Differential Equations

Single Correct Answer Type

1. The family of curves passing through (0,0) and satisfying the differential equation $\frac{y_2}{y_1}=1$ (where $y_n = d^n y/dx^n$) is

a) $y=k$ b) $y=kx$ c) $y=k(e^x+1)$ d) $y=k(e^x-1)$

Key. D

Sol. $\frac{dp}{dx}=P\left(where\ p=\frac{dy}{dx}\right)$

$\ln P = x+c \Rightarrow p=e^{x+c}$

$\frac{dy}{dx}=ke^x$

$y=ke^x+\lambda$

Satisfying $(0,0)$, $So\ \lambda=-k$

$y=k\left(e^x-1\right)$

2. The degree of the differential equation satisfying the relation

$\sqrt{1+x^2}+\sqrt{1+y^2}=\lambda(x\sqrt{1+y^2}-y\sqrt{1+x^2})$ is

a) 1 b) 2 c) 3 d) none of these

Key. A

Sol. On Putting $x=\tan A, y=\tan B$ we get

$\sec A+\sec B=\lambda\left(\tan A\sec B-\tan B\sec A\right)$

$\cos A+\cos B=\lambda\left(\sin A-\sin B\right)$

$\tan\left(\frac{A-B}{2}\right)=\frac{1}{\lambda}$

$\tan^{-1}x-\tan^{-1}y=2\tan^{-1}\frac{1}{\lambda}$

On differentiating $\frac{1}{1+x^2}-\frac{1}{1+y^2}\frac{dy}{dx}=0$

3. S1: The differential equation of parabolas having their vertices at the origin and foci on the x-axis is an equation whose variables are separable

S2: The differential equation of the straight lines which are at a fixed distance p from the origin is an equation of degree 2

S3: The differential equation of all conics whose both axes coincide with the axes of coordinates is an equation of order 2

a) TTT b) TFT c) FFT d) TTF

Key. A

Sol. S_1 - Equation of parabola is $y^2 = \pm 4ax$

$$2y\frac{dy}{dx} = \pm 4a$$

D.E of parabola $\Rightarrow y^2 = 2yx\frac{dy}{dx}$

$$2\frac{dy}{y} = \frac{dx}{x}$$

Which is variable seperable

S_2 - Equation of line which is fixed distance. P from origin can be equation of tangent to circle $x^2 + y^2 = p^2$

Line is $y = mx + p\sqrt{1+m^2}$ $\left(m = \frac{dy}{dx}\right)$

$$\left(y - x\frac{dy}{dx}\right)^2 = P\left(1 + \left(\frac{dy}{dx}\right)^2\right)$$

So, degree is 2

S_3 - Equation of conic whose both axis co-incide with co-ordinate axis is $ax^2 + by^2 = 1$

As there are two constants, so order of D.E is 2

4. The order of the differential equation of all tangent lines to the parabola $y = x^2$ is

a) 1 b) 2 c) 3 d) 4

Key. A

Sol. The parametric form of the given equation is $x = t, y = t^2$. The equation of any tangent at t is $2xt = y + t^2$, Differentiating we get $2t = y_1\left(= \frac{dy}{dx}\right)$ putting this value in the above equation, we have $2x\frac{v_1}{2} = y + \left(\frac{y_1}{2}\right)^2 \Rightarrow 4xy_1 = 4y + y_1{}^2$

The order of this equation is 1

Hence (A) is the correct answer

5. Which of the following transformation reduces the differential equation $\frac{dz}{dx} + \frac{z}{x}\log z = \frac{z}{x^2}(\log z)^2$ to the form $P(x)u = Q(x)$

a) $u = \log z$ b) $u = e^z$ c) $u = (\log z)^{-1}$ d) $u = (\log z)^2$

Key. A or B or C or D

Sol. Given equ. Can be written as

$$\frac{dz}{z(\ln z)^2}+\frac{dx}{x(\ln z)}=\frac{dx}{x^2}$$

Put $u=\frac{1}{\ln z}$

$$\frac{du}{dx}=-\frac{1}{(\ln z)^2}\cdot\frac{1}{z}\cdot\frac{dz}{dx}$$

$$\therefore -\frac{du}{dx}+\frac{1}{x}.u=\frac{1}{x^2}$$

$$\frac{du}{dx}+\left(-\frac{1}{x}\right)u=\left(-\frac{1}{x^2}\right)$$

$$I.F.=e^{\int -\frac{1}{x}dx}=e^{\ln\frac{1}{x}}=\frac{1}{x}$$

$$u.\frac{1}{x}=\int -\frac{1}{x^2}\cdot\frac{1}{x}dx$$

$$\frac{u}{x}=-\int\frac{1}{x^3}dx=+\frac{1}{4x^4}+c$$

6. If $y_1(x)$ is a solution of the differential equation $dy/dx-f(x)y=0$, then a solution of the differential equation $\frac{dy}{dx}+f(x)y=r(x)$

a) $\frac{1}{y_1(x)}\int r(x)y_1(x)dx$ b) $y_1(x)\int\frac{r(x)}{y_1(x)}dx$

c) $\int r(x)y_1(x)dx$ d) none of these

Key. A

Sol. $\frac{dy}{dx}-f(x).y=0$

$$\frac{dy}{y}=f(x)dx$$

$$\ln y=\int f(x)dx$$

$y_1(x)=e^{\int f(x)dx}$ Then for given equation I.F = $e^{\int f(x)dx}$

Hence Solution $y.y_1(x)=\int r(x).y_1(x)dx$

$$y=\frac{1}{y_1(x)}\int r(x).y_1(x)dx$$

7. The solution of $(x+2y^3)(dy/dx)=y$ is (where c is arbitrary constant)

1) $x=y^3+cy$ 2) $x=y^3-cy$

3) $y=x^3-cy$ 4) $y=x^3+cy$

Key. 1 or 2

Sol. $\frac{dx}{dy}-\frac{1}{y}x=2y^2$

This is linear equation taking y as independent variable.

Here, I.F. $=e^{-\int\frac{1}{y}dy}=e^{-\log y}=\frac{1}{y}$

$\therefore$ solution is $x\frac{1}{y}=\int\frac{1}{y}2y^2dy+c$

$\Rightarrow\frac{x}{y}=y^2+c\Rightarrow x=y^3+cy$

8. Solution of the equation $xdy=\left(y+x\frac{f(y/x)}{f^{'}(y/x)}\right)dx$

1) $f\left(\frac{x}{y}\right)=cy$ 2) $f\left(\frac{y}{x}\right)=cx$

3) $f\left(\frac{y}{x}\right)=cxy$ 4) $f\left(\frac{y}{x}\right)=0$

Key. 2

Sol. We have, $xdy=\left(y+\frac{xf(y/x)}{f^{'}(y/x)}\right)dx$

$\Rightarrow\frac{dy}{dx}=\frac{y}{x}+\frac{f(y/x)}{f^{'}(y/x)}$ which is homogenous

Putting $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$,we obtain

$v+x\frac{dv}{dx}=v+\frac{f(v)}{f^{'}(v)}\Rightarrow\frac{f^{'}(v)}{f(v)}dv=\frac{dx}{x}$

Integrating, we get log $f(v)=\log x+\log c$

$\Rightarrow\log f(v)=\log cx\Rightarrow f\left(\frac{y}{x}\right)=cx$

9. The general solution of the differential equation $\frac{dy}{dx} = y\tan x - y^2 \sec x$ is

1) $\tan x = (c + \sec x)y$ 2) $\sec y = (c + \tan y)x$

3) $\sec x = (c + \tan x)y$ 4) None of these

Key. 3

Sol. We have $\frac{dy}{dx} = y\tan x - y^2 \sec x$

$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} - \frac{1}{y}\tan x = -\sec x$

Putting $\frac{1}{y} = v \Rightarrow \frac{-1}{y^2}\frac{dy}{dx} = \frac{dv}{dx}$, we obtain

$\frac{dV}{dx} + \tan x.v = \sec x$ which is linear

$I.F = e^{\int \tan x dx} = e^{\log \sec x} = \sec x$

$\therefore$ The solution is

$v\sec x = \int \sec^2 x\, dx + c \Rightarrow \frac{1}{y}\sec x = \tan x + c$

$\Rightarrow \sec x = y(c + \tan x)$

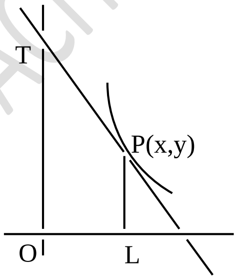
10. The curve $y = f(x)$ is such that the area of the trapezium formed by the coordinate axes, ordinate of an arbitrary point and the tangent at this point equals half the square of its abscissa. The equation of the curve can be

1) $y = cx^2 \pm x$ 2) $y = cx^2 \pm 1$

3) $y = cx \pm x^2$ 4) $y = cx^2 \pm x^2 \pm 1$

Key. 1

Sol. Let $P(x, y)$ be any point on the curve. Length of intercept on y-axis by any tangent at P is



$OT = y - x\frac{dy}{dx}$

$\therefore$ Area of trapezium $OLPTO = \frac{1}{2}(PL + OT)OL$

$= \frac{1}{2}\left(y + y - x\frac{dy}{dx}\right)x = \frac{1}{2}\left(2y - x\frac{dy}{dx}\right)x$

According to question

Area of trapezium $OLPTO = \frac{1}{2}x^2$

$$i.e., \frac{1}{2}\left(2y - x\frac{dy}{dx}\right)x = \pm\frac{1}{2}x^2$$

$$\Rightarrow 2y - x\frac{dy}{dx} = \pm x \text{ or } \frac{dy}{dx} - \frac{2y}{x} = \pm 1$$

Which is linear differential equation and $I.F. = e^{-2\ln x} = \frac{1}{x^2}$

$\therefore$ The solution is $\frac{y}{x^2} = \int \pm\frac{1}{x^2}dx + c = \pm\frac{1}{x} + c$

$\therefore y = \pm x + cx^2$ or $y = cx^2 \pm x$, where c is an arbitrary constant

11. If p and q are order and degree of differential equation $y^2\left(\frac{d^2y}{dx^2}\right)^2 + 3x\left(\frac{dy}{dx}\right)^{\frac{1}{3}} + x^2y^2 = \sin x$, then

1) p > q
2) $\frac{p}{q} = \frac{1}{2}$
3) p = q
4) p < q

Key. 4

Sol. $y^2\left(\frac{d^2y}{dx^2}\right)^2 + x^2y^2 - \sin x = -3x\left(\frac{dy}{dx}\right)^{\frac{1}{3}}$

$$\left(y^2\left(\frac{d^2y}{dx^2}\right)^2 + x^2y^2 - \sin x\right)^3 = -9x^3\left(\frac{dy}{dx}\right)$$

Here order = 2 = p

Degree = 6 = q

$\therefore p < q$

12. The solution of the differential equation $\frac{dy}{dx} - ky = 0, y(0) = 1$, approach zero when $x \to \infty$, if

1) K = 0
2) k > 0
3) k < 0
4) none of these

Key. 3

Sol. $\frac{dy}{dx} - Ky = 0, \frac{dy}{y} = Kdx$

$\ln y = Kx + c$

At $x = 0, y = 1$ $\therefore C = 0$

Now, $\ln y = Kx$

$y = e^{Kx}$

$\lim_{x\to\infty} y = \lim_{x\to\infty} e^{Kx} = 0$

$\therefore K < 0$

13. The solution of $\frac{dv}{dt}+\frac{k}{m}v=-g$ is

1) $v=ce^{-\frac{k}{m}t}-\frac{mg}{k}$
2) $v=c-\frac{mg}{k}e^{-\frac{k}{m}t}$
3) $ve^{-\frac{k}{m}t}=c-\frac{mg}{k}$
4) $ve^{\frac{k}{m}t}=c-\frac{mg}{k}$

Key. 1

Sol. $\frac{dv}{dt}+\frac{K}{m}v=-g$

Integrating factor $(I.F.)=e^{\int\frac{k}{m}dt}=e^{\frac{K}{m}t}$

$\therefore Ve^{\frac{K}{m}t}=-\int g\,e^{K.t/m}+c$

$Ve^{\frac{K}{m}t}=\frac{-gm}{K}e^{\frac{K}{m}t}+c$

$V=C.e^{\frac{-K}{m}t}-\frac{mg}{K}$

14. If $y_1(x)$ is a solution of the differential equation $\frac{dy}{dx}+f(x)y=0$, then a solution of differential equation $\frac{dy}{dx}+f(x)y=r(x)$ is

1) $\frac{1}{y(x)}\int y_1(x)dx$
2) $y_1(x)\int\frac{r(x)}{y_1(x)}dx$
3) $\int r(x)y_1(x)dx$
4) none of these

Key. 2

Sol. i) $\frac{dy_1}{dx}+f(x)y_1=0\Rightarrow f(x)=\frac{-1}{y_1}\frac{dy_1}{dx}$

ii) $\frac{dy}{dx}-\frac{1}{y_1}\frac{dy_1}{dx}.y=r(x)$

$e^{-\int\frac{1}{y_1}\frac{dy}{dx}dx}=e^{-\int\frac{dy_1}{y_1}}=\frac{1}{y_1}$

$\frac{d}{dx}\left(\frac{y}{y_1}\right)=\frac{r(x)}{y_1}\Rightarrow\frac{y}{y_1}=\int\frac{r(x)dx+c}{y_1}$

$y=y_1\int\frac{r(x)dx}{y_1}+cy_1$

15. The solution of the differential equation $y_1y_3=3y_2^2$ is

1) $x=A_1y^2+A_2y+A_3$
2) $x=A_1y+A_2$

3) $x = A_1 y^2 + A_2 y$ 4)none of these

Key. 1

Sol. $y_1 y_3 = 3y_2^2$

$\frac{y_3}{y_2} = 3\frac{y_2}{y_1} \Rightarrow \ln y_2 = 3\ln y_1 + \ln c$

$y_2 = cy_1^3$

$\frac{y_2}{y_1^2} = cy_1$

$-\frac{1}{y_1} = cy + c_2$

$\frac{dx}{dy} = -cy - c_2$

$x = -\frac{cy^2}{2} - c_2 y + c_3$

$\therefore x = A_1 y^2 + A_2 y + A_3$

16. The solution of the differential equation $\left(x^2 \sin^3 y - y^2 \cos x\right)dx + \left(x^3 \cos y \sin^2 y - 2y \sin x\right)dy = 0$ is

1) $x^3 \sin^3 y = 3y^2 \sin x + C$ 2) $x^3 \sin^3 y + 3y^2 \sin x = C$

3) $x^2 \sin^3 y + y^3 \sin x = C$ 4) $2x^2 \sin y + y^2 \sin x = C$

Key.

Sol. $\left(x^2 \sin^2 y - y^2 \cos x\right)dx + \left(x^3 \cos y \sin^2 y - 2y \sin x\right)dy = 0$

$\frac{dy}{dx} = \frac{y^2 \cos x - x^2 \sin^3 y}{x^3 \cos y \sin^2 - 2y \sin x}$

$\left(x^3 \cos y \sin^2 y - 2y \sin x\right)dy$

$= \left(y^2 \cos x - x^2 \sin^3 y\right)dx = 0$

$\left(\frac{x^3}{3} d \sin^3 y - \sin dy^2\right) - \sin^3 yd\left(\frac{x^3}{3}\right) + y^2 d \sin x = 0$

$\frac{x^3}{3} d \sin^2 y + \sin^3 yd\left(\frac{x^3}{3}\right) - \left(\sin dy^2 + y^2 d \sin x\right)$

$d\left(\frac{x^3}{3} \sin^3 y\right) - d\left(y^2 \sin x\right) = 0$

$\frac{x^3}{3} \sin^3 y - y^2 \sin x = c$

17. The differential equation whose solution is $\left(x-h\right)^2 + \left(y-k\right)^2 = a^2$ is (a is a constant)

1) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\frac{d^2y}{dx^2}$ 2) $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$

3) $\left[1+\left(\frac{dy}{dx}\right)\right]^3 = a^2\left(\frac{d^2y}{dx^2}\right)^2$ 4) none of these

Key.

Sol. $(x-h)^2+(y-k)^2=a^2$ ....(1)

$2(x-h)+2(y-k)\frac{dy}{dx}=0$ ....(2)

$1+\left(\frac{dy}{dx}\right)^2+(y-k)\frac{d^2y}{dx^2}=0$ ....(3)

From (3) we have (y-k), use in (2) to get (x-h) and put(x-h) and(y-k) in(1)

18. An equation of the curve in which subnormal varies as the square of the ordinate is (k is constant of proportionality)

1) $y=Ae^{kx}$ 2) $y=e^{kx}$ 3) $y^2/2+kx=A$ 4) $y^2+kx^2=A$

Key. 1

Sol. According to the given condition $y\frac{dy}{dx}=ky^2$

$\Rightarrow \frac{dy}{y}=kdx$ (variables separable equation)

$\Rightarrow \log|y|=kx+C \Rightarrow |y|=Be^{kx} \Rightarrow y=Ae^{kx}$ where $A=\pm B$ and k is the constant of proportionality.

19. The solution of $\frac{dy}{dx}=\frac{ax+b}{cy+d}$ represents a parabola if

1) $a=0,c=0$ 2) $a=1,b=2$ 3) $a=0,c\neq 0$ 4) $a=1,c=1$

Key. 3

Sol. The given equation is with separable variables so $(cy+d)dy=(ax+b)dx$. Integrating we have $\frac{cy^2}{2}+dy+K=\frac{ax^2}{2}+bx, K$ being the constant of integration. The last equation represents a parabola if $c=0,a\neq 0$ or $a=0,c\neq 0$.

20. The equation of the curve passing through $(3,9)$ which satisfies $dy/dx=x+1/x^2$ is

1) $6xy=3x^2-6x+29$ 2) $6xy=3x^2-29x+6$

3) $6xy=3x^3+29x-6$ 4) None of these

Key. 3

Sol. The given differential equation has variable separable so integrating, we have $y = x^2/2 - 1/x + C.$ This will pass through $(3,9)$ if $9 = 9/2 - 1/3 + C \Rightarrow C = 29/6.$

Hence the required equation is $6xy = 3x^3 + 29x - 6$

21. Solution of the differential equation $2y\sin x\dfrac{dy}{dx} = 2\sin x\cos x - y^2\cos x$ satisfying $y(\pi/2) = 1$ is given by

1) $y^2 = \sin x$
2) $y = \sin^2 x$
3) $y^2 = \cos x + 1$
4) $y^2 \sin x = 4\cos^2 x$

Key. 1

Sol. The given equation can be written as $2y\sin x\dfrac{dy}{dx} + y^2\cos x = \sin 2x$

$\Rightarrow \dfrac{d}{dx}\left(y^2\sin x\right) = \sin 2x \Rightarrow y^2\sin x = (-1/2)\cos 2x + C.$

So $\left(y(\pi/2)\right)^2\sin(\pi/2) = (-1/2)\cos(2\pi/2) + C \Rightarrow C = 1/2.$

Hence $y^2\sin x = (1/2)(1 - \cos 2x) = \sin^2 x \Rightarrow y^2 = \sin x$

22. An equation of the curve satisfying $xdy - ydx = \sqrt{x^2 - y^2}\,dx$ and $y(1) = 0$ is

1) $y = x^2\log|\sin x|$
2) $y = x\sin(\log|x|)$
3) $y^2 = x(x-1)^2$
4) $y = 2x^2(x-1)$

Key. 2

Sol. The equation can be written as $x^2\dfrac{xdy - ydx}{x^2} = x\sqrt{1 - (y/x)^2}\,dx$

$\Rightarrow \dfrac{d(y/x)}{\sqrt{1 - (y/x)^2}} = \dfrac{dx}{x} \Rightarrow \sin^{-1} y/x = \log|x| + \text{const}$

Since $y(1) = 0$ so const =0. Hence $y = x\sin(\log|x|).$

23. If for the differential equation $y^1 = \dfrac{y}{x} + \phi\left(\dfrac{x}{y}\right)$ the general solution is $y = \dfrac{x}{\log|Cx|}$ then $\phi(x/y)$ is given by

1) $-x^2/y^2$
2) y^2/x^2
3) x^2/y^2
4) $-y^2/x^2$

Key. 4

Sol. Putting $v = y/x$ so that $\dfrac{xdv}{dx} + v = \dfrac{dy}{dx}$, we have

$\dfrac{xdv}{dx} + v = v + \phi(1/v) \Rightarrow \dfrac{dv}{\phi(1/v)} = \dfrac{dx}{x}$

$\Rightarrow \log|Cx| = \int\dfrac{dv}{\phi(1/v)}$ (being constant of integration.)

