Quadratic Equations & Theory of Equations

Single Correct Answer Type

1. Let α and β be the roots of $x^2-6x-2=0$ with $\alpha>\beta$ if $a_n=\alpha^n-\beta^n$ for $n\geq 1$ then the value of $\frac{a_{10}-2a_8}{3a_9}=$

1) 1 2) 2
3) 3 4) 4

Key. 2

Sol. $\alpha^2-6\alpha-2=0$ $\beta^2-6\beta-2=0$

$\Rightarrow\alpha^{10}-6\alpha^9-2\alpha^8=0$ ....(1)

$\Rightarrow\beta^{10}-6\beta^9-2\beta^8=0$ ....(2)

subtract (2) from (1)

2. If a,b,c are positive real numbers such that $a+b+c=1$ then the least value of $\frac{(1+a)(1+b)(1+c)}{(1-a)(1-b)(1-c)}$ is

1) 16 2) 8
3) 4 4) 5

Key. 2

Sol. $a=1-b-c$

$\Rightarrow 1+a=(1-b)+(1-c)\geq 2\sqrt{(1-b)(1-c)}$

$\therefore(1+a)(1+b)(1+c)\geq 8(1-a)(1-b)(1-c)$

3. The range of values of $'a'$ for which all the roots of the equation $(a-1)(1+x+x^2)^2=(a+1)(1+x^2+x^4)$ are imaginary is

1) $(-\infty,-2]$ 2) $(2,\infty)$
3) $(-2,2)$ 4) $[2,\infty)$

Key. 3

Sol. The given equation can be written as $(x^2+x+1)(x^2-ax+1)=0$

4. If α,β are the roots of the equation $ax^2+bx+c=0$ and $S_n=\alpha^n+\beta^n$ then $aS_{n+1}+bS_n+cS_{n-1}=\ (n\geq 2)$

1) 0 2) $a+b+c$
3) $(a+b+c)n$ 4) $n^2\ abc$

Key. 1

Sol. $S_{n+1}=\alpha^{n+1}+\beta^{n+1}$

$=(\alpha+\beta)(\alpha^n+\beta^n)-\alpha\beta(\alpha^{n-1}+\beta^{n-1})$

$=-\frac{b}{a}.S_n-\frac{c}{a}.S_{n-1}$

5. A group of students decided to buy a Alarm Clock priced between Rs. 170 to Rs 195. But at the last moment, two students backed out of the decision so that the remaining students had to pay 1 Rupee more than they had planned. If the students paid equal shares, the price of the Alarm Clock is

1) 190 2) 196
3) 180 4) 171

Key. 3

Sol. Let cost of clock = x
number of students $= n$
then $\frac{x}{n-2} = \frac{x}{n} + 1 \Rightarrow x = \frac{n^2 - 2n}{2}$
$\Rightarrow 170 \le \frac{n^2 - 2n}{2} \le 195$

6. If $\tan A, \tan B$ are the roots of $x^2 - Px + Q = 0$ the value of $\sin^2(A+B) =$ $(where\ P, Q \in R)$

1) $\frac{P^2}{P^2 + (1-Q)^2}$ 2) $\frac{P^2}{P^2 + Q^2}$

3) $\frac{Q^2}{P^2 + (1-Q)^2}$ 4) $\frac{P^2}{(P+Q)^2}$

Key. 1

Sol. $\tan(A+B) = \frac{P}{1-Q}$ then $\sin^2(A+B) = \frac{\tan^2(A+B)}{1+\tan^2(A+B)}$

7. The number of solutions of $\left|[x] - 2x\right| = 4$ where $[x]$ is the greatest integer $\le x$ is

1) 2 2) 4
3) 1 4) Infinite

Key. 2

Sol. If $x = n \in Z,\ |n - 2n| = 4 \Rightarrow n = \pm 4$
If $x = n + K$ where $0 < K < 1$ then $|n - 2(n+k)| = 4$, it is possible if $K = \frac{1}{2}$
$\Rightarrow |-n-1| = 4$
$\therefore n = 3, -5$

8. Let a, b and c be real numbers such that $a + 2b + c = 4$ then the maximum value of $ab + bc + ca$ is

1) 1 2) 2 3) 3 4) 4

Key. 4

Sol. Let $ab + bc + ca = x$
$\Rightarrow 2b^2 + 2(c-2)b - 4c + c^2 + x = 0$
Since $b \in R$,
$\therefore c^2 - 4c + 2x - 4 \le 0$
Since $c \in R$

$\therefore x \le 4$

9. For the equation $3x^2 + px + 3 = 0, p > 0$, if one root is the square of the other then value of P is

1) $\frac{1}{3}$ 2) 1

3) 3 4) $\frac{2}{3}$

Key. 3

Sol. $\alpha + \alpha^2 = -\frac{p}{3}$

$\alpha^3 = 1$

10. If the equations $2x^2 + kx - 5 = 0$ and $x^2 - 3x - 4 = 0$ have a common root, then the value of k is

1) -2 2) -3

3) $\frac{27}{4}$ 4) $-\frac{1}{4}$

Key. 2

Sol. If 'α' is the common root then $2\alpha^2 + k\alpha - 5 = 0, \alpha^2 - 3\alpha - 4 = 0$ solve the equations.

11. If α and β are the roots of the equation $x^2 - x + 1 = 0$ then $\alpha^{2009} + \beta^{2009} =$

1) 1 2) 2

3) -1 4) -2

Key. 1

Sol. $x = \frac{1 \pm i\sqrt{3}}{2}$

$\therefore \alpha = -\omega, \beta = -\omega^2$

12. If $P(Q-r)x^2 + Q(r-P)x + r(P-Q) = 0$ has equal roots then $\frac{2}{Q} =$

$(where\ P, Q, r \in R)$

1) $\frac{1}{P} + \frac{1}{r}$ 2) $\frac{1}{P} - \frac{1}{r}$

3) $P + r$ 4) Pr

Key. 1

Sol. Product of the roots $= 1$

13. If $(1+K)\tan^2 x - 4\tan x - 1 + K = 0$ has real roots $\tan x_1$ and $\tan x_2$ then

1) $k^2 \le 5$ 2) $k^2 \ge 6$

3) $k = 3$ 4) $k > 10$

Key. 1

Sol. Discriminate ≥ 0

14. α,β are the roots of $ax^2+bx+c=0$ and γ,δ are the roots of $px^2+qx+r=0$ and D_1,D_2 be the respective discriminants of these equations. If α,β,γ and δ are in A.P. then $D_1:D_2=$ (where $\alpha,\beta,\gamma,\delta\in R$ & $a,b,c,p,q,r\in R$)

1) $a^2:p^2$ 2) $a^2:b^2$

3) $c^2:r^2$ 4) $a^2:r^2$

Key. 1

Sol. $\beta=\alpha+d,\ \gamma=\alpha+2d,\ \delta=\alpha+3d$

$d^2=\frac{D_1}{a^2}=\frac{D_2}{p^2}$

15. If $x^2+4y^2-8x+12=0$ is satisfied by real values of x and y then $'y'\in$

1) $[2,6]$ 2) $[2,5]$

3) $[-1,1]$ 4) $[-2,-1]$

Key. 3

Sol. $x^2-8x+\left(4y^2+12\right)=0$ is a quadratic in $'x'$, $'x'$ is real then discriminate ≥ 0

16. For $x>0, 0\le t\le 2\pi, K>\frac{3}{2}+\sqrt{2}$, K being a fixed real number the minimum value of $x^2+\frac{K^2}{x^2}-2\left\{(1+\cos t)x+\frac{K(1+\sin t)}{x}\right\}+3+2\cos t+2\sin t$ is

a) $\left\{\sqrt{K}-\left(1+\frac{1}{\sqrt{2}}\right)\right\}^2$ b) $\frac{1}{2}\left\{\sqrt{K}-\left(1+\frac{1}{\sqrt{2}}\right)\right\}^2$

c) $3\left\{\sqrt{K}-\left(1+\frac{1}{\sqrt{2}}\right)\right\}^2$ d) $2\left\{\sqrt{K}-\left(1+\frac{1}{\sqrt{2}}\right)\right\}^2$

Key. D

Sol. Given expansion = $\{x-(1+\cos t)\}^2+\left\{\frac{K}{x}-(1+\sin t)\right\}^2$

17. Let $\phi(x)=\frac{(x-b)(x-c)}{(a-b)(a-c)}f(a)+\frac{(x-c)(x-a)}{(b-c)(b-a)}f(b)+\frac{(x-a)(x-b)}{(c-a)(c-b)}f(c)-f(x)$

Where $a<c<b$ and $f^{11}(x)$ exists at all points in (a,b). Then, there exists a real number $\mu, a<\mu<b$ such that

$\frac{f(a)}{(a-b)(a-c)}+\frac{f(b)}{(b-c)(b-a)}+\frac{f(c)}{(c-a)(c-b)}=$

a) $f^{11}(\mu)$ b) $2f^{11}(\mu)$ c) $\frac{1}{2}f^{11}(\mu)$ d) $\frac{1}{3}f^{111}(\mu)$

Key. C

Sol. Apply RT's, twice

18. If α, β, γ are the roots of the equation $x^3 + px + q = 0$, then the value of the determinant $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is

(A) 4 (B) 2 (C) 0 (D) –2

Key. C

Sol. Since α, β, γ are the roots of $x^3 + px + q = 0$

$\therefore \quad \alpha+\beta+\gamma = 0$

Applying $C_1 \to C_1 + C_2 + C_3$, then

$$\begin{vmatrix} \alpha+\beta+\gamma & \beta & \gamma \\ \alpha+\beta+\gamma & \gamma & \alpha \\ \alpha+\beta+\gamma & \alpha & \beta \end{vmatrix} = \begin{vmatrix} 0 & \beta & \gamma \\ 0 & \gamma & \alpha \\ 0 & \alpha & \beta \end{vmatrix} = 0$$

19. The number of points (p, q) such that $p, q \in \{1,2,3,4\}$ and the equation $px^2 + qx + 1 = 0$ has real roots is

A. 7 B. 8 C. 9 D. None of these

Key. A

Sol. $px^2 + qx + 1 = 0$ has real roots if $q^2 - 4p \geq 0$ or $q^2 \geq 4p$

Since $p, q \in \{1,2,3,4\}$

The required points are(1,2), (1,3),(1, 4), (2,3),(2,4),(3,4),(4,4)

So the required number is 7

20. The value of *b* and *c* for which the identity $f(x+1) - f(x) = 8x + 3$ is satisfied, where $f(x) = bx^2 + cx + d$ are

(A) b = 2, c = 1 (B) b = 4, c = –1

(C) b = – 1, c = 4 (D) b = – 1, c = 1

Key. B

Sol. $\because f(x+1) - f(x) = 8x + 3$

$\Rightarrow \quad \{b(x+1)^2 + c(x+1) + d\} - \{bx^2 + cx + d\} = 8x + 3$

$\Rightarrow \quad b\{(x+1)^2 - x^2\} + c = 8x + 3$

$\Rightarrow \quad b(2x+1) + c = 8x + 3$ on comparing

2b = 8 and b + c = 3

Then, b = 4 and c = –1

21. Let $f(x) = ax^2 + bx + c$, $g(x) = ax^2 + px + q$ where a, b, c, q, p, $\in$ R and $b \neq p$. If their discriminants are equal and f(x) = g(x) has a root α, then

1) α will be A.M. of the roots of f(x) = 0, g(x) = 0
2) α will be G.M of all the roots of f(x) = 0, g(x) = 0
3) α will be A.M of the roots of f(x) = 0 or g(x) = 0
4) α will be G.M of the roots of f(x) = 0 or g(x) = 0

Key. 1

Sol. $a\alpha^2 + b\alpha + c = a\alpha^2 + p\alpha + q \Rightarrow \alpha = \frac{q-c}{b-p} \rightarrow (i)$

And $b^2 - 4ac = p^2 - 4aq$

$\Rightarrow b^2 - p^2 = 4a(c-q)$

$\Rightarrow b + p = \frac{4a(c-q)}{b-p} = -4a\alpha \qquad (\text{from}(i))$

$\alpha = \frac{-(b+p)}{4a} = \frac{\frac{-b}{a} - \frac{p}{a}}{4}$ which is A.M of all the roots of f(x) = 0 and g(x) = 0

22. If the equations $x^2 + 2\lambda x + \lambda^2 + 1 = 0,\ \lambda \in R$ and $ax^2 + bx + c = 0$ where a, b, c are lengths of sides of triangle have a common root, then the possible range of values of λ is

1) (0, 2) 2) $(\sqrt{3}, 3)$ 3) $(2\sqrt{2}, 3\sqrt{2})$ 4) $(0, \infty)$

Key. 1

Sol. $(x+\lambda)^2 + 1 = 0$ has clearly imaginary roots

So, both roots of the equations are common

$\therefore \frac{a}{1} = \frac{b}{2\lambda} = \frac{c}{\lambda^2 + 1} = k(say)$

Then a = k, b = $2\lambda k$, c = $(\lambda^2 + 1)$ k

As a, b, c are sides of triangle

$a + b > c \Rightarrow 2\lambda + 1 > \lambda^2 + 1 \Rightarrow \lambda^2 - 2\lambda < 0$

$\Rightarrow \lambda \in (0, 2)$

The other conditions also imply same relation.

23. The number of real or complex solutions of $x^2 - 6|x| + 8 = 0$ is

1) 6 2) 7 3) 8 4) 9

Key. 1

Sol. If x is real, $x^2 - 6|x| + 8 = 0 \Rightarrow |x|^2 - 6|x| + 8 = 0 \Rightarrow |x| = 2, 4 \Rightarrow x = \pm 2, \pm 4$

If x is non – real, say $x = \alpha + i\beta$, then

$(\alpha + i\beta)^2 - 6\sqrt{\alpha^2 + \beta^2} + 8 = 0 \qquad \left(|\alpha + i\beta| = \sqrt{\alpha^2 + \beta^2}\right)$

$\left(\alpha^2 - \beta^2 + 8 - 6\sqrt{\alpha^2 + \beta^2}\right) + 2i\alpha\beta = 0$

Comparing real and imaginary parts,

$\alpha\beta = 0 \Rightarrow \alpha = 0$ (if $\beta = 0$ then x is real.)

& $-\beta^2 + 8 - 6\sqrt{\beta^2} = 0$

$\beta^2 \pm 6\beta - 8 = 0 \Rightarrow \beta = \frac{\mp 6 \pm \sqrt{68}}{2}$

ie., $\beta = \pm(3 - \sqrt{17})$

Hence $\pm(3 - \sqrt{17})i$ are non-real roots.

24. If $x_1, x_2\left(x_1 > x_2\right)$ are abscissae of points P, Q lying on $y = 2x^2 - 4x - 5$ such that the tangents drawn at these points pass through the point (0, -7), then $3x_1 - 2x_2$ equals to

1) 4 2) 5 3) 6 4) 7

Key. 2

Sol. Let (α, β) be point on the curve such that the tangent drawn at (α, β) passes through (0, 7)

$y^1 = 4x - 4 \Rightarrow y^1_{(\alpha,\beta)} = 4\alpha - 4$

Tangent at (α, β) is $y - \beta = (4\alpha - 4)(x - \alpha)$ pass through (0, -7) $\Rightarrow -7 - \beta = (4\alpha - 4)(0 - \alpha)$

But $\beta = 2\alpha^2 - 4\alpha - 5 \therefore$ It follows that α^2 = 1

$\Rightarrow \alpha = \pm 1$

So, $x_1 = 1, x_2 = -1$

So, $3x_1 - 2x_2 = 5$.

25. Let $f(x) = x^2 + 5x + 6$, then the number of real roots of $\left(f(x)\right)^2 + 5f(x) + 6 - x = 0$ is

1) 1 2) 2 3) 3 4) 0

Key. 4

Sol. Use "f(x) = x has non real roots $\Rightarrow$ f(f(x)) = x also has non-real roots"

26. Sum of the roots of the equation is $4^x - 3\left(2^{x+3}\right) + 128 = 0$

1) 5 2) 6 3) 7 4) 8

Key. 3

Sol. Put $2^x = y.$ Equation becomes

$y^2 - 3(8y) + 128 = 0 \Rightarrow y^2 - 24y + 128 = 0$

$\Rightarrow (y - 8)(y - 16) = 0 \Rightarrow y = 16, 8$

$\Rightarrow 2^x = 16, 8 \Rightarrow x = 4, 3$

$\therefore$ Sum of the roots is 7.

27. The number of solutions of $\sqrt{3x^2 + x + 5} = x - 3$ is

1) 0 2) 1 3) 2 4) 4

Key. 1

Sol. Note that we must have $3x^2 + x + 5 \geq 0$ and $x - 3 \geq 0$ or $x \geq 3.$

$\sqrt{3x^2 + x + 5} = x - 3$.... (1)

Squaring both sides of (1), we get

$3x^2 + x + 5 = x^2 - 6x + 9$

$\Rightarrow 2x^2 + 7x - 4 = 0 \Rightarrow (2x - 1)(x + 4) = 0$

$\Rightarrow x = 1/2, -4$

None of these satisfy the inequality $x \geq 3.$ Thus, (1) has no solution.

28. The value of a for which one root of the quadratic equation.

$\left(a^2 - 5a + 3\right)x^2 + (3a - 1)x + 2 = 0$ is twice as large as other, is

1) $-2/3$ 2) $1/3$ 3) $-1/3$ 4) $2/3$

Key. 4

Sol. $\left(a^2 - 5a + 3a\right)x^2 + (3a - 1)x + 2 = 0$...... (1)

Let α and 2α be the roots of (1), then

$\left(a^2-5a+3\right)\alpha^2+\left(3a-1\right)\alpha+2=0$ .... (2)

and $\left(a^2-5a+3\right)\left(4\alpha^2\right)+\left(3a-1\right)\left(2\alpha\right)+2=0$ ..... (3)

Multiplying (2) by 4 and subtracting it form (3) we get $\left(3a-1\right)\left(2\alpha\right)+6=0$

Clearly $a\neq 1/3.$ Therefore, $\alpha=-3/\left(3a-1\right)$

Putting this value in (2) we get

$\left(a^2-5a+3\right)\left(9\right)-\left(3a-1\right)^2\left(3\right)+2\left(3a-1\right)^2=0$

$\Rightarrow 9a^2-45a+27-\left(9a^2-6a+1\right)=0\Rightarrow -39a+26=0$

$\Rightarrow a=2/3.$

For $x=2/3,$ the equation becomes $x^2+9x+18=0,$ whose roots are $-3,-6.$

29. If $f\left(x\right)=x^2+2bx+2c^2$ and $g\left(x\right)=-x^2-2cx+b^2$ are such that $\min f\left(x\right)>\max g\left(x\right)$, then relation between b and c, is

1) no relation 2) $0<c<b/2$ 3) $|c|<\dfrac{|b|}{\sqrt{2}}$ 4) $|c|>\sqrt{2}|b|$

Key. 4

Sol. $f\left(x\right)=\left(x+b\right)^2+2c^2-b^2$

$\Rightarrow \min f\left(x\right)=2c^2-b^2$

Also $g\left(x\right)=-x^2-2cx+b^2=b^2+c^2-\left(x+c\right)^2$

$\Rightarrow \max g\left(x\right)=b^2+c^2$

As $\min f\left(x\right)>\max g\left(x\right)$, we get $2c^2-b^2>b^2+c^2$

$\Rightarrow c^2>2b^2\Rightarrow |c|>\sqrt{2}|b|$

30. The equation $\left(\cos p-1\right)x^2+\left(\cos p\right)x+\sin p=0$ in variable x has real roots, if p belongs to the interval

1) $\left(0,2\pi\right)$ 2) $\left(-\pi,0\right)$ 3) $\left(-\pi/2,\pi/2\right)$ 4) $\left(0,\pi\right)$

Key. 4

Sol. $\left(\cos p-1\right)x^2+\left(\cos p\right)x+\sin p=0$ ...... (1)

Discriminant of (1) is given by

$D=\cos^2 p-4\left(\cos p-1\right)\sin p=\cos^2 p+4\left(1-\cos p\right)\sin p$

Note that $\cos^2 p\geq 0,1-\cos p\geq 0.$ Thus, $D\geq 0$ if $\sin p\geq 0$ *i.e.* if $p\in\left(0,\pi\right).$

31. If $x^2+2ax+10-3a>0$ for each $x\in R,$ then

1) $a<-5$ 2) $-5<a<2$ 3) $a>5$ 4) $2<a<5$

Key. 2

Sol. $x^2+2ax+10-3a>0\forall x\in R$

$\Rightarrow\left(x+a\right)^2-\left(a^2+10-3a\right)>0\forall x\in R$

$\Rightarrow a^2+3a-10<0$

$\Rightarrow (a+5)(a-2)<0$

$\Rightarrow -5<a<2$

32. Sum of all the values of x satisfying the equation $\log_{17}\log_{11}\left(\sqrt{x+11}+\sqrt{x}\right)=0$ is

1) 25 2) 36 3) 171 4) 0

Key. 1

Sol. $\log_{17}\log_{11}\left(\sqrt{x+1}+\sqrt{x}\right)=0$ …….. (1)

Equation (1) is defined if $x\ge 0$.

We can rewrite (1) as $\log_{11}\left(\sqrt{x+11}+\sqrt{x}\right)=17^0=1$

$\Rightarrow \sqrt{x+11}+\sqrt{x}=11^1=11$

$\Rightarrow \sqrt{x+11}=11-\sqrt{x}$

Squaring both sides we get $x+11=121-22\sqrt{x}+x$

$\Rightarrow 22\sqrt{x}=110 \Rightarrow \sqrt{x}=5$ or $x=25$

This clearly satisfies (1). Thus, sum of all the values satisfying (1) is 25.

33. The number of solutions of the equations of the equation $x^2+[x]-4x+3=0$ is Where [ ] denotes G.I.F.

1) 0 2) 1 3) 2 4) 3

Key. 1

Sol. Given equation can be written as $(x^2-3x+3)-f=O$ where $f=x-[x]$ *and* $O\le f<1$

$\therefore O\le x^2-3x+3<1$

solving $x^2-3x+3=O$; roots are Imaginary

$\therefore x^2-3x+3\ge O\,\forall x\in R$

solving $x^2-3x+3<1 \Rightarrow 1<x<2$

if $1<x<2; [x]=1.$

putting $[x]=1$ in the given equation and solving we get $x=2$. But $1<x<2$ $\therefore$ the given equation has no solution.

34. The number of values of $'a'$ for which the equation $(x-1)^2=|x-a|$ has exactly three solutions is

1) 1 2) 2 3) 3 4) 4

Key. 3

Sol. $|x-a|=(x-1)^2$ Iff $a=x\pm(x-1)^2$

No of solutions = no of intersection its between

$y=a$; $f(x)=x^2-x+1$ and $g(x)=-x^2+3x-1$. clearly the graphs of $f(x), g(x)$ are tangents to each other at $A(1,1)$. The line $y=a$ intersects the two graphs at three points

Iff it passes through one of the three pts A,B, C. Here $B=\left(\frac{1}{2},\frac{3}{4}\right)$ vertex of f

and $C=\left(\frac{3}{2},\frac{5}{4}\right)$ vertex of 'g' i.e if $a\in\left\{\frac{3}{4},\frac{5}{4},1\right\}$

35. If *a, b, c* are positive numbers such that a>b>c and the equation $(a+b-2c)x^2+(b+c-2a)x+(c+a-2b)=0$ has a root in the interval $(-1,0)$, then

A) b cannot be the G.M. of a, c B) b may be the G.M. of a, c
C) b is the G.M. of a, c D) none of these

Key. A

Sol. Let $f(x) = (a+b-2c)x^2 + (b+c-2a)x + (c+a-2b)$

According to the given condition, we have

$$f(0)\,f(-1) < 0$$

i.e. $(c+a-2b)(2a-b-c) < 0$

i.e. $(c+a-2b)(a-b+a-c) < 0$

i.e. $c+a-2b < 0$ $\quad [a > b > c,\ given \Rightarrow a-b > 0, a-c > 0]$

i.e. $b > \frac{a+c}{2}$

$\Rightarrow$ *b* cannot be the G.M. of *a, c,* since G.M < A.M. always.

36. Let α, β (a < b) be the roots of the equation $ax^2+bx+c=0$. If $\lim_{x\to m} \frac{|ax^2+bx+c|}{ax^2+bx+c} = 1,$ then

A) $\frac{|a|}{a} = -1, m < \alpha$ B) $a > 0, \alpha < m < \beta$ C) $\frac{|a|}{a} = 1, m > \beta$ D) $a < 0, m > \beta$

Key. C

Sol. According to the given condition, we have

$$|am^2+bm+c| = am^2+bm+c$$

i.e. $am^2+bm+c > 0$

$\Rightarrow$ if *a< 0,* the *m* lies in (α, β)

and if *a>0,* then *m* does not lies in (α, β)

Hence, option (c) is correct, since

$$\frac{|a|}{a} = 1 \Rightarrow a > 0$$

And in that case *m* does not lie in (α, β).

37. Let $f(x)$ be a function such that $f(x) = x - [x]$, where $[x]$ is the greatest integer less than or equal to x. Then the number of solutions of the equation $f(x) + f\left(\frac{1}{x}\right) = 1$ is (are)

A) 0 B) 1 C) 2 D) infinite

Key. D

Sol. Given, $f(x) = x - [x],\ x \in R - \{0\}$

Now $f(x) + f\left(\frac{1}{x}\right) = 1$ $\quad\therefore\quad x - [x] + \frac{1}{x} - \left[\frac{1}{x}\right] = 1$

$\Rightarrow \left(x + \frac{1}{x}\right) - \left([x] + \left[\frac{1}{x}\right]\right) = 1$ $\quad \Rightarrow \left(x + \frac{1}{x}\right) = [x] + \left[\frac{1}{x}\right] + 1$ ...(i)

Clearly ,R.H.S is an integer $\quad \therefore$ L. H. S. is also an integer

Let $x + \frac{1}{x} = k$ an integer $\quad \Rightarrow x^2 - kx + 1 = 0$

$\therefore x=\dfrac{k\pm\sqrt{k^2-4}}{2}$

For real values of *x*, $k^2-4\geq 0\Rightarrow k\geq 2$ or $k\leq -2$

We also observe that *k=2* and -2 does not satisfy equation *(i)*

$\therefore$ The equation *(i)* will have solutions if $k>2$ or $k<-2$, where $k\in z$.

Hence equation *(i)* has infinite number of solutions.

38. If both the roots of $(2a-4)9^x-(2a-3)3^x+1=0$ are non-negative, then

A) $0<a<2$ B) $2<a<\dfrac{5}{2}$ C) $a<\dfrac{5}{4}$ D) $a>3$

Key. B

Sol. Putting $3^x=y,$ we have

$$(2a-4)y^2-(2a-3)y+1=0$$

This equation must have real solution

$\Rightarrow$ $(2a-3)^2-4(2a-4)\geq 0$

$\Rightarrow$ $4a^2-20a+25\geq 0$

$\Rightarrow$ $(2a-5)^2\geq 0.$ This is true.

$y=1$ satisfies the equation

Since 3^x is positive and $3^x\geq 3^0,\ y\geq 1$

Product of the roots $=1\times y>1$

$\Rightarrow$ $\dfrac{1}{2a-4}>1$

$\Rightarrow$ $2a-4<1\Rightarrow a<\dfrac{5}{2}$

Sum of the roots $=\dfrac{2a-3}{2a-4}>1$

$\Rightarrow$ $\dfrac{(2a-3)-(2a-4)}{2a-4}>0$

$\Rightarrow$ $\dfrac{1}{2a-4}>0\Rightarrow a>2$

$\Rightarrow$ $2<a<\dfrac{5}{2}$

39. If the equation $x^2+9y^2-4x+3=0$ is satisfied for real values of x and y then

A) $x\in[1,3],y\in[1,3]$ B) $x\in[1,3],y\in\left[\dfrac{-1}{3},\dfrac{1}{3}\right]$

C) $x\in\left[\dfrac{-1}{3},\dfrac{1}{3}\right],y\in[1,3]$ D) $x\in\left[\dfrac{-1}{3},\dfrac{1}{3}\right],y\in\left[\dfrac{-1}{3},\dfrac{1}{3}\right]$

Key. B

Sol. Given equation is $x^2+9y^2-4x+3=0$ …(i)

Or, $x^2-4x+9y^2+3=0.$

Since x is real $\therefore \left(-4\right)^2-4\left(9y^2+3\right)\geq 0$

Or, $16-4\left(9y^2+3\right)\geq 0$ or, $4-9y^2-3\geq 0$

Or, $9y^2-1\leq 0$ or, $9y^2\leq 1$ or, $y^2\leq\frac{1}{9}$

Now $y^2\leq\frac{1}{9}\Leftrightarrow -\frac{1}{3}\leq y\leq\frac{1}{3}$ ...(ii)

Equation (i) can also be written as

$9y^2+0y+x^2-4x+3=0$ ...(iii)

Since y is real $\therefore 0^2-4.9\left(x^2-4x+3\right)\geq 0$

Or, $x^2-4x+3\leq 0$

$\Rightarrow x\in[1,3]$

40. The equation $a_8x^8+a_7x^7+a_6x^6+...+a_0=0$ has all its roots positive and real $\left(where\ a_8=1, a_7=-4, a_0=1/2^8\right)$, then

A) $a_1=\frac{1}{2^8}$ B) $a_1=-\frac{1}{2^4}$ C) $a_2=\frac{7}{2^5}$ D) $a_2=\frac{7}{2^8}$

Key. B

Sol. Let the roots be $\alpha_1,\alpha_2,....,\alpha_8$

$\Rightarrow$ $\alpha_1+\alpha_2+....+\alpha_8=4$

$\alpha_1\alpha_2.....\alpha_8=\frac{1}{2^8}$

$\Rightarrow$ $\left(\alpha_1\alpha_2.....\alpha_8\right)^{1/8}=\frac{1}{2}=\frac{\alpha_1+\alpha_2+...+\alpha_8}{8}$

$\Rightarrow$ AM=GM$\Rightarrow$ all the roots are equal to $\frac{1}{2}$.

$\Rightarrow$ $a_1=-{}^8C_7\left(\frac{1}{2}\right)^7=-\frac{1}{2^4}$

$a_2={}^8C_6\left(\frac{1}{2}\right)^6=-\frac{7}{2^4}$

$a_3=-{}^8C_5\left(\frac{1}{2}\right)^5$

41. If every root of a polynomial equation (of degree ‘n’) $f\left(x\right)=0$ with leading coefficient “1” is real and distinct, then the equation $f''\left(x\right)f\left(x\right)-\left\{f'\left(x\right)\right\}^2=0$ has.

(A) at least one real root (B) no real root
(C) at most one real root (D) exactly two real roots

Key. B

Sol. Let $f(x)=(x-a_1)(x-a_2)..............(x-an)$ where $a_1,a_2.........a_{n\in R}$ take log both sides and differentiate. Then

$$\frac{f'(x)}{f(x)}=\frac{1}{x-a_1}+\frac{1}{x-a_2}+..........+\frac{1}{x-a_n}$$

Again diff w.r.t. 'x'

$$\frac{f\ f''-(f')^2}{f^2}=-\left[\frac{1}{(x-a_1)^2}+\frac{1}{(x-a_2)^2}+......\frac{1}{(x-a_n)^2}\right]$$

$$<0\forall x\in R$$

$\Rightarrow f\ f''-(f')^2=0$ has no real root

42. If $f(x)$ is a polynomial of least degree such that $f(r)=\frac{1}{r}, r=1,2,3,$ ___9, then $f(10)$ = ___

A. 1 B. $\frac{1}{2}$ C. $\frac{1}{10}$ D. $\frac{1}{5}$

Key. D

Sol. $xf(x)-1=0$ has roots 1,2,3 ____9

$xf(x)-1=A(x-1)(x-2)$ _____x-9

Put $x=0\Rightarrow A=\frac{1}{9!}$

Put $x=10\Rightarrow 10f(10)-1=1\Rightarrow f(10)=\frac{1}{5}$

43. The number of ordered pairs of integers (x, y) satisfying the equation $x^2+6x+y^2=4$ is

A. 2 B. 8 C. 6 D. 10

Key. B

Sol. $(x+3)^2+y^2=13$

$x+3=\pm2, y=\pm3 or\ x+3=\pm3, y=\pm2$

44. The number of non-negative integer solutions of $x+y+2z=20$ is

A. 76 B. 84 C. 112 D. 121

Key. D

Sol. $x+y=20-2Z,\ Z=0,1,2,...10$

The number of solutions (non –ve) is $\sum_{Z=0}^{10}(20-2Z+1)_{C_1}=121$

45 If $a+b+c=0$ for $a,b,c\in R$, then the equation $3ax^2+2bx+c=0$ has

A. Atleast one root in $[0,1]$

B. One root in $[2,3]$ and another root in $[-2,-1]$

C. Imaginary roots

D. Atleast one root in $[1,2]$

Key. A

Sol. Let $f(x)=ax^3+bx^2+cx$. Then f is continuous and differentiable in $[0,1]$, $f(0)=f(1)=0$. Hence by Rolle's theorem there exists $k\in(0,1)$ such that $3ak^2+2bk+c=0$

46. If a,b,c be the sides of a triangle ABC and if roots of the equation a(b – c)x^2 + b(c – a) x + c(a – b) = 0 are equal, then $\sin^2\left(\frac{A}{2}\right),\sin^2\left(\frac{B}{2}\right),\sin^2\left(\frac{C}{2}\right)$ are in

(A) AP (B)GP (C) HP (D) AGP

Key. C

Sol. $\because$ a(b – c) + b(c – a) + c (a – b) = 0

$\therefore$ x = 1 is a root of the equation

a(b – c)x^2 + b(c – a)x + c(a – b) =0

Then, other root = 1 ($\because$ roots are equal)

$\therefore \quad \alpha\times\beta=\frac{c(a-b)}{a(b-c)}$

$\Rightarrow$ ab – ac = ca – bc

$\therefore \quad b=\frac{2ac}{a+c}$

$\therefore$ a, b, c are in HP

Then, $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in AP.

$\Rightarrow \frac{s}{a},\frac{s}{b},\frac{s}{c}$ are in AP

$\Rightarrow \frac{s}{a}-1,\frac{s}{b}-1,\frac{s}{c}-1$ are in AP.

$\Rightarrow \frac{(s-a)}{a},\frac{(s-b)}{b},\frac{(s-c)}{c}$ are in AP.

Multiplying in each by $\frac{abc}{(s-a)(s-b)(s-c)}$

Then $\frac{bc}{(s-b)(s-c)},\frac{ca}{(s-c)(s-a)},\frac{ab}{(s-a)(s-b)}$ are in AP.

$\Rightarrow \frac{(s-b)(s-c)}{bc},\frac{(s-c)(s-a)}{ca},\frac{(s-a)(s-b)}{ab}$ are in HP.

Or $\sin^2\left(\frac{A}{2}\right), \sin^2\left(\frac{B}{2}\right), \sin^2\left(\frac{C}{2}\right)$ are in HP

47. If α, β, γ are the roots of the equation $x^3 + px + q = 0$, then the value of the determinant $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is

(A) 4 (B)2 (C)0 (D) –2

Key. C

Sol. Since α, β, γ are the roots of $x^3 + px + q = 0$

$\therefore \quad \alpha+\beta+\gamma = 0$

Applying $C_1 \rightarrow C_1 + C_2 + C_3$, then

$$\begin{vmatrix} \alpha+\beta+\gamma & \beta & \gamma \\ \alpha+\beta+\gamma & \gamma & \alpha \\ \alpha+\beta+\gamma & \alpha & \beta \end{vmatrix} = \begin{vmatrix} 0 & \beta & \gamma \\ 0 & \gamma & \alpha \\ 0 & \alpha & \beta \end{vmatrix} = 0$$

48. The value of *b* and *c* for which the identity $f(x+1) - f(x) = 8x + 3$ is satisfied, where $f(x) = bx^2 + cx + d$ are

(A) b = 2, c = 1 (B) b = 4, c = –1

(C) b = – 1, c = 4 (D) b = – 1, c = 1

Key. B

Sol. $\because f(x+1) - f(x) = 8x + 3$

$\Rightarrow \quad \left\{b(x+1)^2 + c(x+1) + d\right\} - \left\{bx^2 + cx + d\right\} = 8x + 3$

$\Rightarrow \quad b\left\{(x+1)^2 - x^2\right\} + c = 8x + 3$

$\Rightarrow \quad b(2x+1) + c = 8x + 3$ on comparing

2b = 8 and b + c = 3

Then, b = 4 and c = –1

49. If *a, b, c* are positive numbers such that a>b>c and the equation $(a+b-2c)x^2 + (b+c-2a)x + (c+a-2b) = 0$ has a root in the interval $(-1, 0)$, then

A) b cannot be the G.M. of a, c B) b may be the G.M. of a, c

C) b is the G.M. of a, c D) none of these

Key. A

Sol. Let $f(x) = (a+b-2c)x^2 + (b+c-2a)x + (c+a-2b)$

According to the given condition, we have

$f(0)\,f(-1) < 0$

i.e. $(c+a-2b)(2a-b-c) < 0$

i.e. $(c+a-2b)(a-b+a-c) < 0$

i.e. $c+a-2b < 0 \qquad \left[a > b > c,\ given \Rightarrow a-b > 0, a-c > 0\right]$

i.e. $b > \frac{a+c}{2}$

$\Rightarrow$ *b* cannot be the G.M. of *a, c,* since G.M < A.M. always.

50. The values of 'a' for which the quadratic expression $ax^2+(a-2)x-2$ is negative for exactly two integral values of x, belongs to

(A) $[-1,1]$ (B) $[1,2)$

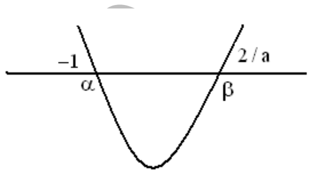
(C) $[3,4]$ (D) $[-2,-1)$

Key. B

Sol. Let $f(x)=ax^2+(a-2)x-2$

$f(x)$ is negative for two integral values of x, so graph should be vertically upward parabola i.e., $a>0$

Let two roots of $f(x)=0$ are α and β then $\alpha,\beta=\dfrac{-(a-2)\pm(a+2)}{2a}$



$\Rightarrow \alpha=-1, \beta=\dfrac{2}{a}\Rightarrow 1<\beta\le 2\Rightarrow 1<\dfrac{2}{a}\le 2\Rightarrow a\in[1,2]$

51. Let $f(x)$ be a function such that $f(x)=x-[x]$, where $[x]$ is the greatest integer less than or equal to x. Then the number of solutions of the equation $f(x)+f\left(\dfrac{1}{x}\right)=1$ is (are)

A) 0 B) 1 C) 2 D) infinite

Key. D

Sol. Given, $f(x)=x-[x],\ x\in R-\{0\}$

Now $f(x)+f\left(\dfrac{1}{x}\right)=1$ $\therefore$ $x-[x]+\dfrac{1}{x}-\left[\dfrac{1}{x}\right]=1$

$\Rightarrow\left(x+\dfrac{1}{x}\right)-\left([x]+\left[\dfrac{1}{x}\right]\right)=1$ $\Rightarrow\left(x+\dfrac{1}{x}\right)=[x]+\left[\dfrac{1}{x}\right]+1$ ...(i)

Clearly ,R.H.S is an integer $\therefore$ L. H. S. is also an integer

Let $x+\dfrac{1}{x}=k$ an integer $\Rightarrow x^2-kx+1=0$

$\therefore x=\dfrac{k\pm\sqrt{k^2-4}}{2}$

For real values of *x*, $k^2-4\ge 0\Rightarrow k\ge 2$ or $k\le -2$

We also observe that *k=2* and -2 does not satisfy equation *(i)*

$\therefore$ The equation *(i)* will have solutions if $k>2$ or $k<-2$,where $k\in z$.

Hence equation *(i)* has infinite number of solutions.

52. If both the roots of $(2a-4)9^x-(2a-3)3^x+1=0$ are non-negative, then

A) $0<a<2$ B) $2<a<\dfrac{5}{2}$ C) $a<\dfrac{5}{4}$ D) $a>3$

Key. B

Sol. Putting $3^x=y$, we have

$$(2a-4)y^2-(2a-3)y+1=0$$

This equation must have real solution

$\Rightarrow \quad (2a-3)^2 - 4(2a-4) \geq 0$

$\Rightarrow \quad 4a^2 - 20a + 25 \geq 0$

$\Rightarrow \quad (2a-5)^2 \geq 0.$ This is true.

$y=1$ satisfies the equation

Since 3^x is positive and $3^x \geq 3^0$, $y \geq 1$

Product of the roots $=1 \times y > 1$

$\Rightarrow \quad \frac{1}{2a-4} > 1$

$\Rightarrow \quad 2a-4 < 1 \Rightarrow \quad a < \frac{5}{2}$

Sum of the roots $= \frac{2a-3}{2a-4} > 1$

$\Rightarrow \quad \frac{(2a-3)-(2a-4)}{2a-4} > 0$

$\Rightarrow \quad \frac{1}{2a-4} > 0 \Rightarrow a > 2$

$\Rightarrow \quad 2 < a < \frac{5}{2}$

53. If the equation $x^2 + 9y^2 - 4x + 3 = 0$ is satisfied for real values of x and y then

A) $x \in [1,3], y \in [1,3]$ B) $x \in [1,3], y \in \left[\frac{-1}{3}, \frac{1}{3}\right]$

C) $x \in \left[\frac{-1}{3}, \frac{1}{3}\right], y \in [1,3]$ D) $x \in \left[\frac{-1}{3}, \frac{1}{3}\right], y \in \left[\frac{-1}{3}, \frac{1}{3}\right]$

Key. B

Sol. Given equation is $x^2 + 9y^2 - 4x + 3 = 0$ …(i)

Or, $x^2 - 4x + 9y^2 + 3 = 0.$

Since x is real $\therefore \quad (-4)^2 - 4(9y^2+3) \geq 0$

Or, $16 - 4(9y^2+3) \geq 0$ or, $4 - 9y^2 - 3 \geq 0$

Or, $9y^2 - 1 \leq 0$ or, $9y^2 \leq 1$ or, $y^2 \leq \frac{1}{9}$

Now $y^2 \leq \frac{1}{9} \Leftrightarrow -\frac{1}{3} \leq y \leq \frac{1}{3}$ ...(ii)

Equation (i) can also be written as

$9y^2 + 0y + x^2 - 4x + 3 = 0$ ...(iii)

Since y is real $\therefore 0^2 - 4.9(x^2 - 4x + 3) \geq 0$

Or, $x^2 - 4x + 3 \leq 0$

$\Rightarrow x \in [1,3]$

54. The equation $a_8x^8 + a_7x^7 + a_6x^6 + ... + a_0 = 0$ has all its roots positive and real $\left(where\ a_8 = 1, a_7 = -4, a_0 = 1/2^8\right)$, then

A) $a_1 = \frac{1}{2^8}$ B) $a_1 = -\frac{1}{2^4}$ C) $a_2 = \frac{7}{2^5}$ D) $a_2 = \frac{7}{2^8}$

Key. B

Sol. Let the roots be $\alpha_1, \alpha_2, ..., \alpha_8$

$\Rightarrow \quad \alpha_1 + \alpha_2 + ... + \alpha_8 = 4$

$\alpha_1\alpha_2.....\alpha_8 = \frac{1}{2^8}$

$\Rightarrow \quad (\alpha_1\alpha_2.....\alpha_8)^{1/8} = \frac{1}{2} = \frac{\alpha_1 + \alpha_2 + ... + \alpha_8}{8}$

$\Rightarrow$ AM=GM$\Rightarrow$ all the roots are equal to $\frac{1}{2}$.

