Trigonometry

Single Correct Answer Type

1. $\sec h^{-1}(\sin\theta)=$

1) $\log\tan\dfrac{\theta}{2}$ 2) $\log\sin\dfrac{\theta}{2}$ 3) $\log\cos\dfrac{\theta}{2}$ 4) $\log\cot\dfrac{\theta}{2}$

Key. 4

Sol. $\log_e\left[\dfrac{1+\sqrt{\cos^2\theta}}{\sin\theta}\right]$

$=\log_e\cot\theta/2$

2. The value of the expression $\sec h^2\left(\operatorname{Tan} h^{-1}(1/2)\right)+\operatorname{cosech}^2\left(\cot h^{-1}3\right)$ is

A) $\dfrac{35}{9}$ B) $\dfrac{43}{4}$ C) $\dfrac{35}{4}$ D) $\dfrac{43}{9}$

Key. 3

Sol. Conceptual

3. If $x=\log\left[\cot\left(\dfrac{\pi}{4}+\theta\right)\right]$ *then* $\sinh x=$

1) $\tan 2\theta$ 2) $\cot 2\theta$ 3) $-\tan 2\theta$ 4) $-\cot 2\theta$

Key. 3

Sol. $x=\log\left[\cot(\pi/4+\theta)\right]$

$=\log\left[\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right]\Rightarrow e^x=\dfrac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$

$\sinh x=\dfrac{e^x-e^{-x}}{2}=\dfrac{1}{2}\left[\dfrac{(\cos\theta-\sin\theta)^2-(\cos\theta+\sin\theta)^2}{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}\right]$

$=\dfrac{1}{2}\left[\dfrac{-4\cos\theta\sin\theta}{\cos^2\theta-\sin^2\theta}\right]=\dfrac{-\sin 2\theta}{\cos 2\theta}=-\tan 2\theta$

4. If $Sinh^{-1}\ 2x=2Cosh^{-1}y, then$

1) $x^2+y^2=x^4$ 2) $x^2+y^2=4$

3) $x^2+y^2=y^4$ 4) $x^2=y^2$

Key. 3

Sol. $\sinh^{-1}2x=2\cosh^{-1}y$

$2x=\sinh\left(2\cosh^{-1}y\right)=2\sinh\left(\cosh^{-1}y\right)\cosh\left(\cosh^{-1}y\right)$

$=2\sinh\left(\sinh^{-1}\left(\sqrt{y^2-1}\times y\right)\right)$

$2x=2y\sqrt{y^2-1}$

$\Rightarrow x^2+y^2=y^4$

5. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60^0. He moves away from the pole along the line BC to a point D such that CD =7 m. From D the angle of elevation of the point A is 45^0. Then the height of the pole is

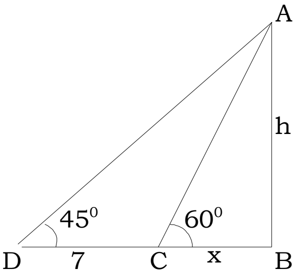
1) $\frac{7\sqrt{3}}{2}\left(\sqrt{3}+1\right)m$ 2) $\frac{7\sqrt{3}}{2}\left(\sqrt{3}-1\right)m$ 3) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}+1}m$ 4) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}-1}m$

Key. 1

Sol. $x = h\cot 60^0 = h/\sqrt{3}$

$x + 7 = h\cot 45^0 \Rightarrow h = h - h/\sqrt{3} = 7$

$\Rightarrow h = \frac{7\sqrt{3}}{\sqrt{3}-1}$



6. The angle of elevation of an object from a point P on the level ground is α. Moving d metres on the ground towards the object, the angle of elevation is found to be β. Then the height (in metres) of the object is

1) d tan α 2) d cot β

3) $\frac{d}{\cot\alpha+\cot\beta}$ 4) $\frac{d}{\cot\alpha-\cot\beta}$

Key. 4

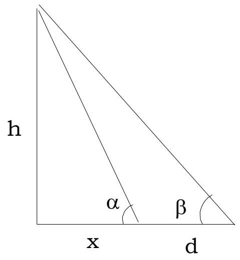
Sol. $\text{Tan}\alpha = \frac{h}{x+d}$

$\Rightarrow x + d = h\cot\alpha$

$\text{Tan}\beta = \frac{h}{x} \Rightarrow x = h\cot\beta$

$x + d - x = h[\cot\alpha - \cot\beta]$

$h = \frac{d}{\cot\alpha - \cot\beta}$



7. The angle of elevation of a cloud from a point h mt above the surface of a lake is θ and the angle of depression of its reflection in the lake is φ. The height of the cloud is

1) $\frac{h\sin(\varphi+\theta)}{\sin(\varphi-\theta)}$ 2) $\frac{h\sin(\varphi-\theta)}{\sin(\varphi+\theta)}$ 3) $\frac{h\sin(\theta+\varphi)}{\sin(\theta-\varphi)}$ 4) $\frac{h\sin(\theta-\varphi)}{\sin(\theta+\varphi)}$

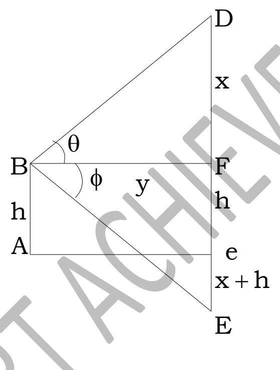
Key. 1

Sol. $\text{Tan}\theta = \frac{x}{y}$

$\text{Tan}\phi = \frac{2h+x}{y}$

$\Rightarrow x = \frac{2h}{\cot\theta.\tan\phi - 1}$

$CD = h + x = \frac{h\sin(\phi+\theta)}{\sin(\phi-\theta)}$



8. If $\tan\left(\frac{\pi}{4}+\frac{y}{2}\right)=\tan^3\left(\frac{\pi}{4}+\frac{x}{2}\right)$, then $\sin x\left(\frac{3+\sin^2 x}{1+3\sin^2 x}\right)$ equals

(A) cos y (B) sin y
(C) sin 2y (D) 0

Key. B

Sol. $\frac{1+\tan\frac{y}{2}}{1-\tan\frac{y}{2}} = \left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)^3$

Square both sides, we get

$\frac{1+\sin y}{1-\sin y} = \frac{(1+\sin x)^3}{(1-\sin x)^3}$

Using componendo and dividendo

$$\frac{2\sin y}{2} = \frac{(3+\sin^2 x)}{1+3\sin^2 x}\sin x$$

9. If $x = \sin\frac{2\pi}{7} + \sin\frac{4\pi}{7} + \sin\frac{8\pi}{7}$ and $y = \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{8\pi}{7}$ then $x^2 + y^2 =$

A. 1 B. 2 C. 3 D. 4

KEY. B

SOL. $x^2 + y^2 = 3 + 2\left(\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}\right) = 2$

10. If $0 < A < B < \pi, \sin A - \sin B = \frac{1}{\sqrt{2}}, \cos A - \cos B = \sqrt{\frac{3}{2}}$ then A+B=

A. $\frac{2\pi}{3}$ B. $\frac{5\pi}{6}$ C. π D. $\frac{4\pi}{3}$

KEY. D

SOL. $(\sin A - \sin B)^2 + (\cos A - \cos B)^2 = 2 \Rightarrow B = A + \frac{\pi}{2}$ *and* $A = \frac{5\pi}{12}$

11. $\cot\frac{7\pi}{6} + 2\cot\frac{3\pi}{8} + \cot\frac{15\pi}{16} =$

A. -4 B. 4 C. 1 D. 0

KEY. A

SOL. $\tan\frac{\pi}{16} - \cot\frac{\pi}{16} + 2\cot\left(\frac{3\pi}{8}\right) = -2\cot\frac{\pi}{8} + 2\tan\frac{\pi}{6} = -4$

12. Tan $\frac{4\pi}{5} - Tan\frac{2\pi}{15} + \sqrt{3}\,Tan\frac{4\pi}{5}\,Tan\frac{2\pi}{15} =$

A. $\sqrt{3}$ B. $\frac{1}{\sqrt{3}}$ C. $-\sqrt{3}$ D. $-\frac{1}{\sqrt{3}}$

KEY. C

SOL. $TanA - TanB - TanATanBTan(A-B) = Tan(A-B)$

13. If $x_1, x_2, x_3, ...x_n$ are in A.P. Whose common difference is α, then the value of

$\sin\alpha[\sec x_1 \sec x_2 + \sec x_2 \sec x_3 + ... + \sec x_{n-1} \sec x_n]$ =

A. $\frac{\sin n\alpha}{\cos x_n \cos x_1}$ B. $\frac{\sin(n-1)\alpha}{\cos x_n \cos x_1}$ C. $\frac{\sin(n+1)\alpha}{\cos x_n \cos x_1}$ D. $\frac{\cos(n-1)\alpha}{\cos x_n \cos x_1}$

KEY. B

SOL. $= \frac{\sin(x_2 - x_1)}{\cos x_1 \cos x_2} + \frac{\sin(x_3 - x_2)}{\cos x_2 \cos x_3} + ... + \frac{\sin(x_n - x_{n-1})}{\cos x_{n-1} \cos x_n}$

$= \tan x_2 - tax_1 + \tan x_3 - \tan x_2 + ... + \tan x_n - \tan x_{n-1}$

$= \tan x_n - \tan x_1 = \frac{\sin(x_n - x_1)}{\cos x_n \cos x_1} = \frac{\sin(n-1)\alpha}{\cos x_n \cos x_1}$

14. If $a\sin^2 x + b\cos^2 x = c, b\sin^2 y + a\cos^2 y = d$ and atanx = btany then $\frac{a^2}{b^2} =$

A. $\frac{(a-d)(c-a)}{(b-c)(d-b)}$ B. $\frac{(a+d)(c+a)}{(b+c)(d+b)}$ C. $\frac{(a-d)(b-a)}{(a-c)(c-b)}$ D. $\frac{(d-a)(c-a)}{(b-c)(d-b)}$

KEY. A

SOL. $a\tan^2 x + b = c(1+\tan^2 x)$

$$\Rightarrow \tan^2 x = \left(\frac{c-b}{a-c}\right), \tan^2 y = \left(\frac{d-a}{b-d}\right)$$

$$\frac{a^2}{b^2} = \frac{\tan^2 y}{\tan^2 x} = \frac{(a-d)(c-a)}{(b-c)(d-b)}$$

15. If $\cos^3 x \sin 2x = \sum_{r=0}^{n} a_r \sin(rx), \forall x \in R$ then

A. $n = 5, a_1 = \frac{1}{2}$ B. $n = 5, a_1 = \frac{1}{4}$ C. $n = 5, a_2 = \frac{1}{8}$ D. $n = 5, a_2 = \frac{1}{4}$

KEY. B

SOL. $\cos^3 x \sin 2x = \cos^2 x . \cos x \sin 2x$

$$= \left(\frac{1-\cos 2x}{2}\right)\left(\frac{2\sin 2x \cos x}{2}\right) = \frac{1}{4}(1-\cos 2x)(\sin 3x + \sin x)$$

$$= \frac{1}{4}[\sin 3x + \sin x - \frac{1}{2}(2\sin 3x \cos 2x) - \frac{1}{2}(2\cos 2x \sin x)]$$

$$= \frac{1}{4}[\sin 3x + \sin x - \frac{1}{2}(\sin 5x + \sin x) - \frac{1}{2}(\sin 3x - \sin x)] = \frac{1}{4}[\sin x + \frac{1}{2}\sin 3x - \frac{1}{2}\sin 5x]$$

$a_1 = \frac{1}{4}; a_3 = \frac{1}{8}; n = 5$

16. If , $\cos\theta = \frac{a\cos\phi + b}{a + b\cos\phi}$ then $\tan\theta/2$ is equal to

A. $\sqrt{\left(\frac{a-b}{a+b}\right)}\tan(\phi/2)$ B. $\sqrt{\left(\frac{a+b}{a-b}\right)}\cos(\phi/2)$ C. $\sqrt{\left(\frac{a-b}{a+b}\right)}\sin(\phi/2)$ D. none of these

Key. A

Sol. $\tan\theta/2 = \sqrt{\left(\frac{1-\cos\theta}{1+\cos\theta}\right)}$

$$= \sqrt{\frac{1-\left(\frac{a\cos\phi+b}{a+b\cos\phi}\right)}{1+\left(\frac{a\cos\phi+b}{a+b\cos\phi}\right)}}$$

$$= \sqrt{\frac{(a-b)(1-\cos\phi)}{(a+b)(1+\cos\phi)}}$$

$$= \sqrt{\frac{(a-b)}{(a+b)}}\tan(\phi/2)$$

17. If in a triangle ABC, $\cos 3A + \cos 3B + \cos 3C = 1,$ then one angle must be exactly equal to

A. $\frac{\pi}{3}$ B. $\frac{2\pi}{3}$ C. π D. $\frac{\pi}{6}$

Key. B

Sol. $\therefore \cos 3A + \cos 3B + \cos 3C = 1$

$$\Rightarrow \cos 3A + \cos 3B + \cos 3C - 1 = 0$$

$$\Rightarrow \cos 3A + \cos 3B + \cos 3C + \cos 3\pi = 0$$

$$\Rightarrow 2\cos\left(\frac{3A+3B}{2}\right)\cos\left(\frac{3A-3B}{2}\right) + 2\cos\left(\frac{3\pi+3C}{2}\right)\cos\left(\frac{3\pi-3C}{2}\right) = 0$$

$$\Rightarrow 2\cos\left(\frac{3\pi-3C}{2}\right)\left\{\cos\left(\frac{3A-3B}{2}\right) + \cos\left(\frac{3\pi+3C}{2}\right)\right\} = 0$$

$$\Rightarrow 2\cos\left(\frac{3\pi}{2} - \frac{3C}{2}\right)2\cos\left(\frac{3\pi+3C+3A-3B}{4}\right).\cos\left(\frac{3\pi+3C-3A+3B}{4}\right) = 0$$

$$\Rightarrow 2\cos\left(\frac{3\pi}{2} - \frac{3C}{2}\right)2\cos\left(\frac{3\pi}{2} - \frac{3B}{2}\right).\cos\left(\frac{3\pi}{2} - \frac{3A}{2}\right) = 0$$

$\Rightarrow \quad -4\sin\left(\frac{3A}{2}\right)\sin\left(\frac{3B}{2}\right)\sin\left(\frac{3C}{2}\right)=0$

$\Rightarrow \quad \sin\left(\frac{3A}{2}\right)\sin\left(\frac{3B}{2}\right)\sin\left(\frac{3C}{2}\right)=0$

$\therefore \frac{3A}{2}=\pi$ or $\frac{3B}{2}=\pi$ or $\frac{3C}{2}=\pi$

$\therefore A=\frac{2\pi}{3}$ or $B=\frac{2\pi}{3}$ or $C=\frac{2\pi}{3}$

18. The value of $\sum_{r=0}^{10}\cos^3\frac{\pi r}{3}$ is equal to

(A) $\frac{-9}{2}$ (B) $\frac{-7}{2}$

(C) $\frac{-9}{8}$ (D) $\frac{-1}{8}$

Key. D

Sol. $I=\sum_{r=0}^{10}\frac{1}{4}\left(\cos 3\frac{\pi r}{3}+3\cos\frac{\pi r}{3}\right)$

$=\sum_{r=0}^{10}\frac{1}{4}\left(\cos \pi r+3\cos\frac{\pi r}{3}\right)$

$=\frac{1}{4}(I_1+I_2)$

$\therefore \quad I_1=\sum_{r=0}^{10}\cos\pi r=1-1+1-1+.....-1+1=1$

$I_2=3\sum_{r=0}^{10}\cos\frac{\pi r}{3}=\frac{3\cos\left(\frac{10}{2}\frac{\pi}{3}\right)\sin\frac{11\pi}{3}}{\sin\frac{\pi}{6}}=-\frac{1\times 3}{2}=-\frac{3}{2}$

$\Rightarrow \quad I=\frac{1}{4}\left(1-\frac{3}{2}\right)=-\frac{1}{8}$

19. The number of distinct real roots of the equation $\tan x=mx, m>1$ in the interval $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ is

A) 1 B) 2 C) 3 D) 0

Key. C

Sol. Conceptual

20. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points in the XY-Plane whose co-ordinates satisfy the equation $\cot^2(x+y)+\tan^2(x+y)+y^2+2y-1=0$. The minimum distance between P and Q is

A) $\pi/4$ B) $\pi/2$ C) $3\pi/4$ D) π

Key. B

Sol. $[\cot(x+y)-\tan(x+y)]^2+(y+1)^2=0$

$\therefore \tan^2(x+y)=1$ and $y=-1$

21. If α is the angle which each side of a regular polygon of n sides subtends at its centre then $1+\cos\alpha+\cos 2\alpha+\cos 3\alpha+....+\cos(n-1)\alpha$ is equal to

(a) n (b) 0 (c) 1 (d) n – 1

Key. B

Sol. $\cos\alpha+\cos(\alpha+\beta)+.....+\cos(\alpha+(n-1)\beta)=\dfrac{\sin\frac{nB}{2}}{\sin\frac{B}{2}}\sin\left(\alpha+\dfrac{(n-1)\beta}{2}\right)$

22. If $\lfloor C = 90^0$ in ΔABC, then $\tan^{-1}\left(\dfrac{a}{b+c}\right)+\tan^{-1}\left(\dfrac{b}{c+a}\right)$ is equal to

a) $\dfrac{\pi}{2}$ b) $\dfrac{\pi}{4}$ c) $\dfrac{\pi}{3}$ d) π

Ans. b

Sol. $\tan^{-1}\left(\dfrac{\frac{a}{b+c}+\frac{b}{c+a}}{1-\frac{a}{b+c}\cdot\frac{b}{c+a}}\right)$ as $\dfrac{ab}{(b+c)(c+a)}<1$

But in right angled ΔABC

$c^2 = a^2 + b^2$

$\therefore \tan^{-1}(1)=\dfrac{\pi}{4}$

23. In a ΔABC, $\dfrac{a^2+b^2+c^2}{\Delta}$ is always

a) $\geq 6\sqrt{3}$ b) $\geq 4\sqrt{3}$ c) $\geq 8\sqrt{3}$ d) $\geq 12\sqrt{3}$

Ans. b

Sol. $\dfrac{a^2+b^2+c^2}{\Delta}\geq 4\sqrt{3}$: use the fact that $\Delta \leq \dfrac{(a+b+c)^2}{12\sqrt{3}}$

24. In triangle ABC, the value of the expression $\sum_{r=0}^{n} {}^nC_r\, a^r b^{n-r}\cos\left(rB-(n-r)A\right)$ is equal to

a) C^n b) Zero c) a^n d) b^n

Ans. a

Sol. It is the expansion of $(a\cos B + b\cos A)^n = C^n$

25. Total number of solution of $2^{\cos x} = |\sin x|$ in $[-2\pi, 5\pi]$ is equal to

a) 12 b) 14 c) 16 d) 15

Ans. b

Sol. Drawn the graphs of both. Total intersection points are 14.