But $y = \dfrac{x}{\log|Cx|}$ is the general solution so

$\dfrac{x}{y} = \dfrac{1}{v} = \int\dfrac{dv}{\phi(1/v)} \Rightarrow \phi(1/v) = -1/v^2 \Rightarrow \phi(x/y) = -y^2/x^2.$

24. The solution $y(x)$ of the differential equation $\frac{d^2y}{dx^2} = \sin 3x + e^x + x^2$ when $y_1(0) = 1$ and $y(0) = 0$ is

1) $-\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x - 1$
2) $-\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x$
3) $-\frac{\cos 3x}{3} + e^x + \frac{x^4}{12} + \frac{1}{3}x + 1$
4) None of these

Key. 1

Sol. Integrating the given differential equation, we have $\frac{dy}{dx} = -\frac{\cos 3x}{3} + e^x + \frac{x^3}{3} + C_1$

but $y_1(0) = 1$ so $1 = -\frac{1}{3} + 1 + C_1 \Rightarrow C_1 = \frac{1}{3}$

Again integrating, we get $y = -\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x + C_2$

but $y(0) = 0$ so $0 = 1 + C_2 \Rightarrow C_2 = -1$. Thus $y = -\frac{\sin 3x}{9} + e^x + \frac{x^4}{12} + \frac{1}{3}x - 1.$

25. The orthogonal trajectories of the family of curves $a^{n-1}y = x^n$ are given by

1) $x^n + n^2 y =$ constant 2) $ny^2 + x^2 =$ constant
3) $n^2 x + y^n =$ constant 4) $n^2 x - y^n =$ constant

Key. 2

Sol. Differentiating, we have $a^{n-1}\frac{dy}{dx} = nx^{n-1} \Rightarrow a^{n-1} = nx^{n-1}\frac{dx}{dy}$

Putting this value in the given equation, we have $nx^{n-1}\frac{dx}{dy}y = x^n$

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$, we have $ny = -x\frac{dx}{dy}$

$\Rightarrow nydy + xdx = 0 \Rightarrow ny^2 + x^2 =$ constant. Which is the required family of orthogonal trajectories.

26. The solution of $\left(y\left(1+x^{-1}\right)+\sin y\right)dx + \left(x + \log x + x\cos y\right)dy = 0$ is

1) $\left(1+y^{-1}\sin y\right) + x^{-1}\log x = C$
2) $\left(y + \sin y\right) + xy\log x = C$
3) $xy + y\log x + x\sin y = C$
4) None of these

Key. 3

Sol. The given equation can be written as

$y\left(1+x^{-1}\right)dx + \left(x + \log x\right)dy + \sin y dx + x\cos y dy = 0$

$\Rightarrow d\left(y\left(x+\log x\right)\right) + d\left(x\sin y\right) = 0$

$\Rightarrow y\left(x + \log x\right) + x\sin y = C$

27. The degree of the differential equation satisfying

$\sqrt{1+x^2}+\sqrt{1+y^2}=A\left(x\sqrt{1+y^2}-y\sqrt{1+x^2}\right)$ is

1) 2 2) 3 3) 4 4) None of these

Key. 4

Sol. Put $x=\tan\theta$ and $y=\tan\phi$. Then $\sqrt{1+x^2}=\sec\theta, \sqrt{1+y^2}=\sec\phi,$ and the equation becomes $\sec\theta+\sec\phi=A(\tan\theta\sec\phi-\tan\phi\sec\theta)$

$$\Rightarrow \frac{\cos\phi+\cos\theta}{\cos\theta\cos\phi}=A\left(\frac{\sin\theta-\sin\phi}{\cos\theta\cos\phi}\right)\Rightarrow 2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2}=2A\sin\frac{\theta-\phi}{2}\cos\frac{\theta+\phi}{2}$$

$$\Rightarrow \cot\frac{\theta-\phi}{2}=A\Rightarrow\theta-\phi=2\cot^{-1}A$$

$$\Rightarrow \tan^{-1}x-\tan^{-1}y=2\cot^{-1}A.$$

Differentiating this, we get $\frac{1}{1+x^2}-\left(\frac{1}{1+y^2}\right)\frac{dy}{dx}=0$, which is a differential equation of degree 1.

28. A solution of $y=2x\left(\frac{dy}{dx}\right)+x^2\left(\frac{dy}{dx}\right)^4$ is

1) $y=2c^{1/2}x^{1/4}+c$ 2) $y=2\sqrt{c}x^2+c^2$ 3) $y=2\sqrt{c}\,(x+1)$ 4) $y=2\sqrt{cx}+c^2$

Key. 4

Sol. Writing $p=\frac{dy}{dx}$ and differentiating w.r.t.x, we have

$$p=2p+2x\frac{dp}{dx}+2xp^4+4p^3x^2\frac{dp}{dx}\Rightarrow 0=p\left(1+2xp^3\right)+2x\frac{dp}{dx}\left(1+2p^3x\right)$$

$$\Rightarrow p+2x\frac{dp}{dx}=0\Rightarrow 2\frac{dp}{p}=-\frac{dx}{x}$$

$$\Rightarrow 2\log p+\log x=\text{const}\Rightarrow p^2x=c \text{ or } p=\sqrt{\frac{c}{x}}$$

Substituting this value in the given equation, we get $y=2\sqrt{cx}+c^2$

29. The solution of $y_2-7y_1+12y=0$ is

1) $y=C_1e^{3x}+C_2e^{4x}$ 2) $y=C_1xe^{3x}+C_2e^{4x}$ 3) $y=C_1e^{3x}+C_2xe^{4x}$ 4) None of these

Key. 1

Sol. The given equation can be written as $\left(\frac{d}{dx}-3\right)\left(\frac{dy}{dx}-4y\right)=0$ ...... (i)

If $\frac{dy}{dx}-4y=u$ then (I) reduces to $\frac{du}{dx}-3u=0$

$\Rightarrow \frac{du}{u} = 3dx \Rightarrow u = C_1 e^{3x}$. Therefore, we have $\frac{dy}{dx} - 4y = C_1 e^{3x}$ which is a linear equation

whose $I.F.$ is e^{-4x}. So $\frac{d}{dx}\left(ye^{-4x}\right) = C_1 e^{-x}$

$\Rightarrow ye^{-4x} = -C_1 e^{-x} + C_2 \Rightarrow y = C_1 e^{3x} + C_2 e^{4x}$

30. The curves satisfying the differential equation $\left(1-x^2\right)y^1 + xy = ax$ are

1) ellipses and hyperbolas
2) ellipses and parabola
3) ellipses and straight lines
4) circles and ellipses

Key. 1

Sol. The given equation is linear in y and can be written as

$$\frac{dy}{dx} + \frac{x}{1-x^2}y = \frac{ax}{1-x^2}$$

Its integrating factor is $e^{\int \frac{x}{1-x^2}dx} = e^{-(1/2)\log\left(1-x^2\right)} = \frac{1}{\sqrt{1-x^2}}$ if $-1 < x < 1$ and if $x^2 > 1$ then

$$I.F. = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(y\frac{1}{\sqrt{1-x^2}}\right) = \frac{ax}{\left(1-x^2\right)^{3/2}} = -\frac{1}{2}a\frac{-2x}{\left(1-x^2\right)^{3/2}}$$

$$\Rightarrow y\frac{1}{\sqrt{1-x^2}} = \frac{a}{\sqrt{1-x^2}} + C \Rightarrow y = a + C\sqrt{1-x^2}$$

$$\Rightarrow \left(y-a\right)^2 = C^2\left(1-x^2\right) \Rightarrow \left(y-a\right)^2 + C^2x^2 = C^2$$

Thus if $-1 < x < 1$ the given equation represents an ellipse. If $x^2 > 1$ then the solution is of the form $-\left(y-a\right)^2 + C^2x^2 = C^2$ which represents a hyperbola.

31. Solution of the differential equation : $\frac{dy}{dx} = \frac{3x^2y^4 + 2xy}{x^2 - 2x^3y^3}$ is

(A) $x^2y^2 + \frac{x^2}{y} = c$
(B) $x^3y^2 + \frac{x^2}{y} = c$
(C) $x^3y^2 + \frac{y^2}{x} = c$
(D) $x^2y^3 + \frac{x^2}{y} = c$

Key. B

Sol.

$$x^2dy - 2x^3y^3dy = 3x^2y^4dx + 2xydx$$
$$\Rightarrow x^2dy - 2xydx = 3x^2y^4dx + 2x^3y^3dy$$
$$\Rightarrow \frac{2xydx - x^2dy}{y^2} + 3x^2y^2dx + 2x^3ydy = 0$$
$$\Rightarrow d\left(\frac{x^2}{y}\right) + d\left(x^3y^2\right) = 0$$
$$\Rightarrow \frac{x^2}{y} + x^3y^2 = C$$

32. Solution to the differential equation $\frac{x + \frac{x^3}{3!} + \frac{x^5}{5!} + ...}{1 + \frac{x^2}{2!} + \frac{x^4}{4!} + ...} = \frac{dx - dy}{dx + dy}$ is

(A) $2y\ e^{2x} = C.e^{2x} + 1$ (B) $2y\ e^{2x} = C.e^{2x} - 1$

(C) $y\ e^{2x} = C.e^{2x} + 2$ (D) $2x\ e^{2y} = C.e^{x} - 1$

Key. B

Sol. Applying C and D, we get

$\frac{dy}{dx} = \frac{e^{-x}}{e^{x}} = e^{-2x} \Rightarrow 2y = -e^{-2x} + C$

or $2y\ e^{2x} = C.e^{2x} - 1$.

33. The curve for which the x-intercept of the tangent drawn at any point P on the curve is three times the x-coordinate of the point P, is

(A) $xy = c$ (B) $xy^2 = c$ (C) $xy^3 = c$ (D) $x^2y = c$

Key. B

Sol. $x - y\frac{dx}{dy} = 3x \Rightarrow \frac{dx}{x} + 2\frac{dy}{y} = 0$

$\Rightarrow \ln(xy^2) = k \Rightarrow xy^2 = c$.

34. A curve is such that the mid point of the portion of the tangent intercepted between the point where the tangent is drawn and the point where the tangent meets y-axis, lies on the line y = x. If the curve passes through (1, 0), then the curve is

(A) $2y = x^2 - x$ (B) $y = x^2 - x$ (C) $y = x - x^2$ (D) $y = 2(x - x^2)$

Key. C

Sol. The point on y-axis is $\left(0,\ y - x\frac{dy}{dx}\right)$.

According to given condition,

$\frac{x}{2} = y - \frac{x}{2}\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = 2\frac{y}{x} - 1$

putting $\frac{y}{x} = v$ we get

$x\frac{dv}{dx} = v-1 \Rightarrow \ln\left|\frac{y}{x}-1\right| = \ln|x|+c \Rightarrow 1-\frac{y}{x} = x$ (as f (1) = 0).

35. The family of curves passing through (0,0) and satisfying the differential equation $\frac{y_2}{y_1} = 1$ (where $y_n = d^n y/dx^n$) is

a) $y = k$ b) $y = kx$ c) $y = k(e^x+1)$ d) $y = k(e^x-1)$

Key. D

Sol. $\frac{dp}{dx} = P\left(\text{where } p = \frac{dy}{dx}\right)$

$\ln P = x + c \Rightarrow p = e^{x+c}$

$\frac{dy}{dx} = ke^x$

$y = ke^x + \lambda$

Satisfying $(0,0), So\ \lambda = -k$

$y = k\left(e^x - 1\right)$

36. The degree of the differential equation satisfying the relation $\sqrt{1+x^2}+\sqrt{1+y^2} = \lambda(x\sqrt{1+y^2}-y\sqrt{1+x^2})$ is

a) 1 b) 2 c) 3 d) none of these

Key. A

Sol. On Putting $x = \tan A, y = \tan B$ we get

$\sec A + \sec B = \lambda\left(\tan A \sec B - \tan B \sec A\right)$

$\cos A + \cos B = \lambda\left(\sin A - \sin B\right)$

$\tan\left(\frac{A-B}{2}\right) = \frac{1}{\lambda}$

$\tan^{-1}x - \tan^{-1}y = 2\tan^{-1}\frac{1}{\lambda}$

On differentiating $\frac{1}{1+x^2} - \frac{1}{1+y^2}\frac{dy}{dx} = 0$

37. S1: The differential equation of parabolas having their vertices at the origin and foci on the x-axis is an equation whose variables are separable

S2: The differential equation of the straight lines which are at a fixed distance p from the origin is an equation of degree 2

S3: The differential equation of all conics whose both axes coincide with the axes of coordinates is an equation of order 2

a) TTT b) TFT c) FFT d) TTF

Key. A

Sol. S_1 - Equation of parabola is $y^2 = \pm 4ax$

$$2y\frac{dy}{dx} = \pm 4a$$

D.E of parabola $\Rightarrow y^2 = 2yx\frac{dy}{dx}$

$$2\frac{dy}{y} = \frac{dx}{x}$$

Which is variable seperable

S_2 - Equation of line which is fixed distance. P from origin can be equation of tangent to circle $x^2 + y^2 = p^2$

Line is $y = mx + p\sqrt{1+m^2}$ $\left(m = \frac{dy}{dx}\right)$

$$\left(y - x\frac{dy}{dx}\right)^2 = P\left(1 + \left(\frac{dy}{dx}\right)^2\right)$$

So, degree is 2

S_3 - Equation of conic whose both axis co-incide with co-ordinate axis is $ax^2 + by^2 = 1$

As there are two constants, so order of D.E is 2

38. The order of the differential equation of all tangent lines to the parabola $y = x^2$ is

a) 1 b) 2 c) 3 d) 4

Key. A

Sol. The parametric form of the given equation is $x = t, y = t^2$. The equation of any tangent at t is $2xt = y + t^2$, Differentiating we get $2t = y_1\left(= \frac{dy}{dx}\right)$ putting this value in the above equation, we have $2x\frac{v_1}{2} = y + \left(\frac{y_1}{2}\right)^2 \Rightarrow 4xy_1 = 4y + y_1^2$

The order of this equation is 1

Hence (A) is the correct answer

39. The solution of the differential equation $\frac{\sqrt{x}dx + \sqrt{y}dy}{\sqrt{x}dx - \sqrt{y}dy} = \sqrt{\frac{y^3}{x^3}}$ is given by

a) $\frac{3}{2}\log\left(\frac{y}{x}\right)+\log\left|\frac{x^{3/2}+y^{3/2}}{x^{3/2}}\right|+\tan^{-1}\left(\frac{y}{x}\right)^{3/2}+c=0$

b) $\frac{2}{3}\log\left(\frac{y}{x}\right)+\log\left|\frac{x^{3/2}+y^{3/2}}{x^{3/2}}\right|+\tan^{-1}\frac{y}{x}+c=0$

c) $\frac{2}{3}\left(\frac{y}{x}\right)+\log\left(\frac{x+y}{x}\right)+\tan^{-1}\left(\frac{y^{3/2}}{x^{3/2}}\right)+c=0$

d) None of these

Key. D

Sol. We have $\frac{\sqrt{x}dx+\sqrt{y}dy}{\sqrt{x}dx-\sqrt{y}dy}=\sqrt{\frac{y^3}{x^3}}$

$\Rightarrow \frac{d(x^{3/2})+d(y^{3/2})}{d(x^{3/2})-d(y^{3/2)}}=\frac{y^{3/2}}{x^{3/2}}$

$\Rightarrow \frac{du+dv}{du-dv}=\frac{v}{u}$, where $u=x^{3/2}$ and $v=y^{3/2}$

$\Rightarrow udu+udv=vdu-vdv \Rightarrow udu+vdv=vdu-udv$

$\Rightarrow \frac{udu+vdv}{u^2+v^2}=\frac{vdu-udv}{u^2+v^2}$

$\Rightarrow \frac{d(u^2+v^2)}{u^2+v^2}=-2d\tan^{-1}\left(\frac{v}{u}\right)+c$

On integrating, we get

$\log(u^2+v^2)=-2\tan^{-1}\left(\frac{v}{u}\right)+c$

$\Rightarrow \frac{1}{2}\log(x^3+y^3)+\tan^{-1}\left(\frac{y}{x}\right)^{3/2}=\frac{c}{2}$

40. A function $y=f(x)$ has a second order derivative $f''=6(x-1)$. If its graph passes through the point(2,1) and at that point the tangent to the graph is $y=3x-5$, then the function is

a) $(x-1)^2$ b) $(x-1)^3$ c) $(x+1)^2$ d) $(x+1)^3$

Key. B

Sol. Since $f''(x)=6(x-1)$

$\Rightarrow f'(x)=3(x-1)^2+c$ (integrating) ----(i)

Also, at the point (2,1), the tangent to the graph is $y=3x-5$ and slope of the tangent = 3

$\Rightarrow f'(2)=3$

$3(2-1)^2+c=3$ [from Eq (i)

$\Rightarrow 3+c=3 \Rightarrow c=0$

From Eq (i) we have

$f'(x)=3(x-1)^2$

$\Rightarrow f(x)=(x-1)^2+k$ (Integrating) ----(ii)

$\therefore 1=(2-1)^2+k \Rightarrow k=0$

Hence the equation of the function is $f(x)=(x-1)^3$.

