Complex Numbers

Single Correct Answer Type

1. If z_1, z_2, z_3 and z_4 be the consecutive vertices of a square, then $z_1^2 + z_2^2 + z_3^2 + z_4^2$ equals

(a) $z_1z_2 + z_2z_3 + z_3z_4 + z_4z_1$

(b) $z_1z_2 + z_1z_3 + z_1z_4 + z_2z_3 + z_2z_4 + z_3z_4$

(c) 0 (d) None of the above

Key. A

Sol. We know that, $\frac{z_2 - z_1}{z_4 - z_1} = \frac{|z_2 - z_1|}{|z_4 - z_1|} e^{i\pi/2} = i$ (as $|z_2 - z_1| = |z_4 - z_1|$)

$\Rightarrow (z_4 - z_1)^2 + (z_2 - z_1)^2 = 0$

similarly $\frac{z_4 - z_3}{z_2 - z_3} = i$

$\Rightarrow (z_4 - z_3)^2 + (z_2 - z_3)^2 = 0$

On adding Eqs (i) and (ii), we get

$2\left[z_1^2 + z_2^2 + z_3^2 + z_4^2 - z_1z_2 - z_4z_1 - z_4z_3 - z_2z_3\right] = 0$

$z_1^2 + z_2^2 + z_3^2 + z_4^2 = z_1z_2 + z_2z_3 + z_3z_4 + z_4z_1$

2. If z_1, z_2 and z_3 are the vertices of an isosceles right angled triangle, right angled at the vertex z_2, then $(z_1 - z_2)^2 + (z_3 - z_2)^2$ equals

(a) 0 (b) $(z_1 - z_3)^2$ (c) $\left(\frac{z_1 + z_3}{2}\right)^2$ (d) None of these

Key. A

Sol. we know that $\frac{z_3 - z_2}{z_1 - z_2} = \frac{|z_3 - z_2|}{|z_1 - z_2|} . e^{-\frac{i\pi}{2}}$

$\Rightarrow z_3 - z_2 = -i(z_1 - z_2)$

$\Rightarrow (z_3 - z_2)^2 + (z_1 - z_2)^2 = 0$

3. Let $C = \cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{8\pi}{7}$ and $S = \sin\frac{2\pi}{7} + \sin\frac{4\pi}{7} + \sin\frac{8\pi}{7}$, then

(a) $C = \frac{1}{2}$ (b) $S = \frac{1}{2}$

(c) $C = \frac{\sqrt{7}}{2}$ (d) $S = \frac{\sqrt{7}}{2}$

Key. D

Sol. $C + is = e^{i\theta} + e^{i(2\theta)} + e^{i(4\theta)}$, where $\theta = \frac{2\pi}{7}$

i.e $C + iS = \alpha + \alpha^2 + \alpha^4$, where $\alpha = e^{i\theta}$ ....(i)

so, $C - iS = \bar{\alpha} + (\bar{\alpha}^2) + (\bar{\alpha}^4) = \alpha^6 + \alpha^5 + \alpha^3$ ....(ii)

$\left(\sin ce\ \bar{a} = \dfrac{a\bar{a}}{a} = \dfrac{1}{a} = \dfrac{a^7}{a} = a^6\ \ etc\right)$

From Eqs (i) and (ii) , we get

$$2C = a + a^2 + a^3 + a^4 + a^5 + a^6 = \frac{a\left(a^6 - 1\right)}{a - 1}$$

$$\Rightarrow\ 2C = \frac{1 - a}{a - 1}\left(\because\ a^7 = 1\right)$$

$$\Rightarrow\ C = -\frac{1}{2}$$

Again $(C + iS)(C - iS) = \left(a + a^2 + a^4\right)\left(a^6 + a^5 + a^3\right)$

$$\Rightarrow\ C^2 + S^2 = 1 + a^6 + a^4 + a + 1 + a^5 + a^3 + a^2 + 1 \qquad \left(a^7 = 1\right)$$

$$\Rightarrow\ \left(-\frac{1}{2}\right)^2 + S^2 = 2 + \left(1 + a + a^2 + a^3 + a^4 + a^5 + a^6\right)$$

$$\Rightarrow\ S^2 = 2 + 0 - \frac{1}{4} = \frac{7}{4}$$

$$\Rightarrow\ S = \frac{\sqrt{7}}{2}$$

4. The point of intersection of the curves $\arg(z - 3i) = \dfrac{3\pi}{4}$ and $\arg(2z + 1 - 2i) = \dfrac{\pi}{4}$ is

(a) $\frac{1}{4}(3 + 9i)$ (b) $\frac{1}{4}(3 - 9i)$ (c) $\frac{1}{2}(3 + 2i)$ (d) None of these.

Key. D

Sol. Clearly the two eqns represent two rays which are not intersecting. Hence no point of intersection.

5. If z_1, z_2, z_3 are non-zero complex numbers representing the points A, B, C such that $\dfrac{2}{z_1} = \dfrac{1}{z_2} + \dfrac{1}{z_3}$. Then

(a) A, B, C are collinear.

(b) Circle passes through points A, B, C has centre at origin O

(c) Circle passes through A, B, C passes through origin.

(d) None of these.

Key. C

Sol. $\dfrac{2}{z_1} = \dfrac{1}{z_2} + \dfrac{1}{z_3} \Rightarrow\ \arg\left(\dfrac{z_2 - z_1}{z_3 - z_1}\right) = \arg\left(-\dfrac{z_2}{z_3}\right) = \arg\left(\dfrac{z_2}{z_3}\right) \pm \pi$

$$\Rightarrow\ \arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) = \arg\left(\frac{z_2 - 0}{z_3 - 0}\right) \pm \pi$$

$$\Rightarrow\ \arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) - \arg\left(\frac{z_2 - 0}{z_3 - 0}\right) = \pm\pi$$

Sum of angles at A and origin is $\pm\pi$. Hence points O, B, A, C are concyclic.

6. If $|2z - 4 - 2i| = |z|\sin\left(\dfrac{\pi}{4} - \arg z\right)$, then locus of z is/an

(a) Ellipse (b) Circle (c) Parabola (d) Pair of straight line

Key. A

Sol. Let $z = x + iy = r(\cos\theta + i\sin\theta)$, then the equation is

$$|2(x-2)+2i(y-1)| = r\left(\frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta\right) = \frac{1}{\sqrt{2}}(r\cos\theta - r\sin\theta)$$

$$\Rightarrow \sqrt{(x-2)^2+(y-1)^2} = \frac{1}{\sqrt{2}}\left(\frac{x-y}{\sqrt{2}}\right)$$

It is an ellipse with focus at $(2,1)$ and directrix $x - y = 0$ and eccentricity $= \frac{1}{\sqrt{2}}$.

7. If $|z-3i| = 3$, (where i = $\sqrt{-1}$) and arg z $\in$ (0, π/2), then cot (arg (z)) $-\frac{6}{z}$ is equal to

a) 0 b) –i c) i d) none of these

Key. C

Sol. Conceptual

8. If the imaginary part of the expression $\frac{z-1}{e^{i\theta}} + \frac{e^{i\theta}}{z-1}$ be zero, then the locus of z can be

(a) a straight line parallel to x-axis.
(b) a parabola
(c) a circle of radius 1
(d) none of these.

Key. C

Sol. Conceptual

9. If $\cos\alpha + \cos\beta + \cos\gamma = 0 = \sin\alpha + \sin\beta + \sin\gamma$ then $\frac{\sin 3\alpha + \sin 3\beta + \sin 3\gamma}{\sin(\alpha+\beta+\gamma)}$ is equal to

(a) 0 (b) 1 (c) 2 (d) 3

Key. D

Sol. Let $a = e^{i\alpha}, b = e^{i\beta}, c = e^{i\gamma}$ clearly $a+b+c = 0 \Rightarrow \frac{1}{a}+\frac{1}{b}+\frac{1}{c} = 0$

$a+b+c = 0 \Rightarrow a^3+b^3+c^3 = 3abc$

$\Rightarrow \sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha+\beta+\gamma)$.

10. If z is a complex number satisfying $z^4 + z^3 + 2z^2 + z + 1 = 0$, then the set of possible values of $|z|$ is

(a) $\{1,2\}$ (b) $\{1\}$ (c) $\{1,2,3\}$ (d) $\{1,2,3,4\}$

Key. B

Sol. The given equation is $(z^2+z+1)(z^2+1) = 0$.

$z = \pm i, w, w^2$, w being an imaginary cube root of unity. Thus $|z| = 1$.

11. Let A, B and C represents the complex numbers z_1, z_2 and z_3 in the Argand plane. If circumcentre of the triangle ABC is at the origin, then the complex number corresponding to orthocentre is

(a) $\frac{1}{4}(z_1+z_2+z_3)$ (b) $\frac{1}{3}(z_1+z_2+z_3)$ (c) $\frac{1}{2}(z_1+z_2+z_3)$ (d) $z_1+z_2+z_3$

Key. D

Sol. Centroid of $\mathrm{D}ABC$ is at $\dfrac{z_1 + z_2 + z_3}{3}$.

