Vectors

Multiple Correct Answer Type

1. Let $\bar{a}$ and $\bar{b}$ be two non collinear unit vectors. If $\bar{u} = \bar{a} - (\bar{a}.\bar{b})\bar{b}$ and $\bar{v} = \bar{a} \times \bar{b}$ then $|\bar{v}| =$

   a) $|\bar{u}|$ b) $|\bar{u}| + |\bar{u}.\bar{a}|$

   c) $|\bar{u}| + |\bar{u}.\bar{b}|$ d) $|\bar{u}| + \bar{u}.(\bar{a} + \bar{b})$

Key. A,C

Sol. Given $\bar{v} = \bar{a} \times \bar{b} \Rightarrow |\bar{v}| = |\bar{a}||\bar{b}|\sin\theta = \sin\theta$

$\bar{u} = \bar{a} - (\bar{a}.\bar{b})\bar{b}$ = $\bar{a} - \bar{b}\cos\theta$

$\Rightarrow |\bar{u}|^2 = (\bar{a} - \bar{b}\cos\theta)^2 = |\bar{a}|^2 + |\bar{b}|^2\cos^2\theta - 2\bar{a}.\bar{b}\cos\theta$

$= 1 + \cos^2\theta - 2\cos^2\theta$

$= 1 - \cos^2\theta$

$= \sin^2\theta$

$\Rightarrow |\bar{u}| = |\bar{v}|$

Again $\bar{u}.\bar{b} = \bar{a}.\bar{b} - (\bar{a}.\bar{b})(\bar{b}.\bar{b}) = 0$

$\Rightarrow |\bar{u}.\bar{b}| = 0$

2. Three vectors $\bar{a}\left(|\bar{a}| \neq 0\right)$, $\bar{b}$ and $\bar{c}$ are such that $\bar{a} \times \bar{b} = 3\bar{a} \times \bar{c}$. Also $|\bar{a}| = |\bar{b}| = 1$ and $|\bar{c}| = \frac{1}{3}$. If the angle between $\bar{b}$ and $\bar{c}$ is 60^0, then.

   a) $\bar{b} = 3\bar{c} + \bar{a}$ b) $\bar{b} = 3\bar{c} - \bar{a}$ c) $\bar{a} = 6\bar{c} + 2\bar{b}$ d) $\bar{a} = 6\bar{c} - 2\bar{b}$

Key. A,B

Sol. $\bar{a} \times (\bar{b} - 3\bar{c}) = \bar{0}$

$\Rightarrow \bar{b} - 3\bar{c} = \lambda\bar{a}$

$\Rightarrow |\bar{b} - 3\bar{c}| = |\lambda\bar{a}|$

$\Rightarrow 1 + 1 - 6.1.\frac{1}{3}.\frac{1}{2} = |\lambda| \Rightarrow \lambda = \pm 1$

$\therefore \bar{b} - 3\bar{c} = \pm\bar{a}$

3. If $\vec{a}$ and $\vec{b}$ are two unit vectors perpendicular to each other and $\vec{c} = \lambda_1\vec{a} + \lambda_2\vec{b} + \lambda_3(\vec{a} \times \vec{b})$, then the following is (are) true

a) $\lambda_1 = \vec{a}.\vec{c}$ b) $\lambda_2 = \left|\vec{b}\times\vec{a}\right|$

c) $\lambda_3 = \left|\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right|$ d) $\lambda_1+\lambda_2+\lambda_3 = \left(\vec{a}+\vec{b}+\vec{a}\times\vec{b}\right).\vec{c}.$

Key. A,D

Sol. (a) is proved if we take dot product of both sides with $\vec{a}$.

(b) If we take dot produuct with $\vec{b}$, we get

$$\lambda_2 = \vec{b}.\vec{c}$$

$\Rightarrow$ Choice (b) is not true.

(c) If we take dot product of both sides with $\vec{a}\times\vec{b}$, we get $[\vec{c}\,\vec{b}\,\vec{a}] = \lambda_3[\vec{a}\times\vec{b}]^2$

$\Rightarrow \lambda_3 = [\vec{a}\,\vec{b}\,\vec{c}]$ OR $\vec{c}.(\vec{a}\times\vec{b})$

$\Rightarrow$ Choice (c) is wrong.

(d) is correct since $\lambda_1+\lambda_2+\lambda_3 = \vec{c}.\vec{a}+\vec{b}.\vec{c}+[\vec{a}\,\vec{b}\,\vec{c}]$.

4. $\vec{a} = (\cos q)\vec{i} - (\sin q)\vec{j}$, $\vec{b} = (\sin q)\vec{i} + (\cos q)\vec{j}$, $\vec{c} = \vec{k}$, $\vec{r} = 7\vec{i} + \vec{j} + 10\vec{k}$

if $\vec{r} = x\vec{a} + y\vec{b} + z\vec{c}$, then

a) min. of x + y + z = 0 b) min. of x + y + z = 5

c) max. of x + y + z = 15 d) max. of x + y + z = 20

Key. A,D

Sol. $x = 7\cos q - \sin q,\ y = 7\sin q + \cos q,\ z = 10$

$x + y + z = 8\cos q + 6\sin q + 10$

min value = $10 - \sqrt{8^2 + 6^2} = 0$, max value = 10 + 10 = 20

5. If a vector $\vec{r}$ satisfies the equation $\vec{r}\times\left(\hat{i}+2\hat{j}+\hat{k}\right)=\hat{i}-\hat{k}$, then $\vec{r}$ is equal to

(A) $\hat{i}+3\hat{j}+\hat{k}$

(B) $3\hat{i}+7\hat{j}+3\hat{k}$

(C) $\hat{j}+t\left(\hat{i}+2\hat{j}+\hat{k}\right)$ where t is any scalar

(D) $\hat{i}+(t+3)\hat{j}+\hat{k}$ where t is any scalar

Key. A,B,C

Sol. $\vec{r}\times\left(\hat{i}+2\hat{j}+\hat{k}\right)=\hat{i}-\hat{k}$

Let $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$\therefore \quad \left(x\hat{i}+y\hat{j}+z\hat{k}\right)\times\left(\hat{i}+2\hat{j}+\hat{k}\right)=\hat{i}-\hat{k}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 2 & 1 \end{vmatrix}=\hat{i}-\hat{k}$$

Put values from options and check.

6. If $\bar{a}$ *and* $\bar{b}$ are unit vectors and $\bar{c}$ is a vector such that $\bar{c}=\bar{a}\ x\ \bar{c}+\bar{b}$ then

(A) $\left[\bar{a}\ \bar{b}\ \bar{c}\right]=\bar{b}.\bar{c}-\left(\bar{a}.\bar{b}\right)^2$

(B) $\left[\bar{a}\ \bar{b}\ \bar{c}\right]$ = 0

(C) Maximum value of $\left[\bar{a}\ \bar{b}\ \bar{c}\right]=\frac{1}{2}$

(D) Minimum value of $\left[\bar{a}\ \bar{b}\ \bar{c}\right]$ is $\frac{1}{2}$

Key. A,C

Sol. $\bar{c}.\bar{a}=\left(\left(\bar{a}\,x\bar{c}\right)+\bar{b}\right).\bar{a}=\bar{b}.\bar{a}$

$\bar{b}\ x\bar{c}=\left(\bar{b}.\bar{c}\right)+\bar{a}-\left(\bar{a}-\bar{b}\right).\bar{c}$

$\therefore \left[\bar{a}\,\bar{b}\,\bar{c}\right]=\bar{b}.\bar{c}-\left(\bar{a}-\bar{b}\right).\left(\bar{a}.\bar{c}\right)$

Also $\bar{c}.\bar{b}=1-\left[\bar{a}\,\bar{b}\,\bar{c}\right]$

$\therefore 2\ \left[\bar{a}\,\bar{b}\,\bar{c}\right]=1-\left(\bar{a}.\bar{b}\right)^2\le 1$

$\therefore \left[\bar{a}\,\bar{b}\,\bar{c}\right]\le\frac{1}{2}$

7. If a vector $\vec{r}$ satisfies the equation $\vec{r}\times\left(\hat{i}+2\hat{j}+\hat{k}\right)=\hat{i}-\hat{k}$, then $\vec{r}$ is equal to

(A) $\hat{i}+3\hat{j}+\hat{k}$

(B) $3\hat{i}+7\hat{j}+3\hat{k}$

(C) $\hat{j}+t\left(\hat{i}+2\hat{j}+\hat{k}\right)$ where t is any scalar

(D) $\hat{i}+(t+3)\hat{j}+\hat{k}$ where t is any scalar

Key. A,B,C

Sol. $\vec{r}\times\left(\hat{i}+2\hat{j}+\hat{k}\right)=\hat{i}-\hat{k}$

Let $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$\therefore \quad \left(x\hat{i}+y\hat{j}+z\hat{k}\right)\times\left(\hat{i}+2\hat{j}+\hat{k}\right)=\hat{i}-\hat{k}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 2 & 1 \end{vmatrix}=\hat{i}-\hat{k}$$

Put values from options and check.

8. In a four-dimensional space where unit vectors along axes are $\hat{i}, \hat{j}, \hat{k}$ and $\hat{\ell}$ and $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ are four non zero vectors such that no vector can be expressed as linear combination of others and $(\lambda-1)(\vec{a}_1-\vec{a}_2)+\mu(\vec{a}_2+\vec{a}_3)+\gamma(\vec{a}_3+\vec{a}_4-2\vec{a}_2)+\vec{a}_3+\delta\vec{a}_4=\vec{o}$ then

(A) $\lambda=1$ (B) $\mu=-\frac{2}{3}$ (C) $\lambda=\frac{2}{3}$ (D) $\delta=\frac{1}{3}$

Key. A,B,D

Sol. (a, b, d)

$$(\lambda-1)(\vec{a}_1-\vec{a}_2)+\mu(\vec{a}_2+\vec{a}_3)+\gamma(\vec{a}_3+\vec{a}_4-2\vec{a}_2)+\vec{a}_3+\delta\vec{a}_4=\vec{o}$$

i.e $(\lambda-1)\vec{a}_1+(1-\lambda+\mu-2\gamma)\vec{a}_2+(\mu+\gamma+1)\vec{a}_3+(\gamma+\delta)\vec{a}_4=0$

since $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$ are linearly independent

$\therefore$ $\lambda-1=0, 1-\lambda+\mu-2\gamma=0, \mu+\lambda+1=0$ $\gamma+\delta=0$

i.e. $\lambda=1, \mu=2\gamma, \mu+\gamma+1=0, \gamma+\delta=0$

i.e. $\lambda=1, \mu=-\frac{2}{3}, \gamma=-\frac{1}{3}, \delta=\frac{1}{3}$

9. A vector $(\vec{d})$ is equally inclined to three vectors $\vec{a}=\hat{i}-\hat{j}+\hat{k}, \vec{b}=2\hat{i}+\hat{j}$ and $\vec{c}=3\hat{j}-2\hat{k}$. Let $\vec{x}, \vec{y}, \vec{z}$ be three vector in the plane of $\vec{a}, \vec{b}; \vec{b}, \vec{c}; \vec{c}, \vec{a}$ respectively then

(A) $\vec{x}.\vec{d}=14$ (B) $\vec{y}.\vec{d}=3$

(C) $\vec{z}.\vec{d}=0$ (D) $\vec{r}.\vec{d}=0$ where $\vec{r}=\lambda\vec{x}+\mu\vec{y}+\delta\vec{z}$

Key. C,B

Sol. (c, d)

since $[\vec{a}, \vec{b}, \vec{c}]=0$

$\therefore$ $\vec{a}, \vec{b}$ and $\vec{c}$ are complanar vectors

Further since $\vec{d}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$

$\therefore$ $\vec{d}.\vec{a}=\vec{d}.\vec{b}=\vec{d}.\vec{c}=0$

$\therefore$ $\vec{d}.\vec{r}=0$

10. Identify the statement(s) which is/are incorrect ?

(A) $\vec{a}\times[\vec{a}\times(\vec{a}\times\vec{b})]=(\vec{a}\times\vec{b})(\vec{a}^2)$

(B) If $\vec{a}, \vec{b}, \vec{c}$ are non-zero, non coplanar vector and $\vec{v}.\vec{a}=\vec{v}.\vec{b}=\vec{v}.\vec{c}=0$ then $\vec{v}$ must be a null vector.

(C) If $\vec{a}$ and $\vec{b}$ lie in a plane normal to the plane contaning the vectors $\vec{c}$ and $\vec{d}$ then $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=\vec{o}$

(D) If $\vec{a},\vec{b},\vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vectors then $\vec{a}.\vec{b}'+\vec{b}.\vec{c}'+\vec{c}.\vec{a}'=3$

Key. A,C,D

Sol. (a, c, d)

(A) $\vec{a}\times[\vec{a}\times(\vec{a}\times\vec{b})]=\vec{a}\times[(\vec{a}\,.\,\vec{b})\vec{a}-(\vec{a}.\vec{a})\,b]=-(\vec{a}.\vec{a})(\vec{a}\times\vec{b})$

(A) is not correct

(B) Let $\vec{a},\vec{b},\vec{c}$ ne no coplanar vector

then $\vec{v}=\alpha\vec{a}+\beta\vec{b}+\gamma\vec{c}$

now $\vec{v}.\vec{a}=0$

$\Rightarrow$ $\alpha(\vec{a}.\vec{a})+\beta(\vec{b}.\vec{a})+\gamma(\vec{c}.\vec{a})=0$ ......(1)

and similarly

$\alpha(\vec{a}.\vec{b})+\beta(\vec{b}.\vec{b})+\gamma(\vec{c}.\vec{b})=0$ ....(2)

$\alpha(\vec{a}.\vec{c})+\beta(\vec{b}.\vec{c})+\gamma(\vec{c}.\vec{c})=0$ ....(3)

here $\begin{vmatrix}\vec{a}.\vec{a} & \vec{b}.\vec{a} & \vec{c}.\vec{a}\\ \vec{a}.\vec{b} & \vec{b}.\vec{b} & \vec{c}.\vec{b}\\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c}\end{vmatrix}=[\vec{a}\,\vec{b}\,\vec{c}]\neq 0$

Equation (1) (2) and (3) will have only one solution i.e. $\alpha=\beta=\gamma=0$

$\therefore$ (B) is true

(C) Let $\vec{a}.\vec{b}$ lie in the plane P_1

$\therefore$ $\vec{a}\times\vec{b}\perp P_1$

Let $\vec{c},\vec{d}$ lie in the plane P_2

$\therefore$ $\vec{c}\times\vec{d}\perp P_2$

as P_1 & P_2 are $\perp\ \perp$ to each other.

$\therefore$ $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})=0\ \&\ (\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})\neq 0$

(D) $\vec{a}.\vec{b}'+\vec{b}.\vec{c}'+\vec{c}.\vec{a}=0$ (property of reciprocal system)

11. The equation of a plane is 2x – y – 3z = 5 and A(1, 1, 1), B(2, 1, –3), C(1, –2, –2) and D(–3, 1, 2) are four points. Which of the following line segments are intersected by the plane?

(A) AD (B) AB (C) AC (D) BC

Key. B,C

Sol. For A(1, 2, 3), 2x – y – 3z – 5 = 2 – 1 – 3 – 5 < 0

For B(2, 1, –3), 2x – y – 3z – 5 = 4 – 1 + 9 – 5 > 0

For C(1, –2, –2), 2x – y – 3z – 5 = 2 + 2 + 6 – 5 > 0

For D(–3, 1, 2), 2x – y – 3z – 5 = – 6 – 1 – 6 – 5 < 0

$\therefore$ AD are on one side of the plane and B, C are on the other side

$\therefore$ the line segments AB, AC, BD, CD intersect the plane.

12. If $\vec{a}$, $\vec{b}$, $\vec{c}$ be three non zero vectors satisfying the condition $\vec{a}\times\vec{b}=\vec{c}$ & $\vec{b}\times\vec{c}=\vec{a}$ then

(A) $\vec{a}$, $\vec{b}$, $\vec{c}$ are orthogonal in pairs (B) $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = |\vec{a}|^2$

(C) $\left[\vec{a}\ \vec{b}\ \vec{c}\right] = |\vec{c}|^2$ (D) $|\vec{b}| = |\vec{c}|$

Key. B,C

Sol. Clearly $\vec{a}\cdot\vec{c} = 0$ & $\vec{b}\cdot\vec{c} = 0$ Also $\vec{a}\cdot\vec{b} = 0 \Rightarrow$ A

Again $\left.\begin{array}{l}|\vec{a}|\ |\vec{b}| = |\vec{c}| \\ |\vec{b}|\ |\vec{c}| = |\vec{a}|\end{array}\right] \Rightarrow \dfrac{|\vec{a}|}{|\vec{c}|} = \dfrac{|\vec{c}|}{|\vec{a}|} \Rightarrow |\vec{a}| = |\vec{c}|$ & $|\vec{b}| = 1$

$\Rightarrow \vec{a}\times\vec{b}\cdot\vec{c} = |\vec{a}||\vec{b}||\vec{c}| = |\vec{a}|^2 = |\vec{c}|^2$]

13. If $\vec{r}=\hat{i}+\hat{j}+\lambda\left(2\hat{i}+\hat{j}+4\hat{k}\right)$ and $\vec{r}\cdot\left(\hat{i}+2\hat{j}-\hat{k}\right) = 3$ are the equations of a line and a plane respectively then which of the following is incorrect?

(A) line is perpendicular to the plane

(B) line lies in the plane

(C) line is parallel to the plane but does not lie in the plane

(D) line cuts the plane obliquely

Key. A,C,D

Sol. Since $(2\hat{i}+\hat{j}+4\hat{k}).(\hat{i}+2\hat{j}-\hat{k})=0$ and, $(\hat{i}+\hat{j}).(\hat{i}+2\hat{j}-\hat{k})$ 1 + 2 = 3 $\Rightarrow$ line lies in the plane

14. If $\bar{r}$ is a vector satisfying $\bar{r}\times\left(\hat{i}+j+2k\right)=\hat{i}-j$ then $|\bar{r}|$ can be

A) π B) e C) $\dfrac{1}{3}$ D) $\dfrac{1}{\sqrt{5}}$

Key. A,B

Sol. Solving the equation we get $\bar{r}=\hat{i}+\hat{j}+\hat{k}+\lambda\left(\hat{i}+\hat{j}+2\hat{k}\right)$, $\lambda\in R$

15. If each of $\bar{a},\bar{b},\bar{c}$ is orthogonal to the sum of the other two vectors and $|\bar{a}|=3,\ |\bar{b}|=4,\ |\bar{c}|=5$ then which of the following statement(s) is/are true

a) if $\vec{a}$ makes angles of equal measures with x,y,z axes, then tangent of this angle is $\pm\sqrt{2}$

b) range of $|\bar{a}-\bar{b}|$ is $[1,\ 7]$ c) range of $|\bar{b}-\bar{c}|$ is $[1,\ 9]$ d) $|\bar{a}+\bar{b}+\bar{c}| = 2\sqrt{5}$

Sol : ans: a

a)according to the given condition

$a_1 = a_2 = a_3$ $\quad\quad a_1 = \pm\frac{1}{\sqrt{3}}$

$\cos\alpha = \pm\frac{1}{\sqrt{3}} \Rightarrow \tan\alpha = \pm\sqrt{2}$

b) $|\bar{a}-\bar{b}|^2 = 1 \text{ or } 49$ c) $|\bar{b}-\bar{c}|^2 = b^2 + c^2 - 2.\bar{b}.\bar{c} = 1 \text{ or } 81$

d) $|\bar{a}+\bar{b}+\bar{c}|^2 = 50+0 \Rightarrow |\bar{a}+\bar{b}+\bar{c}| = 5\sqrt{2}$

16. The position vector of a point P is $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, when x, y, z $\in$ N and $\vec{a} = \hat{i} + \hat{j} + \hat{k}$. If $\vec{r}.\vec{a} = 10$, the number of possible position of P is

(A) 36 (B) 72

(C) 66 (D) ${}^{9}C_2$

Key : A, D

Sol : $\because \vec{r}.\vec{a} = 10$

$\therefore$ x + y + z = 10; x ≥ 1, y ≥ 1, z ≥ 1

The required number of positions

$= {}^{10-1}C_{3-1} = {}^{9}C_2 = 36$

17. Let $\bar{a}$ and $\bar{b}$ be two non collinear unit vectors. If $\bar{u} = \bar{a} - (\bar{a}.\bar{b})\bar{b}$ and $\bar{v} = \bar{a}\times\bar{b}$ then $|\bar{v}|$ =

a) $|\bar{u}|$ b) $|\bar{u}| + |\bar{u}.\bar{a}|$ c) $|\bar{u}| + |\bar{u}.\bar{b}|$ d) $|\bar{u}| + \bar{u}.(\bar{a}+\bar{b})$

Key. A,C

Sol. Given $\bar{v} = \bar{a}\times\bar{b} \Rightarrow |\bar{v}| = |\bar{a}||\bar{b}|\sin\theta = \sin\theta$

$\bar{u} = \bar{a} - (\bar{a}.\bar{b})\bar{b} = \bar{a} - \bar{b}\cos\theta$

$\Rightarrow |\bar{u}|^2 = (\bar{a} - \bar{b}\cos\theta)^2 = |\bar{a}|^2 + |\bar{b}|^2\cos^2\theta - 2\bar{a}.\bar{b}\cos\theta$

$= 1 + \cos^2\theta - 2\cos^2\theta$

$= 1 - \cos^2\theta$

$= \sin^2\theta$

$\Rightarrow |\bar{u}| = |\bar{v}|$

Again $\bar{u}.\bar{b} = \bar{a}.\bar{b} - (\bar{a}.\bar{b})(\bar{b}.\bar{b}) = 0$

$\Rightarrow |\bar{u}.\bar{b}| = 0$

18. If the unit vectors $\vec{a}$ and $\vec{b}$ are inclined at an angle 2θ such that $|\vec{a}-\vec{b}| < 1$ and $0 \le \theta \le \pi$, then θ lies in the interval

(A) [0, π/6) (B) (5π/6, π]

(C) (π/6, π/2] (D) [π/2, 5π/6)

Key. A,B

Sol. Since, $\vec{a}$ and $\vec{b}$ are unit vectors, we have

$$\left|\vec{a}-\vec{b}\right| = \sqrt{\left(\vec{a}-\vec{b}\right)^2}$$

$$\therefore \sqrt{\left(\vec{a}\right)^2+\left(\vec{b}\right)^2-2\vec{a}.\vec{b}} = \sqrt{1+1-2\cos 2\theta} = 2\left|\sin\theta\right|$$

Therefore, $\left|\vec{a}-\vec{b}\right|<1$

$\Rightarrow \quad 2\left|\sin\theta\right|<1$

$\left|\sin\theta\right|<\frac{1}{2}$

$\Rightarrow \quad \theta\in\left[0,\frac{\pi}{6}\right)$

or $\left(\frac{5\pi}{6},\pi\right]$

19. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are non-zero, non-collinear vectors such that a vector $\vec{p}$ = a b cos $\left(2\pi-\left(\vec{a} \wedge \vec{b}\right)\right)$ $\vec{c}$ and a vector $\vec{q}$ = a c cos $\left(\pi-\left(\vec{a} \wedge \vec{c}\right)\right)$ $\vec{b}$ then $\vec{p}$ + $\vec{q}$ is

(A) parallel to $\vec{a}$
(B) perpendicular to $\vec{a}$
(C) coplanar with $\vec{b}$ & $\vec{c}$
(D) coplanar with $\vec{a}$ and $\vec{c}$

Key. B,C

Sol. $\vec{p}=ab\cos(2\pi-\theta)\vec{c}$ where θ is the angle between $\vec{a}$ and $\vec{b}$ and

$\vec{q}=ac\cos(\pi-\varphi)\vec{b}$ where ϕ is the angle between $\vec{a}$ and $\vec{c}$

Now $\vec{p}+\vec{q}$ = $(ab\cos\theta)\vec{c}-ac\cos\varphi\vec{b}$ = $(\vec{a}.\vec{b})\vec{c}-(\vec{a}.\vec{c})\vec{b}$ = $\vec{a}\times(\vec{c}\times\vec{b})$ $\Rightarrow$ B and C

20. Given three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ such that they are non – zero, non – coplanar vectors, then which of the following are coplanar.