41. The differential equation representing the family of curves $y^2=2c(x+\sqrt{c})$, where $c>0$ is a parameter, is of order and degree as follows :

a) order 1, degree 3 b) order 2, degree 3

c) order 1, degree 2 d) order 2, degree 1

Key. A

Sol. Differentiating, we get

$2yy'=2c \Rightarrow c=yy'$

$\therefore y^2=2yy'(x+\sqrt{yy'}) \Rightarrow (y^2-2yy'x)^2=4(yy')^3$

$\Rightarrow$ degree= 3 and order =1

42. The normal to a curve at $P(x,y)$ meets the $x-axis$ at G. If the distance of G from the origin is twice the abscissa of P, then the curve is

a) a parabola b) a circle c) a hyperbola d) an ellipse

Key. C

Sol. The slope of the tangent = $\frac{dy}{dx}$

$\therefore$ the slope of the normal $=-\frac{dx}{dy}$

$\therefore$ the equation of the normal is $Y-y=-\frac{dx}{dy}(X-x)$

This meets the x-axis $(Y=0)$, where

$$-y=\frac{-dx}{dy}(X-x)\Rightarrow X=x+y\frac{dy}{dx}$$

$\therefore$ G is $\left(x+y\frac{dy}{dx},0\right)$

$\therefore$ $OG=2x\Rightarrow x+y\frac{dy}{dx}=2x$

$\Rightarrow y\frac{dy}{dx}=x\Rightarrow ydy=xdx$ [variable separable integrating, we get]

$$\frac{y^2}{2}=\frac{x^2}{2}+constant$$

$\Rightarrow x^2-y^2=c$, which is a hyperboal

43. Which of the following transformation reduces the differential equation $\frac{dz}{dx}+\frac{z}{x}\log z=\frac{z}{x^2}(\log z)^2$ to the form $P(x)u=Q(x)$

a) $u=\log z$ b) $u=e^z$ c) $u=(\log z)^{-1}$ d) $u=(\log z)^2$

Key. C

Sol. Given equ. Can be written as

$$\frac{dz}{z(\ln z)^2}+\frac{dx}{x(\ln z)}=\frac{dx}{x^2}$$

Put $u=\frac{1}{\ln z}$

$$\frac{du}{dx}=-\frac{1}{(\ln z)^2}.\frac{1}{z}.\frac{dz}{dx}$$

$$\therefore -\frac{du}{dx}+\frac{1}{x}.u=\frac{1}{x^2}$$

$$\frac{du}{dx}+\left(-\frac{1}{x}\right)u=\left(-\frac{1}{x^2}\right)$$

$$I.F.=e^{\int -\frac{1}{x}dx}=e^{\ln\frac{1}{x}}=\frac{1}{x}$$

$$u.\frac{1}{x}=\int -\frac{1}{x^2}.\frac{1}{x}dx$$

$$\frac{u}{x}=-\int \frac{1}{x^3}dx=+\frac{1}{4x^4}+c$$

44. If $y_1(x)$ is a solution of the differential equation $dy/dx-f(x)y=0$, then a solution of the differential equation $\frac{dy}{dx}+f(x)y=r(x)$

a) $\frac{1}{y_1(x)}\int r(x)y_1(x)dx$

b) $y_1(x)\int \frac{r(x)}{y_1(x)}dx$

c) $\int r(x)y_1(x)dx$

d) none of these

Key. A

Sol. $\frac{dy}{dx}-f(x).y=0$

$$\frac{dy}{y}=f(x)dx$$

$$\ln y=\int f(x)dx$$

$y_1(x)=e^{\int f(x)dx}$ Then for given equation I.F= $e^{\int f(x)dx}$

Hence Solution $y.y_1(x)=\int r(x).y_1(x)dx$

$$y=\frac{1}{y_1(x)}\int r(x).y_1(x)dx$$

45. Solution of the differential equation : $\frac{dy}{dx}=\frac{3x^2y^4+2xy}{x^2-2x^3y^3}$ is

(A) $x^2y^2+\frac{x^2}{y}=c$

(B) $x^3y^2+\frac{x^2}{y}=c$

(C) $x^3y^2+\frac{y^2}{x}=c$

(D) $x^2y^3+\frac{x^2}{y}=c$

Key. B

Sol.

$$x^2dy - 2x^3y^3dy = 3x^2y^4dx + 2xydx$$

$$\Rightarrow x^2dy - 2xydx = 3x^2y^4dx + 2x^3y^3dy$$

$$\Rightarrow \frac{2xydx - x^2dy}{y^2} + 3x^2y^2dx + 2x^3ydy = 0$$

$$\Rightarrow d\left(\frac{x^2}{y}\right) + d\left(x^3y^2\right) = 0$$

$$\Rightarrow \frac{x^2}{y} + x^3y^2 = C$$

46. Solution to the differential equation $\dfrac{x+\dfrac{x^3}{3!}+\dfrac{x^5}{5!}+\ldots\ldots}{1+\dfrac{x^2}{2!}+\dfrac{x^4}{4!}+\ldots\ldots} = \dfrac{dx-dy}{dx+dy}$ is

(A) $2y\,e^{2x} = C.e^{2x} + 1$ (B) $2y\,e^{2x} = C.e^{2x} - 1$

(C) $y\,e^{2x} = C.e^{2x} + 2$ (D) $2x\,e^{2y} = C.e^{x} - 1$

Key. B

Sol. Applying C and D, we get

$\dfrac{dy}{dx} = \dfrac{e^{-x}}{e^{x}} = e^{-2x} \Rightarrow 2y = -e^{-2x} + C$

or $2y\,e^{2x} = C.e^{2x} - 1$.

47. The curve for which the x-intercept of the tangent drawn at any point P on the curve is three times the x-coordinate of the point P, is

(A) $xy = c$ (B) $xy^2 = c$ (C) $xy^3 = c$ (D) $x^2y = c$

Key. B

Sol. $x - y\dfrac{dx}{dy} = 3x \Rightarrow \dfrac{dx}{x} + 2\dfrac{dy}{y} = 0$

$\Rightarrow \ln(xy^2) = k \Rightarrow xy^2 = c.$

48. A curve is such that the mid point of the portion of the tangent intercepted between the point where the tangent is drawn and the point where the tangent meets y-axis, lies on the line $y = x$. If the curve passes through (1, 0), then the curve is

(A) $2y = x^2 - x$ (B) $y = x^2 - x$ (C) $y = x - x^2$ (D) $y = 2(x - x^2)$

Key. C

Sol. The point on y-axis is $\left(0,\ y - x\dfrac{dy}{dx}\right)$.

According to given condition,

$\dfrac{x}{2} = y - \dfrac{x}{2}\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = 2\dfrac{y}{x} - 1$

putting $\frac{y}{x} = v$ we get

$x\frac{dv}{dx} = v - 1 \Rightarrow \ln\left|\frac{y}{x} - 1\right| = \ln|x| + c \Rightarrow 1 - \frac{y}{x} = x$ (as f (1) = 0).

49. The solution curves of the differential equation $(xdx + ydy)\sqrt{x^2 + y^2} = (xdy - ydx)\left(\sqrt{1 - x^2 - y^2}\right)$ are

a) circles of radius 1, passing through the origin

b) circles of radius $1/2$, passing through the origin

c) circles not passing through origin

d) solution curve is not a circle

Key: B

Hint: $x = r\cos\theta,\ x = r\cos\theta$

$\frac{dr}{\sqrt{1 - r^2}} = d\theta \Rightarrow \sin^{-1} r = \theta + \alpha$

$r = \sin(\theta + \alpha)$

$\Rightarrow x^2 + y^2 - x\sin\alpha - y\cos\alpha = 0$

$\therefore \text{radius} = \frac{1}{2}$

50. A conic C satisfies the differential equation $(1 + y^2)dx - xydy = 0$ and passes through the point (1,0). An ellipse E which is confocal with C has its eccentricity $\sqrt{2/3}$. The angle of intersection of the curves C and E is

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

Key: D

Hint The conic C is $x^2 - y^2 = 1$ *and* E *is* $x^2 + 3y^2 = 3$, Angle of intersection is $\pi/2$

51. The solution of the differential equation $y^2dx + (x^2 - xy + y^2)dy = 0$ is

(A) $\tan^{-1}\left(\frac{x}{y}\right) + \ln y + C = 0$ (B) $2\tan^{-1}\left(\frac{x}{y}\right) + \ln x + C = 0$

(C) $\ln\left(y + \sqrt{x^2 + y^2}\right) + \ln y + C = 0$ (D) $\ln\left(x + \sqrt{x^2 + y^2}\right) + C = 0$

(Where C is arbitrary constant)

Key: A

Hint: $\frac{dx}{dy} + \frac{x^2}{y^2} - \frac{x}{y} + 1 = 0$

Put $x = vy$ where v is a function of 'y'

$\Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$

$\therefore v + y\frac{dv}{dy} + v^2 - v + 1 = 0 \Rightarrow -y\frac{dv}{dy} = \left(1+v^2\right) \Rightarrow \frac{dv}{v^2+1} = -\frac{dy}{y}$

$\tan^{-1}v + C = -\ln y \Rightarrow \tan^{-1}\left(\frac{x}{y}\right) + \ln y + C = 0$

Where C is arbitrary constant

52. The solution of the differential equation $2x^3ydy + \left(1-y^2\right)\left(x^2y^2+y^2-1\right)dx = 0$

[Where c is a constant]

(A) $x^2y^2 = (cx+1)\left(1-y^2\right)$ (B) $x^2y^2 = (cx+1)\left(1+y^2\right)$

(C) $x^2y^2 = (cx-1)\left(1-y^2\right)$ (D) none of these

Key : C

Hint : $\frac{2y}{\left(1-y^2\right)^2}\cdot\frac{dy}{dx} + \frac{y^2}{1-y^2}\frac{1}{x} = \frac{1}{x^3}$

Put $\frac{y^2}{1-y^2} = t \Rightarrow \frac{2y}{\left(1-y^2\right)^2}\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow \frac{dt}{dx} + \frac{t}{x} = \frac{1}{x^3}$

$\Rightarrow t\cdot x = \int\frac{1}{x^2}dx + c$

$\Rightarrow x^2y^2 = (cx-1)\left(1-y^2\right).$

53. The solution of differential equation $(x^5 + x + 2x^2y)\,dy + (3x^4y - y)\,dx = 0$ is

(A) $x^4y + xy^2 + y = cx$ (B) $x^4y^2 + xy + y = cx$

(C) $x^4y + x^2y^2 + xy = c$ (D) $x^4y - xy^2 - y = cx$

Key : A

Sol : Divide both sides by x^2

$x^3dy + y.3x^2dx + \frac{xdy - ydx}{x^2} + 2ydy = 0$

$\Rightarrow d(x^3y) + d\left(\frac{y}{x}\right) + d(y^2) = 0$

$\Rightarrow x^4y + xy^2 + y = cx$

$\Rightarrow$ 2. The equation to the curve which is such that portion of the axis of *x* cutoff

$\Rightarrow$ between the origin and the tangent at any point is proportional to the

⇒ ordinate of that point is (constant of proportionality is K)

⇒ a) x=y(C-K log y)

b) log x = Ky^2+C

⇒ c) x^2=y(C- K log y)

d) None of these

⇒ Key: A

⇒ Hint $x - y\frac{dx}{dy} = ky$

54. Let a function f(x) be such that f ′′(x) = f′(x) + e^x and f(0) = 0, f ′(0) = 1 , then ln$\left(\frac{(f(2))^2}{4}\right)$ equal to

(A) $\frac{1}{2}$ (B) 1 (C) 2 (D) 4

⇒ Key: D

⇒ Hint f′′(x) – f′(x) = e^x

⇒ put f′(x) = v

⇒ $\frac{dv}{dx} + v(-1) = e^x$

⇒ ⇒ $ve^{-x} = \int e^x.e^{-x}dx$

⇒ ve^{-x} = x + C_1 , f ′ (0) = 1 ⇒ C_1 = 1

⇒ f′(x) = $xe^x + e^x$

⇒ f(x) = $xe^x + C_2$

⇒ ⇒ f(0) = 0 ⇒ C_2 = 0

⇒ ⇒ f(x) = xe^x ⇒ f(2) = $2e^2$

⇒ ln $\left(\frac{(f(2))^2}{4}\right)$ = 4.

55. The solution of $\frac{dy}{dx} + x(x+y) = x^3(x+y)^3 - 1, y(0) = 1;$ is given by $2(1+x^2) - \frac{1}{(x+y)^2} = f(x)$ where $f(x)$ =

a) e^{-x} b) e^{-x^2} c) e^x d) e^{x^2}

Key. D

Sol. Put $x + y = Y$ then equation given becomes $\frac{dY}{dx} + Y = x^3Y^3$.

$$\Rightarrow \frac{1}{Y^3}\frac{dY}{dx}+\frac{x}{Y^2}=x^3 \ \ putting\ \ z=\frac{1}{Y^2}\ \ makes\, it\, \frac{dz}{dx}-2xz=-2x^3$$

$$\Rightarrow z=2+2x^2+ce^{x^2}$$

$$\Rightarrow \frac{1}{(x+y)^2}=2+2x^2+ce^{x^2}\ putting\ x=0, y=1\, gives\, c=-1$$

56. The solution of the differential equation $\dfrac{\sqrt{x}dx+\sqrt{y}dy}{\sqrt{x}dx-\sqrt{y}dy}=\sqrt{\dfrac{y^3}{x^3}}$ is given by

a) $\dfrac{3}{2}\log\left(\dfrac{y}{x}\right)+\log\left|\dfrac{x^{3/2}+y^{3/2}}{x^{3/2}}\right|+\tan^{-1}\left(\dfrac{y}{x}\right)^{3/2}+c=0$

b) $\dfrac{2}{3}\log\left(\dfrac{y}{x}\right)+\log\left|\dfrac{x^{3/2}+y^{3/2}}{x^{3/2}}\right|+\tan^{-1}\dfrac{y}{x}+c=0$

c) $\dfrac{2}{3}\left(\dfrac{y}{x}\right)+\log\left(\dfrac{x+y}{x}\right)+\tan^{-1}\left(\dfrac{y^{3/2}}{x^{3/2}}\right)+c=0$

d) None of these

Key. D

Sol. We have $\dfrac{\sqrt{x}dx+\sqrt{y}dy}{\sqrt{x}dx-\sqrt{y}dy}=\sqrt{\dfrac{y^3}{x^3}}$

$$\Rightarrow \frac{d(x^{3/2})+d(y^{3/2})}{d(x^{3/2})-d(y^{3/2})}=\frac{y^{3/2}}{x^{3/2}}$$

$$\Rightarrow \frac{du+dv}{du-dv}=\frac{v}{u},\ \text{where } u=x^{3/2} \text{ and } v=y^{3/2}$$

$$\Rightarrow udu+udv=vdu-vdv \Rightarrow udu+vdv=vdu-udv$$

$$\Rightarrow \frac{udu+vdv}{u^2+v^2}=\frac{vdu-udv}{u^2+v^2}$$

$$\Rightarrow \frac{d(u^2+v^2)}{u^2+v^2}=-2d\tan^{-1}\left(\frac{v}{u}\right)+c$$

On integrating, we get

$$\log(u^2+v^2)=-2\tan^{-1}\left(\frac{v}{u}\right)+c$$

$$\Rightarrow \frac{1}{2}\log(x^3+y^3)+\tan^{-1}\left(\frac{y}{x}\right)^{3/2}=\frac{c}{2}$$

57. A function $y = f(x)$ has a second order derivative $f'' = 6(x-1)$. If its graph passes through the point(2,1) and at that point the tangent to the graph is $y = 3x-5$, then the function is

a) $(x-1)^2$ b) $(x-1)^3$ c) $(x+1)^2$ d) $(x+1)^3$

Key. B

Sol. Since $f''(x) = 6(x-1)$

$\Rightarrow f'(x) = 3(x-1)^2 + c$ (integrating) ----(i)

Also, at the point (2,1), the tangent to the graph is $y = 3x-5$ and slope of the tangent = 3

$\Rightarrow f'(2) = 3$

$3(2-1)^2 + c = 3$ [from Eq (i)

$\Rightarrow 3 + c = 3 \Rightarrow c = 0$

From Eq (i) we have

$f'(x) = 3(x-1)^2$

$\Rightarrow f(x) = (x-1)^2 + k$ (Integrating) ----(ii)

$\therefore 1 = (2-1)^2 + k \Rightarrow k = 0$

Hence the equation of the function is $f(x) = (x-1)^3$.

58. The differential equation representing the family of curves $y^2 = 2c(x+\sqrt{c})$, where $c > 0$ is a parameter, is of order and degree as follows :

a) order 1, degree 3 b) order 2, degree 3

c) order 1, degree 2 d) order 2, degree 1

Key. A

Sol. Differentiating, we get

$2yy' = 2c \Rightarrow c = yy'$

$\therefore y^2 = 2yy'(x+\sqrt{yy'}) \Rightarrow (y^2 - 2yy'x)^2 = 4(yy')^3$

$\Rightarrow$ degree= 3 and order =1

59. The normal to a curve at $P(x, y)$ meets the $x - axis$ at G. If the distance of G from the origin is twice the abscissa of P, then the curve is

a) a parabola b) a circle c) a hyperbola d) an ellipse

Key. C or D

Sol. The slope of the tangent = $\dfrac{dy}{dx}$

$\therefore$ the slope of the normal $= -\dfrac{dx}{dy}$

$\therefore$ the equation of the normal is $Y - y = -\dfrac{dx}{dy}(X - x)$

This meets the x-axis $(Y = 0)$, where

$$-y = \frac{-dx}{dy}(X - x) \Rightarrow X = x + y\frac{dy}{dx}$$

$\therefore$ G is $\left(x + y\dfrac{dy}{dx}, 0\right)$

$\therefore$ $OG = 2x \Rightarrow x + y\dfrac{dy}{dx} = 2x$

$\Rightarrow y\dfrac{dy}{dx} = x \Rightarrow ydy = xdx$ [variable separable integrating, we get]

$$\frac{y^2}{2} = \frac{x^2}{2} + constant$$

$\Rightarrow x^2 - y^2 = c$, which is a hyperboal

60. The general solution of $x\dfrac{dy}{dx} + (\log x)y = x^{1-\frac{1}{2}\log x}$ is

a) $y = x^{1-\frac{1}{2}\log x} + cx^{-\frac{1}{2}\log x}$

b) $y.x^{\frac{1}{2}\log x} = x^{\frac{1}{2}\log x} + c$

c) $y = e^{\frac{(\log x)^2}{2}}(x + c)$

d) $y = e^{\frac{1}{2}(\log x)^2}\left(x^{1-\frac{1}{2}(\log x)} - x^{-\frac{1}{2}(\log x)}\right) + c$

Key. A

Sol. $\dfrac{dy}{dx} + \dfrac{\log x}{x}y = x^{-\frac{1}{2}\log x}$

$$I.F = e^{\int \frac{\log x}{x}dx} = e^{\frac{(\log x)^2}{2}} = \left(e^{\log x}\right)^{\frac{\log x}{2}} = x^{\frac{1}{2}\log x}$$

$$G.S is\, x^{\frac{1}{2}\log x}.y = \int dx$$

$$y\,x^{\frac{1}{2}\log x} = x + c$$

61. Solution of $\dfrac{x + y\dfrac{dy}{dx}}{y - x\dfrac{dy}{dx}} = x^2 + 2y^2 + \dfrac{y^4}{x^2}$

a) $\frac{y}{x}-\frac{1}{2\left(x^2+y^2\right)}=c$ b) $\frac{y}{x}+\frac{1}{2\left(x^2+y^2\right)}=c$

c) $\frac{x}{y}+\frac{1}{2\left(x^2+y^2\right)}=c$ d) $\frac{x}{y}-\frac{1}{2\left(x^2+y^2\right)}=c$

Key. A

Sol. $\frac{xdx+ydy}{ydx-xdy}=\frac{x^4+2x^2y^2+y^4}{x^2}$

$\frac{2xdx+2ydy}{2\left(x^2+y^2\right)^2}=-d\left(\frac{y}{x}\right)$

$\frac{xdx+ydy}{\left(x^2+y^2\right)^2}=\frac{ydx-xdy}{x^2}$

$\frac{-1}{x^2+y^2}=-2\frac{y}{x}+c$

$2\frac{y}{x}-\frac{1}{x^2+y^2}=c$

62. Solution of $\frac{y+\sin x\cos^2(xy)}{\cos^2(xy)}dx+\left(\frac{x}{\cos^2(xy)}+\sin y\right)dy=0$

a) $\tan(xy)+\cos x+\cos y=c$ b) $\tan(xy)-\cos x-\cos y=c$

c) $\tan(xy)+\cos x-\cos y=c$ c) $\tan(xy)-\cos x+\cos y=c$

Key. B

Sol. $\frac{y\,dx+x\,dy}{\cos^2(xy)}+\sin xdx+\sin ydy=0$

$\int\frac{d(xy)}{\cos^2(xy)}+\int\sin x\,dx+\int\sin y\,dy=0$

$\tan(xy)-\cos x-\cos y=c$

63. The equation of the curve which is passing through (1, 1) and whose differential equation is $\frac{dy}{dx}+\frac{y}{x}=y^3$ is

(A) $2x^2y^2-xy^2=1$ (B) $2xy^2+x^2y^2=1$

(C) $2x^2y^2+xy^2=1$ (D) $2xy^2-x^2y^2=1$

Key. D

Sol. $\frac{dy}{dx}+\frac{y}{x}=y^3 \Rightarrow \frac{1}{y^3}.\frac{dy}{dx}+\frac{1}{xy^2}=1$

Put t = $\frac{1}{y^2}$

$\Rightarrow \frac{dt}{dx}-\frac{2t}{x}=-2$

$\Rightarrow$ Solution is t. $\frac{1}{x^2}$ = $-2\int\frac{1}{x^2}dx$

$\Rightarrow 2xy^2+cx^2y^2=1$

Since this curve passes through (1, 1)

$\Rightarrow$ c = 1.