Orthocentre divides Centroid andcircumcentre in 2:3 externally.

12. If $z = x + iy$ then the equation $\left|\dfrac{2z - i}{z + 1}\right| = m$ does not represent a circle when

(a) $m = 1/2$ (b) m =1 (c) m = 2 (d) m = 3

Key. C

Sol. The given equation is $\left|\dfrac{z - \frac{i}{2}}{z + 1}\right| = \dfrac{m}{2}$, which does not represent a circle when $\dfrac{m}{2} = 1$.

13. α, β, γ are the roots of $x^3 - 3x^2 + 3x + 7 = 0$ (w is cube root of unity) then $\left(\dfrac{\alpha - 1}{\beta - 1} + \dfrac{\beta - 1}{\gamma - 1} + \dfrac{\gamma - 1}{\alpha - 1}\right)$ is

(A) $\dfrac{3}{\omega}$ (B) ω^2 (C) $2\omega^2$ (D) 3ω

Key. A

Sol. We have $x^3 - 3x^2 + 3x + 7 = 0$

$\Rightarrow$ $(x - 1)^3 + 8 = 0$

$\Rightarrow$ $\left(\dfrac{(x - 1)}{-2}\right)^3 = 1$

$\Rightarrow$ $\left(\dfrac{x - 1}{-2}\right) = 1, \omega, \omega^2$

$\Rightarrow$ $x = -1;\ 1 - 2\omega;\ 1 - 2\omega^2$

$\therefore$ $\alpha = -1, \beta = 1 - 2\omega; \gamma = 1 - 2\omega^2$

$\therefore$ required expression = $3\omega^2$.

14. The complex number $3 + 4i$ is rotated about origin by an angle of $p/4$ and then stretched 2- times. The complex number corresponding to new position is

(a) $\sqrt{2}(-3 + 4i)$ (b) $\sqrt{2}(-1 + 7i)$ (c) $\sqrt{2}(3 - 4i)$ (d) $\sqrt{2}(-1 - 7i)$

Key. B

Sol. The new complex number is $2(3 + 4i)e^{ip/4} = \sqrt{2}(-1 + 7i)$.

15. If $(a + ib)^5 = \alpha + i\beta$ then $(b + ia)^5$ is equal to

(A) β - iα (B) β + iα (C) α - β (D) -α - iβ

Key. B

Sol. $(a + ib)^5 = \alpha + i\beta$

Taking complex conjugate

$(a - ib)^5 = \alpha - i\beta$

$(-i^2 a - ib)^5 = \alpha - i\beta$

$(-i)^5 (b + ai)^5 = \alpha - i\beta$

$(b + ai)^5 = -\dfrac{\alpha}{i} + \beta$

$= \alpha i + \beta$

16. The complex number a + i, a – i, 1 + ai, 1 – ai where a $\in$ R taken in that order on the Argand plane represent the vertices of a parallelogram if

(A) a = 1 (B) a = – 1 (C) a = 0 (D) none of these

Key. B

Sol. Diagonals of Parallelogram intersects at midpoint

Solution : $\frac{a+1}{2} = \frac{a+1}{2}$

$\frac{-1-a}{2} = \frac{a+1}{2}$

2a = -2

a = -1

17. If (1 + i) (1 + 2i) (1 + 3i) .....(1 + ni) = α + iβ, then $2 \cdot 5 \cdot 10.....(1+n^2)$ is equal to $(where\, \alpha, \beta, n \in R)$

(A) α -iβ (B) $\alpha^2 - \beta^2$ (C) $\alpha^2 + \beta^2$ (D) none of these

Key. C

Sol. (1 + i) (1 + 2i) (1 + 3i) ……….(1 + ni) = α + iβ

α - iβ = (1 – i) (1 – 2i) (1 – 3i)…….. (1 – ni)

$\alpha^2 + \beta^2 = 2 \cdot 5 \cdot 10......(1+n^2)$

18. If $z = \left(1+\sqrt{3}i\right)^{10} + \left(1-\sqrt{3}i\right)^{10}$, then arg z is

(A) $\frac{\pi}{2}$ (B) π (C) $\frac{\pi}{4}$ (D) none of these

Key. B

Sol. $z = a + \bar{a}$

= always real

$\Rightarrow$arg z = 0 or π.

19. If α, β, γ are the cube roots of p(< 0), then $\frac{x\alpha + y\beta + z\gamma}{x\beta + y\gamma + z\alpha}$ for any x, y, z is equal to (where ω is complex cube root of unity)

(A) 1 (B) 0 (C) ω^2 (D) 3

Key. C

Sol. $x = -p$

$x^{1/3} = p^{1/3}\left(-1\right)^{1/3}$

$\alpha = -p^{1/3}$ $\beta = -p^{1/3}w$ $g = -p^{1/3}w^2$

$= \frac{1}{w}\frac{xw + yw^2 + z}{xw + yw^2 + z} = w^2$

20. If the complex numbers z_1, z_2, z_3 satisfying $3z_1 = 5z_2 - 2z_3$, then z_1, z_2 and z_3 lie in a

(A) circle (B) parabola (C) line (D) hyperbola

Key. C

Sol. $3z_1 = 5z_2 - 2z_3$

$z_1 = \frac{5z_2 - 2z_3}{5-2}$

$\Rightarrow$ z_1 divides line joining z_2 and z_3 externally in ration 5 : 2

$\Rightarrow$ z_1, z_2, z_3 are collinear.

21. If z_1 and z_2 are two complex numbers satisfying the equation $\left|\frac{z_1+z_2}{z_1-z_2}\right| = 1$, then $\frac{z_1}{z_2}$ is a number which is

(A) Positive real (B) Negative real

(C) Zero or purely imaginary (D) None of these

Key. C

Sol. $\left|\frac{z_1+z_2}{z_1-z_2}\right| = 1$

$\left|\frac{\frac{z_1}{z_2}+1}{\frac{z_1}{z_2}-1}\right| = 1$

$\left|\frac{z_1}{z_2}+1\right| = \left|\frac{z_1}{z_2}-1\right|$

$\left|\frac{z_1}{z_2}+1\right| = \left|\frac{z_1}{z_2}-1\right|$

$\Rightarrow$ $\frac{z_1}{z_2} = 0$ or purely imaginary

22. If $|z_1| = 2$, $|z_2| = 3$, $|z_3| = 4$ and $|z_1 + z_2 + z_3| = 5$ then $|4z_2z_3 + 9z_3z_1 + 16z_1z_2|$ =

a) 20 b) 24 c) 48 d) 120

Key. D

Sol. $|4z_2z_3 + 9z_3z_1 + 16z_1z_2|$

= $|z_1\overline{z_1}z_2z_3 + \overline{z_2}z_2z_3z_1 + \overline{z_3}z_3z_1z_2|$

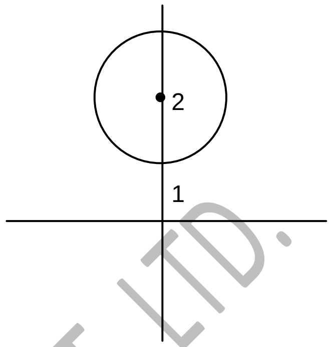
= $\|z_1||z_2||z_3\||z_1 + z_2 + z_3| = 120$

23. The value of $\sin\left[\log_e\left\{\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)^z\right\}\right]$ is, where z satisfies the equation $|z - 2i| = 1$ and has least modulus

(a) 1 (b) 0 (c) -1 (d) $\frac{1}{2}$.

Key. C

Sol.



$$A = \sin\left[\log\left\{\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)^z\right\}\right]$$

$$= \sin\left[\log e^{z\pi/2i}\right]$$

$$= \sin\left(\frac{z\pi}{2}i\right)$$

Again |z – 2i| = 1 is a circle centered at (0, 2) with radius 1.

Therefore a point on circle of least modulus is z = i.

$\therefore$ By equation θ

$$A = \sin\left(-\frac{\pi}{2}\right)$$

= - 1

24. If $c^2+s^2=1$, then $\frac{1+c+is}{1+c-is}$ is equal to

(a) $c-is$ (b) $c+is$ (c) $s+ic$ (d) $s-ic$.

