Ellipse

Single Correct Answer Type

1. If a variable tangent of the circle $x^2 + y^2 = 1$ intersect the ellipse $x^2 + 2y^2 = 4$ at P and Q then the locus of the points of intersection of the tangents at P and Q is

A. a circle of radius 2 units
B. a parabola with fouc as (2, 3)
C. an ellipse with eccentricity $\frac{\sqrt{3}}{4}$
D. an ellipse with length of latus rectrum is 2 units

Key. D

Sol. $x^2 + y^2 = 1; x^2 + 2y^2 = 4$

Let $R(x_1, y_1)$ is pt of intersection of tangents drawn at P,Q to ellipse

$\Rightarrow PQ$ is chord of contact of $R(x_1, y_1)$

$\Rightarrow xx_1 + 2yy_1 - 4 = 0$

This touches circle $\Rightarrow r^2(\ell^2 + m^2) = n^2$

$\Rightarrow 1(x_1^2 + 4y_1^2) = 16$

$\Rightarrow x^2 + 4y^2 = 16$ is ellipse $e = \frac{\sqrt{3}}{2}; LL^1 = 2$

2. A circle S = 0 touches a circle $x^2 + y^2 - 4x + 6y - 23 = 0$ internally and the circle $x^2 + y^2 - 4x + 8y + 19 = 0$ externally. The locus of centre of the circle S = 0 is conic whose eccentricity is k then $\left[\frac{1}{k}\right]$ is where [.] denotes G.I.F

A. 7
B. 2
C. 0
D. 3

Key. A

Sol. $c_1(2, -3) r_1 = 6$

$c_2(2, -4) r_2 = 1$

Let C is the center of S = 0

$\therefore \left.\begin{array}{l} cc_1 = r_1 - r \\ cc_2 = r_1 + r \end{array}\right\} \Rightarrow cc_1 + cc_2 = r_1 + r_2$

$\therefore$ Locus is an ellipse whose foci are (2, -3) & (2, -4)

$e = \frac{2ae}{2a} = \frac{c_1c_2}{r_1 + r_2} = \frac{1}{7} \Rightarrow k = \frac{1}{7}$

3. If circum centre of an equilateral triangle inscribed in $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with vertices having eccentric angles α, β, γ respectively is (x_1, y_1) then $\sum \cos\alpha \cos\beta + \sum \sin\alpha \sin\beta =$

A. $\frac{9x_1^2}{a^2} + \frac{9y_1^2}{b^2} + \frac{3}{2}$ B. $9x_1^2 - 9y_1^2 + a^2b^2$ C. $\frac{9x_1^2}{2a^2} + \frac{9y_1^2}{2b^2} - \frac{3}{2}$ D. $\frac{9x_1^2}{a^2} + \frac{9y_1^2}{b^2} + 3$

Key. C

Sol. $(x_1, y_1) = \left(\frac{a\sum \cos\alpha}{3}, \frac{b\sum \sin\alpha}{3} \right)$

$\sum \cos\alpha = \frac{3x_1}{a}$ ....(1)

$\sum \sin\alpha = \frac{3y_1}{b}$ ....(2)

Squarding & adding

4. The ratio of the area enclosed by the locus of mid-point of PS and area of the ellipse where P is any point on the ellipse and S is the focus of the ellipse, is

A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{1}{5}$ D. $\frac{1}{4}$

Key. D

Sol. Ellipse equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, Area = πab

Let $P = (a\cos\theta, b\sin\theta)$

$S = (ae, 0)$

M(h,k) mid point of PS

$\Rightarrow h = \frac{ae + a\cos\theta}{2}; k = \frac{b\sin\theta}{2}$

$= \frac{h - \frac{ae}{2}}{a/2} + \frac{k^2}{(b^2/4)} = 1$, locus of (h,k) is ellipse

Area $=\pi\left(\frac{a}{2}\right)\left(\frac{b}{2}\right)=\frac{1}{4}\pi ab$

5. How many tangents to the circle $x^2+y^2=3$ are there which are normal to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$

A) 3 B) 2 C) 1 D) 0

Key. D

Sol. Equation of normal at $p(3\cos\theta, 2\sin\theta)$ is $3x\sec\theta-2y\operatorname{cosec}\theta=5$

$$\frac{5}{\sqrt{9\sec^2\theta+4\operatorname{cosec}^2\theta}}=\sqrt{3}$$

But Min. of $9\sec^2\theta+4\operatorname{cosec}^2\theta=25$

$\therefore$ no such $\overset{-1}{\theta}$ exists.

6. If the ellipse $\frac{x^2}{a^2-3}+\frac{y^2}{a+4}=1$ is inscribed in a square of side length $a\sqrt{2}$ then a is

A) 4 B) 2 C) 1 D) None of these

Key. D

Sol. Sides of the square will be perpendicular tangents to the ellipse so, vertices of the square will lie on director circle. So diameter of director circle is

$$2\sqrt{(a^2-3)+(a+4)}=\sqrt{2a^2+2a^2}$$

$$2\sqrt{a^2+a+1}=2a\Rightarrow a=-1$$

But for ellipse $a^2>3\ \&\ a>-4$

So a cannot take the value $'-1'$

7. Let 'O' be the centre of ellipse for which A,B are end points of major axis and C,D are end points of minor axis, F is focus of the ellipse. If in radius of ΔOCF is '1' then $|AB|\times|CD|=$

A) 65 B) 52 C) 78 D) 47

Key. A

Sol. $r=\frac{\Delta}{S}\Rightarrow\Delta=S$

$$\frac{1}{2}(ae)b=\frac{ae+b+\sqrt{a^2e^2+b^2}}{2}$$

$$ae = 6 \Rightarrow 6b = 6 + b + \sqrt{36 + b^2} \Rightarrow b = \frac{5}{2}$$

$$\Rightarrow a^2\left(1 - e^2\right) = \frac{25}{4} \Rightarrow a^2 - 36 = \frac{25}{4} \Rightarrow a = \frac{13}{2}$$

8. If the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ meet the ellipse $\frac{x^2}{1} + \frac{y^2}{a^2} = 1$ in four distinct points and $a = b^2 - 10b + 25$, then the value b does not satisfy

1. $(-\infty, 4)$ 2. (4, 6) 3. $(6, \infty)$ 4. [4, 6]

Key. 4

Sol. a > 1

9. The perimeter of a triangle is 20 and the points (-2, -3) and (-2, 3) are two of the vertices of it. Then the locus of third vertex is :

1. $\frac{(x-2)^2}{49} + \frac{y^2}{40} = 1$ 2. $\frac{(x+2)^2}{49} + \frac{y^2}{40} = 1$ 3. $\frac{(x+2)^2}{40} + \frac{y^2}{49} = 1$ 4. $\frac{(x-2)^2}{40} + \frac{y^2}{49} = 1$

Key. 3

Sol. PA + PB + AB = 20 where A & B are foci

10. Tangents are drawn from any point on the circle $x^2+y^2=41$ to the Ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ then the angle between the two tangents is

1. $\frac{\pi}{4}$ 2. $\frac{\pi}{3}$ 3. $\frac{\pi}{6}$ 4. $\frac{\pi}{2}$

Key. 4

Sol. Director circle

11. The area of the parallelogram formed by the tangents at the points whose eccentric angles are $\theta, \theta + \frac{\pi}{2}, \theta + \pi, \theta + \frac{3\pi}{2}$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

1. ab 2. 4ab 3. 3ab 4. 2ab

Key. 2

Sol. Put $\theta = 0^0$

12. A normal to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the axes in L and M. The perpendiculars to the axes through L and M intersect at P. Then the equation to the locus of P is

1. $a^2x^2-b^2y^2=\left(a^2+b^2\right)^2$
2. $a^2x^2+b^2y^2=\left(a^2+b^2\right)^2$
3. $b^2x^2-a^2y^2=\left(a^2-b^2\right)^2$
4. $a^2x^2+b^2y^2=\left(a^2-b^2\right)^2$

Key. 4

Sol. $P=\left(x_1,y_1\right),\frac{x}{x_1}+\frac{y}{y_1}=1$ Apply normal condition

13. The points of intersection of the two ellipse $x^2+2y^2-6x-12y+23=0$, $4x^2+2y^2-20x-12y+35=0$

1. Lie on a circle centered at $\left(\frac{8}{3},3\right)$ and of radius $\frac{1}{3}\sqrt{\frac{47}{2}}$
2. Lie on a circle centered at $\left(\frac{8}{3},-3\right)$ and of radius $\frac{1}{3}\sqrt{\frac{47}{3}}$
3. Lie on a circle centered at (8,9) and of radius $\frac{1}{3}\sqrt{\frac{47}{2}}$
4. Are not concyclic

Key. 1

Sol. If S_1 = 0 and S_2 = 0 are the equations, Then $\lambda S_1+S_2=0$ is a second degree curve passing through the points of intersection of S_1 = 0 and S_2 = 0

$$\Rightarrow(\lambda+4)x^2+2(\lambda+1)y^2-2(3\lambda+10)x-12(\lambda+1)y+(23\lambda+35)=0$$

For it to be a circle, choose λ such that the coefficients of x^2 and y^2 are equal $\therefore\lambda=2$

This gives the equation of the circle as

$$6\left(x^2+y^2\right)-32x-36y+81=0\{u\sin g(1)\}$$

$$\Rightarrow x^2+y^2-\frac{16}{3}x-6y+\frac{27}{2}=0$$

Its centre is $C\left(\frac{8}{3},3\right)$ and radius is

r = $\sqrt{\frac{64}{9}+9-\frac{27}{2}}=\frac{1}{3}\sqrt{\frac{47}{2}}$

14. In a model, it is shown that an arc of a bridge is semi elliptical with major axis horizontal. If the length of the base is 9m and the highest part of the bridge is 3m from the horizontal; then the height of the arch, 2m from the centre of the base is (in meters)

1. $\frac{8}{3}$ 2. $\frac{\sqrt{65}}{3}$ 3. $\frac{\sqrt{56}}{3}$ 4. $\frac{9}{3}$

Key. 2

Sol. Let the equation of the semi elliptical are be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\left(y>0\right)$

Length of the major axis = 2a = 9 $\Rightarrow$ a = 9/2

So the equation of the arc becomes $\frac{4x^2}{81}+\frac{y^2}{9}=1$

If x=2, then $y^2=\frac{65}{9}\Rightarrow y=\frac{1}{3}\sqrt{65}$

15. If a tangent of slope 2 of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is normal to the circle $x^2+y^2+4x+1=0$ then the maximum value of ab is

1. 2 2. 4 3. 6 3. Can n't be found

Key. 2

Sol. A tangent of slope 2 is $y=2x\pm\sqrt{4a^2+b^2}$ this is normal to $x^2+y^2+4x+1=0$ then

$0=-4\pm\sqrt{4a^2+b^2}\Rightarrow 4a^2+b^2=16$ using $Am\geq GM$

$ab\leq 4$

16. The distance between the polars of the foci of the Ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ w.r.to itself is

1. $\frac{25}{2}$ 2. $\frac{25}{9}$ 3. $\frac{25}{8}$ 4. $\frac{25}{3}$

Key. 1

Sol. $\frac{2a}{e}$

17. An ellipse passing through origin has its foci at (5, 12) and (24, 7). Then its eccentricity is

1. $\frac{\sqrt{386}}{38}$ 2. $\frac{\sqrt{386}}{39}$ 3. $\frac{\sqrt{386}}{47}$ 4. $\frac{\sqrt{386}}{51}$

Key. 1

Sol. Conceptual

18. If $e=\frac{\sqrt{3}}{2}$, its length of latusrectum is

1. $\frac{1}{2}$ (length of major axis)
2. $\frac{1}{3}$ (length of major axis)
3. $\frac{1}{4}$ (length of major axis)
4. Length of major axis

Key. 3

Sol. $L.L.R=\frac{2b^2}{a}$

19. Number of normals that can be drawn from the point (0, 0) to $3x^2+2y^2=30$ are

1. 2
2. 4
3. 1
4. 3

Key. 2

Sol. It is centre

20. A tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ cuts the axes in M and N. Then the least length of MN is

1. a + b
2. a – b
3. $a^2 + b^2$
4. $a^2 – b^2$

Key. 1

Sol. Standard

21. $p(\theta),\ D\left(\theta+\frac{\pi}{2}\right)$ are two points on the Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ Then the locus of point of intersection of the two tangents at P and D to the ellipse is

1. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{4}$
2. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=4$
3. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$
4. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{2}$

Key. 3

Sol. $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$ → 1eq

$\frac{x}{a}\cos\left(\frac{\pi}{2}+\theta\right)+\frac{y}{b}\sin\left(\frac{\pi}{2}+\theta\right)=1$ → 2eq

Eliminate θ from 1 and 2

22. The abscissae of the points on the ellipse $9x^2+25y^2-18x-100y-116=0$ lie between

1. 3, -5
2. -4, 6
3. 5, 7
4. 2, 5

Key. 2

Sol. $-5\geq x-1\leq 5$

23. Tangents to the ellipse $b^2x^2+a^2y^2=a^2b^2$ makes angles $\theta_1 \, and \, \theta_2$ with major axis such that $\cot\theta_1+\cot\theta_2=k$. Then the locus of the point of intersection is

1. $xy=2k(y^2+b^2)$ 2. $2xy=k(y^2-b^2)$ 3. $4xy=k(y^2-b^2)$ 4. $8xy=k(y^2-b^2)$

Key. 2

Sol. Apply sum of the slopes = $\frac{2x_1y_1}{x_1^2-a^2}$

24. The equation $\frac{x^2}{10-a}+\frac{y^2}{4-a}=1$ represents an ellipse if

1. a < 4 2. a > 4 3. 4 <a < 10 4. a > 10

Key. 1

Sol. 10 – a > 0, 4 – a > 0

25. The locus of the feet of the perpendiculars drawn from the foci of the ellipse S=0 to any tangent to it is

1. a circle 2. an ellipse 3. a hyperbola 4. not a conic

Key. 1

Sol. Standard

26. If the major axis is "n" (n>1) times the minor axis of the ellipse, then eccentricity is

1. $\frac{\sqrt{n-1}}{n}$ 2. $\frac{\sqrt{n-1}}{n^2}$ 3. $\frac{\sqrt{n^2-1}}{n^2}$ 4. $\frac{\sqrt{n^2-1}}{n}$

Key. 4

Sol. 2a = n(2b)

$$\Rightarrow n=\frac{a}{b}$$

$$\therefore e=\sqrt{\frac{a^2-b^2}{a^2}}=\sqrt{1-\frac{b^2}{a^2}}=$$

$$\sqrt{1-\frac{1}{n^2}}=\frac{\sqrt{n^2-1}}{n}$$

27. If $\left(\sqrt{3}\right)bx+ay=2ab$ is tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then eccentric angle θ is

1. $\frac{\pi}{4}$ 2. $\frac{\pi}{6}$ 3. $\frac{\pi}{2}$ 4. $\frac{\pi}{3}$

Key. 2

Sol. Equation of tangent at a point $(a\cos\theta, b\sin\theta)$ is $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$

But, it is the same as $\frac{x}{a}\frac{\sqrt{3}}{2}+\frac{y}{b}.\frac{1}{2}=1$

$\therefore \cos\theta=\frac{\sqrt{3}}{2}, \sin\theta=\frac{1}{2} \Rightarrow \theta=\frac{\pi}{6}$

28. If PSQ is a focal chord of the ellipse $16x^2+25y^2=400$ such that SP = 8 then the length of SQ =

1. 2 2. $\frac{11}{3}$ 3. 16 4. 25

Key. 1

Sol. $\frac{1}{SP}+\frac{1}{SQ}=\frac{2a}{b^2}$

29. A man running round a race course notes that the sum of the distances of two flag posts from him is 8 meters. The area of the path he encloses in square meters if the distance between flag posts is 4 is

1. $15\sqrt{3}\pi$ 2. $12\sqrt{3}\pi$ 3. $18\sqrt{3}\pi$ 4. $8\sqrt{3}\pi$

Key. 4

Sol. Area = πab

30. The locus of point of intersection of the two tangents to the ellipse $b^2x^2+a^2y^2=a^2b^2$ which makes an angle 60^0 with one another is

1. $4\left(x^2+y^2-a^2-b^2\right)^2=3\left(b^2x^2+a^2y^2-a^2b^2\right)$

2. $3\left(x^2+y^2-a^2-b^2\right)^2=4\left(b^2x^2+a^2y^2-a^2b^2\right)$

3. $3\left(x^2+y^2-a^2-b^2\right)^2=2\left(b^2x^2+a^2y^2-a^2b^2\right)$

4. $3\left(x^2+y^2-a^2-b^2\right)^2=\left(b^2x^2+a^2y^2-a^2b^2\right)$

Key. 2

Sol. $Tan\theta=\frac{2ab\sqrt{S_{11}}}{{x_1}^2+{y_1}^2-a^2-b^2}$

31. If the equation of the chord joining the points $P(\theta)$ and $D\left(\theta+\frac{\pi}{2}\right)$ on $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $x\cos\alpha+y\sin\alpha=p$ then $a^2\cos^2\alpha+b^2\sin^2\alpha=$

1. $4p^2$ 2. p^2 3. $\frac{p^2}{2}$ 4. $2p^2$

Key. 4

Sol. $\frac{x}{a}\cos\left(\frac{\theta+\theta+\frac{\pi}{2}}{2}\right)+\frac{y}{b}\sin\left(\frac{\theta+\theta+\frac{\pi}{2}}{2}\right)$

$=\cos\left(\frac{\theta-\theta-\frac{\pi}{2}}{2}\right)$ → 1eq

$x\cos\alpha+y\sin\alpha=P$ → 2eq

(1) = (2)

32. The locus of mid point of chords of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which passes through the foot of the directrix from focus is

1. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{ex}{a^2}$ 2. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{x}{ae}$ 3. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{x}{a^2e}$ 4. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{x}{ae^2}$

Key. 2

Sol. $S_1 = S_{11}$ passes through $\left(\frac{a}{e},0\right)$

33. Consider two points A and B on the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$, circles are drawn having segments of tangents at A and B in between tangents at the two ends of major axis of ellipse as diameter, then the length of common chord of the circles is

A) 8 B) 6 C) 10 D) $4\sqrt{2}$

Key. A

Sol. All such circles pass through foci $\therefore$ The common chord is of the length 2ae

$10\times\frac{4}{5}=8$

34. If 'CF' is the perpendicular from the centre C of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ on the tangent at any point P and G is the point where the normal at P meets the major axis, then CF.PG is

A) b^2 B) $2b^2$ C) $\frac{b^2}{2}$ D) $3b^2$

Key. A

Sol. $CF=\frac{ab}{\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}}$ $PG=\frac{b}{a}\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}$

35. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2+9y^2=9,$ meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin 'O' is

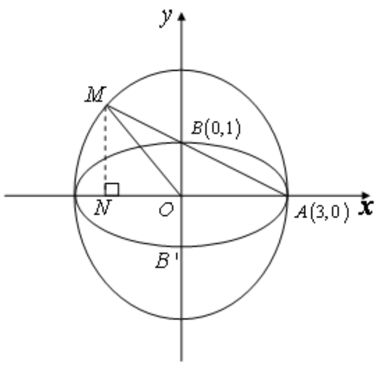
A) $\frac{31}{10}$ B) $\frac{29}{10}$ C) $\frac{21}{10}$ D) $\frac{27}{10}$

Key. D

Sol. Equation of given ellipse is $\frac{x^2}{9}+\frac{y^2}{1}=1$

Equation of auxiliary circle is $x^2+y^2=9.........(1)$

Equation of line AB is $\frac{x}{3}+\frac{y}{1}=1 \Rightarrow x=3(1-y)$



Putting this in (1), we get $9(1-y)^2+y^2=9 \Rightarrow 10y^2-18y=0 \Rightarrow y=0,\frac{9}{5}$

Thus, y coordinate of 'M' is $\frac{9}{5}$

$$\Delta OAM=\left(\frac{1}{2}\right)(OA)(MN)=\frac{1}{2}(3)\frac{9}{5}=\frac{27}{10}$$

36. The normal at an end of a latus rectum of the ellipse $x^2/a^2+y^2/b^2=1$ passes through an end of the minor axis if

(a) $e^4+e^2=1$ (b) $e^3+e^2=1$ (c) $e^2+e=1$ (d) $e^3+e=1$

Key. A

Sol. Given ellipse equation is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Let $P\left(ae,\frac{b^2}{a}\right)$ be one end of latus rectum.

Slope of normal at $P\left(ae,\frac{b^2}{a}\right)=\frac{1}{e}$

Equation of normal is

$$y=\frac{b^2}{a}=\frac{1}{e}(x-ae)$$

It passes through $B^{'}(0,b)$ then

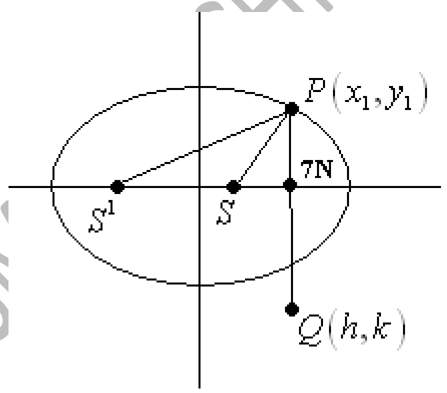
$$b-\frac{b^2}{a}=-a$$
$$a^2-b^2=-ab$$
$$a^4e^4=a^2b^2$$
$$e^4+e^2=1$$

37. From any point P lying in first quadrant on the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, PN is drawn perpendicular to the major axis such that N lies on major axis. Now PN is produced to the point Q such that NQ equals to PS, where S is a focus. The point Q lies on which of the following lines

(A) $2y-3x-25=0$ (B) $3x+5y+25=0$

(C) $2x-5y-25=0$ (D) $2x-5y+25=0$

Key. B



Sol.

$$a^2=25$$
$$b^2=16$$
$$e=\sqrt{\frac{25-16}{25}}=\frac{3}{5}$$

Let point Q be (h, k), where K < 0

Given that $|K|=a+eh\left(as\ x_1=h\right)$

$$-y=a+ex$$

$$-y=5+\frac{3}{5}x$$
$$3x+5y+25=0$$

38. A circle of radius 'r' is concentric with the Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Then inclination of common tangent with major axis is ________(b<r<a)

1. $\tan^{-1}\left(\frac{b}{a}\right)$ 2. $\tan^{-1}\left(\frac{rb}{a}\right)$ 3. $\tan^{-1}\sqrt{\frac{r^2-b^2}{a^2-r^2}}$ 4. $\frac{\pi}{2}$

Key. 3

Sol. The tangent of Ellipse is $y=mx+\sqrt{a^2m^2+b^2}$, this line touches $x^2+y^2=r^2$

Condition is $\left|\frac{\sqrt{a^2m^2+b^2}}{\sqrt{m^2+1}}\right|=r$

$$a^2m^2+b^2=r^2m^2+r^2$$

$$m^2\left(a^2-r^2\right)=r^2-b^2 \Rightarrow m^2=\frac{r^2-b^2}{a^2-r^2}$$

$$m=\sqrt{\frac{r^2-b^2}{a^2-r^2}}$$

Inclimation is $\tan^{-1}\sqrt{\frac{r^2-b^2}{a^2-r^2}}$

39. A circle cuts the X-axis and Y-axis such that intercept on X-axis is a constant a and intercept on Y-axis is a constant b. Then eccentricity of locus of centre of circle is

1. 1 2. $\frac{1}{2}$ 3. $\sqrt{2}$ 4. $\frac{1}{\sqrt{2}}$

Key. 3

Sol. Locus of centre of circle is a rectangular hyperbola hence its eccentricity is $\sqrt{2}$

40. Consider two points A and B on the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$, circles are drawn having segments of tangents at A and B in between tangents at the two ends of major axis of ellipse as diameter, then the length of common chord of the circles is

A) 8 B) 6 C) 10 D) $4\sqrt{2}$

Key. A

Sol. All such circles pass through foci $\therefore$ The common chord is of the length 2ae

$$10\times\frac{4}{5}=8$$

41. If 'CF' is the perpendicular from the centre C of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ on the tangent at any point P and G is the point where the normal at P meets the major axis, then CF.PG is

A) b^2 B) $2b^2$ C) $\frac{b^2}{2}$ D) $3b^2$

Key. A

Sol. $CF=\frac{ab}{\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}}$ $PG=\frac{b}{a}\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}$

42. The line passing through the extremity A of the major axis and extremity B of the minor axis
of the ellipse $x^2+9y^2=9,$ meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin 'O' is

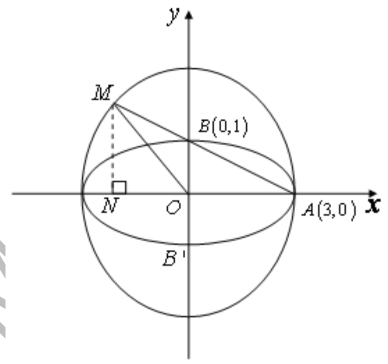
A) $\frac{31}{10}$ B) $\frac{29}{10}$ C) $\frac{21}{10}$ D) $\frac{27}{10}$

Key. D

Sol. Equation of given ellipse is $\frac{x^2}{9}+\frac{y^2}{1}=1$

Equation of auxiliary circle is $x^2+y^2=9.........(1)$

Equation of line AB is $\frac{x}{3}+\frac{y}{1}=1\Rightarrow x=3(1-y)$



Putting this in (1), we get $9(1-y)^2+y^2=9\Rightarrow 10y^2-18y=0\Rightarrow y=0,\frac{9}{5}$

Thus, y coordinate of 'M' is $\frac{9}{5}$

$$\Delta OAM=\left(\frac{1}{2}\right)(OA)(MN)=\frac{1}{2}(3)\frac{9}{5}=\frac{27}{10}$$

43. If $2x^2+y^2-24y+80=0$ then maximum value of x^2+y^2 is

A. 20 B. 40

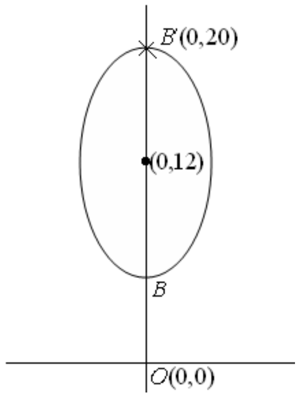
C. 200 D. 400

Key. D

Sol. Given equation is $2x^2+y^2-24y+80=0$

$$2x^2+(y-12)^2=64$$

$$\frac{x^2}{32}+\frac{(y-12)^2}{64}=1$$



If is an ellipse with center (0, 12)

If (x, y) is any point on this distance from origin is $\sqrt{x^2+y^2}$

x^2+y^2 is max If $\sqrt{x^2+y^2}$ is

max

$B^1(1,\infty)$ is at max distance from 0

$\therefore \max(x^2+y^2)=400$

44. An ellipse whose foci $(2,4)(14,9)$ touches x –axis then its eccentricity is

A. $\frac{13}{\sqrt{313}}$ B. $\frac{1}{\sqrt{313}}$

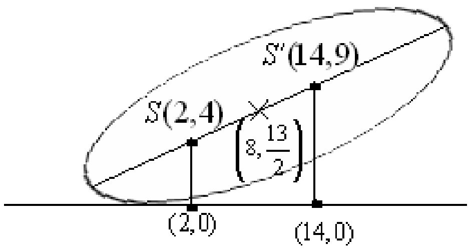
C. $\frac{2}{\sqrt{313}}$ D. $\frac{1}{\sqrt{13}}$

Key. A

Sol. Equation of aurally circle $(x-8)^2+\left(y-\frac{13}{2}\right)^2=a^2$

(2, 0) lies on it

$$36+\frac{169}{4}=a^2 \Rightarrow \frac{313}{4}=a^2$$



$a = \frac{\sqrt{313}}{2}$

But $SS' = 2ae$

$\sqrt{144 + 25} = 2ae$

$13 = 2ae$

$e = \frac{13}{2a} = \frac{13}{\sqrt{313}}$

45. A circle of radius 2 is concentric with the ellipse $\frac{x^2}{7} + \frac{y^2}{3} = 1$ then inclination of common tangent with X-axis

A. $\frac{\pi}{2}$ B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$ D. $\frac{\pi}{6}$

Key. D

Sol. tangent is $y = mx + \sqrt{7m^2 + 3}$

$\frac{x^2}{7} + \frac{y^2}{3} = 1$

$x^2 + y^2 = 4$

It is also touching $x^2 + y^2 = 4$

$\left| \frac{\sqrt{7m^2 + 3}}{\sqrt{m^2 + 1}} \right| = 2$

$7m^2 + 3 = 4m^2 + 4$

$m^2 = \frac{1}{3} \Rightarrow m = \frac{1}{\sqrt{3}}$

$\therefore \tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$

46. The points of intersection of two ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1$ be at the extremeties of conjugate diameters of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then $\frac{a^2}{\alpha^2}+\frac{b^2}{\beta^2}=$

A. 1 B. 2 C. 3 D. 4

Key. B

Sol. Clearly $P(a\cos\theta, b\sin\theta)$ $Q(-a\sin\theta, b\cos\theta)$ are extremities of conjugate diameters of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

P and Q lies m $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1$

$$\frac{a^2\cos^2\theta}{\alpha^2}+\frac{b^2\sin^2\theta}{\beta^2}=1$$

$$\frac{a^2\sin^2\theta}{\alpha^2}+\frac{b^2\cos^2\theta}{\beta^2}=1$$

(+) $\frac{a^2}{\alpha^2}+\frac{b^2}{\beta^2}=2$

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47. From the focus $(-5,0)$ of the ellipse $\frac{x^2}{45}+\frac{y^2}{20}=1$ a ray of light is sent which makes angle $\cos^{-1}\left(\frac{-1}{\sqrt{5}}\right)$ with the positive direction of X-axis upon reacting the ellipse the ray is reflected from it. Slope of the reflected ray is

A) $-3/2$ B) $-7/3$ C) $-5/4$ D) $-2/11$

Key. D

Sol. Let $\theta=\cos^{-1}\left(\frac{-1}{\sqrt{5}}\right)\Rightarrow\cos\theta=\frac{-1}{\sqrt{5}}\Rightarrow\tan\theta=-2$

Foci are $(\pm5,0)$

Equation of line through (-5, 0) with slope –2 is $y=-29x+5)=-2x-10$

This line meets the ellipse above X-axis at $(-6,2)$

$\therefore$ Slope = $\frac{2-0}{-6-5}=-\frac{2}{11}$.

