Differential Calculus

Single Correct Answer Type

1. Let $f(x)=4x+8\cos x-\ln\{\cos x(1+\sin x)\}+\tan x-2\sec-6$. If $f(x)>0\ \forall\, x\in(0,a)$ then

a) $a=\frac{\pi}{6}$ b) $a=\frac{\pi}{3}$ c) $a=\frac{\pi}{2}$ d) none of these

Ans. a

Sol. $f'(x)=4-8\sin x-\frac{(-\sin x+\cos^2 x-\sin^2 x)}{\cos x(1+\sin x)}+\sec^2 x-\sec x\tan x$

$=4(1-2\sin x)+\sec^2 x(1-2\sin x)-4\sec(1-2\sin x)$

$=f(x)=(\sec x-2)^2(1-2\sin x)$

If $f(x)>0\ \forall\, x\in(0,a)$, then f(x) is increasing in (0, a) $\Rightarrow a=\frac{\pi}{6}$

2. If f(x) is continuous for all real values of x, then $\sum_{r=1}^{n}\int_0^1 f(r-1+x)\,dx=$

a) $\int_0^n f(x)\,dx$ b) $\int_0^1 f(x)\,dx$ c) $n\int_0^1 f(x)\,dx$ d) $(n-1)\int_0^1 f(x)\,dx$

Ans. a

Sol. $\sum_{r=1}^{n}\int_0^1 f(r-1+x)\,dx=\int_0^1 f(x)\,dx+\int_0^1 f(1+x)\,dx+\int_0^1 f(2+x)\,dx+......+\int_0^1 f(n-1+x)\,dx$

= $\int_0^1 f(x)\,dx+\int_1^2 f(x)\,dx+\int_2^3 f(x)\,dx+......+\int_{n-1}^{n} f(x)\,dx=\int_0^n f(x)\,dx$

3. The coordinates of the point on the curve $x^3 = y(x-a)^2$, a > 0 where the ordinate is minimum

a) (2a, 8a) b) $\left(-2a,\frac{-8a}{9}\right)$ c) $\left(3a,\frac{27a}{4}\right)$ d) $\left(-3a,\frac{-27a}{16}\right)$

Ans. c

The ordinates of any point on the curve is given by $y=\frac{x^3}{(x-a)^2}$

Sol. $\frac{dy}{dx}=\frac{x^2(x-3a)}{(x-a)^3}$

Now, $\frac{dy}{dx}=0\Rightarrow x=0\ or\ x=3a$

$\left.\frac{d^2y}{dx^2}\right|_{x=0}=0\ and\ \left.\frac{d^2y}{dx^2}\right|_{x=3a}=\frac{72a^5}{(2a)^6}>0$

Hence y is minimum at x = 3a and is equal to $\frac{27a}{4}$

4. Let $I_n = \int_0^{\frac{\pi}{2}} \frac{\sin(2n-1)x}{\sin x} dx$, $J_n = \int_0^{\frac{\pi}{2}} \frac{\sin^2 nx}{\sin^2 x} dx$, $n \in N$, then

a) $J_{(n+1)} - J_n = I_n$ b) $J_{(n+1)} - J_n = I_{(n+1)}$ c) $J_{n+1} + J_n = J_n$ d) $J_{n+1} + J_{n+1} = J_n$

Ans. b

Sol. $J_n - J_{n-1} = \int_0^{\frac{\pi}{2}} \frac{\sin^2 nx - \sin^2(n-1)x}{\sin^2 x} dx - \int_0^{\frac{\pi}{2}} \frac{\sin(2n-1)x - \sin x}{\sin^2 x} dx = I_n$

i.e $J_n - J_{n-1} = I_n \Rightarrow J_{n+1} - J_n = I_{n+1}$

5. A curve whose concavity is directly proportional to the logarithm of its x-coordinates at any of the curve, is given by

a) $c_1 x^2 (2\log x - 3) + c_2 x + c_3$ b) $c_1 x^2 (2\log x + 3) + c_2 x + c_3$

c) $c_1 x^2 (2\log x) + c_2$ d) none of these

Ans. a

Sol. $\frac{d^2 y}{dx^2} = k \log x \Rightarrow \frac{dy}{dx} = k(x\log x - x) + A$

$\Rightarrow y = k\left[\frac{1}{2}x^2 \log x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} - \frac{x^2}{2} dx\right] + Ax + B$

$\Rightarrow y = \frac{k}{4}\{2x^2 \log x - x^2 - 2x^2] + Ax + B$

$\Rightarrow y = c_1(2\log x - 3)x^2 + c_2 x + c_3$

6. The domain of the function $f(x) = \sqrt{3 - 2^x - 2^{1-x}} + \sqrt{\sin^{-1} x}$ is

a) $[-1, 0]$ b) $[0, 1]$ c) $\left[\frac{1}{2}, 1\right]$ d) [1, 2]

Ans. b

Sol. $\sin^{-1} x \geq 0 \Rightarrow 0 \leq x \leq 1$

and $2^x + 2^{1-x} \leq 3 \Rightarrow 2^x + 2.2^{-x} - 3 \leq 0$

Put $2^x = t$, then $t^2 - 3t + 2 \leq 0 \Rightarrow (t-2)(t-1) \leq 0$

$\Rightarrow 1 \leq t \leq 2$ *i.e* $1 \leq 2^x \leq 2$

$\Rightarrow 0 \leq x \leq 1$

7. Area bounded by the curve y = sinx, y = cosx, $x = -\frac{\pi}{3}$, $x = 2\pi$

a) $4\sqrt{2} - \left(\frac{\sqrt{3}+1}{2}\right)$ b) $\sqrt{2} + \left(\frac{\sqrt{3}+1}{2}\right)$ c) $\sqrt{2} - \left(\frac{\sqrt{3}+2}{2}\right)$ d) $4\sqrt{2} + \left(\frac{\sqrt{3}+1}{2}\right)$

Ans. d

Sol. $A = \int_{-\pi/3}^{2\pi} |\sin x - \cos x| dx \Rightarrow 4\sqrt{2} + \frac{\sqrt{3}+1}{2}$

8. Let f(1) = 1 and $f(n) = 2\sum_{r=1}^{n-1} f(r)$, then $\sum_{n=1}^{m} f(n)$ is equal to

a) $3^{m-1} - 1$ b) 3^{m-1} c) $3^m - 1$ d) none of these

Ans. b

Sol. $f(n) = 2\left(f(1) + f(2) + ... + f(n-1)\right)$

$\therefore$ f(n + 1) = 2(f(1) + f(2) + ……+ f(n))

$\Rightarrow$ f(n + 1) = 3f(n) for $n \geq 2$

Also f(2) = 2f(1) = 2

f(3) = 3f(2) = $2 \cdot 3$

$$\sum_{n=1}^{m} f(n) = f(1) + f(2) + ... + f(m)$$

= 1 + 2 + 2.3 + 2.3^2 + …. + 2.3^{m-2} = 1 + 2(1 + 3 + 3^2 + …. + 3^{m-2})

9. $I = \int \frac{2+3\cos\theta}{\sin\theta + 2\cos\theta + 3}\, d\theta$, then

a) $I = \frac{6\theta}{5} + \frac{3}{5}\log\left|\sin\theta + 2\cos\theta + 3\right| - \frac{8}{5}\tan^{-1}\left(\frac{\tan\left(\frac{\theta}{2}\right)+1}{2}\right) + c$

b) $I = \frac{6\theta}{5} - \frac{3}{5}\log\left|\sin\theta + 2\cos\theta + 3\right| - \frac{8}{5}\tan^{-1}\left(\frac{\tan\left(\frac{\theta}{2}\right)+1}{2}\right) + c$

c) $I = \frac{6\theta}{5} - \frac{3}{5}\log\left|\sin\theta + 2\cos\theta + 3\right|$

d) none of these

Ans. a

Sol. $2 + 3\cos\theta = I\left(\sin\theta + 2\cos\theta + 3\right) + m\left(\cos\theta - 2\sin\theta\right) + n$, then integrate

10. The value of $\lim_{n\to\infty}\left(\frac{\sqrt{n}}{\left(3+4\sqrt{n}\right)^2} + \frac{\sqrt{n}}{\sqrt{2}\left(3\sqrt{2}+4\sqrt{n}\right)^2} + ... + \frac{1}{49n}\right)$ is equal to

a) $\frac{1}{14}$ b) $\frac{2}{7}$ c) $\frac{3}{7}$ d) none of these

Ans. a

Sol. $\lim_{n\to\infty}\sum_{r=1}^{n}\frac{\sqrt{n}}{\sqrt{r}\left(3\sqrt{r}+4\sqrt{n}\right)^2}$

Put $\frac{r}{n} = x \Rightarrow \frac{1}{n} = dx$

$= \int_0^1 \frac{dx}{\sqrt{x}\left(3\sqrt{x}+4\right)^2} = \frac{1}{14}$

11. Solution of differential equation $xdy - \left(y + xy^3\left(1+\log x\right)\right)dx = 0$

a) $\frac{-x^2}{y^2} = \frac{2x}{3}\left(\frac{2}{3} + \log x\right) + c$ b) $\frac{x^2}{y^2} = \frac{2x^2}{3}\left(\frac{2}{3} + \log x\right) + c$

c) $\frac{-x^2}{y^2}=\frac{2x^3}{3}\left(\frac{2}{3}+\log x\right)+c$ d) none of these

Ans. c

Sol. $-d\left(\frac{x}{y}\right)=xy(1+\log x)dx$

$\int -\frac{x}{y}d\left(\frac{x}{y}\right)=\int x^2(1+\log x)dx$ gives solution

12. Let $f:R\to R$ and $g:R\to R$ be twice differentiable function satisfying $f''(x)=g''(x)$, $2f'(1)=g'(1)=4$ and 3f(2) = g(2) = 9. The value of f(4) – g(4) is equal to

a) – 6 b) – 16 c) – 10 d) – 8

Ans. c

Sol. $f'(x)=g(x)-2$

f(x) = g(x) – 2x – 2

f(u) – g(u) = - 10

13. Let a, b, c be three real numbers such that a < b < c. Let f(x) be continuous $\forall\ x\in[a,c]$ and differentiable $\forall\ x\in(a,c)$. If $f''(x)>0\ \forall\ x\in(a,c)$ then

a) (c – b) f(a) + (b – a) f(c) > (c – a) f(b)
b) (c – b) f(a) + (a – c) f(b) < (a – b) f(c)
c) f(a) < f(b) < f(c)
d) none of these

Ans. a

Sol. By LMVT

$\frac{f(b)-f(a)}{b-a}>\frac{f(c)-f(b)}{c-b}$

14. The solution of $y^5x+y-x\frac{dy}{dx}=0$ is

a) $\frac{x^4}{4}+\frac{1}{5}\left(\frac{x}{y}\right)^5=c$ b) $\frac{x^5}{5}+\frac{1}{4}\left(\frac{x}{y}\right)^4=c$ c) $\left(\frac{x}{y}\right)^5+\frac{x^4}{4}=c$ d) $(xy)^4+\frac{x^5}{5}=c$

Ans. b

Sol. $y^5xdx+ydx-xdy=0$, multiply by x^3/y^5

$\Rightarrow x^4dx+\frac{x^3}{y^3}(d(x/y))=0$

$\Rightarrow \frac{x^5}{5}+\frac{1}{4}\left(\frac{x}{y}\right)^4=c$

15. A point P lying inside the curve $y=\sqrt{2ax-x^2}$ is moving such that its shortest distance from the curve at any position is greater than its distance from x-axis. The point P enclose a region whose area is equal to

a) $\frac{\pi a^2}{2}$ b) $\frac{a^2}{3}$ c) $\frac{2a^2}{3}$ d) $\left(\frac{3\pi-4}{6}\right)a^2$

Ans. c

Sol.

$y=\sqrt{2ax-x^2} \Rightarrow (x-a)^2+y^2=a^2$

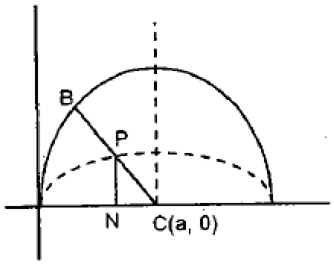
Let P(h, k) be a point then BP > PN

For the boundary condition BP = PN = k

Now $AP=a-k=\sqrt{(h-a)^2+k^2} \Rightarrow k=h-\frac{h^2}{2a}$

$\therefore$ boundary of the region is $y=x-\frac{x^2}{2a}$

Required area = $2\int_0^a\left(x-\frac{x^2}{2a}\right)dx=\frac{2a^2}{3}$



16. If $\log_x\left(\log_y k\right)>0$ where $x,k\in(0,1)$ then $y\in$

a) (0, x) b) (0, k) c) (k, 1) d)R^+

Ans. c

Sol. $\log_y k < 1$

case 1 : if y > 1 $\Rightarrow$ k < y

for $\log_y k > 0 \Rightarrow$ k > 1 which is not possible

case 2 : if y < 1 $\Rightarrow$ k > y

and for $\log_y k > 0 \Rightarrow$ k < 1 which is true

17. Period of $f(x)=x-[x+\lambda]-\mu$ where $\lambda,\ \mu\in R$ and $[\cdot]$ denotes the g.i.f is

a) λ b) μ c) $|\lambda-\mu|$ d) 1

Ans. d

Sol. $f(x)=x-[x+\lambda]-\mu=x+\lambda-[x+\lambda]-(\lambda+\mu)$

$=\{x+\lambda\}-(\lambda+\mu)$

$\therefore$ Period of f(x) = 1

18. If $f(x)=2\sin^3 x-3\sin^2 x+12\sin x+5\ \forall\ x\in\left(0,\frac{\pi}{2}\right)$, then

a) f is increasing in $\left(0,\frac{\pi}{2}\right)$ b) f is decreasing in $\left(0,\frac{\pi}{2}\right)$

c) f is increasing $\left(0,\frac{\pi}{4}\right)$ and decreasing in $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

d) f is decreasing is $\left(0,\frac{\pi}{4}\right)$ and increasing in $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

Ans. a

Sol. $f'(x)=6\cos x\left(\sin^2 x-\sin x+2\right)>0\ \forall\in\left(0,\frac{\pi}{2}\right)$

Thus f(x) is increasing in $\left(0,\frac{\pi}{2}\right)$

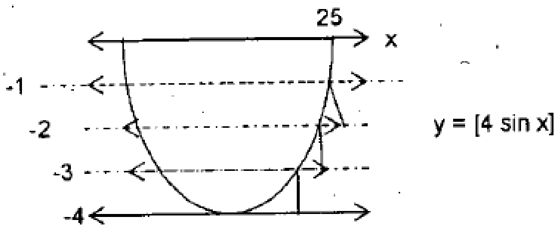
19. Total number of points of non-differentiability of $f(x)=[3+4\sin x]$ in $[\pi,2\pi]$ where $[\cdot]$ denote the g.i.f are

a) 5 b) 6 c) 8 d) 9

Ans. c

Sol.

f(x) = 3 + [4 sinx]
f(x) is non-differentiable where g(x) =[4sinx] is non differentiable
In [π, 2 π], g(x) is clearly non-differentiable at 8 points.



