Matrices & Determents

Single Correct Answer Type

1. A and B are two non singular matrices so that $A^6 = I$ and $AB^2 = BA(B \neq I)$. A value of K so that $B^K = I$ is

a) 31 b) 32

c) 63 d) 64

Key. C

Sol. $A^5(AB^2) = A^5BA.$

$\Rightarrow B^2 = A^5BA$

$\Rightarrow B^4 = (A^5BA)(A^5BA) = A^5B^2A = A^5(A^5BA)A$

$\Rightarrow B^4 = A^4BA^2$

$\Rightarrow B^8 = (A^4BA^2)(A^4BA^2) = A^4B^2A^2 = A^4(A^5BA)A^2$

$\Rightarrow B^8 = A^3BA^3$

$\Rightarrow B^{16} = (A^3BA^3)(A^3BA^3) = A^3B^2A^3 = A^3(A^5BA)A^3 = A^2BA^4$

$A^{32} = (A^2BA^4)(A^2BA^4) = A^2B^2A^4 = A^2(A^5BA)A^4 = ABA^5$

$A^{64} = (ABA^5)(ABA^5) = AB^2A^5 = A(A^5BA)A^5 = B \Rightarrow A^{63} = I$

2. For each real number x such that $-1 < x < 1$, let A (x) be the matrix $(1-x)^{-1}\begin{bmatrix} 1 & -x \\ -x & 1 \end{bmatrix}$ and $z = \frac{x+y}{1+xy}$. Then,

(A) A(z) = A(x) + A(y) (B) $A(z) = A(x)\,[A(y)]^{-1}$

(C) A(z) = A(x) A(y) (D) A(z) = A(x) – A(y)

Key. C

Sol. $A(z) = A\left(\frac{x+y}{1+xy}\right) = \left[\frac{1+xy}{(1-x)(1-y)}\right]\begin{bmatrix} 1 & -\left(\frac{x+y}{1+xy}\right) \\ -\left(\frac{x+y}{1+xy}\right) & 1 \end{bmatrix}$

$\therefore A(x).A(y) = A(z)$

3. A and B are two non singular matrices so that $A^6 = I$ and $AB^2 = BA(B \neq I)$. A value of K so that $B^K = I$ is

a) 31 b) 32 c) 63 d) 64

Key. C

Sol. $A^5\left(AB^2\right)=A^5BA.$

$\Rightarrow B^2=A^5BA$

$\Rightarrow B^4=\left(A^5BA\right)\left(A^5BA\right)=A^5B^2A=A^5\left(A^5BA\right)A$

$\Rightarrow B^4=A^4BA^2$

$\Rightarrow B^8=\left(A^4BA^2\right)\left(A^4BA^2\right)=A^4B^2A^2=A^4\left(A^5BA\right)A^2$

$\Rightarrow B^8=A^3BA^3$

$\Rightarrow B^{16}=\left(A^3BA^3\right)\left(A^3BA^3\right)=A^3B^2A^3=A^3\left(A^5BA\right)A^3=A^2BA^4$

$A^{32}=\left(A^2BA^4\right)\left(A^2BA^4\right)=A^2B^2A^4=A^2\left(A^5BA\right)A^4=ABA^5$

$A^{64}=\left(ABA^5\right)\left(ABA^5\right)=AB^2A^5=A\left(A^5BA\right)A^5=B\Rightarrow A^{63}=I$

4. If matrix A = $[a_{ij}]_{3\times3}$, matrix B = $[b_{ij}]_{3\times3}$ where $a_{ij}+a_{ji}=0$ and $b_{ij}-b_{ji}=0$, then $A^4 . B^3$ is

(A) skew-symmetric matrix (B) singular

(C) symmetric (D) zero matrix

Key. B

Sol. Since matrix A is skew-symmetric,

$\therefore$ | A | = 0

$\therefore$ $| A^4 . B^3 | = 0$

5. If A = $\begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$, then det (Adj (Adj A)) is

(A) $(14)^4$ (B) $(14)^6$ (C) $(14)^9$ (D) $(14)^2$

Key. A

Sol. | A | = = (1 + 2) – 2 (– 1 – 4) – (1 – 2)

= 3 + 10 + 1 = 14

$\therefore$ det (Adj (Adj A)) = $|\text{Adj } A|^2 = |A|^4 = (14)^4$

6. In the expansion of $\left(\sqrt{\frac{q}{p}}+\sqrt[10]{\frac{p^7}{q^3}}\right)^n$, there is a term similar to pq, then that term is equal to

(A) 210 pq (B) 252 pq (C) 120 pq (D) 45 pq

Key. B

7. Let x, y, z be real numbers such that 3x, 4y and 5z form a geometric progression while x, y, z form an H.P. Then the value of $\frac{x}{z}+\frac{z}{x}=\frac{m}{n}$ where *m* and *n* are relatively prime then, (m + n) is equal to

(A) 29 (B) 39 (C) 49 (D) 59

Key. C

8. If A is a square matrix fo order 3 much that $|A| = 2$ then $\left|\left(adj\ A^{-1}\right)^{-1}\right|$ is

(A) 1 (B) 2 (C) 4 (D) 8

Key. C

9. Let A and B be square matrices of same order satisfying AB = A and BA = B. Then A^2B^2 equals, (O being the zero matrix of the same order as B)

(A) A (B) B (C) I (D) O

Key: A

Hint Conceptual

10. If A and B are square matrices of the same order and A is non-singular, then for a positive integer *n,* $(A^{-1}BA)^n$ is equal to

A) $A^{-n} B^n A^n$ B) $A^n B^n A^{-n}$ C) $A^{-1} B^n A$ D) $n(A^{-1} BA)$

Key: C

Hint: $(A^{-1}BA)^2 = (A^{-1}BA)(A^{-1}BA) = A^{-1}B(AA^{-1})BA = A^{-1}BIBA = A^{-1}B^2A$

$\Rightarrow (A^{-1}BA)^3 = (A^{-1}B^2A)(A^{-1}BA) = A^{-1}B^2(AA^{-1})BA = A^{-1}B^2IBA = A^{-1}B^3A$ and so on

$\Rightarrow (A^{-1}BA)^n = A^{-1}B^nA$

11. If A is a skew–symmetric matrix of order 3, then the matrix A^4 is

(A) skew symmetric (B) symmetric (C) diagonal (D) none of those

Key: B

Hint: We have $A^T = -A$

$(A^4)^T = (A.A.A.A.)^T = A^T A^T A^T A^T$

$\Rightarrow (-A)(-A)(-A)(-A)$

$= (-1)^4 A^4 = A^4$

12. If A and B are symmetric matrices of same order and X= AB + BA and Y = AB – BA, then $(XY)^T$ is equal to

(A) XY (B)YX (C) – YX (D)none of these

Key: C

Hint: X = AB + BA $\Rightarrow X^T = X$

and Y = AB – BA $\Rightarrow Y^T = -Y$

Now, $(XY)^T = Y^T \times X^T = -YX.$

13. If A and B are any two different square matrices of order n with A – B is non-singular $A^3 = B^3$ and A(AB) = B(BA) , then

(A) $A^2 + B^2 = O$ (B) $A^2 + B^2 = I$ (C) $A^2 + B^3 = I$ (D) $A^3 + B^3 = O$

Key: A

Hint: $A^3 = B^3$.......(i)

$A^2B = B^2A$.................(ii)

$$\left(A^2+B^2\right)(A-B)=0$$

$$\because |A-B| \neq 0$$

$$A^2+B^2=0$$

14. A square matrix A is said to be nilpotent of index m. If $A^m = 0$, now, if for this A $(I - A)^n = I + A + A^2 + ... + A^{m-1}$, then n is equal to

(A) 0 (B) m (C) - m (D) -1

Key: D

Hint: Let $B = I + A + A^2 + ... + A^{m-1}$

$\Rightarrow B (I - A) = (I + A + A^2 + ... + A^{m-1}) (I - A)$

$= I - A^m = I$

$\Rightarrow B = (I - A)^{-1} \Rightarrow n = -1.$

15. If $A=\begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ and $B=\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then A^8 equals

(a) 4B (b)128 B (c) –128 B (d) –64 B

Key: b

Hint: We have A = iB

$\Rightarrow A^2 = (iB)^2 = i^2B^2 = -B^2 = -\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = -2B$

$\Rightarrow A^4 = (-2B)^2 = 4B^2 = 4(2B) = 8B$

$\Rightarrow (A^4)^2 = (8B)^2 \Rightarrow A^8 = 64B^2 = 128B$

16. The number of positive integral solutions of the equation $\begin{vmatrix} y^3+1 & y^2z & y^2x \\ yz^2 & z^3+1 & z^2x \\ yx^2 & x^2z & x^3+1 \end{vmatrix} = 11$ is

(A) 1 (B) 2 (C) 3 (D) 4

Key: C

Hint: Multiply by y,z and x in rows 1,2 and 3 respectively and then take common y, z and x from column 1,2 and 3 respectively, then

$$\begin{vmatrix} y^3+1 & y^3 & y^3 \\ z^3 & z^3+1 & z^3 \\ x^3 & x^3 & x^3+1 \end{vmatrix} = 11$$

$$\Rightarrow \begin{vmatrix} 1 & 0 & y^3 \\ -1 & 1 & z^3 \\ 0 & -1 & x^3+1 \end{vmatrix} = 11 \qquad \left(C_1 \to C_1 - C_2 \text{ and } C_2 \to C_2 - C_3\right)$$

$$\Rightarrow 1\left(x^3+1+z^3\right)+y^3(1)=11 \Rightarrow x^3+y^3+z^3=10$$

So solution are (1,1,2) , (1,2,1) or (2,1,1)

17. If $a - 2b + c = 1$, then the value of $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ is

(A) x (B) $-x$ (C) -1 (D) 1

Key. C

Sol. $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ $\quad a-2b+c=1$, $(a-b)+(c-b)=1$

Apply the operation,

$R_1 \to R_1 - 2R_2 + R_3$

$R_3 \to R_3 - R_2$, the determinant reduces to

$$\begin{vmatrix} 0 & 0 & 1 \\ x+2 & x+3 & x+b \\ 1 & 1 & c-b \end{vmatrix} = -1$$

18. If A is involutary matrix, then which of the following is/are correct?

(A) I + A is idempotent (B) I – A is idempotent

(C) (I + A)(I – A) is singular (D) $\frac{I+A}{3}$ is idempotent

Key. C

Sol. $A^2 = I$

$(I + A)(I - A) = I - A^2 = I - I = O$

19. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, $C = ABA^T$, then A^TC^nA equals to $\left(n \in I^+\right)$

(A) $\begin{bmatrix} -n & 1 \\ 1 & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -n \\ 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 1 \\ 1 & -n \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$

Key. D

Sol. $A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$

$AA^T = I$ (i)

Now, $C = ABA^T$

$\Rightarrow$ $A^TC = BA^T$ (ii)

Now $A^TC^nA = A^TC.C^{n-1}A = BA^TC^{n-1}A$ (from (ii))

$= BA^TC.C^{n-2}A = B^2A^TC^{n-2}A =$ ....

$= B^{n-1}A^TCA = B^{n-1}BA^TA = B^n = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix}$

20. If $p+q+r=0$ and $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = k\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$, then K=

1) 0 2) abc 3) pqr 4) a+b+c

Key. 3

Sol. $p+q+r=0 \Rightarrow p^3+q^3+r^3=3pqr$

$$\begin{vmatrix} pa & qb & rc \\ qc & rb & pb \\ rb & pc & qa \end{vmatrix} = pqr(a^3+b^3+c^3-3abc)$$

$$pqr\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} \Rightarrow k = pqr$$

21. If $a = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$, then $\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix}$ is

1) purely real 2) purely imaginary

3) a complex number 4) a

Key. 2 or 3

Sol. $a = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} = w^2$

$\therefore \begin{vmatrix} 1 & 1 & 1 \\ 1 & w^2 & w \\ 1 & w & w^2 \end{vmatrix} = 3(w-w^2)$ purely imaginary

22. $\begin{vmatrix} {}^xC_r & {}^xC_{r+1} & {}^xC_{r+2} \\ {}^yC_r & {}^yC_{r+1} & {}^yC_{r+2} \\ {}^zC_r & {}^zC_{r+1} & {}^zC_{r+2} \end{vmatrix} - \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix} =$

1) 0 2) 2^n 3) ${}^{x+y+z}C_r$ 4) ${}^{x+y+z}C_{r+2}$

Key. 1

Sol. $\begin{vmatrix} {}^xC_r & {}^xC_{r+1} & {}^xC_{r+2} \\ {}^yC_r & {}^yC_{r+1} & {}^yC_{r+2} \\ {}^zC_r & {}^zC_{r+1} & {}^zC_{r+2} \end{vmatrix} = \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+1}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+1}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+1}C_{r+2} \end{vmatrix}$

By applying $C_2 \to C_2 + C_1, C_3 \to C_3 + C_2$

Now apply $C_3 \to C_3 + C_2$, $\begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix}$

$\therefore$ Ans = 0

23. If A is an orthogonal matrix of order n, then the value of |adj.(adj A)| is
(A) 0 (B) ± 1
(C) n (D) n – 2

Key. B

Sol. $AA' = I$
$\Rightarrow$ $|A| = \pm 1$
$\therefore$ |adj. (adj. A)|
$= |A|^{(n-1)^2} = \pm 1.$

24. If a, b, c, d > 0; x ∈ R and $(a^2 + b^2 + c^2)x^2 - 2(ab + bc + cd)x + b^2 + c^2 + d^2 \le 0$, then
$$\begin{vmatrix} 33 & 14 & \log a \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} =$$
(A) 1 (B) – 1
(C) 0 (D) none of these

Key. C

Sol. We have
$(a^2 + b^2 + c^2)x^2 - 2(ab + bc + cd)x + b^2 + c^2 + d^2 \le 0$
$\Rightarrow$ $(ax - b)^2 + (bx - c)^2 + (cx - d)^2 \le 0$
$\Rightarrow$ $(ax - b)^2 + (bx - c)^2 + (cx - d)^2 = 0$
$\Rightarrow$ $\frac{b}{a} = \frac{c}{b} = \frac{d}{c} = x$
$\Rightarrow$ $b^2 = ac$ or $2\log b = \log a + \log c$,

Now, $$\begin{vmatrix} 33 & 14 & \log a \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} = \begin{vmatrix} 130 & 54 & \log a + \log c \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix}$$ [Apply $R_1 \to R_1 + R_3$]

$$\begin{vmatrix} 0 & 0 & 0 \\ 65 & 27 & \log b \\ 97 & 40 & \log c \end{vmatrix} = 0$$ [Apply $R_1 \to R_1 - 2R_2$]

25. A square matrix P satisfies $P^2 = I - P$, where I is an identity matrix of order as order of P. if $P^n = 5I - 8P$, then n =
(a) 4 (b) 5
(c) 6 (d) 7

Key. C

SOL. SINCE $P^2 = I - P$ (GIVEN) ——(1)
$P^3 = P(I - P)$
$P^3 = P - P^2 = P - (I - P)$ (USING) —— (II)
$P^3 = 2P - I$
SIMILARLY $P^4 = 2P^2 - P = 2I - 3P$ AND $P^5 = 5P - 3I$
$P^6 = 5P^2 - 3P = 5I - 8P$
$\therefore$ n = 6

26. If $Y = SX, Z = tX$ all the variables being differentiable functions of x and lower suffices denote the derivative with respect to x and $\begin{vmatrix} X & Y & Z \\ X_1 & Y_1 & Z_1 \\ X_2 & Y_2 & Z_2 \end{vmatrix} \div \begin{vmatrix} S_1 & t_1 \\ S_2 & t_2 \end{vmatrix} = X^n$, then $n =$
a) 1 b) 2 c) 3 d) 4

Key. C

Sol. $$\Delta = \begin{vmatrix} X & SX & tX \\ X_1 & SX_1 + S_1X & tX_1 + t_1X \\ X_2 & SX_2 + 2S_1X_1 + S_2X & tX_2 + 2t_1X_1 + t_2X \end{vmatrix}$$

$$\begin{pmatrix} C_2 \leftarrow C_2 - SC_1 \\ C_3 \leftarrow C_3 - C_1 \end{pmatrix}$$

$$= \Delta = \begin{vmatrix} X & 0 & 0 \\ X_1 & S_1X & t_1X \\ X_2 & 2S_1X_1 + S_2X & 2t_1X_1 + t_2X \end{vmatrix}$$

$$= S^2 \begin{vmatrix} S_1 & t_1 \\ 2S_1X_1 + S_2X & 2t_1X_1 + t_2X \end{vmatrix}$$

$$= = X^3 \le \begin{vmatrix} S_1 & t_1 \\ S_2 & t_2 \end{vmatrix} \left(R_2 \leftarrow R_2 - 2X_1R_1\right)$$

$\therefore n = 3.$

27. If A and B are two non singular matrices and both are symmetric and commute each other then

a) Both $A^{-1}B$ and $A^{-1}B^{-1}$ are symmetric.
b) $A^{-1}B$ is symmetric but $A^{-1}B^{-1}$ is not symmetric
c) $A^{-1}B^{-1}$ is symmetric but $A^{-1}B$ is not symmetric
d) Neither $A^{-1}B$ nor $A^{-1}B^{-1}$ are symmetric

Key. A

Sol. $AB = BA$

Previous & past multiplying both sides by A^{-1}.

$$A^{-1}(AB)A^{-1} = A^{-1}(BA)A^{-1}$$

$$\left(A^{-1}A\right)\left(BA^{-1}\right) = A^{-1}B\left(AA^{-1}\right)$$

$$\Rightarrow \left(BA^{-1}\right)^1 = \left(A^{-1}B\right)^1 = \left(A^{-1}\right)^1 B^1 \text{(reversal laws)}$$

$$= A^{-1}B\left(\text{ as } B = B^1\right)$$

$\left(A^{-1}\right)^1 = A^{-1} \Rightarrow A^{-1}B$ is symmetric

Similarly for $A^{-1}B^{-1}$.