48. If $f(x)$ is a decreasing function for all $x\in R$ and $f(x)>0\ \forall x\in R$ then the range of *K* so that the equation $\frac{x^2}{f(K^2+2K+5)}+\frac{y^2}{f(K+11)}=1$ represents an ellipse whose major axis is the X-axis is

A) $(-2,3)$ B) $(-3,2)$

C) $(-\infty,-3)\cup(2,\infty)$ D) $(-\infty,-2)\cup(3,\infty)$

Key. B

Sol. Conceptual

49. P,Q are points on the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ such that PQ is a chord through the point $R(3,0)$. If $|PR|=2$ then length of chord PQ is

A) 8 B) 6 C) 10 D) 4

Key. C

Sol. Conceptual

50. Let $Q=(3,\sqrt{5}),R=(7,3\sqrt{5})$. A point *P* in the XY-plane varies in such a way that perimeter of ΔPQR is 16. Then the maximum area of ΔPQR is

A) 6 B) 12 C) 18 D) 9

Key. B

Sol. *P* lies on the ellipse for which Q,R are foci and length of major axis is 10 and eccentricity is 3/5.

51. *O* is the centre of ellipse for which A,B are end points of major axis and C,D are end points of minor axis. *F* is a focus of the ellipse. If $|OF|=6$ and inradius of ΔOCF is 1, then $|AB|\times|CD|=$

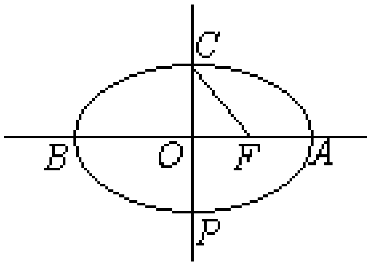
A) 65 B) 52 C) 78 D) 47

Key. A

Sol. $b^2=\frac{25}{4}\Rightarrow a^2-a^2e^2=\frac{25}{4}\Rightarrow a^2=\frac{25}{4}+36=\frac{169}{4}\Rightarrow a=\frac{13}{2}$

$$|OF| = ae = 6 \Rightarrow \frac{abe}{2} = 1 \times \frac{(ae + b + \sqrt{a^2e^2 + b^2})}{2}$$

$$6b = 6 + b + \sqrt{b^2 + 36} \Rightarrow (5b - 6)^2 = b^2 + 36 \Rightarrow 24b^2 = 60b \Rightarrow b = 5/2$$



52. A triangle is formed by a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the coordinate axes. The area of the triangle cannot be less than

a) $\frac{a^2 + b^2}{2}$ sq units

b) $\frac{a^2 + ab + b^2}{3}$ sq units

c) $\frac{a^2 + 2ab + b^2}{2}$ sq units

d) ab sq units

Key. D

Sol. Equation of tangent at θ is $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$

Area with the axes is $\frac{ab}{\sin 2\theta} \geq ab$

53. Equation of circle of minimum radius which touches both the parabolas $y = x^2 + 2x + 4$ and $x = y^2 + 2y + 4$ is

a) $2x^2 + 2y^2 - 11x - 11y - 13 = 0$

b) $4x^2 + 4y^2 - 11x - 11y - 13 = 0$

c) $3x^2 + 3y^2 - 11x - 11y - 13 = 0$

d) $x^2 + y^2 - 11x - 11y - 13 = 0$

Key. B

Sol. Circle will be touching both parabolas. Circles centre will be on the common normal

54. Image of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ in the line x + y = 10 is :

a) $\frac{(x-10)^2}{16} + \frac{(y-10)^2}{25} = 1$

b) $\frac{(x-10)^2}{25} + \frac{(y-10)^2}{16} = 1$

c) $\frac{(x-5)^2}{16} + \frac{(y-5)^2}{25} = 1$

d) $\frac{(x-5)^2}{25} + \frac{(y-5)^2}{16} = 1$

Key. A

Sol. Conceptual

55. Length of common tangent to $x^2 + y^2 = 16$ and $\frac{x^2}{25} + \frac{y^2}{7} = 1$

a) $\frac{9}{4\sqrt{2}}$ b) $\frac{9}{4}$ c) $\frac{9}{2\sqrt{2}}$ d) $\frac{9}{2}$

Key. B

Sol. $y = -x + 4\sqrt{2}$ is a common tangent to two curves in the 1st quadrant. Touching the curves at $P\left(2\sqrt{2}, 2\sqrt{2}\right) \& Q\left(\frac{25}{4\sqrt{2}}, \frac{7}{4\sqrt{2}}\right)$

PQ = length of common tangent.

56. An ellipse having foci S (3, 4) & S' (6, 8) passes through the point P(0, 0). The equation of the tangent at P to the ellipse is

a) $4x + 3y = 0$ b) $3x + 4y = 0$ c) $x + y = 0$ d) $x - y = 0$

Key. B

Sol. Normal at a point is bisector of angle SPS'

57. The angle subtended at the origin by a common tangent of the ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2x}{c} = 0$ and $\frac{x^2}{b^2} + \frac{y^2}{a^2} + \frac{2x}{c} = 0$, is

a) $\pi/6$ b) $\pi/4$ c) $\pi/3$ d) $\pi/2$

Key. D

Sol. Conceptual

58. Let a hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. The transverse and conjugate axes of this hyperbola coincide with the major and minor axes of the given ellipse, also the product of eccentricities of given ellipse and hyperbola is 1, then

a) The equation of hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = 1$ b) The equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{25} = 1$

c) Focus of hyperbola is (5, 0) d) vertex of hyperbola is $\left(5\sqrt{3}, 0\right)$

Key. C

Sol. Conceptual

59. If the normals at 4 points having eccentric angles $\alpha, \beta, \gamma, \delta$ on an ellipse be concurrent, then $\left(\sum \cos\alpha\right)\left(\sum \sec\alpha\right) =$

a) 4 b) $(\alpha\beta\gamma\delta)^{\frac{1}{4}}$

c) $\dfrac{\alpha+\beta+\gamma+\delta}{4}$ d) None of these

Key. A

Sol. Conceptual

60. If the length of the major axis intercepted between the tangent & normal at a point $(a\cos\theta, b\sin\theta)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to the length of semi-major axis, then , eccentricity of the ellipse is,

a) $\dfrac{\cos\theta}{\sqrt{1-\cos\theta}}$ b) $\dfrac{\sqrt{1-\cos\theta}}{\cos\theta}$

c) $\dfrac{\sqrt{1-\cos\theta}}{\sin\theta}$ d) $\dfrac{\sin\theta}{\sqrt{1-\sin\theta}}$

Key. B

Sol. $\dfrac{a}{\cos\theta}-\dfrac{(a^2-b^2)}{a}\cos\theta = a \Rightarrow e^2\cos^2\theta = 1-\cos\theta \Rightarrow e=\dfrac{\sqrt{1-\cos\theta}}{\cos\theta}$

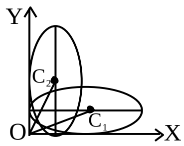
61. An ellipse with major and minor axes of lengths $10\sqrt{3}$ and 10 respectively slides along the co-ordinate axes and always remains confined in the first quadrant. The length of the arc of the locus of the centre of the ellipse is

(A) 10π (B) 5π

(C) $\dfrac{5\pi}{4}$ (D) $\dfrac{5\pi}{3}$

Key. D

Sol. The locus of the centre of the ellipse is director circle ie $x^2+y^2=100$



$C_1OC_2=\theta$

$\Rightarrow \dfrac{\pi}{2}-2\tan^{-1}\left(\dfrac{5}{5\sqrt{3}}\right)=\dfrac{\pi}{6}$

$\therefore$ arc length = $10.\dfrac{\pi}{6}=\dfrac{5\pi}{3}$

62. Tangents drawn to the ellipse $\frac{x^2}{36}+\frac{y^2}{9}=1$ from the point P meet the co-ordinate axes at concyclic points . The locus of the point P is

(A) $x^2+y^2=7$ (B) $x^2+y^2=25$

(C) $x^2-y^2=7$ (D) $x^2-y^2=25$

Key. C

Sol. Let $P=(h,k)$

Equation of any tangent is $y=mx\pm\sqrt{16m^2+9}$

$\Rightarrow k=mh\pm\sqrt{16m^2+9}$

$\Rightarrow m^2\left(h^2-16\right)-2mhk+\left(k^2-9\right)=0$

Let m_1,m_2 are the slope of the tangents $m_1m_2=\dfrac{k^2-9}{h^2-16}$

For concyclic points $m_1m_2=1$

$\Rightarrow h^2-16=k^2-9$

$\Rightarrow h^2-k^2=7 \quad \Rightarrow x^2-y^2=7$

63. The line $2px+y\sqrt{1-p^2}=1(|p|<1)$ for different value of p touches.

(A) An ellipse of eccentricity $\dfrac{2}{\sqrt{3}}$ (B) An ellipse of eccentricity $\dfrac{\sqrt{3}}{2}$

(C) Hyperbola of eccentricity 2 (D) None

Key. B

Sol. $y=\dfrac{-2p}{\sqrt{1-p^2}}x+\dfrac{1}{\sqrt{1-p^2}}$

$m=-\dfrac{2p}{\sqrt{1-p^2}}\Rightarrow p^2=\dfrac{m^2}{4+m^2}$

$y=mx+\dfrac{1}{\sqrt{1-\dfrac{m^2}{4+m^2}}}\Rightarrow y=mx+\sqrt{\dfrac{4+m^2}{4}}$

$\Rightarrow y=mx+\sqrt{1+\dfrac{1}{4}m^2}$

It touches $\dfrac{x^2}{1/4}+\dfrac{y^2}{1}=1, e=\dfrac{\sqrt{3}}{2}$

64. The normal to the curve x^2+3y^2-4-0 at the point $P\left(\pi/6\right)$ intersects the curve again at the point $Q(\theta),\theta$ being the eccentric angle at the point Q then $\theta=$ __ .

A) 0 B) $\pi/2$ C) π D) $3\pi/2$

Key. D

Sol. Given curve is $\dfrac{x^2}{4}+\dfrac{y^2}{4/3}=1$ point $P\left(2\cos\pi/6, 2/\sqrt{3}\sin\pi/6\right)=\left(\sqrt{3},\dfrac{1}{\sqrt{3}}\right)$.

Equation of the normal at P is $x - y = \sqrt{3} - \frac{1}{\sqrt{3}}$ it passes through

$Q(\theta) = \left(2\cos\theta, \frac{2}{\sqrt{3}}\sin\theta\right) \Rightarrow \theta = \frac{3\pi}{2}$

65. If tangents PQ and PR are drawn from a point on the circle $x^2 + y^2 = 25$ to the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1 \ (b < 4)$ so that the fourth vertex 'S' of parallelogram PQSR lies on the circum circle of triangle PQR, then the eccentricity of the ellipse is

A) $\sqrt{5}/4$ B) $\sqrt{7}/3$ C) $\sqrt{7}/4$ D) $\sqrt{5}/3$

Key. C

Sol. A cyclic parallelogram will be rectangle or square. $\therefore \angle QPR = 90^O \Rightarrow 'P'$ lies on director circle

$\Rightarrow b^2 = 9 \therefore e = \sqrt{7}/4 \ \left(b^2 = a^2\left(1 - e^2\right)\right)$

66. If A and B are foci of ellipse $(x - 2y + 3)^2 + (8x + 4y + 4)^2 = 20$ and P is any point on it, then PA+PB=__.

A) 2 B) 4 C) $\sqrt{2}$ D) $2\sqrt{2}$

Key. B

Sol. $\frac{\left(\frac{x - 2y + 3}{\sqrt{5}}\right)}{4} + \frac{\left(\frac{2x - y + 1}{\sqrt{5}}\right)}{1/4} = 1 \Rightarrow PA + PB = 2a = 4$

67. The ratio of the area enclosed by the locus of the midpoint of PS and area of the ellipse is ___(P-be any point on the ellipse and S, its focus)

A) $1/2$ B) $1/3$ C) $1/5$ D) $1/4$

Key. D

Sol. mid point of PS is (h,k) & $h = \frac{a\cos\theta + ae}{2} \Rightarrow \cos\theta = \frac{2h - ae}{a}$; $k = \frac{b\sin\theta}{2}$

$\frac{(2h - ae)^2}{a^2} + \frac{4k^2}{b^2} = 1 \Rightarrow \frac{\left(h - \frac{ae}{2}\right)^2}{a^2/4} + \frac{k^2}{b^2/4} = 1$ its area $\Rightarrow \pi . \frac{a}{2} . \frac{b}{2} = \frac{\pi ab}{4}$. $\therefore$ ratio $= 1/4$

68. The normal to the curve $x^2 + 3y^2 - 4 = 0$ at the point $P(\pi/6)$ intersects the curve again at the point $Q(\theta)$, θ being the eccentric angle of the point Q, then θ =

(A) 0 (B) $\pi/2$

(C) π (D) $3\pi/2$

Key. D

Sol. The given curve is $\frac{x^2}{4} + \frac{y^2}{4/3} = 1$. Point P is $\left(2\cos\frac{\pi}{6}, \frac{2}{\sqrt{3}}\sin\frac{\pi}{6}\right) \equiv \left(\sqrt{3}, \frac{1}{\sqrt{3}}\right)$

On differentiating the given equation, w.r.t. x, we get x + 3y $\frac{dy}{dx} = 0$ ⇒ $\left[\frac{dy}{dx}\right]_P = \left[-\frac{x}{3y}\right]_P = -1$

The equation of normal is y - $\frac{1}{\sqrt{3}} = 1\left(x - \sqrt{3}\right)$ ⇒ x - y = $\sqrt{3} - \frac{1}{\sqrt{3}}$. Normal passes through Q(θ). Hence 2cosθ - $\frac{2}{\sqrt{3}}\sin\theta = \sqrt{3} - \frac{1}{\sqrt{3}}$

⇒ $2\sqrt{3}$ cosθ - 2sinθ = 2 ⇒ $\sqrt{3}$ cosθ - sinθ = 1 ⇒ θ = $\frac{3\pi}{2}$.

69. If the curve $x^2 + 3y^2 = 9$ subtends an obtuse angle at the point (2α, α), then a possible value of α^2 is

(A) 1 (B) 2

(C) 3 (D) 4

Key. B

Sol. The given curve is $\frac{x^2}{9} + \frac{y^2}{3} = 1$, whose director circle is $x^2 + y^2 = 12$. For the required condition (2α, α) should lie inside the circle and outside the ellipse i.e.,

$(2\alpha)^2 + 3\alpha^2 - 9 > 0$ and $(2\alpha)^2 + \alpha^2 - 12 < 0$ i.e., $\frac{9}{7} < \alpha^2 < \frac{12}{5}$.

70. If the tangent at Point P to the ellipse $16x^2 + 11y^2 = 256$ is also the tangent to the circle $x^2 + y^2 - 2x = 15$, then the eccentric angle of point P is

(A) $\pm\frac{\pi}{2}$ (B) $\pm\frac{\pi}{4}$

(C) $\pm\frac{\pi}{3}$ (D) $\pm\frac{\pi}{6}$

Key. C

Sol. The equation of tangent at point $P\left(4\cos\theta, \frac{16}{\sqrt{11}}\sin\theta\right)$ to the ellipse

$16x^2 + 11y^2 = 256$ is

$16x(4\cos\theta) + 11y\left(\frac{16}{\sqrt{11}}\sin\theta\right) = 256$

$4x\cos\theta + \sqrt{11}\, y\sin\theta = 16$

This touches the circle

$(x + 1)^2 + y^2 = 16$

So, $\frac{|4\cos\theta - 16|}{\sqrt{16\cos^2\theta + 11\sin^2\theta}} = 4$

⇒ $(\cos\theta - 4)^2 = 11 + 5\cos^2\theta$

$4\cos^2\theta + 8\cos\theta - 5 = 0$

$\therefore \cos\theta = \dfrac{1}{2}$

$\therefore \theta = \pm \dfrac{\pi}{3}$

71. From the focus $(-5,0)$ of the ellipse $\dfrac{x^2}{45}+\dfrac{y^2}{20}=1$ a ray of light is sent which makes angle $\cos^{-1}\left(\dfrac{-1}{\sqrt{5}}\right)$ with the positive direction of X-axis upon reacting the ellipse the ray is reflected from it. Slope of the reflected ray is

A) $-3/2$ B) $-7/3$ C) $-5/4$ D) $-2/11$

Key. D

Sol. Let $\theta=\cos^{-1}\left(\dfrac{-1}{\sqrt{5}}\right) \Rightarrow \cos\theta=\dfrac{-1}{\sqrt{5}} \Rightarrow \tan\theta=-2$

Foci are $(\pm 5,0)$

Equation of line through (-5, 0) with slope –2 is $y=-29x+5)=-2x-10$

This line meets the ellipse above X-axis at $(-6,2)$

$\therefore$ Slope = $\dfrac{2-0}{-6-5}=-\dfrac{2}{11}$.

72. If $f(x)$ is a decreasing function for all $x\in R$ and $f(x)>0\ \forall x\in R$ then the range of *K* so that the equation $\dfrac{x^2}{f(K^2+2K+5)}+\dfrac{y^2}{f(K+11)}=1$ represents an ellipse whose major axis is the X-axis is

A) $(-2,3)$ B) $(-3,2)$

C) $(-\infty,-3)\cup(2,\infty)$ D) $(-\infty,-2)\cup(3,\infty)$

Key. B

Sol. Conceptual

73. P,Q are points on the ellipse $\dfrac{x^2}{25}+\dfrac{y^2}{16}=1$ such that PQ is a chord through the point $R(3,0)$. If $|PR|=2$ then length of chord PQ is

A) 8 B) 6 C) 10 D) 4

Key. C

Sol. Conceptual

74. Let $Q=(3,\sqrt{5}), R=(7,3\sqrt{5})$. A point *P* in the XY-plane varies in such a way that perimeter of ΔPQR is 16. Then the maximum area of ΔPQR is

A) 6 B) 12 C) 18 D) 9

Key. B

Sol. *P* lies on the ellipse for which Q, R are foci and length of major axis is 10 and eccentricity is 3/5.

75. If the curve $x^2+3y^2=9$ subtends as obtuse angle at the point $(2\alpha,\alpha)$ ($(\alpha \in \text{integer})$, then a possible value of α^2 is

A) 1 B) 2 C) 3 D) 4

Key. B

Sol. The generated curve is $\frac{x^2}{9}+\frac{y^2}{3}=1$, whose director circle is $x^2+y^2=12$. For the required condition $(2\alpha,\alpha)$ should lie inside the circle and out side the ellipse i.e.

$$(2\alpha)^2+3\alpha^2-9>0 \,\&\, (2\alpha)^2+\alpha^2-12<0 \Rightarrow \frac{9}{7}<\alpha^2<\frac{12}{5}$$

76. Tangent at any point $'P'$ of ellipse $9x^2+16y^2-144=0$ is drawn. Eccentric angle of $'P'$ is $\theta=\frac{1}{2}\sin^{-1}\left(\frac{1}{7}\right)$. If $'N'$ is the foot of perpendicular from centre $'O'$ to this tangent then $\angle PON$ is

A) $\tan^{-1}\left(\frac{1}{12}\right)$ B) $\tan^{-1}\left(\frac{1}{24}\right)$ C) $\frac{\pi}{12}$ D) $\frac{\pi}{3}$

Key. B

Sol.
$$\tan\phi=\sin 2\theta\left(\frac{a^2-b^2}{2ab}\right)$$

$$|\tan\phi|=\frac{16-9}{2\times 4\times 3}\times\frac{1}{7} \Rightarrow \phi=\tan^{-1}\left(\frac{1}{24}\right)$$

77. If there are exactly two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, whose distance from its centre is same and is equal to $\sqrt{\frac{a^2+2b^2}{2}}$, then eccentricity of the ellipse is

A) $\frac{1}{2}$ B) $\frac{1}{\sqrt{2}}$ C) $\frac{1}{\sqrt{3}}$ D) $\frac{1}{2\sqrt{2}}$

Key. C

Sol.
$$a=\sqrt{\frac{a^2+2b^2}{2}}$$

78. An ellipse slides between two perpendicular straight lines $x = 0$ and $y = 0$ then, locus of its foci is

(A) a parabola (B) an ellipse

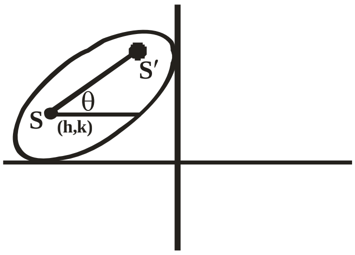
(C) a circle (D) none of these

Key. D

Sol. $(h + 2ae\cos\theta)h = b^2$ ….. (1)

$(k + 2ae\sin\theta)k = b^2$ …. (2)

$$2ae\cos\theta = \frac{b^2 - h^2}{h}$$



$$2ae\sin\theta = \frac{b^2 - k^2}{k}$$

$$4a^2e^2(\cos^2\theta + \sin^2\theta) = \left(\frac{b^2 - h^2}{h}\right)^2 + \left(\frac{b^2 - k^2}{k}\right)^2$$

79. If a variable tangent to the circle $x^2 + y^2 = 1$ intersects the ellipse $x^2 + 2y^2 = 4$ at points P and Q, then the locus of the point of intersection of tangents to the ellipse at P and Q is a conic whose

a) eccentricity is $\frac{\sqrt{3}}{2}$ b) eccentricity is $\frac{\sqrt{5}}{2}$

c) latus -rectum is of length 2 units d) foci are $\left(\pm 2\sqrt{5}, 0\right)$

Key: A,C

Hint: A tangent to the circle $x^2 + y^2 = 1$ is $x\cos\theta + y\sin\theta = 1. R(x_o, y_o)$ is the point of intersection of the tangents to the ellipse at P and Q $\Leftrightarrow x\cos\theta + y\sin\theta = 1$ and $x_o x + 2y_o y = 4$ represent the same line

$$\Leftrightarrow x_o = 4\cos\theta \text{ and } y_o = 2\sin\theta$$

$\Leftrightarrow \frac{x_0^2}{16} + \frac{y_0^2}{4} = 1.$ Hence, locus of P is the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$

80. From a point P, perpendicular tangents PQ and PR are drawn to ellipse $x^2 + 4y^2 = 4$. Locus of circumcentre of triangle PQR is

(A) $x^2 + y^2 = \frac{16}{5}(x^2 + 4y^2)^2$ (B) $x^2 + y^2 = \frac{5}{16}(x^2 + 4y^2)^2$

(C) $x^2 + 4y^2 = \frac{16}{5}(x^2+y^2)^2$ (D) $x^2 + 4y^2 = \frac{5}{16}(x^2+y^2)^2$

Key: B

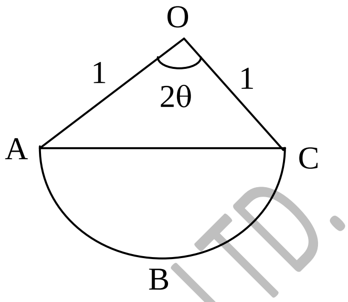
Hint

$x^2 + 4y^2 = 4$

P lies on $x^2 + y^2 = 5$

Let P($\sqrt{5}\cos\theta$, $\sqrt{5}\sin\theta$)

Comparing chord of contact with chord with middle point



$$\frac{xh}{h^2+4k^2}+\frac{y4k}{h^2+4k^2}=1$$

$$\frac{x\sqrt{5}\cos\theta}{4}+\frac{y\sqrt{5}\sin\theta}{1}=1$$

Eliminating θ

$\Rightarrow x^2 + y^2 = \frac{5}{16}(x^2+4y^2)^2$

81. Let PQ be a chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, which subtends an angle of $\pi/2$ radians at the centre. If L is the foot of perpendicular from (0, 0) to PQ, then

(A) locus of L is an ellipse

(B) locus of L is circle concentric with given ellipse

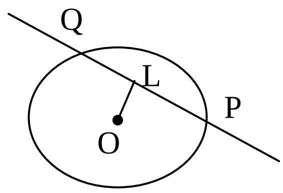
(C) locus of L is a hyperbola concentric with given ellipse

(D) a square concentric with given ellipse

Key: B

Hint

PQ : $x\cos\alpha + y\sin\alpha - p = 0$ ... (A)



Homogenising $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ with (A)

$\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$ = constant

82. If the chords of contact of tangents from two points (x_1, y_1) and (x_2, y_2) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ are at right angles then $\left(\frac{x_1x_2}{y_1y_2}\right)$ is equal to

(A) $\frac{a^2}{b^2}$ (B) $-\frac{b^2}{a^2}$ (C) $-\frac{a^4}{b^4}$ (D) $-\frac{b^4}{a^4}$

Key: C

Hint: Chord of contact from (x_1, y_1) is

$$\frac{xx_1}{a^2}+\frac{yy_1}{b^2}=1$$

Whose slope is $-\frac{b^2}{a^2}\frac{x_1}{y_1}$

Similarly slope of another chord of contact is $-\frac{b^2}{a^2}\frac{x_2}{y_2}$

We have $\left(-\frac{b^2}{a^2}\frac{x_1}{y_1}\right)\times\left(-\frac{b^2}{a^2}\frac{x_2}{y_2}\right)=-1\Rightarrow\frac{x_1x_2}{y_1y_2}=-\frac{a^4}{b^4}$

83. If $\frac{x^2}{f(4a)}+\frac{y^2}{f(a^2-5)}=1$ represents an ellipse with major axis as y-axis and f is a decreasing function positive for all 'a' then a belongs to

A) (0,6) B) (-1,1) C) (-1,5) D) (5, ∞)

Key: C

Hint: $f\left(a^2-5\right)>f(4a)\Rightarrow a^2-5<4a\Rightarrow a\in(-1,5)$

84. An ellipse whose focii are $(2,4)$ and $(14,9)$ and touches X-axis then its eccentricity is

A) $\frac{\sqrt{13}}{213}$ B) $\frac{13}{\sqrt{179}}$ C) $\frac{13}{\sqrt{313}}$ D) $\frac{1}{13}$

Key: C

Hint: $2ae=13$

$b^2=36$

85. An ellipse has the point (1, -1) and (2, -1) as its foci and x + y =5 as one of its tangent then value of a^2 + b^2 where a, b are the length of semimajor and semiminor axis of ellipse respectively, is

a) $\frac{41}{2}$ b) 10 c) 19 d) $\frac{81}{4}$

Key: D

Hint: 2ae = SS^1 = 1

p_1 p_2 = b^2, where p_1 & p_2 are the length of perpendicular from S & S^1 to the tangent

$\frac{5}{\sqrt{2}}.\frac{4}{\sqrt{2}}=b^2\Rightarrow b^2=10\Rightarrow b^2=10=a^2-e^2a^2\Rightarrow a^2=\frac{41}{4}$

86. If circumcentre of an equilateral triangle inscribed in $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with vertices having eccentric angles α,β,γ respectively is $\left(x_1,y_1\right)$ then $\sum\cos\alpha.\cos\beta+\sum\sin\alpha.\sin\beta$ is

(A) $\frac{9x_1^2}{2a^2}+\frac{9y_1^2}{2b^2}-\frac{3}{2}$ (B) $\frac{x_1^2}{2a^2}+\frac{y_1^2}{2b^2}-\frac{5}{2}$

(C) $\frac{x_1^2}{9a^2}+\frac{y_1^2}{9b^2}-\frac{5}{9}$ (D) $\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-\frac{1}{2}$

Key: A

Hint: $A(a\cos\alpha, b\sin\alpha)$, $B(a\cos\beta, b\sin\beta)$, $C(a\cos\gamma, b\sin\gamma)$

Controid = circumcentre $=(x_1, y_1)=\left[\frac{\sum a\cos\alpha}{3}, \frac{\sum b\sin\alpha}{3}\right]$

$\frac{3x_1}{a}=\sum\cos\alpha, \frac{3y_1}{b}=\sum\sin\alpha$

$\left(\frac{9x_1^2}{a^2}+\frac{9y_1^2}{b^2}-3\right)=2\left(\sum\cos\alpha\cos\beta+\sum\sin\alpha\sin\beta\right)$

$\Rightarrow\frac{9x^2}{2a^2}+\frac{9y^2}{2b^2}-\frac{3}{2}$

87. The inclination to the major axis of the diameter of an ellipse the square of whose length is the harmonic mean between the squares of the major and minor axes is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$

c) $\frac{2\pi}{3}$ d) $\frac{\pi}{2}$

KEY : A

HINT: $4(a^2\cos^2\theta+b^2\sin^2\theta)=\frac{2(4a^2)(4b^2)}{4a^2+4b^2}$

88. An ellipse slides between two perpendicular straight lines $x=0$ and $y=0$ then, locus of its foci is

(A) a parabola (B) an ellipse

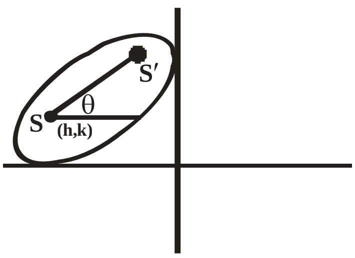
(C) a circle (D) none of these

Key. D

Sol. $(h+2ae\cos\theta)h=b^2$ ….. (1)

$(k+2ae\sin\theta)k=b^2$ …. (2)

$2ae\cos\theta=\frac{b^2-h^2}{h}$



$$2ae\sin\theta = \frac{b^2 - k^2}{k}$$

$$4a^2e^2(\cos^2\theta + \sin^2\theta) = \left(\frac{b^2 - h^2}{h}\right)^2 + \left(\frac{b^2 - k^2}{k}\right)^2$$

89. A circle of radius 'r' is concentric with the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Then inclination of common tangent with major axis is _______(b<r<a)

1. $\tan^{-1}\left(\frac{b}{a}\right)$ 2. $\tan^{-1}\left(\frac{rb}{a}\right)$ 3. $\tan^{-1}\sqrt{\frac{r^2 - b^2}{a^2 - r^2}}$ 4. $\frac{\pi}{2}$

Key. 3

Sol. The tangent of Ellipse is $y = mx + \sqrt{a^2m^2 + b^2}$, this line touches $x^2 + y^2 = r^2$

Condition is $\left|\frac{\sqrt{a^2m^2 + b^2}}{\sqrt{m^2 + 1}}\right| = r$

$$a^2m^2 + b^2 = r^2m^2 + r^2$$

$$m^2\left(a^2 - r^2\right) = r^2 - b^2 \Rightarrow m^2 = \frac{r^2 - b^2}{a^2 - r^2}$$

$$m = \sqrt{\frac{r^2 - b^2}{a^2 - r^2}}$$

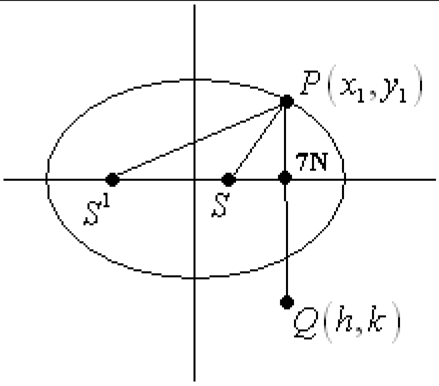
Inclimation is $\tan^{-1}\sqrt{\frac{r^2 - b^2}{a^2 - r^2}}$

90. From any point P lying in first quadrant on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, PN is drawn perpendicular to the major axis such that N lies on major axis. Now PN is produced to the point Q such that NQ equals to PS, where S is a focus. The point Q lies on which of the following lines

(A) $2y - 3x - 25 = 0$ (B) $3x + 5y + 25 = 0$

(C) $2x - 5y - 25 = 0$ (D) $2x - 5y + 25 = 0$

Key. B



Sol.