(A) $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$
(B) $\vec{a}-\vec{b}$, $\vec{b}+\vec{c}$, $\vec{c}+\vec{a}$
(C) $\vec{a}+\vec{b}$, $\vec{b}-\vec{c}$, $\vec{c}+\vec{a}$
(D) $\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}-\vec{a}$

Key. B,C,D

Sol. Verify $\vec{v}_1+\vec{v}_2=\vec{v}_3$ in order to quickly answer

21. Let OABC be a tetrahedron whose four faces are equilateral triangles of unit side. Let $\overrightarrow{OA}=\vec{a}$, $\overrightarrow{OB}=\vec{b}$ and $\overrightarrow{OC}=\vec{c}$, then

(A) $\vec{c}=\frac{1}{3}\left(\vec{a}+\vec{b}\pm2\sqrt{2}\,\vec{a}\times\vec{b}\right)$
(B) $\vec{c}=\frac{1}{2}\left(\vec{a}+\vec{b}\pm2\sqrt{3}\,\vec{a}\times\vec{b}\right)$
(C) volume of the tetrahedron is $\frac{1}{2\sqrt{3}}$
(D) $\left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|=\frac{1}{\sqrt{2}}$

Key. A,D

Sol. Let $\vec{C}=x\vec{a}+y\vec{b}+z(\vec{a}\times\vec{b})$. Taking succecive dots with $\vec{a},\vec{b},\vec{c}$ and $\vec{a}\times\vec{b}$ we get x = y = $\frac{1}{3}$ and z = ± $\frac{2\sqrt{2}}{3}$.

22. If $\vec{a},\vec{b},\vec{c},\vec{d}$ are unit vectors such that $\left(\vec{a}\times\vec{b}\right).\left(\vec{c}\times\vec{d}\right)=1$ and $\vec{a}.\vec{c}=\frac{1}{2}$ then

(A) $\vec{a},\vec{b},\vec{c}$ are non coplanar
(B) $\vec{b},\vec{d}$ are non parallel
(C) $\vec{b},\vec{c},\vec{d}$ are coplanar
(D) $\vec{a},\vec{d}$ are parallel and $\vec{b},\vec{c}$ are parallel

Key. B,C

Sol. $\left(\vec{a}\times\vec{b}\right).\left(\vec{c}\times\vec{d}\right)=1 \Rightarrow \sin\alpha\ \sin\beta\ ((\hat{n}_1.\hat{n}_2)=1 \Rightarrow \sin\alpha\ \sin\beta\ \cos\theta = 1$

$\Rightarrow \sin\alpha = 1,\ \sin\beta = 1$ and $\cos\theta = 1 \Rightarrow \alpha = \beta = \pi/2,\ \theta = 0$ i.e., $\hat{n}_1 \| \hat{n}_2$

So, $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are coplanar. Again $\vec{a}.\vec{c}=\frac{1}{2} \Rightarrow \cos\gamma = \frac{1}{2} \Rightarrow \gamma = \pi/3$

So, no two of vectors $\vec{a},\vec{b},\vec{c},\vec{d}$ are parallel.

23. *ABCDEFGH* is a regular octagon. If $\overline{AB}=\overline{a},\overline{BC}=\overline{b},\overline{CD}=l\overline{a}+m\overline{b}$ and $\overline{DE}=p\overline{a}+q\overline{b}$, then

A) *m, p* are irrational
B) l,q are rational
C) $m+p=0$
D) $l-q=0$

Key. A,B,C

Sol. $\overline{CD}=-\overline{a}+\sqrt{2}\overline{b}$

$\overline{DE}=-\sqrt{2}\overline{a}+\overline{b}$.

24. In a triangle ABC, the point D divides BC in the ratio 3:4 and the point E divides BA in the ratio 4:3.If AD and CE intersects at F, then

a) AF:FD =21 :16
b) AF:FD = 2:1
c) CF:FE= 28:9
d) CF:FE=9:28

Key. A,C

Sol. Using Menelau's theorem or by vectors

$$\frac{AF}{DF}=\frac{21}{16},\frac{CF}{FE}=\frac{28}{9}$$

25. If $A_1 B_1 C_1$ and $A_2B_2C_2$ are two coplanar triangles such that perpendicular from A_1, B_1, C_1 to the sides B_2C_2, C_2A_2, A_2B_2 of the triangles $A_2B_2C_2$ are concurrent, then

(A) $\Sigma\ \vec{a}_1\ (\vec{c}_2-\vec{b}_2)=0$
(B) $\Sigma\ \vec{a}_1\,\vec{b}_2\vec{c}_2=0$
(C) $\Sigma\ \vec{a}_1\ (\vec{c}_2+\vec{b}_2)=0$
(D) $\Sigma\ \vec{a}_2\ (\vec{c}_1-\vec{b}_1)=0$

Key. A,D

Sol. Let H be the point of concurrency

$A_1H \perp B_2C_2 \Rightarrow (\vec{h}-\vec{a}_1)\ (\vec{c}_2-\vec{b}_2) = 0$

$B_1H \perp C_2A_2 \Rightarrow (\vec{h}-\vec{b}_1)\ (\vec{a}_1-\vec{c}_1) =$

$C_1H \perp A_1B_1 \Rightarrow (\vec{h}-\vec{c}_1)\ (\vec{b}_2-\vec{a}_2)=0$

$\Rightarrow \Sigma\ \vec{a}_1\ (\vec{c}_2-\vec{b}_2)=0$

26. $\overline{a},\overline{b},\overline{c}$ are unit vectors which are linearly dependent. $\overline{d}$ is a unit vector perpendicular to the plane containing $\overline{a},\overline{b},\overline{c}$. If $(\overline{a}\times\overline{b})\times(\overline{c}\times\overline{d})=\frac{1}{6}(i-2j+2k)$ and the angle between $\overline{a},\overline{b}$ is $\frac{\pi}{6}$ then $\overline{c}$ can be

A) $\frac{i-2j+2k}{3}$ B) $\frac{2i+j-k}{3}$ C) $\frac{-2i-2j+k}{3}$ D) $\frac{-i+2j-2k}{3}$

Key. A,D

Sol. Conceptual

27. If $\bar{r}=x\bar{a}\times(\bar{a}\times\bar{b})+y\bar{a}\times\bar{b}$ and $\bar{r}$ satisfies the conditions $\bar{r}.\bar{b}=1;[\bar{r}\ \bar{a}\ \bar{b}]=1$ and also $\bar{a}.\bar{b}\neq 0$ then

A) $\bar{r}.\bar{a}=0$ B) $x=\frac{-1}{(\bar{a}\times\bar{b})^2}$ C) $x=\frac{\bar{a}.\bar{b}}{(\bar{a}\times\bar{b})^2}$ D) $x+y=0$

Key. A,B,D

Sol. Conceptual

28. $\vec{u}=\hat{i}-\hat{j}+\hat{k},\vec{v}=\alpha\hat{i}+\alpha\hat{j}+(\beta+1)\hat{k}$, $\vec{w}=\beta\hat{i}+\beta\hat{j}+(2\alpha+1)\hat{k}$. If it is possible to construct a parallelo piped using $\vec{u},\vec{v},\vec{w}$ as its 3-coterminus sides for any value of α, then which of the following is/are false.

A) $\frac{-1-\sqrt{2}}{2\sqrt{2}}<\beta<\frac{\sqrt{2}-1}{2\sqrt{2}}$ B) $\frac{-1-\sqrt{2}}{2\sqrt{2}}<\beta<\frac{1-\sqrt{2}}{2\sqrt{2}}$

C) $\frac{-1+\sqrt{2}}{2\sqrt{2}}<\beta<\frac{1+\sqrt{2}}{2\sqrt{2}}$ D) $\frac{1-\sqrt{2}}{2\sqrt{2}}<\beta<\frac{1+\sqrt{2}}{2\sqrt{2}}$

Key. C,D

Sol. $[\vec{u}\ \ \vec{v}\ \ \vec{w}]\neq 0\Rightarrow 2\alpha^2+\alpha-\beta^2-\beta\neq 0$

$\therefore D<0$

29. Let $\vec{a}\,\&\,\vec{c}$ are unit vectors and $\left|\vec{b}\right|=4$ with $\vec{a}\times\vec{b}=2\vec{a}\times\vec{c}$. The angle between $\vec{a}\,\&\,\vec{c}$ is $\cos^{-1}(1/4)$. If $\vec{b}-2\vec{c}=\lambda\vec{a}$, then λ equals

A) 1/3 B) 1/4 C) -4 D) 3

Key. C,D

Sol. $\left|\vec{b}\right|=\left|2\vec{c}+\lambda\vec{a}\right|$

30. Unit vectors $\vec{a}$ and $\vec{b}$ are perpendicular and unit vector $\vec{c}$ be inclined at an angle θ to both $\vec{a}$ and $\vec{b}$. If $\vec{c}=\alpha\vec{a}+\beta\vec{b}+\gamma(\vec{a}\times\vec{b})$ then

(A) $\alpha=\beta$ (B) $1-2\alpha^2=\gamma^2$ (C) $\gamma^2=1-2\cos^2\theta$ (D) $\alpha^2-\beta^2=\gamma^2$

Key. A,B,C

Sol. $\vec{c}=\alpha\vec{a}+\beta\vec{b}+\gamma(\vec{a}\times\vec{b})$

$\vec{c}.\vec{a}=\alpha\Rightarrow\cos\theta=\alpha\rightarrow(1)$

$\vec{c}.\vec{b}=\beta\Rightarrow\cos\theta=\beta\rightarrow(2)$

Also $2\cos^2\theta+\cos^2\left(\vec{c},\vec{a}\times\vec{b}\right)=1$

$\Rightarrow\gamma^2=1-2\alpha^2\rightarrow(3)$

From (1) , (2) and (3) it follows

31. If ABCD be a tetrahedron with G as centroid and position vectors of A,B,C,D are $\vec{a},\vec{b},\vec{c},\vec{d}$ respectively then volume of the tetrahedron GABC =

(A) $\frac{1}{6}\left|\left[\vec{a}\,\vec{b}\,\vec{c}\right]\right|$ (B) $\frac{1}{6}\left|\left[\vec{b}\,\vec{c}\,\vec{d}\right]\right|$ (C) $\frac{1}{3}\left|\left[\vec{b}\,\vec{c}\,\vec{d}\right]\right|$ (D) $\frac{1}{3}\left|\left[\vec{a}\,\vec{b}\,\vec{c}\right]\right|$

Key. A,B

Sol. Conceptual

32. If $\bar{a}$ and $\bar{c}$ are unit vectors and $|\bar{b}| = 4$ with $\bar{a}\times\bar{b} = 2\bar{a}\times\bar{c}$. The angle between $\bar{a}$ and $\bar{c}$ is $\cos^{-1}\left(\frac{1}{4}\right)$, $\bar{b}-2\bar{c} = \lambda\bar{a}$, then λ =

(a) 3 (b) -3 (c) 4 (d) -4

Key. A,D

Sol. $|\bar{a}| = |\bar{c}| = 1, |\bar{b}| = 4$

$\bar{a}\times\bar{b} = 2(\bar{a}\times\bar{c}); (\bar{a},\bar{c}) = \cos^{-1}\left(\frac{1}{4}\right)$

Now $\bar{a}.\bar{c} = |\bar{a}||\bar{c}|\frac{1}{4} = \frac{1}{4} \Rightarrow \bar{a}.\bar{c} = \frac{1}{4} \rightarrow$ (1)

Given $\bar{b}-2\bar{c} = \lambda\bar{a}$

$|\bar{b}-2\bar{c}|^2 = \lambda^2|\bar{a}|^2$

$\Rightarrow b^2 + 4c^2 - 4\bar{b}\cdot\bar{c} = \lambda^2$

$\Rightarrow 4\bar{b}\cdot\bar{c} = 20-\lambda^2$

$\Rightarrow \bar{b}\cdot\bar{c} = \frac{20-\lambda^2}{4}$

$\bar{b}\cdot\bar{c} - 2\bar{c}\cdot\bar{c} = \lambda\bar{a}\cdot\bar{c}$

$\frac{20-\lambda^2}{4} - 2 = \frac{\lambda}{4}$

$\Rightarrow 20-\lambda^2-8 = \lambda$

$\Rightarrow \lambda^2+\lambda-12 = 0$

$\Rightarrow (\lambda-3)(\lambda+4) = 0$

$\Rightarrow \lambda = 3 \ or \ -4$

33. If (x, y, z) $\neq$ (0, 0, 0) and $\left(\hat{i}+j+3\hat{k}\right)x+\left(3\hat{i}-3j+\hat{k}\right)y+\left(-4\hat{i}+5j\right)z = \lambda\left(x\hat{i}+yj+z\hat{k}\right)$ where $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the coordinate axes, then

(a) λ = 0 (b) λ = 2 (c) λ = 1 (d) λ = -1

Key. A,D

Sol. Here $\left(\hat{i}+\hat{j}+3\hat{k}\right)x+\left(3\hat{i}-3\hat{j}+\hat{k}\right)y+\left(-4\hat{i}+5\hat{j}\right)z = \lambda\left(x\hat{i}+y\hat{j}+z\hat{k}\right)$

On equating we obtain

$(1-\lambda)x+3y-4z = 0$

$x-(3+\lambda)y+5z = 0$

$3x+y-\lambda z = 0$

Since the equation have non trivial solutions

Hence $\begin{vmatrix} 1-\lambda & 3 & -4 \\ 1 & -(3+\lambda) & 5 \\ 3 & 1 & -\lambda \end{vmatrix} = 0 \quad \Rightarrow \lambda = 0 \text{ or } -1$

34. If $\bar{a} = \bar{i} + \bar{j} + \bar{k}$, $\bar{b} = 4\bar{i} + 3\bar{j} + 4\bar{k}$ and $\bar{c} = \bar{i} + \alpha\bar{j} + \beta\bar{k}$ are linearly dependent vectors and $|\bar{c}| = \sqrt{3}$ then

(a) $\beta = \pm 1$ (b) $\beta = 1$ (c) $\alpha = 1$ (d) $\alpha = -1$

Key. B,C,D

Sol. If $\bar{a}, \bar{b}, \bar{c}$ are linearly independent vectors, then $\bar{c}$ should be a linear combination of $\bar{a}$ and $\bar{b}$

$\bar{c} = p\bar{a} + q\bar{b}$ for some scalars p and q

i.e., $\bar{i} + \alpha\bar{j} + \beta\bar{k} = p(\hat{i} + \hat{j} + k) + q(4\hat{i} + 3\hat{j} + 4k)$

$\Rightarrow 1 = p + 4q \quad \alpha = p + 3q \qquad \beta = p + 4q$

$\Rightarrow \beta = 1$ Now $|\bar{c}| = \sqrt{3} \qquad \Rightarrow 1 + \alpha^2 + \beta^2 = 3$

$\Rightarrow 1 + \alpha^2 + 1 = 3$

$\Rightarrow \alpha^2 = 1 \qquad \Rightarrow \alpha = \pm 1$

35. Let $\bar{a}, \bar{b}, \bar{c}$ be three non coplanar vectors such that $\bar{r}_1 = \bar{a} - \bar{b} + \bar{c}$, $\bar{r}_2 = \bar{b} + \bar{c} - \bar{a}$, $\bar{r}_3 = \bar{a} + \bar{b} + \bar{c}$, $\bar{r}_4 = 2\bar{a} - 3\bar{b} + 4\bar{c}$. If $\bar{r}_4 = p_1\bar{r}_1 + p_2\bar{r}_2 + p_3\bar{r}_3$ then

(a) $p_1 = 7$ (b) $p_1 + p_3 = 3$

(c) $p_1 + p_2 + p_3 = 4$ (d) $p_2 + p_3 = 0$

Key. B,C

Sol. $\bar{r}_4 = p_1\bar{r}_1 + p_2\bar{r}_2 + p_3\bar{r}_3$

$\Rightarrow 2\bar{a} - 3\bar{b} + 4\bar{c} = (p_1 - p_2 + p_3)\bar{a} + (-p_1 + p_2 + p_3)\bar{b} + (p_1 + p_2 + p_3)\bar{c}$

Since $\bar{a}, \bar{b}, \bar{c}$ are non coplanar

$\Rightarrow p_1 - p_2 + p_3 = 2, \quad -p_1 + p_2 + p_3 = -3, \quad p_1 + p_2 + p_3 = 4$

Solving $\Rightarrow p_1 = \frac{7}{2}, \quad p_2 = 1, \quad p_3 = -\frac{1}{2}$

36. P is the point $\vec{i} + x\vec{j} + 3\vec{k}$ .The vector $\overrightarrow{OP}$ ('O' is the origin) is rotated about the point 'O' through an angle θ.Q is the point $4\vec{i} + (4x-2)\vec{j} + 2\vec{k}$ on the new support of $\overrightarrow{OP}$ such that OQ = 2OP. Then *x* value is

a) 2 b) $\frac{2}{3}$ c) $\frac{1}{3}$ d) $\frac{-2}{3}$

Key. A,D

Sol. $\overrightarrow{OP}=\vec{i}+x\vec{j}+3\vec{k}$

$\overrightarrow{OQ}=\overrightarrow{4i}+(4x-2)\vec{j}+2\vec{k}$, $OQ=2OP\Rightarrow 16+(4x-2)^2+4=4(1+x^2+9)$

$\Rightarrow 12x^2-16x-16=0 \quad \Rightarrow 3x^2-4x-4=0 \quad \Rightarrow (3x+2)(x-2)=0$

$\Rightarrow x=2,\frac{-2}{3}$

37. If $\bar{a}\times\bar{b}=\bar{c}$ and $\bar{b}\times\bar{c}=\bar{a}$ then

a) $|\bar{a}|=1$

b) $|\bar{b}|=1$

c) $|\bar{a}|=|\bar{c}|$

d) $|\bar{b}|=|\bar{c}|$

Key. B,C

Sol. $\bar{a}\times\bar{b}=\bar{c}\Rightarrow\bar{c}$ is perpendicular to $\bar{a}$ and $\bar{b}$.

$\bar{b}\times\bar{c}=\bar{a}\Rightarrow\bar{a}$ is perpendicular to $\bar{b}$ and $\bar{c}$

$\Rightarrow$ $\bar{a},\bar{b},\bar{c}$ are mutually perpendicular

Again $\bar{a}\times\bar{b}=\bar{c}\Rightarrow|\bar{a}\times\bar{b}|=|\bar{c}|\Rightarrow|\bar{a}||\bar{b}|=|\bar{c}| \quad \to(1)$

$\bar{b}\times\bar{c}=\bar{a}\Rightarrow|\bar{b}\times\bar{c}|=|\bar{a}|\Rightarrow|\bar{b}||\bar{c}|=|\bar{a}| \quad \to(2)$

$\therefore$ from (1) & (2) $|\bar{c}|=|\bar{a}|\ \&\ |\bar{b}|=1$

38. The lines whose vector equations are $\vec{r}=2\hat{i}-3j+7k+\lambda\left(2\vec{i}+p\vec{j}+5\vec{k}\right)$ and $\vec{r}=\hat{i}+2j+3k+\mu\left(3\vec{i}-p\vec{j}+p\vec{k}\right)$ are perpendicular for all values of □ and □, if

a) p = - 6 b) p = - 1 c) p = 1 d) p = 6

Ans. b, d

Sol. Given lines are perpendicular if $2\vec{i}+p\vec{j}+5\vec{k}$ and $3\vec{i}-p\vec{j}+p\vec{k}$ are perpendicular.

$\Rightarrow 2\cdot 3+p(-p)+5p=0\Rightarrow p=-1,6$

39. The vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are of the same length and taken pair wise they form equal angles. If $\vec{a}=\hat{i}+j$ and $\vec{b}=j+k$, the coordinates of $\vec{c}$ can be

a) (1, 0, 1) b) (-1, 1, 2) c) $\left(-\frac{1}{3},\frac{4}{3},-\frac{1}{3}\right)$ d) $\left(\frac{1}{3},0,\frac{2}{3}\right)$

Ans. a,c

Sol. Let $\vec{c}=c_1\hat{i}+c_2j+c_3k$. Then $|\vec{c}|=\sqrt{2}=\sqrt{c_1^2+c_2^2+c_3^2}$ - (1)

And = $\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}=\frac{1}{2}\Rightarrow\frac{1}{2}=\frac{\vec{a}\cdot\vec{c}}{|\vec{a}||\vec{c}|}=\frac{c_1+c_2}{\sqrt{2}\cdot\sqrt{2}}\Rightarrow c_1+c_2=1$

and $\frac{1}{2}=\frac{\vec{b}\cdot\vec{c}}{|\vec{b}||\vec{c}|}=\frac{c_2+c_3}{2}\Rightarrow c_2+c_3=1$

From (1)

$2=(1-c_2)^2+c_2^2+(1-c_2)^2$

$\Rightarrow 3c_2^2-4c_2=0$

$\Rightarrow c_2 = 0 \ or \ c_2 = \frac{4}{3}$

Therefore, the points are (1, 0, 1) and $\left(-\frac{1}{3}, \frac{4}{3}, -\frac{1}{3}\right)$

Vectors

Assertion Reasoning Type

1. Statement- 1: If $\vec{a}=3\hat{i}-3\hat{j}+\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}+\hat{k}$ and $\vec{c}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{d}=2\hat{i}-\hat{j}$, then there exist real numbers α,β,γ such that $\vec{a}=\alpha\vec{b}+\beta\vec{c}+\gamma\vec{d}$

Statement- 2: $\vec{a},\vec{b},\vec{c},\vec{d}$ are four vectors in a 3 - dimensional space. If $\vec{b},\vec{c},\vec{d}$ are non-coplanar, then there exist real numbers α,β,γ such that $\vec{a}=\alpha\vec{b}+\beta\vec{c}+\gamma\vec{d}$

Key. B

Sol. Both the statements are ture and statement-2 is the not correct explanation of statement-1

Because $\vec{b},\vec{c},\vec{d}$ in statement - 1 are coplanar.

2. Statement - 1: Let $\vec{a},\vec{b},\vec{c}\,\&\,\vec{d}$ are position vector four points A, B, C & D and $3\vec{a}-2\vec{b}+5\vec{c}-6\vec{d}=\vec{0}$, then points A, B, C and D are coplaner.

Statement -2: Three nonzero, linearly dependent co-initial vectors $(\overrightarrow{PQ},\overrightarrow{PR}\,\&\,\overrightarrow{PS})$ are coplnar.

Key. A

Sol. $3\vec{a}-2\vec{b}+5\vec{c}-6\vec{d}=(2\vec{a}-2\vec{b})+(-5\vec{a}+5\vec{c})+(6\vec{a}-6\vec{d})=-2\overrightarrow{AB}+5\overrightarrow{AC}-6\overrightarrow{AD}=\vec{o}$

$\therefore$ $\overrightarrow{AB},\overrightarrow{AC}$ and $\overrightarrow{AD}$ are linearly dependent, hence by statement-2,the statement -1 is true.

C) Statement - 1 is true, Statement - 2 is false D) Statement - 1 is false, Statement - 2 is true

3. Given 3 vectors

$\vec{V_1}=a\hat{i}+b\hat{j}+c\hat{k}$, $\vec{V_2}=b\hat{i}+c\hat{j}+a\hat{k}$, $\vec{V_3}=c\hat{i}+a\hat{j}+b\hat{k}$

Where a, b, c are distirict +ve real numbers

Statement – 1: $\vec{V_1}$, $\vec{V_2}$ and $\vec{V_3}$ are linearly dependent vectors.

Statement – 2: $\left[\vec{V_1}\ \vec{V_2}\ \vec{V_3}\right]\neq 0$

Key : D

Sol. conceptual question

4. Let B_1, C_1, D_1 are points on AB, AC, AD of the parallelogram ABCD such that $\overrightarrow{AB_1}=k_1\overrightarrow{AB}$, $\overrightarrow{AC_1}=k_2\overrightarrow{AC}$ and $\overrightarrow{AD_1}=k_3\overrightarrow{AD}$ where k_1, k_2 and k_3 are scalars

STATEMENT-1 : k_1, $2k_2$ and k_3 are in harmonic progression if B_1, C_1 and D_1 are collinear

STATEMENT-2 : . $\dfrac{\overrightarrow{AB_1}}{k_1}+\dfrac{\overrightarrow{AD_1}}{k_3}=\dfrac{\overrightarrow{AC_1}}{k_2}$

KEY-A

Sol :

Equation of line B_1D_1

$\vec{r} = \overrightarrow{AB_1} + \lambda\left(\overrightarrow{B_1D_1}\right)$

$\Rightarrow \quad \vec{r} = \overrightarrow{AB_1} + \lambda\left(\overrightarrow{AD_1} - \overrightarrow{AB_1}\right)$

If points B_1, C_1 and D_1 are collinear, then

$$\overrightarrow{AC_1} = \overrightarrow{AB_1} + \lambda\left(\overrightarrow{AD_1} - \overrightarrow{AB_1}\right)$$

Since $\dfrac{\overrightarrow{AB_1}}{k_1} + \dfrac{\overrightarrow{AD_1}}{k_3} = \dfrac{\overrightarrow{AC_1}}{k_2}$ $\quad\left(\because \overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}\right)$

$\Rightarrow \quad \dfrac{k_2}{k_1}\overrightarrow{AB_1} + \dfrac{k_2}{k_3}\overrightarrow{AD_1} = (1-\lambda)\overrightarrow{AB_1} + \lambda\overrightarrow{AD_1}$

Since, AB and AD form linearly independent system of vectors.

