_2 y_3 y_4 = c^4$

Key. A,B,C,D

Sol. Take point on $xy = c^2$ as $\left(t, \frac{c}{t}\right)$

21. The coordinates of a point common to a directrix and an asymptote of the hyperbola $\frac{x^2}{25}-\frac{y^2}{16}=1$ are

A) $\left(\frac{25}{\sqrt{41}},\frac{20}{\sqrt{41}}\right)$ B) $\left(-\frac{25}{\sqrt{41}},-\frac{20}{\sqrt{41}}\right)$ C) $\left(\frac{25}{3},\frac{20}{3}\right)$ D) $\left(-\frac{25}{3},-\frac{20}{3}\right)$

Key. A,B

Sol. Equation of directrices are $x=\pm\frac{25}{\sqrt{41}}$ --- (1)

Equation of asymptotes of hyperbola are $\frac{x^2}{25}-\frac{y^2}{16}=0$ --- (2)

Solving (1) & (2) we get $\left(\pm\frac{25}{\sqrt{41}},\pm\frac{20}{\sqrt{41}}\right)$

22. If the foci of hyperbola lies on the line $y=x$, one asymptote is $y=2x$ and it is passing through the point (3, 4), then

A) Equation of hyperbola is $2x^2-xy+2y^2=38$

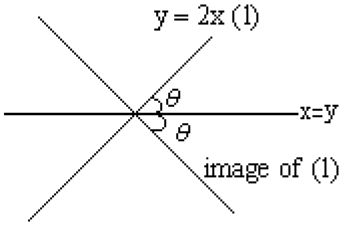
B) Equation of hyperbola is $2x^2-5xy+2y^2+10=0$

C) Eccentricity of hyperbola is $\sqrt{17}/4$

D) Eccentricity of hyperbola is $\sqrt{10}/3$

Key. B,D

Sol. Other asymptote is the image of $y=2x$ in the line $x=y$ .i.e, $x=2y$



$\Rightarrow$ Hyperbola is $(x-2y)(2x-y)=K$

$\because$ It passes through (3, 4) $\Rightarrow K=-10$

$\because$ angle between asymptotes $=2\sec^{-1}e$

$\Rightarrow \tan^{-1}\left(\frac{3}{4}\right)=2\sec^{-1}e \Rightarrow e=\frac{\sqrt{10}}{3}$

23. The equation $16x^2-3y^2-32x-12y-44=0$ represents a hyperbola with

A) length of transverse axis $=2\sqrt{3}$ B) length of conjugate axis = 8

C) centre at $(1,-2)$ D) eccentricity $=\sqrt{19}$

Key. A,B,C

Sol. $\frac{(x-1)^2}{\left(\sqrt{3}\right)^2}-\frac{(y+2)^2}{4^2}=1$

$2a=2\sqrt{3}, 2b=8$

Centre $(1,-2)$

$e=\sqrt{19/3}$

24. The equation of tangents to the hyperbola $3x^2 - y^2 = 3$ parallel to $y = 2x + 4$ is

A) $y = 2x + 3$ B) $y = 2x + 1$ C) $y = 2x - 1$ D) $y = 2x + 2$

Key. B,C

Sol. $y = mx \pm \sqrt{m^2 - 3}$

$m = 2$

$y = 2x \pm 1$

25. The locus of the point of intersection of two perpendicular tangents to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

A) director circle B) $x^2 + y^2 = a^2$ C) $x^2 + y^2 = a^2 + b^2$ D) $x^2 + y^2 = a^2 - b^2$

Key. A,D

Sol. Equation of director circle of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $x^2 + y^2 = a^2 - b^2$

---

26. If foci of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ concide with the focii of $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and eccentricity of the hyperbola is 2, then

A) $a^2 + b^2 = 16$

B) there is no director circle to the hyperbola

C) centre of the director circle is (0, 0)

D) length of latus rectum of the hyperbola = 12

Key. A,B,D

Sol. For the ellipse: a = 5 & $e = \sqrt{\frac{25-9}{25}} = \frac{4}{5}$

$\therefore$ ae = 4

$\therefore$ the focii are (- 4, 0) and (4, 0)

For the hyperbola

ae = 4, e = 2

$\therefore$ e = 2

$b^2 = 4(4-1) = 12$

$b = \sqrt{12}$

27. If (5, 12) and (24, 7) are the focii of a conic passing through the origin then the eccentricity of conic is

A) $\sqrt{386}/12$ B) $\sqrt{386}/13$ C) $\sqrt{386}/25$ D) $\sqrt{386}/38$

Key. A,D

Sol. Let A(5, 12) and B(24, 7) be two fixed points,

So, $|OA - OB| = 12$ $|OA + OB| = 38$

If conic is ellipse $e = \frac{\sqrt{386}}{38}$ $\left\{2ae = \sqrt{386} \text{ and } a = 19\right\}$

and it conic is hyperbola $e = \frac{\sqrt{386}}{12}$ $\left\{2ae = \sqrt{386} \text{ and } a = 6\right\}$

28. For the hyperbola $9x^2 - 16y^2 - 18x + 32y - 151 = 0$

A) one of the directrix is $x = \frac{21}{5}$ B) Length of latus rectum $= \frac{9}{2}$

C) Focii are (6, 1) and (- 4, 1) D) eccentricity is $\frac{5}{4}$

Key. A,B,C,D

Sol. Given hyperbola can be written as $\frac{(x-1)^2}{16}-\frac{(y-1)^2}{9}=1$

$\Rightarrow \quad \frac{X^2}{16}-\frac{Y^2}{9}=1$ {where X = x – 1, Y = y = 1}

$e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{9}{16}}=\frac{5}{4}$

Directrices are $X=\pm\frac{a}{e}$

$\Rightarrow \quad x-1=\pm\frac{16}{5} \quad \Rightarrow \quad x=\frac{21}{5}$ and $x=-\frac{11}{5}$

Length of Latus rectum $=\frac{2b^2}{a}=\frac{9}{2}$

and focii are

$\Rightarrow \quad X=\pm ae, Y=0 \quad \Rightarrow$ (6, 1) and (- 4, 1)

29. If (a sec θ, b tan θ) and (a sec ϕ, b tan ϕ) are the ends of a focal chord of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then $\tan\frac{\theta}{2}\tan\frac{\phi}{2}$ equal to

A) $\frac{e-1}{e+1}$ B) $\frac{1-e}{1+e}$ C) $\frac{1+e}{1-e}$ D) $\frac{e+1}{e-1}$

Key. C

Sol. Equation of chord joining θ and ϕ

$\frac{x}{a}\cos\frac{\theta-\phi}{2}-\frac{y}{b}\sin\frac{\theta+\phi}{2}=\cos\frac{\theta+\phi}{2}$

It passes through (ae, 0)

$\therefore \quad e\cos\frac{\theta-\phi}{2}=\cos\frac{\theta+\phi}{2}$

$\therefore \quad \frac{\cos\frac{\theta-\phi}{2}}{\cos\frac{\theta+\phi}{2}}=\frac{1}{e}$

$\frac{\cos\frac{\theta-\phi}{2}-\cos\frac{\theta+\phi}{2}}{\cos\frac{\theta-\phi}{2}+\cos\frac{\theta+\phi}{2}}=\frac{1-e}{1+e}$

$\frac{2\sin\frac{\theta}{2}\sin\frac{\phi}{2}}{2\cos\frac{\theta}{2}\cos\frac{\phi}{2}}=\frac{1-e}{1+e} \quad \Rightarrow \quad \tan\frac{\theta}{2}\tan\frac{\phi}{2}=\frac{1-e}{1+e}$

Since the chord also passes thru (- ae, 0)

Similarly as above, we get $\tan\frac{\theta}{2}\tan\frac{\phi}{2}=\frac{1+e}{1-e}$

Hyperbola

Assertion Reasoning Type

a) Both A and R are true and R is correct explanation of A.

b) Both A and R are true but R is not correct explanation of A.

c) A is true, R is false. d) A is false, R is true.

1. Assertion (A): The distance of a focus of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ to an asymptote of the hyperbola is *b*.

Reason (R): The product of distances of any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ from the asymptotes of the hyperbola is $\frac{a^2b^2}{a^2+b^2}$.

Key. B

Sol. Conceptual

2. Assertion (A): The distance of a focus of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ to an asymptote of the hyperbola is *b*.

Reason (R): The product of distances of any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ from the asymptotes of the hyperbola is $\frac{a^2b^2}{a^2+b^2}$.

Key. B

Sol. Conceptual

3. Let $S:\frac{x^2}{9}-\frac{y^2}{16}=1 \quad C:x^2+y^2=7$

Statement I : Tangents drawn from any point $\left(\sqrt{7}\cos\theta,\sqrt{7}\sin\theta\right)$ ($0\le\theta\le2\pi$) to S are perpendicular.

Statement II : Two common tangents can be drawn to S and C

Key: D

Hint: If $a<b$ perpendicular tangents cannot be drawn to $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

4. Assertion: The pair of asymptotes of $\frac{x^2}{10}-\frac{y^2}{4}=1$ and the pair of asymptotes of $\frac{x^2}{10}-\frac{y^2}{4}=-1$ coincide.

Reason : A hyperbola and its conjugate hyperbola possess the same pair of asymptotes

1) Both A and R are true and R is the correct explanation of A
2) Both A and R are true but R is not correct explanation of A
3) A is true R is false
4) A is false R is true

Key. 1

Sol. conceptual

5. Statement I: The equation of the director circle of the hyperbola $\frac{x^2}{4}-\frac{y^2}{9}=-1$ is $x^2+y^2 = 5$

Statement II: If a< b the director circle of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ $x^2+y^2 =b^2-a^2$

Key. C

Sol. Equation of director circle of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1$ is $x^2+y^2 = b^2 - a^2$ so statement I is true and II is false

6. STATEMENT–1
If a line intersect a hyperbola at (–2, –6) and (4, 2) and one of the asymptote at (1, –2) then the centre of the hyperbola is (1, –2)
because
STATEMENT–2
Mid point of the chord intercepted by hyperbola is same as mid–point of the chord intercepted between asymptotes

Key. A

Sol. Statement 2: is correct
$\Rightarrow \frac{h+1}{2}=\frac{-2+4}{2}, \frac{k-2}{2}=\frac{-6+2}{2}$ $\Rightarrow$ (h, k) ≡ (1, –2)
$\Rightarrow$ line intersect asymptote at one point only. Hence it is the centre of the hyperbola.

7. STATEMENT-1: Sum of the ordinates of the feet of normals drawn from a point (h, k) to be rectangular hyperbola $xy = c^2$ is k.
because
STATEMENT-2: Only two normals can be drawn from origin and sum of ordinates is 0.

Key. B

Sol. Normal at $\left(ct, \frac{c}{t}\right)$ through (h, k)

$ct^4 - ht^3 + kt - c = 0$

$\sum t_1 = \frac{n}{c}$ etc.

$K = C\sum \frac{1}{t_1}$

Hence sum of ordinates of normal points is k.

8. Statement – 1 : Exactly two common tangents can be drawn from (2, 1) to $\frac{x^2}{2}-\frac{y^2}{4}=1$

Because

Statement – 2 : No tangents can be drawn from interior point of hyperbola to the hyperbola.

Key. D

Sol. as (2, 1) lie inside the hyperbola
$\Rightarrow$ no tangent can be drawn.