$a^2 = 25$

$b^2 = 16$

$e = \sqrt{\frac{25-16}{25}} = \frac{3}{5}$

Let point Q be (h, k), where K < 0

Given that $|K| = a + eh(as\ x_1 = h)$

$-y = a + ex$

$-y = 5 + \frac{3}{5}x$

$3x + 5y + 25 = 0$

91. The normal at an end of a latus rectum of the ellipse $x^2/a^2 + y^2/b^2 = 1$ passes through an end of the minor axis if

(a) $e^4 + e^2 = 1$ (b) $e^3 + e^2 = 1$ (c) $e^2 + e = 1$ (d) $e^3 + e = 1$

Key. A

Sol. Given ellipse equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Let $P\left(ae, \frac{b^2}{a}\right)$ be one end of latus rectum.

Slope of normal at $P\left(ae, \frac{b^2}{a}\right) = \frac{1}{e}$

Equation of normal is

$y = \frac{b^2}{a} = \frac{1}{e}(x - ae)$

It passes through $B^{'}(0, b)$ then

$b - \frac{b^2}{a} = -a$

$a^2 - b^2 = -ab$

$a^4e^4 = a^2b^2$

$e^4 + e^2 = 1$

92. Area of the greatest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

1. $\sqrt{ab}$ 2. a/b 3. $2ab$ 4. ab

Key. 3

Sol. Let the vertices of the rectangle be $(\pm a\cos\theta, \pm b\sin\theta)$, then the Area of the rectangle is $4ab\sin\theta\cos\theta = 2ab\sin 2\theta$. The maximum value of which is $2ab$ as $\sin 2\theta \le 1$.

93. The number of values of C such that the straight line $y = 4x + c$ touches the curve $x^2/4 + y^2 = 1$ is

1. 0 2. 1 3. 2 4. Infinite

Key. 3

Sol. We know that $y = mx + c$ touches the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $c^2 = a^2m^2 + b^2$

Here $m = a^2 = 4, b^2 = 1$ so $c^2 = 4 \times 4^2 + 1 \quad \Rightarrow \quad c = \pm\sqrt{65}$

94. The normal at an end of a latus rectum of the ellipse $x^2/a^2 + y^2/b^2 = 1$ passes through an end of the minor axis if

1. $e^4 + e^2 = 1$ 2. $e^3 + e^2 = 1$
3. $e^2 + e = 1$ 4. $e^3 + e = 1$

Key. 1

Sol. Let at end of a latus rectum be $(ae, \sqrt{1-e^2})$, then the equation of the normal at this end is

$$\frac{x - ae}{ae/a^2} = \frac{y - b\sqrt{1-e^2}}{b\sqrt{1-e^2}/b^2}$$

It will pass through the end $(0, -b)$ if

$$-a^2 = \frac{-b^2(1+\sqrt{1-e^2})}{\sqrt{1-e^2}} \text{ or } \frac{b^2}{a^2} = \frac{\sqrt{1-e^2}}{1+\sqrt{1-e^2}}$$

Or $(1-e^2)\left[1+\sqrt{1-e^2}\right] = \sqrt{1-e^2}$

Or $\sqrt{1-e^2} + 1 - e^2 = 1$ or $e^4 + e^2 = 1$.

95. The locus of the middle points of the portions of the tangents of the ellipse $x^2/a^2 + y^2/b^2 = 1$ included between the axes is the curve.

1. $x^2/a^2 + y^2/b^2 = 4$ 2. $a^2/x^2 + b^2/y^2 = 4$

3. $a^2x^2 + b^2y^2 = 4$ 4. $b^2x^2 + a^2y^2 = 4$

Key. 2

Sol. Equation of a tangent to the ellipse can be written as $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$ which meets the axes at A $(a/\cos\theta,0)$ and $B(0,b/\sin\theta)$. If (h,k) is the middle point of AB , then

$$h=a/2\cos\theta,\ k=b/2\sin\theta$$

Eliminating θ we get $(a/2h)^2+(b/2k)^2=1$

$\Rightarrow$ locus of $P(h,k)$ is $a^2/x^2+b^2/y^2=4$.

96. The locus of the points of intersection of the tangents at the extremities of the chords of the ellipse $x^2+2y^2=6$ which touch the ellipse $x^2+4y^2=4$ is

1. $x^2+y^2=4$ 2. $x^2+y^2=6$ 3. $x^2+y^2=9$ 4. None of these

Key. 3

Sol. We can write $x^2+4y^2=4$ as $\frac{x^2}{4}+\frac{y^2}{1}=1$ (i)

Equation of a tangent to the ellipse (i) is

$$\frac{x}{2}\cos\theta+y\sin\theta=1 \quad \text{(ii)}$$

Equation of the ellipse $x^2+2y^2=6$ can be written as

$$\frac{x^2}{6}+\frac{y^2}{3}=1 \quad \text{(iii)}$$

Suppose (ii) meets the ellipse (iii) at P and Q and the tangents at P and Q to the ellipse (iii) intersect at (h,k), then (ii) is the chord of contact of (h,k) with respect to the ellipse (iii) and thus its equation is $\frac{hx}{6}+\frac{ky}{3}=1$ (iv)

Since (ii) and (iv) represent the same line

$$\frac{h/6}{(\cos\theta)/2}=\frac{k/3}{\sin\theta}=1$$

$\Rightarrow$ $h=3\cos\theta, k=3\sin\theta$

And the locus of (h,k) is $x^2+y^2=9$

97. A tangent at any point to the ellipse $4x^2+9y^2=36$ is cut by the tangent at the extremities of the major axis at T and T^1. The circle on $T\ T^1$ as diameter passes through the point.

1. $\left(0,\sqrt{5}\right)$ 2. $\left(\sqrt{5},0\right)$ 3. $(2,1)$ 4. $\left(0,-\sqrt{5}\right)$

Key. 2

Sol. Any point on the ellipse is $P(3\cos\theta, 2\sin\theta)$

Equation of the tangent at P is $\frac{x}{3}\cos\theta + \frac{y}{2}\sin\theta = 1$

Which meets the tangents $x = 3$ and $x = -3$ at the extremities of the major axis at

$T\left(3, \frac{2(1-\cos\theta)}{\sin\theta}\right)$ and $T^1\left(3, \frac{2(1+\cos\theta)}{\sin\theta}\right)$

Equation of the circle on TT^1 as diameter is

$$(x-3)(x+3) + \left(y - \frac{2(1-\cos\theta)}{\sin\theta}\right)\left(y - \frac{2(1+\cos\theta)}{\sin\theta}\right) = 0$$

$\Rightarrow x^2 + y^2 - \frac{4}{\sin\theta}y - 5 = 0$,which passes through $(\sqrt{5}, 0)$

98. If $y = x$ and $3y + 2x = 0$ are the equations of a pair of conjugate diameters of an ellipse, then the eccentricity of the ellipse is

1. $\sqrt{2/3}$ 2. $1/\sqrt{3}$ 3. $1/\sqrt{2}$ 4. $1/\sqrt{5}$

Key. 2

Sol. Let the equation of the ellipse be $x^2/a^2 + y^2/b^2 = 1$

Slope of the given diameters are $m_1 = 1, m_2 = -2\sqrt{3}$.

$\Rightarrow \quad m_1 m_2 = -2/3 = -b^2/a^2$

[using the condition of conjugacy of two diameters]

$3b^2 = 2a^2 \Rightarrow 3a^2(1-e^2) = 2a^2$,

$1 - e^2 = 2/3 \Rightarrow e^2 = 1/3 \Rightarrow e = 1/\sqrt{3}$

99. On the ellipse $4x^2 + 9y^2 = 1$, the point at which the tangent is parallel to the line $8x = 9y$ is

1. $(2/5, 1/5)$ 2. $(-2/5, 1/5)$ 3. $(-2/5, -1/5)$ 4. None of these

Key. 2

Sol. Let the point be $((1/2)\cos\theta, (1/3)\sin\theta)$,then the slope of the tangent is $-\frac{1/3}{1/2}\cot\theta = \frac{8}{9}$

$\Rightarrow \tan\theta = -\frac{3}{4} \Rightarrow \sin\theta = \pm\frac{3}{5}$ and $\cos\theta = \mp\frac{4}{5}$

And the required point can be $\left(-\frac{4}{5}\times\frac{1}{2},\frac{3}{5}\times\frac{1}{3}\right)=\left(-\frac{2}{5},\frac{1}{5}\right)$

100. The circle $x^2+y^2=c^2$ contains the ellipse $x^2/a^2+y^2/b^2=1\ (a>b)$ if

1. $c<a$
2. $c<b$
3. $c>a$
4. $c>b$

Key. 3

Sol. Radius of the circle must be greater than the major axis of the ellipse.

101. In an ellipse, if the lines joining a focus to the extremities of the minor axis make an equilateral triangle with the minor axis, then the eccentricity of the ellipse is

1. 3/4
2. $\sqrt{3}/2$
3. 1/2
4. 2/3

Key. 2

Sol. $a^2e^2+b^2=(2b)^2 \Rightarrow a^2e^2=3a^2(1-e^2) \Rightarrow e^2=3/4.$

102. If $(5,12)$ and $(24,7)$ are foci of an ellipse passing through origin, then the eccentricity of the ellipse is

(A) $\frac{\sqrt{386}}{24}$
(B) $\frac{\sqrt{386}}{38}$
(C) $\frac{\sqrt{386}}{25}$
(D) $\frac{1}{\sqrt{2}}$

Key. 2

Sol. Let $S(5,12)$ $S'(24,7)$ O is origin

$SO=13 \quad S'O=25 \quad SS'=\sqrt{386}$

$e=\frac{\sqrt{386}}{13+25}=\frac{\sqrt{386}}{38}$

103. A variable point P on the ellipse of eccentricity e is joined to foci S and S^1. The eccentricity of the locus of the in centre of triangle PSS^1 is

(A) $\sqrt{\frac{2e}{1+e}}$
(B) $\sqrt{\frac{e}{1+e}}$
(C) $\sqrt{\frac{1-e}{1+e}}$
(D) $\frac{e}{2(1+e)}$

Key. 1

Sol. Let any point be $P(a\cos\theta, b\sin\theta)$.

$SP=a[1-e\cos\theta] \quad S'P=a[1+e\cos\theta] \quad SS^1=2ae$

(h,k) be in centre of $\Delta PSS'$ upon solving we get

$\Rightarrow h=ae\cos\theta$

$k=\frac{b\sin\theta e}{1+e}$

Eliminating 'θ'

$$\frac{h^2}{a^2e^2}+\frac{k^2}{\frac{e^2b^2}{(1+e)^2}}=1$$

Locus of (h,k) $\frac{x^2}{a^2e^2}+\frac{y^2}{\frac{e^2b^2}{(1+e)^2}}=1$

$$\Rightarrow e_1^2=1-\frac{e^2b^2}{(1+e)^2e^2a^2}=\frac{2e}{1+e}$$

$$e_1=\sqrt{\frac{2e}{1+e}}$$

104. An ellipse is drawn with major and minor axis of lengths 10 and 8 respectively. Using one focus as centre, a circle is drawn that is tangent to the ellipse, with no part of circle being outside the ellipse. The radius of circle is

(A) 4 (B) 5 (C) 2 (D) 1

Key. 3

Sol. The circle must touch the end of major axis

$\because$ radius $=a-ae=a-\sqrt{a^2-b^2}=5-\sqrt{5^2-4^2}=2$

105. If the length of the major axis intercepted between the tangent & normal at a point $(a\cos\theta, b\sin\theta)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to the length of semi-major axis, then , eccentricity of the ellipse is,

a) $\frac{\cos\theta}{\sqrt{1-\cos\theta}}$ b) $\frac{\sqrt{1-\cos\theta}}{\cos\theta}$ c) $\frac{\sqrt{1-\cos\theta}}{\sin\theta}$ d) $\frac{\sin\theta}{\sqrt{1-\sin\theta}}$

Key. B

Sol. $\frac{a}{\cos\theta}-\frac{(a^2-b^2)}{a}\cos\theta=a\Rightarrow e^2\cos^2\theta=1-\cos\theta\Rightarrow e=\frac{\sqrt{1-\cos\theta}}{\cos\theta}$

106. P_1, P_2 are the lengths of the perpendicular from the foci on the tangent to the ellipse and P_3, P_4 are perpendiculars from extremities of major axis and P from the centre of the ellipse on the same tangent, then $\frac{P_1P_2-P^2}{P_3P_4-P^2}$ equals (where e is the eccentricity of the ellipse)

(A) e (B) $\sqrt{e}$

(C) e^2 (D) none of these

Key. C

Sol. Let equation of tangent y = mx + c

$$P=\frac{|c|}{\sqrt{1+m^2}} \qquad P_1=\frac{|c+aem|}{\sqrt{1+m^2}}$$

$$P_2=\frac{|c-aem|}{\sqrt{1+m^2}} \qquad P_3=\frac{|c+am|}{\sqrt{1+m^2}}$$

$P_4 = \frac{|c - am|}{\sqrt{1+m^2}}$

So, $\frac{P_1P_2 - P^2}{P_3P_4 - P^2} = \frac{c^2 - a^2e^2m^2 - c^2}{c^2 - a^2m^2 - c^2}$
$= e^2$

107. If $\frac{x^2}{f(4a)} + \frac{y^2}{f(a^2-5)} = 1$ represents an ellipse with major axis as y-axis and f is a decreasing function positive for all 'a' then a belongs to

a) (0,5) b)(-1,1) c) (-1,5) d) $(5, \infty)$

Key. C

Sol. $f\left(a^2-5\right) > f(4a) \Rightarrow a^2 - 5 < 4a \Rightarrow a \in (-1,5)$

108. If a tangent of slope 2 of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is normal to the circle $x^2+y^2+4x+1 = 0$, then the maximum value of ab is

a)4 b)2 c) 1 d)0

Key. A

Sol. Equation of tangent with slope

$y = 2x \pm \sqrt{4a^2 + b^2}$ is a normal to the circle $\therefore 0 = -4 \pm \sqrt{4a^2 + b^2} \Rightarrow 4a^2 + b^2 = 16$

Max of ab = 4

109. The eccentric angle of a point p lying in the first quadrant on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is θ. If OP makes an angle ϕ with x –axis, then $\theta - \phi$ will be maximum when $\theta =$

a) $\tan^{-1}\sqrt{\frac{a}{b}}$ b) $\tan^{-1}\sqrt{\frac{b}{a}}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{3}$

Key. A

Sol. $\tan\theta = \frac{b}{a}\tan\theta$

$y = \frac{a-b}{a\cot\theta + b\tan\theta}$ if will be maximum

If $\tan^2\theta = \frac{a}{b} \Rightarrow \tan\theta = \sqrt{\frac{a}{b}}$

110. From the focus $(-5,0)$ of the ellipse $\frac{x^2}{45}+\frac{y^2}{20}=1$ a ray of light is sent which makes angle $\cos^{-1}\left(\frac{-1}{\sqrt{5}}\right)$ with the positive direction of X-axis upon reacting the ellipse the ray is reflected from it. Slope of the reflected ray is

A) $-3/2$ B) $-7/3$ C) $-5/4$ D) $-2/11$

Key. D

Sol. Let $\theta=\cos^{-1}\left(\frac{-1}{\sqrt{5}}\right)\Rightarrow \cos\theta=\frac{-1}{\sqrt{5}}\Rightarrow \tan\theta=-2$

Foci are $(\pm 5,0)$

Equation of line through (-5, 0) with slope –2 is $y=-29x+5)=-2x-10$

This line meets the ellipse above X-axis at $(-6,2)$

$\therefore$ Slope = $\frac{2-0}{-6-5}=-\frac{2}{11}$.

111. If $f(x)$ is a decreasing function for all $x\in R$ and $f(x)>0\ \forall x\in R$ then the range of *K* so that the equation $\frac{x^2}{f(K^2+2K+5)}+\frac{y^2}{f(K+11)}=1$ represents an ellipse whose major axis is the X-axis is

A) $(-2,3)$ B) $(-3,2)$ C) $(-\infty,-3)\cup(2,\infty)$ D) $(-\infty,-2)\cup(3,\infty)$

Key. B

Sol. Conceptual

112. P,Q are points on the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ such that PQ is a chord through the point $R(3,0)$. If $|PR|=2$ then length of chord PQ is

A) 8 B) 6 C) 10 D) 4

Key. C

Sol. Conceptual

113. Let $Q=(3,\sqrt{5}),R=(7,3\sqrt{5})$. A point *P* in the XY-plane varies in such a way that perimeter of ΔPQR is 16. Then the maximum area of ΔPQR is

A) 6 B) 12 C) 18 D) 9

Key. B

Sol. *P* lies on the ellipse for which Q,R are foci and length of major axis is 10 and eccentricity is 3/5.

114. *O* is the centre of ellipse for which A,B are end points of major axis and C,D are end points of minor axis. *F* is a focus of the ellipse. If $|OF|=6$ and inradius of ΔOCF is 1, then $|AB|\times|CD|=$

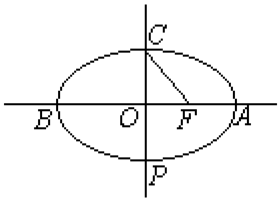
A) 65 B) 52 C) 78 D) 47

Key. A

Sol. $b^2=\frac{25}{4}\Rightarrow a^2-a^2e^2=\frac{25}{4}\Rightarrow a^2=\frac{25}{4}+36=\frac{169}{4}\Rightarrow a=\frac{13}{2}$

$|OF|=ae=6\Rightarrow \frac{abe}{2}=1\times\frac{(ae+b+\sqrt{a^2e^2+b^2})}{2}$

$6b=6+b+\sqrt{b^2+36}\Rightarrow(5b-6)^2=b^2+36\Rightarrow 24b^2=60b\Rightarrow b=5/2$



115. The angle of intersection between the curves $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2-k^2}-\frac{y^2}{k^2-b^2}=1,(a>k>b>0)$ is

a) $\tan^{-1}\left(\frac{b}{a}\right)$ b) $\tan^{-1}\left(\frac{b}{ka}\right)$ c) $\tan^{-1}\left(\frac{a}{kb}\right)$ d) None of these

Key. D

Sol. Confocal ellipse and hyperbola cut at right angles

116. Image of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ in the line x + y = 10 is :

a) $\frac{(x-10)^2}{16}+\frac{(y-10)^2}{25}=1$ b) $\frac{(x-10)^2}{25}+\frac{(y-10)^2}{16}=1$

c) $\frac{(x-5)^2}{16}+\frac{(y-5)^2}{25}=1$ d) $\frac{(x-5)^2}{25}+\frac{(y-5)^2}{16}=1$

Key. A

Sol. Conceptual

117. Length of common tangent to $x^2+y^2=16$ and $\frac{x^2}{25}+\frac{y^2}{7}=1$

a) $\frac{9}{4\sqrt{2}}$ b) $\frac{9}{4}$ c) $\frac{9}{2\sqrt{2}}$ d) $\frac{9}{2}$

Key. B

Sol. $y=-x+4\sqrt{2}$ is a common tangent to two curves in the 1st quadrant. Touching the curves at $P\left(2\sqrt{2},2\sqrt{2}\right)\&Q\left(\frac{25}{4\sqrt{2}},\frac{7}{4\sqrt{2}}\right)$

PQ = length of common tangent.

118. An ellipse having foci S (3, 4) & S' (6, 8) passes through the point P(0, 0). The equation of the tangent at P to the ellipse is

a) $4x+3y=0$ b) $3x+4y=0$ c) $x+y=0$ d) $x-y=0$

Key. B

Sol. Normal at a point is bisector of angle SPS'

119. The angle subtended at the origin by a common tangent of the ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{2x}{c}=0$ and $\frac{x^2}{b^2}+\frac{y^2}{a^2}+\frac{2x}{c}=0$, is

a) $\pi/6$ b) $\pi/4$ c) $\pi/3$ d) $\pi/2$

Key. D

Sol. Conceptual

120. If the normals at 4 points having eccentric angles $\alpha,\beta,\gamma,\delta$ on an ellipse be concurrent, then $\left(\sum\cos\alpha\right)\left(\sum\sec\alpha\right)=$

a) 4 b) $(\alpha\beta\gamma\delta)^{\frac{1}{4}}$ c) $\frac{\alpha+\beta+\gamma+\delta}{4}$ d) None of these

Key. A

Sol. Conceptual

121. If $\frac{x^2}{f(4a)}+\frac{y^2}{f(a^2-5)}=1$ represents an ellipse with major axis as y-axis and f is decreasing function, then

A) $a\in(-\infty,1)$ B) $a\in(5,\infty)$ C) $a\in(1,4)$ D) $a\in(-1,5)$

Key. D

Sol. $f(4a)<f(a^2-5)\Rightarrow 4a>a^2-5[\because f \text{ is} \downarrow \text{fn}]$

$\therefore a\in(-1,5)$

Ellipse

Multiple Correct Answer Type

1. An ellipse whose major axis is parallel to x-axis is such that the segments of a focal chord are 1 and 3 units. The lines ax + by + c = 0 are the chords of the ellipse such that a,b,c are in A.P and bisected by the point at which they are concurrent. The equation of auxiliary circle is $x^2+y^2+2\alpha x+2\beta y-2\alpha-1=0$. Then

   A. The locus of perpendicular tangents to the ellipse is $x^2+y^2=7$
   B. Length of the double ordinate which is conjugate to directrix is 3
   C. Area of an auxillary circle is 2π
   D. Eccentricity of the ellipse is $\frac{1}{2}$

Key. B,D
Sol. a,b,c are in A.P $\Rightarrow ax+by+c=0$ are concurrent at (1,2)

$\therefore$ centre of auxiliary circle $=(-\alpha,-\beta)=(1,-2)$

Radius of auxiliary circle = 2

Length of major axis = 4 = 2a

$\because \frac{1}{SP}+\frac{1}{SQ}=\frac{2a}{b^2}\Rightarrow b=\sqrt{3}$, hence $e=\frac{1}{2}$

2. If foci of an ellipse be $(-1,2)$ and $(-2,3)$ and its tangent at a point A is $2x+3y+9=0$

   A. Length of the minor axis of the ellipse will be $2\sqrt{14}$
   B. Co-ordinate of the point 'A' will be $\left(\frac{-32}{9},\frac{-17}{27}\right)$
   C. Distance between the foci is $2\sqrt{2}$
   D. Product of the perpendiculars from foci to any tangent is 56

Key. A,B
Sol. 28 to 29 P(-1,2) and Q(-2,3)

Hence image of point P from lie 2x + 3y + 9 = 0

$$\frac{x+1}{2}=\frac{y-2}{3}=-2\left(\frac{-2+6+9}{13}\right)$$

X = -5 ; y = -4

$P\in(-5,-4)$

Now length of PQ $=\sqrt{9+49}=\sqrt{58}=2a \Rightarrow a=\frac{\sqrt{58}}{2}$

We know that, 2ae = PQ

$$2ae=\sqrt{2}\Rightarrow ae=\frac{1}{\sqrt{2}}\Rightarrow a^2e^2=\frac{1}{2}$$

$$b^2=a^2(1-e^2)$$

$$b^2=\frac{58}{4}-\frac{1}{2}\Rightarrow b^2=\frac{58-2}{4}\Rightarrow b^2=\frac{56}{4}\Rightarrow b=\sqrt{14}$$

$b^2=14$ length of minor axis $=2\sqrt{14}=\sqrt{56}$

Equation of line PQ is

$$(y-3)=\frac{4-3}{-3}(x+2)$$

3y – 9 = 72 + 14

7x – 3y + 23 = 0

On solving 7x – 3y + 23 = 0 and 2x + 3y + 9 = 0 we get

$$A=\left(-\frac{32}{9},-\frac{17}{27}\right)$$

3. Let $P(x_1,y_1)$, $Q(x_2,y_2)$, $y_1>0,\ y_2>0$ be the end points of the latus rectum of the ellipse $3x^2+4y^2=12$. The equations of the parabolas with latus rectum PQ are

A) $x^2-2y-2=0$ B) $x^2-2y+2=0$ C) $x^2+2y-4=0$ D) $x^2-2y+4=0$

Key. B,C

Sol. $\frac{x^2}{4}+\frac{y^2}{3}=1$ $e=\frac{1}{2}$

Ends of latus rectum $=\left(\pm1,\pm\frac{3}{2}\right)$

Let $P=\left(1,\frac{3}{2}\right)$, $Q=\left(1-,\frac{3}{2}\right)$

Focus of the parabola $S=\left(0,\frac{3}{2}\right)$

LLR of parabola = 2, Distance between the focus and vertex = ½

$A=(0,1)\&A'=(0,2)$

$\therefore$ Equation of parabolas $x^2=2(y-1),\ x^2=-2(y-2)$

4. If the chord through the points whose eccentric angles are θ and ϕ on the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ passes through a focus then $\tan\frac{\theta}{2}.\tan\frac{\phi}{2}$ is

A) $\frac{1}{9}$ B) -9 C) $-\frac{1}{9}$ D) 9

Key. C,D

Sol. Equation of the chord $\frac{x}{5}\cos\left(\frac{\theta+\phi}{2}\right)+\frac{y}{3}\sin\left(\frac{\theta+\phi}{2}\right)=\cos\left(\frac{\theta-\phi}{2}\right)$

If it passes through $(4,0)$, $\frac{\cos\left(\frac{\theta+\phi}{2}\right)}{\cos\left(\frac{\theta-\phi}{2}\right)}=\frac{5}{4}\Rightarrow\tan\frac{\theta}{2}.\tan\frac{\phi}{2}=-\frac{1}{9}$

If it passes through $(-4,0)$ then $\tan\frac{\theta}{2}.\tan\frac{\phi}{2}=-9$

5. If a tangent of slope $\frac{1}{3}$ of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (a > b) is normal to the circle $x^2+y^2+2x+2y+1=0$, then

A) maximum value of ab is $\frac{2}{3}$

B) $a\in\left(\sqrt{\frac{2}{5}},2\right)$

C) $a\in\left(\frac{2}{3},2\right)$

D) maximum value of ab is 1

Key. A,B

Sol. $y=mx\pm\sqrt{a^2m^2+b^2}$ $\quad y=\frac{1}{3}x\pm\sqrt{\frac{a^2}{9}+b^2}$

$3y=x\pm\sqrt{a^2+9b^2}$ passes through (-1, -1)

$-3=-1\pm\sqrt{a^2+9b^2}$ $\quad -2=\pm\sqrt{a^2+9b^2}$ $\quad a^2+9b^2=4$

$\frac{a^2+9b^2}{2}\ge\sqrt{9a^2b^2}$

$2\ge 3ab,\ \ ab\le\frac{2}{3}$ $\quad \because a>b$ $\quad a^2+9a^2>4$ $\quad a^2>\frac{4}{10}$

$a>\sqrt{\frac{2}{5}}$ but $a\le 2$

6. If a quadrilateral formed by four tangents to the ellipse $3x^2 + 4y^2 = 12$ is a square, then

A) The vertices of the square lie on y $= \pm x$

B) The vertices of the square lie on $x^2 + y^2 = 7$

C) The area of all such squares is constant

D) Only two such squares are possible

Key. B,C

Sol. Vertices of the squares will lie on the director circle ie, on $x^2 + y^2 = 4+3$ and have the area of the squares is 2(4+3)=14. only one such square is possible (same area for all squares)

7. The distance of a point P on the ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$ from centre C is 2, then eccentric angle of P may be