$\Rightarrow \quad 1-\lambda = \dfrac{k_2}{k_1}$ and $\lambda = \dfrac{k_2}{k_3}$

$\Rightarrow \quad 1 - \dfrac{k_2}{k_3} = \dfrac{k_2}{k_1}$

$\Rightarrow \quad \dfrac{1}{k_2} = \dfrac{1}{k_1} + \dfrac{1}{k_3}.$

5. Statement - 1: If $\bar{a} \not\parallel \bar{b}$ and $\bar{r} + \bar{r}\times\bar{a} = \bar{b}$ then $|\bar{r}| = \sqrt{\dfrac{(\bar{a}.\bar{b})^2 + \bar{b}^2}{1+\bar{a}^2}}$.

Statement - 2: If $\bar{a} \not\parallel \bar{b}$ and $\bar{r} + \bar{r}\times\bar{a} = \bar{b}$ then $\bar{r} = \dfrac{(\bar{a}.\bar{b})\bar{a} + \bar{b} + \bar{a}\times\bar{b}}{1+\bar{a}.\bar{a}}$.

(given that $\bar{r}, \bar{a}, \bar{b}$ are vectors)

KEY : A

HINT Conceptual Question

6. Statement-I: If p>q>r>0 then

$$\cot^{-1}\left(\frac{1+pq}{p-q}\right) + \cot^{-1}\left(\frac{1+qr}{q-r}\right) + \cot^{-1}\left(\frac{1+rp}{r-p}\right) = \pi$$

Statement-II: $\cot^{-1}(x) = \tan^{-1}\left(\dfrac{1}{x}\right) \quad \forall x \in R - \{0\}$

KEY : C

HINT: $\cot^{-1}(x) = \pi + \tan^{-1}\left(\dfrac{1}{x}\right)$ when x<0

$= \tan^{-1}\left(\dfrac{1}{x}\right)$ when x>0

7. Let a plane S be parallel to the line $\bar{r} = \bar{u} + t\bar{v}$, t is a variable parameter and passing through the points P, Q whose position vectors are $\bar{p}$ and $\bar{q}$ where $\overrightarrow{PQ}, \bar{v}$ are non collinear, then

STATEMENT-1: equation of S is $[\bar{r}\ \ \bar{u}\ \ \bar{v}] = [\bar{p}\ \ \bar{u}\ \ \bar{v}]$

STATEMENT-2: A vector along the normal to S is $\left(\bar{p}-\bar{q}\right)\times\bar{v}$

KEY : D

HINT : $\left(\bar{r}-\bar{p}\right).\left(\left(\bar{p}-\bar{q}\right)\times\bar{v}\right)=0$

8. STATEMENT-1: Unit vector coplanar with $i+2j-k$ and $2i-j+k$ and orthogonal to vector $i+3j+5k$ is $\frac{10j-6k}{2\sqrt{34}}$.

STATEMENT-2: $\bar{a}\times\left(\bar{b}\times\bar{c}\right)$ is a vector perpendicular to $\bar{a}$ and coplanar with $\bar{b},\bar{c}$.

KEY : A

HINT : $\bar{a}\times\left(\bar{b}\times\bar{c}\right).\bar{a}=\left[\bar{a}\ \bar{b}\times\bar{c}\ \bar{a}\right]=0$

$\&\bar{a}\times\left(\bar{b}\times\bar{c}\right)$ is a vector coplanar with $\bar{b},\bar{c}$ and perpendicular to $\bar{a}$

9. STATEMENT -1: If $\vec{u}\ \&\ \vec{v}$ are unit vectors inclined at an angle α and $\vec{x}$ is a unit vector bisecting the angle between them, then $\vec{x}=\frac{\vec{u}+\vec{v}}{2\cos\frac{\alpha}{2}}$

because

STATEMENT-2: If ΔABC is an isosceles triangle with AB = AC = 1, then vector representing bisector of angle A is given by $\overrightarrow{AD}=\frac{\vec{AB}+\vec{AC}}{2}$

KEY : A or B

HINT : Option (A) is correct

In an isosceles triangle ABC is which AB = AC, the median and bisector from A must be same line $\Rightarrow$ statement 2 is true.

Now $\overrightarrow{AD}=\frac{\vec{u}+\vec{v}}{2}$

& $|\overrightarrow{AD}|^2=\frac{1}{2}2\cos^2\frac{\alpha}{2}$, So $|\overrightarrow{AD}|$ = cos $\frac{\alpha}{2}$

$\Rightarrow$ unit vector along AD i.e. x is given by $\vec{x}=\frac{\overrightarrow{AD}}{|\overrightarrow{AD}|}=\frac{\vec{u}+\vec{v}}{2\cos\frac{\pi}{2}}$

10. STATEMENT -1: The value of expression $\hat{i}(\hat{j}\times\hat{k})+\hat{j}(\hat{k}\times\hat{i})+\hat{k}(\hat{i}\times\hat{j})=3$

because

STATEMENT-2: $\hat{i}(\hat{j}\times\hat{k})=[\hat{i}.\hat{j}.\hat{k}]=1$

KEY : A

HINT : $\hat{i}(\hat{j}\times\hat{k})+\hat{j}(\hat{k}\times\hat{i})+\hat{k}(\hat{i}\times\hat{j})$

= $\hat{i}.\hat{i}+\hat{j}.\hat{j}+\hat{k}.\hat{k}$ = 1 + 1 + 1 = 3.

11. STATEMENT 1: $\vec{a}=2\vec{i}+\vec{j}-\vec{k}$ and $\vec{b}=\vec{i}+3\vec{k}$ If $\vec{c}$ is a unit vector, then the

maximum value of the scalar triple product $[\vec{a}\ \vec{b}\ \vec{c}]$ is $\sqrt{63}$

STATEMENT 2: If $\vec{u}$ and $\vec{v}$ and two vectors then the scalar product $\vec{u}.\vec{v} \le |\vec{u}||\vec{v}|$

Key: D

Hint $\left[\bar{a}\ \bar{b}\ \bar{c}\right] = \left(\vec{a}\times\vec{b}\right).\vec{c} \le |\vec{a}\times\vec{b}||\vec{c}|$

12. Statement - 1: If $\bar{u}$ and $\bar{v}$ are unit vectors inclined at angle 'α' and '$\bar{x}$' is a unit vector bisecting the angle between them, then $\bar{x} = \dfrac{\bar{u}+\bar{v}}{2\sin\alpha/2}$

Statement - 2: If ABC is an isosceles triangle with AB = AC = 1, then the vector representing bisector of angle 'A' is given by $\overline{AD} = \dfrac{\overline{AB}+\overline{AC}}{2}$

Key. D

Sol. In an isosceles triangle ABC in which AB=AC, the median and bisector from 'A' must be same line

$\Rightarrow$ Reason 'R' is true

Now $\overline{AD} = \dfrac{\bar{u}+\bar{v}}{2}$ and $|\overline{AD}|^2 = \dfrac{1}{4}\left(|\bar{u}|^2 + |\bar{v}|^2 + 2|\bar{u}||\bar{v}|\cos\alpha\right)$

$$= \frac{1}{4}(1+1+2\cos\alpha)$$

$$\Rightarrow |\overline{AD}| = \cos\alpha/2$$

$\Rightarrow$ Unit vector along AD is $\bar{x} = \dfrac{\bar{u}+\bar{v}}{2\cos\alpha/2}$

13. Assertion (A): Two straightlines in space which are neither parallel nor intersecting are called as skew lines.

Reason (R): If θ is angle between $\bar{r} = \bar{a} + \lambda\bar{b}$ and $\bar{r}\cdot\bar{n} = d$ then $\cos\theta = \dfrac{\bar{b}\cdot\bar{n}}{|\bar{b}||\bar{n}|}$

Key. C

Sol. Conceptual

14. Statement–1 : If $\vec{u}$ and $\vec{v}$ are unit vectors inclined at an angle α and $\vec{x}$ is a unit vector bisecting the angle between them then $\vec{x} = \dfrac{\vec{u}+\vec{v}}{2\cos\frac{\alpha}{2}}$.

Statement–2 : If ABC be an isosceles triangle with AB = AC = 1 then vector representing bisector of angle A is given by $\overrightarrow{AD} = \dfrac{\overrightarrow{AB}+\overrightarrow{AC}}{2}$.

Key. A

Sol. In an isosceles triangle ABC in which AB = AC, the median and bisector from A must be same line.

Now $\overrightarrow{AD} = \frac{\vec{u}+\vec{v}}{2}$ and

$|\overrightarrow{AD}|^2 = \frac{1}{4}\left(|\vec{u}|^2 + |\vec{v}|^2 + 2\vec{u}\cdot\vec{v}\right) = \frac{1}{4}[1+1+2\cos\alpha] = \frac{1}{2}.2\cos^2\frac{\alpha}{2} = \cos\frac{\alpha}{2}$

$\Rightarrow$ Unit vector along AD i.e. x is given by $\vec{x} = \frac{\overrightarrow{AD}}{|\overrightarrow{AD}|} = \frac{\vec{u}+\vec{v}}{2\cos\frac{\alpha}{2}}$.

15. STATEMENT–1

For the real numbers α, β, γ; $(\cos\alpha + \cos\beta + \cos\gamma)^2 \le 3(\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

because

STATEMENT–2

For two non–zero vectors $\vec{A}$ and $\vec{B}$, $\left(\vec{A}.\vec{B}\right)^2 \le |\vec{A}|^2|\vec{B}|^2$

Key. A

Sol. Consider $\vec{A} = \hat{i}+\hat{j}+\hat{k}$

$\vec{B} = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}$

$(\vec{A}.\vec{B})^2 \le |\vec{A}|^2\,|\vec{B}|^2$

$\Rightarrow (\cos\alpha + \cos\beta + \cos\gamma)^2 \le (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$ (3).

16. Let $\bar{a}, \bar{b}, \bar{c}$ be non coplanar vectors and $\bar{r} = \left(\bar{a}\times\bar{b}\right)\times\left(\bar{a}\times\bar{c}\right)$

Statement - 1: $\bar{r}$ and $\bar{a}$ are linearly dependent

Statement - 2: $\bar{r}$ is perpendicular to each of the three vectors $\bar{a}, \bar{b}, \bar{c}$

Key. C

Sol. $\bar{r} = \left(\bar{a}\times\bar{b}\right)\times\left(\bar{a}\times\bar{c}\right) = \left[\bar{a}\bar{b}\bar{c}\right]\bar{a} \Rightarrow \bar{r}\text{ and }\bar{a}$ are collinear

17. Statement - 1: If $A(\vec{a}), B(\vec{b}), C(\vec{c})$ are the vertices of a triangle with circum centre at the origin, then centre of its nine point circle will be $\frac{\vec{a}+\vec{b}+\vec{c}}{2}$.

Statement - 2: If $A(\vec{a}), B(\vec{b}), C(\vec{c})$ are the vertices of a triangle with circum centre at the origin, then centroid of ΔABC is $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$.

Key. A

Sol. Conceptual

18. Statement - 1: $\vec{b}\ \&\ \vec{c}$ are two non-collinear vectors, such that $\vec{a}.(\vec{b}+\vec{c}) = 4$ and $\vec{a}\times(\vec{b}\times\vec{c}) = (x^2-2x+6)\vec{b} + (\sin y)\vec{c}$, where $x, y \in R$. The point (x, y) lies on $x = 1$.

Statement - 2: The vector $\vec{a}$ lies in the plane of $\vec{b}\times\vec{c}$.

Key. C

Sol. Conceptual

19. Statement - 1: Let $\vec{a}\,\&\,\vec{b}$ are two perpendicular unit vectors. If $\vec{c}$ is another vector equally inclined at angle θ to the vectors $\vec{a}\,\&\,\vec{b}$, then set of exhaustive value of θ in $[0, 2\pi]$ is $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$.

Statement - 2: $\cos 2\theta \le 0$.

Key. A

Sol. Conceptual

20. Assertion (A) : The line of intersection of planes $\vec{r}.\left(\vec{i}+2\vec{j}+3\vec{k}\right)=0$ and $\vec{r}.\left(3\vec{i}+2\vec{j}+\vec{k}\right)=0$ is equally inclined to $\vec{i}$ and $\vec{k}$

Reason (R) : The angle between two planes is angle between their normals

Key. B

Sol. Conceptual

21. Assertion (A) : If $\vec{u}$ and $\vec{v}$ are unit vectors inclined at an angle α and $\vec{x}$ is a unit vector bisecting the angle between them then $\vec{x}=\dfrac{\vec{u}+\vec{v}}{2\cos\frac{\alpha}{2}}$

Reason (R) : If triangle ABC is an isosceles triangle with $AB = AC = 1$ then vector representing bisector of angle A is given by $\dfrac{\overrightarrow{AB}+\overrightarrow{AC}}{2}$

Key. A

Sol. $\left|\vec{u}+\vec{v}\right| = 2\cos\dfrac{\alpha}{2}$

22. Statement-I: The position vector of the foot of the perpendicular from the point $(4,6,2)$ on the line $\bar{r}=(2\bar{i}+2\bar{j}+2\bar{k})+t(3\bar{i}+2\bar{j}+\bar{k})$ is (5,4,3)

Statement-II: Position vector of the foot of the perpendicular from the point $\bar{c}$ on the line $\bar{r}=\bar{a}+t\bar{b}$ is $\bar{a}-\dfrac{(\bar{c}-\bar{a})\cdot\bar{b}}{\left|\bar{b}\right|^2}\bar{b}$

Key. C

Sol. $\bar{a}=2\bar{i}+2\bar{j}+2\bar{k}$

$\bar{c}-\bar{a}=2\bar{i}+4\bar{j}$

$\bar{b}=3\bar{i}+2\bar{j}+\bar{k}$

Foot of the $\perp^{er}$ from $\bar{c}$ on $\bar{r}=\bar{a}+t\bar{b}$ is $\bar{a}+\dfrac{(\bar{c}-\bar{a})\cdot\bar{b}}{\left|\bar{b}\right|^2}\,\bar{b}$

$$(2i+2j+2k)+\frac{(2\bar{i}+4\bar{j}).(3i+2j+\bar{k})}{\left|3i+2j+\bar{k}\right|^2}(3\bar{i}+2\bar{j}+\bar{k})$$

$$=(2i+2j+2\bar{k})+(3i+2j+k)$$

$$=5\bar{i}+4\bar{j}+3\bar{k}$$

23. Statement-I: $\bar{a}\times(\bar{b}\times\bar{c}),\ \bar{b}\times(\bar{c}\times\bar{a}),\ \bar{c}\times(\bar{a}\times\bar{b})$ are non coplanar

Statement-II: If $\bar{a},\bar{b},\bar{c}$ are non coplanar then $\bar{a}\times\bar{b},\ \bar{b}\times\bar{c},\ \bar{c}\times\bar{a}$ are also non coplanar

Key. D

Sol. $\bar{a}\times(\bar{b}\times\bar{c})+\bar{b}\times(\bar{c}\times\bar{a})+\bar{c}\times(\bar{a}\times\bar{b})=\bar{O}$

Hence $\bar{a}\times(\bar{b}\times\bar{c}),\bar{b}(\bar{c}\times\bar{a}),\bar{c}\times(\bar{a}\times\bar{b})$ are coplanar.

24. Statement – 1: The position vectors of A and B are $\bar{a}$ and $\bar{b}$ respectively and the position vector

of C is $\frac{3\bar{a}}{4}+\frac{\bar{b}}{2}$ then 'C' is inside ΔOAB.

Statement – 2: The position vector of a point which divides $\bar{a},\bar{b}$ in the ratio m : n internally is

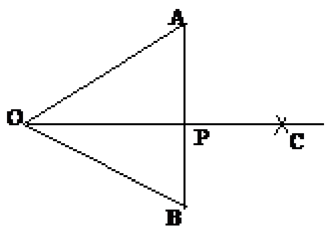
$$\frac{m\bar{b}+n\bar{a}}{m+n}$$

Key. D

Sol. $\overline{OC}=\frac{3\bar{a}}{4}+\frac{\bar{b}}{2}=\frac{3\bar{a}+2\bar{b}}{4}=\frac{5}{4}\left(\frac{3\bar{a}+2\bar{b}}{5}\right)>\frac{3\bar{a}+2\bar{b}}{5}=\overline{OP}$ say

$\therefore$ C lies on the extended line OP

Where 'P' is a point which divides AB in the ratio 2 : 3



25. Statement – 1: The points $2\bar{a}+\bar{b}-\bar{c}$, $5\bar{a}-\bar{b}+2\bar{c}$ and $8\bar{a}-3\bar{b}+5\bar{c}$ are collinear.

Statement – 2: If the points whose position vectors are $\bar{a},\bar{b},\bar{c}$ collinear iff $\exists$ scalars

$x,y,z \ni x\bar{a}+y\bar{b}+z\bar{c}=\bar{0}$ where $x+y+z=0$

Key. A

Sol. Conceptual

26. Statement – 1: If $A(\bar{a}),B(\bar{b}),C(\bar{c}),D(\bar{d})$ are the vertices of a parallelogram ABCD then

$\bar{a}+\bar{c}=\bar{b}+\bar{d}$

Statement – 2: If the vectors $\bar{a},\bar{b},\bar{c},\bar{d}$ be the sides of a parallelogram then

$\bar{a}+\bar{c}=\bar{b}+\bar{d}=\bar{0}$

Key. B or C

Sol. Conceptual

27. Statement – 1: Let $\bar{a},\bar{b}$ are position vectors of two points A and B respectively with respect

to origin 'O'. If the point 'C' is on OA is such that $2\overline{AC}=\overline{CO}$, $\overline{CD}$ is parallel to

$\overline{OB}$ and $|\overline{CD}|=3|\overline{OB}|$ then $\overline{AD}=3\bar{b}-\frac{\bar{a}}{3}$

Statement – 2: If $\bar{a},\bar{b}$ are the position vectors of A and B then $\frac{m\bar{b}+n\bar{a}}{m+n}$ lies on $\overline{AB}$

Key. B or D

Sol. $\overline{OA}=\bar{a},\ \overline{OB}=\bar{b}$

$2\overline{AC}=\overline{CO}\ \Rightarrow 2\left(\overline{OC}-\overline{OA}\right)=-\overline{OC}\quad \Rightarrow 3\overline{OC}=2\overline{OA}$ ....(1)

$\overline{CD}=3\overline{OB}\ \Rightarrow \overline{OD}-\overline{OC}=3\overline{OB}$ ....(2)

$\overline{AD}=\overline{OD}-\overline{OA}=3\overline{OB}+\overline{OC}-\overline{OA}$

$=3\bar{b}-\frac{1}{3}\bar{a}$

Statement – 2 clearly $\frac{m\bar{b}+n\bar{a}}{m+n}$ divides A(a), B(b) in the ratio m : n internally.

28. STATEMENT-1 : Let the vector $\vec{a}=\vec{i}+\vec{j}+\vec{k}$ be vertical. The line of greatest slope on a plane with normal $\vec{b}=2\vec{i}-\vec{j}+\vec{k}$ is along the vector $\vec{i}-4\vec{j}+2\vec{k}$.

STATEMENT-2 : If $\vec{a}$ is vertical, then the line of greatest slope on a plane with normal $\vec{b}$ is along the vector $\left(\vec{a}\times\vec{b}\right)\times\vec{b}$

Key. D

Sol. $\vec{a}\times\vec{b}=2\vec{i}+\vec{j}-3\vec{k},(\vec{a}\times\vec{b})\times\vec{b}=-2\vec{i}-8\vec{j}-4\vec{k}$

Which is along $\vec{i}+4\vec{j}+2\vec{k}$ $\therefore$ A is false and R is true

29. STATEMENT-1: The volume of the parallelepiped formed by the vectors $\hat{i}+a\hat{j};\ a\hat{i}+\hat{j}+k$ and $\hat{j}+ak$ is maximum when a $=-\frac{1}{\sqrt{3}}$

STATEMENT-2 : The volume of the parallelepiped having three coterminous edges $\bar{a},\bar{b}$ and $\bar{c}=\left|\left[\bar{a}\ \bar{b}\ \bar{c}\right]\right|$

Key. D

Sol. $V=\begin{vmatrix}1 & a & 0\\ a & 1 & 1\\ 0 & 1 & a\end{vmatrix}=a-1-a^3 \qquad \therefore \frac{dV}{da}=1-3a^2=0 \qquad \therefore a=\pm\frac{1}{\sqrt{3}}$

$\Rightarrow \frac{d^2V}{da^2}=-6a,\ \left(\frac{d^2V}{da^2}\right)_{a=\frac{1}{\sqrt{3}}}=-\frac{1}{\sqrt{3}}\ \therefore V$ is maximum at $a=\frac{1}{\sqrt{3}}$

30. Statement – 1: Let $A\left(\vec{a}\right),B\left(\vec{b}\right) and\, C\left(\vec{c}\right)$ be three points such that $\vec{a}=2\hat{i}+j+k,\ \vec{b}=3\hat{i}-j+3k$ and $\vec{c}=-\hat{i}+7j-5k$ then OABC is a tetrahedron.

Statement – 2: Let $A\left(\vec{a}\right)$, $B\left(\vec{b}\right)$ and $C\left(\vec{c}\right)$ be three points such that $\vec{a}$, $\vec{b}$ and $\vec{c}$ are non-coplanar, then OABC is tetrahedron, where O is the origin.

a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for statement-1

b) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for statement-1

c) Statement-1 is true, Statement-2 is false d) Statement-1 is false, Statement-2 is true

Ans. d

Sol. Since $\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)=0$ $\therefore$ a, b, c are coplanar.

Vectors

Comprehension Type

Paragraph – 1

Consider the equations of planes $P_1 \equiv \bar{r}.\left(\bar{i}+2\bar{j}+\bar{k}\right)-3=0$ $P_2 \equiv \bar{r}.\left(2\bar{i}-\bar{j}+\bar{k}\right)-5=0$

1. The equation of plane passing through the intersection of $P_1=0$, $P_2=0$ and through the point $A(1,1,1)$ is

a) $\bar{r}.\left(5\bar{i}-4\bar{j}+5\bar{k}\right)=6$ b) $\bar{r}.\left(5\bar{i}+5\bar{j}-4\bar{k}\right)=6$

c) $\bar{r}.\left(5\bar{i}+5\bar{j}+4\bar{k}\right)=14$ d) None of these

Key. C

2. The line of intersection of planes $P_1=0$, $P_2=0$ is parallel to

a) $3\bar{i}-5\bar{j}-\bar{k}$ b) $3\bar{i}+\bar{j}-5\bar{k}$ c) $2\bar{i}-\bar{j}-\bar{k}$ d) None of these

Key. B

Sol. 61. The required plane is x+2y+z-3+k(2x-y+z-5)=0 for some k. since it passes A, k=1/3

$\therefore$ The equation of plane is 5x+5y+4z-14=0, i.e. $\bar{r}\left(5\bar{i}+5\bar{j}+4\bar{k}\right)=14$

62. The line of intersection of planes $\bar{r}.\overline{n_1}=d_1$ and $\bar{r}.\overline{n_1}=d_2$ is parallel to $\overline{n_1}\times\overline{n_2}$

Paragraph – 2

Let $\vec{a}=2\hat{i}+3\hat{j}-6\hat{k}, \vec{b}=2\hat{i}-3\hat{j}+6\hat{k}$ and $\vec{c}=-2\hat{i}+3\hat{j}+6\hat{k}$. let $\vec{a}_1$ be projection of $\vec{a}$ on $\vec{b}$ and $\vec{a}_2$ be the projection of $\vec{a}_1$ on $\vec{c}$. Then

3. $\vec{a}_2=$

(A) $\frac{943}{49}(2\hat{i}-3\hat{j}-6\hat{k})$ (B) $\frac{943}{49^2}(2\hat{i}-3\hat{j}-6\hat{k})$

(C) $\frac{943}{49}(-2\hat{i}+3\hat{j}+6\hat{k})$ (D) $\frac{943}{49^2}(-2\hat{i}+3\hat{j}+6\hat{k})$

Key. B

Sol. $\vec{a}_1=\left[(2\hat{i}+3\hat{j}-6\hat{k}).\frac{(2\hat{i}-3\hat{j}+6\hat{k})}{7}\right]\frac{2\hat{i}-3\hat{j}+6\hat{k}}{7}=\frac{-41}{49}(2\hat{i}-3\hat{j}+6\hat{k})$

$\vec{a}_2=\frac{-41}{49}\left((2\hat{i}+3\hat{j}-6\hat{k}).\frac{(2\hat{i}-3\hat{j}+6\hat{k})}{7}\right)\frac{(-2\hat{i}+3\hat{j}+6\hat{k})}{7}$

$\frac{-41}{(49)^2}(-4-9+36)(-2\hat{i}+3\hat{j}+6\hat{k})=\frac{943}{49^2}(2\hat{i}-3\hat{j}-6\hat{k})$

4. $\vec{a}_1.\vec{b}$

(A) - 41 (B) $-\frac{41}{7}$

(C) 41 (D) 287

Key. A

Sol. $\vec{a}_1.\vec{b} = \frac{-41}{49}(2\hat{i}-3\hat{j}+6\hat{k}).(2\hat{i}-3\hat{j}+6\hat{k}) = -41$

5. Which of the following is true

(A) $\vec{a}$ and $\vec{a}_2$ are collinear (B) $\vec{a}_1$ and $\vec{c}$ are collinear

(C) $\vec{a}, \vec{a}_1, \vec{b}$ are coplanar (D) $\vec{a}, \vec{a}_1, \vec{a}_2$ are coplanar

Key. C

Sol. $\vec{a}, \vec{a}_1, \vec{b}$ are coplanar, because $\vec{a}_1.\vec{b}$ are collinear.