26. If $2\sec 2\alpha = \tan\beta + \cot\beta$, then one positive value of $\alpha + \beta$ is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) 0

Ans. b

Sol. $2\sec 2\alpha = \left(\frac{1}{\sin\beta\cos\beta}\right)$

$\Rightarrow 2\alpha = \frac{\pi}{2} - 2\beta \Rightarrow \alpha + \beta = \frac{\pi}{4}$

27. If in a triangle $\frac{s-a}{11} = \frac{s-b}{12} = \frac{s-c}{13}$ and $\lambda \tan^2(A/2) = 455$, then λ must be

a) 1155 b) 1551 c) 5511 d) 1515

Ans. a

Sol. $\frac{s-a}{11} = \frac{s-b}{12} = \frac{s-c}{13} = \frac{s}{36}$ calculate $\tan^2(A/2) = \frac{13}{33}$

$\lambda = 1155$

28. The value of $\sin^3 10^0 + \sin^3 50^0 - \sin^3 70^0$ is equal to

a) $-\frac{3}{2}$ b) $\frac{3}{4}$ c) $-\frac{3}{4}$ d) $-\frac{3}{8}$

Ans. d

Sol. We have $\sin^3 10^0 + \sin^3 50^0 - \sin^3 70^0$

$= \frac{1}{4}\left[\left(3\sin 10^0 - \sin 30^0\right) + \left(3\sin 50^0 - \sin 150^0\right) - \left(3\sin 70^0 - \sin 120^0\right)\right]$

$= \frac{1}{4}\left[3\left(\sin 10^0 + \sin 50^0 - \sin 70^0\right) - \frac{3}{2}\right]$

$= \frac{1}{4}\left[3\left(\sin 10^0 - 2\cos 60^0 \cdot \sin 10^0\right) - \frac{3}{2}\right] = -\frac{3}{8}$

29. If $\tan(\alpha - \beta) = \frac{\sin 2\beta}{3 - \cos 2\beta}$, then

a) $\tan\alpha = 2\tan\beta$ b) $\tan\beta = 2\tan\alpha$ c) $2\tan\alpha = 3\tan\beta$ d) $3\tan\alpha = 2\tan\beta$

Ans. a

Sol. We have $\frac{\sin 2\beta}{3 - \cos 2\beta} = \frac{2\sin\beta \cdot \cos\beta}{2 - 2\cos 2\beta + 1 + \cos 2\beta}$

$= \frac{2\sin\beta \cdot \cos\beta}{4\sin^2\beta + 2\cos^2\beta} = \frac{\tan\beta}{1 + 2\tan^2\beta} = \frac{2\tan\beta - \tan\beta}{1 + 2\tan^2\beta}$

$$= \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \cdot \tan\beta} = \frac{2\tan\beta - \tan\beta}{1 + 2\tan^2\beta}$$

$$\therefore \tan\alpha = 2\tan\beta$$

30. In a triangle ABC, if angle C is obtuse and angles A and B are given by roots of the equation $\tan^2 x + p \tan x + q = 0$, then the value of q is

a) greater than 1 b) less than 1 c) equal to 1 d) 0

Ans. b

Sol. We have A + B = π – C

$$= \tan(A + B) = -\tan C$$

$$= \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B} > 0 \qquad [\because \tan A > 0, \tan B > 0, \tan C < 0]$$

$$= \tan A \cdot \tan B < 1 \Rightarrow q < 1$$

31. If 2sinx – cos2x = 1, then $\cos^2 x + \cos^4 x$ is equal to

a) 1 b) – 1 c) $-\sqrt{5}$ d) $\sqrt{5}$

Ans. a

Sol. Given 2 sinx + $2\sin^2 x$ – 1 = 1

Or, $\sin^2 x$ + sinx – 1 = 0

$$\therefore \sin x = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 + \sqrt{5}}{2}$$

$$\Rightarrow \sin^2 x = \frac{3 - \sqrt{5}}{2} \Rightarrow \cos^2 x = \frac{\sqrt{5} - 1}{2}$$

$$\therefore \cos^2 x(1 + \cos^2 x) = \frac{\sqrt{5} - 1}{2} \times \frac{\sqrt{5} + 1}{2} = 1$$

32. If ABCD is a cyclic quadrilateral such that 13cosA + 12 = 0 and 3tanB – 4 = 0, then the quadratic equation whose roots are tanC and cosD is

a) $15x^2 + 60x - 11 = 0$ b) $60x^2 + 11x - 15 = 0$

c) $11x^2 + 60x - 15 = 0$ d) none of these

Ans. b

Sol. In a cyclic quadrilateral, no angle is greater than 180^0

Here $\cos A = -\frac{12}{13} \Rightarrow \frac{\pi}{2} < A < \pi$ and 0 < C < π/2 (since A + C = 180^0)

$$\therefore \tan A = -\frac{5}{12} \Rightarrow \tan C = \frac{5}{12}$$

Also $\tan B = \frac{4}{3} \Rightarrow 0 < B < \frac{\pi}{2}$ and $\frac{\pi}{2} < D < \pi$ (since B + D = 180^0)

$$\therefore \cos B = \frac{3}{5} \Rightarrow \cos D = -\frac{3}{5}$$

Now, the required equation is

$$x^2 - \left(\frac{5}{12} - \frac{3}{5}\right)x + \left(\frac{5}{12}\right)\left(-\frac{3}{5}\right) = 0$$

$$\Rightarrow 60x^2 + 11x - 15 = 0$$

33. If A, B, C are the angles of a triangle such that $\cot\frac{A}{2} = 3\tan\frac{C}{2}$, then sinA, sinB, sinC are in

a) A.P b) G.P c) H.P d) none of these

Ans. a

Sol. Given $\cot\frac{A}{2}\cdot\cot\frac{C}{2} = 3$

$$\Rightarrow \frac{\cos\frac{A}{2}\cdot\cos\frac{C}{2}}{\sin\frac{A}{2}\cdot\sin\frac{C}{2}} = 3 \Rightarrow \frac{\cos\frac{A-C}{2}}{\cos\frac{A+C}{2}} = 2$$ (using componendo and dividendo)

$$\Rightarrow \frac{2\sin\frac{A+C}{2}\cos\frac{A-C}{2}}{2\sin\frac{A+C}{2}\cdot\cos\frac{A+C}{2}} = 2$$

= 2sinB = sinA + sinC

34. If $\frac{2\tan\alpha}{1+\sec\alpha+\tan\alpha} = \lambda,$ then $\frac{2\tan\alpha/2}{1+\tan\alpha/2}$ is equal to

a) $\frac{1}{\lambda}$ b) λ c) $1-\lambda$ d) $1+\lambda$

Ans. b

Sol. We have $\frac{2\tan\alpha}{1+\sec\alpha+\tan\alpha} = \frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}$

$$= 2\frac{2\tan\alpha/2}{\left(1+\tan^2\alpha/2\right)+\left(1-\tan^2\alpha/2\right)+2\tan\alpha/2} = \frac{2\tan\alpha/2}{1+\tan\alpha/2}$$

35. In ΔABC, if $b^2 + c^2 = 2a^2$, then the value of $\frac{\cot A}{\cot B+\cot C}$ is

a) 1/2 b) 3/2 c) 5/2 d) 5/3

Ans. a

Sol. $$\frac{\cot A}{\cot B+\cot C} = \frac{\frac{R\left(b^2+c^2-a^2\right)}{abc}}{\frac{R\left(a^2+c^2-b^2\right)}{abc}+\frac{R\left(a^2+b^2-c^2\right)}{abc}} = 1/2$$

36. If $0 \le A, B, C \le \pi$ and A + B + C = π, then the minimum value of sin 3A + sin 3B + sin 3C is

a) -2 b) $-\frac{3\sqrt{3}}{2}$ c) 0 d) none of these

Ans. a

Sol. Since A + B + C = π

$\Rightarrow$ all of sin3A, sin3B, sin3C can't be negative

Let us take sin3A = - 1 $\Rightarrow$ A = π/2

$\Rightarrow$ sin3A = - 1, sin3B = - 1 and sin 3C = 0

So minimum value is – 2 1. Let $\theta \in (0, \pi/4)$ and $t_1 = (\tan\theta)^{\tan\theta}$, $t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}$, $t_4 = (\cot\theta)^{\cot\theta}$ then

a) $t_1 > t_2 > t_3 > t_4$ b) $t_4 > t_3 > t_1 > t_2$

c) $t_3 > t_1 > t_2 > t_4$ d) $t_2 > t_3 > t_1 > t_4$

Key. B

Sol. $\theta \in \left(0, \frac{\pi}{4}\right)$

therefore, $\tan\theta < \cot\theta$

since $\tan\theta < 1 \& \cot\theta > 1$

therefore, $(\tan\theta)^{\cot\theta} < 1$ and $(\cot\theta)^{\tan\theta} > 1$

therefore, $t_4 > t_1$

2. If $\theta = \frac{2\pi}{7}$ then the value of $\tan\theta\tan2\theta + \tan2\theta\tan4\theta + \tan4\theta\tan\theta$ is

a) –1 b) 0 c) $\frac{1}{8}$ d) - 7

Key. D

Sol. $7\theta = 2\pi$

$\theta + 2\theta + 4\theta = 2\pi$

$\cos(\theta + 2\theta + 4\theta) = 1$

Expanding and dividing with $\cos\theta\cos2\theta\cos4\theta$ we have

$$\tan\theta\tan2\theta + \tan2\theta\tan4\theta + \tan4\theta\tan\theta = 1 - \frac{1}{\cos\theta\cos2\theta\cos4\theta} = 1 - \frac{1}{\left(\frac{1}{8}\right)} = -7$$

$$\left(\because \cos\theta\cos2\theta\cos4\theta = \frac{\sin8\theta}{8\sin\theta} = \frac{1}{8}\right)$$

3. If $k_1 = \tan27\theta - \tan\theta$ and $k_2 = \frac{\sin\theta}{\cos3\theta} + \frac{\sin3\theta}{\cos9\theta} + \frac{\sin9\theta}{\cos27\theta}$ then

a) $k_1 = 2k_2$ b) $k_1 = k_2$ c) $k_1 = -k_2$ d) $2k_1 = k_2$

Key. A

Sol. $\tan3\theta - \tan\theta = \frac{\sin2\theta}{\cos3\theta\cos\theta} = \frac{2\sin\theta}{\cos3\theta}$ (1)

$\tan9\theta - \tan3\theta = \frac{2\sin3\theta}{\cos9\theta}$ (2)

$\tan27\theta - \tan9\theta = \frac{2\sin9\theta}{\cos27\theta}$ (3)

Adding (1), (2), (3) $k_1 = 2k_2$

4. If $\frac{\cos x}{a} = \frac{\cos(x+\theta)}{b} = \frac{\cos(x+2\theta)}{c} = \frac{\cos(x+3\theta)}{d}$ then $\frac{a+c}{b+d}$ is equal to

a) $\frac{a}{d}$ b) $\frac{c}{b}$

c) $\frac{b}{c}$ d) $\frac{d}{a}$

Key. C

Sol. For each of the ratio be k

$$\frac{a+c}{b+d}=\frac{k\cos x+k\cos(x+2\theta)}{k\cos(x+\theta)+k\cos(x+3\theta)}=\frac{2\cos(x+\theta)\cos\theta}{2\cos(x+2\theta)\cos\theta}$$

$$=\frac{\cos(x+\theta)}{\cos(x+2\theta)}=\frac{k\cos(x+\theta)}{k\cos(x+2\theta)}=\frac{b}{c}$$

$\Rightarrow$ (c) is correct.

5. If $\cos\alpha=\frac{2\cos\beta-1}{2-\cos\beta}(0<\alpha,\beta<\pi)$, then $\frac{\tan\frac{\alpha}{2}}{\tan\frac{\beta}{2}}$ is equal to

a) 1 b) $\sqrt{2}$

c) $\sqrt{3}$ d) $\frac{1}{\sqrt{3}}$

Key. C

Sol. Take $\beta = 120^o$, then

$$\cos\alpha=\frac{2\left(-\frac{1}{2}\right)-1}{2-\left(-\frac{1}{2}\right)}=\frac{2}{5/2}=\frac{4}{5}$$

$$\cos\alpha=-\frac{4}{5}\Rightarrow\tan\alpha=\frac{3}{4}$$

If $\tan\alpha=-\frac{3}{4}$, then we get $\frac{\tan\alpha/2}{\tan\beta/2}=\sqrt{3}$ and $\tan\frac{\alpha}{2}=3$

$$\Rightarrow\frac{3}{4}=\frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}\Rightarrow\tan\frac{\alpha}{2}=3\Rightarrow\frac{\tan\frac{\alpha}{2}}{\tan\frac{\beta}{2}}=\frac{\frac{1}{3}}{\frac{1}{\sqrt{2}}}=\sqrt{3}$$

6. Cot 16^0 Cot 44^0 + Cot44^0 Cot 76^0 – Cot 76^0 Cot 16^0 =

a) 1 b) 2 c) 3 d) 4

Key. C

Sol.

$$\frac{\cos16\cos44}{\sin16.\sin44}-1+\frac{\cos44.\cos76}{\sin44\sin76}-1-\frac{\cos76\cos16}{\sin76.\sin16}-1+3$$

$$=\frac{\cos60}{\sin16.\sin44}+\frac{\cos120}{\sin44\sin76}-\frac{\cos60}{\sin76.\sin16}+3=\frac{1}{2}\left(\frac{\sin76-\sin16}{\sin16.\sin44.\sin76}\right)-\frac{1}{2\sin76.\sin16}+3=3.$$

37. The value of x which satisfies equation $2\tan^{-1}2x=\sin^{-1}\frac{4x}{1+4x^2}$ is

a) $\left[\frac{1}{2},\infty\right)$ b) $\left(-\infty,-\frac{1}{2}\right]$ c) $[-1,\ 1]$ d) $\left[-\frac{1}{2},\ \frac{1}{2}\right]$

Ans. d

Sol. $-\frac{\pi}{2} \le 2\tan^{-1} 2x \le \frac{\pi}{2}$

$\Rightarrow -\frac{1}{2} \le x \le \frac{1}{2}$

38. Number of integral solutions of the equation $3\tan^{-1} x + \cos^{-1}\left(\frac{1-3x^2}{\left(1+x^2\right)^{3/2}}\right) = 0$ is

a) 1 b) 2 c) 0 d) infinite

Ans. b

Sol. Let $\tan^{-1} x = \theta, \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$3\theta + \cos^{-1}(\cos 3\theta) = 0$

$\cos^{-1}(\cos 3\theta) = -3\theta \Rightarrow -\pi \le 3\theta \le 0$

$\Rightarrow -\frac{\pi}{3} \le \theta \le 0$

$\Rightarrow x \in \left[-\sqrt{3}, 0\right]$, so number of integral solutions is 2.