64. The solution of $\frac{dy}{dx}+x(x+y)=x^3(x+y)^3-1, y(0)=1;$ is given by

$2(1+x^2)-\frac{1}{(x+y)^2}=f(x)$ where $f(x)=$

a) e^{-x} b) e^{-x^2} c) e^{x} d) e^{x^2}

Key. D

Sol. Put $x+y=Y$ then equation given becomes $\frac{dY}{dx}+Y=x^3Y^3.$

$\Rightarrow \frac{1}{Y^3}\frac{dY}{dx}+\frac{x}{Y^2}=x^3$ putting $z=\frac{1}{Y^2}$ makes it $\frac{dz}{dx}-2xz=-2x^3$

$\Rightarrow z=2+2x^2+ce^{x^2}$

$\Rightarrow \frac{1}{(x+y)^2}=2+2x^2+ce^{x^2}$ putting $x=0, y=1$ gives $c=-1.$

65. If the solution of differential equation $\frac{dy}{dx}=\frac{ax+3}{2y+f}$ represents a circle of non zero radius then

a) $a=2;9+4f^2>4c$ b) $a=-2;9+4f^2<4c$

c) $a=2;9+4f^2<4c$ d) $a=-2;9+4f^2>4c$

Key. D

Sol. $(2y+f)dy=(ax+3)dx$ on integration

$$y^2+fy+c=\frac{ax^2}{2}+3x \Rightarrow -\frac{ax^2}{2}+y^2+fy-3x+c=0$$

$$\Rightarrow a=-2,\ 9+4f^2-4c>0.$$

66. Solution of $\left(xe^{\frac{y}{x}}-y\text{Sin}\frac{y}{x}\right)dx+x\text{Sin}\frac{y}{x}dy=0$

a) $\log x=c+\frac{1}{2}e^{\frac{-y}{x}}\left(\text{Sin}\frac{y}{x}+\text{Cos}\frac{y}{x}\right)$ b) $\log x=c+\frac{1}{2}e^{\frac{y}{x}}\left(\text{Sin}\frac{y}{x}-\text{Cos}\frac{y}{x}\right)$

c) $\log x=c+\frac{1}{2}e^{\frac{y}{x}}\left(\text{Sin}\frac{y}{x}+\text{Cos}\frac{y}{x}\right)$ d) $\log x=c+\frac{1}{2}e^{\frac{-y}{x}}\left(\text{Sin}\frac{y}{x}-\text{Cos}\frac{y}{x}\right)$

Key. A

Sol. Put y =vx $\Rightarrow \int\frac{dx}{x}+\int e^{-v}\text{Sin}v\,dv=c$

$$\Rightarrow \int e^{-v}\sin v\,dv=-\frac{e^{-v}}{2}(\text{Sin}v+\text{Cos}v)$$

$$\Rightarrow \log x = c + \frac{1}{2} e^{\frac{-y}{x}} \left(\sin\frac{y}{x} + \cos\frac{y}{x} \right)$$

67. The solution of differential equation $xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$

a) $y = \tan\left(\frac{c - x - y}{2}\right)$
b) $y = x\tan\left(\frac{c - x^2 - y^2}{2}\right)$
c) $y = x\tan\left(\frac{c + x^2 + y^2}{2}\right)$
d) $y = x\tan\left(\frac{c + x^2 - y^2}{2}\right)$

Key. B

Sol. Given equation

$$\frac{1}{2}d\left(x^2 + y^2\right) + d\left(\tan^{-1}\frac{y}{x}\right) = 0 \Rightarrow y = x\tan\left(\frac{c - x^2 - y^2}{2}\right)$$

68. The differential equation of all circles in a plane must be

a) $y_3(1 + y_1^{\,2}) - 3y_1 y_2^2 = 0$
b) of order 3 and degree 3
c) of order 3 and degree 2
d) $y_3(1 - y_1^{\,2}) - 3y_1 y_2^2 = 0$

Key. A

Sol. Equation of circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

Differentiating two times w.r.t. x

we have f $= -\frac{1 + yy_2 + y_1^2}{y_2}$

Again differentiating

$$\frac{y_2\left[2y_1y_2 + y_1y_2 + yy_3\right] - \left[y_3(1 + y_1^2 + yy_2)\right]}{y_2^2} = 0$$

$$\Rightarrow \quad 3y_1 yy_1^2 + yy_2y_3 - y_3 - y_3y_1^2 - yy_2y_3 = 0$$

$$\Rightarrow \quad 3y_1 y_2^2 = y_3\left[1 + y_1^2\right]$$

69. If the population of a country doubles in 50 years in how many years will it become thrice the original, assume the rate of increase is proportional to the number of inhabitants

a) 75
b) $50\log_2 3$
c) $50\log_3 2$
d) 100

Key. B

Sol. P – Population, y – population after 't' years

$$\frac{dy}{dt}\alpha y \Rightarrow \frac{dy}{dt} = ky \Rightarrow \int\frac{dy}{y} = \int kdt \Rightarrow \log y = kt + c$$

$$t = 0 \,\&\, y = p \Rightarrow \log p = 0 + c$$

$$t = 0 \,\&\, y = 2p \Rightarrow \log 2p = 50k + \log p$$

$$\Rightarrow \log\left(\frac{2p}{p}\right) = 50k \Rightarrow k = \frac{\log(2)}{50}$$

$$t = ? \,\&\, y = 3p \Rightarrow kt = \log y - c$$

$$\left(\frac{\log 2}{50}\right)t = \log 3p - \log p$$

$$t=\frac{\log 3}{(\log 2/50)}=50\log_2 3$$

70. The equation of the curve not passing through the origin and having the portion of the tangent included between the coordinate axes is bisected at the point of contact is

a) a parabola
b) an ellipse or a straight line
c) a circle or an ellipse
d) a hyperbola or a straight line

Key. D

Sol. Equation of line passing through P (x_1, y_1)

Y- y_1 = $\frac{dy}{dx}$(X- x_1), x-int = x_1 - $\frac{y_1}{m}$, y-int = y_1 - x_1 m,

according to condition

$$\left(x-\left(x-y\frac{dx}{dy}\right)\right)^2+y^2=\left(y-\left(y-x\frac{dy}{dx}\right)\right)^2+x^2$$

$$\Rightarrow\left(x\frac{dy}{dx}-y\right)\left(x\frac{dy}{dx}+y\right)=0$$

that is y=cx or xy=c

71. Solution of the differential equation : $\frac{dy}{dx}=\frac{3x^2y^4+2xy}{x^2-2x^3y^3}$ is

(A) $x^2y^2+\frac{x^2}{y}=c$
(B) $x^3y^2+\frac{x^2}{y}=c$
(C) $x^3y^2+\frac{y^2}{x}=c$
(D) $x^2y^3+\frac{x^2}{y}=c$

Key. B

Sol.

$$x^2dy-2x^3y^3dy=3x^2y^4dx+2xydx$$

$$\Rightarrow x^2dy-2xydx=3x^2y^4dx+2x^3y^3dy$$

$$\Rightarrow\frac{2xydx-x^2dy}{y^2}+3x^2y^2dx+2x^3ydy=0$$

$$\Rightarrow d\left(\frac{x^2}{y}\right)+d\left(x^3y^2\right)=0$$

$$\Rightarrow\frac{x^2}{y}+x^3y^2=C$$

72. Solution to the differential equation $\frac{x+\frac{x^3}{3!}+\frac{x^5}{5!}+........}{1+\frac{x^2}{2!}+\frac{x^4}{4!}+......}=\frac{dx-dy}{dx+dy}$ is

(A) 2y e^{2x} = C.e^{2x} + 1
(B) 2y e^{2x} = C.e^{2x} – 1
(C) y e^{2x} = C.e^{2x} + 2
(D) 2x e^{2y} = C.e^{x} – 1

Key. B

Sol. Applying C and D, we get

$\frac{dy}{dx} = \frac{e^{-x}}{e^{x}} = e^{-2x} \Rightarrow 2y = -e^{-2x} + C$

or $2y\, e^{2x} = C.e^{2x} - 1$.

73. The curve for which the x-intercept of the tangent drawn at any point P on the curve is three times the x-coordinate of the point P, is

(A) $xy = c$ (B) $xy^2 = c$ (C) $xy^3 = c$ (D) $x^2y = c$

Key. B

Sol. $x - y\frac{dx}{dy} = 3x \Rightarrow \frac{dx}{x} + 2\frac{dy}{y} = 0$

$\Rightarrow \ln(xy^2) = k \Rightarrow xy^2 = c$.

74. A curve is such that the mid point of the portion of the tangent intercepted between the point where the tangent is drawn and the point where the tangent meets y-axis, lies on the line y = x. If the curve passes through (1, 0), then the curve is

(A) $2y = x^2 - x$ (B) $y = x^2 - x$ (C) $y = x - x^2$ (D) $y = 2(x - x^2)$

Key. C

Sol. The point on y-axis is $\left(0,\ y - x\frac{dy}{dx}\right)$.

According to given condition,

$\frac{x}{2} = y - \frac{x}{2}\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = 2\frac{y}{x} - 1$

putting $\frac{y}{x} = v$ we get

$x\frac{dv}{dx} = v - 1 \Rightarrow \ln\left|\frac{y}{x} - 1\right| = \ln|x| + c \Rightarrow 1 - \frac{y}{x} = x$ (as f (1) = 0).

75. The solution of $\frac{dy}{dx}\sqrt{1+x+y} = x+y-1$ is

1) $2\left[\sqrt{1+x+y} + \frac{1}{3}\log\left|\sqrt{1+x+y}-1\right| - \frac{4}{3}\log\left|\sqrt{1+x+y}+2\right|\right] = x+c$

2) $2\left[\sqrt{1+x+y} + \frac{1}{3}\log\left|\sqrt{1+x+y}\right| - \frac{4}{3}\log\left|\sqrt{1+x+y}+2\right|\right] = x+c$

3) $2\left[\sqrt{1+x+y} + \frac{1}{3}\log\left|\sqrt{1+x+y}\right| - \frac{4}{3}\log\left|\sqrt{1+x+y}\right|\right] = x+c$

4) $\left[\sqrt{1+x+y} + \frac{1}{3}\log\left|\sqrt{1+x+y}\right| - \frac{4}{3}\log\left|\sqrt{1+x+y}\right|\right] = x+c$

Key. 1

Sol. Putting $\sqrt{1+x+y} = v,$ we have,

$x+y-1 = v^2 - 2 \Rightarrow 1 + \frac{dy}{dx} = 2v\frac{dv}{dx}$

Then the given equation transforms to

$$\left(2v\frac{dv}{dx}-1\right)v=v^2-2\Rightarrow\frac{dv}{dx}=\frac{v^2+v-2}{2v^2}$$

$$\Rightarrow\int\frac{2v^2}{v^2+v-2}dv=\int dx\Rightarrow 2\int\left[1+\frac{1}{3(v-1)}-\frac{4}{3(v+2)}\right]dv=\int dx$$

$$\Rightarrow 2\left[v+\frac{1}{3}\log|v-1|-\frac{4}{3}\log|v+2|\right]=x+c$$

Where $v=\sqrt{1+x+y}$

76. The solution of $\left(x+2y^3\right)\left(dy/dx\right)=y$ is (where c is arbitrary constant)

1) $x=y^3+cy$
2) $x=y^3-cy$
3) $y=x^3-cy$
4) $y=x^3+cy$

Key. 1 or 2

Sol. $\frac{dx}{dy}-\frac{1}{y}x=2y^2$

This is linear equation taking y as independent variable.

Here, I.F. $=e^{-\int\frac{1}{y}dy}=e^{-\log y}=\frac{1}{y}$

$\therefore$ **solution is** $x\frac{1}{y}=\int\frac{1}{y}2y^2dy+c$

$$\Rightarrow\frac{x}{y}=y^2+c\Rightarrow x=y^3+cy$$

77. The solution of $(1+x)\frac{dy}{dx}+1=e^{x-y}$ is

1) $e^y(x+1)=c$
2) $e^y(x+1)=e^x+c$
3) $e^y(x+1)=ce^x$
4) $(x+1)=e^x+c$

Key. 2

Sol. $(x+1)\frac{dy}{dx}+1=\frac{e^x}{e^y}\Rightarrow e^y(x+1)\frac{dy}{dx}+e^y=e^x$

$$\int\frac{d}{dx}\left(e^y(x+1)\right)dx=\int e^x dx\Rightarrow e^y(x+1)=e^x+c$$

78. Solution of the equation $xdy=\left(y+x\frac{f(y/x)}{f'(y/x)}\right)dx$

1) $f\left(\frac{x}{y}\right)=cy$
2) $f\left(\frac{y}{x}\right)=cx$
3) $f\left(\frac{y}{x}\right)=cxy$
4) $f\left(\frac{y}{x}\right)=0$

Key. 2

Sol. We have, $xdy=\left(y+\frac{xf(y/x)}{f'(y/x)}\right)dx$

$\Rightarrow \frac{dy}{dx}=\frac{y}{x}+\frac{f(y/x)}{f'(y/x)}$ which is homogenous

Putting $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$,we obtain

$v+x\frac{dv}{dx}=v+\frac{f(v)}{f'(v)} \Rightarrow \frac{f'(v)}{f(v)}dv=\frac{dx}{x}$

Integrating, we get log $f(v)=\log x+\log c$

$\Rightarrow \log f(v)=\log cx \Rightarrow f\left(\frac{y}{x}\right)=cx$

79. The general solution of the differential equation $\frac{dy}{dx}=y\tan x-y^2\sec x$ is

1) $\tan x=(c+\sec x)y$
2) $\sec y=(c+\tan y)x$
3) $\sec x=(c+\tan x)y$
4) None of these

Key. 3

Sol. We have $\frac{dy}{dx}=y\tan x-y^2\sec x$

$\Rightarrow \frac{1}{y^2}\frac{dy}{dx}-\frac{1}{y}\tan x=-\sec x$

Putting $\frac{1}{y}=v \Rightarrow \frac{-1}{y^2}\frac{dy}{dx}=\frac{dv}{dx}$**, we obtain**

$\frac{dV}{dx}+\tan x.v=\sec x$ **which is linear**

$I.F=e^{\int \tan x\,dx}=e^{\log\sec x}=\sec x$

$\therefore$ The solution is

$v\sec x=\int \sec^2 x\,dx+c \Rightarrow \frac{1}{y}\sec x=\tan x+c$

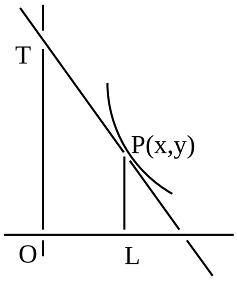
$\Rightarrow \sec x=y(c+\tan x)$

80. The curve $y=f(x)$ is such that the area of the trapezium formed by the coordinate axes, ordinate of an arbitrary point and the tangent at this point equals half the square of its abscissa. The equation of the curve can be

1) $y=cx^2\pm x$
2) $y=cx^2\pm 1$
3) $y=cx\pm x^2$
4) $y=cx^2\pm x^2\pm 1$

Key. 1

Sol. Let $P(x,y)$ be any point on the curve. Length of intercept on y-axis by any tangent at P is



$OT = y - x\frac{dy}{dx}$

$\therefore$ Area of trapezium $OLPTO = \frac{1}{2}(PL + OT)OL$

$= \frac{1}{2}\left(y + y - x\frac{dy}{dx}\right)x = \frac{1}{2}\left(2y - x\frac{dy}{dx}\right)x$

According to question

Area of trapezium $OLPTO = \frac{1}{2}x^2$

$i.e., \frac{1}{2}\left(2y - x\frac{dy}{dx}\right)x = \pm\frac{1}{2}x^2$

$\Rightarrow 2y - x\frac{dy}{dx} = \pm x \ or \ \frac{dy}{dx} - \frac{2y}{x} = \pm 1$

Which is linear differential equation and $I.F. = e^{-2\ln x} = \frac{1}{x^2}$

$\therefore$ The solution is $\frac{y}{x^2} = \int \pm\frac{1}{x^2}dx + c = \pm\frac{1}{x} + c$

$\therefore y = \pm x + cx^2 \ or \ y = cx^2 \pm x$, where c is an arbitrary constant

81. If p and q are order and degree of differential equation $y^2\left(\frac{d^2y}{dx^2}\right)^2 + 3x\left(\frac{dy}{dx}\right)^{\frac{1}{3}} + x^2y^2 = \sin x$, then

1) p > q

2) $\frac{p}{q} = \frac{1}{2}$

3) p = q

4) p < q

Key. 4

Sol. $y^2\left(\frac{d^2y}{dx^2}\right)^2 + x^2y^2 - \sin x = -3x\left(\frac{dy}{dx}\right)^{\frac{1}{3}}$

$\left(y^2\left(\frac{d^2y}{dx^2}\right)^2 + x^2y^2 - \sin x\right)^3 = -9x^3\left(\frac{dy}{dx}\right)$

Here order = 2 = p

Degree = 6 = q

$\therefore p < q$

82. The solution of the differential equation $\frac{dy}{dx} - ky = 0, y(0) = 1$, approach zero when $x \to \infty$, if

1) K = 0

2) k > 0

3) k < 0 4) none of these

Key. 3

Sol. $\frac{dy}{dx}-Ky=0,\ \frac{dy}{y}=Kdx$

$\ln y=Kx+c$

At $x=0, y=1$ $\therefore\ C=0$

Now, $\ln y=Kx$

$y=e^{Kx}$

$\lim_{x\to\infty} y=\lim_{x\to\infty} e^{Kx}=0$

$\therefore K<0$

83. The solution of $\frac{dv}{dt}+\frac{k}{m}v=-g$ is

1) $v=ce^{-\frac{k}{m}t}-\frac{mg}{k}$ 2) $v=c-\frac{mg}{k}e^{-\frac{k}{m}t}$

3) $ve^{-\frac{k}{m}t}=c-\frac{mg}{k}$ 4) $ve^{\frac{k}{m}t}=c-\frac{mg}{k}$

Key. 1

Sol. $\frac{dv}{dt}+\frac{K}{m}v=-g$

Integrating factor $(I.F.)=e^{\int\frac{k}{m}dt}=e^{\frac{K}{m}t}$

$\therefore Ve^{\frac{K}{m}t}=-\int g\,e^{K.t/m}+c$

$Ve^{\frac{K}{m}t}=\frac{-gm}{K}e^{\frac{K}{m}t}+c$

$V=C.e^{\frac{-K}{m}t}-\frac{mg}{K}$

84. If $y_1(x)$ is a solution of the differential equation $\frac{dy}{dx}+f(x)y=0$, then a solution of differential equation $\frac{dy}{dx}+f(x)y=r(x)$ is

1) $\frac{1}{y(x)}\int y_1(x)dx$ 2) $y_1(x)\int\frac{r(x)}{y_1(x)}dx$

3) $\int r(x)y_1(x)dx$ 4) none of these

Key. 2

Sol. i) $\frac{dy_1}{dx}+f(x)y_1=0\Rightarrow f(x)=\frac{-1}{y_1}\frac{dy_1}{dx}$

ii) $\frac{dy}{dx}-\frac{1}{y_1}\frac{dy_1}{dx}.y=r(x)$

$$e^{-\int \frac{1}{y_1}\frac{dy}{dx}dx} = e^{-\int \frac{dy_1}{y_1}} = \frac{1}{y_1}$$

$$\frac{d}{dx}\left(\frac{y}{y_1}\right) = \frac{r(x)}{y_1} \Rightarrow \frac{y}{y_1} = \int \frac{r(x)dx + c}{y_1}$$

$$y = y_1 \int \frac{r(x)dx}{y_1} + cy_1$$

85. The solution of the differential equation $\left(x^2 \sin^3 y - y^2 \cos x\right)dx + \left(x^3 \cos y \sin^2 y - 2y \sin x\right)dy = 0$ is

1) $x^3 \sin^3 y = 3y^2 \sin x + C$

2) $x^3 \sin^3 y + 3y^2 \sin x = C$

3) $x^2 \sin^3 y + y^3 \sin x = C$

4) $2x^2 \sin y + y^2 \sin x = C$

Key.