28. If $f(x) = ax^2 + bx + c \quad a, b, c \in R$ and the equation $f(x) - x = 0$ has imaginary roots α and β and γ and δ be the roots of $f(f(x)) - x = 0$, then $\begin{vmatrix} 2 & \alpha & \delta \\ \beta & 0 & \alpha \\ \gamma & \beta & 1 \end{vmatrix}$ is

a) 0 b) purely real c) purely imaginary d) none of these

Key. B

Sol. $f(x) - x > 0 \text{ or, } f(x) - x < 0 \forall x \in R$

$$f(f(x)) - f(x) > 0 \text{ or } f(f(x)) - f(x) < 0$$

Adding, $f(f(x)) - x > 0 \text{ or, } f(f(x)) - x < 0$

$\Rightarrow$ roots of $f(f(x)) - x = 0$ are imaginary.

Let $z = \begin{vmatrix} 2 & \alpha & \delta \\ \beta & 0 & \alpha \\ \gamma & \beta & 1 \end{vmatrix}$

$$\bar{z} = \begin{vmatrix} 2 & \bar{\alpha} & \bar{\delta} \\ \bar{\beta} & 0 & \bar{\alpha} \\ \bar{\gamma} & \bar{\beta} & 1 \end{vmatrix} = \begin{vmatrix} 2 & \beta & \gamma \\ \alpha & 0 & \beta \\ \delta & \alpha & 1 \end{vmatrix} = z$$

29. Suppose a Matrix A satisfies $A^2 - 5A + 7I = 0$ If $A^5 = aA + bI$, then the values of $2a+b$ is.

a) -87 b) -105 c) 1453 d) 1155

Key. A

Sol. $A^3 = AA^2 = A(5A - 7I)$

$= 5A^2 - 7A = 5(5A - 7I) - 7A = 18A - 35I$

$A^4 = A.A^3 = A(18A - 35I) = 18(5A - 7I) - 35A$

$A^5 = 149A - 385I \qquad = 55A - 126I$

$A^5 = 149A - 385I$

$a = 149, b = -385$

30. The digits A, B, C are such that the three digit numbers A88, 6B8, 86C are divisible by 72, then the determinant $\begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 8 & 8 & C \end{vmatrix}$ is divisible by

a) 76 b) 144 c) 216 d) 276

Key. B

Sol. $100A + 80 + 8 = 72\lambda_1$

$600 + 10B + 8 = 72\lambda_2 \quad \lambda_1, \lambda_2, \lambda_3 \in I.$

$800 + 60 + C = 72\lambda_3$

$\begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 8 & 8 & C \end{vmatrix} (R_3 \leftarrow R_3 + 10R_2 + 100R_1)$

$= \begin{vmatrix} A & 6 & 8 \\ 8 & B & 6 \\ 72\lambda_1 & 72\lambda_2 & 72\lambda_3 \end{vmatrix}$

A88 is div. by 72

$\Rightarrow$ A88 is div. by 9

$\Rightarrow$ A+8+8 is div. by 9

$\therefore A = 2$

$6B8$ is div. by 9 $\Rightarrow B = 4.$

31. If the matrix $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ is invertible, then the planes $a_{11}x + a_{12}y + a_{13}z = 0$, $a_{21}x + a_{22}y + a_{23}z = 0$ and $a_{31}x + a_{32}y + a_{33}z = 0$ ($a_{ij} \in R, \forall i, j$)

(A) intersect in a point (B) intersect in a line
(C) have no common point (D) are same

Key. A

Sol. Given matrix A is invertible $\Rightarrow$ det A $\neq 0$
$\Rightarrow$ the given system of equation has only one solution
i.e., (0, 0, 0). Hence option (A) is correct.

32. If A is a skew–symmetric matrix of order 3, then the matrix A^4 is
(A) skew symmetric (B) symmetric
(C) diagonal (D) none of those

Key. B

Sol. We have $A^T = -A$
$(A^4)^T = (A.A.A.A.)^T = A^T A^T A^T A^T$
$\Rightarrow (-A)(-A)(-A)(-A)$
$= (-1)^4 A^4 = A^4$

33. If 'α' is a root of $x^4 = 1$ with negative principal argument, then the principal argument of $\Delta(\alpha)$ where

$$\Delta(\alpha) = \begin{vmatrix} 1 & 1 & 1 \\ \alpha^n & \alpha^{n+1} & \alpha^{n+3} \\ \frac{1}{\alpha^{n+1}} & \frac{1}{\alpha^n} & 0 \end{vmatrix}$$ is

(A) $\frac{5\pi}{14}$ (B) $-\frac{3\pi}{4}$
(C) $\frac{\pi}{4}$ (D) $-\frac{\pi}{4}$

Key. B

Sol. Clearly $\alpha = -i$ where $i^2 = -1$

So $$\Delta(\alpha) = \alpha^n \frac{1}{\alpha^n}\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^3 \\ \frac{1}{\alpha} & 1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -i & i \\ i & 1 & 0 \end{vmatrix}$$ $= 1(-i) + 1(i^2) + (1 + i^2) = -1 - i$

So, principal argument of $\Delta(\alpha)$ is $-\frac{3\pi}{4}$

34. If z is a complex number and $l_1, l_2, l_3, m_1, m_2, m_3$ are all real, then

$$\begin{vmatrix} l_1 z + m_1\bar{z} & m_1 z + l_1\bar{z} & m_1 z + l_1 \\ l_2 z + m_2\bar{z} & m_2 z + l_2\bar{z} & m_2 z + l_2 \\ l_3 z + m_3\bar{z} & m_3 z + l_3\bar{z} & m_3 z + l_3 \end{vmatrix}$$ is equal to

(A) $|z|^2$ (B) 3
(C) $(l_1 l_2 l_3 + m_1 m_2 m_3)^2 |z|^2$ (D) 0

Key. D

Sol. $$\begin{vmatrix} l_1 & m_1 & 0 \\ l_2 & m_2 & 0 \\ l_3 & m_3 & 0 \end{vmatrix} \times \begin{vmatrix} z & \bar{z} & 0 \\ \bar{z} & z & 0 \\ 1 & z & 0 \end{vmatrix} = 0$$

35. Let A, B be square matrix such that A B = 0 and B is non singular then
(A) |A| must be zero but A may non zero (B) A must be zero matrix
(C) nothing can be said in general about A (D) none of these

Key. B

Sol. $A.B = 0 \Rightarrow A.B.B^{-1} = 0.B^{-1}$
$\Rightarrow A.I = 0$
$\Rightarrow A = 0$

36. The value of $\begin{vmatrix} \sqrt{6} & 2i & 3+\sqrt{6} \\ \sqrt{12} & \sqrt{3}+\sqrt{8}i & 3\sqrt{2}+\sqrt{6}i \\ \sqrt{18} & \sqrt{2}+\sqrt{12}i & \sqrt{27}+2i \end{vmatrix}$ is

a) an integer b) a rational number c) an irrational number d) an imaginary number

Key. A

Sol. Take $\sqrt{6}$ common from C_1 and apply $C_3 \to C_3 - 3C_1$, $C_2 \to C_2 - 2iC_1$

37. If p + q + r = 0 and $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = K \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ then the value of K is

a) p + q –r b) p + q + r c) pqr d) - pqr

Key. D

Sol. $pqr\left(a^3+b^3+c^3\right) - abc\left(p^3+q^3+r^3\right)$

$\Rightarrow pqr\left(a^3+b^3+c^3-3abc\right) - abc\left(p^3+q^3+r^3-3pqr\right)$

$\Rightarrow pqr\left(a^3+b^3+c^3-3abc\right) - abc\left(p+q+r\right)\left(p^2+q^2+r^2-pq-qr-rp\right)$

$= pqr\left(a^3+b^3+c^3-3abc\right)$

38. If f(x), g(x), h(x) are polynomials of degree 4 and $\begin{vmatrix} f(x) & g(x) & h(x) \\ a & b & c \\ p & q & r \end{vmatrix} =$

$mx^4+nx^3+rx^2+5x+t$ be an identity in x, then the value of

$\begin{vmatrix} f'''(0)-f''(0) & g'''(0)-g''(0) & h'''(0)-h''(0) \\ a & b & c \\ p & q & r \end{vmatrix}$ is

a) $(3n-r)$ b) $2(3n-r)$ c) $3(3n-r)$ d) $3n+r$

Key. B

Sol. LHS = $(24mx+6n)-(12mx^2+6nx+2r)$

$x=0 \Rightarrow 6n-2r$

$\Rightarrow 2(3n-r)$

39. Let x > 0, y > 0, z > 0 are respectively the 2nd, 3rd, 4th, terms of a G.P and

$$\Delta=\begin{vmatrix} x^k & x^{k+1} & x^{k+2} \\ y^k & y^{k+1} & y^{k+2} \\ z^k & z^{k+1} & z^{k+2}\end{vmatrix}=(r-1)^2\left(1-\frac{1}{r^2}\right)$$ (where r is the common ratio) then

a) $k=-1$ b) $k=1$ c) $k=0$ d) None of these

Key. A

Sol. $$x^k y^k z^k\begin{vmatrix} 1 & ar & a^2r^2 \\ 1 & ar^2 & a^2r^4 \\ 1 & ar^3 & a^3r^6\end{vmatrix}$$

$$a^{3(k+1)}.r^{3(2k+1)}\left[(r-1)(r^4-1)-(r^2-1)^2\right] \Rightarrow k=-1$$

40. If $f(x)=\begin{vmatrix} x^2-4x+6 & 2x^2+4x+10 & 3x^2-2x+16 \\ x-2 & 2x+2 & 3x-1 \\ 1 & 2 & 3\end{vmatrix}$. Then the value of

$\int_{-3}^{3}\frac{x^2\sin x}{1+x^6}.f(x)dx$ is

a) 6 b) 3 c) 0 d) $\frac{\pi}{2}$

Key. C

Sol. $f(x)$ is const.

Hence = 0

41. If A and B are square matrices of order 3 such that $|A|=-1, |B|=3$ then $|3AB|$ is equals to

A) –9 B) –81 C) –27 D) 81

Key. B

Sol. $|3AB|=3^3|A||B|=27\times-1\times3=-81$

42. A = $[a_{ij}]_{n\times n}$ and a_{ij} = i^2–j^2 then A is necessarily

a) a unit matrix b) symmetric matrix c) skew symmetric matrix d) zero matrix

Key. C

Sol. $a_{ji}=j^2-i^2=(i^2-j^2)=-a_{ij}$

43. If $A=\frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y\end{bmatrix}$ is an orthogonal matrix then value of x+y is equal to

a) –3 b) 0 c) 1 d) 3

Key. A

Sol. $A.A^T = I \Rightarrow \frac{1}{9}\begin{bmatrix} 9 & 0 & x+4+2y \\ 0 & 9 & 2x+2-2y \\ x+4+2y & 2x+2-2y & x^2+y^2+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

x+4+2y = 0, 2x+2–2y = 0 $\Rightarrow$ x = –2, y = –1

44. If $A=\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ then A^{16} =

a) $\begin{bmatrix} 0 & 256 \\ 256 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$

c) $\begin{bmatrix} -16 & 0 \\ 0 & -16 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 16 \\ 16 & 0 \end{bmatrix}$

Key. B

Sol. $A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$, $A^4 = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$ $A^8 = \begin{bmatrix} 16 & 0 \\ 0 & 16 \end{bmatrix}$, $A^{16} = \begin{bmatrix} 256 & 0 \\ 0 & 256 \end{bmatrix}$

45. Let a, b, c be positive real numbers. Then the following system of equations in x, y, z

$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1, \frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1, -\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ has

a) no solution b) unique solution

c) infinite solution d) finitely many solution

Key. D

Sol. Let $\frac{x^2}{a^2}=X$, $\frac{y^2}{b^2}=Y$, $\frac{z^2}{c^2}=Z$

$X+Y-Z=1$, $X-Y+Z=1$, $-X+Y+Z=1$ on solving $X=Y=Z=1$

$\Rightarrow x=\pm a, y=\pm b, z=\pm c$ $\Rightarrow$ 8 solution

46. The value of determinant $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$ is equal to

a) $(1-a^2-b^2)^3$ b) $(a+b+1)^2(ab+b+a)$ c) $(1+a^2+b^2)^3$ d) $(1-a^2+b^2)^3$

Key. C

Sol. $\Delta = \frac{1}{ab}\begin{vmatrix} b(1+a^2-b^2) & 2ab^2 & -2b^2 \\ 2a^2b & a(1-a^2+b^2) & 2a^2 \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$ by $(R_1\times b, R_2\times a)$

$= \begin{vmatrix} 1+a^2-b^2 & 2b^2 & -2b^2 \\ 2a^2 & 1-a^2+b^2 & 2a^2 \\ 2 & -2 & 1-a^2-b^2 \end{vmatrix}$ by $\left(\frac{C_1}{b}, \frac{C_2}{a}\right)$

$\begin{vmatrix} 1+a^2+b^2 & 0 & -2b^2 \\ 1+a^2+b^2 & 1+a^2+b^2 & 2a^2 \\ 0 & -(1+a^2+b^2) & 1-a^2-b^2 \end{vmatrix}(C_1\to C_1+C_2, C_2\to C_2+C_3)$

$=(1+a^2+b^2)^3$

47. If $\begin{vmatrix} a^2+\lambda^2 & ab+c\lambda & ca-b\lambda \\ ab-c\lambda & b^2+\lambda^2 & bc+a\lambda \\ ac+b\lambda & bc-a\lambda & c^2+\lambda^2 \end{vmatrix}\begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix}=(1+a^2+b^2+c^2)^3$ then λ is equal to

a) 0 b) 1 c) –1 d) ±1

Key. B

Sol. If $\Delta=\begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix}$ other determinant (say Δ^1) is the cofactor determinant

$\Delta\Delta^1=\Delta^3$ (for 3rd order det)

$\Delta=\lambda(\lambda^2+a^2+b^2+c^2)$ by comparing $\lambda=1$

48. Constant term in f(x) = $\begin{vmatrix} x & (1+\sin x)^3 & \cos x \\ 1 & \ln(1+x) & 2 \\ x^2 & (1+x)^2 & 0 \end{vmatrix}$ when f(x) is expressed polynomial in x, is

a) 0 b) –1 c) 1 d) 2

Key. C

Sol. $f(0)=+1$

49. If A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, B = $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, a, b ∈ N, then number of matrix 'B' such that AB = BA are

a) 0 b) 1 c) finitely many d) infinite

Key. D

Sol. $AB=\begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}, BA=\begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix}$

AB= BA ⇒ a= b

50. If $f(x)=\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then f(α+β) =

a) f(α) + f(β) b) f(α).f(β) c) f(α)–f(β) d) 0

Key. B

Sol. f(α).f(β) = $\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}=\begin{bmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & 0 \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

51. The system of equations x + ky + 3z = 0, 3x + ky – 2z = 0, 2x + 3y – 4z = 0 possess a non-trivial solution over the set of rationals, then 2k is an integral element of the interval

A) [10, 20] B) (20, 30) C) [30, 40] D) (40, 50)

Key. C

Sol. For the given system to have a non-trival solution, we must have

$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 3 & -4 \end{vmatrix}=0 \Rightarrow \begin{vmatrix} 1 & k & 3 \\ 0 & -2k & -11 \\ 0 & 3-2k & -10 \end{vmatrix}=0$

[Applying $R_2\to R_2-3R_1, R_3\to R_3-2R_1$]

$\Rightarrow \quad 20k + 11(3-2k) = 0 \quad \Rightarrow \quad k = \frac{33}{2}$

52. If p + q + r = 0 = a + b + c, then the value of the determinant $\begin{vmatrix} pa & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix}$ is

A) 0 B) pq + qb + rc C) 1 D) none of these

Key. A

Sol. $\begin{vmatrix} pq & qb & rc \\ qc & ra & pb \\ rb & pc & qa \end{vmatrix} = pqr(a^3+b^3+c^3) - abc(p^3+q^3+r^3) = pqr(3\,abc) - abc(3\,pqr) = 0$

53. Let A and B are two non-singular square matrices, A^T and B^T are the transpose matrices of A and B respectively, then which of the following is correct

A) B^TAB is symmetric matrix if and only if A is symmetric

B) B^TAB is symmetric matrix if and only if B is symmetric

C) B^TAB is skew symmetric matrix for every matrix A

D) B^TAB is skew symmetric matrix if B is skew symmetric

Key. A

Sol. $(B^TAB)^T = B^TA^T(B^T)^T = B^TA^TB$

$= B^TAB$ iff A is symmetric

$\therefore$ B^TAB is symmetric iff A is symmetric

Also $(B^TAB)^T = B^TA^TB = (-B)A^TB$

$\therefore$ B^TAB is not skew symmetric if B is skew symmetric

54. If A and B are two square matrices of order 3 × 3 which satisfy AB = A and BA = B then $(A+B)^7$ is

A) 7(A + B) B) $7.I_{3\times3}$ C) 64(A + B) D) $128I_{3\times3}$

Key. C

Sol. AB = A, BA = B $\Rightarrow$ $A^2 = A$ and $B^2 = B$

$(A+B)^2 = A^2 + B^2 + AB + BA$

= A + B + A + B = 2 (A + B)

$(A+B)^3 = (A+B)^2(A+B) = 2(A+B)^2 = 2^2(A+B)$

$\therefore$ $(A+B)^7 = 2^6(A+B) = 64(A+B)$

55. $|A_{3\times3}| = 3, |B_{3\times3}| = -1$ and $|C_{2\times2}| = +2$ then |2ABC| =

A) $2^3(6)$ B) $2^3(-6)$ C) 2(-6) D) none of these

Key. D

Sol. 2ABC is not defined

$\therefore$ there is no solution

56. If A is a non-diagonal involutory matrix, then

A) A – I = 0 B) A + I = 0

C) A – I is non zero singular D) none of these

Key. C

Sol. $A^2 = I \Rightarrow A^2 - I = O$

$\Rightarrow$ (A + I) (A – I) = O

$\therefore$ either |A + I| = 0 or

|A – I| = 0

If |A – I| $\neq$ 0, then (A + I) (A – I) = O $\Rightarrow$ A + I = O which is not so

$\therefore$ |A – I| = 0 and A – I $\neq$ O.

