3D-Geometry

Single Correct Answer Type

1. In a three dimensional co - ordinate system P, Q and R are images of a point A(a, b, c) in the x - y the y - z and the z - x planes respectively. If G is the centroid of triangle PQR then area of triangle AOG is (O is the origin)

a) 0 b) $a^2 + b^2 + c^2$ c) $\frac{2}{3}\left(a^2 + b^2 + c^2\right)$ d) none of these

Key. A

Sol. Point A is (a, b, c)

$\Rightarrow$Points P, Q, R are (a, b, –c), (-a, b, c) and (a, –b, c) respectively.

$\Rightarrow$centroid of triangle PQR is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

$\Rightarrow G \equiv \left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

$\Rightarrow$A, O, G are collinear $\Rightarrow$ area of triangle AOG is zero.

2. The four lines drawing from the vertices of any tetrahedron to the centroid of the opposite faces meet in a point whose distance from each vertex is 'k' times the distance from each vertex to the opposite face, where k is

a) $\frac{1}{3}$ b) $\frac{1}{2}$ c) $\frac{3}{4}$ d) $\frac{5}{4}$

Key. C

Sol. Let A $\left(x_1, y_1, z_1\right)$ B $\left(x_2, y_2, z_2\right)C\left(x_3, y_3, z_3\right)$ D$\left(x_4, y_4, z_4\right)$ be the vertices of tetrahydron. If E is the centroid of face BCD and G is the centroid of A B C D the AG=$3/4\left(AE\right) \therefore K = 3/4$

3. The coordinates of the circumcentre of the triangle formed by the points (3, 2, -5), (-3, 8, -5) (-3, 2, 1) are

a) (-1, 4, -3) b) (1, 4, -3) c) (-1, 4, 3) d) (-1, -4, -3)

Key. A

Sol. Triangle formed is an equilateral $\Rightarrow$ Circum centre = centroid = (-1, 4, -3)

4. The volume of a right triangular prism $ABCA_1B_1C_1$ is equal to 3. Than the co-ordinates of the vertex A_1, if the co-ordinates of the base vertices of the prism are A(1, 0, 1), B(2, 0, 0) and C(0, 1, 0)

a) (–2, 2, 2) or (0, –2, 1) b) (2, 2, 2) or (0, –2, 0)

c) (0, 2, 0) or (1, –2, 0) d) (3, –2, 0) or (1, –2, 0)

Key. B

Sol. Volume = Area of base $\times$ height

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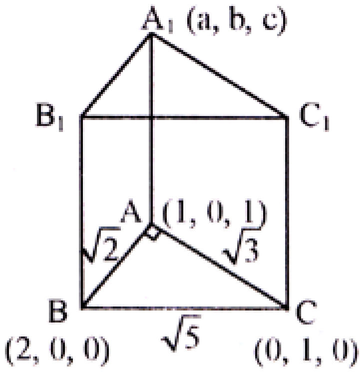

$3 = \frac{1}{2} \times \sqrt{2} \times \sqrt{3} \times h$

h = $\sqrt{6}$

$(A_1A)^2 = h^2 = 6$

$\overrightarrow{A_1A}.\overrightarrow{AB} = 0$

$\overrightarrow{A_1A}.\overrightarrow{AC} = 0$

$\overrightarrow{AA_1}.\overrightarrow{BC} = 0$

solving we get position vector of A_1 are (0, –2, 0) or (2, 2, 2)

5. If $\vec{a},\vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a}+\vec{b}+\vec{c}$ is also a unit vector and θ_1,θ_2 *and* θ_3 are angles between the vectors $\vec{a},\vec{b};\vec{b},\vec{c}$ *and* $\vec{c},\vec{a}$, respectively, then among θ_1, θ_2 *and* θ_3.

a) all are acute angles b) all are right angles

c) at least one is obtuse angle d) None of these

Key. C

Sol. Since $\left|\vec{a}+\vec{b}+\vec{c}\right| = 1 \Rightarrow \left(\vec{a}+\vec{b}+\vec{c}\right).\left(\vec{a}+\vec{b}+\vec{c}\right) = 1 \Rightarrow \vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a} = -1$

$\Rightarrow \cos\theta_1 + \cos\theta_2 + \cos\theta_3 = -1$

So, at least one of $\cos\theta_1, \cos\theta_2$ *and* $\cos\theta_3$ must be negative

6. Given that the points $A(3,2,-4), B(5,4,-6)$ *and* $C(9,8,-10)$ are collinear, the ratio in which B divides $\overline{AC}$ is :

1)1 : 2 2)2 : 1 3)3 : 2 4)2 : 3

Key. 1

Sol. $\left(\frac{9m+3n}{m+n}, \frac{8m+2n}{m+n}, \frac{-10m-4n}{m+n}\right) = (5,4,-6)$

$$\frac{m}{n} = \frac{1}{2}$$

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7. If $A(0,1,2), B(2,-1,3)\, and\ C(1,-3,1)$ are the vertices of a triangle, then its circumcentre and orthocenter are situated at a distance of

1)3 units 2)2 units 3)3/2 units 4) $3/\sqrt{2}$ units

Key. 4

Sol. ortho center- $(2,-1,3)$

Circum center- $\left(\frac{1}{2},-1,\frac{3}{2}\right)$

8. Equation of the plane passing through the origin and perpendicular to the planes $x+2y+z=1, 3x-4y+z=5$ is

1) $x+2y-5z=0$ 2) $x-2y-3z=0$ 3) $x-2y+5z=0$ 4) $3x+y-5z=0$

Key. 4

Sol. $\begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 3 & -4 & 1 \end{vmatrix}=0$

$A=3i+j-5k$

$\Rightarrow 3x+y-5z=0$

9. If θ is the angle between $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda}z+4=0$ and is such that $\sin\theta=1/3$, the value of λ =

1) $-\frac{4}{3}$ 2) $\frac{4}{3}$ 3) $-\frac{3}{5}$ 4) $\frac{5}{3}$

Key. 4

Sol. $Sin\theta=\left|\frac{2-2+2\sqrt{\lambda}}{3\sqrt{5+\lambda}}\right|=\frac{1}{3}$

$\lambda=\frac{5}{3}$

10. The image of the point $(-1,3,4)$ in the plane x-2y=0 is

1) $(15,11,4)$ 2) $\left(-\frac{17}{3},-\frac{19}{3},1\right)$ 3) $\left(\frac{9}{5},-\frac{13}{5},4\right)$ 4) $\left(-\frac{17}{3},-\frac{19}{3},4\right)$

Key. 3

Sol. $\frac{h+1}{1}=\frac{k-3}{-2}=\frac{p-4}{0}=-2\left(\frac{-1-6}{5}\right)$

$(h,k,p)=\left(\frac{9}{5},\frac{-13}{5},4\right)$

11. The plane passing through the points $(-2,-2,2)$ and containing the line joining the points $(1,1,1)$ and $(1,-1,2)$ makes intercepts on the coordinates axes, the sum of whose lengths is

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1. 3 2. 4 3. 6 4. 12

Key. 4

Sol. Equation of the plane be $a(x+2)+b(y+2)+c(z-2)=0.$ As it passes through $(1,1,1)$ and $(1,-1,2), \frac{a}{1}=\frac{b}{-3}=\frac{c}{-6}$. Equation of the plane is $\frac{x}{-8}+\frac{y}{8/3}+\frac{z}{8/6}=1$ and the required sum $=12.$

12. An equation of the plane containing the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and the point $(0,7,-7)$ is

1. $x+y+z=0$
2. $x+2y-3z=35$
3. $3x-2y+3z+35=0$
4. $3x-2y-z=21$

Key. 1

Sol. Equation of the plane is $A(x+1)+B(y-3)+C(z+2)=0$ where $3A+2B+1=0$ and $A+B(7-3)+C(-7+2)=0$

13. The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the coordinate axes at A,B,C respectively. D and E are the mid-points of AB and AC respectively. Coordinates of the mid-point of DE are

1. $(a,b/4,c/4)$ 2. $(a/4,b,c/4)$ 3. $(a/4,b/4,c)$ 4. $(a/2,b/4,c/4)$

Key. 4

Sol. $A(a,0,0), B(0,b,0), C(0,0,c), D(a/2,b/2,0), E(a/2,0,c/2)$ so midpoint of DE is $(a/2,b/4,c/4).$

14. The coordinates of a point on the line $x=4y+5, z=3y-6$ at a distance $3\sqrt{26}$ from the point $(5,0,-6)$ are

1. $(17,3,3)$ 2. $(-7,3,-15)$ 3. $(-17,-3,-3)$ 4. $(7,-3,15)$

Key. 1

Sol. Line is $\frac{x-5}{4/\sqrt{26}}=\frac{y}{1/\sqrt{26}}=\frac{z+6}{3/\sqrt{26}}$. A point on this line at a distance $3\sqrt{26}$ from $(5,0,-6)$ is $(5\pm(3\times4),\pm3,-6\pm9)=(17,3,3)$ or $(-7,-3,-15).$

15. The points $(0,7,10),(-1,6,6)$ and $(-4,9,6)$ are the vertices of

1. A right angled isosceles triangle 2. Equilateral triangle
3. An isosceles triangle 4. An obtuse angled triangle

Key. 1

Sol. Length of the sides are 18, 18 and 36.

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16. Equation of a plane bisecting the angle between the planes $2x-y+2z+3=0$ and $3x-2y+6z+8=0$ is

1. $5x-y-4z-45=0$
2. $5x-y-4z-3=0$
3. $23x+13y+32z-45=0$
4. $23x-13y+32z+5=0$

Key. 2

Sol. Equations of the planes bisecting the angle between the given planes are

$$\frac{2x-y+2z+3}{\sqrt{2^2+(-1)^2+2^2}}=\pm\frac{3x-2y+6z+8}{\sqrt{3^2+(-2)^2+6^2}}$$

$$\Rightarrow\ 7(2x-y+2z+3)=\pm3(3x-2y+6z+8)$$

$\Rightarrow$ $5x-y-4z-3=0$ taking the $+ve\ sign,$ and $23x-13y+32z+45=0$ taking the – ve sign.

17. If the perpendicular distance of a point P other than the origin from the plane $x+y+z=p$ is equal to the distance of the plane from the origin, then the coordinates of P are

1. $(p,2p,0)$
2. $(0,2p,-p)$
3. $(2p,p,-p)$
4. $(2p,-p,2p)$

Key. 3

Sol. The perpendicular distance of the origin $(0,0,0)$ from the plane $x+y+z=p$ is

$$\left|\frac{-p}{\sqrt{1+1+1}}\right|=\frac{|p|}{\sqrt{3}}.$$

If the coordinates of P are (x,y,z), then we must have

$$\left|\frac{x+y+z-p}{\sqrt{3}}\right|=\frac{|p|}{\sqrt{3}}$$

$$\Rightarrow\ |x+y+z-p|=|p|$$

Which is satisfied by (c)

18. If p_1, p_2, p_3 denote the distances of the plane $2x-3y+4z+2=0$ from the planes $2x-3y+4z+6=0, 4x-6y+8z+3=0$ and $2x-3y+4z-6=0$ respectively, then

1. $p_1+8p_2-p_3=0$
2. $p_3^2=16p_2^2$
3. $8p_2^2=p_1^2$
4. $p_1+2p_2+3p_3=\sqrt{29}$

Key. 1 or 4

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Sol. Since the planes are all parallel planes, $p_1 = \frac{|2-6|}{\sqrt{2^2+3^2+4^2}} = \frac{4}{\sqrt{4+9+16}} = \frac{4}{\sqrt{29}}$

Equation of the plane $4x-6y+8z+3=0$ can be written as $2x-3y+4z+3/2=0$

So $p_2 = \frac{|2-3/2|}{\sqrt{2^2+3^2+4^2}} = \frac{1}{2\sqrt{29}}$ and $p_3 = \frac{|2+6|}{\sqrt{2^2+3^2+4^2}} = \frac{8}{\sqrt{29}}$

$\Rightarrow \quad p_1 + 8p_2 - p_3 = 0$

19. The radius of the circle in which the sphere $x^2+y^2+z^2+2x-2y-4z-19=0$ is cut by the plane $x+2y+2z+7=0$ is

1. 2 2. 3 3. 4 4. 1

Key. 2

Sol. Centre of the sphere is $(-1,1,2)$ and its radius is $\sqrt{1+1+4+19}=5.$

Length of the perpendicular from the centre on the plane is $|\frac{-1+2+4+7}{\sqrt{1+4+4}}|=4$

Radius of the required circle is $\sqrt{5^2-4^2}=3.$

20. The shortest distance from the plane $12x+4y+3z=327$ to the sphere $x^2+y^2+z^2+4x-2y-6z=155$ is

1. $11\frac{3}{4}$ 2. 13 3. 39 4. 26

Key. 2

Sol. The centre of the sphere is $(-2,1,3)$ and its radius is $\sqrt{4+1+9+155}=13$

Length of the perpendicular from the centre of the sphere on the plane is

$$\left|\frac{-24+4+9-327}{\sqrt{144+16+9}}\right| = \frac{338}{13} = 26$$

So the plane is outside the sphere and the required distance is equal to $26-13=13.$

21. An equation of the plane passing through the line of intersection of the planes $x+y+z=6$ and $2x+3y+4z+5=0$ and the point $(1,1,1)$ is

1. $2x+3y+4z=9$ 2. $x+y+z=3$ 3. $x+2y+3z=6$ 4. $20x+23y+26z=69$

Key. 4

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Sol. Equation of any plane through the line of intersection of the given planes is
$2x+3y+4z+5+\lambda(x+y+z-6)=0$

It passes through $(1,1,1)$ if $(2+3+4+5)+\lambda(1+1+1-6)=0 \Rightarrow \lambda=14/3$ and the required equation is therefore, $20x+23y+26z=69.$

22. The volume of the tetrahedron included between the plane $3x+4y-5z-60=0$ and the coordinate planes is

1. 60 2. 600 3. 720 4. None of these

Key. 2

Sol. Equation of the given plane can be written as $\frac{x}{20}+\frac{y}{15}+\frac{z}{-12}=1$

Which meets the coordinates axes in points $A(20,0,0), B(0,15,0)$ and $C(0,0,-12)$ and the coordinates of the origin are $(0,0,0).$

$\therefore$ the volume of the tetrahedron $OABC$ is

$$\frac{1}{6}\begin{vmatrix} 0 & 0 & 0 & 1 \\ 20 & 0 & 0 & 1 \\ 0 & 15 & 0 & 1 \\ 0 & 0 & -12 & 1 \end{vmatrix} = \left|\frac{1}{6}\times 20\times 15\times(-12)\right| = 600.$$

23. Two lines $x=ay+b, z=cy+d$ and $x=a^1y+b^1, z=c^1y+d^1$ will be perpendicular, if and only if

1. $aa^1+bb^1+cc^1=0$
2. $(a+a^1)(b+b^1)(c+c^1)=0$
3. $aa^1+cc^1+1=0$
4. $aa^1+bb^1+cc^1+1=0$

Key. 3

Sol. Lines can be written as $\frac{x-b}{a}=\frac{y}{1}=\frac{z-d}{c}$ and $\frac{x-b^1}{a^1}=\frac{y}{1}=\frac{z-d^1}{c^1}$ which will be perpendicular if and only if $aa^1+1+cc^1=0$

24. A tetrahedron has vertices at $O(0,0,0), A(1,2,1), B(2,1,3)$ and $C(-1,1,2).$ Then the angle between the faces OAB and ABC will be

1. $\cos^{-1}(17/31)$ 2. 30^0 3. 90^0 4. $\cos^{-1}(19/35)$

Key. 4

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Sol. Let the equation of the face OAB be $ax+by+cz=0$ where

$a+2b+c=0$ and $2a+b+3c=0 \Rightarrow \frac{a}{5}=\frac{b}{-1}=\frac{c}{-3}$

25. If the angle θ between the lines $\frac{x+1}{1}=\frac{y-1}{2}=\frac{z-2}{2}$ and the plane $2x-y+\sqrt{\lambda}z+4=0$ is such that $\sin\theta=1/3$, then the value of λ is

1. $3/4$ 2. $-4/3$ 3. $5/3$ 4. $-3/5$

Key. 3

Sol. Since the line makes an angle θ with the plane in makes an angle $\pi/2-\theta$ with normal to the plane

$$\therefore \cos(\frac{\pi}{2}-\theta)=\frac{2(1)+(-1)(2)+(\sqrt{\lambda})(2)}{\sqrt{1+4+4}\times\sqrt{4+1+\lambda}}$$

$$\Rightarrow \frac{1}{3}=\frac{2\sqrt{\lambda}}{3\sqrt{\lambda+5}} \Rightarrow \lambda+5=4\lambda$$

$$\Rightarrow \lambda=5/3$$

26. The ratio in which the yz plane divides the segment joining the points $(-2,4,7)$ and $(3,-5,8)$ is

1. $2:3$ 2. $3:2$ 3. $4:5$ 4. $-7:8$

Key. 1

Sol. Let yz plane divide the segment joining $(-2,4,7)$ and $(3,-5,8)$ in the ration $\lambda:1$. Then $\Rightarrow \frac{3\lambda-2}{\lambda+1}=0 \Rightarrow \lambda=\frac{2}{3}$ and the required ratio is $2:3$.

27. The coordinates of the point equidistant from the points $(a,0,0),(0,a,0),(0,0,a)$ and $(0,0,0)$ are

1. $(a/3,a/3,a/3)$ 2. $(a/2,a/2,a/2)$ 3. (a,a,a) 4. $(2a,2a,2a)$

Key. 2

Sol. Let the coordinates of the required point be (x,y,z) then

$x^2+y^2+z^2=(x-a)^2+y^2+z^2=x^2+(y-a)^2+z^2=x^2+y^2+(z-a)^2$

$\Rightarrow x=a/2=y=z.$ Hence the required point is $(a/2,a/2,a/2)$.

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28. Algebraic sum of the intercepts made by the plane $x+3y-4z+6=0$ on the axes is

1. $-13/2$ 2. $19/2$ 3. $-22/3$ 4. $26/3$

Key. 1

Sol. Equation of the plane can be written as $\frac{x}{-6}+\frac{y}{-2}+\frac{z}{3/2}=1$

So the intercepts on the coordinates axes are $-6,-2,3/2$ and the required sum is

$-6-2+3/2=-13/2.$

29. If a plane meets the co-ordinate axes in A,B,C such that the centroid of the triangle ABC is the point $(1,r,r^2)$, then equation of the plane is

1. $x+ry+r^2z=3r^2$ 2. $r^2x+ry+z=3r^2$ 3. $x+ry+r^2z=3$ 4. $r^2x+ry+z=3$

Key. 2

Sol. Let an equation of the required plane be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

This meets the coordinates axes in $A(a,0,0),B(0,b,0)$ and $C(0,0,c)$.

So that the coordinates of the centroid of the triangle ABC are

$(a/3,b/3,c/3)=(1,r,r^2)\,(given)\Rightarrow a=3,b=3r,3r^2$ and the required equation of the plane is

$\frac{x}{3}+\frac{y}{3r}+\frac{z}{3r^2}=1$ or $r^2x+ry+z=3r^2.$

30. An equation of the plane passing through the point $(1,-1,2)$ and parallel to the plane $3x+4y-5z=0$ is

1. $3x+4y-5z+11=0$ 2. $3x+4y-5z=11$ 3. $6x+8y-10z=1$ 4. $3x+4y-5z=2$

Key. 1

Sol. Equation of any plane parallel to the plane $3x+4y-5z=0$ is $3x+4y-5z=K$

If it passes through $(1,-1,2)$, then $3-4-5(2)=K\Rightarrow K=-11$

So the required equation is 3x+4y-5z+11=0.

31. Equations of a line passing through $(2,-1,1)$ and parallel to the line whose equations are $\frac{x-3}{2}=\frac{y+1}{7}=\frac{z-2}{-3}$,is

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1. $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-1}{2}$
2. $\frac{x-2}{2}=\frac{y+1}{7}=\frac{z-1}{-3}$
3. $\frac{x-2}{2}=\frac{y-7}{-1}=\frac{z+3}{1}$
4. $\frac{x-3}{2}=\frac{y+1}{-1}=\frac{z-2}{1}$

Key. 2

Sol. The required line passes through $(2,-1,1)$ and its direction cosines are proportional to $2,7,-3$ so its equation is $\frac{x-2}{2}=\frac{y+1}{7}=\frac{z-1}{-3}$

32. The ratio in which the plane $2x-1=0$ divides the line joining $(-2,4,7)$ and $(3,-5,8)$ is

1. $2:3$ 2. $4:5$ 3. $7:8$ 4. $1:1$

Key. 4

Sol. Let the required ratio be $k:1$, then the coordinates of the point which divides the join of the points $(-2,4,7)$ and $(3,-5,8)$ in this ratio are given by $\left(\frac{3k-2}{k+1},\frac{-5k+4}{k+1},\frac{8k+7}{k+1}\right)$

As this point lies on the plane $2x-1=0$.

$\Rightarrow \frac{3k-2}{k+1}=\frac{1}{2}\Rightarrow k=1$ and thus the required ratio as $1:1$.

33. If l_1, m_1, n_1 and l_2, m_2, n_2, are d.c.'s of $\overrightarrow{OA}$, $\overrightarrow{OB}$ such that $\angle AOB=\theta$ where 'O' is the origin, then the d.c.'s of the internal bisector of the angle $\angle AOB$ are

(A) $\frac{l_1+l_2}{2\sin\theta/2},\frac{m_1+m_2}{2\sin\theta/2},\frac{n_1+n_2}{2\sin\theta/2}$
(B) $\frac{l_1+l_2}{2\cos\theta/2},\frac{m_1+m_2}{2\cos\theta/2},\frac{n_1+n_2}{2\cos\theta/2}$
(C) $\frac{l_1-l_2}{2\sin\theta/2},\frac{m_1-m_2}{2\sin\theta/2},\frac{n_1-n_2}{2\sin\theta/2}$
(D) $\frac{l_1-l_2}{2\cos\theta/2},\frac{m_1-m_2}{2\cos\theta/2},\frac{n_1-n_2}{2\cos\theta/2}$

Key. B

Sol. Let OA and OB be two lines with d.c's l_1, m_1, n_1 and l_2, m_2, n_2. Let OA = OB = 1. Then, the coordinates of A and B are (l_1, m_1, n_1) and (l_2, m_2, n_2), respectively. Let OC be the bisector of $\angle AOB$. Then, C is the mid point of AB and so its coordinates are $\left(\frac{l_1+l_2}{2},\frac{m_1+m_2}{2},\frac{n_1+n_2}{2}\right)$.

$\therefore$ d.r's of OC are $\frac{l_1+l_2}{2},\frac{m_1+m_2}{2},\frac{n_1+n_2}{2}$

We have, $OC=\sqrt{\left(\frac{l_1+l_2}{2}\right)^2+\left(\frac{m_1+m_2}{2}\right)^2+\left(\frac{n_1+n_2}{2}\right)^2}$

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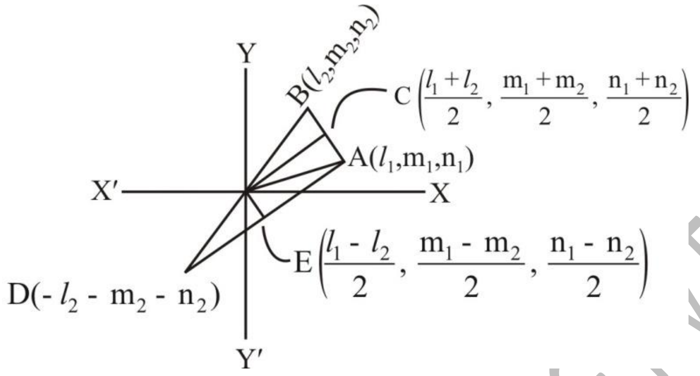
$$= \frac{1}{2}\sqrt{\left(l_1^2 + m_1^2 + n_1^2\right) + \left(l_2^2 + m_2^2 + n_2^2\right) + 2\left(l_1 l_2 + m_1 m_2 + n_1 n_2\right)}$$

$$= \frac{1}{2}\sqrt{2 + 2\cos\theta} \qquad \left[\because \cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2\right]$$

$$= \frac{1}{2}\sqrt{2\left(1 + \cos\theta\right)} = \cos\left(\frac{\theta}{2}\right)$$



$\therefore$ d.c's of OC are $\frac{l_1 + l_2}{2(OC)}, \frac{m_1 + m_2}{2(OC)}, \frac{n_1 + n_2}{2(OC)}$

34. A line is drawn from the point $P(1,1,1)$ and perpendicular to a line with direction ratios $(1,1,1)$ to intersect the plane $x + 2y + 3z = 4$ at Q. The locus of point Q is

A) $\frac{x}{1} = \frac{y-5}{-2} = \frac{z+2}{1}$ B) $\frac{x}{-2} = \frac{y-5}{1} = \frac{z+2}{1}$

C) $x = y = z$ D) $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$

Key. A

Sol. Locus of 'Q' is the line of intersection of the plane $x + 2y + 3z = 4$ and $1(x-1) + 1(y-1) + 1(z-1) = 0 \Rightarrow$ then the line is $\frac{x}{1} = \frac{y-5}{-2} = \frac{z+2}{1}$

35. A line is drawn from the point P(1, 1, 1) and perpendicular to a line with direction ratios $(1,1,1)$ to intersect the plane $x + 2y + 3z = 4$ at Q. The locus of point Q is

A) $\frac{x}{1} = \frac{y-5}{-2} = \frac{z+2}{1}$ B) $\frac{x}{-2} = \frac{y-5}{1} = \frac{z+2}{1}$ C) $x = y = z$ D) $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$

Key: A

Hint: Locus of Q is the line of intersection of the plane $x + 2y + 3z = 4$ and $1(x-1) + 1(y-1) + 1(z-1) = 0 \Rightarrow$ then line is $\frac{x}{1} = \frac{y-5}{-2} = \frac{z+2}{1}$

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36. If a line with direction ratios 2 : 2: 1 intersects the line $\frac{x-7}{3}=\frac{y-5}{2}=\frac{z-3}{1}$ and $\frac{x-1}{2}=\frac{y+1}{4}=\frac{z+1}{3}$ at A and B then AB=.

a) $\sqrt{2}$ b) 2 c) $\sqrt{3}$ d) 3

Key:

Hint $A(7+3\alpha,5+2\alpha,3+\alpha),B(1+2\beta,-1+4\beta,-1+3\beta)$

Dr's of AB are 2:2:1

$$\frac{6+3\alpha-2\beta}{2}=\frac{3+\alpha-2\beta}{1}=\frac{4+\alpha-3\beta}{1}$$

$\alpha=-2,\beta=1$

A(1,1,1)B(3,3,2)

AB = 3

37. A, B, C are the points on x, y and z axes respectively in a three dimensional co-ordinate system with O as origin. Suppose the area of triangles OAB, OBC and OCA are 4, 12 and 6 respectively, then the area of the triangle ABC equals

(A) 16 (B) 14 (C) 28 (D) 32

Key: B

Hint $[ABC]=\sqrt{[OAB]^2+[OBC]^2+[OCA]^2}$

where $[ABC]$ = area of triangle ABC

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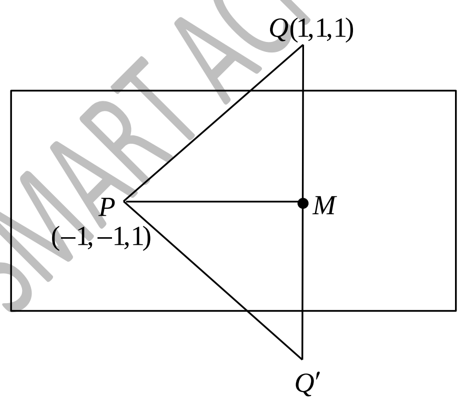
38. The area of the figure formed by the points $(-1,-1,1);(1,1,1)$ and their mirror images on the plane $3x+2y+4z+1=0$ is

(a) $\frac{5\sqrt{33}}{29}$ (b) $\frac{4\sqrt{33}}{29}$ (c) $\frac{20\sqrt{33}}{27}$ (d) $\frac{20\sqrt{33}}{29}$

Key. D



Sol.