Sol. $\left(x^2 \sin^2 y - y^2 \cos x\right)dx + \left(x^3 \cos y \sin^2 y - 2y \sin x\right)dy = 0$

$$\frac{dy}{dx} = \frac{y^2 \cos x - x^2 \sin^3 y}{x^3 \cos y \sin^2 - 2y \sin x}$$

$$\left(x^3 \cos y \sin^2 y - 2y \sin x\right)dy$$

$$= \left(y^2 \cos x - x^2 \sin^3 y\right)dx = 0$$

$$\left(\frac{x^3}{3} d \sin^3 y - \sin dy^2\right) - \sin^3 yd\left(\frac{x^3}{3}\right) + y^2 d \sin x = 0$$

$$\frac{x^3}{3} d \sin^2 y + \sin^3 yd\left(\frac{x^3}{3}\right) - \left(\sin dy^2 + y^2 d \sin x\right)$$

$$d\left(\frac{x^3}{3}\sin^3 y\right) - d\left(y^2 \sin x\right) = 0$$

$$\frac{x^3}{3}\sin^3 y - y^2 \sin x = c$$

86. The differential equation whose solution is $(x-h)^2 + (y-k)^2 = a^2$ is (a is a constant)

1) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = a^2 \frac{d^2 y}{dx^2}$

2) $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = a^2 \left(\frac{d^2 y}{dx^2}\right)^2$

3) $\left[1 + \left(\frac{dy}{dx}\right)\right]^3 = a^2 \left(\frac{d^2 y}{dx^2}\right)^2$

4) none of these

Key.

Sol. $(x-h)^2+(y-k)^2=a^2$ ....(1)

$2(x-h)+2(y-k)\frac{dy}{dx}=0$ ....(2)

$1+\left(\frac{dy}{dx}\right)^2+(y-k)\frac{d^2y}{dx^2}=0$ ....(3)

From (3) we have (y-k), use in (2) to get (x-h) and put(x-h) and(y-k) in(1)

87. The solution of the differential equation $\frac{\sqrt{x}dx+\sqrt{y}dy}{\sqrt{x}dx-\sqrt{y}dy}=\sqrt{\frac{y^3}{x^3}}$ is given by

a) $\frac{3}{2}\log\left(\frac{y}{x}\right)+\log\left|\frac{x^{3/2}+y^{3/2}}{x^{3/2}}\right|+\tan^{-1}\left(\frac{y}{x}\right)^{3/2}+c=0$

b) $\frac{2}{3}\log\left(\frac{y}{x}\right)+\log\left|\frac{x^{3/2}+y^{3/2}}{x^{3/2}}\right|+\tan^{-1}\frac{y}{x}+c=0$

c) $\frac{2}{3}\left(\frac{y}{x}\right)+\log\left(\frac{x+y}{x}\right)+\tan^{-1}\left(\frac{y^{3/2}}{x^{3/2}}\right)+c=0$

d) None of these

Key. D

Sol. We have $\frac{\sqrt{x}dx+\sqrt{y}dy}{\sqrt{x}dx-\sqrt{y}dy}=\sqrt{\frac{y^3}{x^3}}$

$$\Rightarrow \frac{d(x^{3/2})+d(y^{3/2})}{d(x^{3/2})-d(y^{3/2})}=\frac{y^{3/2}}{x^{3/2}}$$

$$\Rightarrow \frac{du+dv}{du-dv}=\frac{v}{u}, \text{ where } u=x^{3/2} \text{ and } v=y^{3/2}$$

$$\Rightarrow udu+udv=vdu-vdv \Rightarrow udu+vdv=vdu-udv$$

$$\Rightarrow \frac{udu+vdv}{u^2+v^2}=\frac{vdu-udv}{u^2+v^2}$$

$$\Rightarrow \frac{d(u^2+v^2)}{u^2+v^2}=-2d\tan^{-1}\left(\frac{v}{u}\right)+c$$

On integrating, we get

$$\log(u^2+v^2)=-2\tan^{-1}\left(\frac{v}{u}\right)+c$$

$$\Rightarrow \frac{1}{2}\log(x^3+y^3)+\tan^{-1}\left(\frac{y}{x}\right)^{3/2}=\frac{c}{2}$$

88. A function $y=f(x)$ has a second order derivative $f''=6(x-1)$. If its graph passes through the point(2,1) and at that point the tangent to the graph is $y=3x-5$, then the function is

a) $(x-1)^2$ b) $(x-1)^3$ c) $(x+1)^2$ d) $(x+1)^3$

Key. B

Sol. Since $f''(x)=6(x-1)$

$\Rightarrow f'(x)=3(x-1)^2+c$ (integrating) ----(i)

Also, at the point (2,1), the tangent to the graph is $y=3x-5$ and slope of the tangent = 3

$\Rightarrow f'(2)=3$

$3(2-1)^2+c=3$ [from Eq (i)

$\Rightarrow 3+c=3 \Rightarrow c=0$

From Eq (i) we have

$f'(x)=3(x-1)^2$

$\Rightarrow f(x)=(x-1)^2+k$ (Integrating) ----(ii)

$\therefore 1=(2-1)^2+k \Rightarrow k=0$

Hence the equation of the function is $f(x)=(x-1)^3$.

89. The differential equation representing the family of curves $y^2=2c(x+\sqrt{c})$, where $c>0$ is a parameter, is of order and degree as follows :

a) order 1, degree 3 b) order 2, degree 3

c) order 1, degree 2 d) order 2, degree 1

Key. A

Sol. Differentiating, we get

$2yy'=2c \Rightarrow c=yy'$

$\therefore y^2=2yy'(x+\sqrt{yy'}) \Rightarrow (y^2-2yy'x)^2=4(yy')^3$

$\Rightarrow$ degree= 3 and order =1

90. The normal to a curve at $P(x,y)$ meets the $x-axis$ at G. If the distance of G from the origin is twice the abscissa of P, then the curve is

a) a parabola b) a circle c) a hyperbola d) an ellipse

Key. C or D

Sol. The slope of the tangent = $\frac{dy}{dx}$

$\therefore$ the slope of the normal $= -\frac{dx}{dy}$

$\therefore$ the equation of the normal is $Y - y = -\frac{dx}{dy}(X - x)$

This meets the x-axis $(Y = 0)$, where

$$-y = \frac{-dx}{dy}(X - x) \Rightarrow X = x + y\frac{dy}{dx}$$

$\therefore$ G is $\left(x + y\frac{dy}{dx}, 0\right)$

$\therefore$ $OG = 2x \Rightarrow x + y\frac{dy}{dx} = 2x$

$\Rightarrow y\frac{dy}{dx} = x \Rightarrow ydy = xdx$ [variable separable integrating, we get]

$$\frac{y^2}{2} = \frac{x^2}{2} + constant$$

$\Rightarrow x^2 - y^2 = c$, which is a hyperboal

91. Which of the following transformation reduces the differential equation

$$\frac{dz}{dx} + \frac{z}{x}\log z = \frac{z}{x^2}(\log z)^2$$ to the form $P(x)u = Q(x)$

a) $u = \log z$ b) $u = e^z$ c) $u = (\log z)^{-1}$ d) $u = (\log z)^2$

Key. A or B or C or D

Sol. Given equ. Can be written as

$$\frac{dz}{z(\ln z)^2} + \frac{dx}{x(\ln z)} = \frac{dx}{x^2}$$

Put $u = \frac{1}{\ln z}$

$$\frac{du}{dx} = -\frac{1}{(\ln z)^2}.\frac{1}{z}.\frac{dz}{dx}$$

$\therefore -\frac{du}{dx}+\frac{1}{x}.u=\frac{1}{x^2}$

$\frac{du}{dx}+\left(-\frac{1}{x}\right)u=\left(-\frac{1}{x^2}\right)$

$I.F.=e^{\int -\frac{1}{x}dx}=e^{\ln\frac{1}{x}}=\frac{1}{x}$

$u.\frac{1}{x}=\int -\frac{1}{x^2}.\frac{1}{x}dx$

$\frac{u}{x}=-\int\frac{1}{x^3}dx=+\frac{1}{4x^4}+c$

92. If $y_1(x)$ is a solution of the differential equation $dy/dx-f(x)y=0$, then a solution of the differential equation $\frac{dy}{dx}+f(x)y=r(x)$

a) $\frac{1}{y_1(x)}\int r(x)y_1(x)dx$

b) $y_1(x)\int\frac{r(x)}{y_1(x)}dx$

c) $\int r(x)y_1(x)dx$

d) none of these

Key. A

Sol. $\frac{dy}{dx}-f(x).y=0$

$\frac{dy}{y}=f(x)dx$

$\ln y=\int f(x)dx$

$y_1(x)=e^{\int f(x)dx}$ Then for given equation I.F= $e^{\int f(x)dx}$

Hence Solution $y.y_1(x)=\int r(x).y_1(x)dx$

$y=\frac{1}{y_1(x)}\int r(x).y_1(x)dx$

93. Solution of the differential equation $(x^2\sin^3 y-y^2\cos x)dx+(x^3\cos y\sin^2 y-2y\sin x)dy=0$, is

(A) $\frac{(x\sin y)^2}{2}-y^2\sin x=c$

(B) $\frac{(x\sin y)^3}{3}-y^2\sin x=c$

(C) $x\sin y-y^2\sin x=c$

(D) $\frac{(x\sin y)^4}{4}-y^2\sin x=c$

Key. B

Sol. The given differential equation can be written as

$x^2 \sin^2 y(\sin y\, dx + x \cos y\, dy) - (y^2 \cos x\, dx + 2y \sin x\, dy) = 0$

$\Rightarrow (x \sin y)^2 d(x \sin y) - d(y^2 \sin x) = 0$

On integrating, we get

$$\int (x\sin y)^2 d(x\sin y) - \int 1.d(y^2 \sin x) = 0$$

$\Rightarrow \frac{(x\sin y)^3}{3} - y^2 \sin x + C = 0$ which is the required solution.

94. Solution of the differential equation : $\frac{dy}{dx} = \frac{3x^2y^4 + 2xy}{x^2 - 2x^3y^3}$ is

(A) $x^2y^2 + \frac{x^2}{y} = c$ (B) $x^3y^2 + \frac{x^2}{y} = c$

(C) $x^3y^2 + \frac{y^2}{x} = c$ (D) $x^2y^3 + \frac{x^2}{y} = c$

Key. B

Sol. The given differential equation can be written as

$(3x^2y^4 + 2xy)\, dx = (x^2 - 2x^3y^3)dy$

$\Rightarrow 3x^2 y^4dx + 2x^3y^3dy + 2xy\, dx - x^2dy = 0$

$\Rightarrow y^2(3x^2 y^2dx + 2x^3 y\, dy) + 2 xy\, dx - x^2 dy = 0$

$\Rightarrow y^2[y^2d(x)^3 + x^3d(y)^2] + yd(x)^2 - x^2dy = 0$

$\Rightarrow y^2d(x)^3 + x^3d(y^2) + \frac{yd(x)^2 - x^2dy}{y^2} = 0$

$$\Rightarrow d(x^3y^2) + d\left(\frac{x^2}{y}\right) = 0$$

On integration, we obtain $x^3y^2 + \frac{x^2}{y} = C,$

as the required solution.

95. If for a curve ratio of the distance between the normal at any of its points and the origin to the distance between the same normal and the point (a, b) is equal to the constant k (k >0, k ≠1), then the curve is a

(A) circle (B) parabola

(C) ellipse (D) hyperbola

Key. A

Sol. Let y = f(x) be the curve and let P (x, y) be any point on the curve. The equation of the normal at P(x, y) to the given curve is

$$Y - y = -\frac{1}{\frac{dy}{dx}}(X - x)$$

The distance of (i) from the origin is

$$d_1 = \left|\frac{y + \frac{x}{\frac{dy}{dx}}}{\sqrt{1 + \frac{1}{(dy/dx)^2}}}\right| = \left|\frac{x + y\frac{dy}{dx}}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}\right| \qquad d_2 = \left|\frac{(a-x) + (b-y)\frac{dy}{dx}}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}\right|$$

Now $d_1 = kd_2$

On integrating, we get

$$\frac{x^2}{2}+\frac{y^2}{2}=\pm k\left[-\frac{(a-x)^2}{2}-\frac{(b-y)^2}{2}\right]+C$$

96. Solution of $\left(\frac{x+y-1}{x+y-2}\right)\frac{dy}{dx}=\left(\frac{x+y+1}{x+y+2}\right)$, given that y = 1 when x = 1 is

(A) $\ln\left|\frac{(x-y)^2-2}{2}\right|=2(x+y)$ (B) $\ln\left|\frac{(x+y)^2-2}{2}\right|=2(x-y)$

(C) $\ln\left|\frac{(x-y)^2+2}{2}\right|=2(x+y)$ (D) $\ln\left|\frac{(x+y)^2+2}{2}\right|=2(x-y)$

Key. B

Sol. $\because \quad \left(\frac{x+y-1}{x+y-2}\right)\frac{dy}{dx}=\left(\frac{x+y+1}{x+y+2}\right)$

Put x + y = ν

$\therefore \quad \left(\frac{\nu-1}{\nu-2}\right)\left(\frac{d\nu}{dx}-1\right)=\left(\frac{\nu+1}{\nu+2}\right)$

$\Rightarrow \quad \frac{d\nu}{dx}-1=\frac{(\nu+1)(\nu-2)}{(\nu-1)(\nu+2)}=\frac{\nu^2-v-2}{v^2+v-2}$

or $\quad \frac{d\nu}{dx}=\frac{2\nu^2-4}{(\nu^2-\nu-2)}$

Given that y = 1, when x = 1

$\therefore \quad 0+\frac{1}{2}\ln 2=c$

$$(y-x)+\frac{1}{2}\left|\frac{\ln(x+y)^2-2}{2}\right|=0$$

or $\quad \ln\left|\frac{(x+y)^2-2}{2}\right|=2(x-y)$

97. Solution of the differential equation $\left\{\frac{1}{x}-\frac{y^2}{(x-y)^2}\right\}dx+\left\{\frac{x^2}{(x-y)^2}-\frac{1}{y}\right\}dy=0$ is

(A) $\ln\left|\frac{x}{y}\right|+\frac{xy}{(x-y)}=c$ (B) $\ln|xy|+\frac{xy}{(x-y)}=c$

(C) $\frac{xy}{(x-y)}=ce^{x/y}$ (D) $\frac{xy}{(x-y)}=ce^{xy}$

Key. A

Sol. The given equation can be written as

$$\left(\frac{dx}{x}-\frac{dy}{y}\right)+\frac{(x^2dy-y^2dx)}{(x-y)^2}=0$$

$$\Rightarrow \quad \left(\frac{dx}{x}-\frac{dy}{y}\right)+\frac{\left(\frac{dy}{y^2}-\frac{dx}{x^2}\right)}{\left(\frac{1}{y}-\frac{1}{x}\right)^2}=0$$

$$\Rightarrow \quad \left(\frac{dx}{x}-\frac{dy}{y}\right)+\frac{\frac{dy}{y^2}-\frac{dx}{x^2}}{\left(\frac{1}{x}-\frac{1}{y}\right)^2}=0$$

Integrating, we get

$$\ln|x|-\ln|y|-\frac{1}{\left(\frac{1}{x}-\frac{1}{y}\right)}=c$$

or $$\ln\left|\frac{x}{y}\right|-\frac{xy}{(y-x)}=c = \ln\left|\frac{x}{y}\right|+\frac{xy}{x-y}=c$$

Differential Equations

Multiple Correct Answer Type

1. A normal is drawn at a point $P(x, y)$ of a curve. It meet the x-axis at Q. If PQ is of constant length k. Then

a) The differential equation describing such curves is $y\frac{dy}{dx} = \pm\sqrt{k^2 - y^2}$

b) The curve is passing through (0, k)

c) The curve is passing through (k, 0)

d) The equation of the curve represents circle with centre as origin

Key. A

Sol. Equation of the normal at a point $P(x, y)$ is given by

$$Y - y = -\frac{1}{dy/dx}(X - x) \quad \text{---(1)}$$

Let the point Q at the x-axis be $(x_1, 0)$. From (1) we get

$$y\frac{dy}{dx} = x_1 - x \quad \text{--(2)}$$

Now given that $PQ^2 = k^2$

We have $(x - x_1)^2 + y^2 = k^2$

or $x - x_1 = \pm\sqrt{k^2 - y^2}$,

Hence using (2) we obtain $y\frac{dy}{dx} = \pm\sqrt{k^2 - y^2}$ ---(3)

(3) is the required differential equation for such curves

Now solving (3) we get $\int \frac{-ydy}{\sqrt{k^2 - y^2}} = \int -dx$

or $x^2 + y^2 = k^2$ which passes through (0,k)

2. The orthogonal trajectories of the system of curves $\left(\frac{dy}{dx}\right)^2 = \frac{a}{x}$ are

a) $9a(y + c)^2 = 4x^3$

b) $y + c = \frac{-2}{3\sqrt{a}}x^{3/2}$

c) $y + c = \frac{2}{3\sqrt{a}}x^{3/2}$

d) none of these

Key. A,B,C

Sol. Replacing $\frac{dy}{dx} by - dx/dy$, we get

$$\left(\frac{dy}{dx}\right)^2 = x/a \Rightarrow dy = \pm(x^{1/2}/a^{1/2})dx$$

Integrating we get $y + c = \pm\frac{2}{3a^{1/2}}x^{3/2}$

$$\Rightarrow 9a(y+c)^2 = 4x^3$$

3. The curve whose sub tangent is 'n' times the abscissa of the point of contact and passes through the point (2, 3), then

a) for n=1 equation of the curve is $2y = 3x$

b) for n=1 equation of the curve is $2y^2 = 9x$

c) for n=2 equation of the curve is $2y = 3x$

d) for n=2 equation of the curve is $2y^2 = 9x$

Key. A,D

Sol. If (x, y) is any point on the curve, the sub tangent at (x,y)

$$= y\frac{dx}{dy}$$

$$\therefore y\frac{dx}{dy} = nx \qquad \text{(given)}$$

or $$n\frac{dy}{y} = \frac{dx}{x}$$

Integrating $n\log y = \log x + \log c$

or $\log y^n = \log cx$

or $y^n = cx......(i)$ which is the required equation of the family of curves.