$\Rightarrow \quad a_1 = -{}^8C_7\left(\frac{1}{2}\right)^7 = -\frac{1}{2^4}$

$a_2 = {}^8C_6\left(\frac{1}{2}\right)^6 = -\frac{7}{2^4}$

$a_3 = -{}^8C_5\left(\frac{1}{2}\right)^5$

55. If f(x) = $\prod_{i=1}^{i=3}(x - a_i) + \sum_{i=1}^{3} a_i - 3x$, where $a_i < a_{i+1}$, then f(x) = 0 has

(A) only one real root (B) three real roots of which two of them are equal

(C) three distinct real roots (D) three equal roots

KEY : C

SOL : f(x) = $(x - a_1)(x - a_2)(x - a_3) + (a_1 - x) + (a_2 - x) + (a_3 - x)$

Now f(x) $\to -\infty$ as x $\to -\infty$ and f(x) $\to \infty$ are x $\to \infty$.

Again f(a_1) = ($a_2 - a_1$) + ($a_3 - a_1$) > 0 $[\because a_1 < a_2 < a_3]$

$\Rightarrow$ One root belongs to $(-\infty, a_1)$

Also, $f(a_3) = (a_1 - a_3) + (a_2 - a_3) < 0$

$\Rightarrow$ One root belongs to (a_1, a_3)

So f(x) = 0 has three distinct real roots.

56. If a, b and c are numbers for which the equation $\frac{x^2+10x-36}{x(x-3)^2} = \frac{a}{x}+\frac{b}{x-3}+\frac{c}{(x-3)^2}$ is an identity, then a + b + c equals

(A) 2 (B) 3 (C) 10 (D) 8

Key. A

Sol. =

hence $x^2 + 10x - 36 = a(x-3)^2 + b(x-3)x + cx$

put x = 0; $-36 = 9a \Rightarrow a = -4$

$x^2 + 10x - 36 = x^2(-4 + b) + x(24 - 3b + c) + (-36)$

comparing coefficients

also, $-4 + b = 1 \Rightarrow b = 5$ $24 - 15 + c = 10 \Rightarrow 9 + c = 10 \Rightarrow c = 1$

a = – 4; b = 5; c = 1 i.e. a + b + c = 2

57. If one root of equation $x^2 - 4ax + a + f(a) = 0$ is three times of the other then minimum value of $f(a)$ is

A) $\frac{-1}{6}$ B) $\frac{-1}{10}$ C) $\frac{-1}{5}$ D) $\frac{-1}{12}$

Key. D

Sol. Let roots are α and 3α, then $4\alpha = 4a \Rightarrow \alpha = a$ and

$a^2 - 4a^2 + f(a) = 0 \Rightarrow f(a) = 3a^2 - a$

$f'(a) = 6a - 1,\ f''(a) = 6$, then minimum value of $f'(a) = 6a - 1,\ f''(a) = 6$

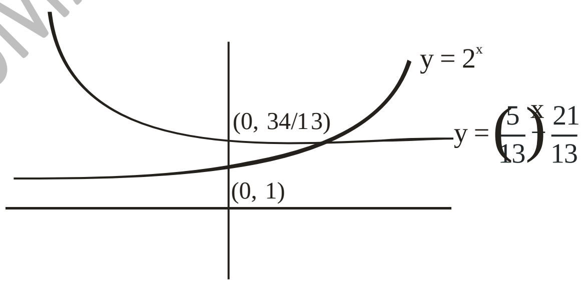
58. The number of real roots of $\left(\frac{5}{13}\right)^x + \frac{21}{13} = 2^x$ is

(A) Two (B) Infinitely many

(C) only one (D) zero

Key. C

Sol.



Both graphs cut at only one point

59. For a non zero polynomial P, the equation $|P(x)| = e^x$ has

(A) At least one solution (B) No solution

(C) Exactly 2 solution (D) Exactly 1 solution

Key. A

Sol. $\lim\limits_{x\to\infty} e^{-x}|P(x)|=0$

and $\mathop{\mathrm{Lt}}\limits_{x\to-\infty} e^{-x}|P(x)|=\infty$

consequently there is an $x_0 \in R$ such that $e^{-x_0}|P(x_0)|=1$

60. A continuous function $y=f(x)$ is defined in a closed interval $[-7,5]$. $A(-7,-4)$, $B(-2,6)$, $C(0,0)$, $D(1,6)$, $E(5,-6)$ are consecutive points on the graph of 'f' and AB, BC, CD, DE are line segments. The minimum number of real roots of the equation $f[f(x)]=6$ is

A) 6 B) 4 C) 2 D) 0

Key. A

Sol. $f[f(x)]=6 \Rightarrow f(x)=-2$ (or) $f(x)=1$

$f(x)=-2$, has two roots and $f(x)=1$ has four roots.

61. If $f(x)=-3x+\prod\limits_{i=1}^{3}(x-a_i)+\sum\limits_{i=1}^{3}a_i$, where $a_i<a_{i+1}$, then $f(x)=0$ has

A) Only one real root

B) Three real roots of which two of them are equal

C) Three distinct real roots

D) Three equal roots

Key. C

Sol. $f(x)=(x-a_1)(x-a_2)(x-a_3)+(a_1-x)+(a_2-x)+(a_3-x)$

Now, $f(x)\to-\infty$ as $x\to-\infty$ and $f(x)\to\infty$ are $x\to\infty$

Again $f(a_1)=(a_2-a_1)+(a_3-a_1)>0\ [\because a_1<a_2<a_3]$

$\Rightarrow$ One root belongs to $(-\infty,a_1)$

Also, $f(a_3)=(a_1-a_3)+(a_2-a_3)<0$

$\Rightarrow$ One root belongs to (a_1,a_3)

So, $f(x)=0$ has three distinct real roots.

62. The number of real values of $'m'$ from for which the equation $z^3+(3+i)z^2-3z-(m+i)=0$ has atleast one real root is

A) 1 B) 3 C) Infinite D) 2

Key. D

Sol. $z^3+(3+i)z^2-3z-(m+i)=0$

$(z^3+3z^2-3z-m)+i(z^2-1)=0$

If $'z'$ is a real root, then $z^3+3z^2-3z-m=0$ and $z^2-1=0$

$\therefore\ z=\pm 1$

$z=1 \Rightarrow m=1$

$z=-1 \Rightarrow m=5$

63. Number of all integral values of x, so that $x^2 + 19x + 89$ is a perfect square is

a) 0 b) 1 c) 2 d) 3

Key : C

Sol. Let $x^2+19x+89=\lambda^2$

$\Rightarrow x^2+19x+(89-\lambda^2)=0$ should have integral roots

$\therefore$ D should be a perfect square.

$\Rightarrow\quad (19)^2-4(89-\lambda^2)=\text{Perfect square}$

$\Rightarrow\quad (19)^2-4(89-\lambda^2)=\text{Perfect square}$

$\Rightarrow\quad (m^2-4\lambda^2)=5\Rightarrow(m-2\lambda)(m+2\lambda)=5$

$\therefore\quad (m-2\lambda=5, m+2\lambda=1)$

or $\quad (m-2\lambda=-5, m+2\lambda=-1)$

$\Rightarrow\quad (m-2\lambda=-5, m+2\lambda=-1)$

$\Rightarrow\quad m=3,-3, \lambda=1,-1$

For $\lambda=\pm1$ equation becomes $x^2+19x+88=0$

$\Rightarrow\quad (x+11)(x+8)=0$

$\Rightarrow\quad x=-8,-11.$

Thus, required values of x are –8, –11.

64. Let f(x) = x^2 + bx + c, b is negative odd integer, f(x) = 0 has two distinct prime number as roots, and b + c = 15, then least value of f(x) is

(A) $\frac{-233}{4}$ (B) $\frac{233}{4}$

(C) $-\frac{225}{4}$ (D) none of these

Key: C

Hint: $f(x) = (\sin^2\theta)x^3 + \frac{1}{2}\sin 2\theta\, x^2 - 2\sin^2\theta .\, x - \sin 2\theta$

$f'(x) = (3\sin^2\theta)x^2 + \sin 2\theta x - 2\sin^2\theta$

Then D > 0 and product of roots < 0

So f(x) has local maxima at some $x \in R^-$

and local minima at some $x \in R^+$

65. Let $f(x) = x^2 + \lambda x + \mu \cos x$, λ being an integer and μ a real number. The number of ordered pairs (λ, μ) for which the equations $f(x) = 0$ and $f(f(x)) = 0$ have the same (non empty) set of real roots is

(A) 4 (B) 6

(C) 8 (D) infinite

Key: A

Hint: Let α be a root of $f(x) = 0$, so we have $f(\alpha) = 0$ and thus $f(f(\alpha)) = 0$,

$\Rightarrow f(0) = 0 \Rightarrow \mu = 0$.

We then have $f(x) = x(x + \lambda)$ and thus $\alpha = 0, -\lambda$.

$f(f(x)) = x(x+\lambda)(x^2 + \lambda x + \lambda)$

We want λ such that $x^2 + \lambda x + \lambda$ has no real roots besides 0 and $-\lambda$. We can easily find that $0 \le \lambda < 4$.

66. If $ax^2 + bx + c$; $a, b, c \in R$ has no real zeroes, and if c < 0, then

(a) a < 0 (b) a + b + c > 0 (c) 4a + 2b + c > 0 (d) a – b + c > 0

Key: a

Hint: Let $f(x) = ax^2 + bx + c$. Since f(x) has no real zeroes, either f(x) > 0 or f (x) < 0 for all $x \in R$. since f(0) = c < 0, we get f(x) < 0 for all $x \in R$. Therefore, a < 0 as the parabola y = f(x) must open downward. Obviously f(1), f(–1) and f(2) < 0.

67. The quadratic equation $(4 + \cos\theta)\, x^2 - (2\sin\theta)\, x + (3 - \cos\theta) = 0$ has

(A) Real and distinct roots for all θ

(B) Real or complex roots for depending upon θ

(C) Equal roots for all θ

(D) Complex roots for all θ

Key : D

Sol : Discriminant $= 4\sin^2\theta - 4(4 + \cos\theta)(3 - \cos\theta)$

$= 4[\sin^2\theta - (12 - \cos\theta - \cos^2\theta)]$

$= 4[-11 + \cos\theta] < 0 \quad \forall \theta \in R$.

68. If $\alpha_1, \alpha_2, \ldots \alpha_n$ are roots of the equation $x^n + ax + b = 0$, then $(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)(\alpha_1 - \alpha_4) \ldots (\alpha_1 - \alpha_n)$ is equal to

(A) n (B) $n\alpha_1^{n-1}$

(C) $n\alpha_1 + b$ (D) $n\alpha_1^{n-1} + a$

KEY : D

SOL : $x^n + ax + b = (x - \alpha_1)(x - \alpha_2) \ldots (x - \alpha_n)$

differentiate both sides w.r.t. x

$nx^{n-1} + a = (x - \alpha_2) \ldots (x - \alpha_n) + (x - \alpha_1)\left(\frac{d}{dx}(x - \alpha_2) \ldots (x - \alpha_n)\right)$

put $x = \alpha_1$ $n\alpha_1^{n-1} + a = (\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3) \ldots (\alpha_1 - \alpha_n)$

69. The equation $|2ax - 3| + |ax + 1| + |5 - ax| = \frac{1}{2}$ possesses

(A) infinite number of real solution for some $a \in R$

(B) finite number of real solutions for some $a \in R$

(C) no real solution for some $a \in R$

(D) no real solution for all $a \in R$

Key: D

Hint: The equation $|2ax - 3| + |ax + 1| + |5 - ax|$......

$$|2ax - 3| + |ax + 1| + |5 - ax| \ge |2ax - 3 + (-ax - 1) + 5 - ax| \ge 1$$

So no solution for $\frac{1}{2}$

70. Let P(x) be a polynomial with degree 2009 and leading co-efficient unity such that P(0)=2008, P(1)=2007, P(2)=2006,....P(2008)=0 then the value of P(2009)= $(\underline{|n})-a$ where n and a are natural number then value of $(n+a)$

(A) 2010 (B) 2009

(C) 2011 (D) 2008

Key: A

Hint: $P(x) - 2008 + x = x(x-1)(x-2)(x-3).....(x-2008)$

Put x = 2009

$P(2009) + 1 = (2009)!$

71. (L-2)If $f(x) = ax^2 + bx + c = 0$ has real roots and its coefficients are odd positive integers then

a) f(x) = 0 always has irrational roots

b) $\left|f\left(\frac{p}{q}\right)\right| \ge \frac{1}{q^2}$ where $p, q \in I$

c) If a.c = 1, then equation must have exactly one root α such that $[\alpha] = -1,$ where [.] is greatest integer function

d) equation has rational roots

Key ; a, b

Sol : An equation with odd coefficients cannot have rational roots

$\therefore f(x)=0$ has irrational roots.

$f\left(\frac{p}{q}\right)=\frac{ap^2+bpq+cq^2}{a^2}\ge\frac{1}{a^2}$ ($\therefore$ a, b, c are odd integers p, q are integers)

72. (L-1)Let a, b, c be real numbers with $a\ne 0$ and let α,β be the roots of the equation $ax^2+bx+c=0$. Then one of the roots of the equation $a^3x^2+abcx+c^3=0$ in terms of α,β are

a) $\frac{\alpha^2}{\beta}$ b) α^3

c) β^3 d) $\alpha\beta^2$

Key: d

Sol : We have $\alpha+\beta=-\frac{b}{a},\ \alpha\beta=\frac{c}{a}$

Let γ,δ be the roots of $a^3x^2+abcx+c^3=0$.

Then $\gamma,\delta=\frac{-abc\pm\sqrt{(abc)^2-4a^3c^3}}{2a^3}=\frac{ac\left\{-b\pm\sqrt{b^2-4ac}\right\}}{2a^3}=\frac{c}{2a}\left\{-\frac{b}{a}\pm\sqrt{\left(\frac{b}{a}\right)^2-4\frac{c}{a}}\right\}$

$=\frac{1}{2}(\alpha\beta)\left\{(\alpha+\beta)\pm\sqrt{(\alpha+\beta)^2-4\alpha\beta}\right\}$

$=\frac{1}{2}(\alpha\beta)\left\{(\alpha+\beta)\pm(\alpha-\beta)\right\}=\alpha^2\beta,\alpha\beta^2$

Thus, roots of $a^3x^2+abcx+c^3=0$ are $\alpha^2\beta \text{ and } \alpha\beta^2$

73. (L-2)If α,β are the roots of $x^2-3x+\lambda=0\ (\lambda\in R)$ and $\alpha<1<\beta$, then the true set of values of λ equals

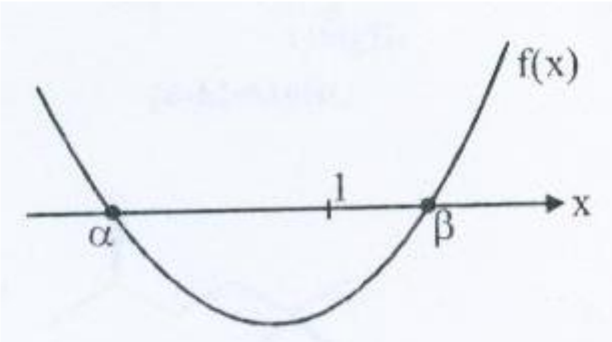
a) $\lambda\in\left(2,\frac{9}{4}\right]$ b) $\lambda\in\left(-\infty,\frac{9}{4}\right]$

c) $\lambda\in(2,\infty)$ d) $\lambda\in(-\infty,2)$

Key : d

Sol : Let $f(x)=x^2-3x+\lambda$

Clearly f(1) < 0



$\Rightarrow 1-3+\lambda<0 \Rightarrow \lambda<2 \Rightarrow \lambda \in (-\infty, 2)$

74. (L-1)Let $2^{y-x}(x+y)=1$ and $(x+y)^{x-y}=2$ then ordered pair (x, y) can be

a) $\left(\frac{3}{2}, \frac{1}{2}\right)$
b) $\left(-\frac{1}{4}, \frac{3}{4}\right)$
c) $\left(\frac{3}{2}, \frac{3}{4}\right)$
d) $\left(-\frac{1}{4}, \frac{1}{2}\right)$

Key : a

Sol : Put x = 3/2, y = ½ in given equations.

75. (L-1)The equation $|2ax-3|+|ax+1|+|5-ax|=\frac{1}{2}$ possesses

a) infinite number of real solution for some $a \in R$
b) finite number of real solutions for some $a \in R$
c) no real solution for some $a \in R$
d) no real solution for all $a \in R$

Key : d

Sol : $|2ax-3|+|ax+1|+|5-ax| \geq |2ax-3-ax-1+5-ax|$

$=1 \neq \frac{1}{2}$

Hence it has no solution

76. (L-1)If $x^2+5=2x-4\cos(a+bx)$ where $a, b \in (0,5)$, is satisfied for at least one real x, then the maximum value of (a + b) is

a) π
b) 2π
c) 3π
d) none of these

Key : c

Sol : $x^2-2x+5=-4\cos(a+bx)$

$-4\cos(a+bx)\ge 4 \to \cos(a+bx)\le -1$

$\therefore \cos(a+b)=-1$

$\therefore a+b=\pi or 3\pi$

77. (L-2)If the equation $x^n+a_1x^{n-1}+a_2x^{n-2}+.....+a_n=5,$ with integral co-efficients, has four distinct integral roots then the number of integral roots of the equation

a) 0 b) 1 c) 2 d) 4

KEY : a

Sol : Let $\alpha_i\ i=1,2,3,4 the 4$ integral roots of $x^n+a_1x^{n-1}+...+a_n=5$ and let K be an integral root of $x^n+a_1x^{n-1}+..+a_n=7$

$\Rightarrow (x-\alpha_1)(x-\alpha_2)(x-\alpha_3)(x-\alpha_4)=2$ has an integral root K.

$\Rightarrow (K-\alpha_1)(K-\alpha_2)(K-\alpha_3)(K-\alpha_4)=2$

$K-\alpha_i$, i = 1,2,3,4 are all integers and are distinct which is impossible

($\because$ product of 4 district integers cannot be 2).

Hence $x^n+a_1x^{n-1}+a_2x^{n-2}+...+a_n=7$ has no integral roots.

24. (L-1)The set of values of 'a' for which

$x^2+ax+\sin^{-1}(x^2-4x+5)+\cos^{-1}(x^2-4x+5)=0$ has at least one real solution is given by

a) $(-\infty,-\sqrt{2\pi}]\cup[\sqrt{2\pi},\infty)$ b) $\frac{-\pi-8}{4}$

c) R d) $\frac{\pi-8}{4}$

Key : b

Sol : Charly $x^2-4x+5=(x-2)^2+1, lies b|w -1,1.$ $\Rightarrow x=2$ is the only point of the domain, It must be the solution. $\therefore 4+2a+\frac{\pi}{2}=0\Rightarrow a\Rightarrow\frac{-\pi-8}{4}$

78. (L-1)If ax^2 + bx + c = 0 and 5x^2 + 6x + 12 = 0 have a common root where a , b and c are sides of a triangle ABC, then

a) ΔABC is obtuse angled b) Δ ABC is acute angled

c) Δ ABC is right angled d) none of these

Key : d

sol : 5x^2 + 6x + 12 = 0

(has complex roots only)

79. (L-1)If 0 < a < 5, 0 < b < 5 and $\frac{x^2+5}{2}=x-2\cos(a+bx)$ is satisfied for atleast one real x, then value of a + b may be equal to

a) π b) $\frac{\pi}{2}$ c) 3π d) 4π

Key : a

sol : cos (a + bx) = $-1-\frac{(x-1)^2}{4}$ exists only when x = 1

at x = 1; a + b = π

cos (a + bx) = $\frac{-(x^2-2x+5)}{4}=-1-\frac{(x-1)^2}{4}$

$\Rightarrow$ x = 1

$\Rightarrow$ a + b = 5

80. (L-1)Number of integral values of x satisfying $3x^2+8x<2\sin^{-1}\sin 4-\cos^{-1}\cos 4$ is

a) one b) two

c) three d) infinite

Key : a

Sol : $3x^2+8x<2\sin^{-1}\sin 4-\cos^{-1}\cos 4$

$3x^2+8x<2(\pi-4)-(2\pi-4)$

$<2\pi-8-2\pi+4$

<-4

$\Rightarrow 3x^2+8x+4<0$ has one solution

81. The value of 'a' for which one root of the quadratic equation $(a^2-5a+3)x^2+(3a-1)x+2=0$ is twice as large as the other, is

(A) $\frac{2}{3}$ (B) $-\frac{2}{3}$ (C) $\frac{1}{3}$ (D) $-\frac{1}{3}$

Key. A

Sol. Let the roots are α and 2α

$\Rightarrow$ $\alpha+2\alpha=\frac{1-3a}{a^2-5a+3}$ and $\alpha.2\alpha=\frac{2}{a^2-5a+3}$

$\Rightarrow$ $2\left[\frac{1}{9}\frac{(1-3a)^2}{(a^2-5a+3)^2}\right]=\frac{2}{a^2-5a+3}$

$\Rightarrow$ $9a^2-6a+1=9a^2-45a+27$

$\Rightarrow$ $39a=26$

$\Rightarrow$ $\frac{2}{3}$

82. (L-1)If a, b and c are each positive, and a + b + c = 6 then the minimum value of

$\left(a+\frac{1}{b}\right)^2+\left(b+\frac{1}{c}\right)^2+\left(c+\frac{1}{a}\right)^2$ is

a) $\frac{75}{2}$ b) $\frac{75}{4}$

c) $\frac{65}{4}$ d) $\frac{65}{2}$

Key: b

Sol: Using the $AM \ge HM$ of $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ we get, $\frac{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}{3} \ge \frac{3}{a+b+c}=\frac{3}{6}=\frac{1}{2}$

So, $\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \ge \frac{3}{2}$

Now,

$$\frac{\left(a+\frac{1}{b}\right)^2+\left(b+\frac{1}{c}\right)^2+\left(c+\frac{1}{a}\right)^2}{3} \ge \left(\frac{a+\frac{1}{b}+b+\frac{1}{c}+c+\frac{1}{a}}{3}\right)^2 \ge \left(\frac{6+\frac{3}{2}}{3}\right)^2=\left(\frac{5}{2}\right)^2=\frac{25}{4}$$

$$\therefore \left(a+\frac{1}{b}\right)^2+\left(b+\frac{1}{c}\right)^2+\left(c+\frac{1}{a}\right)^2 \ge \frac{75}{4}$$

83. (L-2)Given positive real numbers a, b and c such that a + b + c = 1, then maximum value of $a^a b^b c^c + a^b b^c a^a + a^c b^a c^b$ is

a) 1 b) 2 c) 3 d) 4

Key: a

Sol : Using the weighted AM – GM in equality we get,

$$\frac{c.a+a.b+b.c}{c+a+b} \ge \left(a^c b^a c^b\right)^{\frac{1}{a+b+c}}$$

$$\frac{b.a+c.b+a.c}{b+c+a} \ge \left(a^b.b^c.c^a\right)^{\frac{1}{a+b+c}}$$

$$\frac{a.a+b.b+c.c}{a+b+c} \ge \left(a^a b^b c^c\right)^{\frac{1}{a+b+c}}$$

Adding these inequalities together we get,

$$\frac{a^2+b^2+c^2+2(ab+bc+ca)}{a+b+c} \geq \left(a^a.b^b.c^c\right)+\left(a^cb^ac^b\right)+\left(a^bb^cc^a\right)[\because a+b+c=1]$$

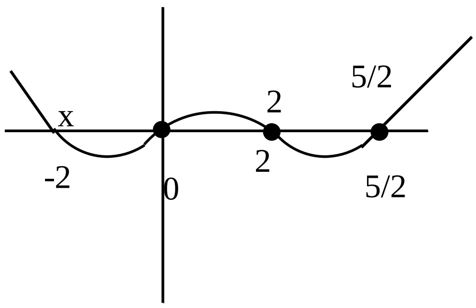
$$1=\frac{(a+b+c)^2}{a+b+c} \geq \left(a^a.b^b.c^c\right)+\left(a^c.b^a.c^b\right)+\left(a^bb^cc^a\right)$$

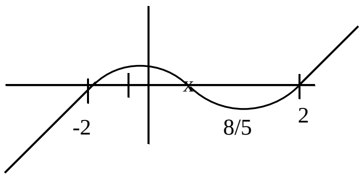
84. (L-2)The solution of $\left|\frac{x^2-5x+4}{x^2-4}\right| \leq 1$ is

a) $\left[0,\frac{8}{5}\right]\cup\left[\frac{5}{2},+\infty\right)$ b) $\left[0,\frac{5}{8}\right]\cup\left[\frac{5}{2},+\infty\right)$ c) $\left[0,\frac{5}{8}\right]\cup\left[\frac{8}{5},\infty\right)$ d) None of these

Key:A

Hint: $-1 \leq \frac{x^2-5x+4}{x^2-4} \leq 1$





$$\frac{x^2-5x+4}{x^2-4}+1 \geq 0$$

$$\frac{2x^2-5x}{x^2-4} \geq 0$$

$$\frac{x^2-5x+4}{x^2-4}-1 \leq 0$$

$$x\left(x-\pi_2\right)(x-2)(x+2) \geq 0$$

$$\frac{x^2-5x+4-x^2+4}{x^2-4} \leq 0$$

$$\frac{8-5x}{x^2-4} \leq 0$$

$$(8-5x)\left(x^2-4\right) \leq 0$$

$$(x+2)(5x-8)(x-2) \geq 0$$

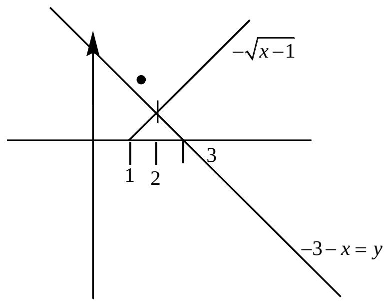
85. (L-2)Complete solution set of the inequation $\sqrt{x-1} \geq 3-x$ is

a) $2 \leq x \leq 5$ b) $2 \leq x \leq 3$

c) $1 \leq x \leq 3$ d) $x \leq 2$

Key: B

Hint:



86. (L-2) The least value of k such that the equation (ln x) + k = e^{x-k} has a solution is

a) e

b) $\frac{1}{e}$

c) 1

d) none of these

Key : c

Sol : $f(x) = e^{x-k}$ then inverse of $f(x);\ f^{-1}(x) = (\ln x) + k$

and also both functions are increasing, therefore

$f(x) = f^{-1}(x)$ is equivalent to $f(x) = f^{-1}(x) = x$

$\Rightarrow \ln x + k = x$ should have a solution

$\Rightarrow k = x - \ln x$

Now, let g(x) = x – ln x

has least value 1 as $g'(x) = 1 - \frac{1}{x}$ has a minimum at x = 1

and $\lim_{x\to 0^+} g(x),\ \lim_{x\to\infty} g(x)$ both approach to ∞.

87. (L-2)f(x) be a polynomial of degree n and $f(x) = x^n f\left(\frac{1}{x}\right)$ then f(x) = 0

a) a reciprocal equation of second type

b) not a reciprocal equation

c) a reciprocal equation of first type

d) nothing can be say.

Key : c

Sol : Let $f(x) = a_0x^n + a_1x^{n-1} + ... + a_n$

Then $x^n f\left(\frac{1}{x}\right) = x^n\left(\frac{a_0}{x^n} + \frac{a_1}{x^{n-1}} + ... + a_n\right)$

$= a_0 + a_1x + ... + a_nx^n$

Since, $f(x) = x^n f\left(\frac{1}{x}\right)$,

$\therefore a_0 = a_n, a_1 = a_{n-1}, ..., a_n = a_0$

$\therefore$ f(x) = 0 is a reciprocal equation of first type.

88. (L-2)Reduced the equation $3x^6 + x^5 - 27x^4 + 27x^2 - x - 3 = 0$ in standard reciprocal form is

a) $3x^4 + x^3 - 24x^2 + x + 3 = 0$ b) $3x^4 + x^3 + 24x^2 + x + 3 = 0$

c) $3x^4 - x^3 + 24x^2 - x + 3 = 0$ d) none of these

Key ; a

Sol : $\therefore 3x^6 + x^5 - 27x^4 + 27x^2 - x - 3 = 0$

This can be written as,

$3(x^6 - 1) + x(x^4 - 1) - 27x^2(x^2 - 1) = 0$

or, $(x^2 - 1)\{3(x^4 + x^2 + 1) + x(x^2 + 1) - 27x^2\} = 0$

or, $(x^2 - 1)\{3x^4 - 24x^2 + x^3 + x + 3\} = 0$

or, $(x^2 - 1)\{3x^4 + x^3 - 24x^2 + x + 3\} = 0$

So, $3x^4 + x^3 - 24x^2 + x + 3 = 0$ is a reciprocal equation of even degree (i.e. 4) and first type

Hence it is standard form of reciprocal equation.

89. (L-2)The polynomial $x^3 - 3x^2 - 9x + c$ can be written in the form $(x - \alpha)^2 (x - \beta)$ if value of c is

a) 5 b) -7 c) 25 d) 27

Key: d

Sol: The polynomial $x^3 - 3x^2 - 9x + c$ can be written in the form of $(x - \alpha)^2 (x - \beta)$ if the equation $x^3 - 3x^2 - 9x + c = 0$ has two equal roots. Let these be α, α, β.

We have $\alpha + \alpha + \beta = 3$ or $2\alpha + \beta = 3$ …. (1)

$\alpha\alpha + \alpha\beta + \alpha\beta = -9$ or $2\alpha\beta + \alpha^2 = -9$ …. (2)

Putting value of β in (2) we get

$2\alpha(3 - 2\alpha) + \alpha^2 = -9$

or $6\alpha - 3\alpha^2 = -9$

$\Rightarrow \alpha^2 - 2\alpha - 3 = 0$

$\Rightarrow (\alpha - 3)(\alpha + 1) = 0 \Rightarrow \alpha = -1, 3$

When $\alpha = -1, \beta = 5$ and when $\alpha = 3, \beta = -3$. We also have $\alpha^2\beta = -c$

When $\alpha = -1, \beta = 5, c = -5$ when $\alpha = 3, \beta = -3, c = 27$

90. (L-1)The smallest positive value of p for which the equation cos (p sin α) = (p cos α) has a solution $\forall \alpha \in [0, 2\pi]$ is

a) $\frac{\pi}{\sqrt{2}}$ b) $\pi\sqrt{2}$ c) $\frac{\pi\sqrt{2}}{4}$ d) $\frac{\pi}{4\sqrt{2}}$

Key : c

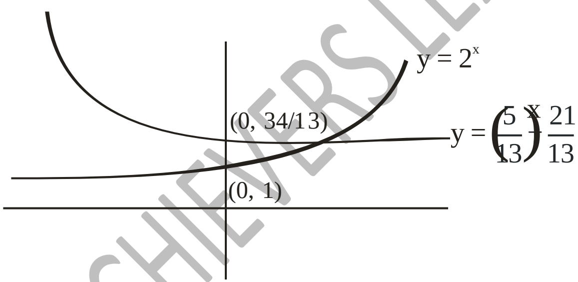
Sol : $\sin\left(\pi + \frac{\pi}{4}\right) = 1 \Rightarrow$ P is minimum

$\Rightarrow P = \frac{\pi}{2\sqrt{2}}$

91. The number of real roots of $\left(\frac{5}{13}\right)^x + \frac{21}{13} = 2^x$ is

(A) Two (B) Infinitely many
(C) only one (D) zero

Key. C



Sol.
Both graphs cut at only one point

92. For a non zero polynomial P, the equation $|P(x)| = e^x$ has

(A) At least one solution (B) No solution
(C) Exactly 2 solution (D) Exactly 1 solution

Key. A

Sol. $\lim_{x\to\infty} e^{-x}|P(x)| = 0$

and $\text{Lt}_{x\to-\infty} e^{-x}|P(x)| = \infty$

consequently there is an $x_0 \in R$ such that $e^{-x_0}|P(x_0)| = 1$

93. Number of rational roots of the equation $\left|x^2 - 2x - 3\right| + 4x = 0$ is

a) 0 b) 1 c)2 d) 4

Key. B

Sol.
$x^2 - 2x - 3 \geq 0 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow x = -3$
$x^2 - 2x - 3 < 0 \Rightarrow x^2 - 6x - 3 = 0$ *no rational root*

94. If the equations $2x^2 - 7x + 1 = 0$ and $ax^2 + bx + 2 = 0$ have a common root, then

a) a=2,b=-7 b) $a = \frac{-7}{2}, b = 1$ c) a = 4, b = –14 d) a = –4, b = 1

Key. C

Sol. First equation has irrational roots.:. both roots common

95. If p,q,r I R and the quadratic equation $px^2 + qx + r = 0$ has no real root, then

a) $p(p+q+r) > 0$ b) $p(p+q+r) < 0$

c) $q(p+q+r) > 0$ d) $q(p+q+r) < 0$

Key. A

Sol. $p\left(px^2 + qx + r\right) > 0 \quad for\ x \in R.$ Take x=1

96. For $x^2 - (\alpha + 2)|x| + 9 = 0$ to have real solutions, the range of 'α' is

(A) $(-\infty, 4]$ (B) $[4, \infty)$

(C) $(-\infty, 7] \cup [11, \infty)$ (D) $[-4, \infty)$

Key. B

Sol. $\alpha = \frac{x^2 + 9}{|x|} - 2 = |x| + \frac{9}{|x|} - 2$

$\alpha \geq 4$.

97. The number of solution(s) of the equations $e^x = x^2$ and $e^x = x^3$ are respectively

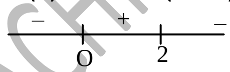
(A) 1 and 2 (B) 1 and 0

(C) 3 and 2 (D) 2 and 1

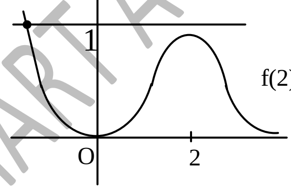
Key. A

Sol. Let $f(x) = e^{-x} x^k$, $f'(x) = e^{-x} x^{k-1} (k - x)$

For k = 2, f′(x) :



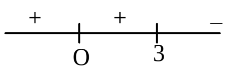
f(x) :



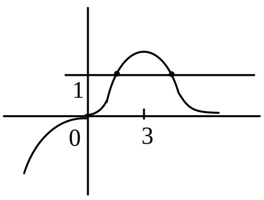
$f(2) = \left(\frac{2}{e}\right)^2 < 1$

So, one solution.

For k = 3, f′(x) :



f(x) :



$f(3) = \left(\frac{3}{e}\right)^3 > 1$

So, two solutions.

98. If a,b,c,d are four positive numbers in G.P. then the minimum value of $\frac{c+d}{b}$ is

(A) $\frac{3b^{\frac{1}{3}}c^{\frac{1}{3}}+a^{2/3}}{a^{2/3}}$ (B) $\frac{3(bc)^{\frac{1}{3}}-2a^{2/3}}{a^{2/3}}$

(C) $\frac{3(bc)^{\frac{1}{3}}+3a^{2/3}}{a^{2/3}}$ (D) $\frac{3b^{\frac{1}{3}}c^{\frac{1}{3}}-a^{2/3}}{a^{2/3}}$

Key. D

Sol. Let b = ar, c = ar^2 , d = ar^3

$$\frac{c+d}{b}=r+r^2$$

$$\frac{3b^{\frac{1}{3}}c^{\frac{1}{3}}-a^{2/3}}{a^{2/3}}=3r-1$$

Since $(r-1)^2 \geq r^2-2r+1 \geq 0 \Rightarrow r^2+r \geq 3r-1 \Rightarrow \frac{c+d}{b} \geq \frac{3b^{\frac{1}{3}}c^{\frac{1}{3}}-a^{2/3}}{a^{2/3}}$

99. Three distinct positive real numbers a, b, c are in H.P. then for the quadratic equation $x^2 - kx + 2b^{101} - a^{101} - c^{101} = 0$, $k \in R$ has

(a) roots of same sign (b) roots of opposite sign

(c) roots of imaginary (d) roots are real and equal

Key. B

SOL. IF α, β ARE ROOTS

THEN $\alpha\beta = 2B^{101} - A^{101} - C^{101}$

NOW $\frac{a^{101}+c^{101}}{2} \geq (\sqrt{ac})^{101} \geq b^{101}$

$\Rightarrow$ $2B^{101} - A^{101} - C^{101} < 0$

$\Rightarrow$ $\alpha\beta < 0$

$\Rightarrow$ roots are opposite in sign.

100. If α and β, α and γ , α and δ are the roots of the equations $ax^2+2bx+c=0, 2bx^2+cx+a=0$ and $cx^2+ax+2b=0$ respectively where a, b, c are positive real numbers, then $\alpha+\alpha^2=$

a) -1 b) 1 c) 0 d) abc

Key. A

Sol. $a\alpha^2+2b\alpha+c=0$

$a+2b\alpha^2+c\alpha=0$ then $(a+2b+c)(1+\alpha+\alpha^2)=0$

$a\alpha+2b+c\alpha^2=0$

$\because$ a, b, c$\in R^+$ then $\alpha+\alpha^2=-1$

101. If a, b, c are in geometric progression and the roots of the equations $ax^2+2bx+c=0$ are α and β and those of $cx^2+2bx+a=0$ are γ and δ then

a) $\alpha \neq \beta \neq \gamma \neq \delta$ b) $\alpha \neq \beta$ and $\gamma \neq \delta$

c) $a\alpha = a\beta = c\gamma = c\delta$ d) $\alpha = \beta; \gamma \neq \delta$

Key. C

Sol. $\because b^2 = ac;$ the roots of both the equations are equal.

$\therefore \alpha=\beta$; and $\gamma=\delta$. But $\gamma=\frac{1}{\alpha}:\ \delta=\frac{1}{\beta}$ as the given equations are reciprocal to each other

$\therefore \gamma\delta=\frac{a}{c}$ then $c\gamma=a\beta$

$a\alpha=a\beta=c\gamma=c\delta$

102. If $f(x)=(x^2+3x+2)(x^2-7x+a)$ and $g(x)=(x^2-x-12)(x^2+5x+b)$ then the values of a and b, If $(x+1)(x-4)$ is HCF of $f(x)$ and $g(x)$

a) $a=10:b=6$ b) $a=4:b=12$
c) $a=12:b=4$ d) $a=6:b=10$

Key. C

Sol. x^2-7x+a is divisible by $x-4$ & x^2+5x+b is divisible by $x+1$
$\therefore a=12; b=4$

103. The equation $(x^2+3x+4)^2+3(x^2+3x+4)+4=x$ has

a) all its solutions real but not all positive b) only two of its solutions real
c) two of its solutions positive and two negative d) none of solutions real.

Key. D

Sol. $f(x)=ax^2+bx+c$: If $f(x)=x$ has no real solution then $f(f(x))=x$ also has no real solution:

104. Let A be a square Matrix all of whose entries are integers. Then which of the following is True?

a) If $\det A=\pm1$, then A^{-1} exists but all it entries are not necessarily integers.
b) If $\det A\neq\pm1$, then A^{-1} exists and all it entries are non integers.
c) If $\det A=\pm1$, then A^{-1} exists ad all its entries are integers.
d) If $\det A=\pm1$, A^{-1} need not exist.

Key. C

Sol. Conceptual

105. The values of a for which the roots of the equation $(a+1)x^2-3ax+4a=0\ (a\neq-1)$ are real and greater than 1

a) $\left[-\frac{10}{7},1\right]$ b) $\left[-\frac{12}{7},0\right]$ c) $\left[-\frac{16}{7},-1\right)$ d) $\left(-\frac{16}{7},0\right)$

Key. C

Sol. $D=9a^2-16a(a+1)\geq0, x_1>1, x_2>1$

Where $x_1+x_2=\frac{3a}{a+1}, x_1x_2=\frac{4a}{a+1}\Rightarrow x_1+x_2-1>0\ \&\ (x_1-1)(x_2-1)>0$

$\Rightarrow a(7a+16)\leq0 \quad (1)$

$\frac{a-2}{a+1}>0 \quad (2)$

$\frac{2a+1}{a+1}>0 \quad (3)$

Solving $-\frac{16}{7}\leq a<-1$.

106. If the equation $x^4-4x^3+ax^2+bx+1=0$ has four positive roots then (a, b) is given by
(A) (4, 6) (B) (6, –4)

(C) (–4, –6) (C) (2, 3)

Key. B

Sol. Let the roots of the equation be x_1, x_2, x_3, x_4 then $x_1 + x_2 + x_3 + x_4 = 4$
and $x_1 x_2 x_3 x_4 = 1$
As A.M ≥ G.M and equality sign holds only when numbers are equal.
We have $1 = \frac{x_1 + x_2 + x_3 + x_4}{4} \geq (x_1 x_2 x_3 x_4)^{\frac{1}{4}} = 1$
$\Rightarrow x_1 = x_2 = x_3 = x_4 = 1$
$\Rightarrow x^4 - 4x^3 + ax^2 + bx + 1 = (x-1)^4 \Rightarrow a = 6, b = -4$.

107. If roots of the equation $ax^2 + bx + c = 0$; $a, b, c \in R^+$ be non–real numbers, lying inside the unit circle, centered at origin, then

(A) $b > 0$ (B) $b < a$

(C) $c < a$ (D) none of these

Key. C

Sol. Let z_1 be one of the root, then the other root is $\bar{z}_1$

$|z_1|^2 = \frac{c}{a} \Rightarrow \frac{c}{a} < 1 \Rightarrow c < a$

108. If both the roots of the equation $x^2 + 2bx + \log_3(b^2 - 4b + 4) = 0$ are of opposite sign then 'b' belongs to

(A) (1, 3) (B) $(-\infty, 1) \cup (3, \infty)$

(C) [1, 3] (D) $(1, 2) \cup (2, 3)$

Key. D

Sol. Let $f(x) = x^2 + 2bx + \log_3(b^2 - 4b + 4)$
For both roots to be of opposite sign
$f(0) < 0 \Rightarrow \log_3(b^2 - 4b + 4) < 0$
$\Rightarrow b^2 - 4b + 4 < 1$
$\Rightarrow b^2 - 4b + 3 < 0$
$\Rightarrow (b-1)(b-3) < 0 \Rightarrow 1 < b < 3$
But $b \neq 2$
$\therefore b \in (1, 2) \cup (2, 3)$.

109. Let $f(x) = x^3 + ax^2 + bx + c$ and x_1, x_2 be the roots of $f'(x) = 0$, if $x_1 < x_2$ then $f(x) = 0$ will have

a) No real root if $f(x_1) < 0$ or $f(x_2) > 0$

b) Only one real root if $f(x_1) < 0$ or $f(x_2) > 0$

c) Three real roots if $f(x_1) < 0$ or $f(x_2) > 0$

d) cannot say any thing

Key. B

Sol. Since coefficient of x^3 is Positive .

$\therefore$ local maximum is at x_1 and local minimum is at x_2 . case (i) : If $f(x_1) < 0$ then $f(x_2) < f(x_1) < 0$ then the only real root will be in (x_2, ∞) case (ii) : If $f(x_2) > 0$ then $f(x_1) > f(x_2) > 0$ then equation will have only one real root in the interval $(-\infty, x)$.