Req. area = ΔPQQ^1

$=2\Delta PQM$

$=2\cdot\frac{1}{2}\cdot QM\cdot PM$

39. If a plane passes through the point $(1,1,1)$ and is perpendicular to the line $\frac{x-1}{3}=\frac{y-1}{0}=\frac{z-1}{4}$ then its perpendicular distance from the origin is

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$ (C) $\frac{7}{5}$ (D) 1

Key: C

Hint: The d.r of the normal to the plane is 3, 0, 4 . The equation of the plane is $3x+0y+4z+d=0$ since it passes through $(1,1,1)$ so; $d=-7$

Now distance of the plane $3x+4z-7=0$ from $(0,0,0)$ is $\frac{7}{\sqrt{3^2+4^2}}=\frac{7}{5}$ unit

40. Three straight lines mutually perpendicular to each other meet in a point P and one of them intersects the x-axis and another intersects the y-axis, while the third line passes through a fixed point (0, 0, c) on the z-axis. Then the locus of P is

A) $x^2+y^2+z^2-2cx=0$ B) $x^2+y^2+z^2-2cy=0$

C) $x^2+y^2+z^2-2cz=0$ D) $x^2+y^2+z^2-2c(x+y+z)=0$

Key: C

Hint: Let L_1, L_2, L_3 be the mutually perpendicular lines and $P(x_0, y_0, z_0)$ be their point of concurrence. If L_1 cuts the x-axis at A(a, 0, 0), L_2 meets the y-axis at B(0, b, 0) and C(0, 0, c) $\in L_3$, then $L_1 11(x_0-a, y_0, z_0)$, $L_2 11(x_0, y_0-b, z_0)$ and $L_3 11(x_0, y_0, z_0-c)$. Hence

$$\left.\begin{aligned} x_0(x_0-a)+y_0(y_0-b)+z_0^2=0 \\ x_0^2+(y_0-b)y_0+z_0(z_0-c)=0 \end{aligned}\right\}$$

$$x_0(x_0-a)+y_0^2+z_0(z_0-c)=0$$

Eliminating a and b from the equations, we get

$$x_0^2+y_0^2+z_0^2-2cz_0=0$$

41. The centroid of the triangle formed by (0, 0, 0) and the point of intersection of $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z-1}{1}$ with $x=0$ *and* $y=0$ is

(a) $(1,1,1)$ (b) $\left(\frac{1}{6},\frac{-1}{3},\frac{1}{6}\right)$ (c) $\left(\frac{-1}{6},\frac{1}{3},\frac{-1}{6}\right)$ (d) $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

Key. B

Sol. Any point on the given line $(K+1, 2K+1, K+1)$

but $x=0 \quad \Rightarrow A(0,-1,0)$

$y=0 \quad \Rightarrow B(\frac{1}{2},0,\frac{1}{2}); 0(0,0,0)$

42. The plane $x-y-z=4$ is rotated through 90^o about its line of intersection with the plane $x+y+2z=4$ and equation in new position is $Ax+By+Cz+D=0$ where A,B,C are least positive integers and $D<0$ then

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(a) $D=-10$ (b) $ABC=-20$

(c) $A+B+C+D=0$ (d) $A+B+C=10$

Key. D

Sol. Given planes are $x-y-z=4$ ---------- (1) and $x+y+2z=4$ ---------- (2)

Since required plane passes through the line of intersection (1) & (2)

$\Rightarrow$ Its equation is $(x-y-z-4)+\alpha(x+y+2z-4)=0$

$\Rightarrow(1+\alpha)x+(\alpha-1)y+(2\alpha-1)z-(4\alpha+4)=0$ ---------- (3)

Since (1) & (3) are perpendicular

$\Rightarrow 1(1+\alpha)-1(\alpha-1)-1(2\alpha-1)=0$

$1+\alpha-\alpha+1-2\alpha+1=0 \qquad \Rightarrow \alpha=3/2$

$\Rightarrow$ Its equations is $(x-y-z-4)+\frac{3}{2}(x+y+2z-4)=0$

$5x+y+4z-20=0$

43. Three lines $y-z-1=0=x;\ z+x+1=0=y;\ x-z-1=0=y$ intersect the xy plane at A, B, C then orthocenter of triangle ABC is

(a) $(0,1,0)$ (b) $(-1,0,0)$ (c) $(0,0,0)$ (d) $(1,1,1)$

Key. A

Sol. Intersection of $y-z-1=0=x$ with xy plane gives $A(0,1,0)$ similarly $B(-1,0,0)$, $C(1,0,0)$

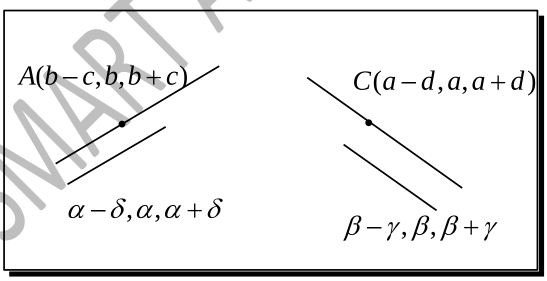
$\therefore$ orthocentre is $(0,1,0)$

44. The lines $\frac{x-a+d}{\alpha-\delta}=\frac{y-a}{\alpha}=\frac{z-a-d}{\alpha+\delta};\frac{x-b+c}{\beta-r}=\frac{y-b}{\beta}=\frac{z-b-c}{\beta+r}$ are coplanar and the equation of the plane in which they lie is

(a) $x+y+z=0$ (b) $x-y+z=0$ (c) $x-2y+z=0$ (d) $x+y-2z=0$

Key. C

Sol.



45. The reflection of the point P(1, 0, 0) in the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ is

(a) (3, –4, –2) (b) (5, –8, –4) (c) (1, –1, –10) (d) (2, –3, 8)

Key: b

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Hint: Coordinates of any point Q on the given line are

(2r + 1, –3r – 1, 8r – 10) for some $r \in R$

So the direction ratios of PQ are 2r, –3r – 1, 8r – 10

Now PQ is perpendicular to the given line

if 2(2r) – 3 (–3r – 1) + 8(8r – 10) = 0

$\Rightarrow$ 77r – 77 = 0 $\Rightarrow$ r = 1

and the coordinates of Q, the foot of the perpendicular from P on the line are (3, –4, –2).

Let R(a, b, c) be the reflection of P in the given lines when Q is the mid-point of PR

$\Rightarrow \frac{a+1}{2} = 3, \frac{b}{2} = -4, \frac{c}{2} = -2$

$\Rightarrow$ a = 5, b = –8, c = –4

and the coordinates of the required point are $(5, -8, -4)$.

46. Reflection of plane 2x + 3y + 4z + 1 = 0 in plane x + 2y + 3z – 2 = 0 is

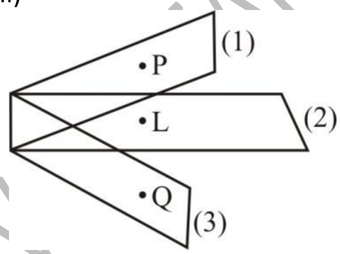
(A) 6x – 19y + 32z = 47 (B) 6x + 19y + 32z = 47

(C) 6x + 19y + 16z = 47 (D) 3x + 19y + 16z = 47

Key. B

Sol. 2x + 3y + 4z + 1 = 0 .....(i)

x + 2y + 3z – 2 = 0 .....(ii)



(iii) is reflection of plane

reflection of ax + by + cz + d = 0 in $a'x + b'y + c'z + d' = 0$

$= (aa' + bb' + cc')(a'x + b'y + c'z + d')$

$= (a'^2 + b'^2 + c'^2)(ax + by + cz + d)$

$2(2+6+12)(x+2y+3z-2) = (1^2 + 2^2 + 3^2)(2x+3y+4z+1)$

$4(x+2y+32-2) = 14(2x+3y+4z+1)$

$12x + 38y + 64z = 94$

$\Rightarrow 6x + 19y + 32z = 47$

47. The reciprocal of the distance between two points, one on each of the lines $\frac{x-2}{3} = \frac{y-4}{2} = \frac{z-5}{5}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

(A) cannot be less than 9 (B) having minimum value $5\sqrt{3}$

(C) cannot be greater than 78 (D) cannot be $2\sqrt{19}$

Key. D

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Sol. The shortest distance (SD) = $\dfrac{\left|\begin{vmatrix} 2-1 & 4-2 & 5-3 \\ 2 & 3 & 4 \\ 3 & 2 & 5 \end{vmatrix}\right|}{\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 2 & 5 \end{vmatrix}\right|} = \dfrac{1}{\sqrt{78}}$

So, $\dfrac{1}{SD} = \sqrt{78}$

48. Equation of the plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the straight lines $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is

(A) x + 2y – 2z = 0 (B) 3x + 2y – 2z = 0
(C) x – 2y + z = 0 (D) 5x + 2y – 4z = 0

Key. C

Sol. Vector along the required plane is $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix} = 8\hat{i} - \hat{j} - 10\hat{k}$

So, normal vector $(\vec{n})$ to the plane is $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & -1 & -10 \\ 2 & 3 & 4 \end{vmatrix} = 26\hat{i} - 52\hat{j} + 26\hat{k}$.

So, equation of the plane is $\vec{r}.\vec{n} = 0 \Rightarrow x - 2y + z = 0$.

49. The distance between the plane x – 2y + z –6 = 0 and the plane containing the sets of points (1 + 2λ, 2 + 3λ, 3 + 4λ) and (2 + 3μ, 3 + 4μ, 4 + 5μ), where λ, μ are parameters, is

(A) $\sqrt{3/2}$ (B) $\sqrt{6}$
(C) $\sqrt{12}$ (D) $2\sqrt{6}$

Key. B

Sol. Normal vector : $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i} + 2\hat{j} - \hat{k}$

equation of plane: $-1(x - 1) + 2(y - 2) - 1(z - 3) = 0$
$\Rightarrow x - 2y + z = 0$

So, required distance = $\dfrac{|6|}{\sqrt{1+4+1}} = \sqrt{6}$

50. If the point (0, λ, 1) lies within the triangular prism formed by the planes x = 0, 2y – z + 2 = 0 and 2y + 3z – 6 = 0 then the set of values of λ is

(A) (–2, 2) (B) $\left(-\frac{1}{2}, \frac{3}{2}\right)$

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(C) $\left(-4,-\frac{4}{3}\right)$ (D) (0, 4)

Key. B

Sol. The planes are 2y + z = 0, 5x – 12y = 13 and 3x + 4z = 10

Solving we get z = $\frac{11}{2}$

51. Number of lattice point (x, y, z all being integers) inside the tetrahedron (not on the surface) having vertices (0, 0, 0), (21, 0, 0), (0, 21, 0), (0, 0, 21) is

(A) 1140 (B) 4000

(C) 2024 (D) none of these

Key. A

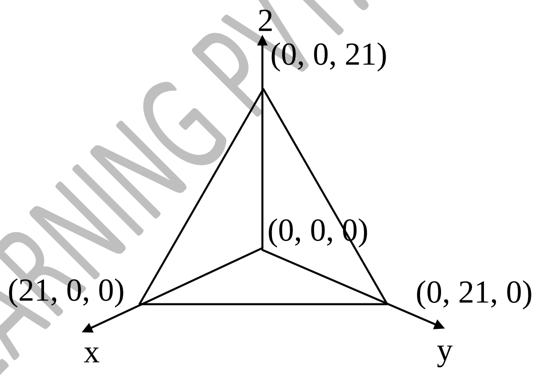
Sol.

Tetrahedron is bounded by $x \geq 0$, $y \geq 0$, $z \geq 0$ and

x + y + z = 21

Total no. of lattice point in side the tetrahedron is

= 1140



52. The equations of hypotenuse of a right angled isosceles triangle are $\frac{x+6}{5}=\frac{y+10}{3}=\frac{z+14}{8}$ and the centroid of the triangle is $\left(\frac{20}{3},-1,\frac{16}{3}\right)$. If (α,β,γ) is the circumcentre of the triangle then $\gamma =$

A) 6 B) -4 C) 5 D) 3

Key. A

Sol. Let $\bar{a}=5i+3j+8k$ (vector parallel to given line)

$G=\left(\frac{20}{3},-1,\frac{16}{3}\right), P=(5\lambda-6,3\lambda-10,8\lambda-14)$

P is the circumcentre $\overrightarrow{GP}.\bar{a}=0$.

53. The distance of the point of intersection of lines $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$ and $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ from $(7,-4,7)$ is

A) 6 B) $\sqrt{24}$ C) $\sqrt{14}$ D) 5

Key. C

Sol. Point of intersection = $(5,-7,6)$

54. Let ABCD be a tetrahedron in which position vectors of A, B, C & D are $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+\hat{j}+2\hat{k}$, $3\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+3\hat{j}+2\hat{k}$. If ABC be the base of tetrahedron then height of tetrahedron is

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A) $\sqrt{\frac{3}{2}}$ B) $\sqrt{\frac{3}{5}}$ C) $\frac{2\sqrt{2}}{\sqrt{3}}$ D) $\frac{1}{\sqrt{3}}$

Key. C

Sol. $\overrightarrow{AB}\times\overrightarrow{AC}=-\hat{i}+2\hat{j}+\hat{k}$

Height $=\frac{\left|\overrightarrow{AD}.(\overrightarrow{AB}\times\overrightarrow{AC})\right|}{\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|}=\frac{2\sqrt{3}}{\sqrt{3}}$

55. The plane passing through the point whose position vector is $i+j-k$ and parallel to the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x-1}{-1}=\frac{y+1}{-2}=\frac{z-1}{1}$ has l,m,n as direction cosines of its normal then $|l+m+n|=$

A) $1/\sqrt{3}$ B) $1/\sqrt{2}$ C) $1/\sqrt{5}$ D) $1/\sqrt{6}$

Key. C

Sol. $a+2b+3c=0$

$-a-2b+c=0$

$\Rightarrow a:b:c=2:-1:0$

56. If a line with direction ratios 2 : 2 : 1 intersects the lines $\frac{x-7}{3}=\frac{y-5}{2}=\frac{z-3}{1}$ and $\frac{x-1}{2}=\frac{y+1}{4}=\frac{z+1}{3}$ at A and B then AB=

A) $\sqrt{2}$ B) 2 C) $\sqrt{3}$ D) 3

Key. D

Sol. Let $A(7+3\alpha,5+2\alpha,3+\alpha),\ B(1+2\beta,-1+4\beta,-1+3\beta)$

D.R's of AB are in 2 : 2 : 1

$\therefore \frac{6+3\alpha-2\beta}{2}=\frac{3+\alpha-2\beta}{1}=\frac{4+\alpha-3\beta}{1}$

$\therefore \alpha=-2,\beta=1,\ A(1,1,1),\ B(3,3,2)$

57. The two lines whose direction cosines are connected by the relations $al+bm+cn=0$ and $ul^2+vm^2+wn^2=0$ are perpendicular then

(a) $a^2(v-w)+b^2(w-u)+c^2(u-v)=0$

(b) $\frac{a^2}{u}+\frac{b^2}{v}+\frac{c^2}{w}=0$

(c) $a(v^2+w^2)+b(w^2+u^2)+c(u^2+v^2)=0$

(d) $a^2(v+w)+b^2(w+u)+c^2(u+v)=0$

Key. D

Sol. Given relations are

$al+bm+cn=0$ ---------- (1)

$ul^2+vm^2+wn^2=0$ ---------- (2)

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Eliminating 'n' between the given relations we get $ul^2 + vm^2 + w\left(\frac{al+bm}{-c}\right)^2 = 0$

$$c^2ul^2 + c^2vm^2 + wa^2l^2 + wb^2m^2 + 2abwlm = 0$$

$$\left(c^2u + wa^2\right)\frac{l^2}{m^2} + 2abw\frac{l}{m} + \left(b^2w + c^2v\right) = 0 \rightarrow 1$$

The above is quadratic equation in $\frac{l}{m}$, whose roots are $\frac{l_1}{m_1}, \frac{l_2}{m_2}$

$$\frac{l_1l_2}{m_1m_2} = \frac{b^2w + c^2v}{c^2u + wa^2}$$

$$\frac{l_1l_2}{b^2w + c^2v} = \frac{m_1m_2}{c^2u + wa^2} = \frac{n_1n_2}{a^2v + b^2u}$$

If the lines are perpendicular, then $l_1l_2 + m_1m_2 + n_1n_2 = 0$

$$b^2w + c^2v + c^2u + wa^2 + a^2v + b^2u = 0$$

$$a^2(v+w) + b^2(u+w) + c^2(u+v) = 0$$

58. $f(x)$ be a polynomial in x satisfying the condition $f(x)f\left(\frac{1}{x}\right) = f(x) + f\left(\frac{1}{x}\right)$ and $f(2) = 9$. Then the direction cosines of the ray joining the origin and point $\left(f(0), f(1), f(-1)\right)$ are given by

a) $\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0\right)$ b) $(1,2,0)$ c) $(0,1,-1)$ d) $\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$

Key. A

Sol. $f(x) = x^n + 1$. $f(2) = 9$ imply $f(x) = x^3 + 1$ and $f(0) = 1$ $f(1) = 2, f(-1) = 0$,

Dc's of ray joining $(0,0,0)$ & $(1,2,0)$ is $\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0\right)$.

59. The plane $x - y - z = 4$ is rotated through 90^o about its line of intersection with the plane $x + y + 2z = 4$ and equation in new position is $Ax + By + Cz + D = 0$ where A,B,C are least positive integers and $D < 0$ then

a) $D = -10$ b) $ABC = -20$ c) $A + B + C + D = 0$ d) $A + B + C = 10$

Key. D

Sol. Given planes are $x - y - z = 4$ ---------- (1) and $x + y + 2z = 4$ ---------- (2)

Since required plane passes through the line of intersection (1) & (2)

$\Rightarrow$ Its equation is $(x - y - z - 4) + \alpha(x + y + 2z - 4) = 0$

$\Rightarrow (1+\alpha)x + (\alpha - 1)y + (2\alpha - 1)z - (4\alpha + 4) = 0$ ---------- (3)

Since (1) & (3) are perpendicular

$\Rightarrow 1(1+\alpha) - 1(\alpha - 1) - 1(2\alpha - 1) = 0 \Rightarrow 1 + \alpha - \alpha + 1 - 2\alpha + 1 = 0 \quad \Rightarrow \alpha = 3/2$

$\Rightarrow$ Its equation is $(x - y - z - 4) + \frac{3}{2}(x + y + 2z - 4) = 0 \Rightarrow 5x + y + 4z - 20 = 0$

60. The equation of motion of a point in space is $x = 2t, y = -4t, z = 4t$. where it is measured in hour and the coordinates of moving point in kilometers.The distance of the point from the starting point O(0,0,0) in 10 hours is

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a) 20 km b) 40 km c) 55 km d) 60 km

Key. D

Sol. Eliminating 't' from the equation we get the equation of the path, $\frac{x}{2}=\frac{y}{-4}=\frac{z}{4}$

Thus the path represents a straight line through the origin. For t= 10 h, we have x= 20, y=-40,z=40 and $|\vec{r}|=|O\vec{M}|=\sqrt{(x^2+y^2+z^2)}=\sqrt{400+1600+1600}$ =60 km

61. A mirror and a source of light are situated at the origin O and a point on OX respectively. A ray of light from the source strikes the mirror and is reflected. If the DRs of normal to the plane of mirror are 1,-1,1, then DCs for the reflected ray are

a) $\frac{1}{3},\frac{2}{3},\frac{2}{3}$ b) $-\frac{1}{3},-\frac{2}{3},\frac{2}{3}$ c) $-\frac{1}{3},-\frac{2}{3},-\frac{2}{3}$ d) $-\frac{1}{3},\frac{2}{3},\frac{2}{3}$

Key. B

Sol. DCs of the reflected ray are $-\frac{1}{3},-\frac{2}{3},\frac{2}{3}$

62. Through a point P(a, b, c) a plane is drawn at right angles to OP to meet the co-ordinate axes in A, B and C. If OP = p, then the area of ΔABC is

(A) $\frac{p^2ab}{c^2}$ (B) $\frac{p^3c}{3ab}$ (C) $\frac{p^2c^2}{2ab}$ (D) $\frac{p^5}{2abc}$

Key. D

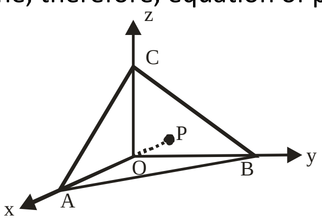
Sol. Here $OP=\sqrt{h^2+k^2+l^2}=p$

$\therefore$ DRs of OP are :

$$\frac{h}{\sqrt{h^2+k^2+l^2}},\frac{k}{\sqrt{h^2+k^2+l^2}},\frac{l}{\sqrt{h^2+k^2+l^2}}$$

or $\frac{h}{p},\frac{k}{p},\frac{l}{p}$

Since OP is normal to the plane, therefore, equation of plane is



$$\frac{h}{p}x+\frac{k}{p}y+\frac{l}{p}z=p \text{ or } hx+ky+lz=p^2$$

$$\therefore A\left(\frac{p^2}{h},0,0\right),B\left(0,\frac{p^2}{k},0\right),C\left(0,0,\frac{p^2}{l}\right)$$

Now, Area of $\Delta ABC, \Delta=\sqrt{A_{xy}^2+A_{yz}^2+A_{zx}^2}$

Where, A_{xy}^2 is area of projection of ΔABC on xy plane = area of ΔAOB

Now, $A_{xy}=\frac{1}{2}\begin{vmatrix} p^2/h & 0 & 1 \\ 0 & p^2/k & 1 \\ 0 & 0 & 1 \end{vmatrix}=\frac{p^4}{2|hk|}$

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Similarly, $A_{yz} = \frac{p^4}{2|kl|}$ and $A_{zx} = \frac{p^4}{2|lh|}$

$$\therefore \Delta^2 = A_{xy}^2 + A_{yz}^2 + A_{zx}^2, \Delta = \frac{p^5}{2hkl}$$

63. If $l_i^2 + m_i^2 + n_i^2 = 1 \forall i \in \{1,2,3\}$ and $l_il_j + m_im_j + n_in_j = 0 \ \forall i, j \in \{1,2,3\}$ $(i \neq j)$

$\Delta = \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}$ then

(A) $|\Delta| = 3$ (B) $|\Delta| = 2$ (C) $|\Delta| = 1$ (D) $\Delta = 0$

Key. C

Sol. We have,

$$\Delta^2 = \Delta\Delta = \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}$$

$$= \begin{vmatrix} l_1^2 + m_1^2 + n_1^2 & l_1l_2 + m_1m_2 + n_1n_2 & l_1l_3 + m_1m_3 + n_1n_3 \\ l_1l_2 + m_1m_2 + n_1n_2 & l_2^2 + m_2^2 + n_2^2 & l_2l_3 + m_2m_3 + n_1n_3 \\ l_1l_3 + m_1m_3 + n_1n_3 & l_2l_3 + m_2m_3 + n_2n_3 & l_3^2 + m_3^2 + n_3^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \Rightarrow \Delta = \pm 1 \Rightarrow |\Delta| = 1$$

64. Equation of the straight line in the plane $\vec{r}.\vec{n} = d$ which is parallel to $\vec{r} = \vec{a} + \lambda\vec{b}$ and passes through the foot of perpendicular drawn from the point $P(\vec{a})$ to the plane $\vec{r}.\vec{n} = d$. (where $\vec{n}.\vec{b} = 0$) is

A) $\vec{r} = \vec{a} + \left(\frac{d - \vec{a}.\vec{n}}{n^2}\right)\vec{n} + \lambda\vec{b}$ B) $\vec{r} = \vec{a} + \left(\frac{d - \vec{a}.\vec{n}}{n}\right)\vec{n} + \lambda\vec{b}$

C) $\vec{r} = \vec{a} + \left(\frac{\vec{a}.\vec{n} - d}{n^2}\right)\vec{n} + \lambda\vec{b}$ D) $\vec{r} = \vec{a} + \left(\frac{\vec{a}.\vec{n} - d}{n}\right)\vec{n} + \lambda\vec{b}$

Key. A

Sol. Foot perpendicular from point $A(\vec{a})$ on the plane $\vec{r}.\vec{n} = d$ is $\vec{a} + \frac{(d - \vec{a}.\vec{n})}{|\vec{n}|^2}\vec{n}$

$\therefore$ Equation of line parallel to $\vec{r} = \vec{a} + \lambda\vec{b}$ in the plane $\vec{r}.\vec{n} = d$ is given by

$$\vec{r} = \vec{a} + \frac{(d - \vec{a}.\vec{n})}{|\vec{n}|^2}\vec{n} + \lambda\vec{b}$$

65. If the foot of the perpendicular from the origin to a plane is P(a, b, c), the equation of the plane is

A) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$ B) ax + by + cz = 3

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C) ax + by + cz = $a^2+b^2+c^2$ D) ax + bx + cz = a + b + c

Key. C

Sol. Direction ratios of OP are $\langle a,b,c\rangle$

$\therefore$ equation of the plane is

e(x – a) + b(y – b) + c(z – c) = 0

i.e. xa + yb + zc $=a^2+b^2+c^2$

66. Equation of line in the plane $\pi=2x-y+z-4=0$ which is perpendicular to the line *l* whose equation is $\frac{x-2}{1}=\frac{y-2}{-1}=\frac{z-3}{-2}$ and which passes through the point of intersection of *l* and π is

A) $\frac{x-2}{3}=\frac{y-1}{5}=\frac{z-1}{-1}$ B) $\frac{x}{3}=\frac{y-3}{5}=\frac{z-5}{-1}$

C) $\frac{x+2}{2}=\frac{y+1}{-1}=\frac{z+1}{1}$ D) $\frac{x+2}{2}=\frac{y-1}{-1}=\frac{z-1}{1}$

Key. B

Sol. Let direction ratios of the line by $\langle a,b,c\rangle$, then

2a – b + c = 0

a – b – 2c = 0

i.e. $\frac{a}{3}=\frac{b}{5}=\frac{c}{-1}$

$\therefore$ direction ratios of the line are $\langle 3,5,-1\rangle$

Any point on the line is $(2+\lambda,2-\lambda,3-2\lambda)$. It lies on the plane π if

$2(2+\lambda)-(2-\lambda)+(3-2\lambda)=4$

i.e. $4+2\lambda-2+\lambda+3-2\lambda=4$

i.e. $\lambda=-1$

$\therefore$ The point of intersection of the line and the plane is (1, 3, 5)

$\therefore$ equation of the required line is $\frac{x-1}{3}=\frac{y-3}{5}=\frac{z-5}{-1}$

67. Equation of plane which passes through the point of intersection of lines $\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ at greatest distance from the point (0, 0, 0) is

A) 4x + 3y + 5z = 25 B) 4x + 3y + 5z = 50

C) 3x + 4y + 5z = 49 D) x + 7y – 5z = 2

Key. B

Sol. Let a point $(3\lambda+1,\lambda+2,2\lambda+3)$ of the first line also lies on the second line

Then $\frac{3\lambda+1-3}{1}=\frac{\lambda+2-1}{2}=\frac{2\lambda+3-2}{3}\Rightarrow\lambda=1$

Hence the point of intersection P of the two lines is (4, 3, 5)

Equation of plane perpendicular to OP where O is (0, 0, 0) and passing through P is

4x + 3y + 5z = 50

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68. Let $A(\vec{a})$ and $B(\vec{b})$ be points on two skew lines $\vec{r}=\vec{a}+\lambda\vec{p}$ and $\vec{r}=\vec{b}+u\vec{q}$ and the shortest distance between the skew lines is 1, where $\vec{p}$ and $\vec{q}$ are unit vectors forming adjacent sides of a parallelogram enclosing an area of $\frac{1}{2}$ units. If an angle between AB and the line of shortest distance is 60°, then AB =

A) $\frac{1}{2}$ B) 2 C) 1 D) $\lambda \in R-\{0\}$

Key. B

Sol. $1=\left|(\vec{b}-\vec{a}).\frac{(\vec{p}\times\vec{q})}{|\vec{p}\times\vec{q}|}\right| \Rightarrow |\vec{b}-\vec{a}|\cos 60° = \frac{1}{2}AB \quad \Rightarrow \quad AB=2$

69. If plane 2x + 3y + 6z + k = 0 is tangent to the sphere $x^2+y^2+z^2+2x-2y+2z-6=0$, then a value of k is

A) 26 B) 16 C) – 26 D) none of these

Key. A

Sol. Centre and radius of the sphere are (- 1, 1, - 1) and 3 respectively.