20. If $f(x)+2f(1-x)=x^2+1\ \forall x\in R$ and $\int_0^k f(x)dx=0$, then k equals to

a) 3 b) 2 c) 4 d) none of these

Ans. a

Sol. Putting (1 – x) for x and subtracting we get $f(x)=\frac{x^2-4x+3}{3}$

Now $\int_0^k \frac{x^2-4x+3}{3}dx=0 \Rightarrow \frac{k^3}{3}-2k^2+3k=0$

$\Rightarrow k=3$

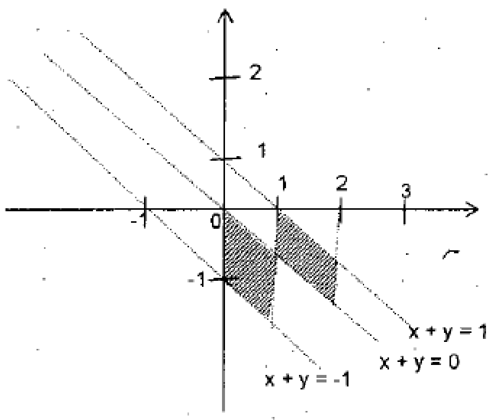
21. A point p(x, y) moves is such a way that [x + y + 1] = [x] (where [ ] denotes g.i.f) and $x\in(0,2)$. Then the area representing all the possible positions of P equals

a) $\sqrt{2}$ sq. units b) $2\sqrt{2}$ sq. units c) $4\sqrt{2}$ sq. units d) none of these

Ans. d

Sol.

If $x\in(0,1)$
Then $-1\le x+y<0$
and if $x\in(1,2)$
$0\le x+y<1$
Required area =
$4\left(\frac{1}{2}\cdot1\cdot\sqrt{2}\sin\frac{\pi}{4}\right)=2\,sq\,units$



22. Let f(x) be a polynomial with real coefficients satisfies $f(x)=f'(x)\times f'''(x)$. If f(x)=0 satisfies x = 1, 2, 3 only then the value of $f'(1)\times f'(2)\times f'(3)$ =

a) positive b) negative c) 0 d) inadequate data

Ans. c

Sol. $f(x)=f'(x)\times f'''(x)$ is satisfied by only the polynomial of degree 4.
Since f(x) = 0 satisfies x = 1, 2, 3 only. It is clear one of the root is twice repeated.
$\Rightarrow f'(1)f'(2)f'(3)=0$

23. The value of $\lim_{n\to\infty}\left(\frac{n!}{(mn)^n}\right)^{1/n}$ is

a) em b) $\frac{e}{m}$ c) $\frac{1}{em}$ d) none of these

Ans. c

Sol. $L=\lim_{n\to\infty}\frac{1}{m}\left(\frac{1}{n}\cdot\frac{2}{n}\cdot\frac{3}{n}.....\frac{n}{n}\right)^{1/n}$

$$In\ L=\lim_{n\to\infty}\left[\ln\left(\frac{1}{m}\right)+\frac{1}{n}\left(\ln\frac{1}{n}+\ln\frac{2}{n}+....+\ln\frac{n}{n}\right)\right]$$

$$=\ln m+\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\ln\left(\frac{r}{n}\right)=-\ln m+\int_0^1\ln x\,dx=-\ln m-1=\ln\left(\frac{1}{em}\right)$$

$$\therefore L=\frac{1}{em}$$

24. Let A = {1, 2, 3, 4, 5} and f : A $\to$ A be an into function such that $f(i)\neq i\ \forall\ i\in A$, then number of such functions f are

a) 1024 b) 904 c) 984 d) none of these

Ans. d

Sol. Total number of functions for which $f(i)\neq i=4^5$

and number of onto functions in which $f(i)\neq i=44$

$\Rightarrow$ required numbers of functions = 980

25. The area of the region bounded between the curves $y=e\|x|\ln|x\|$, $x^2+y^2-2(|x|+|y|)+1\geq 0$ and x-axis where $|x|\leq 1$, if α is the x-coordinate of the point of intersection of curves is 1st quadrant, is

a) $4\left[\int_0^{\alpha}ex\ln x dx+\int_{\alpha}^{1}\left(1-\sqrt{1-(x-1)^2}\right)dx\right]$ b) $\left[\int_0^{\alpha}ex\ln x dx-\int_1^{\alpha}\left(1-\sqrt{1-(x-1)^2}\right)dx\right]$

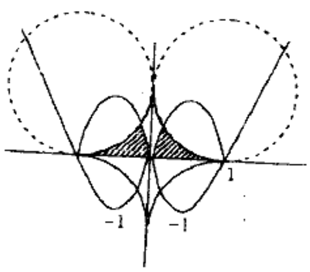
c) $2\left[-\int_0^{\alpha}ex\ln x dx+\int_{\alpha}^{1}\left(1-\sqrt{1-(x-1)^2}\right)dx\right]$ d) $2\left[\int_0^{\alpha}ex\ln x dx+\int_{\alpha}^{1}\left(1-\sqrt{1-(x-1)^2}\right)dx\right]$

Ans. c

Sol.

Required area is

$$2\left[\int_0^{\alpha}ex\ln x dx+\int_1^{\alpha}\left(1-\sqrt{1-(x-1)^2}\right)dx\right]$$



26. The value of $\lim_{n\to\infty}n\left[\frac{1}{3n^2+8n+4}+\frac{1}{3n^2+16n+16}+.....n\ terms\right]$ is

a) $\frac{1}{4}\ln\left(\frac{9}{5}\right)$ b) $\frac{1}{5}\ln\left(\frac{9}{5}\right)$ c) $\frac{1}{4}\ln\left(\frac{8}{5}\right)$ d) $\frac{1}{4}\ln\left(\frac{9}{7}\right)$

Ans. a

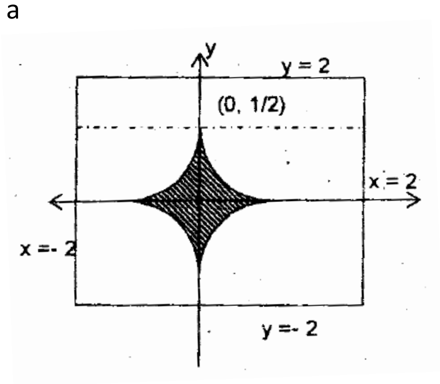
Sol. Use definite integral of first principal as a limit of sum

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{4\left(1+\frac{r}{n}\right)^2-1}\cdot\frac{1}{n}$$

27. The area of the region containing the points satisfying $|y|+\frac{1}{2}\le e^{-|x|}$, $\max\left(|x|,|y|\right)\le 2$ is

a) $2\log\left(\frac{e}{2}\right)$ b) $2\log\left(\frac{2e}{3}\right)$ c) $3\log\left(\frac{e}{2}\right)$ d) $3\log\left(\frac{2e}{3}\right)$

Ans. a



28. If $y=2^{-2^{\frac{1}{1-x}}}$; then $\lim_{x\to1^+} y$ is

a) – 1 b) 1 c) 0 d) $\frac{1}{2}$

Ans. b

Sol. $\lim_{h\to0} 2^{-2^{\frac{1}{1-(1+h)}}}=2^{-0}=1$

29. If $y=\frac{2x+5}{3x+10}$, then $2\left(\frac{dy}{dx}\right)\left(\frac{d^3y}{dx^3}\right)$ is equal to

a) $\left(\frac{d^2y}{dx^2}\right)^2$ b) $3\frac{d^2y}{dx^2}$ c) $3\left(\frac{d^2y}{dx^2}\right)^2$ d) $3\frac{d^2x}{dy^2}$

Ans. c

Sol. 3xy + 10y = 2x + 5, now differentiate 3 times.

30. If the number of solutions of $\ln|\sin x|=-x^2+2x$ when $x\in(0,\pi)$ is m and when $x\in\left[-\frac{\pi}{2},\frac{3\pi}{2}\right]$ is n, then (m + n) is equal to

a) 2 b) 4 c) 6 d) 1

Ans. a

Sol. m = 0, n = 2

31. If x, {x} and 2[x] represent the segments of a focal chord and length of latus rectum of an ellipse respectively, then length of major axis of ellipse is always greater than (where xÏ Z)

a) 7 b) 5 c) 8 d) 2

Ans. d

Sol. Clearly, x, [x] and {x} are in H.P = $\left[x\right]=\frac{2x\left\{x\right\}}{x+\left\{x\right\}}\Rightarrow\left[x\right]=1$

$$\Rightarrow\frac{b^2}{a}=1\Rightarrow a\left(1-e^2\right)=1\Rightarrow 2a>2 \qquad \text{[since } 0 < e < 1\text{]}$$

32. The value of $\int_3^6\left(\sqrt{x+\sqrt{12x-36}}+\sqrt{x-\sqrt{12x-36}}\right)dx$ is equal to

a) $6\sqrt{3}$ b) $4\sqrt{3}$ c) $12\sqrt{3}$ d) none of these

Ans. a

Sol. $I=\int_3^6\left(\left(\sqrt{x-3}+\sqrt{3}\right)+\left(\sqrt{3}-\sqrt{x-3}\right)\right)dx=6\sqrt{3}$

33. If integral $\int\frac{dx}{\left(\sec x+\operatorname{cosec} x+\tan x+\cot x\right)^2}=\frac{x}{a}+\frac{\sqrt{2}\cos\left(x+\frac{\pi}{4}\right)}{b}+\frac{\cos 2x}{c}+d$, then a + b + c is equal to

a) – 2 b) – 4 c) 2 d) none of these

Ans. b

Sol. Clearly,

$$I=\int\frac{\sin^2 x\cos^2 x}{\left(\sin x+\cos x+1\right)^2}dx=\frac{1}{4}\int\frac{\left(\left(\sin x+\cos x\right)^2-1\right)^2}{\left(\sin x+\cos x+1\right)}dx=\frac{1}{4}\int\left(\sin x+\cos x-1\right)^2dx$$

On simplifying a + b + c = - 4

34. If $I_n=\int_{-n}^{n}\left(\left\{x+1\right\}\left\{x^2+2\right\}+\left\{x^2+3\right\}\left\{x^2+4\right\}\right)dx$, (where {.} denotes the fractional part) then I_1 is equal to

a) $-\frac{1}{3}$ b) $-\frac{2}{3}$ c) $\frac{1}{3}$ d) none of these

Ans. b

Sol. $I_1=\int_{-1}^{1}\left(\left\{x\right\}+\left\{x^3\right\}\right)\left\{x^2\right\}dx=-2\int_0^1\left\{x^2\right\}dx=-2\times\left.\frac{x^3}{3}\right|_0^1=-\frac{2}{3}$

35. Area bounded by y = $f^{-1}(x)$ and tangent and normal drawn to it at the points with abscissae π and 2π, where f(x) = sinx – x is

a) $\frac{\pi^2}{2}-1$ b) $\frac{\pi^2}{2}-2$ c) $\frac{\pi^2}{2}-4$ d) $\frac{\pi^2}{2}$

Ans. b

Sol. Required area A = $\int_{\pi}^{2\pi}\left(\left(\sin x-x\right)+2\pi\right)dx=\frac{\pi^2}{2}-2\ sq.units$

36. Let a curve y = f(x), $f(x)\geq 0\ \forall\ x\in R$ has property that for every point P on the curve length of subnormal is equal to abscissa of P. If f(1) = 3, then f(4) is equal to

a) $-2\sqrt{6}$ b) $2\sqrt{6}$ c) $3\sqrt{5}$ d) none of these

Ans. b

Sol. Given $y\frac{dy}{dx}=x$

$y\,dy = x\,dx$

$y^2 = x^2 + c$

$f(1)=3 \Rightarrow 9-1+c \Rightarrow c=8$

$\Rightarrow y^2 = x^2 + 8$

$f(x)=\sqrt{x^2+8}$

$f(4)=\sqrt{16+8}=2\sqrt{6}$

37. Range of $f(x)=\cos^{-1}\left(\frac{x^2+x+1}{x^4+1}\right)$ is

a) $\left[0, \frac{\pi}{2}\right]$ b) $\left[0, \frac{\pi}{2}\right)$ c) $\left(0, \frac{\pi}{2}\right]$ d) $[0, \pi]$

Ans. b

Sol. Let $g(x)=\frac{x^2+x+1}{x^4+1}$

$\Rightarrow 0 < g(x) \le 1$

So range of f(x) is $\left[0, \frac{\pi}{2}\right)$

38. If f(x) = 0 is a cubic equation with positive and distinct roots α, β, γ such that β is H.M of the roots of $f'(x)=0$, then α, β and γ are in

a) A.P b) G.P c) H.P d) none of these

Ans. b

Sol. $f(x)=(x-\alpha)(x-\beta)(x-\gamma)$

$\Rightarrow f'(x)=3x^2-2x(\alpha+\beta+\gamma)+\alpha\beta+\beta\gamma+\gamma\alpha$

$\Rightarrow \beta=\frac{2\alpha_1\beta_1}{\alpha_1+\beta_1}$ (where α_1, β_1 are the roots of $f'(x)$ = 0)

$\Rightarrow \beta^2=\gamma\alpha$

39. Let a curve y = f(x), $f(x) \ge 0\ \forall\ x \in R$ has property that for every point P on the curve, the length of subnormal is equal to abscissa of P. If f(1) = 3, then f(4) is equal to

a) $-2\sqrt{6}$ b) $2\sqrt{6}$ c) $3\sqrt{5}$ d) none of these

Ans. b

Sol. $y\frac{dy}{dx}=x \Rightarrow y^2=x^2+c$

$f(x)=\sqrt{x^2+8} \Rightarrow f(4)=2\sqrt{6}$

40. If $\int_0^{\pi/2}\frac{dx}{a^2\cos^2 x+b^2\sin^2 x}=\frac{\pi}{2ab}$, then the value of $\int_0^{\pi/2}\frac{dx}{\left(4\cos^2 x+9\sin^2 x\right)^2}$ is equal to

a) $\frac{11\pi}{864}$ b) $\frac{13\pi}{864}$ c) $\frac{17\pi}{864}$ d) none of these