A) $\frac{\pi}{4}$ B) $\frac{\pi}{6}$ C) $\frac{3\pi}{4}$ D) $\frac{7\pi}{4}$

Key. A,C,D

Sol. $\cos^2\theta = \frac{d^2 - b^2}{a^2 - b^2} = \frac{4-2}{6-2} = \frac{1}{2} \Rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$

8. A point on the ellipse $x^2 + 3y^2 = 37$ where the normal is parallel to the line $6x - 5y = 2$ is

(A) (5, -2) (B) (5, 2) (C) (-5, 2) (D) (-5, -2)

Key. B,D

Sol. Let $P(x_1, y_1)$ be any point on the ellipse

i.e., $x_1^2 + 3y_1^2 = 37$.................(1)

$\frac{dy}{dx(x_1, y_1)} = \frac{-x_1}{3y_1}$ (Slope of tan)

Slope of normal : $\frac{3y_1}{x_1} = \frac{6}{5}$

$x_1 = \frac{15y_1}{6}$

$\Rightarrow y_1 = \pm 2$ (from (1) $\Rightarrow x_1 = \pm 5$

9. The distance of a point P on the ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$ from centre C is 2, then eccentric angle of P may be

A) $\frac{\pi}{4}$ B) $\frac{\pi}{6}$ C) $\frac{3\pi}{4}$ D) $\frac{7\pi}{4}$

Key. A,C,D

Sol. $\cos^2 q = \frac{d^2 - b^2}{a^2 - b^2} = \frac{4- 2}{6- 2} = \frac{1}{2}$ Þ $\cos q = \pm \frac{1}{\sqrt{2}}$ Þ $q = \frac{p}{4}, \frac{3p}{4}, \frac{7p}{4}$

10. If a tangent of slope $\frac{1}{3}$ of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (a > b) is normal to the circle $x^2 + y^2 + 2x + 2y + 1 = 0$,then

A) maximum value of ab is $\frac{2}{3}$

B) $a \in \left(\sqrt{\frac{2}{5}}, 2\right)$

C) $a \in \left(\frac{2}{3}, 2\right)$

D) maximum value of ab is 1

Key. A,B

Sol. $y = mx \pm \sqrt{a^2m^2 + b^2}$ $\quad y = \frac{1}{3}x \pm \sqrt{\frac{a^2}{9} + b^2}$

$3y = x \pm \sqrt{a^2 + 9b^2}$ passes through (-1, -1)

$-3 = -1 \pm \sqrt{a^2 + 9b^2}$ $\quad -2 = \pm\sqrt{a^2 + 9b^2}$ $\quad a^2 + 9b^2 = 4$

$\frac{a^2 + 9b^2}{2} \geq \sqrt{9a^2b^2}$

$2 \geq 3ab, \ ab \leq \frac{2}{3}$ $\quad \because a > b$ $\quad a^2 + 9a^2 > 4$ $\quad a^2 > \frac{4}{10}$

$a > \sqrt{\frac{2}{5}}$ but $a \leq 2$

11. If a quadrilateral formed by four tangents to the ellipse $3x^2 + 4y^2 = 12$ is a square, then

A) The vertices of the square lie on $y = \pm x$

B) The vertices of the square lie on $x^2 + y^2 = 7$

C) The area of all such squares is constant

D) Only two such squares are possible

Key. B,C

Sol. Vertices of the squares will lie on the director circle ie, on $x^2 + y^2 = 4 + 3$ and have the area of the squares is 2(4+3)=14. only one such square is possible (same area for all squares)

12. The focal chord to $y^2 = 16x$ is tangent to $(x-6)^2 + y^2 = 2$, then slope of focal chord is

A. 1

B. $\frac{1}{2}$

C. $-\frac{1}{2}$

D. -1

Key. A,D

Sol. $(x-6)^2+y^2=2 \rightarrow$ tangent is $y=m(x-6)+\sqrt{2m^2+2}$

It is passing through (4, 0) focus of parabola

$$0=-2m+\sqrt{2m^2+2} \Rightarrow 2m^2+2=4m^2$$

$$m^2=1 \Rightarrow m=\pm 1$$

13. A point on the ellipse $x^2+3y^2=37$ where the normal is parallel to the line 6x-5y=2

A. (5, -2)   B. (5, 2)

C. (-5, 2)   D. (-5, -2)

Key. B,D

Sol. $\frac{x^2}{37}+\frac{y^2}{\frac{37}{3}}=1$

Normal at($(x_1\ y_1)$ is $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$

$$\frac{37x}{x_1}-\frac{37y}{3y_1}=37-\frac{37}{3}$$

$$\frac{x}{x_1}-\frac{y}{3y_1}=\frac{2}{3}$$

It is parallel to $6x-5y=2$ $\frac{3y_1}{x_1}=\frac{6}{5}$ $\frac{y_1}{x_1}=\frac{2}{5}$

But $x_1^2+3y_1^2=37$

$$x_1^2+3\left(\frac{4x_1^2}{25}\right)=37$$

$$x_1^2\left(1+\frac{12}{25}\right)=37$$

$$x_1^2=25 \Rightarrow x_1=\pm 5$$

$\therefore y_1 = \pm 2$ $\therefore$ (5, 2) and (-5, -2) are two points

14. P and Q are two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose eccentric angles are differ by 90^0 then

A. Locus of point of intersection of tangets at P and Q is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$

B. Locus of Mid point (P, Q) is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{2}$

C. Product of slopes of OP and OQ where O is the centre is $\frac{-b^2}{a^2}$

D. Max area of $\Delta^{le} OPQ$ is $\frac{1}{2}ab$

Key. A,B,C,D

Sol. $P=(a\cos\theta, b\sin\theta)\quad Q=(-a\sin\theta, b\cos\theta)$

Tangent at P $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1\rightarrow(1)$

$\frac{-x\sin\theta}{a}+\frac{y\cos\theta}{b}=1\rightarrow(2)$

Elimination θ $(1)^2+(2)^2\Rightarrow\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$ $\therefore$ (A) is correct

Now mid(PQ) = $\left(\frac{a(\cos\theta-\sin\theta)}{2}\quad\frac{b(\sin\theta+\cos\theta)}{2}\right)=(x,y)$

$\cos\theta-\sin\theta=\frac{2x}{a}\rightarrow(3)$

$\cos\theta+\sin\theta=\frac{2y}{b}\rightarrow(4)$

$(3)^2+(4)^2\Rightarrow 2=4\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)\Rightarrow\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{2}$ $\therefore$ (B) is correct

Slope of OP = $\frac{b\sin\theta}{a\cos\theta}=m_1$

Slope of OQ = $\frac{-b\cos\theta}{a\sin\theta} = m_2$

Now $m_1 m_2$ = $\frac{-b^2}{a^2}$ $\therefore$ (C) is correct

Now are of triangle OPQ = $\frac{1}{2}\begin{vmatrix} a\cos\theta & b\sin\theta \\ -a\sin\theta & b\cos\theta \end{vmatrix}$

$\frac{1}{2}ab(\cos^2\theta + \sin 2\theta) = \frac{1}{2}ab$ $\therefore$ (D) is correct

15. The equations of the common tangents of the curves $x^2+4y^2=8$ and $y^2=4x$ are

A) $x+2y+4=0$ B) $x-2y+4=0$ C) $2x+y=4$ D) $2x-y+4=0$

Key. A,B

Sol. $\frac{x^2}{8}+\frac{y^2}{2}=1,\ \ y^2=4x$

Any tangent to parabola is $y=mx+\frac{1}{m}$

If this line is tangent to ellipse then $\frac{1}{m^2}=8m^2+2 \Rightarrow 8m^4+2m^2-1=0$

$m^2=\frac{-2\pm\sqrt{4+32}}{16}=\frac{-2\pm 6}{16}$

$\Rightarrow m^2=\frac{1}{4} \Rightarrow m=\pm\frac{1}{2}$

$y=\frac{x}{2}+2$ or $y=-\frac{x}{2}-2$

$x-2y+4=0$ or $x+2y+4=0$

16. If latus rectum of the ellipse $x^2\tan^2\alpha + y^2\sec^2\alpha = 1$ is ½, then $\alpha\left(0<\alpha<\pi\right)$ is equal to

(A) $\pi/12$ (B) $\pi/6$

(C) $5\pi/12$ (D) none of these

Key : A, C

Sol : $x^2\tan^2\alpha + y^2\sec^2\alpha = 1$

$\Rightarrow \frac{x^2}{\cot^2\alpha}+\frac{y^2}{\cos^2\alpha}=1$

$\because \cos^2\alpha = \cot^2\alpha\left(1-e^2\right)$

$\Rightarrow \sin^2\alpha = \left(1-e^2\right)$

$\therefore \quad e^2=\cos^2\alpha\left(\alpha\neq 90^\circ\right)$

$e = \cos\alpha$

$\because$ Latustrectum = 1/2 = $2b^2/a$

$\Rightarrow\ a = 4b^2$

$\Rightarrow \cot\alpha = 4\cos^2\alpha$

$\Rightarrow \frac{1}{\sin\alpha} = 4\cos\alpha$

$\Rightarrow \sin 2\alpha = \frac{1}{2}$

$\sin 2\alpha = \frac{1}{2}$

$2\alpha = n\pi + (-1)^n \frac{\pi}{6}$

$\alpha = \frac{n\pi}{2} + (-1)^n \frac{\pi}{12}$

for $n = 0$

$\alpha = \frac{\pi}{12}$

and

for n = 1

$\alpha = \frac{\pi}{2} - \frac{\pi}{12} = \frac{5\pi}{12}$

17. An ellipse passes through the point (4, - 1) and it's axes are along the axes of coordinates. If the line x + 4y – 10 = 0 is a tangent to it then it's equation can be

a) $\frac{x^2}{100} + \frac{y^2}{5} = 1$ b) $\frac{x^2}{80} + \frac{y^2}{5/4} = 1$ c) $\frac{x^2}{20} + \frac{y^2}{5} = 1$ d) none of these

Key. B,C

Sol. Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, Using the point (4,-1) and the tangent x+4y -10 = 0 ;

$\frac{16}{a^2} + \frac{1}{b^2} = 1; a^2 + 16b^2 = 100$ solving we get options (b),(c)

18. The normal at an end of latusrectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ pass through an end of minor axis if

a) $e^4 + e^2 = 1$ b) $e = \sqrt{\frac{\sqrt{5}-1}{2}}$ c) e = 2 sin18° d) e = 2 cos36°

Key. A,B

Sol. L$\left(ae \frac{b^2}{a}\right)$ normal is $\frac{a^2x}{ae} - \frac{b^2y}{b^2a} = a^2e^2$ passes through (0,-b)

$\therefore e^4 + e^2 = 1$ solving $e = \frac{\sqrt{\sqrt{5}-1}}{2}$

19. The eccentric angle of a point on the ellipse $x^2 + 3y^2 = 6$ at a distance 2 unit from the origin is

(A) $\frac{\pi}{4}$

(B) $\frac{3\pi}{4}$

(C) $\frac{5\pi}{4}$

(D) None of these

Key. A,B,C

Sol. A point on the ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$ is $\left(\sqrt{6}\cos\theta, \sqrt{2}\sin\theta\right)$

$6\cos^2\theta + 2\sin^2\theta = 4$

$\Rightarrow \quad 4\cos^2\theta = 2$

$\cos^2\theta = \frac{1}{2}$

$\cos^2\theta = 0$

$2\theta = \left(2x+1\right)\frac{\pi}{2},\ \theta = \left(2n+1\right)\frac{\pi}{4}, n \in I$.

20. If $x + y = 2$ is a tangent to the ellipse foci are (2 ,3) & (3 ,5) is

A) length of minor axis is 6

B) length of minor axis is 3

C) length of major axis is $\frac{\sqrt{41}}{2}$

D) eccentricity of ellipse is $\frac{\sqrt{5}}{\sqrt{41}}$

Key. A,D

Sol. (a,d) $b^2 = \left(\text{semi minor axis}\right)^2 = \frac{2+3-2}{\sqrt{2}} . \frac{3+5-2}{\sqrt{2}} = 9$; b=3; $2ae = \sqrt{5} \Rightarrow e = \sqrt{5/41}$ $\&\ a = \frac{\sqrt{41}}{2}$

21. If the ellipse $\frac{x^2}{4} + y^2 = 1$ meets the ellipse $x^2 + \frac{y^2}{a^2} = 1$ in four points and $a = b^2 - 5b + 7$ then $b \in$ ____.

A) $(-\infty, 0)$

B) $[4,5]$

C) $[2,3]$

D) $[0,\infty]$

Key. A,B

Sol. (a,b) Both the ellipse have their centre at (0,0). The major axis of first is along x-axis and in case the two ellipse meet in 4 distinct points, then the major axis of second ellipse should be along the y-axis. $\Rightarrow a^2 > 1 \& b^2 - 5b + 7 > 1 ; \Rightarrow b < 2 \text{ or } b > 3$

22. AB and CD are two equal and parallel chords of their ellipse $x^2/a^2 + y^2/b^2 = 1$. Tangents to the ellipse at A and B intersect at P and at C and D at Q,then line PQ

A) passes through the origin

B) is bisected at origin

C) cannot pass through the origin

D) is not bisected at the origin

Key. A,B

Sol. (ab) let $P(\alpha,\beta), Q(\alpha_1,\beta_1)$. Equations of AB and Cd are $\frac{x}{a}\alpha+\frac{y}{b}\beta=1$ & $\frac{x}{a}\alpha_1+\frac{y}{b}\beta_1=1$

$\alpha/\alpha_1=\beta/\beta_1=k \Rightarrow \frac{\alpha^2}{a^2}+\frac{\beta^2}{b^2}=\frac{\alpha_1^2}{a^2}+\frac{\beta_1^2}{b^2} \Rightarrow \frac{\alpha}{\alpha_1}=\frac{\beta}{\beta_1}=-1$; 'PQ' passes through origin and is bisected at origin.

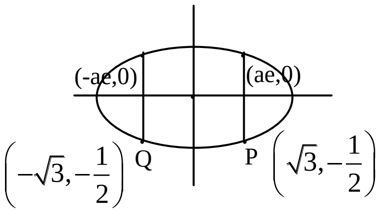
23. Let P (x_1, y_1) and $Q(x_2, y_2)$, $y_1 < 0$, $y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are

(A) $x^2 + 2\sqrt{3}\ y = 3 + \sqrt{3}$ (B) $x^2 - 2\sqrt{3}\ y = 3 + \sqrt{3}$

(C) $x^2 + 2\sqrt{3}\ y = 3 - \sqrt{3}$ (D) $x^2 - 2\sqrt{3}\ y = 3 - \sqrt{3}$

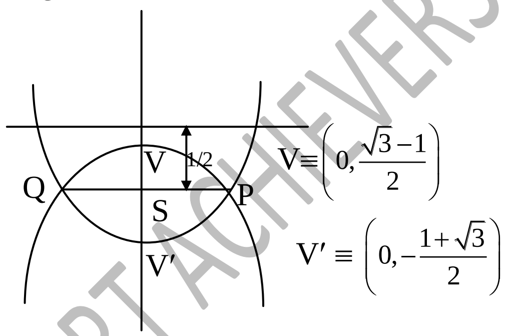
Key. B,C

Sol. Given ellipse is $\frac{x^2}{4}+\frac{y^2}{1}=1 \Rightarrow e = \sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}$



$P \equiv (ae, -b^2/a) = \left(\sqrt{3},-\frac{1}{2}\right)$, $Q \equiv (-ae, -b^2/a) = (-\sqrt{3}, -\frac{1}{2})$

length of PQ = $2\sqrt{3}$



$VS = SV' = \frac{PQ}{4} = \frac{\sqrt{3}}{2}$

$\therefore$ Equations of parabolas are

$x^2 = -2\sqrt{3}\left(y-\frac{\sqrt{3}-1}{2}\right) \Rightarrow x^2 + 2\sqrt{3}\ y = 3 - \sqrt{3}$

and $x^2 = 2\sqrt{3}\left(y+\frac{1+\sqrt{3}}{2}\right) \Rightarrow x^2 - 2\sqrt{3}\,y = 3 + \sqrt{3}$

24. $x^2+y^2=1$ intersects the ellipse $x^2+2y^2=4$ at points P and Q, then the locus of the point of intersection of tangents to the ellipse at P and Q is a conic whose

a) eccentricity is $\frac{\sqrt{3}}{2}$ b) eccentricity is $\frac{\sqrt{5}}{2}$

c) latus -rectum is of length 2 units d) foci are $\left(\pm 2\sqrt{5}, 0\right)$

Key. A,C

Sol. A tangent to the circle $x^2 + y^2 = 1$ is $x\cos\theta + y\sin\theta = 1. R\left(x_o, y_o\right)$ is the point of intersection of the tangents to the ellipse at P and Q $\Leftrightarrow x\cos\theta + y\sin\theta = 1$ and $x_o x + 2y_o y = 4$ represent the same line

$\Leftrightarrow x_o = 4\cos\theta$ and $y_o = 2\sin\theta$

$\Leftrightarrow \frac{x_0^2}{16} + \frac{y_0^2}{4} = 1.$ Hence, locus of P is the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$

25. Consider the ellipse $\frac{x^2}{\tan^2\alpha} + \frac{y^2}{\sec^2\alpha} = 1$ where $\alpha \in \left(0, \pi/2\right)$ which of the following quantities would vary as α varies ?

(A) eccentricity (B) ordinate of the vertex

(C) ordinates of the foci (D) length of the latus rectum

Key: A,B,C,D

Hint: $a^2 = b^2\left(1 - e^2\right)$

$\left(\sec^2\alpha\right)e^2 = 1$

$e = \cos\alpha$

$l = \frac{2a^2}{b}$

26. If P(α, β), the point of intersection of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1$ and the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{a^2(E^2-1)} = \frac{1}{4}$, is equidistant from the foci of the two curves (all lying in the right of y-axis) then

(A) $2\alpha = a(2e + E)$ (B) $a - e\alpha = E\alpha - a/2$

(C) $E = \frac{\sqrt{e^2 + 24} - 3e}{2}$ (D) $E = \frac{\sqrt{e^2 + 12} - 3e}{2}$

Key: B,C

Hint: $S_1P = S_2P \Rightarrow a - e\alpha = E\alpha - \left(\frac{a}{2}\right)$. Also, $\alpha = \frac{ae + \frac{a}{2}E}{2}$

Eliminating α we get $E^2 + 3eE + (2e^2 - 6) = 0 \Rightarrow E = \frac{\sqrt{e^2 + 24} - 3e}{2}$.

27. If a tangent of slope $\frac{1}{3}$ of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (a > b) is normal to the circle $x^2+y^2+2x+2y+1=0$ then

a) maximum value of ab is $\frac{2}{3}$

b) $a\in\left(\sqrt{\frac{2}{5}},2\right)$

c) $a\in\left(\frac{2}{3},2\right)$

d) maximum value of ab is 1

Key: A, B

Hint: Equation of tangent $y=\frac{1}{3}x\pm\sqrt{\frac{a^2}{9}+b^2}$, it passes through (-1, -1)

$-3=-1\pm\sqrt{a^2+9b^2}$

$a^2+9b^2=4$

$AM\ge GM \qquad \frac{4}{2}\ge a.3b\Rightarrow ab\le\frac{2}{3}$

$a^2+9a^2\left(1-e^2\right)=4\Rightarrow e^2=\frac{10a^2-4}{9a^2}$

$0<e^2<1\Rightarrow a\in\left(\sqrt{\frac{2}{5}},2\right)$

28. A point on the ellipse $x^2+3y^2=37$ where the normal is parallel to the line $6x-5y=2$ is

(A) (5, -2) (B) (5, 2) (C) (-5, 2) (D) (-5, -2)

Key. B,D

Sol. Let $P\left(x_1,y_1\right)$ be any point on the ellipse

i.e., $x_1^2+3y_1^2=37$.................(1)

$\frac{dy}{dx\left(x_1,y_1\right)}=\frac{-x_1}{3y_1}$ (Slope of tan)

Slope of normal : $\frac{3y_1}{x_1}=\frac{6}{5}$

$x_1=\frac{15y_1}{6}$

$\Rightarrow y_1=\pm2$ (from (1) $\Rightarrow x_1=\pm5$

29. The number of values of c such that the straight line $y=4x+c$ touches the curve $x^2/4+y^2=1$ is K then K = ………………

Key. 2

Sol. If $y=mx+c$ is tangent to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then

$C^2=a^2m^2+b^2$

$y = 4x + c, \quad \frac{x^2}{4} + \frac{y^2}{1} = 1$

$C = \pm\sqrt{65}$

30. Tangent is drawn to ellipse $x^2/27 + y^2 = 1$ at $\left(3\sqrt{3}\cos\theta, \sin\theta\right)$ (where $\theta \in \left(0, \pi/2\right)$). Then the value of θ such that sum of intercepts on coordinate axes made by this tangent is least is $\frac{\pi}{K}$ then K =

Key. 6

Sol. $\frac{x^2}{27} + \frac{y^2}{1} = 1,\ P\left(3\sqrt{3}\cos\theta, \sin\theta\right)$

$\frac{3\sqrt{3}\cos\theta}{27} + \frac{\sin\theta y}{1} = 1$

$A\left(\frac{3\sqrt{3}\cos\theta}{27}, 0\right), B = \left(0, \frac{1}{\sin\theta}\right)$

$f(\theta) = 3\sqrt{3}\sec\theta + \cos ec\theta$

$f^1(\theta) = \frac{3\sqrt{3}\sin\theta}{\cos^2\theta} - \frac{\cos\theta}{\sin^2\theta} = 0$

$\Rightarrow \tan^3\theta = \frac{1}{3\sqrt{3}} = \left(\frac{1}{\sqrt{3}}\right)^3$

$\theta = \frac{\pi}{6}$

31. If P(α, β), the point of intersection of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1$ and the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{a^2(E^2-1)} = \frac{1}{4}$, is equidistant from the foci of the two curves (all lying in the right of y-axis) then

(A) $2\alpha = a(2e + E)$

(B) $a - e\alpha = E\alpha - a/2$

(C) $E = \frac{\sqrt{e^2 + 24} - 3e}{2}$

(D) $E = \frac{\sqrt{e^2 + 12} - 3e}{2}$

Key. B,C

Sol. $S_1P = S_2P \Rightarrow a - e\alpha = E\alpha - \left(\frac{a}{2}\right)$. Also, $\alpha = \frac{ae + \frac{a}{2}E}{2}$

Eliminating α we get $E^2 + 3eE + (2e^2 - 6) = 0 \ \Rightarrow\ E = \frac{\sqrt{e^2 + 24} - 3e}{2}$.

32. The equations of the common tangents of the curves $x^2 + 4y^2 = 8$ and $y^2 = 4x$ are

A) $x + 2y + 4 = 0$ B) $x - 2y + 4 = 0$ C) $2x + y = 4$ D) $2x - y + 4 = 0$

Key. A,B

Sol. $\frac{x^2}{8}+\frac{y^2}{2}=1,\ y^2=4x$

Any tangent to parabola is $y=mx+\frac{1}{m}$

If this line is tangent to ellipse then $\frac{1}{m^2}=8m^2+2\Rightarrow 8m^4+2m^2-1=0$

$m^2=\frac{-2\pm\sqrt{4+32}}{16}=\frac{-2\pm 6}{16}$

$\Rightarrow m^2=\frac{1}{4}\Rightarrow m=\pm\frac{1}{2}$

$y=\frac{x}{2}+2$ or $y=-\frac{x}{2}-2$

$x-2y+4=0$ or $x+2y+4=0$

33. If the tangent at the point $\left(4\cos\theta,\frac{16}{\sqrt{11}}\sin\theta\right)$ to the ellipse $16x^2+11y^2=256$ is also a tangent to the circle $x^2+y^2-2x=15$, then θ is

A) $\pi/3$ B) $2\pi/3$ C) $-\pi/3$ D) $5\pi/3$

Key. A,C,D

Sol. Equation at $\left(4\cos\theta,\frac{16}{\sqrt{11}}\sin\theta\right)$ to $16x^2+11y^2=256$ is $4\cos\theta+\sqrt{11}\sin\theta y=16$

$\frac{|4\cos\theta-16|}{\sqrt{16\cos^2\theta+11\sin^2\theta}}=4$

$\Rightarrow \cos\theta=\frac{1}{2}$ or $-\frac{5}{2}$ (not possible)

$\therefore \theta=\pm\frac{\pi}{3},\frac{5\pi}{3}$

34. For the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ which of the following are correct?

(a) The point of intersection of tangents drawn at the points $\left(ae,\frac{b^2}{a}\right)$ and $\left(ae,\frac{-b^2}{a}\right)$ is $\left(\frac{a}{e},0\right)$.

(b) The foot of the perpendicular drawn from focus onto a tangent lies on $x^2+y^2=a^2$

(c) equation of the directrix corresponds to the focus (ae,0) is $x-\frac{a}{e}=0$

(d) If F_1,F_2 are the two foci and B is one end of the minor axis such that ΔBF_1F_2 is equilateral then the eccentricity of the ellipse is $\frac{1}{2}$

Key. A,B,C,D

Sol. b) $y=mx+\sqrt{a^2m^2+b^2}$ is a tangent

Line perpendicular to this line and passing through the focus (ae,o) is $y=\frac{-1}{m}(x-ae)$

$\therefore (y-mx)^2+(x+my)^2=a^2m^2+b^2+a^2e^2=a^2m^2+a^2$

$\therefore x^2+y^2=a^2$

d) $BF_1=BF_2=F_1F_2$ and $F_1F_2=2ae$

$BF_1+BF_2=2(F_1F_2)$

2a = 2 (2ae)

$\therefore e=\frac{1}{2}$

35. Equation of the locus at point of intersection of perpendicular tangents to $\frac{(x+y-2)^2}{9}+\frac{(x-y)^2}{16}=1$ is

A) $(x-1)^2+(y-1)^2=\frac{25}{2}$ B) $(x-1)^2+(y-1)^2=50$

C) $(x-1)^2+(y-1)^2=25$ D) $(x+y-2)^2+(x-y)^2=25$

Key. A,D

Sol. $\frac{\left(\frac{x+y-2}{\sqrt{2}}\right)^2}{\frac{9}{2}}+\frac{\left(\frac{x-y}{\sqrt{2}}\right)^2}{8}=1$ (which is in standard form)

$\Rightarrow (x-1)^2+(y-1)^2=\frac{9}{2}+8$ or $\frac{(x+y-2)^2+(x-y)^2}{2}=\frac{9}{2}+8$ is required locus.

36. An ellipse whose major axis is parallel to x-axis is such that the segments of a focal chord are 1 and 3 units. The lines $ax+by+c=0$ are the chords of the ellipse such that a, b, c are in A.P and bisected by the point at which they are concurrent. The equation of auxiliary circle is $x^2+y^2+2\alpha x+2\beta y-2\alpha-1=0$. Then make all the correct alternative

A) The equation of the auxiliary circle is $x^2+y^2-2x+4y+1=0$

B) Eccentricity of the ellipse is $1/2$

C) Lengths of major and minor axes are $4,\sqrt{3}$

D) Eccentricity of the ellipse is $\sqrt{3}/2$

Key. A,B,C

Sol. a,b,c are in A.P $\Rightarrow ax+by+c=0$ are concurrent at $(1,-2)$

$\therefore$ centre of auxiliary circle $=(-\alpha,-\beta)=(1,-2)$

Radius of aux. circle = 2; Length of major axis = 4 = 2A

$\because \frac{1}{SP}+\frac{1}{SQ}=\frac{2A}{B^2}\Rightarrow B=\sqrt{3}$, hence $e=\frac{1}{2}$

29. The equations of the common tangents of the curves $x^2+4y^2=8$ and $y^2=4x$ are

A) $x+2y+4=0$ B) $x-2y+4=0$ C) $2x+y=4$ D) $2x-y+4=0$

Key. A,B

Sol. $\frac{x^2}{8}+\frac{y^2}{2}=1,\ \ y^2=4x$

Any tangent to parabola is $y=mx+\frac{1}{m}$

If this line is tangent to ellipse then $\frac{1}{m^2}=8m^2+2\Rightarrow 8m^4+2m^2-1=0$

$m^2=\frac{-2\pm\sqrt{4+32}}{16}=\frac{-2\pm 6}{16}$

$\Rightarrow m^2=\frac{1}{4}\Rightarrow m=\pm\frac{1}{2}$

$y=\frac{x}{2}+2$ or $y=-\frac{x}{2}-2$

$x-2y+4=0$ or $x+2y+4=0$

Ellipse

Assertion Reasoning Type

a) Both A and R are true and R is correct explanation of A.

b) Both A and R are true but R is not correct explanation of A.

c) A is true, R is false. d) A is false, R is true.

1. Statement - 1: Product of the lengths of the perpendicular drawn from the points (4, 2) and $(4,-6)$ to any tangent of $\frac{(x-4)^2}{9}+\frac{(y+2)^2}{25}=1$ is 9.

Statement - 2: Product of the lengths of the perpendicular drawn from the foci of an ellipse to any tangent of the ellipse is square on the semi minor axis of the ellipse.

Key. A

Sol. Conceptual

2. Statement - 1: The equation $x^2\cos^2\theta+y^2\cot^2\theta=1$ represents a family of confocal ellipses.

Statement - 2: The equation $x^2\sec^2\theta-y^2\text{cosec}^2\theta=1$ represents a family of confocal hyperbolas.

Key. B

Sol. Conceptual

3. Assertion (A): The distance of a focus of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ to an asymptote of the hyperbola is *b*.

Reason (R): The product of distances of any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ from the asymptotes of the hyperbola is $\frac{a^2b^2}{a^2+b^2}$.

Key. B

Sol. Conceptual

4. Assertion (A): Maximum area of the triangle whose vertices lie on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b>0)$ is $\frac{3\sqrt{3}ab}{4}$.

Reason (R): The area of a triangle inscribed in a circle of given radius is maximum when the triangle is equilateral.