Distance of (- 1, 1, - 1) from the plane is $\left|\frac{-2+3-6+k}{\sqrt{4+9+36}}\right|$

Since the plane is tangent to the sphere

$\therefore \quad \left|\frac{k-5}{7}\right|=3 \qquad$ is |k – 5| = 21

$\therefore \quad$ k = - 16, 26

70. If $P_1:\vec{r}.\vec{n}_1-d_1=0,\ P_2:\vec{r}.\vec{n}_2-d_2=0$ and $P_3:\vec{r}.\vec{n}_3-\vec{d}_3=0$ are three planes and $\vec{n}_1.\vec{n}_2$ and $\vec{n}_3$ are three non-coplanar vectors then, the three lines $P_1=0, P_2=0; P_2=0, P_3=0$ and $P_3=0, P_1=0$ are

A) parallel lines B) coplanar lines C) coincident lines D) concurrent lines

Key. D

Sol. $P_1=P_2=0,\ P_2=P_3=0$ and $P_3=P_1=0$ are lines of intersection of the three planes P_1, P_2 and P_3. As $\vec{n}_1, \vec{n}_2$ and $\vec{n}_3$ are non-coplanar, planes $P_1, P_2\ P_3$ will intersect at unique point. So the given lives will pass through a fixed point.

71. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors equally inclined to each other at an angle α. Then the angle between $\vec{a}$ and plane of $\vec{b}$ and $\vec{c}$ is

A) $\theta=\cos^{-1}\left(\frac{\cos\alpha}{\cos\frac{\alpha}{2}}\right)$ B) $\theta=\sin^{-1}\left(\frac{\cos\alpha}{\cos\frac{\alpha}{2}}\right)$

C) $\theta=\cos^{-1}\left(\frac{\sin\frac{\alpha}{2}}{\sin\alpha}\right)$ D) $\theta=\sin^{-1}\left(\frac{\sin\frac{\alpha}{2}}{\sin\alpha}\right)$

Key. A

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Sol. Let θ be the required angle then θ will be the angle between $\vec{a}$ and $\vec{b}+\vec{c}$ ($\vec{b}+\vec{c}$ lies along the angular bisector of $\vec{a}$ and $\vec{b}$)

$$\cos\theta = \frac{\vec{a}.\left(\vec{b}+\vec{c}\right)}{\left|\vec{a}\right|\left|\vec{b}+\vec{c}\right|}$$

$$= \frac{2\cos\alpha}{\sqrt{2+2\cos\alpha}} = \frac{\cos\alpha}{\cos\frac{\alpha}{2}}$$

$$\theta = \cos^{-1}\left(\frac{\cos\alpha}{\cos\alpha/2}\right)$$

72. The reflection of the point P(1, 0, 0) in the line $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ is

A) (3, - 4, - 2) B) (5, - 8, - 4) C) (1, - 1, - 10) D) (2, - 3, 8)

Key. B

Sol. Let reflection of P(1, 0, 0) in the line

$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ be (α, β, γ)

Then $\left(\frac{\alpha+1}{2}, \frac{\beta}{2}, \frac{\gamma}{2}\right)$ lies on the line

and $(\alpha-1)\hat{i}+\beta\hat{j}+\gamma\hat{k}$ is perpendicular to $2\hat{i}-2\hat{j}+8\hat{k}$

$$\therefore \quad \frac{\frac{\alpha+1}{2}}{2} = \frac{\frac{\beta}{2}+1}{-3} = \frac{\frac{\gamma}{2}+10}{8} = \lambda$$

And $2(\alpha-1) - 3(\beta) + \gamma(8) = 0$

$\Rightarrow \quad \alpha = 5, \beta = -8, \gamma = -4$

73. Let A(1, 1, 1), B (2, 3, 5), C (- 1, 0, 2) be three points, then equation of a plane parallel to the plane ABC which is at a distance 2 is

A) 2x – 3y + z + $2\sqrt{14} = 0$ B) 2x – 3y + z $-\sqrt{14} = 0$

C) 2x – 3y + z + 2 = 0 D) 2x – 3y + z – 2 = 0

Key. A

Sol. A(1, 1, 1), B(2,3,5), C (- 1, 0, 2) directions ratios of AB are < 1, 2, 4> direction ratios of AC are < - 2, - 1, 1 >

$\therefore$ direction ratios of normal to plane ABC are < 2, - 3, 1 >

$\therefore$ Equation of the plane ABC is 2x – 3y + z = 0

Let the equation of the required plane be 2x – 3y + z = k, then $\left|\frac{k}{\sqrt{4+9+1}}\right| = 2$

$$k = \pm 2\sqrt{14}$$

$\therefore$ Equation of the required plane is $2x - 3y + z + 2\sqrt{14} = 0$

74. The points A(2 – x, 2, 2), B(2, 2 – y, 2), C(2, 2, 2 – z) and D(1, 1, 1) are coplanar, then locus of P(x, y, z) is

A) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 1$ B) x + y + z = 1

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C) $\frac{1}{1-x}+\frac{1}{1-y}+\frac{1}{1-z}=1$ D) None of these

Key. A

Sol. Here $\overrightarrow{AB}=x\hat{i}-y\hat{j}$

$\overrightarrow{AC}=x\hat{i}-z\hat{k}$

$\overrightarrow{AD}=(x-1)\hat{i}-\hat{j}-\hat{k}$

As these vectors are coplanar $\Rightarrow \begin{vmatrix} x & -y & 0 \\ x & 0 & -z \\ x-1 & -1 & -1 \end{vmatrix}=0 \Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$

75. The equation of plane through (1, 2, 3) and at the maximum distance from origin is

A) x + 2y + 3z = 14 B) x + y + z = 6

C) x + 2y + 3z + 14 = 0 D) 3x

Key. A

Sol. Direction rations of normal to the plane is (1, 2, 3)

$\Rightarrow$ Equation of plane (x – 1)1 + (y – 2).2 + (z – 3). 3 = 0

$\Rightarrow$ x + 2y + 3z = 14

76. If P(α, β, γ) be a vertex of an equilateral triangle PQR where vertex Q and R are (-1,0,1) and (1, 0, -1) respectively then P will lie on the plane

a) x + y + z + 6 = 0 b) 2x + 4y + 3z + 10 = 0

c) x – y + z + 12 = 0 d) $x+y+z+3\sqrt{2}=0$

Ans. **d**

$QR=2\sqrt{2}$ = OP = 6

77. The length of the perpendicular from (1, 0, 2) on the line $\frac{x+1}{3}=\frac{y-2}{-2}=\frac{z+1}{-1}$ is

a) $\frac{3\sqrt{6}}{2}$ b) $\frac{6\sqrt{3}}{5}$ c) $3\sqrt{2}$ d) $2\sqrt{3}$

Ans. **a**

$$PM=\sqrt{\left(1-\frac{1}{2}\right)^2+(0-1)^2\left(2+\frac{3}{2}\right)^2}=\frac{3\sqrt{6}}{2}$$

78. In triangle OAB, B = (3, 4). If $H\equiv(1,4)$ be the orthocenter of the triangle, then the coordinates of A are (where O is the origin)

a) $\left(0,\frac{15}{4}\right)$ b) $\left(0,\frac{17}{4}\right)$ c) $\left(0,\frac{21}{4}\right)$ d) $\left(0,\frac{19}{4}\right)$

Ans. **d**

Sol. Let $A=(h,k)$, slope of $AH=\frac{k-4}{h-1}$, slope of OB = $\frac{4}{3}$

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$$\Rightarrow \frac{4(k-4)}{3(h-1)} = -1$$

$$\Rightarrow 4k + 3h = 19$$

Slope of OA = $\frac{k}{h}$, slope of BH = 0 As $OA \perp BH$

$\therefore h = 0,$ put in (1)

$$k = \frac{19}{4}$$

79. In an acute angles triangle ABC, AA_1, AA_2 are the median and altitude respectively. Then A_1A_2 is equal to

a) $\frac{|a^2 - c^2|}{2b}$ b) $\frac{|a^2 - b^2|}{2c}$ c) $\frac{|b^2 - c^2|}{2a}$ d) none of these

Ans. **c**

Sol. $A_2C = AB\cos B = c\cos B = \frac{a^2 + c^2 - b^2}{2a}$

Also $A_1B = \frac{a}{2}$ and $A_2A_1 = BA_1 - BA_2 = \left|\frac{a}{2} - \frac{a^2 + c^2 - b^2}{2a}\right|$

$= \left|\frac{b^2 - c^2}{2a}\right|$

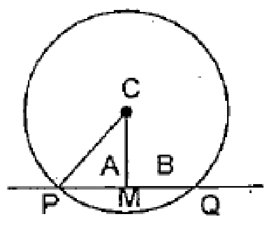
80. If a chord of the circle $x^2 + y^2 - 4x - 2y - c = 0$ is trisected at the points $\left(\frac{1}{3}, \frac{1}{3}\right)$ and $\left(\frac{8}{3}, \frac{8}{3}\right)$, then c =

a) 10 b) 20 c) 30 d) 40

Ans. **b**

Sol. Cut A : $\left(\frac{1}{3}, \frac{1}{3}\right)$ and $B\left(\frac{8}{3}, \frac{8}{3}\right)$. Also C(2, 1).

Then equation of AB is y = x, and length AB = $\frac{7\sqrt{2}}{3}$



If PQ be the chord, then

Length $PQ = 7\sqrt{2}$

Now $CP^2 = PM^2 + CM^2$

$$\Rightarrow 4 + 1 + c = \left(\frac{7\sqrt{2}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 25 \Rightarrow c = 20$$

81. From a point on hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ tangents are drawn to circle $x^2 + y^2 = 9$ then locus of midpoint of chord of contact

a) $9(9x^2 - 4y^2) = 4(x^2 + y^2)^2$ b) $9(4x^2 - 9y^2) = 4(x^2 + y^2)^2$

c) $5(9x^2 - 4y^2) = 4(x^2 + y^2)^2$ d) $9(9x^2 - 5y^2) = 4(x^2 - y^2)^2$

Ans. **b**

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Sol. Equation of chord of contact is $3x\sec\theta + 2y\tan\theta = 9$ - (1)

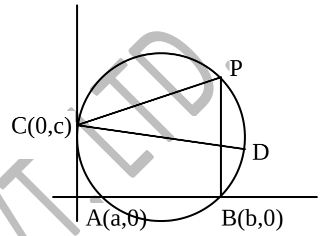
Let midpoint of chord of contact be (h, k) then hx + ky = $h^2 + k^2$ - (2)

(1) and (2) are identical

$$\sec\theta = \frac{9h}{3\left(h^2+k^2\right)}, \tan\theta = \frac{9k}{2\left(h^2+k^2\right)}$$

Then $\sec^2\theta - \tan^2\theta = 1$

82. In figure shown two points A and B are given on x-axis and third point C on y-axis. Then locus of P such that four A, B, P and C lie on a circle



a) $\left(x-\frac{a+b}{2}\right)^2+\left(y-\frac{c^2+ab}{2c}\right)^2=\frac{c^4+a^2b^2+b^2c^2+c^2a^2}{4c^2}$

b) $\left(x+\frac{a+b}{2}\right)^2+\left(y-\frac{c^2+ab}{2c}\right)^2=\frac{c^4+a^2b^2+b^2c^2+c^2a^2}{4c^2}$

c) $\left(x-\frac{a+b}{2}\right)^2+\left(y+\frac{c^2+ab}{2c}\right)^2=\frac{c^4+a^2b^2+b^2c^2+c^2a^2}{4c^2}$

d) none of these

Ans. a

Sol. Let equation of circle be $(x-\alpha)^2+(y-\beta)^2=r^2$

Since it passes through A, B, C

$r^2=(a-\alpha)^2+\beta^2$

$r^2=(b-\alpha)^2+\beta^2$ on solving get equation

$r^2=\alpha^2+(c-\beta)^2$

83. Let A be the fixed point (0, 4) and B be a moving point (2t, 0), M be the midpoint of AB and let the perpendicular bisector of AB meet the y-axis at R. The locus of the midpoint of MR is

a) $x^2=-(y-2)$ b) $x^2+(y-2)^2=1/4$ c) $x^2+1/4=(y-2)^2$ d) none of these

Ans. a

Sol. M(t, 2) $\Rightarrow$ equation of MR is $y-2=\frac{t}{2}(x-t)$

$\Rightarrow R\equiv\left(0, 2-t^2/2\right)$, let midpoint be (h, k)

$\Rightarrow h=t/2,\ k=2-t^2/4$

84. If P be a point inside an equilateral ΔABC such that PA = 3, PB = 4 and PC = 5, then the side length of the equilateral ΔABC is

a) $\sqrt{25-12\sqrt{3}}$ b) 13 c) $\sqrt{25+12\sqrt{3}}$ d) 17

Ans. c

Sol.

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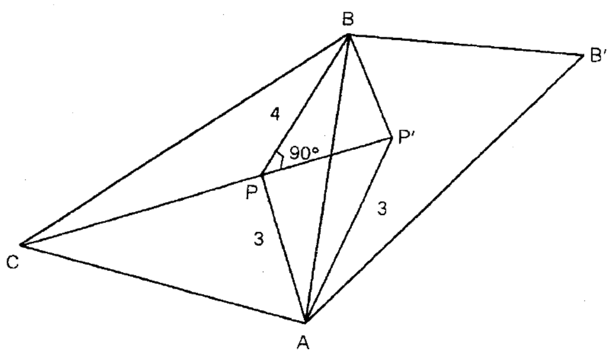
Rotate the triangle in clockwise direction through an angle 60^0. Let the points A, B, C and P will be A, B', B and P' respectively after the rotation. We have $PA = P'A = 3$ and $\angle PAP' = 60^0 \Rightarrow PP' = 3$. Also $CP = BP' = 5$. So $\Delta BPP'$ is right angle triangle which $\angle BPP' = 90^0$. Now apply cosine rule in ΔBPA because $\angle BPA = 90^0 + 60^0 = 150^0$, PA = 3 and BP = 4, we can get AB.



85. Consider $A \equiv (3,4)$, $B \equiv (7,13)$. If P be a point on the line y = x such that PA + PB is minimum, then the coordinate of P are

a) $\left(\frac{13}{7}, \frac{13}{7}\right)$ b) $\left(\frac{23}{7}, \frac{23}{7}\right)$ c) $\left(\frac{31}{7}, \frac{31}{7}\right)$ d) $\left(\frac{33}{7}, \frac{33}{7}\right)$

Ans. **c**

Sol. Let A, be the reflection of A in $y = x \Rightarrow A_1 \equiv (4,3)$

Now PA + PB = A_1P + PB, which is minimum when A_1, P, B are collinear

Equation of A_1B is $(y-3) = \frac{13-3}{7-4}(x-4) \Rightarrow 3y = 10x - 31$ and y = x gives $P \equiv \left(\frac{31}{7}, \frac{31}{7}\right)$

86. In triangle ABC, equation of the side BC is x – y = 0. Circumcentre and orthocenter of the triangle are (2, 3) and (5, 8) respectively. Equation of the circumcircle of the triangle is

a) $x^2 + y^2 - 4x + 6y - 27 = 0$ b) $x^2 + y^2 - 4x - 6y - 27 = 0$
c) $x^2 + y^2 + 4x + 6y - 27 = 0$ d) $x^2 + y^2 + 4x - 6y - 27 = 0$

Ans. **b**

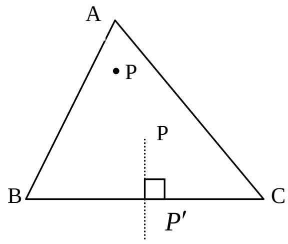
Sol.

Reflection P in BC will lie on BC

$\therefore$ Equation of circumcircle is

$$(x-2)^2 + (y-3)^2 = (8-2)^2 + (5-3)^2 \text{ or}$$

$x^2 + y^2 - 4x - 6y - 27 = 0$



87. The locus of the midpoints of the chords of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ that pass through the origin is

a) $x^2 + y^2 + 2gx + 2fy = 0$ b) $x^2 + y^2 + gx + fy + c = 0$
c) $x^2 + y^2 + gx + fy = 0$ d) $2(x^2 + y^2 + gx + |y| + c = 0$

Ans. **c**

Sol. $T = S_1 \Rightarrow xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

It passes through (0, 0)

$\therefore x_1^2 + y_1^2 + gx_1 + fy_1 = 0$

$\therefore$ Locus is $x^2 + y^2 + gx + fy = 0$

88. Circles C_1 and C_2 having centres G_1 and G_2 respectively intersect each other at the points A and B, secants L_1 and L_2 are drawn to the circles C_1 and C_2 to intersect them in the points A_1, B_1 and A_2, B_2 respectively. If the secants L_1 and L_2 intersect each other at a point P in the exterior

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region of circles C_1 and C_2 and $PA_1 \times PB_1 = PA_2 \times PB_2$ then which of the following statement is false

a) points P, A and B are collinear
b) line joining G_1 and G_2 is perpendicular to line joining P and A
c) $PA_1 \times PB_1 = PA \times PB$
d) $PA = PA_1$

Ans. **d**

Sol. Line joining PAB will be the radical axis of the two circles so a, b and c are correct

89. Distance between centres of circles which pass through A(a, a) and B(2a, 2a) and touch the y-axis is

a) 4a b) $2\sqrt{2}a$ c) $4\sqrt{2}\,a$ d) $\sqrt{2}\,a$

Ans. **c**

Sol. Let $(\alpha, 3a-\alpha)$, $(\beta, 3a-\beta)$ be the centres of the circle

$\Rightarrow \alpha, \beta$ are the roots of equation $(x-a)^2 + (2a-x)^2 = x^2$

$\Rightarrow \alpha + \beta = 6a,\ \alpha\beta = 5a^2$

$\Rightarrow |\alpha - \beta| = 4a$

$\Rightarrow C_1C_2 = 4a\sqrt{2}$

90. The locus of the centre of a circle which cuts orthogonally the parabola $y^2 = 4x$ at (1, 2) will pass through points

a) (3, 4) b) (4, 3) c) (5, 3) d) (2, 4)

Ans. **a**

Sol. Tangent to parabola $y^2 = 4x$ at (1, 2) will be the locus

i.e $y \cdot 2 = 2(x+1)$

y = x + 1

91. Let AB be any chord of the circle $x^2 + y^2 - 4x - 4y + 4 = 0$ which subtends an angle of 90^0 at the point (2, 3), then the locus of the midpoint of AB is circle whose centre is

a) (1, 5) b) $\left(1, \frac{5}{2}\right)$ c) $\left(1, \frac{3}{2}\right)$ d) $\left(2, \frac{5}{2}\right)$

Ans. **d**

Sol. Let midpoint of AB is M(h, k)
AB subtends 90^0 at (2, 3)

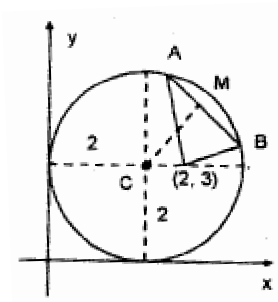
$\Rightarrow AM = MB$

$\Rightarrow \sqrt{(h-2)^2 + (k-3)^2}$

Also, $CM^2 + MB^2 = CB^2$

$\Rightarrow (h-2)^2 + (k-2)^2 + (h-2)^2 + (k-3)^2 = 4$

$\Rightarrow x^2 + y^2 - 4x - 5y + \frac{17}{2} = 0$



92. If line y = 2x + c neither cuts the circle $(x-2)^2 + (y-3)^2 = 4$ nor the ellipse $x^2 + 6y^2 = 6$, then the range of c is

a) [-5, 5] b) $(-\infty, -5) \cup (5, \infty)$ c) (-4, 4) d) none of these

Ans. **b**

Sol. Since the given line does not meet the given ellipse and circle.
$c^2 > 6 \times 2^2 + 1$ [From $c^2 > a^2m^2 + b^2$]

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and $c^2 > 4(1 + 4)$ [From $c^2 > a^2(1 + m^2)$]

$\Rightarrow c^2 > 25$

$\therefore c \in (-\infty, -5) \cup (5.\infty)$

93. If the eccentricity of the hyperbola $x^2 \cos ec^2\theta - y^2 \sec^2\theta = 5$ is 5 times the eccentricity of the ellipse $x^2 \cos ec^2\theta + y^2 \sec^2\theta = 25$, then θ =

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\cot^{-1}\left(\frac{\pm 2}{\sqrt{3}}\right)$ d) $\tan^{-1}\left(\frac{4}{5}\right)$

Ans. c

Sol. $\frac{x^2}{5\sin^2\theta} - \frac{y^2}{5\cos^2\theta} = 1,\ \frac{x^2}{25\sin^2\theta} + \frac{y^2}{25\cos^2\theta} = 1$

$e_H^2 = 1 + \cot^2\theta$

$e_e^2 = 1 - \cot^2\theta$

$1 + \cot^2\theta = 5(1 - \cot^2\theta)$

$6\cot^2\theta = 4$ = $\cot^2\theta = \frac{2}{3}$

$\cot\theta = \pm\sqrt{\frac{2}{3}}$

$\theta = \cot^{-1}\left(\frac{+2}{\sqrt{3}}\right)$

94. Area enclosed by ellipse $x^2 + \sin^4\alpha\, y^2 = \sin^2\alpha$, $\alpha \in \left(0, \frac{\pi}{2}\right)$ is

a) 2π b) π c) 1 d) none of these

Ans. **b**

Sol. Area = πab – π sinα cosecα = π.

95. Find the eccentricity of the conic formed by the locus of the point of intersection of the lines $\sqrt{3}x - y - 4\sqrt{3}k = 0$ and $\sqrt{3}kx + ky - 4\sqrt{3} = 0$

a) 4 b) 1/4 c) 2 d) 1/2

Ans. c

Sol. $\sqrt{3}x - y - 4\sqrt{3}k = 0,\ \sqrt{3}kx + ky - 4\sqrt{3} = 0$

In order to find the locus of point of intersection

We have to eliminate k

$\therefore \frac{\sqrt{3}x - y}{4\sqrt{3}} = k$ put this k in another

$\left(\sqrt{3}x - y\right)\left(\sqrt{3}x + y\right) = \left(4\sqrt{3}\right)^2$

or $3x^2 - y^2 = 48$

$\frac{x^2}{16} - \frac{y^2}{48} = 1 \quad \therefore a^2 = 16,\ b^2 = 48$

Clearly locus form a hyperbola.

$b^2 = a^2(e^2 - 1)$

$48 = 10(e^2 - 1)$

∴ e = 2

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96. If a pair of variable straight lines $x^2+4y^2+\alpha xy=0$ (where α is a real parameter) cut the ellipse $x^2+4y^2=4$ at two points A and B, then locus of point of intersection of tangents at A and B is

a) $x^2-4y^2+8xy=0$ b) $(2x-y)(2x+y)=0$

c) $x^2-4y^2+4xy=0$ d) $(x-2y)(x+2y)=0$

Ans. **d**

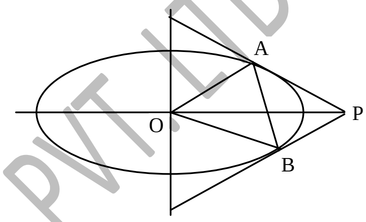
Sol. Let the point of intersection of tangents at A and B be P(h, k) then equation of AB is

$$\frac{xh}{4}+\frac{yk}{1}=1 \qquad - \qquad (1)$$

Homogenizing the ellipse with (1)

$$\frac{x^2}{4}+\frac{y^2}{1}=\left(\frac{xh}{4}+\frac{yk}{1}\right)^2$$

$$\Rightarrow x^2\left(\frac{h^2-4}{16}\right)+y^2\left(k^2-1\right)+\frac{2hk}{4}xy=0 \qquad - \qquad (2)$$



Given, equation of OA and OB is

$x^2 + 4y^2 + \alpha xy = 0$ - (3)

(2) and (3) are same

$\Rightarrow$ (h – 2k) (h + 2k) = 0

Therefore locus is (x – 2y) (x + 2y) = 0

97. A man starts from point P(-3, 4) and reaches the point Q(0, 1) touching x-axis at R, such that PR + RQ is minimum, then the coordinates of point R is

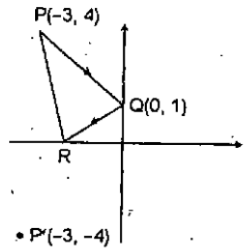
a) $\left(-\frac{3}{5},0\right)$ b) (1, 0) c) (- 1, 0) d) $\left(\frac{3}{5},0\right)$

Ans. **a**

Sol. Let $P'(-3,-4)$ be the image of P with respect to x-axis PR + RQ minimum

$\Rightarrow P'R+RQ$ is minimum

$\Rightarrow P'RQ$ should be collinear



98. Let A, B and C are any three points on the ellipse $36x^2+\frac{y^2}{192}=1$, then the maximum area of the triangle ABC is

a) 1 b) 2 c) 3 d) 4

Ans. **c**

Sol. Area of the triangle inscribed in the ellipse is maximum in difference of the eccentric angles of the point A, B, C is $\frac{2\pi}{3}$

So maximum area of the inscribed triangle is $\frac{3\sqrt{3}}{4}\cdot\frac{1}{6}\cdot 8\sqrt{3}=3\,sq.units$

99. If $P(x_1,y_1), Q(x_2,y_2), R(x_3,y_3)$ and $S(x_4,y_4)$ are four concyclic points on the rectangular hyperbola $xy=c^2$, then the coordinates of the orthocenter of ΔPQR are

a) $(x_4,-y_4)$ b) (x_4,y_4) c) $(-x_4,-y_4)$ d) $(-x_4,y_4)$

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Ans. c

Sol. Orthocentre and (x_4, y_4) are the images of each other with respect to the origin.

100. If two of the lines given by $3x^3 + 3x^2y - 3xy^2 + dy^3 = 0$ are at right angled, then the slope of the third line is

a) – 1 b) 1 c) 3 d) – 3

Ans. a

Sol. Let the lines be $y = m_1x$, $y = m_2x$, $y = m_3x$

$$\therefore m_1m_2m_3 = -\frac{3}{d}$$

Let $m_1m_2 = -1$ (two of the lines are perpendicular)

$$\therefore m_3 = \frac{3}{d}$$

$y = \frac{3}{d}x$ satisfying given equation

$$\Rightarrow d\left(\frac{3}{d}\right)^3 - 3\left(\frac{3}{d}\right)^2 + 3\left(\frac{3}{d}\right) + 3 = 0$$

$$\Rightarrow d = -3$$

$\therefore$ The given equation $x^3 + x^2y - xy^2 - y^3 = 0$

$$\Rightarrow (x+y)(x^2 - y^2) = 0$$

$\therefore$ slopes of other 2 lines are 1, - 1

101. If the angle between tangents drawn to $x^2 + y^2 - 6x - 8y + 9 = 0$ at the points where it is cut by the line $y = 3x + k$ is $\frac{\pi}{2}$, then

a) $k = -5 \pm 2\sqrt{5}$ b) $k = -5 \pm 3\sqrt{5}$ c) $k = 2\sqrt{5} + \sqrt{2}$ d) none of these

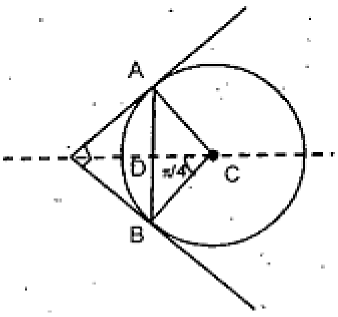
Ans. a

Sol. $\text{CD} = \text{CB}\cos\frac{\pi}{4} = \sqrt{2}$

$$\sqrt{2} = \frac{|4 - 9 - k|}{\sqrt{1^2 + 3^2}} = \frac{|-5 - k|}{\sqrt{10}}$$

$$20 = (5 + k)^2$$

$$\Rightarrow k = -5 \pm 2\sqrt{5}$$



102. If directions of two sides of a triangle are fixed and length of third side is constant and is sliding between these sides, then locus of the orthocenter of the triangle is

a) circle b) ellipse c) straight line d) hyperbola

Ans. a

Sol. Let fixed directions be OA and OB inclined at a constant angle α and AB = c.