Key. B

Sol. $\frac{1+c+is}{1+c-is}=\frac{(1+c)+is}{(1+c)-is}\times\frac{(1+c)+is}{(1+c)+is}$

$$=\frac{\left((1+c)^2-s^2\right)+i\left(s(1+c)+s(1+c)\right)}{(1+c)^2+s^2}$$

$$=\frac{\left(1+c^2+2c-s^2\right)+i\left(2s(1+c)\right)}{(1+c)^2+s^2}$$

$$=\frac{2c(c+1)+i2s(c+1)}{2+2c}$$

= c + is

25. If $\omega\neq 1$ be a cube root of unity and $(1+\omega)^7 = l+m\omega$, then the value of $l+m=$

(a) 0 (b) 1 (c) 2 (d) -1

Key. C

Sol. $\because \omega$ is cube root of unity

$\therefore\ 1+\omega+\omega^2=0$

$\Rightarrow\ 1+\omega=-\omega^2$

Now if $(1+\omega)^7=l+m\omega$

$\Rightarrow\ (-\omega^2)^7=l+m\omega$

$\Rightarrow\ -\omega^{14}=l+m\omega$

$\Rightarrow\ -\omega^{12}\cdot\omega^2=l+m\omega$

$\Rightarrow\ -(\omega^3)^4\omega^2=l+m\omega$

$\Rightarrow\ -\omega^2=l+m\omega$

$\Rightarrow\ 1+\omega=l+m\omega$

can comparison l = 1, m = 1

26. One vertex of an equilateral triangle is at the origin and the other two vertices are, roots of $2z^2+2z+k=0$, then k is

(A) 1 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{1}{2}$.

Key. C

Sol. $2z^2+2z+k=0$

$z=\frac{-2\pm\sqrt{4-8k}}{4}$

Since 'z' is a complex number

4 – 8k will be negative

$\Rightarrow$k> ½

(0, 0), $\left(\frac{-1}{2},\frac{\sqrt{2k-1}}{2}\right)\left(\frac{-1}{2},\frac{-1}{2}\sqrt{2k-1}\right)$

Since triangle is equilateral

$\therefore \frac{1}{4}(2k-1)+\frac{1}{4}=(2k-1)$

$\Rightarrow$ k = 2/3.

27. If $\log_{\tan 30^\circ}\left(\frac{2|z|^2+2|z|-3}{|z|+1}\right)<-2$ then

a) $|z|<\frac{3}{2}$ b) $|z|>\frac{3}{2}$ c) $|z|>2$ d) $|z|<2$

Key. C

Sol. $\log_{\tan 30^\circ}\left(\frac{2|z^2|+2|z|-3}{|z+1|}\right)<-2$

$\Rightarrow\frac{2|z^2|+2|z|-3}{|z|+1}>3$

$\Rightarrow\left((|z|-2)(2|z|)+3\right)>0$

$\Rightarrow$ |z| > 2

28. The number of common roots of the equations $x^3 + 2x^2 + 2x + 1 = 0$ and $x^{2012} + x^{2014} + 1 = 0$ is

(a) 1 (b) 2 (c) 3 (d) 4

Key. D

Sol. $x^3 + 2x^2 + 2x + 1 = 0 \Rightarrow x = -1, w, w^2$

But $x = w, w^2$ will only satisfy $x^3 + 2x^2 + 2x + 1 = 0$ and $x^{2012} + x^{2014} + 1 = 0$.

29. If $|z| = \min\{|z-1|, |z+1|\}$ then $|z + \bar{z}| =$

(a) 1 (b) 2 (c) 3 (d) 4

Key. A

Sol. If $|z| = |z-1|$

Then $|z|^2 = |z-1|^2$

$\Rightarrow z + \bar{z} = 1$

If $|z| = |z+1|$

Then $|z|^2 = |z+1|^2$

$\Rightarrow z + \bar{z} = 1$

$\Rightarrow |z + \bar{z}| = 1$

30. If the roots of $z^3 + iz^2 + 2i = 0$ represent the vertices of a ΔABC in the argand plane then the area of the triangle is (in square units)

A) 3 B) 1 C) 4 D) 2

Key. D

Sol. $(z-i)(z^2 + 2iz - 2) = 0 \Rightarrow z = i, 1-i, -1-i$

Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} -1 & 1 \\ 2 & 2 \end{vmatrix} = 2$ square units.

31. If $n \geq 3$ and $1, \alpha_1, \alpha_2, \alpha_3 ......\alpha_{n-1}$ are n roots of unity, then value of $\sum_{1 \leq i < j \leq n-1} \alpha_i \alpha_j$ is

(a) 0 (b) 1 (c) -1 (d) $(-1)^n$

Key. B

Sol. $x^n - 1 = (x-1)(x-\alpha_1)(x-\alpha_2)............(x-\alpha_{n-1})$

$= x^n - x^{n-1}(1 + \alpha_1 + ... + \alpha_{n-1}) + x^{n-2}\left(\sum_{i+j} \alpha i\,\alpha_j + \alpha_1 + \alpha_2 ..... + \alpha_{n-1}\right)$+..........-1 = 0

$\Rightarrow \sum_{i+j} \alpha i\,\alpha_j + \alpha_1 + \alpha_2 ..... + \alpha_n = 0$

$\sum_{i+j} \alpha i\,\alpha_j = 1$

32. Let $z=\cos\theta+i\sin\theta$. Then, the value of $\sum_{m=1}^{15} lm\left(z^{2m-1}\right)$ at $\theta=2^0$ is

(A) $\frac{1}{2^0}$ (B) $\frac{1}{3\sin 2^0}$ (C) $\frac{1}{2\sin 2^0}$ (D) $\frac{1}{4\sin 2^0}$

Key. D

Sol. Given that $z=\cos\theta+isin\theta=e^{i\theta}$

$$\therefore \sum_{m=1}^{15}\left(z^{m-1}\right)=\sum_{m=1}^{15} lm\left(e^{i\theta}\right)^{2m-1}$$

$$=\sum_{m=1}^{15} lm\, e^{i(2m-1)\theta}$$

$$=\sin\theta+\sin3\theta+\sin5\theta+.......+\sin29\theta$$

$$=\frac{\sin\left(\frac{\theta+29\theta}{2}\right)\sin\left(\frac{15\times2\theta}{2}\right)}{\sin\left(\frac{2\theta}{2}\right)}$$

$$=\frac{\sin(15\theta)\sin(15\theta)}{\sin\theta}=\frac{1}{4\sin 2^0}$$

33. If z_1 is a root of the equation $a_0z^n+a_1z^{n-1}+.....+a_{n-1}z+a_n=3$, where $|a_i|<2$ for $i=0,1,.....n$. Then

(A) $|z_1|>\frac{1}{3}$ (B) $|z_1|<\frac{1}{4}$ (C) $|z_1|>\frac{1}{4}$ (D) $|z|<\frac{1}{3}$

Key. A

Sol. $a_0z^n+a_1z^{n-1}+a_2z^{n-2}+..............+a_{n-1}z+a_n=3$

$|3|=\left|a_0z^n+a_1z^{n-1}+a_2z^{n-2}+..............+a_{n-1}z+a_n\right|$

$3\le|a_0||z|^n+|a_1||z|^{n-1}+.......+a_{n-1}|z|+|a_n|$

$3<2\left(|z|^n+|z|^{n-1}+.......+|z|+1\right)$

$\frac{3}{2}<1+|z|+|z|^2+......+|z|^n$

$\frac{1-|z|^{n+1}}{1-|z|}>\frac{3}{2}$

$2-2|z|^{n+1}<3|z|-1$

$3|z|-1>0$

$|z|>\frac{1}{3}$

34. If x = a+ib is a complex number such that x^2 =3+4i and x^3=2+11i where i = $\sqrt{-1}$ then a+b =______________________________

1. 1 2. 2 3. 3 4. 4

Key. 3

Sol. $x=\frac{x^3}{x^2}=\frac{2+11i}{3+4i}=\frac{(2+11i)(3-4i)}{25}$

$\therefore a+ib=\frac{6+44+25i}{25}=2+i$

$\Rightarrow a=2, b=1 \Rightarrow a+b=3$

35. If the complex number Z satisfying $Z+|Z|=2+8i$ then value of $|Z|$ =

1. 8 2.17 3. 15 4. 24

Key. 2

Sol. Let $z=a+ib$

$\Rightarrow a+ib+\sqrt{a^2+b^2}=2+8i$

$\Rightarrow b=8,\ a+\sqrt{a^2+64}=2$

$a^2+64=a^2-4a+4$

$\Rightarrow -4a=60 \Rightarrow a=-15$

$\therefore |z|=\sqrt{a^2+b^2}=\sqrt{225+64}=\sqrt{289}=17$

36. If $|Z+2-i|=5$ then maximum value of $|3Z+9-7i|=$ ______________

1. 20 2.15 3. 5 4. 16

Key. 1

Sol. $|3Z+9-7i|=|3Z+6-3i+3-4i|$

$\leq |3(z+2-i)+|=|3-4i|$

$\leq 3|z+2-i|+\sqrt{3^2+4^2}$

$3(5)+5=20$

37. If Z-lies on the circle $|z-2i|=2\sqrt{2}$ *then value of* $\arg\left(\frac{Z-2}{Z+2}\right)$ *is*

1. $\frac{\pi}{3}$ 2. $\frac{\pi}{4}$ 3. $\frac{\pi}{6}$ 4. $\frac{\pi}{2}$

Key. 2

Sol. Circle with centre (0,2) cuts X-axis at A(-2,0) and B(2,0). Now AB subtends an angle 90^0 at the center C.