Key. A

Sol. Conceptual

5. Statement I: The radius of the largest circle with center (1, 0) that can be inscribed in the ellipse $x^2+4y^2=16$ is $\sqrt{\frac{11}{3}}$.

Statement II: The normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ at a point 'θ' is

$$\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$$

Key. A

Sol. Equation of any normal to $\frac{x^2}{16}+\frac{y^2}{4}=1$ is (4 sec θ) x-(2cosec θ) y = 12

Putting (1,0) we have 4sec θ = 12 $\Rightarrow \cos\theta=\frac{1}{3},\sin\theta=\frac{2\sqrt{2}}{3}$

Hence the point of contact is $\left(\frac{4}{3},\frac{4\sqrt{2}}{3}\right)$

Req rad. = $=\sqrt{\left(\frac{4}{3}-1\right)^2+\left(\frac{4\sqrt{2}}{3}\right)^2}=\sqrt{\frac{11}{3}}$

Statement I is true

Statement II is also true but not a correct explanation of statement –I

6. Statement – 1 : The sum of eccentric angles of four co-normal points of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is an odd multiple of π(π radian = 180^0)

Statement – 2 : The sum of the eccentric angles of the points in which a circle cuts an ellipse is an even multiple of π (π radius = 180)

Key. B

Sol. Conceptual

7. Statement I: The angle of intersection between the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$& the circle $x^2+y^2=ab$ is $\tan^{-1}\left(\frac{b-a}{\sqrt{ab}}\right)$.

Statement II : The point of intersection of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$& $x^2+y^2=ab$ is $\left(\sqrt{\frac{ab}{a+b}},\sqrt{\frac{ab}{a+b}}\right)$

Key. C

Sol. POI is , $\left(\sqrt{\frac{a^2b}{a+b}},\sqrt{\frac{ab^2}{a+b}}\right)$, with

$$m_1=\frac{-b^2}{a^2}\sqrt{\frac{a}{b}}4m_2=-\sqrt{\frac{a}{b}}.\Rightarrow\theta=\tan^{-1}\left(\frac{m_1-m_2}{1+m_1m_2}\right)=\tan^{-1}\left(\frac{b-a}{\sqrt{ab}}\right).$$

8. STATEMENT-1 : The condition on a & b for which two distinct chords of the ellipse $\frac{x^2}{2a^2}+\frac{y^2}{2b^2}=1$ passing through $(a,-b)$ are bisected by $x+y=b$ is $a^2+6ab-7b^2>0$.

STATEMENT-2 : Equation of the chord of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose mid point (x_1,y_1) is of the form $T=S_1$. i.e. $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1=\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1$.

Key. A

Sol. Let the mid point $(t,b-t)\frac{tx}{2a^2}+\frac{(b-t)y}{2b^2}=\frac{t^2}{2a^2}+\frac{(b-t)}{2b^2}$

It passes through $(a,-b)\frac{ta}{2a^2}-\frac{b(b-t)}{2b^2}=\frac{t^2}{2a^2}+\frac{(b-t)^2}{2b^2}$

$t^2\left(a^2+b^2\right)-ab(3a+b)t+2a^2b^2=0$ For real t, $a^2b^2(3a+b)^2-4\left(a^2+b^2\right)2a^2b^2>0$

$9a^2+6ab+b^2-8a^2-8b^2>0$

$a^2+6ab-7b^2>0$

9. Statement - 1: The angle of intersection, of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the circle $x^2+y^2=ab$ is $\tan^{-1}\left(\frac{b-a}{\sqrt{ab}}\right)$.

Statement - 2: The point of intersection, of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $x^2+y^2=ab$ is $\left(\sqrt{\frac{ab}{a+b}},\sqrt{\frac{ab}{a+b}}\right)$

Key. C

Sol. point of intersection is $\left(\sqrt{\frac{a^2b}{a+b}},\sqrt{\frac{ab^2}{a+b}}\right)$ with $m_1=\frac{-b^2}{a^2}.\sqrt{a/b},m_2=-\sqrt{a/b}$

$\Rightarrow\theta=\tan^{-1}\left(\frac{b-a}{\sqrt{ab}}\right)$

10. Statement - 1: If the point (x , y) lies on the curve $2x^2+y^2-24y+80=0$ then the maximum value of x^2+y^2 is 400.

Statement - 2: The point (x , y) is at a distance of $\sqrt{x^2+y^2}$ from origin.

Key. A

Sol. given equation represents ellipse $\frac{x^2}{32}+\frac{(y-12)^2}{64}=1$; The maximum value of $\sqrt{x^2+y^2}$ is the distance between (0, 0) & (0, 20).

11. Consider $\frac{x^2}{25}+\frac{y^2}{9}=1$ and $x^2+y^2=9$

Statement - 1: There can be some points on the ellipse, from which normals drawn are tangents to the given circle.

Statement - 2: If the radius of the circle is greater than the difference of length of semi major axis and semi minor axis of the ellipse which can be tangent to the circle.

Key. C

Sol. Conceptual

12. Statement - 1: Product of the lengths of the perpendicular drawn from the points (4, 2) and $(4,-6)$ to any tangent of $\frac{(x-4)^2}{9}+\frac{(y+2)^2}{25}=1$ is 9.

Statement - 2: Product of the lengths of the perpendicular drawn from the foci of an ellipse to any tangent of the ellipse is square on the semi minor axis of the ellipse.

Key. A

Sol. Conceptual

13. Assertion (A): Maximum area of the triangle whose vertices lie on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 (a>b>0)$ is $\frac{3\sqrt{3}ab}{4}$.

Reason (R): The area of a triangle inscribed in a circle of given radius is maximum when the triangle is equilateral.

Key. A

Sol. Conceptual

14. Statement - 1: If the point (x , y) lies on the curve $2x^2+y^2-24y+80=0$ then the maximum value of x^2+y^2 is 400.

Statement - 2: The point (x , y) is at a distance of $\sqrt{x^2+y^2}$ from origin.

Key. A

Sol. given equation represents ellipse $\frac{x^2}{32}+\frac{(y-12)^2}{64}=1$; The maximum value of $\sqrt{x^2+y^2}$ is the distance between (0, 0) & (0, 20).

15. Consider $\frac{x^2}{25}+\frac{y^2}{9}=1$ and $x^2+y^2=9$

Statement - 1: There can be some points on the ellipse, from which normals drawn are tangents to the given circle.

Statement - 2: If the radius of the circle is greater than the difference of length of semi major axis and semi minor axis of the ellipse which can be tangent to the circle.

Key. D

Sol. conceptual

16. STATEMENT – 1 : In ellipse the sum of the distances between the foci is always less than the sum of the focal distances of any point on it.
STATEMENT – 2 : The eccentricity of any ellipse is less than 1.

Key. A

Sol. distance between foci is 2a e. Sum of the focal distance is 2a.
$ae < a, e < 1.$

17. Statement 1: If the length of the latus rectum of an ellipse is 1/3 of the major axis, then the eccentricity of the ellipse is $\sqrt{2/3}$.
Statement 2: If a focus of an ellipse is at the origin, directrix is the line $x = 4$ and the eccentricity is $\sqrt{2/3}$, then the length of the semi major axis is $4.\sqrt{6}.$

A)Statement I is True, Statement II is True and Statement II is correct explanation of Statement I

B)Statement I is True, Statement II is True but Statement II is not correct explanation of Statement I

C)Statement I is True, Statement II is False

D)Statement I is False, Statement II is True

Key. B

Sol. In statement-1 , if the equation of the ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then

$\frac{2b^2}{a} = \frac{1}{3} \times 2a \Rightarrow \frac{b^2}{a^2} = \frac{1}{3}$

$\Rightarrow 1 - e^2 = 1/3 \Rightarrow e^2 = 2/3$. So the statement -1 true.

In statement -2, Equation of the ellipse is

$x^2 + y^2 = (2/3)(4 + x)^2$ (by definition of ellipse)

$\Rightarrow 3(x^2 + y^2) = 2(16 - 8x + x^2)$

$\Rightarrow x^2 + 16x + 3y^2 = 32$

$\Rightarrow (x + 8)^2 + 3y^2 = 96$

$\Rightarrow \frac{(x+8)^2}{96} + \frac{y^2}{32} = 1$

Length of the semi-major axis $=\sqrt{96}=4\sqrt{6}.$

So the statement-2 is also true but does not lead to statement-1

18. Statement 1: If the normal at an end of a latus-rectum of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the major axis at G, O is the centre of the ellipse, then $OG = ae^3, e$ being the eccentricity of the ellipse

Statement 2: Equation of the normal at a point $(a\cos\theta, b\sin\theta)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{ax}{\cos\theta}+\frac{by}{\sin\theta}=a^2+b^2$

Key. C

Sol. Statement -2 is false , equation of the normal is $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$

In statement-1, $L(ae, b^2/a)=(a\cos\theta, b\sin\theta)$

$\Rightarrow \quad \cos\theta = e$

So normal at $L, \frac{ax}{e}-\frac{by}{\sqrt{1-e^2}}=a^2e^2$

Which meets the major axis $y=0$ at $x=ae^3$ and the statement -1 is True.

19. Statement I: The angle of intersection between the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ & the circle $x^2+y^2=ab$ is $\tan^{-1}\left(\frac{b-a}{\sqrt{ab}}\right)$.

Statement II : The point of intersection of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ & $x^2+y^2=ab$ is $\left(\sqrt{\frac{ab}{a+b}}, \sqrt{\frac{ab}{a+b}}\right)$

Key. C

Sol. POI is , $\left(\sqrt{\frac{a^2b}{a+b}}, \sqrt{\frac{ab^2}{a+b}}\right)$, with

$$m_1=\frac{-b^2}{a^2}\sqrt{\frac{a}{b}}4m_2=-\sqrt{\frac{a}{b}}.\Rightarrow \theta=\tan^{-1}\left(\frac{m_1-m_2}{1+m_1m_2}\right)=\tan^{-1}\left(\frac{b-a}{\sqrt{ab}}\right).$$

20. Consider the curve $4x^2 + 9y^2 = 36$. A tangent is drawn at any arbitrary point P of the above curve and meets the line passing through the point Q$\left(\sqrt{5},0\right)$ and perpendicular to the above tangent at R.

STATEMENT–1
Point R lies on the curve $x^2 + y^2 = 9$
because
STATEMENT–2
Tangents drawn from the point (2, 3) to the curve $4x^2 + 9y^2 = 36$ are perpendicular to each other.

Key. B

Sol. $\left(\sqrt{5},0\right)$ is the focus of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$

$\Rightarrow$ lines PR and QR meets at the auxillary circle of the given ellipse i.e. on the curve $x^2 + y^2 = 9$.

Also $x^2 + y^2 = 13$ is the director circle of the curve $4x^2 + 9y^2 = 36$.

21. Statement – 1 : If any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ intercepts equal length 'l' on the axes,

then $l=\sqrt{a^2+b^2}$

Because

statement – 2 : $lx+my=n$ is a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ if $a^2l^2+b^2m^2=n^2$

Key. A

Sol. $a^2l^2+b^2m^2=n^2$

$$\Rightarrow a^2.1+b^2.1=l^2 \Rightarrow l=\sqrt{a^2+b^2}$$

22. Statement - 1: Product of the lengths of the perpendicular drawn from the points (4, 2) and $(4,-6)$ to any tangent of $\frac{(x-4)^2}{9}+\frac{(y+2)^2}{25}=1$ is 9.

Statement - 2: Product of the lengths of the perpendicular drawn from the foci of an ellipse to any tangent of the ellipse is square on the semi minor axis of the ellipse.

Key. A

Sol. Conceptual

23. Statement - 1: The equation $x^2\cos^2\theta+y^2\cot^2\theta=1$ represents a family of confocal ellipses.

Statement - 2: The equation $x^2\sec^2\theta-y^2\text{cosec}^2\theta=1$ represents a family of confocal hyperbolas.

Key. B

Sol. Conceptual

24. Statement – 1 : The sum of eccentric angles of four co-normal points of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is an odd multiple of π (π radian = 180^0)

Statement – 2 : The sum of the eccentric angles of the points in which a circle cuts an ellipse is an even multiple of π (π radius = 180)

Key. B

Sol. Conceptual

25. Statement-I: In a ΔABC, if base BC is fixed and perimeter of the triangle is also fixed, Then vertex A moves on an ellipse, because

Statement-II: Locus of a moving point is an ellipse if sum of its distances from two fixed points is a positive constant (where all the points are coplanar)

Key. C

Sol. Conceptual

26. Statement - I : The angle between the tangents drawn from the point (2,3) to the ellipse $9x^2+16y^2=144$ is 90^0

Statement - II : Locus of the point of intersection of perpendicular tangents to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is the circle $x^2+y^2=a^2+b^2$

Key. D

Sol. Tangent is $y=mx+\sqrt{a^2m^2+b^2}$

This passes through $(2,3)\Rightarrow$

$$3=2m+\sqrt{16m^2+9}$$

$$(3-2m)^2=16m^2+9$$

$$12m^2+12m=0$$

$$m=0,-1$$

$$\therefore \text{Angle between the tangents is not } 90^0$$

27. Statement - I :P is a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose eccentricity is e and foci $F_1 \text{ and } F_2$.

If The angle $PF_1F_2=\theta_1$ *and angle* $PF_2F_1=\theta_2$, then $\left(\tan\frac{\theta_1}{2}\right)\left(\tan\frac{\theta_2}{2}\right)=\frac{e-1}{1+e}$

Statement - II : In ΔABC, $\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ where s is the semiperimeter of ΔABC

Key. A

Sol. $F_1=(ae,0),\ F_2=(-ae,0)$

Equation of a chord joining α and β is

$$\frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right)+\frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right)=\cos\left(\frac{\alpha-\beta}{2}\right)$$

This passes through (ae,0)

$$\therefore e\cos\left(\frac{\alpha+\beta}{2}\right)=\cos\frac{\alpha-\beta}{2}$$

$$\therefore e=\frac{\cos\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha+\beta}{2}\right)}$$

Use componendo and divedendo

28. Statement - I : The eccentricity of the ellipse $3x^2+4y^2+6x-8y-5=0$ is $\frac{1}{3}$

Statement - II : The eccentricity e of the ellipse (a>b) is given by $b^2=a^2(1-e^2)$

Key. D

Sol. The given ellipse is $3(x+1)^2+4(y-1)^2=12$

$$\frac{(x+1)^2}{4}+\frac{(y-1)^2}{3}=1$$

$$3=4(1-e^2)\Rightarrow e^2=1-\frac{3}{4}=\frac{1}{4}$$

29. Statement - I :If P(x,y) is a point on the ellipse $16x^2+25y^2=400$ and $F_1=(3,0)$ and $F_2=(-3,0)$, Then PF_1+PF_2 is 10

Statement - II : In an ellipse, the sum of the distances of a point on the ellipise from the foci is always constant

Key. A

Sol. $\frac{X^2}{25}+\frac{Y^2}{16}=1$

$$PF_1+PF_2=2a=10$$

Ellipse

Comprehension Type

Paragraph – 1

A sequence of ellipse $E_1, E_2..........E_n$ are constructed as follows: Ellipse E_n is drawn so as to touch ellipse E_{n-1} as the extremities of the major axis of E_{n-1} and to have its foci at the extremities of the minor axis of E_{n-1}

1. If E_n is independent of n then eccentricity of the ellipse E_{n-2}

A) $\frac{3-\sqrt{5}}{2}$ B) $\frac{\sqrt{5}-1}{2}$ C) $\frac{2-\sqrt{3}}{2}$ D) $\frac{\sqrt{3}-1}{2}$

Key. B

Sol. $\frac{x^2}{a_n^2}+\frac{y^2}{b_n^2}=1\left(a_n > b_n\right),\ b_n^2=a_n^2\left(1-e_n^2\right)$ ....(1)

For $E_{n-1},\ a_{n-1}^2=b_{n-1}^2\left(1-e_{n-1}^2\right),\ b_n=b_{n-1}e_{n-1},\ a_{n-1}=a_n$

$b_{n-1}^2e_{n-1}^2=a_n^2\left(1-e_n^2\right)=a_{n-1}^2\left(1-e_n^2\right)$

$b_{n-1}^2e_{n-1}^2=b_{n-1}^2\left(1-e_{n-1}^2\right)\left(1-e_n^2\right)$

Let all the eccentricities are 'e'

$e^2=\left(1-e^2\right)\left(1-e^2\right)\Rightarrow e^4-3e^2+1=0\Rightarrow e=\frac{\sqrt{5}-1}{2}$

2. If eccentricity of ellipse E_n is independent of $'n'$ then the locus of the mid point of chords of slope $'-1'$ of E_n (If the axis of E_n is along Y-axis)

A) $\left(\sqrt{5}-1\right)x=2y$ B) $\left(\sqrt{5}+1\right)x=2y$

C) $\left(3-\sqrt{5}\right)x=2y$ D) $\left(3+\sqrt{5}\right)x=2y$

Key. B

Sol. $T=S_1\Rightarrow\frac{xx_1}{a_n^2}+\frac{yy_1}{b_n^2}=\frac{x_1^2}{a_n^2}+\frac{y_1^2}{b_n^2}$

$\Rightarrow\frac{-b_n^2x_1}{a_n^2y_1}=-1\Rightarrow b_n^2x_1=a_n^2y_1\Rightarrow x_1\left(1-e^2\right)=y_1$

$x_1=y_1\left(1-\frac{3-\sqrt{5}}{2}\right)\Rightarrow 2x_1=\left(\sqrt{5}-1\right)y_1$

$$2y_1 = \left(\sqrt{5}+1\right)x_1$$

Paragraph – 2

$C_1 : x^2 + y^2 = r^2$ and $C_2 : \dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$ intersect at four distinct points A, B, C, D and D. Their common tangents form a parallelogram $A'B'C'D'$.

3. If ABCD is a square then r is equal to

(a) $\dfrac{12}{5}\sqrt{2}$ (b) $\dfrac{12}{5}$ (c) $\dfrac{12}{5\sqrt{5}}$ (d) None of these

Key. A

4. If $A'B'C'D'$ is a square then r is equal to

(a) $\sqrt{20}$ (b) $\sqrt{12}$ (c) $\sqrt{15}$ (d) None of these

Key. D

5. If $A'B'C'D'$ is a square, then the ratio of area of the circle C_1 to the area of the circumcircle of $\Delta A'B'C'$ is

(a) $\dfrac{9}{16}$ (b) $\dfrac{3}{4}$ (c) $\dfrac{1}{2}$ (d) None of these

Key. C

Sol. 3. $x^2 + y^2 = r^2$, $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$

$$\frac{r^2 - y^2}{16} + \frac{y^2}{9} = 1$$

$$9r^2 - 9y^2 + 16y^2 = 144$$

$$y^2 = \frac{144 - 9r^2}{7}$$

$$x^2 = r^2 - y^2 \Rightarrow \frac{16r^2 - 144}{7} = x^2$$

If ABCD is square $x^2 = y^2$

$$\Rightarrow \frac{16r^2 - 144}{7} = \frac{144 - 9r^2}{7}$$

$$25r^2 = 288$$

$$r = \pm\frac{12}{5}\sqrt{2}$$

4. $y = mx \pm \sqrt{16m^2 + 9}$ is equation to $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$

$y = mx + r\sqrt{1+m^2}$ is equation to $x^2 + y^2 = r^2$

$$r^2\left(1+m^2\right) = 25$$

$$2r^2 = 25$$

$$r^2 = \frac{25}{2} \qquad r = \pm\frac{5}{\sqrt{2}}$$

5. $A^1B^1C^1D^1$ is square then common tangent is $y=\pm x\pm 5$

$y=x+5, y=x-5, y=-x+5, y=-x-5$

$y=x+5$

$y=-x+5 \Rightarrow y=5$

$A^1(0,5) \qquad C^1(0,-5)$

$A^1C^1=10$

Radius of circum circle of $\Delta A^1B^1C=5$

Area of circle $C_1=\frac{25\pi}{2}$

Ratio = $\frac{1}{2}$

Paragraph – 3

Consider the circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$ where $b>a>0$. Let $A(-a,0); B(a,0)$. A parabola passes through the points A, B and its directrix is a tangent to $x^2+y^2=b^2$. If the locus of focus of the parabola is a conic then

6. The eccentricity of the conic is

A) $2a/b$ B) b/a C) a/b D) 1

Key. C

7. The foci of the conic are

A) $(\pm 2a,0)$ B) $(0,\pm a)$ C) $(\pm a,2a)$ D) $(\pm a,0)$

Key. D

8. Area of triangle formed by a latusrectum and the lines joining the end points of the latusrectum and the centre of the conic is

A) $\frac{a}{b}\left(b^2-a^2\right)$ B) $2ab$ C) $ab/2$ D) $4ab/3$

Key. A

Sol. 6 – 8:

$x^2+y^2=a^2;\ x^2+y^2=b^2;\ b>a>0,\ A=(-a,0);\ \ B=(a,0)$

Let (h,k) be a point on the locus. Any tangent to circle $x^2+y^2=b^2$ is $x\cos\theta+y\sin\theta=b$

$\therefore$ Equation of parabola is $\sqrt{(x-h)^2+(y-K)^2}=|x\cos\theta+y\sin\theta-b|$

i.e., $(x-h)^2+(y-K)^2=(x\cos\theta+y\sin\theta-b)^2$

The points $(\pm a,0)$ satisfy this equation

$\therefore (a-h)^2+K^2=(a\cos\theta-b)^2$ --- (1)

$(a+h)^2+K^2=(a\cos\theta+b)^2$ ---- (2)

(2) – (1) $\Rightarrow h=b\cos\theta$

$\therefore$ Required locus is $(a+x)^2+y^2=\left(\frac{ax}{b}+b\right)^2$

i.e., $\frac{x^2}{b^2}+\frac{y^2}{b^2-a^2}=1$ which is an ellipse.

Paragraph – 4

If $\alpha,\beta,\gamma,\delta$ are eccentric angles of 4 – points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ the normals at which are concurrent then

9. $\alpha+\beta+\lambda+\delta=$

A. $2n\pi, n\in z$ B. $(2n+1)\frac{\pi}{2}, n\in z$

C. $(2n+1)\pi, n\in z$ D. $(2n+1)\frac{\pi}{4}, n\in z$

Key. C

10. $\cos(\alpha+\beta)+\cos(\alpha+\lambda)+\cos(\alpha+\delta)+\cos(\beta+\gamma)+\cos(\beta+\delta)+\cos(\lambda+\delta)=$

A. 6 B. 3 C. 0 D. 1

Key. C

11. $\sin(\alpha+\beta)+\sin(\beta+\lambda)+\sin(\lambda+\delta)=$

A. O B. 1 C. -1 D. 2

Key. A

Sol.

Let Z=cisθ $\frac{1}{Z}=\cos\theta-i\sin\theta$

$2\cos\theta=Z+\frac{1}{Z}$, $\cos\theta=\frac{Z^2+1}{2Z}$

$\sin\theta=\frac{Z^2+1}{2iZ}$

Equation of normal is $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$

It is drawn from (x_1,y_1).

$\frac{ax_1}{\cos\theta}-\frac{by_1}{\sin\theta}=a^2-b^2$

$$\frac{ax_1}{\left(\frac{Z^2+1}{2t}\right)}-\frac{by_1}{\frac{Z^2-1}{2\,iZ}}=a^2-b^2$$

$$(a^2-b^2)Z^4-2(ax_1-iby_1)Z^3+2(ax_1+iby_1)Z-(a^2-b^2)=0\rightarrow(1)$$

9. Roots are Z_1,Z_2,Z_3,Z_4

$$Z_1\,Z_2\,Z_3\,Z_4=-1 \qquad cis\alpha.cis\beta.cis\gamma.cis\delta=-1$$

$$cis(\alpha+\beta+\gamma+\delta)=-1$$

$$\cos(\alpha+\beta+\gamma+\delta)=-1,\quad \sin(\alpha+\beta+\gamma+\delta)=0$$

$$\alpha+\beta+\gamma+\delta=(2n+1)\pi$$

10. $\sum Z_1Z_2=0$

$$\sum cis\alpha.cis\beta=0$$

$$\sum cis(\alpha+\beta)=0$$

$$\cos(\alpha+\beta)+\cos(\alpha+\gamma)+\cos(\alpha+\delta)+\cos(\beta+\gamma)+\cos(\beta+\delta)+\cos(\gamma+\delta)=0$$

11. Ily, $\sin(\alpha+\beta)+\sin(\alpha+\gamma)+\sin(\alpha+\delta)+\sin(\beta+\gamma)+\sin(\beta+\delta)+\sin(\gamma+\delta)=0$

$$\sin(\alpha+\beta)=\sin(\gamma+\delta)$$

$$\sin(\beta+\gamma)=\sin(\alpha+\delta)$$

$$\sin(\gamma+\alpha)=\sin(\beta+\delta)$$

$$2(\sin(\alpha+\beta)+\sin(\beta+\gamma)+\sin(\gamma+\delta))=0$$

$$\sin(\alpha+\beta)+\sin(\beta+\gamma)+\sin(\gamma+\delta)=0$$

Paragraph – 5

Let P, Q, R be three points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and let P', Q', R' be their corresponding points on it's auxiliary circle, then

12. The maximum area of the triangle PQR is

a) $\frac{3\sqrt{3}}{4}ab$ b) $\frac{3\sqrt{3}}{2}ab$ c) $\frac{\sqrt{3}}{4}ab$ d) πab

Key. A

Sol. Let P = (a cos α, b sin α) P^1 = (a cos α, a sin α)
P = (a cos β, b sin β) Q^1 = (a cos β, a sin β)
R= (a cos γ, b sin γ) R^1 = (a cos γ, a sin γ).
Area of ΔPQR is 2ab $\sin\frac{\alpha-\beta}{2}\sin\frac{\beta-\gamma}{2}\sin\frac{\gamma-\alpha}{2}$ its max value is 2ab
$\left(\frac{\sqrt{3}}{2}\right)\left(\sqrt{3}/2\right)\left(\sqrt{3}/2\right)=\frac{3\sqrt{3}ab}{4}$

13. $\frac{Area\,of\ \Delta PQR}{Area\,of\ \Delta P'Q'R'}=$

a) $\frac{a}{b}$ b) $\frac{b}{a}$

c) $\frac{1}{2}$ d) depends on points taken

Key. B

Sol. $$\frac{\text{Area of }\Delta PQR}{\text{Area of }\Delta P^1Q^1R^1}=\frac{\frac{1}{2}ab\begin{vmatrix}\cos\alpha-\cos\gamma & \sin\alpha-\sin\gamma\\ \cos\alpha-\cos\beta & \sin\alpha-\sin\beta\end{vmatrix}}{\frac{1}{2}a^2\begin{vmatrix}\cos\alpha-\cos\gamma & \sin\alpha-\sin\gamma\\ \cos\alpha-\cos\beta & \sin\alpha-\sin\beta\end{vmatrix}}=\frac{b}{a}$$

14. When the area of triangle PQR is maximum, the centroid of triangle $P'Q'R'$ lies at

a) one focus b) one vertex c) centre d) on one directrix

Key. C

Sol. Area of ΔPQR is max when $\alpha-\beta=\beta-\gamma=\gamma-d=120^0$ is $\Delta P^1Q^1R^1$ is equilateral hence its centroid is (0,0) centre of the ellipse

Paragraph – 6

If 'P' is any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$ S_1 and S_2 are foci of the ellipse

15. Locus of incentre of triangle PS_1S_2 will be

a) a straight line b) a circle c) a parabola d) an ellipse

Key. D

16. If $e=\frac{1}{2}$ and $\lfloor PS_1S_2=\alpha, \lfloor PS_2S_1=\beta, \lfloor S_1PS_2=\gamma,$ then $\cot\frac{\alpha}{2}, \cot\frac{\gamma}{2}$ and $\cot\frac{\beta}{2}$ are in

a) A.P b) G.P

c) H.P d) None

Key. A

17. Maximum area of the triangle PS_1S_2 is equal to

a) b^2e sq.units b) a^2e sq.units

c) ab sq.units d) abe sq.units

Key. D

Sol. 15. $\frac{PS_2}{S_2G}=\frac{PS_1}{GS_1}=\frac{PS_2+PS_1}{S_2G+GS_1}=\frac{2a}{2ae}=\frac{1}{e}$

So $PI:IG=1:e$

16. $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{1-e}{1+e}=\frac{1}{3}$

17. Base S_1S_2 fixed and PS_2+PS_2 is fixed, Hence area will be maximum if $PS_1=PS_2$

Paragraph – 7

Consider the conic defined by the equation :

$$\left|\sqrt{(x-1)^2+(y-2)^2}-\sqrt{(x-5)^2+(y-5)^2}\right|=3$$

18. The equation of an axis of the conic is

a) $6x+8y=45$ b) $3x-4y-5=0$

c) $8x+6y=45$ d) $3x+4y+5=0$

Key. C

Sol. Given equation represents a hyperbola having foci $S(1,2)$ and $S'(5,5)$ & $2a=3$

transverse axis : line SS': $3x-4y+5=0$

Conjugate axis : perpendicular bisector of $SS':8x+6y=45$

19. The distance between the directrices of the conic is

a) $9/5$ b) $3/5$

c) $5/3$ d) $5/9$

Key. A

Sol. Distance between diretrices = $=\frac{2a}{e}=\frac{3}{5/3}=\frac{9}{5}$

20. The eccentricity of the conic conjugate to the given one, is

a) $5/3$ b) $5/4$ c) $5/2$ d) 5

Key. B

Sol. let e' be the ecc. of conjugate hyperbola then $\frac{1}{e^2}+\frac{1}{e'^2}=1 \Rightarrow e'^2=\frac{25}{16}$

Paragraph – 8

An ellipse E has its centre C (1,3), focus at S (6, 3) and passes through the point P (4, 7). Then

21. The product of the perpendicular distances of foci from tangent at P to the ellipse, is

a) 20 b) 45 c) 40 d) 60

Key. A

22. The point of intersection of the lines joining each focus to the foot of the perpendicular from the other focus upon the tangent at P, is

a) $\left(\frac{5}{3},5\right)$ b) $\left(\frac{4}{3},3\right)$

c) $\left(\frac{8}{3},3\right)$ d) $\left(\frac{10}{3},5\right)$

Key. D

23. If the normal at a variable point on the ellipse (E) meets its axes in Q and R, then the locus of the midpoint of QR is a conic with eccentricity =

a) $3/\sqrt{10}$ b) $\sqrt{5}/3$

c) $3/\sqrt{5}$ d) $\sqrt{10}/3$

Key. B

Sol. $CS=ae=5$

$S'=(-4,5)$

$PS+PS'=2a=6\sqrt{5}$

$\Rightarrow e=\frac{\sqrt{5}}{3}$

23. Product $=b^2$

Paragraph – 9

The equation $ax^2+2hxy+by^2=1, h^2 \neq ab$ represents ellipse or a hyperbola accordingly as $h^2<ab(or)h^2>ab$. The length of the axis of the conic are related with the roots of the quadratic $\left(ab-h^2\right)t^2-(a+b)t+1=0$. If t_1, t_2 are positive, then, lengths of the axes are $2\sqrt{t_1}\ \&\ 2\sqrt{t_2}$. If $t_1>0$ & $t_2<0$, then, lengths of the transverse and conjugtate axes are $2\sqrt{t_1}\ \&\ 2\sqrt{-t_2}$. The equation to the axes of the conic are $(at_1-1)x+ht_1y=0\ \&\ (at_2-1)x+ht_2y=0$.