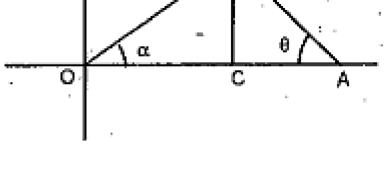
Let $\angle BAO = \theta$ then BC = $c\sin\theta$ and AC = $c\cos\theta$.

$$\therefore OC = c\sin\theta \cdot \cot\alpha$$

Equation of the line passing through A and perpendicular to OB is $y = -\cot\theta(x - c\sin\theta\cot\theta - c\cos\theta)$ and equation of BC is x = $c\sin\theta \cdot \cot\alpha$

$\therefore$ orthocenter is $(c\sin\theta \cdot \cot\theta, c\cos\theta \cdot \cot\alpha)$

$\Rightarrow$ Required locus is $x^2 + y^2 = c^2\cot^2\alpha$, which is the



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equation of a circle.

103. The number of triangles having two vertices are (1, 2) and (6, 2) and incentre (4,6) is

a) 2 b) 1 c) infinite d) 0

Ans. **d**

Sol. Equation of BC is y = 2, which is parallel to x-axis

$\therefore \tan\frac{B}{2}=\frac{4}{3}\Rightarrow B>\frac{\pi}{2}$ and $\tan\frac{C}{2}=2\Rightarrow C>\frac{\pi}{2}$

In a triangle two angles cannot be greater than 90^0 and hence there is no such triangle.

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3D-Geometry

Multiple Correct Answer Type

1. Consider the planes $P_1: 2x+y+z+4=0,\ P_2: y-z+4=0$ and $P_3: 3x+2y+z+8=0.$

Let L_1, L_2, L_3 be the lines of intersection of the planes P_2 and P_3, P_3 and P_1, and P_1 and P_2 respectively. Then,

A) Atleast two of the lines L_1, L_2 and L_3 are non–parallel

B) Atleast two of the lines L_1, L_2 and L_3 are parallel

C) The three planes intersect in a line

D) The three planes form a triangular prism

Key. B,C

Sol. Observe that the lines L_1, L_2 and L_3 are parallel to the vector $(1, -1, -1)$

Also, $\Delta = 0 = \Delta_1$ and $b_1c_2 - b_2c_1 \neq 0$

$\therefore$ The three planes intersect in a line

2. The plane passing through the point (-2, -2, 2) and containing the line joining points (1, 1, 1) and (1, -1, 2) makes intercepts of lengths a, b, c respectively on the axes of x, y and z then

a) a = 3b b) b = 2c c) a + b + c = 12 d) a + 3b + 3c = 20

Key. A,B,C,D

Sol. Equation of plane passing through (-2, -2, 2) is l(x+2) + m (y+2) + n(z-2) = 0

Where l, m, n are dr's of normal to the plane

Since it contains the line joining (1, 1, 1) and (1, -1, 2) these points also lie on the planes

$\Rightarrow 3l+3m-n=0$ and $3l+m=0$

$$\Rightarrow \frac{l}{1}=\frac{m}{-3}=\frac{n}{-6}$$

$\Rightarrow$ equation of the plane is (x+2) – 3(y+2) – 6(z-2) = 0

(or) x – 3y -6z + 8 = 0

$$\Rightarrow \frac{x}{-8}=\frac{y}{8/3}=\frac{z}{8/6}=1$$

$\Rightarrow$ a= 8, b= 8/3, c=8/6

3. $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ intersects the co-ordinate axes at points A, B and C respectively. If ΔPQR has mid-points A, B and C then

(A) centroids of ΔABC and ΔPQR coincide

(B) foot of normal to ΔABC from O is circumcentre of ΔPQR

(C) $ar(\Delta PQR) = 2\sqrt{a^2b^2+b^2c^2+c^2a^2}$

(D) incentres of ΔABC and ΔPQR coincide

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Key: A, B, C

Hint: (a), (b), (c)

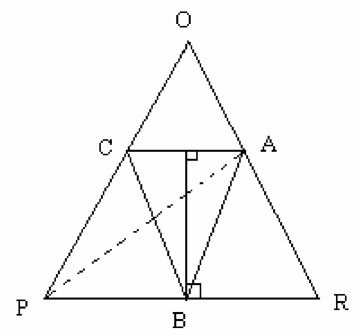
$AC \| PR$ and $2AC = PR$

So, ABPC is a parallelogram comparing the coordinates

of mid-point of diagonals, we get

$P(-a,b,c)$ and $Q(a,-b,c)$ and $R(a,b,-c)$

Also, AP is median of ΔABC and ΔPQR so centroids are Coinciding. The perpendicular bisector of PR is also perpendicular to AC. Therefore circumcentre of ΔPQR is orthocenter of ΔABC



$ar\Delta PQR = 4 ar\Delta ABC = 4\sqrt{(OAB)^2 + (OBC)^2 + (OAC)^2}$

Where OAB is the area of the projection of ΔABC on the plane XOZ etc.

4. The projection of line $3x - y + 2z - 1 = 0 = x + 2y - z - 2$ on the plane $3x + 2y + z = 0$ is

(A) $\frac{x+1}{11} = \frac{y-1}{-9} = \frac{z-1}{-15}$ (B) $3x - 8y + 7z + 4 = 0 = 3x + 2y + z$

(C) $\frac{x+12}{11} = \frac{y+8}{-9} = \frac{z+14}{15}$ (D) $\frac{x+12}{11} = \frac{y+8}{-9} = \frac{z+14}{-15}$

Key: A,B

Hint: Equation of a plane passing through the line $3x - y + 2z - 1 = 0 = x + 2y - z - 2$ is

$3x - y + 2z - 1 + \lambda(x + 2y - z - 2) = 0$

Since it is perpendicualr to the given plane

$\therefore \lambda = -\frac{3}{2}$

$\therefore$ Equation of the line of projection is

$3x - 8y + 7z + 4 = 0 = 3x + 2y + z$

Its direction ratios are < 11, $-$ 9 , $-$15 > and the point ($-$1,1,1) lies on the line

$\therefore \frac{x+1}{11} = \frac{y-1}{-9} = \frac{z-1}{-15}$ is also the equation of the line of projection.

5. The equation of three planes are $x - 2y + z = 3,\ 5x - y - z = 8,\ and\ x + y - z = 7$ then

a) they form a triangular prism

b) all three plane have a common line of intersection

c) line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ is parallel to each plane

d) line $\frac{x}{1} = \frac{y}{3} = \frac{z}{4}$ intersect all three plane

Key: A, C

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Hint: $\text{Augment matrix}\begin{bmatrix}1 & -2 & 1 & 3\\ 5 & -1 & -1 & 8\\ 1 & 1 & -1 & 7\end{bmatrix}\approx\begin{bmatrix}1 & -2 & 1 & 3\\ 0 & 9 & -6 & -7\\ 0 & 3 & -2 & 4\end{bmatrix}\approx\begin{bmatrix}1 & -2 & 1 & 3\\ 0 & 9 & -6 & -7\\ 0 & 0 & 0 & 19\end{bmatrix}$

System of equation has no solution. $|A|=0\Rightarrow\left[\vec{n_1}\ \ \vec{n_2}\ \ \vec{n_3}\right]=0\Rightarrow$ normals of plane are coplanar hence they are not intersecting at any point and forming a triangular prism.

$(x,y,z)=(r,2r,3r)$ does not satisfy by any plane for any value of 'r' hence $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ is parallel to each plane. (x, y, z) = (r, 3r, 4r) satisfy by plane (1) & plane (2) for some value of 'r' but not satisfy by plane '3' for any value of r. hence line $\frac{x}{1}=\frac{y}{3}=\frac{z}{4}$ does not interest plane '3'.

06. The line $\frac{x-2}{3}=\frac{y-1}{2}=\frac{z-1}{-1}$ intersects the curve $x^2-y^2=a^2;z=0$ if a is equal to ______

a) 4 b) $\sqrt{5}$ c) -4 d) 3

Key. A,C

Sol. For the point where the line intersects the curve, we have z = 0, so

$$\frac{x-2}{3}=\frac{y-1}{2}=\frac{0-1}{-1}\Rightarrow x=5 \text{ and } y=3$$

Putting these values in $x^2-y^2=a^2$ we get $a^2=16$ i.e., $a=\pm4$

7. If P_1,P_2,P_3 denote the distances of the plane $2x-3y+4z+2=0$ from the planes $2x-3y+4z+6=0$, $4x-6y+8z+3=0$ and $2x-3y+4z-6=0$ respectively, then

A) $P_1+8P_2-P_3=0$ B) $P_3=16P_2$ C) $8P_2=P_1$ D) $P_1+2P_2+3P_3=\sqrt{29}$

Key. A,B,C,D

Sol. Since planes are parallel planes

$P_1=\frac{4}{\sqrt{29}},P_3=\frac{8}{\sqrt{29}},P_2=\frac{1}{2\sqrt{29}}$

$P_3=16P_2=2P_1$

$P_1+2P_2+3P_3=\sqrt{29}$

$P_1+8P_2-P_3=0$

8. If lines $\frac{x-1}{2}=\frac{y-2}{x_1}=\frac{z-3}{x_2}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ lies in same plane then for equation $x_1t^2+(x_2+2)t+a=0$

A) $2x_1-x_2=2$ B) Sum of roots of above equation = -2

C) $2x_1+x_2=-4$ D) Sum of roots = 0

Key. A,B

Sol. $\begin{vmatrix}1 & 1 & 1\\ 2 & x_1 & x_2\\ 3 & 4 & 5\end{vmatrix}=0\Rightarrow 2x_1-x_2=2$

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9. A plane passing through (1, 1, 1) cuts +ve direction of co-ordinate axes at A, B & C, then the volume of tetrahedron OABC (V) satisfies

A) $V < \frac{9}{2}$ B) $V = \frac{9}{2}$ C) $V > \frac{9}{2}$ D) $V \le \frac{9}{2}$

Key. B,C

Sol. Let plane equation be $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

$\therefore \frac{1}{a}+\frac{1}{b}+\frac{1}{c}=1$ ($\because (1,1,1)$ lies on it)

$A.M \ge H.M \Rightarrow abc \ge 27$

10. The plane $3x+4y=0$ is rotated through an angle of $\pi/4$ about its line of intersection with the xy-plane. The equation of the plane in the new position is

A) $3x+4y-z=0$ B) $3x+4y+z=0$

C) $3x+4y+5z=0$ D) $3x+4y-5z=0$

Key. C,D

Sol. Required plane is $3x+4y+\lambda(Z)=0$

11. If the median through A of ΔABC having vertices $A\equiv(2,3,5)$, $B\equiv(-1,3,2)$ and $C\equiv(\lambda,5,\mu)$ is equally inclined to the axes then

(a) $\lambda=7$ (b) $\mu=10$ (c) $\lambda=10$ (d) $\mu=7$

Key. A,B

Sol. Mid point of $BC=\left(\frac{\lambda-1}{2},4,\frac{\mu+2}{2}\right)$

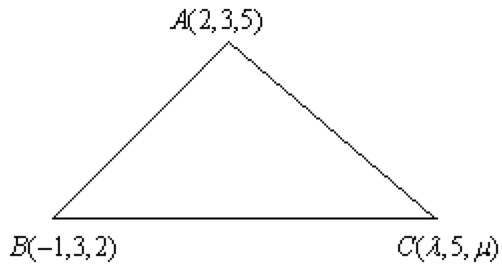
dr's median through A are

$\left(\frac{\lambda-1}{2}-2,\ 4-3,\ \frac{\mu+2}{2}-5\right)$

$=\left(\frac{\lambda-5}{2},1,\ \frac{\mu-8}{2}\right)$

The median is equally inclined to axis so the direction ratios must be equal; so

$\frac{\lambda-5}{2}=1=\frac{\mu-8}{2}\Rightarrow\lambda=7,\mu=10$



12. The equation of the line x + y + z – 1 = 0, 4x + y – 2z + 2 = 0 written in the symmetrical form is

A) $\frac{x+1}{1}=\frac{y-2}{-2}=\frac{z-0}{1}$ B) $\frac{x}{1}=\frac{y}{-2}=\frac{z-1}{1}$

C) $\frac{x+1/2}{1}=\frac{y-1}{-2}=\frac{z-1/2}{1}$ D) $\frac{x-1}{2}=\frac{y+2}{-1}=\frac{z-2}{2}$

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Key. A,B,C

Sol. x + y + z – 1 = 0

4x + y – 2z + 2 = 0

∴ direction ratios of the line are < - 3, 6, - 3 >

i.e. < 1, - 2, 1 >

Let z = k, then x = k – 1, y = 2 – 2k

i.e. (k – 1, 2 – 2k, k) is any point on the line

∴ (-1, 2, 0), (0, 0, 1) and $\left(-\frac{1}{2},1,\frac{1}{2}\right)$ are points on the line

∴ (A), (B) and (C) are correct options

13. Consider the planes 3x – 6y + 2z + 5 = 0 and 4x – 12y + 3z = 3. The plane 67x – 162y + 47z + 44 = 0 bisects that angle between the given planes which

A) Contains origin B) is acute C) is obtuse D) none of these

Key. A,B

Sol. 3x – 6y + 2z + 5 = 0 …(i)

- 4x + 12y – 3z + 3 = 0 …(ii)

$$\frac{3x-6y+2z+5}{\sqrt{9+36+4}}=\frac{-4x+12y-3z+3}{\sqrt{16+144+9}}$$

Bisects the angle between the planes that contains the origin

13(3x – 6y + 2z + 5) = 7(- 4x + 12y – 3z + 3)

39x – 78y + 26z + 65 = 0 28x + 84y – 2 1z + 21

67x – 162y + 47z + 44 = 0 …(iii)

Further 3 x (-4)+ (-6) (12) + 2 x (- 3) < 0

∴ origin lies in acute angle

14. The plane $lx + my = 0$ is rotated about its line of intersection with the plane z = 0, through an angle α, then equation of plane in its new position may be

A) $lx+my+z\sqrt{l^2+m^2}\tan\alpha=0$ B) $lx+my-z\sqrt{l^2+m^2}\tan\alpha=0$

C) data is not sufficient D) None of these

Key. A,B

Sol. Equation of required plane is

$lx + my + \lambda z = 0$

angle between (i) & $lx + my = 0$ is α.

$$\Rightarrow \quad \cos\alpha=\frac{l^2+m^2}{\sqrt{l^2+m^2}\sqrt{l^2+m^2+\lambda^2}}$$

$$\Rightarrow \quad \cos^2\alpha=\frac{l^2+m^2}{l^2+m^2+\lambda^2} \quad \Rightarrow \quad \lambda=\pm\sqrt{l^2+m^2}\tan\alpha$$

Hence equation of plane is

$$lx+my\pm z\sqrt{l^2+m^2}\tan\alpha=0$$

15. If p_1, p_2, p_3 denotes the distance of the plane 2x – 3y + 4z + 2 = 0 from the planes 2x – 3y + 4z + 6 = 0, 4x – 6y + 8z + 3 = 0 and 2x – 3y + 4z – 6 = 0 respectively, then

a) $p_1 + 8p_2 - p_3 = 0$ b) $p_3 = 16p_2$

c) $8p_2 = p_1$ d) $p_1 + 2p_2 + 3p_3 = \sqrt{29}$

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Ans. a,b,c,d

Since all planes are parallel

$$P_1 = \frac{4}{\sqrt{29}},\ P_2 = \frac{1}{2\sqrt{29}},\ P_3 = \frac{8}{\sqrt{29}}$$

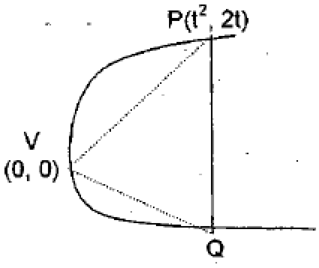
16. Let PQ be the chord of the parabola $y^2 = 4x$. A circle drawn with PQ as a diameter passes through the vertex V of the parabola. If area of triangle PVQ = 20, then the coordinates of P are

a) (16, 8) b) (16, -8) c) (-16, 8) d) (-16, -8)

Ans. a,b

Sol. Slope of PV = $\frac{2t-0}{t^2-0} = \frac{2}{t}$

$\therefore$ equation of QV is $y = -\frac{t}{2}\cdot x$



Solving it with $y^2 = 4x,\ Q \equiv \left(\frac{16}{t^2}, -\frac{8}{t}\right)$

Area $(\Delta PVQ) = \frac{1}{2}(PV)(VQ) = 20$

$$PV^2 \cdot VQ^2 = 40^2 \ or\ (t^2)^2 + (2t)^2\left\{\left(\frac{16}{t^2}\right)^2 + \left(-\frac{8}{t}\right)^2\right\} = 1600$$

$$\Rightarrow (t^2-16)(t^2-1) = 0 \Rightarrow t = \pm 4, \pm 1$$

17. The line y = x + 5 touches

a) the parabola $y^2 = 20x$ b) the circle $x^2 + y^2 = 25$

c) the ellipse $9x^2 + 16y^2 = 144$ d) the hyperbola $\frac{x^2}{29} - \frac{y^2}{4} = 1$

Ans. a,c,d

Sol. y = x + 5 = y = mx + c $\Rightarrow$ m = 1, c = 5

It touches the parabola $y^2 = 20x$ since $c = \frac{a}{m}$

It touches the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, since $c^2 = a^2m^2 + b^2$

It touches the hyperbola $\frac{x^2}{29} - \frac{y^2}{4} = 1$, since $c^2 = a^2m^2 - b^2$

It does not touches the circle $x^2 + y^2 = 25$, since $c^2 \neq (1 + m^2)$

18. Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ has least area but contain the circle $(x-1)^2 + y^2 = 1$

a) equation of ellipse is $2x^2 + 6y^2 = 9$ b) equation of ellipse is $6x^2 + 2y^2 = 9$

c) eccentricity of ellipse is $e = \sqrt{\frac{2}{3}}$ d) eccentricity of ellipse is e=1/2

Ans. a, c

Sol. On solving $(b^2 - a^2)x^2 + 2a^2x - a^2b^2 = 0$

Now D = 0

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$a^2 - \left(a^2e^2b^2\right) = 0 \Rightarrow b = \frac{1}{e}$

Also $a^2 = \frac{b^2}{1-e^2} \Rightarrow a^2 = \frac{1}{e^2\left(1-e^2\right)} \Rightarrow a = \frac{1}{e\sqrt{1-e^2}}$

$s = \pi ab = \frac{\pi}{e^2\sqrt{1-e^2}}$

$\frac{ds}{de} = \pi\left(\frac{e\left(3e^2-2\right)}{e^4\left(1-e^2\right)^{3/2}}\right)$ since 0 < e < 1

$\Rightarrow$ s is least when $e = \sqrt{\frac{2}{3}}$

$\therefore$ ellipse is $2x^2 + 6y^2 = 9$

19. $\sqrt{x} + \sqrt{y} = 1$ is a part of parabola whose

a) focus is $\left(\frac{\sqrt{2}+1}{4}, \frac{\sqrt{2}+1}{4}\right)$ b) directrix is $x + y = \frac{\sqrt{2}-1}{2}$

c) latus rectum is 2 unit d) vertex is $\left(\frac{1}{4}, \frac{1}{4}\right)$

Ans. a,b,c,d

Sol. $(y - x - 1)^2 = 4x \Rightarrow x^2 - 2xy + y^2 - 2x - 2y + 1 = 0$

$\Rightarrow \left(x - y + \lambda\right)^2 = 2x + 2y - 1 + 2\lambda x - 2\lambda y + \lambda^2,\ \lambda \in R$

We choose λ such that

$x - y + \lambda = 0$ and $2x + 2y - 1 + 2\lambda x - 2\lambda y + \lambda^2 = 0$ are perpendicular lines

$\Rightarrow \lambda = 0$ now solve it.

20. If the line y = mx, is one of the bisector of the lines $x^2 + 4xy - y^2 = 0$, then the value of m is equal to

a) $\frac{-1+\sqrt{5}}{2}$ b) $\frac{1+\sqrt{5}}{2}$ c) $\frac{-1-\sqrt{5}}{2}$ d) $\frac{1-\sqrt{5}}{2}$

Ans. a,c

Sol. $\frac{x^2-y^2}{1-(-1)} = \frac{xy}{2}$ put y = mx and solve $m = \frac{-1\pm\sqrt{5}}{2}$

21. The point of intersection of the lines $\sqrt{3}x - y - 4\sqrt{3} = 0$ and $\sqrt{3}mx + my - 4\sqrt{3} = 0$ describes

a) an ellipse b) a hyperbola

c) conic with eccentricity $\frac{1}{2}$ d) conic with eccentricity 2

Ans. b, d

Sol. Eliminating m, we get $3x^2 - y^2 = 48$, which is a hyperbola.

Its eccentricity = $\sqrt{1+\frac{48}{16}} = 2$

22. If P is a point inside a convex quadrilateral ABCD such that $PA^2 + PB^2 + PC^2 + PD^2$ is twice the area of the quadrilateral then which of the following statements are correct.

a) PA, PB, PC, PD are all equal

b) ABCD must be a square and P must be its centre

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c) ABCD must be a square

d) ABCD may be any quadrilateral

Ans. a,b,c

Sol. $PA^2 + PB^2 = 2PA \cdot PB$

$\Rightarrow \sum PA^2 \geq 2\,area\left(PAB + PBC + PCB + PDA\right)$

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3-D Geometry

Assertion Reasoning Type

1. Statement 1: Consider the planes $3x-6y-2z=15$ and $2x+y-2z=5.$

The parametric equations of the line of intersection of the given planes are $x=3+14t, y=1+2t, z=15t; t$ being the parameter

Statement 2: The vector $14\hat{i}+2\hat{j}+15\hat{k}$ is parallel to the line of intersection of the given planes.

1)Statement I is True, Statement II is True and Statement II is correct explanation of Statement I
2)Statement I is True, Statement II is True but Statement II is not correct explanation of Statement I
3)Statement I is True, Statement II is False
4)Statement I is False, Statement II is True

Key. 4

Sol. Equation of the line in statement-1 can be written as $\frac{x-3}{14}=\frac{y-1}{2}=\frac{z-0}{15}=t.$ This is the line of intersection of the planes, then the point $(3,1,0)$ which lies on the line must be on both the planes which is not true and hence the statement-1 is false. Direction ratios of the line of intersection of the given planes is $(-6)(-2)-(-2)(1),(-2)(2)-(-2)(3),3(1)-(2) i.e. 14,2,15;$ showing that the vector in statement-2 is parallel to the line of intersection of the planes and thus statement-2 is True.

2. Statement 1: $L_1:\frac{x+1}{3}=\frac{y+2}{1}=\frac{z+1}{2}, L_2:\frac{x-2}{1}=\frac{y+2}{2}=\frac{z-3}{3}.$

The unit vector perpendicular to both L_1 and L_2 is $\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}.$

Statement 2: The distance of the point $(1,1,1)$ from the plane passing through the point $(-1,-2,-1)$ and whose normal is perpendicular to both the lines L_1 and L_2 is $23/5\sqrt{3}.$

1)Statement I is True, Statement II is True and Statement II is correct explanation of Statement I
2)Statement I is True, Statement II is True but Statement II is not correct explanation of Statement I
3)Statement I is True, Statement II is False
4)Statement I is False, Statement II is True

Key. 3

Sol. L_1 and L_2 are parallel to the vectors $a=3i+j+2k$ and $b=i+2j+3k$ respectively. The vector perpendicular to both L_1 and L_2 is $a\times b$ and the required unit vector is $\frac{-i-7j+5k}{\sqrt{1+49+25}}$, so

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statement-1 is true. In satatement-2, equation of the plane is $-(x+1)-7(y+2)+5(z+1)=0$ whose distance from $(1,1,1)$ is $13/5\sqrt{3}$, so the statement-2 is false.

3. Statement 1: If $x, y, z \in R$ and $3x+4y+5z=10\sqrt{2}$ then the least value of $x^2+y^2+z^2$ is 4.

Statement 2: If π is a given plane and $'P'$ is a given point then the point on plane which is nearest to $'P'$ is the foot of the perpendicular from $'P'$ to the plane.

Key. A

Sol. Conceptual

4. STATEMENT–1: The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{3}$ and $\frac{x-1}{-3}=\frac{y-4}{2}=\frac{z-5}{1}$ are skew lines.

STATEMENT–2: Two non-parallel, non-intersecting lines are skew lines

Key: D

Hint: Since $\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & 3 \\ -3 & 2 & 1 \end{vmatrix}=0$

So; the two lines are not skew

5. Statement - 1: The lines $\frac{x-1}{1}=\frac{y}{-1}=\frac{z+1}{1}$ and $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z}{3}$ are coplanar and equation of the plane containing them is 5x+ 2y – 3z-8 = 0.

Statement - 2: The line $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z}{3}$ is perpendicular to the plane 3x+6y +9z-8=0 and parallel to the plane x+y-z=0

Key. B

Sol. $\begin{vmatrix} 1-2 & 0+1 & -1-0 \\ 1 & -1 & 1 \\ 1 & 2 & 3 \end{vmatrix}=0 \Rightarrow$ given lines are coplanar

Equation of the plane is $\begin{vmatrix} x-1 & y & z+1 \\ 1 & -1 & 1 \\ 1 & 2 & 3 \end{vmatrix}=0$

i.e., 5x + 2y – 3z - 8 = 0

Since $\frac{1}{3}=\frac{2}{6}=\frac{3}{9} \Rightarrow \frac{x-2}{1}=\frac{y+1}{2}=\frac{z}{3}$ is perpendicular to the plane

And also 1(1) + 2(1) +3 (-1) = 0

$\Rightarrow \frac{x-2}{1}=\frac{y+1}{2}=\frac{z}{3}$ is parallel to x+ y –z = 0

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6. Assertion (A): The area of the triangle whose vertices are $A(1,2,3); B(-2,1,-4); C(3,4,-2)$ is $\frac{\sqrt{1218}}{2}$ square units.

Reason (R): If A is area of $\Delta ABC; A_x, A_y, A_z$ are areas of projections of ΔABC on yz, zx, xy planes respectively then area of $\Delta ABC = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Key. A

Sol. Conceptual

7. Statement-1: P is a point (a, b, c). Let A, B, C be the images of P in yz, zx ad xy planes respectively, then equation of the plane passing through the points A, B and C is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1.$

Statement-2: The image of a point P in a plane is the foot of the perpendicular drawn from P on the plane.

Key: C

Hint: Statement-2 is not true because image of P in a plane is a point M such that PM is perpendicular to the plane and the mid-point of PM lies on the plane.

The points A, B, C are respectively (–a, b, c), (a, –b, c) and (a, b, –c) which lies on the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ and thus statement-1 is true.

8. Consider the planes 3x – 6y – 2z = 15 and 2x + y – 2z = 5.

Statement-1: The parametric equation of the line of intersection of the given planes is x = 3 + 14t, y = 1 + 2t, z = 15t, t being the parameter.

Statement-2: The vector $14\hat{i}+2\hat{j}+15\hat{k}$ is parallel to line of intersection of the given planes.

Key: d

Hint: Equation of the line in statement-1 can be written as $\frac{x-3}{14}=\frac{y-1}{2}=\frac{z-0}{15}=t$.

If this represents the line of intersection of the given planes, then the point (3, 1, 0) which lies on the line must be on both the planes which is not true. So statement-1 is false. The direction ratios of the line of intersection of the planes is

$$(-6)(-2)-(-2)(1),\ (-2)(2)-(-2)(3),(3)(1)-(2)(-6)$$

i.e. 14, 2, 15showing that the vector $14\hat{i}+2\hat{j}+15\hat{k}$ is parallel to the line of intersection of the given planes and hence the statement-2 is true.

9. Consider the lines L_1 : $\frac{x-3}{2}=\frac{y+1}{-3}=\frac{z+2}{1}$ and $L_2:\frac{x-7}{-3}=\frac{y}{1}=\frac{z+7}{2}$

STATEMENT–1

L_1 and L_2 are coplanar

because

STATEMENT–2

L_1 and L_2 intersect.