110. Let $f_1(x)$ and $f_2(x)$ be continuous and differentiable functions. If

$f_1(0)=f_1(2)=f_1(4), f_1(1)+f_1(3)=f_2(0)=f_2(2)=f_2(4)=0$ and if $f_1(x)=0$ and

$f_2^1(x)=0$ do not have common root, then the minimum number of zeros of,

$f_1^1(x)f_2^1(x)+f_1(x)f_2^{11}(x)$ in $[0,4]$, is

a) 2 b) 4 c) 5 d) 3

Key. D

Sol. $f_1(x)=0$ has mini two sols in $[0,4]$

$f_2(x)=0$ has mini 3 sols in [0,4]

$f_2^1(x)=0$ has mini 2 sol in [0,4]

$f_1(x)f_2^1(x)=0$ has minimum 4 sols in [0,4]

$\frac{d}{dx}\left(f_1(x)f_2^1(x)\right)=0$ has mini 3 sols in [0.4]

111. For $x^2-(\alpha+2)|x|+9=0$ to have real solutions, the range of 'α' is

(A) $[-\infty, 4]$ (B) $[4, \infty)$

(C) $(-\infty, 7] \cup [11, \infty)$ (D) $[-4, \infty)$

Key. B

Sol. $\alpha=\frac{x^2+9}{|x|}-2=|x|+\frac{9}{|x|}-2$

$\Rightarrow \quad \alpha \geq 4.$

112. $0 < c < b < a$ and α, β are roots of equation $cx^2 + bx + a = 0$ if α, β are non real then

(A) $\frac{|\alpha|+|\beta|}{2}=|\alpha||\beta|$ (B) $\frac{2}{|\alpha|}=\frac{1}{|\beta|}$

(C) $\frac{1}{|\alpha|}+\frac{1}{|\beta|}<2$ (D) $|\alpha|+\frac{1}{|\beta|}<2$

Key. C

SOL. $\alpha\beta=\frac{a}{c}>1$

$|\alpha|\,|\beta| > 1$

$\Rightarrow \quad |\alpha|^2 > 1$

$\Rightarrow \quad |\alpha| > 1$

$\Rightarrow \quad |\beta| > 1$

$\Rightarrow \quad \frac{1}{|\alpha|}+\frac{1}{|\beta|}<2$

113. If two roots of the equation $(P-1)(x^2+x+1)^2-(p+1)(x^4+x^2+1)=0$ are real and distinct and $f(x)=\frac{1-x}{1+x}$ then $f(f(x))+f\left(f\left(\frac{1}{x}\right)\right)$ is equal to ___

a) P b) $-P$ c) $2P$ d) $-2P$

Key. A

Sol. $\frac{p+1}{p-1}=\frac{x^2+x+1}{x^2-x+1} \Rightarrow \frac{2p}{2}=\frac{2\left(x^2+1\right)}{2x} \Rightarrow p=x+\frac{1}{x}$

As $f\left(x\right)=\frac{1-x}{1+x} \Rightarrow f\left(f\left(x\right)\right)+f\left(f\left(\frac{1}{x}\right)\right)=x+\frac{1}{x}$

$\Rightarrow f\left(f\left(x\right)\right)+f\left(f\left(\frac{1}{x}\right)\right)=p$

114. If α_1, α_2, ... α_n are roots of the equation $x^n + ax + b = 0$, then $(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)(\alpha_1 - \alpha_4) \ldots (\alpha_1 - \alpha_n)$ is equal to

(A) n (B) $n\alpha_1^{n-1}$

(C) $n\alpha_1 + b$ (D) $n\alpha_1^{n-1}+a$

Key. D

Sol. $x^n + ax + b = (x - \alpha_1)(x - \alpha_2) \ldots (x - \alpha_n)$

differentiate both sides w.r.t. x

$nx^{n-1} + a = (x - \alpha_2) \ldots (x - \alpha_n) + (x - \alpha_1)\left(\frac{d}{dx}(x - \alpha_2) \ldots (x - \alpha_n)\right)$

put $x = \alpha_1$

$n\alpha_1^{n-1}+a = (\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3) \ldots (\alpha_1 - \alpha_n)$

115. ω is a non real complex cube root of unity and $a,b\in R$ .If ω,ω^2 are roots of $\frac{1}{a+x}+\frac{1}{b+x}=\frac{3}{x}$ then *a,b* are roots of

a) $3x^2-6x+2=0$ b) $6x^2-3x+2=0$

c) $2x^2-3x+6=0$ d) $6x^2-2x+3=0$

Key. B

Sol. The given equation simplifies $x^2+2x\left(a+b\right)+3ab=0$, whose roots are given table ω,ω^2

Hence $a+b=\frac{1}{2},ab=\frac{1}{3}$ .So a,b are roots of $x^2-x\left(\frac{1}{2}\right)+\frac{1}{3}=0$

116. If the function $f(x)=x^3+3(a-7)x^2+3(a^2-9)x-1$ has a point of maximum at positive values of x then

(a) $a\in\left(-\infty,\frac{29}{7}\right)$ (b) $a\in\left(-\infty,7\right)$

(c) $a\in\left(-\infty,-3\right)\cup\left(3,\frac{29}{7}\right)$ (d) $a\in\left(3,\infty\right)\cup\left(-\infty,-3\right)$

Key. C

Sol. $f(x)=x^3+3(a-7)x^2+3(a^2-9)x-1$

$f^{'}(x)=3x^2+6(a-7)x+3(a^2-7)$

The roots of $f'(x)=0$ positive and distinct which is possible if

(i) $b^2-4ac>0 \Rightarrow 6(a-7)^2-4(3)(3)(a^2-9)>0$

$\Rightarrow a < \frac{29}{7}$

(ii) Product of Roots > 0 $a^2 - 9 > 0$

(iii) Sum of Roots > 0 $a - 7 < 0$

$a < 7$

$\Rightarrow$ From i, ii, iii $a \in (-\infty, -3) \cup (3, \frac{29}{7})$

117. If α, β are the roots of $x^2 - px + q = 0$ then value of $\frac{\alpha^2 + \beta^2}{\alpha^{-2} + \beta^{-2}} =$

(A) p (B) q (C) p^2 (D) q^2

Key. D

Sol. $\alpha^2\beta^2 = q^2$

118. For $p > 0$ and $3x^2 + px + 3 = 0$ one root of above equation is square of the other then p is

(A) – 6 (B) 10 (C) 2 (D) 3

Key. D

Sol. $\alpha + \alpha^2 = \frac{-1}{3}; \alpha^3 = 1$

$\alpha = 1, \omega, \omega^2$

If $\alpha = 1$

P=-6 as P>0 neglected

$if \alpha = \omega; P = 3$

119. If one root or the equation $x^2 - 2x + k = 0$ is 1+2i and $k \in R$ then the value of k is

(A) -3 (B) -5 (C) 5 (D) 3

Key. C

Sol. $b^2 = 4ac => 4m^2 = 4(8m - 15)$

$m^2 - 8m + 15 = 0; m = +3, +5$

120. If $\left|\frac{12x}{4x^2 + 9}\right| \le 1$ then

(A) $x \in R$ (B) $x \in \phi$

(C) $x \in \{1\}$ (D) $x \in C$ where C is set of complex numbers

Key. A

Sol. $12|x| \le 4x^2 + 9$

$(2x - 3)^2 \ge 0$; $x \in R$

121. If α, β are roots of $3x^2 + 2bx + c = 0$ whose descriminant is $\Delta_1; \alpha + \delta, \beta + \delta$ are roots of $9x^2 + 2Bx + C = 0$ whose descriminant is Δ_2 then $\frac{\Delta_1}{\Delta_2}$ is

(A) $\frac{1}{9}$ (B) 9 (C) 3 (D) $\frac{1}{3}$

Key. A

Sol. $\alpha-\beta=\frac{\sqrt{\Delta_1}}{3}$

$(\alpha+\delta)-(\beta+\delta)=\frac{\sqrt{\Delta_2}}{9}$

$\frac{\Delta_1}{9}=\frac{\Delta_2}{81};\frac{\Delta_1}{\Delta_2}=\frac{1}{9}$

122. If the sum of the roots of the equation $5x^2-4x+2+k(4x^2-2x-1)=0$ is 6 , then k =

(A) $13/17$ (B) $17/13$ (C) $-17/13$ (D) $-13/11$

Key. D

Sol. sum of the roots =6

$\frac{2k+4}{5+4k}=6 => k=\frac{-13}{11}$

123. If $\tan\alpha$, $\tan\beta$, $\tan\gamma$ are the roots of the equation $x^3-px^2-r=0$ then the value of $(1+\tan^2\alpha)(1+\tan^2\beta)(1+\tan^2\gamma)$ is equal to

a) $(p-r)^2$ b) $1+(p-r)^2$ c) $1-(p-r)^2$ d) none

Key. B

Sol. $(1+\tan^2\alpha)(1+\tan^2\beta)(1+\tan^2\gamma)$

$=1+(\tan^2\alpha+\tan^2\beta+\tan^2\gamma)+(\tan^2\alpha\tan^2\beta+\tan^2\beta\tan^2\gamma+\tan^2\gamma\tan^2\alpha)+\tan^2\alpha\tan^2\beta\tan^2\gamma$

$=1-(p-r)^2$ $\because x^2y^2+y^2z^2+z^2x^2$

$=(xy+yz+zx)^2-2xyz(x+y+z)$

124. If the equation $x^2+9y^2-4x+3=0$ is satisfied for real values of x and y then

A) $x\in[1,3], y\in[1,3]$ B) $x\in[1,3], y\in\left[\frac{-1}{3},\frac{1}{3}\right]$

C) $x\in\left[\frac{-1}{3},\frac{1}{3}\right], y\in[1,3]$ D) $x\in\left[\frac{-1}{3},\frac{1}{3}\right], y\in\left[\frac{-1}{3},\frac{1}{3}\right]$

Key. B

Sol. (B)Given equation is $x^2+9y^2-4x+3=0$ …(i)

Or, $x^2-4x+9y^2+3=0.$

Since x is real $\therefore$ $(-4)^2-4(9y^2+3)\geq 0$

Or, $16-4(9y^2+3)\geq 0$ or, $4-9y^2-3\geq 0$

Or, $9y^2-1\leq 0$ or, $9y^2\leq 1$ or, $y^2\leq\frac{1}{9}$

Now $y^2\leq\frac{1}{9}\Leftrightarrow -\frac{1}{3}\leq y\leq\frac{1}{3}$ …(ii)

Equation (i) can also be written as

$$9y^2+0y+x^2-4x+3=0 \qquad ...(iii)$$

Since y is real $\therefore 0^2-4.9\left(x^2-4x+3\right)\geq 0$

Or, $x^2-4x+3\leq 0$

$\Rightarrow x\in[1,3]$

125. The equation $a_8x^8+a_7x^7+a_6x^6+...+a_0=0$ has all its roots positive and real $\left(where\ a_8=1, a_7=-4, a_0=1/2^8\right)$, then

A) $a_1=\frac{1}{2^8}$ B) $a_1=-\frac{1}{2^4}$ C) $a_2=\frac{7}{2^5}$ D) $a_2=\frac{7}{2^8}$

Key. B

Sol. (B) Let the roots be $\alpha_1,\alpha_2,....,\alpha_8$

$\Rightarrow \quad \alpha_1+\alpha_2+....+\alpha_8=4$

$\alpha_1\alpha_2.....\alpha_8=\frac{1}{2^8}$

$\Rightarrow \quad \left(\alpha_1\alpha_2.....\alpha_8\right)^{1/8}=\frac{1}{2}=\frac{\alpha_1+\alpha_2+...+\alpha_8}{8}$

$\Rightarrow$ AM=GM $\Rightarrow$ all the roots are equal to $\frac{1}{2}$.

$\Rightarrow \quad a_1=-{}^8C_7\left(\frac{1}{2}\right)^7=-\frac{1}{2^4}$

$a_2={}^8C_6\left(\frac{1}{2}\right)^6=\frac{7}{2^4}$

$a_3=-{}^8C_5\left(\frac{1}{2}\right)^5$

126. If a, b, c are positive numbers such that a>b>c and the equation $\left(a+b-2c\right)x^2+\left(b+c-2a\right)x+\left(c+a-2b\right)=0$ has a root in the interval $\left(-1,0\right)$, then

A) b cannot be the G.M. of a, c B) b may be the G.M. of a, c

C) b is the G.M. of a, c D) none of these

Key. A

Sol. Let $f\left(x\right)=\left(a+b-2c\right)x^2+\left(b+c-2a\right)x+\left(c+a-2b\right)$

According to the given condition, we have

$f\left(0\right)f\left(-1\right)<0$

i.e. $\left(c+a-2b\right)\left(2a-b-c\right)<0$

i.e. $\left(c+a-2b\right)\left(a-b+a-c\right)<0$

i.e. $c+a-2b<0 \qquad \left[a>b>c,\ given\Rightarrow a-b>0, a-c>0\right]$

i.e. $b>\frac{a+c}{2}$

$\Rightarrow$ *b* cannot be the G.M. of *a, c,* since G.M < A.M. always.

127. Let α, β (a < b) be the roots of the equation $ax^2+bx+c=0$. If $\lim_{x\to m}\frac{|ax^2+bx+c|}{ax^2+bx+c}=1$, then

A) $\frac{|a|}{a}=-1, m<\alpha$ B) $a>0, \alpha<m<\beta$ C) $\frac{|a|}{a}=1, m>\beta$ D) $a<0, m>\beta$

Key. C

Sol. According to the given condition, we have

$$|am^2+bm+c|=am^2+bm+c$$

i.e. $am^2+bm+c>0$

$\Rightarrow$ if *a< 0,* the *m* lies in (α,β)

and if *a>0,* then *m* does not lies in (α,β)

Hence, option (c) is correct, since

$$\frac{|a|}{a}=1\Rightarrow a>0$$

And in that case *m* does not lie in (α,β).

128. Let $f(x)$ be a function such that $f(x)=x-[x]$, where $[x]$ is the greatest integer less than or equal to x. Then the number of solutions of the equation $f(x)+f\left(\frac{1}{x}\right)=1$ is (are)

A) 0 B) 1 C) 2 D) infinite

Key. D

Sol. Given, $f(x)=x-[x],\ x\in R-\{0\}$

Now $f(x)+f\left(\frac{1}{x}\right)=1$ $\therefore$ $x-[x]+\frac{1}{x}-\left[\frac{1}{x}\right]=1$

$\Rightarrow\left(x+\frac{1}{x}\right)-\left([x]+\left[\frac{1}{x}\right]\right)=1$ $\Rightarrow\left(x+\frac{1}{x}\right)=[x]+\left[\frac{1}{x}\right]+1$ ...(i)

Clearly ,R.H.S is an integer $\therefore$ L. H. S. is also an integer

Let $x+\frac{1}{x}=k$ an integer $\Rightarrow x^2-kx+1=0$

$\therefore x=\frac{k\pm\sqrt{k^2-4}}{2}$

For real values of *x*, $k^2-4\ge 0\Rightarrow k\ge 2$ or $k\le -2$

We also observe that *k=2* and -2 does not satisfy equation *(i)*

$\therefore$ The equation *(i)* will have solutions if $k>2$ or $k<-2$, where $k\in z$.

Hence equation *(i)* has infinite number of solutions.

129. If both the roots of $(2a-4)9^x-(2a-3)3^x+1=0$ are non-negative, then

A) $0<a<2$ B) $2<a<\frac{5}{2}$ C) $a<\frac{5}{4}$ D) $a>3$

Key. B

Sol. Putting $3^x=y$, we have

$$(2a-4)y^2-(2a-3)y+1=0$$

This equation must have real solution

$\Rightarrow \quad (2a-3)^2 - 4(2a-4) \ge 0$

$\Rightarrow \quad 4a^2 - 20a + 25 \ge 0$

$\Rightarrow \quad (2a-5)^2 \ge 0.$ This is true.

$y=1$ satisfies the equation

Since 3^x is positive and $3^x \ge 3^0,\ y \ge 1$

Product of the roots $=1 \times y > 1$

$\Rightarrow \quad \frac{1}{2a-4} > 1$

$\Rightarrow \quad 2a-4 < 1 \Rightarrow \ a < \frac{5}{2}$

Sum of the roots $= \frac{2a-3}{2a-4} > 1$

$\Rightarrow \quad \frac{(2a-3)-(2a-4)}{2a-4} > 0$

$\Rightarrow \quad \frac{1}{2a-4} > 0 \Rightarrow a > 2$

$\Rightarrow \quad 2 < a < \frac{5}{2}$

130. Let α and β be the roots of $x^2-6x-2=0$ with $\alpha > \beta$ if $a_n = \alpha^n - \beta^n$ for $n \ge 1$ then the value of $\frac{a_{10}-2a_8}{3a_9} =$

1) 1 2) 2 3) 3 4) 4

Key. 2

Sol. $\alpha^2 - 6\alpha - 2 = 0$ $\quad\quad \beta^2 - 6\beta - 2 = 0$

$\Rightarrow \alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0$ ....(1)

$\Rightarrow \beta^{10} - 6\beta^9 - 2\beta^8 = 0$ ....(2)

subtract (2) from (1)

131. If a,b,c are positive real numbers such that $a+b+c=1$ then the least value of $\frac{(1+a)(1+b)(1+c)}{(1-a)(1-b)(1-c)}$ is

1) 16 2) 8 3) 4 4) 5

Key. 2

Sol. $a = 1-b-c$

$\Rightarrow 1+a = (1-b)+(1-c) \ge 2\sqrt{(1-b)(1-c)}$

$\therefore (1+a)(1+b)(1+c) \ge 8(1-a)(1-b)(1-c)$

132. The range of values of $'a'$ for which all the roots of the equation $(a-1)(1+x+x^2)^2 = (a+1)(1+x^2+x^4)$ are imaginary is

1) $(-\propto,-2]$ 2) $(2,\propto)$

3) $(-2,2)$ 4) $[2,\infty)$

Key. 3

Sol. The given equation can be written as $(x^2+x+1)(x^2-ax+1)=0$

133. If α,β are the roots of the equation $ax^2+bx+c=0$ and $S_n=\alpha^n+\beta^n$ then $aS_{n+1}+bS_n+cS_{n-1}=\quad (n\ge 2)$

1) 0 2) $a+b+c$

3) $(a+b+c)n$ 4) $n^2\ abc$

Key. 1

Sol. $S_{n+1}=\alpha^{n+1}+\beta^{n+1}$

$=(\alpha+\beta)(\alpha^n+\beta^n)-\alpha\beta(\alpha^{n-1}+\beta^{n-1})$

$= -\frac{b}{a}.S_n-\frac{c}{a}.S_{n-1}$

134. A group of students decided to buy a Alarm Clock priced between Rs. 170 to Rs 195. But at the last moment, two students backed out of the decision so that the remaining students had to pay 1 Rupee more than they had planned. If the students paid equal shares, the price of the Alarm Clock is

1) 190 2) 196

3) 180 4) 171

Key. 3

Sol. Let cost of clock = x

number of students $=n$

then $\frac{x}{n-2}=\frac{x}{n}+1\Rightarrow x=\frac{n^2-2n}{2}$

$\Rightarrow 170\le\frac{n^2-2n}{2}\le 195$

135. If $\tan A,\tan B$ are the roots of $x^2-Px+Q=0$ the value of $\sin^2(A+B)=$ $(where\ P,Q\in R)$

1) $\frac{P^2}{P^2+(1-Q)^2}$ 2) $\frac{P^2}{P^2+Q^2}$

3) $\frac{Q^2}{P^2+(1-Q)^2}$ 4) $\frac{P^2}{(P+Q)^2}$

Key. 1

Sol. $\tan(A+B)=\frac{P}{1-Q}$ then $\sin^2(A+B)=\frac{\tan^2(A+B)}{1+\tan^2(A+B)}$

136. The number of solutions of $|[x]-2x|=4$ where $[x]$ is the greatest integer $\le x$ is

1) 2 2) 4

3) 1 4) Infinite

Key. 2

Sol. If $x = n \in Z, \quad |n-2n| = 4 \Rightarrow n = \pm 4$

If $x = n + K$ where $0 < K < 1$ then $|n - 2(n+k)| = 4$, it is possible if $K = \frac{1}{2}$

$\Rightarrow |-n-1| = 4$

$\therefore n = 3, -5$

137. Let a, b and c be real numbers such that $a + 2b + c = 4$ then the maximum value of $ab + bc + ca$ is

1) 1 2) 2 3) 3 4) 4

Key. 4

Sol. Let $ab + bc + ca = x$

$\Rightarrow 2b^2 + 2(c-2)b - 4c + c^2 + x = 0$

Since $b \in R$,

$\therefore c^2 - 4c + 2x - 4 \le 0$

Since $c \in R$

$\therefore x \le 4$

138. For the equation $3x^2 + px + 3 = 0, p > 0$, if one root is the square of the other then value of P is

1) $\frac{1}{3}$ 2) 1 3) 3 4) $\frac{2}{3}$

Key. 3

Sol. $\alpha + \alpha^2 = -\frac{p}{3}$

$\alpha^3 = 1$

139. If the equations $2x^2 + kx - 5 = 0$ and $x^2 - 3x - 4 = 0$ have a common root, then the value of k is

1) -2 2) -3

3) $\frac{27}{4}$ 4) $-\frac{1}{4}$

Key. 2

Sol. If 'α' is the common root then $2\alpha^2 + k\alpha - 5 = 0,\ \alpha^2 - 3\alpha - 4 = 0$ solve the equations.

140. If α and β are the roots of the equation $x^2 - x + 1 = 0$ then $\alpha^{2009} + \beta^{2009} =$

1) 1 2) 2

3) -1 4) -2

Key. 1

Sol. $x=\frac{1\pm i\sqrt{3}}{2}$

$\therefore \alpha=-\omega,\ \beta=-\omega^2$

141. If $P(Q-r)x^2+Q(r-P)x+r(P-Q)=0$ has equal roots then $\frac{2}{Q}=$

$(where\ P,Q,r\in R)$

1) $\frac{1}{P}+\frac{1}{r}$ 2) $\frac{1}{P}-\frac{1}{r}$

3) $P+r$ 4) Pr

Key. 1

Sol. Product of the roots $=1$

142. The solution of the differential equation $y_1y_3=3y_2^2$ is

1) $x=A_1y^2+A_2y+A_3$ 2) $x=A_1y+A_2$

3) $x=A_1y^2+A_2y$ 4)none of these

Key. 1

Sol. $y_1y_3=3y_2^2$

$\frac{y_3}{y_2}=3\frac{y_2}{y_1}\Rightarrow \ln y_2=3\ln y_1+\ln c$

$y_2=cy_1^3$

$\frac{y_2}{y_1^2}=cy_1$

$-\frac{1}{y_1}=cy+c_2$

$\frac{dx}{dy}=-cy-c_2$

$x=-\frac{cy^2}{2}-c_2y+c_3$

$\therefore x=A_1y^2+A_2y+A_3$

143. If $(1+K)\tan^2 x-4\tan x-1+K=0$ has real roots $\tan x_1$ and $\tan x_2$ then

1) $k^2\le 5$ 2) $k^2\ge 6$

3) $k=3$ 4) $k>10$

Key. 1

Sol. Discriminate ≥ 0

144. Let $f(x)$ be a real valued function satisfying $a.f(x)+b\,f(-x)=px^2+qx+r,\forall x\in R$. Where $p,q,r\in R-\{0\}$ and $a,b\in R$ such that $|a|\ne|b|$. Then the condition that $f(x)=0$ will have real roots is

A) $\left(\frac{a+b}{a-b}\right)^2 \le \frac{q^2}{4pr}$ B) $\left(\frac{a+b}{a-b}\right)^2 \le \frac{4pr}{q^2}$

C) $\left(\frac{a+b}{a-b}\right)^2 \ge \frac{q^2}{4pr}$ D) $\left(\frac{a+b}{a-b}\right)^2 \ge \frac{4pr}{q^2}$

Key. D

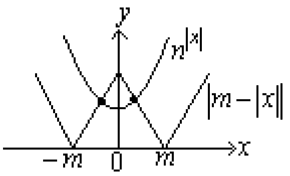
Sol. Using hypothesis we get $f(x)-f(-x)=\frac{2qx}{a-b}$

145. The number of solutions of the equations $n^{-|x|}.\left|m-|x|\right|=1$(where $m,n>1 \& n>m$) is

A) 0 B) 1 C) 2 D)4

Key. C

Sol.



•+• = two solutions

146. The values of 'a' for which the equation $x^3+ax+1=0$ and $x^4+ax^2+1=0$ have a common root

A) 2 B) –2 C) 0 D) 1

Key. B

Sol. Let α be a common root

Then $\alpha^3+a\alpha+1=0$ --- (1)

And $\alpha^4=a\alpha^2+1=0$ --- (2)

$\alpha\times(1)-(2)\Rightarrow \alpha-1=0\Rightarrow \alpha=1$

So, from $x^3+ax+1=0\Rightarrow 1+a+1=0\Rightarrow a=-2$

147. If the roots of the equation $ax^2+bx+c=0$ are of the form $\frac{\alpha}{\alpha-1}$ and $\frac{\alpha+1}{\alpha}$, then value of $(a+b+c)^2$ is

A) $2b^2-ac$ B) b^2-2ac D) b^2-4ac D) $4b^2-2ac$

Key. C

Sol. By hypothesis $\frac{\alpha}{\alpha-1}+\frac{\alpha+1}{\alpha}=-\frac{b}{a}$ and $\frac{\alpha}{\alpha-1}.\frac{\alpha+1}{\alpha}=\frac{c}{a}$

$\Rightarrow \frac{2\alpha^2-1}{\alpha^2-\alpha}=-\frac{b}{a}$ and $\alpha=\frac{c+a}{c-a}$

$\Rightarrow (c+a)^2+2b(c+a)+b^2=b^2-4ac \Rightarrow (a+b+c)^2=b^2-4ac$

148. The value of a , for which one root of the equation $(a-5)x^2-2ax+(a-4)=0$ is smaller than 1 and the other greater than 2 is ____________

A) $a\in(5,24)$ B) $a\in\left(\frac{20}{3},\infty\right)$ C) $a\in(5,\infty)$ D) $a\in(-\infty,\infty)$

Key. A

Sol. (i) $D>0$

$4a^2-4(a-5)(a-4)>0$

$9a-20>0 \Rightarrow a>\frac{20}{9} \Rightarrow a \in \left(\frac{20}{9},\infty\right)$ --- (1)

(ii) $(a-5)f(1)<0;\ (a-5)f(2)<0$

$\Rightarrow (a-5)(a-5-2a+a-4)<0$

$\Rightarrow a>5 \Rightarrow a \in (5,\infty)$ --- (2)

and $(a-5)\left\{(a-5).4-4a+a-4\right\}<0$

$\Rightarrow (a-5)(a-24)<0 \Rightarrow 5<a<24$

$\Rightarrow a \in (5,24)$ --- (3)

Using (1) , (2) & (3)

The common condition is $a \in (5,24)$

149. If the equations $ax^2-2bx+c=0, bx^2-2cx+a=0$ and $cx^2-2ax+b=0$ have only positive roots then

A) $a>b>c$ B) $a<b<c$ C) $a=b=c$ D) $a>b; b<c$

Key. C

Sol. Roots of equation $ax^2-2bx+c=0$ are +ve then discriminent $\geq 0 \Rightarrow b^2 \geq ac$

Sum of roots $=\frac{b}{a}>0$, product of roots $=\frac{c}{a}>0$

Similarly for other two equations, we get $c^2 \geq ab \Rightarrow \frac{c}{b}>0, \frac{a}{b}>0$ and

$a^2 \geq bc \Rightarrow \frac{a}{c}>0 \& \frac{b}{c}>0$

Using above conditions a,b,c are all +ve (or) all are –ve.

Multiplying we get $c^2a^2 \geq ab^2c$

$\Rightarrow ac(b^2-ac) \leq 0 \Rightarrow b^2-ac \leq 0 (\because ac>0)$

Also $a^2-bc \leq 0$ & $c^2-ab \leq 0$

And all, we get $a^2+b^2+c^2-ab-bc-ca \leq 0$

$\Rightarrow \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]=0$

$3x^2+px+3=0, p>0,$ (or) P (or) $\frac{1}{3}$

$\frac{2}{3}$

$\alpha+\alpha^2=-\frac{P}{3}$

150. If α is a root of $ax^2+bx+c=0$; β is a root fo $-ax^2+bx+c=0$ and γ is a root of $ax^2+2bx+2c=0$ then

A) $\gamma<\alpha<\beta$ B) $\alpha<\beta<\gamma$ C) $\alpha<\gamma<\beta$ D) $\frac{\alpha}{\beta}<\gamma<\frac{\beta}{\alpha}$

Key. C

Sol. Let $f(x)=ax^2+2bx+2c$

Then, we have $f(\alpha)=a\alpha^2+2b\alpha+2c=-a\alpha^2+2(a\alpha^2+b\alpha+c)$

$=-a\alpha^2$ [$\because \alpha$ is a root of $ax^2+bx+c=0. \therefore a\alpha^2+b\alpha+c=0$]

Also we have, $f(\beta)=a\beta^2+2b\beta+2c=3a\beta^2+2(-a\beta^2+b\beta+c)$

$=3a\beta^2$ [$\because \beta$ is a root of $-ax^2+bx+c=0. \therefore a^2\beta-b\beta-c=0$]

Now. $f(\alpha)f(\beta)=-3a^2\alpha^2\beta^2<0$ which implies that $f(\alpha), f\left(\beta\right)$ are of opposite signs and hence, proves that the curve represented by $y=f(x)$ cuts the X-axis somewhere between α and β.

In other words $f(x)=0$ has a root lying between α and β.

151. If for any real x, we have $-1\le\frac{x^2+nx-2}{x^2-3x+4}\le 2$ then the value of n is

A) $n\in\left[-1,\sqrt{40}-6\right]$ B) $n\in[-1,3)$ C) $n\in\left[-\sqrt{40}-6,-1\right]$ D) $n\in\left[1,\sqrt{40}+6\right]$

Key. A

Sol. $\frac{x^2+nx-2}{x^2-3x+4}-2\le 0$

$\Rightarrow x^2-(n+6)x=10\ge 0$, true $\forall x\in R$ then

$D\le 0\Rightarrow (n+6)^2-40\le 0\Rightarrow -\sqrt{40}-6\le n\le\sqrt{40}-6$ --- (1)

Similarly $\frac{x^2+nx-2}{x^2-3x+4}+1\ge 0\Rightarrow 2x^2+(x-3)x+2\ge 0$

$\Rightarrow D\le 0\Rightarrow (n-3)^2-16\le 0\Rightarrow -1\le n\le 7$ --- (2)

Combined (1) & (2) we get $n\in\left[-1,\sqrt{40}-6\right]$

Quadratic Equations & Theory of Equations

Multiple Correct Answer Type

1. Consider the fraction $\dfrac{x^3-ax^2+19x-a-4}{x^3-(a+1)x^2+23x-a-7}$

a) The value of '*a*' at which the above fraction admits of reduction is 8

b) The value of '*a*' at which the above fraction admits of reduction is 4

c) The lowest admitted reduction form of the fraction is $\dfrac{x-4}{x-5}$

d) The lowest admitted reduction form of the fraction is $\dfrac{x-3}{x-4}$

Key. A,C

Sol. subtracting numerator from denominator, we get

x^2-4x+3 i.e $(x-1)(x-3)$.

Thus it is concluded that numerator and denominator must be completely divisible by $(x-1)$ or $(x-3)$ in other words both must vanish when $x=1$ or when $x=3$, if $x=3$ we get, $a=8$

And fraction becomes

$$\frac{x^3-8x^2+19x-12}{x^3-9x^2+23x-15}=\frac{x^2-7x+12}{x^2-8x+15}=\frac{x-4}{x-5}$$

If we put $x=1$, we get also that $a=8$.

2. Two numbers are such that their sum multiplied by the sum of their squares is 5500 and their difference multiplied by the difference of the squares is 352. Then the numbers are

a) Prime numbers only b) odd positive integers

c) prime but not odd d) odd but not prime

Key. B

Sol. Let the two number be x,y then

$$(x+y)(x^2+y^2)=5500 \quad \text{---(i)}$$

and $$(x-y)(x^2-y^2)=352 \quad \text{--(ii)}$$

After solving these two equations

$$\left.\begin{matrix} x=9 \, or \;\; 13 \\ y=13 \, or \; 9 \end{matrix}\right\}$$

3. If by eliminating x between the equations $x^2+ax+b=0$ and $xy+l(x+y)+m=0$, a quadratic equation in y is formed whose roots are the same as those original quadratic in x, then

a) $a=2l$ b) $b=m$ c) $b+m=al$ d) $a+b=l$

Key. A,B,C

Sol. Given equation are

$x^2 + ax + b = 0$ --(1)

$xy + l(x + y) + m = 0$ --(2)

From (2), we get, $x(y+1) = -(m + ly)$

$\therefore x = -\left(\dfrac{m+ly}{y+l}\right)$

Substituting this value in (1), we have

$\left(\dfrac{m+ly}{y+l}\right)^2 - a\left(\dfrac{m+ly}{y+l}\right) + b = 0$

or $(m+ly)^2 - a(m+ly)(y+l) + b(y+l)^2 = 0$

or $(y^2l^2 + b - al) + y(2lm + 2bl - al^2 - am) + m^2 - alm + bl^2 = 0$

Since this equation is equivalent to (1)

$\therefore \dfrac{l^2 - al + b}{l} = \dfrac{2lm - al^2 - am + 2bl}{a} = \dfrac{m^2 - alm + bl^2}{b}$

From 1st and third fraction

$b(l^2 - al + b) = m^2 - alm + bl^2$

$i.e\ al(b-m) - (b^2 - m^2) = 0$

$or\ (b-m)(al - b - m) = 0$

$\therefore$ either $b = m$ or $b + m = al$

From 1st and second fraction, putting $b = m$

$al^2 - a^2l + am = 4lm - al^2 - am$

or $2al^2 - a^2l - 4lm - 2am = 0$

or $a^2l - 2a(l^2 + m) + 4lm = 0$

or $(a - 2l)(al - 2m) = 0$

$\therefore a = 2l\ or\ al = 2m$

Thus either $b = m$ and $a = 2l$

$b = m$ and $al = 2m$

4. If α and β are the roots of the equation $x^2 + px + q = 0$ and α^4 and β^4 are the roots of $x^2 - rx + s = 0$, the roots of $x^2 - 4qx + 2q^2 - r = 0$ are always

a) both real

b) both positive

c) both negative d) one positive & one negative

Key. A,D

Sol. We have $\alpha+\beta = -p, \alpha\beta = q, \alpha^4+\beta^4 = r$ and $\alpha^4\beta^4 = s$

Therefore, $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = p^2-2q,$ so that

$r = \alpha^4+\beta^4 = (\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2 = (p^2-2q)^2 - 2q^2$

i.e., $(p^2)^2 - 4q(p^2) + 2q^2 - r = 0$

This shows that p^2 is one root of $x^2 - 4qx + 2q^2 - r = 0$. If its other root is γ, we have $\gamma + p^2 = 4q$, i.e., $\gamma = 4q - p^2$. Further the discriminant of this quadratic equation is

$(4q)^2 - 4(2q^2 - r) = 8q^2 + 4\left[(p^2-2q)^2 - 2q^2\right] = 4(p^2-2q)^2 \ge 0$

So that both roots, p^2 and $-p^2+4q$ are real. Since α and β are real $p^2 - 4q \ge 0,$ i.e., $-p^2 + 4q \le 0$. Thus the roots of $x^2 - 4qx + 2q^2 - r = 0$ are positive and negative

5. Let $|a| < |b|$ and a, b are the roots of the equation $x^2 - |\alpha| x - |\beta| = 0.$ If $|\alpha| < b - 1,$ then the equation $\log_{|a|}\left(\frac{x}{b}\right)^2 - 1 = 0$ has at least one

A) root lying between $(-\infty, a)$ B) roots lying between (b, ∞)

C) negative root D) positive root

Key. A,B,C,D

Sol. $|\alpha| =$ sum of roots $= b + a$

$-|\beta| =$ product of root $= ab$

Because $|a| < |b|$ so *a* is negative and *b* is positive.

Now, $|\alpha| < b - 1 \Rightarrow a + b < b - 1 = a < -1.$

Because *a* is negative so magnitude of $'a'$ is greater than one and magnitude of *b* is greater than $1 + |\alpha|$ or say greater than 2.

Now, $\log_{|a|}\left(\frac{x}{b}\right)^2 - 1 = 0 \Rightarrow \left(\frac{x}{b}\right)^2 = |a|$

$\Rightarrow \quad x = \pm b\sqrt{|a|}$

Magnitude of x is greater than $'a'$ as well as greater than $'b'$

$\Rightarrow$ one root lies in $\Rightarrow (-\infty, a)$ and other root lies in (b, ∞).

6. The value of $'x'$ satisfying the equation $x^4 - 2\left(x\sin\left(\frac{\pi}{2}x\right)\right)^2 + 1 = 0$

A) 1 B) -1 C) 0 D) No value of $'x'$

Key. A,B

Sol. $x^4 - 2\left(x\sin\left(\frac{\pi}{2}x\right)\right)^2 + 1 = 0$

$\Rightarrow \quad x^4+1=2x^2\sin^2\left(\frac{\pi}{2}x\right)$

$\Rightarrow \quad x^2+\frac{1}{x^2}=2\sin^2\left(\frac{\pi}{2}x\right)$

Now, LHS ≥ 2 where as RHS ≤ 2

So, equality holds when

$$x^2+\frac{1}{x^2}=2 \text{ and } 2\sin^2\left(\frac{\pi}{2}x\right)=2 \Rightarrow x=\pm 1$$

7. In a ΔABC, $\tan A$ and $\tan B$ satisfy the inequation $\sqrt{3}x^2-4x+\sqrt{3}<0$. Then

A) $a^2+b^2-ab<c^2$ B) $a^2+b^2>c^2$ C) $a^2+b^2+ab>c^2$ D) All of the above

Key. A,C

Sol. $\left(x-\sqrt{3}\right)\left(x\sqrt{3}-1\right)<0$

$\Rightarrow$ x lies between $\frac{1}{\sqrt{3}}$ and $\sqrt{3} \Rightarrow$ Both $\tan A$ *and* $\tan B$ lie between $\frac{1}{\sqrt{3}}$ and $\sqrt{3}$

Both A and B lie between 30° and 60°.

$\Rightarrow$ 60°<C<120°

$\Rightarrow \quad -\frac{1}{2}<\frac{a^2+b^2-c^2}{2ab}<\frac{1}{2}$

8. Let $f(x)=\frac{3}{x-2}+\frac{4}{x-3}+\frac{5}{x-4}$, then $f(x)=0$ has

A) exactly one real root in $(2,3)$ B) exactly one real root in $(3,4)$

C) at least one real root in $(2,3)$ D) None of these

Key. A,B,C

Sol. $f(x)=\frac{3}{x-2}+\frac{4}{x-3}+\frac{5}{x-4}$

$$\left.\begin{array}{l}\because \quad f(2^+)\to\infty \\ and \quad f(3^-)\to-\infty\end{array}\right\}$$

$\Rightarrow f(x)=0$ has exactly one root in (2,3)

Again $\left.\begin{array}{l}\because \quad f(3^+)\to\infty \\ and \quad f(4^-)\to-\infty\end{array}\right\}\Rightarrow f(x)=0$

Has exactly one root in (3,4)

9. If $x_1>x_2>x_3$ and x_1,x_2,x_3 are roots of $\frac{x-a}{b}+\frac{x-b}{a}=\frac{b}{x-a}+\frac{a}{x-b};(a,b,>0)$ and $x_1-x_2-x_3=c$, then a, c, b are in.

A) A.P. B) G.P. C) H.P. D) None

Key. C

Sol. Given equation can be written as

$$\frac{x-a}{b}-\frac{b}{x-a}+\frac{x-b}{a}-\frac{a}{x-b}=0$$

$$=\frac{(x-a)^2-b^2}{b(x-a)}+\frac{(x-b)^2-a^2}{a(x-b)}=0$$

$$\Rightarrow (x-a-b)\left[\frac{x-a+b}{b(x-a)}+\frac{x-b+a}{a(x-b)}\right]=0$$

$$\Rightarrow (x-a-b)\left\{\frac{a\left[x^2-bx-ax+ab+bx-b^2\right]+b\left[x^2-ax-bx+ab+ax-a^2\right]}{ab(x-a)(x-b)}\right\}=0$$

$$\Rightarrow (x-a-b)\left(ax^2-a^2x+a^2b-ab^2+bx^2-b^2x+ab^2-a^2b\right)$$

$$\Rightarrow x(x-a-b)\{x(a+b)-(a^2+b^2)\}=0$$

$\therefore$ roots will be x=0, $a+b, \frac{a^2+b^2}{a+b}$

Let $x_1=a+b$, $x^2=\frac{a^2+b^2}{a+b}$ and $x_3=0$

$\because x_1-x_2-x_3=c$ (given)

$$\therefore (a+b)-\frac{a^2+b^2}{a+b}-0=c$$

$$\Rightarrow \frac{(a+b)^2-(a^2+b^2)}{a+b}=c\Rightarrow \frac{2ab}{a+b}=c$$

i.e a,c,b are in H. P

10. If the equation whose roots are the squares of the roots of the cubic $x^3-ax^2+bx-1=0$ is identical with the given cubic equation, then

A) *a,b* are roots of $x^2+x+2=0$ B) *a* = *b* = 0

C) *a* = *b* = 3 D) *a* = 0, *b* = 3

Key. A,B,C

Sol. If roots of the equation be α,β,γ then

$$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)=a^2-2b$$

$$\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2=(\alpha\beta+\beta\gamma+\gamma\alpha)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma)$$

$$=b^2-2a$$

$$\alpha^2\beta^2\gamma^2=1.$$

So, the equation whose roots are $\alpha^2,\beta^2,\gamma^2$ is

$$x^3-(a^2-2b)x^2+(b^2-2a)x-1=0$$

It is identical to $x^3 - ax^2 + bx - 1 = 0$

$\therefore a^2 - 2b = a$ and $b^2 - 2a = b,$ eliminating b, we get

$$\frac{\left(a^2 - a\right)^2}{4} - 2a = \frac{a^2 - a}{2}$$

$\Rightarrow \quad a\left\{a\left(a-1\right)^2 - 8 - 2\left(a-1\right)\right\} = 0$

$\Rightarrow \quad a\left(a^3 - 2a^2 - a - 6\right) = 0$

or $\quad a\left(a-3\right)\left(a^2 + a + 2\right) = 0$

$\therefore \quad a = 0 \; or \; a = 3 \; or \; a^2 + a + 2 = 0$

Which give $b = 0 \, or \, b = 3 \, or \, b^2 + b + 2 = 0$

So, $\quad a = b = 0$ or $a = b = 3$

Or *a, b* are roots of $x^2 + x + 2 = 0$

11. $\dfrac{\pi^e}{x-e} + \dfrac{e^\pi}{x-\pi} + \dfrac{\pi^\pi + e^e}{x-\pi-e} = 0$ has

A) One real root in (e, π) and other in $(\pi - e, e)$

B) One real root in (e, π) and other in $(\pi, \pi, + e)$

C) Two real roots in $(\pi - e, \pi + e)$ D) No real roots

Key. B,C

Sol. Given equation can be expressed as

$$\pi^e (x-\pi)(x-\pi-e) + e^\pi (x-e)(x-\pi-e) + (\pi^\pi + e^e)(x-e)(x-\pi) = 0$$

Let $f(x) = \pi^e (x-\pi)(x-\pi-e) + e^\pi (x-e)(x-\pi-e) + (\pi^\pi + e^e)(x-e)(x-\pi)$

$f(e) = \pi^e (e-\pi)(-\pi) > 0$

and $f(\pi) = e^\pi (\pi - e)(-e) < 0$

hence given equation has a real root in (e, π)

again $f(\pi + e) = (\pi^\pi + e^e)\pi \cdot e > 0$

$\because \pi + e > \pi$, it concludes it has a real root in $(\pi, \pi + e)$

Also $\because \pi - e < e$

hence $f(x)$ has two real roots in $(\pi - e, \pi + e)$

12. Consider the fraction $\dfrac{x^3 - ax^2 + 19x - a - 4}{x^3 - (a+1)x^2 + 23x - a - 7}$

a) The value of '*a*' at which the above fraction admits of reduction is 8

b) The value of '*a*' at which the above fraction admits of reduction is 4

c) The lowest admitted reduction form of the fraction is $\dfrac{x-4}{x-5}$

d) The lowest admitted reduction form of the fraction is $\frac{x-3}{x-4}$

Key. A,C

Sol. subtracting numerator from denominator, we get

x^2-4x+3 i.e $(x-1)(x-3)$.