AB subtends an angle $\frac{\pi}{4}$ at any point z on the major arc circle arg $\left(\frac{z-2}{z+2}\right)=\frac{\pi}{4}$

38. If $\lambda \in R$ and non real roots of $2Z^2 + 2Z + \lambda = 0$ and (0,0) forms vertices of an equilateral triangle then λ =

1. 1 2. $\frac{1}{2}$ 3. $\frac{1}{3}$ 4. $\frac{2}{3}$

Key. 4

Sol. Let $z_{1,} z_2$ be roots of $2z^2 + 2z + \lambda = 0$

$z_1 + z_2 = -1 \quad z_1 z_2 = \frac{\lambda}{2}$

When origin , $z_1 z_2$ forms equilateral Δ^{le}

We have $z_1^2 + z_2^2 = z_1 z_2$

$(z_1 + z_2)^2 = 3z_1 z_2$

$1 = \frac{3.\lambda}{2} \Rightarrow \lambda = \frac{2}{3}$

39. The greatest positive argument of z satisfying $|Z-4| = \text{Re}(Z)$

1. $\frac{\pi}{3}$ 2. $\frac{2\pi}{3}$ 3. $\frac{\pi}{2}$ 4. $\frac{\pi}{4}$

Key. 4

Sol. $|x + iy - 4| = x$

$(X-4)^2 + y^2 + x^2$

$y^2 - 8x + 16 = 0$

z lies on the parabola with vertex (2,0) focus (4,0) and tangents from (0,0) ie a point on the directrix in x always include 90^0

$\therefore$ greatest arg(z) is $45^0 = \frac{\pi}{4}$

40. If Z and W are two complex numbers such that $\bar{z} + i\,\overline{w} = 0$ and arg (Zw)= π then arg (Z) = ________

1. $\frac{\pi}{4}$ 2. $\frac{\pi}{2}$ 3. $\frac{3\pi}{4}$ 4. $\frac{5\pi}{4}$

Key. 3

Sol. $\bar{z} + i\bar{w} \Rightarrow z - iw = 0 \Rightarrow z = iw$

Arg $(zw) = \pi \Rightarrow \arg(z) + \arg(w) = \pi$

$\arg(iw) + \arg w = \pi$

$\arg i + 2\arg w = \pi$

$\frac{\pi}{2}+2\arg w=\pi$

$2\arg w=\frac{\pi}{2}$

$\arg w=\frac{\pi}{4}\Rightarrow\arg(z)=\frac{3\pi}{4}$

41. If A (Z_1) B(Z_2) C(Z_3) are vertices of a triangle such that $Z_3=\left(\frac{Z_2-iZ_1}{1-i}\right)$ $and\ |Z_1|=3, |Z_2|=4\ and\ |Z_2+iZ_1|=|Z_1|+|Z_2|$ then area of triangle ABC is

1. $\frac{5}{2}$ 2. 0 3. $\frac{25}{2}$ 4. $\frac{25}{4}$

Key. 4

Sol. $|z_2+iz_1|=|z_1|+|z_2|\Rightarrow z_2, iz_1, 0$ are collinear.

$\therefore \arg(iz_1=\arg z_2)$

$\Rightarrow \arg i+\arg z_1=\arg z_2$

$\Rightarrow \arg z_2-\arg z_1=\frac{\pi}{2}$

$z_3=\frac{z_2-iz_1}{l-i}$

$(l-i)z_3=z_2-iz_1$

$z_3-z_2=i(z_3-z_1)$

$\frac{z_3-z_2}{z_3-z_1}=i\Rightarrow \arg\left(\frac{z_3-z_2}{z_3-z_1}\right)=\frac{\pi}{2}$ and $|z_3-z_2|=|z_3-z_1|$

$\therefore$ AB=BC, $\therefore\ AB^2=AC^2+BC^2$

$25=2AC^2$

$\Rightarrow AC=\frac{5}{\sqrt{2}}$

Required area=$\frac{1}{2}\times\frac{5}{\sqrt{2}}\times\frac{5}{\sqrt{2}}=\frac{25}{4}$ sq. units

42. The radius of the circle given by arg $\left(\frac{Z-5+4i}{Z+3-2i}\right)=\frac{\pi}{4}$

1. $5\sqrt{2}$ 2.5 3. $\frac{5}{\sqrt{2}}$ 4. $\sqrt{2}$

Key. 1

Sol. A(5,-4) B(-3,2) subtends an angle $\frac{\pi}{4}$ at C(z) on the circle hence $\frac{\pi}{2}$ at centre

$M \rightarrow M.dAB \therefore AM = \frac{AB}{2}$

$= \frac{\sqrt{64+36}}{2} \frac{10}{2} = 5$

Radius $= \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$

43. f(x) = $2x^3 + 2x^2 - 7x + 72$ then $f\left(\frac{3-5i}{2}\right)$ = ___________

1. 1 2. 2 3.3 4.4

Key. 4

Sol. Let $x = \frac{3-5i}{2}$

$2x = 3 - 5i$

$(2x-3)^2 = 5i$

$4x^2 - 12x + 9 = 25i^2$

$\Rightarrow 2x^2 - 12x + 34 = 0 \Rightarrow 2x^2 - 6x + 17 = 0$

$2x^2 - 6x + 17)2x^3 + 2x^2 = 7x + 72(x + 4$

$2x^3 - 6x^2 + 17x$

$8x^2 - 24x + 72$

$8x^2 - 24x + 68$

4

44. If $Cos\,\alpha + Cos\,\beta + Cos\gamma = 0 = Sin\,\alpha + Sin\,\beta + Sin\gamma$ then $Sin^2\,\alpha + Sin^2\,\beta + Sin^2\gamma$ = _______

1. $\frac{1}{2}$ 2. $\frac{3}{2}$ 3. 4 4. 1

Key. 2

Sol. Let $x = cis\alpha$ $y = cis\beta$ $z = cis\gamma$

Clearly $x+y+z=0, \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$

$$x^2+y^2+z^2=(x+y+z)^2-2xyz\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=0$$

$$=cis2\alpha+cis2\beta+cis2\gamma=0$$

$$\Rightarrow \cos 2\alpha+\cos 2\beta+\cos 2\gamma=0$$

$$1-2\sin^2\alpha+1-2\sin^2\beta+1-2\sin^2\gamma=0$$

$$\Rightarrow \sin^2\alpha+\sin^2\beta+\sin^2\gamma=\frac{3}{2}$$

45. If Z_1 *and* Z_2 are two complex numbers such that $Z_1{}^2+Z_2{}^2\in R$ *and* $Z_1(Z_1{}^2-3Z_2{}^2)=2$ $Z_2(3Z_1{}^2-Z_2{}^2)=11$ then $Z_1{}^2+Z_2{}^2=$

A) 5 2.125 3. 25 4. 15

Key. 1

Sol. $z_1\left(z_1^2-3z_2^2\right)=2$

$$z_1^2\left(z_1^4+9z_2^4-6z_1^2z_2^2\right)=4$$

$$\left(z_1^2\right)^3+9z_1^2z_2^4-6z_1^4z_2^2=4 \longrightarrow ①$$

$$z_2^2\left(3z_1^2-z_2^2\right)^2=|121|$$

$$\Rightarrow\left(z_2^2\right)^3+9z_2^2z_1^4-6z_1^2z_2^4=121 \longrightarrow ②$$

① + ② $\Rightarrow\left(z_1^2+z_2^2\right)^3 125$

$$z_1^2+z_2^2=5$$

46. Let 'C' denote the set of complex numbers and define A & B by

$A=\{(z,w); z,w\in C \text{ and } |z|=|w|\}$

$B=\{(z,w); z,w\in C; \text{ and } z^2=w^2\}$ then

A) $A=B$ B) $A\subset B$ C) $B\subset A$ D) none

Key: C

Hint: Conceptual

47. If $\left|z-|z+1|\right|=\left|z+|z-1|\right|$ where z is a complex number on the complex plane, then which of the following lies on the locus of z

A) line $y=0$ B) line $x=2$

C) circle $x^2+y^2=1$ D)line $x=0$ or on a line segment joining $(-1,0)$ to $(1,0)$

Key: D

Hint: $\left|z-|z+1|\right|^2=\left|z+|z-1|\right|^2$

$\Rightarrow(z+\bar{z})(|z+1|+|z-1|-2)=0$

$\Rightarrow$ z lies on y-axis or

Z lies on line segment joining the points $(-1,0)$ and $(1,0)$

48. If Z_1, Z_2 are two complex numbers such that $|Z_1|=1$, $|Z_2|=1$ then the maximum value of $|Z_1+Z_2|+|Z_1-Z_2| is$

a) 2 b) $2\sqrt{2}$ c) 4 d) none of these

Key: B

$Z_1=\cos\alpha+i\sin\alpha,\quad Z_2=\cos\beta+i\sin\beta$

$|Z_1+Z_2|+|Z_1-Z_2|=\sqrt{2+2\cos(\alpha-\beta)}+\sqrt{2-2\cos(\alpha-\beta)}$

Hint: $let\,\alpha-\beta=\theta$

$2\cos\frac{\theta}{2}+2\sin\frac{\theta}{2}=2\sqrt{2}\sin\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$

49. If $|z-2|=\min\{|z-1|,|z-5|\}$, where Z is a complex number then

(A) $\text{Re}(z)=\frac{3}{2}\text{only}$ (B) $\text{Re}(z)=\frac{7}{2}\text{only}$

(C) $\text{Re}(z)\in\left\{\frac{3}{2},\frac{7}{2}\right\}$ (D) $\text{Re}(z)\in\left\{\frac{1}{2},\frac{3}{2}\right\}$

Key: C

Hint: draw the locus Z in argand plane.