39. In a triangle ABC, with $A = \frac{\pi}{7}, B = \frac{2\pi}{7}, C = \frac{4\pi}{7}$, then $a^2 + b^2 + c^2$ is (R = circumradius of ΔABC)

a) $4R^2$ b) $6R^2$ c) $7R^2$ d) $8R^2$

Ans. c

Sol. $a^2 + b^2 + c^2 = 4R^2\left(\sin^2 A + \sin^2 B + \sin^2 C\right)$

$= 2R^2\left(1 - \cos 2A + 1 - \cos 2B + 1 - \cos 2C\right) = 2R^2\left[3 - \left(\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{8\pi}{7}\right)\right]$

$= 2R^2\left[3 - \left(\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7}\right)\right]$

$= 2R^2\left[3 - \left(2\sin\frac{\pi}{7}\cdot\cos\frac{2\pi}{7} + 2\sin\frac{\pi}{7}\cdot\cos\frac{4\pi}{7} + 2\sin\frac{\pi}{7}\cdot\cos\frac{6\pi}{7}\right)\frac{1}{2\sin\frac{\pi}{7}}\right]$

$= 2R^2\left[3 - \frac{1}{2\sin\frac{\pi}{7}}\left(\sin\frac{3\pi}{7} - \sin\frac{\pi}{7} + \sin\frac{5\pi}{7} - \sin\frac{3\pi}{7} + \sin\frac{7\pi}{7} - \sin\frac{5\pi}{7}\right)\right]$

$= 2R^2\left[3 + \frac{1}{2}\right] = 7R^2$

40. For which value of x, sin(cot^{-1}(x + 1)) = cos(tan^{-1} x)

a) $\frac{1}{2}$ b) 0 c) 1 d) $-\frac{1}{2}$

Ans. d

Sol. $\sin\left(\cot^{-1}(x+1)\right)=\sin\left(\sin^{-1}\left(\frac{1}{\sqrt{x^2+2x+2}}\right)\right)$

$\Rightarrow \sin\left(\cot^{-1}(x+1)\right)=\frac{1}{\sqrt{x^2+2x+2}}$

$\cos\left(\tan^{-1}x\right)=\cos\left(\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right)=\frac{1}{\sqrt{1+x^2}}$

$\Rightarrow \frac{1}{x^2+2x+2}=\frac{1}{1+x^2}$

41. If the equation x^2 + 12 + 3 sin(a + bx) + 6x = 0 has atleast one real solution where $a,b\in[0,\,2\pi]$, then value of cosθ where θ is least positive value of a + bx is

a) π b) 2π c) 0 d) $\frac{\pi}{2}$

Ans. c

Sol. $(x+3)^2+3+3\sin(a+bx)=0$

x = - 3, sin(a + bx) = - 1

$\Rightarrow \sin(a-3b)=-1$

$a-3b=(4n-1)\frac{\pi}{2},\ n\in 1$

n = 1

a – 3b = 3π/2

cos(a – 3b) = 0

42. In any ΔABC, which is not right angled, $\sum\cos A\,\mathrm{cosec}B\,\mathrm{cosec}C$ is

a) constant b) less than 1 c) greater than 2 d) none of these

Ans. a

Sol. $\sum\frac{\cos A}{\sin B\sin C}=\frac{-\sum\cos(B+C)}{\sin B\sin C}=\sum(1-\cot B\cot C)=3-\sum\cot A\cot B=2$

1. Range of $f(x)=\sin^6x+\cos^6x$ is

(A) [0, 1] (B) $[0,\sqrt{2}]$

(C) $\left[\frac{1}{\sqrt{2}},\frac{3}{4}\right]$ (D) $\left[\frac{1}{4},1\right]$

Key. D

Sol. $f(x)=(\sin^2x+\cos^2x)^3-3\sin^2x\cos^2x(\sin^2x+\cos^2x)\ =1-\frac{3}{4}\sin^2 2x$

Range of $\sin^2 2x$ is $[0,1]$

Range of $f(x)$ is $\left[\frac{1}{4},1\right]$.

Note: Certain questions are better done by avoiding derivatives. Derivatives is one of the tools to determine extrema.

20. The maximum value of $4\sin^2 x + 3\cos^2 x + \sin\frac{x}{2} + \cos\frac{x}{2}$ is

a) $4+\sqrt{2}$ b) $3+\sqrt{2}$ c) 9 d) 4

Key. A

Sol. Maximum value of $4\sin^2 x + 3\cos^2 x$ i.e., $\sin^2 x + 3$ is 4 and that of $\sin\frac{x}{2} + \cos\frac{x}{2}$ is $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$, both attained at $x = \pi/2$. Hence the given function has maximum value of $4+\sqrt{2}$

21. If $\sin\theta + \sin 2\theta + \sin 3\theta = \sin\alpha$ and $\cos\theta + \cos 2\theta + \cos 3\theta = \cos\alpha$, then θ is equal to

a) $\frac{\alpha}{2}$ b) α c) 2α d) $\frac{\alpha}{6}$

Key. A

Sol. $\sin\theta + \sin 3\theta + \sin 2\theta = \sin\alpha$

$\Rightarrow \quad 2\sin 2\theta \cos\theta + \sin 2\theta = \sin\alpha$

$\Rightarrow \quad \sin 2\theta(2\cos\theta + 1) = \sin\alpha$ ....(1)

Now $\cos\theta + \cos 3\theta + \cos 2\theta = \cos\alpha$

$\cos 2\theta(2\cos\theta + 1) = \cos\alpha$ ....(2)

From (1) and (2),

$\tan 2\theta = \tan\alpha \Rightarrow 2\theta = \alpha \Rightarrow \theta = \frac{\alpha}{2}$

22. If $\pi < 2\theta < \frac{3\pi}{2}$, then $\sqrt{2+\sqrt{2+2\cos 4\theta}}$ equals to

a) $-2\cos\theta$ b) $-2\sin\theta$ c) $2\cos\theta$ d) $2\sin\theta$

Key. D

Sol. $\sqrt{2+2(1+\cos 4\theta)} = \sqrt{2+2|\cos 2\theta|}$

$= \sqrt{2(1-\cos 2\theta)}$

$= 2|\sin\theta| = 2\sin\theta$ as $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$

23. $\cos^2\alpha + \cos^2(\alpha + 120^0) + \cos^2(\alpha - 120^0)$ is equal to

a) $\frac{3}{2}$ b) 1 c) $\frac{1}{2}$ d) 0

Key. A

Sol. $\cos^2\alpha + \cos^2(\alpha + 120^0) + \cos^2(\alpha - 120^0)$

$= \cos^2\alpha + \left\{\cos(\alpha + 120^0) + \cos(\alpha - 120^0)\right\}^2 - 2\cos(\alpha + 120^0)\cos(\alpha - 120^0)$

$= \cos^2\alpha + \left\{2\cos\alpha\cos 120^0\right\}^2 - 2\left\{\cos^2\alpha - \sin^2 120^0\right\}$

$= \cos^2\alpha + \cos^2\alpha - 2\cos^2\alpha + 2\sin^2 120^0$

$= 2\sin^2 120^0 = 2\times\frac{3}{4} = \frac{3}{2}$

Trigonometry

Multiple Correct Answer Type

1. In ΔPQR , if $3\sin P+4\cos Q=6$ and $4\sin Q+3\cos P=1$ then $\angle R$ can be

A) $\frac{\pi}{6}$ B) $\frac{3\pi}{4}$ C) $\frac{5\pi}{6}$ D) $\frac{\pi}{4}$

Key. A

Sol. Given equations $\Rightarrow 16+9+24\sin\left(P+Q\right)=37$ $\Rightarrow P+Q=\frac{5\pi}{6}$ *or* $\frac{\pi}{6}$

If $P+Q=\frac{\pi}{6}$ then $R=\frac{5\pi}{6}$

If $P<\frac{\pi}{6}, 3\sin P<\frac{1}{2}$ then $3\sin P+4\cos Q<\frac{1}{2}+4<6$

$\therefore P+Q=\frac{\pi}{6}$ is not possible $\therefore R=\frac{\pi}{6}$

2. If $\left(\frac{\sin\theta}{\sin\phi}\right)^2=\frac{\tan\theta}{\tan\phi}=3$ then

A. $\tan\phi=\frac{1}{\sqrt{3}}$ B. $\tan\phi=-\frac{1}{\sqrt{3}}$ C. $\tan\theta=\sqrt{3}$ D. $\tan\theta=-\sqrt{3}$

KEY. A,B,C,D

SOL. $\Rightarrow \frac{\sin\theta}{\sin\phi}\frac{\sin\theta}{\sin\phi}=\frac{\sin\theta}{\sin\phi}\frac{\cos\phi}{\cos\theta}$

$\Rightarrow \frac{\sin\theta}{\sin\phi}=\frac{\cos\phi}{\cos\theta} \Rightarrow \sin 2\theta=\sin 2\phi$

$\Rightarrow \frac{2\tan\theta}{1+\tan^2\theta}=\frac{2\tan\phi}{1+\tan^2\phi} \Rightarrow \frac{6\tan\phi}{1+9\tan^2\phi}=\frac{2\tan\phi}{1+\tan^2\phi} \Rightarrow \tan^2\phi=\frac{1}{3}, \tan\theta=\pm\sqrt{3}$

3. For $\alpha=\frac{\pi}{7}$ which of the following hold (s) good?

A. $\tan\alpha\tan 2\alpha\tan 3\alpha=\tan 3\alpha-\tan 2\alpha-\tan\alpha$

B. $\cos ec\alpha=\cos ec2\alpha+\cos ec4\alpha$

C. $\cos\alpha-\cos 2\alpha+\cos 3\alpha=\frac{1}{2}$

D. $8\cos\alpha\cos 2\alpha\cos 4\alpha=1$

KEY. A,B,C

SOL. (A) $3\alpha=2\alpha+\alpha$

$\tan 3\alpha=\tan(2\alpha+\alpha)$

$\tan 3\alpha-\tan 2\alpha-\tan\alpha=\tan\alpha\tan 2\alpha\tan 3\alpha$

(B) $RHS = \frac{\sin 4\alpha + \sin 2\alpha}{\sin 2\alpha \sin 4\alpha} = \frac{2\sin 3\alpha \cos\alpha}{\sin 2\alpha \sin 4\alpha} = \frac{2\sin\frac{3\pi}{7}\cos\frac{\pi}{7}}{\sin\frac{2\pi}{7}\sin\frac{4\pi}{7}} = \frac{1}{\sin\alpha} = \operatorname{cosec}\alpha$

(C) $\cos\alpha - \cos 2\alpha + \cos 3\alpha = \cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} = \frac{1}{2}$

(D) $8\cos\alpha\cos 2\alpha\cos 4\alpha = -1$

4. Which of the following quantities are rational?

A. $\sin\left(\frac{11\pi}{12}\right)\sin\left(\frac{5\pi}{12}\right)$

B. $\operatorname{cosec}\left(\frac{9\pi}{10}\right)\sec\left(\frac{4\pi}{5}\right)$

C. $\sin^4\left(\frac{\pi}{8}\right)+\cos^4\left(\frac{\pi}{8}\right)$

D. $\left(1+\cos\frac{2\pi}{9}\right)\left(1+\cos\frac{4\pi}{9}\right)\left(1+\cos\frac{8\pi}{9}\right)$

KEY. A,B,C,D

SOL. (A) $=\sin\frac{11\pi}{12}\sin\frac{5\pi}{12} = \sin\left(\frac{\pi}{12}\right)\cos\left(\frac{\pi}{12}\right) = \frac{1}{2}\sin\left(\frac{1}{4}\right) \in Q$

(B) $= \operatorname{cosec}\left(\frac{9\pi}{10}\right)\sec\left(\frac{4\pi}{5}\right) = -\operatorname{cosec}\left(\frac{\pi}{10}\right)\sec\left(\frac{\pi}{5}\right) = -4 \in Q$

(C) $= 1 - 2\sin^2\left(\frac{\pi}{8}\right)\cos^2\left(\frac{\pi}{8}\right) = 1 - \frac{1}{4} = \frac{3}{4} \in Q$

(D) $\left(2\cos^2\frac{\pi}{9}\right)\left(2\cos^2\frac{2\pi}{9}\right)\left(2\cos^2\frac{4\pi}{9}\right) = \frac{1}{8} \in Q$

5. In ΔPQR, if $3\sin P + 4\cos Q = 6$ and $4\sin Q + 3\cos P = 1$ then $\angle R$ can be

A) $\frac{\pi}{6}$ B) $\frac{3\pi}{4}$ C) $\frac{5\pi}{6}$ D) $\frac{\pi}{4}$

Key. A

Sol. Given equations $\Rightarrow 16 + 9 + 24\sin(P+Q) = 37$ $\Rightarrow P + Q = \frac{5\pi}{6}$ *or* $\frac{\pi}{6}$

If $P + Q = \frac{\pi}{6}$ then $R = \frac{5\pi}{6}$

If $P < \frac{\pi}{6}, 3\sin P < \frac{1}{2}$ then $3\sin P + 4\cos Q < \frac{1}{2} + 4 < 6$

$\therefore P + Q = \frac{\pi}{6}$ is not possible $\therefore R = \frac{\pi}{6}$

6. A solution (x, y) of the system of equations $x-y=\frac{1}{3}$ and $\cos^2(\pi x)-\sin^2(\pi y)=\frac{1}{2}$

A. $\left(\frac{7}{6},\frac{5}{6}\right)$ B. $\left(\frac{2}{3},\frac{1}{3}\right)$ C. $\left(\frac{-5}{6},\frac{-7}{6}\right)$ D. $\left(\frac{13}{6},\frac{11}{6}\right)$

KEY. A,C,D

SOL. $x-y=\frac{1}{3}$ and $\cos\{\pi(x+y)\}\cos\{\pi(x-y)\}=\frac{1}{2}$ $\Rightarrow x+y=2n, n\in z$

7. For $0\le x\le 2\pi$ then $2^{\cos ec^2 x}\sqrt{\frac{y^2}{2}-y+1}\le\sqrt{2}$ is

A. satisfied by exactly one value of y B. satisfied by exactly two values of x
C. satisfied by x for which cosx = 0 D. satisfied by x for which sinx = 0

KEY. A,B,C

SOL. $2^{\cos ec^2 x}\sqrt{(y-1)^2+1}\le\sqrt{2}$

$\Rightarrow \cos ec^2 x=1$ and y = 1

$\Rightarrow x=\frac{\pi}{2},\frac{3\pi}{2}, y=1$

8. If $x\cos\alpha+y\sin\alpha=x\cos\beta+y\sin\beta=2a$ and $2\sin\left(\frac{\alpha}{2}\right)\sin\left(\frac{\beta}{2}\right)=1$ then

A. $y^2=4a(a-x)$ B. $\cos\alpha+\cos\beta=\cos\alpha\cos\beta$

C. $\cos\alpha.\cos\beta=\frac{4a^2+y^2}{x^2+y^2}$ D. $\cos\alpha+\cos\beta=\frac{4ax}{x^2+y^2}$

KEY. A,B,D

SOL. α and β satisfy $x\cos\theta+y\sin\theta=2a$

$\Rightarrow (x^2+y^2)\cos^2\theta-4ax\cos\theta+(4a^2-y^2)=0$

$\cos\alpha+\cos\beta=\frac{4ax}{x^2+y^2}, \cos\alpha.\cos\beta=\frac{4a^2-y^2}{x^2+y^2}$

$2\sin\left(\frac{\alpha}{2}\right)\sin\left(\frac{\beta}{2}\right)=1\Rightarrow 4\sin^2\left(\frac{\alpha}{2}\right)\sin^2\left(\frac{\beta}{2}\right)=1 \Rightarrow \cos\alpha+\cos\beta=\cos\alpha.\cos\beta$

9. If $\frac{\tan 3A}{\tan A}=k(k\ne 1)$ then

A. $\dfrac{\cos A}{\cos 3A}=\dfrac{k-1}{2}$ B. $\dfrac{\sin 3A}{\sin A}=\dfrac{2k}{k-1}$ C. $k<\dfrac{1}{3}$ D. k > 3

KEY. A,B,C,D

SOL. $\dfrac{k+1}{k-1}=2\cos 2A$

$$\frac{\sin 3A}{\sin A}=1+2\cos 2A=\frac{2k}{k-1}$$

sw

Also $k=\dfrac{3-\tan^2 A}{1-3\tan^2 A}\Rightarrow k<\dfrac{1}{3},k>3$

10. Let $f_n(\theta)=\tan\dfrac{\theta}{2}(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)....(1+\sec 2^n\theta)$, then