Answer the following.

24. The eccentricity of the conic $x^2+xy+y+y^2=1$ is

a) $\frac{1}{\sqrt{3}}$ b) $\frac{3}{5}$ c) $\frac{\sqrt{2}}{3}$ d) $\frac{2}{\sqrt{6}}$

Key. D

Sol. Conceptual

25. Area enclosed by the ellipse $5x^2-6xy+5y^2=8$ is,

a) $\pi\sqrt{2}$ b) 2π c) $\pi\sqrt{3}$ d) $\frac{4\pi}{3}$.

Key. B

Sol. Conceptual

26. If the line $\frac{x}{a}+\frac{y}{b}=1$ is the transverse axis of the hyperbola $(x+1)^2+4(x+1)(y-1)+(y-1)^2=4$, then, $a+b=$

a) 0 b) -1 c) 2 d) -3.

Key. A

Sol. Conceptual

Paragraph – 10

A sequence of ellipse $E_1, E_2, ...E_n$ is constructed as follows : Ellipse E_n is drawn so as to touch ellipse E_{n-1} as the extremities of the major axis of E_{n-1} and to have its foci at the extremities of the minor axis of E_{n-1}.

27. If E_n is independent of n then the eccentricity of the ellipse E_{n-2} .

(A) $\frac{3-\sqrt{5}}{2}$ (B) $\frac{\sqrt{5}-1}{2}$

(C) $\frac{2-\sqrt{3}}{2}$ (D) $\frac{\sqrt{3}-1}{2}$

Key. B

Sol. $\frac{x^2}{a_n^2}+\frac{y^2}{b_n^2}=1, \quad a_n>b_n$

$b_n^2 = a_n^2\left(1-b_n^2\right)$ ...(i)

$b_n = b_{n-1}$ .....(ii), $a_{n-1} = a_n b_n$ ....(iii)

For $E_{n-1}, a_{n-1} = b_{n-1}^2\left(1-e_{n-1}^2\right)$

From (i) & (ii) $b_{n-1}^2 = an^2\left(1-e_{n-1}^2\right)$

$\therefore \quad a_n^2 b_n^2 = a_n^2\left(1-e_n^2\right)\left(1-e_{n-1}^2\right)$

Let all the eccentricities are e

$\therefore \quad e^2 = \left(1-e^2\right)^2 \Rightarrow e^4 - 3e^2 + 1 = 0$

$e^2 = \frac{3 \pm \sqrt{5}}{2} \Rightarrow e = \frac{\sqrt{5}-1}{2}$

28. If eccentricity of ellipse E_n is e_n then locus of $\left(e_n^2, e_{n-1}^2\right)$ is a

(A) parabola (B) An ellipse

(C) Circle (D) A rectangular hyperbola

Key. D

Sol. $e_n^2 = \left(1-e_n^2\right)\left(1-e_{n-1}^2\right)$

$\Rightarrow \quad h = (1-h)(1-k) \qquad h = e_n^2$

$\Rightarrow \quad x = 1 - x - y + xy \qquad k = e_{n-1}^2$

$\Rightarrow \quad xy - 2x - y + 1 = 0$

A rectangular hyperbola.

29. If eccentricity of E_n is independent of n then the locus of the mid point of chords of slope - 1 of E_n (If axis of E_n is along y-axis)

(A) $\left(\sqrt{5}-1\right)x = 2y$ (B) $\left(\sqrt{5}+1\right)x = 2y$

(C) $\left(3-\sqrt{5}\right)x = 2y$ (D) $\left(3+\sqrt{5}\right)x = 2y$

Key. B

Sol. $T = S_1 \Rightarrow \frac{xx_1}{a_n^2} + \frac{yy_1}{b_n^2} = \frac{x_1^2}{a_n^2} + \frac{y_1^2}{b_n^2} - \frac{b_n^2 x_1}{a_n^2 y_1} = -1$

If eccentricity of E_n is independent of n

$e = \frac{\sqrt{5}-1}{2} \Rightarrow e^2 = \frac{3-\sqrt{5}}{2}$

$b_n^2 x_1 = a_n^2 y_1$

$x_1 = \left(1 - \frac{\left(3-\sqrt{5}\right)}{2}\right) y_1 \Rightarrow 2x_1 = \left(\sqrt{5}-1\right) y_1$

$\Rightarrow 2x_1\left(\sqrt{5}+1\right)=4y_1$

$2y = x\left(\sqrt{5}+1\right)$

Paragraph – 11

Suppose than an ellipse and a circle are respectively given by the equation

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ..(1) and

$x^2+y^2+2gx+2fy+c=0$ ...(2)

The equation, $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)+\lambda(x^2+y^2+2gx+2fy+c)=0$ ...(3)

Represents a curve which passes through the common points of the ellipse (1) and the circle (2).

We can choose λ so that the equation (3) represents a pair of straight lines. In general we get three values of λ, indicating three pair of straight lines can be drawn through the points. Also when (3) represents a pair of straight lines they are parallel to the lines $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\lambda(x^2+y^2)=0$ ¸which represents a pair of lines equally inclined to axes (the term containing xy is absent). Hence two straight lines through the points of intersection of an ellipse and any circle make equal angles with the axes. Above description can be applied identically for a hyperbola and a circle.

30. The radius of the circle passing through the point of intersection of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ & $x^2-y^2=0$ is

(A) $\frac{ab}{\sqrt{a^2+b^2}}$ (B) $\frac{\sqrt{2}\,ab}{\sqrt{a^2+b^2}}$

(C) $\frac{a^2-b^2}{\sqrt{a^2+b^2}}$ (D) $\frac{a^2+b^2}{\sqrt{a^2+b^2}}$

Key. B

Sol. $x^2+y^2=\frac{a^2b^2}{a^2+b^2}$

$\therefore$ radius of the circle

$\sqrt{\frac{2a^2b^2}{a^2+b^2}}=\frac{\sqrt{2}ab}{\sqrt{a^2+b^2}}$

31. Suppose two lines are drawn through the common points of intersection of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ & $x^2+y^2+2gx+2fy+c=0$. If these lines are inclined at an angle α,β to $x-$ axis then

(A) $\alpha=\beta$ (B) $\alpha+\beta=\frac{\pi}{2}$

(C) $\alpha+\beta=\pi$ (D) $\alpha+\beta=2\tan^{-1}\left(\frac{b}{a}\right)$

Key. C

Sol. As the lines joining common point of intersection must be equally inclined to the axis

$\tan\alpha=-\tan\beta\Rightarrow\alpha+\beta=T_1$

32. The no. of pair of St. lines through the points of intersection of $x^2-y^2=1$ and $x^2+y^2-4x-5=0$.

(A) 0 (B) 1

(C) 2 (D) 3

Key. C

Sol. Any curve through their point of intersection

$x^2+y^2-4x-5+\lambda(x^2-y^2-1)\Rightarrow(1+\lambda)x^2+(1-\lambda)y^2-4x-5-\lambda=0$

$(1+\lambda)(1-\lambda)(-5-\lambda)+0-(1+\lambda).0-(1-\lambda).4+(5+\lambda).0=0$

$(\lambda-1)(\lambda+3)^2=0\Rightarrow\lambda=1,-3$

$\therefore$ Two pair of St.lines can be drawn.

Paragraph – 12

The points P,Q,R are taken on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with eccentricities $\theta,\ \theta+\alpha,\ \theta+2\alpha$ then

33. Area of the triangle PQR is independent of

A) θ B) α C) θ & α both D) none

Key. A

34. If the area of triangle PQR is maximum, then

A) $\alpha=\pi/3$ B) $\alpha=\pi/2$ C) $\alpha=2\pi/3$ D) none

Key. C

35. If A_1 be the area of triangle PQR and A_2 be the area of the triangle formed by corresponding points on the auxiliary circle then $\frac{A_1}{A_2}$ is ___.

A) 1 B) a/b C) b/a D) none

Key. C

Sol. 33,34 & 35- P-III.

(28) a ; 29) c ; 30) c)

$$A_1=\Delta=\frac{1}{2}\begin{vmatrix} a\cos\theta & b\sin\theta & 1\\ a\cos(\theta+\alpha) & b\sin(\theta+\alpha) & 1\\ a\cos(\theta+2\alpha) & b\sin(\theta+2\alpha) & 1\end{vmatrix}=ab(1-\cos\alpha)\sin\alpha$$

Δ is max $\Rightarrow\alpha=2\pi/3$;

$$A_1 = \frac{1}{2}\begin{vmatrix} a\cos\alpha & b\sin\alpha & 1 \\ a\cos\beta & b\sin\beta & 1 \\ a\cos\gamma & b\sin\gamma & 1 \end{vmatrix} = A_2 = \frac{1}{2}\begin{vmatrix} a\cos\alpha & b\sin\alpha & 1 \\ a\cos\beta & b\sin\beta & 1 \\ a\cos\gamma & b\sin\gamma & 1 \end{vmatrix} \quad \therefore A_1/A_2 = b/a$$

Paragraph – 13

P is any point of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. S and S′ are foci and e is the eccentricity of ellipse. $\angle PSS' = \alpha$ and $\angle PS'S = \beta$

36. $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}$ is equal to

(A) $\frac{2e}{1-e}$ (B) $\frac{1+e}{1-e}$

(C) $\frac{1-e}{1+e}$ (D) $\frac{2e}{1+e}$

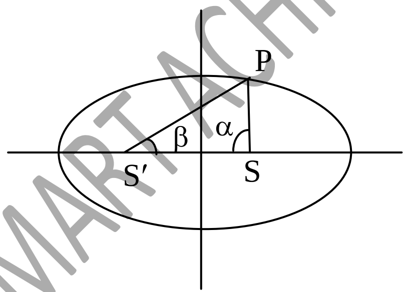
Key. C

Sol. $\frac{PS}{\sin\beta}=\frac{PS'}{\sin\alpha}=\frac{2ae}{\sin(\pi-(\alpha+\beta)}$

or, $\frac{2a}{\sin\alpha+\sin\beta}=\frac{2ae}{\sin(\alpha+\beta)}$

or, $\frac{1}{e}=\frac{2\sin\frac{\alpha+\beta}{2}.\cos\frac{\alpha-\beta}{2}}{2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}}$

$\therefore \frac{1-e}{1+e}=\tan\frac{\alpha}{2}\tan\frac{\beta}{2}$



37. Locus of incentre of triangle PSS′ is

(A) an ellipse (B) hyperbola

(C) parabola (D) circle

Key. A

Sol. $y - 0 = \tan\frac{\beta}{2}(x + ae)$ ... (i)

$y - 0 = -\tan\frac{\alpha}{2}(x - ae)$ ... (ii) or, $y^2 = -\left(\frac{1-e}{1+e}\right)[x^2 - a^2e^2]$

or, $\left(\frac{1-e}{1+e}\right)x^2 + y^2 = \left(\frac{1-e}{1+e}\right)a^2e^2$

or, $\frac{x^2}{a^2e^2} + \frac{y^2}{\left(\frac{1-e}{1+e}\right)a^2e^2} = 1$

which is clearly an ellipse.

38. Eccentricity of conic, which is locus of incentre of triangle PSS′

(A) $\sqrt{\frac{e}{1+e}}$ (B) $\sqrt{\frac{2e}{1+e}}$

(C) $\sqrt{\frac{2e}{1-e}}$ (D) $\sqrt{\frac{e}{1-e}}$

Key. B

Sol. $e' = \sqrt{1-\frac{1-e}{1+e}}$

$= \sqrt{\frac{2e}{1+e}}$

Paragraph – 14

Consider the conic defined by $x^2 + y^2 = (3x+4y+10)^2$.

39. If (α, β) is the centre of the conic then $4\alpha + 3\beta =$

A) –8 B) –10 C) –6 D) –9

Key. B

40. If (p, q) is a vertex of the conic then $2p - q =$

A) –1 B) 1 C) –3 D) 2

Key. A

41. The number of points through which a pair of real perpendicular tangents can be drawn to the conic is

A) infinite B) 1 C) 0 D) 4

Key. C

Sol. (39 – 41)

The given equation can be expressed as $\sqrt{x^2 + y^2} = 5\frac{|3x+4y+10|}{5}$

Hence it is Hyperbola with eccentricity 5.

Focus is (0, 0)

Directrix is $3x + 4y + 10 = 0$

And hence the axis is $4x - 3y = 0$

Paragraph – 15

The points P,Q,R are taken on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with eccentricities $\theta,\ \theta+\alpha,\ \theta+2\alpha$ then

42. Area of the triangle PQR is independent of

A) θ B) α C) θ & α both D) none

Key. A

43. If the area of triangle PQR is maximum, then

A) $\alpha=\pi/3$ B) $\alpha=\pi/2$ C) $\alpha=2\pi/3$ D) none

Key. C

44. If A_1 be the area of triangle PQR and A_2 be the area of the triangle formed by corresponding points on the auxiliary circle then $\frac{A_1}{A_2}$ is ____.

A) 1 B) a/b C) b/a D) none

Key. C

Sol. 42,43 & 44

$$A_1=\Delta=\frac{1}{2}\begin{vmatrix} a\cos\theta & b\sin\theta & 1\\ a\cos(\theta+\alpha) & b\sin(\theta+\alpha) & 1\\ a\cos(\theta+2\alpha) & b\sin(\theta+2\alpha) & 1\end{vmatrix}=ab(1-\cos\alpha)\sin\alpha$$

Δ is max $\Rightarrow \alpha=2\pi/3$;

$$A_1=\frac{1}{2}\begin{vmatrix} a\cos\alpha & b\sin\alpha & 1\\ a\cos\beta & b\sin\beta & 1\\ a\cos\gamma & b\sin\gamma & 1\end{vmatrix}=A_2=\frac{1}{2}\begin{vmatrix} a\cos\alpha & b\sin\alpha & 1\\ a\cos\beta & b\sin\beta & 1\\ a\cos\gamma & b\sin\gamma & 1\end{vmatrix}\quad \therefore A_1/A_2=b/a$$

Paragraph – 16

Consider the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with centre “O” where a >b>0. Tangent at any point P on the ellipse meets the co-ordinate axis at X and Y and N is the root of the perpendicular from the origin on the tangent at P. Minimum length of XY is 24 and maximum length of PN is 8.

45. The eccentricity of the ellipse is

a) 2/5 b) 3/5 c) $\frac{\sqrt{3}}{2}$ d) ¾

46. maximum area of an isosceles triangle inscribed in the ellipse of one of its vertex coincides with one end of the major axis of the ellipse is

a) $196\sqrt{3}$ b) $96\sqrt{3}$ c) 96 d) $3\sqrt{96}$

47. Maximum area of the triangle OPN is

a) 96 b) $96\sqrt{3}$ c) $196\sqrt{3}$ d) 48

Sol. 45. (c) $a+b=24$, $a-b=8$

46. (b) $\frac{3\sqrt{3}}{4}ab$

47. (d) $\frac{a^2-b^2}{4}$

Paragraph – 17

To find out the lengths and positions of the axes of the conic whose equation is $ax^2+2hxy+by^2=1$ ---(1), where the axes of co-ordinates being rectangular, consider a circle of radius ‘r’ with its’ centre at the centre of the conic, whose equation is $\frac{x^2+y^2}{r^2}=1$---(2). Subtracting (2) from (1), we obtain $\left(a-\frac{1}{r^2}\right)x^2+2hxy+\left(b-\frac{1}{r^2}\right)y^2=0$. ---(3), which represents a pair of straight lines through origin and the intersection of (1) and (2). Theses straight lines will be coincident when and only when they lie along the axes of the conic, the condition for which is $\left(a-\frac{1}{r^2}\right)\left(b-\frac{1}{r^2}\right)=h^2$ ----(4). If r_1^2 and r_2^2 be the root and both be +ve, then the conic is an ellipse with $2r_1$ and $2r_2$ as the length of its axes.

Given a conic $5x^2-6xy+5y^2+22x-26y+29=0$, the axes being rectangular. Now answer the following questions.

48. Length of major axis is

a) 4 b) 6 c) 3 d) 2

49. Lengths of minor axis is

a) 3 b) 4 c) 2 d) 1

50. Equation of major and minor axes respectively are

a) $2x+y-1=0, x-2y+3=0$ b) $2x-y+3=0, x+2y+4=0$

c) $x-y+5=0, x+y+3=0$ d) $x-y+3=0, x+y-1=0$

Sol. 48-50. (A) (C) (D) The centre is given by

$5x-3y+11=0,$

$-3x+5y-13=0,$

from which we find x = -1, y = 2.

On transeferring the origin to this point we find that the equation of the conic becomes

$5x^2-6xy+5y^2-8=0$,

that is $\frac{5}{8}x^2-2\left(\frac{3}{8}\right)xy+\frac{5}{8}y^2=1,$

so that $a=\frac{5}{8},h=-\frac{3}{8},b=\frac{5}{8}$

the lengths of the semi-axes are then given by

$$\left(\frac{5}{8}-\frac{1}{r^2}\right)\left(\frac{5}{8}-\frac{1}{r^2}\right)=\frac{9}{64}$$

$$\therefore r^3=4\ or\ L$$

these re of course the equations of the axes of the ellipse referred to the new axes of coordinates. The equation of the major axis referred to the original axes will be

$(x+1)-(y-2)=0,$ that is $x-y+3=0,$ and of the minor axis.

$(x+1)+(y-2)=0,$ That is

$x+y-1=0,$ and of the minor axis.

Paragraph – 18

A conic "c" satisfies the differential equation, $\left(1+y^2\right)dx-xydy=0$ and passes through the point (1,0) An ellipse ''E' which is confocal with "c" having its eccentricity equal to $\sqrt{\frac{2}{3}}$

51. Length of the latus rectum of the conic "C" is

a)1 b)2 c)3 d)4

52. Equation of the ellipse "E" is

a) $\frac{x^2}{9}+\frac{y^2}{4}=1$ b) $\frac{x^2}{4}+\frac{y^2}{9}=1$ c) $\frac{x^2}{1}+\frac{y^2}{3}=1$ d) $\frac{x^2}{3}+\frac{y^2}{1}=1$

53. Locus of the point of intersection of the perpendicular tangents to the ellipse E is

a) $x^2+y^2=4$ b) $x^2+y^2=8$ c) $x^2+y^2=10$ d) $x^2+y^2=12$

Sol. 51 – 53. (B) (D) (A)

$$\left(1+y^2\right)dx=xydy$$

$$2\log x=\log\left(1+y^2\right)+1$$

$$x=1,y=0\Rightarrow c=0$$

$$eqn^n of\ 'c' is\ x^2+y^2$$

$$e=\sqrt{2}$$

51. $2a=2$

52. $b^2=a^2\left(1-e^2\right)=1$

ellipse $\frac{x^2}{3}+\frac{y^2}{1}=1$

53. $x^2+y^2=4$

Paragraph – 19

Consider the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with centre "O" where a >b>0. Tangent at any point P on the ellipse meets the co-ordinate axis at X and Y and N is the root of the perpendicular from the origin on the tangent at P. Minimum length of XY is 24 and maximum length of PN is 8.

54. The eccentricity of the ellipse is

a) 2/5 b) 3/5 c) $\frac{\sqrt{3}}{2}$ d) ¾

55. maximum area of an isosceles triangle inscribed in the ellipse of one of its vertex coincides with one end of the major axis of the ellipse is

a) $196\sqrt{3}$ b) $96\sqrt{3}$ c) 96 d) $3\sqrt{96}$

56. Maximum area of the triangle OPN is

a) 96 b) $96\sqrt{3}$ c) $196\sqrt{3}$ d) 48

Sol. 54. (C) $a+b=24$, $a-b=8$

55. (B) $\frac{3\sqrt{3}}{4}ab$

56. (D) $\frac{a^2-b^2}{4}$

Paragraph – 20

An ellipse whose major axis is parallel to x –axis such that the segments of the focal chords are 1 and 3 units. The lines $ax+by+c=0$ are the chords of the ellipse such that a,b,c are in A.P. and bisected by the point at which they intersect. The equation of its auxiliary circle is $x^2+y^2+2\alpha x+2\beta y-2\alpha-1=0$ then

57. The centre of the ellipse is

A) $(1,1)$ B) $(1,2)$ C) $(1,-2)$ D) $(-2,1)$

Key. C

Sol. Conceptual

58. Equation of the auxiliary circle is

A) $x^2+y^2-2x+4y+1=0$ B) $x^2+y^2+2x+2y-3=0$

C) $x^2+y^2+2x+4y+1=0$ D) $x^2+y^2-4x+2y-3=0$

Key. A

Sol. Conceptual

59. Length of major and minor axis are

A) $4, 2\sqrt{3}$ B) $4, \sqrt{3}$ C) $2, \sqrt{3}$ D) $3, 2\sqrt{3}$

Key. A
Sol. Conceptual

Paragraph – 21

A, B, C, D are consecutive vertices of a rectangle whose area is 2006. An ellipse with area 2006π passes through A and C and has foci at B and D.

60. The perimeter of the rectangle is

A) $8\sqrt{2006}$ B) $8\sqrt{1003}$ C) $6\sqrt{1003}$ D) $6\sqrt{2006}$

61. The eccentricity of the ellipse is

A) $\sqrt{\frac{2006}{4009}}$ B) $\sqrt{\frac{3009}{4012}}$ C) $\frac{3}{11}$ D) $\sqrt{\frac{2006}{2009}}$

62. The radius of director circle of the ellipse is

A) $\sqrt{5015}$ B) $\sqrt{4014}$ C) $\sqrt{3003}$ D) $\sqrt{2009}$

Key: B-B-A
Hint: Question nos: 60 – 62
Let 2a, 2b respectively be the lengths of major axis and minor axis of the ellipse. Let the dimensions of the rectangle be x, y then by hypothesis ab = 2006 = xy and $x^2 + y^2 = 4(a^2 - b^2)$.

Paragraph – 22

An ellipse whose distance between focii S & S^1 is 4 units is inscribed in the ΔABC, touching the sides AB, AC and BC at P, Q and R. If centre of ellipse is at origin 'O' and major axis along x-axis and $SP + S^1P = 6$ then

63. If $\angle BAC = 90^0$ then locus of vertex A is

A. $x^2 + y^2 = 12$ B. $x^2 + y^2 = 16$ C. $x^2 + y^2 = 14$ D. $x^2 + y^2 = 25$

Key. C
Sol. 2a = 6, 2ae = 4

$$\therefore e = \frac{2}{3}$$

Ellipse equation is $\frac{x^2}{9} + \frac{y^2}{5} = 1$

$\angle BAC = 90^0 \Rightarrow$ A lies on director circle $x^2 + y^2 = 14$

$\angle POQ = 90^0 \Rightarrow$ Let A(h, k)

Equation of PQ is $S_1 = 0$

$\frac{hx}{9} + \frac{ky}{5} = 1$

Homoginising

$\frac{x^2}{9} + \frac{y^2}{5} = \left(\frac{hx}{9} + \frac{ky}{5}\right)^2$

$coefficient\ of\ \ x^2 + coefficient\ of\ \ y^2 = 1$

$25h^2 + 81k^2 = 630$

$AB = BC, \angle B = 90^0 \Rightarrow \angle A = 45^0$

$\tan 45^0 = \frac{2ab\sqrt{S_{11}}}{x_1^2 + y_1^2 - a^2 - b^2}$

$(x_1^2 + y_1^2 - a^2 - b^2)^2 = (2ab\sqrt{b^2x_1^2 + ay_1^2 - a^2b^2})^2$

$(x^2 + y^2 - 14)^2 = 20x^2 + 36y^2 - 180$

64. If $\angle POQ = 90^0$ then locus of vertex A is

A. $25x^2 + 81y^2 = 330$ B. $81x^2 + 25y^2 = 630$ C. $25x^2 + 81y^2 = 630$ D. $25x^2 + 81y^2 = 230$

Key. C

Sol. 2a = 6, 2ae = 4

$\therefore e = \frac{2}{3}$

Ellipse equation is $\frac{x^2}{9} + \frac{y^2}{5} = 1$

$\angle BAC = 90^0 \Rightarrow$ A lies on director circle $x^2 + y^2 = 14$

$\angle POQ = 90^0 \Rightarrow$ Let A(h, k)

Equation of PQ is $S_1 = 0$

$$\frac{hx}{9}+\frac{ky}{5}=1$$

Homoginising

$$\frac{x^2}{9}+\frac{y^2}{5}=\left(\frac{hx}{9}+\frac{ky}{5}\right)^2$$

$$coefficient\ of\ \ x^2+coefficient\ of\ \ y^2=1$$

$$25h^2+81k^2=630$$

$$AB=BC,\angle B=90^0\Rightarrow\angle A=45^0$$

$$\tan 45^0=\frac{2ab\sqrt{S_{11}}}{x_1^2+y_1^2-a^2-b^2}$$

$$(x_1^2+y_1^2-a^2-b^2)^2=(2ab\sqrt{b^2x_1^2+ay_1^2-a^2b^2})^2$$

$$(x^2+y^2-14)^2=20x^2+36y^2-180$$

65. If AB = BC and $\angle B=90^0$ then locus of vertex A is

A. $(x^2+y^2-14)^2=20x^2+36y^2-180$
B. $(x^2+y^2+14)^2=20x^2+36y^2-180$
C. $(x^2+y^2-14)=20x^2-36y^2-180$
D. $(x^2+y^2-14)^2=20x^2-36y^2-180$

Key. A

Sol. 2a = 6, 2ae = 4

$$\therefore e=\frac{2}{3}$$

Ellipse equation is $\frac{x^2}{9}+\frac{y^2}{5}=1$

$\angle BAC=90^0\Rightarrow$ A lies on director circle $x^2+y^2=14$

$\angle POQ=90^0\Rightarrow$ Let A(h, k)

Equation of PQ is $S_1=0$

$$\frac{hx}{9}+\frac{ky}{5}=1$$

Homoginising

$$\frac{x^2}{9}+\frac{y^2}{5}=\left(\frac{hx}{9}+\frac{ky}{5}\right)^2$$

$$coefficient\ of\ \ x^2+coefficient\ of\ \ y^2=1$$

$$25h^2+81k^2=630$$

$$AB=BC,\angle B=90^0\Rightarrow\angle A=45^0$$

$$\tan 45^0=\frac{2ab\sqrt{S_{11}}}{x_1^2+y_1^2-a^2-b^2}$$

$$(x_1^2+y_1^2-a^2-b^2)^2=(2ab\sqrt{b^2x_1^2+ay_1^2-a^2b^2})^2$$

$$(x^2+y^2-14)^2=20x^2+36y^2-180$$

Paragraph – 23

$C_1:x^2+y^2=r^2$ and $C_2:\frac{x^2}{16}+\frac{y^2}{9}=1$ intersect at four distinct points A, B, C, D and D. Their common tangents form a parallelogram $A^{'}B^{'}C^{'}D^{'}$.