9. Assertion (A): The distance of a focus of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ to an asymptote of the hyperbola is *b*.

Reason (R): The product of distances of any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ from the asymptotes of the hyperbola is $\dfrac{a^2b^2}{a^2+b^2}$.

Key. B

Sol. Conceptual

10. Statement-I: In a central conic any 4 co-normal points can lie on a rectangular hyperbola, because

Statement-II: In a central conic sum of the eccentric angles of any 4 conormal points is always an odd multiple of π

Key. A

Sol. Let normals at $(x_i, y_i), i = 1,2,3,4$ be concurrent at (h, k)

Normal at (x_1, y_1) $\dfrac{a^2x}{x_1} - \dfrac{b^2y}{y_1} = a^2 - b^2$ passes through (h, k)

$$\Rightarrow \frac{a^2h}{x_1} - \frac{b^2k}{y_1} = a^2 - b^2$$

$$\Rightarrow x_1y_1(a^2 - b^2) - a^2hy_1 + b^2kx_1 = 0$$

$\Rightarrow (x_i, y_i)$ satisfy an equation of the type $xy(a^2 - b^2) - a^2hy + b^2kx = 0$

Which represents a hyperbola

11. Statement – 1: Ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$ and $12x^2 - 4y^2 = 27$ intersect each other at right angle

Statement – 2: Whenever con focal conics intersect, they intersect each other orthogonally.

Key. A

Sol. $e = \dfrac{3}{5}$ $a = 5$ $\therefore$ focii are $(\pm 3, 0)$

For hyperbola $\dfrac{x^2}{\frac{27}{12}} - \dfrac{y^2}{\frac{27}{4}} = 1$

$e = \sqrt{\dfrac{12+4}{4}} = 2$ $a = \dfrac{3}{2}$ $\therefore$ focii are $(\pm 3, 0)$

$\therefore$ The two conics are confocal

12. Statement – 1: A bullet is fired and hit a target. An observer in the same plane heard two sounds the crack of the rifle and the thud of the bullet striking the target at the same instant, then locus of the observer is hyperbola where velocity of sound is smaller than velocity of bullet.

Statement – 2: If difference of distances of a point 'P' from the two fixed points is constant and less than the distance between the fixed points then locus of 'P' is a hyperbola

Key. A

Sol. Let P be the position of the gun and Q be the position of the target

Let u be the velocity of sound, v be the velocity of bullet

and R be the position of the man then we have

$$\frac{PR}{u} = \frac{QR}{u} + \frac{PQ}{v}$$

i.e. $\dfrac{PR}{u} - \dfrac{QR}{u} = \dfrac{PQ}{v}$

i.e. PR – QR = $\dfrac{u}{v}$. PQ = constant and $\dfrac{u}{v}PQ < PQ$

$\therefore$ locus of R is a hyperbola

13. Statement – 1: With respect to a hyperbola $\frac{x^2}{9}-\frac{y^2}{16}=1$ perpendicular are drawn from a point (5, 0) on the lines $3y\pm4x=0$, then their feet lie on circle $x^2+y^2=16$.
Statement – 2: If from any foci of a hyperbola perpendicular are drawn on the asymptotes of the hyperbola then their feet lie on auxiliary circle.

Key. D

Sol. (5, 0) is a focus of the hyperbola $\frac{x^2}{9}-\frac{y^2}{16}=1$
and $3y\pm4x=0$ are asymptotes.
the auxiliarly circle is $x^2+y^2=9$
$\therefore$ the feet lie on $x^2+y^2=9$
$\therefore$ Statement – 1 is false
Statement – 2 is true

14. Statement – 1: If eccentricity of a hyperbola is 2 then eccentricity of its conjugate hyperbola is $\frac{2}{\sqrt{3}}$.
Statement – 2: If e and e' are the eccentricities of a hyperbola and its conjugate hyperbola then $\frac{1}{e^2}+\frac{1}{e'^2}=1$.

Key. A

Sol. Statement – 2 is true
Since $\frac{1}{2^2}+\left(\frac{\sqrt{3}}{2}\right)^2=1$
$\therefore$ Statement – 1 is also true

15. Statement – 1: If a circle S = 0 intersects a hyperbola xy = 4 at four points. Three of them are (2, 2) (4, 1) and (6, 2/3) then co-ordinates of the fourth point are (1/4,16)
Statement – 2: If a circle S = 0 intersects a hyperbola $xy=c^2$ at t_1,t_2,t_3,t_4, then $t_1,t_2,t_3,t_4=1$.

Key. D

Sol. Statement – 2 is true
For the point (2, 2), $t_1=1$
For the point (4, 1), $t_2=2$
For the point (6, 23), $t_3=3$
For the point (1/4, 16), $t_2=\frac{1}{8}$
Now, $t_1,t_2,t_3,t_4=\frac{3}{4}\neq1$ $\therefore$ statement – 1 is false

16. Statement – 1: If a tangent is drawn to a hyperbola $16x^2-9y^2=144$ at a point (15/4,3) then another tangent at the point (-15/4, -3) will be parallel to the previous tangent.
Statement – 2: Two parallel tangents to a hyperbola touches the hyperbola at the extremities of a diameter and converse is also true.

Key. A

Sol. Statement – 2 is true
Since $\left(\frac{15}{4},3\right)and\left(-\frac{15}{4},-3\right)$ are extremities of a diameter
$\therefore$ tangents at the points are parallel.

Hyperbola

Comprehension Type

Paragraph – 1

$H: x^2 - y^2 = 9; P: y^2 = 4(x-5), L: x = 9$

1. If L is the chord of contact of the hyperbola H, then the equation of the corresponding pair of tangents is

(A) $9x^2 - 8y^2 + 18x - 9 = 0$ (B) $9x^2 - 8y^2 + 18x + 9 = 0$

(C) $9x^2 - 8y^2 - 18x + 9 = 0$ (D) $9x^2 - 8y^2 - 18x + 9 = 0$

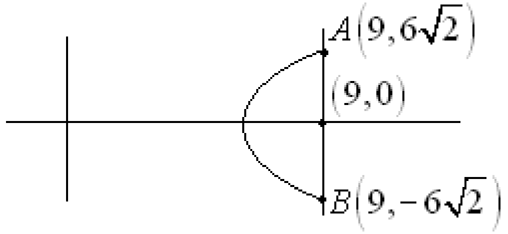
Key. C

2. If R is the point of intersection of the tangents to H at the extremities of the chord L, then equation of the chord of contact of R with respect to the parabola P is

(A) x = 7 (B) x = 9 (C) y = 7 (D) y = 9

Key. B

Sol. 1.

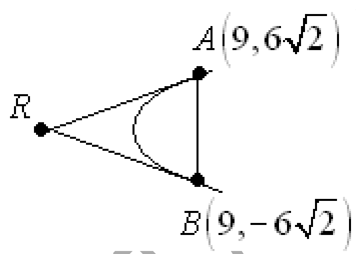


Equation of tan at $A(9, 6\sqrt{2}): x(9) - y(6\sqrt{2}) - 9 = 0$

Equation of tan at $B(9, -6\sqrt{2}): x(9) + y(6\sqrt{2}) - 9 = 0$

$\Rightarrow 9x^2 - 8y^2 - 18x + 9 = 0$

2.



Equation of chord of contact: x = 9

Paragraph – 2

In hyperbola portion of tangent intercept between asymptoes is bisected at the point of contact. Consider a hyperbola whose centre is at origin. A line x + y = 2 touches this hyperbola at P(1,1) and interests the asymptotes at A and B such that $AB = 6\sqrt{2}$ units.

3. Equation of asymptotes are

A. $5xy + 2x^2 + 2y^2 = 0$ B. $3x^2 + 2y^2 + 6xy = 0$

C. $2x^2 + 2y^2 - 5xy = 0$ D. $2x^2 + 3y^2 + 5x = 0$

Key. A

Sol. Equation of tangent in paramedic form is given by

$$\Rightarrow \frac{x-1}{-1/\sqrt{2}} = \frac{y-1}{1/\sqrt{2}} = \pm 3\sqrt{2}$$

$$\Rightarrow A \equiv (4,-2), B \equiv (-2,4)$$

Equation of asymptotes (OA and OB) are given by

$$y+2 = \frac{-2}{4}(x-4) \Rightarrow 2y+x=0 \text{ and}$$

$$y-4 = \frac{4}{-2}(x+2) \Rightarrow 2x+y=0$$

Hence, the combined equation of asymptotes is $2x^2+2y^2+5xy=0$

4. Equation of the tangent to the hyperbola at $\left(-1, \frac{7}{2}\right)$ is

A. 5x + 2y = 2 B. 3x + 2y = 4 C. 3x + 4y = 11 D. 3x + 2y = 6

Key. B

Sol. Let the equation of the hyperbola be

$$2x^2+2y^2+5xy+\lambda=0$$

It passes through (1,1).

$$\Rightarrow \lambda = -9$$

So, the hyperbola is $2x^2+2y^2+5xy=9$

Equation of the tangent at $\left(-1, \frac{7}{2}\right)$ is 3x + 2y = 4

Paragraph – 3

The normal at any point (x_1, y_1) of curve is a line perpendicular to tangent at the point (x_1, y_1). In case of parabola $y^2 = 4ax$ the equation of normal is $y = mx - 2am - am^3$ (m is slope of normal). In case of rectangular hyperbola $xy = c^2$ the equation of normal at (ct, c/t) is $xt^3 - yt - ct^4 + c = 0$. The shortest distance between any two curves always exist along the common normal.

5. If normal at (5, 3) of rectangular hyperbola xy – y – 2x – 2 = 0 intersect it again at a point

(A) (–1, 0) (B) (–1, 1) (C) (0, –2) (D) (3/4, -14)

Key. (D)

Sol. xy – y – 2x – 2 = 0

(x – 1) (y – 2) = 4

XY = 4

Normal at (ct, c/t) intersect it again at (ct′, c/t′) then $t' = -1/t^3$

2t = 4

t = 2

$(X', Y') \equiv \left(-\frac{1}{4}, -16\right)$

$(x', y') \equiv (3/4, -14)$

6. The shortest distance between the parabola $2y^2 = 2x - 1$, $2x^2 = 2y - 1$ is

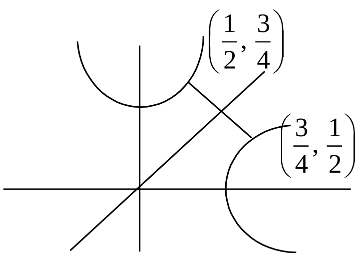
(A) $2\sqrt{2}$ (B) $\frac{1}{2\sqrt{2}}$ (C) 4 (D) $\sqrt{\frac{36}{5}}$

Key. (B)

Sol. $2y\frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{2y} = 1 \Rightarrow y = \frac{1}{2}$

$d = \sqrt{\frac{1}{16} + \frac{1}{16}} = \frac{1}{2\sqrt{2}}$



7. Number of normals drawn from $\left(\frac{7}{6}, 4\right)$ to parabola $y^2 = 2x - 1$ is

(A) 1 (B) 2 (C) 3 (D) 4

Key. (A)

Sol. $y^2 = 2(x - \frac{1}{2})$

$Y^2 = 2X$

For 3 normals $X > 1$

$x > 3/2$

$\Rightarrow$ only one normal can be drawn.