Ans. b

Sol. $I=\int_0^{\pi/2}\frac{dx}{a^2\cos^2 x+b^2\sin^2 x}=\frac{\pi}{2ab}$

$\frac{dI}{da}=\frac{-\pi}{2a^2b}$

$\Rightarrow\int_0^{\pi/2}\frac{\cos^2 xdx}{\left(a^2\cos^2 x+b^2\sin^2 x\right)}=\frac{\pi}{4a^3b}$

differentiating with respect to b

$\int_0^{\pi/2}\frac{\sin^2 xdx}{\left(a^2\cos^2 x+b^2\sin^2 x\right)}=\frac{\pi}{4a^3b}$

$\Rightarrow\int_0^{\pi/2}\frac{dx}{\left(a^2\cos^2 x+b^2\sin^2 x\right)}=\frac{\pi}{2ab}\left[\frac{1}{a^2}+\frac{1}{b^2}\right]=\frac{\pi}{24}\left[\frac{1}{4}+\frac{1}{9}\right]=\frac{13\pi}{864}$

41. If $\int\frac{dx}{\cos^3 x-\sin^3 x}=A\tan^{-1}\left(\sin x+\cos x\right)+B$ ln f(x) + C, then A is equal to

a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $-\frac{2}{3}$ d) none of these

Ans. a

Sol. $I=\int\frac{dx}{\left(\cos x-\sin x\right)\left(1+\frac{\sin 2x}{2}\right)}=\int\frac{\cos x-\sin x}{\left(\cos x-\sin x\right)^2\left(1+\frac{\sin 2x}{2}\right)}dx$

Put cosx + sinx = t

$I=\frac{2}{3}\tan^{-1}\left(\sin x+\cos x\right)-\frac{2}{3\sqrt{2}}\ln f\left(x\right)+c$

42. Solution of the differential equation $y\left(2x^4+y\right)\frac{dy}{dx}=\left(1-4xy^2\right)x^2$ is given by

a) $3\left(x^2y\right)^2+y^3-x^3=c$ b) $xy^2+\frac{y^3}{3}-\frac{x^3}{3}+c=0$

c) $\frac{2}{5}yx^5+\frac{y^3}{3}=\frac{x^3}{3}-\frac{4xy^3}{3}+c$ d) none of these

Ans. a

Sol. Given equation can be written as

$2x^2y(x^2dy + 2xy\,dx) + y^2\,dy - x^2\,dx = 0$

or $2x^2y\,d(x^2y) + y^2dy - x^2dx = 0$

Integrating, we get

$3(x^2y)^2 + y^3 - x^3 = c$

43. If $I=\int_0^{\pi}\frac{\cos x}{\left(x+2\right)^2}dx,$ then $\int_0^{\pi}\frac{\sin x}{x+1}dx$ is equal to

a) 21 b) $\frac{1}{\pi+2}-\frac{1}{2}-1$ c) 0 d) $\frac{1}{\pi+2}+\frac{1}{2}-1$

Ans. d

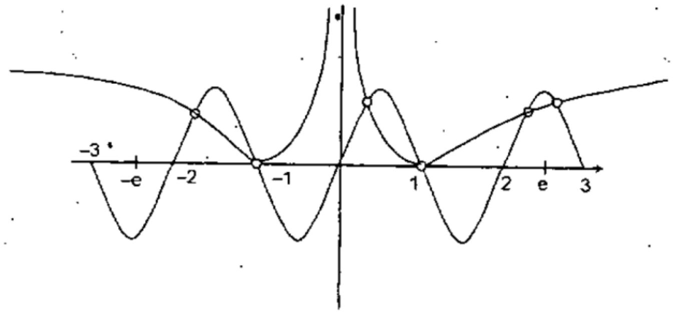
Sol. $I=\int_0^{\pi}\cos x\,d\left(-\frac{1}{x+2}\right)=\left[\frac{-\cos x}{x+2}\right]_0^{\pi}-\int_0^{\pi}\frac{\sin x}{x+2}dx$

$$= \frac{1}{\pi+2} + \frac{1}{2} - \int_0^{\pi/2} \frac{\sin 2x}{x+1} dx$$

44. The number of solutions of $\sin \pi x = \left| \log |x| \right|$ is

a) infinite b) 8 c) 6 d) 0

Ans. c



45. If $f(x) = \left| x^2 + (k-1)|x| - k \right|$ is non differentiable at five real points, then k will lie in

a) $(-\infty, 0)$ b) $(0, \infty)$ c) $(-\infty, 0) - \{-1\}$ d) $(0, \infty) - \{1\}$

Ans. c

Sol. $f(x) = \left| x^2 + (k-1)|x| - k \right| = \left| (|x|-1)(|x|+k) \right|$

Both roots of (x – 1) (x + k) = 0 should be positive and distinct

$\Rightarrow k \in (-\infty, 0) - \{-1\}$

46. Let $g(x) = \int_a^x f(t)dt$ and $f(x)$ satisfies the following condition

$f(x+y) = f(x) + f(y) + 2xy - 1, \forall x, y \in R$ and $f'(0) = \sqrt{3+a-a^2}$, then the exhaustive set of values of x where $g(x)$ increases is

a) $\left(-\infty, -\frac{3}{2}\right)$ b) $\left(-\frac{3}{2}, 0\right)$ c) $(0, \infty)$ d) $(-\infty, \infty)$

Ans. d

Sol. $f(x) = x^2 + \left(\sqrt{3+a-a^2}\right)x + 1$

$g'(x) = f(x) > 0, \ \forall x \in R$

47. Number of positive continuous function $f(x)$ defined in $[0,1]$ for which

$\int_0^1 f(x)dx = 1, \int_0^1 xf(x)dx = 2, \int_0^1 x^2 f(x)dx = 4,$ is

a) 1 b) 4 c) infinite d) none of these

Ans. d

Sol. Multiplying these three integral by 4, - 4, 1 and adding we get $\int_0^1 f(x)(x-2)^2 dx = 0$.

Hence there does not exist any function satisfying these conditions.

48. Tangents are drawn at the point of intersection P of ellipse $x^2+2y^2=50$ and hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$, in the first quadrant. The area of the circle passing through the point P which cuts the intercept of 2 unit length each from these tangents, is

a) 2π b) $\sqrt{2}\pi$ c) 4π d) 6π

Ans. a

Sol. Given conic are confocal so they cut orthogonally.

49. Let $f(x)=x^3+\frac{1}{x^3},\ x \neq 0$. If the intervals in which f(x) increases are $(-\infty, a]$ and $[b, \infty)$ then min(b – a) is equal to

a) 0 b) 2 c) 3 d) 4

Ans. b

Sol. Here $f'(x)=3x^2-\frac{3}{x^4}\geq 0 \Rightarrow x^6-1\geq 0 \Rightarrow x\in(-\infty.1]\cup[1,\infty)$

$\therefore$ min (b – a) = min(b) – max(a) = 1 – (- 1) = 2

50. Let y = f(x), f : R $\rightarrow$ R be an odd differentiable function such that $f'''(x) > 0$ and g(α, β) = sin⁸α + cos⁸β + 2 – 4 sin²α cos²β . If $f'''(g(\alpha,\beta)) = 0$, then $\sin^2\alpha + \sin^2\beta$ is equal to

a) 0 b) 1 c) 2 d) 3

Ans. b

Sol. $f''(x)$ is odd function $\Rightarrow g(\alpha,\beta)=0$

$\Rightarrow(\sin^4\alpha-1)^2+(\cos^4\beta-1)^2+2(\sin^2\alpha-\cos^2\beta)^2=0$

$\Rightarrow \sin^2\alpha+\sin^2\beta=1$

51. If $f(x)=\int_0^x (f(t))^2\,dt,\ f:R\rightarrow R^+$ be differentiable function and f(g(x)) is differentiable function at x = a, then

a) g(x) must be differentiable at x = a b) g(x) may be non-differentiable at x=a
c) g(x) may be discontinuous at x = a d) none of these

Ans. a

Sol. Here, $f'(x)=(f(x))^2>0$; $\frac{d}{dx}f(g(x))|_{x=a}=f'(g(x))\lim_{x\rightarrow a}\frac{g(x)-g(a)}{x-a}$

As $f'(g(x))\neq 0$

g(x) must be differentiable at x = a.

52. A polynomial of 6th degree f(x) satisfies f(x) = f(2 – x) $\forall\ x\in R$, if f(x) = 0 has 4 distinct and two equal roots, then sum of roots of f(x) = 0 is

a) 4 b) 5 c) 6 d) 7

Ans. c

Sol. Let α be the root of f(x) = 0 $\Rightarrow f(\alpha)=f(2-\alpha)=0$

f(x) has 4 distinct and two equal roots.

$\therefore$ sum of roots = 6

53. The number of integral solutions of equation $4\int_0^{\infty}\frac{\ln t\,dt}{x^2+t^2}-\pi\ln 2=0$; x > 0 is

a) 0 b) 1 c) 2 d) 3

Ans. c

Sol. We have on putting $t=\frac{x^2}{2}$ and solving

$$\int_0^{\infty}\frac{\ln x}{x^2+t^2}dt=\frac{2\pi\ln x}{x}\Rightarrow\frac{\ln x}{x}=\frac{\ln 2}{2}$$

$\Rightarrow$ x = 2 and 4; two solutions.

54. If $f(x)=\begin{cases} e^{x-1}, & 0\le x\le 1 \\ x+1-\{x\}, & 1<x<3\end{cases}$ and g(x) = x^2 – ax + b, such that f(x). g(x) is continuous in [0, 3) then the values of a and b is

a) 2, 3 b) 3, 2 c0 $\frac{3}{2},1$ d) none of these

Ans. b

Sol. Clearly f(x) is discontinuous at x = 1 and 2, for f(x) g(x) to be continuous at x = 1 and 2 ; g(1) and g(2) = 0 $\Rightarrow$ a = 3 and b = 2

55. $\int_0^{16n^2/\pi}\cos\frac{\pi}{2}\left[\frac{x\pi}{n}\right]dx$ is

a) 0 b) 1 c) 2 d) 3

Ans. a

Sol. Let $\frac{x\pi}{n}=t\Rightarrow\int_0^{\frac{16n^2}{\pi}}\cos\frac{\pi}{2}\left[\frac{\pi x}{n}\right]dx=\frac{n}{\pi}\int_0^{16n}\cos\frac{\pi}{2}[t]dt=\frac{4n^2}{\pi}\int_0^{4}\cos\frac{\pi}{2}[t]dt=0$

56. If $\int_2^{-1}(ax^2-5)dx=0$ and $5+\int_1^{2}(bx+c)dx=0$ then

a) ax^2 – bx + c = 0 has atleast one root in (1, 2)
b) ax^2 – bx + c = 0 has atleast one root in (-2, -1)
c) ax^2 + bx + c = 0 has atleast one root in (-2, -1)
d) none of these

Ans. b

Sol. We have $\int_{-2}^{-1}(ax^2-5)dx+\int_1^{2}(bx+c)dx+5=\int_{-2}^{-1}(ax^2-5-bx+c+5)dx=0$

$\Rightarrow ax^2-bx+c=0$ has atleast one root in (-2, -1)

Differential Calculus

Multiple Correct Answer Type

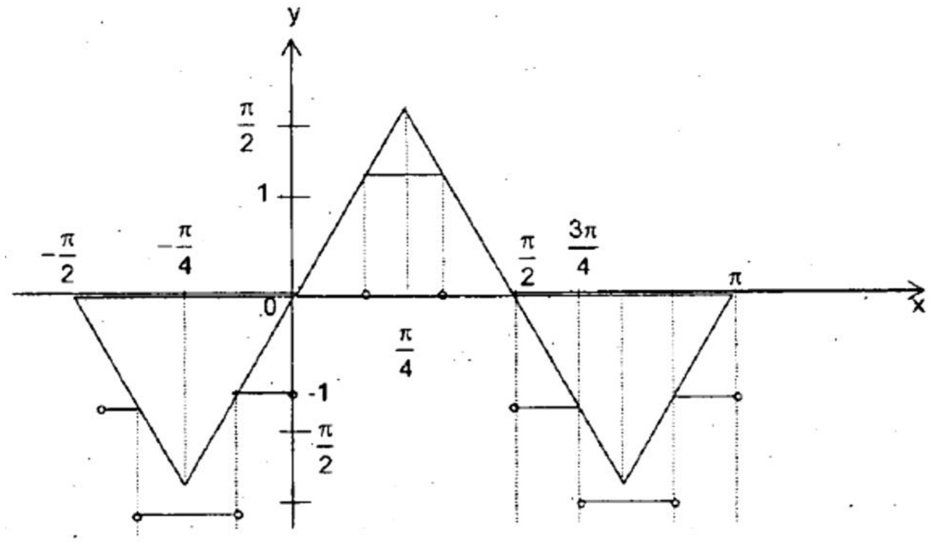
1. If $f(x)=\left[\sin^{-1}(\sin 2x)\right]\left([\cdot]\, denote\ g.i.f\right)$, then

a) $\int_0^{\frac{\pi}{2}} f(x)\,dx=\frac{\pi}{2}-\sin^{-1}(\sin 1)$ b) f(x) is periodic with period π

c) $\lim\limits_{x\to\frac{\pi}{2}} f(x)=-1$ d) none of these

Ans. a,b,c

Sol.



2. The equation $(x-n)^m+\left(x-n^2\right)^m+\left(x-n^3\right)^m+.....+\left(x-n^m\right)^m=0$, where m is odd integer has

a) all real roots b) both complex and real roots

c) one real and (m – 1) complex roots d) (m – 1) real and 01 complex roots

Ans. b, c

Sol. $f'(x)=m\left((x-n)^{m-1}+\left(x-n^2\right)^{m-1}+.....+\left(x-m^m\right)^{m-1}\right)>0$

$\Rightarrow f'(x)\neq 0$

$\Rightarrow f(x)=0$ has exactly one real root.