Paragraph – 3

Three vector $\hat{a}, \hat{b}$ and $\hat{c}$ are forming a right handed system, if $\vec{a}\times\vec{b}=\vec{c}, \vec{b}\times\vec{c}=\vec{a}$, $\vec{c}\times\vec{a}=\vec{b}$. If vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are forming a right handed system, then answer the following question.

6. If $\vec{x}=\hat{a}+\hat{b}-\hat{c}$, $\vec{y}=-\hat{a}+\hat{b}-2\hat{c}, \vec{z}=-\hat{a}+2\hat{b}-\hat{c}$, then a unit vector normal to the vector $\vec{x}+\vec{y}$ and $\vec{y}+\vec{z}$ is

(A) $\vec{a}$ (B) $\vec{b}$

(C) $\vec{c}$ (D) None of these

Key. D

Sol. $\vec{x}+\vec{y}=2\vec{b}-\vec{c}$ and $\vec{y}+\vec{z}=-2\vec{a}+3\vec{b}-3\vec{c}$

$$\therefore \quad (\vec{x}+\vec{y})\times(\vec{y}+\vec{z}) = \begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ 0 & 2 & -3 \\ -2 & 3 & -3 \end{vmatrix} = 3\vec{a}+6\vec{b}+4\vec{c}$$

$\therefore$ required unit vector = $\frac{3\vec{a}+6\vec{b}+4\vec{c}}{\sqrt{61}}$

7. Vector $2\hat{a}-3\hat{b}+4\hat{c}$, $\hat{a}+2\hat{b}-\hat{c}$ and $x\hat{a}-\hat{b}+2\hat{c}$ are coplanar, then x =

(A) (B)

(C) 0 (D) None of these

Key. A

Sol. $\begin{vmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ x & -1 & 2 \end{vmatrix} = 0 \quad \Rightarrow 2(4-1)+3(2+x)+4(-1-2x)=0 \Rightarrow x=\frac{8}{5}$

8. Let $\vec{x}=\hat{a}+\hat{b}$, $\vec{y}=2\hat{a}-\hat{b}$, then the point of intersection of straight lines $\vec{r}\times\vec{x}=\vec{y}\times\vec{x}, \vec{r}\times\vec{y}=\vec{x}\times\vec{y}$ is

(A) $\frac{8}{5}$ (B) $\frac{5}{8}$

(C) $3\vec{a}$ (D) None of these

Key. C

Sol. $\vec{r}\times\vec{x}=\vec{y}\times\vec{x} \Rightarrow (\vec{r}-\vec{y})\times\vec{x}=0 \Rightarrow \vec{r}=\vec{y}+\lambda\vec{x}$

$\vec{r}\times\vec{y}=\vec{x}\times\vec{y} \Rightarrow (\vec{r}-\vec{x})\times\vec{y}=0 \Rightarrow \vec{r}=\vec{x}+\mu\vec{y}$

$\vec{y}+\lambda\vec{x}=\vec{x}+\mu\vec{y}$

$(2\hat{a}-\hat{b})++\lambda(\hat{a}+\hat{b})=(\hat{a}+\hat{b})+\mu(2\hat{a}-\hat{b})$

$\Rightarrow \quad 2+\lambda=1+2\mu, -1+\lambda=1-\mu \quad \Rightarrow \mu=1, \lambda=1$

The point of intersection is $3\vec{a}$.

9. $\hat{a}.(\hat{b}\times\hat{c})+\hat{b}.(\hat{c}\times\hat{a})+\hat{c}.(\hat{a}\times\hat{b})$ is equal to

(A) 1 (B) 3

(C) 0 (D) None of these

Key. B

Sol. Sol. $\hat{a}\times\hat{b}=\hat{c} \quad \Rightarrow \hat{c}.\hat{a}\times\hat{b}=\hat{c}.\hat{c}=1 \quad \Rightarrow \quad [\hat{a}\ \hat{b}\ \hat{c}]=1$

$\hat{a}.(\hat{b}\times\hat{c})+\hat{b}.(\hat{c}\times\hat{a})+\hat{c}.(\hat{a}\times\hat{b})$ = 3

Paragraph – 4

The vertices of a triangle ABC are A ≡(2, 0, 2), B ≡ (-1,1 ,1) and C ≡ (1, -2 , 4). the points D and E divide the side AB and CA in the ratio 1 : 2 respectively. Another point F is taken in space such that perpendicular drawn from F on ΔABC, meets the triangle at the point of intersection of the line segment CD and BE, say P. if the distance of F from the plane of the Δ ABC is units $\sqrt{2}$, then

10. The position vector of P, is

(A) $\hat{i}+\hat{j}\,3\hat{k}$ (B) $\hat{i}-\hat{j}+3\hat{k}$ (C) $2\hat{i}-\hat{j}-3\hat{k}$ (D) none

Key. B

11. The vector, is :

(A) $7\hat{j}+7\hat{k}$ (B) $\frac{7}{\sqrt{2}}(\hat{j}+\hat{k})$ (C) $(\hat{j}+\hat{k})$ (D) none

Key. C

12. The volume of the tetrahedron ABCF, is :

(A) 7 cubic units (B) 3/5 cubic units (C) 7/3 cubic units (D) none

Key. C

13. The equation of the line AF, is :

(A) $\vec{r}=\left(2\hat{i}+2k\right)+\lambda\left(\hat{i}+2k\right)$ (B) $\vec{r}=\left(2\hat{i}+2k\right)+\lambda\left(\hat{i}-2k\right)$

(C) $\vec{r}=\left(\hat{i}+k\right)+\lambda\left(\hat{i}+2k\right)$ (D) $\vec{r}=\left(2\hat{i}+2k\right)+\lambda\left(-\hat{i}+2k\right)$

Key. D

Sol. 10 to 13

The position vectors of D and E are marked in figure.

The vector equation of CD and BE are

$\vec{r}=\left(\hat{i}-2j+4k\right)+\frac{\lambda}{3}\left(7j-k\right)$ ....(i)

and $\vec{r}=\left(-\hat{i}+j+k\right)+\frac{\mu}{3}\left(7\hat{i}-7j+7k\right)$ ....(ii)

respectively.

CD and BE intersect at point P. At their point of intersection, we must have

$$\left(\hat{i}-2j+4k\right)+\frac{\lambda}{3}\left(7j-7k\right)=\left(-\hat{i}+j+k\right)+\frac{\mu}{3}\left(7i-7j+7k\right)$$

$$\Rightarrow 1=-1+\frac{7\mu}{3},-2+\frac{7\lambda}{3}=1-\frac{7\mu}{3}$$

and $4-\frac{7\lambda}{3}=1+7\frac{\mu}{3}$ $\Rightarrow \mu=6/7$ and $\lambda=3/7$

Substituting the value of λ in (i) or that of μ in (ii), we obtain the position vector $\vec{r}_1$ of point P as, $\vec{r}_1=\hat{i}-j+3k$

Now, $\Delta=\text{area of }\Delta ABC=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$

$=\frac{1}{2}\left|\left(-3\hat{i}+j-k\right)\times-\hat{i}-2j+2k\right|$ $=\frac{7\sqrt{2}}{2}\text{ sq unit.}$

$\therefore$ Volume of the tetrahedron ABCF

$=\frac{1}{3}$ (area of the base) height

$=\frac{1}{3}\cdot\frac{7\sqrt{2}}{2}\cdot\sqrt{2}=\frac{7}{3}$ cubic units

We have, $\overrightarrow{AB}\times\overrightarrow{AC}=7j+7k$

Since, $\overrightarrow{PF}$ is parallel to $\overrightarrow{AB}\times\overrightarrow{AC}$ and $PF=\sqrt{2}$ units.

$$\therefore \overrightarrow{PF}=\sqrt{2}\frac{\left(7j+7k\right)}{\sqrt{49+49}}=j+k$$

$$\Rightarrow P.V\text{ of }\vec{F}=j+k$$

$$\Rightarrow P.V\text{ of }\vec{F}=\left(j+k\right)+\left(\hat{i}-j+3k\right)=\hat{i}+4k$$

$\therefore$ Vector equation of AF is,

$$\vec{r}=\left(2\hat{i}=2k\right)+\lambda\left(\hat{i}+4k-2\hat{i}-2k\right)$$

$\vec{r} = \left(2\hat{i} + 2k\right) + \lambda\left(-\hat{i} + 2k\right)$.

Paragraph – 5

Let $\vec{r}$ be the variable point satisfying $\vec{r} - \vec{n}_2 = d_1$, $\vec{r} - \vec{n}_2 = d_2$, $\vec{r} - \vec{n}_2$,$= d_3$,where $\vec{n}_1, \vec{n}_2 \text{ and } \vec{n}_3$ are

non– coplanar vectors. Then

14. The position vector of the point of intersection of three planes, is :

(A) $\frac{1}{[\vec{n}_1\vec{n}_2\vec{n}_3]}\left[d_3\left(\vec{n}_1 \times \vec{n}_2\right) + d_1\left(\vec{n}_2 \times \vec{n}_3\right) + d_2\left(\vec{n}_1 \times \vec{n}_3\right)\right]$

(B) $\frac{4}{[\vec{n}_1\vec{n}_2\vec{n}_3]}\left[d_3\left(\vec{n}_1 \times \vec{n}_2\right) + d_1\left(\vec{n}_2 \times \vec{n}_3\right) + d_2\left(\vec{n}_1 \times \vec{n}_3\right)\right]$

(C) $\frac{-4}{[\vec{n}_1\vec{n}_2\vec{n}_3]}\left[d_3\left(\vec{n}_1 \times \vec{n}_2\right) + d_1\left(\vec{n}_2 \times \vec{n}_3\right) + d_2\left(\vec{n}_1 \times \vec{n}_3\right)\right]$

(D) none of these

Key. A

15. If the planes $\vec{r}.\vec{n}. = d_1$, $\vec{r}.\vec{n}_2 = d_2$ and $\vec{r}.\vec{n}_3$ d_3 , have a common lien of intersection, then is $d_1\left(\vec{n}_2 \times \vec{n}_3\right) + d_2\left(\vec{n}_3 \times \vec{n}_1\right) + d_3\left(\vec{n}_1 \times \vec{n}_2\right)$

(A) $\left[\vec{n}_1\vec{n}_2\vec{n}_3\right]$ (B) $4\left[\vec{n}_1\vec{n}_2\vec{n}_3\right]$ (C) $2\left[\vec{n}_1\vec{n}_2\vec{n}_3\right]$ (D) none

Key. D

Sol. 14 and 15

$\vec{n}_1, \vec{n}_2, \vec{n}_3$ are non-coplaner vectors. Therefore vectors $\vec{n}_2 \times \vec{n}_2, \vec{n}_2 \times \vec{n}_3 \text{ and } \vec{n}_3 \times \vec{n}_1$ are also non-coplaner

Let $\vec{\alpha}$ be the position vector of the mid point of intersection of the given planes. Then,

$\vec{\alpha} \cdot \vec{n}_1 = d_1, \vec{\alpha} \cdot \vec{n}_2 = d_2$ and $\vec{\alpha} \cdot \vec{n}_3 = d_3$

We know that any vector in space can be written as the linear combination of three non-coplaner

vectors. So, let

$\Rightarrow \vec{\alpha} = x\left(\vec{n}_1 \times \vec{n}_2\right) + y\left(\vec{n}_2 \times \vec{n}_3\right) + z\left(\vec{n}_3 \times \vec{n}_1\right)$ ....(i)

Now, $\vec{\alpha} \cdot \vec{n}_1 = d_1$

$\Rightarrow \left\{x\left(\vec{n}_1 \times \vec{n}_2\right) + y\left(\vec{n}_2 \times \vec{n}_3\right) + z\left(\vec{n}_3 \times \vec{n}_1\right)\right\} \cdot \vec{n} = d_1$

$\Rightarrow y\left\{\left(\vec{n}_2 \times \vec{n}_3\right) \cdot \vec{n}_1\right\} = d_1$ $\Rightarrow y = \frac{d_1}{[n_1\, n_2\, n_3]}$

Similarly, we have

$\vec{\alpha} \cdot \vec{n}_2 = d_2$ and $\vec{\alpha} \cdot \vec{n}_3 = d_3$

$\Rightarrow z=\frac{d_2}{\left[\vec{n}_1\vec{n}_2\vec{n}_3\right]}$ and $x=\frac{d_3}{\left[\vec{n}_1\vec{n}_2\vec{n}_3\right]}$

$\therefore$ Positive vector of the point of intersection of three planes, is

$\Rightarrow \frac{1}{\left[\vec{n}_1\vec{n}_2\vec{n}_3\right]}\left\{d_1\left(\vec{n}_2\times\vec{n}_3\right)+d_2\left(\vec{n}_1\times\vec{n}_3\right)+d_3\left(\vec{n}_1\times\vec{n}_2\right)\right\}$

Also, the equation fo a plane pasing through the line of intersection of the planes

$\vec{r}\cdot\vec{n}_1=d_1$ and $\vec{r}\cdot\vec{n}_2=d_2$ is

$\vec{r}\cdot\left(\vec{n}_1+\vec{n}_2\lambda\right)=d_1+\lambda d_2$, , where λ uis parameter since, three palnes have a common line of intersection.

$\therefore$ The above equation should be identical to for some value of

Thus for some value of λ, we have

$\vec{n}_1+\lambda\vec{n}_2=\mu\vec{n}_3$ .....(ii)

$and\ \ \vec{d}_1+\lambda\vec{d}_2=\mu d_3$ ....(iii)

Now, $\vec{n}_1+\lambda\vec{n}_2=\mu\vec{n}_3$

$\Rightarrow\left(\vec{n}_1+\lambda\vec{n}_2\right)\times\vec{n}_3=\mu\left(\vec{n}_3\times\vec{n}_3\right)$

$\Rightarrow\vec{n}_1+\vec{n}_3+\lambda\left(\vec{n}_2\times\vec{n}_3\right)=0$ ....(iv)

$\Rightarrow\lambda=-\frac{\left(\vec{n}_1\times\vec{n}_3\right)}{\left(\vec{n}_3\times\vec{n}_3\right)}$

Again, $\vec{n}_1+\lambda\vec{n}_2=\mu\vec{n}_3$

$\Rightarrow\left(\vec{n}_1+\lambda\vec{n}_2\right)\times\vec{n}_2=\mu\left(\vec{n}_3\times\vec{n}_2\right)$

$\mu\left(\vec{n}_2\times\vec{n}_3\right)=-\left(\vec{n}_1\times\vec{n}_2\right)$ ....(v)

Now, $d_1+\lambda d_2=\mu d_3$

$\Rightarrow\left(d_1+\lambda d_2\right)\left(\vec{n}_2\times\vec{n}_3\right)=d_3\left\{\mu\left(\vec{n}_2\times\vec{n}_3\right)\right\}$

$\Rightarrow d_1\left(\vec{n}_2\times\vec{n}_3\right)+d_2\left(\vec{n}_3\times\vec{n}_1\right)+d_3\left(\vec{n}_1\times\vec{n}_2\right)=0$

{using (iv) and (v)}

Paragraph – 6

If $\vec{a},\vec{b},\vec{c}$ are three given non – coplanar vectors and any arbitrary vectors $\vec{r}$ is space,

where $\Delta_1=\begin{vmatrix}\vec{r}.\vec{a} & \vec{b}.\vec{a} & \vec{c}.\vec{a}\\ \vec{r}.\vec{b} & \vec{b}.\vec{b} & \vec{c}.\vec{b}\\ \vec{r}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c}\end{vmatrix}$ $\Delta_2=\begin{vmatrix}\vec{a}.\vec{a} & \vec{r}.\vec{a} & \vec{c}.\vec{a}\\ \vec{a}.\vec{b} & \vec{r}.\vec{b} & \vec{c}.\vec{b}\\ \vec{a}.\vec{c} & \vec{r}.\vec{c} & \vec{c}.\vec{c}\end{vmatrix}$

$$\Delta_3 = \begin{vmatrix} \vec{a}.\vec{a} & \vec{b}.\vec{a} & \vec{r}.\vec{a} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} & \vec{c}.\vec{b} \\ \vec{r}.\vec{c} & \vec{b}.\vec{c} & \vec{r}.\vec{c} \end{vmatrix} \qquad \Delta = \begin{vmatrix} \vec{a}.\vec{a} & \vec{b}.\vec{a} & \vec{c}.\vec{a} \\ \vec{a}.\vec{b} & \vec{b}.\vec{b} & \vec{c}.\vec{b} \\ \vec{a}.\vec{c} & \vec{b}.\vec{c} & \vec{c}.\vec{c} \end{vmatrix}$$

16. The vector is expressible in the form :

(A) $\vec{r} = \frac{\Delta_1}{\Delta}\vec{a} + \frac{\Delta_2}{\Delta}\vec{b} + \frac{\Delta_3}{\Delta}\vec{c}$

(B) $\vec{r} = \frac{2\Delta_1}{\Delta}\vec{a} + \frac{2\Delta_2}{\Delta}\vec{b} + \frac{2\Delta_3}{\Delta}\vec{c}$

(C) $\vec{r} = \frac{\Delta}{\Delta_1}\vec{a} + \frac{\Delta}{\Delta_2}\vec{b} + \frac{\Delta}{\Delta_3}\vec{c}$

(D) $\vec{r} = \frac{\Delta_1}{\Delta}\vec{a} + \frac{\Delta_2}{\Delta}\vec{b} + \frac{\Delta_3}{\Delta}\vec{c}$

Key. D

17. the vector is expressible as :

(A) $\vec{r} = \frac{[\vec{r}\,\vec{b}\,\vec{c}]}{2[\vec{a}\,\vec{b}\,\vec{c}]}\ \vec{a} + \frac{[\vec{r}\,\vec{c}\,\vec{a}]}{2[\vec{a}\,\vec{b}\,\vec{c}]}\vec{b} + \frac{[\vec{r}\,\vec{a}\,\vec{b}]}{2[\vec{a}\,\vec{b}\,\vec{c}]}\ \vec{c}$

(B) $\vec{r}\ \frac{2[\vec{r}\,\vec{b}\,\vec{c}]}{[\vec{a}\,\vec{b}\,\vec{c}]} = \vec{a} + \frac{2[\vec{r}\,\vec{c}\,\vec{a}]}{[\vec{a}\,\vec{b}\,\vec{c}]}\vec{b} + \frac{2[\vec{r}\,\vec{a}\,\vec{b}]}{[\vec{a}\,\vec{b}\,\vec{c}]}\ \vec{c}$

(C) $\vec{r} = [\vec{a}\,\vec{b}\,\vec{c}]\left(\frac{\vec{a}}{[\vec{r}\,\vec{b}\,\vec{c}]} + \frac{\vec{b}}{[\vec{r}\,\vec{c}\,\vec{a}]} + \frac{\vec{c}}{[\vec{r}\,\vec{a}\,\vec{b}]}\right)$

(D) none

Key. D

18. If vector is expressible as, $\vec{r} = x\,\vec{a} + y\,\vec{b} + z\,\vec{c}$ then

(A) $\vec{a} = \frac{\vec{a}.\vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}(\vec{b}\times\vec{c}) + \frac{\vec{a}.\vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}(\vec{c}\times\vec{a}) + \frac{\vec{c}.\vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}(\vec{a}\times\vec{b})$

(B) $\vec{a} = \vec{a}.\vec{a}(\vec{b}\times\vec{c}) + \vec{a}.\vec{b}(\vec{c}\times\vec{a}) + \vec{c}.\vec{a}(\vec{a}\times\vec{b})$

(C) $\vec{a} = [\vec{a}\,\vec{b}\,\vec{c}](\vec{b}\times\vec{c}) + [\vec{a}\,\vec{b}\,\vec{c}](\vec{c}\times\vec{a}) + [\vec{a}\,\vec{b}\,\vec{c}](\vec{a}\times\vec{b})$

(D) none

Key. A

19. The value for $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}.\vec{p} & \vec{b}.\vec{p} & \vec{c}\cdot\vec{p} \\ \vec{a}\cdot\vec{q} & \vec{b}\cdot\vec{q} & \vec{c}.\vec{q} \end{vmatrix}$, is :

(A) $(\vec{p}\times\vec{q})[\vec{a}\times\vec{b}\ \vec{b}\times\vec{c}\ \vec{c}\times\vec{a}]$

(B) $2(\vec{p}\times\vec{q})[\vec{a}\times\vec{b}\ \vec{b}\times\vec{c}\ \vec{c}\times\vec{a}]$

(C) $4(\vec{p}\times\vec{q})[\vec{a}\times\vec{b}\ \vec{b}\times\vec{c}\ \vec{c}\times\vec{a}]$

(D) $(\vec{p}\times\vec{q})\sqrt{[\vec{a}\times\vec{b}\ \vec{b}\times\vec{c}\ \vec{c}\times\vec{a}]}$

Key. D

Sol. 16 to 19

Since $\vec{a},\vec{b},\vec{c}$ are three non-coplaner vectors.