Key. A

Sol. Any point on the line L_1 is $(2r_1 + 3, -3r_1 - 1, r_1 - 2)$

Any point on the line L_2 is $(-3r_2 + 7, r_2, 2r_2 - 7)$

Let the lines L_1 and L_2 intersect at P

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$\therefore$ $2r_1 + 3 = -3r_2 + 7 \Rightarrow 2r_1 + 3r_2 = 4$ .... (i)
Also $-3r_1 - 1 = r_2 \Rightarrow -3r_1 - r_2 = 1$ ... (ii)
and $r_1 - 2 = 2r_2 - 7 \Rightarrow r_1 - 2r_2 = -5$ ... (iii)
Solving (i) & (iv), we get $r_1 = = -1, r_2 = 2$
Clearly $r_1 = -1$ and $r_2 = 2$ satisfy equation (iii)
$\therefore$ lines L_1 and L_2 intersect $\Rightarrow L_1$ and L_2 are coplanar.

10. Statement - 1: If the planes $x = cy + bz$, $y = az + cx$ and $z = bx + ay$ pass through a line, then $a^2 + b^2 + c^2 + 2abc = 1$.
Statement - 2: $\begin{vmatrix} 1 & -c & -b \\ -c & 1 & -a \\ -b & -a & 1 \end{vmatrix} = 0$.

Key. A
Sol. Conceptual

11. Statement - 1: Let A, B, C be points with position vectors $r_1 = 2\hat{i} - \hat{j} + \hat{k},\ r_2 = \hat{i} + 2\hat{j} + 3\hat{k}$ and $r_3 = 3\hat{i} + \hat{j} + 2\hat{k}$ relative to origin 'O'. The shortest distance between the point B and plane OAC is $\sqrt{5/7}$.
Statement - 2: Shortest distance $= \dfrac{(\overrightarrow{OA} \times \overrightarrow{OC}).\overrightarrow{OB}}{|\overrightarrow{OA} \times \overrightarrow{OC}|}$.

Key. D
Sol. Conceptual

12. Statement - 1: The lines $\dfrac{x+1}{1} = \dfrac{y-2}{3} = \dfrac{z}{K}, \dfrac{x-1}{2} = \dfrac{y+1}{-1} = \dfrac{z-2}{3}$ will be coplanar for more than one value of K.
Statement - 2: Two lines in a plane will be either parallel or intersecting.

Key. D
Sol. $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

13. Statement-I: The line $\dfrac{x-2}{3} = \dfrac{y-3}{4} = \dfrac{z-4}{5}$ is parallel to the plane $2x + y - 2z = 0$
Statement-II: The normal of the given plane is $\perp^{er}$ to the given line
Key. A
Sol. dr's of normal to the given plane is (2, 1, -2)
Drs of the given line = (3, 4, 5)
(2)(3) + 1 (4) + (-2) (5) = 0
$\therefore$ line is parallel to the given plane.

14. STATEMENT-1: The distance of the point of intersection of the line $\dfrac{x-3}{1} = \dfrac{y-4}{2} = \dfrac{z-5}{2}$ and the plane $x + y + z = 17$ from the point $(3, 4, 5)$ is 3
STATEMENT-2 : The distance from the point (x_1, y_1, z_1) to the plane $ax + by + cz + d = 0$

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measured parallel to the line $\dfrac{x-x_1}{\cos\alpha} = \dfrac{y-y_1}{\cos\beta} = \dfrac{z-z_1}{\cos\gamma}$ is

$$\frac{|a x_1 + b y_1 + c z_1 + d\,|}{\sqrt{a\,Cos\,\alpha + b\,Cos\,\beta + c\,Cos\,\gamma}}$$

Key. C
Sol. Conceptual

15. Statement - 1: The lines $\dfrac{x-1}{1} = \dfrac{y}{-1} = \dfrac{z+1}{1}$ and $\dfrac{x-2}{1} = \dfrac{y+1}{2} = \dfrac{z}{3}$ are coplanar and equation of the plane containing them is 5x+ 2y – 3z-8 = 0.

Statement - 2: The line $\dfrac{x-2}{1} = \dfrac{y+1}{2} = \dfrac{z}{3}$ is perpendicular to the plane 3x+6y +9z-8=0 and parallel to the plane x+y-z=0

Key. B

Sol. $\begin{vmatrix} 1-2 & 0+1 & -1-0 \\ 1 & -1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow$ given lines are coplanar

Equation of the plane is $\begin{vmatrix} x-1 & y & z+1 \\ 1 & -1 & 1 \\ 1 & 2 & 3 \end{vmatrix} = 0$ i.e., 5x + 2y – 3z - 8 = 0

Since $\dfrac{1}{3} = \dfrac{2}{6} = \dfrac{3}{9} \Rightarrow \dfrac{x-2}{1} = \dfrac{y+1}{2} = \dfrac{z}{3}$ is perpendicular to the plane

And also 1(1) + 2(1) +3 (-1) = 0 $\Rightarrow \dfrac{x-2}{1} = \dfrac{y+1}{2} = \dfrac{z}{3}$ is parallel to x+ y –z = 0

16. Statement – 1: line $\dfrac{x-1}{3} = \dfrac{y-2}{11} = \dfrac{z+1}{11}$ lies in the plane 11x – 3z – 14 = 0

Statement – 2: A straight line lies in a plane if the line is parallel to the plane and a point of the line lies in the plane

Key. A

Sol. S-1: (1, 2, – 1) is a point on the line and 11 + 3 – 14 = 0

$\therefore$ The point lies on the plane 11 x – 3z – 14 = 0

Further 3 × 11 + 11 (- 3) = 0

$\therefore$ The line lies in the plane

S-2: trivially true

17. Statement – 1: Let $A\left(\vec{i}+\vec{j}+\vec{k}\right)$ and $B\left(\vec{i}-\vec{j}+\vec{k}\right)$ be two points, then point $P\left(2\vec{i}+3\vec{j}+\vec{k}\right)$ lies exterior to the sphere with AB as one of its deameters.

Statement – 2: If A and B are any two points and P is a point in space such that $\vec{PA}\,.\,\vec{PB} > 0$, then the point P lies exterior to the sphere with AB as one of its diameters.

Key. A

Sol. Statement – 1 $\overrightarrow{PA}.\,\overrightarrow{PB} = 9 > 0$

$\therefore$ P is exterior to the sphere

Statement – 2: is true (standard result)

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18. Statement – 1: If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then equation $\vec{r} \times \left(2\hat{i} - \hat{j} + 3\hat{k}\right) = 3\hat{i} + \hat{k}$ represents a straight line.

Statement – 2: If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then equation $\vec{r} \times \left(\hat{i} + 2\hat{j} - 3\hat{k}\right) = 2\hat{i} - \hat{j}$ represents a st. line

Key. D

Sol. Statement – 2: $\vec{r} \times \left(\hat{i} - 2\hat{j} - 3\hat{k}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 2 & -3 \end{vmatrix}$

$$\hat{i}(-3y - 2z) - \hat{j}(-3x - z) + \hat{k}(2x - y)$$

$\therefore$ -3y – 2z = 2, 3x + z = - 1, 2x – y = 0

i.e. - 6x – 2z = 2, 3x + z = - 1

$\therefore$ straight lien 2x – y = 0, 3x + z = 1

Statement – 1: $\vec{r} \times \left(2\hat{i} - \hat{j} + 3\hat{k}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 2 & -1 & 3 \end{vmatrix}$

$$= \hat{i}(3y + z) - \hat{j}(3x - 2z) + \hat{k}(-x - 2y)$$

$\therefore$ 3y + z = 3, 3x – 2z = 0, - x – 2y = 1

3x – 2(3 – 3y) = 0

$\Rightarrow$ 3x + 6y = 6 $\Rightarrow$ x + 2y = 2

Now x + 2y = - 1, x + 2y = 2 are parallel planes

$\therefore$ $\vec{r} \times \left(2\hat{i} - \hat{j} + 3\hat{k}\right) = 3\hat{i} + \hat{k}$ is not a straight line

19. Statement – 1: Let θ be the angle between the line $\frac{x-2}{2} = \frac{y-1}{-3} = \frac{z+2}{-2}$ and the plane x + y – z = 5.

Then $\theta = \sin^{-1}\frac{1}{\sqrt{51}}$

Statement – 2: Angle between a st. line and a plane is the complement of angle between the line and normal to the plane.

Key. D

Sol. $\sin\theta \left|\frac{2-3+2}{\sqrt{4+9+4}\sqrt{3}}\right| = \frac{1}{\sqrt{51}}$

$\therefore$ Statement – 1 is true. Statement – 2 is true by definition

20. Statement – 1: A point on the straight line 2x + 3y -4z = 5 and 3x – 2y + 4z = 7 can be determined by taking x = k and then solving the two equations for y and z, where k is any real number.

Statement – 2: If $c' \neq kc$, then the straight line ax + by + cz + d = 0, kax + kby + $c'z + d' = 0$, does not intersect the plane $z = \alpha$, where α is any real number.

Key. B

Sol. Statement – 1

3y – 4z = 5 – 2k

- 2y + 4z = 7 – 3k

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$\therefore$ x = k, y = 12 – 5k, $z = \frac{31-13k}{4}$ is a point on the line for all real values of k

Statement is true

Statement – 2

direction ratios of the straight line are $< bc' - kbc, kac - ac', 0 >$

direction ratios of normal to be plane < 0, 0, 1 >

Now $0 \times (bc' - kbc) + 0 \times (kac - ac') + 1 \times 0 = 0$

$\therefore$ the straight line is parallel to the plane

$\therefore$ statement is true but does not explain statement – 1

21. Statement – I: If A_x, A_y, A_z be projection of an area A on yz, zx, xy planes respectively then $A_x^2 + A_y^2 + A_z^2 = A^2$

Statement – II : If l, m, n be direction cosines of normal to the area A then A_x = lA, A_y= mA and A_z = nA

a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1

b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1

c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. a

Sol. Conceptual

22. Statement – 1: The distance between the lines represented by $x^2 + 2\sqrt{2}xy + 2y^2 + 4\sqrt{2}x + 4y + 1 = 0$ is 2

Statement – 2: Distance between the lines ax + by + c = 0 and ax + by + c_1 = 0 is $\frac{|c - c_1|}{\sqrt{a^2 + b^2}}$

a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for statement-1

b) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for statement-1

c) Statement-1 is true, Statement-2 is false d) Statement-1 is false, Statement-2 is true

Ans. d

Sol. The given conic is a parabola not pair of straight lines.

23. Statement – I: Lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ are concurrent if $\frac{a_1}{c_1} + \frac{a_2}{c_2} + \frac{a_3}{c_3}$

Statement – II : Aforesaid lines will be concurrent if $\left(\frac{a_1}{c_1}, \frac{b_1}{c_1}\right), \left(\frac{a_2}{c_2}, \frac{b_2}{c_2}\right)$ and $\left(\frac{a_3}{c_3}, \frac{b_3}{c_3}\right)$ are collinear

a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1

b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1

c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. a

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Sol. Above lines are concurrent as $\frac{a_1}{c_1} = \frac{a_2}{c_2} = \frac{a_3}{c_3}$ implies all the points lie on the line parallel to y-axis and hence collinear.

24. Statement – I: If the circumference of the circle $x^2 + y^2 - 2x + 8y - q^2 = 0$ is bisected by the circle $x^2 + y^2 + 4x + 22y + p^2 = 0$, then pq can not exceed 25
Statement – II : Common chord of two circles must be equidistant from the centres of both the circles
a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1
b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1
c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. c

Sol. Equation of common chord is $6x + 14y + p^2 + q^2 = 0$
It must pass through the centre of the first circle
$\therefore p^2 + q^2 = 50$
Now $\frac{p^2 + q^2}{2} \geq pq \Rightarrow pq \leq 25$

25. a, b, c are positive numbers and the chord of contact of the tangents drawn from any point on the circle $x^2 + y^2 = a^2$ to the circle $x^2 + y^2 = b^2$ touches the circle $x^2 + y^2 = c^2$
Statement – I: $a - 2bk + ck^2$ is non-negative for every $k \in R$
Statement – II : The given circles are concentric
a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1
b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1
c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. b

Sol. Chord of contact of any point $(a\cos\theta, a\sin\theta)$ with respect to $x^2 + y^2 = b^2$ is
$x\cos\theta + y\sin\theta = \frac{b^2}{a}$ which touches $x^2 + y^2 = c^2 \Rightarrow b^2 = ac \Rightarrow (2b)^2 = 4ac$
Thus $cx^2 - 2bx + a \geq 0 \;\forall\; x \in R$

26. Statement – I: Through $(\lambda, \lambda + 1)$ there cannot be more than one normal to parabola $y^2 = 4x$ if $\lambda < 2$
Statement – II : The points $(\lambda, \lambda + 1)$ lies outside the parabola for all $\lambda \neq 1$
a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1
b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1
c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. b

Sol. Any normal to the parabola $y^2 = 4x$ is $y + tx = 2t + t^3$
If this passes through $(\lambda, \lambda+1)$
$\lambda + 1 + t\lambda = 2t + t^3$
$t^3 + t(2-\lambda) - \lambda - 1 = 0$
$f(t) = t^3 + t(2-\lambda) - \lambda - 1$

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If $\lambda < 2,\ f'(t) = 3t^2 + (2-\lambda) > 0$

f(t) = 0 will have only one real root

$\Rightarrow$ statement 1 is true

Statement 2 is also true since $(\lambda+1)^2 > 4\lambda$ is true for all $\lambda \neq 1$.

Statement 1 is true but not follow from statement 2.

27. Statement – I: $\frac{5}{3}\, and\, \frac{5}{4}$ are the eccentricities of a hyperbola and its conjugate.

Statement – II : If e and e_1 are the eccentricities then $ee_1 > 1$

a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1

b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1

c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. b

Sol. Use $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$

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3D-Geometry

Comprehension Type

Passage – 1:

Let PQRS be a rectangle of size 9 × 3, if it is folded along QS such that plane PQS is perpendicular to plane QRS and point R moves to point T.

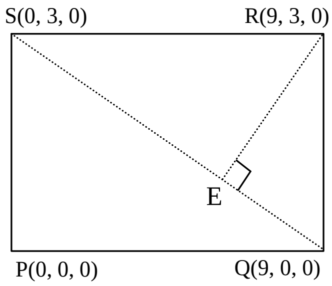
1. Distance between the points P and T will be

(A) $\sqrt{90}$ (B) $\frac{3}{5}\sqrt{205}$

(C) $\frac{4}{5}$ (D) none of these

Key: B

Hint: Equation of line QS in 2-D will be x + 3y - 9 = 0, RE = $\frac{9}{\sqrt{10}}$ and E ≡ $\left(\frac{81}{10},\frac{3}{10},0\right)$, so point T will be $\left(\frac{81}{10},\frac{3}{10},\frac{9}{\sqrt{10}}\right)$,



Hence PT = $\frac{3}{5}\sqrt{205}$

2. If θ is angle between the line QP and QT then tanθ is equal to

(A) $\frac{3}{10}$ (B) $\frac{10}{3}$

(C) $\frac{\sqrt{91}}{3}$ (D) none of these

Key: C

Hint: Direction ratio of QP = 9, 0, 0 direction ratio of QT ≡ $\frac{9}{10},\frac{-3}{10},\frac{-9}{\sqrt{10}}$

So, cosθ = $\frac{3}{10}$ ⇒ tanθ = $\frac{\sqrt{91}}{3}$

3. Shortest distance between the edges PQ and TS is

(A) $3\sqrt{\frac{10}{19}}$ (B) $\sqrt{\frac{10}{19}}$

(C) $2\sqrt{\frac{10}{19}}$ (D) none of these

Key: A

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Hint: Shortest distance between the lines $\bar{r}=\bar{a}+\lambda\bar{\alpha}$ and $\bar{r}=\bar{b}+\mu\bar{\beta}$ is given by $\frac{|(\bar{a}-\bar{b}).(\bar{\alpha}\times\bar{\beta})|}{|\bar{\alpha}\times\bar{\beta}|}$ = $3\sqrt{\frac{10}{19}}$

Passage – 2:

Let a plane P_1 passes through the point $(1,-2,3)$ and is parallel to the plane P_2 given by $2x-2y+z=0$.

4. The distance of the point $(-1,2,0)$ from the plane P_1 is

(A) 2 units (B) 3 units (C) 5 units (D) 7 units

5. The coordinate of the foot of perpendicular drawn from point $(1,-2,3)$ to the plane P_2 is

(A) $(0,0,0)$ (B) $(-1,0,2)$ (C) $(1,0,-2)$ (D) $(2,0,-4)$

6. The distance between parallel planes P_1 and P_2 is

(A) 2 units (B) 3 units (C) 5 units (D) 7 units

Key: C-B-B

Hint: The equation of the plane P_1 is $2x-2y+z=\lambda$

Since, it passes through $(1,-2,3)$

Then $\lambda=9$

So, P_1 is $2x-2y+z=9$

Its distance from point $(-1,2,0)$ is $\frac{|2\times(-1)-2\times(2)+0-9|}{3}=5$

Now the line perpendicular to plane P_2 and passing through $(1,-2,3)$ is given by $\frac{x-1}{2}=\frac{y+2}{-2}=\frac{z-3}{1}$

Any point on this line can be taken as $(2t+1, -2t-2, t+3)$

If it lies on plane P_2 then we have

$2(2t+1)-2(-2t-2)+t+3=0$

$\Rightarrow 9t+9=0$

$\Rightarrow t=-1$

So, the coordinate of the foot of perpendicular drawn from point $(1,-2,3)$ to the plane P_2 is $(-1,0,2)$.

Again the distance between the parallel planes $2x-2y+z-9=0$ and $2x-2y+z=0$ is given by $\frac{|9-0|}{\sqrt{2^2+(-2)^2+1}}=\frac{9}{3}=3$ units

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Passage – 3:

Consider the planes $S_1 : 2x - y + z = 5,\ S_2 : x + 2y - z = 4$ having normals $N_1\ and\ N_2$ respectively. $P(2,-1,0)\ and\ Q(1,1,-1)$ are points on S_1 and S_2 respectively.

7. A vector of magnitude $\sqrt{140}$ units and lies along the line of intersection of $S_1\ and\ S_2$ is

A) $2(5i+3j-k)$ B) $2(i+3j+5k)$ C) $2i-6j-10k$ D) $2(3i-j+5k)$

8. The distance of the origin from the plane passing through the point (1, 1, 1) and whose normal is perpendicular to N_1 and N_2 is

A) $\frac{9}{\sqrt{61}}$ B) $\frac{11}{\sqrt{35}}$ C) $\frac{10}{\sqrt{61}}$ D) $\frac{7}{\sqrt{35}}$

9. Let L_1 be the line passing through P and parallel to N_1, L_2 be the line passing through Q and parallel to N_2. The shortest distance between $L_1\ and\ L_2$ is

A) $\frac{2}{\sqrt{35}}$ B) $\frac{8}{\sqrt{35}}$ C) $\frac{14}{\sqrt{35}}$ D) $\frac{17}{\sqrt{35}}$

Key: C-D-A

Hint: Q7, 8, 9

Unit vector along line of intersection of S_1 and

$$S_2 = \pm\frac{(2i-j+k)\times(i+2j-k)}{|(2i-j+k)\times(i+2j-k)|} = \pm\frac{(-i+3j+5k)}{\sqrt{35}}$$

7. $\pm 2\sqrt{35}\times\frac{(-i+3j+5k)}{\sqrt{35}} = \pm 2(-i+3j+5k)$

8. Equation of plane is $-1(x-1)+3(y-1)+5(z-1)=0$

9. $L_1 : \frac{x-2}{2} = \frac{y+1}{-1} = \frac{z}{1},\ L_2 : \frac{x-1}{1} = \frac{y-1}{2} = \frac{z+1}{-1}$ are skew lines

Passage – 4:

If P = (1, 6, 3) be the given point, $L = \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ be the given line, $\pi : 2x+3y-z=7$ be the given plane.

10. Equation of the plane passing through 'P' and perpendicular to the plane π is

a) $x+y+5z=1$ b) $5x+3z=39$ c) $3x-2y=15$ d) $3x+y+9z=36$

Key. D

Sol. Conceptual

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11. If θ is the angle between the plane π and the line L is given by

a) $\cos\theta = \frac{5}{14}$ b) $\sin\theta = \frac{5}{14}$ c) $\cos\theta = \frac{1}{14}$ d) $\sin\theta = \frac{1}{14}$

Key. B

Sol. Conceptual

12. Length of perpendicular from 'p' on the 'L' is:

a) $\sqrt{13}$ b) $\sqrt{14}$ c) $\sqrt{46}$ d) $\frac{10}{\sqrt{14}}$

Key. A

Sol. Conceptual

Passage – 5:

$(a_1a_2 + b_1b_2 + c_1c_2)^2 \le (a_1^2 + b_1^2 + c_1^2)(a_2^2 + b_2^2 + c_2^2)$ and equality holds when $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

13. If (l, m, n) are direction cosines of a line then the range of values of '3l + 4m – 5n' is

(A) $[-5\sqrt{2}, 5\sqrt{2}]$ (B) $[-6\sqrt{2}, 6\sqrt{2}]$

(C) $[-7\sqrt{2}, 7\sqrt{2}]$ (D) $[-8\sqrt{2}, 8\sqrt{2}]$

Key. A

14. If '3l + 4m – 5n' takes its maximum value, then the value of |lm + mn + nl| is equal to

(A) 0.12 (B) 0.46

(C) 1.08 (D) 1.72

Key. B

15. If ax + by + cz = $\sqrt{a+b+c}$ (a, b, c are fixed +ve real nos.) then the minimum value of $ax^2 + by^2 + cz^2$ is

(A) 1 (B) a + b + c

(C) $a^2 + b^2 + c^2$ (D) $(a + b + c)^2$

Key. A

Sol. 13. $(3\ell + 4m - 5n)^2 \le (3^2 + 4^2 + (-5)^2)(\ell^2 + m^2 + n^2) = 50 \times 1$

14. $\frac{\ell}{3} = \frac{m}{4} = \frac{n}{-5} = \frac{3\ell + 4m - 5n}{3^2 + 4^2 + (-5)^2} = \frac{5\sqrt{2}}{50} = \frac{1}{5\sqrt{2}}$

15. $(ax + by + cz)^2 = (\sqrt{a}\,.\,\sqrt{a}x + \sqrt{b}\,.\,\sqrt{b}\,y + \sqrt{c}\,.\,\sqrt{c}\,y)^2 \le (a+b+c)(ax^2 + by^2 + cz^2)$

$\Rightarrow a + b + c \le (a+b+c)(ax^2 + by^2 + cz^2) \Rightarrow ax^2 + by^2 + cz^2 \ge 1$

Passage – 6:

L is the line of intersection of two non-parallel planes π_1, π_2. L_1 is a straight line which is perpendicular to *L* and points on L_1 are equidistant from the planes π_1, π_2. Equation of π_1 is $2x + 3y + z = 1$ and equations of L_1 are $6x = 3y = 2z$

16. The direction ratios of *L* are

A) $(6, -3, 0)$ B) $(7, -5, 1)$ C) $(5, -1, -1)$ D) $(11, -1, -3)$

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Key. B

17. The direction ratios of normal to the plane containing L, L_1 are

A) $(12, -3, 4)$ B) $(17, 20, -19)$ C) $(13, -2, -3)$ D) $(14, -5, 2)$

Key. B

18. The X-intercept of plane π_2 is

A) $-7/3$ B) 5 / 2 C) 2 D) 8 / 3

Key. A

Sol. **16 – 18**

Vector parallel to *L* is $\begin{vmatrix} i & j & k \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

Equations of plane containing L, L_1 is $\begin{vmatrix} x & y & z \\ 1 & 2 & 3 \\ 7 & -5 & 1 \end{vmatrix} = 0$. i.e., $17x + 20y - 19z = 0$

$17x + 20y - 19z = 0$ bisects an angle between π_1, π_2

Passage – 7:

Two lines whose equations are $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{\lambda}$ and $\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-2}{3}$ lie in the same plane, then

19. The value of $\sin^{-1}(\sin \lambda)$ is equal to

A) 3 B) $\pi - 3$

C) 4 D) $\pi - 4$

Key. D

20. Point of intersection of the lines lies on

A) $3x + y + z = 20$ B) $3x + y + z = 25$

C) $3x + 2y + z = 24$ D) $3x + 2y + z = 25$

Key. B

21. Equation of plane containing both lines is

A) $x + 5y - 3z = 10$ B) $x + 6y + 5z = 20$

C) $x + 6y - 5z = 10$ D) $x + 6y + 5z = 10$

Key. C

Sol. 19. $\begin{vmatrix} 2 & 3 & \lambda \\ 3 & 2 & 3 \\ 1 & -1 & -1 \end{vmatrix} = 0 \Rightarrow \lambda = 4$

20. Let $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{4} = r_1$

$(3 + 2r_1, 2 + 3r_1, 1 + 4r_1)$ lies on $\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-2}{3}$

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21. $\begin{vmatrix} x-3 & y-2 & z-1 \\ 2 & 3 & 4 \\ 3 & 2 & 3 \end{vmatrix} = 0$

Passage – 8:

T is the region of the plane $x+y+z=1$ with $x,y,z>0$. *S* is the set of points (a,b,c) in *T* such that just two of the following three inequalities hold:

$$a \le \frac{1}{2}, b \le \frac{1}{3}, c \le \frac{1}{6}$$

22. Area of region *T* is

A) $\sqrt{3}/4$ B) $\sqrt{3}/2$ C) $\sqrt{3}$ D) $1/2$

Key. B

23. Area of region *S* is

A) $\sqrt{3}/72$ B) $7\sqrt{3}/36$ C) $\sqrt{3}/4$ D) $1/2$

Key. B

24. The difference of region *T* and region *S* consists of

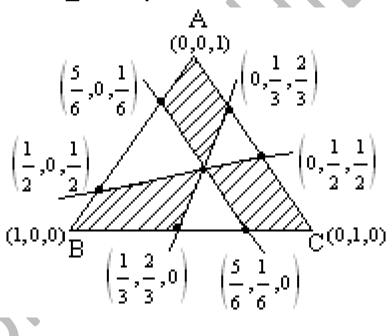
A) Three parallelograms B) Three equilateral triangles
C) Three rectangles D) Three squares

Key. B

Sol. 23 – 24:

T is an equilateral triangle with vertices at (1, 0, 0), (0, 1, 0) and (0, 0, 1).

$S = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{4}(a^2+b^2+c^2)$ where a,b,c are sides of 3 small equilateral triangle.



Passage – 9:

Let $L_1 : \bar{r} = (i-j)+t_1(2i+3j+k)$, $L_2 : \bar{r} = (-i+2j+2k)+t_2(5i+j)$, then

25. The unit vector perpendicular to both the lines L_1 and L_2 is

A) $\frac{3i+4j}{5}$ B) $\frac{5i+j-13k}{\sqrt{195}}$ C) $\frac{-i+5j-13k}{\sqrt{195}}$ D) $\frac{4i-3k}{5}$

Key. C

26. The shortest distance between L_1 and L_2 is

A) $4/\sqrt{195}$ B) 17 / 5 C) $9/\sqrt{195}$ D) 7 / 5

Key. C

27. The distance of origin from the plane passing through the point $(1,-1,1)$ and whose normal is perpendicular to both L_1 and L_2 is

A) 14 / 5 B) $19/\sqrt{195}$ C) $11/\sqrt{195}$ D) 24 / 5

Key. B

Sol. **25 – 27**

$$L_1 : \frac{x-1}{2}=\frac{y+1}{3}=\frac{z-0}{1};\ \ L_2 : \frac{x+1}{5}=\frac{y-2}{1}=\frac{z-2}{0}$$

L_1 and L_2 are skew lines

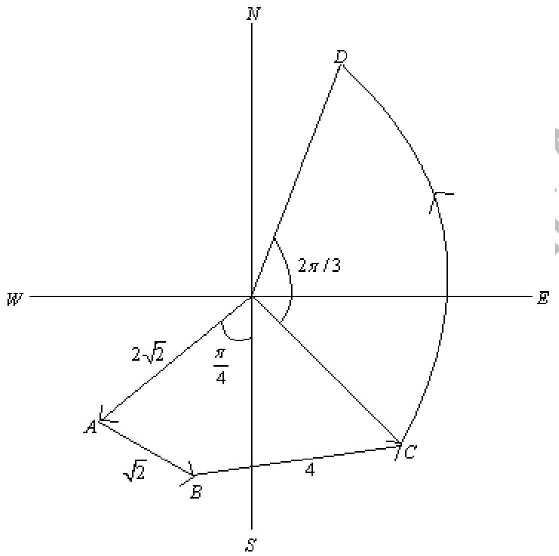
Let a,b,c be D.C's of a line perpendicular to both L_1 and L_2

$2a+3b+c=0$

$5a+b+ac=0$

$$\frac{a}{-1}=\frac{b}{5}=\frac{c}{-13}$$

$$\therefore (l,m,n)=\left(-\frac{1}{\sqrt{195}},\frac{5}{\sqrt{195}},-\frac{13}{\sqrt{195}}\right)$$



Passage – 10:

Let S_1 be the plane which contains the lines $\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}$, $\frac{x-4}{2}=\frac{y}{0}=\frac{z+1}{3}$. Let S_2 be the plane which bisects perpendicularly the line joining the points $(2,1,-1)$ and $(2,-1,3)$ at P. S_1 and S_2 intersects at P.