Thus it is concluded that numerator and denominator must be completely divisible by $(x-1)$ or $(x-3)$ in other words both must vanish when $x=1$ or when $x=3$, if $x=3$ we get, $a=8$

And fraction becomes

$$\frac{x^3-8x^2+19x-12}{x^3-9x^2+23x-15}=\frac{x^2-7x+12}{x^2-8x+15}=\frac{x-4}{x-5}$$

If we put $x=1$, we get also that $a=8$.

13. Two numbers are such that their sum multiplied by the sum of their squares is 5500 and their difference multiplied by the difference of the squares is 352. Then the numbers are

a) Prime numbers only b) odd positive integers

c) prime but not odd d) odd but not prime

Key. B,D

Sol. Let the two number be x,y then

$$(x+y)(x^2+y^2)=5500 \qquad \text{---(i)}$$

and $$(x-y)(x^2-y^2)=352 \qquad \text{--(ii)}$$

After solving these two equations

$$\left.\begin{array}{l} x=9\,or\ \ 13 \\ y=13\,or\ 9 \end{array}\right\}$$

14. If by eliminating x between the equations $x^2+ax+b=0$ and $xy+l(x+y)+m=0$, a quadratic equation in y is formed whose roots are the same as those original quadratic in x, then

a) $a=2l$ b) $b=m$ c) $b+m=al$ d) $a+b=l$

Key. A,B,C

Sol. Given equation are

$x^2+ax+b=0$ --(1)

$xy+l(x+y)+m=0$ --(2)

From (2), we get, $x(y+1)=-(m+ly)$

$\therefore x=-\left(\dfrac{m+ly}{y+l}\right)$

Substituting this value in (1), we have

$$\left(\frac{m+ly}{y+l}\right)^2-a\left(\frac{m+ly}{y+l}\right)+b=0$$

or $(m+ly)^2-a(m+ly)(y+l)+b(y+l)^2=0$

or $(y^2l^2+b-al)+y(2lm+2bl-al^2-am)+m^2-alm+bl^2=0$

Since this equation is equivalent to (1)

$$\therefore \frac{l^2-al+b}{l}=\frac{2lm-al^2-am+2bl}{a}=\frac{m^2-alm+bl^2}{b}$$

From 1st and third fraction

$b(l^2-al+b)=m^2-alm+bl^2$

$i.e\ al(b-m)-(b^2-m^2)=0$

$or\ (b-m)(al-b-m)=0$

$\therefore$ either $b=m$ or $b+m=al$

From 1st and second fraction, putting $b=m$

$al^2-a^2l+am=4lm-al^2-am$

or $2al^2-a^2l-4lm-2am=0$

or $a^2l-2a(l^2+m)+4lm=0$

or $(a-2l)(al-2m)=0$

$\therefore a=2l\ or\ al=2m$

Thus either $b=m$ and $a=2l$

$b=m$ and $al=2m$

15. If α and β are the roots of the equation $x^2+px+q=0$ and α^4 and β^4 are the roots of $x^2-rx+s=0$, the roots of $x^2-4qx+2q^2-r=0$ are always

a) both real
b) both positive
c) both negative
d) one positive & one negative

Key. A,D

Sol. We have $\alpha+\beta=-p,\alpha\beta=q,\alpha^4+\beta^4=r$ and $\alpha^4\beta^4=s$

Therefore, $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=p^2-2q,$ so that

$r=\alpha^4+\beta^4=(\alpha^2+\beta^2)^2-2\alpha^2\beta^2=(p^2-2q)^2-2q^2$

i.e., $(p^2)^2-4q(p^2)+2q^2-r=0$

This shows that p^2 is one root of $x^2-4qx+2q^2-r=0$. If its other root is γ, we have $\gamma+p^2=4q$, i.e., $\gamma=4q-p^2$. Further the discriminant of this quadratic equation is

$(4q)^2-4\left(2q^2-r\right)=8q^2+4\left[(p^2-2q)^2-2q^2\right]=4(p^2-2q)^2\geq 0$

So that both roots, p^2 and $-p^2+4q$ are real. Since α and β are real $p^2-4q\geq 0$, i.e., $-p^2+4q\leq 0$. Thus the roots of $x^2-4qx+2q^2-r=0$ are positive and negative

16. Let $|a|<|b|$ and a,b are the roots of the equation $x^2-|\alpha|x-|\beta|=0.$ If $|\alpha|<b-1,$ then the equation $\log_{|a|}\left(\frac{x}{b}\right)^2-1=0$ has at least one

A) root lying between $(-\infty,a)$ B) roots lying between (b,∞)

C) negative root D) positive root

Key. A,B,C,D

Sol. $|\alpha|=$ sum of roots $=b+a$

$-|\beta|=$ product of root $=ab$

Because $|a|<|b|$ so *a* is negative and *b* is positive.

Now, $|\alpha|<b-1\Rightarrow a+b<b-1=a<-1.$

Because *a* is negative so magnitude of 'a' is greater than one and magnitude of *b* is greater than $1+|\alpha|$ or say greater than 2.

Now, $\log_{|a|}\left(\frac{x}{b}\right)^2-1=0\Rightarrow\left(\frac{x}{b}\right)^2=|a|$

$\Rightarrow$ $x=\pm b\sqrt{|a|}$

Magnitude of x is greater than 'a' as well as greater than 'b'

$\Rightarrow$ one root lies in $\Rightarrow(-\infty,a)$ and other root lies in (b,∞).

17. The value of 'x' satisfying the equation $x^4-2\left(x\sin\left(\frac{\pi}{2}x\right)\right)^2+1=0$

A) 1 B) -1 C) 0 D) No value of 'x'

Key. A,B

Sol. $x^4-2\left(x\sin\left(\frac{\pi}{2}x\right)\right)^2+1=0$

$\Rightarrow$ $x^4+1=2x^2\sin^2\left(\frac{\pi}{2}x\right)$

$\Rightarrow$ $x^2+\frac{1}{x^2}=2\sin^2\left(\frac{\pi}{2}x\right)$

Now, LHS ≥ 2 where as RHS ≤ 2

So, equality holds when

$$x^2+\frac{1}{x^2}=2 \text{ and } 2\sin^2\left(\frac{\pi}{2}x\right)=2 \Rightarrow x=\pm 1$$

18. In a $\Delta ABC,$ $\tan A$ and $\tan B$ satisfy the inequation $\sqrt{3}x^2-4x+\sqrt{3}<0.$ Then

A) $a^2+b^2-ab<c^2$ B) $a^2+b^2>c^2$ C) $a^2+b^2+ab>c^2$ D) All of the above

Key. A,C

Sol. $\left(x-\sqrt{3}\right)\left(x\sqrt{3}-1\right)<0$

$\Rightarrow$ x lies between $\frac{1}{\sqrt{3}}$ and $\sqrt{3} \Rightarrow$ Both $\tan A$ *and* $\tan B$ lie between $\frac{1}{\sqrt{3}}$ and $\sqrt{3}$

Both A and B lie between 30° and 60°.

$\Rightarrow$ 60°<C<120°

$$\Rightarrow \quad -\frac{1}{2}<\frac{a^2+b^2-c^2}{2ab}<\frac{1}{2}$$

19. Let $f(x)=\frac{3}{x-2}+\frac{4}{x-3}+\frac{5}{x-4}$, then $f(x)=0$ has

A) exactly one real root in $(2,3)$ B) exactly one real root in $(3,4)$

C) at least one real root in $(2,3)$ D) None of these

Key. A,B,C

Sol. $f(x)=\frac{3}{x-2}+\frac{4}{x-3}+\frac{5}{x-4}$

$$\left.\begin{aligned}\because \quad & f(2^+)\to\infty \\ and \quad & f(3^-)\to-\infty\end{aligned}\right\}$$

$\Rightarrow f(x)=0$ has exactly one root in (2,3)

Again $\left.\begin{aligned}\because \quad & f(3^+)\to\infty \\ and \quad & f(4^-)\to-\infty\end{aligned}\right\}\Rightarrow f(x)=0$

Has exactly one root in (3,4)

20. If $b^2\geq 4ac$ for the equation $ax^4+bx^2+c=0$, then all the roots of the equation will be real if

(A) b > 0, a < 0, c > 0 (B) b < 0, a > 0, c > 0

(C) b < 0, a > 0, c < 0 (D) b > 0, a < 0, c < 0

Key. B,D

Sol. $x^2=t, \quad t\geq 0$

$at^2+bt+c=0, \quad t\geq 0$

$-\frac{b}{a} > 0$ ….. (1)

$\frac{c}{a} > 0$ ….. (2)

21. Let x, y, z be positive reals. Then

A) $\frac{4}{x}+\frac{9}{y}+\frac{16}{z} \geq 81$ if $x+y+z=1$

B) $\frac{x}{y+z}+\frac{y}{z+x}+\frac{z}{x+y} \geq \frac{3}{2}$

C) If $xyz=1$, then $(1+x)(1+y)(1+z) < 8$

D) If $x+y+z=1$, then $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} \geq 9$

Key. A,B,D

Sol. A) $x+y+z=1 \Rightarrow \frac{4}{x}+\frac{9}{y}+\frac{16}{z} = \left(\frac{4}{x}+\frac{9}{y}+\frac{16}{z}\right)(x+y+z)$ 0

$= 29+\left(\frac{4y}{x}+\frac{9x}{y}\right)+\left(\frac{16y}{z}+\frac{9z}{y}\right)+\left(\frac{4z}{x}+\frac{16x}{z}\right)$

Use $A.M. \geq G.M.$

B) $\frac{(y+z)+(z+x)+(x+y)}{3} \geq \sqrt[3]{(y+z)(z+x)(x+y)}$

$\therefore \frac{2}{3}(x+y+z) \geq \sqrt[3]{(y+z)(z+x)(x+y)}$ ……….(1)

Similarly, $\frac{\frac{1}{y+z}+\frac{1}{z+x}+\frac{1}{x+y}}{3} \geq \left[(y+z)(z+x)(x+y)\right]^{\frac{-1}{3}}$ ………(2)

On multiplication of (1) & (2) and expanding, we get the desired result.

D) $(x+y+z)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right) \geq 3^2$

22. Given
$|ax^2+bx+c| \leq |Ax^2+Bx+C|$, $\forall x \in R$, a,b,c A,B,C $\in R$ and $d=b^2-4ac > 0$ and $D=B^2-4AC > 0$. Then which of the following statements are true

a) $|a| \leq |A|$ b) $|d| \leq |D|$ c) $|a| \geq |A|$

d) if D,d are not necessarily positive then roots of $ax^2+bx+c=0$ and $Ax^2+Bx+C=0$

may not be equal

Sol : ans: a,b,d

Let $\alpha \& \beta$ are the roots of

$Ax^2 + Bx + c = 0$

$\because |ax^2 + bx + c| \le |Ax^2 + Bx + c| \quad \forall \in R$

$\Rightarrow ax^2 + bx + c = 0$ also has α, β as roots

$\Rightarrow |ax^2 + bx + c| = |a|\ |x - \alpha||x - \beta| = |A||x - \alpha||x - \beta|$

$\Rightarrow |a| \le |A|$

&

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta \Rightarrow \frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4AC}{A^2} \Rightarrow |d| \le |D|$$

23. A continuous function $y = f(x)$ is defined in a closed interval $[-7, 5]$. $A(-7, -4)$, $B(-2, 6)$, $C(0, 0)$, $D(1, 6)$, $E(5, -6)$ are consecutive points on the graph of f and AB, BC, CD, DE are line segments. The number of real roots of the equation $f[f(x)] = 6$ is

A) 6 B) 4 C) 2 D) 0

KEY : A

HINT

$f[f(x)] = 6 \Rightarrow f(x) = -2 \text{ or } f(x) = 1$

$f(x) = -2$ has two roots and $f(x) = 1$ has four roots.

24. If both the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ in the variable x are less than 3 then a can be

A) 2 B) 5/2 C) $\sqrt{3}$ D) -7

KEY : C,D

HINT: $\text{disc} \ge 0$, $a < 3$ and $f(3) > 0$ where $f(x) = x^2 - 2ax + a^2 + a - 3$

25. The equation $x^7 + 3x^3 + 4x - 9 = 0$ has

a) no real root b) all its roots real

c) a unique rational root d) a unique irrational root

KEY : D

HINT: Let $f(x) = x^7 + 3x^3 + 4x - 9$

$f'(x) = 7x^6 + 9x^2 + 4 > 0$

$\therefore$ f is increasing in R. Hence there exists only one real root.Observe that f(1).f(2) < 0. That is a root should lie in (1, 2). If that root is a rational number then coefficient of x^7 can not be 1. Hence only one irrational root exists.

26. The coefficient of x^{30} in the polynomial $(x - 1)(x^2 - 2)(x^3 - 3)(x^4 - 4)(x^5 - 5)(x^6 - 6)(x^7 - 7)(x^8 - 8)$ is

a) –1 b) 1 c) 0 d) 4

KEY : B

HINT: Coefficient of x^{30} is (–6) + (–1)(–5) + (–2)(–4) + (–1)(–2)(–3)

= –6 + 5 + 8 – 6

= 1

27. If the equation $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ (a, b, c are unequal non zero real) have a common root then $f(x) = bx^3 + cx^2 + ax - 5$ always passes through fixed point

(A) (1, –5) (B) (0, –5)

(C) (–1, –5) (D) (0, 5)

KEY : A,B

HINT: and $bx^2 + cx + a = 0$ have a common root $\Rightarrow a^3 + b^3 + c^3 - abc = 0$

$\frac{1}{2}(a+b+c)\,[(a-b)^2 + (b-c)^2\,(c-a)^2] = 0 \Rightarrow a + b + c = 0$

$f(x) = bx^3 + cx^2 + ax - 5$

$f(0) = -5$

$f(A) = a + b + c - 5 = 5$

$\Rightarrow$ f(x) will always pass through (0, –5) and (1, –5)

Hence (a, b)

28. Let $f(x) = x^2 + \lambda x + \mu \cos x$, λ being an integer and μ a real number. The number of ordered pairs (λ, μ) for which the equations $f(x) = 0$ and $f(f(x)) = 0$ have the same (non empty) set of real roots is

(A) 4 (B) 6 (C) 8 (D) Infinite

KEY : A

HINT: Let α be a root of $f(x) = 0$, so we have $f(\alpha) = 0$ and thus $f(f(\alpha)) = 0$,

$\Rightarrow f(0) = 0 \Rightarrow \mu = 0$.

We then have $f(x) = x(x+\lambda)$ and thus $\alpha = 0, -\lambda$.

$f(f(x)) = x(x+\lambda)(x^2 + \lambda x + \lambda)$

We want λ such that $x^2 + \lambda x + \lambda$ has no real roots besides 0 and $-\lambda$. We can easily find that $0 \le \lambda < 4$.

29. If α, β, γ are the roots of the equation $9x^3 - 7x + 6 = 0$ then the equation $x^3 + Ax^2 + Bx + C = 0$ has roots $3\alpha + 2,\ 3\beta + 2,\ 3\gamma + 2$, where

(A) $A = 6$ (B) $B = -5$

(C) $C = 24$ (D) $A + B + C = 23$

KEY : C, D

HINT : Let $P = 3\alpha + 2$

$\Rightarrow \alpha = \frac{P-2}{3}$

Since $9\alpha^3 - 7\alpha + 6 = 0$

$$\Rightarrow \frac{9(P-2)^3}{27} - \frac{7}{3}(P-2) + 6 = 0$$

$$\Rightarrow \frac{1}{3}\left(P^3 - 8 - 6P^2 + 12P\right) - \frac{7}{3}P + \frac{14}{3} + 6 = 0$$

$$\Rightarrow P^3 - 6P^2 + 12P - 8 - 7P + 14 + 18 = 0$$

$$\Rightarrow P^3 - 6P^2 + 5P + 24 = 0$$

So, the equation $x^3 - 6x^2 + 5x + 24 = 0$ has roots $3\alpha + 2,\ 3\beta + 2,\ 3\gamma + 2$

30. If α, β are the roots of the equation $x^2 + ax + 1 = 0$ then the equation whose roots are $-\left(\alpha + \frac{1}{\beta}\right),\ -\left(\frac{1}{\alpha} + \beta\right)$

(A) $x^2 = 0$ (B) $x^2 + 2ax + 4 = 0$

(C) $x^2 - 2ax + 4 = 0$ (D) $x^2 - ax + 1 = 0$

KEY : C

31. If 0 < c < b < a and the roots α, β of the equation $cx^2 + bx + a = 0$ are imaginary, then

(A) $\frac{|\alpha| + |\beta|}{2} = |\alpha||\beta|$ (B) $\frac{1}{|\alpha|} = \frac{1}{|\beta|}$

(C) $\frac{1}{|\alpha|} + \frac{1}{|\beta|} < 2$ (D) $\frac{1}{|\alpha|} + \frac{1}{|\beta|} > 2$

KEY : C, B

HINT : Since roots are imaginary.

So, discriminant < 0

$$\alpha = \frac{-b + i\sqrt{4ac - b^2}}{2c}$$

$$\beta = \frac{-b - i\sqrt{4ac - b^2}}{2c},\ |\alpha| = |\beta| = \sqrt{\frac{b^2}{4c^2} + \frac{4ac - b^2}{4c^2}} = \sqrt{\frac{a}{c}} > 1$$

32. Suppose a, b > 0 and x_1, x_2, x_3 ($x_1 > x_2 > x_3$) are roots of $\frac{x-b}{a} + \frac{x-a}{b} = \frac{b}{x-a} + \frac{a}{x-b}$ and $x_1 - x_2 - x_3 = c$, then

(A) a, c, b are in H.P. and $x_1 = a + b$ (B) a, c, b are in A.P. and $x_2 = a + b$

(C) a, c, b are in A.P. and $x_3 = 0$ (D) a, c, b are in H.P. and $x_3 = 0$

KEY : A, D

HINT : $\frac{x-b}{a} - \frac{b}{x-a} = \frac{a}{x-b} - \frac{x-a}{b}$

$$(x^2 - (a + b)x)[(b + a)x - (a^2 + b^2)] = 0$$

$$x = 0,\ a + b,\ \frac{a^2 + b^2}{a + b}$$

$$x_3 = 0,\ x_2 = \frac{a^2 + b^2}{a + b},\quad x_1 = a + b$$

$$(a+b)-(a+b)+\frac{2ab}{a+b}-0=c$$

$$c=\frac{2ab}{a+b}$$

A, C B ARE IN H.P.

33. The values of a for which $\frac{x^3-6x^2+11x-6}{x^3+x^2-10x+8}+\frac{a}{30}=0$ does not have a real solution is

1) -10 2) 12 3) 5 4) -30

KEY : 2,3,4

SOL : $\frac{x^3-6x^2+11x-6}{x^3+x^2-10x+8}=\frac{(x-1)(x-2)(x-3)}{(x-1)(x-2)(x+4)}$

$\therefore x\neq 1,2-4$ then $f(x)=\frac{x-3}{x+4}$

Range of $f(x)=R-\left\{1,-\frac{2}{5},-\frac{1}{6}\right\}$

So Equation does not have a solution if $\frac{a}{30}=-1,\frac{2}{5},\frac{1}{6}$

$\Rightarrow a=-30,12,5$

34. The Quadratic polynomials defined on real coefficients

$P(x)=a_1x^2+2b_1x+c_1;Q(x)=a_2x^2+2b_2x+c_2;$ where

$a_1\neq 0,a_2\neq 0$ and $P(x)$ and $Q(x)$ both take positive values $\forall x\in R$.

$g(x)=a_1a_2x^2+b_1b_2x+c_1c_2$ then

A) $g(x)$ takes +ve values only B) $g(x)$ takes negative values only

C) $g(x)$ takes both +ve and –Ve values D) nothing can be said about $g(x)$

KEY : A

SOL : $D_1=4b_1^2-4a_1c_1<0,D_2=4b_2^2-4a_2c_2<0$

$\Rightarrow a_1a_2c_1c_2>b_1^2b_2^2$ the $D_3=(b_1b_2)^2-4a_1a_2c_1c_2<0$

35. If a, b, $c\in R$ and $a+b+c=0$, then the quadratic equation $3ax^2+2bx+c=0$ has

(A) at least one root in [0,1] (B) at least one root in [–1,1]

(C) at least one root in [0,2] (D) none of these

Key : A, B, C

Sol : Let f(x) = ax^3+bx^2+cx+d

f is continuos and derivable on R. Also, f(0) = d and f(1) = a +b + c +d = d. By the Rolle’s theorem, there exists at least one $\alpha\in(0,1)$ such that

$f'(\alpha)=0 \Rightarrow 3a\alpha^2+2b\alpha+c=0$

Thus, $3ax^2+2bx+c=0$ has at leas one root in [0,1].

Also, $[0,1]\subseteq[-1,1]$ and $[0,1] \subseteq [0,2]$

36. $\cos\alpha$ is a root of the equation $169x^2-26x-35=0,\ -1<x<0,$ then $\sin 2\alpha$ is

a) $\frac{144}{169}$ b) $-\frac{144}{169}$

c) $\frac{144}{169}$ d) $-\frac{120}{169}$

Key: c, d

Sol : $169x^2-26x-35=0 \Rightarrow (13x-7)(13x+5)=0$

$\Rightarrow x=\frac{7}{13} \text{ or } x=\frac{-5}{13}$

$\therefore \cos\alpha=\frac{-5}{13} \Rightarrow \sin 2\alpha = 2\times\frac{5}{13}\times\pm\frac{12}{13}=\pm\frac{120}{169}$

37. The values of a, for which $\frac{x^3-6x^2+11x-6}{x^3+x^2-10x+8}+\frac{a}{30}=0$ doesn't have a real solution, are

a) -10 b) 12 c) 5 d) -30

Key : b, c, d

Sol : Let $f(x)+\frac{a}{30}=0$

Where $f(x)=\frac{x^3-6x^2+11x-6}{x^3+x^2-10x+8}=\frac{(x-1)(x-2)(x-3)}{(x-1)(x-2)(x+4)}=\frac{x-3}{x+4}$

$x \neq 1, 2, -4$

Range of $f(x)=R-\left\{1,\frac{-2}{5},\frac{-1}{6}\right\}$

$\therefore \frac{a}{30}\neq -1,\frac{2}{5},\frac{1}{6}$

$\Rightarrow a\neq -30, 12, 5$

38. The value of $\frac{\sin x\cos 3x}{\cos x\sin 3x}$, when ever defined never lies between

a) 0 and 1 b) -1 and 1

c) $\frac{1}{3}$ and 3   d) $\frac{1}{2}$ and 2

Key : c,d

Sol : $y=\frac{\sin x\cos 3x}{\cos x\sin 3x}=\frac{\tan x}{\tan 3x}$

Let tan x = t

$\therefore y=\frac{t\left(1-3t^2\right)}{3t-t^3}=\frac{1-3t^2}{3-t^2}$ as $t\neq 0$ ($\because$ t = 0 will make by indeterminate)

$\therefore y\left(3-t^2\right)=1-3t^2$

or $t^2=\frac{3y-1}{y-3}=+ve=\frac{(3y-1)(y-3)}{(y-3)^2}=\frac{3\left(y-\frac{1}{3}\right)(y-3)}{(y-3)^2}$

Above will be +ve if $y<\frac{1}{3}$ or $y>3$

$\therefore$ y cannot lie between $\frac{1}{3}$ and 3

39. Complete set of real values of a for the equation $9^x+a.3^x+1=0$ has

a) two real solutions, is $(-\infty,-2)$ b) no real solution, is $(-2,\infty)$

c) exactly one real solution, is {-2}   d) at least one real solution, is $(-\infty,-2]$

Key : a, c, d

Sol : $t^2+at+1=0\Rightarrow 3^x=\frac{-a\pm\sqrt{a^2-4}}{2}$

has Z solutions a < 0 and a < - 2 or a > 2

two solutions $a\varepsilon(-\infty,-2)$

no solutions if $a\varepsilon(-2,-\infty)$

exactly one solution if a a = {-2}

at least one real solution if $a\varepsilon(-\infty,-2)$

40. If $x^2+2x-\lambda>0$ for all real values of ‘x’, then value of λ may be:

a) – 1   b) 1   c) – 3   d) – 5

Key: C, D

Hint: $b^2-4ac\lambda D$

22. (L-1)The equation $(x+1)^4 = a(x^4+1)$ is a reciprocal equation for

a) a = 1 b) $a \neq 1$

c) a = -2 d) all values of a

Key : b, c

Sol : f(x) = $(x+1)^4 - a(x^4+1)$

when a = 1, f(0) = 0 and therefore f(x) = 0 is not a reciprocal equation.

41. If $a_1 < a_2 < a_3 < a_4 < a_5 < a_6$, then the equation

$(x-a_1)(x-a_3)(x-a_5)+3(x-a_2)(x-a_4)(x-a_6)=0$ has

a) three reals roots b) a root in $(-\infty, a_1)$

c) a root in (a_1, a_2) d) a root in (a_5, a_6)

Key : a, c, d

Sol : Let $f(x)=(x-a_1)(x-a_3)(x-a_5)+3(x-a_2)(x-a_4)(x-a_6)$

Note that, $f(x) \to -\infty$ as $x \to -\infty$

$f(a_1)=3(a_1-a_2)(a_1-a_4)(a_1-a_6)<0$

Similarly, $f(a_2)>0, f(a_3)>0, f(a_4)<0, f(a_5)<0, f(a_6)<0$

Thus, f(x) = 0 has a root in each of the following intervals $(a_1,a_2),(a_3,a_4)\&(a_5,a_6)$. Thus f(x) = 0 has three real roots.

42. If $b^2 \geq 4ac$ for the equation $ax^4+bx^2+c=0$, then all the roots of the equation will be real if

(A) b > 0, a < 0, c > 0 (B) b < 0, a > 0, c > 0

(C) b < 0, a > 0, c < 0 (D) b > 0, a < 0, c < 0

Key. B,D

Sol. $x^2 = t, \quad t \geq 0$

$at^2+bt+c=0, \ t \geq 0$

$-\dfrac{b}{a} > 0$ ….. (1)

$\dfrac{c}{a} > 0$ ….. (2)

43. If 0 < c < b < a and the roots α, β of the equation $cx^2 + bx + a = 0$ are imaginary, then

(A) $\frac{|\alpha|+|\beta|}{2}=|\alpha||\beta|$ (B) $\frac{1}{|\alpha|}=\frac{1}{|\beta|}$

(C) $\frac{1}{|\alpha|}+\frac{1}{|\beta|}<2$ (D) $\frac{1}{|\alpha|}+\frac{1}{|\beta|}>2$

Key. B,C

Sol. Since roots are imaginary.

So, discriminant < 0

$$\alpha=\frac{-b+i\sqrt{4ac-b^2}}{2c}$$

$$\beta=\frac{-b-i\sqrt{4ac-b^2}}{2c},\ |\alpha|=|\beta|=\sqrt{\frac{b^2}{4c^2}+\frac{4ac-b^2}{4c^2}}=\sqrt{\frac{a}{c}}>1$$

44. The equation $x^3 - 3x + 1 = 0$ has

(a) three real roots (b) three irrational roots

(c) one rational and two irrational roots (d) atleast one negative root

Key. A,B,D

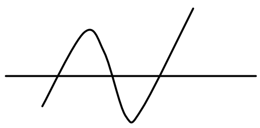
Sol. $f(x) = x^3 - 3x + 1$

$f'(x) = 3x^2 - 3$

$f'(x) = 0 \Rightarrow x = \pm 1$

$\therefore f(-1) . f(1) < 0$

Hence (a), (b), (d) are correct answer.



45. If a,b,c are positive integers such that a>b>c and the quadratic equation (a+b-2c) x^2 +(b+c-2a)x+ (c+a-2b) = 0 has a root in the interval (-1,0) then

a) b+c>a b) c+a <2b

c) both roots of the given equation are rational

d) the equation $ax^2+2bx+c = 0$ has both negative real roots.

Key. B,C,D

Sol. Clearly 1 is a root of the given equation .Given that 2nd root lies in (-1,0) $\Rightarrow$ Product of roots < 0

Is $\frac{c+a-2b}{a+b-2c}<0\Rightarrow c+a-zb<0(\because a+b-2c>0)$

The roots of the equation are both rational for the equation ax^2 +zbx+c = 0 we have f(0) = C>0

F(-1) = c+a-2b< 0. hence one root is –ve

Also for an equation with +ve real coefficients all roots are –ve hence 2 nd root is also –ve.

46. Which of the following is/are correct

(A) between any two roots of $e^x \cos x = 1$ there exists atleast one root of tan x = 1

(B) between any two roots of $e^x \sin x = 1$ there exists atleast one root of tan x = –1

(C) between any two roots of $e^x \cos x = 1$ there exists atleast one root of $e^x \sin x = 1$

(D) between any two roots of $e^x \sin x = 1$ there exists atleast one root of $e^x \cos x = 1$

Key. A,B,C,D

Sol. (a) Let $f(x) = e^x \cos x - 1$

$f'(x) = e^x (\cos x - \sin x) = 0$

$\Rightarrow \tan x = 1$ between two roots of f(x) (Rolle's theorem)

(b) $g(x) = e^x \sin x - 1$, $g'(x) = e^x (\sin x + \cos x) = 0 \Rightarrow \tan x = -1$ between two roots of g(x).

(c) $h(x) = e^{-x} - \cos x$, $h'(x) = -e^{-x} + \sin x = 0 \Rightarrow e^{-x} = \sin x$ between two roots of h(x).

47. If 0 < c < b < a and the roots α, β of the equation $cx^2 + bx + a = 0$ are imaginary, then

(A) $\frac{|\alpha|+|\beta|}{2} = |\alpha||\beta|$ (B) $\frac{1}{|\alpha|} = \frac{1}{|\beta|}$

(C) $\frac{1}{|\alpha|} + \frac{1}{|\beta|} < 2$ (D) $\frac{1}{|\alpha|} + \frac{1}{|\beta|} > 2$

Key. A,B,C

Sol. Since roots are imaginary.

So, discriminant < 0

$$\alpha = \frac{-b + i\sqrt{4ac - b^2}}{2c}$$

$$\beta = \frac{-b - i\sqrt{4ac - b^2}}{2c} ,$$

$$|\alpha| = |\beta| = \sqrt{\frac{b^2}{4c^2} + \frac{4ac - b^2}{4c^2}} = \sqrt{\frac{a}{c}} > 1$$

48. Suppose a, b > 0 and x_1, x_2, x_3 ($x_1 > x_2 > x_3$) are roots of $\frac{x-b}{a} + \frac{x-a}{b} = \frac{b}{x-a} + \frac{a}{x-b}$ and $x_1 - x_2 - x_3 = c$, then

(A) a, c, b are in H.P. and $x_1 = a + b$ (B) a, c, b are in A.P. and $x_2 = a + b$

(C) a, c, b are in A.P. and $x_3 = 0$ (D) a, c, b are in H.P. and $x_3 = 0$

Key. A,D

SOL. $\frac{x-b}{a} - \frac{b}{x-a} = \frac{a}{x-b} - \frac{x-a}{b}$

$(X^2 - (A + B)X)[(B + A)X - (A^2 + B^2)] = 0$

$x = 0,\ a + b,\ \frac{a^2 + b^2}{a + b} \quad (a + b) - \frac{ab}{a + b}$

$x_3 = 0,\ x_2 = \frac{a^2 + b^2}{a + b}, \quad x_1 = a + b$

$(a + b) - (a + b) + \frac{2ab}{a + b} - 0 = c$

$c = \frac{2ab}{a + b}$

a, c b are in H.P.

49. If a, b, c are +Ve and $a = 2b + 3c$, then roots of the equation $ax^2 + bx + c = 0$ are real for

a) $\left|\frac{a}{c} - 11\right| \ge 4\sqrt{7}$ b) $\left|\frac{c}{a} - 11\right| > 4\sqrt{7}$ c) $\left|\frac{b}{c} - 4\right| \ge 2\sqrt{7}$ d) $\left|\frac{c}{b} - 4\right| \ge 2\sqrt{7}$

Key. A,C

Sol. $\Delta \geq 0 \Rightarrow \left(\frac{a-3c}{2}\right)^2 - 4ac \geq 0$

$$\left(\frac{a}{c}\right)^2 - 22\left(\frac{a}{c}\right) + 9 \geq 0 \Rightarrow \left|\frac{a}{c} - 11\right| \geq 4\sqrt{7}$$

50. If $(a, 0)$ is a point on a diameter of the circle $x^2 + y^2 = 4$ then $x^2 - 4x - a^2 = 0$ has

(a) Exactly one real root in (-1, 0] (b) Exactly one real root in [2, 5]

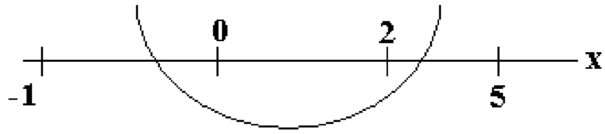
(c) Distinct roots greater than -1 (d) Distinct roots less than '5'

Key. A,B,C,D

Sol. Since (a, 0) is a point on the diameter of the circle $x^2 + y^2 = 4$

Maximum value of a^2 is 4

Let $f(x) = x^2 - 4x - a^2$



$f(-1) = 5 - a^2 > 0$

$f(0) = -a^2 < 0$

$f(2) = 4 - 8 - a^2$

$= -(a^2 + 4) < 0$

and $f(5) = 5 - a^2 > 0$

51. For $y = ax^3 + bx^2 + cx + d\ (a \neq 0)\ a, b, c, d \in R$ which of the following is true?

a) For $b^2 < 3ac$ y has no critical points

b) If y has two distinct critical points then they are bisected by their point of inflexion.

c) If y has one critical point then it is the point of inflexion.

d) y has no points of inflexion.

Key. A,B,C

Sol. $y^1 = 3ax^2 + 2bx + c, y^{11} = 6ax + 2b = 0 \Rightarrow x = -\frac{b}{3a}$. If $b^2 < 3ac$ then $y^1 = 0$ has no real roots hence y has no real roots hence y has no critical points.

If $b^2 > 3ac$ then $y^1 = 0$ has two distinct roots say x_1, x_2 then

$x_1 + x_2 = -\frac{2b}{3a}$ or $\frac{x_1 + x_2}{2} = -\frac{b}{3a}$.

If $b^2 = 3ac$ then $y^1 = 0$ only for one value of $x = -\frac{b}{3a}$

52. If $\left|x^2 + 2x - 8\right| + x - 2 = 0$ then

(A) Number of roots are 3 (B) Sum of roots is - 6

(C) Product of roots is 30 (D) Number of roots are 4

Key. A,B,C

Sol. $\left|x^2 + 2x - 8\right| + x - 2 = 0$

$x^2 + 3x - 10 = 0$ if $x \in (-\infty, -4) \cup (2, \infty)$

$(x+5)(x-2) = 0$

$x = -5 \& 2$

$x = -5$ is one root

$-x^2 - 2x + 8 + x - 2 = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

x = -3 or 2

x = -3 is other root

$\therefore$ x = 2 is also a root

No. of roots is 3

Sum of roots is -6

Product of roots is 30

53. Let $f(x) = A.x^2 + B.x + C$ when A,B, $C \in R$ If x is an integer then f(x) is an integer then

(A) C is an integer
(B) A+B is an integer
(C) B is an integer
(D) 2A is an integer

Key. A,B,D

Sol. $f(0) = C$

As $f(x)$ is an integer for $x \in Z$

$\therefore C \in Z$

$f(1) = A + B + C$

$f(-1) = A - B + C$

$f(1) + f(-1) = 2(A + C)$

$\therefore 2A$ is an integer

A + B is also an integer

54. The roots of the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ are

(A) $\frac{c(a-b)}{a(b-c)}$
(B) 1
(C) $\frac{c(a-b)}{a(b-c)}, \frac{b(c-a)}{a(b-c)}$
(D) $a; \frac{c(a-b)}{a(b-c)}$

Key. A,B

Sol. Roots of $Ax^2 + Bx + C = 0$ are 1 and C/A

If A+B+C=0 $\therefore$ $\text{roots} = 1, \frac{c(a-b)}{a(b-c)}$

55. If the equation whose roots are the squares of the roots of the cubic $x^3 - ax^2 + bx - 1 = 0$ is identical with the given cubic equation, then

A) *a,b* are roots of $x^2 + x + 2 = 0$
B) *a* = *b* = 0
C) *a* = *b* = 3
D) *a* = *0, b* = 3

Key. A,B,C

Sol. (ABC) If roots of the equation be α, β, γ then

$$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha) = a^2 - 2b$$

$$\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2=(\alpha\beta+\beta\gamma+\gamma\alpha)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma)$$
$$=b^2-2a$$
$$\alpha^2\beta^2\gamma^2=1.$$

So, the equation whose roots are $\alpha^2,\beta^2,\gamma^2$ is

$$x^3-(a^2-2b)x^2+(b^2-2a)x-1=0$$

It is identical to $x^3-ax^2+bx-1=0$

$\therefore a^2-2b=a$ and $b^2-2a=b,$ eliminating b, we get

$$\frac{(a^2-a)^2}{4}-2a=\frac{a^2-a}{2}$$

$\Rightarrow$ $a\{a(a-1)^2-8-2(a-1)\}=0$

$\Rightarrow$ $a(a^3-2a^2-a-6)=0$

or $a(a-3)(a^2+a+2)=0$

$\therefore$ $a=0$ *or* $a=3$ *or* $a^2+a+2=0$

Which give $b=0 \, or \, b=3 \, or \, b^2+b+2=0$

So, $a=b=0$ or $a=b=3$

Or *a, b* are roots of $x^2+x+2=0$

56. $\frac{\pi^e}{x-e}+\frac{e^\pi}{x-\pi}+\frac{\pi^\pi+e^e}{x-\pi-e}=0$ has

A) One real root in (e,π) and other in $(\pi-e,e)$

B) One real root in (e,π) and other in $(\pi,\pi,+e)$

C) Two real roots in $(\pi-e,\pi+e)$ D) No real roots

Key. B,C

Sol. Given equation can be expressed as

$$\pi^e(x-\pi)(x-\pi-e)+e^\pi(x-e)(x-\pi-e)+(\pi^\pi+e^e)$$
$$(x-e)(x-\pi)=0$$

Let

$$f(x)=\pi^e(x-\pi)(x-\pi-e)+e^\pi(x-e)(x-\pi-e)+(\pi^\pi+e^e)(x-e)(x-\pi)$$

$$f(e)=\pi^e(e-\pi)(-\pi)>0$$

and $f(\pi)=e^\pi(\pi-e)(-e)<0$

hence given equation has a real root in (e,π)

again $f(\pi+e)=(\pi^\pi+e^e)\pi\cdot e>0$

$\because \pi+e>\pi$, it concludes it has a real root in $(\pi,\pi+e)$

Also $\because \pi-e<e$

hence $f(x)$ has two real roots in $(\pi - e, \pi + e)$

57. Let $|a| < |b|$ and a, b are the roots of the equation $x^2 - |\alpha| x - |\beta| = 0$. If $|\alpha| < b - 1$, then the equation $\log_{|a|}\left(\frac{x}{b}\right)^2 - 1 = 0$ has at least one

A) root lying between $(-\infty, a)$
B) roots lying between (b, ∞)
C) negative root
D) positive root

Key. A,B,C,D

Sol. $|\alpha| =$ sum of roots $= b + a$

$-|\beta| =$ product of root $= ab$

Because $|a| < |b|$ so *a* is negative and *b* is positive.

Now, $|\alpha| < b - 1 \Rightarrow a + b < b - 1 = a < -1.$

Because *a* is negative so magnitude of $'a'$ is greater than one and magnitude of *b* is greater than $1 + |\alpha|$ or say greater than 2.

Now, $\log_{|a|}\left(\frac{x}{b}\right)^2 - 1 = 0 \Rightarrow \left(\frac{x}{b}\right)^2 = |a|$

$\Rightarrow \quad x = \pm b\sqrt{|a|}$

Magnitude of x is greater than $'a'$ as well as greater than $'b'$

$\Rightarrow$ one root lies in $\Rightarrow (-\infty, a)$ and other root lies in (b, ∞).

58. The value of $'x'$ satisfying the equation $x^4 - 2\left(x\sin\left(\frac{\pi}{2}x\right)\right)^2 + 1 = 0$

A) 1
B) -1
C) 0
D) No value of $'x'$

Key. A,B

Sol. $x^4 - 2\left(x\sin\left(\frac{\pi}{2}x\right)\right)^2 + 1 = 0$

$\Rightarrow \quad x^4 + 1 = 2x^2\sin^2\left(\frac{\pi}{2}x\right)$

$\Rightarrow \quad x^2 + \frac{1}{x^2} = 2\sin^2\left(\frac{\pi}{2}x\right)$

Now, LHS ≥ 2 where as RHS ≤ 2

So, equality holds when

$$x^2 + \frac{1}{x^2} = 2 \text{ and } 2\sin^2\left(\frac{\pi}{2}x\right) = 2 \Rightarrow x = \pm 1$$

59. In a ΔABC, $\tan A$ and $\tan B$ satisfy the inequation $\sqrt{3}x^2 - 4x + \sqrt{3} < 0$. Then

A) $a^2 + b^2 - ab < c^2$
B) $a^2 + b^2 > c^2$
C) $a^2 + b^2 + ab > c^2$
D) All of the above

Key. A,C

Sol. $\left(x - \sqrt{3}\right)\left(x\sqrt{3} - 1\right) < 0$

$\Rightarrow$ x lies between $\frac{1}{\sqrt{3}}$ and $\sqrt{3} \Rightarrow$ Both $\tan A$ *and* $\tan B$ lie between $\frac{1}{\sqrt{3}}$ and $\sqrt{3}$

Both A and B lie between 30° and 60°.

$\Rightarrow$ 60°<C<120°

$$\Rightarrow \quad -\frac{1}{2} < \frac{a^2+b^2-c^2}{2ab} < \frac{1}{2}$$

60. Let $f(x) = \frac{3}{x-2} + \frac{4}{x-3} + \frac{5}{x-4}$, then $f(x) = 0$ has

A) exactly one real root in $(2,3)$ B) exactly one real root in $(3,4)$

C) at least one real root in $(2,3)$ D) None of these

Key. A,B,C

Sol. $f(x) = \frac{3}{x-2} + \frac{4}{x-3} + \frac{5}{x-4}$

$$\left.\begin{array}{ll} \because & f(2^+) \to \infty \\ and & f(3^-) \to -\infty \end{array}\right\}$$

$\Rightarrow f(x) = 0$ has exactly one root in (2,3)

Again $\left.\begin{array}{ll} \because & f(3^+) \to \infty \\ and & f(4^-) \to -\infty \end{array}\right\} \Rightarrow f(x) = 0$

Has exactly one root in (3,4)

61. If $x_1 > x_2 > x_3$ and x_1, x_2, x_3 are roots of $\frac{x-a}{b} + \frac{x-b}{a} = \frac{b}{x-a} + \frac{a}{x-b}; (a,b,>0)$ and $x_1 - x_2 - x_3 = c$, then a, c, b are in.