Paragraph – 4

A point P moves such that sum of the slopes of the normals drawn from it to the hyperbola xy = 16 is equal to the sum of ordinates of feet of normals. The locus of P is a curve C.

8. The equation of the curve C is

(A) $x^2 = 4y$ (B) $x^2 = 16y$
(C) $x^2 = 12y$ (D) $y^2 = 8x$

9. If the tangent to the curve C cuts the co–ordinate axis in A and B, then the locus of the middle point of AB is

(A) $x^2 = 4y$ (B) $x^2 = 2y$
(C) $x^2 + 2y = 0$ (D) $x^2 + 4y = 0$

10. Area of the equilateral triangle inscribed in a curve C having one vertex is the vertex of curve C.

(A) $772\sqrt{3}$ sq. units (B) $776\sqrt{3}$ sq. units
(C) $760\sqrt{3}$ sq. units (D) $768\sqrt{3}$ sq. units

Sol. 8. (b)

Any point on the hyperbola xy = 16 is $\left(4t, \frac{4}{t}\right)$ of the normal passes through P(h, k), then $k - 4/t = t^2(h - 4t)$

$\Rightarrow \quad 4t^4 - t^3h + tk - 4 = 0$

$\therefore \quad \sum t_1 = \frac{h}{4}$

$\sum t_1t_2 = 0$

$\sum t_1t_2t_3 = -\frac{k}{4}$ and $t_1t_2t_3t_4 = -1$

$\therefore \quad \frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \frac{1}{t_4} = \frac{k}{4} \quad \Rightarrow \quad y_1 + y_2 + y_3 + y_4 = k$

from questions

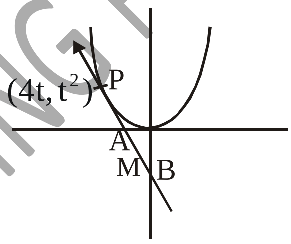
$t_1^2 + t_2^2 + t_3^2 + t_4^2 = \frac{h^2}{16} = k$

$\Rightarrow$ Locus of (h, k) is $x^2 = 16y$.

9. (c)

$x^2 = 16y$

Equation of tangent of P is



$x \,.\, 4t = \frac{16(y + t^2)}{2}$

$4tx = 8y + 8t^2$

$tx = 2y + 2t^2$

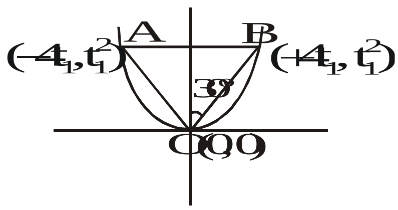
$A = (2t, 0), B = (0, -t^2)$

$M(h, k)$ is the middle point of AB.

$h = t, k = -\frac{t^2}{2} \Rightarrow 2k = -h^2$

Locus of $M(h, k)$ is $x^2 + 2y = 0$.

10. (d)



$\tan 30° = \frac{4t_1}{t_1^2} = \frac{4}{t_1}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{4}{t_1} \Rightarrow t_1 = 4\sqrt{3}$

$AB = 8t_1 = 32\sqrt{3}$

Area of $\Delta OAB = \frac{\sqrt{3}}{4} \times 32\sqrt{3} \times 32\sqrt{3} = 768\sqrt{3}$ sq.units

Paragraph – 5

The normal at any point (x_1, y_1) of curve is a line perpendicular to tangent at the point (x_1, y_1). In case of parabola $y^2 = 4ax$ the equation of normal is $y = mx - 2am - am^3$ (m is slope of normal). In case of rectangular hyperbola $xy = c^2$ the equation of normal at (ct, c/t) is $xt^3 - yt - ct^4 + c = 0$. The shortest distance between any two curve always exist along the common normal.

11. If normal at (5, 3) of rectangular hyperbola $xy - y - 2x - 2 = 0$ intersect it again at a point

(A) (–1, 0) (B) (–1, 1)
(C) (0, –2) (D) (3/4, -14)

12. The shortest distance between the parabola $2y^2 = 2x - 1$, $2x^2 = 2y - 1$ is

(A) $2\sqrt{2}$ (B) $\frac{1}{2\sqrt{2}}$

(C) 4 (D) $\sqrt{\frac{36}{5}}$

13. Number of normals drawn from $\left(\frac{7}{6},4\right)$ to parabola $y^2 = 2x - 1$ is

(A) 1 (B) 2

(C) 3 (D) 4

KEY : D-B-A

HINT

11. $xy - y - 2x - 2 = 0$

$(x - 1)(y - 2) = 4$

$XY = 4$

Normal at $(ct, c/t)$ intersect it again at $(ct', c/t')$ then $t' = -1/t^3$

$2t = 4$

$t = 2$

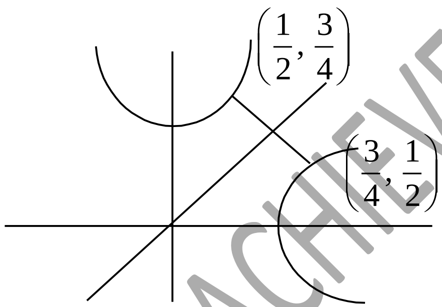
$(X', Y') \equiv \left(-\frac{1}{4},-16\right)$

$(x', y') \equiv (3/4, -14)$

12. $2y\frac{dy}{dx}=1$

$\frac{dy}{dx}=\frac{1}{2y}=1 \Rightarrow y = \frac{1}{2}$

$d = \sqrt{\frac{1}{16}+\frac{1}{16}}=\frac{1}{2\sqrt{2}}$



13. $y^2 = 2(x - \frac{1}{2})$

$Y^2 = 2X$

For 3 normals $X > 1$

$x > 3/2$

$\Rightarrow$ only one normal can be drawn.

Paragraph – 6

A straight line drawn through the point $p(-1,2)$ meets the hyperbola $xy = c^2$ at the points A and B (points A and B lie on the same side of P)

14. A point Q is chosen on this line such that PA, PQ and PB are in A.P, then locus of point Q is.

a) $x = y(1+2x)$ b) $x = y(1+x)$

c) $2x = y(1+2x)$ d) None of these

Key. C

15. If PA, PQ and PB are in G.P., then locus of Q is

a) $xy-y+2x-c^2=0$
b) $xy+y-2x+c^2=0$
c) $xy+y+2x+c^2=0$
d) $xy-y-2x-c^2=0$

Key. B

16. If PA, PQ and PB are in H.P. then locus of Q is

a) $2x-y=2c^2$
b) $x-2y=2c^2$
c) $2x+y+2c^2=0$
d) $x+2y=2c^2$

Key. A

Sol. 14. $x=\gamma\cos\theta-1,\ y=\gamma\sin\theta+2$

$xy=c^2$

$\Rightarrow \sin\theta\cos\theta\gamma^2+(2\cos\theta-\sin\theta)\gamma-2-c^2=0$

$\frac{PA+PB}{2}=PQ\Rightarrow -\frac{2\cos\theta-\sin\theta}{2\sin\theta\cos\theta}=\gamma$

15. $(PA)(PB)=\frac{-(2+c^2)}{\sin\theta\cos\theta}=\gamma^2$

16. $\frac{2}{PQ}=\frac{2}{\gamma}=\frac{PA+PB}{PA.PB}=\frac{\sin\theta-2\cos\theta}{-(2+c^2)}$

Paragraph – 7

If the axis of the rectangular hyperbola $x^2-y^2=a^2$ are rotated through an angle of $\frac{\pi}{4}$ in clock wise direction, then the equation $x^2-y^2=a^2$ reduces to $xy=c^2$ where $c=\frac{a}{\sqrt{2}}$. Parametric equation of $xy=c^2$ are $x=ct, y=\frac{c}{t}$ Where 't' is the parameter.

Answer the following.

17. If $t_1\ \&\ t_2$ are the roots of the equation $x^2-8x+4=0$, then, the point of intersection of tangents at $t_1\ \&\ t_2$ on $xy=c^2$ is.

a) (c,c)
b) $\left(c,\frac{c}{2}\right)$
c) $\left(c,\frac{c}{4}\right)$
d) $\left(\frac{c}{4},\frac{c}{4}\right)$

Key. C

Sol. Conceptual

18. If $A(t_1),B(t_2),c(t_3)$ are three points on $xy=c^2$, then , area of triangle ABC is

a) $c^2(t_1-t_2)(t_2-t_3)(t_3-t_1)$
b) $\frac{c^2}{2t_1t_2t_3}(t_1-t_2)(t_2-t_3)(t_3-t_1)$
c) $\frac{c^2}{t_1t_2t_3}(t_1-t_2)(t_2-t_3)(t_3-t_1)$
d) $2c^2t_1t_2t_3(t_1-t_2)(t_2-t_3)(t_3-t_1)$

Key. B

Sol. Conceptual

19. If the normal drawn at $P(t=1)$ to $xy=1$ cuts the curve again at Q, then, length of PQ is

a) 1 b) $2\sqrt{2}$ c) $3\sqrt{2}$ d) $4\sqrt{2}$

Key. B

Sol. Conceptual

Paragraph – 8

If the second degree curve representing by a hyperbola S = 0. The difference between the equations of hyperbola and pair of asymptotes is constant. That is A $\equiv S+\lambda=0$ where λ is constant. By using the condition of pair of straight lines, we get λ. If the equation of conjugate hyperbola of S is represented by S_1. Also asymptotes is the arithmetic mean of S and S_1.

20. Pair of asymptotes of the hyperbola $xy-3y-2x=0$ is

(A) $xy-3y-2x+2=0$ (B) $xy-3y-2x+4=0$

(C) $xy-3y-2x+6=0$ (D) $xy-3y-2x+12=0$

Key. C

Sol. Pair of asymptotes $xy-3y-2x+\lambda=0$ for pair of straight lines

$$0+2.\left(-\frac{3}{2}\right)(-1).\frac{1}{2}-0-0-\lambda\left(\frac{1}{2}\right)^2=0$$

$$\frac{3}{2}=\frac{\lambda}{4}\Rightarrow\lambda=6$$

$$xy-3y-2x+6=0$$

21. If the angle between the asymptotes of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{\pi}{3}$. Then the eccentricity of conjugate hyperbola is

(A) $\sqrt{2}$ (B) 2

(C) $\frac{2}{\sqrt{3}}$ (D) $\frac{4}{\sqrt{3}}$

Key. B

Sol. $2\tan^{-1}\left(\frac{b}{a}\right)=\frac{\pi}{3}$

$$\frac{b}{a}=\frac{1}{\sqrt{3}}$$

$$e^2=1+\frac{1}{3}=\frac{4}{3}$$

$$\frac{1}{e'^2}+\frac{1}{e^2}=1$$

$$\Rightarrow \quad \frac{1}{e'^2}+\frac{3}{4}=1$$

$$\Rightarrow \quad \frac{1}{e'^2}=\frac{1}{4}\Rightarrow e'=2$$

22. A variable chord $x\cos\theta+y\sin\theta=p$ of $\frac{x^2}{a^2}-\frac{y^2}{2a^2}=1$ subtends a right angle at the origin. This chord always touches a curve whose radius is

(A) a (B) $\frac{a}{\sqrt{2}}$

(C) $a\sqrt{2}$ (D) $2a\sqrt{2}$

Key. C

Sol. $\frac{x^2}{a^2}-\frac{y^2}{2a^2}=\left(\frac{x\cos\theta+4\sin\theta}{p}\right)^2$