$\text{Re}(z)\in\left\{\frac{3}{2},\frac{7}{2}\right\}$

50. If Z is a non-real complex number, then the minimum value of $\frac{\text{Im}\,Z^5}{\text{Im}^5 Z}$ is

(A) -1 (B) -2

(C) -4 (D) -5

Key: C

Hint: Let $Z=a+ib, b\neq 0$ where $\text{Im}\,Z=b$

$Z^5=(a+ib)^5=a^5+{}^5C_1a^4bi+{}^5C_2a^3b^2i^2+{}^5C_3a^2b^3i^3+{}^5C_4i^4+i^5b^5$

$\text{Im}\,Z^5=5a^4b-10a^2b^3+b^5$

$y=\frac{\text{Im}\,Z^5}{\text{Im}^5 z}=5\left(\frac{a}{b}\right)^4-10\left(\frac{1}{b}\right)^2+1$

Let $\left(\frac{a}{b}\right)^2 = x(\text{say}), x \in R^+$

$y = 5x^2 - 10x + 1 = 5\left[x^2 - 2x\right] + 1 = 5\left[(x-1)^2\right] - 4$

Hence $y_{min} = -4$.

51. Let $z_r\ (1 \le r \le 4)$ be complex numbers such that $|z_r| = \sqrt{r+1}$ and

$|30z_1 + 20z_2 + 15z_3 + 12z_4| = k|z_1z_2z_3 + z_2z_3z_4 + z_3z_4z_1 + z_4z_1z_2|$

Then the value of k equals

(A) $|z_1z_2z_3|$ (B) $|z_2z_3z_4|$

(C) $|z_4z_1z_2|$ (D) None of these

Key: C

Hint: We have $\left|\frac{z_1}{2} + \frac{z_2}{3} + \frac{z_1}{4} + \frac{z_4}{5}\right| = \frac{k}{60}|z_1z_2z_3z_4|\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \frac{1}{z_4}\right|$

Now, $z_1\bar{z}_1 = 2,\ z_2\bar{z}_2 = 3,\ z_3\bar{z}_3 = 4$ and $z_4\bar{z}_4 = 5$

So, $k = \frac{60}{|z_1z_2z_3z_4|} = \frac{60}{\sqrt{2}\sqrt{3}\sqrt{4}\sqrt{5}} = \sqrt{30} = |z_4z_1z_2|$

Note for objective take $z_1 = \sqrt{2}; z_2 = \sqrt{3}; z_3 = 2; z_4 = \sqrt{5}\,|$

52. If P(z) and A(z_1) two be variable points such that $zz_1 = |z|^2$ and $|z - \bar{z}| + |z_1 + \bar{z}_1| = 10$ then area enclosed by the curve formed by them

(A) 25π (B) 20 π

(C) 50 (D) 100

Key: C

53. A particle starts to travel from a point P on the curve $C_1 : |z - 3 - 4i| = 5$, where $|z|$ is maximum. From P, the particle moves through an angle $\tan^{-1}\frac{3}{4}$ in anticlock wise direction on $|z - 3 - 4i| = 5$ and reaches at point Q. From Q, it comes down parallel to imaginary axis by 2 units and reaches at point R. Complex number corresponding to point R in the Argand plane is

(A) (3+5i) (B) (3+7i) (C) (3+8i) (D) (3+9i)

Key: B

Hint: $|z - 3 - 4i| = 5$

$\Rightarrow (x-3)^2 + (y-4)^2 = 25$

R is (3,7)

54. If $|z_1| = 2$, $|z_2| = 3$, $|z_3| = 4$ and $|2z_1 + 3z_2 + 4z_3| = 4$, then absolute value of $8z_2z_3 + 27z_3z_1 + 64z_1z_2$ equals

(a) 24 (b) 48 (c) 72 (d) 96

Key: D

Hint: $|8z_2z_3 + 27z_3z_1 + 64z_1z_2| =$

$$|z_1||z_2||z_3|\left|\frac{8}{z_1}+\frac{27}{z_2}+\frac{64}{z_3}\right| = (2)(3)(4)\left|\frac{8\bar{z}_1}{|z_1|^2}+\frac{27\bar{z}_2}{|z_2|^2}+\frac{64\bar{z}_3}{|z_3|^2}\right|$$

$$= 24|2\bar{z}_1 + 3\bar{z}_2 + 4\bar{z}_3| = 24|\overline{2z_1 + 3z_2 + 4z_3}| = 24|2z_1 + 3z_2 + 4z_3|$$

= (24) (4) = 96

55. If the ratio $\frac{1-z}{1+z}$ is purely imaginary, then

(a) $0 < |z| < 1$ (b) $|z| = 1$

(c) $|z| > 1$ (d) bounds for $|z|$ can not be decided

Key: b

Hint: $0 = \frac{1-z}{1+z} + \frac{1-\bar{z}}{1+\bar{z}} = \frac{(1-z)(1+\bar{z})+(1-\bar{z})(1+z)}{(1+z)(1+\bar{z})} = \frac{2(1-|z|)^2}{|1+z|^2} \Rightarrow |z| = 1$

56. If P and Q are represented by the numbers z_1 and z_2 such that $\left|\frac{1}{z_2}+\frac{1}{z_1}\right| = \left|\frac{1}{z_2}-\frac{1}{z_1}\right|$, then the circumcentre of ΔOPQ, (where O is the origin) is

(A) $\frac{z_1 - z_2}{2}$ (B) $\frac{z_1 + z_2}{2}$

(C) $\frac{z_1 + z_2}{3}$ (D) $z_1 + z_2$

Key : B

Sol : $\left|\frac{1}{z_2}+\frac{1}{z_1}\right| = \left|\frac{1}{z_2}-\frac{1}{z_1}\right|$

$\Rightarrow$ $|z_1 + z_2| = |z_1 - z_2|$

$\Rightarrow$ $z_1\bar{z}_2 + z_2\bar{z}_1 = 0$

$\Rightarrow$ $\frac{z_1}{z_2}$ is purely imaginary

$\Rightarrow$ $\arg\left(\frac{z_1}{z_2}\right) = \pm\frac{\pi}{2}$

$\Rightarrow$ $\angle POQ = \frac{\pi}{2}$

Circumcentre of ΔPOQ is the mid point of PQ i.e.

57. If α is non real root of $x^7 = 1$, then $1 + 3\alpha + 5\alpha^2 + 7\alpha^3 + ... + 13\alpha^6$ is equal to

(A) 0 (B) $\frac{14}{1-\alpha}$

(C) $\frac{14}{\alpha-1}$ (D) none of these

Key: C

Hint: Let A = 1 + 3α + $5\alpha^2 + 7\alpha^3 + ... + 11\alpha^5 + 13\alpha^6$

$\alpha A = \alpha + 3\alpha^2 + 5\alpha^3 + 7\alpha^5 + ... + 11\alpha^7 + 13\alpha^7$

$(1 - \alpha) A = 1 + 2\alpha + 2\alpha^2 + 2\alpha^3 + ... + 2\alpha^6 - 13\alpha^7$

$= -12 + 2[\alpha + \alpha^2 + ... + \alpha^6) = -14$

$A = -\frac{14}{1-\alpha}$

58. If z_1, z_2 are two complex numbers satisfying the equation $\left|\frac{z_1+z_2}{z_1-z_2}\right|=1$, then $\frac{z_1}{z_2}$ is a number which is

(A) Positive real (B) Negative real

(C) Zero (D) Lying on imaginary axis

Key. D

Sol. $\left|\frac{z_1+z_2}{z_1-z_2}\right|=1 \Rightarrow \frac{z_1+z_2}{z_1-z_2}=\cos\alpha+i\sin\alpha$

where α is the argument of $\frac{(z_1+z_2)}{(z_1-z_2)}$. Applying componendo and dividendo, we get

$\frac{z_1}{z_2}=\frac{1+\cos\alpha+i\sin\alpha}{-1+\cos\alpha+i\sin\alpha}$

$=\frac{2\cos\left(\frac{\alpha}{2}\right)\left[\cos\left(\frac{\alpha}{2}\right)+i\sin\left(\frac{\alpha}{2}\right)\right]}{2i\sin\left(\frac{\alpha}{2}\right)\left[\cos\left(\frac{\alpha}{2}\right)+i\sin\left(\frac{\alpha}{2}\right)\right]}=-i\cot\left(\frac{\alpha}{2}\right)$