A) A.P. B) G.P. C) H.P. D) None

Key. C

Sol. Given equation can be written as

$$\frac{x-a}{b} - \frac{b}{x-a} + \frac{x-b}{a} - \frac{a}{x-b} = 0$$

$$= \frac{(x-a)^2 - b^2}{b(x-a)} + \frac{(x-b)^2 - a^2}{a(x-b)} = 0$$

$$\Rightarrow (x-a-b)\left[\frac{x-a+b}{b(x-a)} + \frac{x-b+a}{a(x-b)}\right] = 0$$

$$\Rightarrow (x-a-b)\left\{\frac{a\left[x^2-bx-ax+ab+bx-b^2\right]+b\left[x^2-ax-bx+ab+ax-a^2\right]}{ab(x-a)(x-b)}\right\} = 0$$

$$\Rightarrow (x-a-b)\left(ax^2-a^2x+a^2b-ab^2+bx^2-b^2x+ab^2-a^2b\right)$$

$$\Rightarrow x(x-a-b)\{x(a+b)-(a^2+b^2)\} = 0$$

$\therefore$ roots will be x=0, $a+b, \dfrac{a^2+b^2}{a+b}$

Let $x_1 = a+b$, $x^2 = \dfrac{a^2+b^2}{a+b}$ and $x_3 = 0$

$\because x_1 - x_2 - x_3 = c$ (given)

$$\therefore (a+b) - \frac{a^2+b^2}{a+b} - 0 = c$$

$$\Rightarrow \frac{(a+b)^2-(a^2+b^2)}{a+b} = c \Rightarrow \frac{2ab}{a+b} = c$$

i.e a, c, b are in H. P

62. Two numbers such that their sum is 9 and the sum of their fourth powers is 2417. Then the numbers are

a) even positive integers
b) odd positive integers
c) one is even & another is odd
d) both are prime

Key. C,D

Sol. Let the two number be x and y

Then $x+y=9$ and $x^4+y^4=2417$

Now $(x+y)^4 = 9^4$

or $x^4+4x^3y+6x^2y^2+4xy^3+y^4=6561$

or $4x^3y+6x^2y^2+4xy^3=6561-2417$

$(\because x^4+y^4=2417)$

or $4xy(x^2+y^2)+6x^2y^2=4144$

or $4xy[(x+y)^2-2xy]+6x^2y^2=4144$

or $4xy[81-2xy]+6x^2y^2=4144$

or $324xy-8x^2y^2+6x^2y^2=4144$

or $2x^2y^2-324xy+4144=0$

or $(xy)^2-162xy+2072=0$

or $(xy-148)(xy-14)=0$

$\therefore xy=148 \, or \, xy=14$

When $xy=14$, and $x+y=9$

Then $x=7, y=2$ the other solution is inadmissible.

Hence the numbers are 7 and 2

63. The equation $|x+1||x-1| = a^2 - 2a - 3$ can have real solutions for x, if a belongs x to

A) $(-\infty,-1]\cup[3,\infty)$ B) $\left[1-\sqrt{5},1+\sqrt{5}\right]$ C) $\left[1-\sqrt{5},-1\right]\cup\left[3,1+\sqrt{5}\right]$ D) $[-1,3]$

Key. A,D

Sol. $|x+1||x-1| = a^2-2a-3 \Rightarrow |x^2-1| = a^2-2a-3$

$\therefore a^2-2a-3 \ge 0$

$\Rightarrow (a+1)(a-3) \ge 0$

$\therefore a \in (-\infty,-1)\cup[3,\infty)$

64. Let $a,b,c \in R$. If $ax^2+bx+c=0$ has two real roots A and B where $A<-1$ and $B>1$, then

A) $1+\left|\frac{b}{a}\right|+\frac{c}{a}<0$ B) $1-\left|\frac{b}{a}\right|+\frac{c}{a}<0$ C) $|c|<|a|$ D) $|c|<|a|-|b|$

Key. A,B

Sol. $a>0, f(-1)<0$ and $f(1)<0$

$\Rightarrow a-b+c<0$ and $a+b+c<0$

$\Rightarrow 1-\frac{b}{a}+\frac{c}{a}<0$ and $1+\frac{b}{a}+\frac{c}{a}<0$

$\Rightarrow 1\pm\left|\frac{b}{a}\right|+\frac{c}{a}<0$

$a<0, f(-1)>0$ and $f(1)>0$

$\Rightarrow a\pm b+c>0$

$\Rightarrow 1\pm\frac{b}{a}+\frac{c}{a}<0$ ($\because a<0$)

$\Rightarrow 1\pm\left|\frac{b}{a}\right|+\frac{c}{a}<0$

65. Let $f(x)=ax^2+bx+c; a,b,c\in R$ and $a\ne 0$. Suppose $f(x)>0$ for all $x\in R$. Let $g(x)=f(x)+f'(x)+f''(x)$. Then

A) $g(x)>0\,\forall x\in R$ B) $g(x)<0\,\forall x\in R$

C) $g(x)=0$ has non real complex roots D) $g(x)=0$ has real roots

Key. A,C

Sol. Since, $f(x)>0, \forall x\in R, a>0$ and $b^2-4ac<0$

We have, $f'(x)=2ax+b$ and $f''(x)=a$

Thus, $g(x)=ax^2+bx+c+2ax+b+2a=ax^2+(2a+b)x+(2a+b+c)$

We have $a>0$ and $D=(2a+b)^2-4a(2a+b+c)$

$=b^2-4ac-4a^2<0$, since $b^2-4ac<0$

Thus, $g(x)>0, \forall x\in R$. Therefore, $g(x)=0$ has non real complex roots.

66. If every pair from among the equations $x^2+px+qr=0, x^2+qx+rp=0$ and $x^2+rx+pq=0$ have a common root, then $\left(\frac{\text{sum of roots}}{\text{product of roots}}\right)$ is

A) $\frac{\sum p}{pqr}$ B) $\sum \frac{1}{pq}$ C) $(p+q+r)^2$ D) $\frac{p}{q}+\frac{q}{r}+\frac{r}{p}$

Key. A,B,C

Sol. The given equations are

$x^2+px+qr=0$ --- (1)

$x^2+qx+rp=0$ --- (2)

$x^2+rx+pq=0$ --- (3)

Let α,β be roots of (1); β,γ or (2), γ,α of (3)

Since β is a common root of (1), (2)

$\therefore \beta^2+p\beta+qr=0$ and $\beta^2+q\beta+rp=0$

$\Rightarrow (p-q)\beta+r(q-p)=0 \Rightarrow \beta=r$

Now $\alpha\beta=qr \Rightarrow \alpha r=qr \Rightarrow \alpha=q$

Similarly from equation (2) and (3), we get $\gamma=p$

$\therefore \alpha+\beta+\gamma=p+q+r$

$(\alpha\beta).(\beta\gamma).(\gamma\alpha)=(qr).(rp).(pq) \Rightarrow (\alpha\beta\gamma)^2=(pqr)^2 \Rightarrow \alpha\beta\gamma=pqr$

$\therefore \frac{\text{sum of roots}}{\text{product of roots}}=\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}=\frac{p+q+r}{pqr}=\frac{\sum p}{pqr}=\sum\frac{1}{pq}$

67. If $a+3b+9c=0, ac<0$ and one root of the equation $ax^2+bx+c=0$ is square of the other, then

A) a and b have same sign B) b and c have opposite sign

C) both roots are rational D) a,b,c are irrational

Key. A,B

Sol. Let $f(x)=ax^2+bx+c$

$f\left(\frac{1}{3}\right)=\frac{a}{9}+\frac{b}{3}+c=\frac{1}{9}(a+3b+9c)=0$

$\therefore \frac{1}{3}$ is the root of the given equation.

Also, product of the roots $=\frac{ac}{a^2}<0$

Therefore, another root must be negative, hence it will be $-1/\sqrt{3}$ and required equation is

$\left(x-\frac{1}{3}\right)\left(x+\frac{1}{\sqrt{3}}\right)=0$ or $3\sqrt{3}x^2+(3-\sqrt{3})x-1=0$

68. If roots of $ax^2+2bx+c=0$ $(a\neq 0)$ are non real complex and $a+c<2b$, then

A) $c>0$ B) $c<0$ C) $4a+c<4b$ D) $4a+c>4b$

Key. B,C

Sol. Roots of $ax^2+2bx+c=0$ are non real complex.

$\therefore f(x)=ax^2+2bx+c>0$ or <0 for all x

But $f(-1)=a-2b+c<0$

$\therefore f(0)$ and $f(-2)$ must be less than zero $f(0)<0 \Rightarrow c<0$

and $f(-2)<0 \Rightarrow 4a+c<4b$

69. $5^x + (2\sqrt{3})^{2x} - 169 \le 0$ is true in the interval

A) $(-\infty, 2)$ B) $(0,2)$ C) $(2,\infty)$ D) $(0,4)$

Key. A,B

Sol. $(25)^{x/2} + (144)^{x/2} \le 169$

Equality holds if $x = 2$

$\therefore$ Eq.(i) is true if $x < 2$.

70. If the equation $ax^2 + bx + c = 0 (a > 0)$ has two roots α and β such that $\alpha < -2$ and $\beta > 2$, then

A) $b^2 - 4ac > 0$ B) $c < 0$

C) $a + |b| + c < 0$ D) $4a + 2|b| + c < 0$

Key. A,B,C,D

Sol. Since, the equation has two distinct roots α and β, the discriminant $b^2 - 4ac > 0$, we must have

$f(x) = ax^2 + bx + c < 0$ for $\alpha < x < \beta$

Since, $\alpha < 0 < \beta$ we must have $f(0) = c < 0$

Also, as $\alpha < -1, 1 < \beta$ we get $f(-1) = a - b + c < 0$

And $f(1) = a + b + c < 0$, i.e., $a + |b| + c < 0$

Since, $\alpha < -2, 2 < \beta$

$f(-2) = 4a - 2b + c < 0$ and $f(2) = 4a + 2b + c < 0$ i.e., $4a + 2|b| + c < 0$

71. If $c \ne 0$ and the equation $\frac{p}{2x} = \frac{a}{x+c} + \frac{b}{x-c}$ has two equal roots, then p can be

A) $\left(\sqrt{a} - \sqrt{b}\right)^2$ B) $\left(\sqrt{a} + \sqrt{b}\right)^2$ C) $a + b$ D) $a - b$

Key. A,B

Sol. $\frac{p}{2x} = \frac{(a+b)x + c(b-a)}{x^2 - c^2}$

or $p(x^2 - c^2) = 2(a+b)x^2 - 2c(a-b)x$

or $(2a + 2b - p)x^2 - 2c(a-b)x$ or $(2a + 2b - p)x^2 - 2c(a-b)x + pc^2 = 0$

Now, $c^2(a-b)^2 - pc^2(2a + 2b - p) = 0$ ($\because$ equal roots)

$\Rightarrow (a-b)^2 - 2p(a+b) + p^2 = 0$ $(\because c^2 \ne 0)$

$\Rightarrow \left[p - (a+b)\right]^2 = (a+b)^2 - (a-b)^2$

$\Rightarrow p = a + b \pm 2\sqrt{ab} = \left(\sqrt{a} \pm \sqrt{b}\right)^2$

72. If $\frac{|x| - 1}{|x| - 2} \ge 0, x \in R,\ x \ne \pm 2,$ then x belongs to

(A) $(-\infty, -2)$ (B) $[-1, 1]$

(C) $(2, \infty)$ (D) $(1, 2)$

Key. A,B,C

Sol. Conceptual

73. If $\frac{(x-2)^2(1-x)(x-3)^3(x-4)^2}{x+1} \le 0$, then x belongs to

(A) $(-1, 1]$ (B) $[3, \infty)$

(C) $(1, 2)$ (D) $(2, 3)$

Key. A,B

Sol. Conceptual

74. $x^2 - 9 < 0$ is valid If x belongs to

(A) $(-\infty, -3)$ (B) $(-3, 0)$

(C) $(0, 3)$ (D) $[3, \infty)$

Key. B,C

Sol. Conceptual

75. If $\log_7 \frac{2x-6}{2x-2} > 0$ then $x \in$

(A) $(-\infty, 0]$ (B) $[1, 2]$

(C) $(2, \infty)$ (D) $\left[0, \frac{1}{2}\right)$

Key. A,D

Sol. Conceptual

30. Let f(x) be a polynomial over real, if 2 + 3i is a root of f(x) = 0 then

(A) 2 –3i is its other root

(B) f(x) is divisible by $x^2 - 4x + 13$

(C) 2 –3i may not be and its other root

(D) the sum of the roots of f(x) = 0 is certainly a real number.

Key. A,B,D

Sol. Obviously 2 – i3 is also its root.

$\therefore$ f(x) is divisible by $\{x-(2+i3)\}\{x-(2-i3)\}$

i.e. $x^2 - 4x + 13$. Sum of the roots = 4 = a real number

$\therefore$ (a), (b), (d) are correct.

31. If $b^2 \geq 4ac$ for the equation $ax^4 + bx^2 + c = 0$, then all roots of the equation will be non zero real if

(A) $b > 0, a > 0, c > 0$ (B) $b < 0, a > 0, c > 0$

(C) $b > 0, a > 0, c < 0$ (D) $b > 0, a < 0, c < 0$

Key. B,D

Sol. All roots of equation $ax^4 + bx^2 + c = 0$ will be real if both roots of $ay^2 + by + c$ will be positive (replace $x^2 = y$)

i.e. sum of roots = $-\frac{b}{a} > 0$

Product of roots $= \frac{c}{a} > 0$

Hence, a and b are of opposite sign, while a and c of same sign.

32. If α is one root of the equation $4x^2 + 2x - 1 = 0$, then its other root is given by

(A) $4\alpha^3 - 3\alpha$ (B) $4\alpha^3 + 3\alpha$

(C) $\alpha - \frac{1}{2}$ (D) $-\alpha - \frac{1}{2}$

Key. A,D

Sol. If other root is $\beta \Rightarrow \alpha + \beta = -2/4$

$\Rightarrow \beta = -\frac{1}{2} - \alpha$ and $4\alpha^2 + 2\alpha - 1 = 0$

$\Rightarrow 4\alpha^2 = 1 - 2\alpha$

$\Rightarrow 4\alpha^3 = \alpha\left(4\alpha^2\right)$

$= \alpha(1-2\alpha) = \alpha - 2\alpha^2$

$= \alpha - 2\left[\frac{1-2\alpha}{4}\right] = 2\alpha - \frac{1}{2}$

$= 4\alpha^3 - 3\alpha = -\alpha - \frac{1}{2} = \beta$

24. If a, b, c $\in$ Q then which of the following equations has rational roots

(A) $ax^2 + bx + c = 0$ where if a + b + c = 0

(B) $(a+c-b)x^2 + 2cx + (b+c-a) = 0$

(C) $abc^2x^2 + 3a^2cx + b^2cx - 6a^2 - 4ab - 2b^2 = 0$

(D) $(a+b-c)x^2 + (a+c-b)x + (b+c-a) = 0$

Key. A,B,C

Sol. (B) $(a+c-b)1^2 + 2c(1) + (b+c-a) = 0$

1 is root of the equation which is rational. 2nd must also be rational

(C) $x = \frac{2}{c}$ satisfies given equation which is rational

25. Which of the following statements are true

(A) If a^2, b^2, c^2 are in A.P. then $b+c, c+a$ and $a+b$ are in H.P.

(B) If $p^{th}, q^{th}, r^{th}, s^{th}$ terms of an A.P. are in G.P. then p-q, q-r, r-s are in G.P.

(C) If b is HM of a & c then $\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{a} + \frac{1}{c}$

(D) If b is HM of $a \& c$ then $\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{a} - \frac{1}{c}$

Key. A,B,C

Sol. If b+c, c+a, a+b are in H.P

(A) $\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a} \Rightarrow b^2 - a^2 = c^2 - b^2$

$\Rightarrow a^2, b^2, c^2$ are in A.P.

(B) Let A + (p-1)d, A + (q-1)d, A + r-1) d, A + (s -1) d

p^{th}, q^{th}, r^{th}, s^{th} term of A.P. it satisfies the given condition

(C) a, b, c, are in H.P

$$= \frac{1}{\frac{2ac}{a+c} - a} + \frac{1}{\frac{2ac}{a+c} - c} = \frac{a+c}{ac - a^2} + \frac{a+c}{ac - c^2} = \frac{1}{a} + \frac{1}{c}$$

26. The real values of λ for which the equation $x^3 - 3x^2 - 9x + \lambda = 0$ has three distinct real roots, if $\lambda \in$

(A) (–2,0) (B) [0,1]

(C) [1, 2] (D) $(-\infty, \infty)$

Key. A,B,C

Sol. $f'(x) = 3x^2 - 6x - 9 = 0 \Rightarrow x = +3, -1$

equation has three distinct real rots if f(3) f(–1) < 0

$\Rightarrow \lambda \in (-5, 27)$

27. Few identical balls are arranged in a form whose base is an equilateral triangle and one side of the base triangle contains n balls then

(A) Number of balls in base triangle are $n^2 + n$

(B) Number of balls in base triangle are $\frac{1}{2}(n^2 + n)$

(C) Total number of balls in pyramid are $\frac{n(n+1)(n+2)}{6}$

(D) Total number of balls in pyramid are $\frac{n(n+1)(n+3)}{8}$

Key. B,C

Sol. Total no. of balls in base triangle = $\sum n = \frac{1}{2}(n^2 + n)$

total no of balls in pyramid = $\frac{1}{2}\left(\sum n^2 + \sum n\right) = \frac{n(n+1)(n+2)}{6}$

28. Which of the following is/are true ?

(A) $a^{\log_a x} = x$ if a > 2 and x > 0 (B) $a^{\log_b c} = c^{\log_b a}$ if a > 0, b > 0 & c > 0

(C) $\log_a b = \frac{\log_m b}{\log_m a}$ if a > 0, b > 0 & m >0 (D) $\log_a b = \frac{\log_m a}{\log_m b}$ if a > 0, b > 0 & m >0

Key. A

Sol. Basic properties of log.

Quadratic Equations & Theory of Equations

Assertion Reasoning Type

1. Statement-I : The greatest integral value of λ for which $(2\lambda-1)x^2-4x+(2\lambda-1)=0$ has real roots is 2

Statement-II : For real root of $ax^2+bx+c=0, D\geq 0$

Key. D

Sol. For real roots

$\Rightarrow (-4)^2-4(2\lambda-1)(2\lambda-1)\geq 0$

$\Rightarrow (2\lambda-1)^2\leq 4$

$\Rightarrow -2\leq 2\lambda-1\leq 2$

$\Rightarrow -\dfrac{1}{2}\leq \lambda\leq \dfrac{3}{2}$

$\therefore$ Integral values of λ are 0 and 1

Hence, the greatest integer value of $\lambda=1$

2. Statement-I : Let $f(x)$ be a quadratic expression such that $f(0)+f(1)=0$. If -2 is one of the roots of $f(x)=0$. Then the Sum of roots is 3/5

Statement-II : If α and β are the zeros of $f(x)=ax^2+bx+c$, then the sum of zeros = $-b/a$ and the product of zeros = c/a

Key. D

Sol. Since $x=-2$ is a root of f(x)

$\therefore$ f(x)=(x+2)(ax+b)

But $f(0)+f(1)=0$

$\therefore 2b+3a+3b=0 \Rightarrow -\dfrac{b}{a}=\dfrac{3}{5}$

3. Consider the equation $x^3-3x+k=0,\ k\in R.$

Statement I There is no value of K for which the given equation has two distinct roots in $(0,1)$.

Statement II Between two consecutive roots of $f'(x)=0$, ($f(x)$ is a polynomial). $f(x)=0$ must have one root.

Key. C

Sol. By Rolle's Theorem between the roots of $f'(x)=0$. If there exists a root of $f(x)=0$, $[f(x) is\ polynomial]$, then it must be unique.

4. Statement I All the real roots of the equation $x^4-3x^3-2x^2-3x+1=0$ lie in the interval $[0,3]$

Statement II The equation reduces to quadratic equation in the variable t, by substituting $x+\frac{1}{x}=t.$

Key. D

Sol. Dividing by x^2 , we get

$$x^2-3x-2-\frac{3}{x}+\frac{1}{x^2}=0$$

$$x^2+\frac{1}{x^2}-3\left(x+\frac{1}{x}\right)-2=0$$

$$\Rightarrow t^2-3t-4=0 \Rightarrow t=-1,4$$

$$x+\frac{1}{x}=-1 \Rightarrow x^2+x+1=0$$

$\therefore\ x=\omega,\omega^2$, the complex cube roots of unity

$$x+\frac{1}{x}=4 \Rightarrow x=2\pm\sqrt{3}$$

$\therefore$ one root is outside [0,3]

5. Let a,b,c,p,q be real numbers, suppose α,β are the roots of equation $x^2+2px+q=0$ and $\alpha,\frac{1}{\beta}$ are roots of equation $ax^2+2bx+c=0$ where $\beta^2 \notin \{-1,0,1\}$

Statement I $(p^2-q)(b^2-ac)\geq 0$

Statement II $b \neq pa\ or\ c \neq qa$

Key. B

Sol. If the roots are imaginary, then $\beta=\bar{\alpha},\frac{1}{\beta}=\bar{\alpha} \Rightarrow \beta^2=1$, contradiction

$\therefore$ The roots are real

$\Rightarrow (p^2-q)$ and $(b^2-ac)\geq 0$

suppose b = pa and c= qa then the second equation becomes identical with the first equation.

$\therefore \beta=\frac{1}{\beta} \Rightarrow \beta^2=1$, contradiction

$\therefore$ Either $b\neq pa$ or $c\neq qa$.

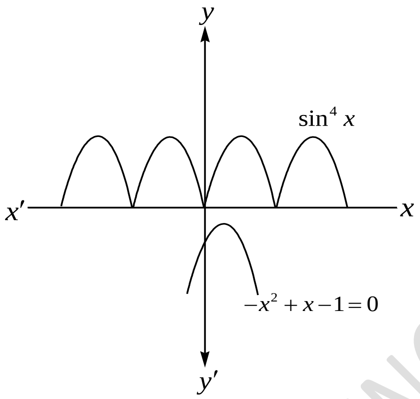
6. Statement-I : The equation $-x^2+x-1=\sin^4 x$ has only one solution

Statement-II : If the curves $y = f(x)$ and $y = g(x)$ cut at one point, the number of solution is 1

Key. D

Sol. Let $f(x) = -x^2 + x - 1$

Here $a < 0$ and $D = (1)^2 - 4(1) < 0$, then $f(x) < 0$



But $\sin^4 x \geq 0$

$\therefore -x^2 + x - 1 \neq \sin^4 x$

Hence the number of solutions is 0

7. Consider the equation $x^3 - 3x + k = 0,\ k \in R.$

Statement I There is no value of K for which the given equation has two distinct roots in $(0,1)$.

Statement II Between two consecutive roots of $f'(x) = 0$, ($f(x)$ is a polynomial). $f(x) = 0$ must have one root.

Key. C

Sol. By Rolle's Theorem between the roots of $f'(x) = 0$. If there exists a root of $f(x) = 0$, $\left[f(x) is\ polynomial\right]$, then it must be unique.

8. STATEMENT-I: The differential equation of all circles in a plane must be of order 3.

because

STATEMENT-II: If three points are non collinear, then only one circle always passes through these points.

Key. A

Sol. Conceptual

9. Statement I All the real roots of the equation $x^4 - 3x^3 - 2x^2 - 3x + 1 = 0$ lie in the interval $[0,3]$

Statement II The equation reduces to quadratic equation in the variable *t*, by substituting $x+\frac{1}{x}=t.$

Key. D

Sol. Dividing by x^2 , we get

$$x^2-3x-2-\frac{3}{x}+\frac{1}{x^2}=0$$

$$x^2+\frac{1}{x^2}-3\left(x+\frac{1}{x}\right)-2=0$$

$$\Rightarrow t^2-3t-4=0\Rightarrow t=-1,4$$

$$x+\frac{1}{x}=-1\Rightarrow x^2+x+1=0$$

$\therefore\ x=\omega,\omega^2$, the complex cube roots of unity

$$x+\frac{1}{x}=4\Rightarrow x=2\pm\sqrt{3}$$

$\therefore$ one root is outside [0,3]

10. Let a,b,c,p,q be real numbers, suppose α,β are the roots of equation $x^2+2px+q=0$ and $\alpha,\frac{1}{\beta}$ are roots of equation $ax^2+2bx+c=0$ where $\beta^2\notin\{-1,0,1\}$

Statement I $\left(p^2-q\right)\left(b^2-ac\right)\ge 0$

Statement II $b\ne pa\ or\ c\ne qa$

Key. B

Sol. If the roots are imaginary, then $\beta=\bar{\alpha},\frac{1}{\beta}=\bar{\alpha}\Rightarrow\beta^2=1$, contradiction

$\therefore$ The roots are real

$\Rightarrow(p^2-q)$ and $(b^2-ac)\ge 0$

suppose b = pa and c= qa then the second equation becomes identical with the first equation.

$\therefore\beta=\frac{1}{\beta}\Rightarrow\beta^2=1$, contradiction

$\therefore$ Either $b\ne pa$ or $c\ne qa$.

11. STATEMENT–1: The equation $ax^2 + bx + c = 0$ cannot have rational roots, if a, b, c are odd integers.

STATEMENT–2: If an odd number does not leave remainder 1 when divided by 8, then it cannot be a perfect square.

Key: A

Hint : The reason R is true since the square of an odd number $2l + 1$ is given by $(2l + 1)^2 = 4l^2 + 4l + 1 = 4l(l + 1) + 1 = 8k + 1$ (since $l(l + 1)$ is a multiple of 2)

$\Rightarrow$ Square of odd number leaves remainder 1 when divided by 8.
The assertion A is true, if all the coefficients are odd
Let a = 2l + 1, b = 2m + 1, c = 2n + 1
Then $b^2 - 4ac = (2m + 1)^2 - 4(2l + 1)(2n + 1)$
$= 4m^2 + 4m - 16ln - 8l - 8n - 3$
$= 8\left[\frac{m(m+1)}{2} - 2ln - l - n\right] - 3 = 8k - 3 \quad \left(\because \frac{m(m+1)}{2} \text{ is an integer}\right)$

$\Rightarrow$ $b^2 - 4ac$ is an odd number which cannot be a perfect square.

$\Rightarrow$ roots $= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{\text{rational} + \text{irrational}}{\text{rational}} = \text{irrational}$

$\Rightarrow$ Assertion is true.

12. (L-1)Statement-1 : If f(x) = 3(x – 2) (x – 6) + 4(x – 3) (x – 7), then f(x) = 0 has two different and real roots

Statement-2 : If f(x) = 3(x – a) (x – c) + 4(x – b) (x – d) and 0 < a < b < c < d, then f(x) = 0 has two different and real roots.

Key : A

Sol : $f(2) > 0, f(3) > 0; f(6) < 0, f(7) > 0$

Hence f(x) has two different real roots.
$\therefore$ statement – I is true
Statement – II is also true and is correct explanation of I

13. (L-1)Statement-1 : If one of root of $x^4 - 4x^3 + 4x - \lambda = 0$ is $2 + \sqrt{3}$, where $\lambda \in Q$, then the value of λ is 1

Statement-2 : n^{th} degree polynomial has even number of irrational zeros

Key : C

Sol : 2^{ND} root must be $2 - \sqrt{3}$ hence $x^2 - 4x + 1$ is a factor of $x^4 - 4x^3 + 4x - d = 0 \Rightarrow \lambda = 1$
Statement I is true
Statement – 2 is false because n degree polynomial to have even number of irrational zero of should
Have rational coefficients.

14. (L-1)Statement-1 : Given a real quadratic, $\left(ax^2 + bx + c, a, b, c \in R\right)$ if the sum and the product of the roots are both positive, then its roots must be positive real numbers.

Statement-2 : If the product of real roots of a real quadratic is positive the roots must be of like sign and if their sum is also positive, each of them must be positive.

Key : D

Sol : $\alpha\beta > 0 \Rightarrow \alpha > 0, \beta > 0 \, or \, \alpha < 0, \beta < 0$ and $\alpha + \beta > 0 \Rightarrow \alpha, \beta$ are +ve.

15. (L-1)In a triangle ABC if $\Delta = r$, then

Statement-1 : $a^2+b^2+c^2+2abc<2$ and

Statement-2 : As $s=1 \Rightarrow 1-a, 1-b, 1-c>0$

Key : B

Sol : $\frac{(1-a)+(1-b)+(1-c)}{3} \geq ((1-a)(1-b)(1-c))^{1/3}$

16. Let the equation $4ax^2-2bx-4c=0$ where $a,b,c \in R$ *and* $a \neq 0$ does not possess real roots and $c>4a-b$ then

Statement I : $2c>2a+b$

Statement II : Graph of y = $4ax^2-2bx-4c$ lies completely below the x-axis.

KEY : A

HINT : $16a-4b-4c<0 \Rightarrow f(2)<0 \Rightarrow f(x)<0 \; \forall x \in R$

$f(-1)<0 \Rightarrow 4a+2b-4c<0 \Rightarrow 2c>2a+b$

17. STATEMENT-1: All the real roots of the equation $x^4-3x^3-2x^2-3x+1=0$ lie the interval $[0,3]$.

STATEMENT-2: The equation $x^4-3x^3-2x^2-3x+1=0$ is reciprocal equation.

KEY : D

HINT : The given equation is a reciprocal equation

$\therefore x+\frac{1}{x}=t \Rightarrow t^2-3t-4=0 \Rightarrow (t-4)(t+1)=0$

Let $x+\frac{1}{x}=t \Rightarrow t^2-3t-4=0 \Rightarrow (t-4)(t+1)=0$

$x+\frac{1}{x}=4$ or $x+\frac{1}{x}=-1$

$\Rightarrow x=2\pm\sqrt{3}$.

18. $x \in Z$ *and* $a,b,c,d \in Z \, (a<b \leq c<d)$

STATEMENT-1: If $(x-a)(x-b)(x-c)(x-d)=2009$ has 4 integer roots of which exactly two are equal then sum of other two roots is ± 42

STATEMENT-2: 2009 is a prime number

KEY : C

19. (L-1)Statement – 1 : $\tan\left(\frac{\pi}{4}\left(\frac{1+\sin^2 x}{1+\sin^2 y}\right)\right)+\tan\left(\frac{\pi}{4}\left(\frac{1+\cos^2 x}{1+\cos^2 y}\right)\right)>1$ for $x, y \in \left(0, \frac{\pi}{2}\right)$

Statement – 2 :

If $f(x,y)=\left(\frac{1+\sin^2 x}{1+\sin^2 y}-1\right)$, then $f(x,y).f\left(\frac{\pi}{2}-x,\frac{\pi}{2}-y\right)\le 0\,\forall x,y\in\left(0,\frac{\pi}{2}\right)$

Key : C

Sol : Let $x>y$ $\frac{1+\sin^2 x}{1+\sin^2 y}-1=\frac{\sin^2 x-\sin^2 y}{1+\sin^2 y}>0$

$\therefore \frac{1+\sin^2 x}{1+\sin^2 y}>1 \text{parallely } \frac{1+\cos^2 x}{1+\cos^2 y}-1<0$

$\therefore$ A is false R is true

20. Assertion: Let (a_1,a_2,a_3,a_4,a_5) denote a re-arrangement of $(1,-4,6,7,-10)$. Then the equation $a_1x^4+a_2x^3+a_3x^2+a_4x+a_5=0$ has at least two real roots

Reason: If $ax^2+bx+c=0$ and $a+b+c=0$, then $x=1$ is root of $ax^2+bx+c=0$

Key. A

Sol. $\sum a_1=0$

$\Rightarrow x=1$ is a root of $a_1x^4+a_2x^3+a_3x^2+a_4x+a_5=0$ max. root=4. & complex roots are in pair form. Hence the given equation has at least two real roots.

21. Statement–1 : 3 is a multiple root of order 2 of the equation $x^3-5x^2+3x+9=0$.

Statement–2 : If $f(x)=x^3-5x^2+3x+9$, then $f''(3)=0$

Key. C

Sol. $f(x)=(x-3)^2(x+1)$ $\Rightarrow$ 3 is a multiple root of order 2.

$f'(x)=3x^2-10x+3$

$f''(x)=6x-10$

$f'(3)=0,\ f''(3)\ne 0$

22. Statement–1 : If $f(x)=ax^2-bx+2; a+b+2<0$, then exactly one root lies between –1 and 0.

Statement–2 : $ab<0$.

Key. C

Sol. $f(x)=ax^2-bx+2$

$f(0)=2$

$f(-1)=a+b+2<0$ $(\because a+b+c<0)$

$\therefore$ $f(0)f(-1)<0$

$\therefore$ one roots lie between (–1, 0)

Nothing can be said about ab.

23. Let $a,b,c\in R$

Statement–1 : The equation $a^2x^3-3abx^2+3b^2x+c=0$ has only one real root.

Statement–2 : Any cube function f(x) has exactly one real root if the product of the maximum and minimum values of the function f(x) is positive.

Key. B

Sol. Let f(x) = $a^2x^3-3abx^2+3b^2x+c=0$

$f'(x)=3a^2x^2-6abx+3b^2=3(ax-b)^2\ge 0,\forall x\in R$

$\Rightarrow$ f(X) is an increasing function so y = f(x) will cut the x–axis at once or we can say f(x) = 0 has only one real root.

24. Statement – 1 : Let f(x) be a polynomial with real co- efficients such that $f(x) = f'(x)\, f'''(x),\ f(x) = 0$ is satisfied by x = 1, 2, 3, only then the value of $f'(1)\times f'(2)\times f'(3)$ is o.

Because

Statement – 2 : If $f\left(x^2-6x+6\right)+f\left(x^2-4x+4\right)=2x\ \forall\ \ x\in R$ then $f(-3)+f(9)$ is 14.

Key. B

Sol. $f(x)$ is a polynomial of degree so either $x=1\ or\ x=2\ or,\ x=3$ is a repeating root of f(x)

$\therefore\ f'(1).f'(2).f'(3)=0$

Statement – 1 and (2) not related in any sence but both are correct

25. Statement – 1 : The values of 'a' for which the point of local minima of $f(x) = x^3 - 3ax^2 + 3\left(a^2-1\right)x + 1$ is less than 4 and point of local maxima is greater then – 2 belongs to $(-1, 3)$.

Because

Statement – 2 : The roots of $f'(x) = 0$ are real and different and lie in the internal $(-2, 4)$.

Key. A

Sol. $f'(x)=3\left(x^2-2ax+a^2-1\right)$

The roots of the equation $f'(x)=0$ must be real distinct and lie in the interval (-2, 4) .

$\therefore D>0 \Rightarrow a\in R\ -(i),$

$f'(-2)>0\ \Rightarrow a<-3\ or\ \ a>-1-(ii)$

$f'(4)>0 \Rightarrow\qquad a>5\ or\ a<3\ -(iii)$

And $-2<-\dfrac{B}{2A}<4\ \ \Rightarrow\ -2<a<4-(iv)$

From $(i),(ii),(iii)$ and (iv) $\Rightarrow -1<a<3$.

26. Statement – 1 : The equation $x^2+bx+ca=0$ and $x^2+cx+ab=0$ have a common root, then their other roots are given by $x^2+ax+bc=0$

Because

Statement – 2 : If 'S' be the sum and ' P' be the product of the roots of a quadratic equation then $x^2-Sx+P=0$

Key. A

Sol. $\dfrac{x^2}{a\left(b^2-c^2\right)}=\dfrac{x}{a(c-b)}=\dfrac{1}{c-b}$

$a = -(b+c)$

27. STATEMENT–1:
If a , b , c , d $\in$ R such that a < b < c < d, then the equation
(x – a) (x – c) + 2(x – b) (x – d) = 0 are real and distinct.
STATEMENT–2:
If f(x) = 0 is a polynomial equation and a, b are two real numbers such that f(a) f(b) < 0 has at least one real root.

Key. A

Sol. Let f(x) = (x – a) (x – c) + 2 (x – b) (x – d)
Then f(a) = 2 (a – b) (a – d) > 0
f(b) = (b – a) (b – c) < 0
f(d) = (d – a) (d – b) > 0
Hence a root of f(x) = 0 lies between a & b and another root lies between (b & d).
Hence the roots of the given equation are real and distinct.

28. Assertion (A): If both roots of the equation $4x^2 - 2x + a = 0, a \in R$, lie in the interval (-1, 1), then $-2 < a \le 1/4$.
Reason (R): If $f(x) = 4x^2 - 2x + a$, then $D \ge 0, f(-1) > 0, f(1) > 0 \Rightarrow -2 < a \le 1/4$.

Key. A

Sol. Conceptual

29. Statement - 1: If $ax^2 + bx + c = 0$ and $x^2 + 2x + 3 = 0$ have a common root then other root is also common
Statement - 2: If $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ are having a common root then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Key. C

Sol. Roots of $x^2 + 2x + 3 = 0$ are imaginary then both roots are common

30. Statement - 1: The number of solutions of $\sin e^x = 5^x + 5^{-x}$ are zero
Statement - 2: $x + \frac{1}{x}$ is always greater then or equal to two if x is positive

Key. A

Sol. sin $e^x = 5^x + \frac{1}{5^x}$

$$= \left(\sqrt{5^x} - \frac{1}{\sqrt{5^x}}\right)^2 + 2$$

≥ 2

Which is impossible .

31. Statement - 1: If α, β are roots of $ax^2 + bx + c = 0$ then $\left(\frac{\alpha}{\alpha\beta + b}\right)^3 - \left(\frac{\beta}{a\alpha + b}\right)^3 = 0$

Statement - 2: If α, β are roots of $ax^2+bx+c=0$ then

$$a\alpha^2+b\alpha+c=0 \, and \, a\beta^2+b\beta+c=0$$

Key. D

Sol. $a\alpha^2+b\alpha+c=0$

$$\alpha(a\alpha+b)=-c$$

$$a\alpha+b=\frac{-c}{\alpha}$$

$$a\beta+b=-\frac{c}{\beta}$$

$$\frac{\alpha}{a\beta+b}=\frac{\alpha}{\frac{-c}{\beta}}=\frac{-\alpha\beta}{c}$$

32. Statement - 1: If $a,b,c\in C$, $a\neq 0$ and $ax^2+bx+c=0$ then the roots of above equation are always conjugate complex numbers

Statement - 2: If $a,b,c\in R$ and $a\neq 0$ then roots of $ax^2+bx+c=0$ are always conjugate complex numbers if $b^2-4ac<0$

Key. D

Sol. $a,b,c\in C; a\neq 0$

Roots of $ax^2+bx+c=0$

Need not be conjugate complex numbers.

33. Consider the equation $x^3-3x+k=0,\ k\in R.$

Statement I There is no value of K for which the given equation has two distinct roots in $(0,1)$.

Statement II Between two consecutive roots of $f'(x)=0$, ($f(x)$ is a polynomial). $f(x)=0$ must have one root.

Key. C

Sol. By Rolle's Theorem between the roots of $f'(x)=0$. If there exists a root of $f(x)=0$, $[f(x) is\ polynomial]$, then it must be unique.

34. Statement I All the real roots of the equation $x^4-3x^3-2x^2-3x+1=0$ lie in the interval $[0,3]$

Statement II The equation reduces to quadratic equation in the variable *t*, by substituting $x+\frac{1}{x}=t.$

Key. D

Sol. Dividing by x^2, we get

$$x^2-3x-2-\frac{3}{x}+\frac{1}{x^2}=0$$

$$x^2+\frac{1}{x^2}-3\left(x+\frac{1}{x}\right)-2=0$$

$$\Rightarrow t^2-3t-4=0 \Rightarrow t=-1,4$$

$x+\frac{1}{x}=-1 \Rightarrow x^2+x+1=0$

$\therefore$ $x=\omega,\omega^2$, the complex cube roots of unity

$x+\frac{1}{x}=4 \Rightarrow x=2\pm\sqrt{3}$

$\therefore$ one root is outside [0,3]

35. Let a,b,c,p,q be real numbers, suppose α,β are the roots of equation $x^2+2px+q=0$ and $\alpha,\frac{1}{\beta}$ are roots of equation $ax^2+2bx+c=0$ where $\beta^2 \notin \{-1,0,1\}$

Statement I $\left(p^2-q\right)\left(b^2-ac\right)\geq 0$

Statement II $b\neq pa$ *or* $c\neq qa$

Key. B

Sol. If the roots are imaginary, then $\beta=\bar{\alpha},\frac{1}{\beta}=\bar{\alpha}\Rightarrow \beta^2=1$, contradiction

$\therefore$ The roots are real

$\Rightarrow (p^2-q)$ and $(b^2-ac)\geq 0$

suppose b = pa and c= qa then the second equation becomes identical with the first equation.

$\therefore \beta=\frac{1}{\beta}\Rightarrow \beta^2=1$, contradiction

$\therefore$ Either $b\neq pa$ or $c\neq qa$.

36. Statement-I : The greatest integral value of λ for which $(2\lambda-1)x^2-4x+(2\lambda-1)=0$ has real roots is 2

Statement-II : For real root of $ax^2+bx+c=0, D\geq 0$

Key. D

Sol. For real roots

$\Rightarrow (-4)^2-4(2\lambda-1)(2\lambda-1)\geq 0$

$\Rightarrow (2\lambda-1)^2\leq 4$

$\Rightarrow -2\leq 2\lambda-1\leq 2$

$\Rightarrow -\frac{1}{2}\leq\lambda\leq\frac{3}{2}$

$\therefore$ Integral values of λ are 0 and 1

Hence, the greatest integer value of $\lambda=1$

37. Statement-I : Let $f(x)$ be a quadratic expression such that $f(0)+f(1)=0$. If -2 is one of the roots of $f(x)=0$. Then the Sum of roots is 3/5

Statement-II : If α and β are the zeros of $f(x)=ax^2+bx+c$, then the sum of zeros = $-b/a$ and the product of zeros = c/a

Key. D

Sol. Since $x=-2$ is a root of f(x)

$\therefore$ f(x)=(x+2)(ax+b)

But $f(0)+f(1)=0$

$\therefore 2b+3a+3b=0 \Rightarrow -\frac{b}{a}=\frac{3}{5}$

11. **Statement – 1** : $1 \le x \le 2$, then $\sqrt{x+2\sqrt{x-1}}+\sqrt{x-2\sqrt{x-1}}=2$

Statement – 2 : If $1 \le x \le 2$, then $(x-1) > 1$

Key. C

Sol. Since $1 \le x \le 2$

$\therefore 0 \le x-1 \le 1$

$\sqrt{x+x\sqrt{x-1}}+\sqrt{x-2\sqrt{x-1}}$

$=\sqrt{\left(\sqrt{1}+\sqrt{x-1}\right)^2}+\sqrt{\left(\sqrt{1}-\sqrt{x-1}\right)^2}$

= $1+\sqrt{x-1}+1-\sqrt{x-1}=2$

12. **Statement – 1** : For all $x\in R$, $x^2+3|x|+2=0$ has no real root.

Statement – 2 : For all $x\in R$, $|x| \ge 0$.

Key. A

Sol. Since $x^2 \ge 0$, $|x| \ge 0$

$\therefore x^2+3|x|+2 \ne 0$.