57. If $A^3 = O$, then $I + A + A^2$ equals

A) I – A B) $(I - A)^{-1}$ C) $(I + A)^{-1}$ D) none of these

Key. B

Sol. $A^3 = 0$

$(I + A + A^2)(I - A) = I - A^3 = I$

$\therefore \quad I + A + A^2 = (I - A)^{-1}$

58. If a determinant of order 3 × 3 is formed by using the numbers 1 or – 1 then minimum value of determinant is

A) – 2 B) – 4 C) 0 D) – 8

Key. B

Sol. Let $\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$c_2 \to c_2 - \frac{a_{12}}{a_{11}} C_1 \qquad C_3 \to \quad C_3 - \frac{a_{13}}{a_{11}} C_1$

$$\begin{vmatrix} a_{11} & 0 & 0 \\ a_{21} & \left(a_{22} - \frac{a_{12}}{a_{11}} \times a_{21}\right) & \left(a_{23} - \frac{a_{13}}{a_{11}} a_{21}\right) \\ a_{31} & \left(a_{32} - \frac{a_{12}}{a_{11}} \times a_{31}\right) & \left(a_{32} - \frac{a_{13}}{a_{11}} \times a_{31}\right) \end{vmatrix}$$ so minimum value = - 4

59. If A is a diagonal matrix of order 3 × 3 is commutative with every square matrix of order 3 × 3 under multiplication and trace (A) = 12, then

A) |A| = 64 B) |A| = 16 C) |A| = 12 D) |A| = 0

Key. A

Sol. A diagonal matrix is commutative with every square matrix if it is scalar matrix so every diagonal element is 4.

$\therefore$ |A| = 64

60. If A is a square matrix of order 3 such that |A| = 2 then $\left|\left(\text{adj } A^{-1}\right)^{-1}\right|$ is

A) 1 B) 2 C) 4 D) 8

Key. C

Sol. $\left|\text{adj}A^{-1}\right| = \left|A^{-1}\right|^2 = \frac{1}{|A|^2}$

$\left|\left(\text{adj}A^{-1}\right)^{-1}\right| = \frac{1}{\left|\text{adj } A^{-1}\right|} = |A|^2 = 2^2 = 4$

61. If A and B are two matrices such that AB = B and BA = A, then

A) $\left(A^6 - B^5\right)^3 = A - B$ B) $\left(A^5 - B^5\right)^3 = A^3 - B^3$

C) A – B is idempotent D) A – B is nilpotent

Key. D

Sol. Since AB = B and BA = A

$\therefore$ A and B both are idempotent

$\left(A-B\right)^2 = A^2 - AB - BA + B^2 = A - B - A + B = 0$

$\therefore$ A – B is nilpotent

62. Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ are two matrices such that AB = BA and $c \neq 0$, then value of $\frac{a-d}{3b-c}$ is:

A) 0 B) 2 C) – 2 D) – 1

Key. D

Sol. $AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a+2c & b+2d \\ 3a+4c & 3b+4d \end{bmatrix}$

$BA = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a+3b & 2a+4b \\ c+3d & 2c+4d \end{bmatrix}$

If AB = BA, then a + 2c = a + 3b

$\Rightarrow$ 2c = 3b $\Rightarrow b \neq 0$

b + 2d = 2a + 4b

$\Rightarrow$ 2a – 2d = - 3b

$$\frac{a-d}{3b-c} = \frac{-\frac{3}{2}b}{3b - \frac{3}{2}b} = -1$$

63. Let $f\left(\alpha\right) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then $\left(f\left(\alpha\right)\right)^{-1}$ is equal to

A) $f\left(\alpha\right)$ B) $f\left(-\alpha\right)$ C) $f\left(\alpha - 1\right)$ D) none

Key. B

Sol. $\left(f\left(\alpha\right)\right)^{-1} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} = f\left(-\alpha\right)$

64. A and B are square matrices and A is non-singular matrix, $\left(A^{-1}BA\right)^n$, $n \in I^+$, is equal to

A) $A^{-n}B^nA^n$ B) $A^nB^nA^{-n}$ C) $A^{-1}B^nA$ D) $A^{-n}BA^n$

Key. C

Sol. For n = 2 $\Rightarrow$ $\left(A^{-1}BA\right)\left(A^{-1}BA\right) = A^{-1}B^2A$

$\left(A^{-1}B^3A\right)$ and soon

Thus $\left(A^{-1}BA\right)^n = A^{-1}B^nA$

65. If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then A =

A) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ D) $-\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

Key. A

Sol. $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix} (-1)$

$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

23. If $A = \begin{bmatrix} 5 & -6 \\ 1 & -1 \end{bmatrix}$ then determinant of $A^{1003} - 5A^{1002}$ is

(A) 1 (B) 2
(C) 4 (D) 6

Key. D

Sol. $\left|A^{1003} - 5A^{1002}\right| = \left|A^{1002}\left(A - 5I\right)\right|$

$= \left|A^{1002}\right| \left|A - 5I\right|$

$= \left|A\right|^{1002} \left|A - 5I\right|$

$= 1 \times \begin{vmatrix} 0 & -6 \\ 1 & -6 \end{vmatrix} = 6$

Matrices & Determents

Multiple Correct Answer Type

1. If $\Delta = \begin{vmatrix} a & c & c-a & a+c \\ c & b & b-c & b+c \\ a-b & b-c & 0 & a-c \\ x & y & z & 1+x+y \end{vmatrix} = 0$, then

a) a,b,c are in A.P b) b,a,c are in A.P c) a,c,b are in G.P d) a,c,b are in H.P

Key. A,C

Sol. $c_3 \to c_3 - (c_2 - c_1)$

$c_4 \to c_4 - (c_1 + c_2)$

$\begin{vmatrix} a & c & 0 & 0 \\ c & b & 0 & 0 \\ a-b & b-c & a+c-2b & 0 \\ x & y & z+x-y & 1 \end{vmatrix} = (a+c-2b)(ab-c^2)$

2. If 'A' is a matrix of size $n \times n$ such that $A^2 + A + 2I = 0$, then

a) A is non-singular b) A is symmetric

c) $|A| \neq 0$ d) $A^{-1} = \frac{-1}{2}(A+I)$

Key. A,C,D

Sol. $A(A+I) = -2I$

$|A(A+I)| = |-2I|$

$|A||A+I| = (-2)^n \neq 0$

3. If A and B are two invertible matrices of the same order, then adj(AB) is equal to

a) $adj(B)\ adj(A)$ b) $|B|\ |A|\ B^{-1}A^{-1}$ c) $|B|\ |A|.A^{-1}B^{-1}$ d) $|A|\ |B|(AB)^{-1}$

Key. A,B,D

Sol. $adj(AB) = |AB|\ (AB)^{-1}$

$=|A||B|\left(B^{-1}A^{-1}\right)$

4. If the sum of two idempotent matrices is idempotent then

a) $AB+BA=0$ b) $AB=BA=I$

c) $AB\neq BA$ d) $AB=BA=0$

Key. A,D

Sol. $(A+B)^2=A^2+AB+BA+B^2=A+B$

$\Rightarrow AB+BA=0$

$A^2B+ABA=0\Rightarrow A^2B+(-BA)A=0$

$\Rightarrow AB-BA=0$

5. If a square matrix $A=\left[a_{ij}\right], a_{ij}=i^2-j^2$ is of even order, then

(A) A is a skew symmetric matrix

(B) $|A|$ is a perfect square

(C) A is a symmetric and $|A|=0$

(D) A is neither symmetric nor skew symmetric

Key. A,B

Sol. Let A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}=\begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$

$A'\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}=-\begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}=-A$

$\therefore$ A is skew-symmetric and $|A|=\begin{vmatrix} 0 & -3 \\ 3 & 0 \end{vmatrix}=9$ is a perfect square.

6. If the sum of two idempotent matrices is idempotent then

a) $AB+BA=0$ b) $AB=BA=I$

c) $AB\neq BA$ d) $AB=BA=0$

Key. A,D

Sol. $(A+B)^2=A^2+AB+BA+B^2=A+B$

$\Rightarrow AB+BA=0$

$A^2B+ABA=0\Rightarrow A^2B+(-BA)A=0$

$\Rightarrow AB-BA=0$

7. If matrix $A=\{a_{ij}\}_{n\times n}$, where $a_{ij}=i^2-j^2$, then

(A) A is skew symmetric matrix (B) trace (A) = 0

(C) A is singular if n is odd (D) A is singular " n Î N

Key. A,B,C

Sol. $a_{ij} = i^2 - j^2$

$a_{ji} = j^2 - i^2 = -(i^2 - j^2) = -a_{ij}$

$\therefore$ A is skew symmetric, trace of A = 0 and singular if n is odd

8. A square matrix A with elements from the set of real numbers is said to be orthogonal if

$A^T = A^{-1}$. If A is an orthogonal matrix, then

(A) A^T is orthogonal (B) A^{-1} is orthogonal

(C) Adj A = A’ (D) $|A^{-1}| = 1$

Key. A,B

Sol. $A^T = A^{-1}$ AA’ = I = A’A since A is orthogonal

A’(A’)’ = A’A = I from above

hence A’ is orthogonal

A^{-1} = A’ $\therefore$ A^{-1} is also orthogonal

9. Let then

(A*) (B)

(C) is not invertible (D*) is invertible

Sol Also, is invertible

is invertible.

10. If $\left(a^{\log_b x}\right)^2 - 5\, x^{\log_b a} + 6 = 0$, where a > 0, b > 0 & ab ¹1, then the value of x can be equal to

(A) $2^{\log_b a}$ (B) $3^{\log_a b}$ (C) $b^{\log_a 2}$ (D) $a^{\log_b 3}$

Key. B,C

11. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then

(A) adj(adj A)=A (B) |adj(adjA)|=1 (C) |adj A| = 1 (D) non of these

Key. A,B,C

12. P is a non singular matrix and A,B are two matrices such that $B = P^{-1}AP$ then the true statements among the following are

A) A is invertible iff B is invertible

B) $B^n = P^{-1}A^nP \;\forall\; n \in N$

C) $\forall\ \lambda \in R,\ B-\lambda I = P^{-1}(A-\lambda I)P$ (I is the identity matrix)

D) A,B are both singular matrices

Key. A,B,C

Sol. Conceptual

13. If $x, y, z, w \in R$ satisfy the following system of equations $x+y+z+w=1$; $x+2y+4z+8w=16$; $x+3y+9z+27w=81$ and $x+4y+16z+64w=256$, then the pairs which has $H.C.F.$ as 2 is

A) $(|w|,|z|)$ B) $(|z|,|y|)$ C) $(|y|,|x|)$ D) $(|z|,|x|)$

Key. C

Sol. Observe that $1,2,3,4$ are roots of

$n^4 - wn^3 - zn^2 - yn - x = 0$

$w=10,\ z=-35, y=50, x=-24$

14. Which of the following functions will not have absolute minimum value?

A) $\cot(\sin x)$ B) $\tan(\log x)$

C) $x^{2005} - x^{1947} + 1$ D) $x^{2006} + x^{1947} + 1$

Key. A,B,C

Sol. Even degree polynomial with leading coefficient positive will have absolute minimum.

15. Let A_n is a $n \times n$ matrix in which diagonal elements are $1,2,3,.....,n$ $(i.e., a_{11}=1, a_{22}=2, a_{33}=3,....., a_{ii}=i,......a_{nn}=n)$ and all other elements are equal to 'n' then

A) A_n is singular for all 'n' B) A_n is nonsingular for all 'n'

C) $\det.A_5 = 120$ D) $\det.A_n = 0$

Key. B,C

Sol.
$$A_n = \begin{pmatrix} 1 & n & n.... & n \\ n & 2 & n.... & n \\ n & n & 3.... & n \\ n & n & n.... & n \end{pmatrix}$$

$\therefore$ A_n is nonsingular for all 'n'

$\left|A_n\right| = (-1)^{n+1} . n\ !$

16. If $A = \left[a_{ij}\right]$ is a square matrix such that $a_{ij} = i^2 - j^2$, then

a) |A| is a perfect square if A is of even order

b) |A|=0 if A is of odd order

c) A is singular for any square matrix

d) we cannot say any thing about |A|

Key: A,B

Hint $a_{ij} = i^2 - j^2 \Rightarrow a_{ij} = j^2 - i^2 = -\left(i^2 - j^2\right) = -a_{ij}$

$\therefore a_{ij} = -a_{ji}$

A is skew symentic

17. If $\Delta(x) = \begin{vmatrix} x^2+4x-3 & 2x+4 & 13 \\ 2x^2+5x-9 & 4x+5 & 26 \\ 8x^2-6x+1 & 16x-6 & 104 \end{vmatrix} = ax^3 + bx^2 + cx + d$, then

(A) a = 3 (B) a = 0 (C) c = 0 (D) b = 2

Key: B, C

Hint: $\Delta'(x) = \begin{vmatrix} 2x+4 & 2x+4 & 13 \\ 4x+5 & 4x+5 & 26 \\ 16x-6 & 16x-6 & 104 \end{vmatrix} + \begin{vmatrix} x^2+4x-3 & 2 & 13 \\ 2x^2+5x-9 & 4 & 26 \\ 8x^2-6x+1 & 16 & 104 \end{vmatrix}$

= 0 + 2 × 13 × (0) = 0

$\Rightarrow$ Δ(x) = constant

$\Rightarrow$ a = 0, b = 0, c = 0

18. Let $a_1, a_2, a_3........$ be real numbers which are in arithmetic progression with common difference $d \neq 0$. Then

A) $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_5 & a_6 & a_7 \end{bmatrix}$ is singular

B) $B = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_2 & a_3 & a_4 \\ a_2 & a_4 & a_6 \end{bmatrix}$ is non-singular

C) The system of equations $a_1x + a_2y + a_3z = 0$

$a_3x + a_1y + a_2z = 0$

$a_4x + a_5y + a_6z = 0$ has unique solution

D) The system of equations $a_1x + a_2y + a_3z = 0$

$a_4x + a_5y + a_6z = 0$

$a_7x + a_8y + a_9z = 0$ has infinitely many solutions

Key: A, C, D

Hint: Determinants in (A), (B) and (D) are zero, and in (C) the determinant is non zero.

19. Let $A=\{1^2,3^2,5^2,........\}$.If 9 elements selected from set A to make a 3×3 matrix then det (A) will be divisible by

(A) 9 (B) 36

(C) 8 (D) 64

Key: C,D

Hints: $(2n+1)^2-(2m+1)^2=4(m+n+1)(n-m)=\text{multiple of } 8$

20. If $A=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$ and $A^{2012}=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, then which of the following is/are correct?

(A) a = d (B) a + b + c + d = 4026

(C) $a^2+b^2+d^2=2$ (D)b = 2012

Key. A,B,C

Sol. $A^2 = \begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 4 & 1\end{bmatrix}$

Hence, $A^n = \begin{bmatrix}1 & 0\\ 2n & 1\end{bmatrix}$

21. Which of the following statements are *FALSE*?

(A) If A and B are square matrices of the same order such that ABAB = 0, it follows that BABA = 0.

(B) Let A and B be different $n \times n$ matrices with real numbers. If $A^3 = B^3$ and

$A^2B = B^2A$, then $A^2 + B^2$ is invertible.

(C) If A is a square, non-singular and symmetric matrix, then $\left(\left(A^{-1}\right)^{-1}\right)^{-1}$ is skew symmetric.

(D) The matrix of the product of two invertible square matrices of the same order is also invertible.

Key. A,B,C

Sol. (A) $A=\begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 0\\ 0 & 1 & 0\end{bmatrix};\ B=\begin{bmatrix}0 & 0 & 1\\ 1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$

is a counter example. Therefore, (A) is wrong.

(B) We have $(A^2 + B^2)(A - B) = A^3 - B^3 - A^2B + B^2A = 0$

and $A-B\neq 0 \Rightarrow A^2 + B^2$ is not invertible. Therefore (B) is also wrong.

(C) $A^T = A$

$$\left(\left(A^{-1}\right)^{-1}\right)^{-1}=A^{-1}$$

$$\left(A^{-1}\right)^{T}=\left(A^{T}\right)^{-1}=A^{-1}$$

So, $\left(\left(A^{-1}\right)^{-1}\right)^{-1}$ is also symmetric. Therefore (C) is also wrong.

(D) $|A|\neq 0, |B|\neq 0 \Rightarrow |AB|\neq 0$

So, AB is invertible. Therefore (D) is correct.

22. If $f(x) = \begin{vmatrix} (1+x)^a & (1+2x)^b & 1 \\ 1 & (1+x)^a & (1+2x)^b \\ (1+2x)^b & 1 & (1+x)^a \end{vmatrix}$, where a, b being positive integers, then

(A) constant term of $f(x)$ is 0
(B) coefficient of x in $f(x)$ is 0
(C) constant term in $f(x)$ is $(a-b)$
(D) coefficient of x in f(x) is (a – b)

Key. A,B

Sol. Let $\begin{vmatrix} (1+x)^a & (1+2x)^b & 1 \\ 1 & (1+x)^a & (1+2x)^b \\ (1+2x)^b & 1 & (1+x)^a \end{vmatrix} = A + Bx + Cx^2 + \ldots$

putting $x = 0$, we get $A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$

Now differentiating both sides w.r.t. x and putting $x = 0$, we get

$$B = \begin{vmatrix} a & 2b & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 0 & a & 2b \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 2b & 0 & a \end{vmatrix} = 0$$

Hence coefficient of x is 0.

