Logs & Surds

Single Correct Answer Type

1. Number of ordered triplets of natural number (a, b, c) for which abc ≤ 11 is

(A) 52 (B) 53 (C) 55 (D) 56

Key. D

Sol. abc = 1 in 1 ways
abc = 2, 3, 5, 7, 11 in 15 ways
abc = 4, 9 in 12 ways
abc = 8 in 10 ways
abc = 6, 10 in 18 ways
So, total number of solution is 56

2. The value of $\sqrt{5-\sqrt{10}-\sqrt{15}+\sqrt{6}}$ is

(A) $\frac{\sqrt{5}-\sqrt{3}-\sqrt{2}}{\sqrt{2}}$ (B) $\frac{\sqrt{3}+\sqrt{2}-\sqrt{5}}{\sqrt{2}}$

(C) $\frac{\sqrt{3}+\sqrt{2}-\sqrt{5}}{2}$ (D) $\frac{\sqrt{5}-\sqrt{3}-\sqrt{2}}{2}$

Key. B

Sol. $5-\sqrt{10}-\sqrt{15}+\sqrt{6}$ can be written as

$$\frac{3+2+5-2\sqrt{2}\sqrt{5}-2\sqrt{5}\sqrt{3}+2\sqrt{3}\sqrt{2}}{2}$$

$$=\left(\frac{\sqrt{3}+\sqrt{2}-\sqrt{5}}{\sqrt{2}}\right)^2$$

3. $a>0(a\neq 1), b>0(b\neq 1)$ such that $a^{\left(\log_a^b\right)^x}=b^{\left(\log_b^a\right)^x}$ then $x=$

(A) 1 (B) -1 (C) $\frac{1}{2}$ (D) 2

Key: C

Hint: Taking $\log_b$ both sides we get

$$\left(\log_a^b\right)^x \log_b^a=\left(\log_b^a\right)^x$$

$$\therefore\left(\log_a^b\right)^x=\left(\log_b^a\right)^{x-1}$$

$$\therefore 1-x=x\Rightarrow x=\frac{1}{2}$$

4. Given that $\log_{10}^{5} = 0.70$ and $\log_{10}^{3} = 0.48$ then the value of $\log_{30}^{8}$ (correct upto 2 places of decimal) is

(A) 0.56 (B) 0.61 (C) 0.68 (D) 0.73

Key: B

5. The value of $\sqrt{5-\sqrt{10}-\sqrt{15}+\sqrt{6}}$ is

(A) $\frac{\sqrt{5}-\sqrt{3}-\sqrt{2}}{\sqrt{2}}$ (B) $\frac{\sqrt{3}+\sqrt{2}-\sqrt{5}}{\sqrt{2}}$

(C) $\frac{\sqrt{3}+\sqrt{2}-\sqrt{5}}{2}$ (D) $\frac{\sqrt{5}-\sqrt{3}-\sqrt{2}}{2}$

Key. B

Sol. $5-\sqrt{10}-\sqrt{15}+\sqrt{6}$ can be written as

$$\frac{3+2+5-2\sqrt{2}\sqrt{5}-2\sqrt{5}\sqrt{3}+2\sqrt{3}\sqrt{2}}{2}$$

$$=\left(\frac{\sqrt{3}+\sqrt{2}-\sqrt{5}}{\sqrt{2}}\right)^2$$

6. The value of $\sqrt{5-\sqrt{10}-\sqrt{15}+\sqrt{6}}$ is

(A) $\frac{\sqrt{5}-\sqrt{3}-\sqrt{2}}{\sqrt{2}}$ (B) $\frac{\sqrt{3}+\sqrt{2}-\sqrt{5}}{\sqrt{2}}$

(C) $\frac{\sqrt{3}+\sqrt{2}-\sqrt{5}}{2}$ (D) $\frac{\sqrt{5}-\sqrt{3}-\sqrt{2}}{2}$

Key. B

Sol. $5-\sqrt{10}-\sqrt{15}+\sqrt{6}$ can be written as

$$\frac{3+2+5-2\sqrt{2}\sqrt{5}-2\sqrt{5}\sqrt{3}+2\sqrt{3}\sqrt{2}}{2}$$

$$=\left(\frac{\sqrt{3}+\sqrt{2}-\sqrt{5}}{\sqrt{2}}\right)^2$$

7. There exist positive integers A, B and C with no common factors greater than 1, such that $A \log_{200}5 + B \log_{200}2 = C$. The sum A + B + C equals

(A) 5 (B) 6 (C) 7 (D) 8

Key. B

Sol. $A \log_{200}5 + B \log_{200}2 = C$

= C

$A \log 5 + B \log 2 = C \log 200 = C \log(5^2\ 2^3) = 2C \log 5 + 3 C \log 2$

hence, A = 2C and B = 3C

for no common factor greater than 1, C = 1

$\therefore$ A = 2; B = 3 $\Rightarrow$ A + B + C = 6 Ans.