13. **Statement – 1** : The set of all real numbers 'a' such that a^2+6a, a^2+2a+3 and $3a^2+2a+11$ are the sides of a triangle is (2, 4).

Statement – 2 : In a triangle the sum of any two sides is greater than the third side and also the sides are always positive.

Key. A

Sol. In a triangle sum of two sides greater than the other.

$\Rightarrow a^2-6a+a^2+2a+3 > 3a^2+2a+11$

$\Rightarrow a^2+6a+8 < 0$

$\Rightarrow 2 < a < 4$

$\therefore$ (for positive 'a', $3a^2+2a+11$ is the greatest side).

14. **STATEMENT -1:** $(x-1)^3(x+2)^5(x-3)^4 \ge 0$ is true for $[1,\infty)\cup(-\infty,-2]$

STATEMENT-2: Statement 1 is evident from wavy Curve method

Key. A

Sol. Conceptual

Quadratic Equations & Theory of Equations

Comprehension Type

Passage – 1:

Let x_1, x_2, x_3, x_4 be the roots (real or complex) of the equation

$x^4 + ax^3 + bx^2 + cx + d = 0$

If $x_1 + x_2 = x_3 + x_4$ and $a, b, c, d \in R$, then

1. If $a = 2,$ then the value of $b - c$ is

A) -1 B) 1 C) -2 D) 2

Key. B

2. If $b < 0$ then how many different real values of *'a'* we may have?

A) 3 B) 2 C) 1 D) 0

Key. C

3. If $b + c = 1$ and $a \neq -2,$ then for real values of $'a'$ the value of $c \in$

A) $\left(-\infty, \frac{1}{4}\right)$ B) $(-\infty, 3)$ C) $(-\infty, 1)$ D) $(-\infty, 4)$

Key. A

Sol. Let $x^4 + ax^3 + bx^2 + cx + d$

$= (x - x_1)(x - x_2)(x - x_3)(x - x_4)$

Let $(x - x_1)(x - x_2) = x^2 + px + q$

and $(x - x_3)(x - x_4) = x^2 + px + r$

$\therefore\ q = x_1x_2$ and $r = x_3x_4$

$\therefore\ x^4 + ax^3 + bx^2 + cx + d$

$= x^4 + 2px^3 + (p^2 + q + r)x^2 + p(q + r)x + qr$

$\therefore\ a = 2p, b = p^2 + q + r, c = p(q + r), d = qr$

Clearly, $a^3 - 4ab + 8c = 0$

1. (B)If $a = 2 \Rightarrow b - c = 1$

2. (C) Investigating the nature of the cubic equation of $'a'$.

Let $f(a) = a^3 - 4ab + 8c$

$f'(a) = 3a^2 - 4b$

If $b < 0 \Rightarrow f'(a) > 0$

$\therefore$ The equation $a^3 - 4ab + 8c = 0$ hence only one real root.

3. (A) Substituting $c = 1 - b$ in Eq. (i) we have

$(a + 2)\left[(a - 1)^2 + 3 - 4b\right] = 0 \Rightarrow 4b - 3 > 0$

$\Rightarrow \quad b > \frac{3}{4} \quad \Rightarrow \quad c < \frac{1}{4}$

Passage – 2:

If roots of the equation $x^4 - 12x^3 + bx^2 + cx + 81 = 0$ are positive, then

4. Value of b is

A) -54 B) 54 C) 27 D) -27

Key. B

5. Value of c is

A) 108 B) -108 C) 54 D) -54

Key. B

6. Root of equation $2bx + c = 0$ is

A) $-\frac{1}{2}$ B) $\frac{1}{2}$ C) 1 D) -1

Key. C

Sol. Let $\alpha, \beta, \gamma, \delta$ be roots of the given equation

$\alpha + \beta + \gamma + \delta = 12$ ----(1)

$\Sigma\alpha\beta = b$ ----(2)

$\Sigma\alpha\beta\gamma = -c$ ----(3)

$\alpha\beta\gamma\delta = 81$ ----(4)

As $AM \geq GM$

$$\therefore \frac{\alpha + \beta + \gamma + \delta}{4} \geq (\alpha\beta\gamma\delta)^{1/4}$$

$$\frac{12}{4} \geq (81)^{1/4}$$

But as AM = GM

$\therefore \alpha = \beta = \gamma = \delta = 3$

4. (B) $b = \Sigma\alpha\beta = 6 \times 9 = 54$

5. (B) $c = \Sigma\alpha\beta\gamma = 4 \times -27 = -108$

6. (C) $2bx + c = 0$

$108x - 108 = 0$

$\Rightarrow x = 1$

Passage – 3:

Consider the equation $\sin^2 x + a\sin x + b = 0,\ x \in (0,\pi)$

7. The above equation has exactly two roots and both are equal then

(a) $a = 1$ (b) $a = -1$

(c) $b = 1$ (d) $b = -1$

Key. C

8. The above has exactly three distinct solutions then

(a) $b \in (-1,0)$ (b) $b \in (0,1)$

(c) $b \in [-1,0]$ (d) $b \in [0,1]$

Key. B

9. The above equation has four solutons then which of the following are not true

(a) $a \in (-2, 0)$ (b) $b \in (0, 1)$

(c) $a^2 - 4b > 0$ (d) $b \in (-1, 0)$

Key. D

Sol. 7. Ans: c

Sol : If the given equation should have two equal roots, both should be equal to $\frac{\pi}{2}$

$\therefore \sin x = 1$ $\therefore$ product of roots = $\frac{b}{1} = 1$ $\Rightarrow b = 1$

8. Ans : b

Sol : If the given equation should have three solutions, one root should infinitely be $\frac{\pi}{2}$.

$\therefore \sin x_1 = 1$

Now, we should get two more roots and thus $\sin x \in (0, 1)$

$\therefore$ product of roots = b = $\sin x_1 . \sin x = 1 . \sin x = \sin x$

$\therefore b \in (0, 1)$

9. Ans : d

Sol : If the above equation has four roots, $\sin x \in (0, 1)$

$\therefore$ sum = $-a \in (0, 2)$ $= a \in (-2, 0)$

product = $b \in (0, 1)$

discriminant = $a^2 - 4b > 0$

$\therefore b \in (-1, 0)$ is the wrong option.

Passage – 4:

If f : R ~{–1} → R and f is differentiable function that satisfies the equation $f(x + f(y) + xf(y)) = y + f(x) + y f(x)$, $\forall$ x, y $\in$ R – {–1} and $f(x) \neq x$, then

10. f(x) equals,

(A) $\frac{x}{1+x}$ (B) $-\left(\frac{x}{1+x}\right)$ (C) $\frac{-1}{1+x}$ (D) none of these

Key. B

11. $\int_0^1 \left\{ f(x) + f\left(\frac{1}{x}\right) \right\} dx$ equals

(A) 2 (B) – 2

(C) – 1 (D) 1

Key. C

12. The number of solutions of the equation f(x) = c is

(A) one if $c \neq -1$ (B) one if c = –1

(C) more than one if $c \neq -1$ (D) more than one if c = –1

Key. A

SOL. 10 TO 12.

DIFFERENTIATING BOTH SIDE WITH RESPECT TO X AND THEN W.R.T. TO Y AND THEN DIVIDING THE RESULT OBTAINED IN BOTH CASES.

WE GET $f'(x) = \pm\frac{(1+f(x))}{1+x}$

$\Rightarrow \quad \frac{1+f(x)}{c} = (1+x)^{\pm 1}$

NOW, PUTTING X = 0, Y = 0, WE GET

F(C – 1) = (C – 1)

$\Rightarrow$ C = 0, 1

$\therefore \quad f(x) = -\frac{x}{1+x}$

Passage – 5:

P(x) be polynomial of degree at most 5 which leaves remainders -1 and 1 upon division by $(x-1)^3$ and $(x+1)^3$ respectively.

13. Numbers of real roots of P(x) = 0

a) 1 b) 3 c) 5 d) 2

14. The maximum value of $y = P^n(x)$ can be obtained at x =

a) $-\frac{1}{\sqrt{3}}$ b) 0 c) $\frac{1}{\sqrt{3}}$ d) 1

15. The sum of pairwise product of all roots (real and complex) of P(x) = 0 is

a) $-\frac{5}{3}$ b) $-\frac{10}{3}$ c) 2 d) -5

13 – 15. (A,C,B)

P(x) + 1 = 0 has a thrice repeated root at x = 1 P'(x) ahs a twice repeated root at x = 1 similarly, P'(x) has a twice repeated root at $x = -1$.

$\Rightarrow P'(x)$ is divisible by $(x-1)^2(x+1)^2$

$\therefore P'(x) = K(x-1)^2(x+1)^2$ where 'K' is any constant

$\therefore P(x) = K\left(\frac{x^5}{5} - \frac{2}{3}x^3 + x\right) + c$

Now, $P(1) = -1$ and $P(-1) = 1$

$\therefore K = \frac{-15}{8}$ and e = 0

$\therefore P(x) = \frac{-3}{8}x^5 + \frac{5}{4}x^3 - \frac{15}{8}x$

Passage – 6:

Let f (x) = $x^4 + ax^3 + bx^2 + ax + 1$ be a polynomial where a and b are real numbers, then

16. If f (x) = 0 has two different pairs of equal roots, then the value of a + b is

a) 0 b) – 4 c) – 2 d) 4

Key : d

Sol : Let $x^4 + ax^3 + bx^2 + ax + 1 = (x^2 + kx + 1)^2$

comparing $2k = a$; $b = k^2 + 2 \Rightarrow a + b = (k + 1)^2 + 1$ it can be 4

17. If f (x) = 0 has two different negative roots and two equal positive roots, then the least integral value of a is

a) 1 b) 2 c) 3 d) 4

Key : a

Sol : The two equal +ve roots must be 1, 1, and let the –ve roots be $\alpha, \frac{1}{\alpha}(\alpha \neq 1)$

Now $-a = 2 + \alpha + \frac{1}{\alpha} \Rightarrow a = -\alpha - \frac{1}{\alpha} - 2 > 0$

$\therefore$ The least integral value is '1'

18. If all the roots are imaginary and b = -1 then number of all possible integral values of a is

a) 0 b) 1 c) 2 d) 4

Key : b

Sol : Given equation is $t^2 + at - 3 = 0$ where $t = x + \frac{1}{x}$ both roots must lie between -2, 2

$\Rightarrow \frac{-1}{2} < a < \frac{1}{2}$

$\Rightarrow a = 0$

Passage – 7:

$y = ax^2 + bx + c = 0$, $\forall$ a, b, c $\in$ R with $a \neq 0$ is a quadratic equation which has real roots if and only if $b^2 - 4ac \geq 0$. If F(x, y) = 0 is a second degree equation, then using above fact we can get the range of x and y by treating it as quadratic equation in y or x. Similarly $ax^2 + bx + c \geq 0\ \forall x \in R$ if $a > 0$ and $b^2 - 4ac \leq 0$.

19. If $0 < \alpha, \beta < 2\pi$, then the number of ordered pairs (α, β) satisfying

$\sin^2(\alpha + \beta) - 2\sin\alpha \sin(\alpha + \beta) + \sin^2\alpha + \cos^2\beta = 0$ is

(A) 2 (B) 0 (C) 4 (D) 6

Key. C

Sol. Solving it, we get $\sin(\alpha + \beta) = \sin\alpha \pm \sqrt{-\cos^2\beta}$

$\Rightarrow \cos\beta = 0 \Rightarrow \beta = \frac{\pi}{2}, \frac{3\pi}{2}$

(i) If $\beta = \frac{\pi}{2} \Rightarrow \tan\alpha = 1, \ \alpha \in \left\{\frac{\pi}{4}, \frac{5\pi}{4}\right\}$

(ii) If $\beta = \frac{3\pi}{2} \Rightarrow \tan\alpha = -1, \ \alpha \in \left\{\frac{3\pi}{4}, \frac{7\pi}{4}\right\}$

20. Let x, y, z be real variables satisfying the equations x + y + z = 6 and xy + yz + zx = 7, then the range of x is

(A) $\left[\frac{6-\sqrt{5}}{3}, \frac{6+\sqrt{15}}{3}\right]$ (B) $\left[\frac{6-2\sqrt{15}}{3}, \frac{6+2\sqrt{15}}{3}\right]$

(C) $\left[\frac{6-\sqrt{15}}{2}, \frac{6+\sqrt{15}}{2}\right]$ (D) $\left[\frac{6-\sqrt{15}}{7}, \frac{6+2\sqrt{15}}{7}\right]$

Key. B

Sol. We have x + y + z = 6 …..(i)

$xy + yz + zx = 7$ …..(ii)

From (i), $z = 6 - x - y$ & putting it in (ii),
we get $xy + y(6 - x - y) + x(6 - x - y) = 7$
or $y^2 + (x - 6)y + (x^2 - 6x + 7) = 0$
Since y is real, $(x - 6)^2 - 4(x^2 - 6x + 7) \geq 0$
$\Rightarrow 3x^2 - 12x - 8 \leq 0$

$$\Rightarrow \frac{6-2\sqrt{15}}{3} \leq x \leq \frac{6+2\sqrt{15}}{3}$$

21. If $9^{x+1} + (a^2 - 4a - 2)3^x + 1 > 0 \ \forall \ x \in R$, then
(A) $a \in R$ (B) $a \in R^+$ (C) $a \in [1, \infty)$ (D) $a \in R - \{2\}$

Key. D

Sol. $$3^x\left(9.3^x + \frac{1}{3^x} + (a^2 - 4a - 2)\right) > 0$$

$$\Rightarrow 3^x\left(\left(3.3^{x/2} - \frac{1}{3^{x/2}}\right)^2 + (a-2)^2\right) > 0$$

$a \in R - \{2\}$

Passage – 8:

Consider the inequation $9^x - a.3^x - a + 3 \leq 0$ where 'a' is a real parameter. The given inequation has

22. At least one negative solution if
a) $a \in (2,3)$ b) $a \in (2,\infty)$ c) $a \in (-\infty,2)$ d) $a \in (-\infty,3)$

Key. A

23. At least one positive solutions if
a) $a \in (-\infty,2)$ b) $a \in (0,2)$ c) $a \in (-\infty,-6)$ d) $a \in (2,\infty)$

Key. D

24. At least one solution in $(1,2)$ if
a) $a \in (3,\infty)$ b) $a \in \left(3, \frac{84}{10}\right)$ c) $a \in \left(\frac{84}{10}, \infty\right)$ d) $a \in R$

Key. B

Sol. 22,23,24. Let $3^x = t \Rightarrow t^2 - ta - a + 3 \leq 0 : t > 0$
Let $f(t) = t^2 - at + 3 - a$

Discriminate of $f(t) = 0$ is $a^2 - 4(3-a)a$

$i.e., a^2 + 4a - 12.$

$D \geq 0 \Rightarrow a \leq -6$ or $a \geq 2.$

22. $f(t) \leq 0$. Has at least one positive solution.

If $x < 0$ then at least one t of $f(t) = 0$ lies in .

Case I: exactly one $t \in (0,1)$ then $D \geq 0$ and $f(0)f(1) < 0$ then $a \in (2,3)$

Case II: both rots lines in $(0,1)$ then $(1)D\geq 0\ (2)\ f(0)>0\ (3)\ f(1)>0$

$(4)0<\frac{a}{2}<1$

Then $a\in\phi$

$\therefore a\in(2,3)$

23. $f(t)\leq 0.$ has at least one positive solution

i.e., $x>0\Rightarrow t>1$

Case I: exactly one root is greater than 1

$(1)D>0(2)f(1)<0$ then $a>2$

Case II: both roots greater than 1

$(1)D\geq 0(2)\ f(1)>0(3)\frac{a}{2}>1$

Then $a\in\phi$

$\therefore\ a\in(2,\infty)$

24. Similarly $x\in(1,2)$ then $t\in(3,9)$

Similar to the above the question

$f(q)=(a^2+b^2+c^2)\cos^2 q$

$f^1(q)=(a^2+b^2+c^2)^3\times(-\sin 2q)$

Passage – 9:

If $abc=m$ and $\det P=\begin{vmatrix}a & b & c\\ c & a & b\\ b & c & a\end{vmatrix}$, where P is an orthogonal matrix.

25. The value of $a+b+c$ is

a) $\pm m$ b) ± 1 c) 0 d) m^2

Key. B

26. The cubic equation whose roots are a^{-1},b^{-1} and c^{-1} can be

a) $mt^3+t-1=0$ b) $mt^3-t+1=0$ c) $mt^3+m^2t^2+t+1=0$ d) $mt^3+m^2t^2+m+1=0$

Key. B

27. The value of $a^{-10}b^{-12}c^{-12}+a^{-12}b^{-10}c^{-12}+a^{-12}b^{-12}c^{-10}$ can be

a) 1 b) m^{-12} c) m^{-10} d) m^{10}

Key. B

Sol. 25. $PP^T=\begin{bmatrix}a & b & c\\ c & a & b\\ b & c & a\end{bmatrix}\begin{bmatrix}a & c & b\\ b & a & c\\ c & b & a\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

$$\Rightarrow\begin{bmatrix}a^2+b^2+c^2 & ab+bc+ca & ab+bc+ca\\ ab+bc+ca & a^2+b^2+c^2 & ab+bc+ca\\ ab+bc+ca & ab+bc+ca & a^2+b^2+c^2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

26. We know the cubic equation whose roots are α, β, γ

$t^3-(\alpha+\beta+\gamma)t^2+(\alpha\beta+\beta\gamma+\alpha\gamma)+\alpha\beta\gamma=0,$ where t is origin. The equation whose roots are a^{-1}, b^{-1}, c^{-1} is

$$t^3-\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)t^2+\left(\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}\right)t+\frac{1}{abc}=0$$

$$\Rightarrow t^3-(0)t^2+\left(\frac{\pm 1}{n}\right)t+\frac{1}{m}=0 \Rightarrow mt^3 \pm t+1=0$$

27. $$\frac{a^2+b^2+c^2}{a^{12}b^{12}c^{12}}=\frac{a^2+b^2+c^2}{(abc)^{12}}=\frac{1}{m^{12}}=m^{-12}.$$

Passage – 10

Let $\left(a+\sqrt{b}\right)^{Q(x)}+\left(a-\sqrt{b}\right)^{Q(x)-2\lambda}=A$ Where $\lambda \in N, A \in R$ and $a^2-b=1$

$\therefore \left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right)=1 \Rightarrow \left(a+\sqrt{b}\right)=\left(a-\sqrt{b}\right)^{-1}$ and $\left(a-\sqrt{b}\right)=\left(a+\sqrt{b}\right)^{-1}$

i.e., $\left(a\pm\sqrt{b}\right)=\left(a+\sqrt{b}\right)^{\pm 1}$ or $\left(a-\sqrt{b}\right)^{\pm 1}$

28. If α, β are the roots of the equation $1!+2!+3!+\ldots\ldots\ldots\ldots+(x-1)!+x!=k^2$ and k Î I, Where $\alpha<\beta$ and If $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are the roots of the equation

$\left(a+\sqrt{b}\right)^{x^2-\left[1+2\alpha+3\alpha^2+4\alpha^3+5\alpha^4\right]}+\left(a-\sqrt{b}\right)^{x^2+[-5\beta]}=2a$.

Where $a^2-b=1$ and $[.]$ denotes G.I.F, then the value of $\left|\alpha_1+\alpha_2+\alpha_3+\alpha_4-\alpha_1\alpha_2\alpha_3\alpha_4\right|$ is

a) 216 b) 221 c) 224 d) 209

Key. C

29. If $\left(\sqrt{\left(49+20\sqrt{6}\right)}\right)^{\sqrt{a\sqrt{a\sqrt{a\ldots\ldots\infty}}}}+\left(5-2\sqrt{6}\right)^{x^2+x-3-\sqrt{x\sqrt{x\sqrt{x\ldots\ldots\ldots\infty}}}}=10$ where $a=x^2-3,$ then x is

a) $-\sqrt{2}$ b) $\sqrt{2}$ c) -2 d) 2

Key. D

30. If α, β are the roots of the equation x^2-4x+1=0, where $\alpha>\beta,$ then the number of real solutions of the equation $\alpha^{y^2-2y+1}+\beta^{y^2-2y-1}=\frac{101}{10\beta}$ are

a) 0 b) 2 c) 4 d) 6

Key. B

Sol. 28. For $x\geq 4,$ the last digit of $1!+2!+\ldots.+x!$ is 3

For $x<4,$ the given equation has only solutions

$x=1, K=\pm 1$ and $x=3, K=\pm 3$

$\alpha=1, \beta=3$

$\left(a+\sqrt{b}\right)^{x^2-15}+\left(a-\sqrt{b}\right)^{x^2-15}=2a$

$\therefore x^2-15=\pm 1 \Rightarrow x=\pm 4, \pm\sqrt{14}$

$\alpha_1 = -4, \alpha_2 = 4, \alpha_3 = -\sqrt{14}, \alpha_4 = \sqrt{14}.$

$\therefore |\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 - \alpha_1\alpha_2\alpha_3\alpha_4| = |0 - 16 \times 14| = 224$

29. $a^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+.......} = a$

$\sqrt{x\sqrt{x\sqrt{x......\infty}}} = x$ and $\sqrt{49 + 20\sqrt{6}} = 5 + 2\sqrt{6}$

$x^2 - 3 > 0$ and $x > 0$

$\Rightarrow x > \sqrt{3}$

$\left(5+2\sqrt{6}\right)^{\sqrt{a\sqrt{a\sqrt{a......\infty}}}} + \left(5-2\sqrt{6}\right)^{x^2+x-3-\sqrt{x\sqrt{x\sqrt{x...\infty}}}} = 10$

$\Rightarrow \left(5+2\sqrt{6}\right)^{x^2-3} + \left(5-2\sqrt{6}\right)^{x^2-3} = 10$

$\therefore x^2 - 3 = 1 \Rightarrow x = 2 \left(\because x > \sqrt{3}\right)$

30. $x = 2 \pm \sqrt{3}$

$\left(2+\sqrt{3}\right)^{y^2-2y+1} + \left(2-\sqrt{3}\right)^{y^2-2y+1} = \dfrac{101}{10\left(2-\sqrt{3}\right)}$

$\Rightarrow \left(2+\sqrt{3}\right)^{y^2-2y} + \left(2-\sqrt{3}\right)^{y^2-2y} = \dfrac{101}{10}$

Passage – 11

Let consider quadratic equation $ax^2 + bx + c = 0$ .....(i)

Where $a, b, c \in R$ and $a \neq 0$. If Eq. (i) has roots, α, β

$\alpha + \beta = -\dfrac{b}{a}, \alpha\beta = \dfrac{c}{a}$ and Eq. (i) can be written as $ax^2 + bx + c = a(x-\alpha)(x-\beta)$

Also, if $a_1, a_2, a_3, a_4, ...$ are in AP, then $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = ... \neq 0$ and if $b_1, b_2, b_3, b_4, ...$ are in

GP, then $\dfrac{b_2}{b_1} = \dfrac{b_3}{b_2} = \dfrac{b_4}{b_3} = ... \neq 1$ Now, if $c_1, c_2, c_3, c_4, ...$ are in HP, then

$\dfrac{1}{c_2} - \dfrac{1}{c_1} = \dfrac{1}{c_3} - \dfrac{1}{c_2} = \dfrac{1}{c_4} - \dfrac{1}{c_3} = ... \neq 0$

31. Let p and q be roots of the equation $x^2 - 2x + A = 0$ and let r and s be the roots of the equation $x^2 - 18x + B = 0$. If p < q < r < s are in arithmetic progression. Then the values of A and B respectively are.

(A) $-5, 67$ (B) $-3, 77$

(C) $67, -5$ (D) $77, -3$

Key. B

32. Let α_1, α_2 be the roots of $x^2 - x + p = 0$ and α_3, α_4 be the roots of $x^2 - 4x + q = 0$. If $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are in GP, then the integral values, of p and q respectively are

(A) $-2, -32$ (B) $-2, 3$

(C) $-6,3$ (D) $-6,-32$

Key. A

33. Given that β_1,β_3 be roots of the equation $Ax^2-4x+1=0$ and β_2,β_4 the roots of the equation $Bx^2-6x+1=0$.

If $\beta_1,\beta_2,\beta_3,\beta_4$ are in HP; then the integral value of A and B respectively are

(A) $-3,8$ (B) $-3,16$

(C) $3,8$ (D) $3,16$

Key. C

Sol. 31. p+q=2, pq=A and
r+s = 18, rs = B
p,q,r,s are in AP.
Then q= p+D , r=p+2D and s= p+3D

32. $\alpha_1+\alpha_2=1,\alpha_1\alpha_2=p$ (i) and $\alpha_3+\alpha_4=4,\alpha_3\alpha_4=q$ (ii)

$\alpha_1,\alpha_2,\alpha_3,\alpha_4$ are in GP

$\alpha_2=\alpha_1R,\alpha_3=\alpha_1R^2,\alpha_4=\alpha_1R^3$

33. $\beta_1+\beta_3=\dfrac{4}{A},\beta_1\beta_3=\dfrac{1}{A}$ (i)

$\beta_2+\beta_4=\dfrac{6}{B},\beta_2\beta_4=\dfrac{1}{B}$ (ii)

From eq. (i) , $\dfrac{\beta_1\beta_3}{(\beta_1+\beta_3)}=\dfrac{1}{4}$ (iii)

And eq. (ii) , $\dfrac{\beta_2\beta_4}{(\beta_2+\beta_4)}=\dfrac{1}{6}$

Passage – 12

If x_1,x_2 be the roots of the equation $x^2-3x+A=0$ and x_3,x_4 be the roots of $x^2-12x+B=0$ and x_1,x_2,x_3,x_4 be an increasing G.P, also $x^2-8x+C=0$ where the product of the roots is half the sum of the roots, on the basis of above information answer the following

33. The equation of the plane through intersection of the planes $x-Ay+3z+C=0$ and $Ax-3y+Cz-7=0$ and the point $(1,-1,1)$ is

(a) $9x-13y+17z=B+7$ (b) $Ax-By+Cz=7$

(c) $2Ax-13y+17z=B$ (d) $9x-13y+17z=A+B+C-1$

Key. A

34. Equation of the plane perpendicular to $x+2y+Cz+B=0,Ax+2y-3z+2009=0$ and passing through (A,B,C) is

(a) $14x+11y+2z-316=0$ (b) $14x-11y+2z+316=0$

(c) $14x+11y-2z+316=0$ (d) $14x-11y+2z-316=0$

Key. B

35. The image of the plane $\pi_1=Ax-3y+Cz+9=0$ in the plane mirror $\pi_2=Cx-Ay+z-5=0$ is

(a) $34x-3y-16z-123=0$ (b) $3x-34y+16z+123=0$

(c) $2x-3y+4z+17=0$ (d) $4x-11y+17z-39=0$

Key. A

Sol. 33. Clearly A=2, B=32, C=4

$\pi_1+\lambda\pi_2=0$

34. dr's of normal to the required plane is (14, -11, 2)

Equation of reqd plane is $14(x-2)-11(y-32)+2(z-4)=0$

$14x-11y+2z-316=0$

Passage – 13

$$\text{Max}\{f(x),g(x)\}=\frac{f(x)+g(x)}{2}+\left|\frac{f(x)-g(x)}{2}\right|$$

$$\text{Min}\{f(x),g(x)\}=\frac{f(x)+g(x)}{2}-\left|\frac{f(x)-g(x)}{2}\right|$$

Let $f(x)=f_1(x)-2f_2(x)$. Where $f_1(x)=\min\{x^2,|x|\}$, for $-1\le x\le 1$

$=\max\{x^2,|x|\}$, for $|x|>1$

$f_2(x)=\max\{x^2,|x|\}$, for $-1\le x\le 1$

$=\min\{x^2,|x|\}$, for $|x|>1$

And $g(x)=\begin{cases}\min\{f(t):-3\le t\le x,\ -3\le x<0\}\\ \max\{f(t):0\le t\le x,\ 0\le x\le 3\}\end{cases}$

35. For $-3\le x\le -1$, range of g(x) is

a) $[-1,3]$ b) $[-1,-15]$ c) $[-1,10]$ d) None of these

Key. A

36. For $x \in (-1,0)$, $f(x)-g(x)$ is

a) x^2-2x+1 b) x^2+2x-1

c) x^2+2x+1 d) x^2-2x-1

Key. C

37. The range of a for which the equation $f(x)=|x|+a$ has 4 solutions is

a) $\left[-\frac{9}{4},1\right)$ b) $\left(-\frac{9}{4},0\right)$

c) $\left(-\frac{9}{4},2\right)$ d) $\left[-\frac{9}{4},1\right]$

Key. B

Sol. 35. $f_1(x)=x^2$

37. $f_2(x)=|x|$

Draw graph.

Passage – 14

The general form of quadratic equation is given by $ax^2+bx+c=0$ where $a \neq 0$ and $a,b,c \in C$

38. The number of real solutions of the equation $x^2-7|x|+12=0$ is

(A)-7 (B) 8 (C) 5 (D) 4

Key. D

39. If sum of roots of $ax^2+bx+c=0$ is same as that of their squares then

(A) $b^2+ab=2ac$ (B) $b^2+ac=2ab$

(C) $c^2+ab=2bc$ (D) $a^2+b^2+c^2=ab+bc+ca$

Key. A

40. If $(1-P)$ is a root of $x^2+Px+(1-P)$=0 then roots are

(A) 0,1 (B) – 1, 1 (C) 0, -1 (D) -1,2

Key. C

Sol. 38. $(|x|-3)(|x|-4)=0$

39. $\alpha+\beta=\alpha^2+\beta^2$

$\alpha+\beta=(\alpha+\beta)^2-2\alpha\beta$

40. Let other root is α

$(1-P)\alpha=(1-P)$

$\alpha \neq 1$

$\therefore P=1$

$x^2-x=0; x=0,-1$

Passage – 15

If α, β are roots of $ax^2+bx+c=0$ then $ax^2+bx+c=a(x-\alpha)(x-\beta)$

41. If the difference of the roots of the equation $x^2-bx+c=0$ is equal to the difference of the roots of the equation $x^2-cx+b=0$ and $b \neq c$, then b+c =

(A) 0 (B) 2 (C) 4 (D) -4

Key. D

42. If each root of the equation $3x^2-7x+4=0$ is increased by 2, then the resulting equation is

(A) $3x^2-19x+30=0$ (B) $3x^2+5x+2=0$ (C) $3x^2-19x+2=0$ (D) $3x^2-19x+20=0$

Key. A

43. If $\sin\theta$, $\cos\theta$ are the roots of the equation $ax^2+bx+c=0$ then

(A) $a^2-b^2+2ac=0$ (B) $a^2+b^2+2ac=0$ (C) $a-b+2ac=0$ (D) $a+b+2c=0$

Key. A

Sol. 41.

$$\frac{\Delta}{\Delta_1}=\frac{a^2}{p^2}=>\frac{b^2-4ax}{c^2-4b}=1$$

$$b^2-c^2=4c-4b; b+c=-4$$

42.

$$3(x-2)^2-7(x-2)+4=0$$

$$3x^2-12x+12-7x+14+4=0$$

$$3x^2-19x+3=0$$

43. $sin\theta+\cos\theta=-b/a; \sin\theta\cos\theta=c/a$

Passage – 16

If the quadratic equation $ax^2+bx+c=0$ is satisfied by more than two values of x then it must be an identity for which $a=b=c=0$.

44. If p,q are the roots of $x^2+px+q=0$ then

(A) p = 1 (B) p = 1 or zero (C) p = -2 (D) p = -2 or zero

Key. B

45. The solution of $\left|3+\frac{1}{x}\right|=2$ is

(A) $0,-1.-\frac{1}{5}$ (B) 2, -1 (C) 0,-1 (D) $-1.-\frac{1}{5}$

Key. D

46. If α, β are roots of $x^2-(1+n^2)x+\frac{1+n^2+n^4}{2}=0$ then $\alpha^2+\beta^2$ is

(A) n^2+2 (B) $-n^2$ (C) n^2 (D) $2n^2$

Key. C

Sol. 44. $p+q=-p; pq=q$

$q=0\ \ ;\ \ p=1$

$q=0\ \ \Rightarrow\ p=0$

$p=1\ \ \Rightarrow q=-2$

45. $3+\frac{1}{x}=2;\ \frac{1}{x}=-1;\ x=-1$

$3+\frac{1}{x}=-2;\ \frac{1}{x}=-5;\ x=-\frac{1}{5}$

46. $\alpha+\beta=1+n^2$

$\alpha\beta=\frac{1+n^2+n^4}{2}$

$\alpha^2+\beta^2=\left(1+n^2\right)^2-\left(\frac{1+n^2+n^4}{2}\right)$

$=n^2$

Passage – 17

Let x_1,x_2,x_3,x_4 be the roots (real or complex) of the equation

$x^4+ax^3+bx^2+cx+d=0$

If $x_1+x_2=x_3+x_4$ and $a,b,c,d\in R$, then

47. If $a=2,$ then the value of $b-c$ is

A) -1 B) 1 C) -2 D) 2

Key. B

48. If *b < 0* then how many different real values of *'a'* we may have?

A) 3 B) 2 C) 1 D) 0

Key. C

49. If $b+c=1$ and $a\neq-2,$ then for real values of $'a'$ the value of $c\in$

A) $\left(-\infty,\frac{1}{4}\right)$ B) $(-\infty,3)$ C) $(-\infty,1)$ D) $(-\infty,4)$

Key. A

Sol. Let $x^4+ax^3+bx^2+cx+d$

$=(x-x_1)(x-x_2)(x-x_3)(x-x_4)$

Let $(x-x_1)(x-x_2)=x^2+px+q$

and $(x-x_3)(x-x_4)=x^2+px+r$

$\therefore\ q=x_1x_2$ and $r=x_3x_4$

$\therefore\ x^4+ax^3+bx^2+cx+d$

$=x^4+2px^3+(p^2+q+r)x^2+p(q+r)x+qr$

$\therefore\ a=2p, b=p^2+q+r, c=p(q+r), d=qr$

Clearly, $a^3 - 4ab + 8c = 0$

47. (B)If $a = 2 \quad \Rightarrow b - c = 1$

48. (C) Investigating the nature of the cubic equation of $'a'$.

Let $f(a) = a^3 - 4ab + 8c$

$f'(a) = 3a^2 - 4b$

If $b < 0 \Rightarrow f'(a) > 0$

$\therefore$ The equation $a^3 - 4ab + 8c = 0$ hence only one real root.

49. (A) Substituting $c = 1 - b$ in Eq. (i) we have

$(a+2)\left[(a-1)^2 + 3 - 4b\right] = 0 \Rightarrow 4b - 3 > 0$

$\Rightarrow \quad b > \frac{3}{4} \quad \Rightarrow \quad c < \frac{1}{4}$

Passage – 18

If roots of the equation $x^4 - 12x^3 + bx^2 + cx + 81 = 0$ are positive, then

50. Value of b is

A) -54 B) 54 C) 27 D) -27

Key. B

51. Value of c is

A) 108 B) -108 C) 54 D) -54

Key. B

52. Root of equation $2bx + c = 0$ is

A) $-\frac{1}{2}$ B) $\frac{1}{2}$ C) 1 D) -1

Key. C

Sol. Let $\alpha, \beta, \gamma, \delta$ be roots of the given equation

$\alpha + \beta + \gamma + \delta = 12$ ----(1)

$\Sigma\alpha\beta = b$ ----(2)

$\Sigma\alpha\beta\gamma = -c$ ----(3)

$\alpha\beta\gamma\delta = 81$ ----(4)

As $AM \geq GM$

$\therefore \frac{\alpha + \beta + \gamma + \delta}{4} \geq (\alpha\beta\gamma\delta)^{1/4}$

$\frac{12}{4} \geq (81)^{1/4}$

But as AM = GM

$\therefore \alpha = \beta = \gamma = \delta = 3$

50. (B) $b = \Sigma\alpha\beta = 6 \times 9 = 54$

51. (B) $c = \Sigma\alpha\beta\gamma = 4 \times -27 = -108$

52. (C) $2bx + c = 0$

$108x - 108 = 0$

$\Rightarrow x = 1$

Passage – 19

Let consider the quadratic equation $(1+m)x^2 - 2(1+3m)x + (1+8m) = 0$, where $m \in R \setminus \{-1\}$

On the basis of above information answer the following:

53. The number of integral values of m such that given quadratic equation has imaginary roots are

A) 0 B) 1 C) 2 D) 3

Key. C

54. The set of values of m such that the given quadratic equation has at least one root is negative is

A) $m \in (-\infty, -1)$ B) $m \in \left(-\frac{1}{8}, \infty\right)$ C) $m \in \left(-1, -\frac{1}{8}\right)$ D) $m \in R$

Key. C

55. The set of values of m such that the given quadratic equation has both roots are positive is

A) $m \in R$ B) $m \in (-1, 3)$ C) $m \in [3, \infty)$ D) $(-\infty, -1) \cup [3, \infty)$

Key. D

Sol. **Q.Nos (53 – 55)**

If α, β are the roots and D be the discriminant of the given quadratic equation, then

$\alpha + \beta = \dfrac{2(1+3m)}{(1+m)}, \alpha\beta = \dfrac{(1+8m)}{(1+m)}$ --- (1)

and $D = 4(1+3m)^2 - 4(1+m)(1+8m) = 4(m^2 - 3m) = 4m(m-3)$

If roots are real, then $D \geq 0$

$\therefore m \in (-\infty, 0] \cup [3, \infty)$ --- (2)

If roots are real, then $D \geq 0$

53. $D < 0$

$\Rightarrow 4m(m-3) < 0 \Rightarrow 0 < m < 3$

$\therefore m = 1, 2$

54. At least one root is negative ie, one root is negative or both roots are negative, then

$\{(\alpha\beta < 0) \cup (\alpha + \beta < 0)\} \cap (D \geq 0)$

$\Rightarrow \left\{\left(\dfrac{(1+8m)}{(1+m)} < 0\right) \cup \left(\dfrac{2(1+3m)}{(1+m)} < 0\right)\right\} \cap m \in (-\infty, 0] \cup [3, \infty)$

$\Rightarrow \left\{m \in \left(-1, -\frac{1}{8}\right)\right\} \cap \{m \in (-\infty, 0] \cup [3, \infty)\}$

ie. $m \in \left(-1, -\frac{1}{8}\right)$

55. $\alpha+\beta>0$ and $\alpha\beta>0$

$\Rightarrow(\alpha+\beta>0)\cap(\alpha\beta>0)\cap(D\le 0)$

$\Rightarrow\left(\frac{2(1+3m)}{1+m}>0\right)\cap\left(\frac{1+8m}{1+m}>0\right)\cap\{4m(m-3)\ge 0\}$

$\therefore m\in\left\{(-\infty,-1)\cup\left(-\frac{1}{3},\infty\right)\right\}\cap\left\{(-\infty,-1)\cup\left(-\frac{1}{8},\infty\right)\right\}\cap\{m\in(-\infty,0]\cup[3,\infty)\}$

$\Rightarrow m\in(-\infty,-1)\cup[3,\infty)$

Passage – 20

Consider the quadratic equation $ax^2-bx+c=0$, $a,b,c\in N$. If the given equation has two real & distinct roots α and β belonging to the interval $(1,2)$ then

56. The value of $(\alpha-1)(\beta-1)(2-\alpha)(2-\beta)\in$

A) R B) $\left(0,\frac{1}{16}\right)$ C) $\left[0,\frac{1}{16}\right]$ D) $(-\infty,0)$

Key. C

57. The minimum value of $(a-b+c)(4a-2b+c)$ is

A) 1 B) 2 C) 4 D) 5

Key. A

58. The minimum value of 'a' is

A) 2 B) 3 C) 4 D) 5

Key. D

Sol. Given $0<\alpha<2;\ 0<\beta<2$

$\Rightarrow 0<\alpha-1<1\ \&\ 0<(\beta-1)<1$

similarly $-2<-\alpha<-1\ \&0<2-\alpha<1$

$0<2-\beta<1$

Apply $AM\ge GM$ for $\alpha-1\ \&\ 2-\alpha$

$\frac{\alpha-1+2-\alpha}{2}\ge\sqrt{(ga-1)(2-\alpha)}$

$\Rightarrow(\alpha-1)(2-\alpha)\le\frac{1}{4}$ similarly $(\beta-1)(2-\beta)\le\frac{1}{4}$

$\therefore 0\le(\alpha-1)(\beta-1)(2-\alpha)(2-\beta)\le\frac{1}{16}$

$\Rightarrow(\alpha-1)(\beta-1)(2-\alpha)(2-\beta)\in\left[0,\frac{1}{16}\right]$

Paragraph for Questions Nos. 18 to 20

For x∈R, f(x) is defined as

f(x) = $\begin{cases} x+2, & 0\le x<2 \\ x-4, & x\ge 2\end{cases}$ For x∈R, $|x|=\begin{cases} x, & x\ge 0 \\ -x, & x<0\end{cases}$

18. For $0\le x\le 1$, the solution set of |x| f(x) > 2 is

(A) ϕ (B) (0, 1)

(C) $\left[\frac{1}{2}, 2\right]$ (D) none of these

Key. A

19. The number of real solutions of |x| + |x – 1| = 5 is

(A) 2 (B) 3

(C) 1 (D) none of these

Key. A

20. For x ≥ 3, the solution set of (f(x) + |x – 2|) f(x) ≤ 0 lies in

(A) (4, ∞) (B) (–∞, 3)

(C) [3, 4] (D) none of these

Key. C

Sol. 18. For $0 \le x \le 1$

$|x|\, f(x) > 2$

$\Rightarrow\ x(x+2) > 2$

$\Rightarrow x^2 + x - 2 > 0$

$\Rightarrow\ (x+2)(x-1) > 0$

$x \in (-\infty, -2) \cup (1, \infty)$

$\because$ there is no solution

19. $|x| + |x-1| = 5$

Case I : $x < 0,\ -x+1-x = 5$

$\Rightarrow -2x = 4$

$\Rightarrow\ \ x = -2$

Case II : $0 \le x < 1,\ \ x+1-x = 5$ (not possible)

Case III : $x \ge 1,\ x+x-1 = 5$

$\Rightarrow\ 2x = 6$

$\Rightarrow x = 3$

$\therefore$ there are two real solutions

20. (f(x) + |x – 2|) f(x) ≤ 0

⇒ (x – 4 + x – 2) (x – 4) ≤ 0

⇒ 2(x – 3) (x – 4) ≤ 0

⇒ 3 ≤ x ≤ 4.

Paragraph for Questions Nos. 21 to 23

In a ΔABC, with vertex A (a, –5), x–coordinates of two points B and C are the roots of $x^2 - bx + 3 = 0$ and their y–coordinates are the roots of the equation $x^2 - x - 6 = 0$. x–coordinate of B is less than the y–coordinate of C and y–coordinate of B is greater than y–coordinate of C, where a is the least positive integer of the inequality $x^2 - 2x - 3 \ge 0$ and b is the greatest negative integer of the inequality |x – 2| ≥ 6. The slope of any line joining two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is = $\frac{y_2 - y_1}{x_2 - x_1}$.