Putting $x = 2$, $y = 3$ in (i), we have $3^n = 2c$ or $c = \frac{3^n}{2}$

Putting this value of c in (i)

$$y^n = \frac{3^n}{2}x \qquad \text{or } 2y^n = 3^n x \qquad \text{(ii)}$$

Which is the particular curve passing through the point (2,3)

Putting n=1 in (ii), we have $2y = 3x$

Which is a straight line

Putting n = 2 in (ii) we have $2y^2 = 9x$

Which is a parabola.

4. If the solution of the equation $\frac{d^2x}{dt^2}+4\frac{dx}{dt}+3x=0$, given that for $t=0$, $x=0\, and\, \frac{dx}{dt}=12$ is in the form $x=Ae^{-3t}+Be^{-t}$ then__

a) A + B = 0 b) A + B =12 c) |AB| = 36 d) |AB| = 49

Key. A,C

Sol. $x=Ae^{-3t}+Be^{-t}$

$\frac{dx}{dt}=-3Ae^{-3t}-Be^{-t}$

$dt\, t=0, x=0 \Rightarrow 0=A+B$ ------1

At $t=0, \frac{dx}{dt}=12 \Rightarrow 12=-3A-B$ -----2

Solving 1 & 2 A =-6,B=6

5. If differential equation is formed to the family of all the central conics centred at origin, then

(A) order = 2 (B) order = 3

(C) degree = 1 (D) degree = order = 3

Key. B,C

Sol. equation of such conics are

$ax^2+by^2+cxy=1$

$\Rightarrow \text{order}=3=\text{parameters}$

$\text{deg ree}=1$ (no parameters is being repeated)

6. Solution of the differential equation : $\frac{x+y\frac{dy}{dx}}{y-x\frac{dy}{dx}}=\frac{x\sin^2(x^2+y^2)}{y^3}$

(A) $-\cot(x^2+y^2)=\left(\frac{x}{y}\right)^2+c$ (B) $\frac{y^2}{x^2+y^2c}=-\tan(x^2+y^2)$

(C) $-\cot(x^2+y^2)=\left(\frac{y}{x}\right)^2+c$ (D) $\frac{y^2+x^2c}{x^2}=-\tan^2(x^2+y^2)$

Key. A,B

Sol.

$$\frac{xdx+ydy}{ydx-xdy}=\frac{x\sin^2\left(x^2+y^2\right)}{y^3}\Rightarrow\frac{1}{2}\operatorname{cosec}^2\left(x^2+y^2\right)d\left(x^2+y^2\right)=\frac{x}{y}d\left(\frac{x}{y}\right)$$

$$\Rightarrow-\frac{1}{2}\cot\left(x^2+y^2\right)=\left(\frac{x}{y}\right)^2+C$$

$$\Rightarrow-\cot\left(x^2+y^2\right)=\frac{x^2}{y^2}+K$$

$$\Rightarrow\frac{y^2}{x^2+ky^2}=-\tan\left(x^2+y^2\right)$$

7. The orthogonal trajectories of the system of curves $\left(\frac{dy}{dx}\right)^2=\frac{a}{x}$ are

a) $9a(y+c)^2=4x^3$

b) $y+c=\frac{-2}{3\sqrt{a}}x^{3/2}$

c) $y+c=\frac{2}{3\sqrt{a}}x^{3/2}$

d) none of these

Key. A,B,C

Sol. Replacing $\frac{dy}{dx}by-dx/dy$, we get

$$\left(\frac{dy}{dx}\right)^2=x/a\Rightarrow dy=\pm(x^{1/2}/a^{1/2})dx$$

Integrating we get $y+c=\pm\frac{2}{3a^{1/2}}x^{3/2}$

$$\Rightarrow 9a(y+c)^2=4x^3$$

8. Two numbers such that their sum is 9 and the sum of their fourth powers is 2417. Then the numbers are

a) even positive integers

b) odd positive integers

c) one is even & another is odd

d) both are prime

Key. C,D

Sol. Let the two number be x and y

Then $x+y=9$ and $x^4+y^4=2417$

Now $(x+y)^4=9^4$

or $x^4+4x^3y+6x^2y^2+4xy^3+y^4=6561$

or $4x^3y+6x^2y^2+4xy^3=6561-2417$

$(\because x^4+y^4=2417)$

or $4xy(x^2+y^2)+6x^2y^2=4144$

or $4xy[(x+y)^2-2xy]+6x^2y^2=4144$

or $4xy[81-2xy]+6x^2y^2=4144$

or $324xy-8x^2y^2+6x^2y^2=4144$

or $2x^2y^2-324xy+4144=0$

or $(xy)^2-162xy+2072=0$

or $(xy-148)(xy-14)=0$

$\therefore\ xy=148\, or\, xy=14$

When $xy=14,$ and $x+y=9$

Then $x=7, y=2$ the other solution is inadmissible.

Hence the numbers are 7 and 2

9. The curve whose sub tangent is 'n' times the abscissa of the point of contact and passes through the point (2, 3), then

a) for n=1 equation of the curve is $2y=3x$

b) for n=1 equation of the curve is $2y^2=9x$

c) for n=2 equation of the curve is $2y=3x$

d) for n=2 equation of the curve is $2y^2=9x$

Key. A,D

Sol. If (x, y) is any point on the curve, the sub tangent at (x,y)

$$=y\frac{dx}{dy}$$

$$\therefore y\frac{dx}{dy}=nx \qquad \text{(given)}$$

or $$n\frac{dy}{y}=\frac{dx}{x}$$

Integrating $n\log y=\log x+\log c$

or $\log y^n=\log cx$

or $y^n=cx......(i)$ which is the required equation of the family of curves.

Putting $x=2$, $y=3$ in (i), we have $3^n = 2c$ or $c = \frac{3^n}{2}$

Putting this value of c in (i)

$y^n = \frac{3^n}{2}x$ or $2y^n = 3^n x$ (ii)

Which is the particular curve passing through the point (2,3)

Putting n=1 in (ii), we have $2y = 3x$

Which is a straight line

Putting n = 2 in (ii) we have $2y^2 = 9x$

Which is a parabola.

10. If the solution of the equation $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = 0$, given that for $t = 0$, $x = 0 \, and \, \frac{dx}{dt} = 12$ is in the form $x = Ae^{-3t} + Be^{-t}$ then__

a) A + B = 0 b) A + B =12 c) |AB| = 36 d) |AB| = 49

Key. A,C

Sol. $x = Ae^{-3t} + Be^{-t}$

$\frac{dx}{dt} = -3Ae^{-3t} - Be^{-t}$

$dt\, t = 0, x = 0 \Rightarrow 0 = A + B$ ------1

At $t = 0, \frac{dx}{dt} = 12 \Rightarrow 12 = -3A - B$ -----2

Solving 1 & 2 A =-6,B=6

11. If differential equation is formed to the family of all the central conics centred at origin, then

(A) order = 2 (B) order = 3 (C) degree = 1 (D) degree = order = 3

Key. B,C

Sol. equation of such conics are

$ax^2 + by^2 + cxy = 1$

$\Rightarrow \text{order} = 3 = \text{parameters}$

$\text{degree} = 1$ (no parameters is being repeated)

12. Solution of the differential equation : $\frac{x + y\frac{dy}{dx}}{y - x\frac{dy}{dx}} = \frac{x\sin^2(x^2+y^2)}{y^3}$

(A) $-\cot(x^2+y^2)=\left(\frac{x}{y}\right)^2+c$ (B) $\frac{y^2}{x^2+y^2c}=-\tan(x^2+y^2)$

(C) $-\cot(x^2+y^2)=\left(\frac{y}{x}\right)^2+c$ (D) $\frac{y^2+x^2c}{x^2}=-\tan^2(x^2+y^2)$

Key. A,B,C

Sol.

$$\frac{xdx+ydy}{ydx-xdy}=\frac{x\sin^2\left(x^2+y^2\right)}{y^3}\Rightarrow\frac{1}{2}\operatorname{cosec}^2\left(x^2+y^2\right)d\left(x^2+y^2\right)=\frac{x}{y}d\left(\frac{x}{y}\right)$$

$$\Rightarrow-\frac{1}{2}\cot\left(x^2+y^2\right)=\left(\frac{x}{y}\right)^2+C$$

$$\Rightarrow-\cot\left(x^2+y^2\right)=\frac{x^2}{y^2}+K$$

$$\Rightarrow\frac{y^2}{x^2+ky^2}=-\tan\left(x^2+y^2\right)$$

13. Let C be a curve such that the normal at any point P on it meets x-axis and y-axis at A and Y respectively. If BP:PA=1:2 (internally) and the curve passes through the point (0,4) then which of the following alternative(s) is/are correct?

(A) The curves passes through $\left(\sqrt{10},-6\right)$

(B) The equation of tangent at $\left(4,4\sqrt{3}\right)$ is $2x+\sqrt{3}y=20$

(C) The differential equation for the curve is $yy'+2x=0$

(D) The curve represent a hyperbola

Key: A,D

Hint:

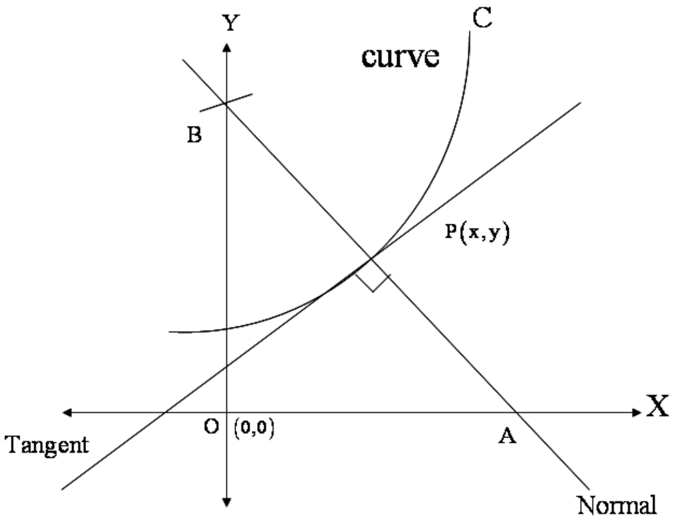
The equation of normal of $P\left(x,y\right)$ is

$$\left(Y-y\right)=\frac{-1}{\frac{dy}{dx}}\left(X-x\right)$$

$$\therefore A\left(x+y\frac{dy}{dx},0\right)$$ and

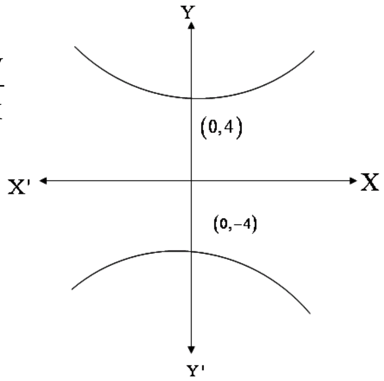
$$B\left(0,y+\frac{x}{\frac{dy}{dx}}\right)$$

Now



$$\frac{1\left(x+y\frac{dy}{dx}\right)+2(0)}{1+2}=x\Rightarrow x+y\frac{dy}{dx}$$

$y\frac{dy}{dx}=2x$ …..(1)



$\Rightarrow\int y dy=\int 2x\,dx\Rightarrow\frac{y^2}{2}=x^2+C$

Also (0,4) satisfy it, so C = 8

$\therefore y^2=2x^2+16$ (equation of curve)

Which represent a hyperbola

Also $\left.\frac{dy}{dx}\right]_{(4,4\sqrt{3})}=\frac{2(4)}{4\sqrt{3}}=\frac{2}{\sqrt{3}}$

14. The general solution of the differential equation $\frac{dy}{dx}+\frac{y\ln y}{x}=\frac{y}{x^2}(\ln y)^2$ is, (c being the constant of integration)

(A) $2x=(1-cx^2)\ln y$ (B) $x=(1+2cx^2)\ln y$

(C) $2x=(1+2cx^2)\ln y$ (D) $x=(1+cx^2)\ln y$

Key : A or C (final key)

Sol : Let $P=\int_0^{\pi/2}\frac{\sin x\cos x}{x+1}dx$

$=\frac{1}{2}\int_0^{\pi/2}\frac{\sin 2x}{x+1}dx$

$=\int_0^{\pi/2}\frac{\sin\ 2x}{2x+2}dx$

Let 2x = t, so that $dx=\frac{1}{2}dt$

$P=\frac{1}{2}\int_0^{\pi}\frac{\sin t}{t+2}dt$

$=\frac{1}{2}\left\{\left[-\frac{\cos t}{t+2}\right]_0^{\pi}-\int_0^{\pi}\frac{\cos t}{(t+2)^2}dt\right\}$

$=\frac{1}{2}\left\{\frac{1}{2}-\frac{-1}{\pi+2}\right\}-\frac{1}{2}I$

$=\frac{1}{4}+\frac{1}{2(\pi+2)}-\frac{1}{2}I$

15. If solution of $x^2\frac{d^2y}{dx^2}-9x\frac{dy}{dx}+21y=0$ is of the from $y=c_1x^m+c_2x^n$ (c_1,c_2 are arbitrary constants, $m,n\in N$) then values of m,n can be

a) m=3,n=0 b) m=3,n=7 c) m=7,n=3 d) m=0,n=7

Key. A,B,C,D

Sol. $y=c_1x^m+c_2x^n \ or \ y_1=mc_1x^{m-1}+nc_2x^{n-1}, y_2=m(m-1)c_1x^{m-2}+n(n-1)c_2x^{n-2}$

substituting gives $m^2-10m+21=0, n^2-10n+21=0$

$\Rightarrow m=3 \ or \ 7 \ and \ n=3 \ or \ 7$

16. A solution of $y=x\frac{dy}{dx}+\frac{4}{\frac{dy}{dx}}$ is

a) $y=x+4$ b) $y=4\sqrt{x}$ c) $y=4x+1$ d) $y=3x+2$

Key. A,B,C

Sol. Given $\frac{\int_{f(y)}^{f(x)} e^t dt}{\int_y^x \frac{1}{t}dt}$ = 1

$\Rightarrow e^{f(x)}-e^{f(y)}=\log x-\log y$

$\Rightarrow e^{f(x)}-\log x=c \Rightarrow f(x)=\log(\log(x+c))$, since $f\left(\frac{1}{e}\right)=0\Rightarrow c=2$

$$f(g(x))=\begin{cases}\log(x+2), & x\ge k\\ \log(2+x^2), & 0<x<k\end{cases}$$

For continuity at x = k

Log $(k+c)=\log(k^2+c) \Rightarrow either \ k=0 \ or$ k = 1.

17. If the solution of $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=x, y(0)=y(1)=1$ is given by $y^2=f(x)$ then

a) f(x) is monotonically increasing $\forall x\in(1,\infty)$ b) f(x) =0 has only one root

c) f(x) is neither even nor odd d) f(x) has 3 real roots

Key. A,B,C

Sol. Given

$$\frac{d}{dx}\left(y\frac{dy}{dx}\right)=x\Rightarrow y\frac{dy}{dx}=\frac{x^2}{2}+c$$

$$\Rightarrow \frac{d}{dx}\left(\frac{y^2}{2}\right)=\frac{x^2}{2}+c$$

$$\Rightarrow y^2=\frac{x^3}{3}+\alpha x+\beta \ given \ y(0)=1, y(1)=1$$

$$\Rightarrow \beta=1, \alpha=\frac{-1}{3}$$

$$\therefore y^2 = f(x) = \frac{x^3 - x + 3}{3},\ f'(x) = 3x^2 - 1 > 0 \ \ for \ \ x > 1$$

$$\therefore f(x) \uparrow$$

$f\left(\frac{1}{\sqrt{3}}\right) f\left(-\frac{1}{\sqrt{3}}\right) > 0\, f(x) = 0$ has only one real root .