23. The value of the determinant $\begin{vmatrix} \cos(\theta+\alpha) & -\sin(\theta+\alpha) & \cos 2\alpha \\ \sin\theta & \cos\theta & \sin\alpha \\ -\cos\theta & \sin\theta & \lambda\cos\alpha \end{vmatrix}$ is

(A) independent of θ for all $\lambda \in R$
(B) independent of θ and α when $\lambda = 1$
(C) independent of θ and α when $\lambda = -1$
(D) independent of λ for all θ

Key. A,C

Sol. We have, $\begin{vmatrix} \cos(\theta+\alpha) & -\sin(\theta+\alpha) & \cos 2\alpha \\ \sin\theta & \cos\theta & \sin\alpha \\ -\cos\theta & \sin\theta & \lambda\cos\alpha \end{vmatrix}$

$$= \frac{1}{\sin\alpha\cos\alpha}\begin{vmatrix} \cos(\theta+\alpha) & -\sin(\theta+\alpha) & \cos 2\alpha \\ \sin\theta\sin\alpha & \cos\theta\sin\alpha & \sin^2\alpha \\ -\cos\theta\cos\alpha & \sin\theta\cos\alpha & \lambda\cos^2\alpha \end{vmatrix}$$

[Multiplying R_2 and R_3 by $\sin\alpha$ and $\cos\alpha$, respectively]

$$= \frac{1}{\sin\alpha\cos\alpha}\begin{vmatrix} 0 & 0 & \cos 2\alpha + \sin^2\alpha + \lambda\cos^2\alpha \\ \sin\theta\sin\alpha & \cos\theta\sin\alpha & \sin^2\alpha \\ -\cos\theta\cos\alpha & \sin\theta\cos\alpha & \lambda\cos^2\alpha \end{vmatrix}$$

[Applying $R_1 \to R_1 + R_2 + R_3$]

$$= \frac{\cos 2\alpha + \sin^2\alpha + \lambda\cos^2\alpha}{\sin\alpha.\cos\alpha}\begin{vmatrix} \sin\theta\sin\alpha & \cos\theta\sin\alpha \\ -\cos\theta\cos\alpha & \sin\theta\cos\alpha \end{vmatrix}$$

$$= (\cos^2\alpha + \lambda\cos^2\alpha)\begin{vmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{vmatrix} = (1+\lambda)\cos^2\alpha$$

Therefore, the given determinant is independent of θ for all real values of λ. Also, $\lambda = -1$, then it is independent of θ and α.

24. Suppose that a, b, c are real numbers such that a + b + c = 1. If the matrix $A = \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ be an orthogonal matrix, then

(A) A is an involutory matrix
(B) |A| is negative
(C) $a^3 + b^3 + c^3 - 3abc = 1$
(D) atleast one of a, b, c is negative

Key. A,B,C,D

Sol. $AA^T = A^TA = I$. Also $A^T = A$, so $A^2 = I \Rightarrow$ A is involutory matrix.

$\Rightarrow$ $|A^2| = |A|^2 = 1$ or, $|A| = \pm 1$.

But $|A| = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = (a + b + c)(ab + bc + ca - a^2 - b^2 - c^2)$

$|A| = ab + bc + ca - a^2 - b^2 - c^2$ $(\because a + b + c = 1)$

$\therefore$ $a^2 + b^2 + c^2 - ab - bc - ca \ge 0$

So $|A| = -1$. Hence $a^3 + b^3 + c^3 - 3abc = 1$. Again $a^2 + b^2 + c^2 - ab - bc - ca = 1$

$\Rightarrow$ $1 - 3(ab + bc + ca) = 1$, so $ab + bc + ca = 0$, $\Rightarrow$ atleast one of a, b, and c is negative.

25. The determinant $\begin{vmatrix} 1 & a & a^2 \\ \cos(n-1)x & \cos nx & \cos(n+1)x \\ \sin(n-1)x & \sin nx & \sin(n+1)x \end{vmatrix} = 0$, if

(A) a = 0
(B) cosx = 0
(C) sinx = 0
(D) $\cos x = \dfrac{1+a^2}{2a}$

Key. C,D

Sol. $\sin x(1 - 2a\cos x + a^2) = 0$

$\Rightarrow$ $\sin x = 0$ $\cos x = \dfrac{1+a^2}{2a}$

26. A, B are any two square matrices of same order such that AB = A & BA = B then $(A + B)^5 =$ ____

(A) 32 (A + B)
(B) 16 (A + B)
(C) 64 (A + B)
(D) 1024 (A + B)

Key. B

Sol. AB = A, BA = B $\Rightarrow A^2 = A$ & $B^2 = B$

Now $(A + B)^2 = 2(A + B)$

Similarly $(A + B)^5 = 16(A + B)$

27. If A is a square Matrix such that $A.(AdjA) = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ then

a) $|A|=4$ b) $|adjA|=16$ c) $\frac{|adj(adjA)|}{|adjA|}=16$ d) $|adj2A|=128$

Key. A,B,C

Sol. $A(adjA)=|A|I$ $\therefore |A|=4$

$|adjA|=|A|^{n-1}=16; |adj(adjA)|=|A|^{(n-1)^2}=|A|^4=256$

$adjKA=K^{n-1}adjA$

$\therefore |adjKA|=(K^{n-1})^n|adjA|$

$\therefore |adj2A|=2^6.16$.

28. If $\Delta(x)=\begin{vmatrix} x^2+4x-3 & 2x+4 & 13 \\ 2x^2+5x-9 & 4x+5 & 26 \\ 8x^2-6x+1 & 16x-6 & 104 \end{vmatrix}=ax^3+bx^2+cx+d$, then

(A) a = 3 (B) a = 0

(C) c = 0 (D) none of these

Key. B,C

Sol. $\Delta'(x)=\begin{vmatrix} 2x+4 & 2x+4 & 13 \\ 4x+5 & 4x+5 & 26 \\ 16x-6 & 16x-6 & 104 \end{vmatrix}+\begin{vmatrix} x^2+4x-3 & 2 & 13 \\ 2x^2+5x-9 & 4 & 26 \\ 8x^2-6x+1 & 16 & 104 \end{vmatrix}$

$= 0 + 2 \times 13 \times (0) = 0$

$\Rightarrow$ $\Delta(x)$ = constant

$\Rightarrow$ a = 0, b = 0, c = 0

29. The system of equation is $x - y\cos\theta + z\cos 2\theta = 0$, $x\cos 2\theta - y + z\cos\theta = 0$ $x\cos 2\theta - y\cos\theta + z = 0$ has non-trivial solution for θ equals to

(A) $\frac{8\pi}{3}$ (B) $\frac{\pi}{6}$

(C) $\frac{2\pi}{3}$ (D) $\frac{\pi}{12}$

Key. A,C

Sol. $\begin{vmatrix} 1 & -\cos\theta & \cos 2\theta \\ \cos 2\theta & -1 & \cos\theta \\ \cos 2\theta & -\cos\theta & 1 \end{vmatrix}=0$

$\Rightarrow \theta=\frac{2\pi}{3}$

30. If a, b, c are real numbers such that a + b + c = 1. If the matrix

$A=\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ be an orthogonal matrix, then

(A) A is an involutory matrix (B) |A| is negative

(C) $a^3 + b^3 + c^3 - 3abc = 1$ (D) atleast one of a, b, c is negative

Key. A,B,C,D

Sol. $AA^T = A^TA = I$. Also $A^T = A$, so $A^2 = I \Rightarrow$ A is involutory matrix.

$\Rightarrow$ $|A^2| = |A|^2 = 1$ or, $|A| = \pm 1$.

But $|A| = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$ = (a + b + c) $(ab + bc + ca - a^2 - b^2 - c^2)$

$|A| = ab + bc + ca - a^2 - b^2 - c^2$ ($\because$ a + b + c = 1)

$\therefore$ $a^2 + b^2 + c^2 - ab - bc - ca \geq 0$

So $|A| = -1$. Hence $a^3 + b^3 + c^3 - 3abc = 1$. Again $a^2 + b^2 + c^2 - ab - bc - ca = 1$

$\Rightarrow$ 1 – 3(ab + bc + ca) = 1, so ab + bc + ca = 0, $\Rightarrow$ atleast one of a, b, and c is negative.

31. Consider the homogeneous system of linear equations in x, y and z : $(\sin 3\theta)x - y + z = 0$, $(\cos 2\theta)x + 4y + 3z = 0$, $2x + 7y + 7z = 0$. The values of 'θ' for which the system of equations has a non-trivial solution are

a) $\{n\pi : n \in I\}$ b) $\left\{m\pi + (-1)^m \frac{\pi}{6} : m \in I\right\}$ c) $\left\{n\pi + (-1)^n \frac{\pi}{3} : n \in I\right\}$ d) none

Key. A,B

Sol. $\Delta = 0$

$$\begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0 \Rightarrow 7\sin 3\theta + 14\cos 2\theta - 14 = 0$$

$$\Rightarrow \sin 3\theta + 2\cos 2\theta - 2 = 0 \Rightarrow \sin\theta\left(4\sin^2\theta + 4\sin\theta - 3\right) = 0$$

$$\sin\theta = 0 \text{ or } \sin\theta = \frac{1}{2}$$

32. If $A = \begin{bmatrix} a & b & c \\ x & y & z \\ p & q & r \end{bmatrix}$ and $B = \begin{bmatrix} q & -b & y \\ -p & a & -x \\ r & -c & z \end{bmatrix}$, then

a) $|A| = |B|$ b) $|A| = -|B|$

c) $|A| = 2|B|$ d) A is invertible if and only if B is invertible

Key. B,D

Sol. $|B| = \begin{vmatrix} q & -b & y \\ -p & a & -x \\ r & -c & z \end{vmatrix} = -\begin{vmatrix} q & b & y \\ -p & -a & -x \\ r & c & z \end{vmatrix}$ $(\text{operate } C_2)$

$$= \begin{vmatrix} q & b & y \\ p & a & x \\ r & c & z \end{vmatrix} = -\begin{vmatrix} p & a & x \\ q & b & y \\ r & c & z \end{vmatrix}$$

$(\text{on } R_2)$ $(\text{on } R_1 \leftrightarrow R_2)$

$$\Rightarrow |B| = -|A|$$

33. If $\Delta = \begin{vmatrix} a & c & c-a & a+c \\ c & b & b-c & b+c \\ a-b & b-c & 0 & a-c \\ x & y & z & 1+x+y \end{vmatrix} = 0$, then

a) a,b,c are in A.P b) b,a,c are in A.P c) a,c,b are in G.P d) a,c,b are in H.P

Key. A,C

Sol. $c_3 \to c_3 - (c_2 - c_1)$

$c_4 \to c_4 - (c_1 + c_2)$

$$\begin{vmatrix} a & c & 0 & 0 \\ c & b & 0 & 0 \\ a-b & b-c & a+c-2b & 0 \\ x & y & z+x-y & 1 \end{vmatrix} = (a+c-2b)(ab-c^2)$$

34. If 'A' is a matrix of size $n \times n$ such that $A^2 + A + 2I = 0$, then

a) A is non-singular b) A is symmetric

c) $|A| \neq 0$ d) $A^{-1} = \frac{-1}{2}(A+I)$

Key. A,C,D

Sol. $A(A+I) = -2I$

$|A(A+I)| = |-2I|$

$|A||A+I| = (-2)^n \neq 0$

35. If A and B are two invertible matrices of the same order, then adj(AB) is equal to

a) $adj(B)\ adj(A)$ b) $|B|\ |A|\ B^{-1}A^{-1}$ c) $|B|\ |A|.A^{-1}B^{-1}$ d) $|A|\ |B|(AB)^{-1}$

Key. A,B,D

Sol. $adj(AB) = |AB|\ (AB)^{-1}$

$= |A||B|(B^{-1}A^{-1})$

36. Let $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then

A) $A^2 - 4A - 5I_3 = 0$ B) $A^{-1} = \frac{1}{5}(A - 4I_3)$ C) A^3 is not invertible D) A^2 is invertible

Key. A,B,D

Sol. $A^2 - 4A - 5I_3 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

$$-4\begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix}-5\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}9&8&8\\8&9&8\\8&8&9\end{bmatrix}+\begin{bmatrix}-4&-8&-8\\-8&-4&-8\\-8&-8&-4\end{bmatrix}+\begin{bmatrix}-5&0&0\\0&-5&0\\0&0&-5\end{bmatrix}$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}=0$$

$\therefore$ $A^2-4A-5I_3=0$

or $A^{-1}A^2-4A^{-1}A-5A^{-1}I_3=0$

or $(A^{-1}A)A-4I_3-5A^{-1}=0$

or $IA-4I_3-5A^{-1}=0$

$\therefore$ $A^{-1}=\frac{1}{5}(A-4I_3)$

Also, $|A^2|=\begin{vmatrix}9&8&8\\8&9&8\\8&8&9\end{vmatrix}=9(81-64)-8(72-64)+8(64-72)$

$= 9 \times 17 - 8 \times 8 + 8 \times (-8)$

$= 133 - 128 = 5 \neq 0$

$\therefore$ A^2 is invertible

and $A^3 = A.\ A^2 = A.\ (4A-5I_3) = 4A^2-5A$

$$=\begin{bmatrix}36&32&32\\32&36&32\\32&32&36\end{bmatrix}+\begin{bmatrix}-5&-10&-10\\-10&-5&-10\\-10&-10&-5\end{bmatrix}=\begin{bmatrix}31&22&22\\22&31&22\\22&22&31\end{bmatrix}$$

$\therefore$ $|A^3|\neq 0$

$\therefore$ A^3 is invertible.

37. If A and B are invertible square matrices of the same order, then which of the following is correct?

A) adj(AB) = (adjB) (adjA)

B) $(\text{adj}A)'=(\text{adj}A')$

C) $|\text{adj}A|=|A|^{n-1}$, where n is the order of matrix A

D) adj (adjB) = $|B|^{n-2}$ B, where n is the order of matrix B

Key. A,B,C,D

Sol. Here, (a), (b), (c), (d) are the properties of adjoint.

38. If $A=\begin{bmatrix}3&-3&4\\2&-3&4\\0&-1&1\end{bmatrix}$, then

A) adj(adj A) = A B) |adj(adj A)| = 1 C) |adj A| = 1 D) None of these

Key. A,B,C

Sol. $\because$ $|A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix} = 3(-3+4) - 2(-3+4) + 0 = 1$

$\because$ $\text{adj}(\text{adj } A) = |A|^{3-2} A = A$ and |adj(adjA)| = |A | = 1

Also, $|\text{adj } A| = |A|^{3-1} = |A|^2 = 1^2 = 1$

39. System of equation x + 3y + 2z = 6
$x + \lambda y + 2z = 7$
x + 3y + 2z = μ has

A) Unique solution if $\lambda = 2, \mu \neq 6$ B) infinitely many solution if $\lambda = 4, \mu = 6$
C) no solution if $\lambda = 5, \mu = 7$ D) no solution if $\lambda = 3, \mu = 5$

Key. B,C,D

Sol. x + 3y + 2z = 6 …(i)
$x + \lambda y + 2z = 7$ …(ii)
x + 3y + 2z = μ …(iii)

(A) If $\lambda = 2$, then D = 0, therefore unique solution is not possible

(B) If $\lambda = 4, \mu = 6$
x + 3y = 6 – 2z
x + 4y + 7 – 2z
$\therefore$ y = 1 and x = 3 – 2z
Substituting in equation (iii)
3 – 2z + 3 + 2z = 6 is satisfied
$\therefore$ Infinite solutions

(C) $\lambda = 5, \mu = 7$
Consider equation (ii) and (iii)
x + 5y = 7 – 2z
x + 3y = 7 – 2z
$\therefore$ y = 0 x = 7 – 2z are solution
Sub. In (i)
7 – 2z + 2z = 6 does not satisfy
$\therefore$ no solution

(D) if $\lambda = 3, \mu = 5$
then equation (i) and (ii) have no solution
$\therefore$ no solution

40. Which of the following statement is always true
A) Adjoint of a symmetric matrix is a symmetric matrix
B) Adjoint of a unit matrix is unit matrix
C) A (adj A) = (adj A) A
D) Adjoint of a diagonal matrix is diagonal matrix

Key. A,B,C,D

Sol. Obvious (using properties)

41. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, then

A) $A^3 = 9A$ B) $A^3 = 27A$ C) $A + A = A^2$ D) A^{-1} does not exist

Key. A,D

Sol. $A^2 = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = 3A$

$A^3 = A^2 A = 3A.A = 3A^2 = 3.(3A) = 9A$ and $|A| = 0$

$\therefore$ A^{-1} does not exist

Matrices & Determents

Assertion Reasoning Type

1. Statement 1 : The determinants of a metrix $A=\left[a_{ij}\right]_{5\times 5}$ where $a_{ij}+a_{ji}=0$ for all i and j is zero

Statement 2 : The determinants of a skew symmetric of odd order is zero

Key. A

Sol. Conceptual

2. Statement 1: For a singular square matrix A, if AB = AC B = C

Statement 2 : If |A| = 0 then A^{-1} dose not exist

Key. A

Sol. exist only for non-singular matrix

AB = AC B = C if exist, If exist.

3. Consider the system of equations $2x-3y+5z=12\,;3x+y+k_1z=k_2\,;\,x-7y+8z=17$

STATEMENT-1: The system of equations will have infinite solutions if $k_1=2\,;k_2=7$.

STATEMENT-2: Let $A=\begin{bmatrix}2 & -3 & 5\\ 3 & 1 & k_1\\ 1 & -7 & 8\end{bmatrix},\ B=\begin{bmatrix}12\\ k_2\\ 17\end{bmatrix}$ then $|A|=0\ and\ (adjA)B=0$ for $k_1=2\ and\ k_2\in R$.

Key: C

Hint: Conceptual

4. STATEMENT-1 : If A is skew symmetric of order 3 then its determinant should be zero

STATEMENT-2 : If A is square matrix, then

Key: B

Hint: The Reson R is false since

det A' = det ($-A'$) is not true A'

Indeed, det ($-A'$) = det $(-1)^3$det A'

Now as A = $-A'$ (A is skew symmetric)

det A = det ($-A'$)

= $-A'$ det (A') = $-A'$ det A

det A = 0

Thus Assertion if true.

5. Statement - 1: If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ then $A^5=\begin{bmatrix}1069 & 1558\\ 2337 & 3406\end{bmatrix}$.

Statement - 2: If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ then $A^2-5A=2I$.