$\therefore$ There exists scalars x,y,z such that

$\vec{b}\times\vec{c},\vec{c}\times\vec{a}$ and $\vec{a}\times\vec{b}$ ....(i)

Taking dot product with $\vec{b}\times\vec{c},\vec{c}\times\vec{a}$ and $\vec{a}\times\vec{b}$ successively, we get

$$\vec{r}\cdot\left(\vec{b}\times\vec{c}\right)=\left(x\vec{a}+y\vec{b}+z\vec{c}\right)\cdot\left(\vec{b}\times\vec{c}\right)$$

$$\vec{r}\cdot\left(\vec{c}\times\vec{a}\right)=\left(x\vec{a}+y\vec{b}+z\vec{c}\right)\cdot\left(\vec{c}\times\vec{a}\right)$$

$$\vec{r}\cdot\left(\vec{a}\times\vec{b}\right)=\left(x\vec{a}+y\vec{b}+z\vec{c}\right)\cdot\left(\vec{a}\times\vec{b}\right)$$

$$\Rightarrow \quad \left[\vec{r}\,\vec{b}\,\vec{c}\right]=x\left[\vec{a}\,\vec{b}\,\vec{c}\right]$$

$$\left[\vec{r}\,\vec{c}\,\vec{a}\right]=y\left[\vec{b}\,\vec{c}\,\vec{a}\right]$$

$$\text{and} \quad \left[\vec{r}\,\vec{a}\,\vec{b}\right]=z\left[\vec{c}\,\vec{a}\,\vec{b}\right]$$

$$x=\frac{\left[\vec{r}\,\vec{b}\,\vec{c}\right]}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]},y\frac{\left[\vec{r}\,\vec{c}\,\vec{a}\right]}{\left[\vec{b}\,\vec{c}\,\vec{a}\right]},z=\frac{\left[\vec{r}\,\vec{a}\,\vec{b}\right]}{\left[\vec{c}\,\vec{a}\,\vec{b}\right]}$$

substituting the values fo x,y,z in (i), we get

$$\vec{r}=\left\{\frac{\left[\vec{r}\,\vec{b}\,\vec{c}\right]}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}\right\}\vec{a}+\left\{\frac{\left[\vec{r}\,\vec{c}\,\vec{a}\right]}{\left[\vec{b}\,\vec{c}\,\vec{a}\right]}\right\}\vec{b}+\left\{\frac{\left[\vec{r}\,\vec{a}\,\vec{b}\right]}{\left[\vec{c}\,\vec{a}\,\vec{b}\right]}\right\}\vec{c}$$

Again, since $\vec{a},b,\vec{c}$ are non-coplaner vectors,

$$\therefore\left[\vec{a}\,\vec{b}\,\vec{c}\right]\neq 0$$

$$\Rightarrow\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2\neq 0$$

$$\Rightarrow\begin{vmatrix}\vec{a}\cdot\vec{a} & \vec{b}\cdot\vec{a} & \vec{c}\cdot\vec{a}\\ \vec{a}\cdot\vec{b} & \vec{b}\cdot\vec{b} & \vec{c}\cdot\vec{b}\\ \vec{a}\cdot\vec{c} & \vec{b}\cdot\vec{c} & \vec{c}\cdot\vec{c}\end{vmatrix}=\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2\neq 0$$

Since any vector $\vec{r}$ in space can be expressed as a liner combination of three non-coplaner vectors. So, let

$\vec{r}=l\vec{a}+m\vec{b}+n\vec{c}$ ...(i)

Taking dot products on both sides successively by $\vec{a},\vec{b},\vec{c}$ we get

$$\left.\begin{array}{l}\vec{r}\cdot\vec{a}=l\vec{a}\cdot\vec{a} \quad m\vec{b}\cdot\vec{a} \quad n\vec{c}\cdot\vec{a}\\ \vec{r}\cdot\vec{b}=l\vec{a}\cdot\vec{b} \quad m\vec{b}\cdot\vec{b} \quad n\vec{c}\cdot\vec{b}\\ \vec{r}\cdot\vec{c}=l\vec{a}\cdot\vec{c} \quad m\vec{b}\cdot\vec{c} \quad n\vec{c}\cdot\vec{c}\end{array}\right\}\begin{array}{l}...(ii)\\ ...(iii)\\ ...(iv)\end{array}$$

On eliminating l,m,n from above four relation, we get

$$\begin{vmatrix} \vec{r} & \vec{a} & \vec{b} & \vec{c} \\ \vec{r}\cdot\vec{a} & \vec{a}\cdot\vec{a} & \vec{b}\cdot\vec{a} & \vec{c}\cdot\vec{a} \\ \vec{r}\cdot\vec{b} & \vec{a}\cdot\vec{b} & \vec{b}\cdot\vec{b} & \vec{c}\cdot\vec{b} \\ \vec{r}\cdot\vec{c} & \vec{a}\cdot\vec{c} & \vec{b}\cdot\vec{c} & \vec{c}\cdot\vec{c} \end{vmatrix} = 0$$

On expanding this determinant along first row, we obtain

$$r = \left(\frac{\Delta_1}{\Delta}\right)\vec{a} + \left(\frac{\Delta_2}{\Delta}\right)\vec{b} + \left(\frac{\Delta_3}{\Delta}\right)\vec{c}$$

we know that,

$$\left[\vec{a}\times\vec{b}\,\vec{b}\times\vec{c}\,\vec{c}\times\vec{a}\right] = \left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$$

$$\Leftrightarrow \left[\vec{a}\times\vec{b}\,\vec{b}\times\vec{c}\,\vec{c}\times\vec{a}\right] \neq 0 \qquad \left\{as\left[\vec{a}\,\vec{b}\,\vec{c}\right]\neq 0\right\}$$

$\Leftrightarrow \vec{a}\times\vec{b}\,\vec{b}\times\vec{c}\,\vec{c}\times\vec{a}$ re non-coplaner

we also know that any vector in space can be expressed as a liner combination of any three non-
coplaner vectors. So, let

$$\vec{a} = l\left(\vec{b}\times\vec{c}\right) + m\left(\vec{c}\times\vec{a}\right) + n\left(\vec{a}\times\vec{b}\right)$$

Taking dot product on both sides seccessively by $\vec{a},\vec{b},\vec{c}$, we get

$$\vec{a}\cdot\vec{a} = l\left\{\vec{a}\cdot\left(b\times\vec{c}\right)\right\}$$

$$\vec{a}\cdot\vec{b} = m\left\{\left(\vec{c}\times\vec{a}\right)\cdot\vec{b}\right\}$$

and $\quad \vec{c}\cdot\vec{a} = n\left\{\vec{c}\cdot\left(\vec{a}\times\vec{b}\right)\right\}$

$$\Rightarrow l = \frac{\vec{a}\cdot\vec{a}}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}, m = \frac{\vec{a}\cdot\vec{b}}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}, n = \frac{\vec{c}\cdot\vec{a}}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]},$$

Again let,

$$\vec{a} = a_1\hat{i} + a_2 j + a_3 k, \qquad \vec{b} = b_1\hat{i} + b_2 j + b_3 k,$$

$$\vec{c} = c_1\hat{i} + c_2 j + c_3 k, \qquad \vec{p} = p_1\hat{i} + p_2 j + p_3 k,$$

and $\quad \vec{q} = q_1\hat{i} + q_2 j + q_3 k,$

Then, $\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}\cdot\vec{p} & \vec{b}\cdot\vec{p} & \vec{c}\cdot\vec{p} \\ \vec{a}\cdot\vec{q} & \vec{b}\cdot\vec{q} & \vec{c}\cdot\vec{q} \end{vmatrix}$

$$= \begin{vmatrix} a_1\hat{i} + a_2 j + a_3 k & b_1\hat{i} + b_2 j + b_3 k & c_1\hat{i} + c_2 j + c_3 k \\ a_1 p_1 + a_2 p_2 + a_3 p_3 & b_1 p_1 + b_2 p_2 + b_3 p_3 & c_1 p_1 + c_2 p_2 + c_3 p_3 \\ a_1 q_1 + a_2 q_2 + a_3 q_3 & b_1 q_1 + b_2 q_2 + b_3 q_3 & c_1 q_1 + c_2 q_2 + c_3 q_3 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & j & k \\ p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \end{vmatrix} \cdot \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= (\vec{p}\times\vec{q})\left[\vec{a}\,\vec{b}\,\vec{c}\right]$$

$$= \sqrt{\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2}\,(\vec{p}\times\vec{q}) \quad .$$

$$= \sqrt{\left[\vec{a}\times\vec{b}\,\vec{b}\times\vec{c}\,\vec{c}\times\vec{a}\right]}\,(\vec{p}\times\vec{q})$$

Paragraph – 7

Let $\vec{x}$, $\vec{y}$, $\vec{z}$ be the vector, such that | $\vec{x}$ | = | $\vec{y}$ | = | $\vec{z}$ | = $\sqrt{2}$ and $\vec{x},\vec{y},\vec{z}$ make angles of 60° with each other also,

$\vec{x}\times(\vec{y}\times\vec{z}) = \vec{a}$

$\vec{y}\times(\vec{z}\times\vec{x}) = \vec{b}$

and $\vec{x}\times\vec{y} = \vec{c}$. Then

20. The value of $\vec{x}$ is :

(A) $\left\{(\vec{a}+\vec{b})\times\vec{c} - (\vec{a}+\vec{b})\right\}$

(B) $\left\{(\vec{a}+\vec{b}) - (\vec{a}+\vec{b})\times\vec{c}\right\}$

(C) $\frac{1}{2}\left\{(\vec{a}+\vec{b})\times\vec{c} - (\vec{a}+\vec{b})\right\}$

(D) none of these

Key. C

21. The value of y is

(A) $\frac{1}{2}\left[(\vec{a}+\vec{b}) + (\vec{a}+\vec{b})\times\vec{c}\right]$

(B) $2\left[(\vec{a}+\vec{b}) + (\vec{a}+\vec{b})\times\vec{c}\right]$

(C) $4\left[(\vec{a}+\vec{b}) + (\vec{a}+\vec{b})\times\vec{c}\right]$

(C) none of these

Key. A

22. The value of z is

(A) $\frac{1}{2}\left[(\vec{b}-\vec{a})\times\vec{c} + (\vec{a}+\vec{b})\right]$

(B) $\frac{1}{2}\left[(\vec{b}-\vec{a}) + (\vec{a}+\vec{b})\times\vec{c}\right]$

(C) $\left[(\vec{b}-\vec{a})\times\vec{c} + (\vec{a}+\vec{b})\right]$

(D) none of these

Key. B

Sol. 20 to 22

We have $|\vec{x}| = |\vec{y}| = |\vec{z}| = \sqrt{2}$ and $\vec{x},\vec{y},\vec{z}$ make angle of 60° with each other.

$$\therefore \vec{x}\cdot\vec{y} = |\vec{x}||\vec{y}|\cos 60^\circ = \sqrt{2}(\sqrt{2})\cdot\frac{1}{2} = 1$$

$\vec{y}\cdot\vec{z} = |\vec{y}||\vec{z}|\cos 60^\circ = \sqrt{2}(\sqrt{2})\left(\frac{1}{2}\right) = 1$ and $\vec{x}\cdot\vec{z} = |\vec{x}||\vec{z}|\cos 60^\circ = \sqrt{2}(\sqrt{2})\left(\frac{1}{2}\right) = 1$

$$\vec{x}\cdot\vec{x} = |\vec{x}|^2 = 2$$

$$\vec{y}\cdot\vec{y}=|\vec{y}|^2=2$$

and $\vec{z}\cdot\vec{z}=|\vec{z}|^2=2$

Now, $\vec{x}\times(\vec{y}\times\vec{z})=\vec{a}$ and $\vec{y}\times(\vec{z}\times\vec{x})=\vec{b}$ $\{\text{given}\}$

$\Rightarrow(\vec{x}\cdot\vec{z})\vec{y}-(\vec{x}\cdot\vec{y})\vec{z}=\vec{a}$ and $(\vec{y}\cdot\vec{x})\vec{z}-(\vec{y}\cdot\vec{z})\vec{x}=\vec{b}$

$\Rightarrow\vec{y}-\vec{z}=\vec{a}$ and $\vec{z}-\vec{x}=\vec{b}$

$\Rightarrow\vec{y}-\vec{x}=\vec{a}+\vec{b}$

Thuse, we have

$\vec{y}-\vec{z}=\vec{a}$ ....(i)

$\vec{z}-\vec{x}=\vec{b}$ ...(ii)

$\vec{y}-\vec{x}=\vec{a}+\vec{b}$ ...(iii)

Now, $\vec{x}\times\vec{y}=\vec{c}$ {given}

$\Rightarrow\vec{x}\times(\vec{x}\times\vec{y})=\vec{x}\times\vec{c}$ {taking cross-product with $\vec{x}$}

$\Rightarrow(\vec{x}\cdot\vec{y})\times\vec{x}-(\vec{x}\cdot\vec{x})\vec{y}=\vec{x}\times\vec{c}$

$\Rightarrow\vec{x}-2\vec{y}=\vec{x}\times\vec{c}$ ....(iv)

Again, $\vec{x}\times\vec{y}=\vec{c}$

$\Rightarrow\vec{y}\times(\vec{x}\times\vec{y})=\vec{y}\times\vec{c}$ {taking cross product with $\vec{y}$}

$\Rightarrow(\vec{y}\cdot\vec{y})\vec{x}-(\vec{y}\cdot\vec{x})\vec{y}=\vec{y}\times\vec{c}$

$\Rightarrow2\vec{x}-\vec{y}=\vec{y}\times\vec{c}$ ....(v)

On subtracting (iv) and (v), we get

$\vec{x}-\vec{y}=(\vec{y}\times\vec{c})-(\vec{x}\times\vec{c})$ $\Rightarrow\vec{x}+\vec{y}=(\vec{y}-\vec{x})\times\vec{c}$

$\Rightarrow\vec{x}+\vec{y}=(\vec{a}+\vec{b})\times\vec{c}$ ....(vi)

Adding (iii) and (vi), we get

$2\vec{y}=(\vec{a}+\vec{b})+(\vec{a}+\vec{b})\times\vec{c}$, $\vec{y}=\frac{1}{2}\left[(\vec{a}+\vec{b})(\vec{a}+\vec{b})\times\vec{c}\right]$

Substituting the value of $\vec{y}$ in (iii) in (i), we get

$\vec{x}=\frac{1}{2}\left[(\vec{a}+\vec{b})+(\vec{a}+\vec{b})\times\vec{c}\right]-(\vec{a}+\vec{b})$ $\Rightarrow\vec{x}=\frac{1}{2}\left[(\vec{a}+\vec{b})\times\vec{c}-(\vec{a}+\vec{b})\right]$

$\vec{z}=\frac{1}{2}\left[(\vec{a}+\vec{b})+(\vec{a}+\vec{b})\times\vec{c}\right]-\vec{a}$

Paragraph – 8

If $\bar{a}$, $\bar{b}$, $\bar{c}$ are the three given non coplanar vectors and any vector $\bar{r}$ in the space where

$\Delta_1 = \begin{vmatrix} \bar{r}.\bar{a} & \bar{b}.\bar{a} & \bar{c}.\bar{a} \\ \bar{r}.\bar{b} & \bar{b}.\bar{b} & \bar{c}.\bar{b} \\ \bar{r}.\bar{c} & \bar{b}.\bar{c} & \bar{c}.\bar{c} \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} \bar{a}.\bar{a} & \bar{r}.\bar{a} & \bar{c}.\bar{a} \\ \bar{a}.\bar{b} & \bar{r}.\bar{b} & \bar{c}.\bar{b} \\ \bar{a}.\bar{c} & \bar{r}.\bar{c} & \bar{c}.\bar{c} \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} \bar{a}.\bar{a} & \bar{b}.\bar{a} & \bar{r}.\bar{a} \\ \bar{a}.\bar{b} & \bar{b}.\bar{b} & \bar{r}.\bar{b} \\ \bar{a}.\bar{c} & \bar{b}.\bar{c} & \bar{r}.\bar{c} \end{vmatrix}$ and

$\Delta = \begin{vmatrix} \bar{a}.\bar{a} & \bar{b}.\bar{a} & \bar{c}.\bar{a} \\ \bar{a}.\bar{b} & \bar{b}.\bar{b} & \bar{c}.\bar{b} \\ \bar{a}.\bar{c} & \bar{b}.\bar{c} & \bar{c}.\bar{c} \end{vmatrix}$ then

23. The vector $\bar{r}$ is expressible in the form

a) $\bar{r} = \frac{\Delta_1}{2\Delta}\bar{a} + \frac{\Delta_2}{2\Delta}\bar{b} + \frac{\Delta_3}{2\Delta}\bar{c}$

b) $\bar{r} = \frac{2\Delta_1}{\Delta}\bar{a} + \frac{2\Delta_2}{\Delta}\bar{b} + \frac{2\Delta_3}{\Delta}\bar{c}$

c) $\bar{r} = \frac{\Delta_1}{\Delta}\bar{a} + \frac{\Delta_2}{\Delta}\bar{b} + \frac{\Delta_3}{\Delta}\bar{c}$

d) $\bar{r} = \frac{\Delta}{\Delta_1}\bar{a} + \frac{\Delta}{\Delta_2}\bar{b} + \frac{\Delta}{\Delta_3}\bar{c}$

Key. C

Sol. Since $\bar{a}, \bar{b}, \bar{c}$ are 3 non coplanar vectors

$\exists$ scalars x, y, z such that $\bar{r} = x\bar{a} + y\bar{b} + z\bar{c}$ ……… (1)

Taking dot product with $\bar{b}\times\bar{c}$, $\bar{c}\times\bar{a}$ and $\bar{a}\times\bar{b}$ successively. We get

$\bar{r}.\left(\bar{b}\times\bar{c}\right) = \left(x\bar{a} + y\bar{b} + z\bar{c}\right).\left(\bar{b}\times\bar{c}\right) = x\left[\bar{a}\bar{b}\bar{c}\right]$

$\Rightarrow \left[\bar{r}\,\bar{b}\,\bar{c}\right] = x\left[\bar{a}\,\bar{b}\,\bar{c}\right]$ Similarly $\left[\bar{r}\,\bar{c}\,\bar{a}\right] = y\left[\bar{b}\,\bar{c}\,\bar{a}\right]$ and $\left[\bar{r}\,\bar{a}\,\bar{b}\right] = z\left[\bar{c}\,\bar{a}\,\bar{b}\right]$

$\Rightarrow x = \frac{\left[\bar{r}\,\bar{b}\,\bar{c}\right]}{\left[\bar{a}\,\bar{b}\,\bar{c}\right]}$, $y = \frac{\left[\bar{r}\,\bar{c}\,\bar{a}\right]}{\left[\bar{a}\,\bar{b}\,\bar{c}\right]}$, $z = \frac{\left[\bar{r}\bar{a}\bar{b}\right]}{\left[\bar{a}\,\bar{b}\,\bar{c}\right]}$

Substitute x, y, z in (1) we get $\bar{r} = \frac{\Delta_1}{\Delta}\bar{a} + \frac{\Delta_2}{\Delta}\bar{b} + \frac{\Delta_3}{\Delta}\bar{c}$

24. If vector $\bar{r}$ is expressible as $\bar{r} = x\bar{a} + y\bar{b} + z\bar{c}$ then

a) $\bar{a} = \frac{\bar{a}.\bar{a}}{\left[\bar{a}\bar{b}\bar{c}\right]}\left(\bar{b}\times\bar{c}\right) + \frac{\bar{a}.\bar{b}}{\left[\bar{a}\bar{b}\bar{c}\right]}\left(\bar{c}\times\bar{a}\right) + \frac{\bar{c}.\bar{a}}{\left[\bar{a}\bar{b}\bar{c}\right]}\left(\bar{a}\times\bar{b}\right)$

b) $\bar{a} = \left(\bar{a}.\bar{a}\right)\left(\bar{b}\times\bar{c}\right) + \left(\bar{a}.\bar{b}\right)\left(\bar{c}\times\bar{a}\right) + \left(\bar{c}.\bar{a}\right)\left(\bar{a}\times\bar{b}\right)$

c) $\bar{a} = \left[\bar{a}\bar{b}\bar{c}\right]\left(\bar{b}\times\bar{c}\right) + \left[\bar{a}\bar{b}\bar{c}\right]\left(\bar{c}\times\bar{a}\right) + \left[\bar{a}\bar{b}\bar{c}\right]\left(\bar{a}\times\bar{b}\right)$

d) $\bar{a} = \left[\bar{a}\bar{b}\bar{c}\right]\bar{a} + \left[\bar{a}\bar{b}\bar{c}\right]\bar{b} + \left[\bar{a}\bar{b}\bar{c}\right]\bar{c}$

Key. A

Sol. W.K.T any vector in the space can be expressed as linear combination of any three non coplanar vectors

So let $\bar{a} = l\left(\bar{b}\times\bar{c}\right) + m\left(\bar{c}\times\bar{a}\right) + n\left(\bar{a}\times\bar{b}\right)$ ………(1)

Taking dot product on both sides successively by $\bar{a}, \bar{b}, \bar{c}$ we get

$l = \frac{\bar{a}.\bar{a}}{\left[\bar{a}\,\bar{b}\,\bar{c}\right]}$, $m = \frac{\bar{a}.\bar{b}}{\left[\bar{a}\,\bar{b}\,\bar{c}\right]}$, $n = \frac{\bar{a}.\bar{c}}{\left[\bar{a}\,\bar{b}\,\bar{c}\right]}$

Substitution of *l*, *m*, *n* in (1) we get

$$\bar{a} = \frac{\bar{a}.\bar{a}}{\left[\bar{a}\bar{b}\bar{c}\right]}\left(\bar{b}\times\bar{c}\right) + \frac{\bar{a}.\bar{b}}{\left[\bar{a}\bar{b}\bar{c}\right]}\left(\bar{c}\times\bar{a}\right) + \frac{\bar{c}.\bar{a}}{\left[\bar{a}\bar{b}\bar{c}\right]}\left(\bar{a}\times\bar{b}\right)$$

25. The value of $\begin{vmatrix} \bar{a} & \bar{b} & \bar{c} \\ \bar{a}.\bar{p} & \bar{b}.\bar{p} & \bar{c}.\bar{p} \\ \bar{a}.\bar{q} & \bar{b}.\bar{q} & \bar{c}.\bar{q} \end{vmatrix}$ is

a) $\left(\bar{p}\times\bar{q}\right)\left[\bar{a}\times\bar{b} \quad \bar{b}\times\bar{c} \quad \bar{c}\times\bar{a}\right]$

b) $2\left(\bar{p}\times\bar{q}\right)\left[\bar{a}\times\bar{b} \quad \bar{b}\times\bar{c} \quad \bar{c}\times\bar{a}\right]$

c) $4\left(\bar{p}\times\bar{q}\right)\left[\bar{a}\times\bar{b} \quad \bar{b}\times\bar{c} \quad \bar{c}\times\bar{a}\right]$

d) $\left(\bar{p}\times\bar{q}\right)\sqrt{\left[\bar{a}\times\bar{b} \quad \bar{b}\times\bar{c} \quad \bar{c}\times\bar{a}\right]}$

Key. D

Sol. Let $\bar{a} = a_1\bar{i} + a_2\bar{j} + a_3\bar{k}$, $\bar{b} = b_1\bar{i} + b_2\bar{j} + b_3\bar{k}$, $\bar{c} = c_1\bar{i} + c_2\bar{j} + c_3\bar{k}$ $\bar{p} = p_1\bar{i} + p_2\bar{j} + p_3\bar{k}$

$\bar{q} = q_1\bar{i} + q_2\bar{j} + q_3\bar{k}$

$$\therefore \begin{vmatrix} \bar{a} & \bar{b} & \bar{c} \\ \bar{a}.\bar{p} & \bar{b}.\bar{p} & \bar{c}.\bar{p} \\ \bar{a}.\bar{q} & \bar{b}.\bar{q} & \bar{c}.\bar{q} \end{vmatrix} = \begin{vmatrix} a_1\bar{i} + a_2\bar{j} + a_3\bar{k} & b_1\bar{i} + b_2\bar{j} + b_3\bar{k} & c_1\bar{i} + c_2\bar{j} + c_3\bar{k} \\ a_1p_1 + a_2p_2 + a_3p_3 & b_1p_1 + b_2p_2 + b_3p_3 & c_1p_1 + c_2p_2 + c_3p_3 \\ a_1q_1 + a_2q_2 + a_3q_3 & b_1q_1 + b_2q_2 + b_3q_3 & c_1q_1 + c_2q_2 + c_3q_3 \end{vmatrix}$$

$$= \begin{vmatrix} i & j & k \\ p_1 & p_2 & p_3 \\ q_1 & q_2 & q_3 \end{vmatrix} \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \left(\bar{p}\times\bar{q}\right)\left[\bar{a}\ \bar{b}\ \bar{c}\right]$$

Passage – 9

If $\bar{a},\bar{b},\bar{c}$ are any three vectors then

$$\bar{a}\times(\bar{b}\times\bar{c}) = (\bar{a}\cdot\bar{c})\bar{b} - (\bar{a}\cdot\bar{b})\bar{c}; (\bar{a}\times\bar{b})\cdot(\bar{c}\times\bar{d}) = \begin{vmatrix} \bar{a}\cdot\bar{c} & \bar{b}\cdot\bar{c} \\ \bar{a}\cdot\bar{d} & \bar{b}\cdot\bar{d} \end{vmatrix}$$

26. The value of 'a' so that the volume of the parallelepiped formed by vectors $i + aj + k; j + ak; ai + k$ becomes minimum is

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{-1}{\sqrt{3}}$ (c) 1 (d) $\pm\frac{1}{\sqrt{3}}$

Key. A

Sol. $V=\begin{vmatrix}1 & a & 1\\ 0 & 1 & a\\ a & 0 & 1\end{vmatrix}=1+a^3-a=0; V\ is\ Minimum; \frac{dv}{da}=0$

$a=\pm\frac{1}{\sqrt{3}}$

27. Let $\bar{a}=2i+3j+4k; \bar{b}=i+5j+2k; \bar{c}=3i+15j+6k$ then the value of $\begin{vmatrix}\bar{a}\cdot\bar{a} & \bar{a}\cdot\bar{b} & \bar{a}\cdot\bar{c}\\ \bar{b}\cdot\bar{a} & \bar{b}\cdot\bar{b} & \bar{b}\cdot\bar{c}\\ \bar{c}\cdot\bar{a} & \bar{c}\cdot\bar{b} & \bar{c}\cdot\bar{c}\end{vmatrix}$ is

(a) 429 (b) 0 (c) 1 (d) – 5

Key. B

Sol. $\begin{vmatrix}\bar{a}\cdot\bar{a} & \bar{a}\cdot\bar{b} & \bar{a}\cdot\bar{c}\\ \bar{b}\cdot\bar{a} & \bar{b}\cdot\bar{b} & \bar{b}\cdot\bar{c}\\ \bar{c}\cdot\bar{a} & \bar{c}\cdot\bar{b} & \bar{c}\cdot\bar{c}\end{vmatrix}=[\bar{a}\bar{b}\bar{c}]^2$

28. If $\bar{a}=i+j+k; \bar{b}=4i+3j+4k; \bar{c}=i+\alpha j+\beta k$ are linearly dependent vectors; $|\bar{c}|=\sqrt{3}$ then

(a) $\beta=-1; \alpha=1$ (b) $\alpha=1; \beta=\pm1$ (c) $\alpha=-1; \beta=\pm1$ (d) $\alpha=\pm1; \beta=1$