Purely imaginary in nature

59. If z_1, z_2 and z_3 are the vertices of $\Delta ABC,$ which is not right angled triangle taken in anti-clock wise direction and z_0 is the circumcentre, then $\left(\frac{z_0-z_1}{z_0-z_2}\right)\frac{\sin 2A}{\sin 2B}+\left(\frac{z_0-z_3}{z_0-z_2}\right)\frac{\sin 2C}{\sin 2B}$ is equal to

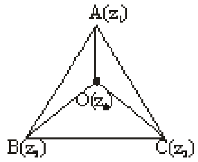
A) 0 B) 1 C) -1 D) 2

Key. C

Sol. Taking rotation at $'O'$

$\frac{z_0-z_1}{z_0-z_2}=\cos 2C-i\sin 2C$

$$\frac{z_0 - z_3}{z_0 - z_2} = \cos 2A + i \sin 2A$$



Now $\left(\frac{z_0 - z_1}{z_0 - z_2}\right)\frac{\sin 2A}{\sin 2B} + \left(\frac{z_0 - z_3}{z_0 - z_2}\right)\frac{\sin 2C}{\sin 2B}$

$$= \frac{\sin 2A \cos 2C - i \sin 2A \sin 2C + \cos 2A \sin 2C + i \sin 2A \sin 2C}{\sin 2B}$$

$$= \frac{\sin(2A + 2C)}{\sin 2B} = -1$$

60. If a complex number 'z' lies in the interior or on the boundary of a circle of radius 3 and centre at $(-4, 0)$, then the greatest and least values of $|z+1|$ are respectively

A) $5, 0$ B) $6, 1$ C) $6, 0$ D) $5, 1$

Key. C

Sol. It is given that $|z+4| \le 3$

Hence the greatest value of $|z+1| is 6$

Since the least value of the modulus of a complex number is zero, therefore

$|z+1| = 0 \Rightarrow z = -1 \quad \Rightarrow |z+4| = |-1+4| = 3$

$\Rightarrow |z+4| \le 3$ is satisfied by $z = -1$

Therefore the least value of $|z+1| is 0$

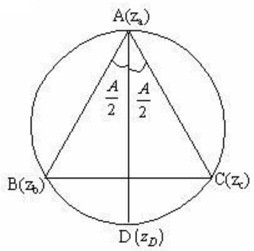
61. $A(z_a), B(z_b), C(z_c)$ be the vertices of ΔABC taken in anticlockwise direction whose circum–circle is $|z| = r$

If the internal angular bisector of angle A meets the circum circle again at $D(z_D)$ then

A) $z_D = z_a z_c$ B) $z_D^2 = z_b z_c$ C) $z_D = \frac{z_b z_c}{z_a}$ D) $z_D = \frac{-z_b z_c}{z_a}$

Key. B

Sol. Let D represents the complex number $z = (z_D)$



$\angle BAD = \angle CAD = A/2$

$$\frac{z-z_a}{|z-z_a|}=\frac{z_b-z_a}{|z_b-z_a|}e^{iA/2}$$

$$\frac{(z-z_a)^2}{|z-z_a|^2}=\frac{(z_b-z_a)^2}{|z_b-z_a|^2}e^{iA} \quad \text{----------- (1)}$$

Similarly $\frac{(z_c-z_a)^2}{|z_c-z_a|^2}=\frac{z-z_a}{|z-z_a|^2}e^{iA}$ ----------- (2)

From (1) & (2)

$$\frac{z-z_a}{\bar{z}-\bar{z}_a}=\frac{z_b-z_a}{\bar{z}_b-\bar{z}_a}e^{iA},\quad \frac{z_c-z_a}{\bar{z}_c-\bar{z}_a}=\frac{z-z_a}{\bar{z}-\bar{z}_a}e^{iA} \quad \Rightarrow \frac{z-z_a}{\bar{z}-\bar{z}_a}\times\frac{\bar{z}_c-\bar{z}_a}{z_c-z_a}=\frac{z_b-z_a}{\bar{z}_b-\bar{z}_a}\,\frac{\bar{z}-\bar{z}_a}{z-z_a}$$

$$\Rightarrow\left(\frac{z-z_a}{\bar{z}-\bar{z}_a}\right)^2=\frac{z_b-z_a}{\bar{z}_b-\bar{z}_a}\cdot\frac{z_c-z_a}{\bar{z}_c-\bar{z}_a}$$

$$\Rightarrow\left(\frac{z-z_a}{\frac{r^2}{z}-\frac{r^2}{z_a}}\right)^2=\left(\frac{z_b-z_a}{\frac{r^2}{z_b}-\frac{r^2}{z_a}}\right)\left(\frac{z_c-z_a}{\frac{r^2}{z_c}-\frac{r^2}{z_a}}\right)\left(\begin{array}{l}\because z_a,z_b,z_c \text{ and } z \text{ lie on } |z|=r\\ \therefore |z_a|=|z_b|=|z_c|=r\end{array}\right)$$

$$\Rightarrow (z\,z_a)^2=(z_az_b)(z_az_c)\Rightarrow z^2=z_bz_c\Rightarrow z_D^2=z_bz_c$$

62. The least positive integer 'n' for which $\left(\frac{1+i}{1-i}\right)^n=\frac{2}{\pi}\sin^{-1}\left(\frac{1+x^2}{2x}\right)$, where $x>0$ and $i=\sqrt{-1}$ is

A) 2 B) 4 C) 8 D) 12

Key. B

Sol. $\because -1\le\frac{1+x^2}{2x}\le 1$

$$\Rightarrow\left|\frac{1+x^2}{2x}\right|\le 1\Rightarrow\frac{1+x^2}{2|x|}\le 1\Rightarrow\frac{1+|x^2|}{2|x|}-1\le 0\Rightarrow\frac{(|x|-1)^2}{|x|}\le 0$$

$$\therefore |x|>0, \therefore (|x|-1)^2\le 0 \quad \Rightarrow (|x|-1)^2=0$$

$\therefore |x| = 1 \qquad \Rightarrow x = \pm 1$

$\therefore x = 1 \left(\because x > 0\right)$

$\left(\frac{1+i}{1-i}\right)^n = \frac{2}{\pi}\cdot\sin^{-1}(1) = \frac{2}{\pi}\cdot\frac{\pi}{2} = 1 \quad \Rightarrow \left(\frac{(1+i)^2}{2}\right)^n = 1$

$(i)^n = 1$

$\Rightarrow (-1)^{n/2} = (-1)^2, (-1)^4, (-1)^6, ... \Rightarrow \frac{n}{2} = 2$

$\therefore n = 4$ (least positive value)

63. If 'a' is a complex number such that $|a| = 1$, then the values of 'a', so that equation $az^2 + z + 1 = 0$ has one purely imaginary root is

A) $a = \cos\theta + i\sin\theta; \theta = \cos^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$

B) $a = \sin\theta + i\cos\theta, \theta = \cos^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$

C) $a = \cos\theta + i\sin\theta; \theta = \cos^{-1}\left(\frac{\sqrt{5}-1}{4}\right)$

D) $a = \sin\theta + i\cos\theta, \theta = \cos^{-1}\left(\frac{\sqrt{5}-1}{4}\right)$

Key. A

Sol. $az^2 + z + 1 = 0 ....(i)$

Taking conjugate of both sides, $\overline{az^2 + z + 1} = \bar{0} \quad \Rightarrow \bar{a}\left(\bar{z}\right)^2 + \bar{z} + \bar{1} = 0$

$\bar{a}z^2 - z + 1 = 0$ (since $\bar{z} = -z$ as 'z' is purely imaginary) ....(ii)

Eliminating 'z' from both the equations, we get $\left(\bar{a} - a\right)^2 + 2\left(a + \bar{a}\right) = 0$

Let $a = \cos\theta + i\sin\theta$ (since $|a| = 1$) so that $(-2i\sin\theta)^2 + 2(2\cos\theta) = 0$

$\Rightarrow \cos\theta = \frac{-1 \pm \sqrt{1+4}}{2}$

Only feasible value of $\cos\theta$ is $\frac{\sqrt{5}-1}{2}$

Hence $a = \cos\theta + i\sin\theta$, where $\theta = \cos^{-1}\left(\frac{\sqrt{5}-1}{2}\right)$

64. The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} - i\cos\frac{2k\pi}{11}\right)$ is

A) -1 B) 0 C) $-i$ D) i

Key. D

Sol. $= -i\left(\cos\frac{2k\pi}{11}+i\sin\frac{2k\pi}{11}\right) = -i\left(e^{i\frac{2\pi}{11}}\right)^k$

Let $e^{i\frac{2\pi}{11}} = z$

$\therefore \sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11}-i\cos\frac{2k\pi}{11}\right) = -i\sum_{k=1}^{10} z^k$