A. $f_2\left(\dfrac{\pi}{16}\right)=1$ B. $f_3\left(\dfrac{\pi}{32}\right)=1$ C. $f_4\left(\dfrac{\pi}{64}\right)=1$ D. $f_5\left(\dfrac{\pi}{128}\right)=1$

Key. A,B,C,D

Sol. $f_n(\theta)=\tan(\theta/2)\prod_{r=0}^{n}(1+\sec 2^r\theta)$

$$=\tan(\theta/2)\prod_{r=0}^{n}\left\{\frac{1+\cos(2^r\theta)}{\cos(2^r\theta)}\right\}$$

$$=\tan(\theta/2)\prod_{r=0}^{n}\frac{2\cos^2(2^{r-1}\theta)}{\cos(2^r\theta)}$$

$$=2^{n+1}.\tan(\theta/2)\prod_{r=0}^{n}\frac{\cos^2(2^{r-1}\theta)}{\cos(2^r\theta)}$$

$$=2^{n+1}.\tan(\theta/2).\cos^2(\theta/2)\prod_{r=0}^{n}\frac{\cos(2^r\theta)}{\cos(2^n\theta)}$$

$$=2^n.\sin\theta.\frac{\sin(2^n\theta)}{2^n.\sin\theta.\cos(2^n\theta)}$$

$$=\tan(2^n\theta)$$

$\therefore$ Alternate. (a) : $f_2\left(\dfrac{\pi}{16}\right)=\tan\left(\dfrac{\pi}{4}\right)=1$

Alternate. (a) : $f_3\left(\frac{\pi}{32}\right)=\tan\left(\frac{\pi}{4}\right)=1$

Alternate. (a) : $f_4\left(\frac{\pi}{64}\right)=\tan\left(\frac{\pi}{4}\right)=1$

Alternate. (a) : $f_5\left(\frac{\pi}{128}\right)=\tan\left(\frac{\pi}{4}\right)=1$

11. Let ABC be a triangle inscribed in a circle of radius r and AB = AC and h is the altitude from A to BC, then (P = perimeter of Δ ABC, Δ = area)

a) $P=2\left(\sqrt{2hr-h^2}\right)$ b) $\Delta=h\sqrt{2hr-h^2}$ c) $\lim\limits_{h\to 0}\frac{\Delta}{p^3}=\frac{1}{128r}$ d) $\lim\limits_{h\to 0}\frac{\Delta}{p^3}=\frac{1}{64r}$

Ans. b, c

Sol. $BC=2BD=2\sqrt{r^2-(h-r)^2}=2\sqrt{2hr-h^2}$

$\Rightarrow AB=\sqrt{2hr}$ so that P =2AB + BC

$=2\left[\sqrt{2hr-h^2}+\sqrt{2hr}\right]$

$\Delta=BD\times AD=h\sqrt{2hr-h^2}$

$\therefore \frac{\Delta}{P^3}=\frac{\sqrt{2r-h}}{8\left(\sqrt{2r-h}+\sqrt{2r}\right)^3}\Rightarrow \lim\limits_{h\to 0}\frac{\Delta}{p^3}=\frac{\sqrt{2r}}{8\left(2\sqrt{2r}\right)^3}=\frac{1}{128r}$

12. Sum of series $\sum\limits_{r=1}^{n}\sin^{-1}\left[\frac{2r+1}{r(r+1)\left(\sqrt{r^2+2r}+\sqrt{r^2-1}\right)}\right]$ is

a) $\frac{\pi}{2}-\sin^{-1}\left(\frac{1}{n+1}\right)$ b) $\cos^{-1}\left(\frac{1}{n+1}\right)$ c) $\cos^{-1}\left(\frac{1}{n+2}\right)$ d) none of these

Ans. a, b

Sol. $T_r=\sin^{-1}\left(\frac{\sqrt{r^2+2r}-\sqrt{r^2-1}}{r(r+1)}\right)$

$T_r=\sin^{-1}\left(\frac{1}{r}\sqrt{1-\frac{1}{(r+1)^2}}-\frac{1}{r+1}\sqrt{1-\frac{1}{r^2}}\right)$

$T_r=\sin^{-1}\left(\frac{1}{r}\right)-\sin^{-1}\left(\frac{1}{r+1}\right)$

$S_n=\cos^{-1}\left(\frac{1}{n+1}\right)$

13. Minimum positive values of x and y such that $x+y=\frac{\pi}{2}$ and $\sec x+\sec y=2\sqrt{2}$

a) $x=\frac{\pi}{4}$ b) $y=\frac{\pi}{4}$ c) $x=-\frac{\pi}{4}$ d) $y=-\frac{\pi}{4}$

Ans. a, b

Sol. $\sec\left(\frac{x+y}{2}\right) \le \frac{\sec x + \sec y}{2}$

14. Which of the following is true for a ΔABC

a) $R^2 \ge \left(\frac{abc}{a+b+c}\right)$

b) r + 2R = s if C = 90^0

c) $\sin 2A + \sin 2B + \sin 2C \le \frac{3\sqrt{3}}{2}$

d) $\frac{2}{R} \le \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$

Ans. a,b,c,d

Sol. $R \ge 2r \Rightarrow R^2 \ge \frac{4\Delta R}{2s} \Rightarrow R^2 \ge \frac{abc}{a+b+c}$

If $\underline{|C} = 90^0 \Rightarrow a^2 + b^2 = c^2, c = 2R$

r + 2R = (s – c) $\tan\frac{C}{2} + c$

= s – c + c = s $\left(\because C = 90^0\right)$

15. Three straight lines are drawn through a point P lying in the interior of the triangle ABC and parallel to its sides. The area of the three resulting triangles with P as the vertex are Δ_1, Δ_2 *and* Δ_3 then area of triangle ABC is

a) $\left(\sqrt{\Delta_1} + \sqrt{\Delta_2} + \sqrt{\Delta_3}\right)^2$

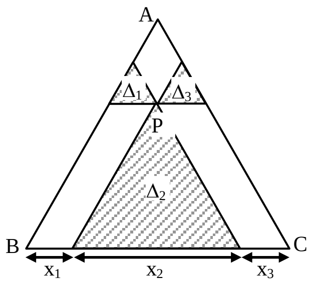
b) $\left(\Delta_1 + \Delta_2 + \Delta_3\right)$

c) $\left(\sqrt{\Delta_1\Delta_2 + \Delta_2\Delta_3 + \Delta_3\Delta_1}\right)$

d) $\Delta_1 + \Delta_2 + \Delta_3 + 2\left(\sqrt{\Delta_1\Delta_2} + \sqrt{\Delta_2\Delta_3} + \sqrt{\Delta_3\Delta_1}\right)$

Ans. a, d

Sol. $\frac{\Delta_1}{\Delta} = \frac{x_1^2}{a^2}$ or $\sqrt{\frac{\Delta_1}{\Delta}} = \frac{x_1}{a}$ = Similarly $\sqrt{\frac{\Delta_2}{\Delta}} = \frac{x_2}{a}$ and $\sqrt{\frac{\Delta_3}{\Delta}} = \frac{x_3}{a}$



7. If $f_n(\theta) = \tan\frac{\theta}{2}(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta).....(1+\sec 2^n\theta)$ then

a) $f_2\left(\frac{\pi}{16}\right) = 1$

b) $f_3\left(\frac{\pi}{32}\right) = 1$

c) $f_4\left(\frac{\pi}{64}\right) = 1$

d) $f_5\left(\frac{\pi}{128}\right) = 1$

Key. A,B,C,D

Sol. $1 + \sec\theta = \frac{\cos\theta + 1}{\cos\theta} = \frac{2\cos^2\left(\frac{\theta}{2}\right)}{\cos\theta}$

Similarly, 1 + sec 2θ = $\frac{2\cos^2\theta}{\cos 2\theta}$ etc.

$$\Rightarrow\ f_n(\theta)=\tan\frac{\theta}{2}\times\frac{2\cos^2(\theta/2)}{\cos\theta}\times\frac{2\cos^2\theta}{\cos 2\theta}\times\frac{2\cos^2 2\theta}{\cos 4\theta}\times.......\times\frac{2\cos^2 2^{n-1}\theta}{\cos 2^n\theta}$$

$$f_2\left(\frac{\pi}{16}\right)=\tan\left(2^2.\frac{\pi}{16}\right)=\tan\frac{\pi}{4}=1$$

$$f_3\left(\frac{\pi}{32}\right)=\tan\left(2^3.\frac{\pi}{32}\right)=\tan\frac{\pi}{4}=1$$

$$f_4\left(\frac{\pi}{64}\right)=\tan\left(2^4.\frac{\pi}{64}\right)=\tan\frac{\pi}{4}=1$$

8. The relation $\frac{\tan 3x}{\tan x}=\tan\left(\frac{\pi}{3}-x\right)\tan\left(\frac{\pi}{3}+x\right)$

a) is an identity for all x   b) is not an identity

c) is an identity if $x\neq\frac{k\pi}{3}$ d) is an identity if $x\neq\frac{k\pi}{6}$

Key. C,D

Sol. $\tan 3x=\frac{3\tan x-\tan^3 x}{1-3\tan^2 x}$

For $x\neq\frac{k\pi}{6}$

$$\frac{\tan 3x}{\tan x}=\frac{3-\tan^2 x}{1-\tan^2 x}=\frac{\left(\sqrt{3}-\tan x\right)\left(\sqrt{3}+\tan x\right)}{\left(1+\sqrt{3}\tan x\right)\left(1-\sqrt{3}\tan x\right)}=\tan\left(\frac{\pi}{3}-x\right).\tan\left(\frac{\pi}{3}+x\right)$$

$\Rightarrow$ (d) is correct.

Now $x\neq\frac{k\pi}{6}\Rightarrow x\neq\frac{k\pi}{3}$

$\Rightarrow$ (c) is also correct.

9. If x sin a + y sin 2a + z sin 3a = sin 4a
x sin b + y sin 2b + z sin 3b = sin 4b
x sin c + y sin 2c + z sin 3c = sin 4c

then the roots of the equation $t^3-\left(\frac{z}{2}\right)t^2-\left(\frac{y+2}{4}\right)t+\left(\frac{z-x}{8}\right)=0,\ \ a,b,c\neq n\pi,$ are

a) sin a, sin b, sin c   b) sin 2a, sin 2b, sin 2c
c) cos a, cosb, cosc   d) cos 2a, cos2b, cos2c

Key. C

Sol. x sin a+ y sin 2a + z sin 3a = sin 4a

$$8\cos^3 a-4z\cos^2 a-(2y+4)\cos a+(z-x)=0$$

Cos a is a root of the given equation

10. If b > a > 0, then the expression E given by

$$E=\frac{1}{\sqrt{b-a}}\frac{\sqrt{\frac{b-a}{a}}\sin x}{\sqrt{\left[1+\left(\sqrt{\frac{b-a}{a}}\sin x\right)^2\right]}}.\sqrt{a+b\tan^2 x}$$ must be equal to

a) tan x b) |tan x|

c) $\frac{\sin x}{|\cos x|}$ d) unity if $x=\frac{11\pi}{4}$

Key. C,D

Sol. $E=\frac{1}{\sqrt{b-a}}\frac{\sqrt{b-a}\sin x}{\sqrt{a+a\left(\frac{b-a}{a}\sin^2 x\right)}}.\sqrt{a+b\tan^2 x}$

$=\frac{\sin x\sqrt{a+b\tan^2 x}}{\sqrt{a+(b-a)\sin^2 x}}=\frac{\sin x\sqrt{a+b\tan^2 x}}{\sqrt{a\cos^2 x+b\sin^2 x}}=\frac{\sin x}{|\cos x|}\left(\frac{\sqrt{a+b\tan^2 x}}{\sqrt{a+b\tan^2 x}}\right)$ = $\frac{\sin x}{|\cos x|}$

$\Rightarrow$ (c) is correct.

at $x=\frac{11\pi}{4}$, $\frac{\sin x}{|\cos x|}=1$

$\Rightarrow$ (d) is correct

24. Which of the following statements are correct

a) sin 2 > sin 2^0 b) tan 2 < 0 c) tan 1 > tan 2 d) tan 2 < tan 1 < 0

Key. A,B,C

Sol. Since 1 radian lies between 57^0 and 58^0 and sin 57^0 > sin 1^0, so sin 1 > sin 1^0.
Again 1 radian is an acute angle and 2 radian is an obtuse angle, tan 1 > 0.
Tan 2 < 0, so that tan 1 > tan 2.

25. If A and B are acute angles such that A + B and A – B satisfy the equation $\tan^2\theta-4\tan\theta+1=0$, then

a) $A=\frac{\pi}{4}$ b) $A=\frac{\pi}{6}$ c) $B=\frac{\pi}{4}$ d) $B=\frac{\pi}{6}$

Key. A,D

Sol. From the given equation, we have
tan(A + B) + tan (A – B) = 4 ....(1)
tan(A + B)tan(A – B) = 1 ....(2)
From (1) and (2) we get

$\tan[A+B+A-B]=\infty\Rightarrow 2A=\frac{\pi}{2}\Rightarrow A=\frac{\pi}{4}$

and from (1) we get

$\frac{1+\tan B}{1-\tan B}+\frac{1-\tan B}{1+\tan B}=4\Rightarrow\frac{(1+\tan B)^2+(1-\tan B)^2}{1-\tan^2 B}=4$

$\Rightarrow\frac{2(1+\tan^2 B)}{1-\tan^2 B}=4\Rightarrow\frac{1-\tan^2 B}{1+\tan^2 B}=\frac{1}{2}$

$\Rightarrow\cos 2B=\frac{1}{2}\Rightarrow 2B=\frac{\pi}{3}\Rightarrow B=\frac{\pi}{6}$

26. If $\cos 5\theta=a\cos\theta+b\cos^2\theta+c\cos^5\theta+d$, then

a) a = 20 b) b = −20 c) c = 16 d) d=5

Key. B,C

Sol. $\cos 5\theta = \cos(4\theta + \theta) = \cos 4\theta \cos\theta - \sin 4\theta \sin\theta$

$= (2\cos^2 2\theta - 1)\cos\theta - 2\sin 2\theta \cos 2\theta \sin\theta$

$= \left[2(2\cos^2\theta - 4\cos^2\theta + 1) - 1\right]\cos\theta - 4\cos\theta(2\cos^2\theta - 1)(1 - \cos^2\theta)$

$= \cos\theta(8\cos^4\theta - 8\cos^2\theta + 1) - 4\cos\theta(3\cos^2\theta - 2\cos^4\theta - 1)$

$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$

Clearly, a = 5, b = −20, c = 16 and d = 0.

27. If $\tan 1^0 \tan 2^0 .... \tan 89^0 = x^2 - 8$, then the value of x can be

a) −1 b) 1 c) -3 d) 3

Key. C,D

Sol. $x^2 - 8 = (\tan 1^0 \tan 89^0)(\tan 2^0 \tan 88^0).....(\tan 44^0 \tan 46^0)\tan 45^0$

$= (\tan 1^0 \cot 1^0)(\tan 2^0 \cot 2^0).....(\tan 44^0 \cot 44^0)\tan 45^0$

$= 1 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$

28. The value of $\sin\frac{\pi}{16}\sin\frac{3\pi}{16}\sin\frac{5\pi}{16}\sin\frac{7\pi}{16} = \frac{\sqrt{a}}{b}$ then

a) a = 2 b) b = 16 c) a + b = 18 d) $\frac{a}{b} = \frac{1}{8}$

Key. A,B,C,D

Sol. We have $\sin\frac{\pi}{16}.\sin\frac{3\pi}{16}.\sin\frac{5\pi}{16}.\sin\frac{7\pi}{16}$

$= \frac{1}{4}\left[2\sin\frac{\pi}{16}.\sin\frac{3\pi}{16}.2\sin\frac{5\pi}{16}.\sin\frac{7\pi}{16}\right]$

$= \frac{1}{4}\left[\left(\cos\frac{\pi}{8} - \cos\frac{\pi}{4}\right)\left(\cos\frac{\pi}{8} - \cos\frac{3\pi}{4}\right)\right]$

$= \frac{1}{4}\left[\left(\cos\frac{\pi}{8} - \frac{1}{\sqrt{2}}\right)\left(\cos\frac{\pi}{8} + \frac{1}{\sqrt{2}}\right)\right]$

$= \frac{1}{4}\left[\left(\cos^2\frac{\pi}{8} - \frac{1}{2}\right)\right] = \frac{1}{8}\left[2\cos^2\frac{\pi}{8} - 1\right]$

$= \frac{1}{8}\left[\cos\frac{\pi}{4}\right] = \frac{1}{8} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{16}$

Trigonometry

Assertion Reasoning Type

1. Statement I: :If $\theta \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ then $\tan\theta^{\cot\theta} > \cot\theta^{\tan\theta}$.

Statement II: The function $x^{1/x}$ decreases for all x>3 .