Key. D

Sol. $\begin{vmatrix}1 & 1 & 1\\ 4 & 3 & 4\\ 1 & \alpha & \beta\end{vmatrix}=0; \sqrt{1+\alpha^2+\beta^2}=\sqrt{3}$

$\alpha^2+\beta^2=2$

$1-\alpha(0)+\beta(-1)=0$

$\beta=1$

Paragraph – 10

The vertices of a ΔABC are A(1, 0 , 2) , B (-2 , 1 , 3) and C (2 , -1, 1) If D is the foot of the perpendicular drawn from A and BC, then

29. The equation of medium of ΔABD passing through the vertex A, is

(A) $\vec{r}=\left(\hat{i}+2\hat{k}\right)+\frac{\lambda}{3}\left(-5\hat{i}+\hat{j}+\tilde{k}\right)$ (B) $\vec{r}=\left(\hat{i}-2\hat{k}\right)+\frac{\lambda}{3}\left(-5\hat{i}+\hat{j}+\tilde{k}\right)$

(C) $\vec{r} = \left(\hat{i} + 2\hat{k}\right) + \frac{\lambda}{3}\left(5\hat{i} - \hat{j} + \tilde{k}\right)$ (D) none

Key. A

30. The vector equation of the bisector of ∠A, is given by :

(A) $\vec{r} - \left(\hat{i} + 2\hat{j}\right) + \lambda\left(\frac{-3\hat{i} + \hat{j} + \hat{k}}{\sqrt{11}} + \frac{-\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}\right)$

(B) $\vec{r} - \left(\hat{i} + 2\hat{k}\right) + \lambda\left(\frac{-3\hat{i} + \hat{j} + \hat{k}}{\sqrt{11}} + \frac{-\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}\right)$

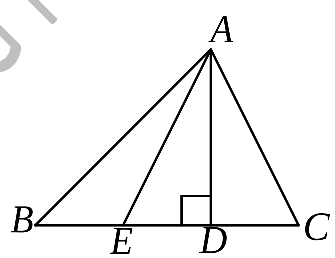
(C) $\vec{r} - \left(\hat{i} + 2\hat{j}\right) + \lambda\left(\frac{3\hat{i} + \hat{j} + \hat{k}}{\sqrt{11}} + \frac{-\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}\right)$

(D) none

Key. B

Sol. 29 and 30

Here, $\overrightarrow{BD} =$ Projection of $\overrightarrow{BA}$ and $\overrightarrow{BC}$



$$= \left(\overrightarrow{BA} \cdot BC\right) BC$$

$$= \left\{\left(\vec{a} - \vec{b}\right) \cdot \frac{\left(\vec{c} - \vec{b}\right)}{\left|\vec{c} - \vec{b}\right|}\right\} \frac{\vec{c} - \vec{b}}{\left|\vec{c} - \vec{b}\right|}$$

$$\Rightarrow \overrightarrow{BD} = \frac{4}{3}\left(2\hat{i} - j - k\right)$$

$$\Rightarrow P.V \text{of } \vec{D} - P.V \text{ of } \vec{B} = \frac{4}{3}\left(2\hat{i} - j - k\right)$$

$$\Rightarrow P.V \text{ of } \vec{D} = \frac{4}{3}\left(2\hat{i} - j - k\right) + \left(-2\hat{i} + j - 3k\right)$$

$$= \frac{1}{3}\left(2\hat{i} - j + 5k\right)$$

Since E is the mid-point of BD.

$$\therefore P.V \text{ of } \vec{E} = \frac{\frac{1}{3}\left(2\hat{i} - j + 5k\right) + \left(-2\hat{i} - j + 3k\right)}{2}$$

$$= \frac{1}{3}\left(-2\hat{i} + j + 7k\right)$$

Equation of line AE is,

$$\vec{r} = \left(\hat{i} + 2k\right) + \lambda\left\{\frac{1}{3}\left(-2\hat{i} + j + 7k\right) - \left(\hat{i} + 2k\right)\right\} \text{ or } \vec{r} = \left(\hat{i} + 2k\right) + \frac{\lambda}{3}\left\{-5\hat{i} + j + k\right\}$$

we have, $\overrightarrow{AB} = -3\hat{i} + j + k$ and $\overrightarrow{AD} = \frac{1}{3}\left(-\hat{i} - j - k\right)$

Vector equation of the bisector of $\angle A$ is given by

$$\vec{r} = \left(\hat{i} + 2k\right) + \lambda\left\{\frac{3\hat{i} + j + k}{\sqrt{11}} + \frac{-\hat{i} - j - k}{\sqrt{3}}\right\}.$$

Paragraph – 11

Let a point P where position vector is $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is called Lattice point if $x, y, z \in N$. If atleast two of x,y,z are equal then this Lattice point is called isosceles Lattice point. If al x,y,z are equal then this Lattice point is called equilateral Lattice point.

31. If a Lattice point is called at random from Lattice points which satisfy $\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) \le 11$, then the probability that the selected Lattice point is equilateral given that it is isosceles Lattice point is

a) $\frac{1}{22}$ b) $\frac{1}{23}$ c) $\frac{2}{33}$ d) $\frac{5}{22}$

Key. B

Sol. Conceptual

32. Area of triangle formed by the isosceles Lattice points lying on the plane $\vec{r}.\left(\hat{i} + \hat{j} + \hat{k}\right) = 4$ is:

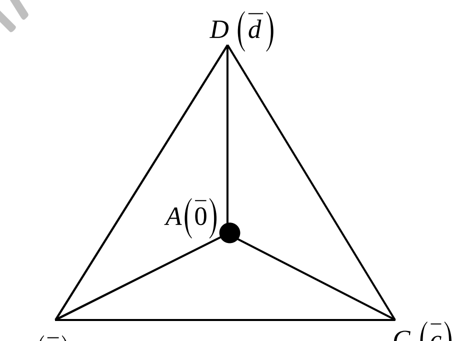
a) $2\sqrt{2}$ b) $\sqrt{2}$ c) $\frac{3}{2}\sqrt{2}$ d) $\frac{\sqrt{3}}{2}$

Key. D

Sol. Conceptual

Paragraph – 12

In tetrahedron ABCD the face ABC is a regular (Equilateral triangle) and the face BCD is perpendicular to it. $\angle DAC = \frac{\pi}{3}, |AD| = 6$ units Angle between the lines $\overline{AD}$ and $\overline{BC}$ is $\cos^{-1}\frac{1}{4}$ and if 'A' origin, $\overline{AD} = \bar{d},\ \overline{AB} = \bar{b},\ \overline{AC} = \bar{c}$



33. Angle between $\bar{d}, \bar{b}$ is

A) $\cos^{-1}\frac{1}{4}$ B) $\cos^{-1}\frac{1}{2}$ C) $\cos^{-1}\frac{1}{\sqrt{2}}$ D) $\cos^{-1}\frac{3}{4}$

Key. A

34. $|\bar{b}|+|\bar{c}|=$

A) 4 B) 6 C) 8 D) 14

Key. B

35. Volume of tetrahedron ABCD (Cu. units)

A) $\frac{27}{8}$ B) $\frac{27}{4}$ C) $\frac{9}{4}$ D) $\frac{8}{3}$

Key. B

Sol. 33. $\alpha=\left(\overline{AD},\overline{BC}\right)=\cos^{-1}\frac{1}{4}\Rightarrow\cos\alpha=\frac{1}{4}=\frac{\overline{AD}.\overline{BC}}{|\overline{AD}||\overline{BC}|}=\frac{\bar{d}.\left(\bar{c}-\bar{b}\right)}{|\bar{d}||\bar{c}-\bar{b}|}$

$\Rightarrow 4\bar{d}.\left(\bar{c}-\bar{b}\right)=|\bar{d}||\bar{c}-\bar{b}|=|\bar{b}||\bar{d}|$

Since $|\bar{b}-\bar{c}|=|\bar{b}|=|\bar{c}|$, Let $\left(\bar{d},\bar{b}\right)=\theta,\ \left(\bar{d},\bar{c}\right)=\frac{\pi}{3}$

$\Rightarrow 4\left(|\bar{d}||\bar{c}|\frac{1}{2}-|\bar{d}||\bar{b}|\cos\theta\right)=|\bar{d}||\bar{b}|$

$\Rightarrow 4\left(\frac{1}{2}-\cos\theta\right)=1\Rightarrow\cos\theta=\frac{1}{4}$

34. $\overline{ABC}\perp^{lr}\overline{DBC}\Rightarrow\left(\overline{AB}\times\overline{AC}\right).\left(\overline{BD}\times\overline{DC}\right)=0$

$\Rightarrow\left(\bar{b}\times\bar{c}\right).\left(\left(\bar{b}-\bar{d}\right)\times\left(\bar{c}-\bar{d}\right)\right)=0$

$\Rightarrow\left(\bar{b}\times\bar{c}\right).\left(\bar{b}\times\bar{c}-\bar{b}\times\bar{d}-\bar{d}\times\bar{c}\right)=0$

$\Rightarrow|\bar{b}\times\bar{c}|^2-\left(\bar{b}\times\bar{c}\right).\left(\bar{b}\times\bar{d}\right)-\left(\bar{b}\times\bar{c}\right).\left(\bar{d}\times\bar{c}\right)=0$

$\Rightarrow|\bar{b}|^2|\bar{c}|^2\frac{3}{4}-\begin{vmatrix}\bar{b}.\bar{b} & \bar{b}.\bar{d}\\ \bar{c}.\bar{b} & \bar{c}.\bar{d}\end{vmatrix}-\begin{vmatrix}\bar{b}.\bar{d} & \bar{b}.\bar{c}\\ \bar{c}.\bar{d} & \bar{c}.\bar{c}\end{vmatrix}=0$

$\Rightarrow\frac{3|\bar{b}|}{4}-6\left(\frac{1}{2}-\frac{1}{8}\right)-6\left(\frac{1}{4}-\frac{1}{4}\right)=0$

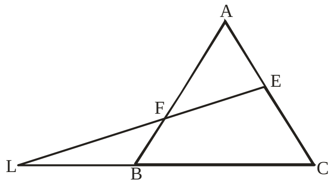
$\Rightarrow|\bar{b}|=3$ and $|\bar{c}|=3\Rightarrow|\bar{b}|+|\bar{c}|=6$.

35. $\left[\bar{d}\,\bar{b}\,\bar{c}\right]^2=\begin{vmatrix}\bar{d}.\bar{d} & \bar{d}.\bar{b} & \bar{d}.\bar{c}\\ \bar{b}.\bar{d} & \bar{b}.\bar{b} & \bar{b}.\bar{c}\\ \bar{c}.\bar{d} & \bar{c}.\bar{b} & \bar{c}.\bar{c}\end{vmatrix}=|\bar{d}|^2|\bar{b}|^2|\bar{c}|^2\begin{vmatrix}1 & 1/4 & 1/2\\ 1/4 & 1 & 1/2\\ 1/2 & 1/2 & 1\end{vmatrix}$

Volume of tetrahedron = $\frac{1}{6}\left[\bar{d}\,\bar{b}\,\bar{c}\right]=\frac{27}{4}$ Cu.units.

Paragraph – 13

In the adjacent figure



Let $\frac{CE}{EA} = \frac{AF}{FB} = \frac{\lambda}{1}$

$\frac{LB}{BC} = \frac{\mu}{1}$

and $\overrightarrow{FL} = \nu\overrightarrow{FE}$ then answer following questions.

36. $\overrightarrow{AB}$ must be equal to

(A) $\frac{\mu\overrightarrow{AC}+\overrightarrow{AL}}{\mu+1}$ (B) $\frac{\mu\overrightarrow{AL}+\overrightarrow{AC}}{\mu+1}$

(C) $\frac{\overrightarrow{AC}+\overrightarrow{AL}}{2}$ (D) $\frac{\overrightarrow{AC}-\overrightarrow{AL}}{2}$

Key. A

37. $\overrightarrow{AL}$ must be equal to

(A) $\frac{\nu(1-\lambda)}{\nu+1}\overrightarrow{AB}+\frac{x}{\nu+1}\overrightarrow{AC}$ (B) $\frac{\lambda(1-\nu)}{\lambda+1}\overrightarrow{AB}+\frac{\nu}{\lambda+1}\overrightarrow{AC}$

(C) $\frac{\nu(1+\lambda)}{\nu+1}\overrightarrow{AB}+\frac{\lambda}{\nu+1}\overrightarrow{AC}$ (D) $\frac{\lambda(1+\nu)}{\lambda+1}\overrightarrow{AB}-\frac{\nu}{\nu+1}\overrightarrow{AC}$

Key. B

38. μ must be equal to

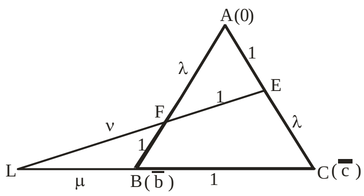
(A) $\frac{1}{\lambda^2+1}$ (B) $\frac{1}{\lambda^2-1}$

(C) $\frac{\lambda}{\lambda+1}$ (D) $\frac{\lambda}{\lambda-1}$

Key. B

Sol. 36. a be the origin and

let position vector of b, c & l are $\bar{b}, \bar{c}$ and $\bar{l}$



$\Rightarrow \quad \frac{\bar{l}+\mu\bar{c}}{\mu+1} = \bar{b}$

$\Rightarrow \quad A\bar{B} = \frac{\mu A\bar{C}+A\bar{L}}{\mu+1}$

37. p.v. of e and f are

$\frac{\bar{c}}{\lambda+1}, \frac{\lambda\bar{b}}{\lambda+1}$ respectively

also p.v. of $F = \frac{\frac{-\nu\bar{c}}{\lambda+1}+\bar{l}}{1-\nu} = \frac{\lambda\bar{b}}{\lambda+1}$

$\Rightarrow \quad \bar{l} = \frac{\lambda(1-\nu)}{\lambda+1}\bar{b}+\frac{\nu}{\lambda+1}\bar{c}$

$\overline{AL} = \frac{\lambda(1-\nu)}{\lambda+1}\overline{AB}+\frac{\nu}{\lambda+1}\overline{AC}$

38. p.v. of E and F are

$$\frac{\bar{c}}{\lambda+1} \text{ and } \frac{\lambda\bar{b}}{\lambda+1} \text{ respectively}$$

p.v. of $L \equiv \frac{(-\nu+1)}{\lambda+1} + \frac{\nu\bar{c}}{\lambda+1} = (\mu+1)\bar{b} - \mu\bar{c}$

as $\bar{b}$ and $\bar{c}$ are non-collinear vectors

$\Rightarrow \quad \frac{(-\nu+1)\lambda}{\lambda+1} = \mu+1 \qquad$ …..(i)

$\frac{-\nu}{\lambda+1} = \mu \qquad$ …..(ii)

from (i) and (ii)

$\mu = \frac{1}{\lambda^2 - 1}$

Paragraph – 14

Let three intersecting lines form a triangle ABC and separate the plane into 7 disjoint regions. Let the region in which the excentres I_1, I_2, I_3 lie be termed as excentral region opposite to angles A,B,C respectively. D be any point in the plane of ABC and O be the origin outside the plane of ΔABC, G is centroid of ΔABC.

Now let the position vectors of A,B,C,D be $\vec{\alpha}, \vec{\beta}, \vec{\gamma}, \vec{\delta}$ respectively. There exist real numbers, p,q,r such that $\vec{\delta} = p\vec{\alpha} + q\vec{\beta} + r\vec{\gamma}$ and $p+q+r=1$

39. If $9\vec{\delta} = 2\vec{\alpha} + 3\vec{\beta} + 4\vec{\gamma}$ then

(a) D is outside the triangle ABC

(b) D is nearer to AB than the centroid G of triangle ABC

(c) D and centroid G are at equal distance from AC

(d) G is nearer than D from BC

Key. C

40. If $3\vec{\delta} = \vec{\alpha} + 3\vec{\beta} - \vec{\gamma}$ then D is

(a) inside the plane ABC (b) on the side BC of ΔABC

(c) in the excentral region opposite to C (d) in the excentral region opposite to B

Key. C

41. $\vec{\delta} = \frac{1}{6}\vec{\alpha} + \frac{1}{3}\vec{\beta} + \frac{1}{2}\vec{\gamma}$ and D is the orthocentre of triangle ABC, then tan B =

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 2

Key. D

Sol. 39. $\vec{\delta} = \frac{2\vec{\alpha} + 3\vec{\beta} + 4\vec{\gamma}}{2+3+4}$

Inside as well as D and G are at equidistant from AC

40. $\vec{\delta} = \frac{\vec{\alpha} + 3\vec{\beta} - \vec{\gamma}}{1+3-1}$

Excentral region opposite to angle C

41. Orthocentre of triangle ABC is

$$\overrightarrow{OH} = \frac{\tan A\vec{a} + \tan B\vec{b} + \tan C\vec{c}}{\tan A + \tan B + \tan C}$$
$$\Rightarrow \tan B = 2$$

Paragraph – 15

Let $\bar{r}$ is position vector of a point in Cartesian OXY plane such that $\bar{r}.(10\hat{j} - 8\hat{i} - \bar{r}) = 40$, max $\left\{|\bar{r} + 2\hat{i} - 3\hat{j}|^2\right\} = l$ and $\min\left\{|\bar{r} + 2\hat{i} - 3\hat{j}|^2\right\} = m$. A tangent line is drawn to the curve $y = \frac{8}{x^2}$ at the point A with abscissa 2. The drawn tangent cuts the x-axis at B then

42. Value of m is
 (A) 9 (B) $2\sqrt{2} - 1$
 (C) $6\sqrt{2} + 3$ (D) $9 - 4\sqrt{2}$

Key. D

43. l + m is equal to
 (A) 2 (B) 10
 (C) 18 (D) 5

Key. C

44. The value of $\overline{AB}.\overline{OB}$ (O is origin) is
 (A) 1 (B) 2
 (C) 3 (D) 4

Key. C

Sol. 42. Let $\bar{r} = xi + yj$

$\bar{r}.(10j - 8i - \bar{r}) = 40$

$\Rightarrow$ $x^2 + y^2 + 8x - 10y + 40 = 0$

$\Rightarrow$ $\bar{r}$ lie on a circle in XOY plane.

l = (min. distance of $\bar{r}$ from $-2i + 3j)^2$

$= 9 + 4\sqrt{2}$

m = (min distance of $\bar{r}$ from $-2i + 3j)^2$

$= 9 - 4\sqrt{2}$

43. $l = 9 + 4\sqrt{2}$

$m = 9 - 4\sqrt{2}$

l + m = 18

44. Clearly point A(2, 2)

Equation of tangent at A(2, 2) is

2x + y – 6 = 0 co-ordinates of B(3, 0)

$\overline{AB} = \hat{i} - 2\hat{j}$

$\overline{OB} = 3\hat{i}$ $\overline{AB}.\overline{OB} = 3$

Paragraph – 16

The maximum value of modulus of dot product of two vectors is the product of moduli of the two vectors and this situation occurs when the two vectors are parallel.

45. If the projection of the vector $12\hat{i} - 4\hat{j} + 3\hat{k}$ on a vector $\vec{a}$ is maximum, then the unit vector along $\vec{a}$ is

(A) $\pm\dfrac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}$ (B) $\pm\dfrac{\hat{i}-2\hat{j}+3\hat{k}}{\sqrt{14}}$

(C) $\pm\dfrac{3\hat{i}+4\hat{j}-5\hat{k}}{5\sqrt{2}}$ (D) $\pm\dfrac{12\hat{i}-4\hat{j}+3\hat{k}}{13}$

Key. D

46. If 'a' is real constant and A, B, C are variable angles and $\sqrt{a^2-4}\tan A + a\tan B + \sqrt{a^2+4}\tan C = 6a$, then the least value of $\tan^2 A + \tan^2 B + \tan^2 C$ is

(A) 6 (B) 12 (C) 24 (D) 36

Key. B

47. In a ΔABC, $\cos 2A + \cos 2B + \cos 2C$ must be

(A) $\ge -\dfrac{3}{2}$ (B) $< -\dfrac{3}{2}$ (C) < -1 (D) ≥ -1

Key. A

Sol. 45. The projection = $|(12\hat{i}-4\hat{j}+3\hat{k}).\,\hat{a}\,|$

$= |12\hat{i}-4\hat{j}+3\hat{k}|\,|\hat{a}|\,|\cos\theta| = 13\,|\cos\theta|$ is maximum where $\theta = 0, \pi$

So, $\hat{a} = \dfrac{12\hat{i}-4\hat{j}+3\hat{k}}{13}$

46. Then given relation can be written as

$$\left(\sqrt{a^2-4}\;\hat{i} + a\hat{j} + \sqrt{a^2+4}\;\hat{k}\right).\left(\tan A\hat{i} + \tan B\hat{j} + \tan C\hat{k}\right) = 6a$$

$\Rightarrow \sqrt{(a^2-4)+a^2+(a^2+4)}.\sqrt{\tan^2 A + \tan^2 B + \tan^2 C}.\cos\theta = 6a$ (as, a.b = |a| |b| cos θ)

$\Rightarrow \sqrt{3}a.\sqrt{\tan^2 A + \tan^2 B + \tan^2 C}\cos\theta = 6a$

$\Rightarrow \tan^2 A + \tan^2 B + \tan^2 C = 12\sec^2\theta$ (1)

$\Rightarrow 12\sec^2\theta \ge 12$ (2) (as $\sec^2\theta \ge 1$)

From (1) and (2), $\tan^2 A + \tan^2 B + \tan^2 C \ge 12$

$\therefore$ least value of $\tan^2 A + \tan^2 B + \tan^2 C = 12$

Paragraph – 17

Let A, B, C, D, E represent vertices of a regular pentagon ABCDE. Given the position vectors of these vertices be $\vec{a}, \vec{a}+\vec{b}, \vec{b}, \lambda\vec{a}$ and $\lambda\vec{b}$ respectively with respect origin O where O is the point of intersection of line AD and EC.

48. The ratio $\dfrac{AD}{BC}$ is equal to

A) $1-\cos\dfrac{3\pi}{5}$ B) $1+2\cos\dfrac{2\pi}{5}$ C) $\cos\dfrac{2\pi}{5}$ D) $1-2\cos\dfrac{2\pi}{5}$

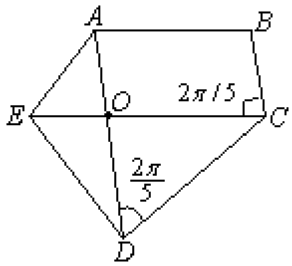
Key. B

49. AD divides EC in the ratio

A) $\cos\frac{2\pi}{5}:1$ B) $\cos\frac{3\pi}{5}:1$ C) $2\cos\frac{2\pi}{5}:1$ D) 1 : 2

Key. C

Sol. 48 – 49



$OA = BC$ & $OC = AB$

$\angle AOC = \angle ABC = \frac{3\pi}{5}$

$\Rightarrow OABC$ is a rhombus

Hence $\angle OAB = \angle OCB = \frac{2\pi}{5}$

$\angle DOC = \frac{2\pi}{5}, \angle EOD = \frac{3\pi}{5}$

Let $OD = \lambda\vec{a}, OE = \lambda\vec{b}$

$|\lambda\vec{a}| = 2|\vec{a}|\cos\frac{2\pi}{5} \Rightarrow \lambda = -2\cos\frac{2\pi}{5}$

$AD : BC = |\lambda - 1| = 1 + 2\cos\frac{2\pi}{5}$

$EO : OC = |\lambda| = 2\cos\frac{2\pi}{5}$

Paragraph – 18

Let $\vec{r}$ is a position vector of a variable point in a Cartesian OXY plane such that $\vec{r}.(10\vec{j} - 8\vec{i} - \vec{r}) = 40$ and $P_1 = \max\left\{\left|\vec{r} + 2\vec{i} - 3\vec{j}\right|^2\right\}, P_2 = \min\left\{\left|\vec{r} + 2\vec{i} - 3\vec{j}\right|^2\right\}$. A tangent line is drawn to the curve $y = \frac{8}{x^2}$ at the point A with abscissa 2. The drawn line cuts x-axis at a point B.