$\Rightarrow \quad \frac{1}{a^2}-\frac{\cos^2\theta}{p^2}+\left(-\frac{1}{2a^2}-\frac{\sin^2\theta}{p^2}\right)=0 \quad \Rightarrow \quad \frac{1}{2a^2}=\frac{1}{p^2}\Rightarrow p=a\sqrt{2}$

$x\cos\theta+y\sin\theta=a\sqrt{2}$ will always touch $x^2+y^2=2a^2$

Paragraph – 9

Consider a hyperbola x y = 4 and a line $y+2x=4$. O is the centre of hyperbola. Tangent at any point P of hyperbola intersect the coordinate axes at A and B

23. Locus of circum centre of triangle OAB is

A) an ellipse with eccentricity $1/\sqrt{2}$ B) an ellipse with eccentricity $1/\sqrt{3}$

C) a hyperbola with eccentricity $\sqrt{2}$ D) a circle

Key. C

24. Shortest distance between the line and hyperbola is

A) $8\sqrt{2}/\sqrt{5}$ B) $\frac{4\left(\sqrt{2}-1\right)}{\sqrt{5}}$ C) $\frac{2\sqrt{2}}{\sqrt{5}}$ D) $\frac{4\left(\sqrt{2}+1\right)}{\sqrt{5}}$

Key. B

25. Let the given line intersects the x-axis at R . If a line through R, intersect the hyperbolas at S and T then, then minimum value of RS$\times$RT is ___

A) 2 B) 4 C) 6 D) 8

Key. D

Sol. 23,24&25

Let $\left(2t, 2/t\right)$ be a point on the hyperbola. Equation of the tangent at this point $x+yt^2=8t$. A=(8t , 0) , B=(0 , 8/t)

Locus of circumcentre of triangle OAB is its eccentricity is $=\sqrt{2}$

Shortest distance exist along the common normal. $t^2=1/2 \Rightarrow t=1/\sqrt{2}$, foot of the perpendicular is

$\left(\sqrt{2},2\sqrt{2}\right)$; shortest distance is $\frac{4\left(\sqrt{2}-1\right)}{\sqrt{5}}$. Let R(2, 0) & $S\left(2+\cos\theta, r\sin\theta\right)$ lies on hyperbola

$|r_1r_2|=8/|\sin 2\theta|$; minimum of $RS\times RT$ is 8

Paragraph – 10

$ABCD$ is a square with $A=(-4,0), B=(4,0)$ and other vertices of the square lie above the x-axis. Let *O* be the origin and O^1 be the mid point of CD. A rectangular hyperbola passes through the points C,D,O and its transverse axis is along the straight line OO^1.

26. The centre of the hyperbola is

A) $(0,4)$ B) $(0,3)$ C) $(0,5)$ D) $(0,2)$

Key. B

27. One of the asymptotes of the hyperbola is

A) $2x+y=3$ B) $y=2x+3$ C) $y=x+3$ D) $y=4-x$

Key. C

28. The area of the larger region bounded by the hyperbola and the square is

A) $20+8\log 3$ B) $44-9\log 3$ C) $44+8\log 3$ D) $44+9\log 3$

Key. D

Sol. (26 – 28)

The equation of Hyperbola is $(y-3)^2-x^2=9$

Paragraph – 11

Consider the conic defined by $x^2+y^2=(3x+4y+10)^2$.

29. If (α,β) is the centre of the conic then $4\alpha+3\beta=$

A) –8 B) –10 C) –6 D) –9

Key. B

30. If (p,q) is a vertex of the conic then $2p-q=$

A) −1 B) 1 C) −3 D) 2

Key. A

31. The number of points through which a pair of real perpendicular tangents can be drawn to the conic is

A) infinite B) 1 C) 0 D) 4

Key. C

Sol. (29 – 31)

The given equation can be expressed as $\sqrt{x^2+y^2}=5\frac{|3x+4y+10|}{5}$

Hence it is Hyperbola with eccentricity 5.

Focus is (0, 0)

Directrix is $3x+4y+10=0$

And hence the axis is $4x-3y=0$

Paragraph – 12

Consider a hyperbola x y = 4 and a line $y+2x=4$. O is the centre of hyperbola. Tangent at any point P of hyperbola intersect the coordinate axes at A and B

32. Locus of circum centre of triangle OAB is

A) an ellipse with eccentricity $1/\sqrt{2}$ B) an ellipse with eccentricity $1/\sqrt{3}$

C) a hyperbola with eccentricity $\sqrt{2}$ D) a circle

Key. C

33. Shortest distance between the line and hyperbola is

A) $8\sqrt{2}/\sqrt{5}$ B) $\frac{4(\sqrt{2}-1)}{\sqrt{5}}$ C) $\frac{2\sqrt{2}}{\sqrt{5}}$ D) $\frac{4(\sqrt{2}+1)}{\sqrt{5}}$

Key. B

34. Let the given line intersects the x-axis at R . If a line through R, intersect the hyperbolas at S and T then, then minimum value of RS×RT is ___

A) 2 B) 4 C) 6 D) 8

Key. D

Sol. 32,33&34

(32) c ; 33) b ; 34) d) Let $\left(2t, 2/t\right)$ be a point on the hyperbola. Equation of the tangent at this point

$x+yt^2=8t$. A=(8t , 0) , B=(0 , 8/t)

Locus of circumcentre of triangle OAB is its eccentricity is $=\sqrt{2}$

Shortest distance exist along the common normal. $t^2=1/2 \Rightarrow t=1/\sqrt{2}$, foot of the perpendicular is

$\left(\sqrt{2}, 2\sqrt{2}\right)$; shortest distance is $\frac{4(\sqrt{2}-1)}{\sqrt{5}}$. Let R(2, 0) & $S(2+\cos\theta, r\sin\theta)$ lies on hyperbola

$|r_1r_2| = 8/|\sin 2\theta|$; minimum of $RS\times RT$ is 8

Paragraph – 13

$H: x^2-y^2=9; P: y^2=4(x-5), L: x=9$

35. If L is the chord of contact of the hyperbola H, then the equation of the corresponding pair of tangents is

(A) $9x^2-8y^2+18x-9=0$ (B) $9x^2-8y^2+18x+9=0$

(C) $9x^2-8y^2-18x+9=0$ (D) $9x^2-8y^2-18x+9=0$

Key. C

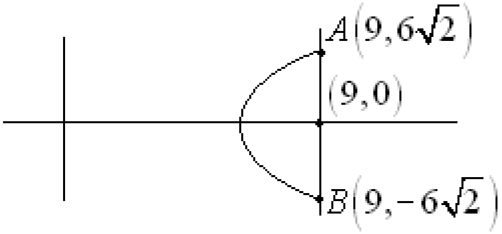
~~36. If R is the point of intersection of the tangents to H at the extremities of the chord L, then equation of the~~

chord of contact of R with respect to the parabola P is

(A) x = 7 (B) x = 9 (C) y = 7 (D) y = 9

Key. B

Sol. 35.

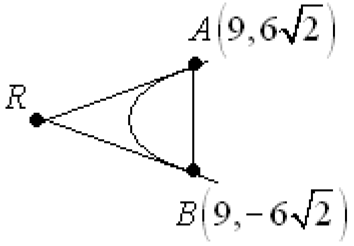


Equation of tan at $A(9,6\sqrt{2}): x(9)-y(6\sqrt{2})-9=0$

Equation of tan at $B(9,-6\sqrt{2}): x(9)+y(6\sqrt{2})-9=0$

$\Rightarrow 9x^2-8y^2-18x+9=0$

36.



Equation of chord of contact: x = 9

Paragraph – 14

Consider the conic defined by $x^2+y^2=(3x+4y+10)^2$

38. If (α,β) is the centre of the conic then $4\alpha+3\beta=$

A) – 8 B) – 10 C) – 6 D) – 9

Key. B

39. If (p,q) is a vertex of the conic then 2p – q =

A) – 1 B) 1 C) – 3 D) 2

Key. A

40. The eccentricity of the conic is

a) 5 B) 4 C) 3 D) 2

Key. A

Sol. 38 to 40.

Given equation

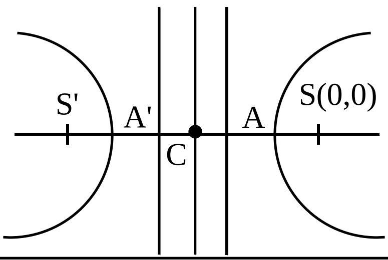
$$\sqrt{x^2+y^2}=5\frac{|3x+4y+10|}{5}$$

Hence it is Hyperbola with eccentricity 5.

Focus is (0, 0)

Directrix is 3x + 4y + 10 = 0

And hence axis is 4x – 3y = 0



e = 5, Slope of the axis = $\text{Tan}\theta = \frac{4}{3}$

$$ZA = \left|ae - \frac{a}{e}\right| = 2 \Rightarrow a = \frac{5}{12}$$

$$ae = \frac{25}{12}$$

$$\frac{x-0}{3/5} = \frac{y-0}{4/5} = \frac{-25}{12} \Rightarrow C = \left(\frac{-5}{4}, \frac{-5}{3}\right) = (\alpha, \beta)$$

$$4\alpha + 3\beta = 10$$

$$CA = a = \frac{5}{12}$$

$$\frac{x+\frac{5}{4}}{\frac{3}{5}} = \frac{y+\frac{5}{3}}{\frac{4}{5}} = \pm\frac{5}{12} \Rightarrow A = \left(-1, \frac{-4}{3}\right), A' = \left(\frac{-3}{2}, -2\right) = (p, q)$$

$$2p - q = -3 + 2 = -1.$$

Paragraph – 15

A point P moves such that sum of the slopes of the normals drawn from it to the hyperbola xy = 16 is equal to the sum of ordinates of feet of normals. The locus of P is a curve C.

41. The equation of the curve C is
(A) $x^2 = 4y$ (B) $x^2 = 16y$
(C) $x^2 = 12y$ (D) $y^2 = 8x$

Key. B

42. If the tangent to the curve C cuts the co–ordinate axis in A and B, then the locus of the middle point of AB is
(A) $x^2 = 4y$ (B) $x^2 = 2y$
(C) $x^2 + 2y = 0$ (D) $x^2 + 4y = 0$

Key. C

43. Area of the equilateral triangle inscribed in a curve C having one vertex is the vertex of curve C.
(A) $772\sqrt{3}$ sq. units (B) $776\sqrt{3}$ sq. units
(C) $760\sqrt{3}$ sq. units (D) $768\sqrt{3}$ sq. units

Key. C

Sol. 41. Any point on the hyperbola xy = 16 is $\left(4t, \frac{4}{t}\right)$ of the normal passes through P(h, k), then

$k - 4/t = t^2(h - 4t)$

$\Rightarrow \quad 4t^4 - t^3h + tk - 4 = 0$

$$\therefore \sum t_1 = \frac{h}{4}$$

$$\sum t_1t_2 = 0$$

$$\sum t_1t_2t_3 = -\frac{k}{4} \quad \text{and } t_1t_2t_3t_4 = -1$$

$$\therefore \quad \frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \frac{1}{t_4} = \frac{k}{4} \quad \Rightarrow \quad y_1 + y_2 + y_3 + y_4 = k$$

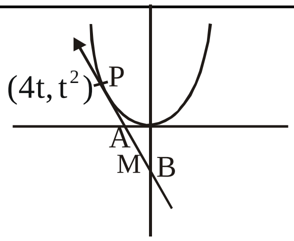
from questions

$$t_1^2 + t_2^2 + t_3^2 + t_4^2 = \frac{h^2}{16} = k$$

$\Rightarrow$ Locus of (h, k) is $x^2 = 16y$.