66. If ABCD is a square then *r* is equal to

(a) $\frac{12}{5}\sqrt{2}$ (b) $\frac{12}{5}$ (c) $\frac{12}{5\sqrt{5}}$ (d) None of these

Key. A

67. If $A'B'C'D'$ is a square then *r* is equal to

(a) $\sqrt{20}$ (b) $\sqrt{12}$ (c) $\sqrt{15}$ (d) None of these

Key. D

68. If $A^{'}B^{'}C^{'}D^{'}$ is a square, then the ratio of area of the circle C_1 to the area of the circumcircle of $\Delta A^{'}B^{'}C^{'}$ is

(a) $\frac{9}{16}$ (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) None of these

Key. C

Sol. 66. $x^2+y^2=r^2$, $\frac{x^2}{16}+\frac{y^2}{9}=1$

$$\frac{r^2-y^2}{16}+\frac{y^2}{9}=1$$

$$9r^2-9y^2+16y^2=144$$

$$y^2=\frac{144-9r^2}{7}$$

$$x^2=r^2-y^2\Rightarrow\frac{16r^2-144}{7}=x^2$$

If ABCD is square $x^2=y^2$

$\Rightarrow \frac{16r^2-144}{7}=\frac{144-9r^2}{7}$

$25r^2=288$

$r=\pm\frac{12}{5}\sqrt{2}$

67. $y=mx\pm\sqrt{16m^2+9}$ is equation to $\frac{x^2}{16}+\frac{y^2}{9}=1$

$y=mx+r\sqrt{1+m^2}$ is equation to $x^2+y^2=r^2$

$r^2\left(1+m^2\right)=25$

$2r^2=25$

$r^2=\frac{25}{2}$ $r=\pm\frac{5}{\sqrt{2}}$

68. $A^1B^1C^1D^1$ is square then common tangent is $y=\pm x\pm 5$

$y=x+5, y=x-5, y=-x+5, y=-x-5$

$y=x+5$

$y=-x+5 \Rightarrow y=5$

$A^1(0,5)$ $C^1(0,-5)$

$A^1C^1=10$

Radius of circum circle of $\Delta A^1B^1C=5$

Area of circle $C_1=\frac{25\pi}{2}$

Ratio = $\frac{1}{2}$

Paragraph – 24

Let the equation $ax^2+2hxy+by^2=1$ represent an ellipse, then $h^2-ab<0$. If the equation of ellipse can be changed to $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1$, by putting $x=X\cos\theta-Y\sin\theta$ and $y=X\sin\theta+Y\cos\theta$ then,

69. $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$ must be equal to

(A) $\frac{1}{a^2}+\frac{1}{b^2}$ (B) $\frac{1}{a}+\frac{1}{b}$ (C) $a+b$ (D) $a-b$

Key. 3

70. $\frac{1}{\alpha^2\beta^2}$ equals

(A) h^2-ab (B) $\sqrt{h^2-ab}$ (C) h^2+ab (D) $ab-h^2$

Key. 4

71. If e is eccentricity, then $e^2=$

(A) $\frac{\sqrt{(a-b)^2+4h^2}}{a+b}$ (B) $\frac{2\sqrt{(a-b)^2+4h^2}}{a+b+\sqrt{(a-b)^2+4h^2}}$

(C) $\frac{\sqrt{(a-b)^2+4h^2}}{a+b+\sqrt{(a-b)^2+4h^2}}$ (D) $\frac{2\sqrt{(a-b)^2+4h^2}}{a+b}$

Key. 2

Sol. 69-71: Equation can be changed to $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1$

$$x = X\cos\theta - Y\sin\theta \quad y = X\sin\theta + Y\cos\theta$$

We get $a(X\cos\theta - Y\sin\theta)^2 + 2h(X\cos\theta - Y\sin\theta)(X\sin\theta + Y\cos\theta) + b(X\sin\theta + Y\cos\theta)^2$

Coeff. $X^2 = a\cos^2\theta + 2h\sin\theta\cos\theta + b\sin^2\theta$

$$= a\left[\frac{1+\cos 2\theta}{2}\right] + 2h\sin\theta\cos\theta + b\left[\frac{1-\cos\theta}{2}\right]$$

$$= \frac{1}{2}\left[(a+b) + (a-b)\cos 2\theta + 2h\sin 2\theta\right]$$

Similarly we get coeff. of $y^2 = \frac{1}{2}\left[(a+b) - (a-b)\cos 2\theta - 2h\sin 2\theta\right]$

$$xy = -2a\sin\theta\cos\theta + 2h[\cos 2\theta] + 2b\sin\theta\cos\theta$$

$$= (b-a)\sin 2\theta + 2h\cos\theta$$

$\Rightarrow$ or $-2h\cos 2\theta = (b-a)\sin 2\theta$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2} = \frac{1}{2}\left[(a+b)+(a-b)\cos 2\theta + 2h\sin 2\theta + (a+b) - (a-b)\cos 2\theta - 2h\sin 2\theta\right]$$

$$= a+b$$

Also $\frac{1}{\alpha^2\beta^2} = \frac{1}{4}\left[(a+b)^2 - \{(a-b)\cos 2\theta + 2h\sin 2\theta\}^2\right]$

$$= \frac{1}{4}\left[(a+b)^2 - \left\{\frac{2h\cos^2 2\theta + 2h\sin^2 2\theta}{\sin 2\theta}\right\}^2\right]$$

$$= \frac{1}{4}\left[(a+b)^2 - 4h^2\operatorname{cosec}^2 2\theta\right]$$

Also $\tan 2\theta = \frac{2h}{a-b}$ we get $\operatorname{cosec}^2 2\theta = \frac{(a-b)^2+4h^2}{4h^2}$

$$= \frac{1}{4}\left[(a+b)^2 - (a-b)^2 - 4h^2\right]$$

$= ab - h^2$

Also eccentricity $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$

$e^2 = \frac{\alpha^2 - \beta^2}{\alpha^2}$

$\alpha^2 + \beta^2 = \frac{a+b}{ab-h^2} \quad \alpha^2\beta^2 = \frac{1}{ab-h^2}$

$e^2 = \frac{2\sqrt{(a-b)^2 + 4h^2}}{a+b+ab-h^2}$

Paragraph – 25

Consider a conic of the form $ax^2 + 2hxy + by^2 = 1 ----(1)$ and a circle

$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1 -----(2)$

(1) –(2) gives $\left(a - \frac{1}{r^2}\right)x^2 + 2hxy\left(b - \frac{1}{r^2}\right)y^2 = 0 -----(3)$

(3) represents a pair of lines passing through the origin and the intersection of the circle and the conic. If these lines coincide they are with either major axis or minor axis.

(3) represent coincident lines if $\left(a - \frac{1}{r^2}\right)\left(b - \frac{1}{r^2}\right) = h^2$. This is a quadratic equation

interms of r^2 whose roots be r_1^2 and r_2^2 : If r_1^2 and r_2^2 are both positive then the conic is an ellipse and lengths of its axes are $2r_1$ and $2r_2$. If r_1^2 is positive and r_2^2 is negative then the conic is a hyperbola and lengths of its axes are $2r_1$ and $2\sqrt{-r_2^2}$.

Given conic is $5x^2 - 6xy + 5y^2 + 22x - 26y + 29 = 0$. The axes being rectangular . Now answer the following questions.

72. Given conic is

a) parabola b) Ellipse

c) Hyperbola whose axes are coordinate axis

d) Hyperbola whose axis are not the coordinate axes.

Key. B

73. .Lenghts of major and minor axes are

a) 8,4 b) 16,4 c)4,2 d) 16,8

Key. C

74. Equations of the major and minor axes are respectively

a) $2x+y-1=0,\ x-2y+3=0$ b) $2x-y+3=1=0,\ x+2y+4=0$

c) $x-y+5=0,\ x+y+3=0$ d) $x-y+3=0,\ x+y-1=0$

Key. D

Sol. **(72 – 74)**

Let $S=5x^2-6xy+5y^2+22x-26y+29=0$

Removing 1st degree terms by shifting the origin to (-1, 2) the equation S= 0 is

$$\frac{5}{8}x^2-\frac{6}{8}xy+\frac{5}{8}y^2=1$$

$$a=\frac{5}{8},\qquad 2h=\frac{-6}{8},\qquad b=\frac{5}{8}$$

$$\left(\frac{5}{8}-\frac{1}{r^2}\right)\left(\frac{5}{8}-\frac{1}{r^2}\right)=\frac{9}{64}$$

$\Rightarrow r^2=4\ or\ 1$ both are +qe

Given conic is an ellipse.

Length of major axis =4,

Length of minor axis =2

Equation of major axis is $\left(\frac{5}{8}-\frac{1}{4}\right)x-\frac{3}{8}y=0\ is\ x-y=0$

Equation of minor axis is $\left(\frac{5}{8}-1\right)x-\frac{3}{8}y=0\ is\ x+y=0$

The equations in original system are x –y +1 =0 and x +y -1=0

Paragraph – 26

If $P(\theta_1)$ and $D(\theta_2)$ be the end point of two semi-conjugate diameters of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ whose centre is C, then answer the following questions

75. $\theta_1-\theta_2=$

a) 45^0 b) 90^0 c) 135^0 d) None

Key. B

76. $CP^2+CD^2=$

a) $\frac{a^2+b^2}{4}$ b) $\left(a^2+b^2\right)$ c) $\frac{b^4+a^4}{b^2+a^2}$ d) $\frac{a^4+b^4}{2\left(b^2+a^2\right)}$

Key. C

77. Locus of midpoint of PD is

a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{2}$ b) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$ c) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{4}$ d) None

Key. A

Sol. 75. $m_1 m_2 = \frac{b^2}{a^2}$

$$\cos(\theta_1 - \theta_2) = 0 \Rightarrow \theta_1 - \theta_2 = \frac{\pi}{2}$$

76. Let p be θ & D be $\frac{\pi}{2} + \theta$

$$Cp^2 + CD^2 = a^2 + b^2$$

77. $2h = a(\cos\theta - \sin\theta)$

$$2k - b(\sin\theta + \cos\theta)$$

$$\frac{4a^2}{a^2} + \frac{4k^2}{b^2} = 2 > \frac{1}{2}$$

Paragraph – 27

Consider an ellipse $\frac{x^2}{4} + y^2 = \alpha,$ (α is parameter > 0) and a parabola $y^2 = 8x$ If a common tangent to the ellipse and the parabola meets the coordinate axes at A and B respectively, then

78. Locus of mid point of AB is

a) $y^2 = -2x$ b) $y^2 = -x$ c) $y^2 = -\frac{x}{2}$ d) $\frac{x^2}{4} + \frac{y^2}{2} = 1$

Key. B

79. If the eccentric angle of a point on the ellipse where the common tangent meets it is $\left(\frac{2\pi}{3}\right)$ ¸then α is equal to______

a) 4 b) 5 c) 26 d) 36

Key. D

80. If two of the three normals drawn from the point (h, 0) on the ellipse to the parabola $y^2 = 8x$ are perpendicular, then

a) h = 2 b) h = 3 c) h = 4 d) h = 6

Key. D

Sol. 78-80. Equation of tangent to $y^2 = 8x$ is $yt - x - 2t^2 = 0 \quad \rightarrow (1)$

Equation of tangent to ellipse is

$$\frac{x\cos\theta}{2\sqrt{\alpha}} + \frac{y\sin\theta}{\sqrt{\alpha}} \rightarrow (2)$$

Comparing $\frac{\sqrt{\alpha}}{\cos\theta} = -t^2; \frac{\sqrt{\alpha}}{\sin\theta} = 2t \rightarrow (3)$

If the tangent meets the coordinate axes at A and B then

A is $\left(\frac{2\sqrt{\alpha}}{\cos\theta},0\right),B\left(0,\frac{\sqrt{\alpha}}{\sin\theta}\right)$

Let mid point of AB is (h,k)

$$h=\frac{\sqrt{\alpha}}{\cos\theta};K=\frac{\sqrt{\alpha}}{2\sin\theta}$$

$$h=-t^2;K=t\Rightarrow K^2-h\,\text{or}\,y^2=-x$$

From (3)

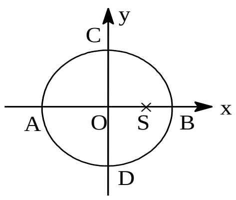
$$\frac{\alpha}{\sin^2\theta}=\frac{-4\sqrt{\alpha}}{\cos\theta}\Rightarrow\sqrt{\alpha}=\frac{-4\sin^2\theta}{\cos\theta}=6$$

Any normal is $y=mx-4m-2m^2$

i.e., $2m^3+(4-h)m=0$ i.e., $h=6$

Paragraph – 28

Consider an ellipse as shown in the adjacent figure, such that OS = 6 units and the in-radius of the triangle OCS is 1 unit, Now answer,



81. The equation of the director circle of the director circle of the ellipse, is

a) $x^2+y^2=\frac{97}{2}$ b) $x^2+y^2=\sqrt{97}$

c) $x^2+y^2=97$ d) $x^2+y^2=\sqrt{\frac{97}{2}}$

Key. C

82. The semi-perimeter of the triangle OCS is

a) 10 b) 5 c) 7.5 d) 12.5 unit

Key. C

83. The area of the ellipse is

a) $\frac{65\pi}{4}$ b) $\frac{64\pi}{5}$ c) 64π d) 65π

Key. A

Sol. Q.81,82,83

Given, OS = ae = 6

Let OB =a , OC = b

In radius of $\Delta OCS,\ r=\dfrac{OS+OC-CS}{2}$

$$\Rightarrow 1=\frac{6+b-a}{2}\Rightarrow a-b=4 \qquad (1)$$

As we know

$$b^2=a^2-a^2e^2$$

$$\Rightarrow (a-4)^2=a^2-36$$

$$\Rightarrow a=13/2\Rightarrow b=5/2$$

Hence, director circle of the given ellipse $\therefore\ x^2+y^2=a^2+b^2$

$\Rightarrow$ the required director circle is : $x^2+y^2=2\left(a^2+b^2\right)$

$$\Rightarrow x^2+y^2=97$$

also, semi-perimeter of $\Delta OCS=\dfrac{OC+CS+OS}{2}=\dfrac{\frac{5}{2}+\frac{13}{2}+6}{2}=\dfrac{15}{2}$

and, area of the ellipse $=\pi ab=\pi\left(\dfrac{65}{4}\right)$

Paragraph – 29

Consider the circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$ where $b>a>0$. Let $A(-a,0);B(a,0)$. A parabola passes through the points A, B and its directrix is a tangent to $x^2+y^2=b^2$. If the locus of focus of the parabola is a conic then

84. The eccentricity of the conic is

A) $2a/b$ B) b/a C) a/b D) 1

Key. C

85. The foci of the conic are

A) $(\pm 2a,0)$ B) $(0,\pm a)$ C) $(\pm a,2a)$ D) $(\pm a,0)$

Key. D

86. Area of triangle formed by a latusrectum and the lines joining the end points of the latusrectum and the centre of the conic is

A) $\dfrac{a}{b}\left(b^2-a^2\right)$ B) $2ab$ C) $ab/2$ D) $4ab/3$

Key. A

Sol. **84 – 86:**

$x^2+y^2=a^2;\ x^2+y^2=b^2;\ b>a>0,\ A=(-a,0);\ \ B=(a,0)$

Let (h,k) be a point on the locus. Any tangent to circle $x^2+y^2=b^2$ is $x\cos\theta+y\sin\theta=b$

$\therefore$ Equation of parabola is $\sqrt{(x-h)^2+(y-K)^2}=\left|x\cos\theta+y\sin\theta-b\right|$

i.e., $(x-h)^2+(y-K)^2=(x\cos\theta+y\sin\theta-b)^2$

The points $(\pm a,0)$ satisfy this equation

$\therefore (a-h)^2+K^2=(a\cos\theta-b)^2$ --- (1)

$(a+h)^2+K^2=(a\cos\theta+b)^2$ ---- (2)

(2) – (1) $\Rightarrow h=b\cos\theta$

$\therefore$ Required locus is $(a+x)^2+y^2=\left(\frac{ax}{b}+b\right)^2$

i.e., $\frac{x^2}{b^2}+\frac{y^2}{b^2-a^2}=1$ which is an ellipse.

Paragraph – 30

If 'P' is any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. S_1 and S_2 are foci of the ellipse

87. Locus of incentre of triangle PS_1S_2 will be

a) a straight line
b) a circle
c) a parabola
d) an ellipse

Key. D

88. If $e=\frac{1}{2}$ and $\angle PS_1S_2=\alpha, \angle PS_2S_1=\beta, \angle S_1PS_2=\gamma$, then $\cot\frac{\alpha}{2}, \cot\frac{\gamma}{2}$ and $\cot\frac{\beta}{2}$ are in

a) A.P
b) G.P
c) H.P
d) None

Key. A

89. Maximum area of the triangle PS_1S_2 is equal to

a) b^2e sq.units
b) a^2e sq.units
c) ab sq.units
d) abe sq.units

Key. D

Sol. 87. $\frac{PS_2}{S_2G}=\frac{PS_1}{GS_1}=\frac{PS_2+PS_1}{S_2G+GS_1}=\frac{2a}{2ae}=\frac{1}{e}$

So $PI:IG=1:e$

88. $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{1-e}{1+e}=\frac{1}{3}$

89. Base S_1S_2 fixed and PS_2+PS_2 is fixed, Hence area will be maximum if $PS_1=PS_2$

Paragraph – 31

An ellipse E has its centre C (1,3), focus at S (6, 3) and passes through the point P (4, 7). Then

90. The product of the perpendicular distances of foci from tangent at P to the ellipse, is
a) 20 b) 45 c) 40 d) 60

Key. A

91. The point of intersection of the lines joining each focus to the foot of the perpendicular from the other focus upon the tangent at P, is
a) $\left(\frac{5}{3},5\right)$ b) $\left(\frac{4}{3},3\right)$
c) $\left(\frac{8}{3},3\right)$ d) $\left(\frac{10}{3},5\right)$

Key. D

92. If the normal at a variable point on the ellipse (E) meets its axes in Q and R, then the locus of the midpoint of QR is a conic with eccentricity =
a) $3/\sqrt{10}$ b) $\sqrt{5}/3$
c) $3/\sqrt{5}$ d) $\sqrt{10}/3$

Key. B

Sol. 90-92. $CS = ae = 5$

$S' = (-4,5)$

$PS + PS' = 2a = 6\sqrt{5}$

$\Rightarrow e = \frac{\sqrt{5}}{3}$

Product $= b^2$

Paragraph – 32

A rod AB of 20 units length with its ends ‘A’ on x-axis and B on the y-axis is sliding between the axes. P is a marked point on AB such that AP = 8 Answer the questions 18, 19, 20

93. The locus of P is :
(a) $\frac{x^2}{144}+\frac{y^2}{64}=1$ (b) $\frac{x^2}{64}+\frac{y^2}{144}=1$ (c) $\frac{x^2}{25}+\frac{y^2}{96}=1$ (d) $\frac{x^2}{169}+\frac{y^2}{144}=1$

Key. A

94. The eccentricity of the conic is :
(a) $\frac{4}{9}$ (b) $\frac{9}{4}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{\sqrt{5}}{3}$

Key. D

95. The locus of the point of inter section of perpendicular tangents is :
(a) $x^2+y^2=200$ (b) $x^2+y^2=208$ (c) $x^2+y^2=313$ (d) $x^2+y^2=41$

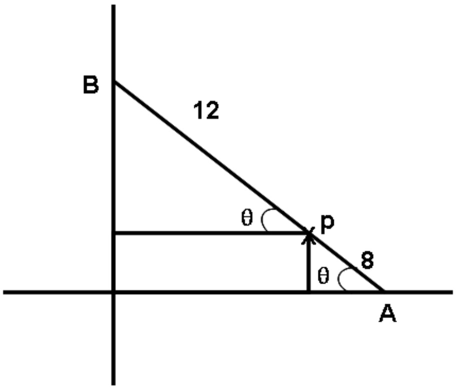
Key. B

Sol. $P=(x_1,y_1) \Rightarrow x_1 = 12\cos\theta,$

$y_1 = 8\sin\theta$

$\therefore \frac{x_1^{\ 2}}{144}+\frac{y_1^{\ 2}}{64}=1$

Locus is $\frac{x^2}{144}+\frac{y^2}{64}=1$



Paragraph – 33

In an ellipse $(x-1)^2+(y-2)^2=\left(\frac{5x+4y-20}{7}\right)^2$

96. The eccentricity is

(A) $\frac{5}{7}$ (B) $\frac{4}{7}$ (C) $\frac{7}{41}$ (D) $\frac{\sqrt{41}}{7}$

Key. D

97. Equation of major axis is

(A) $4x-5y+8=0$ (B) $3x+4y-6=0$ (C) $4x-5y+6=0$ (D) $5x+4y-10=0$

Key. C

98. Length of latus rectum is

(A) 1 (B) 2 (C) 3 (D) 4

Key. B

Sol. 96. $e^2=\frac{41}{49}\Rightarrow e=\frac{\sqrt{41}}{7}$

97. Major axis is perpendicular to directrix and passing through focus (1, 2), i.e., $4(x-1)-5(y-2)=0 \Rightarrow 4x-5y+6=0$.

98. The distance from focus to directrix = $\frac{b^2}{ae}$

$\frac{b^2}{ae}=\left|\frac{5+8-20}{\sqrt{41}}\right|=\frac{7}{\sqrt{41}}\Rightarrow\frac{b^2}{a}=\frac{7}{\sqrt{41}}\times e=1$

L.L.R. = $\frac{2b^2}{a}=2.1=2$

Paragraph – 34

In an ellipse $25(3x-4y+7)^2+16(4x+3y-6)^2=10000$.

99. The length of major axis is

(A) 50 (B) 8 (C) 16 (D) 10

Key. D

100. The length of minor axis is

(A) 5 (B) 8 (C) 40 (D) 50

Key. B

101. Equation of minor axis of ellipse
(A) $x+7y-13=0$ (B) $7x-y+1=0$ (C) $3x-4y+7=0$ (D) $4x+3y-6=0$

Key. D

Sol. 99. Length of major axis = 2b = 2 x 5 = 10

100. Length of minimum axis = 2a = 2 x 4 = 8.

101. Equation of minor axis = 4x + 3y – 6 = 0.

Ellipse

Integer Answer Type

1. Any ordinate MP of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the auxiliary circle of Q; then locus of the point of intersection of normals of P and Q to the respective curves is a circle of radius ___

Key. 8

Sol. The locus is $x^2+y^2=64$

2. The distance between the directrices of the ellipse $(4x-8)^2+16y^2=\left(x+\sqrt{3}y+10\right)^2$ is K then $\frac{K}{2}$ is

Key. 8

Sol. $(x-2)^2+y^2=\left(\frac{1}{2}\right)^2\frac{\left(x+\sqrt{3}y+10\right)^2}{4}$

$(h, k)=(z, 0), e=½$

Perpendicular distance from $(2, 0)$ to $x+\sqrt{3}y+10=0$ is $\frac{a}{e}-ae$

$2a-\frac{a}{2}=6 \Rightarrow a=4$

Distance between directrics = $\frac{2a}{e}=16=K$

3. A circle concentric to an ellipse $\frac{4x^2}{289}+\frac{4y^2}{\lambda^2}=1\left(\lambda<\frac{17}{2}\right)$ passes through foci F_1 and F_2 cuts the ellipse at 'P' such that area of triangle P $F_1 F_2$ is 30 sq.units. If $F_1F_2=13K$ where $K \in Z$ then K =

Key. 1

Sol. Since $F_1 \& F_2$ are the ends of the diameter

Area of $\Delta PF_1F_2=\frac{1}{2}(F_1P)(F_2P)=\frac{1}{2}x(17-x)=30 \Rightarrow x=5 \text{ or } 12 \Rightarrow F_1F_2=13$

4. If F_1, F_2 are the feeet of the perpndiculars from foci S_1, S_2 of the ellipse $16x^2+25y^2=400$ on the tangent at any point P on the ellipse then minimum value of $S_1F_1+S_2F_2$ is

Key. 8

Sol. The minimum perpendiculars from two foci upon any tangent is b^2

$$S_1F_1.S_2F_2 = 16$$

$$AM \geq GM \Rightarrow \frac{S_1F_1 + S_2F_2}{2} \geq \sqrt{S_1F_1 \times S_2F_2} \Rightarrow S_1F_1 + S_2F_2 \geq 8$$

5. The equation of an ellipse is given by $5x^2 + 5y^2 - 6xy - 8 = 0$. If r_1, r_2 are distances of points on the ellipse which are at maximum & minimum distance from origin then $r_1 + r_2 =$

Key. 3

Sol. Any point on ellipse at a distance r from origin is $(r\cos\theta, r\sin\theta)$

$\Rightarrow r^2 = \dfrac{8}{5 - 3\sin 2\theta}$ is maximum if $5 - 3\sin 2\theta$ is minimum $\Rightarrow r^2 = 4$

$r^2 m$ in if $(5 - 3\sin 2\theta)$ is maximum = 8 $\Rightarrow r^2 = 1$

$$r_1 + r_2 = 2 + 1 = 3$$

6. The equation of the curve on reflection of the ellipse $\dfrac{(x-4)^2}{16} + \dfrac{(y-3)^2}{9} = 1$ about the line x–y–2 = 0 is $16x^2 + 9y^2 + ax - 36y + b = 0$ then the value of $a + b - 125 =$

Key. 7

Sol. Let P(4, 0) & Q(0, 3) are two points on given ellipse E_1

P_1 and Q_1 are images of P,Q w.r.to x – y – 2 = 0

$\therefore P_1(2,2)$ $Q_1(5,-2)$ lies on E_2

$\therefore a = -160,\ b = 292$

7. Number of points on the ellipse $\dfrac{x^2}{50} + \dfrac{y^2}{20} = 1$ from which pair of perpendicular tangents are drawn to the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$ is

Key. 4

Sol. Director circle of $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$ is $x^2 + y^2 = 25$

The director circle will cut the ellipse $\dfrac{x^2}{50} + \dfrac{y^2}{20} = 1$ at 4 points.

8. If L be the length of common tangent to the ellipse $\frac{x^2}{25}+\frac{y^2}{4}=1$ and the circle $x^2+y^2=16$ intercepted by the coordinate axis then $\frac{\sqrt{3}L}{2}$ is

Key. 7

Sol. The equation of the tangent at $(5\cos\theta, 2\sin\theta)$ is $\frac{x}{5}\cos\theta+\frac{y}{2}\sin\theta=1$

If it is a tangent to the circle then $\frac{1}{\sqrt{\frac{\cos^2\theta}{25}+\frac{\sin^2\theta}{4}}}=4$

$\Rightarrow \cos\theta=\frac{10}{4\sqrt{7}}, \sin\theta=\frac{\sqrt{3}}{2\sqrt{7}}$

Let A and B be the points where the tangent meets the coordinate axis then $A\left(\frac{5}{\cos\theta}, 0\right)$, $B\left(0, \frac{2}{\sin\theta}\right)$

$L=\sqrt{\frac{25}{\cos^2\theta}+\frac{4}{\sin^2\theta}}=\frac{14}{\sqrt{3}}$

9. An ellipse is sliding along the coordinate axes. If the foci of the ellipse are (1, 1) and (3,3) then the area of the director circle of the ellipse is $K\pi$. Then K = __

Key. 7

Sol. Since axes are tangents, $b^2=3$ and $ae=\sqrt{2} \Rightarrow a^2-b^2=2 \therefore a^2=5$

10. Tangents are drawn from points on the line $x-y+2=0$ to the ellipse $x^2+2y^2=2$, then all the chords of contact pass through the point whose distance from $\left(2, \frac{1}{2}\right)$ is

Key. 3

Sol. Consider any point $(t_1, t+2)$, $t\in R$ on the line $x-y+2=0$

The chord of contact of ellipse with respect to this point is $x(t)+2y(t+2)-2=0$

$\Rightarrow (4y-2)+t(x+2y)=0$, $y=\frac{1}{2}, x=-1$

Hence, the point is $\left(-1, \frac{1}{2}\right)$, Where distance from $\left(2, \frac{1}{2}\right)$ is 3.

11. If P and Q are the ends of a pair of conjugate diameters and C is the centre of the ellipse $4x^2+9y^2=36$ then the area of ΔCPQ in square units.

Key. 3

Sol. $\frac{x^2}{9}+\frac{y^2}{4}=1$, so $P=(3\cos\theta, 2\sin\theta)$ and $Q=\left(3\cos\left(\frac{\pi}{2}+\theta\right), 2\sin\left(\frac{\pi}{2}+\theta\right)\right)$

$$\text{Area of } \Delta CPQ = = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 3\cos\theta & 2\sin\theta & 1 \\ -3\sin\theta & 2\cos\theta & 1 \end{vmatrix} = 3.$$

12. The maximum distance from the origin to any normal chord drawn to the ellipse $\frac{x^2}{25}+\frac{y^2}{4}=1$ is

Key. 3

Sol. The other end of the normal drawn at P(t) in $Q\left(-t-\frac{2}{t}\right)$

If A is the vertex, slope of AP slope AQ = -1

$$\Rightarrow \frac{2}{t}\frac{(-2)}{\left(t+\frac{2}{t}\right)} = -1 \Rightarrow t^2+2=4 \Rightarrow t^2=2$$

13. The area of the quadrilateral formed by the tangents at the end point of latus rectum to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ is 3K. Then K is equal to

Key. 9

Sol. e = 2/3

Equation of tangent at L is $\frac{2x}{9}+\frac{y}{3}=1$ it meets x-axis at $A\left(\frac{9}{2},0\right)$ & y axis at B(0, 3).

$\therefore$ area = $4\left[\frac{1}{2}.\frac{9}{2}.3\right]=27$

14. If a tangent of slope 2 of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is normal to the circle $x^2+y^2+4x+1=0$, then the maximum value of ab is _____________

Key. 4

Sol. A tangent of slope 2 is $y=2x\pm\sqrt{4a^2+b^2} \rightarrow (1)$

This is normal to the circle $x^2+y^2+4x+1=0$

i.e., (1) passes through $(-2,0)\, 4a^2+b^2=16$

Using AM ≥ GM $\Rightarrow \frac{4a^2+b^2}{2} \geq \sqrt{4a^2.b^2}$ $ab \leq 4$

15. If a line through P(a, 2) meets the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ at A and D and meets the axes at B and C, so that PA, PB, PC, PD are in G.P., then the minimum value of $|a|$,is….

Key. 6

Sol. $\frac{x-a}{\cos\theta}=\frac{y-2}{\sin\theta}=r$ $(a+r\cos\theta, 2+r\sin\theta)$ lies on ellipse for A and D.

$$\frac{(a+r\cos\theta)^2}{9}+\frac{(2+r\sin\theta)^2}{4}=1 \Rightarrow r_1r_2=PA.PD$$

PA, PB, PC, PD are in G.P PA. PD = PB. PC. etc.....