Answer the following questions

28. The equation of the plane S_1 is

A) $(\bar{r}-(2i+k)).(3i-j+4k)=0$ B) $\bar{r}.(3i+9j-2k)=4$

C) $\left(\bar{r}-(2i-j+3k)\right).(2i+2j-k)=0$ D) $\left(\bar{r}-(3i+2j-k)\right).(4i-j+k)=0$

Key. B

29. A vector along the line of intersection of S_1 and S_2 is

A) $12i - j + 4k$ B) $4i - 14j + 7k$ C) $-5i + 12j + 6k$ D) $16i - 6j - 3k$

Key. D

30. The distance of origin to the plane S_2 measured along the line $\frac{x}{2} = \frac{y}{2} = \frac{z}{-1}$ is

A) 4 B) 5 C) 3\2 D) 10

Key. C

Sol. 28 – 30

D.R's of normal to $S_1 = \begin{vmatrix} i & j & k \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix} = -3i - 9j + 2k$ and it passes through $P(2,0,1)$

D.R's of line of intersection of S_1 and $S_2 = \begin{vmatrix} i & j & k \\ -3 & -9 & 2 \\ 0 & -2 & 4 \end{vmatrix} = -2(16i - 6j - 3k)$

Point on plane S_2 is $\left(\frac{2r}{3}, \frac{2r}{3}, -\frac{r}{3}\right)$ it lies on S_2. Find *r*.

Passage – 11:

Two lines whose equations are $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{\lambda}$ and $\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-2}{3}$ lie in the same plane then

31. The value of $\sin^{-1}(\sin\lambda)$ is

(a) 3 (b) 4 (c) $\pi - 3$ (d) $\pi - 4$

Key. D

32. Point of intersection of the given lines lie on

(a) $3x + y + z = 20$ (b) $3x + y + z = 25$ (c) $3x + 2y + z = 24$ (d) $3x + 2y + z = 14$

Key. B

33. Equation of the plane containing both the lines is

(a) $x + 5y - 3z = 10$ (b) $x + 6y + 5z = 20$ (c) $x + 6y - 5z = 10$ (d) $x + 2y + 3z = 4$

Key. C

Sol. 31. Both the lines are coplanar $\Rightarrow \begin{vmatrix} 2 & 3 & \lambda \\ 3 & 2 & 3 \\ 1 & -1 & -1 \end{vmatrix} = 0$

$\Rightarrow 2(-2+3) - 3(-3-3) + \lambda(-3-2) = 0$

$2 + 18 - 5\lambda = 0$

$\lambda = 4$

$\sin^{-1}(\sin 4) = \sin^{-1}(\sin(\pi - 4) = \pi - 4$

32. let $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{\lambda} = t$

$\Rightarrow x = 2t + 3, y = 3t + 2, z = \lambda t + 1$

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(x, y, z) lies on $\frac{x-2}{3}=\frac{y-3}{2}=\frac{z-2}{3}$

$\Rightarrow \frac{2t+3-2}{3}=\frac{3t+2-3}{2}=\frac{\lambda t+1-2}{3}$

$\Rightarrow \frac{2t+1}{3}=\frac{3t-1}{2}=\frac{\lambda t-1}{3}$

$\Rightarrow t=1$

$\Rightarrow$ Point of intersection is (5, 5, 5)

33. $\begin{vmatrix} x-3 & y-2 & z-1 \\ 2 & 3 & 4 \\ 3 & 2 & 3 \end{vmatrix}=0$

$(x-3)(9-8)-(y-2)(6-12)+(z-1)(4-9)=0$

$(x-3)(1)-(y-2)(-6)+(z-1)(-5)=0$

$x-3+6y-12-5z+5=0$

$x+6y-5z-10=0$

Passage – 12:

If the direction ratios of two lines are given by (a_1, b_1, c_1) and (a_2, b_2, c_2)

then the acute angle between the lines is $Cos^{-1}\frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

34. If the direction ratios of two non parallel lines are connected by the relation $3a+b+5c=0$ and $5ab+6bc-2ca=0$, then the angle between the lines is

a) $Cos^{-1}\left(\frac{1}{6}\right)$ b) $Cos^{-1}\left(\frac{2}{6}\right)$ c) $Cos^{-1}\left(\frac{3}{6}\right)$ d) $Cos^{-1}\left(\frac{4}{6}\right)$

Key. A

35. If the direction ratios of two non parallel lines are connected by the relation $2a+b+2c=0$ and $3a^2+5b^2-11c^2=0$, then the angle between the lines is

a) $Cos^{-1}\left(\frac{1}{6}\right)$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{2}$

Key. D

36. If the direction ratios of two non parallel lines are connected by the relation $a+b+c=0$ and $2ab-bc+2ca=0$, then the angle between the lines is

a) $Cos^{-1}\left(\frac{1}{6}\right)$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{2}$

Key. B

Sol. 34. put $b=-3a-5c$ in $5ab+6bc-2ca=0$

35 put $b=-2a-2c$ in $3a^2+5b^2-11c^2=0$

36 put $c=-a-b$ in $2ab-bc+2ca=0$

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Passage – 13:

Intersection of a sphere by a plane is called circular section.

(i) If the plane intersects the sphere in more than one different points, then the section is called a circle.

(ii) If the circle of section is of greatest, possible radius, then the circle is called great circle.

(iii) If the radius of circular section is zero, then the section is a point circle.

(iv) If the plane does not meet the sphere at all, then the section is an imaginary circle.

37. Sphere $x^2+y^2+z^2=4$ is intersected by the plane 2x + 3y + 6z + 7 = 0 in

A) a great circle B) a real circle but not great
C) a point circle D) an imaginary circle

Key. B

Sol. Distance of the centre (0, 0, 0) from the plane is $=\frac{7}{\sqrt{4+9+36}}=1<2$

38. Sphere $x^2+y^2+z^2-2x+4y+6z-17=0$ is interected by the plane 3x – 4y + 2z – 5 = 0 in

A) a great circle B) a real circle but not great
C) a point circle D) an imaginary circle

Key. A

Sol. Centre is (1, - 2, - 3). Clearly it lies on
3x – 4y + 2z – 5 = 0 {3 + 8 – 6 – 5 = 0}
$\therefore$ Great circle

39. The sphere $x^2+y^2+z^2+2x+6y-8z-1=0$ is intersected by the plane x + 2y – 3z – 7 = 0 in

A) a great circle B) a real circle but not great
C) a point circle D) an imaginary circle

Key. D

Sol. Distance of the centre (- 1, - 3, 4) from the plane is
$\left|\frac{-1-6-12-7}{\sqrt{1+4+9}}\right|=\frac{26}{\sqrt{14}}>5$ (radius)
$\therefore$ The section is an imaginary circle

Passage – 14:

Let $a_1x+b_1y+c_1z+d_1=0$ and $a_2x+b_2y+c_2z+d_2=0$ be two planes, where $d_1,d_2>0$. Then origin lies in acute angle if $a_1a_2+b_1b_2+c_1c_2<0$ and origin lies in obtuse angle if $a_1a_2+b_1b_2+c_1c_2>0$.

Further point (x_1,y_1,z_1) and origin both lie either in acute angle or in obtuse angle, if $(a_1x_1+b_1y_1+c_1z_1+d_1)(a_2x_1+b_2y_1+c_2z_1+d_2)>0$. One of (x_1,y_1,z_1) and origin lie in acute angle and the other in obtuse angle, if $(a_1x_1+b_1y_1+c_1z_1+d_1)(a_2x_1+b_2y_1+c_2z_1+d_2)<0$

40. Given the planes 2x = 3y – 4z + 7 = 0 and x – 2y + 3z – 5 = 0, if a point P is (1, - 2, 3), then

A) O and P both lie in acute angle between the planes
B) O and P both lie in obtuse angle
C) O lies in acute angle, P lies in obtuse angle
D) O lies in obtus angle, P lies an acute angle.

Key. B

Sol. Equation of the second plane is – x + 2y – 3z + 5 = 0
2(- 1) + 3.2 + (- 4) (- 3) > 0
$\therefore$ Origin lies in obtue angle

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$(2\times1+3(-2)-4\times3+7)(-1+2(-2)-3\times3+5)$

= (2 – 6 – 12 + 7) (- 1 – 4 – 9 + 5) > 0

$\therefore$ P lies in obtuse angle

41. Given the planes x + 2y – 3z + 5 = 0 and 2x + y + 3z + 1 = 0. If a point P is (2, - 1, 2), then
A) O and P both lie in acute angle between the planes
B) O and P both lie in obtuse angle
C) O lies in acute angle, P lies in obtuse angle
D) O lies in obtus angle, P lies an acute angle.

Key. C

Sol. 1 + 2 + 2 × 1 – 3 × 3 < 0

$\therefore$ Origin lies in acute angle

Also (2 + 2(-1) – 3(2) + 5) (2 × 2 – 1 + 3 × 2 + 1)

= (- 1) (10) < 0

$\therefore$ P lies in obtuse angle

42. Given the planes x + 2y – 3z + 2 = 0 and x – 2y + 3z + 7 = 0, if the point P is (1, 2, 2), then

A) O and P both lie in acute angle between the planes
B) O and P both lie in obtuse angle
C) O lies in acute angle, P lies in obtuse angle
D) O lies in obtus angle, P lies an acute angle.

Key. A

Sol. 1 – 4 – 9 < 0

$\therefore$ Origin lies in acute angle

Further

(1 + 4 – 6 + 2) (1 – 4 + 6 + 7) > 0

$\therefore$ The point P lies in acute angle.

Passage – 15:

If $P_1 = 0$ and $P_2 = 0$ are the equations of two planes, then the equation $P_1 + \lambda P_2 = 0$ will represent the equation of family of planes passes through the line of intersection of planes $P_1 = 0$ and $P_2 = 0$ for different values of λ.

43. If the planes $ax + y - z = 0, -x + by + z = 0, x - y + cz = 0$ passes through the same straight line, then value of $a + b + c + abc$ is

a) 0 b) 1 c) -1 d) 3

Ans. a

44. If the plane $x + y = 1$ is rotated about the line of intersection with the plane z = 0 through an angle of $\frac{\pi}{4}$, then the equation of new plane is

a) $x + y - 2\sqrt{2}z = 1$ b) $x + y + 2\sqrt{2}z = 1$ c) $x + y + 3\sqrt{2}z = 1$ d) $x + y + 2z = 1$

Ans. b

45. The line of intersection of planes $x + 2y + 3z = 0$ and $3x + 2y + z = 1$ is equally inclined with vectors

a) $\hat{i}$ *and* j b) $\hat{i} + j$ *and* $\hat{i} - j$ c) $\hat{i}$ *and* k d) $\hat{i} - k$ *and* $j - k$

Ans. c

Equation of the plane passing through the line of intersection of planes ax + y – z=0 and – x + by + z = 0 is (ax + y – z) + λ(- x + by + z) = 0

$\Rightarrow x(a-\lambda) + y(1+b\lambda) + z(-1+\lambda) = 0$ - (1)

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Equation of third plane is x – y + cz = 0 - (2)
Since 1 and 2 represents same plane hence a + b + c + abc = 0
Since the line of intersection of two planes will be perpendicular to the normal vector of plane.
Hence it is parallel to the vector $\left(\hat{i}+2j+3k\right)\times\left(3\hat{i}+2j+k\right)$ or $-4\hat{i}+8j-4k$

Passage – 16:
Common tangent from K are drawn to the parabola $y^2 = 4x$ and the ellipse $3x^2 + 8y^2 = 48$ touching the parabola at A and B and the ellipse at C and D.

46. The point of intersection of tangent is
a) (-3, 0) b) (-4, 0) c) (-4, 1) d) (-4, -1)

Ans. b

47. The area of the quadrilateral ABCD is equal to
a) $55\sqrt{2}$ b) $50\sqrt{2}$ c) $57\sqrt{2}$ d) $62\sqrt{2}$

Ans. a

48. Area of the triangle KAB is equal to
a) $50\sqrt{2}$ b) $55\sqrt{2}$ c) $48\sqrt{2}$ d) none of these

Ans. c

Sol.

Let $y = mx + \frac{1}{m}$ be a tangent to the parabola it touches the ellipse if $\frac{1}{m^2} = 16m^2 + 6$

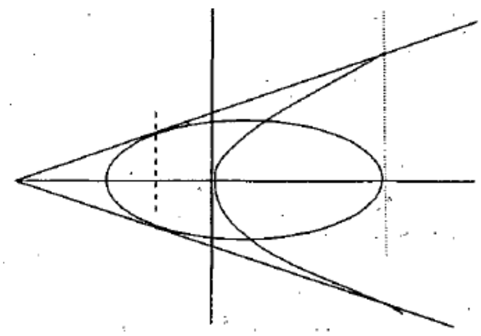
$\left(8m^2-1\right)\left(2m^2+1\right) = 0 \Rightarrow m = \pm\frac{1}{2\sqrt{2}}$

Point of contacts A and B are $\left(8, 4\sqrt{2}\right)$ and $\left(8, -4\sqrt{2}\right)$

Points of contact C, D are

$\frac{a^2m}{\sqrt{a^2m^2+b^2}} \pm \frac{b^2}{\sqrt{a^2m^2+b^2}} \Rightarrow \left(-2 \pm \frac{3}{\sqrt{2}}\right)$

and equation of tangents $y = \frac{1}{2\sqrt{2}}x + 2\sqrt{2}$ and $y = -\frac{1}{2\sqrt{2}}x - 2\sqrt{2}$



$\therefore$ Point of intersection K(-4, 0)

Area of quad ABCD = $\frac{1}{2}\left(AB + CD\right)\times PQ = 55\sqrt{2}$

Area of ΔKAB = $12\times4\sqrt{2} = 48\sqrt{2}$

Passage – 17:
P_1, P_2, P_3, P_4 are the feet of perpendiculars draw from A to the internal and external angle bisectors of angle B and C respectively.

49. Line joining P_2P_4 is
a) parallel to AB b) perpendicular to AC
c) parallel to BC d) perpendicular to AB

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Ans. **c**

50. Area of quadrilateral $P_1P_2P_3P_4$ is equal to

a) $\frac{1}{2}$ area of ΔABC b) area of ΔABC

c) $\frac{1}{4}$ area of ΔABC d) none of these

Ans. **d**

51. If $P_1P_3 = \frac{1}{2}BC$, then ΔABC is an

a) equilateral triangle b) isosceles triangle

c) right angled triangle d) nothing can be said

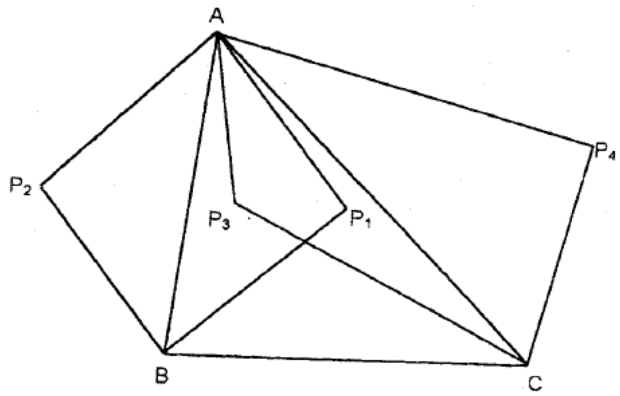
Ans. **a**

Sol.

Quadrilateral AP_2BP_1 is a rectangle

$\Rightarrow$ line joining P_1P_2 will pass through the midpoints of sides AB and AC.

$\Rightarrow$ $P_1P_2P_3P_4$ are collinear and it is parallel to BC.



Passage – 18:

A rectangle ABCD of dimensions r and 2r is folded along the diagonal BD such that planes ABD and CBD are perpendicular to each other. Let the position of the vertex A remains unchanged and C_1 is the new position of C.

52. The distance of C_1 from A is equal to

a) $\frac{2\sqrt{5}\ r}{5}$ b) $\frac{2\sqrt{10}\ r}{5}$ c) $\frac{\sqrt{85}\ r}{5}$ d) $\frac{4r}{5}$

Ans. **c**

53. If $\underline{|ABC_1} = \theta$, then $\cos\theta$ is equal to

a) $\frac{1}{5}$ b) $\frac{2}{5}$ c) $\frac{2}{\sqrt{5}}$ d) $\frac{4}{5}$

Ans. **b**

54. The shortest distance between the edges AB and C_1D is equal to

a) $\frac{\sqrt{5}r}{3}$ b) 2r c) r d) $\frac{4r}{\sqrt{5}}$

Ans. **a**

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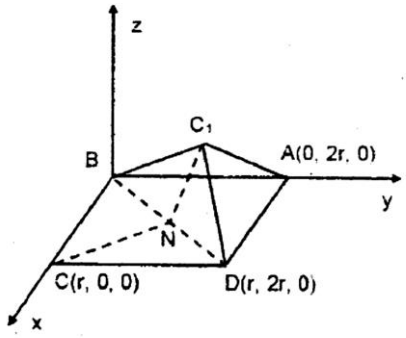
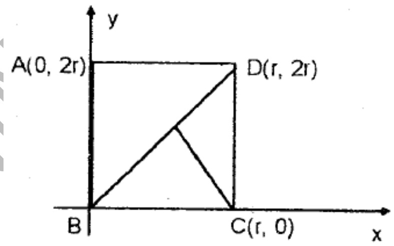
Sol. Let the rectangle ABCD initially lies in xy plane with B lying at origin BC along x-axis and BA along y-axis.
Equation of BD in xy plane is y = 2x. So the coordinates of foot N of C on BD are $\left(\frac{r}{5}, \frac{2r}{5}\right)$ and length $CN = \frac{2r}{\sqrt{5}}$



Clearly CN = C_1N
Hence the coordinates of various points in 3 – D are A(0, 2r, 0), C(r, 0, 0), D(r, 2r, 0), $N\left(\frac{r}{5}, \frac{2r}{5}, 0\right)$ and $C_1\left(\frac{r}{5}, \frac{2r}{5}, \frac{2r}{\sqrt{5}}\right)$

Now $AC_1 = \frac{\sqrt{85}r}{5}$

Direction cosines of $BC_1 = \frac{1}{5}, \frac{2}{5}, \frac{2}{\sqrt{5}} \Rightarrow \cos\theta = \frac{2}{5}$



Any point on AB $(x_1, y_1, z_1) = (0, 0, 0)$
Any point on C_1D $(x_2, y_2, z_2) = (r, 2r, 0)$
Direction cosines of AB = 0, 1, 0 = l_1, m_1, n_1
Direction cosines of C_1D = $\frac{2}{5}, \frac{4}{5}, -\frac{1}{\sqrt{5}} = l_2, m_2, n_2$

Desired shortest distance = $\dfrac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}}{\sqrt{(m_1n_2 - m_2n_1)^2 + (n_1l_2 - n_2l_1)^2 + (l_1m_2 - l_2m_1)^2}}$

$$= \frac{\begin{vmatrix} r & 2r & 0 \\ 0 & 1 & 0 \\ \frac{2}{5} & \frac{4}{5} & -\frac{1}{\sqrt{5}} \end{vmatrix}}{\sqrt{\left(\frac{1}{\sqrt{5}}\right)^2 + \left(\frac{2}{5}\right)^2}} = \frac{\sqrt{5}\,r}{3}\ unit$$

Passage – 19:

A(1, 3) and $C\left(-\frac{2}{5}, \frac{2}{5}\right)$ are the vertices of a ΔABC and the equation of the angle bisector of $\underline{|ABC}$ is x + y = 2

55. Equation of side BC is
a) 4x + 3y – 4 = 0 b) 4x + 3y + 4 = 0 c) 7x – 3y + 4 = 0 d) 7x – 3y – 4 = 0

Ans. **b**

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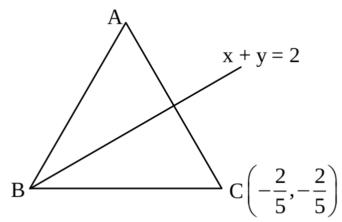
Image of A(1, 3) in x + y = 2 is (-1, 1) lies on BC. So, equation of BC is 7x + 3y+4= 0



56. Coordinates of vertex B is

a) $\left(\frac{3}{10},\frac{17}{10}\right)$ b) $\left(\frac{17}{10},\frac{3}{10}\right)$ c) $\left(-\frac{5}{2},\frac{9}{2}\right)$ d) (1, 1)

Ans. c

By solving 7x + 3y + 4 = 0 and x + y = 2 we get vertex B.

57. Equation of side AB is

a) 3x + 7y = 24 b) 3x + 7y + 24 = 0 c) 13x+7y + 8 = 0 d) 13x – 7y + 8 = 0

Ans. a

Sol. Since $A\equiv(1,3)$ and $B\equiv(-5/2, 9/2)$

So equation of AB is 3x + 7y = 24

Passage – 20:

A tangent is drawn at any point P(t) on the parabola $y^2 = 8x$ and on it is taken a point Q(α, β) from which pair of tangents QA and QB are drawn to the circle $x^2 + y^2 = 4$

58. The locus of the point of concurrency of the chord of contact AB of the circle $x^2 + y^2 = 4$ is

a) $y^2 - 2x = 0$ b) $y^2 - x^2 = 4$ c) $y^2 + 2x = 0$ d) $y^2 - 2x^2 = 4$

Ans. c

Sol. Equation of the tangent at point P of the parabola $y^2 = 8x$ is

$yt = x + 2t^2$ - (1)

Equation of the chord of contact of the circle $x^2 + y^2 = 4$ is $x\alpha + y\beta = 4$ - (2)

$\because(\alpha,\beta)$ lies on (1)

Hence $\beta t = \alpha + 2t^2$ - (3)

$x\alpha + y\left(\frac{\alpha}{t}+2t\right)-4=0$ from (2) and (3)

$2(ty-2)+\alpha\left(x+\frac{y}{y}\right)=0$

For point of concurrency

$x=-\frac{y}{t}$ and $y=\frac{2}{t}$

$\therefore$ locus is $y^2 + 2x = 0$

59. The points from which perpendicular tangents can be drawn both to the given circle and the parabola is/are

a) $\left(4,\pm\sqrt{3}\right)$ b) $\left(-1,\sqrt{2}\right)$ c) $\left(-\sqrt{2},-\sqrt{2}\right)$ d) $\left(-2,\pm\sqrt{2}\right)$

Ans. d

Sol. Required point will lie on the director circle of the given circle as well as on the directrix of parabola.

$\Rightarrow x_1^2 + y_1^2 = 8$ and $x_1 + 2 = 0$

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$\Rightarrow 4 + y_1^2 = 8$

$\Rightarrow y_1 = \pm\sqrt{2}$

$\therefore$ Points are $\left(-2 \pm \sqrt{2}\right)$

60. The locus of circumcentre of ΔAQB if t = 2 is
    a) x – 2y + 4 = 0 b) x + 2y – 4 = 0 c) x – 2y – 4 = 0 d) x + 2y + 4 = 0

Ans. **c**

Sol. Equation of circumcircle of ΔAQB is $x^2 + y^2 - 4 + \lambda\left(x\alpha + y\beta - 4\right) = 0$

$\because$ It passes through (0,0) i.e centre of circle $\Rightarrow \lambda = -1$

Let circumcentre be (h, k)

$\therefore h = \frac{\alpha}{2},\ k = \frac{\beta}{2}$

$\Rightarrow \alpha = 2h,\ \beta = 2k$

Also $\beta t = \alpha + 2t^2$ or $\alpha - 2\beta + 8 = 0$ $\because$ t = 2

Substituting $\alpha = 2h$ and $\beta = 2k$ we get h – 2k + 4 = 0

$\therefore$ locus is x – 2y + 4 = 0

Passage – 21:

To the circle $x^2 + y^2 = 4$ two tangents are drawn from P(-4, 0) which touches the circle at T_1 and T_2 and a rhombus $PT_1P'T_2$ is completed

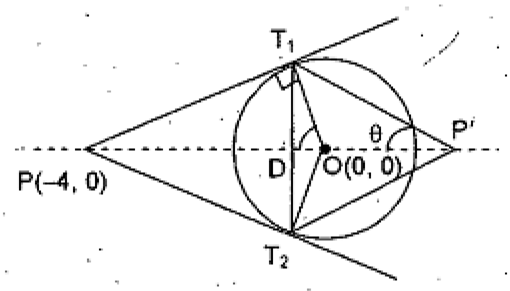
61. Circumcentre of the triangle PT_1T_2 is at
    a) (-2, 0) b) (2, 0) c) $\left(\frac{\sqrt{3}}{2}, 0\right)$ d) none of these

Ans. **a**

Sol. $PT_2 = PT_1 = \sqrt{(-4)^2 + 0^2 - 4} = 2\sqrt{3}$

Circumcentre of triangle PT_1T_2 is midpoint of PO as $\angle PT_1O = \angle PT_2O = 90^0$

So, $\left(\frac{-4+0}{2}, \frac{0+0}{2}\right) = (-2, 0)$



62. Ratio of the area of triangle PT_1P' to that the $P'T_1T_2$ is
    a) 2 : 1 b) 1 : 2 c) $\sqrt{3}:2$ d) none of these

Ans. **d**

63. If P is taken to be at (h, 0) such that P' lies on the circle, the area of the rhombus is
    a) $6\sqrt{3}$ b) $2\sqrt{3}$ c) $3\sqrt{3}$ d) none of these

Ans. **A**

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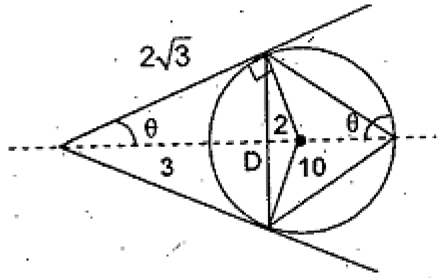
Sol. P' be a point on the circle

$3\theta = \frac{\pi}{2} \Rightarrow \theta = \frac{\pi}{6}$

Area of the rhombus

$= 2\left(3\times 3\sqrt{3}\right) = 6\sqrt{3}$



Passage – 22:

The vertices of a ΔABC lies on a rectangular hyperbola such that the orthocenter of the triangle is (3, 2) and the asymptotes of the rectangular hyperbola are parallel to the coordinate axes. If the two perpendicular tangents of the hyperbola intersect at the point (1,1), then

64. The equation of the asymptotes is

a) xy – 1 = x – y b) xy + 1 = x + y c) 2xy = x + y d) none of these

Ans. **b**

Sol. Perpendicular tangents intersect on the director circle of hyperbola and director circle of rectangular hyperbola is a point circle. Hence centre of hyperbola is (1, 1) and equation of asymptotes are (x – 1) = 0 and y – 1 = 0

65. Equation of the rectangular hyperbola is

a) xy = 2x + y – 2 b) 2xy = x + 2y + 5 c) xy = x + y + 1 d) none of these

Ans. **c**

Sol. Equation of hyperbola is xy – x – y + 1 + λ = 0

It passes through (3, 2) hence λ = - 2

Equation of hyperbola is xy = x + y + 1

66. Number of real normals that can drawn from the point (1, 1) to the rectangular hyperbola is

a) 4 b) 0 c) 3 d) 2

Ans. **d**

Sol. From the centre of hyperbola we can draw two real normals to the rectangular hyperbola.