18. The coordinates of a point P(*x,y*) are functions of time t and satisfy the relations $\frac{dx}{dt} + \frac{dy}{dt} = t \ \ and \ \ \frac{dx}{dt} - 2\frac{dy}{dt} = t^2$ at any instant of time t .The locus of point P(x.y) is a curve given by (assume x(0)=y(0)=0)

a) $(x+y)^3 = (x-2y)^2$
b) $x = \frac{t^2}{3} + \frac{t^3}{9}, y = \frac{t^2}{6} - \frac{t^3}{9}$
c) $9(x+y)^3 = 8(x-2y)^2$
d) $8(x+y)^3 = 9(x-2y)^2$

Key. B,D

Sol. From the given equations we get $x = \frac{t^2}{3} + \frac{t^3}{9} \quad and \quad y = \frac{t^2}{6} - \frac{t^3}{9}$ .Eliminating t gives

$$8(x+y)^3 = 9(x-2y)^2$$

19. If solution of $x^2 \frac{d^2y}{dx^2} - 9x\frac{dy}{dx} + 21y = 0$ is of the from $y = c_1x^m + c_2x^n$ (c_1, c_2 are arbitrary constants, $m, n \in N$) then values of m,n can be

a) m=3,n=0 b) m=3,n=7 c) m=7,n=3 d) m=0,n=7

Key. A,B,C,D

Sol. $y = c_1x^m + c_2x^n \ \ or \ \ y_1 = mc_1x^{m-1} + nc_2x^{n-1}, y_2 = m(m-1)c_1x^{m-2} + n(n-1)c_2x^{n-2}$
substituting gives ssw

$$\Rightarrow m = 3 \ or \ 7 \ and \ n = 3 \ or \ 7$$

20. A curve passes through (2,0) and the slope of tangent at P(x,y) equals

$$\frac{(x+1)^2 + y - 3}{x+1} \quad then$$

a) curve is $y = x^2 - 2x$ b) curve is $y = x^3 - 8$

c) area bounded by the curve and X-axis in fourth quadrant is $\frac{2}{3}$ square units

d) area bounded by the curve and X-axis in fourth quadrant is $\frac{4}{3}$ square units

Key. A,D

Sol. $\frac{dy}{dx} = \frac{(x+1)^2 + y - 3}{x+1} = x + 1 + \frac{y-3}{x+3}$

Putting x+1=X and y-3=Y

$$\frac{dY}{dX} = X + \frac{4}{X}$$

$$\Rightarrow \frac{dY}{dX} - \frac{Y}{X} = X$$

Solution is $\frac{Y}{X} = X + c$

$\Rightarrow y-3=(x+1)^2+(2+1)$

It passes Through $(2,0) \Rightarrow c=-4$

Equation of curve is $y=x^2-2x$

Area bounded= $\int_O^2 (2x-x^2)dx=\frac{4}{3}s.u$

21. The solution of $p^2+(2y\cot x)p=y^2$ *where* $p=\frac{dy}{dx}$ *is*

a) $y(1+\cos x)=c$ b) $y(1-\cos x)=c$

c) $x=2\sin^{-1}\sqrt{c/2y}$ d) $x=2\sin^{-1}(\sqrt{2y})+C$

Key. A,B,C

Sol. $p^2+2py\cot x-y^2=0$

$p=\frac{-2y\cot x\pm\sqrt{4y^2\cot^2 x+4y^2}}{2}$

$\Rightarrow \frac{dy}{dx}=(-\cot x\pm \operatorname{cosec} x)y$

$\Rightarrow \frac{dy}{dx}+y\frac{(\cos x\pm 1)}{\sin x}=0$

On integration

$\log y+2\log\sin\frac{x}{2}=\log c \Rightarrow y(1-\cos x)=2c \,\&\, y(1+\cos x)=c$

$\Rightarrow y\left(2\sin^2 x/2\right)=C \Rightarrow \sin^2 x/2=\frac{C}{2y} \Rightarrow x=2\sin^{-1}\sqrt{\frac{c}{2y}}$

22. The function $f(\theta)=\frac{d}{d\theta}\int_0^\theta \frac{dx}{1-\cos\theta\cos x}$ *satisfies* the differential equation

a) $\frac{df}{d\theta}+2f(\theta)\cot\theta=0$ b) $\frac{df}{d\theta}-2f(\theta)\cot\theta=0$

c) $f(\theta)=1+\cot^2\theta$ d) $f(\theta)=1+\csc^2\theta$

Key. A

Sol. $f(\theta)=\frac{d}{d\theta}\int_0^\theta \frac{dx}{1-\cos x\cos\theta} =\frac{1}{1-\cos\theta}=\csc^2\theta$

$\Rightarrow \frac{df}{d\theta}=-2\csc^2\theta\cot\theta$

$\frac{df}{d\theta}+2f(\theta)\cot\theta=0 \,\&\, f(\theta)=\operatorname{cosec}^2\theta$

23. The orthogonal trajectories of the system of curves $\left(\frac{dy}{dx}\right)^2=\frac{a}{x}$ *are*

a) $9a(y+c)^2=4x^3$ b) $y+c=-\frac{2}{3\sqrt{a}}x^{3/2}$

c) $y+c=\frac{2}{3\sqrt{a}}x^{3/2}$ d) $y+c=\frac{2}{3a^{3/2}}x^{1/2}$

Key. A,B,C

Sol. Replacing $\frac{dy}{dx}$ by $\frac{-dx}{dy}$, we have

$\left(\frac{dy}{dx}\right)^2=\frac{x}{a}\Rightarrow dy=\pm x^{1/2}a^{1/2}\,dx$

On integrating we get $y+c=\pm\frac{2}{3a^{1/2}}x^{3/2}$

$\Rightarrow 9a\left(y+c\right)^2=4x^3$

24. If differential equation is formed to the family of all the central conics centred at origin, then

(A) order = 2 (B) order = 3

(C) degree = 1 (D) degree = order = 3

Key. B,C

Sol. equation of such conics are

$ax^2+by^2+cxy=1$

$\Rightarrow \text{order}=3=\text{parameters}$

$\text{degree}=1$ (no parameters is being repeated)

25. Solution of the differential equation : $\frac{x+y\frac{dy}{dx}}{y-x\frac{dy}{dx}}=\frac{x\sin^2(x^2+y^2)}{y^3}$

(A) $-\cot(x^2+y^2)=\left(\frac{x}{y}\right)^2+c$ (B) $\frac{y^2}{x^2+y^2c}=-\tan(x^2+y^2)$

(C) $-\cot(x^2+y^2)=\left(\frac{y}{x}\right)^2+c$ (D) $\frac{y^2+x^2c}{x^2}=-\tan^2(x^2+y^2)$

Key. A,B

Sol.

$$\frac{xdx+ydy}{ydx-xdy}=\frac{x\sin^2\left(x^2+y^2\right)}{y^3}\Rightarrow\frac{1}{2}\cos ec^2\left(x^2+y^2\right)d\left(x^2+y^2\right)=\frac{x}{y}d\left(\frac{x}{y}\right)$$

$$\Rightarrow-\frac{1}{2}\cot\left(x^2+y^2\right)=\left(\frac{x}{y}\right)^2+C$$

$$\Rightarrow-\cot\left(x^2+y^2\right)=\frac{x^2}{y^2}+K$$

$$\Rightarrow\frac{y^2}{x^2+ky^2}=-\tan\left(x^2+y^2\right)$$

26. The orthogonal trajectories of the system of curves $\left(\frac{dy}{dx}\right)^2=\frac{a}{x}$ are

a) $9a(y+c)^2=4x^3$

b) $y+c=\frac{-2}{3\sqrt{a}}x^{3/2}$

c) $y+c=\frac{2}{3\sqrt{a}}x^{3/2}$

d) none of these

Key. A,B,C

Sol. Replacing $\frac{dy}{dx}by-dx/dy$, we get

$$\left(\frac{dy}{dx}\right)^2=x/a\Rightarrow dy=\pm(x^{1/2}/a^{1/2})dx$$

Integrating we get $y+c=\pm\frac{2}{3a^{1/2}}x^{3/2}$

$$\Rightarrow 9a(y+c)^2=4x^3$$

27. The curve whose sub tangent is 'n' times the abscissa of the point of contact and passes through the point (2, 3), then

a) for n=1 equation of the curve is $2y=3x$

b) for n=1 equation of the curve is $2y^2=9x$

c) for n=2 equation of the curve is $2y=3x$

d) for n=2 equation of the curve is $2y^2=9x$

Key. A,D

Sol. If (x,y) is any point on the curve, the sub tangent at (x,y)

$$=y\frac{dx}{dy}$$

$\therefore y\frac{dx}{dy}=nx$ (given)

or $n\frac{dy}{y}=\frac{dx}{x}$

Integrating $n\log y=\log x+\log c$

or $\log y^n=\log cx$

or $y^n=cx......(i)$ which is the required equation of the family of curves.

Putting $x=2$, $y=3$ in (i), we have $3^n=2c$ or $c=\frac{3^n}{2}$

Putting this value of c in (i)

$y^n=\frac{3^n}{2}x$ or $2y^n=3^nx$ (ii)

Which is the particular curve passing through the point (2,3)

Putting n=1 in (ii), we have $2y=3x$

Which is a straight line

Putting n = 2 in (ii) we have $2y^2=9x$

Which is a parabola.

28. If the solution of the equation $\frac{d^2x}{dt^2}+4\frac{dx}{dt}+3x=0$, given that for $t=0$, $x=0\, and\, \frac{dx}{dt}=12$ is in the form $x=Ae^{-3t}+Be^{-t}$ then__

a) A + B = 0 b) A + B =12 c) |AB| = 36 d) |AB| = 49

Key. A,C

Sol. $x=Ae^{-3t}+Be^{-t}$

$\frac{dx}{dt}=-3Ae^{-3t}-Be^{-t}$

$dt\, t=0, x=0\Rightarrow 0=A+B$ ------1

At $t=0,\frac{dx}{dt}=12\Rightarrow 12=-3A-B$ -----2

Solving 1 & 2 A =-6,B=6

29. Solution of the differential equation : $\frac{x+y\frac{dy}{dx}}{y-x\frac{dy}{dx}}=\frac{x\sin^2(x^2+y^2)}{y^3}$

(A) $-\cot(x^2+y^2)=\left(\frac{x}{y}\right)^2+c$ (B) $\frac{y^2}{x^2+y^2c}=-\tan^2(x^2+y^2)$

(C) $-\cot(x^2+y^2)=\left(\frac{y}{x}\right)^2+c$ (D) $\frac{y^2+x^2c}{x^2}=-\tan^2(x^2+y^2)$

Key. A,B

Sol. The given differential equation can be written as

$$\frac{xdx+ydy}{ydx-xdy}=\frac{x\sin^2(x^2+y^2)}{y^3}$$

$$\Rightarrow\frac{2xdx+2ydy}{ydx-xdy}=\frac{2x\sin^2(x^2+y^2)}{y^3}$$

$$\Rightarrow\frac{d(x^2+y^2)}{\sin^2(x^2+y^2)}=\frac{2x}{y^3}(ydx-xdy)$$

$$\Rightarrow \operatorname{cosec}^2(x^2+y^2)d(x^2+y^2)=2\left(\frac{x}{y}\right)d\left(\frac{x}{y}\right)$$

On integrating, we get

$$\int \operatorname{cosec}^2(x^2+y^2)d(x^2+y^2)=2\int\left(\frac{x}{y}\right)d\left(\frac{x}{y}\right)$$

$$\Rightarrow\ -\cot(x^2+y^2)=\left(\frac{x}{y}\right)^2+C,$$ which is the required solution.

30. Solution of the differential equation :
(3 tan x + 4 cot y – 7) $\sin^2$y dx – (4 tan x + 7 cot y – 5) $\cos^2$xdy = 0 is

(A) $\frac{3}{2}\cot^2 x-7\cot x+\frac{7}{2}\tan^2 y-5\tan y+4\cot x.\tan y=c$

(B) $\frac{3}{2}\tan^2 x-7\tan x+\frac{7}{2}\cot^2 y-5\cot y+4\tan x.\cot y=c$

(C) 3 $\tan^2$y – 14cotx.$\tan^2$y + 7 $\cot^2$x - 10 tany $\cot^2$x + 8 cot x. tan y + 2c $\cot^2$x $\tan^2$y = 0.

(D) 3 $\cot^2$y – 14cotx. $\cot^2$y + 7 $\cot^2$x + 10 coty $\tan^2$x + 8 tan x. cot y = 0.

Key. B,C

Sol. Dividing throughout by $\cos^2$x $\sin^2$y, the given differential equation becomes
(3 tan x + 4 cot y – 7)$\sec^2$x dx – (4 tan x + 7 cot y – 5) cosec^2y dy = 0
$\Rightarrow$ 3 tan x $\sec^2$x dx – 7 $\sec^2$x dx – 7 cot y cosec^2y dy + 5 cosec^2ydy + 4 cot y $\sec^2$x dx – 4 tan x cosec^2y dy = 0
$\Rightarrow$ 3 tan x d(tan x) – 7 d(tan x) + 7 cot y d(cot y) – 5 d(cot y) + 4 d(tan x cot y) = 0
On integrating, we obtain

$$\frac{3}{2}\tan^2 x-7\tan x+\frac{7}{2}\cot^2 y-5\cot y+4\tan x\cot y=C.$$

Differential Equations

Assertion Reasoning Type

1. Statement-I : Order of the differential equation formed from $y = c_1 x + c_2 e^{x} + c_4 e^{-x} + e^{-c_3 x}$ where c_1, c_2, c_4 are arbitrary constants is 3

Statement-II : Order of the differential equation is equal to the number of arbitrary constants involved in the given algebraic equation.

Key. C

Sol. Order of the differential equation is equal to the number of *independent* arbitrary constants involved in the given algebraic equation. So statement II is false.

2. Statement-1: Curve satisfying the differential equation $y^1 = y/2x$ passing through $(2,1)$ is a parabola with focus $(1/4, 0)$

Statement-2: The differential equation $y^1 = y/2x$ is of variable separable.

1) Statement-1 is true, statement-2 is true; Statement-2 is a correct explanation for statement-1
2) Statement-1 is true, statement-2 is true; Statement-2 is not correct explanation for statement-1
3) Statement-1 is true, statement-2 is false
4) Statement-1 is false, statement-2 is true

Key. 4

Sol. $\frac{dy}{dx} = \frac{y}{2x} \Rightarrow \frac{2dy}{y} = \frac{dx}{x}$

$\Rightarrow \log y^2 = \log x + \text{cosnt} \Rightarrow y^2 = Cx$, this passes through $(2,1)$ if $C = 1/2$. Thus $y^2 = 1/2x$ which represents a parabola with focus $(1/8, 0)$.

3. Statement-I : Order of the differential equation formed from $y = c_1 x + c_2 e^{x} + c_4 e^{-x} + e^{-c_3 x}$ where c_1, c_2, c_4 are arbitrary constants is 3

Statement-II : Order of the differential equation is equal to the number of arbitrary constants involved in the given algebraic equation.

Key. C

Sol. Order of the differential equation is equal to the number of *independent* arbitrary constants involved in the given algebraic equation. So statement II is false.

A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct Explanation for Statement-1

B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for

Statement-1

C) Statement-1 is True, Statement-2 is False

D) Statement-1 is False, Statement-2 is True

4. Statement 1:If $'P'$ is a differentiable function of $'x'$ and $\frac{dP}{dx}-3P\le 6\ \forall\ x\ge 0$ and $P(0)=4$ then $P\le 6e^{3x}-2\ \ \forall\ \ x\ge 0$

Statement 2: $\frac{dP}{dx}-3P-6=e^{3x}\frac{d}{dx}\left[(P+2)e^{-3x}\right]$

Key. A

Sol. Conceptual

5. STATEMENT-1 : The degree of the differential equation $\frac{d^2y}{dx^2}+2x\frac{dy}{dx}=\log_e\left(\frac{dy}{dx}\right)$ is 2.

STATEMENT-2 : The degree of a differential equation which can be written as polynomial in the derivatives is the degree of the derivative of the highest order occuring in it.

Key: D

Hint The given equation represents ellipse $\frac{x^2}{32}+\frac{(y-12)^2}{64}=1$. The maximum value of $\sqrt{x^2+y^2}$ is the distance between (0, 0), (0, 20).

6. Statement-I: If a,b,c$\in$ R and 2a+3b+6c=0,then the equation $ax^2+bx+c=0$ has at least one root in (0,1)

Statement-II: If a continuous function f defined on R assumes both positive and negative values then it vanishes at least once.

Key: B

Hint $\frac{a}{3}+\frac{b}{2}+c=0$

$f(x)=\frac{ax^3}{3}+\frac{bx^2}{2}+cx$

f(0)=f(1)=0

Apply Rolle's theorem.

7. STATEMENT- 1: Order of the differential equation of a family of circle of constant radius is 2.

STATEMENT 2: We required two parameters to fix the centre of the circle

Key: A

Hint: Conceptual

8. Statement-1: Curve satisfying the differential equation $y'=\frac{y}{2x}$ passing through (2, 1) is a parabola with focus $\left(\frac{1}{4},0\right)$.

Statement-2: The differential equation $y'=\frac{y}{2x}$ is of variable separable.

Key: D

Hint: $\frac{dy}{dx}=\frac{y}{2x} \Rightarrow 2\frac{dy}{y}=\frac{dx}{x}$

$\Rightarrow \log y^2 = \log x$ + constant $\Rightarrow$ y^2 = Cx, this passes through (2, 1) if C = ½.

Thus $y^2=\frac{1}{2}x$ which represents a parabola with focus $\left(\frac{1}{8},0\right)$.

9. STATEMENT 1: The differential equation of all circles in a plane must be of order 3.

STATEMENT 2: There is only one circle passing through three non collinear points

KEY : A

Sol: CONCEPTUAL

10. STATEMENT–1:

A differential equation $\frac{dy}{dx}+\frac{y}{x}=x^2$ can be solved by finding.

I.F. = $e^{\int Pdx}$ = $e^{\int 1/x dx} = e^{\log x} = x$ then solution y.x = $\int x^3 dx$ + c

because

STATEMENT–2:

Since the given differential equation in of the form dy/dx + py = ϕ wherep, ϕ are function of x

Key. A

Sol. dy/dx + y/x = x^2 ... (1)

This is term of linear differential equation dy/dx + py = ϕ ... (2)

from (1) and (2) p = –1/x , $\phi = x^2$

I.f. $e^{\int Pdx} = e^{\int 1/x dx = x} e$

y.I.f = ∫x×I.fd + c

yx = $\int x^3$ dx + c.

11. Statement - 1: The degree of the differential equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}=\ln\left(\frac{d^2y}{dx^2}\right)$ is 2.

Statement - 2: The degree of a differential equation which can be written as polynomial in the derivatives is the degree of the derivative of the highest order in it.

Key. D

Sol. Conceptual

12. STATEMENT–1 : The differential equation $y^3dy+(x+y^2)\,dx=0$ becomes homogeneous if we put $y^2=t$

STATEMENT–2 : All differential equations of first order and first degree becomes homogeneous if we put $y=tx$.

Key. C

Sol. Statement 2 is false since $\frac{dx}{dx}=\frac{x+y^2}{y+x^2}$ cannot be made Homogeneous

by putting $y=tx$ But if we put y^2 = t in the differential equation in statement 1 then $2y\dfrac{dy}{dx}=\dfrac{dt}{dx}$ And differential equation becomes Homogeneous

13 STATEMENT–1 : The order of the differential equation formed by the family of curve $y=C_1e^x+(C_2+C_3)e^{x+c_4}$ *is* '1' here $C_{1,}C_2,C_3,C_4$ are arbitrary constants

STATEMENT–2 : The order of the differential equation formed by any family of curves is equal to the number of constants present in it

Key. C

Sol. Since order of differential equation = No. of in dependent arbitrary constants

Given equation can be reduced to $y=c_6\cos(x+c_3)+c_7e^x$

where $c_6=c_1+c_2$ and $c_7=c_4e^{c_5}$ So order of equation is 3

Hence the correct choice is (C)

14. STATEMENT–1: The differential equation $x(x^2+y^2+1)\,dx+y(x^2-y^2+1)dy=0$ becomes homogeneous only by putting x^2 = u, y^2 = v

STATEMENT–2: The differential equation $\dfrac{dv}{du}=\dfrac{u+v+1}{u-v+1}$ is reducible to homogeneous differential equation

Key. D

Sol.By putting $x^2=u, y^2=v$, we get $(u+v+1)du+(u-v+1)dv=0$ $\dfrac{du}{dv}=-\dfrac{u+v+1}{u-v+1}$

Which is not homogeneous but is reducible to homogeneous form by putting u =U+α , v = V+β and choose α and β such that $\alpha+\beta+1=0$, $\alpha-\beta+1=0$.