3. Let $f(x)=\int_0^x |\sin t|\,dt$, then

a) f is continuous everywhere b) $f'(x)$ is differentiable at x = π

c) f(π) = 2 d) $f(x)\geq 0$ for all real x

Ans. a,c,d

Sol. $f'(x)=|\sin x|$. Now draw the graphs

4. Let $f(x)=\begin{cases} x+2, & 0\leq x<2 \\ 6-x, & x\geq 2\end{cases}$, $g(x)=\begin{cases} 1+\tan x, & 0\leq x<\frac{\pi}{4} \\ 3-\cot x, & \frac{\pi}{4}\leq x<\pi\end{cases}$

a) fg(x) is continuous in $[0,\pi)$ b) fg(x) is not continuous in $[0,\pi)$

c) fg(x) is differentiable in $[0,\pi)$ d) fg(x) is not differentiable in $[0,\pi)$

Ans. a,d

Sol. $fg(x)=\begin{cases}3+\tan x, & 0\le x<\dfrac{\pi}{4}\\ 3+\cot x, & \dfrac{\pi}{4}\le x<\pi\end{cases}$

fg(x) is continuous in [0, π)

but fg(x) is not differentiable at $x=\dfrac{\pi}{4}$

5. If $f(x)=\int_{x^m}^{x^n}\dfrac{dt}{\ln t}$, $x>0$ and n > m, then

a) $f'(x)=\dfrac{x^{m-1}(x-1)}{\ln x}$ b) f(x) is decreasing for x > 1

c) f(x) is increasing in (0, 1) d) f(x) is increasing for x > 1

Ans. c,d

Sol. $f'(x)=\dfrac{1\cdot x^{n-1}}{(\ln x^n)}-\dfrac{1\cdot mx^{m-1}}{(\ln x^m)}$, clearly c and d are the answers.

6. The triangle formed by the normal to the curve $f(x)=x^2-ax+2a$ at the point (2, 4) and the co-ordinate axes lies in second quadrant if its area is 2 sq. units then a can be

a) 2 b) $\dfrac{17}{4}$ c) 5 d) $\dfrac{19}{4}$

Ans. b, c

Sol. $f'(x)=2x-a$

At (2, 4)

$f'(x)=4-a$

Equation of normal at (2, 4) is

$(y-4)=\dfrac{1}{(4-a)}(x-2)$

Let point of intersection with x and y – axis be A and B respectively then

A ≡ (- 4a + 18, 0), B ≡ $\left(0, \dfrac{4a-18}{a-4}\right)$

Hence $a>\dfrac{9}{2}$ as

$\therefore$ area of triangle = $\dfrac{1}{2}(4a-18)\dfrac{(4a-18)}{(a-4)}=2$

$\Rightarrow(4a-17)(a-5)=0$

$\Rightarrow a=5 \text{ or } \dfrac{17}{4}$

7. Let $I_n = \int_0^{\frac{\pi}{2}} \cos^n x \, dx, \, n \in N$, then

a) $I_{n-2} > I_n$ b) $n\left(I_{n-2} - I_n\right) = I_{n-2}$ c) $I_n - I_{n-1} = \frac{n}{n+1}$ d) none of these

Ans. a,b

Sol. $I_n = \frac{n-1}{n} I_{n-2}$ and $I_n, \, I_{n-1} > 0$ as $\cos^n x \geq 0$ in $\left[0, \frac{\pi}{2}\right]$

$\therefore I_{n-1} > I_n$

Also $I_n = \left(\frac{n-1}{n}\right) I_{n-2}$

8. If f(x) be such that $f\left(x\right) = \max\left\{\left|2-x\right|, 2-x^3\right\}$ then

a) f(x) is continuous $\forall \; x \in R$

b) f(x) is differentiable $\forall \; x \in R$

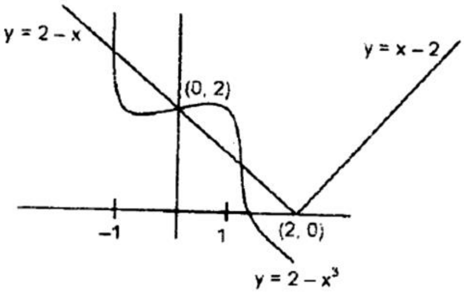
c) f(x) is non-differentiable at one point only

d) f(x) is non-differentiable at 4 points only

Ans. a,d

Sol. Clearly from the graph, f(x) is continuous $\forall \; x \in R$

but not differentiable at – 1, 0, 1, 2, (4 points).



9. Let $I_n = \int_0^{\pi} \frac{\sin^2 n\theta}{\sin^2 \theta} d\theta, \, n \in N$, then

a) $I_4 = 4\pi$ b) $I_5 = 5\pi$ c) $I_4 = 5\pi$ d) $I_5 = 4\pi$

Ans. a, b

Sol. $I_{n+1} - I_n = \int_0^{\pi} \frac{\sin\left(2n+1\right)\theta}{\sin\theta} \, d\theta = \pi \; \forall n \in N$

I_1, I_2, I_3 ….. form an AP with common difference π

$I_n = \pi + \left(n-1\right)\pi \Rightarrow I_n = n\pi$

10. Let $f\left(x\right) = ab\sin x + b\sqrt{1-a^2}\cos x + c$, where $\left|a\right| < 1, \, b > 0$, then

a) maximum value of f(x) is b if c = 0

b) difference of maximum and minimum value of f(x) is 2b

c) f(x) = c if x = $-\cos^{-1}a$ d) f(x) = c if x = $\cos^{-1} a$

Ans. a,b,c

Sol. $f\left(x\right) = \sqrt{a^2b^2 + b^2 - b^2a^2}\,\sin\left(x+\alpha\right) + c = b\sin\left(x+\alpha\right) + c$ where $\tan\alpha = \frac{\sqrt{1-a^2}}{a}$

$\left(f\left(x\right)\right)_{\max} - \left(f\left(x\right)\right)_{\min} = 2b$

Also, at $x = -\cos^{-1} a,\ f(x) = c$

11. $\lim_{x\to 0}\left(\left[n\frac{\sin x}{x}\right]+\left[m\frac{\tan x}{x}\right]\right)$, (where $[\cdot]$ represent greatest integer function) is

a) m + n – 1 if n, m ∈ N
b) m + n – 2 if m ∈ I^-, n ∈ N
c) m + n if m ∈ N, n ∈ I^-
d) m + n – 1 if m, n ∈ I^-

Ans. a,b,c,d

Sol. If $m, n \in N$, and L = n – 1 + m

If $m \in I^-,\ n \in N$, then L = m – 1 + n = m + n – 2

If $m \in N,\ n \in I^-$, then L = n + m

$m,\ n \in I^-$ then L = n + m – 1

12. Let $f : R \to R$, such that $f''(x) - 2f'(x) = 2e^x$ and $f'(x) > 0,\ \forall\ x \in R$, then which of the following can be correct.

a) $\int_2^3 f(x)\,dx = 10$
b) $\int_1^4 f(x)\,dx = -5$
c) $f(4) = 5$
d) $f(-5) = -4$

Ans. a,c

Sol. $\frac{d}{dx}\left(e^{-x}\left(f'(x) - f(x)\right)\right) = 2$

$\Rightarrow e^{-x}\left(f'(x) - f(x)\right) = 2x + c_1$

$\Rightarrow f(x) = \left(x^2 + c_1 x + c_2\right)e^x$ and $f'(x) = \left(x^2 + (c_1 + 2)x + c_1 + c_2\right)e^x$

Given that $f'(x) > 0 \quad \Rightarrow c_1^2 - 4c_2 + 4 < 0$

$\Rightarrow c_1^2 - 4c_2 < 0 \quad \Rightarrow f(x) > 0, \forall x$

Differential Calculus

Assertion Reasoning Type

1. Statement – I: If $2f(x)+f(-x)=\frac{1}{x}\sin\left(x-\frac{1}{x}\right)$ then value of $I=\int_{1/e}^{e} f(x)dx=0$

Statement – II : If $f(2a-x)=-f(x)$, then $\int_{0}^{2a} f(x)dx=0$

a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1

b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1

c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. b

Sol. $2f(x)+f(-x)=2f(-x)+f(x)\Rightarrow f(x)=f(-x)$

$f(x)=\frac{1}{3x}\sin\left(x-\frac{1}{x}\right)$

$I=\int_{1/e}^{e}\frac{1}{3x}\sin\left(x-\frac{1}{x}\right)dx=-\int_{1/e}^{e}\frac{1}{3t}\sin\left(t-\frac{1}{t}\right)dx=-1$

$\Rightarrow I=0$

2. Statement – 1: If y is a function of x such that $y(x-y)^2=x$ then $\int\frac{dx}{x-3y}=\frac{1}{2}\left\{\log(x-y)^2-1\right\}+c$

Statement – 2: $\int\frac{dx}{x-3y}=\log(x-3y)+c$

a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for statement-1

b) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for statement-1

c) Statement-1 is true, Statement-2 is false d) Statement-1 is false, Statement-2 is true

Ans. c

Sol. For statement 1, we will prove that $\frac{d}{dx}(R.H.S)=\frac{1}{x-3y}$

$$R.H.S=\frac{1}{2}\log\left[\frac{x}{y}-1\right]=\frac{1}{2}\left[\log(x-y)-y\right]=\frac{1}{2}\left\{\frac{\log x-\log y}{2}-\log y\right\}=\frac{1}{4}\left\{\log x-3\log y\right\}$$

$$\Rightarrow\frac{d}{dx}(R.H.S)=\frac{1}{4}\left[\frac{1}{x}-\frac{3}{y}\frac{dy}{dx}\right]=\frac{1}{4}\left[\frac{1}{x}-\frac{3}{y}\frac{dy}{dx}\right]=\frac{1}{4}\left[\frac{1}{x}-\frac{3}{y}\left(\frac{-y}{x}\right)\frac{x+y}{x-3y}\right]=\frac{1}{x-3y}$$

$\therefore$ 1 is true

For statement 2 : $\int\frac{dx}{x-3y}=\log(x-3y)+c$, we are assuming that y is constant.

3. Statement – I: The function $f\left(x\right)=\left(3x-1\right)\left|4x^{2}-12x+5\right|\cos\pi x$ is differentiable at $x=\frac{1}{2},\frac{5}{2}$

Statement – II : $\cos\left(2n+1\right)\frac{\pi}{2}=0\ \forall n\in 1$

a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1

b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1

c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. a

Sol. Statement 1 is correct as though $\left|4x^{2}-12x+5\right|$ is non differentiable at $x=\frac{1}{2},\frac{5}{2}$ but

$\cos\pi x=0$ at those points. So $f'\left(\frac{1}{2}\right)$ and $f'\left(\frac{5}{2}\right)$ exists.

4. Statement – I: For the function $f\left(x\right)=\begin{cases}15-x, & x<2\\ 2x-3, & x\geq 2\end{cases}$

x = 2 is neither a maximum, nor a minimum point.

Statement – II : $f'\left(x\right)$ does not exist at x = 2.

a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1

b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1

c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. d

Sol. x = 2 is a point of local minima.

5. Statement – 1: A tangent parallel to x-axis can be drawn for $f\left(x\right)=\left(x-1\right)\left(x-2\right)\left(x-3\right)$ in the interval [1, 3]

Statement – 2: A horizontal tangent cannot be drawn in [1, 3]

a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for statement-1

b) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for statement-1

c) Statement-1 is true, Statement-2 is false d) Statement-1 is false, Statement-2 is true

Ans. c

Sol. Apply Rolle's Theorem.

6. Statement – I: Tangent drawn at (0, 1) to the curve y = x^3 – 3x + 1 meets the curve thrice at one point only.

Statement – II : Tangent drawn at (1, -1) to the curve y = x^3 – 3x + 1 meets the curve thrice at one point only

a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1

b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1

c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. c

Sol. Here $\frac{dy}{dx} = 3x^2 - 3$, statement – 1 $\left.\frac{dy}{dx}\right|_{at\,(0,1)} = -3$

Equation of tangent y = - 3x + 1 meets y = x^3 – 3x + 1 $\Rightarrow$ - 3x + 1 = x^3 – 3x + 1
$\Rightarrow$ x = 0

$\therefore$ tangent meets the curve at one point only $\Rightarrow$ statement -1 is true.

Statement – 2 again $\left.\frac{dy}{dx}\right|_{at\,(1,-1)} = 0$

$\therefore$ equation of tangent is y + 1 = 0(x -1) i.e y = - 1
$\Rightarrow$ - 1 = x^3 – 3x + 1 $\Rightarrow$ (x – 1) (x^2 + x – 1) = 0 $\Rightarrow$ $(x – 1)^2$ (x + 2) = 0
$\Rightarrow$ the tangent meets the curve at two points.

7. Statement – I: The equation $3x^2 + 4ax + b = 0$ has atleast one root in (0, 1) if 3 +4a =0
Statement – II : f(x) = $3x^2 + 4x + b$ is continuous and differentiable in (0, 1)
a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1
b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1
c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. d

Sol. If b < 0, then f(0) = b < 0, f(1) = b < 0
$\therefore$ 0, 1 lie between the roots, statement – 1 is false.

8. Statement – I: If n > 1, then $\int_0^{\infty} \frac{dx}{1+x^n} = \int_0^1 \frac{dx}{\left(1-x^n\right)^{1/n}}$

Statement – II : $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1
b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1
c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. b

Sol. The statement – 1 can be proved by showing that both integrals are equal to a third integral. If we put $x^n = \tan^2\theta$ in the integral on LHS and $x^2 = \sin^2\theta$ in the integral on RHS, then both integrals will be equal to $\frac{2}{n}\int_0^{\pi/2} \tan^{(2/n)-1}\phi\, d\phi$ and $\frac{2}{n}\int_0^{\pi/2} \tan^{(2/n)-1}\theta\, d\theta$ respectively. Since the last two integrals are equal statement – 1 is proved but correct statement – 2 has no role to play here.