16. The number of values of c such that the straight line $y=4x+c$ touches the curve $x^2/4+y^2=1$ is K then K = ...

Key. 2

Sol. If $y=mx+c$ is tangent to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then

$C^2=a^2m^2+b^2$

$y=4x+c, \quad \frac{x^2}{4}+\frac{y^2}{1}=1$

$C=\pm\sqrt{65}$

17. Tangent is drawn to ellipse $x^2/27+y^2=1$ at $\left(3\sqrt{3}\cos\theta, \sin\theta\right)$ (where $\theta\in(0,\pi/2)$). Then the value of θ such that sum of intercepts on coordinate axes made by this tangent is least is $\frac{\pi}{K}$ then K =

Key. 6

Sol. $\frac{x^2}{27}+\frac{y^2}{1}=1, \ P\left(3\sqrt{3}\cos\theta, \sin\theta\right)$

$\frac{3\sqrt{3}\cos\theta}{27}+\frac{\sin\theta y}{1}=1$

$A\left(\frac{3\sqrt{3}\cos\theta}{27}, 0\right), B=\left(0, \frac{1}{\sin\theta}\right)$

$f(\theta)=3\sqrt{3}\sec\theta+\cos ec\theta$

$f^1(\theta)=\frac{3\sqrt{3}\sin\theta}{\cos^2\theta}-\frac{\cos\theta}{\sin^2\theta}=0$

$\Rightarrow \tan^3\theta=\frac{1}{3\sqrt{3}}=\left(\frac{1}{\sqrt{3}}\right)^3$

$\theta=\frac{\pi}{6}$

18. The maximum distance from the origin to any normal chord drawn to the ellipse $\frac{x^2}{25}+\frac{y^2}{4}=1$ is

Key. 3

Sol. The other end of the normal drawn at P(t) in $Q\left(-t-\frac{2}{t}\right)$

If A is the vertex, slope of AP slope AQ = -1

$$\Rightarrow \frac{2}{t}\frac{(-2)}{\left(t+\frac{2}{t}\right)} = -1 \Rightarrow t^2+2=4 \Rightarrow t^2=2$$

19. The area of the quadrilateral formed by the tangents at the end point of latus rectum to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ is 3K. Then K is equal to

Key. 9

Sol. e = 2/3

Equation of tangent at L is $\frac{2x}{9}+\frac{y}{3}=1$ it meets x-axis at $A\left(\frac{9}{2},0\right)$ & y axis at B(0, 3).

$\therefore$ area = $4\left[\frac{1}{2}.\frac{9}{2}.3\right]=27$

20. If a tangent of slope 2 of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is normal to the circle $x^2+y^2+4x+1=0$, then the maximum value of ab is ____________

Key. 4

Sol. A tangent of slope 2 is $y=2x\pm\sqrt{4a^2+b^2}\rightarrow(1)$

This is normal to the circle $x^2+y^2+4x+1=0$

i.e., (1) passes through $(-2,0)\,4a^2+b^2=16$

$$\text{Using AM} \geq \text{GM} \Rightarrow \frac{4a^2+b^2}{2}\geq\sqrt{4a^2.b^2} \quad ab\leq 4$$

21. If a line through P(a, 2) meets the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ at A and D and meets the axes at B and C, so that PA, PB, PC, PD are in G.P., then the minimum value of $|a|$,is….

Key. 6

Sol. $\frac{x-a}{\cos\theta}=\frac{y-2}{\sin\theta}=r$ $(a+r\cos\theta, 2+r\sin\theta)$ lies on ellipse for A and D.

$$\frac{(a+r\cos\theta)^2}{9}+\frac{(2+r\sin\theta)^2}{4}=1\Rightarrow r_1r_2=PA.PD$$

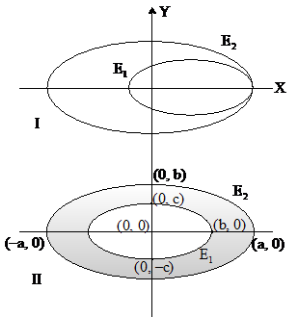
PA, PB, PC, PD are in G.P PA. PD = PB. PC. etc.....

22. Let E_1 and E_2 be two ellipses. The area of the ellipse E_2 is one-third the area of the quadrilateral formed by the tangents at the ends of the latus rectum of the ellipse E_3 (E_3; $5x^2 + 9y^2 = 45$). The eccentricities of E_1, E_2 and E_3 are equal. E_1 is inscribed in E_2 in such a way that both E_1 and E_2 touch each other at one end of their common major-axis. If the length of the major axis of E_1 is equal to the length of the minor axis of E_2 then find the area of the ellipse E_2 outside the ellipse E_1.

Key. 4

Sol. $5x^2 + 9y^2 = 45$

$a = 3,\ b = \sqrt{5},\ e = \frac{2}{3}$, one end of latus rectum in first quadrant $\left(2, \frac{5}{3}\right)$



Equation of tangent $2x + 3y = 9$

It meets axes at $\left(\frac{9}{2}, 0\right)$ and (0, 3)

Area of the quadrilateral

$= 4.\frac{1}{2}.\frac{9}{2}.3 = 27$

Let the equation of the ellipses E_2 and E_1 be

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{b^2} + \frac{x^2}{c^2} = 1$

Figure (I) and figure (II) give the same area which is required.

Area of $E_2 = \pi ab$ and Area of $E_1 = \pi bc$

Required Area $= \pi b (a - c)$

Now, $b^2 = a^2 (1 - e^2)$ and $c^2 = a^2 (1 - e^2)^2$

$\Rightarrow c = a (1 - e^2) \Rightarrow a - c = ae^2$

Required Area $= \pi abe^2$

$= 9 \times \frac{4}{9} = 4$ sq. unit

(because $\pi ab = \frac{1}{3} \times \text{Area}$ of quadratilateral)

23. $P_1, P_2, \ldots, P_i, \ldots P_n$ are the points on the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and $Q_1, Q_2, \ldots Q_i, \ldots Q_n$ are the corresponding points on the auxiliary circle of the ellipse. If the line joining C to Q_i meets the normal at P_i w.r.t. the given ellipse at K_i and $\sum_{i=1}^{n} (CK_i) = 175$, then find the value of n/5.

Key. 5

Sol. Locus of intersection of normal at P_i and Q_i is $n^2 + y^2 = (4 + 3)^2$

$\therefore CK_i = 7$

$\therefore \sum_{i=1}^{n} 7 = 175 \Rightarrow 7n = 175 \Rightarrow n = 25$

24. Coordinates of the vertices $B \,\&\, C$ are $(2,0)$ and $(8,0)$ respectively. The vertex $'A'$ is

varying in such a way that $4\tan\frac{B}{2}\tan\frac{C}{2}=1$. If the locus of '$A$' is an ellipse then the length of its semi major axis is

Key. 5

Sol. $4\tan\frac{B}{2}\tan\frac{C}{2}=1$

$\Rightarrow \sqrt{\frac{(s-c)(s-a)(s-a)(s-b)}{s(s-b)s(s-c)}}=\frac{1}{4}$

$\Rightarrow \frac{s-a}{s}=\frac{1}{4} \Rightarrow \frac{25-a}{a}=\frac{5}{3} \Rightarrow b+c=\frac{5}{3}\times 6=10$

$(\because a=\overline{BC}=6)$

$\therefore$ Locus of A is

$\frac{(x-5)^2}{25}+\frac{y^2}{16}=1$

25. A parabola is drawn through two given points $A(1,0)$ and $B(-1,0)$ such that its directrix always touches the circle $x^2+y^2=4$. If the maximum possible length of semi latus–rectum is 'k' then $[k]$ is (where [.] denotes greatest integer function)

Key. 3

Sol. Any point on circle $x^2+y^2=4$ is $(2\cos\alpha, 2\sin\alpha)$

$\therefore$ Equation of directrix is $x(\cos\alpha)+y(\sin\alpha)-2=0$

Let focus be (x_1, y_1). Then as $A(1,0)$, $B(-1,0)$ lie on parabola we must have

$\left.\begin{matrix}(x_1-1)^2+y_1^2=(\cos\alpha-2)^2\\(x_1+1)^2+y_1^2=(\cos\alpha+2)^2\end{matrix}\right\} \Rightarrow x_1=2\cos\alpha,\ y_1=\pm\sqrt{3}\sin\alpha$

$\therefore$ Length of semi latus–rectum of parabola $=\perp^r$ distance from focus to directrix

$\left|2\pm\sqrt{3}\right|\sin^2\alpha$

Hence, maximum possible length $=2+\sqrt{3}$

26. Let P, Q be two points on the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ whose eccentric angles differ by a right angle. Tangents are drawn at P and Q to meet at R. If the chord PQ divides the joint of C and R in the ratio m : n (C being centre of ellipse), then find m+n(m:n is in simplified form).

Key: 2

Hint: Let P be $(5\cos\theta, 4\sin\theta)$; Q be $(-5\sin\theta, 4\cos\theta)$

Equation of tangent at $P \frac{x}{5}\cos\theta + \frac{y}{4}\sin\theta = 1$ ......(i)

Equation of tangent at $Q - \frac{x}{5}\sin\theta + \frac{y}{4}\cos\theta = 1$ ....(ii)

Solving (i) and (ii) $\Rightarrow R = \left(5(\cos\theta - \sin\theta), 4(\sin\theta + \cos\theta)\right)$

$\therefore$ m : n is 1 : 1

$\Rightarrow$ m + n = 2

Alternate :

Let P(5,0), Q(0,4)

$\Rightarrow$ R(5,4)

Intersection of CR and PQ is $\left(\frac{5}{2}, 2\right)$, which is mid poi8nt of CR

$\Rightarrow m : n = 1 : 1 \Rightarrow m + n = 2$

27. A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + \lambda y^2 = 6$ at P and Q. If tangents at P and Q of ellipse $x^2 + \lambda y^2 = 6$ are at right angles, then $\lambda =$

Key. 2

Sol. Smaller ellipse is $\frac{x^2}{4} + \frac{y^2}{1} = 1$ larger is $\frac{x^2}{6} + \frac{\lambda y^2}{6} = 1$.

Any tangent to smaller is $\frac{x}{2}\cos\theta + y\sin\theta = 1$ ----------- (1)

Let it meet larger at P and Q. Let tangent at P,Q intersect at 'A' (h, k)

PQ is chord of contact wrt A $\therefore PQ = \frac{hx}{6} + \frac{\lambda yk}{6} = 1$ --------- (2)

Comparing (1) and (2) $h = 3\cos\theta \quad k = \frac{6\sin\theta}{\lambda}$

$(h, k) \in x^2 + y^2 = \frac{6+6}{\lambda} \Rightarrow 9\cos^2\theta + \frac{6^2}{\lambda^2}\sin^2\theta = 6 + \frac{6}{\lambda}$

$\Rightarrow \lambda = 2$

28. If the locus of middle points of portions of tangents intercepted between the axes of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{a^2}{x^2} + \frac{b^2}{y^2} = \lambda$, then the numerical value of λ is

Key. 4

Sol. Tangent at 'α' is $\frac{x}{a}\cos\alpha + \frac{y}{b}\sin\alpha = 1$

Let A $(a\sec\alpha, 0)$ $B(0, b\cos ec\alpha)$

(h,k) is mid point

$$h=\frac{1}{2}a\sec\alpha \quad k=\frac{b}{2}\operatorname{cosec}\alpha$$

$$\Rightarrow \frac{a^2}{4x^2}+\frac{b^2}{4y^2}=1$$

29. If PSP^1 is a focal chord of $\frac{x^2}{25}+\frac{y^2}{9}=1$ and SP=3 then $61(S^1P^1)-56=$

Where S, S^1 are foci of ellipse.

Key. 7

Sol. $\because SP+S^1P=2a=10 \Rightarrow S^1P=7$

$$\because \frac{1}{S^1P}+\frac{1}{S^1P^1}=\frac{2}{l}=\frac{2}{\left(\frac{b}{a}\right)^2} \Rightarrow \frac{1}{S^1P^1}=\frac{10}{9}-\frac{1}{7}=\frac{61}{63}$$

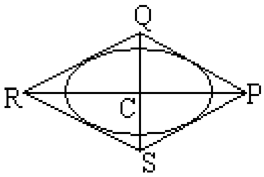
$\therefore 61(S^1P^1)-56=7$

30. Tangents are drawn to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ at ends of latusrectum. If the area of an quadrilateral be λ sq.units, then the value of $\lambda/9=$

Key. 3

Sol. One end of latusrectum $\left(2,\frac{5}{3}\right)$

Equation of tangent at $\left(2,\frac{5}{3}\right)$ is $\frac{2x}{9}+\frac{y}{3}=1$



Area of quadrilateral $=4\times(\Delta CPQ)=4\times\frac{1}{2}\times\frac{9}{2}\times 3=27=\lambda$ (Given)

$\Rightarrow \frac{\lambda}{9}=3$

31. Let P be a point on ellipse $\frac{x^2}{9}+\frac{y^2}{7}=1$ whose parametric angle is $\pi/4$, then abscissa of the incentre of triangle SPS^1 is

Key. 1

Sol. Incentre of ΔSPS^1 is $\left(ae\cos\theta, \frac{be\sin\theta}{1+e}\right)$

$\Rightarrow$ its abscissa $=ae\cos\theta=\sqrt{9-7}\times\frac{1}{\sqrt{2}}=1$

Ellipse

Matrix-Match Type

1. COLUMN - I | COLUMN - II

A) The eccentricity of the conic represented by $x^2 - y^2 - 4x + 4y + 16 = 0$ is — p) 7

B) The foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide ,then b^2 is — q) 0

C) The product of lengths of perpendiculars from any point t of the hyperbola $x^2 - y^2 = 8$ to its asymptotes is — r) $\sqrt{2}$

D) The number of points out side the hyperbola $\frac{x^2}{25} - \frac{y^2}{36} = 1$ from where two perpendicular tangents can be drawn to the hyperbola is /are — s) 4

Key. $A \to R$; $B \to P$; $C \to S$; $D \to Q$

Sol. (a) $h^2 = 0, ab = -1 \Rightarrow h^2 > ab$ and $a + b = 1 - 1 = 0$ rectangular hyperbola eccentricity $= \sqrt{2}$

(b) For ellipse $a^2 = 16$ $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{\frac{16 - b^2}{16}}$

Foci of ellipse $(\pm ae, o) = (\pm\sqrt{16 - b^2}, = o)$

For hyperbola, $a^2 = \left(\frac{12}{5}\right)^2 b^2 = \left(\frac{9}{5}\right)^2 \Rightarrow e = \sqrt{1 + \frac{b^2}{a^2}} = \frac{5}{4}$

So, $16 - b^2 = \left(\frac{12}{5} \times \frac{5}{4}\right)^2 = 9 \Rightarrow b^2 = 7$

(c) $x^2 - y^2 = 8$, $a^2 = 8, b^2 = 8$

product of $\perp^r = \frac{a^2 b^2}{a^2 + b^2} = \frac{8 \times 8}{16} = 4$

(d) $\frac{x^2}{25} - \frac{y^2}{36} = 1$. Equation of direction circle is $x^2 + y^2 = a^2 - b^2 \Rightarrow x^2 + y^2 = -9$

Which in not possible number of points =0

2.

	Column I		Column II
(A)	The coordinates of the point on the parabola $y = x^2 + 7x + 2$, which is nearest to the straight line y = 3x – 3 are	(p)	(2,1)
(B)	y = x + 2 is a tangent to the parabola $y^2 = 8x$. The point on this line, the other tangent from which is perpendicular to this tangent is	(q)	(–2, 0)
(C)	The point on the ellipse $x^2 + 2y^2 = 6$ whose distance from the line x + y = 7 is least is	(r)	$\left(2, \frac{2}{\sqrt{3}}\right)$
(D)	The foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ are S and S'. P is a point on the ellipse whose eccentric angle is $\pi/3$. The incentre of the triangle SPS' is	(s)	(–2,–8)
		(t)	(2, 2)

Key. A – s ; B – q ; C – p ; D – r

Sol. (A) Any point on the parabola is $(x, x^2 + 7x + 2)$ Its distance from the line

y = 3x – 3 is given by

$$P = \left|\frac{3x - (x^2 + 7x + 2) - 3}{\sqrt{9+1}}\right|$$

$$= \left|\frac{x^2 + 4x + 5}{\sqrt{10}}\right|$$

$$= \frac{x^2 + 4x + 5}{\sqrt{10}} (\text{as } x^2 + 4x + 5 > 0 \text{ for all } x \in R)$$

$\frac{dP}{dx} = 0 \Rightarrow x = -2.$ So, the required point is $(-2, -8)$

(B) Let $(x_1 y_1)$ be a point on y = x + 2

Therefore, $y_1 = x_1 + 2$

Equation of the line perpendicular to the given line through (x_1, y_1) is

$$y - (x_1 + 2) = -(x - x_1) \text{i.e.,} y = -x + 2(x_1 + 1)$$

If this line is a tangent to $y^2 = 8x, c = \frac{a}{m}$ gives

$$2(x_1 + 1) = \frac{2}{-1} \text{i.e.,} x_1 + 1 = -1 \Rightarrow x_1 = -2$$

Hence, $y_1 = 0$

Therefore, the required point is (–2, 0)

(C) Given equation of ellipse $\frac{x^2}{6}+\frac{y^2}{3}=1$. Slope of the tangent at any point

$P(x_1, y_1)$ to $\frac{x^2}{6}+\frac{y^2}{3}=1$ is given by $2x+4y\frac{dy}{dx}=0$

$\Rightarrow x = 2y$

$\because \frac{dy}{dx}=\frac{-x}{2y}=-1$

Putting x = 2y in the equation of the ellipse we have y = 1. Evidently, the point lies in the first quadrant

Therefore, y = 1 and x = 2

Hence, required point is (2, 1)

(D) The coordinates of the point P are $\left(\frac{5}{2},\frac{3\sqrt{3}}{2}\right)$. Since $e=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$, so, the coordinates of the foci are S(4, 0) and $S'(-4,0)$ and $SS'=8$.

Also, $SP = a - ex_1 = 5-\frac{4}{5}\times\frac{5}{2}=3$

And $S'P = a + ex_1 = 7$

Therefore, the coordinates of the incentre (x_1, y_1) are

$$x_1=\frac{7\times 4+3\times -4+8\times\frac{5}{2}}{7+3+8}=2$$

$$y_1=\frac{7\times 0+3\times 0+8\times\frac{3\sqrt{3}}{2}}{7+3+8}=\frac{2}{\sqrt{3}}$$

3. Match the following

Column – I	Column II
A. An ellipse passing through the origin has the foci (3,4) (6, 8) then length of minor axis is	p. 8

B. If PQ is focal chord of ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ which passes Through S = (3, 0) and PS =2 then length of chord (PQ) is — q. $10\sqrt{2}$

C. If the line y = x+ k touches the ellipse $9x^2+16y^2=144$ then The difference of values of k is — r. 10

D. Sum of the distances of a point on the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$ from the foci. — s. 12

Key. A-q,B-r, C-r, D-p

Sol. Conceptual

4. *P* is a point on the ellipse $9x^2+25y^2=225$. The tangent at *P* meet the X-axis, Y-axis at T, t respectively and the normal at *P* meet the X-axis, Y-axis at G, g respectively. *C* is the centre of the ellipse and *F* is the foot of the perpendicular from *C* to normal at *P*.

Column – I	Column – II
a) $\lvert PF\rvert\times\lvert PG\rvert=$	p) 25
b) $\lvert PF\rvert\times\lvert Pg\rvert=$	q) 16
c) $\lvert CG\rvert\times\lvert CT\rvert=$	r) 9
d) $\lvert Ct\rvert\times\lvert Cg\rvert=$	s) 24

Key. . a) r; b) p; c) q; d) q

Sol. Conceptual

5.

Column – I	Column – II
a) A tangent to the ellipse $\frac{x^2}{27}+\frac{y^2}{48}=1$ has slope $-\frac{4}{3}$ and the tangent cuts the axes of the ellipse at A, B. Area of ΔOAB is (*O* is the origin)	p) 36
b) Product of perpendiculars drawn from the points $(\pm 3, 0)$ to the line $y=mx-\sqrt{25m^2+16}$ is	q) $10\sqrt{2}$
c) An ellipse passing through (0, 0) has its foci at (3, 4) and (6, 8). Length of its minor axis is	r) 24
d) If *e* is the eccentricity of the conic $\sqrt{x^2+y^2}+\sqrt{(x+3)^2+(y-4)^2}=10$, then $72e=$	s) 16

Key. a) p; b) s; c) q; d) p

Sol. Conceptual

6. Match the following

Column – I	Column II
A. An ellipse passing through the origin has the foci (3,4) (6, 8) then length of minor axis is	p. 8
B. If PQ is focal chord of ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ which passes Through S = (3, 0) and PS =2 then length of chord (PQ) is	q. $10\sqrt{2}$
C. If the line y = x+ k touches the ellipse $9x^2+16y^2=144$ then The difference of values of k is	r. 10
D. Sum of the distances of a point on the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$ from the foci.	s. 12

Key. A-q, B-r, C-r, D-p

Sol. Conceptual

7. Match the following:-

	Column I		Column II
(A)	The locus of the midpoints of chords of an ellipse which are drawn through an end of minor axis, is	(P)	circle
(B)	The locus of an end of latus-recturm of all ellipses having a given major axis, is	(Q)	parabola
(C)	The locus of the foot of perpendicular from a focus of an ellipse on any tangent to it	(R)	ellipse
(D)	The locus of the midpoints of the portions of lines (drawn through a given point) between the co-ordinate axes	(S)	hyperbola

Key. A – R; B – Q; C – P; D – S

Sol. a) Let BC be a chord of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ let B = (0,b) and midpoint be $M=(\alpha,\beta)$ then

$C-(2\alpha,2\beta-b)$ will lie on the ellipse

b) $L\left(ae,\frac{b^2}{a}\right)$ given 2a = constant

$\alpha=ae\Rightarrow e=\frac{\alpha}{a}$

$$\beta = \frac{b^2}{a} = a\left(1 - e^2\right)$$

$$\Rightarrow \beta = a\left(1 - \frac{\alpha^2}{a^2}\right)$$

$$\Rightarrow \alpha^2 = a^2 - a\beta$$

c) Auxilary circle

d) $\frac{x}{a} + \frac{y}{b} = 1$ let midpoint of (a,0) & (o, b), be (α, β) then $2\alpha = a, 2\beta = b$ let all lines pass through a given point (h, k), then $\frac{h}{a} + \frac{k}{b} = 1$.

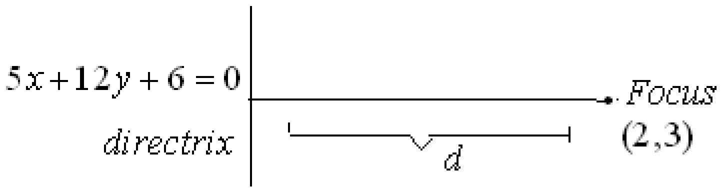
8. Consider the ellipse $(3x-6)^2 + (3y-9)^2 = \frac{4}{169}(5x+12y+6)^2$.

Column I contains the distances associated with this ellipse and Column II gives their value.
Match the expressions/statements in column I with those in column II.

	Column – I		Column – II
(A)	The length of major axis	(P)	$\frac{72}{5}$
(B)	The length of minor axis	(Q)	$\frac{16}{\sqrt{5}}$
(C)	The length of latus rectum	(R)	$\frac{16}{3}$
(D)	The distance between the directrices	(S)	$\frac{48}{5}$

KEY : A-S, B-Q, C-R, D-P

Sol. Rewrite the equation as $(x-2)^2 + (y-3)^2 = \frac{4}{9}\left[\frac{5x+12y+6}{13}\right]^2$



$$d = \frac{5.2 + 12.3 + 6}{\sqrt{5^2 + 12^2}} = \frac{52}{13} = 4$$

Also $e=\frac{2}{3}$

Length of major axis $=\frac{2e}{1-e^2}\ d=\frac{2.2/3}{1-4/9}\times 4=\frac{48}{5}$

Length of minor axis = (Length of major axis) $\sqrt{1-e^2}\ =\frac{16}{\sqrt{5}}$

Length of latusrectum = (Length of major axis) $\left(1-e^2\right)=\frac{16}{3}$

Distance between the directrices = (Length of major axis) $\times$ 1/e = 72/5

9. Match the following:-

	Column I		Column II
(A)	The coordinates of the point on the parabola $y=x^2+7x+2$, which is nearest to the straight line y = 3x – 3 are	(p)	(2,1)
(B)	y = x + 2 is a tangent to the parabola $y^2=8x$. The point on this line, the other tangent from which is perpendicular to this tangent is	(q)	(–2, 0)
(C)	The point on the ellipse $x^2+2y^2=6$ whose distance from the line x + y = 7 is least is	(r)	$\left(2,\frac{2}{\sqrt{3}}\right)$
(D)	The foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ are S and S'. P is a point on the ellipse whose eccentric angle is $\pi/3$. The incentre of the triangle SPS' is	(s)	(–2,–8)
		(t)	(2, 2)

Key. (A-s), (B-q), (C-p), (D-r)

Sol. (A) Any point on the parabola is (x,x^2+7x+2) Its distance from the line y = 3x – 3 is given by

$$P=\left|\frac{3x-(x^2+7x+2)-3}{\sqrt{9+1}}\right|$$

$$=\left|\frac{x^2+4x+5}{\sqrt{10}}\right|$$

$$=\frac{x^2+4x+5}{\sqrt{10}}(\text{as } x^2+4x+5>0 \text{ for all } x\in R)$$

$\frac{dP}{dx}=0\Rightarrow x=-2.$ So, the required point is $(-2,-8)$

(B) Let (x_1y_1) be a point on y = x + 2

Therefore, $y_1=x_1+2$

Equation of the line perpendicular to the given line through (x_1,y_1) is

$y-(x_1+2)=-(x-x_1)\text{i.e.}, y=-x+2(x_1+1)$

If this line is a tangent to $y^2=8x, c=\frac{a}{m}$ gives

$2(x_1+1)=\frac{2}{-1}$ i.e., $x_1+1=-1 \Rightarrow x_1=-2$

Hence, $y_1=0$

Therefore, the required point is (–2, 0)

(C) Given equation of ellipse $\frac{x^2}{6}+\frac{y^2}{3}=1$. Slope of the tangent at any point

$P(x_1, y_1)$ to $\frac{x^2}{6}+\frac{y^2}{3}=1$ is given by $2x+4y\frac{dy}{dx}=0$

$\Rightarrow x=2y$

$\because \frac{dy}{dx}=\frac{-x}{2y}=-1$

Putting x = 2y in the equation of the ellipse we have y = 1. Evidently, the point lies in the first quadrant

Therefore, y = 1 and x = 2

Hence, required point is (2, 1)

(D) The coordinates of the point P are $\left(\frac{5}{2}, \frac{3\sqrt{3}}{2}\right)$. Since $e=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$, so, the coordinates of the foci are S(4, 0) and $S'(-4,0)$ and $SS'=8$.

Also, $SP=a-ex_1=5-\frac{4}{5}\times\frac{5}{2}=3$

And $S'P=a+ex_1=7$

Therefore, the coordinates of the incentre (x_1, y_1) are

$$x_1=\frac{7\times4+3\times-4+8\times\frac{5}{2}}{7+3+8}=2$$

$$y_1=\frac{7\times0+3\times0+8\times\frac{3\sqrt{3}}{2}}{7+3+8}=\frac{2}{\sqrt{3}}$$

10. Match the following loci for the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad (a>b)$

Column I		Column II	
(a)	Locus of point of intersection of two perpendicular tangents, is	(p)	$\left(x^2+y^2\right)^2=a^2x^2+b^2y^2$
(b)	Locus of foot of perpendicular from foci upon any tangent, is	(q)	$4\left(x^2+y^2\right)^2=\left(a^2x^2+b^2y^2\right)$
(c)	Locus of foot of perpendicular from centre on any tangent, is	(r)	$x^2+y^2=a^2$
(d)	Locus of midpoint of segment OM where M is the foot of the perpendicular from O to any tangent (O is centre) , is	(s)	$x^2+y^2=a^2+b^2$

Key. A – s, B – r, C – p, D – q

Sol. (a) Locus must be director circle

(b)Foot of perpendicular lie on auxiliary circle (c)Any tangent to ellipse is $\frac{x}{a}\cos\alpha+\frac{y}{b}\sin\alpha=1$

any lie perpendicular $= y=\left(\frac{a}{b}\tan\alpha\right)x$

$$\tan\alpha=\frac{b}{a}\frac{y}{x}$$

$$\Rightarrow \sin\alpha=\frac{by}{\sqrt{a^2x^2+b^2y^2}} \qquad \cos\alpha=\frac{ax}{\sqrt{a^2x^2+b^2y^2}}$$

$$\Rightarrow \frac{x}{a}\frac{ax}{\sqrt{a^2x^2+b^2y^2}}+\frac{y}{b}\frac{by}{\sqrt{a^2x^2+b^2y^2}}=1$$

$$x^2+y^2=\sqrt{a^2x^2+b^2y^2}$$

$$\left(x^2+y^2\right)^2=a^2x^2+b^2y^2$$

(d) Similar proof take point as $(2x, 2y)$

11. Consider an ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ with centre C and point P on it with eccentric angle $\pi/4$. Normal drawn at P intersects the major and minor axes in A and B respectively N_1 and N_2 are the feet of the perpendiculars from the foci S_1 and S_2 respectively on the tangent at P and N is the foot of the perpendicular from the centre of the ellipse on the normal at P. tangent at 'P' intersect the axis of x at T.

Match the column using the above information.

Column - I	**Column – II**
a) (CA)(CT) is equal to	p) 9
b) (PN)(PB) is equal to	q) 16
c) $(S_1N_1)(S_2N_2)$ is equal to	r) 17
d) $(S_1P)(S_2P)$ is equal to	s) 25
	t) 30

Key. a) q; b) s; c) p; d) r

Sol. a) $(CA)(CT)=\left(e^2 a\cos\theta\right)\left(\frac{a^2}{a\cos\theta}\right)=a^2-b^2=16$

b) $(PN)(PB)=a^2=25$

c) $(S_1N_1)(S_2N_2)=b^2=9$

d) $(S_1P)(S_2P)=[a+e(a\cos\theta)][a-e(a\cos\theta)]=a^2\sin^2\theta+b^2\cos^2\theta=17$