Key: A

Hint: Conceptual Question

6. STATEMENT–1

If $A=\begin{bmatrix}1 & -1\\ 2 & 1\end{bmatrix}$, $B=\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$ and $(A+B)^2=A^2+B^2+2AB$ then $(a,b)=(-1,-2)$

STATEMENT–2

If A and B are two square matrices of same order then $(A+B)^2=A^2+B^2+2AB$.

Key: C

Hint: If $(A+B)^2=A^2+B^2+2AB$ then $AB=BA$

$$AB=\begin{bmatrix}a-b & 2\\ 2a+b & 1\end{bmatrix}$$

$$BA=\begin{bmatrix}a+2 & -a+1\\ b-2 & -b-1\end{bmatrix}$$

So; $a+2=a-b\Rightarrow b=-2$

And $-a+1=2\Rightarrow a=-1$

$(a,b)=(-1,-2)$

7. Statement-1 : If the orthogonal square matrices A and B of same size satisfy det A + det B = 0 then det(A + B) = 0.

Statement-2 : det (X + Y) = det X + det Y, for orthogonal square matrices X and Y of same size.

Key: C

Hint det $A=-1$, det $B=1$, or the other way round

Now $A^T(A+B)B^T=(A+B)^T$

We have on taking determinants on both sides

$(\det A)\det(A+B)\det B=\det(A+B)$

$\Rightarrow -\det(A+B)=\det(A+B)\ \therefore \det(A+B)=0$

8. Statement I: Let A be a $n\times n$ matrix such that $A^n=\alpha A$ where α is a real number $(\neq \pm 1)$ then $A+I_n$ is Inveritable.

Statement I: A square Matrix possess inverse iff it is non-singular.

Key. A

Sol. Let $B=A+I_n \therefore A=B-I_n$

$\because A^n=\alpha A\Rightarrow (B-I_n)^2=\alpha(B-I_n)$

$\Rightarrow B^n-{}^nC_1B^{n-1}+{}^nC_2B^{n-2}+....+(-1)^{n-1}B+(-1)^nI_n=\alpha B-\alpha I_n$

$\Rightarrow B^n-{}^nC_1B^{n-1}+{}^nC_2B^{n-2}+....+(-1)^{n-1}B-\alpha B=-(-1)^nI_n-\alpha I_n$

$\Rightarrow B\left(B^{n-1}-{}^nC_1B^{n-2}+.....+(-1)^{n-1}I_n-\alpha I_n\right)=I_n\left[(-1)^{n+1}-\alpha\right]$

$\because \alpha\neq\pm1$ Determinant of $I_n\left((-1)^{n+1}-\alpha\right)\neq 0$

$\because |B|\neq 0\Rightarrow A+I_n$ is inveritable.

9. Let A and B be two matrices such that AB = O then
STATEMENT–1
If one of them is non singular matrix then the other must be a null matrix
because
STATEMENT–2
At least one of the two matrices must be singular

Key. B

Sol. det (AB) = det(A).det(B) = O. So, II is true
Let det (A) ≠ O, So A^{-1} is defined.
So, A^{-1} (AB) = A^{-1} O ⇒ B = O.

10 Statement - 1: The matrix $3A=\begin{bmatrix}1 & -2 & 2\\ -2 & 1 & 2\\ -2 & -2 & -1\end{bmatrix}$ is an orthogonal matrix

Statement - 2: If A and B are orthogonal, then AB is also orthogonal

Key. D

Sol. for orthogonal matrix

$$AA^T = I_n$$

$$\Rightarrow (3A)\left(3A^T\right) = 9I_n$$

$$\Rightarrow \begin{bmatrix}1 & -2 & 2\\ -2 & 1 & 2\\ -2 & -2 & -1\end{bmatrix}\begin{bmatrix}1 & -2 & -2\\ -2 & 1 & -2\\ 2 & 2 & -1\end{bmatrix} = \begin{bmatrix}9 & 0 & 0\\ 0 & 9 & 0\\ 0 & 0 & 9\end{bmatrix}$$

Hence statement (1) is true

Now if A and B are orthogonal, then

$AA^T = I_n$ and $BB^T = I_n$

$$\Rightarrow (AB)(AB)^T = ABB^TA^T \quad \Rightarrow A\left(BB^T\right)A^T = AI_n\,A^T \qquad \Rightarrow AA^T = I_n$$

∴ Statement 2 is also true but do not explain statement 1.

11. Statement – 1 : If A is a skew-symmetric matrix of order 3×3, then matrix A^3 is also skew-symmetric
Because
Statement – 2 : All positive odd integral powers of a skew-symmetric matrix are skew-symmetric

Key. A

Sol. $\left(A^3\right)^T = (AAA)^T = A^TA^TA^T = (-A)(-A)(-A) = -A^3$

12. Statement – 1 : The system of equations $x+y+z=6, x+2y+3z=14, x+4y+7z=30$ has infinitely many solutions
Because
Statement – 2 : For a system of n-equations in n-unknowns if determinant value of the coefficient matrix be zero, then the system of equations has infinitely many solutions.

Key. C

Sol. Conceptual

13. Statement – 1 : For the given system of non-homogeneous linear equations of the form A x = B, if $|A|=0$, then the system of equations have either no solution or infinite number of solutions

Because

Statement – 2 : For the given system of non-homogeneous linear equations of the form A x = B, if $|A|=0$ & (Adj A) B = 0, then it will have no solution

Key. C

Sol. If $|A|=0 \& (Adj\ A)\ B \neq 0$, no solution

If $|A|=0 \& (Adj\ A)\ B = 0$, infinite solution

14. Assertion (A): $f(x)=\begin{vmatrix}(1+x)^{21} & (1+x)^{22} & (1+x)^{23}\\ (1+x)^{31} & (1+x)^{32} & (1+x)^{33}\\ (1+x)^{41} & (1+x)^{42} & (1+x)^{43}\end{vmatrix}$, then coefficient of *x* in $f(x)$ is zero.

Reason (R): If $F(x)=A_0+A_1x+A_2x^2+.....+A_nx^n$, then $A_1=F^1(0)$ when dash denotes the differential coefficient.

Key. A

Sol. Conceptual

15. Consider the system of equations. x–2y+3z = –1, –x+y–2z = k, x–3y+4z = 1

Statement – I : The system of equation has no solution for k ≠ 3.

Statement – II : $\begin{vmatrix}1 & 3 & -1\\ -1 & -2 & k\\ 1 & 4 & 1\end{vmatrix} \neq 0$ for k ≠ 3

Key. A

Sol. $\Delta=0, \Delta y \neq 0$ for $k \neq 3$

16. Consider {$D_1, D_2, D_3 ......D_n$} be the complete set of third order determinant that can be made with the distinct non-zero real number $a_1, a_2 ....a_9$, then

Statement – I : $\sum_{i=1}^{n} D_i = 0$

Statement – II : $D_i = D_j \quad \forall\ i, j \in n$

Key. C

Sol. If one of the determinant of the given set is type $|C_1 \quad C_2 \quad C_3|$ then this set will also have the determinant of the type $|C_1 \quad C_3 \quad C_2|$, $|C_2 \quad C_3 \quad C_1|$,

$|C_2 \quad C_1 \quad C_3|$, $|C_3 \quad C_1 \quad C_2|$, $|C_3 \quad C_2 \quad C_1|$

If $|C_1 \quad C_2 \quad C_3| = \lambda$ then $|C_1 \quad C_3 \quad C_2| = -\lambda$, and so on so by symmetry $\sum D_i = 0$

17. Statement – I : M is third order matrix such that $M^T M = I$ and det M =1, then det(M-I)=0

Statement – II : If $M \& I$ are third order matrix then $\det(M-I)$ = det M – det I.

Key. C

Sol. $\det(M-I)=\det(M-I)^T=\det(M^T-I^T)$

$=\det(M^T-I)=\det(M^T-M^TM)$

$=\det(M^T)(I-M)=\det M^T.\det(I-M)$

$=-\det(M-I)$

$\Rightarrow 2\det(M-I)=0$

18. Statement – 1: The determinants of a matrix $A=\left[a_{ij}\right]_{5\times5}$ where $a_{ij}+a_{ji}=0$ for I and j is zero

Statement – 2: The determinant of a skew symmetric matrix of odd order is zero.

Key. A

Sol. $A=-A^T \quad \Rightarrow \quad |A|=-|A^T|=-|A|$

$\Rightarrow \quad 2|A|=0 \quad \Rightarrow \quad |A|=0$

19. Statement – 1: The inverse of the matrix $A=\left[a_{ij}\right]_{n\times n}$ where $a_{ij}=0, i\ge j$ is $B=\left[a_{ij}^{-1}\right]_{n\times n}$

Statement – 2: The inverse of singular matrix does not exist.

Key. D

Sol. Statement – 1 is false

$\because \quad A=\left[a_{ij}\right]_{n\times n}$ where $a_{ij}=0, i\ge j$

$\therefore \quad |A|=0$ hence A is singular inverse of A is not defined

Statement – 2 |A| = 0 $\quad\therefore\quad$ inverse of A is not defined

20. Statement – 1: If $f_1(x), f_2(x),...,f_9(x)$ are polynomials whose degree $\ge$ 1, where

$$f_1(\alpha)=f_2(\alpha)=f_2(\alpha)....=f_9(\alpha)=0 \quad \text{and} \quad A(x)=\begin{bmatrix} f_1(x) & f_2(x) & f_3(x) \\ f_4(x) & f_5(x) & f_6(x) \\ f_7(x) & f_8(x) & f_9(x) \end{bmatrix}$$

and $\dfrac{A(x)}{x-\alpha}$ is

Also a matrix of 3 × 3 whose entries are also polynomials

Statement – 2: $x-\alpha$ is a factor of polynomial f(x) if $f(\alpha)=0$

Key. A

Sol. $A(\alpha)=\begin{bmatrix} f_1(\alpha) & f_2(\alpha) & f_3(\alpha) \\ f_4(\alpha) & f_5(\alpha) & f_6(\alpha) \\ f_7(\alpha) & f_8(\alpha) & f_9(\alpha) \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$x-\alpha$ is a factor of $f_1(x), f_2(x)....f_9(x)$

$f(x)=(x-\alpha)\phi(x)$

$f(\alpha)=0 \quad \Rightarrow \quad x-\alpha$ is a factor of f(x)

21. Statement – 1: For a singular square matrix A, if AB = AC $\Rightarrow$ B = C

Statement – 2: If |A| = 0 then A^{-1} does not exist

Key. D

Sol. A^{-1} exist only for non-singular matrix

AB = AC $\quad \Rightarrow \quad$ B = C if A^{-1} exist

If A^{-1} exist

22. Statement – 1: $\left(a_{11}, a_{22}, ..., a_{nn}\right)$ is a diagonal matrix then $A^{-1} = \text{dia}\left(a_{11}{}^{-1}, a_{22}{}^{-1}, ... a_{nn}{}^{-1}\right)$

Statement – 2: If A = dia (2, 1, - 3) and B = dia (1, 1, 2) then det $\left(AB^{-1}\right) = 3$

Key. C

Sol. det $\left(AB^{-1}\right) = \det A. \det B^{-1} = \frac{\det A}{\det B} = \frac{-6}{2} = -3$

Matrices & Determents

Comprehension Type

Paragraph – 1

Let A be a m × n matrix. If there exists a matrix L of type n × m such that LA = I_n, then L is called left inverse of A. Similarly, if there exists a matrix R of type n × m such that AR = I_m, then R is called right inverse of A. For example to find right inverse of matrix

$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}$ we take $R = \begin{bmatrix} x & y & z \\ u & v & w \end{bmatrix}$

and solve $AR = I_3$ i.e.

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x & y & z \\ u & v & w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $x - u = 1$, $y - v = 0$, $z - w = 0$

$x + u = 0$, $y + v = 1$, $z + w = 0$

$2x + 3u = 0$, $2y + 3v = 0$, $2z + 3w = 1$

As this system of equations is inconsistent, we say there is no right inverse for matrix A.

1. Which of the following matrices is NOT left inverse of matrix $\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}$

(A) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ (B) $\begin{bmatrix} 2 & -7 & 3 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ (C) $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 3 & -1 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$

Key. C

2. Which of the following matrices is the right inverse of the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$

(A) $\begin{bmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 7 & 3 \\ 2 & 2 \\ -5 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 5 & 7 \\ -2 & 4 & 9 \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 1 \\ 7 & 1 \\ 3 & 0 \end{bmatrix}$

Key. D

3. The number of left inverses for the matrix $\begin{bmatrix} 1 & 1 \\ -2 & -2 \\ 1 & 1 \end{bmatrix}$ are

(A) 0 (B) 2 (C) 1 (D) infinite

Key. A

Paragraph – 2

Consider the determinant, $\Delta = \begin{vmatrix} p & q & r \\ x & y & z \\ \ell & m & n \end{vmatrix}$

M_{ij} denotes the minor of an element in row and column

C_{ij} denotes the cofactor of an element in row and column

4. The value of $p \cdot C_{21} + q \cdot C_{22} + r \cdot C_{23}$ is

(A) 0 (B) $-\Delta$ (C) Δ (D) Δ^2

Key. A

5. The value of $x \cdot C_{21} + y \cdot C_{22} + z \cdot C_{23}$ is

(A) 0 (B) $-\Delta$ (C) Δ (D) Δ^2

Key. C

6. The vlaue fo is $q \cdot M_{12} - y \cdot M_{22} + m \cdot M_{32}$

(A) 0 (B) $+\Delta^2$ (C) Δ (D) None of these

Key. D

Paragraph – 3

A Pythagorean triple is triplet of positive integers (a, b, c) such that $a^2 + b^2 = c^2$. Define the matrices P, Q and R by

$$P = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}, Q = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -2 \\ 2 & 2 & 3 \end{bmatrix} \text{ and } R = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

7. If we write Pythagorean triples (a, b, c) in matrix form as $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ then which of the following matrix product is not a Pythagorean triplet ?

A) $Q\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ B) $P\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ C) $R\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ D) none of these

8. Which one of the following does not hold good ?

A) $P^{-1} = adj.P$ B) $(PQ)^{-1} = adj.(PQ)$

C) $(QR)^{-1} = adj.(QR)$ D) $(PQR)^{-1} \neq adj.(PQR)$

9. $T_r\left(P + Q^T + 2R\right)$ equals

A) 17 B) 15 C) 14 D) 18

Sol. 7. Ans. (b)

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 26 \\ 20 \\ 29 \end{bmatrix} \Rightarrow 29^2 \neq 20^2 + 26^2$$

Similarly, $Q\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \\ 13 \end{bmatrix}$, and $R\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 15 \\ 8 \\ 17 \end{bmatrix}$

8. Ans. (d)

detP = 1, detQ = 1, detR = 1

det(PQ) = 1, det(QR) = 1, det(RP) = 1

det(PQR) = 1

9 .Ans. (c)

$$T_r\left(P + Q^T + 2R\right) = 14$$

Paragraph – 4

A and *B* are two matrices of same order 3×3, where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 5 & 6 & 8 \end{pmatrix} \text{ and } B = \begin{pmatrix} 3 & 2 & 5 \\ 2 & 3 & 8 \\ 7 & 2 & 9 \end{pmatrix}$$

10. The value of $|adj(adj\ A)|$ is equal to

a) 9 b) 16 c) 25 d) 81

11. The value of $|adj(AB)|$ is equal to

a) 24 b) 24^2 c) 24^3 d) 65

12. The value of $|(adj(adj(adj(adj\ A))))|$ is equal to

a) 2^4 b) 2^9 c) 2^{13} d) 2^{19}

Sol. 10. (B) $|adj(adj\ A)| = |A|^{(n-1)^2} = |A|^4 = 2^4 = 16$

11. (B) $AB = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 5 & 6 & 8 \end{pmatrix}\begin{pmatrix} 3 & 2 & 5 \\ 2 & 3 & 8 \\ 7 & 2 & 9 \end{pmatrix}$

$$= \begin{pmatrix} 28 & 14 & 48 \\ 40 & 21 & 70 \\ 83 & 44 & 145 \end{pmatrix}$$

$$\therefore \quad |\text{AB}| = \begin{vmatrix} 28 & 14 & 48 \\ 40 & 21 & 70 \\ 83 & 44 & 145 \end{vmatrix}$$

Applying $C_1 \to C_1 - 2C_2, C_3 \to C_3 - 3C_3$

Then, $|AB| = \begin{vmatrix} 0 & 14 & 6 \\ -2 & 21 & 7 \\ -5 & 44 & 13 \end{vmatrix}$

$$= \begin{vmatrix} 0 & -4 & 6 \\ -2 & 0 & 7 \\ -5 & 5 & 13 \end{vmatrix} C_2 \to C_2 - 3C_3$$

$$= 0 + 2(-52-30) - 5(-28)$$

$$= -164 + 140 = -24$$

$$|\text{adj } AB| = |AB|^{n-1} = |AB|^2 = (24)^2$$

60. (C) $\left|\left(adj(adj(adj(adjA)))\right)\right| = \left|adj\left(adj\left(2A\right)\right)\right|$

$$= |2A|^{(n-1)^2} = |2A|^4$$

$$= \left(2^3|A|\right)^4 = 2^{12}|A| = 2^{13}$$

Paragraph – 5

If A is 3×3 matrix then a non trivial solution $X = (x \quad y \quad z)^T$ such that $AX = \lambda X \left(\lambda \in R\right)$ yields 3 values of λ say $\lambda_1, \lambda_2, \lambda_3$ .For any such matrix A, λ's are called eigen values and corresponding X's are called eigen vectors .It is known that ,for any 3×3 matrix , $Tr(A) = \lambda_1 + \lambda_2 + \lambda_3$, det A = $\lambda_1\lambda_2\lambda_3$ .Answer the following questions for matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

13. $Tr(A^{-1}) =$

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) $-\frac{1}{3}$

14. $Tr(A^3) =$

(A) 149 (B) 101 (C) 128 (D) 133

15. Which of the following is false?

(A) $\exists$ a nontrivial solution X such that $AX = \left(2+\sqrt{7}\right)X$

(B) $\exists$ a nontrivial solution X such that $AX = X$

(C) $\exists$ a nontrivial solution X such that $A^{-1}X = \left(2-\sqrt{7}\right)X$

(D) The total number of nontrivial solutions X such that $AX = \lambda X$ is 3.