9. Statement – I: If $n \in I^+$, then $\int_0^{n\pi} \left|\frac{\sin x}{x}\right| dx > \frac{2}{\pi}\left[1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}\right]$

Statement – II : $\frac{\sin x}{x} > \frac{2}{\pi}$ in $\left(0, \frac{\pi}{2}\right)$

a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1
b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1

c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. b

Sol. $\int_0^{n\pi}\left|\frac{\sin x}{x}\right|dx=\int_0^{\pi}\left|\frac{\sin x}{x}\right|dx+\int_0^{2\pi}\left|\frac{\sin x}{x}\right|dx+.....+\int_{(n-1)\pi}^{n\pi}\left|\frac{\sin x}{x}\right|dx$

$$=\int_0^{\pi}\frac{\sin x}{x}dx+\int_0^{\pi}\left|\frac{\sin(t+\pi)}{t+\pi}\right|dt+\int_0^{\pi}\left|\frac{\sin(u+2\pi)}{u+2\pi}\right|du+.....$$

$$=\sum_{r=1}^{n}\int_0^{\pi}\frac{\sin x}{x+(r-1)\pi}dx>\sum_{r=1}^{n}\int_0^{\pi}\frac{\sin x}{\pi+(r-1)\pi}dx$$

$$=\sum_{r=1}^{n}\int_0^{\pi}\frac{\sin x}{\pi r}dx=\sum_{r=1}^{n}\frac{2}{\pi r}=\frac{2}{\pi}\left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}\right)$$

10. Statement – I: Function f(x) = sin(x + 3sinx) is periodic

Statement – II : f(g(x)) is periodic if (g(x) is periodic

a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1

b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1

c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. b

Sol. Clearly we have f(x + 2π) = f(x) $\Rightarrow$ 2π is period

Statement 2 is obvious

11. Statement – I: Sum of LHD and RHD of f(x) = $|x^2-5x+6|$ at x = 2 is 0

Statement – II : Sum of LHD and RHD of f(x) = $|(x-a)(x-b)|$ at x = a (a < b) is equal to zero

a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1

b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1

c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. a

Sol. Here $f(x)=\begin{cases}x^2-5x+6, & x\le 2\\ -x^2+5x-6, & 2<x\le 3\\ x^2-5x+6, & x\ge 3\end{cases}$

$$f'(x)=\begin{cases}2x-5, & x<2\\ -2x+5, & 2<x<3\\ 2x-5, & x>3\end{cases}$$

$$f'(2^-)+f'(2^+)=-1+1=0$$

Similarly in statement 2, $f'(a^-)+f'(a^+)=0\Rightarrow$ statement 2 explanations statement 1

12. Statement – I: $\lim_{n\to\infty}\frac{x^n}{n!}=0\ \forall\ x>0$

Statement – II : Every sequence whose nth term contains n! in the denominator, converges to zero

a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1

b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1

c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. c

Sol. The statement – 1 is true for any x > 0, we can choose sufficiently larger n such that $\frac{x^n}{n!}$ is small.

Statement – 2 is false, since $\frac{(n!)^2}{n}$ contains n! in the denominator but diverges to ∞

13. Statement – I: Minimum number of points of discontinuity of the function f(x) = (g(x)) [2x – 1] " $x \hat{I}\ (-3,-1)$, where [.] denotes the greatest integer function and
g(x) = $ax^3 + x^2 + 1$ is zero

Statement – II : f(x) can be continuous at a point of discontinuity, say x = c_1 of [2x – 1] if g(c_1) = 0

a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1

b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1

c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. d

Sol. Clearly, [2x – 1] is discontinuous at three points $x = \frac{-5}{2}, \frac{-3}{2}\, and -2$

f(x) may be continuous if g(x) = $ax^3 + x^2 + x + 1 = 0$ at $x = \frac{-5}{2}, \frac{-3}{2}\, or -2$

g(x) can be zero at only one point for a fixed value of a

$\therefore$ minimum number of points of discontinuity = 2

14. Let $(\sin y)^{\sin\left(\frac{\pi x}{2}\right)} + \frac{\sqrt{3}}{2}\sec^{-1}(2x) + 2^x \tan(x+2) = 0$

Statement – 1: $\frac{dy}{dx}$ at x = 1 will not exist.

Statement – 2: $\left(f(x)\right)^{g(x)}$ is discontinuous if f(x) < 0

a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for statement-1

b) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for statement-1

c) Statement-1 is true, Statement-2 is false d) Statement-1 is false, Statement-2 is true

Ans. a

Sol. Since y < 0 for x = - 1, hence $(\sin y)^{\sin\left(\frac{\pi x}{2}\right)}$ does not exist in neighbourhood of x = - 1

15. Statement – I: Let $f : R - \{1,2,3\} \to R$ be a function defined by $f(x) = \frac{1}{x-1} + \frac{2}{x-2} + \frac{3}{x-3}$.

Then f is many-one function.

Statement – II : If either $f'(x) > 0$ or $f'(x) < 0$ $\forall x \in$ domain of f, then $y = f(x)$ is one-one function.

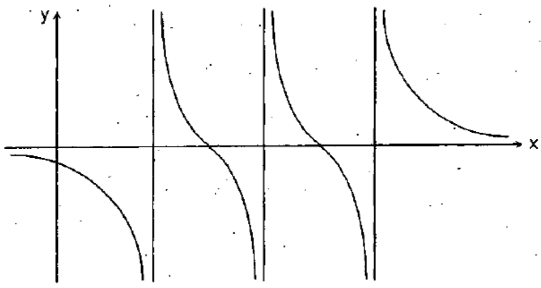
a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1

b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1

c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. c

Sol. From the graph it is clear that f(x) is not one-one statement 1 is true. Also $f'(x) < 0$, $\forall\, x \in$ domain of f but the function is not 1 – 1 so statement – 2 is false.



Differential Calculus

Comprehension Type

Paragraph – 1

Consider the curve given by parametric equation x = t – t^3, y = 1 – t^4, $t \in R$

1. The curve y = f(x) intersect y –axis at

   a) one point only b) 2 point only c) 3 point only d) 4 point only

Ans. b

2. The curve y = f(x) is symmetric about

   a) x –axis b) y-axis c) y = x d) none of these

Ans. b

3. The curve forms a loop of area

   a) 8/35 b) 16/35 c) 31/35 d) none of these

Ans. b

Sol. For x = 0, t = 0, 1, - 1, y = 1, 0

If $\alpha = t - t^3$ replace t with – t, $\beta = 1 - t^4$

$\Rightarrow \alpha_1 = -t + t^3 = -\alpha, \beta = 1 - t^4 = \beta$

$\Rightarrow (\alpha, \beta)$ and $(-\alpha, \beta)$ both lie on curve

Area = $\Rightarrow \left| 2\int_0^1 x dy \right| = \left| 2\int_0^1 (t - t^3)(-4t^3) dt \right| = \frac{16}{35}$

Paragraph – 2

Let n be non-negative integer, $I_n = \int x^n \sqrt{a^2 - x^2} dx,\ a > 0$. Relation between I_{n-2}, I_{n-1}, I_n can be obtain by integrating by parts. Clearly $I_1 = \frac{-1}{3}(a^2 - x^2)^{3/2}$

4. If $I_n = \frac{-x^{n-1}(a^2 - x^2)^{3/2}}{A} + a^2 B I_{n-2}$, where A and B are constants, then A must be equal to

   a) n + 1 b) n – 1 c) n + 2 d) n

Ans. c

5. In the above question, B =

   a) $\frac{n+1}{n+2}$ b) $\frac{n}{n+2}$ c) $\frac{n+2}{n+1}$ d) $\frac{n-1}{n+2}$

Ans. d

6. The value of the integral $\int_0^a x^4 \sqrt{a^2 - x^2} dx$ is equal to

   a) $\frac{\pi a^6}{32}$ b) $\frac{\pi a^4}{16}$ c) $\frac{\pi a^4}{64}$ d) $\frac{\pi a^2}{4}$

Ans. a

Sol. $I_n = -\frac{1}{2}\int x^{n-1}(-2x)\sqrt{a^2 - x^2} dx$

$= -\frac{1}{2}\left[x^{n-1}\frac{2}{3}(a^2 - x^2)^{3/2} - \int (n-1)x^{n-2}\frac{2}{3}(a^2 - x^2)^{3/2} dx \right]$

$$= \frac{-x^{n-1}\left(a^2-x^2\right)^{3/2}}{3} + \frac{n-1}{3}a^2 I_{n-2} - \frac{n-1}{3}I_n$$

$$\therefore I_n = \frac{-x^{n-1}\left(a^2-x^2\right)^{3/2}}{n+2} + \frac{n-1}{n+2}a^2 I_{n-2}$$

$$I_4 = \frac{-x^3\left(a^2-x^2\right)^{3/2}}{6} + \frac{1}{2}a^2 I_2 = \frac{-x^3\left(a^2-x^2\right)^{3/2}}{6} + \frac{a^2}{2}\left[\frac{-x\left(a^2-x^2\right)^{3/2}}{4} + \frac{1}{4}a^2\int\sqrt{a^2-x^2}\,dx\right]$$

$$= \frac{-x^3\left(a^2-x^2\right)^{3/2}}{6} - \frac{a^2x\left(a^2-x^2\right)^{3/2}}{8} + \frac{a^4}{8}\left[\frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$$

$$\therefore \int_0^a x^4\sqrt{a^2-x^2}\,dx = \frac{\pi a^6}{32}$$

Paragraph – 3

An equation of the form $2n\log_a f(x) = \log_a g(x)$, a > 0, $a \neq 1$, $n \in N$ is equivalent to the system $f(x) > 0$ and $\left(f(x)\right)^{2n} = g(x)$

7. Solution set of the equation $\log\left(8-10x-12x^2\right) = 3\log(2x+1)$ is

a) {1} b) {2, 3} c) {5} d) ϕ

Ans. d

Sol. 2x – 1 > 0, 8 – 10x – 12x^2 > 0 and 8 – 10x – 12x^2 = (2x – 1)^3

$\Rightarrow (2x-1)\left(4x^2+2x+9\right) = 0$

No solution

8. Solution set of the equation $\log_{10}(x-9) + 2\log_{10}\sqrt{2x-1} = 2$ is

a) {1} b) {13} c) $\left\{\frac{1}{2}\right\}$ d) ϕ

Ans. b

Sol. x – 9 > 0, 2x – 1 > 0 and (x – 9) (2x – 1) = 100

$\Rightarrow$ x = 13

9. Solution set of $\frac{1}{2}\log_3(x+1) - \log_9(1-x) = \log_9(2x+3)$ is

a) $\left\{\frac{1}{2}\left(\sqrt{5}-1\right)\right\}$ b) $\left\{\frac{\sqrt{5}+1}{2}\right\}$ c) $\left\{\frac{1}{2},\frac{1}{3}\right\}$ d) none of these

Ans. a

Sol. x + 1 > 0, 1 – x > 0, 2x + 3 > 0

$\Rightarrow -\frac{3}{2} < x < 1$

$\therefore$ Equation

$\frac{x+1}{1-x} = 2x+3$

$$\Rightarrow x=\frac{\sqrt{5}-1}{2}$$

Paragraph – 4

If $\dfrac{dy}{dx}=f(x)+\int_0^1 f(x)dx$, then

10. The equation of the curve $y=f(x)$ passing through (0, 1) is

a) $f(x)=\dfrac{2e^x-e+1}{3-e}$ b) $f(x)=\dfrac{3e^x-2e+1}{2(2-e)}$

c) $f(x)=\dfrac{2e^x+e-1}{e+1}$ d) none of these

Ans. a

11. The number of points of discontinuity of y = f(x) in (0, 1) are

a) 4 b) 3 c) 2 d) 0

Ans. d

12. The area bounded by the curve y = f(x), x = 0 and x = 1 is

a) $\dfrac{e-1}{e-3}$ b) $\dfrac{e-1}{3-e}$ c) $-\dfrac{1}{2}$ d) $\dfrac{3(e-1)}{e+1}$

Ans. b

Sol. $f'(x)=f(x)+\int_0^1 f(x)dx \Rightarrow f''(x)=f'(x)\Rightarrow \dfrac{f''(x)}{f'(x)}=1$

$\Rightarrow \log(f'(x))=x+\ln c$

$\Rightarrow \ln\left(\dfrac{f'(x)}{c}\right)=x\Rightarrow f'(x)=ce^x \Rightarrow f(x)=ce^x+D,$ at x = 0, y = 1

So f(x) = ce^x + 1 – c

$\Rightarrow ce^x=e^x+1-c+\int_0^1\left(ce^x+1-c\right)dx$

$\Rightarrow c-1=\left\{ce^x+(1-c)x\right\}_0^1 \Rightarrow c=\dfrac{2}{3-e}$

$\therefore f(x)=\dfrac{2e^x-e+1}{3-e},\ \int_0^1\dfrac{2e^4-e+1}{3-e}dx=\dfrac{e-1}{3-e}$

f(x) is continuous everywhere.

Paragraph – 5

f(x), g(x), h(x) all are continuous and differentiable functions in [a, b] also a < c < b and f(a) = g(a) = h(a). Point of intersection of the tangent at x = c with chord joining x = a and x = b is on the left of c in y = f(x) and on the right in y = h(x). And tangent at x = c is parallel to the chord in case y = g(x). Now answer the following questions.

13. If $f'(x)>g'(x)>h'(x)$ then

a) f(b) < g(b) < h(b) b) f(b) > g(b) > h(b)

c) f(b) ≤ g(b) ≤ h(b) d) f(b) ≥ g(b) ≥ h(b)

Ans. b

14. If f(b) = g(b) = h(b) then

a) $f'(c)=g'(c)=h'(c)$ b) $f'(c)>g'(c)>h'(c)$

c) $f'(c)<g'(c)<h'(c)$ d) none of these

Ans. c

Sol. According to paragraph $\frac{f(b)-f(a)}{b-a}>f'(c),\frac{g(b)-g(a)}{b-a}=g'(c)$ and

$\frac{h(b)-h(a)}{b-a}<h'(c)$

As $f'(x)>g'(x)>h'(x)\Rightarrow\frac{f(b)-f(a)}{b-a}>\frac{g(b)-g(a)}{b-a}>\frac{h(b)-h(a)}{b-a}$

15. If $c=\frac{a+b}{2}$ for each b, then

a) g(x) = Ax^2 + Bx + C b) g(x) = log x c) g(x) = sin x d) g(x) = e^x

Ans. a

Sol. If $g(x)=Ax^2+Bx+C$

$$\Rightarrow\frac{g(b)-g(a)}{b-a}=\frac{A(b^2-a^2)+B(b-a)}{b-a}\Rightarrow 2A\frac{(b+a)}{2}+B=g'\left(\frac{b+a}{2}\right)$$

Paragraph – 6

If $f:R\rightarrow R$ and f(x) = g(x) + h(x) where g(x) is a polynomial and h(x) is a continuous and differentiable bounded function on both sides, then f(x) is onto if g(x) is of odd degree and f(x) is into if g(x) is of even degree. To check whether f(x) is one-one we need to differentiate f(x). If $f'(x)$ changes sign in domain of f then f is many one else one-one.