Key. B

Sol. Clearly f(x) - $x^{1/x}$ decreases when x>e

Clearly $\left(\frac{\pi}{3}, \frac{\pi}{2}\right) \in (0, e) \Rightarrow \text{Tan}\theta > \cot\theta$

$\Rightarrow (\text{Tan}\theta)^{\cot\theta} > (\cot\theta)^{\text{Tan}\theta}$

For x>3, f(x) is decreases but it is not correct explanation for statement I

2. **Statement – 1:** The inequality $\log_{\sin x} 2^{\tan x} > 0$ has no real roots in the interval $\left(0, \frac{\pi}{2}\right)$

Statement – 2: The domain of the function $f(x) = \log_{\sin x} 2^{\tan x}$ is $\bigcup_{n \in Z} \left(2n\pi, 2n\pi + \frac{\pi}{2}\right)$

Key. C

Sol. Conceptual

3. Statement – I: If $I(n) = 2\cos nx$, $n \in N$, then $I(1) \cdot I(n+1) - I(n) = I(n+3)$

Statement – II : $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$ a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1

b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1

c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. d

Sol. $I(1) \cdot I(n+1) - I(n) = 4\cos x \cos(n+1)x - 2\cos nx$

= 2{2cos(n + 1) x. cosx – cos nx}

= 2{cos(n + 2) x + cos nx – cos nx}

= 2cos(n + 2) x

= I(n + 2)

I is false II is true

4. Statement – 1: In any triangle ABC, a cosA + b cos B + c cosC $\leq$ 25

Statement – 2: In any triangle ABC, $\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \leq \frac{1}{8}$

a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for statement-1

b) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for statement-1

c) Statement-1 is true, Statement-2 is false d) Statement-1 is false, Statement-2 is true

Ans. a

Sol. $I \Rightarrow 2R\sin A\cos A + 2R\sin B\cos B + 2R\sin c\cos C \leq 2R(\sin A + \sin B + \sin C)$

or sin2A + sin2B + sin2C $\leq$ 2(sinA + sinB + sinC)

$$\Rightarrow 4\sin A\sin B\sin C \le 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

$$\Rightarrow \sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \le \frac{1}{8}$$

5. Statement – 1: Let O be the orthocentre of ∆ABC and $OA = \frac{a}{2\sin A}$, then the triangle ABC is an isosceles triangle.
Statement – 2: The orthocenter of a triangle is the point of intersection of the attitudes.
a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for statement-1
b) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for statement-1
c) Statement-1 is true, Statement-2 is false d) Statement-1 is false, Statement-2 is true

Ans. d

Sol. OA = 2R cosA

6. Statement – 1: The maximum value of $\sin\sqrt{2}x + \sin ax$ cannot be 2 (a is positive rational number)
Statement – 2: $\frac{\sqrt{2}}{a}$ is irrational.
a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for statement-1
b) Statement-1 is true, Statement-2 is true, Statement-2 is not a correct explanation for statement-1
c) Statement-1 is true, Statement-2 is false d) Statement-1 is false, Statement-2 is true

Ans. a

Sol. The value of $\sin\sqrt{2}x + \sin ax$ can be equal to 2, if $\sin\sqrt{2}x$ and sinax both are equal to one but are not equal to one for any common value of x.

7. Statement – I: If $f(x) = \sin^{-1}x + \operatorname{cosec}^{-1}x + \cos^{-1}x + \sec^{-1}x + \tan^{-1}x$, then f(x) can take every value in the interval $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$
Statement – II : $f(x) = \pi - \tan^{-1}x$ and $-\frac{\pi}{2} < \tan^{-1}x < \frac{\pi}{2}\ \forall\ x$
a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1
b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1
c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. d

Sol. f(x) is defined for x = 1 and – 1 only

8. Statement – I: In ∆ABC if sinA + sinB + sinC $\le$ 1, then min {A+B, B+C, C+A} < 30^0
Statement – II : In ∆ABC, $\frac{A+B+C}{3} \le \min\{A, B, C\}$
a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1
b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1

c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. c

Sol. If we assume $A \geq B \geq C$

Then sinA + sinB + sinC $\geq$ 2sinA

$$A \geq \frac{A+B+C}{3} = 60^0$$

This gives B + C < 30^0

Trigonometry

Comprehension Type

Passage – 1

Let $\cos\frac{\pi}{7}, \cos\frac{3\pi}{7}, \cos\frac{5\pi}{7}$ are the roots of equation $8x^3-4x^2-4x+1=0$

1. The value of $\sec\left(\frac{\pi}{7}\right)+\sec\left(\frac{3\pi}{7}\right)+\sec\left(\frac{5\pi}{7}\right)$ is

A. 2 B. 4 C. 8 D. None

KEY. B

SOL. $\sec\frac{\pi}{7}, \sec\frac{3\pi}{7}, \sec\frac{5\pi}{7}$ are the roots of $x^3-4x^2-4x+8=0$

$$\sec\left(\frac{\pi}{7}\right)+\sec\left(\frac{3\pi}{7}\right)+\sec\left(\frac{5\pi}{7}\right)=4$$

$$8x^3-4x^2-4x+1=8\left(x-\cos\frac{\pi}{7}\right)\left(x-\cos\frac{3\pi}{7}\right)\left(x-\cos\frac{5\pi}{7}\right)$$

2. The value of $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}$ is

A. $\frac{1}{4}$ B. $\frac{1}{8}$ C. $\frac{\sqrt{7}}{4}$ D. $\frac{\sqrt{7}}{8}$

KEY. B

SOL. Put $x=1 \Rightarrow \sin\left(\frac{\pi}{14}\right)\sin\left(\frac{3\pi}{14}\right)\sin\left(\frac{5\pi}{14}\right)=\frac{1}{8}$

3. The value of $\cos\left(\frac{\pi}{14}\right)\cos\left(\frac{3\pi}{14}\right)\cos\left(\frac{5\pi}{14}\right)$ is

A. $\frac{1}{4}$ B. $\frac{1}{8}$ C. $\frac{\sqrt{7}}{4}$ D. $\frac{\sqrt{7}}{8}$

KEY. D

SOL. Put $x=-1 \Rightarrow \cos\left(\frac{\pi}{14}\right)\cos\left(\frac{3\pi}{14}\right)\cos\left(\frac{5\pi}{14}\right)=\frac{\sqrt{7}}{8}$

Passage – 2

Let ABC be a triangle in which the line joining the circumcentre and incentre is parallel to base BC of the triangle, making use of the standard notation r and R.

4. Then range of $\angle A$ is

a) $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$ b) $\left[\frac{\pi}{3}, \frac{\pi}{2}\right)$ c) $\left[\frac{\pi}{3}, \frac{2\pi}{3}\right] - \left\{\frac{\pi}{2}\right\}$ d) $\left[0, \frac{\pi}{2}\right]$

Ans. b

Sol. $\because$ the line joining O and I is parallel to BC

$\therefore$ OI = DE and OD = IE

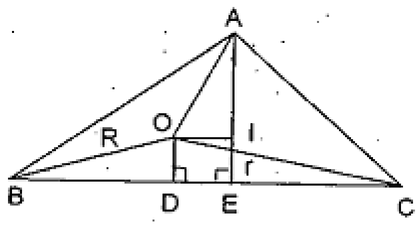
OD = Rsin(90 – A) = RcosA

IE = r $\because$ I is incentre

$\Rightarrow$ R cosA = r

or $\cos A = \frac{r}{R} \le \frac{1}{2} \Rightarrow 0 < \cos A \le \frac{1}{2}$

$\Rightarrow \frac{\pi}{3} \le A < \frac{\pi}{2}$



5. If ODEI is a square where O and I stands for circumcentre and incentre respectively and D and E are the point of perpendicular from O and I on the base BC, then

a) $\frac{r}{R} = \frac{3}{8}$ b) $\frac{r}{R} = 2 - \sqrt{3}\ fr$ c) $\frac{r}{R} = \sqrt{2} - 1$ d $\frac{r}{R} = \frac{1}{4}$

Ans. c

Sol. $\because$ ODEI is a square hence OD = OI

$OI = \sqrt{R^2 - 2Rr}$ OD = RcosA from previous portion is $\sqrt{R^2 - 2Rr} = R\cos A$

$\Rightarrow R^2 - 2Rr = R^2\cos^2 A$

or $1 - \cos^2 A = \frac{2r}{R}$ $Also\ \cos A = \frac{r}{R}$

$\Rightarrow 1 - \left(\frac{r}{R}\right)^2 = \frac{2r}{R}\ or\ \left(\frac{r}{R}\right)^2 + \frac{2r}{R} - 1 = 0$

$\frac{r}{R} = \sqrt{2} - 1$

6. If $\lfloor A = 60^0$, then ΔABC is

a) isosceles b) right angled c) right angled isosceles d) equilateral

Ans. d

Sol. $\because \frac{r}{R} = \cos A = \frac{1}{2} \because A = 60^0$

But $\frac{r}{R} \le \frac{1}{2}$ in any ΔABC

Hence ΔABC is equilateral. **Paragraph for Questions Nos. 15 to 17**

To evaluate an expression in the from

$\sin a + \sin(a+d) + \sin(a+2d) + ... + \sin(a+(n-1)d)$

or $\cos a + \cos(a+d) + \cos(a+2d) + ... + \cos(a+(n-1)d)$

we can multiply each and every term with $2\sin\frac{d}{2}$ *and apply transformation formula to the resulting products and simplify. Using this information answer the following.*

15. For $\alpha = \frac{2\pi}{13}, \cos\alpha + \cos 2\alpha + \cos 3\alpha + \cos 4\alpha + \cos 5\alpha + \cos 6\alpha =$

a) -1 b) $\frac{1}{2}$ c) $-\frac{1}{2}$ d) 1

Key. C

16. For $\alpha=\frac{2\pi}{13},\cos^2\alpha+\cos^2 2\alpha+\cos^2 3\alpha+\cos^2 4\alpha+\cos^2 5\alpha+\cos^2 6\alpha=$

a) $\frac{3}{4}$ b) $\frac{1}{4}$ c) $\frac{7}{4}$ d) $\frac{11}{4}$

Key. D

17. For $\alpha=\frac{2\pi}{13},\cos\alpha\cos 5\alpha+\cos 2\alpha\cos 3\alpha+\cos 4\alpha\cos 6\alpha=$

a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) $\frac{1}{4}$ d) $-\frac{1}{4}$

Key. D

Sol. 16. $\cos^2\alpha+\cos^2 2\alpha+\cos^2 3\alpha+\cos^2 5\alpha+\cos^2 6\alpha+\cos^2 4\alpha$

$$=\frac{6+\left(\cos 2\alpha+\cos 4\alpha+\cos 6\alpha+\cos 10\alpha+\cos 12\alpha+\cos 8\alpha\right)}{2},$$

$$\left(13\alpha=2\pi,12\alpha=2\pi-\alpha,10\alpha=2\pi-3\alpha,8\alpha=2\pi-5\alpha\right)$$

$$=\frac{6+\left(\cos\alpha+\cos 2\alpha+\cos 3\alpha+\cos 4\alpha+\cos 5\alpha+\cos 6\alpha\right)}{2}$$

$$=\frac{11}{4}$$

17. $\cos\alpha\cos 5\alpha+\cos 2\alpha\cos 3\alpha+\cos 4\alpha\cos 6\alpha$

$$=\frac{1}{2}\left[\cos 6\alpha+\cos 4\alpha+\cos 5\alpha+\cos\alpha+\cos 10\alpha+\cos 2\alpha\right] \qquad 10\alpha=2\pi-3\alpha$$

$$=\frac{1}{2}\left(-\frac{1}{2}\right)=\frac{-1}{4}$$

PASSAGE –II

Paragraph for Question Nos.18 to 20

Consider the equation $\frac{Sin^4x}{a}+\frac{Cos^4x}{b}=\frac{1}{a+b},0<x<\frac{\pi}{2}$

Then answer the following questions.

18. $\frac{Sin^{18}x}{a^7}+\frac{Cos^{18}x}{b^7}=$

a) $\frac{1}{(a+b)^8}$ b) $\frac{a^2+b^2}{(a+b)^9}$ c) $\frac{a^2+b^2}{(a+b)^8}$ d) $\frac{1}{(a+b)^7}$

Key. B

19. $\frac{Sin^{16}x}{a^5}+\frac{Cos^{16}x}{b^5}=$

a) $\frac{a^2+b^2}{(a+b)^7}$ b) $\frac{a^2+b^2-ab}{(a+b)^7}$ c) $\frac{1}{(a+b)^5}$ d) $\frac{a^3+b^3}{(a+b)^7}$

Key. B

20. $\dfrac{Sin^{12}x}{a^5}+\dfrac{Cos^{12}x}{b^5}=$

a) $\dfrac{1}{(a+b)^6}$ b) $\dfrac{1}{(a+b)^5}$ c) $\dfrac{a^2+b^2}{(a+b)^5}$ d) $\dfrac{a^2+b^2-ab}{(a+b)^5}$

Key. B

Sol. 18. $\left(\dfrac{a+b}{a}\right)Sin^4x+\left(\dfrac{a+b}{b}\right)Cos^4x=1$

$$S\text{in}^4 x+\ \frac{b}{a}\ Sin^4x\ +\ \frac{a}{b}\ Cos^4x+Cos^4x=1$$

$$\left(S\text{in}^2 x+Cos^2x\right)^2-2Sin^2x\cos^2 x+\frac{b}{a}\ Sin^4x\ +\ \frac{a}{b}\ Cos^4x=1$$

$$\Rightarrow \sqrt{\frac{b}{a}}Sin^2x\ -\ \sqrt{\frac{a}{b}}Cos^2x\ =\ 0\ \Rightarrow\ \tan^2 x\ =\ \frac{a}{b}$$

$$\Rightarrow Sin\,x=\frac{\sqrt{a}}{\sqrt{a+b}}$$

$$Cos\,x=\frac{\sqrt{b}}{\sqrt{a+b}}$$

Now $Sin^{18}x=\dfrac{a^9}{(a+b)^9}$

$Cos^{18}x=\dfrac{b^9}{(a+b)^9}$

Now $\dfrac{Sin^{18}x}{a^7}+\dfrac{Cos^{18}x}{b^7}=\dfrac{a^2+b^2}{(a+b)^9}$

19. $Sin^{16}x=\dfrac{a^8}{(a+b)^8},Cos^{16}x=\dfrac{b^8}{(a+b)^8}$

$\dfrac{Sin^{16}x}{a^5}+\dfrac{Cos^{16}x}{b^5}=\dfrac{a^3+b^3}{(a+b)^8}=\dfrac{a^2+b^2-ab}{(a+b)^7}$

20. $Sin^{12}\theta=\dfrac{a^6}{(a+b)^6},Cos^{12}\theta=\dfrac{b^6}{(a+b)^6}$

$\dfrac{Sin^{12}\theta}{a^5}+\dfrac{Cos^{12}\theta}{b^5}=\dfrac{1}{(a+b)^5}$

PASSAGE –III

Paragraph for Question Nos.21 to 23

Consider the equation $\sin^2 x-\alpha\sin x+\beta=0$

if this equation is satisfied with exactly one value of x in (0, π). Then

21. β will never lie in the interval

a) (–20, –10) b) (–10, –5) c) (5, 10) d) (0, 1)

Key. D

22. α can lie in the interval

a) $\left(1, \frac{3}{2}\right)$ b) $\left(\frac{3}{2}, 2\right)$ c) (1, 2) d) $\left(\frac{5}{2}, 5\right)$

Key. D

23. α and β are related by

a) $\alpha + 1 = \beta$ b) $\beta + 1 = \alpha$ c) $\alpha - \beta = 2$ d) $\beta - \alpha = 2$

Key. B

Sol. 21. As the equation

$\sin^2 x - \alpha \sin x + \beta = 0$ is

satisfied with exactly one value of x in $(0, \pi)$

$\Rightarrow$ sin x = 1, sin x = p with condition $p \geq 1$ or $p \leq 0$

$\Rightarrow$ $(\sin x - 1)(\sin x - p)$ is the factor of $(\sin^2 x - \alpha \sin x + \beta)$

$\therefore$ product of roots $p \times 1 = \beta$

$\Rightarrow$ $\beta \geq 1$ or $\beta \leq 0$

$\Rightarrow$ β will never lie in (0, 1)

22. As $\sin^2 x - \alpha \sin x + \beta = (\sin x - 1)(\sin x - p)$

where $p \geq 1$ or $p \leq 0$

$\alpha = (p+1)$

As $p \geq 1$ or $p \leq 0$

$\Rightarrow \alpha \leq 1$ or $\alpha \geq 2 \Rightarrow \alpha$ will never lie in (1,2)

$\therefore$ option (D) is correct.