42. $x^2 = 16y$

Equation of tangent of P is



$x \cdot 4t = \dfrac{16(y+t^2)}{2}$

$4tx = 8y + 8t^2$

$tx = 2y + 2t^2$

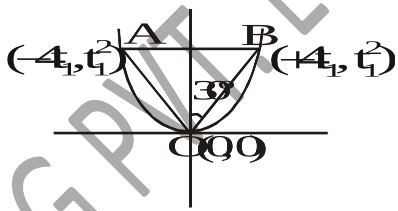
$A = (2t, 0),\ B = (0, -t^2)$

$M(h, k)$ is the middle point of AB.

$h = t,\ k = -\dfrac{t^2}{2} \Rightarrow 2k = -h^2$

Locus of $M(h, k)$ is $x^2 + 2y = 0$.

43. $\tan 30° = \dfrac{4t_1}{t_1^2} = \dfrac{4}{t_1}$



$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{4}{t_1} \Rightarrow t_1 = 4\sqrt{3}$

$AB = 8t_1 = 32\sqrt{3}$

Area of $\Delta OAB = \dfrac{\sqrt{3}}{4} \times 32\sqrt{3} \times 32\sqrt{3} = 768\sqrt{3}$ sq.units

Paragraph – 16

$ABCD$ is a square with $A = (-4, 0), B = (4, 0)$ and other vertices of the square lie above the x-axis. Let *O* be the origin and O^1 be the mid point of CD. A rectangular hyperbola passes through the points C, D, O and its transverse axis is along the straight line OO^1.

44. The centre of the hyperbola is

A) $(0, 4)$ B) $(0, 3)$ C) $(0, 5)$ D) $(0, 2)$

Key. B

45. One of the asymptotes of the hyperbola is

A) $2x + y = 3$ B) $y = 2x + 3$ C) $y = x + 3$ D) $y = 4 - x$

Key. C

46. The area of the larger region bounded by the hyperbola and the square is

A) $20 + 8\log 3$ B) $44 - 9\log 3$ C) $44 + 8\log 3$ D) $44 + 9\log 3$

Key. D

Sol. 43-46. The equation of Hyperbola is $(y-3)^2 - x^2 = 9$

Paragraph – 17

Consider the conic defined by $x^2 + y^2 = (3x + 4y + 10)^2$.

47. If (α, β) is the centre of the conic then $4\alpha + 3\beta =$

A) –8 B) –10 C) –6 D) –9

Key. B

48. If (p, q) is a vertex of the conic then $2p - q =$

A) –1 B) 1 C) –3 D) 2

Key. A

49. The number of points through which a pair of real perpendicular tangents can be drawn to the conic is

A) infinite B) 1 C) 0 D) 4

Key. C

Sol. 47 – 49

The given equation can be expressed as $\sqrt{x^2+y^2}=5\frac{|3x+4y+10|}{5}$

Hence it is Hyperbola with eccentricity 5.
Focus is (0, 0)
Directrix is $3x+4y+10=0$
And hence the axis is $4x-3y=0$

Paragraph – 18

If the normals at $(x_i, y_i), i=1,2,3,4$ on the rectangular hyperbola $xy=c^2$, meet at the point (α,β)
Answer the following questions

50. The value of $\sum y_i$ is

A) $c\beta$ B) $c\alpha$ C) α D) β

Key. D

51. The value of $\sum x_i^2$ is

A) c^2 B) α^2 C) $-c^2$ D) $-\beta^2$

Key. B

52. The value of $\sum y_i^2$ is

A) β^2 B) α^2 C) $-c^2$ D) c^2

Key. A

Sol. 50 – 52

Equation of normal to $xy=c^2$ at $(ct, c/t)$ is $ct^4-t^3x+ty-c=0$

It passes through (α,β), then $ct^4-t^3\alpha+t\beta-c=0$ --- (1)

$\sum t_1=\frac{\alpha}{c}, \sum t_1t_2=0, \sum t_1t_2t_3=-\frac{\beta}{c}, t_1t_2t_3t_4=-1$

50. $\sum y_i = c\sum\frac{1}{t_1}=c\frac{\sum t_1t_2t_3}{t_1t_2t_3t_4}=c\times\frac{\beta}{c}=\beta$

51. $\sum x_i^2=c^2\sum t_1^2=c^2\left\{\left(\sum t_1\right)^2-2\sum t_1t_2\right\}=c^2\left\{\frac{\alpha^2}{c^2}-2.0\right\}=\alpha^2$

52. $\sum y_i^2=\left(\sum y_1\right)^2-2\sum y_1y_2=\beta^2-2c\sum\frac{1}{t_1t_2}=\beta^2=2c\frac{\sum t_1t_2}{t_1t_2t_3t_4}=\beta^2-2c\times 0=\beta^2$

Paragraph – 19

A straight line drawn through the point $p(-1,2)$ meets the hyperbola $xy=c^2$ at the points A and B (points A and B lie on the same side of P)

53. A point Q is chosen on this line such that PA, PQ and PB are in A.P, then locus of point Q is.

a) $x=y(1+2x)$ b) $x=y(1+x)$ c) $2x=y(1+2x)$ d) None of these

Key. C

54. If PA, PQ and PB are in G.P., then locus of Q is

a) $xy-y+2x-c^2=0$ b) $xy+y-2x+c^2=0$

c) $xy+y+2x+c^2=0$ d) $xy-y-2x-c^2=0$

Key. B

55. If PA, PQ and PB are in H.P. then locus of Q is

a) $2x-y=2c^2$ b) $x-2y=2c^2$ c) $2x+y+2c^2=0$ d) $x+2y=2c^2$

Key. A

Sol. 53. $x=\gamma\cos\theta-1,\ y=\gamma\sin\theta+2$

$xy=c^2$

$\Rightarrow \sin\theta\cos\theta\gamma^2+\left(2\cos\theta-\sin\theta\right)\gamma-2-c^2=0$

$\dfrac{PA+PB}{2}=PQ\Rightarrow -\dfrac{2\cos\theta-\sin\theta}{2\sin\theta\cos\theta}=\gamma$

54. $\left(PA\right)\left(PB\right)=\dfrac{-\left(2+c^2\right)}{\sin\theta\cos\theta}=\gamma^2$

55. $\dfrac{2}{PQ}=\dfrac{2}{\gamma}=\dfrac{PA+PB}{PA.PB}=\dfrac{\sin\theta-2\cos\theta}{-\left(2+c^2\right)}$

Paragraph – 20

Consider the conic defined by the equation :

$\left|\sqrt{\left(x-1\right)^2+\left(y-2\right)^2}-\sqrt{\left(x-5\right)^2+\left(y-5\right)^2}\right|=3$

56. The equation of an axis of the conic is

a) $6x+8y=45$ b) $3x-4y-5=0$

c) $8x+6y=45$ d) $3x+4y+5=0$

Key. C

57. The distance between the directrices of the conic is

a) $9/5$ b) $3/5$

c) $5/3$ d) $5/9$

Key. A

58. The eccentricity of the conic conjugate to the given one, is

a) $5/3$ b) $5/4$ c) $5/2$ d) 5

Key. B

Sol. 56. Given equation represents a hyperbola having foci $S\left(1,2\right)\text{and}\,S'\left(5,5\right)\&\,2a=3$

transverse axis : line SS': $3x-4y+5=0$

Conjugate axis : perpendicular bisector of $SS':8x+6y=45$

57. Distance between diretrices = $=\dfrac{2a}{e}=\dfrac{3}{5/3}=\dfrac{9}{5}$

58. let e' be the ecc. of conjugate hyperbola then $\dfrac{1}{e^2}+\dfrac{1}{e'^2}=1\Rightarrow e'^2=\dfrac{25}{16}$

Paragraph – 21

The difference between the second degree curve and pair of asymptotes is constant. If second degree curve represented by a hyperbola $S=0$, then the equation of its asymptotes is $S+\lambda=0$ where λ is constant, which will be a pair of straight lines, then we get λ. Then equation of asymptotes is $A\equiv S+\lambda=0$ and if equation of conjugate hyperbola of S represented by S_1, then A is the arithmetic mean of S and S_1.

59. The asymptotes of a hyperbola having centre at the point (1, 2) are parallel to the lines $2x+3y=0$ and $3x+2y=0$. If the hyperbola passes through the point (5, 3) then its equation is

A) $(2x+3y-3)(3x+2y-5)=256$ B) $(2x+3y-7)(3x+2y-8)=156$
C) $(2x+3y-5)(3x+2y-3)=252$ D) $(2x+3y-8)(3x+2y-7)=154$

Key. D

60. If the angle between the asymptotes of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{\pi}{3}$ then the eccentricity of conjugate hyperbola is
A) $\sqrt{2}$ B) 2 C) $2/\sqrt{3}$ D) $4/\sqrt{3}$

Key. B

61. A hyperbola passing through origin has $3x-4y-1=0$ and $4x-3y-6=0$ as its asymptotes. Then the equation of its transverse and conjugate axes are
A) $x-y-5=0$ and $x+y+1=0$ B) $x-y=0$ and $x+y+5=0$
C) $x+y-5=0$ and $x-y-1=0$ D) $x+y-1=0$ and $x-y-5=0$

Key. C

Sol. 59. Let the asymptotes be $2x+3y+\lambda=0$ and $3x+2y+\mu=0$, it passes through (1, 2)
$\lambda=-8,\mu=-7$
Equation of hyperbola is $(2x+3y-8)(3x+2y-7)+\gamma=0$
It passes through (5, 3) $\gamma=-154$

60. $2\tan^{-1}\frac{b}{a}=\frac{\pi}{3}\Rightarrow a=b\sqrt{3}$
$a^2=b^2(e^2-1)\Rightarrow e=2$

61. $\frac{3x-4y-1}{5}=\pm\frac{4x-3y-6}{5}\Rightarrow x+y-5=0$ and $x-y-1=0$

Paragraph – 22

If the normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at any point $P(a\sec\theta, b\tan\theta)$ meets the transverse and conjugate axes in G and g respectively and F is the foot of perpendicular to the normal at P from centre C,

62. The minimum length of PG is
A) $\frac{b^2}{a}$ B) $\left|\frac{a}{b}(a+b)\right|$ C) $\left|\frac{b}{a}(a-b)\right|$ D) $\left|\frac{a}{b}(a-b)\right|$

Key. A

63. The geometric mean of PF and PG is
A) a B) b C) 2a D) 2b

Key. B

64. The geometric mean of PF and Pg is
A) a B) b C) 2a D) 2b

Key. A

Sol. **(62 – 64)**
$T_P : \frac{x}{a}\sec\theta-\frac{y}{b}\tan\theta=1$
$N_P : ax\cos\theta-by\cot\theta=a^2+b^2$
$G\left(\frac{a^2+b^2}{a}\sec\theta, 0\right), g\left(0, \frac{a^2+b^2}{b}\tan\theta\right)$
$PG^2=\frac{b^2}{a^2}\left\{b^2+(a^2+b^2)\tan^2\theta\right\}, PG_{min}=\frac{b^2}{a}$
$PF.PG=b^2, PF.Pg=a^2$