16. $f:R\rightarrow R$ and f(x) = $a_1x + a_3x^3 + a_5x^5 + ... + a_{2n+1}x^{2n+1} - \cot^{-1}x$ where $0 < a_1 < a_3 < .... < a_{2n+1}$, then the function f(x) is

a) one-one into b) many-one onto c) one-one onto d) many-one into

Ans. c

Sol. f(x) = odd degree polynomial + bounded function $\cot^{-1}x\in(0,\pi)$, also $f'(x)>0$

17. $f:R\rightarrow R$ and $f(x)=\frac{x(x^4+1)(x+1)+x^4+2}{x^2+x+1}$, then f(x) is

a) one-one into b) many-one onto c) one-one onto d) many-one into

Ans. d

Sol. $f(x)=x^4+1+\frac{1}{x^2+x+1}$ = even degree polynomial + bounded function $\frac{1}{x^2+x+1}\in\left(0,\frac{4}{3}\right)$

, $f'(x)=\frac{4x^3(x^2+x+1)^2-2x-1}{(x^2+x+1)^2}$

$\Rightarrow f'(x)=0$ has atleast one root which is repeated odd number of times or it has one root which is not repeated since numerator of $f'(x)$ is a polynomial of degree 7.

$\Rightarrow$ f(x) = 0 has a point of extrema.

18. $f: R \to R$ and f(x) = 2ax + sin2x, then the set of values of a for which $f(x)$ is one-one onto is

a) $a \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ b) $a \in (-1,1)$ c) $a \in R - \left(-\frac{1}{2}, \frac{1}{2}\right)$ d) $a \in R-(-1,1)$

Ans. d

Sol. f(x) = odd degree polynomial + bounded function $\sin 2x \Rightarrow f(x)$ is onto

f(x) is one-one if $f'(x) \geq 0 \ or \ f'(x) \leq 0 \ \forall x$

$\Rightarrow a \geq 1 \cup a \leq -1 \Rightarrow a \in R-(-1,1)$

Paragraph – 7

We are given the curves $y = \int_{-\infty}^{x} f(t)dt$ through the point $\left(0, \frac{1}{2}\right)$ and y = f(x), where f(x) >0 and f(x) is differentiable $\forall \ x \in R$ through (0, 1). If tangents drawn to both the curves at the points with equal abscissae intersect on the same point on the x-axis, then

19. Number of solutions f(x) = 2ex is equal to

a) 0 b) 1 c) 2 d) none of these

Ans. b

20. $\lim_{x \to \infty} (f(x))^{f(-x)}$ is

a) 3 b) 6 c) 1 d) none of these

Ans. c

21. The function f(x) is

a) increasing for all x b) non-monotonic c) decrease for all x d) none of these

Ans. a

Sol. We have the equations of the tangents to the curve $y = \int_{-\infty}^{x} f(t)\,dt$ and y = f(x) at arbitrary points on them are

$Y - \int_{-\infty}^{x} f(t)\,dt = f(x)(X-x)$ - (1)

and $Y - f(x) = f'(x)(X-x)$ - (2)

As (1) and (2), intersect at the same point on x-axis

$\therefore$ Putting Y = 0 and equating X-coordinates we have

$$x - \frac{f(x)}{f'(x)} = x - \frac{\int_{-\infty}^{x} f(t)\,dt}{f(x)}$$

$$\Rightarrow \frac{f(x)}{\int_{-\infty}^{x} f(t)\,dt} = \frac{f'(x)}{f(x)} \Rightarrow \int_{-\infty}^{x} f(t)\,dt = cf(x) \quad - \quad (3)$$

Also f(0) = 1 $\Rightarrow \int_{-\infty}^{0} f(t)\,dt = \frac{1}{2} = c\times 1 \Rightarrow c = \frac{1}{2}$

$\Rightarrow \int_{-\infty}^{x} f(t)\,dt = \frac{1}{2} f(x)$; differentiating both sides and on integrating and using boundary condition.

We get, $f(x) = e^{2x};\ y = 2ex$ is tangent to $y = e^{2x} \Rightarrow$ number of solutions = 1

Clearly f(x) is increasing for all x

$\lim_{x\to\infty}\left(e^{2x}\right)^{e^{-2x}} = 1 \qquad \left(\infty^0 \ \ form\right)$

Paragraph – 8

$f(x) = \int_0^x \left(4t^4 - at^3\right)dt$ and g(x) is quadratic polynomial satisfying g(0) + 6 = $g'(0)$ - c = $g''(0) + 2b = 0$. If y = h(x) and y = g(x) intersect in 4 distinct points with abscissae x_i; i = 1, 2, 3, 4 such that $\sum \frac{i}{x_i} = B$, a, b, c $\in$ R^+, h(x) = $f'(x)$; then

22. Abscissae of point of intersection are in
    a) A.P b) G.P c) H.P d) none of these

Ans. a

23. a is equal to
    a) 6 b) 8 c) 20 d) 12

Ans. c

24. c is equal to
    a) 25 b) $\frac{25}{2}$ c) $\frac{25}{4}$ d) $\frac{25}{8}$

Ans. a

Sol. We have $g(x) = g(0) + xg'(0) + \frac{x^2}{2} g''(0) = -bx^2 + cx - 6$

$h(x) = g(x) = 4x^4 - ax^3 + bx^2 - cx + 6 = 0$ has 4 distinct real roots. Using Descartes rule of sign

$\Rightarrow$ given biquadratic equation has 4 distinct positive roots.

Let x_1, x_2, x_3 and x_4

Now, $\frac{\frac{1}{x_1}+\frac{2}{x_2}+\frac{3}{x_3}+\frac{4}{x_4}}{4} \ge \sqrt[4]{\frac{24}{x_1x_2x_3x_4}}$

$\Rightarrow 2 \ge 2 \Rightarrow \frac{1}{x_1} = \frac{2}{x_2} = \frac{3}{x_3} = \frac{4}{x_4} = k$

$\Rightarrow \frac{1}{x_1}\cdot\frac{2}{x_2}\cdot\frac{3}{x_3}\cdot\frac{4}{x_4} = k^4$

$\Rightarrow \frac{24}{3/2} = k^4 \Rightarrow k = 2$

$\Rightarrow$ Roots are $\frac{1}{2}, 1, \frac{3}{2}, 2$

a = 20, c = 25

Paragraph – 9

Graph of a function y = f(x) is symmetric about the line x = 2 and it is twice differentiable $\forall\ x \in R$. Given $f'(1/2) = f'(1) = 0$, then

25. Minimum number of roots of the equation $f''(x) = 0$ in (0, 4) is/are

a) 2 b) 4 c) 6 d) 8

Ans. b

26. The value of $\int_{-\pi}^{\pi} f(2+x)\sin x dx$ is equal to

a) 1 b) f (2) c) 2π d) none of these

Ans. d

27. If (m , M) be the number of points of minima and maxima respectively of y = f(x) in (0, 4), then m x M is equal to

a) 4 b) 5 c) 6 d) 7

Ans. c

Sol. $f(2-x) = f(2+x)$

$\Rightarrow f(x) = f(4-x)$

$\Rightarrow f'(x) + f'(4-x) = 0$

$\Rightarrow f'(1/2) = f'(1) = f'(3) = f'(7/2) = f'(2) = 0$

Paragraph – 10

Let $f(x) = \dfrac{1}{1+x^2}$, let m be the slope, a be the x intercept and b be the y intercept of a tangent to y = f(x)

28. Abscissa of the point of contact of the tangent for which m is greatest, is

a) $\frac{1}{\sqrt{3}}$ b) 1 c) – 1 d) - $\frac{1}{\sqrt{3}}$

Ans. d

29. Value of b for the tangent drawn to the curve y = f(x) whose slope is greatest, is

a) $\frac{9}{8}$ b) $\frac{3}{8}$ c) $\frac{1}{8}$ d) $\frac{5}{8}$

Ans. a

30. Value of a for the tangent drawn to the curve y = f(x) whose slope is greatest, is

a) - $\sqrt{3}$ b) 1 c) – 1 d) $\sqrt{3}$

Ans. a

Sol. Here we have $f'(x) = \dfrac{-2x}{(1+x^2)^2}$ and $f''(x) = \dfrac{6x^2-2}{(1+x^2)^3}$

$\therefore\ f'(x)$ is maximum at $x = -\dfrac{1}{\sqrt{3}}$

If m is greatest then $m = \frac{3\sqrt{3}}{8}$

y coordinate of the point of contact is $\frac{3}{4}$

$\therefore$ equation of the tangent is $y - \frac{3}{4} = \frac{3\sqrt{3}}{8}\left(x + \frac{1}{\sqrt{3}}\right)$

$\therefore a = -\sqrt{3}$ and $b = \frac{9}{8}$

Paragraph – 11

Consider the function $f(x) = \max\left\{x^2, (1-x)^2, 2x(1-x)\right\}; x \in [0,1]$

31. The interval in which f(x) is increasing is

a) $\left(\frac{1}{3}, \frac{2}{3}\right)$ b) $\left(\frac{1}{3}, \frac{1}{2}\right)$ c) $\left(\frac{1}{3}, \frac{1}{2}\right) \cup \left(\frac{1}{2}, \frac{2}{3}\right)$ d) $\left(\frac{1}{3}, \frac{1}{2}\right) \cup \left(\frac{2}{3}, 1\right)$

Ans. d

32. Let RMVT is applicable for f(x) on (a, b) then a + b + c (where c is the point such that $f'(c) = 0$

a) $\frac{2}{3}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{3}{2}$

Ans. d

33. The interval in which f(x) is decreasing is

a) $\left(\frac{1}{3}, \frac{2}{3}\right)$ b) $\left(\frac{1}{3}, \frac{1}{2}\right)$ c) $\left(0, \frac{1}{3}\right) \cup \left(\frac{1}{2}, \frac{2}{3}\right)$ d) $\left(\frac{1}{3}, \frac{1}{2}\right) \cup \left(\frac{2}{3}, 1\right)$

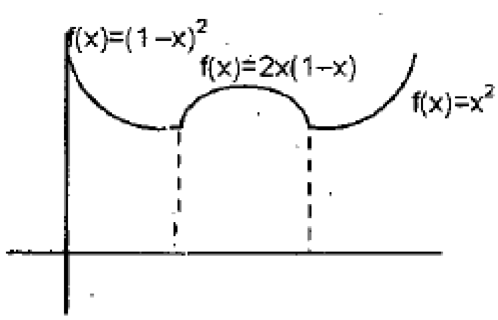
Ans. c

Sol.

We draw the graphs of $f_1(x) = x^2$; $f_2(x) = (1 - x)^2$ and $f_3(x) = 2x(1 - x)$

Here f(x) is redefined as

$$f(x) = \begin{cases} (1-x)^2, & 0 \le x < \frac{1}{3} \\ 2x(1-x), & \frac{1}{3} \le x \le \frac{2}{3} \\ x^2, & \frac{2}{3} < x \le 1 \end{cases}$$



Interval of increase of $f(x)$ is $\left(\frac{1}{3}, \frac{1}{2}\right) \cup \left(\frac{2}{3}, 1\right)$

Interval of decrease of f(x) is $\left(0, \frac{1}{3}\right) \cup \left(\frac{1}{2}, \frac{2}{3}\right)$

Clearly Rolle's theorem is applicable on $\left[\frac{1}{2}, \frac{2}{3}\right]$, where f(x) = 2x(1 – x)

$\Rightarrow f'(c) = 2 - 4c = 0 \Rightarrow c = \frac{1}{2}$

$\Rightarrow a + b + c = \frac{1}{3} + \frac{2}{3} + \frac{1}{2} = \frac{3}{2}$

Paragraph – 12

Let y = f(x) and y = g(x) are two function defined as

$$f(x) = \begin{cases} ax^2 + b, & 0 \le x \le 1 \\ 2bx + 2b, & 1 < x \le 3 \\ (a-1)x + 2a - 3, & 3 < x \le 4 \end{cases} \text{ and } g(x) = \begin{cases} cx^2 + d, & 0 \le x \le 2 \\ dx + 3 - c, & 2 < x \le 3 \\ x^2 + b + 1, & 3 \le x \le 4 \end{cases}$$

34. f(x) is continuous at x = 1 but not differentiable at x = 1, if
a) a = 1, b = 0 b) a = 1, b = 2 c) a = 3, b = 1 d) none of these

Ans. c

Sol. $\lim_{x \to 1} f(x) = a + b$

$\lim_{h \to 1^+} f(x) = 4b$

a + b = 4b $\Rightarrow$ a = 3b

$f'(1^+) \ne f'(1^-)$

$\Rightarrow 2b \ne 2a \Rightarrow a \ne b$

35. g(x) is continuous at x = 2, if
a) c = 1, d = 2 b) c = 2, d = 3 c) c = 1, d = - 1 d) c = 1, d = 4

Ans. a

Sol. $\lim_{x \to 2^-} g(x) = \lim_{x \to 2^+} g(x)$

$\Rightarrow 4c + d = 2d + 3 - c$

$\Rightarrow d = 5c - 3$

36. If f is continuous and differentiable at x = 3, then
a) $a = -\frac{1}{3},\ b = \frac{2}{3}$ b) $a = \frac{2}{3},\ b = -\frac{1}{3}$ c) $a = \frac{1}{3},\ b = -\frac{2}{3}$ d) $a = 2,\ b = \frac{1}{2}$

Ans. d

Sol. $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$

$\Rightarrow 8b = 5a - 6$

$f'(3^-) = 2b$

$f'(3^+) = \lim_{h \to 0^+} \frac{(a-1)(3+h) + 2a - 3 - 8b}{h} = 2b$

$\Rightarrow 5a - 8b - 6 = 0$

$\Rightarrow f'(3^+) = a - 1$

$\Rightarrow a - 1 = 2b \Rightarrow a = 2, b = ½$

Paragraph – 13

Let $y=\int_{u(x)}^{v(x)} f(t)\,dt$, let us define $\frac{dy}{dx}$ as $\frac{dy}{dx}=v'(x)\,f^2(v(x))-u'(x)\,f^2(u(x))$ and the equation of the tangent at (a, b) as $y-b=\left(\frac{dy}{dx}\right)_{(a,b)}(x-a)$

37. If $y=\int_{x}^{x^2} t^2 dt$, then equation of tangent at x = 1 is

a) x + y = 1 b) y = x – 1 c) y = x d) y = x + 1

Ans. b

Sol. At x = 1, y = 0

$\frac{dy}{dx}=2x\cdot\left(x^4\right)^2-\left(x^2\right)^2=1$

Equation of tangent is y = x – 1

38. If $y=\int_{x^2}^{x^4}(\ln t)\,dt$, then $\lim_{x\to 0^+}\frac{dy}{dx}$ is

a) 0 b) 1 c) 2 d) – 1

Ans. a

Sol. $\frac{dy}{dx}=4x^3\left(\ln x^4\right)^2-3x^2\left(\ln x^3\right)^2$

$=64x^3(\ln x)^2-27x^2(\ln x)^2$

$\lim_{x\to 0^+}\frac{dy}{dx}=64\lim_{x\to 0^+}x^3(\ln x)^2-27\lim_{x\to 0^+}x^2(\ln x)^2=0$