23. As $\sin^2 x - \alpha \sin x + \beta = (\sin x - 1)(\sin x - \beta)$

where $p \geq 1$ or $p \leq 0$

$\alpha = p + 1$ (i)

$p = \beta$ (ii)

$\therefore$ $\alpha = \beta + 1$

Trigonometry

Integer Answer Type

1. Let $f(x)=0$ be an equation of degree six, having integer coefficients and whose one root is $2\cos\frac{\pi}{18}$. Then, the sum of all the roots of $f^1(x)=0$, is

Key. 0

Sol. Let $\theta=\frac{\pi}{18}\Rightarrow 6\theta=\frac{\pi}{3}\Rightarrow\cos 6\theta=\frac{1}{2}$

$$\Rightarrow 4\cos^3 2\theta-3\cos 2\theta=\frac{1}{2}\Rightarrow 8\left(2\cos^2\theta-1\right)^3-6\left(2\cos^2\theta-1\right)=1 \text{ let } 2\cos\theta=x$$

$$\Rightarrow 8\left(2.\frac{x^2}{4}-1\right)^3-6\left(2.\frac{x^2}{4}-1\right)=1$$

$$\Rightarrow\left(x^2-2\right)^3-3\left(x^2-2\right)=1$$

$$\Rightarrow x^6-6x^4+9x^2-3=0$$

$$f^1(x)=6x\left(x^4-4x^2+3\right)$$

$$f^1(x)=0\Rightarrow x=0,\pm1,\pm\sqrt{3}$$

2. If $\cos\theta+\cos^2\theta+\cos^3\theta=1$ and $\sin^6\theta=a+b\sin^2\theta+c\sin^4\theta$ then a + b + c =

KEY. 0

SOL. $\cos\theta(1+\cos^2\theta)=\sin^2\theta$

$$(1-\sin^2\theta)[2-\sin^2\theta]^2=\sin^4\theta$$

$$\sin^6\theta=4-8\sin^2\theta+4\sin^4\theta$$

a = 4, b = -8, c =4

a + b + c = 0

3. If $\frac{\sin\theta}{\cos 3\theta}+\frac{\sin 3\theta}{\cos 9\theta}+\frac{\sin 9\theta}{\cos 27\theta}=\frac{1}{A}[\tan B\theta-\tan C\theta]$ then (27A – B – 27C) =

KEY. 0

SOL. $\frac{\sin\theta}{\cos 3\theta}=\frac{2\sin\theta\cos\theta}{2\cos 3\theta\cos\theta}=\frac{\sin 2\theta}{2\cos 3\theta\cos\theta}=\frac{\sin(3\theta-\theta)}{2\cos 3\theta\cos\theta}$

$$\therefore\frac{\sin\theta}{\cos 3\theta}=\frac{1}{2}[\tan 3\theta-\tan\theta]\to(1)$$

$$\frac{\sin 3\theta}{\cos 9\theta}=\frac{1}{2}[\tan 9\theta-\tan 3\theta]\to(2)$$

$$\frac{\sin 9\theta}{\cos 27\theta}=\frac{1}{2}[\tan 27\theta-\tan 9\theta]\to(3)$$

$$\therefore (1)+(2)+(3)\Rightarrow \frac{1}{2}[\tan 27\theta-\tan\theta]$$

A = 2, B = 27, C = 1

27A – B – 27 C = 0

4. If $\frac{\tan x}{2}=\frac{\tan y}{3}=\frac{\tan z}{5}$ and x + y + z = π, $\tan^2 x+\tan^2 y+\tan^2 z=\frac{38}{K}$ then K =

KEY. 3

SOL. tanx = 2t, tany = 3t, tanz = 5t

$$\sum \tan x=\pi(\tan x)\Rightarrow t^2=\frac{1}{3}$$

$$\tan^2 x+\tan^2 y+\tan^2 z=t^2(4+9+25)=38t^2, K=3$$

5. If $\tan\alpha$ is an integral solution of $4x^2-16x+15<0$ and $\cos\beta$ is the slope of the bisector of the angle in the first quadrant between the x and y axis. Then $\sin(\alpha+\beta):\sin(\alpha-\beta)=$

KEY. 1

SOL. $4x^2-16x+15<0$

$$4x^2-10x-6x+15<0$$

$$2x(2x-5)-3(2x-5)<0$$

$$\frac{3}{2}<x<\frac{5}{2}\Rightarrow x=2$$

$$\tan\alpha=2;\cos\beta=1$$

$$\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}=\frac{\tan\alpha+\tan\beta}{\tan\alpha-\tan\beta}=\frac{2+0}{2-0}=1$$

6. If $\sin\theta+\sin^2\theta+\sin^3\theta=1$, then the value of $\cos^6\theta-4\cos^4\theta+8\cos^2\theta$ must be

Key. 4

Sol. We have, $\sin\theta(1+\sin^2\theta)=1-\sin^2\theta$

$$\Rightarrow \quad \sin\theta(2-\cos^2\theta)=\cos^2\theta$$

Squaring both sides, we get

$$\sin^2\theta(2-\cos^2\theta)^2=\cos^4\theta$$

$\Rightarrow \quad (1-\cos^2\theta)(4-4\cos^2\theta+\cos^4\theta)=\cos^4\theta$

$\Rightarrow \quad -\cos^6\theta+5\cos^4\theta-8\cos^2\theta+4=\cos^4\theta$

$\therefore \quad \cos^6\theta-4\cos^4\theta+8\cos^2\theta+4=\cos^4\theta$

7. If $\alpha=\frac{\pi}{14}$, then the value of (tan α tan 2α + tan 2α tan 4α + tan 4α tan α) is

Key. 1

Sol. α + 2α + 4α = 7α = $\frac{\pi}{2}$

$\tan\frac{\pi}{2}=\tan(\alpha+2\alpha+4\alpha)$

$=\frac{\tan\alpha+\tan 2\alpha+\tan 4\alpha-\tan\alpha.\tan 2\alpha.\tan 4\alpha}{1-\tan\alpha.\tan 2\alpha-\tan\alpha.\tan 4\alpha-\tan 2\alpha.\tan 4\alpha}$

tan α.tan 2α + tan2α. tan4α + tanα.tan4α = 1

8. In a □ABC, a = 5, b = 4 and $\cos(A-B)=\frac{31}{32}$, then c must be

Ans. 6

Sol. $\cos(A-B)=\frac{1-\tan^2\left(\frac{A-B}{2}\right)}{1+\tan^2\left(\frac{A-B}{2}\right)}\Rightarrow\tan\left(\frac{A-B}{2}\right)=\frac{1}{\sqrt{63}}$

Use Napier's analogy, we will get $\cot\frac{C}{2}=\frac{9}{\sqrt{63}}$

Then $\cos C=\frac{1}{8}, c=6$

9. In a triangle ABC, if r_1, r_2, r_3 are the ex-radius then $\frac{bc}{r_1}+\frac{ac}{r_2}+\frac{ab}{r_3}=k\frac{abc}{2\Delta}\left[\frac{s}{a}+\frac{s}{b}+\frac{s}{c}-3\right]$ then k is equal to

Ans. 2

Sol. $r_1=\frac{\Delta}{s-a}, r_2=\frac{\Delta}{s-b}, r_3=\frac{\Delta}{s-c}$ substitute this value and take abc common

L.H.S = $\frac{abc}{\Delta}\left[\frac{s-a}{a}+\frac{s-b}{b}+\frac{s-c}{c}\right]=\frac{abc}{\Delta}\left[\sum\frac{s}{a}-3\right]\Rightarrow k=2$

10. In □ABC, if I is incentre then AI + BI + CI ≥ □r then find □□

Ans. 6

Sol. $AI=r\,\mathrm{cosec}\left(\frac{A}{2}\right)$

$$AI+BI+CI=r\left(\cos ec\left(\frac{A}{2}\right)+\cos ec\left(\frac{B}{2}\right)+\cos ec\left(\frac{C}{2}\right)\right)$$

$$A.M \geq G.M$$

$$\cos ec\left(\frac{A}{2}\right)+\cos ec\left(\frac{B}{2}\right)+\cos ec\left(\frac{C}{2}\right)\geq\left(\cos ec\left(\frac{A}{2}\right)+\cos ec\left(\frac{B}{2}\right)+\cos ec\left(\frac{C}{2}\right)\right)^{1/3}$$

$$\geq 3(8)^{1/3}\geq 6$$

11. If $\alpha+\beta+\gamma=\pi$ and $\tan\left[\frac{\alpha+\beta-\gamma}{4}\right]\tan\left[\frac{\gamma+\alpha-\beta}{4}\right]\tan\left[\frac{\gamma+\beta-\alpha}{4}\right]=1$ then the value of $1+\cos\alpha+\cos\beta+\cos\gamma$ is K - 1 where K is

Key. 1

Sol. $A=\frac{\beta+\gamma-\alpha}{4}$ $B=\frac{\gamma+\alpha-\beta}{4}$ $C=\frac{\alpha+\beta-\gamma}{4}$

$$\Rightarrow \tan A\tan B\tan C=1$$

$$\frac{\sin A\sin B}{\cos A\cos B}=\frac{1}{\tan C}\Rightarrow\frac{\sin A\sin B-\cos A\cos B}{\sin A\sin B+\cos A\cos B}=\frac{1-\tan C}{1+\tan C}$$

$$\Rightarrow\frac{-\cos(A+B)}{\cos(A-B)}=\frac{\sin\left(\frac{\pi}{4}-C\right)}{\cos\left(\frac{\pi}{4}-C\right)}$$

$$\Rightarrow 2\sin\left(\frac{\pi}{4}-C\right)\cos(A-B)+2\cos\left(\frac{\pi}{4}-C\right)\cos(A+B)=0$$

$$\Rightarrow\sin\left(\frac{\pi}{4}-C+A-B\right)+\sin\left(\frac{\pi}{4}-C-A+B\right)+\cos\left(\frac{\pi}{4}-C+A+B\right)+\cos\left(\frac{\pi}{4}-C-A-B\right)=0.....(1)$$

$$A-B-C=\frac{\pi}{4}-\alpha\quad B-A-C=\frac{\pi}{4}-\beta\quad C-A-B=\frac{\pi}{4}-\gamma\quad A+B+C=\frac{\pi}{4}$$

$$(1)\Rightarrow\cos\alpha+\cos\beta+\cos\gamma+1=0$$

31. The value of $-2\left(\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}\right)$ is

Key. 1

Sol. $\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{6\pi}{7}=\frac{2\sin\frac{\pi}{7}\cos\frac{2\pi}{7}+2\sin\frac{\pi}{7}\cos\frac{4\pi}{7}+2\sin\frac{\pi}{7}\cos\frac{6\pi}{7}}{2\sin\frac{\pi}{7}}$

$$=\frac{\left(\sin\frac{3\pi}{7}-\sin\frac{\pi}{7}\right)+\left(\sin\frac{5\pi}{7}-\sin\frac{3\pi}{7}\right)+\left(\sin\pi-\sin\frac{5\pi}{7}\right)}{2\sin\frac{\pi}{7}}=\frac{\sin\frac{\pi}{7}}{2\sin\frac{\pi}{7}}=-\frac{1}{2}$$

32. If $A+B+C=180^0$, $\frac{\sin 2A+\sin 2B+\sin 2C}{\sin A+\sin B+\sin C}=k\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ then the value of k is

Key. 8

Sol. From conditional identities we have

$$\frac{\sin 2A+\sin 2B+\sin 2C}{\sin A+\sin B+\sin C}=\frac{4\sin A\sin B\sin C}{4\cos(A/2)\cos(B/2)\cos(C/2)}$$

$$=8\sin(A/2)\sin(B/2)\sin(C/2)$$

$\Rightarrow$ k = 8

33. If A, B and C are the angles of a triangle, then minimum value of

$\left(\tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}\right)$

Key. 1

Sol. We have $\frac{A}{2}+\frac{B}{2}+\frac{C}{2}=\frac{\pi}{2}$, so that

$$\tan\left(\frac{A}{2}+\frac{B}{2}\right)=\tan\left(\frac{\pi}{2}-\frac{C}{2}\right)\Rightarrow\frac{\tan\frac{A}{2}+\tan\frac{B}{2}}{1-\tan\frac{A}{2}\tan\frac{B}{2}}=\frac{1}{\tan\frac{C}{2}}$$

$$\Rightarrow\tan\frac{A}{2}\tan\frac{B}{2}+\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{C}{2}\tan\frac{A}{2}=1$$

$$\Rightarrow\tan^2\frac{A}{2}+\tan^2\frac{B}{2}+\tan^2\frac{C}{2}-1=\frac{1}{2}\left[\sum 2\tan^2\frac{A}{2}-\sum 2\tan\frac{A}{2}\tan\frac{B}{2}\right]$$

$$=\frac{1}{2}\left[\left(\tan\frac{A}{2}-\tan\frac{B}{2}\right)^2+\left(\tan\frac{B}{2}-\tan\frac{C}{2}\right)^2+\left(\tan\frac{C}{2}-\tan\frac{A}{2}\right)^2\right]\geq 0$$

34. The value of $\sqrt{3}\operatorname{cosec}20^0-\sec 20^0$ is

Key. 4

Sol. $L.H.S=\frac{\sqrt{3}}{\sin 20^0}-\frac{1}{\cos 20^0}=\frac{\sqrt{3}\cos 20^0-\sin 20^0}{\sin 20^0\cos 20^0}$

$$=\frac{4.\left(\frac{\sqrt{3}}{2}\cos 20^0-\frac{1}{2}\sin 20^0\right)}{2\sin 20^0\cos 20^0}$$

$$= 4.\frac{\sin 60^0 \cos 20^0 - \cos 60^0 \sin 20^0}{\sin 40^0} = 4\frac{\sin 40^0}{\sin 40^0} = 4$$

35. $\sin 12^0 . \sin 48^0 . \sin 54^0 = \frac{1}{a}$ then the value of a is

Key. 8

Sol. Let $\theta = 12^0$

$$\text{L.H.S} = \frac{1}{\sin 72^0} \sin 12^0 \sin 48^0 \sin 72^0 \sin 54^0$$

$$= \frac{1}{4} \frac{\sin 3(12^0)\sin 54^0}{\sin 72^0} = \frac{\sin 36^0 \sin 54^0}{8 \sin 36^0 \cos 36^0} = \frac{\cos 36^0}{8 \cos 36^0} = \frac{1}{8}$$

36. If $\alpha + \beta + \gamma = \pi$ and $\tan\left(\frac{\beta + \gamma - \alpha}{4}\right)\tan\left(\frac{\gamma + \alpha - \beta}{4}\right)\tan\left(\frac{\alpha + \beta - \gamma}{4}\right) = 1$ then the value of $1 + \cos\alpha + \cos\beta + \cos\gamma$ is

Key. 0

Sol. Let $A = \frac{\beta + \gamma - \alpha}{4}$; $B = \frac{\gamma + \alpha - \beta}{4}$; $C = \frac{\alpha + \beta - \gamma}{4}$

tan A tan B tan C = 1

or $\frac{\sin A \sin B}{\cos A \cos B} = \frac{1}{\tan C}$ or $\frac{\sin A \sin B - \cos A \cos B}{\sin A \sin B + \cos A \cos B} = \frac{1 - \tan C}{1 + \tan A}$

or, $-\frac{\cos(A+B)}{\cos(A-B)} = \frac{\sin\left(\frac{\pi}{4} - C\right)}{\cos\left(\frac{\pi}{4} - C\right)}$

or $2\sin\left(\frac{\pi}{4} - C\right)\cos(A-B) + 2\cos\left(\frac{\pi}{4} - C\right)\cos(A+B) = 0$

or $\sin\left(\frac{\pi}{4}\text{-C+A-B}\right) + \sin\left(\frac{\pi}{4}\text{-C-A+B}\right) + \cos\left(\frac{\pi}{4}\text{-C+A+B}\right) + \cos\left(\frac{\pi}{4}\text{-C-A-B}\right) = 0$ ....(1)

$$A - B - C = \frac{\beta + \gamma - \alpha - \gamma - \alpha + \beta - \alpha - \beta + \gamma}{4} = \frac{\beta + \gamma - 3\alpha}{4} = \frac{\pi - 4\alpha}{4} = \frac{\pi}{4} - \alpha$$

Similarly $B - A - C = \frac{\pi}{4} - \beta$ and $C - A - B = \frac{\pi}{4} - \gamma$

and $C + A + B = \frac{\alpha + \beta + \gamma}{4} = \frac{\pi}{4}$

$\therefore$ Equation (1) reduces to,

$$\sin\left\{\frac{\pi}{4} + (\text{A-B-C})\right\} + \sin\left\{\frac{\pi}{4} + (\text{B-C-A})\right\} + \cos\left\{\frac{\pi}{4} - (\text{C-A-B})\right\} + \cos\left\{\frac{\pi}{4} - (\text{C+A+B})\right\} = 0$$

or $\sin\left(\frac{\pi}{4} + \frac{\pi}{4} - \alpha\right) + \sin\left(\frac{\pi}{4} + \frac{\pi}{4} - \beta\right) + \cos\left(\frac{\pi}{4} - \frac{\pi}{4} + \gamma\right) + \cos\left(\frac{\pi}{4} - \frac{\pi}{4}\right) = 0$

or $\cos\alpha + \cos\beta + \cos\gamma + 1 = 0$.