Passage – 23:

Consider and ellipse $\frac{x^2}{4} + y^2 = \alpha$ (where, α is parameter > 0) and a parabola $y^2 = 8x$. If a common tangent to the ellipse and the parabola meets the coordinate axes at A and B respectively, then

67. Locus of midpoints of AB is

a) $y^2 = -2x$ b) $y^2 = -x$ c) $y^2 = -\frac{x}{2}$ d) $\frac{x^2}{4} + \frac{y^2}{2} = 1$

Ans. **b**

68. If the eccentric angle of a point on the ellipse where the common tangents meets is $\frac{2\pi}{3}$, then α is equal to

a) 4 b) 5 c) 26 d) 36

Ans. **d**

69. If two of the three normals are drawn from (h, 0) on the ellipse to the parabola $y^2 = 8x$ are perpendicular, then

a) h = 2 b) h = 3 c) h = 4 d) h = 6

Ans. **d**

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Sol. Equation of tangent to $y^2 = 8x$ is

$yt - x - 2t^2 = 0$ - (1)

Equation of tangent to ellipse is $\frac{x\cos\theta}{2\sqrt{\alpha}} + \frac{y\sin\theta}{\sqrt{\alpha}} = 1$ - (2)

Comparing (1) and (2)

$\frac{\sqrt{\alpha}}{\cos\theta} = -t^2, \frac{\sqrt{\alpha}}{\sin\theta} = 2t$ - (3)

Let midpoint of AB is (h, k)

$h = \frac{\sqrt{\alpha}}{2\cos\theta}, k = \frac{\sqrt{\alpha}}{2\sin\theta}$

From (3)

$\frac{\alpha}{\sin^2\theta} = \frac{-4\sqrt{\alpha}}{\cos\theta} = \sqrt{\alpha} = \frac{-4\sin^2\theta}{\cos\theta} = 6$

Any normal is y = sinx – cosx – $2m^2$ $\Rightarrow$ h = 6

Passage – 24:

Let P, Q are two points on the curve $y = \log_{1/2}\left(x - \frac{1}{2}\right) + \log_2\sqrt{4x^2 - 4x + 1}$ and P is also on the circle $x^2 + y^2 = 10$, θ lies inside the given circle such that its abscissa is an integer.

70. The coordinates of P are given by

a) (1, 2) b) (2, 4) c) (3, 1) d) (3, 5)

Ans. c

$y = \log_{1/2}\left(x - \frac{1}{2}\right) + \log_2\sqrt{(2x-1)^2}$

P = (3, 1)

71. $\overrightarrow{OP}\cdot\overrightarrow{OQ}$, O being the origin is

a) 4 or 7 b) 4 or 2 c) 2 or 3 d) 7 or 8

Ans. a

$\overrightarrow{OP} = 3i + j, \overrightarrow{OQ}\ i + j\ and\ 2i + j$

$\overrightarrow{OP}\cdot\overrightarrow{OQ} = 4\ or\ 7$

72. Maximum of $\left\{\left|\overrightarrow{PQ}\right|\right\}$ is

a) 5 b) 4 c) 0 d) 2

Ans. d

Sol. $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = -2i\ or\ -i$

$\left|\overrightarrow{PQ}\right| = 2\ or\ 1$

Passage – 25:

In a triangle ABC, the equation of side BC is $2x - y = 3$ and its circumcentre and orthocenter are at (2, 4) and (1, 2) respectively.

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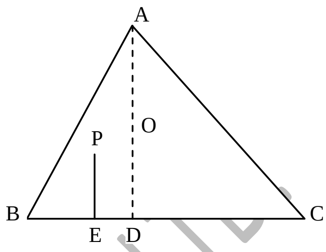
73. Circumradius of triangle ABC is

a) $\sqrt{\frac{61}{5}}$ b) $\sqrt{\frac{51}{5}}$ c) $\sqrt{\frac{41}{5}}$ d) $\sqrt{\frac{43}{5}}$

Ans. **a**

Sol. P(2, 4) and O(1, 2)

$OP^2 = R^2(1 - 8 \cos A \cos B \cos C)$



$$PE = R\cos A = \frac{3}{\sqrt{5}}$$

$$OD = 2R\cos B\cos C = \frac{3}{\sqrt{5}}$$

$$\Rightarrow 5 = R^2 - 4\times\frac{3}{\sqrt{5}}\times\frac{3}{\sqrt{5}} \Rightarrow R = \sqrt{\frac{61}{5}}$$

74. The value of sin B sin C is equal to

a) $\frac{9}{2\sqrt{61}}$ b) $\frac{9}{4\sqrt{61}}$ c) $\frac{9}{\sqrt{61}}$ d) $\frac{9}{3\sqrt{61}}$

Ans. **a**

Sol. $R\cos A = \frac{3}{\sqrt{5}} \Rightarrow -R\cos B\cos C + R\sin B\sin C = \frac{3}{\sqrt{5}}$ and $2R \cos B \cos C = \frac{3}{\sqrt{5}}$

$$\Rightarrow \sin B\sin C = \frac{9}{2\sqrt{61}}$$

75. The distance of orthocenter to vertex A is equal to

a) $\frac{1}{\sqrt{5}}$ b) $\frac{6}{\sqrt{5}}$ c) $\frac{3}{\sqrt{5}}$ d) $\frac{3}{\sqrt{5}}$

Ans. **b**

Sol. Distance of orthocenter from vertex = $2R\cos A = \frac{6}{\sqrt{5}}$

Passage – 26:

Two lines whose equations are $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{\lambda}$ and $\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-2}{3}$ lie in the same plane, then

76. The value of $\sin^{-1}(\sin\lambda)$ is equal to

a) 3 b) $\pi - 3$ c) 4 d) $\pi - 4$

Ans. **d**

Sol. Both lines are coplanar

$$= \begin{vmatrix} 2 & 3 & \lambda \\ 3 & 2 & 3 \\ 1 & -1 & -1 \end{vmatrix} = 0 \Rightarrow \lambda = 4$$

$$\sin^{-1}\sin x = \pi - x$$

77. Point of intersection of the lines lies on

a) 3x + y + z = 20 b) 3x + y + z = 25 c) 3x + 2y + z = 24 d) none of these

Ans. **b**

Sol. Point of intersection is (5, 5, 5)

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78. Equation of plane containing both lines is
a) x + 5y – 3z = 10 b) x + 6y + 5z = 20 c) x + 6y – 5z = 10 d) none of these

Ans. c

Sol. Equation of plane containing both the lines

$$\begin{vmatrix} x-3 & y-2 & z-1 \\ 2 & 3 & 4 \\ 3 & 2 & 3 \end{vmatrix} = 0$$

$$\Rightarrow x+6y-5z=10$$

Passage – 27:

Let A, B, C be the three points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ being eccentric angles α, β, γ respectively

79. Area of triangle PQR formed by corresponding points on auxiliary circle is
a) $\frac{a}{b}\left(area\ of\ \Delta ABC\right)$ b) $\frac{b}{a}\left(area\ of\ \Delta ABC\right)$
c) $\frac{2a}{b}\left(area\ of\ \Delta ABC\right)$ d) $\left(area\ of\ \Delta ABC\right)$

Ans. a

Sol. $$\frac{\Delta ABC}{\Delta PQR}=\frac{\frac{1}{2}\begin{vmatrix} a\cos\alpha & b\sin\beta & 1 \\ a\cos\beta & b\sin\beta & 1 \\ a\cos\gamma & b\sin\gamma & 1 \end{vmatrix}}{\frac{1}{2}\begin{vmatrix} a\cos\alpha & a\sin\beta & 1 \\ a\cos\beta & a\sin\beta & 1 \\ a\cos\gamma & a\sin\gamma & 1 \end{vmatrix}}=\frac{b}{a}$$

80. The centroid of triangle formed by points on the auxiliary circle when area of the ΔABC is maximum is
a) (a, b) b) $\left(\frac{a}{3},\frac{b}{3}\right)$ c) $\left(0,\frac{b}{3}\right)$ d) (0, 0)

Ans. d

Sol. For the maximum area of ΔPQR it must be an equilateral triangle so circumcentre and centroid will coincides.

81. The eccentric angles of the vertices of triangle of maximum area inscribed in an ellipse differ by
a) $\frac{\pi}{3}$ b) $\frac{\pi}{2}$ c) $\frac{2\pi}{3}$ d) π

Ans. c

Sol. ΔPQR will be equilateral if his segment PQ, PR and QR should subtend $\frac{2\pi}{3}$ at centre

circumcentre i.e $\left(\alpha-\beta\right)=\left(\beta-\gamma\right)=\left(\gamma-\alpha\right)=\frac{2\pi}{3}$

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Passage – 28:

Given a hyperbola H : $x^2 - y^2 = 0$

A parabola P : $4(x - 5) = y^2$

and line L : x = 9

82. If L is the chord of contact of hyperbola H, then the equation of corresponding pair of tangents is

a) $9x^2 - 8y^2 + 18x - 9 = 0$ b) $9x^2 - 8y^2 - 18x + 9 = 0$

c) $9x^2 - 8y^2 - 18x - 9 = 0$ d) $9x^2 - 8y^2 + 18x + 9 = 0$

Ans. **b**

Sol. Let $R(x_1, y_1)$ be the point of intersection of tangents to H at the ends of the chords x=9, then the equation of L is $xx_1 - yy_1 = 9$. Comparing we get $x_1 = 1, y_1 = 0$ $\therefore R \equiv (1, 0)$

$$\left(x^2 - y^2 - 9\right)(1 - 0 - 9) = (x - 9)^2$$

$$\Rightarrow -8x^2 + 8y^2 + 72 = x^2 + 81 - 18x$$

$$\Rightarrow 9x^2 - 8y^2 - 18x + 9 = 0$$

83. If R is the point of intersection of the tangents to H at the extremities of the chord L. Then the equation of chord of contact of R with respect to P is

a) x = 7 b) x = 9 c) y = 7 d) y = 9

Ans. **b**

Sol. Equation of the parabola is $y^2 = 4x - 20$

Equation of the chord of contact of the parabola w.r.t R(1, 0) is

$$yy_1 = 2(x+1) - 20 \Rightarrow 2(x+1) - 20 = 0 \Rightarrow x = 9$$

84. If the chord of contact of P with respect to R meets the parabola at T and T', S is the focus of the parabola, then the area of the triangle STT' is equal to

a) 8 sq units b) 9 sq. units c) 12 sq. units d) 16 sq. units

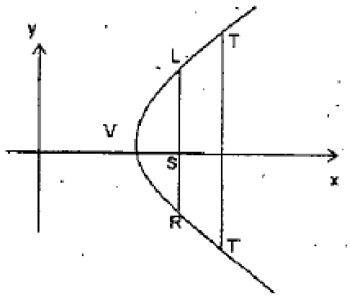
Ans. **c**

Sol. V(5, 0), S(6, 0), LR = 2LS = 4

TT is the chord of contact whose equation is x = 9

$y^2 = 4(x - 5)$, x = 9 $\Rightarrow y = \pm 4$

$$\Rightarrow T \equiv (9, 4), T' = 4(3) = 12\, sq.units$$



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3D-Geometry

Integer Answer Type

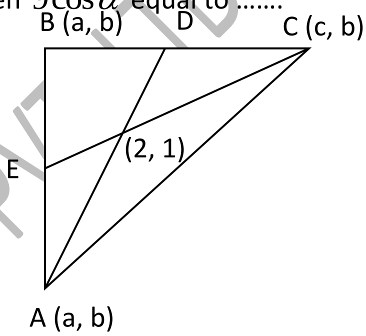
1. The foot of the perpendicular from (1,2,3) to the join of (6,7,7), (9,9,5) is (3,5, λ) then λ =

Key. 9

Sol. Any point of the line joing the given points can be taken as $(6+3t, 7+2t, 7-2t)$ if it is the required foot of the $\perp$ of (1,2,3) we get 3 (5+3t)+2 (5+2t)-2(4-2t) =0 $\Rightarrow t=-1$

2. The plane $2x-2y+z=3$ is rotated about the line where it cuts the xy plane by an acute angle α . If the new position of plane contains the point (3, 1, 1) then $9\cos\alpha$ equal to ....

Key: 7



Hint: Let equation of new plane $2x-2y+z-3+\lambda z=0$

Point (3, 1, 1) lie on $it \Rightarrow \lambda=-2$

Hence equation of new plane 2x – 2y –z = 3

$$\cos\alpha=\frac{4+4-1}{3.3}=\frac{7}{9}$$

3. Shortest distance between the z-axis and the line $x+y+2z-3=0=2x+3y+4z-4$ is ___

Ans: 2.

Hint : Equation of any plane ; continuing the general plane is

$$x+y+2z-3+\lambda(2x+3y+4z-4)=0---(1)$$

$$\text{if plane (1) is parallel to z-axis } \Rightarrow \lambda=-\frac{1}{2}$$

Therefore plane, parallel to z-axis is y+2=0-----(2)

Now, shortest distance between any point on z-axis (0, 0, 0) (say) from plane (2) is 2

4. The point P (1,2,3) is reflected in the xy – plane, then its image Q is rotated by 180^oabout the x – axis to produce R , and finally R is translated in the direction of the positive y – axis through a distance d to produce S (1,3,3). The value of d is

ANS : 3

Hint Reflecting the point (1,2,3) in the xy – plane produces (1,2,-3) . A half turn about the x – axis yields (1,-2,3). Finally translation 5 units will produce (1,3,3)

5. Let A, B, C be three non-collinear points. Then n be the no. of lines lying in plane containing the points A, B, C which are equidistant from all three points then $n+5=$

Key: 8

6. The equation of the plane passing through the intersection of the planes $2x-5y+z=3$ and $x+y+4z=5$ and parallel to the plane $x+3y+6z=1$ is x + 3y + 6z = k, where k is

Key : 7

Sol : Equation of plane passing through the intersection of the planes $2x-5y+z=3$ and $x+y+4z=5$ is

$$(2x-5y+z-3)+\lambda(x+y+4z-5)=0$$

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$\Rightarrow(2+\lambda)x+(-5+\lambda)y+(1+4\lambda)-3-5\lambda=0$ ....(i)

which is parallel to the plane x + 3y + 6z = 1.

Then $\frac{2+\lambda}{1}=\frac{-5+\lambda}{3}=\frac{1+4\lambda}{6}$

Then, $\frac{2+\lambda}{1}=\frac{-5+\lambda}{3}=\frac{1+4\lambda}{6}$

$\therefore \lambda=\frac{-11}{2}$

from eq. (i),

$-\frac{7}{2}x-\frac{21}{2}y-21z+\frac{49}{2}=0$

$\therefore\ x+3y+6z=7$

Hence, k = 7

7. If the distance of a point lying on the plane 2x +3y + 6z = p from the point (3, 0, 1) is unity then the total number of possible values of p, where p is a prime number, is

Key. 6

Sol. $\frac{|2(3)+3+6(1)-p|}{\sqrt{2^2+3^2+6^2}}\le 1$

$\Rightarrow |12-p|\le 7 \Rightarrow -7\le p-12\le 7$

$\Rightarrow 5\le p\le 19 \Rightarrow 5, 7, 11, 13, 17, 19$

i.e. six possible values of p.

8. A line from the origin meets the lines $\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$ and $\frac{x-8/3}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$ at P and Q respectively. If the distance PQ = l then the value of [l] (where [.] represents the greatest integer function), is

Key. 2

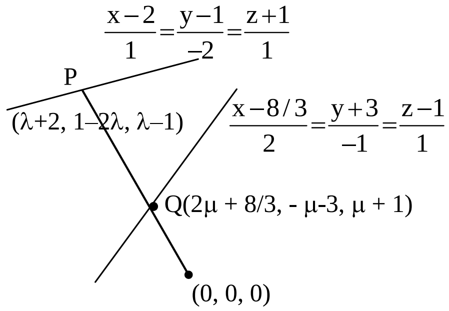
Sol. From the given conditions, we have,

$\frac{2\mu+8/3}{\lambda+2}=\frac{\mu+3}{2\lambda-1}=\frac{\mu+1}{\lambda-1}$

$\Rightarrow \lambda=3,\ \mu=\frac{1}{3}$

$\Rightarrow P\equiv(5,-5,2)\ Q\equiv\left(\frac{10}{3},\frac{-10}{3},\frac{4}{3}\right)$

$\Rightarrow l = PQ = \sqrt{6} \Rightarrow [l]=2$



9. The shortest distance between the z-axis and the line, x + y + 2z – 3 = 0, 2x + 3y + 4z – 4 = 0 is :

Key. 2

Sol. The equation of any plane containing the given line is

$$(x+y+2z-3)+\lambda(2x+3y+4z-4)=0$$

$$\Rightarrow (1+2\lambda)x+(1+3\lambda)y+(2+4\lambda)z-(3+4\lambda)=0 \quad ....(1)$$

If the plane is parallel to z-axis whose direction cosines are 0, 0, 1 ; then the normal to the plane will be perpendicular to z-axis

$$\therefore \quad (1+2\lambda)(0)+(1+3\lambda)(0)+(2+4\lambda)(1)=0$$

$$\Rightarrow \quad \lambda=-\frac{1}{2}$$

Put in eq. (1), the required plane is

$$(x+y+2z-3)-\frac{1}{2}(2x+3y+4z-4)=0 \Rightarrow y+2=0...(2)$$

$\therefore$ S.D. = distance of any point say (0, 0, 0) on z-axis from plane (2)

$$=\frac{2}{\sqrt{(1)^2}}=2$$

10. If equation of the plane through the straight line $\frac{x-1}{2}=\frac{y+2}{-3}=\frac{z}{5}$ and perpendicular to the plane x – y + z + 2 = 0 is ax – by + cz + 4 = 0, then find the value of 10^3a+10^2b+10c

Ans. 1710

Sol. Let equation of a plane containing the line be $l(x-1)+m(y+2)+nz=0$

then $2l-3m+5n=0$ and $l-m+n=0$

$$\therefore \quad \frac{l}{2}=\frac{m}{3}=\frac{n}{1}$$

$\therefore$ the plane is 2(x – 1) + 3(y + 2) + z = 0

i.e. 2x + 3y + z + 6 = 0

$\therefore$ a = 2, b = - 3, c = 1

$\therefore$ $10^3a+10^2b+10c=2000-300+10=1710$ Ans.

11. Find the equation to the line which intersects the lines

x + y + z = 1, 2x – y – z = 2

x + y – z = 3, 2x + 4y – z = 4

and passes through the point (1, 1, 1)

Ans. 19

Sol. The line intersecting the given lines is

$$\left.\begin{aligned}(x+y+z-1)+\lambda\,(2x-y-z-2)=0\\(x-y-z-3)+\mu\,(2x+4y-z-4)=0\end{aligned}\right\} \quad ...(i)$$

If it passes through (1, 1, 1), then we get from (1)

$\lambda=1$ and $\mu=4$

Hence the required equations to the intersecting line are x – 1 = 0 = 9x + 15y – 5z + 19. Ans

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12. Find the shortest distance and the vector equation of the line of shortest distance between the lines given by $\vec{r}=3\vec{i}+8\vec{j}+3\vec{k}+\lambda\left(3\vec{i}-\vec{j}+\vec{k}\right)$ and $\vec{r}=-3\vec{i}-7\vec{j}+6\vec{k}+\mu\left(-3\vec{i}+2\vec{j}+4\vec{k}\right)$.

Ans. $\vec{r}=3\vec{i}+8\vec{j}+3\vec{k}+\lambda\left(-6\vec{i}-15\vec{j}+3\vec{k}\right)$

Sol. $\vec{r}=3\vec{i}+8\vec{j}+3\vec{k}+\lambda\left(3\vec{i}-\vec{j}+\vec{k}\right)$ …(i)

$\vec{r}=-3\vec{i}-7\vec{j}+6\vec{k}+\mu\left(-3\vec{i}+2\vec{j}+4\vec{k}\right)$ …(ii)

Let L and M be points on the line (i) and (ii) respectively
So that LM is perpendicular to both the lines

Let position vector of L be $3\vec{i}+8\vec{j}+3\vec{k}+\lambda_0\left(3\vec{i}-\vec{j}+\vec{k}\right)$

and the position vector of M be $-3\vec{i}-7\vec{j}+6\vec{k}+\mu_0\left(-3\vec{i}+2\vec{j}+4\vec{k}\right)$

then $\overrightarrow{LM}=-6\vec{i}-15\vec{j}+3\vec{k}-\lambda_0\left(3\vec{i}-\vec{j}+\vec{k}\right)+\mu_0\left(-3\vec{i}+2\vec{j}+4\vec{k}\right)$

since $\overrightarrow{LM}$ is perpendicular to both the lines (i) and (ii)

$\therefore$ $\overrightarrow{LM}.\left(3\vec{i}-\vec{j}+\vec{k}\right)=0$ and $\overrightarrow{LM}.\left(-3\vec{i}+2\vec{j}+4\vec{k}\right)=0$

Thus – 18 + 15 + 3 - λ_0 (9 + 1 + 1) + μ_0 (- 9 – 2 + 4) = 0

i.e. $-11\lambda_0-7\mu_0=0$ …(iii)

and 18 – 30 + 12 $-\lambda_0$ (- 9 – 2 + 4) $+\mu_0$ (9 + 4 + 16) = 0

i.e. $7\lambda_0+29\mu_0=0$ …(iv)

from (iii) and (iv) we get

$\lambda_0=\mu_0=0$

$\therefore$ $\overrightarrow{LM}=-6\vec{i}-15\vec{j}+3\vec{k}$

$\therefore$ $\left|\overrightarrow{LM}\right|=\sqrt{36+225+9}=\sqrt{270}=3\sqrt{30}$

Position vector of L is $3\vec{i}+8\vec{j}+3\vec{k}$

$\therefore$ equation of the line of shortest distance (LM) is

$\vec{r}=3\vec{i}+8\vec{j}+3\vec{k}+\lambda\left(-6\vec{i}-15\vec{j}+3\vec{k}\right)$

$\vec{r}=3\vec{i}+8\vec{j}+3\vec{k}+\lambda\left(-6\vec{i}-15\vec{j}+3\vec{k}\right)$

13. If the lengths of external and internal common tangents to two circles $x^2 + y^2 + 14x - 4y + 28 = 0$ and $x^2 + y^2 - 14x + 4y - 28 = 0$ are λ and μ. Then the value of $\left[\frac{\lambda+\mu}{4}\right]$ is equal to (where [.] denotes greatest integer function)

Ans. 4

Sol. $c_1c_2 > r_1+r_2$

External = $\sqrt{d^2-\left(r_2-r_1\right)^2}=14=\lambda$

Internal = $\sqrt{d^2-\left(r_1+r_2\right)}=4=\mu$

$\lambda+\mu=18$ $\left[\frac{\lambda+\mu}{4}\right]=4$

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14. Consider two concentric circle $C_1 : x^2 + y^2 = 1$ and $C_2 : x^2 + y^2 - 4 = 0$. A parabola is drawn through the points where C_1 meet the x-axis and having arbitrary tangent of C_2 as its directrix. Then locus of focus of drawn parabola is $\frac{3}{4}x^2 + y^2 = k$, then value of k is

Ans. 3

Sol. $(h-1)^2 + k^2 = (\cos\theta - 2)^2$ - (1)

$(h+1)^2 + k^2 = (\cos\theta + 2)^2$ - (2)

(2) – (1) gives us $\cos\theta = \frac{h}{2}$

(2) + (1)

$2(h^2 + k^2 + 1) = 2(\cos^2\theta + 4)$

$\frac{3}{4}x^2 + y^2 = 3$

15. All chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ that subtend a right angle at the origin, pass through a fixed point (h, k) then h – k is equal to

Ans. 3

Sol. Let the equation of the chord to y = mx + c

Combined equation of the line joining the point of intersection with origin is

$3x^2 - y^2 - 2(x - 2y)\left(\frac{y - mx}{c}\right) = 0$

$\Rightarrow x^2(3c + 2m) - y^2(c - 4) - 2xy(1 + 2m) = 0$

From the condition of perpendicularity, we get 3c + 2m – c + 4 = 0

$\Rightarrow m + c = -2$

i.e the line y = mx + c, passes through (1, -2)

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3D-Geometry

Matrix-Match Type

1. Match the following:

Column -I		Column -II	
(A)	The vector equation of the plane perpendicular to the line $\frac{x-1}{1}=\frac{y-2}{-1}=\frac{z+1}{2}$ and passing through the point $(3,6,2)$ is	(p)	$\bar{r}.(\hat{i}-3j-2k)=3$
(B)	The vector equation of the plane through the point $(5,-2,4)$ and parallel to the plane $4x-12y-8z=7$	(q)	$\bar{r}.(\hat{i}-j-k)=2$
(C)	The vector equation of the plane containing the line $\bar{r}=2\hat{i}+\lambda(j-k)$ and perpendicular to the plane $\bar{r}.(\hat{i}+k)=3$ is	(r)	$\bar{r}.(\hat{i}-j-k)=0$
(D)	The vector equation of the plane containing the lines $\frac{x-1}{1}=\frac{y-1}{2}=\frac{z}{-1}$; $\frac{x-1}{-1}=\frac{y-1}{1}=\frac{z}{-2}$ is	(s)	$\bar{r}.(\hat{i}-j+2k)=1$
		(t)	$\bar{r}.(\hat{i}-j-k)=1$

Key. A – s ; B – p; C – q ; D – r

Sol. Conceptual

2. Match the following

Column – I		Column – II	
a)	If the plane $ax-by+cz=d$ contains the line $\frac{x-a}{a}=\frac{y-2d}{b}=\frac{z-c}{c}$ then $\frac{b}{d}$ is equal to	p)	1
b)	The distance of the point $(1,-2,3)$ from the plane $x-y+z-5=0$ measured parallel to $\frac{x}{2}=\frac{y}{3}=\frac{z-1}{-6}$ is equal to	q)	2
c)	If the straight lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{4-z}{k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ intersect then k is equal to	r)	0
d)	If a line makes an angle θ with x and y axis then $\cot\theta$ can be equal to	s)	-3

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Key: a) q b) p c) r, s d) p, r

Hint :

a) $a^2-2bd+c^2=d \Rightarrow a^2-b^2+c^2=0 \Rightarrow b^2=a^2+c^2 \Rightarrow b^2=2bd+d$

b) $\frac{x-1}{2}=\frac{y+2}{3}=\frac{z-3}{-6}=r$

$(2r+1,3r-2,3-6r)$

$2r+1-3r+2+3-6r-5=0 \Rightarrow -7r+1=0 \Rightarrow r=1/7$

c) $\frac{x-2}{1}=\frac{y-3}{1}=\frac{4-z}{K}=r_1, \frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}=r_2$

$(r_1+2, r_1+3, 4-Kr_1) \quad (1+Kr_2, 4+2r_2, 5+r_2)$

$r_1+2=1+Kr_2, r_1+3=4+2r_2, 4-Kr_1=5+r_2$. Eliminate K.

d) $\cos^2\theta+\cos^2\theta+\cos^2\gamma=1 \Rightarrow \cos^2\gamma=-\cos 2\theta \ge 0 \Rightarrow \cos 2\theta \le 0$

3. Match the following:-

Column-I		Column-II
A) The distance of the point (1,-2,3) from the plane x-y+z-5=0 measured parallel to $\frac{x}{2}=\frac{y}{3}=\frac{z-1}{-6}$	p)	0
B) If the straight lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{4-z}{K}$ and $\frac{x-1}{K}=\frac{y-4}{2}=\frac{z-5}{1}$ intersect then K is equal to	q)	1
C) The shortest distance between any two opposite edges of the tetrahedron formed by the planes $y+z=0, z+x=0, x+y=0$ and $x+y+z=\sqrt{6}$ is	r)	-3
D) If θ is the angle between line x=y=z and the plane x+y+z=4 then $\tan\frac{\theta}{2}$ is	s)	2

Key. A) q B) p,r C) s D) q

Sol. A) point on the line is (1+2λ,-2+3λ,3-6λ)