39. If $f(x)=\int_{1}^{x} e^{t^2/2}\left(1-t^2\right)dt$, then $\frac{d}{dx}f(x)$ at x = 1 is

a) 0 b) 1 c) 2 d) – 1

Ans. a

Sol. $f(x)=\int_{1}^{x} e^{t^2/2}\left(1-t^2\right)dt$

$f'(x)=\left[e^{x^2/2}\left(1-x^2\right)\right]^2$

$f'(1)=e^{1/2}\cdot 0=0$

Differential Calculus

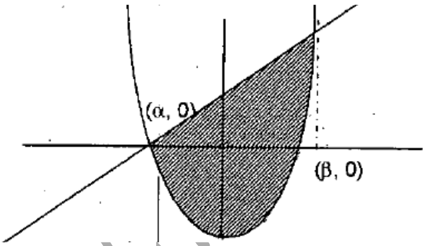
Integer Answer Type

1. If the least value of the area bounded by the line y = mx + 1 and the parabola $y = x^2 + 2x - 3$ is α where m is a parameter then the value of $\frac{6\alpha}{32}$ is

Ans. 2

$A = \int_{\alpha}^{\beta} (y_1 - y_2)\,dx$ where α, β are the roots of $x^2 + 2x - 3 = mx + 1$, on solving we will get $\frac{1}{6}\left(m^2 - 5m + 20\right)^{3/2}$. Hence $\alpha = \frac{32}{3}$

$\Rightarrow \frac{6\alpha}{32} = 2$



2. The value of constant c such that the straight line joining the points (0, 3) and (5, -2) is tangent to the curve $y = \frac{c}{x+1}$

Ans. 4

Equation of line joining (0, 3) and (5, -2) is x + y = 3

Now it touches the curve $y = \frac{c}{x+1}$ at (x_1, y_1)

Hence $\left(\frac{dy}{dx}\right)_{(x_1,\, y_1)} = 1 \Rightarrow (x_1 + 1)^2 = c,\ (x_1 y_1)$ lie on the line. Substituting we get

$\pm\sqrt{c} = 2 \Rightarrow c = 4$

3. Let $f(x) = x^2 + 3x - 3,\ x \ge 0$, if n points $x_1, x_2 \ldots X_n$ are so chosen on the x-axis such that

i) $\frac{1}{n}\sum_{i=1}^{n} f^{-1}(x_i) = f\left(\frac{1}{n}\sum_{i=1}^{n}(x_i)\right)$ ii) $\sum_{i=1}^{n} f^{-1}(x_i) = \sum_{i=1}^{n}(x_i)$

where f^{-1} denote inverse of f. Find A.M. of x_i is

Ans. 1

$f(x) = x$

$x^2 + 3x - 3 = x \Rightarrow x = 1$

4. At how many points in the interval (0, 2), $f(x) = x^2[2x] - x\left[x^2\right]$ is discontinuous (where $[\cdot]$ denotes the greatest integer function)

Ans. 4

Sol. Conceptual

5. $f(x) = \lim_{n\to\infty}\lim_{\alpha\to 1^+} \frac{\alpha^n |\sin x| + |\cos x|\alpha^{-n}}{\alpha^n + \alpha^{-n}}$ then $f\left(\frac{\pi}{2}\right)$ is

Ans. 1

$f(x) = |\sin x|$

$f\left(\frac{\pi}{2}\right) = 1$

6. If $\lim_{x\to 0}\frac{x^n-\sin x^n}{x-\sin^n x}$ exists and has a non-zero value, then n =

Ans. 1

By putting n = 1, the result can easily be obtained.

7. If $\int\frac{1-x^7}{x\left(1+x^7\right)}dx=a\ln|x|+b\ln\left|x^7+1\right|+c$, then $|a+7b|$ =

Ans. 1

Differentiating both sides, we get

$$\frac{1-x^7}{x\left(1+x^7\right)}=\frac{a}{x}+b\cdot\frac{7x^6}{1+x^7}\Rightarrow a=1,\ a+7b=-1$$

8. If $\int_0^{\infty}\left[2e^{-x}\right]dx=\ln k\left(\left[\cdot\right]\ dentoe\ the\ g.i.f\right)$ then k =

Ans. 2

$$\int_0^{\infty}\left[2e^{-x}\right]dx=\int_0^{\ln 2}\left[2e^{-x}\right]dx+\int_{\ln 2}^{\infty}\left[2e^{-x}\right]dx=\int_0^{\ln 2}\left[2e^{-x}\right]dx+0=\ln 2$$

9. The shortest distance between $(1-x)^2+(x-y)^2+(y-z)^2+z^2=\frac{1}{4}$ and 4x + 2y + 4z + 7 = 0 in 3 – dimensional coordinate system is equal to

Ans. 2

Let a = 1 – x
b = x – y
c = y – z
d = z

then a + b + c + d = 1 and $a^2+b^2+c^2+d^2=\frac{1}{4}$

$$\Rightarrow(a-b)^2+(a-c)^2+(a-d)^2+(b-c)^2+(b-d)^2+(c-d)^2=0$$

$$\Rightarrow a=b=c=d$$

$$\therefore x=\frac{3}{4},y=\frac{1}{2},z=\frac{1}{4}$$

So the distance from the point $\left(\frac{3}{4},\frac{1}{2},\frac{1}{4}\right)$ from the plane 4x + 2y + 4z + 7 = 0 is

$$\frac{3+1+1+7}{6}=2$$

Differential Calculus

Matrix-Match Type

1. Match the following:-

Column – I	Column – II
A) The area of the figure bounded by y = x^2 and $y=\sqrt{x}$ is	P) 4/3
B) $\int_0^4 \{\sqrt{x}\} dx$ has the value ({x} denotes fractional part of x}	Q) 5/3
C) The area of the region for which 0 < y < 3 – 2x – x^2 and x > 0 is	R) 7/3
D) $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x}\, dx$ equals	S) 1/3

Ans. A – S ; B – R ; C – Q ; D – P

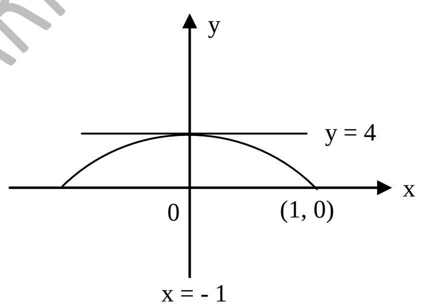
Sol. A) Required area = $\int_0^1 \left(\sqrt{x} - x^2\right) dx = 1/3$

B) $\int_0^4 \left\{\sqrt{x}\right\} dx = \int_0^4 \left(\sqrt{x} - \left[\sqrt{x}\right]\right) dx$

$\int_0^4 \sqrt{x}\; dx = \int_0^1 \left[\sqrt{x}\right] dx - \int_0^4 \left[\sqrt{x}\right] dx = 7/3$

C) Area = $\int_0^1 \left(3 - 2x - x^2\right) dx$

$= \left[3x - x^2 - \dfrac{x^3}{3}\right]_0^1$ = 5/3



D) $2\int_0^{\pi/2} \sqrt{\cos x}\sin x\, dx = 4/3$

2. Match the following

Column – I	Column – II
A) Number of points discontinuity of f(x) = $\tan^2 x - \sec^2 x$ in (0,2π) is	P) 1
B) Number of points at which f(x) = $\sin^{-1}x + \tan^{-1}x + \cot^{-1}x$ is non-differentiable in (-1, 1) is	Q) 2
C) The number of points of discontinuity of y = [sinx]; $x \in [0, 2\pi)$ (where [.] denotes the greatest integer function) is	R) 0
D) Number of points where $y = \left|(x-1)^3\right| + \left|(x-2)^5\right| + \left|x-3\right|$ is non differentiable	S) 3

Ans. A – Q ; B – R ; C – Q ; D – P

Sol. A) $\tan^2 x$ and $\sec^2 x$ are discontinuous at $x = \dfrac{\pi}{2}$ and $\dfrac{3\pi}{2}$

$\therefore$ number of discontinuities = 2

B) Since $f(x)=\sin^{-1}x+\tan^{-1}x+\cot^{-1}x=\sin^{-1}+\frac{\pi}{2}$

$\therefore$ f(x) is differentiable in (-1, 1) $\Rightarrow$ number of points of non-differentiability = 0

C) y = [sin x] is discontinuous at $x=\frac{\pi}{2}$ and π

D) $y=\left|(x-1)\right|^3+\left|(x-2)^5\right|+|x-3|$ is non differentiable at x = 3 only

3. Match the following

Column – I	Column – II
A) The maximum value attained by $y=10-\|x-10\|$, $-1\le x\le 3$ is	P) 3
B) If P(t^2, 2t) ; $t\in[0,2]$ is an arbitrary point on parabola y^2 = 4x, Q is foot of perpendicular from focus S on the tangent at P, then maximum area of ΔPQS is	Q) $\frac{1}{3}$
C) If a + b = 1, a, b > 0 then minimum value of $\sqrt{\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)}$ is	R) 5
D) For real values of x, the greatest and least value of expression $\frac{x+2}{2x^2+3x+6}$ is	S) $-\frac{1}{13}$

Ans. A – P ; B – R ; C – P ; D – Q, S

Sol. A) If y = 10 - $|x-10|$, $x\in[-1,3]$

$-11\le x-10\le -7\Rightarrow 7\le|x-10|\le 11\Rightarrow y\in[-1,3]$

$\therefore y=10-(10-x)=x$

$\therefore$ maximum value of y = 3

B) Equation of tangent at P is ty = x + t^2, it intersects the line x = 0 at Q.

$\therefore$ Coordinates of Q are (0, t)

$$\therefore area\ of\ \Delta PQS=\frac{1}{2}\begin{vmatrix}0 & t & 1\\ 1 & 0 & 1\\ t^2 & 2t & 1\end{vmatrix}=\frac{1}{2}\left(t+t^3\right)$$

$$\Rightarrow\frac{dA}{dt}=\frac{1}{2}\left(3t^2+1\right)>0\ \forall t\in[0,2]$$

$\therefore$ area is maximum for t = 2

$\therefore$ maximum area = 5

C) As a + b = 1 and $\sqrt{\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)}=\sqrt{1+\frac{2}{ab}}$

Again $\sqrt{ab}<\frac{a+b}{2}=\frac{1}{2}\Rightarrow\frac{1}{ab}>4$

$\therefore\ \sqrt{\left(1+\frac{1}{a}\right)\left(1+\frac{1}{b}\right)}\ge\sqrt{1+8}=3$

D) Let $y=\dfrac{x+2}{2x^2+3x+6}\Rightarrow 2yx^2+3xy+6y=x+2$

$\Rightarrow 2yx^2+x(3y-1)+6y-2=0$

$\Rightarrow D\geq 0\Rightarrow (3y-1)^2-8y(6y-2)\geq 0$

$\Rightarrow y\in\left[-\dfrac{1}{13},\dfrac{1}{3}\right]$

4. Match the following:-

Column – I	Column – II
A) If $f(x)=\int_0^{g(x)}\dfrac{dt}{\sqrt{1+t^3}}$, where $g(x)=\int_0^{\cos x}(1+\sin t^2)dt$, then value of $f\left(\dfrac{\pi}{2}\right)$ is	P) – 2
B) If f(x) is a non-zero differentiable function such that $\int_0^x f(t)dt=\{f(x)\}^2\ \forall\ x\in R$ then f(2) is equal to	Q) 2
C) If $\int_a^b(2+x-x^2)dx$ is maximum, then a + b is equal to	R) 1
D) If $\lim\limits_{x\to 0}\left(\dfrac{\sin 2x}{x^3}+a+\dfrac{b}{x^2}\right)=0$, then 3a + b has the value	S) – 1

Ans. A – S ; B – R ; C – R ; D – Q

Sol. We have $f'(x)=\dfrac{g'(x)}{\sqrt{1+g^3(x)}}$ and $g'(x)=\left(1+\sin\left(\cos^2 x\right)\right)(-\sin x)$

Hence $f'(x)=\dfrac{\left(1+\sin\left(\cos^2 x\right)\right)(-\sin x)}{\sqrt{1+g^3(x)}}$, $f'\left(\dfrac{\pi}{2}\right)=\dfrac{1+0}{\sqrt{1+g^3\left(\dfrac{\pi}{2}\right)}}=-1,\ g\left(\dfrac{\pi}{2}\right)=0$

$\therefore\ f'\left(\dfrac{\pi}{2}\right)=-1$

C) Maximum when a = - 1 , b = 2 $\Rightarrow$ a + b = 1

D) If $\lim\limits_{x\to 0}\dfrac{\sin 2x}{x^3}+a+\dfrac{b}{x^2}=0\Rightarrow \lim\limits_{x\to 0}\dfrac{\sin 2x+ax^3+bx}{x^3}=0$

for limit to exist 2 + b = 0 $\Rightarrow$ b = - 2

$\therefore \lim\limits_{x\to 0}\dfrac{\sin 2x+ax^3-2x}{x^3}=0$

Using L.H rule and solving we get $a=\dfrac{4}{3}$

$\therefore$ 3a + b = 2

5. Match the following

Column – I	Column – II
A) The equation x logx = 3 – x has atleast one root in	P) (0, 1)
B) If 27a + 9b + 3c + d = 0, then the equation $4ax^2 + 3bx^2 + 2cx + d = 0$ has atleast one root in	Q) (1, 3)
C) If $c = \sqrt{3}$ and $f(x) = x + \frac{1}{x}$, then interval of x in which LMVT is applicable for f(x) is	R) (0,3)
D) If $c = \frac{1}{2}$ and $f(x) = 2x - x^2$, then interval of x in which LMVT is applicable for f(x) is	S) (-1, 1)

Ans. A – Q ; B – R ; C – Q ; D – P

Sol. A) $f'(x) = \log x - \frac{3}{x} + 1 \Rightarrow f(x) = (x-3)\log x + c$

$\therefore f(1) = f(3)$

B) $f'(x) = 4ax^3 + 3bx^2 + 2cx + d$

$\therefore f(x) = ax^4 + bx^3 + cx^2 + dx + e$

$\therefore f(0) = f(3) \Rightarrow 27a + 9b + 3c + d = 0$

C) $\frac{f(b) - f(a)}{b-a} = f'\left(\sqrt{3}\right) = \frac{2}{3} \Rightarrow \frac{ab-1}{ab} = \frac{2}{3}$