Paragraph – 23

If P is a variable point and F_1 and F_2 are two fixed points such that $|PF_1 - PF_2| = 2a$. Then the locus of the point P is a hyperbola, with points F_1 and F_2 as the two focii $(F_1F_2 > 2a)$. If $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is a hyperbola, then its conjugate hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$. Let P(x, y) is a variable point such that

$$\left|\sqrt{(x-1)^2 + (y-2)^2} - \sqrt{(x-5)^2 + (y-5)^2}\right| = 3$$

65. If the locus of the point P represents a hyperbola of eccentricity e, then the eccentricity e' of the corresponding conjugate hyperbola is

A) $\frac{5}{3}$ B) $\frac{4}{3}$ C) $\frac{5}{4}$ D) $\frac{3}{\sqrt{7}}$

Key. C

Sol. 2a = 3

Distance between the focii (1, 2) and (5, 5) is 5

2ae = 5 $\therefore$ $e = \frac{5}{3}$

$\frac{1}{e^2} + \frac{1}{e'^2} = 1$ $\Rightarrow$ $e' = \frac{5}{4}$

66. Locus of intersection of two perpendicular tangents to the given hyperbola is

A) $(x-3)^2 + \left(y + \frac{7}{2}\right)^2 = \frac{55}{4}$ B) $(x-3)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{25}{4}$

C) $(x-3)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{7}{4}$ D) none of these

Key. D

Sol. Director circle $(x-h)^2 + (y-k)^2 = a^2 - b^2$, where (h, k) is centre

Centre is $\left(\frac{1+5}{2}, \frac{2+5}{2}\right) = \left(3, \frac{7}{2}\right)$

$b^2 = a^2(e^2 - 1) = \left(\frac{3}{2}\right)^2 \left(\left(\frac{5}{3}\right)^2 - 1\right) = 4$

Director circle $(x-3)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{9}{4} - 4$

$$(x-3)^2 + \left(y - \frac{7}{2}\right)^2 = -\frac{7}{4}$$

the does not represent any real point

67. If origin is shifted to point $\left(3, \frac{7}{2}\right)$ and the axes are rotated through an angle θ is clockwise sense so that equation of given hyperbola changes to the standard form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then θ is

A) $\tan^{-1}\left(\frac{4}{3}\right)$ B) $\tan^{-1}\left(\frac{3}{4}\right)$ C) $\tan^{-1}\left(\frac{5}{3}\right)$ D) $\tan^{-1}\left(\frac{3}{5}\right)$

Key. B

Sol. Slope of transverse axis is $\frac{3}{4}$

$\therefore$ angle of rotation $= \theta = \tan^{-1}\frac{3}{4}$

Paragraph – 24

For the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ then normal at P meets the transverse axis AA' in G and the conjugate and BB' in g and CF be perpendicular to the normal from the centre.

68. PF. PG = K CB^2, then K =

A) 2 B) 1 C) $\frac{1}{2}$ D) 4

Key. B

Sol. $PF=\frac{ab}{\sqrt{b^2\sec^2\phi+a^2\tan^2\phi}}$

$PG^2=\frac{b^2}{a^2}\left(b^2\sec^2\phi+a^2\tan^2\phi\right)$

$PF=\frac{ab}{\frac{a}{b}PG}$

$PG.\ PF=b^2=CB^2$

69. PF. Pg equals to

A) CA^2 B) CF^2 C) CB^2 D) CA. CB

Key. A

Sol. $Pg^2=\frac{a^2}{b^2}\left(b^2\sec^2\phi+a^2\tan^2\phi\right)$

70. Locus of middle point of G and g is a hyperbola of eccentricity

A) $\frac{1}{\sqrt{e^2-1}}$ B) $\frac{e}{\sqrt{e^2-1}}$ C) $2\sqrt{e^2-1}$ D) $\frac{e}{2}$

Key. B

Sol. Locus of middle point is $\frac{x^2}{\frac{a^2e^4}{4}}-\frac{y^2}{\frac{a^4e^4}{4b^2}}=1$

$e_1=\sqrt{\frac{\frac{a^2e^4}{4}+\frac{a^4e^4}{4b^2}}{\frac{a^2e^4}{4}}}=\frac{e}{\sqrt{e^2-1}}$

Paragraph – 25

If a circle with centre $C(\alpha,\beta)$ intersects a rectangular hyperbola with centre L(h, k) at four points $P(x_1,y_1)$, $Q(x_2,y_2)$, $R(x_3,y_3)$ and $S(x_4,y_4)$, then the mean of the four points P,Q,R,S is the mean of the points C and L. In other words, the mid-points of CL coincides with the mean point of P,Q,R,S. Analytically, $\frac{x_1+x_2+x_3+x_4}{4}=\frac{\alpha+h}{2}$ and $\frac{y_1+y_2+y_3+y_4}{4}=\frac{\beta+k}{2}$

71. Five points are selected on a circle of radius a. The centres of the rectangular hyperbola, each passing through four of these points, all lie on a circle of radius

A) a B) 2a C) $\frac{a}{\sqrt{2}}$ D) $\frac{a}{2}$

Key. D

Sol. Let the circle is $x^2+y^2=a^2$ and let the centre of rectangular hyperbola is (h, k). Let given points on circle are $\left(a\cos\theta_i,\ a\sin\theta_1\right)$, i = 1, 2, 3, 4, 5 on that $\frac{\sum_{i=1}^{4}a\cos\theta_i}{4}=\frac{O+h}{3}\Rightarrow\sum_{i=1}^{5}a\cos\theta_i-a\cos\theta_5=2h$

Similarly $\sum_{i=1}^{5} a\sin\theta_i - a\sin\theta_5 = 2k$

As the five points are given, $\sum_{i=1}^{5} a\sin\theta_i$ and $\sum_{i=1}^{5} a\cos\theta_i$ are known. Let us assume their values of be μ and λ respectively.

$\therefore\ \lambda - a\cos\theta_5 = 2h$ and $\mu - a\sin\theta_5 = 2k$

$\Rightarrow\ 2h - \lambda = -a\cos\theta_5$ and $2k - \mu = -a\sin\theta_5$

$\Rightarrow (2h-\lambda)^2 + (2h-\mu)^2 = a^2$

$\Rightarrow \left(h - \frac{\lambda}{2}\right)^2 + \left(k - \frac{\mu}{2}\right)^2 = \left(\frac{a}{2}\right)^2$

$\Rightarrow$ centre (h, k) lies on circle of radius $\frac{a}{2}$.

72. A,B,C,D are the points of intersection of a circle and a rectangular hyperbola which have different centres. If AB passes through the centre of the hyperbola, then CD passes through
A) Centre of the hyperbola
B) centre of the circle
C) mid-point of the centres of circle and hyperbola
D) none of the points mentioned in the three options.

Key. B

Sol. Let centre of circle and hyperbola are (α,β) and (h, k) and points are $A(x_1,y_1)$, $B(x_2, y_2)$, $C(x_3,y_3)$ and $D(x_4, y_4)$, then

$$\frac{h+\alpha}{2} = \frac{x_1+x_2+x_3+x_4}{4} \quad \ldots(1)$$

and

$$\frac{k+\beta}{2} = \frac{y_1+y_2+y_3+y_4}{4} \quad \ldots(2)$$

As any chord passing through centre of hyperbola is bisected at the centre.

$\therefore$ AB is bisected at (h, k)

$\Rightarrow \quad \frac{x_1+x_2}{2} = h \quad \ldots(3)$

and $\quad \frac{y_1+y_2}{2} = k \quad \ldots(4)$

From (1) and (3) $\beta = \frac{y_3+y_4}{2}$

$\Rightarrow \quad (\alpha, \beta)$ is mid-point of CD

$\Rightarrow \quad (\alpha, \beta)$ is lies on CD

$\Rightarrow \quad$ centre of circle lies on CD

73. If the normals drawn at four concylic points on a rectangular hyperbola $xy = c^2$ meet at point (α, β) then the centre of the circle has the coordinates

A) (α, β) B) $(2\alpha, 2\beta)$ C) $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$ D) $\left(\frac{\alpha}{4}, \frac{\beta}{4}\right)$

Key. C

Sol. Let the four concylic points at which normals to rectangular hyperbola are concurrent are $A(x_1,y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x_4, y_4)$ and centre of circle be (h, k)

$\therefore \quad \frac{x_1+x_2+x_3+x_4}{4} = \frac{h+o}{2}$ and $\frac{y_1+y_2+y_3+y_4}{4} = \frac{k+o}{2}$

$\Rightarrow \quad x_1+x_2+x_3+x_4 = 2h \quad \ldots(1)$

and $\quad y_1+y_2+y_3+y_4 = 2k \quad \ldots(2)$

Normal to rectangular hyperbola $xy = c^2$ at $\left(ct, \frac{c}{t}\right)$

$ct^4 - xt^3 + yt - c = 0$

As all normal pass through (α, β)

$$\Rightarrow \quad x_1 + x_2 + x_3 + x_4 = c(t_1 + t_2 + t_3 + t_4) = c\left(\frac{\alpha}{c}\right) = \alpha \qquad \ldots(3)$$

and $$y_1 + y_2 + y_3 + y_4 = c\left(\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \frac{1}{t_4}\right) = c\left(\frac{\Sigma t_1 t_2 t_3}{t_1 t_2 t_3 t_{45}}\right)$$

$$= c\left(\frac{-\beta | c}{-c | c}\right) = \beta \qquad \ldots(3)$$

From (1) and (3), $2h = \alpha$

From (2) and (4), $2k = \beta \Rightarrow (h, k) = \left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

Passage-II

$ABCD$ is a square with $A = (-4, 0), B = (4, 0)$ and other vertices of the square lie above the x-axis. Let O be the origin and O^1 be the mid point of CD. A rectangular hyperbola passes through the points C, D, O and its transverse axis is along the straight line OO^1.