Thus statement 1 is false, statement 2 is true

15. STATEMENT–1:- Let a solution $y=y(x)$ of the differential equation $y\sin x+y'\cos x=1$ Satisfying $y(0)=1$ then $y(x)=\sin\left(x+\dfrac{\pi}{4}\right)$

STATEMENT–2 :- The integrating factor of given differential equation is Sec x

Key. D

Sol. It is a linear equation with I.F $e^{\int \tan x dx}=\sec x$

Required solution is y= sinx+cosx $\Rightarrow$ $y=\sqrt{2}\sin\left(x+\dfrac{\pi}{4}\right)$

16. STATEMENT-1: The degree of the differential equation $\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}=\ln\left(\dfrac{d^2y}{dx^2}\right)$ is 2.

STATEMENT-2: The degree of a differential equation which can be written as polynomial in the derivatives is the degree of the derivative of the highest order occurring in it.

Key. D

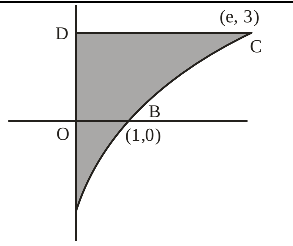
Sol.

Given, $f\left(\dfrac{x}{y}\right)=f(x)-f(y)$

Putting x = y, then

$\Rightarrow$ f(1) = 0

$\therefore$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$



$$= \lim_{h \to 0} \frac{f\left(1 + \frac{h}{x}\right)}{h}$$

$$= \frac{3}{x} \left\{ \because \lim_{x \to 0} \frac{f(1+x)}{x} = 3 \right\}$$

$\therefore$ f(x) = 3 ln x + c

Putting x = 1, then

f(1) = 0 + c = 0

$\Rightarrow$ f(x) = 3 lnx = y (say)

$\therefore$ x = $e^{y/3}$

$\therefore$ Required area $= \int_{-\infty}^{3} x\,dy$

$$= \int_{-\infty}^{3} e^{y/3} dy$$

$= 3(e^{y/3})_{-\infty}^{3} = 3(e - 0) = 3e$ sq. unit.

$\because$ $f''(x) = -\frac{3}{x^2} < 0$

$\Rightarrow$ f(x) is concave down.

Differential Equations

Integer Answer Type

1. If the solution of the differential equation $e^{3x}(p-1)+p^3e^{2y}=0$ is in the form $e^y=ce^x+c^k$ then k = __(Where $p=\dfrac{dy}{dx}$ and 'c' is arbitrary constant)

Key. 3

Sol. (i) put $e^x=X$ and $e^y=Y$

So that $e^x dx=dX$ and $e^y dy=dY$

$\Rightarrow \dfrac{e^y}{e^x}\cdot\dfrac{dy}{dx}=\dfrac{dY}{dX}$ or $\dfrac{Y}{X}p=P$ (say)

or $p=\dfrac{X}{Y}\cdot P$

$\therefore$ The given equation becomes

$$X^3\left(\frac{X}{Y}P-1\right)+\frac{X^3}{Y^3}.Y^2P^3=0$$

or $XP-Y+P^3=0$

or $Y=PX+P^3$

Which is of Clairaut's form.

$\therefore$ the solution is $Y=cX+c^3$

or $e^y=ce^x+c^3$

2. If the solution of the differential equation $y^2(y-xp)=x^4p^2$ is in the form $\dfrac{1}{y}=\dfrac{c}{x}+c^k$ then k = __(Where $p=\dfrac{dy}{dx}$ and 'c' is arbitrary constant)

Key. 2

Sol. (i) Put $x=\dfrac{1}{X}$ and $y=\dfrac{1}{Y}$

$dx=-\dfrac{1}{X^2}dX$ and $dy=-\dfrac{1}{Y^2}dY$

$$\Rightarrow p=\frac{dy}{dx}=\frac{X^2}{Y^2}\frac{dY}{dX}=\frac{X^2}{Y^2}P$$

the given equation becomes

$$\frac{1}{Y^2}\left(\frac{1}{Y}-\frac{1}{X}\cdot\frac{X^2}{Y^2}P\right)=\frac{1}{X^4}\cdot\frac{X^4}{Y^4}P^2$$

$\Rightarrow Y-XP=P^2$ or $Y=PX+P^2$

Which is the Claraut's form

$\therefore$ The solution is $Y = cX + c^2$

or $\frac{1}{y} = \frac{c}{x} + c^2$

3. If the solution of the differential equation $y = 2px + y^2 p^3$ is in the form $y^2 = 2cx + c^k$ then k = __(Where $p = \frac{dy}{dx}$ and 'c' is arbitrary constant)

Key. 3

Sol. The given equation is $y = 2px + y^2 p^3$ ---(i)

Solving for x, $x = \frac{y}{2p} - \frac{1}{2} y^2 p^3$

Differentiating w.r.t y

$$\frac{dx}{dy} = \frac{1}{p} = \frac{1}{2p} - \frac{y}{2p^2} \cdot \frac{dp}{dy} - yp^2 - y^2 p \cdot \frac{dp}{dy}$$

or $2p = p - y\frac{dp}{dy} - 2yp^3 - 2y^2 p^2 \frac{dp}{dy}$

or $p(!+2yp^2) + y\frac{dp}{dy}(1+2yp^2) = 0$

or $(1+2yp^2)\left(p + y\frac{dp}{dy}\right) = 0$

Neglecting the first factor which does not involve $\frac{dp}{dy}$, we have

$$p + y\frac{dp}{dy} = 0$$

$$\Rightarrow \frac{dp}{p} + \frac{dy}{y} = 0$$

Integrating $\log p + \log y = \log c$

or $\log py = \log c$

$\Rightarrow py = c$ ---(ii)

Eliminating p between (i) and (ii)

$$y = 2x \cdot \frac{c}{y} + y^2 \cdot \frac{c^3}{y^3}$$

or $y = \frac{2cx}{y} + \frac{c^3}{y}$

or $y^2 = 2cx + c^3$ which is the required solution

4. If the solution of the differential equation $e^{3x}(p-1)+p^3e^{2y}=0$ is in the form $e^y=ce^x+c^k$ then k = __(Where $p=\frac{dy}{dx}$ and 'c' is arbitrary constant)

Key. 3

Sol. (i) put $e^x=X$ and $e^y=Y$

So that $e^x dx=dX$ and $e^y dy=dY$

$$\Rightarrow \frac{e^y}{e^x}\cdot\frac{dy}{dx}=\frac{dY}{dX} \qquad \text{or } \frac{Y}{X}p=P\,(\text{say})$$

or $p=\frac{X}{Y}\cdot P$

$\therefore$ The given equation becomes

$$X^3\left(\frac{X}{Y}P-1\right)+\frac{X^3}{Y^3}.Y^2P^3=0$$

or $XP-Y+P^3=0$

or $Y=PX+P^3$

Which is of Clairaut's form.

$\therefore$ the solution is $Y=cX+c^3$

or $e^y=ce^x+c^3$

5. If the solution of the differential equation $y^2(y-xp)=x^4p^2$ is in the form $\frac{1}{y}=\frac{c}{x}+c^k$ then k = __(Where $p=\frac{dy}{dx}$ and 'c' is arbitrary constant)

Key. 2

Sol. (i) Put $x=\frac{1}{X}$ and $y=\frac{1}{Y}$

$$dx=-\frac{1}{X^2}dX \text{ and } dy=-\frac{1}{Y^2}dY$$

$$\Rightarrow p=\frac{dy}{dx}=\frac{X^2}{Y^2}\frac{dY}{dX}=\frac{X^2}{Y^2}P$$

the given equation becomes

$$\frac{1}{Y^2}\left(\frac{1}{Y}-\frac{1}{X}\cdot\frac{X^2}{Y^2}P\right)=\frac{1}{X^4}\cdot\frac{X^4}{Y^4}P^2$$

$\Rightarrow Y-XP=P^2$ or $Y=PX+P^2$

Which is the Claraut's form

$\therefore$ The solution is $Y = cX + c^2$

or $\frac{1}{y} = \frac{c}{x} + c^2$

6. If the solution of the differential equation $y = 2px + y^2 p^3$ is in the form $y^2 = 2cx + c^k$ then k = __(Where $p = \frac{dy}{dx}$ and 'c' is arbitrary constant)

Key. 3

Sol. The given equation is $y = 2px + y^2 p^3$ ---(i)

Solving for x, $x = \frac{y}{2p} - \frac{1}{2} y^2 p^3$

Differentiating w.r.t y

$$\frac{dx}{dy} = \frac{1}{p} = \frac{1}{2p} - \frac{y}{2p^2} \cdot \frac{dp}{dy} - yp^2 - y^2 p \cdot \frac{dp}{dy}$$

or $2p = p - y\frac{dp}{dy} - 2yp^3 - 2y^2p^2\frac{dp}{dy}$

or $p(!+2yp^2) + y\frac{dp}{dy}(1+2yp^2) = 0$

or $(1+2yp^2)\left(p + y\frac{dp}{dy}\right) = 0$

Neglecting the first factor which does not involve $\frac{dp}{dy}$, we have

$$p + y\frac{dp}{dy} = 0$$

$$\Rightarrow \frac{dp}{p} + \frac{dy}{y} = 0$$

Integrating $\log p + \log y = \log c$

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$\Rightarrow py = c$ ---(ii)

Eliminating p between (i) and (ii)

$$y = 2x \cdot \frac{c}{y} + y^2 \cdot \frac{c^3}{y^3}$$

or $y = \frac{2cx}{y} + \frac{c^3}{y}$

or $y^2 = 2cx + c^3$ which is the required solution

7. The solution of $x^3\frac{dy}{dx}+4x^2\tan y=e^x\sec y$ satisfying y(1) = 0 is sin y $=e^x(x-1)x^{-k}$ then k =

Key. 4

Sol. Put $\sin y=t\Rightarrow\cos y\frac{dy}{dx}=\frac{dt}{dx}$

$$\therefore\ \frac{dt}{dx}+\frac{4}{x}t=\frac{e^x}{x^3}$$

$$I.F.=e^{\int 4/xdx}=e^{4\ln x}=x^4$$

$$\frac{d(t.x^4)}{dx}=xe^x$$

8. Differential equation, having $y=\left(\sin^{-1}x\right)^2+A\left(\cos^{-1}x\right)+B$ where A and B are arbitary constants is $\left(p-x^2\right)\frac{d^2y}{dx^2}-\frac{xdy}{dx}=q$ then p+q=__

Ans: 3.

$$\frac{dy}{dx}=\frac{2\sin^{-1}x}{\sqrt{1-x^2}}-\frac{A}{\sqrt{1-x^2}}\Rightarrow y^{|}=4y-4B+A^2-2\pi A$$

Hint $\Rightarrow 2\left(1-x^2\right)y^{|}y^{||}-2x\left(y^{|}\right)^2=4y^{|}$

$$\Rightarrow\left(1-x^2\right)y^{||}-xy^{|}=2$$

9. Let $f(x)=\begin{cases}1-|x|,|x|\le 1\\ 0\quad ,|x|>1\end{cases}$ *and* $g(x)=f(x-1)+f(x+1)$. Then the number of points where g is not differentiable is

Ans: 5

Hint
$$f(x)=\begin{cases}1+x\ if\ -1\le x\le 0\\ 1-x\ if\ 0\le x\le 1\\ \quad 0\ if\ \ x>1 or\ x<-1\end{cases}$$

$$g(x)=f(x-1)+f(x+1)=\begin{cases}0 & if\ \ x<-2\\ 1-|x+1| & if\ -2\le x\le 0\\ 1-|x-1| & if\ \ 0\le x\le 2\\ 0 & if\ \ x>2\end{cases}$$

Points where f is not differentiable are – 2, -1 , 0, 1 and 2.

10. For y > 0 and $x\in R$, $ydx+y^2dy=xdy$ where y = f(x). If f(1) = 1, then find f(–3).

Key: 3

Hint: $\frac{xdy-ydx}{y^2}=dy$

$-\int d\left(\frac{x}{y}\right) = \int dy$

$\Rightarrow -\frac{x}{y} = y + c$

At $x = 1, y - 1, c = -2$

At $x = -3, \frac{3}{y} = y - 2$

$\Rightarrow y^2 - 2y - 3 = 0$

$\therefore y = 3$

11. A curve passing through the point $(1,1)$ has the property that the perpendicular distance of the origin from normal at any point 'P' of the curve is equal to the distance of P from the x-axis is a circle with radius

Key: 1

Hint: Equation of normal at the point $p(x, y)$ is $Y - y = -\frac{dx}{dy}(X - x)\left(\text{let } m = \frac{dx}{dy}\right)$

Let , $m = \frac{dx}{dy}$ $\Rightarrow X + mY - (x + my) = 0$

Distance of perpendicular from the origin to line (i) is $\frac{|x + my|}{\sqrt{1 + m^2}} = |y|$

$\Rightarrow \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$

This is homogeneous equation

Let , $y = zx$

$\Rightarrow \frac{dy}{dx} = z + z\frac{dz}{dx}$

$\Rightarrow \frac{2z}{1 + z^2} dz = -\frac{dx}{x}$

Integrating

$\int \frac{2z}{1 + z^2} dz = -\int \frac{dx}{x}$

$\Rightarrow \log\left(1 + z^2\right) = -\log x + c$

$\Rightarrow \left(x^2 + y^2\right) = x.e^c$

This curve passes through $(1,1)$

$\Rightarrow 1+1=1.e^c$

$e^c = 2$

The required equation of the curve is

$\Rightarrow x^2+y^2=2x$

12. The real value of m for which the substitution $y=u^m$ will transform the differential equation $2x^4y\frac{dy}{dx}+y^4=4x^6$ into a homogeneous equation is $\frac{\lambda}{2}$. Then the value of λ is

Ans: 3

Hint: $y=u^m \Rightarrow \frac{dy}{du}=mu^{m-1}$, hence $2x^4.u^m.mu^{m-1}.\frac{du}{dx}=u^{4m}-4x^6$

$\frac{du}{dx}=\frac{4x^6-u^{4m}}{2mx^4u^{2m-1}} \Rightarrow 4m=6 \Rightarrow m=\frac{3}{2}$

13. Let $y=f(x)$ be a curve passing through (e,e^e) which satisfy the differential equation $(2ny+xy\log_e x)dx - x\log_e xdy=0 \quad x>0, y>0$. If $g(x)=\lim_{n\to\infty}(0.020)f(x)$. Then $\int_{\frac{1}{e}}^{e} g(x)dx$ equal to

Ans: 0

Hint: $(2ny+xy\log_e x)dx = x\log_e xdy \Rightarrow \frac{dy}{y}=\left(\frac{2n}{x\log_e x}+1\right)dx$

$\Rightarrow \log(y)=2n\log|\log x|+x+c$ and $c=0$

$$\text{now}, g(x)=\lim_{n\to\infty} f(x)=\begin{cases}\to \infty & \text{if} \quad x<\frac{1}{e}\\ 0 & \text{if} \quad \frac{1}{e}<x<e \\ \to\infty & \text{if} \quad x>e\end{cases} \qquad \therefore \int_{\frac{1}{e}}^{e} g(x)dx=0$$

14. The solution of $x^3\frac{dy}{dx}+4x^2\tan y=e^x\sec y$ satisfying y(1) = 0 is sin y $=e^x(x-1)x^{-k}$ then k =

Key. 4

Sol. Put $\sin y = t \Rightarrow \cos y\frac{dy}{dx}=\frac{dt}{dx}$

$\therefore \frac{dt}{dx}+\frac{4}{x}t=\frac{e^x}{x^3}$

$$\text{I.F.} = e^{\int 4/x\,dx} = e^{4\ln x} = x^4$$

$$\frac{d(t.x^4)}{dx} = xe^x$$

Differential Equations

Matrix-Match Type

1. A function f(x) has continuous third order derivatives every where and satisfies the relation

$$\underset{x\to 0}{Lt}\left(1+x+\frac{f(x)}{x}\right)^{\frac{1}{x}}=e^{3}$$

Column – I	Column – II
a) $f(0)$	p) does not exist
b) $f^{1}(0)$	q) is 0
c) $f^{11}(0)$	r) is 4
d) $\underset{x\to 0}{Lt}\left(1+\frac{f(x)}{x}\right)^{\frac{\ln 2}{x}}$	s) is e^2

Key. a) q b) q c) r d) r

Sol. Conceptual

2. Match the following: -

	Column – I		Column – II
(A)	If order and degree of the differential equation formed by differentiating and eliminating the constants from y = a $\sin^2$x + b $\cos^2$x + c sin 2x + d cos 2x, where a, b, c, d are arbitrary constants are represented by O and D, then	(P)	O + 2D = 5
(B)	The order and degree of the differential equation, whose general solution is given by y = (c_1 + c_2) sin(x + c_3) – $c_4\, e^{x+c_5+c_6}$, where c_1, c_2, c_3, c_4, c_5, c_6 are arbitrary constants, are O and D, then	(Q)	2O + 3D = 5
(C)	The order and degree of the differential equation satisfying $\sqrt{(1+x^2)}+\sqrt{(1+y^2)}=A(x\sqrt{(1+y^2)}+y\sqrt{(1+x^2)})$ are O and D, then	(R)	O = D
(D)	If the order and degree of the differential equation of all parabolas whose axis is x-axis are O and D then	(S)	$O^D + D^O = 4$

Key. **A – P, S; B – P, S; C – Q, R; D – P**

Sol. (A) $\because$ $y=\left(\frac{1-\cos 2x}{2}\right)+b\left(\frac{1+\cos 2x}{2}\right)+c\sin 2x+d\cos 2x$

= A + B sin 2x + C cos 2x

$\therefore$ $\frac{dy}{dx}=2B\cos 2x-2C\sin 2x$

$\Rightarrow$ $\frac{d^2y}{dx^2}=-4B\sin 2x-4C\cos 2x$

$\therefore$ O = 3, D = 1

O + 2D = 5, $O^D + D^O = 4$, $2^O + 3^D = 8 + 3 = 11$ (P, S)

(B) $\because$ y = (c_1 + c_2) sin(x + c_3) – $c_4\, e^{x+c_5+c_6}.e^{x}$

or y = Asin(x + B) + Ce^x .....(i)

$\therefore \quad \frac{dy}{dx} = A\cos(x+B) + Ce^x$ .....(ii)

Subtracting Eq. (i) from Eq. (ii), then

$$\frac{dy}{dx} - y = A\cos(x+B) - A\sin(x+B)$$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = -A\cos(x+B) - A\cos(x+B)$$

$$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$$

O = 3, D = 1

O + 2D = 5, $O^D + D^O = 4$, $2^O + 3^D = 11$ (RS)

(C) Put $x = \tan\theta$, $y = \tan\phi$

Then, $(\sec\theta + \sec\phi) = A(\tan\theta\sec\phi + \tan\phi\sec\theta)$

$\Rightarrow \quad \left(\frac{\cos\theta + \cos\phi}{\cos\theta\cos\phi}\right) = A\left(\frac{\sin\theta + \sin\phi}{\cos\theta\cos\phi}\right)$

$\Rightarrow \quad \cot\left(\frac{\theta+\phi}{2}\right) = A$

$\Rightarrow \quad \frac{\theta+\phi}{2} = \cot^{-1}A$

$\Rightarrow \quad \theta + \phi = 2\cot^{-1}A$

or $\quad \frac{1}{(1+x^2)} + \frac{1}{(1+y^2)}\frac{dy}{dx} = 0$

O = 1, D = 1

Then O = D and 2O + 3D = 5 (QR)