D) $\frac{f(b) - f(a)}{b-a} = f'\left(\frac{1}{2}\right) \Rightarrow a + b = 1$

6. Match the following

Column – I	Column – II
A) $f(x) = \int \frac{x + \sin x}{1 + \cos x} dx$ and f(0) = 0, then $f\left(\frac{\pi}{2}\right)$ is	P) $\frac{\pi}{2}$
B) Let $f(x) = \int e^{\sin^{-1} x}\left(1 - \frac{x}{\sqrt{1 - x^2}}\right)dx$ and f(0) = 1 if $f\left(\frac{1}{2}\right) = \frac{k\sqrt{3}e^{\pi/6}}{\pi}$ then k =	Q) $\frac{\pi}{3}$
C) Let $f(x) = \int \frac{dx}{(x^2+1)(x^2+9)}$ and f(0) = 0 if $f\left(\sqrt{3}\right) = \frac{5}{36}k$, then k is	R) $\frac{\pi}{4}$
D) Let $f(x) = \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ and f(0) = 0 if $f\left(\frac{\pi}{4}\right) = \frac{2k}{\pi}$, then k is	S) π

Ans. A – P ; B – P ; C – R ; D – S

Sol. A) $f(x) = \int \frac{x + \sin x}{1 + \cos x} dx = \int \left(x \times \frac{1}{2}\sec^2 \frac{x}{2} + \tan \frac{x}{2}\right) dx = x \tan \frac{x}{2} + c$

Since f(0) = 0 $\Rightarrow c = 0$ and $f\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$

B) $f(x) = \int e^{\sin^{-1} x}\left(1 - \frac{x}{\sqrt{1-x^2}}\right)dx = \int e^{\sin^{-1} x}\left(\frac{1}{\sqrt{1-x^2}} \cdot \sqrt{1-x^2} - \frac{x}{\sqrt{1-x^2}}\right)dx$

$\Rightarrow f(x) = e^{\sin^{-1} x}\sqrt{1-x^2} + c$

$$\Rightarrow f\left(0\right)=1+c\Rightarrow c=0$$

$$f\left(\frac{1}{2}\right)=e^{\pi/6}\cdot\frac{\sqrt{3}}{2}=\frac{k\sqrt{3}}{\pi}e^{\pi/6}$$

$$\therefore k=\pi/2$$

C) $$f\left(x\right)=\frac{1}{8}\int\left(\frac{1}{x^2+1}-\frac{1}{x^2+9}\right)dx=\frac{1}{8}\left(\tan^{-1}x-\frac{1}{3}\tan^{-1}\frac{x}{3}\right)+c$$

f(0) = 0 = c $\Rightarrow$ c = 0

$$\therefore\frac{1}{8}\left(\frac{\pi}{3}-\frac{1}{3}\frac{\pi}{6}\right)=\frac{5\pi}{144}=\frac{5k}{36}\Rightarrow k=\frac{\pi}{4}$$

D) $$f\left(x\right)=\int\frac{\sqrt{\tan x}}{\sin x\cos x}dx=\int\left(\tan x\right)^{-1/2}\sec^2 xdx=2\sqrt{\tan x}+c$$

f(0) = 0 = c $\Rightarrow$ c = 0

$$\therefore f\left(\frac{\pi}{4}\right)=2=\frac{2k}{\pi}\Rightarrow k=\pi$$

7. Let $f(x)=\begin{cases}[x], & x\in[-2,0)\\ |x|, & x\in[0,2]\end{cases}$; g(x) = secx ; $x\in R-(2n+1)\frac{\pi}{2}$. In the interval $\left(-\frac{3\pi}{2},\frac{3\pi}{2}\right)$ match the following

Column – I	Column – II
A) Limit of fog exist at	P) – 1
B) Limit of gof does not exist at	Q) π
C) Points of discontinuity of fog is/are	R) $\frac{5\pi}{6}$
D) Points of differentiability of fog is/are	S) – π

Ans. A – P, R; B – P ; C – Q,S ; D – P, R

Sol. A) After defining we have $f\left(x\right)=\begin{cases}-2, & x\in\left[-2,-1\right)\\ -1, & x\in\left[-1,0\right)\\ x, & x\in[0,2]\end{cases}$ and

$$g\left(x\right)=\sec x,\ x\in R-\left(2n+1\right)\frac{\pi}{2}$$

Clearly we have $fog=\begin{cases}-2, & x\in\left[-\frac{4}{3},-\frac{2\pi}{3}\right]\cup\left[\frac{2\pi}{3},\frac{4\pi}{3}\right]-\left\{-\pi,\pi\right\}\\ -1, & x=-\pi,\pi\\ \sec x, & x\in\left[-\frac{\pi}{3},\frac{\pi}{3}\right]\end{cases}$

Limit of fog exist at x = - 1

Points of discontinuity of fog are – π, π

Points of differentiability of fog are – 1, $\frac{5\pi}{6}$

$$gof = \begin{cases} \sec(-2), & x \in [-2,-1) - \left\{-\frac{\pi}{2}\right\} \\ \sec(-1), & x \in [-1,0) \\ \sec x, & x \in [0,2] - \left\{\frac{\pi}{2}\right\} \end{cases}$$

Limit of gof does not exist at x = - 1

8. Consider an expression $f(x) = x^n + x^{n+1}$, $n \in N$ f(x) is differentiated successively an aribitrary number of times then multiplied by (x + 1) and again differentiated successively till it attains the form of Ax + B. It is found that A – B is always divisible by a proper integer λ which depends on n. Now in column I different values of n are given and in column II different values of λ are given, Match the corresponding values of n and λ.

Column – I	Column – II
A) 5	P) 15
B) 7	Q) 81
C) 9	R) 49
D) 13	S) 91

Ans. A – P ; B – P, R ; C – P, Q ; D – P,Q, S

Sol. $f(x) = x^n + x^{n+1}$

$$f^k(x) = \frac{n!}{(n-k)!}x^{n-k} + \frac{(n+1)!}{(n+1-k)!}x^{n+1-k}$$

$$(1+x)f^k(x) = \left(\frac{n!}{(n-k)!}x^{n-k} + \frac{(n+1)!}{(n+1-k)!}x^{n+1-k}\right)(1+x) = g(x) \quad \text{(say)}$$

$$g^{(n+1-k)}(x) = \left(\frac{(n+1)!}{(n+1-k)!}(n+2-k)!\right)x + \left(\frac{n!}{(n-k)!} + \frac{(n+1)!}{(n+1-k)!}\right)(n+1-k)!$$

$$\therefore A - B = \frac{(n+1)!}{(n+1-k)!}(n+2-k)! - \left(\frac{n!}{(n-k)!} + \frac{(n+1)!}{(n+1-k)!}\right)(n+1-k)!$$

$$= (n!)n(n+1-k)$$

9. Match the following

Column – I	Column – II
A) $g(x) = 2 - x^{1/3}$ and $f(g(x)) = -x + 5x^{1/3} - x^{2/3}$, the local maximum value of f(x) is	P) 0
B) No. of points of intersection of the curves $\arg\left(\frac{z-3}{z-1}\right)=\frac{\pi}{4}$ and $z(1-i)+\bar{z}(1+i)-4=0$	Q) 1
C) If $f(x) = ax^3 + bx^2 + cx + d$, (a, b, c, d $\in$ Q) and two roots of f(x)=0 are eccentricities of a parabola and a rectangular hyperbola, then a + b + c + d =	R) 2
D) Number of solution of equation $1^x + 2^x + 3^x \ldots + n^x = (n+1)^x$ are	S) 3

Ans. A – S ; B – Q ; C – P ; D – Q

Sol.

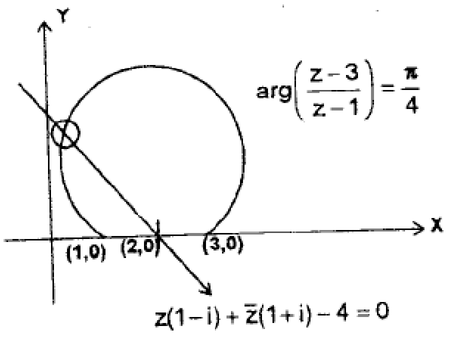
$$x^{1/3} = 2 - g(x)$$

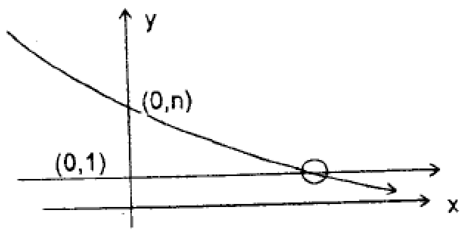
$$\Rightarrow f(x) = 5(2-x) - (2-x)^2 - (2-x)^3$$

$$\Rightarrow x^3 - 7x^2 + 11x - 2 \Rightarrow f_{\max} = 3$$

Clearly 1 is the root $\Rightarrow$ a + b + c + = 0

$$\left(\frac{1}{n+1}\right)^x + \left(\frac{2}{n+1}\right)^x + \ldots\left(\frac{n}{n+1}\right)^x = 1$$





10. If $I_a = \int_0^{\pi/2} \frac{dx}{2\cos x + \sin x + a}$, then the value of I_a for

Column – I	Column – II
A) a = 1	P) log 3
B) a = 3	Q) $\frac{1}{2}\log 3$
C) a = 2	R) $\frac{1}{\sqrt{11}}\left(\tan^{-1}\frac{3}{\sqrt{11}} - \tan^{-1}\frac{1}{\sqrt{11}}\right)$
D) a = 4	S) $\tan^{-1}\left(\frac{1}{3}\right)$

Ans. A – Q ; B – S ; C – P ; D – R

Sol. If $t = \tan\frac{x}{2}$, then $dx = \frac{2dt}{1+t^2}$

A) $a=1,\ I_1=\int_0^1 \frac{dt}{2(1-t^2)+2t-(1+t^2)}$

$I_1=-2\int_0^1 \frac{dt}{t^2-2t-3}=2\int_0^1 \frac{dt}{4-(t-1)^2}=\frac{1}{2}\log 3$

Similarly for others

11. Match the following

Column – I	Column – II
A) $\lim_{x\to 0} \frac{\cos(\tan x)-\cos x}{x^4}$ =	P) 1
B) $\lim_{x\to 0} \frac{1-\cos(1-\cos x)}{x^4}$ =	Q) $\frac{1}{8}$
C) $\lim_{x\to \infty} \frac{\sin^4 x-\sin^2 x+1}{\cos^4 x-\cos^2 x+1}$ =	R) 0
D) Tangents PA and PB are drawn to y = x^2 – x + 1 from $P\left(\frac{1}{2},h\right)$. If area of ΔPAB is maximum, then h =	S) $\frac{1}{3}$

Ans. A – S ; B – Q ; C – P ; D – R

Sol. A) $\lim_{x\to 0} \frac{\cos(\tan x)-\cos x}{x^4}=\lim_{x\to 0} \frac{2\sin\left(\frac{x+\tan x}{2}\right)\sin\left(\frac{x-\tan x}{2}\right)}{x^4}$

$=\frac{1}{2}\lim_{x\to 0} \frac{x^2-\tan^2 x}{x^4}=\lim_{x\to 0} \frac{x^2-\left(x+\frac{x^3}{3}+\frac{2}{15}x^5+....\right)^2}{x^4}=\frac{1}{3}$

B) $\lim_{x\to 0} \frac{1-\cos(1-\cos x)}{x^4}=\lim_{x\to 0} \frac{2\sin^2\left(\sin^2\frac{x}{2}\right)}{x^4}=\frac{1}{8}$

C) $\sin^4 x-\sin^2 x+1=(1-\cos^2 x)^2-(1-\cos^2 x)+1=1+\cos^4 x-\cos^2 x$

D) Vertex $\left(\frac{1}{2},\frac{3}{4}\right)$, equation of the line AB is $\frac{1}{2}(y+h)=x\cdot\frac{1}{2}\cdot\frac{1}{2}\left(x+\frac{1}{2}\right)+1$

$\Rightarrow y=\frac{3}{2}-h$

$\therefore \left(x-\frac{1}{2}\right)^2=\left(\frac{3}{2}-h\right)$

$\Rightarrow x=\frac{1}{2}\pm\sqrt{\frac{3}{4}-h}$

$\Delta APB=\sqrt{\frac{3}{4}-h}\left(\frac{3}{4}-2h\right)=2\left(\frac{3}{4}-h\right)^{3/2}$

So h = 0

12. Match the following

Column – I	Column – II
A) If $f(x) = x^{101} - 2x^{11} + 2x + 1$ and g be inverse of then $g'(1)$ is equal to	P) 0
B) $\lim_{x \to 0} \lvert x(x-1) \rvert^{[\cos 2x]}$ (where $[\cdot]$ denotes the greatest integer function)	Q) 2
C) If the equation $x^4 - 4x^3 + ax^2 + bx + 1 = 0$ has four positive integer roots then a + b	R) $\frac{1}{2}$
D) $f : R \to R$ is defined by $f(x) = x^3 + ax^2 + bx + ce^x$ $(c > 0)$ a, b, c are variable real number be an increasing function, then minimum value of b + c	S) 1

Ans. A – R ; B – S ; C – Q ; D – P

Sol. A) $g\, f(x) = x$

$$g'(f(x)) = \frac{1}{f'(x)}$$

for $g'(1) \Rightarrow f(x) = 1$ at point (0, 1)

$$g'(f(x)) = \frac{1}{2}$$

C) $x_1 + x_2 + x_3 + x_4 = 4$

$x_1x_2x_3x_4 = 1$

$\Rightarrow$ A.M of roots = G.M of roots

$\therefore$ $x_1 = x_2 = x_3 = x_4 = 1$

D) $f'(x) = 3x^2 + 2ax + b + ce^x \geq 0$

$$= 3\left(x^2 + \frac{2a}{3}x + \frac{b}{3} + \frac{a^2}{9} - \frac{a^2}{9}\right) + ce^x$$

$$= 3\left(\left(x + \frac{a}{3}\right)^2 + \left(\frac{3b - a^2}{9} + \frac{ce^x}{3}\right)\right) \geq 0$$

$$\Rightarrow b + ce^x \geq \frac{a^2}{3}$$

at x = 0

$$b + c \geq \frac{a^2}{3}$$

$$b + c \geq 0$$