9. Given real numbers a, b, c > 0 ($\neq$ 1) such that $\log_{\log_c a} e, \log_{(a^{c/2})} e, \log_{(\log_b c)} e$ are in H.P. then c equal to

(a) $\log_a(\log_a b)$ (b) $\log_a(\log_b a)$

(c) $\log_b(\log_b a)$ (d) $\log_b(\log_a b)$

Key. B

SOL. $\log_e(\log_c a), \log_e a^{c/2}, \log_e(\log_b c)$ ARE IN A.P.

$\Rightarrow$ $\log_c a, a^{c/2}, \log_b c$ ARE IN G.P.

$\Rightarrow$ $a^c = \log_c a \log_b c$

$\Rightarrow$ $a^c = \log_b a$

$\Rightarrow$ $c = \log_a(\log_b a)$

Logs & Surds

Multiple Correct Answer Type

1. Which of the following is/are true

(A) $\log_2^3 < \log_5^{17}$

(B) $\log_2^{24}\left(\log_{96}^2\right)^{-1} - \log_2^{192}\left(\log_{12}^2\right)^{-1} = 3$

(C) $\left(\log_2^5\right)^2 > \log_2^{20}$

(D) $\log_{10}^5 .\log_{10}^{20} + \left(\log_{10}^2\right)^2 = 1$

Key: A,B,C,D

Hint: (A) $\log_2^3 - \log_5^{17}$

$= \dfrac{\log_2^3 \log_2^5 - \log_2^{17}}{\log_2^5}$

But $\log_2^{17} > 4$

$\therefore 2^5 > 5^2 \Rightarrow 2^{5/2} > 5$

$\Rightarrow \dfrac{5}{2} > \log_2^5$

$\log_2^5 < \dfrac{5}{2}; 3^5 < 2^8$

$5\log_2^3 < 8$

$\log_2^3 < \dfrac{8}{5}$

$\therefore \log\dfrac{5}{2}.\log\dfrac{3}{2} < 4$

$\therefore \log_2^3 .\log_2^5 - \log_2^{17} < 0 \Rightarrow \log_2^3 < \log_5^{17}$

(B) $GE = \log 24.\log 96 - \log 192.\log 12$

$= (3\log 2 + \log 3)(5\log 2 + \log 3) - (6\log 2 + \log 3)(2\log 2 + \log 3)$

$= 3\left(\log_2^2\right)^2 = 3$

(C) Let $x = \log\dfrac{5}{2} > 2$

$x^2 - x - 2 = (x-2)(x+1) > 0 \because x > 2$

$\therefore x^2 - x - 2 > 0 \Rightarrow \left(\log_2^5\right)^2 > \log_2^5 + 2 = \log_2^{20}$.

(D) $\log_{10}^5\left(\log_{10}^2 + 1\right) + \left(\log_{10}^2\right)^2 = \log_{10}^2\left[\log_{10}^9 + \log_{10}^2\right] + \log_{10}^5$

$= \log_{10}^2 .1 + \log_{10}^5 = \log_{10}^{10} = 1$

2. If $\dfrac{\log_2 x}{4} = \dfrac{\log_2 y}{6} = \dfrac{\log_2 z}{3k}$ and $x^3y^2z = 1$, then k is equal to

(A) – 8 (B) – 4 (C) 0 (D) $\log_2\left(\dfrac{1}{256}\right)$

Key : A, D

Sol : $\dfrac{\log_2 x}{4} = \dfrac{\log_2 y}{6} = \dfrac{\log_2 z}{3k}$

$$= \frac{3\log_2 x + 2\log_2 y + \log_2 z}{12 + 12 + 3k}$$

$$= \frac{\log_2 (x^3 y^2 z)}{24 + 3k}$$

$$= \frac{0}{24 + 3k}$$

Logs & Surds

Assertion Reasoning Type

1. 54. Statement – I : If $\log_{0.2}\left(\frac{x+1}{x}\right)\ge 1$ then $x\in[-1.25,-1)$