21. The value of ab (a + b) is
(A) 12 (B) – 12
(C) 6 (D) none of these

Key. A

22. The slope of BC is
(A) $-\frac{5}{2}$ (B) $\frac{2}{5}$
(C) $-\frac{3}{4}$ (D) $\frac{4}{3}$

Key. A

23. The slope of CA is
(A) $\frac{3}{4}$ (B) $-\frac{3}{4}$
(C) $\frac{1}{2}$ (D) none of these

Key. B

Sol. 21. $x^2 - 2x - 3 \ge 0$
$\Rightarrow (x-3)(x+1) \ge 0$
$\Rightarrow x \le -1,\ x \ge 3$
$x \in (-\infty, -1] \cup [3, \infty)$
$\therefore a = 3$
$|x-2| \ge 6 \Rightarrow x-2 \ge 6$ and $x-2 \le -6$
$x \ge 8$ and $x \le -4$
$x \in (-\infty, -4] \cup [8, \infty)$
$\therefore\ b = -4$
$x^2 - bx + 3 = 0$
$\Rightarrow x^2 + 4x + 3 = 0$
$\Rightarrow x = -3, -1$
$\Rightarrow\ x^2 - x - 6 = 0$
$x = 3, -2$
$A(3,-5), B(-3,3),\ C(-1,-2)$.
$ab(a+b) = -12(-4+3) = 12$
22. Slope of BC = $\frac{-2-3}{-1+3} = \frac{-5}{2}$
23. Slope of CA = $\frac{-2+5}{-1-3} = \frac{-3}{4}$

Paragraph for Questions Nos. 17 to 19

If f(x)=|x – 1|+|x – 3|+|5 – x| $\forall\ x \in R$

17) The set of all values of x for which f increases is
a) $(1, \infty)$ b) $(3, \infty)$ c) $(5, \infty)$ d) $(1, 3)$

Key. B

18) The set of all values of x for which f decreases is

a) $(-\infty,1)$ b) $(-\infty,3)$ c) $(-\infty,5)$ d) $(3,5)$

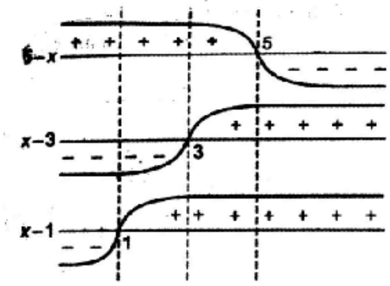
Key. B

19) f(x) is symmetrical about the line $x=\lambda$, then

a) $\lambda = 1$ b) $\lambda = 3$ c) $\lambda = 5$ d) $\lambda = 0$

Key. B

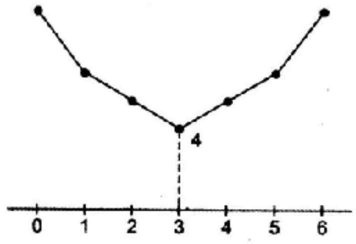
Sol. 17 to 19



$$f(x)=\begin{cases} 9-3x, & x<1 \\ 7-x, & 1\le x<3 \\ x+1, & 3\le x<5 \\ 3x-9, & x\ge 5 \end{cases}$$

$$f'(x)=\begin{cases} -3, & x<1 \\ -1, & 1\le x<3 \\ 1, & 3\le x<5 \\ 3, & x\ge 5 \end{cases}$$

$f'(x)>0,\ x\in(3,\infty)$.
$f'(x)<0,\ x\in(-\infty,3)$.



It is clear from the figure $f(x)$ is symmetrical about the line $x=3$

$\therefore \lambda = 3$

P - 2

(A) The roots of the quadratic equation $ax^2+bx+c=0$ are real, equal and imaginary according as $\Delta=b^2-4ac$ is $>0, =0, <0$

If Δ is positive and perfect square of a rational number then roots are rational.

If Δ is positive but not perfect square of a rational number then roots are irrational.

(B) If a < b, then (x-a) (x-b) is positive if x<a or x> b i.e. , x does not lie between a and b.

It is –ive if a < x < b, i.e., lies between a and b.

Answer the following questions based upon above passage

36. If the equation $ax^2+bx+c=0(a>0)$ has two roots α and β such that $\alpha<-2$ and $\beta>2$ then

(A) $b^2-4ac<0$ (B) $4a-2b-c<0$

(C) $a+|b|+c<0$ (D) c > 0

Key. C

37. The range of values of m for which the equation $(m-5)x^2+2(m-10)x+m+10=0$ has real roots of the same sign, is given by

(A) m > 10 (B) –5 < m < 5

(C) m < –10, 5 < m ≤ 6 (D) –10 < m < 10

Key. C

38. If α and β ($\alpha<\beta$), are the roots of the equation $x^2+bx+c=0$ where $c<0<b$, then

(A) $0<\alpha<\beta$ (B) $\alpha<0<\beta<|\alpha|$

(C) $\alpha<\beta<0$ (D) $\alpha<0<|\alpha|<\beta$

Key. B

Sol. 36. Ans. (c)

f(x)= a (x–α) (x–β) where

$f(x)=0$ at both α and β which are real

$\therefore\ b^2-4ac>0$

for any number lying between α and β say $\pm 2, \pm 1$ and 0 we know that $f(x)$ will be –ive $(a>0)$

$\therefore\ f(\pm 2), f(\pm 1)$ and $f(0)$ all are – ive

$\therefore\ 4a \pm 2b + c < 0$

or $4a + 2|b| + c < 0$

$a \pm b + c < 0$

$\therefore\ a + |b| + c < 0$

$f(0) < 0 \Rightarrow c < 0$

37. Ans (c)

$\Delta \geq 0 \Rightarrow -25m + 150 \geq 0\ \therefore\ m \leq 6$

$P = \frac{m+10}{m-5} = +ive,$ as roots are of same sign.

or $\frac{(m+10)(m-5)}{(m-5)^2} > 0\ \therefore\ m < -10$ or m > 5

$\therefore m < -10 \text{ and } 5 < m \leq 6$

38. Ans (b)

Given $\alpha < \beta, c$ is – ive and b = + ive.

$\alpha + \beta = -b = -ive, \alpha\beta = c = -ive$

$\alpha\beta = -ive \Rightarrow$ one is +ive and other –ive.

Since $\alpha < \beta,$ we must have α is –ive and $\beta + ive.$

Again $\alpha + \beta < 0 \Rightarrow \beta < -\alpha \Rightarrow \beta < |\alpha|$.

Quadratic Equations & Theory of Equations

Integer Answer Type

1. If λ is the minimum value of the expression $|x-p|+|x-15|+|x-p-15|$ for x in the range $p \le x \le 15$ where 0 < *p* < 15. Then $\frac{\lambda}{5}$ =

Key. 3

Sol. $|x-p| = x-p \quad (Since\, x \ge p)$

$|x-15| = 15-x \; (Since\, x \le 15)$

$|x-(p+15)| = (p+15)-x \; (as\; 15+p > x)$

$\therefore$ expression reduces to

$E = x-p+15-x+p+15-x$

$E = 30-x$

$\therefore E_{\min}$ occurs when $x = 15$

$\therefore \lambda = 15$

2. Let $P(x) = x^2+bx+c$, where b and c are integer. If $P(x)$ is a factor of both x^4+6x^2+25 and $3x^4+4x^2+28x+5$, find the value of P(1).

Key. 4

Sol. Since $P(x)$ divides into both of them

Hence P(x) also divides

$(3x^4+4x^2+28x+5)-3(x^4+6x^2+25)$

$= -14x^2+28x-70 = -14(x^2-2x+5)$

Which is a quadratic, Hence $P(x) = x^2-2x+5$

$\therefore P(1) = 4$

3. Largest integral value of m for which the quadratic expression $y = x^2+(2m+6)x+4m+12$ is always positive, $\forall x \in R$, is

Key. 0

Sol. $D < 0 \Rightarrow -3 < m < 1 \Rightarrow m = 0$

4. The number of solution of the equation $e^{2x}+e^{x}+e^{-2x}+e^{-x} = 3(e^{-2x}+e^{x})$ is

Key. 1

Sol. $x = ln2$

5. Let a,b,c be the three roots of the equation $x^3+x^2-333x-1002=0$. If P= $a^3+b^3+c^3$ then the value of $\frac{P}{2006}$ =__

Key. 1

Sol. Let α be the root of the given cubic where α can take values a, b, c

Hence $\alpha^3+\alpha^2-333\alpha-1002=0$ or $\alpha^3=1002+333\alpha-\alpha^2$

$\therefore \Sigma\alpha^3=\Sigma 1002+333\Sigma\alpha-\Sigma\alpha^2=3006+333\Sigma\alpha-\left[(\Sigma\alpha)^2-2\Sigma\alpha_1\alpha_2\right]$

But $\Sigma\alpha=-1;\Sigma\alpha_1\alpha_2=-333$

$\therefore a^3+b^3+c^3=3006-333-[1+666]=3006-333-667=3006-100=2006$ = P

6. The number of the distinct real roots of the equation $(x+1)^5=2(x^5+1)$ is

Key. 3

Sol. $(x+1)^5=2(x^5+1)$

Let $f(x)=\frac{(x+1)^5}{(x^5+1)}$ $(x\neq-1)$

$\Rightarrow$ $f'(x)=\frac{5(x+1)^4(1-x^4)}{(x^5+1)^2}$

$\Rightarrow$ $x=1$ is maximum

As, $f(0)=1$ and $f(1)=16$

And $\lim_{x\to\pm\infty} f(x)=1 \Rightarrow f(x)=2$ has two solutions but given equation has three solutions.

because x = -1 included.

7. The equation $2(\log_3 x)^2-|\log_3 x|+a=0$ has exactly four real solutions if $a\in\left(0,\frac{1}{K}\right)$, then the value of K is __

Key. 8

Sol. on putting $\log_3 x=t,$ we get

$2t^2-|t|+a=0$ ...(i)

If $t>0,$ then $2t^2-t+a=0$ ...(ii)

If $t<0,$ then $2t^2+t+a=0$ ...(iii)

If Eq. (i) has four roots then Eq. (ii) must have both roots positive and Eq. (iii) has both roots negative. Now, Eq. (ii) has both roots positive, if $D>0$

$a/2>0$

$\Rightarrow$ $1-8a>0, a>0$

$\Rightarrow \qquad a \in \left(0, \frac{1}{8}\right)$ on taking intersection.

Again, Eq. (iii) has both roots negative, if $D > 0, a/2 > 0.$

We again get $a \in \left(0, \frac{1}{8}\right) \Rightarrow K = 8$

8. Let α, β be the roots of $x^2 - x + p = 0$ and λ, δ be the roots of $x^2 - 4x + q = 0$ such that $\alpha, \beta, \gamma, \delta$ are in G.P and $p \ge 2.$ If $a, b, c \in \{1, 2, 3, 4, 5\},$ let the number of equation of the form $ax^2 + bx + c = 0$ which have real roots be *r*, then the minimum value of $\frac{pqr}{1536} =$

Key. 1

Sol. $(\alpha + \beta) = 1, \alpha\beta = p, \gamma + \delta = 4, \gamma\delta = q$

Since $\alpha, \beta, \gamma, \delta$ are in G.P

$$\therefore \frac{\beta}{\alpha} = \frac{\delta}{\gamma} \Rightarrow \frac{\beta + \alpha}{\beta - \alpha} = \frac{\delta + \gamma}{\delta - \gamma} \Rightarrow \frac{(\beta + \alpha)^2}{(\beta + \alpha)^2 - 4\alpha\beta} = \frac{(\delta + \gamma)^2}{(\delta + \gamma)^2 - 4\delta\gamma}$$

$$\Rightarrow \frac{1}{1 - 4p} = \frac{16}{16 - 4q} = \frac{4}{4 - q}$$

$$\Rightarrow 4 - q = 4 - 16p$$

Now, $p \ge 2 \ \therefore q \ge 32$

For the given equation $ax^2 + bx + c = 0$ to have real roots $b^2 - 4ac \ge 0$

$$\therefore ac \le \frac{b^2}{4}$$

b	$\frac{b^2}{4}$	Possible values of ac such that $ac \le \frac{b^2}{4}$	No. of possible pairs (a, c)		Value of ac	Possible pairs (a, c)
					1	(1,1)
2	1	1	1		2	(1,2), (2,1)
3	2.25	1,2	3		3	(1,3). (3,1)
4	4	1,2,3,4	8		4	(1,4),(4,1),(2,3)
5	6.25	1,2,3,4,5,6	12		5	(1,5),(5,1)
		Total	24		6	(2,3),(3,2)

Hence number of quadratic equation with real roots, r = 24

Now from (i) and (ii) the minimum value of $pqr = 2.32.24 = 1536$

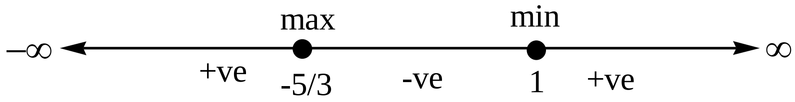
9. Let α, β and γ be the roots of equation $f(x) = 0,$ where $f(x) = x^3 + x^2 - 5x - 1.$ Then the value of $\left|[\alpha] + [\beta] + [\gamma]\right|,$ where $[.]$ denotes the greatest integer function, is equal to

Key. 3

Sol. Given $f(x) = x^3 + x^2 - 5x - 1$

$\therefore f'(x)=3x^2+2x-5$. The roots of $f'(x)=0$ are $-\dfrac{5}{3}$ and 1

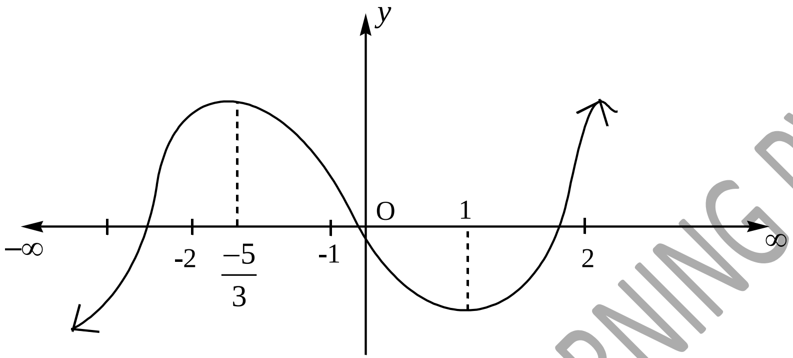
Writing the sign scheme for $f'(x)$,



Also , $f(-\infty)=-\infty<0, f(\infty)=\infty>0$

$f(1)=-4, f\left(-\dfrac{5}{3}\right)=\dfrac{148}{27}$

Now, graph of $y=f(x)$ is as follows



$f(-3)=-27+9+15-1=-4<0$

$f(-2)=-8+4+10-1>0$

$f(-1)=4>0, f(0)=-1<0$

$f(2)=1>0$

$\therefore -3<\alpha<-2, -1<\beta<0, 1<\gamma<2$

$|[\alpha]+[\beta]+[\gamma]|=|-3-1+1|=3$

10. The set of real parameter 'a' for which the equation $x^4-2ax^2+x+a^2-a=0$ has all real solutions, is given by $\left[\dfrac{m}{n},\infty\right)$ where *m* and *n* are relatively prime positive integers, then the value of $(m+n)$ is

Key. 7

Sol. We have $a^2-(2x^2+1)a+x^4+x=0$

$$\therefore a=\frac{(2x^2+1)\pm\sqrt{(2x^2+1)^2-4(x^4+x)}}{2}$$

$$2a=(2x^2+1)\pm(2x-1)$$

On solving +ve & -ve sign we got

$$a\geq\frac{3}{4}$$

$$\therefore m+n=7$$

11. Number of positive integer n for which n^2+96 is a perfect square is

Key. 4

Sol. Suppose m is positive integer such that $n^2+96=m^2$ then
$(m-n)(m+n)=96$
As $m-n<m+n$ and $m-n, m+n$ both must be even
So, the only possibilities are
$m-n=2, m+n=48 : m-n=4, m+n=24$
$m-n=6, m+n=16 : m-n=8, m+n=12$
So, the solutions of (m,n) are $(25,23),(14,10),(11,5),(10,2)$

12. If α,β be the roots of $x^2+px-q=0$ and γ,δ are the roots of $x^2+px+r=0$, $q+r\neq 0$, then $\frac{(\alpha-\gamma)(\alpha-\delta)}{(\beta-\gamma)(\beta-\delta)}$ is equal to

Key. 1

Sol. Here, $\alpha+\beta=-p=\gamma+\delta$
$(\alpha-\gamma)(\alpha-\delta)=\alpha^2-\alpha(\gamma+\delta)+\gamma\delta=\alpha^2-\alpha(\alpha+\beta)+r$
$=-\alpha\beta+r=q+r$
Similarly $(\beta-\gamma)(\beta-\delta)=q+r$
So, ratio is 1

13. Number of real roots of $2x^{99}+3x^{98}+2x^{97}+3x^{96}+.......+2x+3=0$ is

Key. 1

Sol. Given equation can be written as $(2x+3)(x^{98}+x^{96}+......+1)=(2x+3)\frac{(x^{100}-1)}{x^2-1}$

So, the real roots are $x=\pm 1, \frac{-3}{2}$, out of which ± 1 are not roots of given equation.

14. If λ is the minimum value of the expression $|x-p|+|x-15|+|x-p-15|$ for x in the range $p\leq x\leq 15$ where 0 < p < 15. Then $\frac{\lambda}{5}$ =

Key. 3

Sol. $|x-p|=x-p \quad (Since\, x\geq p)$

$|x-15|=15-x \; (Since\, x\leq 15)$

$|x-(p+15)|=(p+15)-x \; (as\; 15+p>x)$

$\therefore \text{exp}\, ression\ reduces\ to$

$E=x-p+15-x+p+15-x$

$E=30-x$

$\therefore E_{\min}\ occurs\ when\ x = 15$

$\therefore \lambda = 15$

15. Let $P(x) = x^2 + bx + c$, where b and c are integer. If $P(x)$ is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, find the value of P(1).

Key. 4

Sol. Since $P(x)$ divides into both of them

Hence P(x) also divides

$(3x^4 + 4x^2 + 28x + 5) - 3(x^4 + 6x^2 + 25)$

$= -14x^2 + 28x - 70 = -14(x^2 - 2x + 5)$

Which is a quadratic, Hence $P(x) = x^2 - 2x + 5$

$\therefore P(1) = 4$

16. Largest integral value of m for which the quadratic expression $y = x^2 + (2m+6)x + 4m + 12$ is always positive, $\forall x \in R$, is

Key. 0

Sol. $D < 0 \Rightarrow -3 < m < 1 \Rightarrow m = 0$

17. For a twice differentiable function $f(x)$, $g(x)$ is defined as $g(x) = f'(x)^2 + f''(x) f(x)$ on $[a, e]$. If for $a < b < c < d < e$, $f(a) = 0$, $f(b) = 2$, $f(c) = -1$, $f(d) = 2$, $f(e) = 0$ then find the minimum number of zeros of g(x).

Key. 6

Sol. $g(x) = (f'(x))^2 + f''(x) f(x) = \frac{d}{dx}(f(x) f'(x))$

Let $h(x) = f(x) f'(x)$

Then, $f(x) = 0$ has four roots namely a, α, β, e

where $b < \alpha < c$ and $c < \beta < d$.

And $f'(x) = 0$ at three points k_1, k_2, k_3 where

$a \square k_1 \square \square, \square \square k_2 \square \square, \square \square k_3 \square e$

[$\because$ Between any two roots of a polynomial function $f\ x\ \square 0$ there lies atleast one root of $f'\ x\ \square 0$]

There are atleast 7 roots of $f\ x\ .f'\ x\ \square 0$

$\square$ There are atleast 6 roots of $\frac{d}{dx}\ f\ x\ f'\ x\ \square 0$ i.e. of $g\ x\ \square 0$

18. $f(x)$ is a polynomial of 6[th] degree and $f(x) = f(2-x)\ \forall\ x \in R$. If $f(x) = 0$ has 4 distinct real roots and two real and equal roots then sum of roots of $f(x) = 0$

Key. 6

Sol. $f(\alpha) = f(2-\alpha) = 0$ sum of roots $= 4$

When $\alpha \neq 2-\alpha$

Where $\alpha = 2-\alpha$ i.e., $\alpha = 1$ sum of roots $= 2$

$\therefore$ Total sum $= 6$

19. $(1+x)(1+x+x^2)(1+x+x^2+x^3)\ldots\ldots(1+x+x^2+\ldots..+x^{100})$

When written in the ascending power of x then (the highest exponent of x) $-$ 5045 is

Key. 5

Sol. Highest exponent of $x = 1 + 2 + 3 + \ldots.. + 100 = \frac{100(101)}{2} : 5050$

20. If the roots of the equation $x^3 - ax^2 + 14x - 8 = 0$ are all real and positive, then the minimum value of [a] (where [a] is the greatest integer of a) is

Key. 6

Sol. $f(x) = x^3 - ax^2 + 14x - 8 = 0$

$\frac{\alpha+\beta+\gamma}{3} \geq (\alpha.\beta\gamma)^{1/3}$

$\frac{a}{3} \geq (8)^{1/3}$

$a \geq 6$

21. The remainder when 2 + [1! + 2(2!) + 3(3!) + ... + 10(10!)] is divided by 11! is

Key. 1

Sol. $n(n!) = (n+1)! - n!$

and proceed

22. The quadratic expression $ax^2 + |2a - 3|x - 6$ is positive for exactly two integral values of x then 2 + [a] (where [.] denotes the greatest integer function) is

Key. 1

Sol. Conceptual

23. If the roots of the equation $x^3+px^2+qx+r=0$, are in G.P. such that geometric mean among the three roots satisfy the equation $px+k_1q=0$ and other two roots satisfy the equation $pqx^2-k_2(q-p^2)qx+p^2r=0$ then the value of k_1+k_2 is

Key. 5

Sol. We have $x_1+x_2+x_3=-p$ ….. (1)

$x_1x_2+x_1x_3+x_2x_3=q$ ….. (2)

$x_1x_2x_3=-r$ ….. (3)

$x_1^2=x_2x_3$ ….. (4)

from (2)

$x_1x_2+x_1x_3+x_1^2=q$

$x_1(x_1+x_2+x_3)=q$

$x_1=-\dfrac{q}{p}$

$\Rightarrow px_1+q=0 \Rightarrow K_1=1$ …. (5)

from (1)

$x_2+x_3=\dfrac{q-p^2}{p}$ ….. (6)

$x_2x_3=\dfrac{rp}{q}$ ….. (7)

Hence, x_2, x_3 satisfy the equation $pqx^2-(q-p^2)qx+p^2r=0$

$\Rightarrow K_2=1$ …. (8)

From (5) and (8)

$K_1+K_2=2$

24. The least integral value of 'a' such that $(a-3)x^2+12x+(a+6)>0\forall x\in R$ is

Key: 7

Hint:

$ax^2+bx+c>0\forall x\in R\Rightarrow a>0, D<0$

$\Rightarrow(i)a-3>0(ii)(a+9)(a-6), a>6$

Least integral value of a = 7

25. Let $p(x)=x^5+x^2+1$ have roots x_1,x_2,x_3,x_4 and x_5, $g(x)=x^2-2$, then the value of $g(x_1)g(x_2)g(x_3)g(x_4)g(x_5)-30g(x_1x_2x_3x_4x_5)$, is

Key: 7

Hint: Given $g(x_1)g(x_2)g(x_3)g(x_4)=A$

$=(x_1^2-2)(x_3^2-2)(x_4^2-2)(X_5^2-2)$

$=-(2-x_1^2)(2-x_2^2)(2-x_3^2)(2-x_4^2)(2-x_5^2)$

$=2^5-\left(\sum x_1^2\right)2^4+\sum x_1^2-x_2^2.2^3-\sum x_1^2.x_2^2.x_3^2.2^2$

$+\sum x_1^2.x_2^2.x_3^2.x_4^2.2-x_1^2.x_2^2.x_3^2.x_4^2.x_3^2]$

$p(x)=x^5+x^2+1=0$ has roots $x_1,x_2,......x_5,$ then that equation $q(x)$ whose roots are square of the roots of $p(x)$ is $q(x)=(\sqrt{y})^5+(\sqrt{y})^2+1=0$; $\alpha=x$ and $y=\alpha^2$

$\Rightarrow(y+1)^2=(-\sqrt{y})^{5\times2}$

$\Rightarrow y^2+2y+1=y^5\Rightarrow q(x)=y^5-y^2-2y-1=0$

Then $\sum x_1^2=\sum y_1=0$

$\sum x_1^2x_2^2=\sum y_1.y_2=0$

$\sum x_1^2x_2^2x_3^2=\sum y_1.y_2.y_3=1$

$\sum x_1^2.x_2^2.x_3^2.x_4^2=\sum y_1.y_2.y_3.y_4=-2$

$\sum x_1^2.x_2^2.x_3^2.x_4^2.x_5^2=\sum y_1.y_2.y_3.y_4.y_5=1$, then

$A=-\left[2^5-0+0-2^2-2.2-1\right]=-[32-4-4-1]=-[32-9]$

= -23

$x_1x_2x_3x_4x_5=-1\Rightarrow g(x_1x_2....x_3)=-1$

$\Rightarrow g(x_1)g(x_2)......g(x_3)-30g(x_1x_2......x_3)=7$

Alternative

Let us form that equation having roots $y=g(x_i)$ i.e., $y=x^2-2$

$x=\sqrt{y+2}$

$\Rightarrow\left(\sqrt{y+2}\right)^5+\left(\sqrt{y+2}\right)^2+1=0$

$\Rightarrow y^5+20y^4+40y^3+79y^2+74y+23=0$

$\therefore g\left(x_1\right).....g\left(x_2\right)=$ Product of roots

= -23

$x_1x_2x_3x_4x_5=-1\Rightarrow g\left(x_1x_2......x_5\right)=-1$

$\Rightarrow g\left(x_1\right)g\left(x_2\right)......g\left(x_5\right)-30g\left(x_1x_2......x_5\right)=7$

26. (L-2)If x, y, z > 0 and $x(1-y)>\frac{1}{4},\ y(1-z)>\frac{1}{4},z(1-x)>\frac{1}{4},$ then the number of ordered triplets (x, y, z) satisfying the above inequalities is/are

Key : 0

Hint : Multiplying we get

$xyz(1-x)(1-y)(1-z)>\frac{1}{64}$ …………(1)

Now $t(1-t)=t-t^2=\frac{1}{4}-\left(\frac{1}{2}-t\right)^2\le\frac{1}{4}$

So $x(1-x)y(1-y)z(1-z)\le\frac{1}{64}$ ……(2)

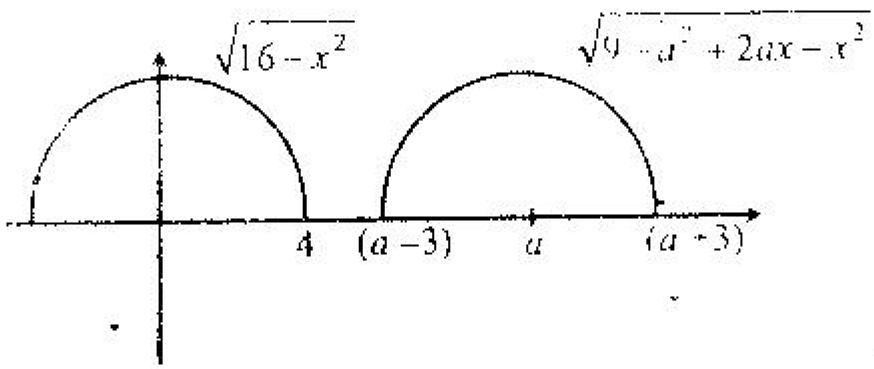
(1) and (2) are contradictory

27. (L-3)Find the least integral value of a such that $\sqrt{9-a^2+2ax-x^2}>\sqrt{16-x^2}$ for at least one positive x.

Key : 6

Sol : $y=\sqrt{9-a^2+2ax-x^2}$

$(x-a)^2+y^2=9$



For given inequality to hold for positive x.

$a - 3 < 4$

$\Rightarrow a < 7 \Rightarrow a = 6$

28. (L-3)Let $f(x) = 30 - 2x - x^3$, then find the number of positive integral values of x which satisfies $f(f(f(x))) > f(f(-x))$.

Key : 2

Sol : $f'(x) = -2 - 3x^2 < 0 \Rightarrow f(x)$ is decreasing

$\therefore f(f(x)) < f(x) \Rightarrow f(x) > -x$

$\Rightarrow 30 - x - x^3 > 0$

$\Rightarrow x^3 + x - 30 < 0$

$\Rightarrow (x+3)(x^2 + 3x + 16) < 0$

$\Rightarrow x < 3$

$\therefore$ No. of values = 2

29. (L-3)Let $p(x) = x^5 + x^2 + 1$ have roots x_1, x_2, x_3, x_4 and x_5, $g(x) = x^2 - 2$, then find the value of $g(x_1)g(x_2)g(x_3)g(x_4)g(x_5) - 30g(x_1x_2x_3x_4x_5)$

Key : 7

Sol : Given $g(x_1)g(x_2)g(x_3)g(x_4)g(x_5) = A$

$= (x_1^2 - 2)(x_2^2 - 2)(x_3^2 - 2)(x_4^2 - 2)(x_5^2 - 2)$

$= -(2 - x_1^2)(2 - x_2^2)(2 - x_3^2)(2 - x_4^2)(2 - x_5^2)$ …. (i)

$$= -\left[2^5 - \left(\sum x_1^2\right)2^4 + \sum x_1^2.x_2^2.2^3 - \sum x_1^2.x_2^2.x_3^2.2^2 + \sum x_1^2.x_2^2.x_3^2.x_4^2.2 - x_1^2.x_2^2.x_3^2.x_4^2.x_5^2\right]$$

$p(x) = x^5 + x^2 + 1 = 0$ has roots $x_1, x_2, ..., x_5$, then that equation q(x) whose roots are square of the roots of p(x) is $q(x) = (\sqrt{y})^5 + (\sqrt{y})^2 + 1 = 0$; $\alpha = x$ and $y = \alpha^2$

$\Rightarrow (y+1)^2 = (-\sqrt{y})^{5\times 2}$

$\Rightarrow y^2 + 2y + 1 = y^5 \Rightarrow q(x) = y^5 - y^2 - 2y - 1 = 0,$

then $\sum x_1^2 = \sum y_1 = 0$

$\sum x_1^2.x_2^2 = \sum y_1.y_2 = 0$

$\sum x_1^2.x_2^2.x_3^2 = \sum y_1.y_2.y_3 = 1$

$\sum x_1^2.x_2^2.x_3^2.x_4^2 = \sum y_1.y_2.y_3.y_4 = -2$

$\sum x_1^2.x_2^2.x_3^2.x_4^2.x_5^2 = \sum y_1.y_2.y_3.y_4.y_5 = 1$, then

$A = -\left[2^5 - 0 + 0 - 2^2 - 2.2 - 1\right] = -[32 - 4 - 4 - 1] = -[32 - 9] = -23$

$x_1x_2x_3x_4x_5 = -1 \Rightarrow g(x_1x_2....x_5) = -1$

$\Rightarrow g(x_1)g(x_2).....g(x_5) - 30g(x_1x_2......x_5) = 7$

Alternative :

Let us form that equation having roots $y = g(x_i)$ i.e., $y = x^2 - 2$

$x = \sqrt{y+2}$

$\Rightarrow \left(\sqrt{y+2}\right)^5 + \left(\sqrt{y+2}\right)^2 + 1 = 0$

$\Rightarrow y^5 + 20y^4 + 40y^3 + 79y^2 + 74y + 23 = 0$

$\therefore g(x_1).....g(x_5) =$ Product of roots = -23

$x_1x_2x_3x_4x_5 = -1 \Rightarrow g(x_1x_2......x_5) = -1$

$\Rightarrow g(x_1)g(x_2)....g(x_5) - 30g(x_1x_2.....x_5) = 7$

30. (L-3)A polynomial equation is said to be a reciprocal equation if the reciprocal of each of its roots is also a root of it.

Therefore a necessary condition for f(x) = 0 to be a reciprocal equation is that 0 is not a root of it i.e. $f(0) \neq 0$.

Let f(x) = 0 be a reciprocal equation of degree n having roots $\alpha_1, \alpha_2, ..., \alpha_n$, none of these zero.

Let ψ (x) = 0 be the equation whose roots are $\frac{1}{\alpha_1}, \frac{1}{\alpha_2}, ..., \frac{1}{\alpha_n}$. Then the equations f(x) = 0 and ψ (x) = 0 are identical.

Let $a_0x^n + a_1x^{n-1} + ... + a_n = 0,\ a_n \neq 0$ be a reciprocal equation. Then it is identical with the equation.

$a_n x^n + a_{n-1}x^{n-1} + ... + a_0 = 0$

Let $a_0 \neq 0$

$\therefore \left(a_0, a_1, ..., a_n\right) = K\left(a_n, a_{n-1}, ..., a_0\right)$ for some $K \neq 0$

$\therefore a_0 = K a_n, a_1 = K a_{n-1}, ...a_n = K a_0$

This implies $K = \pm 1$

If K = 1 then $a_0 = a_n, a_1 = a_{n-1}, ..., a_n = a_0$

This equation is said to be a reciprocal equation of the First type.

If K = -1 then $a_0 = -a_n, a_1 = -a_{n-1}, ..., a_n = -a_0$

This equation is said to be a reciprocal equation of the second type.

A reciprocal equation is said to be of the standard form if it is of the first type and of even degree. Then

31. If the roots of the equation $x^3 - ax^2 + 14x - 8 = 0$ are all real and positive, then the minimum value of [a] (where [a] is the greatest integer of a) is

Key. 6

Sol. $f(x) = x^3 - ax^2 + 14x - 8 = 0$

$\frac{\alpha+\beta+\gamma}{3} \geq \left(\alpha.\beta\gamma\right)^{1/3}$

$\frac{a}{3} \geq (8)^{1/3}$

$a \geq 6$

32. If the roots of the equation $x^3 + px^2 + qx + r = 0$, are in G.P. such that geometric mean among the three roots satisfy the equation $px + k_1 q = 0$ and other two roots satisfy the equation $pqx^2 - k_2(q-p^2)qx + p^2 r = 0$ then the value of $k_1 + k_2$ is

Key. 2

Sol. We have $x_1 + x_2 + x_3 = -p$ ….. (1)

$x_1x_2 + x_1x_3 + x_2x_3 = q$ ….. (2)

$x_1x_2x_3 = -r$ ….. (3)

$x_1^2 = x_2x_3$ ….. (4)

from (2)

$x_1x_2 + x_1x_3 + x_1^2 = q$

$x_1(x_1 + x_2 + x_3) = q$

$x_1 = -\frac{q}{p}$

$\Rightarrow px_1 + q = 0 \Rightarrow K_1 = 1$ …. (5)

from (1)

$x_2 + x_3 = \frac{q - p^2}{p}$ ….. (6)

$x_2 x_3 = \frac{rp}{q}$ ….. (7)

Hence, x_2, x_3 satisfy the equation $pqx^2 - (q - p^2)qx + p^2 r = 0$

$\Rightarrow K_2 = 1$ …. (8)

From (5) and (8)

$K_1 + K_2 = 2$

33. If product of two roots of the equation $x^4 - 18x^3 + kx^2 + 200x - 1984 = 0$ is –32 then the value of $90 - k$ is

Key. 4

Sol. Let α, β, γ, δ be four roots

$\alpha\beta = -32$

$\alpha\beta\gamma\delta = -1984 \Rightarrow \gamma\delta = 62$

$x^4 - 18x^3 + kx^2 + 200x - 1984 = (x - \alpha)(x - \beta)(x - \gamma)(x - \delta)$

$\equiv (x^2 - (\alpha + \beta)x - 32)(x^2 - (\gamma + \delta)x + 62)$

$\alpha + \beta = p \equiv (x^2 - px - 32)(x^2 - qx + 62)$

$\gamma + \delta = q$

equaling co-eff. of x^3, x^2, x

$p + q = 18$ …..(i)

$-62p + 32q = 200$ …..(ii)

$k = 62 + pq - 32$ …..(iii)

from (i) and (ii) p = 4, q = 14

from (iii) k = 86.

34. If the equation $x^4 + px^3 + qx^2 + rx + 5 = 0$ has four positive real roots, then the minimum value of pr/10 is

Key. 8

Sol. Let α, β, γ, δ be four positive real roots of given equation.

Then $\alpha + \beta + \gamma + \delta = -p$

$\Sigma\alpha\beta = q$

$\Sigma\alpha\beta\gamma = -r$

$\alpha\beta\gamma\delta = 5$

using A.M. ≥ G.M.

$$\frac{\alpha + \beta + \gamma + \delta}{4} \ge (\alpha\beta\gamma\delta)^{1/4}$$

$$\frac{\Sigma\alpha\beta\gamma}{4} \ge (\alpha^3\beta^3\gamma^3\delta^3)^{1/4}$$

$$\frac{(\Sigma\alpha).(\Sigma\alpha\beta\gamma)}{16} \ge (\alpha\beta\gamma\delta)$$

$pr \ge 80$

35. The set of real parameter 'a' for which the equation $x^4 - 2ax^2 + x + a^2 - a = 0$ has all real solutions, is given by $\left[\frac{m}{n}, \infty\right)$ where *m* and *n* are relatively prime positive integers, then the value of $(m + n)$ is

Key. 7

Sol. We have $a^2-(2x^2+1)a+x^4+x=0$

$$\therefore a=\frac{(2x^2+1)\pm\sqrt{(2x^2+1)^2-4(x^4+x)}}{2}$$

$$2a=(2x^2+1)\pm(2x-1)$$

On solving +ve & -ve sign we got

$$a\ge\frac{3}{4}$$

$$\therefore m+n=7$$

36. Number of positive integer n for which n^2+96 is a perfect square is

Key. 4

Sol. Suppose m is positive integer such that $n^2+96=m^2$ then

$(m-n)(m+n)=96$

As $m-n<m+n$ and $m-n, m+n$ both must be even

So, the only possibilities are

$m-n=2, m+n=48 : m-n=4, m+n=24$

$m-n=6, m+n=16 : m-n=8, m+n=12$

So, the solutions of (m,n) are $(25,23),(14,10),(11,5),(10,2)$

37. If α,β be the roots of $x^2+px-q=0$ and γ,δ are the roots of $x^2+px+r=0$,

$q+r\ne 0$, then $\dfrac{(\alpha-\gamma)(\alpha-\delta)}{(\beta-\gamma)(\beta-\delta)}$ is equal to

Key. 1

Sol. Here, $\alpha+\beta=-p=\gamma+\delta$

$(\alpha-\gamma)(\alpha-\delta)=\alpha^2-\alpha(\gamma+\delta)+\gamma\delta=\alpha^2-\alpha(\alpha+\beta)+r$

$=-\alpha\beta+r=q+r$

Similarly $(\beta-\gamma)(\beta-\delta)=q+r$

So, ratio is 1

38. Number of real roots of $2x^{99}+3x^{98}+2x^{97}+3x^{96}+.......+2x+3=0$ is

Key. 1

Sol. Given equation can be written as $(2x+3)(x^{98}+x^{96}+......+1)=(2x+3)\dfrac{(x^{100}-1)}{x^2-1}$

So, the real roots are $x=\pm 1,\dfrac{-3}{2}$, out of which ± 1 are not roots of given equation.

39. The number of the distinct real roots of the equation $(x+1)^5=2(x^5+1)$ is

Key. 3

Sol. $(x+1)^5=2(x^5+1)$

Let $f(x)=\frac{(x+1)^5}{(x^5+1)}$ $(x\neq -1)$

$\Rightarrow$ $f'(x)=\frac{5(x+1)^4(1-x^4)}{(x^5+1)^2}$

$\Rightarrow$ $x=1$ is maximum

As, $f(0)=1$ and $f(1)=16$

And $\lim\limits_{x\to\pm\infty} f(x)=1 \Rightarrow f(x)=2$ has two solutions but given equation has three solutions.

because x = -1 included.

40. The equation $2(\log_3 x)^2-|\log_3 x|+a=0$ has exactly four real solutions if $a\in\left(0,\frac{1}{K}\right)$, then the value of K is __

Key. 8

Sol. on putting $\log_3 x=t,$ we get

$2t^2-|t|+a=0$ ...(i)

If $t>0,$ then $2t^2-t+a=0$ ...(ii)

If $t<0,$ then $2t^2+t+a=0$ ...(iii)

If Eq. (i) has four roots then Eq. (ii) must have both roots positive and Eq. (iii) has both roots negative. Now, Eq. (ii) has both roots positive, if $D>0$

$a/2>0$

$\Rightarrow$ $1-8a>0, a>0$

$\Rightarrow$ $a\in\left(0,\frac{1}{8}\right)$ on taking intersection.