B) $\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -K \\ K & 2 & 1 \end{vmatrix}=0$

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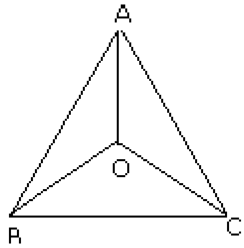
C) Vertices of a tetrahedron are O(0,0,0), A$(\sqrt{6},\sqrt{6},-\sqrt{6})$, $B(\sqrt{6},-\sqrt{6},\sqrt{6})$, $C(-\sqrt{6},\sqrt{6},\sqrt{6})$ find the shortest distance between the lines $\overleftrightarrow{AO}$ & $\overleftrightarrow{BC}$



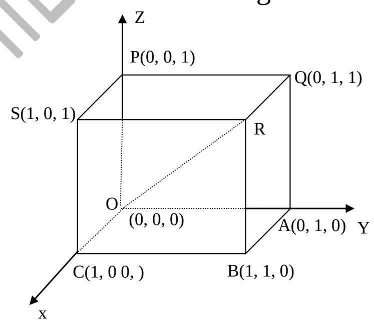
D) line is perpendicular to the plane $\theta = \dfrac{\pi}{2}$

4. Match the following:-

	Column –I		Column –II
(A)	If in a cube, θ is the angle between any two body-diagonals then the value of cosθ is	(p)	1
(B)	If in a cube, θ is the angle between a body-diagonal and a face-diagonal which is skew to it, then the value of sinθ is	(q)	$\frac{1}{\sqrt{2}}$
(C)	If in a cube, θ is the angle between diagonals of two faces through a vertex, then the value of cotθ is	(r)	$\frac{1}{\sqrt{3}}$
(D)	If in a cube, θ is the angle between a body-diagonal and a face-diagonal interesting it then the value of tanθ is	(s)	$\frac{1}{2}$
		(t)	1/3

Key. (A–t), (B–p), (C–r), (D–q)

Sol. Considering the cube as shown in the figure



(A) $\overrightarrow{OR}.\overrightarrow{BP} = (\hat{i}+\hat{j}+\hat{k}).(-\hat{i}-\hat{j}+\hat{k})$

$\Rightarrow \cos\theta = \left|\dfrac{-1-1+1}{\sqrt{3}.\sqrt{3}}\right| = \dfrac{1}{3}$

(B) $\overrightarrow{OR}.\overrightarrow{AC} = (\hat{i}+\hat{j}+\hat{k}).(\hat{i}-\hat{j}) = 0$

$\Rightarrow \theta = 90° \Rightarrow \sin\theta = 1$

(C) $\overrightarrow{OB}.\overrightarrow{OQ} = (\hat{i}+\hat{j}).(\hat{j}+\hat{k})$ $\Rightarrow \cos\theta = \dfrac{1}{\sqrt{2}\sqrt{2}} = \dfrac{1}{2} \Rightarrow \cot\theta = \dfrac{1}{\sqrt{3}}$

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(D) $\overrightarrow{OB}.\overrightarrow{OR} = (\hat{i}+\hat{j}).(\hat{i}+\hat{j}+\hat{k}) \Rightarrow \cos\theta = \frac{1+1}{\sqrt{2}\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{2}}$

5. Match the following: -

Column – I		Column – II	
(A)	$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-1}{3}=\frac{y-3}{4}=\frac{z-5}{5}$ are	(p)	Coincident
(B)	$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-3}{2}=\frac{y-5}{3}=\frac{z-7}{4}$ are	(q)	Parallel and different
(C)	$\frac{x-2}{5}=\frac{y+3}{4}=\frac{5-z}{2}$ and $\frac{x-7}{5}=\frac{y-1}{4}=\frac{z-2}{-2}$ are	(r)	Skew
(D)	$\frac{x-3}{2}=\frac{y+2}{3}=\frac{z-7}{5}$ and $\frac{x-3}{3}=\frac{y-2}{2}=\frac{z-7}{5}$ are	(s)	Intersecting in a point
		(t)	coplanar

Key. A $\rightarrow$ s,t; B $\rightarrow$ p,t; C $\rightarrow$ q; D $\rightarrow$ r

Sol. (A) Both the lines pass through the point (7, 11, 15)

(B) < 2, 3, 4 > are direction ratios of both the lines. Also the point (1, 2, 3) is common to both

$\therefore$ The lines are coincident.

(C) < 5, 4 – 2 > are direction ratios of both the lines

$\therefore$ The lines are parallel.

Also $x = 2+5\lambda, y = -3+4\lambda, z = 5-2\lambda$

$$\therefore \quad \frac{2+5\lambda-7}{5} = \frac{-3+4\lambda-1}{4} = \frac{5-2\lambda-2}{-2}$$

i.e. $\lambda-1 = \lambda-1 = \frac{3-2\lambda}{-2}$

$\therefore$ no value of λ

Thus the lines are parallel and different.

(D) < 2, 3, 5> and < 3, 2, 5 > are direction ratios of first and 2nd line respectively.

$\therefore$ The lines are not parallel

$x = 3+2\lambda, \quad y = -2+3\lambda, \quad z = 4+5\lambda$

$x = 3+3\mu, \quad y = -2+2\mu, \quad z = 7+5\mu$

Are parametric equations of the lines.

Solving $3+2\lambda = 3+3\mu$ and $-2+3\lambda = 2+2\mu$

We get $\lambda = \frac{12}{5}, \mu = \frac{8}{5}$

Now substituting these values in $4+5\lambda = 7+5\mu$

We get

4 + 12 = 7 + 8 i.e. 16 = 15 which is not true.

$\therefore$ The lines do not intersect

Hence the lines are skew.

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6. Match the following: -

	Column – I		Column – II
(A)	Foot of perp. Drawn for point (1, 2, 3) to the line $\frac{x-2}{2}=\frac{y-1}{3}=\frac{z-2}{4}$ is	(p)	$\left(\frac{107}{29},\frac{30}{29},\frac{69}{29}\right)$
(B)	Image of line point (1, 2, 3) in the line $\frac{x-2}{2}=\frac{y-1}{3}=\frac{z-2}{4}$ is	(q)	$\left(\frac{88}{29},\frac{125}{29},\frac{69}{29}\right)$
(C)	Foot of perpendicular from the point (2, 3, 5) to the plane 2x + 3y – 4z + 17 = 0 is	(r)	$\left(\frac{107}{29},\frac{125}{29},\frac{185}{29}\right)$
(D)	Image of the point (2, 5, 1) in the plane 3x – 2y + 4z – 5 = 0 is	(s)	$\left(\frac{68}{29},\frac{44}{29},\frac{78}{29}\right)$
		(t)	$\left(\frac{38}{29},\frac{57}{29},\frac{185}{29}\right)$

Key. A $\rightarrow$ s,t; B $\rightarrow$ p,t; C $\rightarrow$ q; D $\rightarrow$ r

Sol. (A) Let the foot Q of perpendicular be $(2+2\lambda,1+3\lambda,2+4\lambda)$

$\therefore \quad 2(2\lambda+1)+3(3\lambda-1)+4(4\lambda-1)=0$

$29\lambda=5 \qquad \lambda=\frac{5}{29}$

$\therefore \quad \text{Foot } =\left(\frac{68}{29},\frac{44}{29},\frac{8}{29}\right)$

(B) Let the image be the point (a, b, c), then

$\frac{1+a}{2}=\frac{68}{29},\frac{2+b}{2}=\frac{44}{29} \text{ and } \frac{3+c}{2}=\frac{78}{29}$

i.e. $a=\frac{107}{29},b=\frac{30}{29} \text{ and } c=\frac{68}{29}$

(C) $\frac{x-2}{2}=\frac{y-3}{3}=\frac{z-5}{-4}=-\frac{4+9-20+17}{4+9+16}=\frac{-10}{29}$

$\therefore \quad a=\frac{38}{29},b=\frac{57}{29} \text{ and } c=\frac{185}{29}$

(D) $\frac{x-2}{3}=\frac{y-5}{-2}=\frac{z-1}{4}=-2\frac{6-10+4-5}{29}=\frac{10}{29}$

$x=2+\frac{30}{29}=\frac{88}{29},y=5-\frac{20}{29}=\frac{125}{29},z=1+\frac{40}{29}=\frac{69}{29}$

7. Match the following pair of planes with their lines of intersection

Column – I	Column – II
A) x + y = 0 = y + z	P) $\frac{x-2}{0}=\frac{y-2007}{-1}=z+2004$
B) x – 2 = 0 = y – 3	Q) $\frac{x-2}{0}=-y=z-1$
C) x – 2 = 0 = y + z – 3	R) x = - y = z

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D) x – 2 = 0 = x + y + z – 3	S) $\frac{x-2}{0}=\frac{y-3}{0}=z$

Ans. **A – R ; B – S ; C – P ; D – Q**

Sol. Conceptual

8. Match the following.

Column – I	Column – II
A) The volume of the tetrahedron whose vertices are A(3, 7, 4), B(5, -2, 3), C(-4,5,6) and D(1, 2, 3)	P) 1
B) The perpendicular distance between 2x + 2y – z + 1 = 0 and $x+y-\frac{z}{2}+2=0$	Q) 0.74
C) A plane passes through (1, 2, -1) and is perpendicular to two planes 2x – 2y + z = 0 and x – y + 2z = 4. The distance of the plane from the point (1, 2, 2) is	R) 15.33
D) A line is perpendicular to x + 2y + 2z = 0 and passes through (0, 1, 0). The perpendicular distance of this line from (0, 0, 0) is	S) 2.82

Ans. **A – R ; B – P ; C – S ; D – Q**

A) Volume of tetrahedron = $\frac{1}{6}\left\|\begin{matrix}3 & 7 & 4 & 1\\ 5 & -2 & 3 & 1\\ -4 & 5 & 6 & 1\\ 1 & 2 & 3 & 1\end{matrix}\right\|=15.33$

B) Distance = $\frac{|4-1|}{\sqrt{2^2+2^2+1}}=1$

C) Direction ratios of the required plane will be (1, 1, 0). Equation of plane : x + y = λ this plane passes through (1, -2, 1) $\Rightarrow \lambda=-1$

$\Rightarrow x+y+1=0$

Distance of (1, 2, 2) from the plane = $\frac{|4|}{\sqrt{1+1}}=2\sqrt{2}$

9. A variable plane cuts the x-axis, y-axis and z-axis at the points A, B and C respectively such that the volume of the tetrahedron OABC remains constant equal to 32 cubic unit and O is the origin of the coordinate system.

Column – I	Column – II
A) The locus of the centroid of the tetrahedron is	P) xyz = 24
B) the locus of the point equidistant from O, A, B and C is	Q) $\left(x^2+y^2+z^2\right)^3=192xyz$
C) the length of the foot of perpendicular from origin to the plane is	R) xyz = 3
D) If PA, PB and PC are mutually perpendicular then the locus of P is	S) $\left(x^2+y^2+z^2\right)^3=1536xyz$

Ans. **A – R ; B – P ; C – Q ; D – S**

Given $\frac{abc}{6}=32$, where A, B, C are respectively (a, 0, 0), (0, b, 0), (0, 0, c)

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A) Centroid of tetrahedron $(\alpha,\beta,\gamma)\equiv\left(\frac{a}{4},\frac{b}{4},\frac{c}{4}\right)\Rightarrow a=4\alpha, b=4\beta, c=4\gamma$

$\therefore 64\alpha\beta\gamma=32\times6\Rightarrow\alpha\beta\gamma=3$

B) Equidistant point $(\alpha,\beta,\gamma)\equiv\left(\frac{a}{2},\frac{b}{2},\frac{c}{2}\right)\Rightarrow a=2\alpha, b=2\beta, c=2\gamma$

$\therefore 8\alpha\beta\gamma=32\times6\Rightarrow\alpha\beta\gamma=24$

C) The equation of the plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

$\therefore$ foot of perpendicular from the origin $\equiv(\alpha,\beta,\gamma)\equiv\left(\frac{1/a}{\sum\frac{1}{a^2}},\frac{1/b}{\sum\frac{1}{a^2}},\frac{1/c}{\sum\frac{1}{a^2}}\right)$

$\Rightarrow\frac{1}{a\alpha}=\frac{1}{b\beta}=\frac{1}{c\gamma}=t$, where $t=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\sum\frac{1}{a^2}$

$or\ t=\left(\alpha^2+\beta^2+\gamma^2\right)t^2\Rightarrow t=\frac{1}{\alpha^2+\beta^2+\gamma^2}$ and

$a=\frac{\alpha^2+\beta^2+\gamma^2}{\alpha}, b=\frac{\alpha^2+\beta^2+\gamma^2}{\beta}, c=\frac{\alpha^2+\beta^2+\gamma^2}{\gamma}$

Now abc = 6 x 32 $\Rightarrow\left(\alpha^2+\beta^2+\gamma^2\right)^3=192\alpha\beta\gamma$

D) Let P be (α,β,γ) then

$PA\perp PB\Rightarrow\alpha(\alpha-a)+\beta(\beta-b)+\gamma\gamma=0\Rightarrow a\alpha+b\beta=\alpha^2+\beta^2+\gamma^2$

$PB\perp PC\Rightarrow\alpha\alpha+\beta(\beta-b)+\gamma(\gamma-c)=0\Rightarrow b\beta+c\gamma=\alpha^2+\beta^2+\gamma^2$

$\therefore\frac{a}{1/\alpha}=\frac{b}{1/\beta}=\frac{c}{1/\gamma}\Rightarrow a=\frac{\alpha^2+\beta^2+\gamma^2}{2\alpha}, b=\frac{\alpha^2+\beta^2+\gamma^2}{2\beta}, =\frac{\alpha^2+\beta^2+\gamma^2}{2\gamma}$

$\therefore abc=6\times32\Rightarrow\left(\alpha^2+\beta^2+\gamma^2\right)^3=192\times8\alpha\beta\gamma=1536\alpha\beta\gamma$

10. Match the following

Column – I	Column – II
A) Equation of normal to the curve x = y^2 – 6y + 6 which is parallel to the line joining origin to the vertex of parabola is	P) x + y = $4\sqrt{2}$
B) Equation of the line of latus rectum of the curve xy = 4 is	Q) y = 3x – 1
C) A ray of light emanating from the point (4, 3) after getting reflected from the line x + y – 3 = 0 passes through the point (3, 8). The equation of reflected ray is	R) $\sqrt{2}y=2(x-y)$
D) If the normal at (t_1) on the parabola y^2 = 4x having positive slope subtend right angle at the origin, then the equation is	S) 4x + 4y – 3 = 0

Ans. A – S; B – P ; C – Q ; D – R

Sol. A) x = y^2 – 6y + 6 $\Rightarrow$ $(y-3)^2$ = x + 3 $\therefore$ vertex (-3, 3)

$-\frac{dx}{dy}=-(2y_1-6)$

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Slope of line joining vertex to the origin is – 1

$(2y_1 - 6) = 1 \Rightarrow y_1 = \frac{7}{2}$

$\Rightarrow x = \left(\frac{7}{2}\right)^2 - \frac{6.7}{2} + 6 = -\frac{11}{4}$

$\therefore$ Equation of normal is $y - \frac{7}{2} = -\left(x + \frac{11}{4}\right)$

$\Rightarrow 4x + 4y - 3 = 0$

B) xy = 4

Coordinates of foci are $\left(\pm\sqrt{2}c, \pm\sqrt{2}c\right)$ *or* $\left(\pm 2\sqrt{2}, \pm 2\sqrt{2}\right) = 2$

Slope of line of latus rectum is – 1

$\therefore$ Equation is $y - 2\sqrt{2} = -\left(x - 2\sqrt{2}\right)$

$x + y = 4\sqrt{2}$

C) Reflection of (4, 3) w.r.t x + y = 3 is (0, - 1)

$\therefore$ Equation of reflected ray is $y + 1 = \frac{8+1}{3-0}(x - 0)$

$\Rightarrow y = 3x - 1$

D) Equation of normal at (t_1) is $y = -t_1x + 2t_1 + t_1^3$

$\therefore$ it passes through (t_2)

$\Rightarrow t_1 + t_2 = -\frac{2}{t_1}$

Also the normal standard right angle at origin $\Rightarrow t_1t_2 = -1$

$t_1^2 + t_1t_2 = -2$

$t_1^2 = 2$

$\Rightarrow t_1 = -\sqrt{2}$ $\because$ slope of normal has to be positive

$\therefore$ equation is $y = \sqrt{2}x - 2\sqrt{2} - 2\sqrt{2}$

or $\sqrt{2}y = 2(x - y)$

11. A hyperbola has one focus at (1, 2), its corresponding directrix is x + y = 1 and eccentricity is 2. Then

Column – I	Column – II
A) centre of hyperboal	P) $\left(-\frac{5}{3}, -\frac{2}{3}\right)$
B) co-ordinate of other focus	Q) $\left(-\frac{1}{3}, \frac{2}{3}\right)$
C) Equation of conjugate axis	R) 3x + 3y = 1
D) Equation of other directrix	S) 3x + 3y + 1 = 0

Ans. A – Q ; B – P ; C – R ; D – S

Sol. Equation of transverse axis is x – y = - 1

A $\equiv$ (0, 1)

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Also, $\left(ae-\frac{a}{e}\right)=\left|\frac{1+2-1}{\sqrt{2}}\right|=\sqrt{2}\Rightarrow a=\frac{2\sqrt{2}}{3}$ equation of transverse axis in parametric form $\frac{x-1}{1/\sqrt{2}}=\frac{y-2}{1/\sqrt{2}}=r$ - (1)

A) Here $r=-ae=-\frac{2\sqrt{2}}{3}\times 2=-\frac{4\sqrt{2}}{3}$

Centre = $\left(1-\frac{4}{3},\ 2-\frac{4}{3}\right)$

$\equiv\left(-\frac{1}{3},\frac{2}{3}\right)$

B) Here $r=-\left(2ae\right)=-\frac{8\sqrt{2}}{3}$

Other focus = $\left(1-\frac{8}{3},2-\frac{8}{3}\right)\equiv\left(-\frac{5}{3},-\frac{2}{3}\right)$

C) x + y = λ passes through $\left(-\frac{1}{3},\frac{2}{3}\right)$

$-\frac{1}{3}+\frac{2}{3}=\lambda\Rightarrow\lambda=\frac{1}{3}$

x + y = $\frac{1}{3}$ $\Rightarrow$ 3x + 3y = 1

D) Here $r=-\left(ae+\frac{a}{e}\right)=-\left(\frac{4\sqrt{2}}{3},\frac{\sqrt{2}}{3}\right)=\frac{-5\sqrt{2}}{3}$

$A'=\left(1-\frac{5}{3},\ 2-\frac{5}{3}\right)$

$=\left(-\frac{2}{3},\frac{1}{3}\right)$

Other directrix

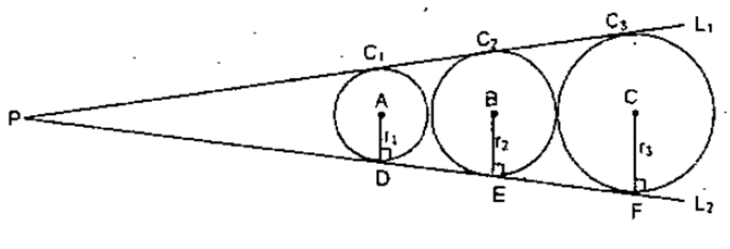
x+ y = λ

$-\frac{2}{3}+\frac{1}{3}=\lambda$

$\lambda=-\frac{1}{3}$

3x + 3y + 1 = 0

12. Let C_1,C_2,C_3 are three circles with radius $r_1,r_2,r_3\left(r_1<r_2<r_3\right)$ and touches each other externally as shown, also let L_1 and L_2 are two direct common tangents to three circles. Let point of intersection of these lines is P and D, E, F are point of contact of L_2 with C_1,C_2,C_3



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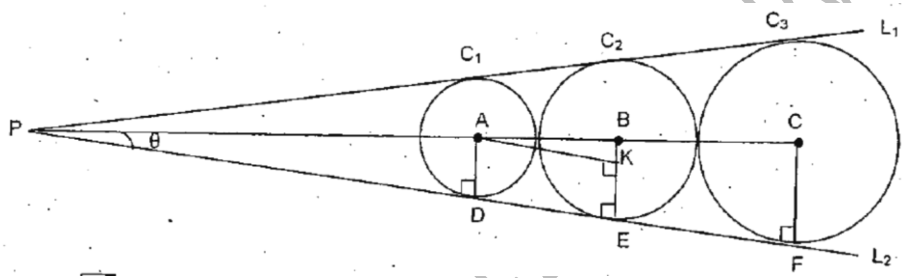
Column – I	Column – II
A) r_3 equals	p) $2\sqrt{r_2}\left(\sqrt{r_1}+\sqrt{r_3}\right)$
B) The distance DE is equal to	q) $\frac{2r_1^{3/2}\cdot r_2^{1/2}}{r_2 r_1}$
C) The distance PD can be given as	r) $\frac{r_2^2}{r_1}$
D) The distance DF is equal to	s) $2\sqrt{r_1 r_2}$
	t) $\frac{r_2^2-r_1^2}{r_1}$

Ans. A – R ; B – S ; C – Q ; D – P

Sol. $DE = AK = \sqrt{\left(r_1+r_2\right)^2-\left(r_2-r_1\right)^2} = 2\sqrt{r_1 r_2}$



$EF = 2\sqrt{r_2 r_3}$

Let PD = x, $\lfloor APD = \theta$

$\Rightarrow \tan\theta = \frac{AD}{PD} = \frac{BE}{PE} = \frac{CF}{PF} \Rightarrow \frac{r_1}{x} = \frac{r_2}{x+2\sqrt{r_1 r_2}} = \frac{r_3}{x+2\sqrt{r_1 r_2}+2\sqrt{r_2 r_3}}$

$\Rightarrow \frac{r_2-r_1}{2\sqrt{r_1 r_2}} = \frac{r_3-r_2}{2\sqrt{r_2 r_3}} \Rightarrow \sqrt{r_3}\left(r_2-r_1\right) = \sqrt{r_1}\left(r_3-r_2\right)$

$\Rightarrow r_2\left(\sqrt{r_3}+\sqrt{r_1}\right) = \sqrt{r_1 r_3}\left(\sqrt{r_1}+\sqrt{r_3}\right) \quad \Rightarrow r_2 = \sqrt{r_1 r_3}$

also $x = \frac{r_1\cdot 2\sqrt{r_1 r_2}}{r_2-r_1}$

also DF = DE + EF

$\Rightarrow 2\sqrt{r_1 r_2}+2\sqrt{r_2 r_3} = 2\sqrt{r_2}\left(\sqrt{r_1}+\sqrt{r_3}\right)$

13. For the ellipse $9(x-4)^2 + 4(y-3)^2 - 36 = 0$ match list I with list II

Column – I	Column – II
A) Equation of the directrix	P) y = 3
B) Equation of the major axis	Q) x = 4
C) Equation of the minor axis	R) $y = 3+\sqrt{5}$
D) Equation of the latus rectum whose distance from the x-axis is more	S) $x = 4+\sqrt{5}$
	T) $y = 3+\frac{9}{\sqrt{5}}$

Ans. A – T ; B – Q ; C – P ; D – R

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Sol. The ellipse is $\frac{x^2}{4}+\frac{y^2}{9}=1$

$a^2=4, b^2=9$ = $e=\frac{\sqrt{5}}{3}$

Equation of the major axis x = 4

Equation of the minor axis y = 3

Equation of the directrix $y=3+\frac{9}{\sqrt{5}}$

Equation of the required latus rectum $y=3+\sqrt{5}$

14. Match the following

Column – I	Column – II
A) The product of length of perpendicular from any point of the hyperbola $x^2 - y^2 = 10$ to its asymptote is	P) 0
B) The number of points on the ellipse $\frac{x^2}{4}+\frac{y^2}{3}=1$ from which mutually perpendicular tangents can be drawn to the hyperbola $\frac{x^2}{4}-\frac{y^2}{1}=1$ is/are	Q) 5
C) The distance between the directrices of the ellipse $(4x-8)^2+16y^2=\left(x+\sqrt{3}y+10\right)^2$ is	R) 16
D) Tangents are drawn from any point on the line y –x+ 2=0 to the parabola $y^2 = 4x$ such that the chords of contact pass through fixed point the sum of whose abscissa and ordinate is	S) 2

Ans. A – Q; B – S ; C – R ; D - P

Sol. A) $x^2 - y^2 = 10$

Equation of asymptotes are $y=\pm x$

Let P_1 and P_2 be length of perpendicular from any point on the asymptotes

$$P_1=\left|\frac{\sqrt{10}\tan\theta-\sqrt{10}\sec\theta}{\sqrt{2}}\right|, \quad P_2=\left|\frac{\sqrt{10}\tan\theta+\sqrt{10}\sec\theta}{\sqrt{2}}\right|$$

$$P_1P_2=\frac{10}{2}\left(\sec^2\theta-\tan^2\theta\right)=5$$

B) Director circle of hyperbola $\frac{x^2}{4}-\frac{y^2}{1}=1$ is $x^2 + y^2 = 3$

Solving this with $\frac{x^2}{4}+\frac{y^2}{3}=1$ gives $3x^2+4\left(3-x^2\right)=12$

$\Rightarrow x=0 \therefore y=\pm\sqrt{3}$ $\therefore$ number of points are 2

C) $(4x-8)^2+16y^2=\left(x+\sqrt{3}y+10\right)^2$

$$(x-2)^2+y^2=\frac{1}{4}\left(\frac{x+\sqrt{3}y+10}{2}\right)^2 \qquad \Rightarrow e=\frac{1}{2}$$

One of the focus is (2, 0) and directrix is $x+\sqrt{3}y+10=0$

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Distance between one of the focus and its corresponding directrix is

$$\frac{a}{e}-ae=a\left(2-\frac{1}{2}\right)=\frac{12}{2}=6 \quad = \quad a=\frac{6\times 2}{4}=4$$

Distance between directrices is $\frac{2a}{e}=\frac{2\times 4}{1/2}=16$

D) Any point on line y – x + 2 = 0 is $(\lambda, \lambda-2)$ equation of chord of contact by $y^2 = 4x$ is

$$y(\lambda-2)=2(x+\lambda)$$

$$=(x+y)-\lambda(y-2)=0 \qquad \Rightarrow y=2 \text{ and } x+y=0$$

15. Let the circle $(x-1)^2 + (y-2)^2 = 25$ cuts the rectangular hyperbola with transverse axis along $y = x$ at four different points A, B, C, D with coordinates (x_i, y_i), i = 1, 2, 3,4 respectively. O being the centre of the hyperbola.

Column – I	Column – II
A) $x_1 + x_2 + x_3 + x_4 =$	P) 2
B) $x_1^2+x_2^2+x_3^2+x_4^2=$	Q) 56
C) $y_1^2+y_2^2+y_3^2+y_4^2 = =$	R) 44
D) $x_1x_2 + x_2x_3 + x_1 + x_4x_1 + x_2x_4 + x_3x_4$	S) – 20

Ans. A – P ; B – R ; C – Q ; D – S

Sol. The circle is $x^2 + y^2 - 2x - 4y - 20 = 0$ let the hyperbola $xy = c^2$ if $\left(ct, \frac{c}{t}\right)$ be the points of intersection, then

$$c^2t^2+\frac{c^2}{t^2}-2ct-\frac{4c}{t}-20=0$$

$$\Rightarrow c^2t^4-2ct^3-20t^2-4ct+c^2=0$$

If t_1, t_2, t_3, t_4 be the roots, then

A) $\sum x_1 = ct_1+ct_2+ct_3+ct_4=c\cdot\frac{2}{c}=2$

B) $\sum x_1x_2 = c^2\sum t_1t_2=-20 \quad = \sum x_1^2=\left(\sum x_1\right)^2-2\sum x_1x_2=44$

C) $\sum y_1=\sum\frac{c}{t_1}=c\frac{\sum t_1t_2t_3}{t_1t_2t_3t_4}=4$

$\sum y_1y_2=c^2\sum\frac{1}{t_1t_2}=-20 \quad = \sum y_1^2=\left(\sum y_1\right)^2-2\sum y_1y_2=56$

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