36. The centre of the hyperbola is

A) $(0, 4)$ B) $(0, 3)$ C) $(0, 5)$ D) $(0, 2)$

Key. B

37. The length of the latus rectum of the hyperbola is

A) 8 B) 7 C) 6 D) 5

Key. C

38. The area of the larger region bounded by the hyperbola and the square is

A) $20 + 8\log 3$ B) $44 - 9\log 3$ C) $44 + 8\log 3$ D) $44 + 9\log 3$

Key. D

Sol. **36– 38**

The equation of Hyperbola is $(y-3)^2 - x^2 = 9$

Hyperbola

Integer Answer Type

1. If P (x, y) satisfy $x^2 + y^2 = 1$. Let maximum value of $(x+y)^2$ is λ then number of tangents from $(\lambda, 0)$ to hyperbola $(x-2)^2 - y^2 = 1$ are

Key. 2

Sol. Let P(x, y) = $(\cos\theta, \sin\theta)$

$$\therefore \lambda = 2$$

No. of tangents from (2, 0) are 0

2. Acute angle between the asymptotes of the hyperbola $x^2 + 2xy - 3y^2 + x + 7y + 9 = 0$ is θ. Then $\tan\theta =$

Key. 2

Sol. Equation of hyperbola is

$$x^2 + 2xy - 3y^2 + x + 7y + 9 = 0$$

The combined equation of asymptotes is $x^2 + 2xy - 3y^2 + x + 7y + K = 0$

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{|a+b|} = \frac{2\sqrt{1+3}}{1-3} = 2$$

3. The equation of Asymptotes of $xy + 2x + 4y + 6 = 0$ is $xy + 2x + 4y + c = 0$, then C = ___

Key. 8

Sol. $xy + 2x + 4y + C = 0$ represents pair of lines $\Rightarrow C = 8$

4. The equation of Asymptotes of $xy + 2x + 4y + 6 = 0$ is $xy + 2x + 4y + c = 0$, then C = ___

Key. 8

Sol. $xy + 2x + 4y + C = 0$ represents pair of lines $\Rightarrow C = 8$

5. Let PN be the ordinate of a point P on the hyperbola $\frac{x^2}{(97)^2} - \frac{y^2}{(79)^2} = 1$ and the tangent at P meets the transverse axis in T, O is the origin. Then $\left[\frac{ON.OT}{2011}\right]$ is equal to (where [.] denotes G.I.F)

Key. 4

Sol. $\text{ON.OT} = 97\cos\theta . 97\sec\theta = 97^2$

$$\therefore \left[\frac{\text{ON.OT}}{2011}\right] = \left[\frac{97^2}{2011}\right] = 4$$

6. If e is the eccentricity of the hyperbola $(5x - 10)^2 + (5y + 15)^2 = (12x - 5y + 1)^2$ then $\frac{25e}{13}$ is equal to ....

Key. 5

Sol. Equation can be rewritten as $\sqrt{(x-2)^2 + (y+3)^2} = \frac{13}{5}\left|\frac{12x - 5y + 1}{13}\right|$

So, e = $\frac{13}{5}$.

7. If a variable tangent of the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$, cuts the circle $x^2 + y^2 = 4$ at point A, B and locus of mid point of AB is $9x^2 - 4y^2 - \lambda (x^2 + y^2)^2 = 0$ then λ is ....

Key. 1

Sol. Equation of chord of circle with mid point (h, k) is $xh + xk = h^2 + k^2$ or

$y=\left(\frac{-h}{k}\right)x+\frac{h^2+k^2}{k}$, it touches the hyperbola

8. If the angle between the asymptotes of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{\pi}{3}$. Then the eccentricity of conjugate hyperbola is

Key. 2

Sol. $2\tan^{-1}\left(\frac{b}{a}\right)=\frac{\pi}{3}$

$\frac{b}{a}=\frac{1}{\sqrt{3}}$

$e^2=1+\frac{1}{3}=\frac{4}{3}$

$\frac{1}{e'^2}+\frac{1}{e^2}=1$

$\Rightarrow \quad \frac{1}{e'^2}+\frac{3}{4}=1$

$\Rightarrow \quad \frac{1}{e'^2}=\frac{1}{4}\Rightarrow e'=2$

9. If PN be the ordinate of a point P on the hyperbola $\frac{x^2}{(97)^2}-\frac{y^2}{(79)^2}=1$ and the tangent at P meets the transverse axis in T, O is the origin; then $\left[\frac{ON.OT}{7999}\right]$ is…… (where [.] denotes greatest integer function).

Key. 1

10. If the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide, then b =

Key. 4

11. The equation $\frac{x^2}{9-\lambda}+\frac{y^2}{4-\lambda}=1$ represents a hyperbola when $a<\lambda<b$ then $\left[\frac{b+a}{b-a}\right]=$

Where [.] denotes greatest integer function.

Key. 2

Sol. $(9-\lambda)(4-\lambda)<0\Rightarrow 4<\lambda<9\Rightarrow\left[\frac{b+a}{b-a}\right]=\left[\frac{13}{5}\right]=2$

12. If CP, CD are semiconjugate diameters of $5(x-2)^2+4(y-3)^2=20$, then $CP^2+CD^2=$

Key. 9

Sol. $CP^2+CD^2=a^2+b^2$

13. If a hyperbola passes through the focus of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and its transverse and conjugate axes coincides with the major and minor axes of the ellipse, and the product of eccentricities is 1, represented by the equation $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, then the value of b^2-a^2 is

Key. 7

Sol. Using the hypothesis, we get equation to hyperbola as $\frac{x^2}{9}-\frac{y^2}{16}=1 \Rightarrow b^2-a^2=7$

14. The product of perpendiculars from any point on the hyperbola $\frac{x^2}{4}-\frac{3y^2}{4}=1$ to its asymptotes is $\frac{1}{K}$, then $K=$

Key. 1

Sol. Product of perpendiculars from any point on the hyperbola to its asymptotes $=\frac{a^2b^2}{a^2+b^2}=\frac{1}{\frac{1}{a^2}+\frac{1}{b^2}}=1$

15. Chords of the hyperbola $x^2-y^2=a^2$ touch the parabola $y^2=4a\,x$. Prove that the locus of their middle points is the curve $y^2(x-a)=x^3$.

Ans. Hence locus is $x^3=y^2(x-a)$.

Sol. Let P(h, k) be midpoint of chords so their equation is $T=S_1$

i.e. $xh-yk=h^2-k^2$ …(i)

Also equation of tangent to the parabola $y^2=4ax$ is

$$y=mx+\frac{a}{m} \quad \ldots(ii)$$

$\therefore$ comparing (i) and (ii), we get

$$m=\frac{h}{k} \text{ and } \frac{a}{m}=\frac{k^2-h^2}{k} \Rightarrow \frac{ak}{h}=\frac{k^2-h^2}{k} \Rightarrow \quad h^3=k^2(h-a)$$

Hence locus is $x^3=y^2(x-a)$.

16. Prove that chord of a hyperbola, which touches the conjugate hyperbola, is bisects at the point of contact.

Sol. Let $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ …(i) be the hyperbola, then its conjugate hyperbola is

$$\frac{y^2}{b^2}-\frac{x^2}{a^2}=1 \quad \ldots(ii)$$

Let any point on (ii) be (($a\tan\theta$, $b\sec\theta$), then equation of the tangent to (ii) at this point is

$$\frac{y\sec\theta}{b}-\frac{x\tan\theta}{a}=1=\sec^2\theta-\tan^2\theta$$

i.e. $$\frac{x\tan\theta}{a}-\frac{y\sec\theta}{b}-1=\frac{a^2\tan^2\theta}{a^2}-\frac{b^2\sec^2\theta}{a^2}-1$$

which is the equation of the chord of (i) whose mid point is $(a\tan\theta, b\sec\theta)$. Hence the result

17. The asymptotes of a hyperbola are parallel to 2 x + 3y = 0 & 3 x + 2 y = 0. Its centre is (1, 2) & it passes through (5, 3). Find the equation of the hyperbola.

Ans. $(2x+3y-8)(3x+2y-7)-154=0$

Sol. Let the asymptotes be $2x+3y+\lambda=0$ and $3x+2y+\mu=0$. Since asymtotes passes through (1, 2), then $\lambda=-8$ and $\mu=-7$

Thus the equation of asymptotes are

$$2x + 3y - 8 = 0 \text{ and } 3x + 2y - 7 = 0$$

Let the equation of hyperbola be

$$(2x + 3y - 8)(3x + 2y - 7) + v = 0$$

It passes through (5, 3), then

$$(10 + 9 - 8)(15 + 6 - 7) + v = 0$$

$\Rightarrow$ $11 \times 14 + v = 0$

$\therefore$ $v = -154$

putting the value of v in (1) we obtain

$$(2x+3y-8)(3x+2y-7)-154=0$$

which is the equation of required hyperbola.

Hyperbola

Matrix-Match Type

1. COLUMN - I — COLUMN - II

A) The length of the latus rectum of the hyperbola $16x^2 - 9y^2 = 144$ is — p) $\frac{2}{3}$

B) The product of the perpendiculars drawn from any point on the hyperbola $x^2 - 2y^2 = 2$ to its asymptotes is — q) 3

C) The length of the transverse axis of the hyperbola xy = 18 — r) $\frac{32}{3}$

D) The product of the lengths of the perpendiculars drawn from the foci of $3x^2 - 4y^2 = 12$ on any of its tangents is — s) 12

Key. $A \to R$; $B \to P$; $C \to S$; $D \to Q$

Sol. (a) If y the parabola a is $\frac{x^2}{2} - \frac{y^2}{1} = 1$

length of the latus rectum = $\frac{2b^2}{a} = \frac{32}{3}$

(b) the hyperbola is is $\frac{x^2}{2} - \frac{y^2}{1} = 1$

Required product = $\frac{a^2b^2}{a^2 + b^2} = \frac{2}{3}$

(c) Given hyperbola is xy =18

length of the latus - rectum = $2\sqrt{2}C$ = 12

(d) the hyperbola is $\frac{x^2}{4} - \frac{y^2}{3} = 1$

Required product = $b^2 = 3$.

2. COLUMN - I — COLUMN - II

A) The eccentricity of the conic represented by $x^2 - y^2 - 4x + 4y + 16 = 0$ is — p) 7

B) The foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide ,then b^2 is — q) 0

C) The product of lengths of perpendiculars from any point t of the hyperbola $x^2 - y^2 = 8$ to its asymptotes is — r) $\sqrt{2}$

D) The number of points out side the hyperbola $\frac{x^2}{25} - \frac{y^2}{36} = 1$ from where two perpendicular tangents can be drawn to the hyperbola is /are — s) 4

Key. $A \to R$; $B \to P$; $C \to S$; $D \to Q$

Sol. (a) $h^2=0, ab=-1 \Rightarrow h^2>ab$ and $a+b=1-1=0$ rectangular hyperbola eccentricity $=\sqrt{2}$

(b) For ellipse $a^2=16$ $e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{\frac{16-b^2}{16}}$

Foci of ellipse $(\pm ae, o)=(\pm\sqrt{16-b^2}, =o)$

For hyperbola, $a^2=\left(\frac{12}{5}\right)^2$ $b^2=\left(\frac{9}{5}\right)^2 \Rightarrow e=\sqrt{1+\frac{b^2}{a^2}}=\frac{5}{4}$

So, $16-b^2=\left(\frac{12}{5}\times\frac{5}{4}\right)^2=9 \Rightarrow b^2=7$

(c) $x^2-y^2=8,\ a^2=8, b^2=8$

product of $\perp^r=\frac{a^2b^2}{a^2+b^2}=\frac{8\times 8}{16}=4$

(d) $\frac{x^2}{25}-\frac{y^2}{36}=1$. Equation of direction circle is $x^2+y^2=a^2-b^2 \Rightarrow x^2+y^2=-9$

Which in not possible number of points = 0

3. The normals at four points (x_i, y_i), $i=1,2,3,4$ on the hyperbola $xy=4$ are concurrent at the point (α, β)

Column – I	Column – II
a) $y_1+y_2+y_3+y_4=$	p) 0
b) $\sum_{1\le i<j\le 4} x_i x_j=$	q) –16
c) $x_1x_2x_3x_4=$	r) $-\beta$
d) $y_1y_2y_3y_4=$	s) β

Key. a) s; b) p; c) q; d) q

Sol. Conceptual

4.