Key: D-B-C

Hint: For the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$, $|A-\lambda I| = 0 \Rightarrow \lambda^3 - 5\lambda^2 + \lambda + 3 = 0 \to (1)$, whose roots are $\lambda_1, \lambda_2, \lambda_3$

$$\lambda_1 + \lambda_2 + \lambda_3 = 5, \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 = 1, \lambda_1\lambda_2\lambda_3 = -3$$

$$\Rightarrow Tr(A) = 5, \det(A) = -3$$

Also $AX = \lambda X \Rightarrow A^{-1}X = \frac{1}{\lambda}X \Rightarrow Tr\left(A^{-1}\right) = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} = \frac{\sum \lambda_1\lambda_2}{\lambda_1\lambda_2\lambda_3} = -\frac{1}{3}$

$$AX = \lambda X \Rightarrow A^3X = \lambda^3 X \Rightarrow Tr(A^3) = \lambda_1^3 + \lambda_2^3 + \lambda_3^3$$

$$= \left(\lambda_1 + \lambda_2 + \lambda_3\right)\left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - \lambda_1\lambda_2 - \lambda_2\lambda_3 - \lambda_3\lambda_1\right) + 3\lambda_1\lambda_2\lambda_3$$

= 5 (25—3(1))-9 =101

Solving (1) gives $\lambda_1 = 1, \lambda_2 = 2+\sqrt{7}, \lambda_3 = 2-\sqrt{7},$ which by theory yield nontrivial solutions .In particular for A^{-1}, the values of λ yieldings nontrivial solutions are $1, \frac{1}{2+\sqrt{7}}, \frac{1}{2-\sqrt{7}}$

i.e $1, \frac{2+\sqrt{7}}{-3}, \frac{2-\sqrt{7}}{-3}$ .Hence (c) is false

Paragraph – 6

Let $\Delta \neq 0$ and Δ^c denotes the determinant of cofactors, then $\Delta^c = \Delta^{n-1}$, where n (>0) is the order of Δ.

On the basis of above information, answer the following questions:

16. If a, b, c are the roots of the equation $x^3 - px^2 + r = 0$, then the value of

$$\begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix}$$

(A) p^2 (B) p^4

(C) p^6 (D) p^9

17. If a, b, c are the roots of the equation $x^3 - 3x^2 + 3x + 7 = 0$, then the value of

$$\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix}$$

(A) 9 (B) 27

(C) 81 (D) 0

18. Suppose a, b, c $\in$ R, a + b + c > 0, A = bc – a^2, B = ca – b^2 and C = ab – c^2 and

$$\begin{vmatrix} A & B & C \\ B & C & A \\ C & A & B \end{vmatrix} = 49, \text{ then } \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \text{ equals}$$

(A) – 7 (B) 7

(C) – 2401 (D) 2401

Key : C – D – A

Sol :

16. a + b + c = p, ab + bc + ca = 0

$\therefore$ $a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca)$

$= p^2 - 0$

$= p^2$

If $\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

$\therefore$ $\Delta^{c} = \begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix} = \Delta^{3-1}$

$$= \Delta^2 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} a & b & c \\ b & c & b \\ c & a & a \end{vmatrix}$$

$$= \begin{vmatrix} a^2+b^2+c^2 & ab+bc+ca & ab+bc+ca \\ ab+bc+ca & a^2+b^2+c^2 & ab+bc+ca \\ ab+bc+ca & ab+bc+ca & a^2+b^2+c^2 \end{vmatrix}$$

$$= \begin{vmatrix} p^2 & 0 & 0 \\ 0 & p^2 & 0 \\ 0 & 0 & p^2 \end{vmatrix} = p^6$$

17. $\because$ $x^3 - 3x^2 + 3x + 7 = 0$

$\Rightarrow$ $(x-1)^3 + 8 = 0$

$\Rightarrow$ $(x-1)^3 = (-2)^3$

$\Rightarrow$ $\left(\dfrac{x-1}{-2}\right)^3 = 1$

$\Rightarrow$ $\dfrac{x-1}{-2} = (1)^{1/3} = 1, \omega, \omega^2$

$\Rightarrow$ $x - 1 = -2, -2\omega, -2\omega^2$

or $x = -1, 1 - 2\omega, 1 - 2\omega^2$

$\therefore$ $a = -1, b = 1 - 2\omega, c = 1 - 2\omega^2$

$$\because \quad \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} -a & c & b \\ -b & a & c \\ -c & b & a \end{vmatrix}$$

(row by row)

$$= \begin{vmatrix} 2bc - a^2 & c^2 & b^2 \\ c^2 & 2ac - b^2 & a^2 \\ b^2 & a^2 & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$$

$$= (a^3 + b^3 + c^3 - 3abc)^2$$

$$= \left\{(a+b+c)(a^2+b^2+c^2-ab-bc-ca)\right\}^2$$

$$= \frac{1}{4}(a+b+c)^2 \left\{(a-b)^2 + (b-c)^2 + (c-a)^2\right\}^2$$

$$= \frac{9}{4}\left\{-12(1+\omega+\omega^2)\right\} = 0$$

18. $$\begin{vmatrix} A & B & C \\ B & C & A \\ C & A & B \end{vmatrix}$$

$$= \begin{vmatrix} bc - a^2 & ca - b^2 & ab - c^2 \\ ca - b^2 & ab - c^2 & bc - a^2 \\ ab - c^2 & bc - a^2 & ca - b^2 \end{vmatrix}$$

$$= \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = 49$$

$$\therefore \quad \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 7$$

Paragraph – 7

$A_0 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ and $B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

If $B_n = adj(B_{n-1})$, $n \in N$ and I is an identity matrix of order 3, then answer the following question

19. Det. $\left(A_0 + A_0^2B_0^2 + A_0^3 + A_0^4B_0^4 + ...10 \text{ terms}\right)$ is equal to

(A) 1000 (B) –800

(C) 0 (D) –8000

Key. C

20. $B_1 + B_2 + ... + B_{49}$ is equal to

(A) B_0 (B) $7B_0$

(C) 49 I (D) $49B_0$

Key. D

Sol. 19 to 20

19. (C)

Use |AB| = |A| |B| and $|A_0| = 0$.

20. (D)

$B_1 = B_2 = B_3......B_{49} = B_0$

Paragraph – 8

Two $n \times n$ square matrices A and B are said to be similar if their exists a non-singular matrix P such that $P^{-1}BP = A$

21. If A & B are similar matrices such that $|A| = |adj(adj(Q))|$, then where, $Q = \begin{bmatrix} 1 & 1 & 0 \\ -2 & 1 & -1 \\ 1 & 2 & 3 \end{bmatrix}$

a) $|B| = 10^{-4}$ b) $|A| + |B| = 2 \times 10^4$ c) $|B| = 10^3$ d) None

Key. B

22. If A and B are similar and B and C are similar, then

a) AB and BC are similar b) A and C are similar

c) A + C and B are similar d) None

Key. B

23. If A and B are two non-singular matrices, then

a) A is similar to B b) AB is similar to BA c) AB is similar to $A^{-1}B$ d) None

Key. B

Sol. 21. $|Q| = 10 \Rightarrow |adj(adjQ)| = |Q|^{(3-1)^2} = 10^4 = |A|$

$P^{-1}BP = A$

$\Rightarrow |B| = |A| \Rightarrow |A| + |B| = 2 \times 10^4$

22. $P^{-1}BP = A$ ($\because$ A & B are similar)

$R^{-1}CR = B$ ($\because$ B & C are similar)

$\Rightarrow P^{-1}\left(R^{-1}CR\right)P = A$

$\Rightarrow \left(P^{-1}R^{-1}\right)C(RP)=A$

$\Rightarrow (RP)^{-1}C(RP)=A$

$\Rightarrow$ A and C are similar

23. $A^{-1}(AB)A=BA$

AB and BA are similar

Paragraph – 9

If A is a square matrix of order n, we can form the matrix $A-\lambda I$, where λ is a scalar and I is the unit matrix of order n. The determinant of this matrix equated to zero (i.e., $|A-\lambda I|=0$) is called as characteristic equation of A. On expanding the determinant, the characteristic equation can be written as a polynomial equation of degree n in λ of the form.
$(-1)^n\lambda^n+k_1\lambda^{n-1}+k_2\lambda^{n-2}+.......k_n=0$. The roots of this equation are called the characteristic roots (or) Eigen values of A. The sum of the Eigen values of matrix A is equal to trace of A. Every square matrix 'A' satisfies its own characteristic equation.
(i.e., $(-1)^n A^n+k_1A^{n-1}+k_2A^{n-2}+.......k_nI=0$) on multiplying the above equation by A^{-1} we can easily obtain the value of A^{-1}. This is the other way of finding A^{-1}.

24. The Eigen values of the matrix $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$

a) 3, -3, 5 b) 3, -3, -5 c) -3, -3, 5 d) -2, 4, -3

Key. C

25. If $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ then A^{-1} =

a) $\frac{1}{4}\left[A^2+6A-9I\right]$ b) $\frac{1}{4}\left[A^2+6A+9I\right]$

c) $\frac{-1}{4}\left[A^2-6A+9I\right]$ d) $\frac{1}{4}\left[A^2-6A+9I\right]$

Key. D

26. If $A=\begin{bmatrix}2 & 1 & 1\\ 0 & 1 & 0\\ 1 & 1 & 2\end{bmatrix}$, find the matrix represented by

$A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$

a) $\begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$ b) $\begin{bmatrix} 8 & 5 & 5 \\ 0 & 0 & 3 \\ 5 & 8 & 5 \end{bmatrix}$ c) $\begin{bmatrix} 5 & 8 & 5 \\ 3 & 0 & 0 \\ 5 & 5 & 8 \end{bmatrix}$ d) $\begin{bmatrix} 8 & -5 & 5 \\ 0 & 0 & -3 \\ 5 & -8 & 5 \end{bmatrix}$

Key. A

Sol. 24. $\lambda^3+\lambda^2-21\lambda-45=0$

$\lambda=-3,-3,5$

25. $\lambda^3-6\lambda^2+9\lambda-4=0$

$\Rightarrow A^2-6A+9I-4A^{-1}=0$

$\Rightarrow A^{-1}=\frac{1}{4}\left(A^2-6A+9I\right)$

26. $A^3-5A^2+7A-3I=0$

$=A^5\left(A^3-5A^2+7A-3I\right)+\left(A^3-5A^2+7A-3I\right)$

$+A^2+A+I=A^2+A+I$

Paragraph – 10

The values of p and q such that the system of equations

$2x+py+6z=8$

$x+2y+qz=5$

$x+y+3z=4$, has

27. unique solution if

a) $p\in R, q\in R$ b) $p\in R-\{2\}, q\in R-\{3\}$ c) $p\in R-\{2\}, q\in R$ d) none of these

Key. B

28. No solution if

a) $p\in R, q\in R$ b) $p\in R-\{2\}, q\in\{3\}$

c) $p\in R-\{2\}, q\in R-\{2\}$ d) $p\in R-\{2\}\ \ q\in R$

Key. B

29. Infinite solution then

a) $p=2$ or q =3 b) $p=2, q=3$

c) $p=2, q\in R$ d) $p\in R-\{2\}, q\in R-\{2\}$

Key. C

Sol. Conceptual

Paragraph – 11

A square matrix A such that $A^\theta=A$ is called Hermitian matrix i.e. $a_{ij}=\overline{a_{ji}}$ for all values of i and j and a square matrix A such that $A^\theta=-A$ is called skew –Hermitian matrix i.e. $a_{ij}=-\overline{a_{ji}}$ for all values of i and j. where A^θ is conjugate transpose matrix.

Let $f: M\to\{1,-1\}$, M is set of all hermitian or Skew–hermitian matrixes, be a function defined as

$$f(A)=\begin{cases}1 & A \text{ is hermitian}\\ -1 & A \text{ is skew hermitian}\end{cases}$$

30. $f(A-A^{\theta})=$

a) +1 b) –1 c) +1 any when f(A) >0 d) –1 any when f(A) >0

Key. B

31. Let $Y=A^n$

a) if f(Y)=1 then f(A)=1 b) if f(Y)= –1 then f(A) = –1

c) f(Y).f(A)=1 if n is odd d) f(Y).f(A)=1 if n is even

Key. B or C

32. Let $A=[a_{ij}]_{4\times4}$ be a matrix such that $\arg(a_{ij})\in\left[0,\frac{\pi}{2}\right]$ and $f(A)=-1$, and if $[b_{ij}]_{2\times2}$ be a matrix defined by $b_{ij}=a_{ii}+a_{jj}\forall i,j$ then

a) B is a unit matrix b) Trace (B) =0

c)B is a null matrix d) Det (B) $\geq$ 0

Key. D

Sol. 30. $X=A-A^{\theta}$ is a skew hermitian

$(\because X^{\theta}=(A-A^{\theta})^{\theta}=A^{\theta}-A=-X)$

$f(X)=-1$

31. $$Y^{\theta}=(A^n)^{\theta}=(A^{\theta})^n=\begin{cases}A^n=Y & \text{if } A \text{ is hermitian}\\ (-A)^n=\begin{cases}Y & \text{if } A \text{ is skew hermitian and } n \text{ is even}\\ -Y & \text{If } A \text{ is skew hermitian and } n \text{ is odd}\end{cases}\end{cases}$$

32. A is skew hermitian $\Rightarrow$ diagonal elements are purely imaginary or zero

Also arg $a_{ij}\in\left[0,\frac{\pi}{2}\right]\Rightarrow \operatorname{Im} a_{ij}\geq 0$

$a_{ij}=iy_{ii}\Rightarrow a_{11}=iy_{11}, a_{22}=iy_{22}, a_{33}=iy_{33}, a_{44}=iy_{44}$ (where $y_{11},y_{22},y_{33},y_{44}\geq 0$)

$$B=\begin{bmatrix}2iy_{11} & iy_{11}+iy_{22}\\ iy_{11}+iy_{22} & 2iy_{22}\end{bmatrix}\Rightarrow |B|=(y_{11}-y_{22})^2\geq 0$$

Paragraph – 12

Consider a system of linear equations in three variables x, y, z

$a_1x+b_1y+c_1z=d_1;$

$a_2x+b_2y+c_2z=d_2;$

$a_3x+b_3y+c_3z=d_3$

The system can be expressed by matrix equation

$$\begin{bmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}d_1\\ d_2\\ d_3\end{bmatrix} \text{ or } AX = B$$

If A is non-singular matrix then the solution of above system can be found by $X=A^{-1}B$. The solution in this case is unique. If A is a singular matrix i.e.

$|A|=0$, then the system will have no unique solution if (Adj A) B = 0 and the system has no solution (i.e. it is inconsistent) if (Adj A) B $\neq 0$

Where Adj A is the adjoint of the matrix A, which is obtained by taking transpose of the matrix obtained by replacing each element of matrix A with corresponding copactors.

Now consider the following matrices

$$A\begin{bmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{bmatrix}, B=\begin{bmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{bmatrix} U=\begin{bmatrix} f \\ g \\ h \end{bmatrix} V=\begin{bmatrix} a^2 \\ 0 \\ 0 \end{bmatrix}, X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

33. If the system AX = U has infinitely many solutions then

(A) $c=d, ab=1$ (B) $c=d, h=g$

(C) $ab=1, h=g$ (D) $a=b, c=d, g=h$

Key. B

34. If AX = U has infinitely many solutions then the equation BX = V has

(A) Unique solution

(B) Infinitely many solution

(C) No solution

(D) Either infinitely many solutions or no solution.

Key. D

35. If AX = U has infinitely many solutions then the equation BX = V is consistent if

(A) abc = 0 (B) bcd = 0 (C) adf = 0 (D) fgh = 0

Key. C

Sol. 33. (B) For infinite solutions or no solution

$$|A|=0 \Rightarrow \begin{vmatrix} a & 1 & 0 \\ 1 & b & d \\ 1 & b & c \end{vmatrix}=0$$

$\Rightarrow (ab-1)(c-d)=0 \Rightarrow ab=1$ or $c=d$

Now cofactors of elements of A in order are bc – bd, $d-c,\ 0; -c, ac, 1-ab; d, -ad, ab-1$,

so cofactor matrix is

$$\begin{bmatrix} bc-bd & d-c & 0 \\ -c & ac & 1-ab \\ d & -ad & ab-1 \end{bmatrix}$$

$$\therefore \operatorname{Adj} A=\begin{bmatrix} bc-bd & -c & d \\ d-c & ac & -ad \\ 0 & 1-ab & ab-1 \end{bmatrix}$$

$$\therefore (\operatorname{Adj} A)U=\begin{bmatrix} bf(c-d)-cg+dh \\ f(d-c)+gac-had \\ g(1-ab)+h(ab-1) \end{bmatrix}$$

Now for infinite solution $(\operatorname{Adj}A)U=0$

$\Rightarrow bf(c-d)-cg+dh=0, f(d-c)+a(cg-dh)=0$ and $(ab-1)(h-g)=0$

All the above holds good if $d = c$ and $g = h$, whether $ab = 1$ or $ab \neq 1$

34. We have $|B| = \begin{vmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{vmatrix}$

[In view of c =d and g = h, C_1 and C_2 are identical]

$\therefore$ The equation has no unique solution. It is either inconsistent or has infinitely many solutions.