50. P_1 is equal to

(A) 9 (B) $2\sqrt{2} - 1$ (C) $6\sqrt{2} + 3$ (D) $9 + 4\sqrt{2}$

Key. D

51. $P_1 + P_2$ is equal to

(A) 2 (B) 10 (C) 18 (D) 5

Key. C

52. $\overrightarrow{AB}.\overrightarrow{OB}$ is

(A) 1 (B) 2 (C) 3 (D) 4

Key. C

Sol. Conceptual

Paragraph – 19

Necessary and sufficient condition for three non-zero vectors $\bar{a},\bar{b},\bar{c}$ to be coplanar is that there exists scalars l, m, n not all zero simultaneously such that $l\bar{a}+m\bar{b}+n\bar{c}=\bar{o},$ then

53. Let α,β,γ be distinct non-negative numbers. If the vectors $\alpha\bar{i}+\alpha\bar{j}+\gamma\bar{k},\bar{i}+\bar{k}$ and $\gamma\bar{i}+\gamma\bar{j}+\beta\bar{k}$ lie in the same plane, then γ is

(a) A.M of α *and* β (b) G.M of α *and* β

(c) H.M of α *and* β (d) Equals to zero

Key. B

54. Let $\bar{a}=2\bar{i}+\bar{j}+\bar{k},\ \bar{b}=\bar{i}+2\bar{j}-\bar{k}$ and a unit vector $\bar{c}$ be coplanar. If $\bar{c}$ is perpendicular to $\bar{a}$ then $\bar{c}=$

(a) $\pm\frac{1}{\sqrt{2}}(-\bar{j}+\bar{k})$ (b) $\pm\frac{1}{\sqrt{2}}(-\bar{j}-\bar{k})$

(c) $\pm\frac{1}{\sqrt{2}}(\bar{i}-2\bar{j})$ (d) $\pm\frac{1}{\sqrt{3}}(\bar{i}-\bar{j}-\bar{k})$

Key. A

55. $\bar{a},\bar{b},\bar{c}$ are three non-zero coplanar vectors. If $\bar{a}$ is not parallel to $\bar{b}$, then $\bar{c}$ =

(a) $\dfrac{\begin{vmatrix}\bar{a}\cdot\bar{c} & \bar{a}\cdot\bar{b}\\ \bar{c}\cdot\bar{b} & \bar{b}\cdot\bar{b}\end{vmatrix}\bar{a}+\begin{vmatrix}\bar{a}\cdot\bar{a} & \bar{c}\cdot\bar{a}\\ \bar{a}\cdot\bar{b} & \bar{c}\cdot\bar{b}\end{vmatrix}\bar{b}}{\begin{vmatrix}\bar{a}\cdot\bar{a} & \bar{a}\cdot\bar{b}\\ \bar{a}\cdot\bar{b} & \bar{b}\cdot\bar{b}\end{vmatrix}}$ (b) $\begin{vmatrix}\bar{c}\cdot\bar{a} & \bar{a}\cdot\bar{b}\\ \bar{c}\cdot\bar{b} & \bar{b}\cdot\bar{b}\end{vmatrix}\bar{a}+\begin{vmatrix}\bar{a}\cdot\bar{a} & \bar{c}\cdot\bar{a}\\ \bar{a}\cdot\bar{b} & \bar{c}\cdot\bar{b}\end{vmatrix}\bar{b}$

(c) $\begin{vmatrix}\bar{a}\cdot\bar{a} & \bar{a}\cdot\bar{b}\\ \bar{a}\cdot\bar{b} & \bar{b}\cdot\bar{b}\end{vmatrix}\bar{a}+\begin{vmatrix}\bar{c}\cdot\bar{a} & \bar{a}\cdot\bar{b}\\ \bar{c}\cdot\bar{b} & \bar{b}\cdot\bar{b}\end{vmatrix}\bar{b}$ (d) $\dfrac{\begin{vmatrix}\bar{a}\cdot\bar{a} & \bar{b}\cdot\bar{b}\\ \bar{a}\cdot\bar{b} & \bar{b}\cdot\bar{b}\end{vmatrix}\bar{a}+\begin{vmatrix}\bar{c}\cdot\bar{a} & \bar{a}\cdot\bar{b}\\ \bar{c}\cdot\bar{b} & \bar{b}\cdot\bar{b}\end{vmatrix}\bar{b}}{\begin{vmatrix}\bar{a}\cdot\bar{a} & \bar{b}\cdot\bar{b}\\ \bar{c}\cdot\bar{c} & \bar{b}\cdot\bar{b}\end{vmatrix}}$

Key. A

Sol. 53. $\begin{vmatrix}\alpha & \alpha & \gamma\\ 1 & 0 & 1\\ \gamma & \gamma & \beta\end{vmatrix}=0$

$\alpha(0-\gamma)-\alpha(\beta-\gamma)+\lambda(\lambda-0)=0$

$-\alpha\gamma-\alpha\beta+\alpha\gamma+\gamma^2=0$

$\gamma^2=\alpha\beta$

54. let $\bar{c}=x\bar{a}+y\bar{b}$

$\bar{c}\cdot\bar{a}=x\bar{a}\cdot\bar{a}+y\bar{a}\cdot\bar{b}$

$\bar{c}\cdot\bar{b}=x\bar{a}\cdot\bar{b}+y\bar{b}\cdot\bar{b}$

Solving $x(\bar{a}\cdot\bar{a})(\bar{a}\cdot\bar{b})+y(\bar{a}\cdot\bar{b})^2=(\bar{a}\cdot\bar{b})(\bar{c}\cdot\bar{a})$

$x(\bar{a}\cdot\bar{a})(\bar{a}\cdot\bar{b})+y(\bar{a}\cdot\bar{b})(\bar{a}\cdot\bar{a})=(\bar{c}\cdot\bar{b})(\bar{a}\cdot\bar{a})$

$y\left((\bar{a}\cdot\bar{b})^2-(\bar{a}\cdot\bar{b})(\bar{a}\cdot\bar{a})\right)=(\bar{a}\cdot\bar{b})(\bar{c}\cdot\bar{a})-(\bar{c}\cdot\bar{b})(\bar{a}\cdot\bar{a})$

$$y=\frac{(\bar{a}\cdot\bar{b})(\bar{c}\cdot\bar{a})-(\bar{c}\cdot\bar{b})(\bar{a}\cdot\bar{a})}{(\bar{a}\cdot\bar{b})^2-(\bar{a}\cdot\bar{b})(\bar{a}\cdot\bar{a})}$$

Paragraph – 20

Let 'S' be the circum centre, 'O' be the orthocentre and N be the centre of nine point circle of triangle ABC, then

56. $\overline{SA}+\overline{SB}+\overline{SC}=$

A) $4\overline{SO}$ B) $2\overline{SO}$ C) $3\overline{SO}$ D) $\overline{SO}$

Key. D

57. $\overline{OA}+\overline{OB}+\overline{OC}=$

A) $\overline{OS}$ B) $2\overline{SO}$ C) $2\overline{OS}$ D) $\overline{O}$

Key. C

Sol. Conceptual

Paragraph – 21

A particle is in equilibrium is subjected to four forces $\overline{F_1}=-10\bar{k}$, $\overline{F_2}=u\left(\frac{4}{13}\bar{i}-\frac{12}{13}\bar{j}+\frac{3}{13}\bar{k}\right)$,

$\overline{F_3}=v\left(\frac{-4}{13}\bar{i}-\frac{12}{13}\bar{j}+\frac{3}{13}\bar{k}\right)$, $\overline{F_4}=w\left(\cos\theta\bar{i}+\sin\theta\bar{j}\right)$ then

58. *u* =

(a) $\frac{65}{3}+\cot\theta$ (b) $\frac{65}{3}+65\cot\theta$ (c) $\frac{65}{3}-65\cot\theta$ (d) $65\cot\theta$

Key. C

59. *v* =

(a) $\frac{65}{3}+\cot\theta$ (b) $\frac{65}{3}+65\cot\theta$ (c) $\frac{65}{3}-65\cot\theta$ (d) $65\cot\theta$

Key. B

60. *w* =

(a) $\csc\theta$ (b) $40\csc\theta$ (c) $40\csc 2\theta$ (d) $-40\csc\theta$

Key. B

Sol. 58 to 60

Since the particle is in equilibrium

$$\Rightarrow \overline{F}_1+\overline{F}_2+\overline{F}_3+\overline{F}_4=0$$

$$\Rightarrow -10\bar{k}+u\left(\frac{4}{13}\bar{i}-\frac{12}{13}\bar{j}+\frac{3}{13}\bar{k}\right)+v\left(\frac{-4}{13}\bar{i}-\frac{12}{13}\bar{j}+\frac{3}{13}\bar{k}\right)+w\left(\cos\theta\bar{i}+\sin\theta\bar{j}\right)=\bar{0}$$

$$\Rightarrow\left(\frac{4u}{13}-\frac{4v}{13}+w\cos\theta\right)\bar{i}+\left(\frac{-12}{13}u-\frac{12}{13}v+w\sin\theta\right)\bar{j}+\left(-10+\frac{3}{13}u+\frac{3}{13}v\right)\bar{k}=\bar{0}$$

$$\Rightarrow \frac{4u}{13}-\frac{4v}{13}+w\cos\theta=0 \quad \text{.........................(1)}$$

$$\Rightarrow \frac{-12}{13}u-\frac{12}{13}v+w\sin\theta=0 \quad \text{...................(2)}$$

$$\Rightarrow -10+\frac{3}{13}u+\frac{3}{13}v=0 \quad \text{........................(3)}$$

From (3) $u+v=\frac{130}{3}$ ....(4)

From (2) $\Rightarrow \frac{12}{13}(u+v)=w\sin\theta$

$$\Rightarrow \frac{12}{13}\left(\frac{130}{3}\right)=w\sin\theta \quad \Rightarrow w=40\csc\theta$$

Substitute (w) in (1) & (2)

$$\Rightarrow \frac{4}{13}(u-v)+40\cot\theta=0$$

$$\Rightarrow \frac{4}{13}(u-v)=-40\cot\theta$$

$$\Rightarrow u-v=-130\cot\theta$$

$$\Rightarrow u+v=\frac{130}{3} \Rightarrow u=\frac{65}{3}-65\cot\theta$$

$$\Rightarrow v=\frac{65}{3}+65\cot\theta$$

Paragraph – 22

Let $\vec{a},\vec{b},\vec{c}$ be three non-coplanar vectors. Define $\vec{a}'=\frac{\vec{b}\times\vec{c}}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}$, $\vec{b}'=\frac{\vec{c}\times\vec{a}}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}$, $\vec{c}'=\frac{\vec{a}\times\vec{b}}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}$. then

61. $\left[\vec{a}\,\vec{b}\,\vec{c}\right]\ \left[\vec{a}'\,\vec{b}'\,\vec{c}'\right]=$

a) 1 b) $2\left[\vec{a}\,\vec{b}\,\vec{c}\right]$ c) 2 d) $\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$

Key. A

62. If $\vec{a}=x\vec{a}'+y\vec{b}'+z\vec{c}'$ (x, y, z are scalars), then $x+y+z$ is equal to

a) 3 b) 0

c) $\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+\left|\vec{c}\right|^2$ d) $\left|\vec{a}\right|^2+\vec{a}.\vec{b}+\vec{a}.\vec{c}$

Key. D

63. $\vec{a}'\times\vec{b}'+\vec{b}'\times\vec{c}'+\vec{c}'\times\vec{a}'$ equals

a) $\vec{0}$ b) $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ c) $\frac{\vec{a}+\vec{b}+\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]}$ d) $\frac{\vec{a}+\vec{b}+\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]^2}$

Key. C

Sol. 61). Observe that $\vec{a}.\vec{a}'.=\vec{b}.\vec{b}'=\vec{c}.\vec{c}'.=1$ and the rest of the dot product will be zero

$$[\vec{a}'\ \vec{b}'\ \vec{c}']=\frac{1}{[\vec{a}\ \vec{b}\ \vec{c}]^3}\{(\vec{b}'\times\vec{c})\times(\vec{c}\times\vec{a}).(\vec{a}\times\vec{b})\}$$

$$=\frac{1}{[\vec{a}\ \vec{b}\ \vec{c}]^3}\{[\vec{b}\ \vec{c}\ \vec{a}]\vec{c}.(\vec{a}\times\vec{b}\}$$

$$\therefore \frac{[\vec{a}\ \vec{b}\ \vec{c}]^2}{[\vec{a}\ \vec{b}\ \vec{c}]^3}=\frac{1}{[\vec{a}\ \vec{b}\ \vec{c}]}$$

62). $x=\vec{a}.\vec{a},y=\vec{a}.\vec{b},z=\vec{a}.\vec{c}$

63). $x=\vec{a}'\times\vec{b}'=\frac{(\vec{b}\times\vec{c})\times(\vec{c}\times\vec{a})}{[\vec{a}\ \vec{b}\ \vec{c}]^2}=\frac{[\vec{b}\ \vec{c}\ \vec{a}\ ]\vec{c}}{[\vec{a}\ \vec{b}\ \vec{c}]^2}$

Paragraph – 23

The vertices of a ΔABC are $A(2,0,2)$, $B(-1,1,1)$ and $C(1,-2,4)$. The points D and E divide the sides AB and AC in the ratio 1 : 2 respectively. Another point F is taken in space such that perpendicular drawn from F on ΔABC meet the Δ at the point of intersection of line segment CD and BE at P. If distance of F from plane of ΔABC is $\sqrt{2}$ units, then

64. The volume of tetrahedron ABCF is

(A) $\frac{7}{3}$ cubic units (B) $\frac{7}{5}$ cubic units

(C) $\frac{3}{5}$ cubic units (D) 7 cubic units

Key. A

65. The vector $\overrightarrow{PF}$ is

(A) $\hat{i}+\hat{j}$ (B) $\hat{j}+\hat{k}$

(C) $7\hat{i}+7\hat{k}$ (D) $\frac{7}{\sqrt{2}}(\hat{j}+\hat{k})$

Key. B

66. The equation of line AF is

(A) $\vec{r}=(\hat{i}+\hat{k})+\lambda(\hat{i}+\hat{j})$ (B) $\vec{r}=(2\hat{i}+2\hat{k})+\lambda(-\hat{i}+2\hat{k})$

(C) $\vec{r}=(2\hat{i}+2\hat{j})+\lambda(-\hat{i}+\hat{j}+\hat{k})$ (D) none of these

Key. D

Sol. 64. (A)Area of $\Delta ABC=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$

$$=\frac{1}{2}|(-3\hat{i}+\hat{j}-\hat{k})\times(-\hat{i}-2\hat{j}+2\hat{k})|$$

= $\frac{1}{2}\left|7\hat{j}+7\hat{k}\right| = \frac{7\sqrt{2}}{2}$ sq. units

Volume of tetrahedron ABCF

$= \frac{1}{3} \times \text{area of base} \times \text{height} = \frac{7}{3}$ cubic units.

65. $\overrightarrow{PF}$ is parallel to $\overrightarrow{AB} \times \overrightarrow{AC}$

$PF = \sqrt{2}$ units

$\overrightarrow{PF} = \sqrt{2}\frac{\left(7\hat{j}+7\hat{k}\right)}{\sqrt{49+49}} = \hat{j}+\hat{k}$

66. $\overrightarrow{PF} = \hat{j}+\hat{k}$

Position vector of $\vec{F} = \left(\hat{j}+\hat{k}\right)+\left(\hat{i}-\hat{j}+3\hat{k}\right) = \hat{i}+4\hat{k}$

Vector equation of AF is

$\vec{r} = \left(2\hat{i}+2\hat{k}\right)+\lambda\left(-\hat{i}+2\hat{k}\right)$

Paragraph – 24

For two vectors $\vec{x}, \vec{y}$, we defined $\vec{x}\cdot\vec{y} = |\vec{x}||\vec{y}|\cos\theta$, $\vec{x}\times\vec{y} = |\vec{x}|\cdot|\vec{y}|\cos\theta n$. Let * is a operation defined by $\vec{x}*\vec{y} = |\vec{x}||\vec{y}|\tan\frac{\theta}{2}$, where θ is angle between $\vec{x}$ and $\vec{y}$.

67. Projection of $\vec{x}$ on $\vec{y}$ will be

a) $\frac{\vec{x}*\vec{y}}{|\vec{y}|}$ b) $\frac{\vec{x}*\vec{y}}{|\vec{x}|}$ c) $|\vec{x}|\left(\frac{|\vec{x}|^2|\vec{y}|^2-\left(\vec{x}*\vec{y}\right)^2}{|\vec{x}|^2|\vec{y}|^2+\left(\vec{x}*\vec{y}\right)^2}\right)$ d) $\left(\frac{|\vec{x}|^2|\vec{y}|^2-\left(\vec{x}*\vec{y}\right)^2}{|\vec{x}|^2|\vec{y}|^2+\left(\vec{x}*\vec{y}\right)^2}\right)$

Ans. c

Sol. Projection of $\bar{x}$ on $\bar{y}$ is $\frac{\bar{x}\cdot\bar{y}}{|\bar{y}|} = |\bar{x}|\cos\theta = |\bar{x}|\left(\frac{1-\tan^2\theta/2}{1+\tan^2\theta/2}\right)$

But $\bar{x}*\bar{y} = |\bar{x}|\cdot|\bar{y}|\tan\frac{\theta}{2}$, so projection of $\bar{x}$ on $\bar{y}$ is $|\bar{x}|\left(\frac{|\bar{x}|^2|\bar{y}|^2-\left(\bar{x}*\bar{y}\right)^2}{|\bar{x}|^2|\bar{y}|^2+\left(\bar{x}*\bar{y}\right)^2}\right)$

68. If $\vec{x}$ and $\vec{y}$ represent the adjacent sides of a parallelogram, then its area is given by

a) $|\vec{x}*\vec{y}|$ b) $\frac{2\left(\vec{x}*\vec{y}\right)|\vec{x}|^2|\vec{y}|^2}{|\vec{x}*\vec{y}|^2+|\vec{x}|^2|\vec{y}|^2}$ c) $\frac{\vec{x}*\vec{y}}{1+\left(\vec{x}*\vec{y}\right)^2}$ d) none of these

Ans. b

Sol. Area of parallelogram = $|\vec{x}||\vec{y}|\sin\theta = |\vec{x}||\vec{y}|\left(\frac{2\tan\theta/2}{1+\tan^2\theta/2}\right)$

69. If $\vec{x}$ and $\vec{y}$ two non-zero linearly independent vectors vectors such that $|\vec{x}\times\vec{y}| = |\vec{x}*\vec{y}|$, then

a) $\vec{x}$ and $\vec{y}$ are parallel b) $\vec{x}$ and $\vec{y}$ are perpendicular

c) angle between $\vec{x}$ and $\vec{y}$ is $\frac{\pi}{4}$ d) none of these

Ans. b

Sol. $\left|\vec{x}\times\vec{y}\right| = \left|\vec{x} * \vec{y}\right|$

$\Rightarrow \left|\vec{x}\right|\left|\vec{y}\right|\sin\theta = \left|\vec{x}\right|\left|\vec{y}\right|\tan\frac{\theta}{2} \Rightarrow \theta = \frac{\pi}{2}$

Vectors

Integer Answer Type

1. If $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar vectors and

$$\left[\left(\vec{a}+\vec{b}\right)\times(\vec{b}-\vec{c})\ \ \left(\vec{b}+\vec{c}\right)\times\left(\vec{c}+\vec{a}\right)\ \ \left(\vec{c}-\vec{a}\right)\times\left(\vec{a}+\vec{b}\right)\right]=K\left[\vec{a}\,\vec{b}\,\vec{c}\right]^2$$ then value of K is ?

Key. 4

Sol. $[(\vec{a}+\vec{b})\times(\vec{b}-\vec{c})\ \ (\vec{b}+\vec{c})\times(\vec{c}+\vec{a})\ \ (\vec{c}-\vec{a})\times(\vec{a}+\vec{b})]$

$=[\vec{a}\times\vec{b}-\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\quad -\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\quad -\vec{a}\times\vec{b}-\vec{b}\times\vec{c}+\vec{c}\times\vec{a}]$

$$=[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}]\begin{vmatrix}1 & -1 & 1\\ -1 & 1 & 1\\ -1 & -1 & 1\end{vmatrix}$$

$=4[\vec{a}\,\vec{b}\,\vec{c}]^2$

2. OABC is regular tetrahedron of unit edge length with volume V then $12\sqrt{2}V=$

Key. 2

Sol. $$\left[\bar{a}\ \ \bar{b}\ \ \bar{c}\right]^2=\begin{vmatrix}\bar{a}\cdot\bar{a} & \bar{a}\cdot\bar{b} & \bar{a}\cdot\bar{c}\\ \bar{b}\cdot\bar{a} & \bar{b}\cdot\bar{b} & \bar{b}\cdot\bar{c}\\ \bar{c}\cdot\bar{a} & \bar{c}\cdot\bar{b} & \bar{c}\cdot\bar{c}\end{vmatrix}=\begin{vmatrix}1 & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & 1 & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2} & 1\end{vmatrix}=\frac{1}{2}$$

$\Rightarrow\left[\bar{a}\ \bar{b}\ \bar{c}\right]=\frac{1}{\sqrt{2}}$ volume $=\frac{1}{6}\left[\bar{a}\ \bar{b}\ \bar{c}\right]=\frac{1}{6\sqrt{2}}$

$12\sqrt{2}V=2$

3. Two points P and Q are given in the rectangular cartesian co-ordinate system on the curve $y=2^{x+2}$, such that $\overrightarrow{OP}.\hat{i}=-1$ and $\overrightarrow{OQ}.\hat{i}=2$. The magnitude of the vector $\overrightarrow{OQ}-4\overrightarrow{OP}$ is $10l$ where l =__ (where O is origin)

Key. 1

Sol. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ then $y_1=2^{x_1+2}$ and $y_2=2^{x_2+2}$ and $\overrightarrow{OP}.\hat{i}=-1$

$\Rightarrow\ (x_1\hat{i}+y_1\hat{i}).\hat{i}=-1 \Rightarrow\ x_1=-1$

and correspondingly $y_1=2^{-1+2}$, ie. $y_1=2$.

4. ABC is any triangle and O is any point in the plane of the same. If AO, BO and CO meet the sides BC, CA and AB in D,E,F respectively, then $\frac{OD}{AD}+\frac{OE}{BE}+\frac{OF}{CF}$ =____.

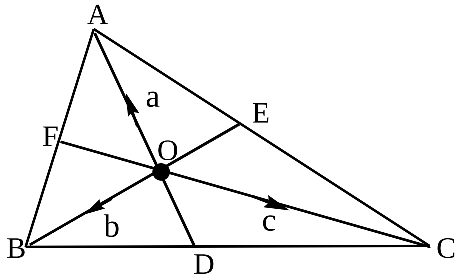
Key. 1

Sol. $\overrightarrow{OD}=x\,\overrightarrow{OA}\Rightarrow\ \vec{r}=-x\vec{a}$

$\because \bar{a},\bar{b},\bar{c}$ are coplanor

$l\vec{x}+m\vec{b}+n\vec{c}=0$

$\because \vec{r}, \vec{b}, \vec{c}$ are collinear $\frac{-l}{x} + m + x = 0 \Rightarrow x = \frac{l}{m+n}$



$\overrightarrow{AD} = -x\vec{a} - \vec{a} = -(x+1)\vec{a}$

$\therefore \frac{OD}{AD} = \frac{+x}{x+1} = \frac{l}{l+m+n}$. Similary $\frac{OE}{BE} = \frac{m}{l+m+n}, \frac{OF}{CF} = \frac{n}{l+m+n}$

$\therefore \frac{OD}{AD} + \frac{OE}{BE} + \frac{OC}{CA} = 1$

5. The vectors $\bar{a}, \bar{b}$ & $\bar{c}$ each two of which are non–collinear. If $\bar{a}+\bar{b}$ is collinear with $\bar{c}, \bar{b}+\bar{c}$ is collinear with $\bar{a}$ & $|\bar{a}| = |\bar{b}| = |\bar{c}| = \sqrt{2}$. Then the value of $|\bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{c} + \bar{c}\cdot\bar{a}| =$

Key. 3

Sol. $\bar{a}+\bar{b} = \lambda\bar{c}, \bar{b}+\bar{c} = m\bar{a}$

$\Rightarrow \bar{a}+\bar{b}+\bar{c} = \bar{0}$

$\Rightarrow |\bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{c} + \bar{c}\cdot\bar{a}| = \left| -\frac{\left(|\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2\right)}{2} \right| = 3$

6. The equation of conic section can also be given by two dimensional vectors. The vector equation of conic must be a relation satisfied by position vectors of all the points on the conic. The position vector of a general point may be taken as $\vec{r}$ .The eccentricity of the conic $|\vec{r} - \hat{i} - \hat{j}| + |\vec{r} + \hat{i} + \hat{j}| = 3$ is "e" then $\left[\sqrt{2}e^{-1}\right]$ where [.] denotes greatest integer function

Key. 1

Sol. e = $2\sqrt{2}/3$

7. Find the distance of the point $\hat{i} + 2\hat{j} + 3k$ from the plane $\vec{r}.(\hat{i} + \hat{j} + k) = 5$ measured parallel to the vector $2\hat{i} + 3\hat{j} - 6k$.