37. In an acute angled triangle ABC, minimum value of $\sum \tan A \tan B$ is

Key. 9

Sol. $(\tan A \tan B - 1) + (\tan B \tan C - 1) + (\tan C \tan A - 1)$

$$\Rightarrow \frac{\tan A+\tan B}{\tan C}+\frac{\tan B+\tan C}{\tan A}+\frac{\tan C+\tan A}{\tan B}$$

$$\Rightarrow \left(\frac{\tan A}{\tan C}+\frac{\tan C}{\tan A}\right)+\left(\frac{\tan A}{\tan B}+\frac{\tan B}{\tan A}\right)+\left(\frac{\tan B}{\tan C}+\frac{\tan C}{\tan B}\right)\geq 6$$

$$\therefore \sum \tan A \tan B \geq 9$$

38. The value of $\sum_{r=0}^{10}\cos^3\frac{r\pi}{3}$ is equal to $\frac{-a}{b}$ then the value of b is (where g. c. d of (a, b)is 1)

Key. 8

Sol. $\sum_{r=0}^{10}\cos^3\frac{r\pi}{3}=\frac{1}{4}\sum_{r=0}^{10}\left(3\cos\frac{r\pi}{3}+\cos r\pi\right)$

$$=\frac{1}{4}\left[3\left(\cos 0+\cos\frac{\pi}{3}+.....+\cos\frac{10\pi}{3}+(1-1+.....-1=1)\right)\right]$$

$$=\frac{\frac{3}{4}\left[\cos\left(\frac{10\pi}{6}\right)\sin\left(\frac{11\pi}{6}\right)\right]}{\sin\frac{\pi}{6}}+\frac{3}{4}=-\frac{1}{8.}$$

Trigonometry
Matrix-Match Type

1. Match the following

Column I	Column II
A. If $\tan\theta$ is the G.M. between $\sin\theta$ and $\cos\theta$ then $2-4\sin^2\theta+3\sin^4\theta-\sin^6\theta=$	P. 1
B. $\sqrt{3}\cot 20^0-4\cos 20^0=$	Q. 0
C. $\cot 16^0\cot 44^0+\cot 44^0\cot 76^0-\cot 76^0\cot 16^0=$	R. 3
D. $\sum_{r=1}^{9}\sin^2\left(\frac{r\pi}{18}\right)=$	S. 5

KEY. A – P; B – P; C – R; D – S

SOL. (A) $\tan^2\theta=\sin\theta\cos\theta \Rightarrow \sin\theta=\cos^3\theta$

$$\therefore (1-\sin^2\theta)+(1-3\sin^2\theta)+3\sin^4\theta-\sin^6\theta$$

$$=\cos^2\theta+(1-\sin^2\theta)^3=\cos^2\theta+\cos^6\theta=\cos^2\theta+\sin^2\theta=1$$

(B) $\sin 40^0=\sin(60^0-20^0)$

$$2\sin 20^0\cos 20^0=\frac{\sqrt{3}}{2}\cos 20^0-\frac{1}{2}\sin 20^0$$

$$4\cos 20^0=\sqrt{3}\cot 20^0-1$$

(C) $\frac{3+\cot 76^0\cot 16^0}{\cot 76^0+\cot 16^0}=\frac{3\sin 76^0\sin 16^0+\cos 76^0\cos 16^0}{\sin(76^0+16^0)}$

$$=\frac{2\sin 76^0\sin 16^0+\cos(76^0-16^0)}{\sin(76^0+16^0)}=\frac{\cos 60^0-\cos 92^0+\cos 60^0}{\sin 92^0}=\frac{1-\cos 92^0}{\sin 92^0}=\tan 46^0$$

$=\cot 44^0$

(D) $\sin^2\left(\frac{\pi}{18}\right)+\sin^2\left(\frac{2\pi}{18}\right)+....+\sin^2\left(\frac{\pi}{2}\right)$ = 5

2. Match the following Trignometric ratios with the equations whose one of the roots is given

Column I	Column II
A. $\cos 20^0$	P. $x^3 - 3x^2 - 3x + 1 = 0$
B. $\sin 10^0$	Q. $32x^5 - 40x^3 + 10x - 1 = 0$
C. $Tan 15^0$	R. $8x^3 - 6x - 1 = 0$
D. $\sin 6^0$	S. $8x^3 - 6x + 1 = 0$

KEY. A – R; B – S; C – P; D – P

SOL. A) $A = 20^0 \Rightarrow 3A = 60^0 \Rightarrow \cos 3A = \frac{1}{2}$

B) $A = 20^0 \Rightarrow 3A = 60^0 \Rightarrow 8x^3 - 6x - 1 = 0$ where $x = \cos 20^0$

C) $A = 10^0 \Rightarrow \sin 3A = \frac{1}{2} \Rightarrow 8x^3 - 6x + 1 = 0$ where $x = \sin 10^0$

D) $A = 6^0 \Rightarrow \sin 5A = \frac{1}{2} \Rightarrow 32x^5 - 40x^3 + 10x - 1 = 0$ where $x = \sin 6^0$

3. Match the following

Column I	Column II
A. The maximum value of $\cos(2A+\theta) + \cos(2B+\theta)$ $(\theta \in R$ and A,B are constants)	P. $2\sin(A+B)$
B. Maximum value of $\cos 2A + \cos 2B$ $\left(A, B \in \left(0, \frac{\pi}{2}\right), A+B\, is\, cons\tan t\right)$	Q. $2\sec(A+B)$
C. Minimum value of $\sec 2A + \sec 2B$ $\left(A, B \in \left(0, \frac{\pi}{4}\right), A+B\, is\, cons\tan t\right)$	R. $2\cos(A+B)$
D. Minimum value of $\sqrt{Tan\theta + \cot\theta - 2\cos(2A+2B)}$ $(\theta \in R$, A,B are constants)	S. $2\cos(A-B)$

KEY. A – S; B – R; C – Q; D – P

SOL. A) $\cos(2A+\theta) + \cos(2B+\theta) = 2\cos(A+B+\theta)\cos(A-B) \le 2\cos(A-B)$

B) $\cos 2A + \cos 2B = 2\cos(A+B)\cos(A-B) \le 2\cos(A+B)$

C) $y = \sec x$ always concave up $\quad \therefore \dfrac{\sec 2A + \sec 2B}{2} \ge \sec(A+B)$

D) $\sqrt{Tan\theta + \cot\theta - 2\cos(2A+2B)} = \sqrt{(Tan\theta - \sqrt{\cot\theta})^2 + 4\sin^2(A+B)}$

$$\ge 2\sin(A+B)$$

4. Match the following: -

	Column I		Column II
(A)	If $\sin\theta = 3\sin(\theta+2\alpha)$, then the value of $\tan(\theta+\alpha)+2\tan\alpha$ is	(p)	0
(B)	If p sin θ + q cos θ = a and p cos θ – q sin θ = b then $\dfrac{p+a}{q+b}+\dfrac{q-b}{p-a}+1$ is equal to	(q)	1
(C)	The value of the expression $\cos\dfrac{\pi}{7}\cos\dfrac{2\pi}{7}\cos\dfrac{10\pi}{7}$ $-\sin\dfrac{\pi}{14}\sin\dfrac{3\pi}{14}\sin\dfrac{5\pi}{14}$ is	(r)	sec θ
(D)	If $\sec\theta+\tan\theta=1$, then one root of the equation $(a-2b+c)x^2+(b-2c+a)x+(c-2a+b)=0$ is	(s)	$-\dfrac{1}{4}$
		(t)	-1/2

Key. A – p ; B – q ; C - s ; D - qr

Sol. (A) Given, $\sin\theta = 3\sin(\theta+2\alpha)$

$\Rightarrow \sin(\theta+\alpha-\alpha) = 3\sin(\theta+\alpha+\alpha)$

$\Rightarrow \sin(\theta+\alpha)\cos\alpha - \cos(\theta+\alpha)\sin\alpha$

$= 3\sin(\theta+\alpha)\cos\alpha + 3\cos(\theta+\alpha)\sin\alpha$

$\Rightarrow -2\sin(\theta+\alpha)\cos\alpha = 4\cos(\theta+\alpha)\sin\alpha$

$\Rightarrow -\dfrac{\sin(\theta+\alpha)}{\cos(\theta+\alpha)} = \dfrac{2\sin\alpha}{\cos\alpha}$

$\Rightarrow -\tan(\theta+\alpha) = 2\tan\alpha$

$\Rightarrow \tan(\theta+\alpha)+2\tan\alpha = 0$

(B) We have, $p\sin\theta + q\cos\theta = a$ ….. (1)

And, $p\cos\theta - q\sin\theta = b$ ….. (2)

Squaring (1) and (2), and then adding, we get

$$(p\sin\theta+q\cos\theta)^2+(p\cos\theta-q\sin\theta)^2=a^2+b^2$$

$$\Rightarrow\ p^2(1)+q^2(1)-a^2-b^2=0$$

$$\Rightarrow\ (p^2-a^2)+(q^2-b^2)=0$$

$$\Rightarrow\ (p+a)(p-a)+(q+b)(q-b)=0$$

$$\Rightarrow\ \frac{p+a}{q+b}+\frac{q-b}{p-a}=0$$

(C) $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}$

$$=\cos\left(\frac{\pi}{2}-\frac{\pi}{14}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{14}\right)\cos\left(\frac{\pi}{2}-\frac{5\pi}{14}\right)$$

$$=\cos\left(\frac{3\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{\pi}{7}\right)$$

$$=-\cos\left(\frac{\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right)$$

Also, $\cos\frac{10\pi}{7}=\cos\frac{4\pi}{7}$

So, $\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{10\pi}{7}-\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}$

$$=2\cos\left(\frac{\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right)=-\frac{1}{4}$$

(D) Clearly, $\sec\theta-\tan\theta=1$

also 1 satisfy the given equation

so the roots of the given equation are 1 & $\sec\theta$.

5. Match the following: -

Column – I		Column II	
(A)	The maximum value of $\sin(\cos x)+\cos(\sin x),\ x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ is	(p)	cos (cos 1)
(B)	The minimum value of $\sin(\cos x)+\cos(\sin x),\ x\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ is	(q)	1 + cos 1
(C)	The maximum value of	(r)	cos 1

	cos (cos(sin x)) is		
(D)	The minimum value of cos(cos (sin x))	(s)	1 + sin 1

KEY : A) $\rightarrow$ S,

(B) $\rightarrow$ R,

(C) $\rightarrow$ P,

(D) $\rightarrow$ Q

SOL : (A) $\rightarrow$ (s); (B) $\rightarrow$ (r); (C) $\rightarrow$ (p), (D) $\rightarrow$ (r)

Let f(x) = sin (cos x) + cos (sin x)

f is an even function. We can take $x \in \left[0, \frac{\pi}{2}\right]$. In $\left[0, \frac{\pi}{2}\right]$, sin x is increasing and cos x is decreasing.

Hence f is a decreasing function. Therefore, maximum value of f is f(0) = sin 1 + 1 and minimum value is

f(π/2) = 0 + cos 1.

Let g(x) = cos(cos(sin x)). Obviously g is an even periodic function of period π. Hence g takes all of its values for $x \in \left[0, \frac{\pi}{2}\right]$.

It can be seen that g is an increasing function in $\left[0, \frac{\pi}{2}\right]$. So maximum value of g = g(π/2) = cos (cos 1), and minimum value of g = g(0) = cos 1.

6.

	Column I		Column II
(A)	If $\sin\theta = 3\sin(\theta+2\alpha)$, then the value of $\tan(\theta+\alpha)+2\tan\alpha$ is	(p)	0
(B)	If p sin θ + q cos θ = a and p cos θ – q sin θ = b then $\frac{p+a}{q+b}+\frac{q-b}{p-a}+1$ is equal to	(q)	1
(C)	The value of the expression $\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{10\pi}{7}$ $-\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}$ is	(r)	sec θ
(D)	If $\sec\theta+\tan\theta=1$, then one root of the equation $(a-2b+c)x^2+(b-2c+a)x+(c-2a+b)=0$ is	(s)	$-\frac{1}{4}$
		(t)	-1/2

Key. (A-p) , (B-q),

(C-s), (D-q,r)

Sol. (A) Given, $\sin\theta = 3\sin(\theta+2\alpha)$

$\Rightarrow \sin(\theta+\alpha-\alpha) = 3\sin(\theta+\alpha+\alpha)$

$\Rightarrow \sin(\theta+\alpha)\cos\alpha - \cos(\theta+\alpha)\sin\alpha$

$= 3\sin(\theta+\alpha)\cos\alpha + 3\cos(\theta+\alpha)\sin\alpha$

$\Rightarrow -2\sin(\theta+\alpha)\cos\alpha = 4\cos(\theta+\alpha)\sin\alpha$

$$\Rightarrow \quad -\frac{\sin(\theta+\alpha)}{\cos(\theta+\alpha)} = \frac{2\sin\alpha}{\cos\alpha}$$

$$\Rightarrow \quad -\tan(\theta+\alpha) = 2\tan\alpha$$

$$\Rightarrow \quad \tan(\theta+\alpha) + 2\tan\alpha = 0$$

(B) We have, $p\sin\theta + q\cos\theta = a$ ….. (1)

And, $p\cos\theta - q\sin\theta = b$ ….. (2)

Squaring (1) and (2), and then adding, we get

$$(p\sin\theta + q\cos\theta)^2 + (p\cos\theta - q\sin\theta)^2 = a^2 + b^2$$

$$\Rightarrow \quad p^2(1) + q^2(1) - a^2 - b^2 = 0$$

$$\Rightarrow \quad (p^2 - a^2) + (q^2 - b^2) = 0$$

$$\Rightarrow \quad (p+a)(p-a) + (q+b)(q-b) = 0$$

$$\Rightarrow \quad \frac{p+a}{q+b} + \frac{q-b}{p-a} = 0$$

(C) $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}$

$$= \cos\left(\frac{\pi}{2} - \frac{\pi}{14}\right)\cos\left(\frac{\pi}{2} - \frac{3\pi}{14}\right)\cos\left(\frac{\pi}{2} - \frac{5\pi}{14}\right)$$

$$= \cos\left(\frac{3\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{\pi}{7}\right)$$

$$= -\cos\left(\frac{\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right)$$

Also, $\cos\frac{10\pi}{7} = \cos\frac{4\pi}{7}$

So, $\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{10\pi}{7} - \sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}$

$$= 2\cos\left(\frac{\pi}{7}\right)\cos\left(\frac{2\pi}{7}\right)\cos\left(\frac{4\pi}{7}\right) = -\frac{1}{4}$$

(D) Clearly, $\sec\theta - \tan\theta = 1$

also 1 satisfy the given equation

so the roots of the given equation are 1 & $\sec\theta$.

7. Match the statements/expressions in Column I with the open intervals in Column II

	Column I		Column II
(A)	If $3\cos\theta + 4\sin\theta = 5$ then $3\sin\theta - 4\cos\theta =$	(p)	$\frac{1}{6}$
(B)	In a triangle ABC, if the ex–radii r_1, r_2 , r_3 are in H.P., then $\frac{a+c}{b} =$	(q)	2
(C)	In a triangle ABC, right angled at A, $\left[\tan^{-1}\left(\frac{c+a}{b}\right) + \tan^{-1}\left(\frac{a+b}{c}\right)\right] =$ (where [.] denotes greatest integer function)	(r)	3
(D)	If a circle is inscribed in an equilateral triangle of side 1, then area of the square inscribed in the circle is…….	(s)	0

		(t)	6

Key. (A–s), (B –q), (C–q), (D –p)

Sol. (A)

$5\cos(\theta - \alpha) = 5 \therefore \theta = \alpha, \quad \cos\alpha = 3/5$

$3\sin\theta - 4\cos\theta = 5\sin(\theta - \alpha) = 0$

(B) $r_2 = \dfrac{2r_1r_3}{r_1 + r_3}$ put $r_1 = \dfrac{\Delta}{s-a}$ etc.