(A)	If the co-ordinates of a point are (4 tan ϕ, 3 sec ϕ) where ϕ is a parameter then the points lies on a conic section whose eccentricity is	(p)	$\sqrt{3}$
(B)	The eccentricity of conic whose conjugate diameter are y = –x & y = 3x is	(q)	$\frac{\sqrt{3}}{2}$
(C)	If AB is a latus rectum of a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ such that ΔOAB (0 is origin) is an equilateral triangle the eccentricity of hyperbola e is	(r)	$\frac{5}{3}$
(D)	If the foci of the ellipse $\frac{x^2}{k^2a^2}+\frac{y^2}{a^2}=1$ and the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{a^2}=1$ coincide then one of the value of k is equal to	(s)	$\sqrt{\frac{2}{3}}$

Key. (A-r), (B-s), (C-q), (D-p)

Sol. (A) Equation of curve is $y^2/9 - x^2/16 = 1$

$\Rightarrow e = 5/3$

(B) We have $F_1P + F_2P = 2a = 10 \Rightarrow a = 5$

$(F_1B_1)(F_2B_2) = b^2 = 16 \Rightarrow b = 4$

$e=\frac{3}{5}$

(C) $\frac{3}{a^2}-\frac{1}{9}=1$

$\frac{1}{a^2}=\frac{10}{27} \Rightarrow a^2=\frac{27}{10}$

Hence $e = \sqrt{\frac{13}{3}}$

(D) focii of the ellipse $\left(\pm a\sqrt{k^2-1},\ 0\right)$, focii of hyperbola $\left(\pm\sqrt{2}\ a,\ 0\right)$ equating the both focii we get $k = \pm\sqrt{3}$, one of the values of $k = \sqrt{3}$.

5. The normals at four points $(x_i, y_i), i = 1,2,3,4$ on the hyperbola xy=16 are concurrent at the point (α, β)

Column I		Column II	
(A)	$x_1x_2x_3x_4 =$	(P)	β
(B)	$y_1y_2y_3y_4 =$	(Q)	0
(C)	$y_1+y_2+y_3+y_4 =$	(R)	- 256
(D)	$\sum\sum_{1\le i<j\le 4} y_iy_j$	(S)	$-\beta$

Key. A –R; B – R; C – P; D – Q
Sol. Conceptual

6. **Column - I** **Column – II**

Length of latus rectum

a) 8 p) $\frac{x^2}{9}+\frac{y^2}{16}=1$

b) 4.5 q) $\frac{x^2}{16}-\frac{y^2}{25}=1$

c) 12.5 r) $x^2-y^2=4$

d) 4 s) $y^2=8(x-4)$

t) $y^2=4(x+5)$

Key. a) s; b) p; c) q; d) r
Sol. Conceptual Question

7. **Column – I** **Column – II**

a) The eccentricity of the hyperbola $7x^2-2y^2+12xy-2x+14y-22=0$ p) 2

b) The eccentricity of the hyperbola whose latusrectum is half of its transverse axis is q) 4/3

c) The eccentricity of the hyperbola $7y^2-9x^2+54x-28y-116=0$ is r) $\sqrt{3/2}$

d) The eccentricity of the hyperbola $12x^2-4y^2-24x+32y-127=0$ is s) 5/4

t) $\sqrt{3}$

Key. a) t; b) r; c) q; d) p
Sol. Conceptual Question

8. Match the following: -

	Column – I		Column – II
(A)	The area of the triangle that a tangent at a point of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ makes with its asymptotes is	(p)	12
(B)	If the line $y=3x+\lambda$ touches the curve $9x^2-5y^2=45$, then $\lvert\lambda\rvert$ is	(q)	6
(C)	If the chord x cos $\alpha+y\sin\alpha=p$ of the hyperbola $\frac{x^2}{16}-\frac{y^2}{18}=1$ subtends a right angle at the centre, then the diameter of the circle, concentric with the hyperbola, to which the given chord is a tangent is	(r)	24
(D)	If λ be the length of the latus rectum of the hyperbola $16x^2-9y^2+32x+36y-164=0$, then 3λ is equal to	(s)	32
		(t)	3

Key. A $\to$ p; B $\to$ q; C $\to$ r; D $\to$ s

Sol. (A) Equation of tangent at (a, 0) is x = a.

$$y=\frac{b}{a}x$$

$\therefore$ Area = a. b = ab

(B) $y=3x+\lambda$ touches $9x^2-5y^2=45$

$\therefore$ $9x^2-5(3x+\lambda)^2=45$

ie $-36x^2-30\lambda x-5\lambda^2-45=0$

ie $36x^2+30\lambda x+5\lambda^2+45=0$ has equal roots

$\therefore$ $900\,\lambda^2-720\lambda^2-180\times36=0$

ie $\lambda^2=36$ $\lambda=\pm6$

$\therefore$ $\lvert\lambda\rvert=6$

(C) $18x^2-16y^2-288=0$

ie $9x^2-8y^2-144=0$

$$9x^2-8y^2-144\left(\frac{x\cos\alpha+y\sin\alpha}{p}\right)^2=0$$

Since these lines are perpendicular to each other.

$\therefore$ $9p^2-8p^2-144\left(\cos^2\alpha+\sin^2\alpha\right)=0$

$p^2=144$ $p=\pm12$

$\therefore$ radius of the circle = 12

$\therefore$ diameter of the circle = 24

(D) $16x^2+32x+16-9\left(y^2-4y+4\right)-144=0$

ie $16(x+1)^2-9(y-2)^2=144$

ie $\frac{(x+1)^2}{9}-\frac{(y-2)^2}{16}=1$

length of latus rectum $=\frac{2\times16}{3}=\frac{32}{3}$

$\therefore$ $3\lambda=32$

9. Match the following: -

Column – I		Column – II	
(A)	A tangent drawn to hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at $P\left(\frac{\pi}{6}\right)$ forms a triangle of area $3a^2$ square units, with coordinate axes, then the square of its eccentricity is equal to	(p)	17
(B)	If the eccentricity of the hyperbola $x^2-y^2\sec^2\theta=5$ $\sqrt{3}$ times the eccentricity of the ellipse $x^2\sec^2\theta+y^2=25$	(q)	32
(C)	For the hyperbola $\frac{x^2}{3}-y^2=3$, acute angle between its asymptotes is $\frac{l\pi}{24}$, then value of 'l' is	(r)	16
(D)	For the hyperbola xy = 8 any tangent of it at P meets co-ordinate axes at Q and R then area of triangle CQR where 'c' is centre of the hyperbola is	(s)	24
		(t)	8

Key. A $\to$ p; B $\to$ s; C $\to$ t; D $\to$ r

Sol. (A) The point $P\left(\frac{\pi}{6}\right)$ is $\left(a\sec\frac{\pi}{6}, b\tan\frac{\pi}{6}\right)$ i.e. $P\left(\frac{2a}{\sqrt{3}}, \frac{b}{\sqrt{3}}\right)$

$\therefore$ Equation of tangent at P is $\frac{x}{\frac{\sqrt{3}a}{2}}-\frac{y}{\sqrt{3}b}=1$

$\therefore$ area of the triangle $=\frac{1}{2}\times\frac{\sqrt{3}a}{2}\times\sqrt{3}b=3a^2$

$\therefore$ $\frac{b}{a}=4$

$\therefore$ $e^2=1+\frac{b^2}{a^2}=17$

(B) eccentricity of the hyperbola $x^2-y^2\sec^2\theta=5$ is $e_1=\sqrt{\frac{1+\sec^2\theta}{\sec^2\theta}}=\sqrt{1+\cos^2\theta}$

Eccentricity of the ellipse $x^2\sec^2\theta+y^2=25$ is $e_2=\sqrt{\frac{\sec^2\theta-1}{\sec^2\theta}}=|\sin\theta|$

$e_1=\sqrt{3}e_2$ $\Rightarrow$ $1+\cos^2\theta=3\sin^2\theta$ $\Rightarrow$ $\cos\theta=\pm\frac{1}{\sqrt{2}}$

$\therefore$ least positive value of θ is $\frac{\pi}{4}$ $\therefore$ $p=24$

(C) Asymptotes are $x=\pm\sqrt{3}\,y$

$\therefore$ angle between the asymptotes is $\frac{\pi}{3}$ $\therefore$ $l=8$

(D) any point of xy = 8 is $P\left(\sqrt{8}t, \frac{\sqrt{8}}{t}\right)$

$\therefore$ equation of the tangent at P is $\frac{x}{\frac{16t}{\sqrt{8}}}+\frac{y}{\frac{16}{\sqrt{8}t}}=1$

$\therefore$ area of the triangle $=\frac{1}{2}.\frac{16t}{\sqrt{8}}.\frac{6}{\sqrt{8}t}=16$

10. Match the following: -

	Column – I		Column – II
(A)	Value of c for which $3x^2 - 5xy - 2y^2 + 5x + 11y + c = 0$ are the asymptotes of the hyperbola $3x^2 - 5xy - 2y^2 + 5x + 11y - 8 = 0$	(p)	3
(B)	If locus of a point, whose chord of contact with respect to the circle $x^2 + y^2 = 4$ is a tangent to the hyperbola xy = 1 is $xy = c^2$, then value of c^2 is	(q)	- 4
(C)	If equation of a hyperbola whose conjugate axis is 5 and distance between its foci is 13, is $ax^2 - by^2 = c$ where a and b are comprime natural numbers, then value of $\frac{ab}{c}$ is	(r)	- 12
(D)	If the vertex of a hyperbola bisects the distance between its centre and the corresponding focus, then ratio of square of its conjugate axis to the square of its transverse axis is	(s)	4
		(t)	- 6

Key. A $\to$ r; B $\to$ s; C $\to$ s; D $\to$ p

Sol. (A) Since $3x^2 - 5xy - 2y^2 + 5x + 11y + c = 0$ are asymptotes

$\therefore$ it represents a pair of a straight lines

$\therefore$ $3(-2)c + 2.\frac{11}{2}\left(\frac{5}{2}\right)\left(\frac{-5}{2}\right) - 3\left(\frac{11}{2}\right)^2 - (-2)\left(\frac{5}{2}\right)^2 - c\left(-\frac{5}{2}\right)^2 = 0$

i.e. $-6c - \frac{275}{4} - \frac{363}{4} + \frac{25}{2} + ...\frac{25}{4}c = 0$ i.e. - 24c – 275 – 363 + 50 – 25 c = 0

i.e. 49c = 0 588

(B) Let the point be (h, k). Then equation of the chord of contact is hx + ky = 4
Since hx + ky = 4 is tangent to xy = 1

$\therefore$ $x\left(\frac{4 - hx}{k}\right) = 1$ has two equal roots

i.e. $hx^2 - 4x + k = 0$ i.e. hk = 4

$\therefore$ locus of (h, k) is xy = 4 i.e. $c^2 = 4$

(C) Equation of the hyperbola is $\frac{x^2}{c/a} - \frac{y^2}{c/b} = 1$

$\therefore$ $\sqrt{\frac{c}{b}} = \frac{5}{2}$ and $\frac{13}{2} = \sqrt{\frac{c}{a}}.\sqrt{\frac{a+b}{b}}$ $\therefore$ $\frac{c}{a} = 36$

$\therefore$ the hyperbola is $25x^2 - 144y^2 = 900$ $\therefore$ a = 25, b = 144, c = 900

$\therefore$ $\frac{ab}{c} = 4$

(D) Let the hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

then 2a = ae i.e. e = 2

$\therefore$ $\frac{b^2}{a^2} = e^2 - 1 = 3$ $\therefore$ $\frac{(2b)^2}{(2a)^2} = 3$