Statement –II : If $0<a<1,\ \log_a^x \ge \log_a^y \Leftrightarrow y\ge x>0$

Key. A

Sol. $0<\frac{x+1}{x}\le 0.2 \Leftrightarrow \frac{x+1}{x}-\frac{1}{5}\le 0$ and $\frac{x+1}{x}>0$

$\Rightarrow \frac{4x+5}{5x}\le 0$ and $\frac{x+1}{x}>0 \Rightarrow x\ne 0; x(4x+5)\le 0; x(x+1)>0$

$\Rightarrow x\ne 0; -\frac{5}{4}\le x\le 0$ and $x<-1$ *or* $x>0$

$\therefore x\in\left[-\frac{5}{4},\ -1\right)$

55. Statement – I : For a, b $\in R$, $\frac{|a+b|}{1+|a+b|}\le\frac{|a|}{1+|a|}+\frac{|b|}{1+|b|}$

Statement – II : a, b $\in R$ $\max\{a,\ b\}=\frac{1}{2}\left(a+b+|a-b|\right)$

Key. B

Sol. If $a=0$ or b=0 or a + b = 0 then inequality is obvious Let $ab(a+b)\ne 0$

$\frac{|a|}{1+|a|}+\frac{|b|}{1+|b|}\ge\frac{|a|}{1+|a|+|b|}+\frac{|b|}{|a|+1+|b|}=\frac{|a|+|b|}{1+|a|+|b|}=\frac{1}{1+\frac{1}{|a|+|b|}}$ .....(1)

But $|a+b|\le|a|+|b|\Rightarrow\frac{1}{|a+b|}\ge\frac{1}{|a|+|b|}\Rightarrow 1+\frac{1}{|a+b|}\ge 1+\frac{1}{|a|+|b|}$

$\Rightarrow\frac{1}{1+\frac{1}{|a+b|}}\le\frac{1}{1+\frac{1}{|a|+|b|}}$..............(2)

From (1) and (2) $\Rightarrow\frac{|a|}{1+|a|}+\frac{|b|}{1+|b|}\ge\frac{1}{1+\frac{1}{|a+b|}}=\frac{|a+b|}{1+|a+b|}$

ST II:- $a\ge b\Rightarrow\frac{1}{2}\left(a+b+|a-b|\right)=\frac{1}{2}\{a+b+a-b\}=a$

$a\le b\Rightarrow\frac{1}{2}\left(a+b+|a-b|\right)=\frac{1}{2}\{a+b-a+b\}=b$

56. Statement – I : The greatest integer satisfying $x^2-6|x|+5<0$ is '4'.

Statement – II: For $k_1,k_2\in R^+,(k_1<k_2)$ and $x\in R$ if $k_1<|x|<k_2$ then $x\in(-k_2,-k_1)\cup(k_1,\ k_2)$.

Key. A

Sol. Conceptual

57. Statement – I : If $(0.5)^{\log_3 \log_{(1/5)}\left(x^2-\frac{4}{5}\right)} > 1$ then $|x| > 1$

Statement – II: The number of solution of inequality

$\sqrt{3x^2+6x+7}+\sqrt{5x^2+10x+14} \le 4-2x-x^2$ are two

Key. C

Sol. St-1 : $\left(\frac{1}{2}\right)^{\log_3 \log_{\frac{1}{5}}\left(x^2-\frac{4}{5}\right)} > \left(\frac{1}{2}\right)^0$

$\Rightarrow \log_3 \log_{\frac{1}{5}}\left(x^2-\frac{4}{5}\right) < 0 \Rightarrow x^2-\frac{4}{5} > \frac{1}{5} \Rightarrow x^2 > 1 \Rightarrow |x| > 1$

St-2 : $\sqrt{3x^2+6x+7} \ge 2$ and $\sqrt{5x^2+10x+4} \ge 3$

$\therefore LHS \ge 5 \qquad RHS \le 5$

$\therefore$ The equation hold only when LHS = RHS = 5

$\therefore$ x = - 1