$\Rightarrow a + c = 2b$

(C) $\left[\pi + \tan^{-1}\dfrac{\left(\dfrac{c+a}{b}\right)+\left(\dfrac{a+b}{c}\right)}{1-\left(\dfrac{c+a}{b}\right)\left(\dfrac{a+b}{c}\right)}\right] = \left[\dfrac{3\pi}{4}\right] = 2$

(D) we have, $s = \dfrac{1}{2}(a+b+c) = \dfrac{3a}{2}$

$\Delta = \dfrac{\sqrt{3}a^2}{4} \therefore r = \dfrac{\Delta}{s} = \dfrac{a}{2\sqrt{3}}$

If x is the length of a side of the square inscribed in incircle of the triangle then

$x^2 + x^2 = (\text{diameter})^2 = (2r)^2$

$\Rightarrow x^2 = 2r^2 = \dfrac{a^2}{6}$

$\Rightarrow$ Area of square $= \dfrac{a^2}{6} = \dfrac{1}{6}\ [\because a = 1]$

8. Match the following

Let $f(n) = \sum_{k=0}^{n} {}^{n}C_k \cos\left(\dfrac{2k\pi}{n}\right)$ and $g(n) = \sum_{k=0}^{n-1} {}^{n-1}C_k \cos\left(\dfrac{2k\pi}{n}\right)$, then

	Column –I		Column –II
(A)	f(n) is	(p)	A rational number
(B)	f(6) is	(q)	An irrational number
(C)	g(6) is	(r)	–27
(D)	g(8) is	(s)	2g(n + 2)
		(t)	2g(n)

Key. (A–t), (B–p, r), (C–p), (D–q)

Sol. $f(6) = \sum_{k=0}^{6} {}^{6}C_k \cos\left(\dfrac{k\pi}{3}\right)$ = 1 + 6 (1/2) + 15 (–1/2) + 20 (–1) + 15 (–1/2) + 6(1/2) + 1 = –27

$g(8) = \sum_{k=0}^{7} {}^{7}C_k \cos\left(\dfrac{k\pi}{4}\right)$ = irrational f(n) = $\sum_{k=0}^{n}\left[\left({}^{n-1}C_{k-1}+{}^{n-1}C_k\right)\right]\cos\frac{2k\pi}{x}=\sum_{k=1}^{n-1}\left({}^{n-1}C_{k-1}\cos\frac{2k\pi}{\pi}\right)+1+g(n)$

= (g (n) – 1) + (1 + g(n) = 2g(n).

9. Match the following: -

Column – 1		Column – 2	
(A)	If $\Delta = a^2 - (b-c)^2$, where Δ is the area of the triangle ABC, then tan A is equal to	(p)	$\frac{1}{2}$
(B)	In a ΔABC, given that tanA : tan B : tan C = 3 : 4 : 5, then the value of sin A sin B sin C is	(q)	$\frac{8}{15}$
(C)	Let $f(\theta)=\frac{1-\sin 2\theta+\cos 2\theta}{2\cos 2\theta}$ and $\alpha+\beta=\frac{5\pi}{4}$, then the value of $f(\alpha)f(\beta)$ is	(r)	$\frac{\pi}{4}$
(D)	The sum of infinite terms of the series $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{2}{9}+\tan^{-1}\frac{4}{33}+....$ is equal to	(s)	$\frac{2\sqrt{5}}{7}$

Key. A - (q); B - (s); C - (p); D - (r)

Sol. (A) We have, $\Delta = a^2-(b-c)^2$

$\Rightarrow \quad \Delta = a^2-b^2-c^2+2bc$

$\Rightarrow \quad b^2+c^2-a^2=2bc-\Delta$

$\Rightarrow \quad 2bc\cos A = 2bc-\frac{1}{2}bc\sin A$

$\Rightarrow \quad 4\cos A+\sin A=4$

$\Rightarrow \quad 4\left(1-2\sin^2\frac{A}{2}\right)+2\sin\frac{A}{2}\cos\frac{A}{2}=4$

$\Rightarrow \quad \tan\frac{A}{2}=\frac{1}{4} \Rightarrow \tan A=\frac{8}{15}$

(B) tan A = 3k, tan B = 4k, tan C = 5 k

$\therefore$ tan A + tan B + tan C = tan A tan B tan C $\Rightarrow$ 12k = 60 k^3 $\Rightarrow$ k = $\frac{1}{\sqrt{5}}$

$\therefore \quad \tan A=\frac{3}{\sqrt{5}},\tan B=\frac{4}{\sqrt{5}},\tan C=\sqrt{5}$

$\therefore \quad \sin A\sin B\sin C=\frac{2\sqrt{5}}{7}$

(C) we have,

$f(\theta)=\frac{2\cos^2\theta-2\sin\theta\cos\theta}{2\left(\cos^2\theta-\sin^2\theta\right)}=\frac{1}{1+\tan\theta}$

$\therefore \quad \tan(\alpha+\beta)=1$

$$f(\alpha) = \frac{1}{1+\tan\alpha},\ f(\beta) = \frac{1}{1+\tan\beta}$$

$\therefore \quad f(\alpha)\,f(\beta) = \frac{1}{2}$

(D) $T_n = \tan^{-1}\left(\frac{2^n - 2^{n-1}}{1+2^n.2^{n-1}}\right) = \tan^{-1}2^n - \tan^{-1}2^{n-1}$

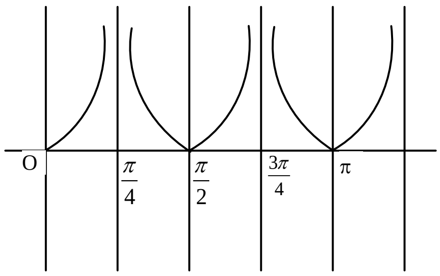
$\therefore \quad S_n = \tan^{-1}2^n - \frac{\pi}{4} \qquad\qquad \Rightarrow \quad S_n = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

10. Match the following

Column – Id	Column – II
A) The number of solutions of the equation $\lvert\tan 2x\rvert = \sin x,\ x\in[0,\pi]$	P) 1
B) The value of $4\tan\frac{\pi}{16} - 4\tan^3\frac{\pi}{16} + 6\tan^2\frac{\pi}{16} - \tan^4\frac{\pi}{16} + 1$ is	Q) 4
C) If the equation tan(pcotx) = cot(p tanx) has a solution in $(0,\pi)-\left\{\frac{\pi}{2}\right\}$, then $\frac{4}{\pi}P_{\max}$ is	R) 3
D) The value of $\frac{2x}{\pi}$ in $[0, 2\pi]$ if $5^{\cos^2 2x + 2\sin^2 x} + 5^{2\cos^2 x + \sin^2 2x} = 126$ has a solution	S) 2

Ans. A – Q ; B – S ; C – P ; D – P, R

Sol. A) Clearly number of solutions of $\lvert\tan 2x\rvert = \sin x$ in $[0,\pi]$ are 4.



B) $$\tan 4A = \frac{2\tan 2A}{1-\tan^2 2A} = \frac{\frac{4\tan A}{1-\tan^2 A}}{1-\left(\frac{2\tan A}{1-\tan^2 A}\right)^2}$$

$$\tan 4A = \frac{4\tan A\left(1-\tan^2 A\right)}{1+\tan^4 A - 6\tan^2 A}$$

$4\tan A - 4\tan^3 A + (6\tan^2 A - \tan^4 A - 1)\tan 4A = 0$

If $A = \frac{\pi}{16}$

$$4\tan\frac{\pi}{16} - 4\tan^3\frac{\pi}{6} + 6\tan^2\frac{\pi}{16} - \tan^4\frac{\pi}{16} - 1 = 0$$

$\therefore$ required value is 2

C) tan (pcotx) = cot(ptanx)

$$\tan(p\cot x)=\tan\left(\frac{\pi}{2}-p\tan x\right)$$

$$p\cot x=n\pi+\frac{\pi}{2}-p\tan x$$

$$p=\frac{n\pi+\frac{\pi}{2}}{\tan x+\cot x}=\frac{\pi}{2}\sin x\cos x \qquad \because x\in[0,\pi]$$

$$P_{\max}=\frac{\pi}{4}$$

$$\frac{4P_{\max}}{\pi}=1$$

D) $5^{\cos^2 2x+2\sin^2 x}+5^{2\cos^2 x+\sin^2 2x}=126$

$$5^y+5^{3-y}=126$$

$$5^y+\frac{125}{5^y}=126$$

$$\Rightarrow 5^y=125,1$$

y= 3, 0

$$\cos^2 2x+2\sin^2 x=3$$

$$\Rightarrow x=\frac{\pi}{2},\ \frac{3\pi}{2}$$

$\cos^2 2x + 2\sin^2 x \neq 0$

$$\frac{2\pi}{\pi}=1,3$$

11. Match the following

Column – I	Column – II
A) Number of solution of equation $\sin^{-1}x+\cos^{-1}x^2=\pi/2$ is	p) 1
B) The number of ordered pairs (x, y) satisfying $\frac{\sin^{-1}x}{x}=\frac{\sin^{-1}y}{y}=2$	q) 2
C) Number of solution of equation $\cos(\cos x)=\sin(\sin x)$ is	r) 0
D) Number of solution of equation $\tan\left(x+\frac{\pi}{6}\right)=2\tan x$	s) 3

Ans. A – Q ; B – R ; C – R ; D – R

Sol. A) $\sin^{-1}x+\cos^{-1}x^2=\pi/2 \Rightarrow \cos^{-1}x^2=\cos^{-1}x$

$$\Rightarrow x=0,1$$

B) $\frac{\sin^{-1}x}{x}$ is increasing for $x\geq 0$ and decreasing for $x\leq 0$

$\Rightarrow \frac{\sin^{-1}x}{x}>1$ and $\frac{\sin^{-1}y}{y}>1 \Rightarrow \frac{\sin^{-1}x}{x}=\frac{\sin^{-1}y}{y}=2$ has no solution

C) $\cos\cos x=\cos\left(\frac{\pi}{2}-\sin x\right)$

$$\Rightarrow \cos x\pm\sin x=2n\pi\pm\pi/2$$

$\Rightarrow$ no solution

D) $\tan\left(x+\frac{\pi}{6}\right)=2\tan x$

Let tan x = y

$\Rightarrow 2y^2-\sqrt{3}y+1=0$

$\Rightarrow$ no solution

12. Match the following

Column – I	Column – II
A) If $\left(\tan^{-1}\frac{x}{3}\right)^2-4\tan^{-1}\frac{x}{3}-5=0$, then x =	P) 2
B) If $\left\{\tan^{-1}(3x+2)\right\}^2+2\tan^{-1}(3x+2)=0$, then x =	Q) – 3 tan 1
C) If $3\left(\tan^{-1}x\right)^2-4\pi\tan^{-1}x+\pi^2=0$, then x =	R) 1
D) Given that $0\le x\le\frac{1}{2}$, the value of $\tan\left[\sin^{-1}\left(\frac{x}{\sqrt{2}}+\sqrt{\frac{1-x^2}{2}}\right)-\sin^{-1}x\right]$	S) $\sqrt{3}$
	T) $-\frac{2}{3}$

Ans. A – Q ; B – T ; C – S ; D – R

Sol. A) $\tan^{-1}\frac{x}{3}=\frac{4\pm\sqrt{16+20}}{2}=5\ or\ -1 \therefore x=-3\tan 1$

B) $\tan^{-1}(3x+2)=-2\ or\ 0\Rightarrow x=-\frac{2}{3}$

C) $\tan^{-1}x=\frac{4\pi\pm\sqrt{16\pi^2-12\pi^2}}{6}=\pi\ or\ \frac{\pi}{3}\Rightarrow x=\sqrt{3}$

D) $0\le x\le\frac{1}{2}\Rightarrow 0\Rightarrow\sin y=x\le\frac{1}{2}\Rightarrow 0\le y\le\frac{\pi}{6}$

$\sin^{-1}\left(-\frac{1}{\sqrt{2}}\sin y+\frac{\cos y}{\sqrt{2}}\right)=\sin^{-1}\left(y+\frac{\pi}{4}\right)$ if $0\le y\le\frac{\pi}{6}$

$\tan\left[\sin^{-1}\left(\frac{x}{\sqrt{2}}+\sqrt{\frac{1-x^2}{2}}\right)-\sin^{-1}x\right]=\tan^{-1}\left(y+\frac{\pi}{4}-y\right)=1$

13. Match the following

Column – I	Column – II
A) If twice the square on the diameter of a circle is equal to the sum of the squares on the sides of the inscribed triangle ABC, then $\sin^2 A+\sin^2 B+\sin^2 C=$	P) 27
B) If in a triangle ABC, $\frac{\sin A}{4}=\frac{\sin B}{5}=\frac{\sin C}{6}$, then the value of cosA + cosB + cosc =	Q) 10
C) If a, b, c, d are the sides of a quadrilateral, the minimum value of	R) 2

$\dfrac{a^2+b^2+c^2}{d^2}$ is	
D) If any triangle ABC, $\Pi\left(\dfrac{\sin^2 A+\sin A+1}{\sin A}\right)$ is always greater than	S) $\dfrac{23}{16}$
	T) $\dfrac{1}{3}$

Ans. A – R ; B – S ; C – T ; D – P,Q,R,S,T

Sol. A)

$$2\cdot(2R)^2=a^2+b^2+c^2=4R^2\left(\sin^2 A+\sin^2 B+\sin^2 C\right)\Rightarrow \sin^2 A+\sin^2 B+\sin^2 C=2$$

B) $$\frac{\sin A}{4}=\frac{\sin B}{5}=\frac{\sin C}{6}\Rightarrow \frac{a}{4}=\frac{b}{5}=\frac{c}{6}\Rightarrow \cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{3}{4},\cos B=\frac{9}{16},\cos C=\frac{1}{8}$$

$$\cos A+\cos B+\cos C=\frac{23}{16}$$

C) $$(a-b)^2+(b-c)^2+(c-a)^2\ge 0\Rightarrow 2\left(a^2+b^2+c^2\right)\ge 2(ab+bc+ca)$$

$$\Rightarrow 3\left(a^2+b^2+c^2\right)\ge a^2+b^2+c^2+2ab+2bc+2ca$$

$$\Rightarrow 3\left(a^2+b^2+c^2\right)<d^2$$

$$\Rightarrow \frac{a^2+b^2+c^2}{d^2}>\frac{1}{3}$$

D) $$\Pi\left(\frac{\sin^2 A+\sin A+1}{\sin A}\right)=\Pi\left(\sin A+\frac{1}{\sin A}+1\right)=\Pi\left\{\left(\sqrt{\sin A}-\frac{1}{\sqrt{\sin A}}\right)^2+3\right\}$$

So, $$\left(\sqrt{\sin A}-\frac{1}{\sqrt{\sin A}}\right)^2+3>3$$

$$\therefore \left\{\left(\sqrt{\sin A}-\frac{1}{\sqrt{\sin A}}\right)^2+3\right\}>3\cdot 3\cdot 3=27$$

29. $\cos\alpha+\cos\beta=\mathrm{a},\ \sin\alpha+\sin\beta=\mathrm{b}$

Column – I	Column – II
a) $\cos(\alpha+\beta)$	p) $2ab/\left(a^2+b^2\right)$
b) $\sin(\alpha+\beta)$	q) b/a
c) $\cos(\alpha-\beta)$	r) $\left(a^2-b^2\right)/\left(a^2+b^2\right)$
d) $\tan\dfrac{\alpha+\beta}{2}$	s) $\left(a^2+b^2-2\right)/2$
	t) $\left(\dfrac{a^2+b^2}{2}\right)-1$

Key. **A $\rightarrow$R,B$\rightarrow$P, C$\rightarrow$S,T D$\rightarrow$Q**

Sol. $$a=2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2},\ b=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}\Rightarrow \tan\frac{\alpha+\beta}{2}=\frac{b}{a}$$

$$\Rightarrow \cos(\alpha+\beta)=\frac{1-\left(b^2/a^2\right)}{1+\left(b^2/a^2\right)}=\frac{a^2-b^2}{a^2+b^2}$$

and $\sin(\alpha+\beta)=\frac{2(b/a)}{1+(b^2/a^2)}=\frac{2ab}{a^2+b^2}$

$a^2+b^2=\sin^2\alpha+\sin^2\beta+\cos^2\alpha+\cos^2\beta+2\cos\alpha\cos\beta+2\sin\alpha\sin\beta$

$\Rightarrow a^2+b^2-2=2\cos(\alpha-\beta)$

$\Rightarrow \cos(\alpha-\beta)=\frac{a^2+b^2-2}{2}$

30. Given $\frac{\sin x}{a}=\frac{\cos x}{b}=\frac{\tan x}{c}=k$

Column – I	Column – II
a) a^2+b^2	p) $\frac{1}{b^2k^4}$
b) $a^2+b^2+c^2$	q) $\frac{1}{k^2}$
c) bc	r) $\frac{1}{ak}$
d) $\frac{1}{ck}+\frac{ak}{1+bk}$	s) $\frac{a}{k}$
	t) $\frac{1}{b^2k^4}-c^2$

Key. **A$\rightarrow$Q,T B$\rightarrow$P, C$\rightarrow$S,D$\rightarrow$R**

Sol. Conceptual