35. The cofactors of elements of B in order are $dh - cg$ = 0,

$cf, -df, g - h = 0,\ ah - f, -ag + f; c - d = 0, -ac, ad$

$\therefore$ cofactor matrix is $\begin{bmatrix} 0 & cf & -df \\ 0 & ad - f & f - ag \\ 0 & -ac & ad \end{bmatrix}$

$\therefore adjB = \begin{bmatrix} 0 & 0 & 0 \\ cf & ad - f & -ac \\ -df & f - ag & ad \end{bmatrix}$

$\therefore (AdjB)V = \begin{bmatrix} 0 \\ a^2cf \\ -a^2df \end{bmatrix}$

$\therefore$ $|B| = 0,$ so for consistent system $(AdjB)V = 0$

$\Leftrightarrow a^2cf = 0$ and $-a^2df = 0 \Leftrightarrow adf = 0 \Leftrightarrow a = 0$ or $d = 0$ or $f = 0$ $(\because c = d)$

Paragraph – 13

Consider the determinant

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix}$$

$M_{ij} =$ Minor of the element of i^{th} row and j^{th} column

$C_{ij} =$ Cofactor of the element of i^{th} row and j^{th} column

36. Value of $b_1.C_{31} + b_2.C_{32} + b_3.C_{33}$ IS

A) 0 B) Δ C) 2Δ D) Δ^2

Key. A

Sol. $b_1.C_{31} + b_2.C_{32} + b_3.C_{33} =$

$b_1\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - b_2\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + b_3\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = 0$

37. If all the elements of the determinant are multiplied by 2, then the value of new determinant is

A) 0 B) 8Δ C) 2Δ D) $2^9\Delta$

Key. B

Sol. Value of new determinant $= 2^3\Delta = 8\Delta$

38. $a_2M_{13} - b_3.M_{23} + d_3.M_{33}$ is equal to

A) 0 B) 4Δ C) 2Δ D) Δ

Key. D

Sol. $a_3M_{13} - b_3.\ M_{23} + d_3.\ M_{33} = a_3C_{13} + b_3.\ C_{23} + d_3.\ C_{33} = \Delta$ by definition

Paragraph – 14

Let A be a m × n matrix. If there exists a matrix L of type n × m such that $LA = I_n$, then L is called left inverse of A. Similarly, if there exists a matrix R of type n × m such that $AR = I_m$, then R is called right inverse of A.

For example to find right inverse of matrix

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix} \text{ we take } R = \begin{bmatrix} x & y & z \\ u & v & w \end{bmatrix}$$

and solve $AR = I_3$ i.e.

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x & y & z \\ u & v & w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$ x – u = 1   y – v = 0   z – w = 0

x + u = 0   y + v = 1   z + w = 0

2x + 3u = 0   2y + 3v = 0   2z + 3w = 1

As this system of equations is inconsistent, we say there is no right inverse for matrix A.

39. Which of the following matrices is NOT left inverse of matrix $\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}$

A) $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ B) $\begin{bmatrix} 2 & -7 & 3 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ C) $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ D) $\begin{bmatrix} 0 & 3 & -1 \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$

Key. C

Sol. As 2^{nd} row of all the options is same, we are to look at the elements of the first row.

Let left inverse be $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$, then

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore$ a + b + 2c = 1

- a + b + 3c = 0 i.e. $b = \frac{1-5c}{2}$, $a = \frac{1+c}{2}$

Thus matrices in the options A, B and D are the inverses and matrix in option C is not the left inverse.

40. The num ber of right inverse for the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$

A) 0   B) 1   C) 2   D) infinite

Key. D

Sol. Let right inverse is

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$$

Now $a - c + 2e = 1$
$b - d + 2f = 0$
$2a - c + e = 0$
$2b - d + f = 1$

Infinite solution
so answer is (D)

41. For which of the following matrices number of left inverses is greater than the number of right inverses

A) $\begin{bmatrix} 1 & 2 & 4 \\ -3 & 2 & 1 \end{bmatrix}$ B) $\begin{bmatrix} 3 & 2 & 1 \\ 3 & 2 & 1 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 4 \\ 2 & -3 \\ 5 & 4 \end{bmatrix}$ D) $\begin{bmatrix} 3 & 3 \\ 1 & 1 \\ 4 & 4 \end{bmatrix}$

Key. C

Sol. By observation there can't be any left inverse for (B) & (D) so we will cheak for (A) & (C) only.

For (A) let left inverse be $\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$, then

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}\begin{bmatrix} 1 & 2 & 4 \\ -3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now $a - 3b = 1$, $2a + 2b = 0$ and $4a + b = 0$ which is not possible.

For (C) $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 2 & -3 \\ 5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ $a + 2b + 5c = 1$, $2a - 3b = 0$ and $5a + 4d = 0$

which is not possible

$\therefore$ There is no right inverse.

Paragraph – 15

Consider the determinant, $\Delta \begin{vmatrix} p & q & r \\ x & y & z \\ l & m & n \end{vmatrix}$

M_{ij} denotes the minor of an element in i^{th} row and j^{th} column

C_{ij} denotes the cofactor of an element in i^{th} row and j^{th} column

42. The value of p. $C_{21} + q.C_{22} + r.\,C_{23} =$

A) 0 B) $-\Delta$ C) Δ D) Δ^2

Key. A

Sol. p,q,r are the entries of first row and C_{21}, C_{22}, C_{23} are cofactors of second row

$\therefore$ $p.\,C_{21} + q.\,C_{22} + r.\,C_{23} = 0$

43. The value of $x.\,C_{21} + y.\,C_{22} + z.\,C_{23} =$

A) 0 B) $-\Delta$ C) Δ D) Δ^2

Key. C

Sol. x,y,z are the entries of second row and C_{21}, C_{22}, C_{23} are cofactors of second row

$\therefore \quad x.\, C_{21} + y.\, C_{22} + z.\, C_{23} = \Delta$

44. The value of $q.\, M_{12} - y.\, M_{22} + m.\, M_{32} =$

A) 0 B) $-\Delta$ C) Δ D) Δ^2

Key. B

Sol. $q.\, M_{12} - y.M_{22} + m.\, M_{32} = -q.\, C_{12} - y.\, C_{22} - m.\, C_{32}$

$= -(q.\, C_{12} + y.C_{22} + m.\, C_{32}) = -\Delta$

{ $\because$ q,y,m are entries of second column and C_{12}, C_{22}, C_{32} are cofactor of second column}

Matrices & Determents

Integer Answer Type

1. If the integers a, b, c inorder are in A.P., lying between 1 and 9 and a23, b53, and c83 are three-digit numbers, then the value of the determinant $\begin{vmatrix} 2 & 5 & 8 \\ a23 & b53 & c83 \\ a & b & c \end{vmatrix}$ is...

Key. 0

Sol. We have,

$$\begin{vmatrix} 2 & 5 & 8 \\ a23 & b53 & c83 \\ a & b & c \end{vmatrix} = \begin{vmatrix} 2 & 5 & 6 \\ 100a+20+3 & 100b+50+3 & 100c+80+3 \\ a & b & c \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 5 & 8 \\ 100a & 100b & 100c \\ a & b & c \end{vmatrix} + \begin{vmatrix} 2 & 5 & 8 \\ 20 & 50 & 80 \\ a & b & c \end{vmatrix} + \begin{vmatrix} 2 & 5 & 8 \\ 3 & 3 & 3 \\ a & b & c \end{vmatrix}$$

$$= 100\begin{vmatrix} 2 & 5 & 8 \\ a & b & c \\ a & b & c \end{vmatrix} + 10\begin{vmatrix} 2 & 5 & 8 \\ 2 & 5 & 8 \\ a & b & c \end{vmatrix} + 3\begin{vmatrix} 2 & 5 & 8 \\ 1 & 1 & 1 \\ a & b & c \end{vmatrix}$$

$$= 0 + 0 + 3\begin{vmatrix} 2 & 3 & 6 \\ 1 & 0 & 0 \\ a & b-a & c-a \end{vmatrix}$$

(Applying $C_2 \to C_2 - C_1,\ C_3 \to C_3 - C_1$)

$$= -3\left[3(c-a) - 6(b-a)\right] = -9\left[c - a - 2b + 2a\right]$$

$$= -9(a - 2b + c) = 0 \qquad \left[\because \text{ a, b, c are in A.P., } \therefore 2b = a + c\right]$$

2. A be set of 3 × 3 matrices formed by entries 0, –1, and 1 only. Also each of 1, -1, 0 occurs exactly three times in each matrix. The number of symmetric matrices with trace (A) = 0 is k, then $\frac{k}{6} = ...$

Key. 6

Sol. For non-diagonal entries, we required even no. of 1, even no. of –1 and even no. of 0, for diagonal three entries are remained, –1, 0, 1. So no. of cases in which trace = 0 are 3! And no. of symmetric matrices for each arrangement of 1, –1, 0 in diagonal = 3!

Total such matrices = 3! × 3! = 36

3. Let A_n, ($n \in N$) be a matrix of order $(2n - 1) \times (2n - 1)$, such that $a_{ij} = 0 \ \forall \ i \neq j$ and $a_{ij} = n^2 + i + 1 - 2n \ \forall \ i = j$ where a_{ij} denotes the element of i^{th} row and j^{th} column of A_n.

Let T_n = $(-1)^n$ × (sum of all the elements of A_n). Find the value of $\left[\dfrac{\sum_{n=1}^{102} T_n}{520200}\right]$, where [.] represents the greatest integer function.

Ans: 2

Hint $a_{ij} = 0 \ \forall \ i \neq j$ and $a_{ij} = (n-1)^2 + i \ \forall \ i = j$

Sum of all the element of $A_n = \sum_{i=1}^{2n-1}[(n-1)^2 + i]$

$= (2n-1)(n-1)^2 + (2n-1)n = 2n^3 - 3n^2 + 3n - 1 = n^3 + (n-1)^3$

So, $T_n = (-1)^n [n^3 + (n-1)^3] = (-1)^n n^3 - (-1)^{n-1}(n-1)^3 = V_n - V_{n-1}$

$$\Rightarrow \sum_{n=1}^{102} T_n = \sum_{n=1}^{102}(V_n - V_{n-1}) = V_{102} - V_0 = (102)^3$$

$$\left[\frac{\sum_{n=1}^{102} T_n}{520200}\right] = 2.$$

4. Find the value of $f\left(\frac{\pi}{6}\right)$, where $f(\theta) = \begin{vmatrix} \cos^2\theta & \cos\theta\sin\theta & -\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$

Key. 1

Sol. Applying $C_1 \to C_1 - \sin\theta C_3$ and $C_2 \to C_2 + \cos\theta C_3$, we get

$$f(\theta) = \begin{vmatrix} 1 & 0 & -\sin\theta \\ 0 & 1 & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$$

Applying $R_3 \to R_3 - \sin\theta R_1 + \cos\theta R_2$, we get

$$f(\theta) = \begin{vmatrix} 1 & 0 & -\sin\theta \\ 0 & 1 & \cos\theta \\ \sin\theta & 0 & \sin^2\theta + \cos^2\theta \end{vmatrix} = 1$$

Thus, $f\left(\frac{\pi}{6}\right) = 1$

5. $\det P = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$, where '$P$' is an orthogonal matrix. Then the value of $|a+b+c|$ is

Key. 1

Sol. $$PP^T = \begin{bmatrix} a & b & c \\ c & a & b \\ b & c & a \end{bmatrix}\begin{bmatrix} a & c & b \\ b & a & c \\ c & b & a \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a^2+b^2+c^2 & ab+bc+ca & ab+bc+ca \\ ab+bc+ca & a^2+b^2+c^2 & ab+bc+ca \\ ab+bc+ca & ab+bc+ca & a^2+b^2+c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

6. If the adjoint of a 3 × 3 matrix P is $\begin{bmatrix} 1 & 2 & 1 \\ 4 & 1 & 1 \\ 4 & 7 & 3 \end{bmatrix}$, then the sum of squares of possible values of determinant of P is

Key. 8

Sol. Adj. P $= \begin{bmatrix} 1 & 2 & 1 \\ 4 & 1 & 1 \\ 4 & 7 & 3 \end{bmatrix}$

$|\text{adj. P}| = 4$

$\Rightarrow |P|^2 = 4 \Rightarrow |P| = \pm 2$

7. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then det($A^{2005}$) equals to

Key. 1

Sol. $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$

$A^3 = A^2A = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 7 & -12 \\ 3 & -5 \end{bmatrix}$

Observing A, A^2, A^3 we can conclude that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

det (A^n) = $\begin{vmatrix} 1+2n & -4n \\ n & 1-2n \end{vmatrix} = 1-4n^2+4n^2 = 1$

$\therefore$ det (A^{2005}) = 1

8. If x, y, z are cube roots of unity and

$$D = \begin{vmatrix} x^2+y^2 & z^2 & z^2 \\ x^2 & y^2+z^2 & x^2 \\ y^2 & y^2 & z^2+x^2 \end{vmatrix},$$ then the real part of D is

Key. 4

Sol. An applying $R_1 \to R_1 - R_2 - R_3$

$$D = \begin{vmatrix} 0 & -2y^2 & -2x^2 \\ x^2 & y^2+z^2 & x^2 \\ y^2 & y^2 & z^2+x^2 \end{vmatrix}$$

$$= -2\begin{vmatrix} 0 & y^2 & x^2 \\ x^2 & z^2 & 0 \\ y^2 & 0 & z^2 \end{vmatrix}$$

$= 4x^2y^2z^2 = 4\ (1.w.w.^2)^2 = 4$

9. Find the coefficient of x in the determinant $\begin{vmatrix} (1+x)^{a_1b_1} & (1+x)^{a_1b_2} & (1+x)^{a_1b_2} \\ (1+x)^{a_2b_1} & (1+x)^{a_2b_2} & (1+x)^{a_2b_3} \\ (1+x)^{a_3b_1} & (1+x)^{a_3b_2} & (1+x)^{a_3b_3} \end{vmatrix}$, whee $a_1, b_1 \in N$

Ans. $\lambda_1 = 0$

Sol. Let $\begin{vmatrix} (1+x)^{a_2b_1} & (1+x)^{a_1b_2} & (1+x)^{a_1b_2} \\ (1+x)^{a_2b_1} & (1+x)^{a_2b_2} & (1+x)^{a_2b_2} \\ (1+x)^{a_2b_2} & (1+x)^{a_2b_2} & (1+x)^{a_2b_2} \end{vmatrix} = \lambda_0 + \lambda_1 x + \lambda_2 x^2 + \lambda_3 x^3 + ...$

For λ_1 differentiate w.r.t. x and put x = 0

so $\lambda_1 = 0$

10. If if $(x) = \begin{vmatrix} \cos(x+\alpha) & \cos(x+\beta) & \cos(x+\gamma) \\ \sin(x+\alpha) & \sin(x+\beta) & \sin(x+\gamma) \\ \sin(\beta-\gamma) & \sin(\gamma-\alpha) & \sin(\alpha-\beta) \end{vmatrix}$ and f(2) = 6, then find $\sum_{r=1}^{25} f(r)$

Ans. 150

Sol. Clearly $f'(x) = 0$

$\therefore \quad f(x) = c = 5$

$\therefore \quad \sum_{r=1}^{25} f(r) = \sum_{r=1}^{25} 6 = 150$ Ans.

11. Let $f(x) = \begin{vmatrix} x & 1 & 1 \\ \sin 2\pi x & 2x^2 & 1 \\ x^3 & 3x^4 & 1 \end{vmatrix}$. If f(x) be an odd function and its odd values is equal g(x), then find the value of λ. If $\lambda f(1) g(1) = 4$

Ans. $\lambda = 1$

Sol. f(- x) = - f(x) = g(x)

$\therefore \quad f(x).g(x) = -(f(x))^2$

or $f(1)g(1) = -(f(1))^2 = -\begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 3 & 1 \end{vmatrix} = -4 \Rightarrow \qquad \lambda = 1$ Ans.

12. If f(x) satisfies the equation $\begin{vmatrix} f(x+1) & f(x+8) & f(x+1) \\ 1 & 2 & -5 \\ 2 & 3 & \lambda \end{vmatrix} = 0$ for all real x. If f is periodic with period 7, then find the value of $|\lambda|$

Ans. 4

Sol. On solving we get

$(2\lambda+15)f(x+1)-(\lambda+10)f(x+8)-f(x+1)=0$

$(2\lambda+14)f(x+1)=(\lambda+10)f(x+8)$

Since f is periodic with period 7

$\therefore$ f(x + 1) = f(x + 8)

$\Rightarrow \quad 2\lambda+14=\lambda+10 \quad \Rightarrow \quad |\lambda|=4$ Ans.

13. If $A=\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$ and B = (adj A) and C = 5A, then find the value of $\frac{|\text{adj } B|}{|C|}$.

Ans. 1

Sol. $\frac{|\text{adj } B|}{|C|} = \frac{|\text{adj}(\text{adj} A)|}{|5A|} = \frac{|A|^{(3-1)^2}}{5^3|A|} = \frac{|A|^3}{125}$

Now $|A| = 5$

$\therefore \quad \frac{|\text{adj } B|}{|C|} = 1$ Ans.

14. If $A=\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$ and $\phi(x)=(1+x)(1-x)^{-1}$, then prove that $\phi(A)=-A$

Ans. $\phi(A)=(I+A)(I-A)^{-1}=\begin{pmatrix} 2 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -\frac{1}{2} & 0 \end{pmatrix}=\begin{pmatrix} -1 & -2 \\ -1 & -1 \end{pmatrix}=-A$

Sol. $I+A\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}+\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}=\begin{pmatrix} 2 & 2 \\ 1 & 2 \end{pmatrix}$ and $I-A=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}-\begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}=\begin{pmatrix} 0 & -2 \\ -1 & 0 \end{pmatrix}$

Now, $|I-A|=\begin{vmatrix} 0 & -2 \\ -1 & 0 \end{vmatrix}=0-2=-2$

adj $(I-A)=\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$

$(I-A)^{-1}=\begin{pmatrix} 0 & -1 \\ -\frac{1}{2} & 0 \end{pmatrix}$

$\therefore \quad \phi(A)=(I+A)(I-A)^{-1}=\begin{pmatrix} 2 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -\frac{1}{2} & 0 \end{pmatrix}=\begin{pmatrix} -1 & -2 \\ -1 & -1 \end{pmatrix}=-A$

15. If $A\begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$, $abc=1$, $A'A=1$, then find the value of $a^3+b^3+c^3$

Ans.