$= -i\left[z+z^2+z^3+.......+z^{10}\right]$

$= -i\left[\frac{z\left(z^{10}-1\right)}{z-1}\right] = -i\left[\frac{z^{11}-z}{z-1}\right] = i$

65. If 'z' lies on the circle $|z-2i| = 2\sqrt{2},$ then the value of $\arg\left(\frac{z-2}{z+2}\right)$ is equal to

A) $\frac{\pi}{3}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{6}$ D) $\frac{\pi}{2}$

Key. B

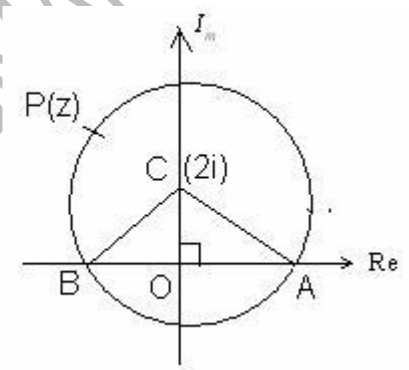
Sol. $CA = CB = 2\sqrt{2}\cdot OC = 2 \Rightarrow OA = OB = 2$

$\Rightarrow A = 2+0i, B = -2+0i$

Clearly, $\angle BCA = \frac{\pi}{2}$

$\Rightarrow \angle BPA = \frac{\pi}{4}$

$\Rightarrow \arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$



66. If 'z' is a complex number such that equation $\left|z-a^2\right|+\left|z-2a\right| = 3$ always represents an

ellipse, then range of $a\left(\in R^{+}\right)$ is

A) $\left(1,\sqrt{2}\right)$ B) $\left[1,\sqrt{3}\right]$ C) $\left(-3,1\right)$ D) $\left(0,3\right)$

Key. D

Sol. $\left|a^{2}-2a\right|<3$

$\Rightarrow -3<a^{2}-2a<3 \Rightarrow -3+1<a^{2}-2a+1<3+1 \Rightarrow -2<\left(a-1\right)^{2}<4$

$\therefore 0\le\left(a-1\right)^{2}<4 \Rightarrow -2<a-1<2$ or $-1<a<3$

But $a\in R^{+}$

$\therefore 0<a<3 \Rightarrow a\in\left(0,3\right)$

67. ω is a non real complex cube root of unity and $a,b\in R$ .If ω,ω^{2} are roots of $\frac{1}{a+x}+\frac{1}{b+x}=\frac{3}{x}$ then *a,b* are roots of

a) $3x^{2}-6x+2=0$ b) $6x^{2}-3x+2=0$

c) $2x^{2}-3x+6=0$ d) $6x^{2}-2x+3=0$

Key. B

Sol. The given equation simplifies $x^{2}+2x\left(a+b\right)+3ab=0$, whose roots are given table ω,ω^{2}

Hence $a+b=\frac{1}{2},ab=\frac{1}{3}$ .So a,b are roots of $x^{2}-x\left(\frac{1}{2}\right)+\frac{1}{3}=0$

68. If z is a complex number such that $\left|z-1\right|$ =1 then *arg* $\left(\frac{1}{z}-\frac{1}{2}\right)$ may be

a) $\frac{\pi}{6}$ b) $-\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $-\frac{\pi}{4}$

Key. B

Sol. Since $\left|z-1\right|=1\Rightarrow z-1=cis\theta\Rightarrow z=\left(1+\cos\theta\right)+i\sin\theta=2\cos\frac{\theta}{2}cis\frac{\theta}{2}$

$\therefore\frac{1}{z}-\frac{1}{2}=\frac{cis\left(-\frac{\theta}{2}\right)}{2\cos\frac{\theta}{2}}-\frac{1}{2}=-\frac{i}{2}\tan\frac{\theta}{2}$ which is purely imaginary

69. $\theta\in\left[0,2\pi\right]$ and z_1,z_2,z_3 are three complex numbers such that they are collinear and $\left(1+\left|\sin\theta\right|\right)z_1+\left(\left|\cos\theta\right|-1\right)z_2-\sqrt{2}\ z_3$ =0 .If at least one of the complex numbers z_1,z_2,z_3 is non- zero then number of possible values of θ is

a) Infinite b) 4 c) 2 d) 8

Key. B

Sol. If z_1, z_2, z_3 are collinear and $az_1 + bz_2 + cz_3 = 0$ then a+b+c =0 .Hence

$$1 + |\sin\theta| + |\cos\theta| - 1 - \sqrt{2} = 0 \Rightarrow |\sin\theta| + |\cos\theta| = \sqrt{2}$$

$$\Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

70. Let $A(z_1), B(z_2), C(z_3)$ be the vertices of a triangle oriented in anti clock wise direction. If $BC : CA : AB = 2 : \sqrt{2} : \sqrt{3}+1$, then the imaginary part of $\left(\frac{z_3 - z_1}{z_2 - z_1}\right)^4$ is

A) 0

B) $-7 + 2\sqrt{6}$

C) $7 - 2\sqrt{6}$

D) cannot be determined

Key. A

Sol. $\cos A = +\frac{1}{\sqrt{2}} \Rightarrow A = \frac{\pi}{4}$

$$\therefore \frac{z_3 - z_1}{z_2 - z_1} = \left|\frac{z_3 - z_1}{z_2 - z_1}\right| \text{cis } (\pi/4)$$

$$\Rightarrow \left(\frac{z_3 - z_1}{z_2 - z_1}\right)^4 = \left(\frac{\sqrt{2}}{\sqrt{3}+1}\right)^4 e^{i\pi} \Rightarrow \left(\frac{z_3 - z_1}{z_2 - z_1}\right)^4 = -\left(\frac{\sqrt{3}-1}{2}\right)^4$$

71. A,B,C are vertices of a triangle inscribed in the circle $|z| = 1$. Altitude from A meets the circumcircle again at D. If D,B,C represents the complex number z_1, z_2, z_3 respectively then the complex number representing the reflection of D in the line BC, is

A) $\frac{z_1 z_2 + z_1 z_3 + z_2 z_3}{z_1}$

B) $\frac{z_1 z_2 + z_2 z_3 + z_1 z_3}{z_1 z_2 z_3}$

C) $\frac{z_1 z_2 + z_1 z_3 - z_2 z_3}{z_1}$

D) $\frac{z_1 z_2 + z_1 z_3 - z_2 z_3}{z_1 z_2 z_3}$

Key. C

Sol. image of D w.r.t sides of triangle is orthocenter

72. A point P representing the complex number z moves in the Argand plane so that it lies always in the region defined by $|z - 1| \le |z - i|$ and $|z - 2 - 2i| \le 1$. If P describes the boundary of this

region then the value of $|z|$ when the arg(z) has least value, is

A) $\sqrt{5}$ B) 7 C) $\sqrt{7}$ D) 5

Key. C

Sol. $|z| = OR = \sqrt{8-1} = \sqrt{7}$

73. Let P(z) be a variable point in the complex plane such that $|z| = \min\{|z-1|, |z+1|\}$ then the value of $\left(z+\bar{z}\right)$ is

A) 1 if Re(z)>0 B) 1 if Re z<0 C) 0 if Re z>0 D) 0 if Re z <0

Key. A

Sol. Let $|z| = |z-1|$ if Re $Z > 0$

$\Rightarrow$ lies line $z = \dfrac{1}{2}$

$\Rightarrow z + \bar{z} = \dfrac{1}{2} + \dfrac{1}{2} = 1$

74. If z is a complex number satisfying $|z|^2 + 2(z+\bar{z}) + 3i(z-\bar{z}) + 4 = 0$, $i = \sqrt{-1}$, then the complex number $z + 3 + 2i$ will lie on a circle with

A) centre $1-5i$, radius 4 B) centre $1+5i$, radius 4

C) centre $1+5i$, radius 3 D) centre $1-5i$, radius 3

Key. C

Sol. Given $|z + (2-3i)| = 3$, Let $w = \left(z+3+2i\right) = z + 2 - 3i + 1 + 5i$

$\Rightarrow |w - (1+5i)| = |z + 2 - 3i| = 3$.