Key. 7

Sol. The distance of the point 'a' from the plane $\vec{r}.\vec{n} = q$ measured in the direction of the unit vector b is $= \frac{q - \vec{a}.\vec{n}}{b.\vec{n}}$

Here $\vec{a}=\hat{i}+2\hat{j}+3k, \vec{n}=\hat{i}+\hat{j}+k$ and q = 5

Also $b=\frac{2\hat{i}+3\hat{j}-6k}{\sqrt{(2)^2+(3)^2+(-6)^2}}=\frac{2\hat{i}+3\hat{j}-6k}{7}$

$\therefore$ The required distance

$$=\frac{5-\left(\hat{i}+2\hat{j}+3k\right).\left(\hat{i}+\hat{j}+k\right)}{\frac{1}{7}\left(2\hat{i}+3\hat{j}-6k\right).\left(\hat{i}+\hat{j}+k\right)}$$

$$\frac{5-(1+2+3)}{\frac{1}{7}(2+3-6)}=7$$

8. If $\vec{a},\vec{b},\vec{c}$ be non-coplanar unit vectors equally inclined to one another at an acute angle θ, and if $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}=p\vec{a}+q\vec{b}+r\vec{c}$ then $p-r=$__ $(p,q,r\in R)$

ans: 0.

Sol. taking dot product with $\vec{a}=\left[\bar{a}\bar{b}\bar{c}\right]=p+q\cos\theta+r\cos\theta---(1)$

taking dot product with $\vec{c}=\left[\bar{a}\bar{b}\bar{c}\right]=p\cos\theta+q\cos\theta+r---(2)$

From (1) and (2) $p=r$.

9. Let A be a point on the line $\bar{r}=\left(-3\hat{i}+6j+3k\right)+t\left(2\hat{i}+3j-2k\right)$ and B be a point on the line $\bar{r}=6j+s\left(2\hat{i}+2j-k\right)$. The least value of the distance AB is

ANS : 5

HINT $Let\ A_o=(-3,6,3), B_o=(0,6,0);\bar{c}=(2,3,-2)\&\bar{d}=(2,2,-1)$

Then $AB_{min}=\left|proj\ of\ \overrightarrow{A_oB_o}\ on\ \vec{c}\times\vec{d}\right|=\frac{|(3,0,-3).(1,-2,-2)|}{3}=3$

10. If $\bar{a},\bar{b},\bar{c}$ are unit vectors such that $\bar{a}$ is perpendicular to plane of $\bar{b}$ and $\bar{c}$ and the angle between $\bar{b}\&\bar{c}$ is $\frac{\pi}{3}$ the $\left|\bar{a}+\bar{b}+\bar{c}\right|$ is

KEY : 2

SOL : $|\bar{a}|=|\bar{b}|=|\bar{c}|=1\&\bar{a}.\bar{b}=0\&\bar{a}.\bar{c}=0$

$\bar{b}.\bar{c}=|\bar{b}||\bar{c}|\cos\frac{\pi}{3}=\frac{1}{2}.$

$\therefore\left|\bar{a}+\bar{b}+\bar{c}\right|^2=3+2.0+2.0+1=4$

$\therefore\left|\bar{a}+\bar{b}+\bar{c}\right|=2$

11. Find the distance of the point $\hat{i}+2\hat{j}+3k$ from the plane $\vec{r}.\left(\hat{i}+\hat{j}+k\right)=5$ measured parallel to the vector $2\hat{i}+3\hat{j}-6k$.

Key. 7

Sol. The distance of the point 'a' from the plane $\vec{r}.\vec{n} = q$ measured in the direction of the unit vector b is $= \dfrac{q - \vec{a}.\vec{n}}{b.\vec{n}}$

Here $\vec{a} = \hat{i} + 2\hat{j} + 3k, \vec{n} = \hat{i} + \hat{j} + k$ and q = 5

Also $b = \dfrac{2\hat{i} + 3\hat{j} - 6k}{\sqrt{(2)^2 + (3)^2 + (-6)^2}} = \dfrac{2\hat{i} + 3\hat{j} - 6k}{7}$

$\therefore$ The required distance

$$= \frac{5 - (\hat{i} + 2\hat{j} + 3k).(\hat{i} + \hat{j} + k)}{\frac{1}{7}(2\hat{i} + 3\hat{j} - 6k).(\hat{i} + \hat{j} + k)}$$

$$\frac{5 - (1 + 2 + 3)}{\frac{1}{7}(2 + 3 - 6)} = 7$$

12. The projection length of a variable vector $x\hat{i} + y\hat{j} + z\hat{k}$ on the vector $\vec{p} = \hat{i} + 2\hat{j} + 3\hat{k}$ is 6. Let ℓ be the minimum projection length of the vector $x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ on the vector $\vec{p}$, then the value of $\sqrt[3]{l^2 + 15^2}$ is

Key. 9

Sol. Projection length = $|\vec{a}.\vec{p}|$

So, $\dfrac{|x + 2y + 3z|}{\sqrt{14}} = 6$

$\Rightarrow |x + 2y + 3z| = 6\sqrt{14}$

$\Rightarrow |(x\hat{i} + \sqrt{2}y\hat{j} + \sqrt{3}z\hat{k}).(\hat{i} + \sqrt{2}\hat{j} + \sqrt{3}\hat{k})| = 6\sqrt{14}$

$\Rightarrow (x^2 + 2y^2 + 3z^2)(1 + 2 + 3)\cos^2\theta = (6\sqrt{14})^2$

$\Rightarrow \dfrac{x^2 + 2y^2 + 3z^2}{\sqrt{14}} \geq 6\sqrt{14} \Rightarrow l = 6\sqrt{14}$

So, $(l^2 + 15^2)^{1/3} = (504 + 225)^{1/3} = (729)^{1/3} = 9$.

13. Non-zero vectors $\vec{a}, \vec{b}, \vec{c}$ satisfy $\vec{a}.\vec{b} = 0$, $(\vec{b} - \vec{a}).(\vec{b} + \vec{c}) = 0$ and $2|\vec{b} + \vec{c}| = |\vec{b} - \vec{a}|$. If $\vec{a} = \mu\vec{b} + 4\vec{c}$ then the value of μ is

Key. 0

Sol. $\vec{c} = \dfrac{\vec{a} - \mu\vec{b}}{4}$ and $\vec{a}.\vec{b} = 0$

Now, $(\vec{b} - \vec{a}).(\vec{b} + \vec{c}) = 0 \Rightarrow (\vec{b} - \vec{a}).\left(\vec{b} + \dfrac{\vec{a} - \mu\vec{b}}{4}\right) = 0$

$\Rightarrow (4 - \mu)b^2 = a^2$ ($\therefore$ $\mu < 4$) ... (i)

Again $4|\vec{b} + \vec{c}|^2 = |\vec{b} - \vec{a}|^2 \Rightarrow 4\left|\dfrac{(4 - \mu)\vec{b} + \vec{a}}{4}\right|^2 = |\vec{b} - \vec{a}|^2$

$\Rightarrow 4\left(\dfrac{4 - \mu}{4}\right)^2 b^2 + \dfrac{a^2}{4} = b^2 + a^2 \Rightarrow ((4 - \mu)^2 - 4)b^2 = 3a^2$ ... (ii)

(i) & (ii) we get $\frac{(4-\mu)^2-4}{4-\mu} = \frac{3}{1} \Rightarrow \mu^2 - 5\mu = 0$

$\Rightarrow \mu = 0$ or 5 but as $\mu < 4$, so, $\mu = 0$.

14. Angle θ is made by line of intersection of planes $\vec{r}\cdot\left(\hat{i}+2j+3k\right)=0$ and $\vec{r}\cdot\left(3\hat{i}+3j+k\right)=0$ with j, where $\cos\theta=\sqrt{\frac{\lambda}{3}}$, then λ is

Ans. 2

Sol. Conceptual

Vectors

Matrix-Match Type

1. Column I — Column II

(A) The area of the triangle whose vertices are the points, with ractangular cartesian coordinates (1, 2, 3), (-2, 1, 4), (3, 4, -2) is — (P) 0

(B) The value of $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})+(\vec{b}\times\vec{c})\ .(\vec{a}\times\vec{d})+(\vec{c}\times\vec{a}).(\vec{b}\times\vec{d})$ is — (Q) 1

(C) A square PQRS is side length P is folded along the diagonal PR so that planes PRQ and PRS are perpendicular to one another, the shortest distance between PQ and RS is , $\frac{P}{k\sqrt{2}}$ then k = — (R) $\frac{\sqrt{1218}}{2}$

(D) $\vec{a}=2\hat{i}+3\hat{j}-\hat{k},\vec{b}=-\hat{i}+2\hat{j}-4\hat{k}\ ,\vec{c}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{d}=3\hat{i}+2\hat{j}+\hat{k}$ then $(\vec{a}\times\vec{b}).(\vec{c}\times\vec{d})$ = — (S) 21

Key. (A - R) , (B- P) , (C-Q), (D-S)

Sol. (A) $\overrightarrow{OA}=\hat{i}+2\hat{j}+3\hat{k},\overrightarrow{OB}=-2\hat{i}+\hat{j}-4\hat{k},\overrightarrow{OC}=3\hat{i}+4\hat{j}-2\hat{k}$

area = $\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|=\frac{\sqrt{1218}}{2}$

(B) $((\vec{a}\times\vec{b})\times\vec{c})+((\vec{b}\times\vec{c})\times\vec{a}).\vec{d}+((\vec{c}+\vec{a})\times\vec{b}).\vec{d}=0$

(C) Taking P as origin position vector of Q' R and S are $P\hat{i}+P\hat{j},\ P\hat{k}$ equations of PQ' and RS $\vec{r}=tP\hat{i}$ are, $\vec{r}=P\hat{i}+P\hat{j}$ $\vec{r}=P\hat{i}+P\hat{j}+\lambda(P\hat{i}+P\hat{j}-P\hat{k})$ shortest distance = $\frac{P}{\sqrt{2}}$.

(D) $(\vec{a}.\vec{c})(\vec{b}.\vec{d})-(\vec{b}.\vec{c})(\vec{a}.\vec{d})=21$

2. Match the following.

Column - I — Column - II

(A) $\vec{a},\vec{b},\vec{c}$ are any three vectors then $\vec{a}\cdot\left(\left(\vec{b}+\vec{c}\right)\times\left(\vec{a}+\vec{b}+\vec{c}\right)\right)$ is equal to — (p) 2

(B) If $\vec{a}=\hat{i}+\hat{j}+k,\vec{b}=4\hat{i}-3\hat{j}+4k,\quad \vec{c}=\hat{i}+\alpha\hat{j}+\beta k$ are linearly dependent and $\left|\vec{c}\right|=\sqrt{3}$ then $\alpha+\beta$ is equal to — (q) -1

(C) If $\vec{a}=\hat{i}+\hat{j}+k,\quad \vec{a}\cdot\vec{b}=1$ and $\vec{a}\times\vec{b}=\hat{j}-k$ and $\hat{b}=\alpha\hat{i}+b\hat{j}+\gamma k$ then $\alpha-\beta$ is equal to — (r) 0

Key. (A) $\rightarrow$ (r), (B) $\rightarrow$ (p,r), (C) $\rightarrow$ (q)

Sol. (A)$\rightarrow$(r), (B)$\rightarrow$(r,s), (C)$\rightarrow$(q)

(B) $\alpha=\pm1,\beta=1$

(C) $\beta-\gamma=0, \gamma-\alpha=1, \alpha-\beta=-1$

3. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at angle α to each other, then

Column I		Column II	
(A)	$\vert\vec{a}+\vec{b}\vert<1$ if	(P)	$\frac{2\pi}{3}<\alpha\le\pi$
(B)	$\vert\vec{a}-\vec{b}\vert=\vert\vec{a}+\vec{b}\vert$ if	(Q)	$\frac{\pi}{2}<\alpha\le\pi$
(C)	$\vert\vec{a}+\vec{b}\vert<\sqrt{2}$ if	(R)	$\alpha=\frac{\pi}{2}$
(D)	$\vert\vec{a}-\vec{b}\vert<\sqrt{2}$ if	(S)	$0\le\alpha<\frac{\pi}{2}$

Key: A – P, B – R, C- Q, P D – S

Hint: A – P, B – R, C- P,Q, D – S

If $|\vec{a}+\vec{b}|<1$ then $(\vec{a}+\vec{b}).(\vec{a}+\vec{b})<1$

So $|\vec{a}|^2+|\vec{b}|^2+2\vec{a}.\vec{b}<1 \Rightarrow \vec{a}.\vec{b}<-\frac{1}{2}$

$\Rightarrow \cos\alpha<-\frac{1}{2} \Rightarrow \frac{2\pi}{3}<\alpha<\pi$

If $|\vec{a}-\vec{b}|=|\vec{a}+\vec{b}|$ then $\vec{a}.\vec{b}=0 \Rightarrow \alpha=\frac{\pi}{2}$

If $|\vec{a}+\vec{b}|<\sqrt{2}$ then cos α < 0 which is true if $\frac{\pi}{2}<\alpha\le\pi$

If $|\vec{a}-\vec{b}|<\sqrt{2}$ then cos α > 0 which is true if $0\le\alpha<\pi$.

4. Match the following

	Column – I		Column – II
A)	If $\bar{a}=x\bar{i}+(x-1)\bar{j}+\bar{k}$ and $\bar{b}=(x+1)\bar{i}+\bar{j}+a\bar{k}$ always make an acute angle with each other for all $x\in R$, then number of non positive integral values of 'a' is	p)	1
B)	Let $\bar{a},\bar{b},\bar{c}$ be unit vectors such that $\bar{a}+\bar{b}+\bar{c}=\bar{x}$, $\bar{a}.\bar{x}=1$, $\bar{b}.\bar{x}=\frac{3}{2}$, $\lvert\bar{x}\rvert=2$ and 'θ' is angle between $\bar{c}$ and $\bar{x}$ then $[2\cos\theta+2]$ is ($[\cdot]$ denotes G.I.F).	q)	0
C)	If $\bar{a}=\bar{i}+\bar{j}+\bar{k}$, $\bar{b}=4\bar{i}+3\bar{j}+4\bar{k}$, $\bar{c}=\bar{i}+p\bar{j}+q\bar{k}$ are linearly dependent and $\bar{c}=\sqrt{3}$ then $p^2-q^2=$	r)	2

D)	If $\bar{a},\bar{b},\bar{c}$ are non coplanar and $\bar{a}+\bar{b}+\bar{c}=\alpha\bar{d}$, $\bar{b}+\bar{c}+\bar{d}=\beta\bar{a}$ then $\left\|\bar{a}+\bar{b}+\bar{c}+\bar{d}\right\|=$	s)	3

Key. A-Q, B-S, C-Q, D-Q

Sol. A) $\bar{a}.\bar{b}>0\Rightarrow x^2+2x+a-1>0$

$\Rightarrow\Delta<0\Rightarrow a>2$

B) $\left(\bar{a}+\bar{b}+\bar{c}\right).\bar{x}=\bar{x}.\bar{x}\Rightarrow 1+\frac{3}{2}+\bar{c}.\bar{x}=4$

$\Rightarrow\bar{c}.\bar{x}=\frac{3}{2}$

$\Rightarrow\left|\bar{c}\right|\left|\bar{x}\right|\cos\theta=\frac{3}{2}\Rightarrow\cos\theta=\frac{3}{4}\Rightarrow\left[2\cos\theta+2\right]=2$

C) $\begin{vmatrix}1 & 1 & 1\\4 & 3 & 4\\1 & p & q\end{vmatrix}=0\Rightarrow q=1 \quad \left|\bar{c}\right|=\sqrt{3}\Rightarrow p^2=1$

Hence $p^2-q^2=0$

5. Observe the following columns:

Column – I	**Column – II**
(A) If $\vec{a}+\vec{b}+\vec{c}=\alpha\vec{d},\vec{b}+\vec{c}+\vec{d}=\beta\vec{a}$ and $\vec{a},\vec{b},\vec{c}$ are non- coplanar, then the $\|\vec{a}+\vec{b}+\vec{c}+\vec{d}\|$ is	p. $\frac{2\pi}{3}$
(B) If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle θ to each other and $\|\vec{a}+\vec{b}\|$ <1, then θ can be equal to	q. $\frac{3\pi}{4}$
(C) If $\vec{a}$ is unit vector perpendicular to another unit vector $\vec{b}$, then $\|\vec{a}\times[\vec{a}\times\{\vec{a}\times(\vec{a}\times\vec{b})\}]\|$ is equal to	r. $\frac{5\pi}{6}$
(D) Let $\vec{a},\vec{b},\vec{c}$ be three unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then the angle between $\vec{a}$ and $\vec{b}$ is equal to	s. 0
	t. 1

Key. A – s; B – q, r;
C – t; D – p

Sol. (A) $\vec{a}+\vec{b}+\vec{c}+\vec{d}=(\alpha+1)\vec{d}=(\beta+1)\vec{a}$

If $\alpha\neq-1$, then $\vec{d}=\left(\frac{\beta+1}{\alpha+1}\right)\vec{a}$

$\Rightarrow \vec{a}+\vec{b}+\vec{c}=\alpha\vec{d}=\alpha\left(\frac{\beta+1}{\alpha+1}\right)\vec{a}$

$\Rightarrow \left\{1-\alpha\left(\frac{\beta+1}{\alpha+1}\right)\right\}\vec{a}+\vec{b}+\vec{c}=0 \Rightarrow \vec{a},\vec{b},\vec{c}$

are coplanar, which is against the given conditions,

so $\alpha=-1$ and hence $\vec{a}+\vec{b}+\vec{c}+\vec{d}=\vec{0}$

(B) $|\vec{a}+\vec{b}|<1 \Rightarrow |\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta<1$

$\Rightarrow \cos\theta<-\frac{1}{2}$

So, $\frac{2\pi}{3}<\theta<\pi$

(C) $\vec{a}\times\left(\vec{a}\times\vec{b}\right)=\left(\vec{a}.\vec{b}\right)\vec{a}-\left(\vec{a}.\vec{a}\right)\vec{b}=-\vec{b}$

$a\times\left\{\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right\}=\vec{a}\times-\vec{b}=-\vec{a}\times\vec{b}$

$a\times[\vec{a}\times\{\vec{a}\times(\vec{a}\times\vec{b})\}=\vec{a}\times(-\vec{a}\times\vec{b})$

$=(\vec{a}.\vec{a})\vec{b}-\left(\vec{a}.\vec{b}\right)\vec{a}=\vec{b}$

(D) $\vec{a}+\vec{b}=-\vec{c} \Rightarrow |\vec{a}+\vec{b}|^2=|\vec{c}|^2=1$

$\Rightarrow \vec{a}.\vec{b}=-\frac{1}{2} \Rightarrow \theta=\frac{2\pi}{3}$

6. Match the following

Column – I	Column – II
A) $\vec{a}$ and $\vec{b}$ are unit vectors and $\vec{a}+2\vec{b}$ is $\perp$ to $5\vec{a}-4\vec{b}$, then $2\left(\vec{a}.\vec{b}\right)$ is equal to	p) 0
B) The points $(1,0,3),(-1,3,4),(1,2,1)$ and $(k,2,5)$ are coplanar when k is equal to	q) 1
C) The vectors $(1,1,m),(1,1,m+1)$ and $(1,-1,m)$ are coplanar then the number of values of m	r) 1
D) $\vec{a}\times\left(\vec{b}\times\vec{c}\right)+\vec{b}\times\left(\vec{c}\times\vec{a}\right)+\vec{c}\left(\vec{a}\times\vec{b}\right)$ is equal to	s) 2

Ans. A – R ; B – Q ; C – P ; D – P

Sol. A) $\left(\vec{a}+2\vec{b}\right)\cdot\left(5\vec{a}-4\vec{b}\right)=0$

$\Rightarrow \vec{a}\cdot\vec{b}=\frac{1}{2}$ *or* $2\left(\vec{a}\cdot\vec{b}\right)=1$

B) $\vec{a}=\hat{i}+3k,\ \vec{b}=-\hat{i}+3j+4k,\ \vec{c}=\hat{i}+2j+k,\ \vec{d}=\hat{i}+2j+5k$ are coplanar

$\left[\vec{d}\quad\vec{b}\quad\vec{c}\right]+\left[\vec{d}\quad\vec{c}\quad\vec{a}\right]+\left[\vec{d}\quad\vec{a}\quad\vec{b}\right]=\left[\vec{a}\quad\vec{b}\quad\vec{c}\right]$

$\Rightarrow k=-1$

C) For no value of m the vectors are coplanar.

D) $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=\left(\vec{a}\cdot\vec{c}\right)\vec{b}-\left(\vec{a}\cdot\vec{b}\right)\vec{c}$

$\vec{b}\times\left(\vec{c}\times\vec{a}\right)=\left(\vec{b}\cdot\vec{a}\right)\vec{c}-\left(\vec{c}\cdot\vec{b}\right)\vec{a}$

$\vec{c}\times\left(\vec{a}\times\vec{b}\right)=\left(\vec{c}\cdot\vec{b}\right)\vec{a}-\left(\vec{c}\cdot\vec{a}\right)\vec{b}$

Sum is zero.

7. Match the following

Column – I	Column – II
A) Let $\vec{a}$ and $\vec{b}$ unit vectors such that $\left\|\vec{a}+\vec{b}\right\|=\sqrt{3}$, then the value of $\left(2\vec{a}+5\vec{b}\right)\cdot\left(3\vec{a}+\vec{b}+\vec{a}\times\vec{b}\right)$ is equal to	P) $\sqrt{2}$
B) Let P be any arbitrary point on the circum circle of an equilateral triangle of side length 2. Then $\left\|\overline{PA}\right\|^2+\left\|\overline{PB}\right\|^2+\left\|\overline{PC}\right\|^2$ is equal to	Q) 5
C) Let $\vec{a}=3, \vec{a}\times\vec{c}=\vec{b}$ and $\vec{c}=\frac{1}{3}\left(p\hat{i}+qj+tk\right)$ then (p + q – t) is equal to	R) 8
D) Let $\vec{a},\vec{b},\vec{c}$ be three unit vectors such that $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}=0$. If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{4}$, then $\vec{a}=\lambda\left(\vec{b}\times\vec{c}\right)$, where λ is equal to	S) $\frac{39}{2}$
	T) $-\sqrt{2}$

Ans. A – S ; B – R ; C – Q ; D – P, T

Sol. A) $\left(2\vec{a}+5\vec{b}\right)\cdot\left(3\vec{a}+\vec{b}+\vec{a}\times\vec{b}\right)=6\vec{a}\cdot\vec{a}+17\vec{a}\cdot\vec{b}+5\vec{b}\cdot\vec{b}=11+17\vec{a}\cdot\vec{b}$

$\left|\vec{a}+\vec{b}\right|=\sqrt{3}\Rightarrow\left|\vec{a}+\vec{b}\right|=3\Rightarrow\vec{a}\cdot\vec{b}=\frac{1}{2}$

B) $\left|\vec{p}\right|=\left|\vec{b}\right|=\left|\vec{a}\right|=\left|\vec{c}\right|=\frac{2}{\sqrt{3}},\left|\overrightarrow{PA}\right|^2=\left|\vec{a}-\vec{p}\right|^2=\left|\vec{a}\right|^2+\left|\vec{p}\right|^2-2\vec{p}\cdot\vec{a}$

$\left|\overrightarrow{PB}\right|=\left|\vec{b}\right|^2+\left|\vec{p}\right|^2-2\vec{p}\cdot\vec{b}, \left|\overrightarrow{PC}\right|=\left|\vec{c}\right|^2+\left|\vec{p}\right|^2-2\vec{p}\cdot\vec{c}$

$\Rightarrow\sum\left|\overrightarrow{PA}\right|^2=6\cdot\frac{4}{3}-2\vec{p}\cdot\left(\vec{a}+\vec{b}+\vec{c}\right)=8$ as $\frac{\vec{a}+\vec{b}+\vec{c}}{3}=0$

C) $\vec{a}\times\vec{c}=b\Rightarrow\vec{a}\times\left(\vec{a}\times\vec{c}\right)=\vec{a}\times\vec{b}\Rightarrow\left(\vec{a}\cdot\vec{c}\right)\vec{a}-\left(\vec{a}\cdot\vec{a}\right)\vec{c}=\vec{a}\times\vec{b}$

$\vec{a}\cdot\vec{a}=3, \vec{a}\times\vec{b}=-2\hat{i}+j+k\Rightarrow3\vec{a}-3\vec{c}=\vec{a}\times\vec{b}=-2\hat{i}+j+k$

$3\vec{c}=5\hat{i}+2j+2k$

$\therefore\vec{c}=\frac{1}{3}\left(5\hat{i}+2j+2k\right)$

D) $\vec{a}=\lambda\left(\vec{b}\times\vec{c}\right)\Rightarrow\left|\vec{a}\right|=\left|\lambda\right|\left|\vec{b}\times\vec{c}\right|=\left|\lambda\right|\left|\vec{b}\right|\left|\vec{c}\right|\sin\frac{\pi}{4}\Rightarrow\left|\lambda\right|=\sqrt{2}$

$\Rightarrow\lambda=\pm\sqrt{2}$