Sol. $A'A = I$

$\therefore \quad |A'A| = |I| \quad \Rightarrow \quad |A| = \pm 1$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \pm 1$$

$\Rightarrow \quad 3abc - a^3 - b^3 - c^3 = \pm 1 \quad \Rightarrow \quad a^3 + b^3 + c^3 = 4$ and 2

16. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $A^2 = 8A + KI_2$, then find the value of |k|

Ans. 7

Sol. Here $|A - \lambda| = 0$

$$\begin{vmatrix} 1-\lambda & 0 \\ -1 & 7-\lambda \end{vmatrix} = 0$$

$\Rightarrow \quad (1-\lambda)(7-\lambda) = 0 \quad \Rightarrow \quad \lambda^2 - 8\lambda + 7 = 0$

$\Rightarrow \quad A^2 - 8A + 7I_2 = 0 \quad \Rightarrow \quad A^2 = 8A - 7I_2$

$\Rightarrow \quad k = -7 \quad \Rightarrow \quad |k| = 7$ Ans.

17. Compute A^{-1} if $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$. Hence solve the system of equations

$$\begin{bmatrix} 3 & 0 & 3 \\ 2 & 1 & 0 \\ 4 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2y \\ z \\ 3y \end{bmatrix}$$

Ans. $\begin{bmatrix} 3 & 0 & 3 \\ 2 & 1 & 0 \\ 4 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2y \\ z \\ 3y \end{bmatrix}$

Sol. Compute A^{-1}. If $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$. Hence solve the system of equations

$$\begin{bmatrix} 3 & 0 & 3 \\ 2 & 1 & 0 \\ 4 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2y \\ z \\ 3y \end{bmatrix}$$

Matrices & Determents

Matrix-Match Type

1. Let $\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix} = ax^5+bx^4+cx^3+dx^2+ex+f$

Column – I		Column – II	
a)	Value of f is	p)	0
b)	Value of e is	q)	1
c)	Value of $a+c$ is	r)	-1
d)	Value of $b+d$ is	s)	3

Key: a) q b) s c) r d) q

Hint: a) Put $x=0 \Rightarrow$ find f

b) Diff. both sides and put $x=0$ find e.

c, d) put $x=1, x=-1$ solve

2. Let $p(\theta)=\begin{vmatrix} -\sqrt{2} & \sin\theta & \cos\theta \\ 1 & \cos\theta & \sin\theta \\ -1 & \sin\theta & -\cos\theta \end{vmatrix}$, $q(\theta)=\begin{vmatrix} \sin 2\theta & -1 & 1 \\ \cos 2\theta & 4 & -3 \\ 2 & 7 & -5 \end{vmatrix}$,

$r(\theta)=\begin{vmatrix} \cos\theta & \sin\theta & \cos\theta \\ -\sin\theta & \cos\theta & \sin\theta \\ -\cos\theta & -\sin\theta & \cos\theta \end{vmatrix}$ and $s(\theta)=\begin{vmatrix} \sec^2\theta & 1 & 1 \\ \cos^2\theta & \cos^2\theta & \operatorname{cosec}^2\theta \\ 1 & \cos^2\theta & \cot^2\theta \end{vmatrix}$

Match the functions on the left with their range on the right.

Column I		Column II	
(A)	$p(\theta)$	(P)	[0, 1]
(B)	$q(\theta)$	(Q)	$\left[0, 2\sqrt{2}\right]$
(C)	$r(\theta)$	(R)	[–2, 2]
(D)	$s(\theta)$	(S)	$\left[-\sqrt{5}-2, \sqrt{5}-2\right]$

Key: A-Q, B-S, C-R, D-P

Hint: (A) Expand along C_1 to obtain

$$p(\theta)=\left(-\sqrt{2}\right)(-1)+(-1)(-2\sin\theta\cos\theta)+(-1)\left(\sin^2\theta-\cos^2\theta\right)$$

$$=\sqrt{2}+\sin 2\theta+\cos 2\theta=\sqrt{2}+\sqrt{2}\sin\left(2\theta+\frac{\pi}{4}\right)$$

$\therefore$ range of p(θ) is $\left[0, 2\sqrt{2}\right]$.

(B) Applying $R_2 \to R_2+4R_1, R_3 \to R_3+7R_1$, we get

$$q(\theta)=\begin{vmatrix}\sin 2\theta & -1 & 1\\ \cos 2\theta+4\sin 2\theta & 0 & 1\\ 2+7\sin 2\theta & 0 & 2\end{vmatrix}$$ = 2 cos 2 θ + 8 sin 2θ - 2 – 7 sin 2θ

= 2 cos 2θ + sin 2θ - 2

As 2 cos 2θ + sin 2θ lies between $-\sqrt{5}$ to $\sqrt{5}$, we get range of q(θ) is $\left[-\sqrt{5}-2, \sqrt{5}-2\right]$.

(C) Using $C_1 \to C_1 + C_3$, we get

$$r(\theta)=2\cos\theta\begin{vmatrix}1 & \sin\theta & \cos\theta\\ 0 & \cos\theta & \sin\theta\\ 0 & -\sin\theta & \cos\theta\end{vmatrix}=2\cos\theta$$

∴ range of r(θ) is [–2, 2]

(D) Taking $\sec^2\theta$ common from R_1, we get

$$s(\theta)=\sec^2\theta\begin{vmatrix}1 & \cos^2\theta & \cos^2\theta\\ \cos^2\theta & \cos^2\theta & \operatorname{cosec}^2\theta\\ 1 & \cos^2\theta & \cot^2\theta\end{vmatrix}$$

$R_3 \to R_3 - R_1$, we get

$$s(\theta)=\sec^2\theta\begin{vmatrix}1 & \cos^2\theta & \cos^2\theta\\ \cos^2\theta & \cos^2\theta & \operatorname{cosec}^2\theta\\ 0 & 0 & \cot^2\theta-\cos^2\theta\end{vmatrix}$$

$$=\sec^2\theta\left(\cot^2\theta-\cos^2\theta\right)\left(\cos^2\theta-\cos^4\theta\right)$$

= $\left(\cot^2\theta-\cos^2\theta\right)\sin^2\theta$ = $\cos^2\theta-\cos^2\theta\sin^2\theta=\cos^4\theta$

∴ range of s(θ) is [0, 1]

3.

	Column I		Column II
(A)	A is a matrix such that $A^2 = A$. If $(I + A)^8 = I + \lambda A$, then $\lambda + 1$ is equal to	(P)	64
(B)	If A is a square matrix of order 3 such that |A| = 2, then $\left\|\left(\text{adj}A^{-1}\right)^{-1}\right\|$ is equal to	(Q)	1
(C)	Let $\|A\|=\|a_{ij}\|_{3\times3}\neq 0$. Each element a_{ij} is multiplied by λ^{i-j}. Let $\|B\|$ the resulting determinant, where $\|A\|=\lambda\|B\|$, then λ is equal to	(R)	256
(D)	If A is a diagonal matrix of order 3 × 3 is commutative with every square matrix of order 3 × 3 under multiplication and trace (A) = 12, then |A| =	(S)	4

Key. A – R, B – S, C – Q, D – P

Sol. (A) $(I+A)^8={}^8C_0I+{}^8C_1IA+{}^8C_2IA^2+\ldots\ldots\ldots+{}^8C_8IA^8$

$={}^8C_0I+{}^8C_1A+{}^8C_2A+\ldots\ldots+{}^8C_8A^8$

$= I + A\left({}^8C_1 + {}^8C_2 + ... + {}^8C_8\right)$

= I + A($2^8 - 1$) $\Rightarrow \lambda = 2^8 - 1$

(B) $\left|adj\left(A^{-1}\right)\right| = |A^{-1}|^2 = \dfrac{1}{|A|^2}$

$\left|\left(adj\left(A^{-1}\right)\right)^{-1}\right| = \dfrac{1}{|adjA^{-1}|} = |A|^2 = 2^2 = 4$

(C) $|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$\Rightarrow \ |B| = \begin{vmatrix} a_{11} & \lambda^{-1}a_{12} & \lambda^{-2}a_{13} \\ \lambda a_{21} & a_{22} & \lambda^{-1}a_{23} \\ \lambda^2 a_{31} & \lambda a_{32} & a_{33} \end{vmatrix} = \dfrac{1}{\lambda^3}\begin{vmatrix} \lambda^2 a_{11} & \lambda a_{12} & a_{13} \\ \lambda^2 a_{21} & \lambda a_{22} & a_{23} \\ \lambda^2 a_{31} & \lambda a_{32} & a_{33} \end{vmatrix} = |A|$

Hence, |A| = |B| $\Rightarrow \lambda$ = 1.

(D) A diagonal matrix is commutative with every square matrix, if it is a scalar matrix.

So every diagonal element is 4.

$\therefore$ |A| = 64.

Match the following

4. A, B are two matrices of 3×3 order. Such that

a) A is non singular and AB =0 then	(p) A=0
b) B is non singular and AB =0 then	(q) B =0
c) A & B both are non zero matrix and AB =0 then	(r) det A =0
d) $A^n = 0$ for some $n \geq 2$ then	(s) det B =0

Key. a – qs; b – pr; c– rs; d– r

Sol. a) $AB = 0 \Rightarrow |A| \neq 0 \Rightarrow A^{-1}$ exist. $A^{-1}AB = 0 \Rightarrow IB = 0 \Rightarrow B = 0$

b) AB = 0, $|B| \neq 0 \Rightarrow B^{-1}$ exist

$ABB^{-1} = 0 \Rightarrow AI = 0 \Rightarrow A = 0$

(c) AB =0 $\Rightarrow |A||B| = 0 \Rightarrow$ three cases

(i) either $|A| = 0, |B| \neq 0$ or (ii) $|A| \neq 0, |B| = 0$ or (iii) $|A| = 0, |B| = 0$

(i) $|B| \neq 0$, AB =0 $\Rightarrow$ A =0 but in question $A \neq 0$. $\Rightarrow$ I[st] case is not possible similarity

(ii) case is also not possible hence both $|A|, |B|$ should be zero.

d) $A^n = 0 \Rightarrow |A^n| = 0 \Rightarrow |A| = 0$

5. A is Non zero matrix

a) If A is Hermitian matrix and X =iA then X is	(p) Hemitian matrix
b) If A is Hermitian matrix and $X = A - A^T$ then X is	(q) Skew Hermitian
c) A is symmetric matrix and $X = iA + iA^\theta$ then X is	(r) symmetric
d) If A is skew Hermitian and X = i(A+A^T) then X is	(s) Skew symmetric

Key. a – q; b – ps; c – q,r; d – p, r

Sol. (a) A is hermitian then A^θ = A

$X^\theta = (iA)^\theta = \left(\overline{iA}\right)^T = \left(-i\overline{A}\right)^T = -i\left(\overline{A}\right)^T = -iA = -X$

$\Rightarrow$ X is skew hermition

6. Match the parameter on which the value of the determinant does not depend upon

a) $\begin{vmatrix} 1 & x & x^2 \\ \cos(a-b)y & \cos ay & \cos(a+b)y \\ \sin(a-b)y & \sin ay & \sin(a+b)y \end{vmatrix}$ (p) a

b) $\begin{vmatrix} x^2+y^2 & ax+by & x+y \\ ax+by & a^2+b^2 & a+b \\ x+y & a+b & 2 \end{vmatrix}$ (q) b

c) $\begin{vmatrix} 1 & a & a^2+b \\ 1 & b & b^2+a \\ 1 & 1 & 1+ab \end{vmatrix}$ (r) x

(s) y

d) $\dfrac{1}{(a^2+b^2+x^2)^3}\begin{vmatrix} a^2+(b^2+x^2)\cos y & ab(1-\cos y) & ax(1-\cos y) \\ ab(1-\cos y) & b^2+(x^2+a^2)\cos y & bx(1-\cos y) \\ ax(1-\cos y) & bx(1-\cos y) & x^2+(a^2+b^2)\cos y \end{vmatrix}$

Key. a – p; b – pqrs; c – rs; d – pqr

Sol. (i) $C_1 \to C_1 + C_3 - 2\cos by\, C_2 \Rightarrow \begin{vmatrix} 1+x^2+2x\cos by & x & x^2 \\ 0 & \cos ay & \cos(a+b)y \\ 0 & \sin ay & \sin(a+b)y \end{vmatrix}$

$= (1+x^2+2x\cos by)(\sin by)$

(ii) $\Delta = \begin{vmatrix} x & y & 0 \\ a & b & 0 \\ 1 & 1 & 0 \end{vmatrix} . \begin{vmatrix} x & a & 1 \\ y & b & 1 \\ 0 & 0 & 0 \end{vmatrix} = 0$

(iv) (i) multiply a, b, x in C_1, C_2, C_3 respectively and after that take a, b, x common from R_1, R_2, R_3 respectively

(ii) $C_1 \to C_1 + C_2 + C_3$ (iii) $R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$\Delta = \dfrac{(a^2+b^2+x^2)^3}{(a^2+b^2+x^2)^3}.\cos^2 y = \cos^2 y$

7. Match the following: -

	Column – I		Column – II
(A)	A is a real skew symmetric matrix such that $A^2 + I = 0$. Then	(p)	BA – AB
(B)	A is a matrix such that $A^2 = A$. If $(I+A)^n = I + \lambda A$, then λ equals $(n \in N)$	(q)	A is of even order
(C)	If for a matrix $A, A^2 = A$, and B = I – A, then AB + BA	(r)	A

	+ I - $\left(I-A\right)^2$ equals		
(D)	A is a matrix with complex entries and A^* stands for transpose of complex conjugate of A. If $A^*=A$ & $B^*=B$, then $\left(AB-BA\right)^*$ equals	(s)	2^n-1
		(t)	${}^nC_1+{}^nC_2+...+{}^nC_n$

Key. A $\to$ q; B $\to$ s,t; C $\to$ r; D $\to$ p

Sol. (A) $A^2=-I$ $\therefore$ A is of even order

(B) $\left(I+A\right)^n=C_0I^n+C_1IA+C_2IA^2+...+C_nIA^n$

$=C_0I+C_1A+C_2A+...+C_nA$

$\therefore$ $\lambda=2^n-1$

(C) $A^2=A$ AND B = I – A

AB + BA + I - $\left(I+A^2-2A\right)$

= AB + BA – A + 2A + AB + BA + A

= A (I – A) + (I – A) A + A

= A – A + A – A + A = A

(D) $A^*=A,\ B^*=B$

$\left(AB-BA\right)^*=B^*A^*-A^*B^*=BA-AB$

8. Match the following: -

	Column – I		**Column – II**
(A)	Let $\lvert A\rvert=\lvert a_{ij}\rvert_{3\times3}\neq0$. Each element a_{ij} is multiplied by k^{i-j}. Let $\lvert B\rvert$ the resulting determinant, where $k_1\lvert A\rvert+k_2\lvert B\rvert=0$. Then $k_1+k_2=$	(p)	0
(B)	The maximum value of a third order determinant each of its entries are ±1 equals	(q)	4
(C)	$\begin{vmatrix}1 & \cos\alpha & \cos\beta\\ \cos\alpha & 1 & \cos\gamma\\ \cos\beta & \cos\gamma & 1\end{vmatrix}=\begin{vmatrix}0 & \cos\alpha & \cos\beta\\ \cos\alpha & 0 & \cos\gamma\\ \cos\beta & \cos\gamma & 0\end{vmatrix}$ if $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=$	(r)	1
(D)	$\begin{vmatrix}x^2+x & x+1 & x-2\\ 2x^2 3x-1 & 3x & 3x-3\\ x^2+2x+3 & 2x-1 & 2x-1\end{vmatrix}=Ax+B$ where A and B	(s)	
		(t)	$\begin{vmatrix}1 & 2\\ 2 & 4\end{vmatrix}$

Key. A $\to$ p,t; B $\to$ q; C $\to$ r; D $\to$ p,t

Sol. (A) $\lvert A\rvert=\begin{vmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{vmatrix}$

$$|B| = \begin{vmatrix} a_{11} & k^{-1}a_{12} & k^{-2}a_{13} \\ ka_{21} & a_{22} & k^{-1}a_{23} \\ k^2a_{31} & ka_{32} & a_{33} \end{vmatrix} = \frac{1}{k^3}\begin{vmatrix} k^2a_{11} & ka_{12} & a_{13} \\ k^2a_{21} & ka_{22} & a_{23} \\ k^2a_{31} & ka_{32} & a_{33} \end{vmatrix} = |A|$$

$k_1|A| + k_2|B| = 0$

$k_1 + k_2 = 0$

(B) $\begin{vmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = 4$

(c) $\begin{vmatrix} 1 & \cos\alpha & \cos\beta \\ \cos\alpha & 1 & \cos\gamma \\ \cos\beta & \cos\gamma & 1 \end{vmatrix} = \begin{vmatrix} 0 & \cos\alpha & \cos\beta \\ \cos\alpha & 0 & \cos\gamma \\ \cos\beta & \cos\gamma & 0 \end{vmatrix}$

$\Rightarrow$ $\sin^2\gamma - \cos\alpha(\cos\alpha - \cos\beta\cos\gamma) + \cos\gamma(\cos\alpha\cos\gamma - \cos\beta)$

$= -\cos\alpha(-\cos\beta\cos\gamma) + \cos\beta(\cos\alpha\cos\gamma)$

$\Rightarrow$ $\sin^2\gamma - \cos^2\alpha + 2\cos\alpha\cos\beta\cos\gamma - \cos^2\beta = 2\cos\alpha\cos\beta\cos\gamma$

$\Rightarrow$ $\sin^2\gamma = \cos^2\alpha + \cos^2\beta \Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

(D) $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix}$

$R_2 \to R_2 \to (R_1 + R_3)$

$= \begin{vmatrix} x^2+x & x+1 & x+2 \\ -4 & 0 & 0 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = 4\begin{vmatrix} x+1 & x-2 \\ 2x-1 & 2x-1 \end{vmatrix} = \begin{vmatrix} x+1 & -3 \\ 2x-1 & 0 \end{vmatrix} = (24x-12)$

$\therefore$ A = 24, B = - 12

$\therefore$ A + 2B = 0