Again, Eq. (iii) has both roots negative, if $D>0, a/2>0.$

We again get $a\in\left(0,\frac{1}{8}\right)\Rightarrow K=8$

41. Let α,β be the roots of $x^2-x+p=0$ and λ,δ be the roots of $x^2-4x+q=0$ such that $\alpha,\beta,\gamma,\delta$ are in G.P and $p\geq 2.$ If $a,b,c\in\{1,2,3,4,5\},$ let the number of equation of the form $ax^2+bx+c=0$ which have real roots be *r*, then the minimum value of $\frac{pqr}{1536}=$

Key. 1

Sol. $(\alpha+\beta)=1, \alpha\beta=p, \gamma+\delta=4, \gamma\delta=q$

Since $\alpha,\beta,\gamma,\delta$ are in G.P

$$\therefore \frac{\beta}{\alpha}=\frac{\delta}{\gamma}\Rightarrow\frac{\beta+\alpha}{\beta-\alpha}=\frac{\delta+\gamma}{\delta-\gamma}\Rightarrow\frac{(\beta+\alpha)^2}{(\beta+\alpha)^2-4\alpha\beta}=\frac{(\delta+\gamma)^2}{(\delta+\gamma)^2-4\delta\gamma}$$

$$\Rightarrow\frac{1}{1-4p}=\frac{16}{16-4q}=\frac{4}{4-q}$$

$\Rightarrow 4-q=4-16p$

Now, $p\geq 2 \therefore q\geq 32$

For the given equation $ax^2+bx+c=0$ to have real roots $b^2-4ac\geq 0$

$\therefore ac\leq \frac{b^2}{4}$

b	$\frac{b^2}{4}$	Possible values of ac such that $ac\leq\frac{b^2}{4}$	No. of possible pairs (a,c)		Value of ac	Possible pairs (a,c)
					1	(1,1)
2	1	1	1		2	(1,2), (2,1)
3	2.25	1,2	3		3	(1,3). (3,1)
4	4	1,2,3,4	8		4	(1,4),(4,1),(2,3)
5	6.25	1,2,3,4,5,6	12		5	(1,5),(5,1)
		Total	24		6	(2,3),(3,2)

Hence number of quadratic equation with real roots, r = 24

Now from (i) and (ii) the minimum value of $pqr=2.32.24=1536$

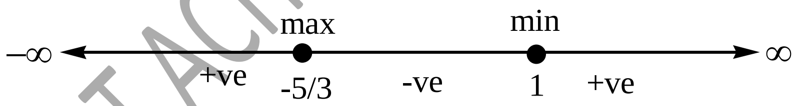
42. Let α,β and γ be the roots of equation $f(x)=0,$ where $f(x)=x^3+x^2-5x-1.$ Then the value of $\left|[\alpha]+[\beta]+[\gamma]\right|$, where $[.]$ denotes the greatest integer function, is equal to

Key. 3

Sol. Given $f(x)=x^3+x^2-5x-1$

$\therefore f'(x)=3x^2+2x-5$. The roots of $f'(x)=0$ are $-\frac{5}{3}$ and 1

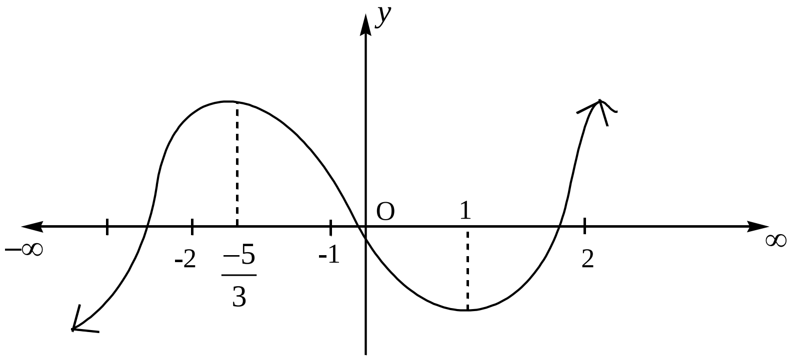
Writing the sign scheme for $f'(x)$,



Also , $f(-\infty)=-\infty<0, f(\infty)=\infty>0$

$f(1)=-4, f\left(-\frac{5}{3}\right)=\frac{148}{27}$

Now, graph of $y=f(x)$ is as follows



$f(-3) = -27 + 9 + 15 - 1 = -4 < 0$

$f(-2) = -8 + 4 + 10 - 1 > 0$

$f(-1) = 4 > 0,\ f(0) = -1 < 0$

$f(2) = 1 > 0$

$\therefore -3 < \alpha < -2, -1 < \beta < 0, 1 < \gamma < 2$

$|[\alpha] + [\beta] + [\gamma]| = |-3 - 1 + 1| = 3$

43. The number of integral values of k for which $x^2 - 2(4k-1)x + 15k^2 - 2k - 7 \geq 0$ hold for all x is

Key. 3

Sol. $D < 0 \Rightarrow k^2 - 6k + 8 < 0 \Rightarrow 2 \leq k \leq 4$

$\Rightarrow k = 2, 3, 4 \Rightarrow 3$ values

44. If roots x_1 and x_2 of $x^2 + 1 = x/a$ satisfy $\left|x_1^2 - x_2^2\right| > \frac{1}{a}$, then $a \in \left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{\sqrt{k}}\right)$ the numerical quantity k must be equal to

Key. 5

Sol. $\left|x_1 + x_2\right|\left|x_1 - x_2\right| > \frac{1}{a} \Rightarrow \left|-\frac{1}{a}\right|\left|\sqrt{\frac{1}{a^2} - 4}\right| > \frac{1}{a}.(*)$

The inequation (*) has meaning if $\frac{1}{a^2} - 4 > 0$

If $a \in \left(-\frac{1}{2}, 0\right)$ then (*) is automatically satisfied

If $a \in \left(0, \frac{1}{2}\right)$ then (*) becomes equivalent to $\sqrt{\frac{1}{a^2} - 4} > 1$ (on canceling $\frac{1}{a} > 0$)

$\Rightarrow -\frac{1}{\sqrt{5}} < a < \frac{1}{\sqrt{5}}$

but $a \in \left(0, \frac{1}{2}\right)$ was assumed $\Rightarrow a \in \left(0, \frac{1}{\sqrt{5}}\right)$

Thus all the values of a lie in the interval $\left(-\frac{1}{2}, 0\right) \cup \left(0, \frac{1}{\sqrt{5}}\right) \Rightarrow k = 5$

45. The integral part of positive value of a for which, the least value of $4x^2 - 4ax + a^2 - 2a + 2$ on $[0, 2]$ is 3, is

Key. 8

Sol. Conceptual Question

46. The least positive integer x such that the three distinct numbers a, b, c are in GP and $a + b + c = xb$ is

Key. 4

Sol. $b^2 = ac$

If $a = 0$, then $b = 0$ a contradiction ($\because a \neq 0$ similarly $b \neq 0$)

If $a \neq 0$, then $c = \frac{b^2}{a}$

On putting in the given relation $a+b+\frac{b^2}{a}=xb \Rightarrow x=\frac{a}{b}+\frac{b}{a}+1$

Now $\frac{a}{b}+\frac{b}{a}\geq 2 \text{ or } \leq -2 \Rightarrow x\geq 3 \text{ or } \leq -3$

But as x has to be positive, x must be ≥ 3

But $x=3$ when $a=b(a\neq b$ is given)

$\Rightarrow x$ should be integer greater than 3.

$\Rightarrow x=4$

47. The sum of all the real roots of the equations $|x-2|^2+|x-2|-2=0$ is ….

Key. 4

Sol. $|x-2|^2+|x-2|-2=0$

$\Rightarrow \left(|x-2|+2\right)\left(|x-2|-1\right)=0 \Rightarrow |x-2|=-2,1$

$\therefore |x-2|=1 \text{ or } x=3,1$

$\Rightarrow$ sum of the roots = 4

48. The number of real solutions of the system of equations $x+y+z=1, 2xy-z^2=1$ is

Key. 1

Sol. $\because x+y+z=1$ and $2xy-z^2=1$

$\because AM\geq GM$

$\Rightarrow \frac{x+y}{2}\geq \sqrt{(xy)} \Rightarrow \left(\frac{1-z}{2}\right)\geq \sqrt{\left(\frac{1+z^2}{2}\right)} \Rightarrow (1-z)^2\geq 2(1+z^2)$

$\Rightarrow z^2+2z+1\leq 0 \Rightarrow (z+1)^2\leq 0$

$\therefore z+1=0 \Rightarrow z=-1$

Then $x+y=2$ and $xy=1$

Hence $x=y=1$

49. Sum of all roots of the equation $\underbrace{\sqrt{x+2\sqrt{x+2\sqrt{x+.....+2\sqrt{x+2\sqrt{3x}}}}}}_{n\text{ radical signs}}=x$ must be equal to

Key. 3

Sol. Rewrite the given equation $\sqrt{x+2\sqrt{x+2\sqrt{x+............+2\sqrt{x+2\sqrt{x+2x}}}}}=x$ --- (1)

On replacing the last letter x on the LHS of eq.(1) by the value of x expressed by Eq.(1) we obtain

$x=\sqrt{x+2\sqrt{x+2\sqrt{x+............+2\sqrt{x+2x}}}}$ (2n radicals)

further, let us replace the last letter x by the same expression

we can write $x=\sqrt{x+2\sqrt{x+2\sqrt{x+.............}}}$

$=\lim_{N\to\infty}\sqrt{x+2\sqrt{x+2\sqrt{x+........+2\sqrt{x+2x}}}}$

It follows that $x=\sqrt{x+2\sqrt{x+2\sqrt{x+.............}}}=\sqrt{x+2\left(\sqrt{x+2\sqrt{x+.........}}\right)}=\sqrt{x+2x}$

Hence $x=\sqrt{x+2x}\Rightarrow x^2=3x\Rightarrow x(x-3)=0$

$x = 0 \ (or) \ x = 3$

$\therefore$ Sum of roots = 3

50. If the roots of $10x^3 - cx^2 - 54x - 27 = 0$ are in harmonic progression, then the value of c must be equal to

Key. 9

Sol. $\because$ Roots of $10x^3 - cx^2 - 54x - 27 = 0$ are in HP

Replacing x by $\frac{1}{x}$, then we get $\frac{10}{x^3} - \frac{c}{x^2} - \frac{54}{x} - 27 = 0$

Or $27xh3 + 54x^2 + cx - 10 = 0$ --- (i)

Now, roots of Eq (i) are in AP

Let roots $\alpha - \beta, \alpha, \alpha + \beta$, then $\alpha - \beta + \alpha + \alpha + \beta = -\frac{54}{27} = -2$ or $\alpha = -\frac{2}{3}$

$\because \alpha = -\frac{2}{3}$ is a root of Eq.(i) then $27\left(-\frac{2}{3}\right)^3 + 54\left(-\frac{2}{3}\right)^2 + c\left(-\frac{2}{3}\right) - 10 = 0$

or $-8 + 24 - \frac{2c}{3} - 10 = 0$

$\therefore c = 9$

32. If the equation $ax^2 - bx + 12 = 0$ where a and b are +ve integers not exceeding 10, has roots both greater than 2 then the number of ordered pair (a, b) is ______.

Key. 1

Sol. Imposing the conditions; $\frac{b}{2a} > 2$, $b^2 \geq 48a$ and f(2) i.e., $2a - b + 12 > 0$ there is only one solution for $(a, b) \equiv (1, 7)$

31. If α, β are the roots of the equation $\lambda(x^2 - x) + x + 5 = 0$ and if $\lambda_1 \,\&\, \lambda_2$ are two values of λ for which the roots α, β are related by $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{5}$, then the value of $\frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1}$ = $\left(255 - k^2\right)$ then $|k|$ is

Key. 1

Sol. $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{4}{5} \Rightarrow \lambda^2 - 16\lambda + 1 = 0$

Now $\frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1} = \frac{\lambda_1^2 + \lambda_2^2}{\lambda_1\lambda_3} = 254$

32. The sum of squares of all integral values of $a > -5$ for which the inequality $x^2 - ax + 6a < 0$ is satisfied for all $x \in (-1, 1)$ must be equal to 6k then k is

Key. 5

Sol. $f(x) = x^2 - ax + 6a$

$D > 0, \qquad f(1) < 0, \ f(-1) < 0$

value of a are $-4,-3,-2,-1$

$(-4)^2+(-3)^2+(-2)^2+(-1)^2=30$

33. The solution set of the inequality $\left(\frac{1}{3}\right)^{\log_{\left(\frac{1}{9}\right)}\left(x^2-\frac{10}{3}x+1\right)} \le 1$ is written as $x\in\left[0,\frac{1}{a}\right)\cup\left(a,\frac{10}{a}\right]$ then find a.

Key. 3

Sol. $0<x^2-\frac{10x}{3}+1\le 1$

$\Rightarrow x\in[0,\frac{1}{3})\cup(3,\frac{10}{3}]$

34. If α,β,γ are the roots of the equation $x^3+3x+c=0$ then find the value of $\frac{1}{27}\sum(\alpha-\beta)^2(\beta-\gamma)^2$.

Key. 3

Sol. Let $\partial_1=(\alpha-\beta)(\beta-\gamma)$

$\partial_2=(\beta-\gamma)(\gamma-\alpha)$

$\partial_3=(\gamma-\alpha)(\alpha-\beta)$

required part is $\frac{\partial_1^2+\partial_2^2+\partial_3^2}{27}$

$\partial_1\partial_2+\partial_2\partial_3+\partial_2\partial_1=0$ & $\partial_1+\partial_2+\partial_3=9$

$\frac{\partial_1^2+\partial_2^2+\partial_3^2}{27}=\frac{81}{27}=3$

35. If a, $b\in R$ and equations $ax^2+30x+b=0$ and $x^2+3x+4=0$ have a common root, then

4a – b is

Key. 0

Sol. $x^2+3x+4=0$ has imaginary roots so both roots are common

$\because \frac{a}{1}=\frac{30}{3}=\frac{b}{4}$

$a+b=50$

36. If $f(x)=\frac{ax+1}{x^2-1}$ gives all real values, then find sum of square of all integral values of a given that $-2\le a\le -1$

Key. 4

Sol. $yx^2-y=ax+1\Rightarrow yx^2-ax-y-1=0$

$\Rightarrow a^2+4(y)(y+1)\ge 0$

$\Rightarrow 4y^2+4y+a^2\ge 0$

$\Rightarrow 16-4\times 4a^2 \leq 0$

$\Rightarrow 1-a^2 \leq 0$

$\Rightarrow a^2-1 \geq 0$

$a \in (-\infty,-1] \cup [1,\infty)$ but $-1 \geq a \geq -5$

so $a=-5,-4,-3,-2,-1$

Quadratic Equations & Theory of Equations

Matrix-Match Type

1. The function f(x) = $\sqrt{ax^3+bx^2+cx+d}$ has its non–zero local minimum and maximum values at
x = −2 and x = 2 respectively if `a' is root of $x^2-x-6=0$ then

Column – I		Column – II	
(A)	The value(s) of `a' is (are)	(p)	0
(B)	Value(s) of 'b' is (are)	(q)	24
(C)	Value(s) of 'c' is (are)	(r)	> 32
(D)	Value(s) of 'd' is (are)	(s)	−2

Key. A-s ; B-p ; C-q ; D-r

Sol. a < 0 $\Rightarrow a=-2$, then g(x) = $-2x^3+6x^2+cx+d$; $g'(x)=-6(x+2)(x-2)$

$\Rightarrow$ b = 0, c = 24, also d > 32 which can be evaluated by $f(-2)=\sqrt{-8a+4b-2c+d}$

2. For the following questions, match the items in column-I to one or more items in column-ii

Column I	Column II	
A) If $^8C_{k+2}+2.^8C_{k+3}+^8C_{k+4}>^{10}C_{4,}$ then the Quadratic equations whose roots are α,β and α^k,β^k have *m* common roots, then *m*=	P)	1
B) If the number of solutions of the equation $\left|2x^2-5x+3\right|+(x-1)=0$ is (are) *n*, then *n*=	Q)	2
C) If the constant term of the quadratic expression $\sum_{k=1}^{n}\left(x-\frac{1}{k+1}\right)\left(x-\frac{1}{k}\right)$ as $n\to\infty$ is *p*, then *p*=	R)	0
	S)	-1
D) The equation $x^2+4a^2=1-4ax$ and $x^2+4b^2=1-4bx$ have only one root in common, then the value of $\left|a-b\right|$ is	T)	-2

Key. A-q;B-p;C-p;D-p

Sol. Given $^8C_{k+2}+2^8C_{k+3}+{}^8C_{k+4}>{}^{10}C_4$

$$\Rightarrow\left({}^8C_{k+2}+{}^8C_{k+3}\right)+\left({}^8C_{k+3}+{}^8C_{k+4}\right)>{}^{10}C_4$$

$$\Rightarrow{}^9C_{k+3}+{}^9C_{k+4}>{}^{10}C_4$$

$\Rightarrow{}^{10}C_{k+4}>{}^{10}C_4$ only ${}^{10}C_5>{}^{10}C_4$ Þ $K+4=5$ Þ $K=1$

$$\therefore\ \alpha^k=\alpha \text{ and } \beta^k=\beta$$

Hence quadratic equation having roots α and β and α^k and β^k are identical and have both roots common.

$\therefore\ m=2$

(B) For $1 \le x < \frac{3}{2} or \frac{3}{2} \le x < \infty, x-1>0$

Therefore no solution is possible

For $x \le 1$, given equation is $\left(2x^2-5x+3\right)+x-1=0$

$\therefore\ 2x^2-4x+2=0 \Rightarrow x^2-2x+1=0 \Rightarrow x=1.$

$\therefore$ The equation has only one solution

$\therefore\ n=1.$

(C) Constant term $C=\frac{1}{1.2}+\frac{1}{2.3}+............+\frac{1}{n(n+1)}$

$$t_n=\frac{1}{n(n+1)}=\frac{1}{n}-\frac{1}{n+1}$$

$$\therefore \qquad C=\sum_{x=1}^{n} t_n=1-\frac{1}{n+1} \qquad \therefore p=1$$

D)

$$(x+2a)^2=1$$

$$(x+2b)^2=1$$

$$x=\pm1-2a, x=\pm1-2b$$

$$1-2a=-1-2b \Rightarrow b-a=-1$$

$$-1-2a=1-2b \Rightarrow b-a=1$$

$$\Rightarrow \qquad |a-b|=1$$

3. Match the statements of Column I with values of Column II.

Column I		Column II
A) The least positive integral values of λ for which $(\lambda-2)x^2+8x+(\lambda+4)>0,$ for all real x is	P)	3
B) The equation $x^2+2(a^2+1)x+(a^2-14a+48)=0$ possesses roots of opposite signs then x value of $'a'$ can be	Q)	5
C) If the equation $ax^2+2bx+4c=16$ has no real roots and $a+c>b+4,$ then integral value of c can be equal to	R)	7
D) If N be the number of solution of the equation	S)	12
$\lvert x^2-x-6\rvert=x+2$ then the value of N is	T)	20

Key. A-q;B-r;C-qrst;D-p

Sol. A) $\lambda>2$

$64-4(\lambda-2)(\lambda+4)<0$

$\Rightarrow \quad (\lambda+6)(\lambda-4)>0$

$\lambda<-6 \text{ or } \lambda>4$

$\therefore$ The least positive integral value of λ is 5

(B) Roots are of opposite signs

$\Rightarrow \quad a^2-14a+48<0$

$(a-6)(a-8)<0$, so a can be 7

The equation is $x^2+100x-1=0$

$\therefore$ discriminant = D = $100^2+4>0$

$\therefore$ Roots are real

C)

Let $f(x)=ax^2+2bx+4c-16$

Clearly $f(-2)=4a-4b+4c-16$

$=4(a-b+c-4)>0$

$=f(x)>0, \forall x\in R$

$\Rightarrow \quad f(0)>0 \Rightarrow 4c-16>0$

$\Rightarrow \quad c>4$

(D) $\because \lvert x^2-x-6\rvert=x+2$

$\Rightarrow \quad \lvert(x-3)(x+2)\rvert=x+2$

$$\Rightarrow \quad |x-3||x+2| = x+2$$

$$\Rightarrow \quad \begin{cases} (x-3)(x+2) = x+2, & x < -2 \\ -(x-3)(x+2) = x+2, & -2 \le x < 3 \\ (x-3)(x+2) = x+2, & x > 3 \end{cases}$$

$$\Rightarrow \quad \begin{cases} x = 4, & x < -2 \\ x = -2, 2 & -2 \le x < 3 \\ x = 4, & x < 3 \end{cases}$$

Hence, $x = -2, 2, 4$

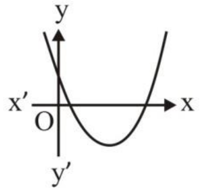
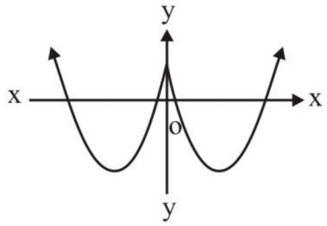
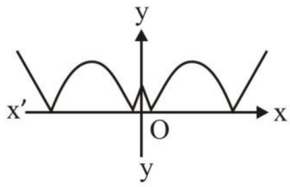
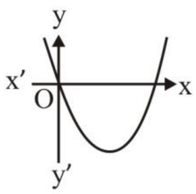
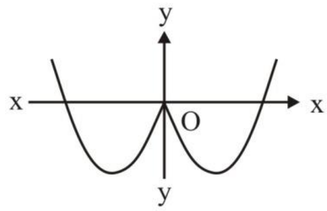
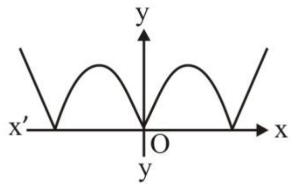
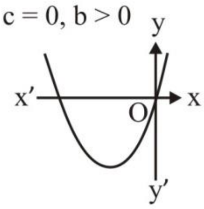
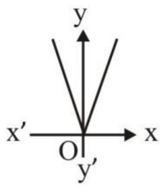
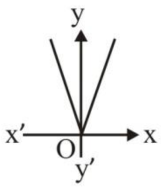
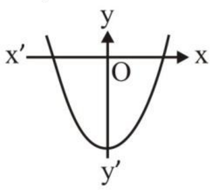
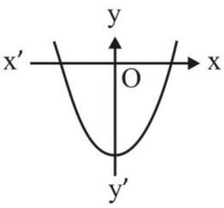
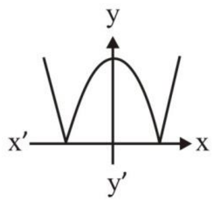
N =3

4. Consider the function f (x) = $x^2 + bx + c$, where D = $b^2 - 4c > 0$

Column – I Condition on b and c		Column – II Number of points of non-differentiability of g(x) = $\|f(\|x\|)\|$	
(A)	b < 0, c > 0	(p)	1
(B)	c = 0, b < 0	(q)	2
(C)	c = 0, b > 0	(r)	3
(D)	b = 0, c < 0	(s)	5

Key. (A→ s), (B→ r), (C→ p), (D→ q)

Sol.

$g(x) = x^2 + bx + c$	$g(\|x\|) = x^2 + b\|x\| + c$	$f(x) = \|g(\|x\|)\| = \|x^2 + b\|x\| + c\|$
$b < 0, c > 0$ 		
$c = 0, b < 0$ 		
$c = 0, b > 0$ 		
$b = 0, c < 0$ 		

5. Match the following:-

Column – I	Column – II
A) $f(x) = x^2 + 2x + 8$	p) positive integral roots
B) $f(x) = x^2 + 4x - 1$	q) $Min(f(x)) = 7$
C) $x^2 + 6x + 5 = 0$	r) $Max(f(x)) = 3$
D) $x^3 - 6x^2 + 11x - 6 = 0$	s) negative integral roots

Key : A – q; B – r; c – s; d – p

Sol : (a) coefficient x^2 is + ve min $(f(x)) = \frac{4ac - b^2}{4a} = 7$

(b) coefficient x^2 is + ve max $(f(x)) = \frac{4ac - b^2}{4a} = 3$

(c) roots are ale – 5, - 1

(d) The roots ale 1,2,3, only +ve into roots.

6. If $x^4 - 6x^3 + 8x^2 + 4ax - 4a^2 = 0$, $a \in R$, then match the following

Column – I	Column – II
A) Equation will have 4 real and distinct roots for $a \in$	p) (0, 1)
B) Equation will have 2 distinct real roots for $a \in$	q) (3, 4)
C) Equation will have at least one negative root for $a \in$	r) $(-2, -1)$
D) Equation will have 2 equal and 2 distinct real roots for $a \in$	s) {2}

Key : A-p; B – q, r; C – p, q, r, s; D – s

Sol : $(x^2 - 2x - 2a)(x^2 - 4x + 2a) = 0$

now $D_1 = 4(1 + 2a)$

$D_2 = 8(2-a)$

7. Match the statements/expressions in Column I with the open intervals in Column II

	Column I		Column II
(A)	If a, b > 0 and a.b = 2a + 3b minimum value of ab	(p)	−1
(B)	Number of real roots of equation $x^2 - 4x + 6 = 2\sin\left(\frac{\pi x}{4}\right)$ is/are	(q)	0
(C)	the equation $x^3 - 6x^2 + 9x + \lambda = 0$ have exactly one root in (1, 3) then [λ + 3] is, where [ ] is GIF	(r)	1
(D)	If $x^2 + 3\lambda x + 2 = 0$ & $(b-c)x^2 + (c-a)x + (a-b) = 0$ have both roots common then [λ + 1] is	(s)	2
		(t)	24

Key. (A - t), (B – r), (C – pqrs), (D - q)

Sol. (A) A.M. ≥ G.M.

$$\frac{2a+3b}{2} \geq \sqrt{2a.3b}$$

Or $\frac{ab}{2} \geq \sqrt{6ab}$

(B) L.H.S Max = R.H.S. Min when x = 2

(C) $f(x) = x^3 - 6x^2 + 9x + \lambda$

$f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3)$

$f'(x) \leq 0 \;\; x$ in $(1,3)$

For f(x) = 0 to have exactly one root in (1, 3)

f(1) f(3) < 0

$\lambda(\lambda+4) < 0 \;\; \therefore \;\; -4 < \lambda < 0$

$[\lambda+3] = -1, 0, 1, 2$

(D) $(b-c)x^2 + (c-a)x + a - b = 0$ have 1 as a both root.

Therefore, $1 + 3\lambda + 2 = 0$

Therefore, $\lambda = -1$

8. Match the following: -

COLUMN –I		COLUMN – II	
A	If $a^2 - 4a - 3 = 0$, then the value of $\frac{a^3 - a^2 + a - 1}{a^2 - 1}\left(a^2 \neq 1\right) =$	P	3
B	The number of value (s) of x satisfying the equation $\sqrt[4]{\lvert x-3\rvert^{x+1}} = \sqrt[3]{\lvert x-3\rvert^{x-2}}$ is	Q	2

C	The number of value (s) of x satisfying the equation $3^x+1-\left|3^x-1\right|=2\log_5\left|6-x\right|$ is	R	4
D	If the sum of the first 2n terms of the A.P 2,5,8,………, is equal to the sum of the first n terms of the A.P., 57,59,61,…, then n equals	S	11

Key. A – R; B – R; C – Q; D – S

Sol. (A) Given $a^2-4a+1=4 \Rightarrow a^2+1=4(1+a)$

$$y=\frac{(a-1)(a^2+1)}{a^2-1}=\frac{a^2+1}{a+1}=\frac{4(a+1)}{a+1}=4$$

(B) $\sqrt[4]{|x-3|^{x+1}}=\sqrt[3]{|x-3|^{x-2}}$ .taking $\log \frac{x+1}{4}\log|x-3|=\frac{x-2}{3}\log|x-3|$

$\Rightarrow \log|x-3|=0$ or $\frac{x+1}{4}-\frac{x-2}{3}=0$

$\Rightarrow \log|x-3|=0$ or $\frac{x+1}{4}-\frac{x-2}{3}=0$

$\Rightarrow x=4,2$ or $x=11$ and x = 3

(C) critical pts $x=0,6$

Case – I: $x\ge 6$ $3^x+1-\left(3^x-1\right)=2\log_5(6-x)\Rightarrow x=11$

Case – II: $0\le x\le 6$ $3^x+1-\left(3^x-1\right)=2\log_5(6-x)\Rightarrow x=1$

Case – III: $x<0\Rightarrow 3^x+1+3^x-1=2\log_5(6-x)$ $\Rightarrow 3^x=\log_5(6-x)$ $\Rightarrow$ no solution

(D) $\frac{2n}{2}\left(4+(2n-1)3\right)=\frac{n}{2}\left(114+(n-1)2\right)$

$\Rightarrow n=11$

9. Let f : R → R, f(x) = $2x^3-3(k+2)x^2+12kx-7$, $-4\le k\le 6$, $k\in I$ then the exhaustive set of values of k for f(x)

Column – I		Column – II	
(A)	to have only one real root	(p)	{–1}
(B)	to have two equal roots	(q)	{0, 1, 2, 3, 4, 5}
(C)	to be invertible	(r)	{–4, –3, –2, 6}
(D)	to have three real and distinct roots	(s)	{2}

Key. (A – q); (B – p); (C – s); (D – r)

Sol. $f(x)=2x^3-3(k+2)x^2+12kx-7$

$f'(x)=6[x^2-(k+2)x+2a]=6(x-k)(x-2)$

(A) For f(x) to have only one real root k = 2 or f(k) f(2) > 0 $\Rightarrow$ k = 0, 1, 2,3, 4, 5

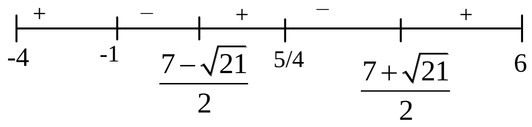
(B) For f(x) to have two equal roots, $k\ne 2$ and f(k) f(2) = 0 $\Rightarrow$ k = –1.

(C) for f(x) to be invertible $f'(x)\ge 0\ \forall\ x\in R \Rightarrow k=2$

(D) for f(x) to have three real and distinct roots, $k\ne 2$ and f(k) f(2) < 0

$(2k^3-3(k+2)k^2+12k^2-7)(16-12(k+2)+24k-7)<0$

$\Rightarrow (k^3-6k^2+7)(4k-5)>0 \Rightarrow (k+1)(k^2-7k+7)(4k-5)>0.$



$\Rightarrow$ k = −4, − 3, −2, 6

10. Match the following: -

Column – I	Column – II
a) If β be a root of the equation $x^5-1=0$, then $\beta^{15}+\beta^{16}+....+\beta^{50}$ is	p) 4
b) If $2f(x^2)+3f\left(\frac{1}{x^2}\right)=x^2-1$ then $f(1)$ is	q) 1
c) The no.of solutions of $\|x+1\|=\|x-1\|$ is	r) 3
d) The least positive integer for which $4^x+8^{\frac{2}{3}(x-2)}-72-4^{x-\frac{3}{2}}$ is non-negative	s) 0

Key. a) q b) s c) q d) p

Sol. Conceptual

11. For the following questions, match the items in column-I to one or more items in column-ii

Column I		Column II
A) If ${}^8C_{k+2}+2.{}^8C_{k+3}+{}^8C_{k+4}>{}^{10}C_4$, then the Quadratic equations whose roots are α,β and α^k,β^k have *m* common roots, then *m*=	P)	1
B) If the number of solutions of the equation $\left\|2x^2-5x+3\right\|+(x-1)=0$ is (are) *n*, then *n*=	Q)	2
C) If the constant term of the quadratic expression	R)	0
$\sum_{k=1}^{n}\left(x-\frac{1}{k+1}\right)\left(x-\frac{1}{k}\right)$ as $n\to\infty$ is *p*, then *p*=	S)	-1
D) The equation $x^2+4a^2=1-4ax$ and $x^2+4b^2=1-4bx$ have only one root in common, then the value of $\|a-b\|$ is	T)	-2

Key. A-q;B-p;C-p;D-p

Sol. Given ${}^8C_{k+2}+2{}^8C_{k+3}+{}^8C_{k+4}>{}^{10}C_4$

$$\Rightarrow\left({}^8C_{k+2}+{}^8C_{k+3}\right)+\left({}^8C_{k+3}+{}^8C_{k+4}\right)>{}^{10}C_4$$

$$\Rightarrow{}^9C_{k+3}+{}^9C_{k+4}>{}^{10}C_4$$

$\Rightarrow{}^{10}C_{k+4}>{}^{10}C_4$ only ${}^{10}C_5>{}^{10}C_4$ Þ $K+4=5$ Þ $K=1$

$\therefore\ \alpha^k=\alpha$ and $\beta^k=\beta$

Hence quadratic equation having roots α and β and α^k and β^k are identical and have both roots common.

$\therefore\ m=2$

(B) For $1 \le x < \frac{3}{2} or \frac{3}{2} \le x < \infty, x-1>0$

Therefore no solution is possible

For $x \le 1,$ given equation is $\left(2x^2-5x+3\right)+x-1=0$

$\therefore\ 2x^2-4x+2=0 \Rightarrow x^2-2x+1=0 \Rightarrow x=1.$

$\therefore$ The equation has only one solution

$\therefore\ n=1.$

(C) Constant term $C=\frac{1}{1.2}+\frac{1}{2.3}+............+\frac{1}{n(n+1)}$

$$t_n=\frac{1}{n(n+1)}=\frac{1}{n}-\frac{1}{n+1}$$

$$\therefore \qquad C=\sum_{x=1}^{n} t_n = 1-\frac{1}{n+1} \qquad \therefore p=1$$

D)

$$(x+2a)^2=1$$

$$(x+2b)^2=1$$

$$x=\pm 1-2a, x=\pm 1-2b$$

$$1-2a=-1-2b \Rightarrow b-a=-1$$

$$-1-2a=1-2b \Rightarrow b-a=1$$

$$\Rightarrow \qquad |a-b|=1$$

12. Match the statements of Column I with values of Column II.

Column I		Column II
A) The least positive integral values of λ for which $(\lambda-2)x^2+8x+(\lambda+4)>0,$ for all real x is	P)	3
B) The equation $x^2+2(a^2+1)x+(a^2-14a+48)=0$ possesses roots of opposite signs then x value of $'a'$ can be	Q)	5
C) If the equation $ax^2+2bx+4c=16$ has no real roots and $a+c>b+4,$ then integral value of c can be equal to	R)	7
D) If N be the number of solution of the equation $\left\|x^2-x-6\right\|=x+2$ then the value of N is	S)	12
	T)	20

Key. A-q;B-r;C-qrst;D-p

Sol. A) $\lambda>2$

$64-4(\lambda-2)(\lambda+4)<0$

$\Rightarrow \quad (\lambda+6)(\lambda-4)>0$

$\lambda<-6 \text{ or } \lambda>4$

$\therefore$ The least positive integral value of λ is 5

(B) Roots are of opposite signs

$\Rightarrow \quad a^2-14a+48<0$

$(a-6)(a-8)<0$, so a can be 7

The equation is $x^2+100x-1=0$

$\therefore$ discriminant = D = $100^2+4>0$

$\therefore$ Roots are real

C)

Let $f(x)=ax^2+2bx+4c-16$

Clearly $f(-2)=4a-4b+4c-16$

$=4(a-b+c-4)>0$

$=f(x)>0, \forall x\in R$

$\Rightarrow \quad f(0)>0 \Rightarrow 4c-16>0$

$\Rightarrow \quad c>4$

(D) $\because \left|x^2-x-6\right|=x+2$

$\Rightarrow \quad \left|(x-3)(x+2)\right|=x+2$

$$\Rightarrow \quad |x-3||x+2| = x+2$$

$$\Rightarrow \quad \begin{cases} (x-3)(x+2) = x+2, & x < -2 \\ -(x-3)(x+2) = x+2, & -2 \le x < 3 \\ (x-3)(x+2) = x+2, & x > 3 \end{cases}$$

$$\Rightarrow \quad \begin{cases} x = 4, & x < -2 \\ x = -2, 2 & -2 \le x < 3 \\ x = 4, & x < 3 \end{cases}$$

Hence, $x = -2, 2, 4$

N =3

13. Let $y = \dfrac{x^2 - 12x + 100}{x^2 + 12x + 100}, x \in R$

Match the following

Column – I	**Column – II**
a) Greatest value of y	p) -10
b) Least value of y	q) 10
c) Greatest value of y is attained at	r) $1/4$
d) Least value of y is attained at	s) 4
	t) -4

Key. A $\rightarrow$ s; B $\rightarrow$ r; C $\rightarrow$ p; D $\rightarrow$ q

Sol. $(x^2 + 12x + 100)y = x^2 - 12x + 100$

$\Rightarrow (y-1)x^2 + 12(y+1)x + 100(y-1) = 0$

As x is real $36(y+1)^2 - 100(y-1)^2 \ge 0$

$\Rightarrow (y - 1/4)(y-4) \le 0 \Rightarrow 1/4 \le y \le 4$

For $y = 4$, we get $x = -10$

For $y = -4$, we get $x = 10$

14. The number of rational roots of

Column – I	**Column – II**
a) $x^3 - px^2 + 1 = 0, p > 2$	p) 3
b) $x^{10} - x^9 - 2 = 0$	q) 2
c) $(x+1)(x+2)(x+3)(x+4) = 24$	r) 1
d) $(\log_3 x)^2 + \log_{3x}(3/x) = 1$	s) 0

Key. A $\rightarrow$ q,r,s; B $\rightarrow$ r; C $\rightarrow$ q; D $\rightarrow$ p

Sol. (a) Any rational root of $x^3 - px^2 + 1 = 0$ must be an integer. But for $a \in I, a^2(p-a) = 1$ is not possible if $p > 2$

(b) As in (i) any root of $x^{10} - x^9 - 2 = 0$ must be an integer. Clearly $x = -1$ satisfies the given equation. For, $x \ne -1, x \in I, x^9(x-1) = 2$ is not possible

(c) The given equation can be written as $(x^2 + 5x + 4)(x^2 + 5x + 6) = 24$

Put $x^2 + 5x = t$ to obtain $t^2 + 10t = 0 \Rightarrow t = 0, t = -10$

For $t = 0, x^2 + 5x = 0 \Rightarrow x = 0, x = -5$

For $t=-10, x^2+5x=-10$ does not have rational roots

(d) Put $\log_3 x = t$ to obtain $t^2+\frac{1-t}{1+t}=1 \Rightarrow t^3+t^2-2t=0$

$\Rightarrow t(t+2)(t-1)=0 \Rightarrow t=0,1,-2$

This gives $x=1,3,1/9$

15. Let α, β be roots of $ax^2+bx+c=0$.
Match the equation on the left with its roots on the right

Column - I	Column – II
a) $(x-b)^2+b(x-b)+ac=0$	p) $2\alpha, 2\beta$
b) $ax^2+2bx+4c=0$	q) $-\alpha/a, -\beta/a$
c) $4a^2x^2-b^2+4ac=0$	r) $a\alpha+b,\ a\beta+b$
d) $a^3x^2-abx+c=0$	s) $\alpha+\frac{b}{2a},\ \beta+\frac{b}{2a}$

Key. A $\rightarrow$ r; B $\rightarrow$ p; C $\rightarrow$ s; D $\rightarrow$ q

Sol. (a) Write equation as $a\left(\frac{x-b}{a}\right)^2+b\left(\frac{x-b}{a}\right)+c=0 \Rightarrow \frac{x-b}{a}=\alpha,\beta$

(b) $a\left(\frac{x}{2}\right)^2+b\left(\frac{x}{2}\right)+c=0 \Rightarrow \frac{x}{2}=\alpha,\beta$

(c) $x=\frac{\pm\sqrt{b^2-4ac}}{2a}=\alpha+\frac{b}{2a}, \beta+\frac{b}{2a}$

(d) $a(-ax)^2+b(-ax)+c=0 \Rightarrow -ax=\alpha,\beta \Rightarrow x=-\alpha/a, -\beta/a$

16. Match the following for the equation $x^2+a|x|+1=0$ where a is a parameter

Column - I	Column – II
a) No real root	p) $a<-2$
b) Two real roots	q) $a=-2$
c) Three real roots	r) ϕ
d) Four real roots	s) $a\geq 0$
	t) $a<-5$

Key. A $\rightarrow$ s; B $\rightarrow$ q; C $\rightarrow$ r; D $\rightarrow$ p,t

Sol. If $x>0$ then $x^2+ax+1=0 \Rightarrow x=\frac{-a\pm\sqrt{a^2-4}}{2}$ --- (A)

If $x<0$, then $x^2+ax+1=0 \Rightarrow x=\frac{a\pm\sqrt{a^2-4}}{2}$ --- (B)

We must have $a^2-4\geq 0$ for real roots

Now both roots in (A) are negative if $a>0$

$\Rightarrow$ Original equation does not have roots.

Again both rotos in (B) are positive if $a>0$

$\Rightarrow$ Original equation does not have roots.

If $a=-2$ then equation is $x^2-2|x|+1=0$ or $(|x|-1)^2=0 \Rightarrow x=1$ *or* -1

$\Rightarrow$ Two real roots.

Now equation has four real roots if $a<-2$, since both roots given by (A) or (B) will satisfy the respective assumptions.

Finally the equation cannot have three real roots for any a.

40. Match the positive value of x on the left with the value on the right

	Column-I		Column II
(A)	$5^2 5^4 5^6 \ldots . 5^{2x} = (0.04)^{-28}$	(p)	3 $\log_3 5$
(B)	$x^2 = (0.2)^{\log_{\sqrt{5}}\left(\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\ldots\right)}$	(q)	4
(C)	$x = (0.16)^{\log_{2.5}\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\right)}$	(r)	2
(D)	$3^{x-1}+3^{x-2}+3^{x-3}+\ldots$	(s)	7
	$=2\left(5^2+5+1+\frac{1}{5}+\frac{1}{5^2}+\ldots\right)$	(t)	even integer

Key. A→s; B→r,t; C-q,t; D→p

Sol. A) $5^{2+4+6+\ldots 2x} = (25)^{28}$

$\Rightarrow 5^{x(x+1)} = 5^{56}$

$\Rightarrow x^2 + x - 56 = 0 \Rightarrow x = 7 \text{ as } x > 0$

B) $2\log_5 x = \log_{\sqrt{5}}\left(\frac{1/4}{1-1/2}\right)\log_5(0.2)$

$= \log_{\sqrt{5}}\left(\frac{1}{2}\right)\log_5\left(\frac{1}{5}\right)$

$= -\frac{\log_5\left(\frac{1}{2}\right)}{\log_5\sqrt{5}} = \log_5 4$

$\Rightarrow x = 2$

C) log x = $\log_{2.5}\left(\frac{1/3}{1-1/3}\right)\log(0.16)$

$=\log_{5/2}(1/2)\log(2/5)^2$

$=\log 4$

$\Rightarrow$ x = 4

D) $3^x \frac{(1/3)}{1-1/3} = \frac{2(5^2)}{1-1/5}$

$\Rightarrow \frac{1}{2}(3^x) = \frac{1}{2}(5^3)$

$\Rightarrow x = 3\log_3 5$

29.

	Column I		**Column II**
(A)	The number of integral solution of $\frac{x+2}{x^2+1} > \frac{1}{2}$ is	(p)	2
(B)	If $x \in Z$ (the set of integers) such that $x^2 - 3x < 4$, then the number of possible values x is	(q)	4

(C)	The number of integral values of x satisfying $\|\|x-1\|-1\|\le 1$	(r)	5
(D)	The number of solutions of $\|[x]-2x\| = 4$, where [x] is the greatest integer $\le$ x, is	(s)	3

Key. A→s; B→q; C→r; D→q

Sol. (A) $\dfrac{x+2}{x^2+1} > \dfrac{1}{2}$

$\Rightarrow 2x+4 > x^2+1$

$\Rightarrow -x^2+2x+3>0 \quad \because x^2+1>0$

$\Rightarrow -1<x<3$

(by sign scheme)

But x is an integer $\therefore$ x =0, 1, 2

$\therefore$ there are 3 values of x

$\therefore$ A – s

(B) $x^2-3x<4$

$\Rightarrow x^2-3x-4<0$

$\Rightarrow (x-4)(x+1)<0$

$\Rightarrow x-4<0, x+1>0$

or x – 4 < 0, x + 1 > 0

$\Rightarrow$ x > 4, x < –1 (not possible)

or x < 4, x > –1 $\Rightarrow$ – 1 < x < 4

But xis an integer $\therefore$ x = 0, 1, 2, 3.

$\therefore$ number of values of x = 4

$\therefore$ B – q

(C) $\left\|\left|x-1\right|-1\right\| \le 1$

$\Rightarrow 1-1 \le |x-1| \le 1+1$

$\Rightarrow 0 \le |x-1| \le 2$

$\Rightarrow$ 1-2 $\le$ x $\le$ 1 + 2

$\Rightarrow$ –1 $\le$ x $\le$ 3

$\Rightarrow x \in [-1,3] . \therefore C-p$

(D) If $x = n \in Z, |n-2n| = 4 \;\; \therefore n = \pm 4.$

If x = n + k, $n \in Z, 0<k<1$ then $|n-2(n+k)|$ = 4

$\therefore |-n-2k| = 4.$ It is possible if k = $\dfrac{1}{2}$

then $|-n-1| = 4$ i.e. n + 1 = $\pm$ 4

$\therefore$ n = 3, –5

$\therefore$ there are 4 values of x.

30.

	Column I		Column II
(A)	The number real solutions of the equation $x^2 - \|x\| - 2 = 0$ is	(p)	0
(B)	For the equation $3x^2 + px + 3 = 0, p > 0,$ if one of the root is square of the other, then p is	(q)	1
(C)	The number of real values of k for which the system of equations $(k+1)x + 8y = 4k$, $kx + (k+3)y = 3k-1$ has infinitely many solution is	(r)	2
(D)	Number of roots of the equation $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$ is	(s)	3

Key. A$\rightarrow$ r; B$\rightarrow$ s; C$\rightarrow$ q; D$\rightarrow$ p

Sol. A $|x|^2 - |x| - 2 = 0 \Rightarrow (|x|+1)(|x|+2) = 0$ $|x| = 2 \Rightarrow$ x = ±2

B Let α, α^2 be toots

product of root $\alpha.\alpha^2 = \frac{3}{3}$

$\Rightarrow \alpha = 1, \omega, \omega^2$

If $\alpha = 1$ then p = –6 not acceptable as p > 0

if $\alpha = \omega, \alpha^2 = \omega^2$ then p = 3

C. $\frac{K+1}{K} = \frac{8}{K+3} = \frac{4K}{3K-1} \Rightarrow K = 1$

D. $x - \frac{2}{x-1} = 1 - \frac{2}{x-1} \Rightarrow x = 1$ but at x = 1, $\frac{2}{x-1}$ is not defined.