75. The value of $i\log_e(x-i) + i^2\pi + i^3\log_e(x+i) + i^4\left(2\tan^{-1}x\right)$, $x > 0$, $i = \sqrt{-1}$ is

A) 0 B) 1 C) 2 D) 3

Key. A

Sol. Let $i\log\dfrac{x-i}{x+i} - \pi + 2\tan^{-1}x = k$

$\Rightarrow \log\left(\dfrac{x+i}{x-i}\right) = \left(k + \pi - 2\tan^{-1}x\right)i = i\theta$

$\Rightarrow \dfrac{x+i}{x-i} = e^{i\theta} \quad \Rightarrow x = \cot\dfrac{\theta}{2} \quad \Rightarrow \theta = 2\cot^{-1}x$

$\therefore k + \pi - 2\tan^{-1}x = 2\cot^{1}x \quad \Rightarrow k = 0$

76. If $\left|\frac{z_1}{z_2}\right| = 1$ and arg $(z_1\ z_2) = 0$, then

A) $z_1 = z_2$ B) $|z_2|^2 = z_1 z_2$ C) $z_1 z_2 = 1$ D) $z_1 z_2 = 2$

Key. B

Sol. $\left|\frac{z_1}{z_2}\right| = 1 \Rightarrow |z_1| = |z_2| = r_1$ as $z_1 = r_1(\cos\theta_1 + \sin\theta)$

$\arg(z_1 z_2) = 0 \quad \Rightarrow z_2 = r_1(\cos(-\theta_1) + i\sin(-\theta_2))$

$\Rightarrow z_2 = \overline{z}_1 \quad \Rightarrow \overline{z}_2 = z_1 \Rightarrow |z_2|^2 = z_1 . z_2$

77. Let z be a complex number and $a_k, b_k (k = 1, 2, 3)$ are real numbers then the value of
$\begin{vmatrix} a_1 z + b_1\overline{z} & a_2 z + b_2\overline{z} & a_3 z + b_3\overline{z} \\ b_1 z + a_1\overline{z} & b_2 z + a_2\overline{z} & b_3 z + a_3\overline{z} \\ b_1 z + a_1 & b_2 z + a_2 & b_3 z + a_3 \end{vmatrix} =$

A) $(a_1 a_2 a_3 + b_1 b_2 b_3)|z|^2$ B) $(a_1 a_2 a_3 - b_1 b_2 b_3)|z|^2$

C) $a_1^{\ 2} - a_1^{\ 2}$ D) $|z|^2$

Key. C

Sol. $\begin{vmatrix} z & \overline{z} & 1 \\ \overline{z} & z & 1 \\ 1 & z & \overline{z} \end{vmatrix} \begin{vmatrix} a_1 & b_1 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & 0 \end{vmatrix} = 0$

78. If z_1, z_2, z_3 are the vertices of an equilateral triangle inscribed in the circle $|z| = 1$ then area of region common to given triangle and another triangle having vertices $-z_1, -z_2, -z_3$, is

A) $\frac{\sqrt{3}}{2}$ B) $\frac{\sqrt{3}}{4}$ C) $\frac{7\sqrt{3}}{4}$ D) $\frac{5\sqrt{3}}{4}$

Key. A

Sol. Area of common region

= Area of ΔABC - 3 Area of $AB'C'$

= $3\frac{\sqrt{3}}{4} - 3.\frac{1}{2}.\frac{1}{2}\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{2}$.

79. If $az^2 + bz + 1 = 0$, $a, b, z \in C$ and $|a| = \frac{1}{2}$, have a root α such that $|\alpha| = 1$ then $|a\overline{b} - b| =$

A) $\frac{1}{4}$ B) $\frac{1}{2}$ C) $\frac{5}{4}$ D) $\frac{3}{4}$

Key. D

Sol. $a\alpha^2 + b\alpha + 1 = 0$

$\bar{a}\bar{\alpha}^2 + \bar{b}\bar{\alpha} + 1 = 0$

$\Rightarrow \alpha^2 + \bar{b}\alpha + \bar{a} = 0$

$\frac{\alpha^2}{\bar{a}b - \bar{b}} = \frac{\alpha}{1-|a|^2} = \frac{1}{a\bar{b} - b} \Rightarrow |a\bar{b} - b| = 1 - |a|^2 = 1 - \frac{1}{4} = \frac{3}{4}$

80. If $|z-1| \le 2$ & $|wz - 1 - w^2| = a$ (where 'w' is a cube root of unity) then complete set of values of a is

a) $0 \le a \le 2$ b) $\frac{1}{2} \le a \le \frac{\sqrt{3}}{2}$

c) $\frac{\sqrt{3}}{2} - \frac{1}{2} \le a \le \frac{1}{2} + \frac{\sqrt{3}}{2}$ d) $0 \le a \le 4$

Key. D

Sol. $|wt - 1 - w^2| = a$

$\Rightarrow |w||z+1| = a$

$\Rightarrow |z-1| + 2 \ge a$

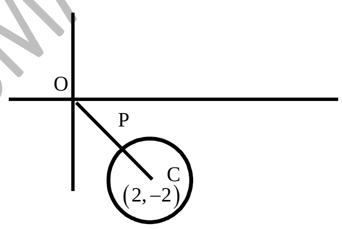
81. If z is a complex number having least absolute value and $|z - 2 + 2i| = 1$ then z =

a) $\left(2 - \frac{1}{\sqrt{2}}\right)(1-i)$ b) $\left(2 - \frac{1}{\sqrt{2}}\right)(1+i)$

c) $\left(2 + \frac{1}{\sqrt{2}}\right)(1-i)$ d) $\left(2 + \frac{1}{\sqrt{2}}\right)(1+i)$

Key. A

Sol. OP = OC - CP



$= 2\sqrt{2} = 1$

$\therefore (0,0)\ (2,-2)$

$2\sqrt{2} - 1 : 1$

$\frac{2(2\sqrt{2}-1)}{2\sqrt{2}}, \frac{-2(2\sqrt{2}-1)}{2\sqrt{2}}$

$= \left(\left(2-\frac{1}{\sqrt{2}}\right), -\left(2-\frac{1}{\sqrt{2}}\right)\right)$

82. Sum of common roots of the equation $z^3+2z^2+2z+1=0$ and $z^{1985}+z^{100}+1=0$ is

a) -1 b) 1 c) 0 d) 1

Key. A

Sol. $(z+1)(z^2+z+1)$

$\Rightarrow z=-1, w, w^2$

Let $f(z)=z^{1985}+z^{100}+1$

$f(-1)\neq 0, f(w)=f(w)=0$

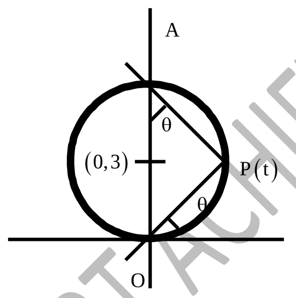
$\therefore w+w^2=-1$

83. Let z be a complex number having the argument $\theta,\ 0<\theta<\frac{\pi}{2}$ and satisfying the equation, $|z-3i|=3$. Then $\cot\theta-\frac{6}{z}=$

a) i b) – i c) 2i d) – 2i

Key. A

Sol. $r=0A\sin\theta=6\sin\theta$



$z=6\sin\theta(\cos\theta+i\sin\theta)$

$\Rightarrow \cot\theta-\frac{6}{z}=i$

84. If $a^2+b^2, ab+bc$ and b^2+c^2 are in G.P. then a, b, c are in

a) A.P b) G.P

c) H.P d) None

Key. B

Sol. $(ab+bc)^2=(a^2+b^2)(b^2+c^2)$

85. If $H_n=1+\frac{1}{2}+....+\frac{1}{n}$ then the value of $S_n=1+\frac{3}{2}+\frac{5}{3}+....+\frac{99}{50}$ is___

a) $H_{50}+50$ b) $100-H_{50}$

c) $49+H_{50}$ d) $H_{50}+100$

Key. B

Sol. $S_n=(2-1)+\left(2\frac{-1}{2}\right)+\left(2-\frac{1}{3}\right)+...+\left(2-\frac{1}{50}\right)$

$=100-H_{50}$

86. Let z be a complex number satisfying $|z+16|=4|z+1|$ then

a) $|z|=4$ b) $|z|=5$

c) $|z|=6$ d) $3<|z|<6$

Key. A

Sol. $|z+16|^2=16|z+1|^2 \Rightarrow (z+16)(\bar{z}+16)=16(z+1)(\bar{z}+1)$

$\Rightarrow z\bar{z}+16z+16\bar{z}+256=16z\bar{z}+16z+16\bar{z}+16$

$\Rightarrow z\bar{z}=16 \Rightarrow |z|^2=16 \Rightarrow |z|=4$

87. Let z_1 and z_2 be any two complex numbers then $\left|z_1+\sqrt{z_1^2-z_2^2}\right|+\left|z_1-\sqrt{z_1^2-z_2^2}\right|$ is equal to

a) $|z_1^2-z_2^2|+|z_1^2+z_2^2|$ b) $|z_1-z_2|+|z_1^2+z_2^2|$

c) $|z_1+z_2|+|z_1^2+z_2^2|$ d) $|z_1+z_2|+|z_1-z_2|$

Key. D

Sol. If $z_1+\sqrt{z_1^2-z_2^2}=u$ and $z_1-\sqrt{z_1^2-z_2^2}=v$,

We have

$|u|^2+|v|^2=\frac{1}{2}|u+v|^2+\frac{1}{2}|u-v|^2$

$=2|z_1|^2+2|z_1^2-z_2^2|$

And so

$(|u|+|v|)^2=2\left\{|z_2|^2+|z_z^1-z_2^2|+|z_2|^2\right\}$

$=|z_1z_2|^2+|z_2-z_2|^2+2|z_1^2-z_2^2|$

$=(|z_1+z_2|+|z_1-z_2|)^2$

88. Both the roots of the equation $z^2+az+b=0$ are of unit modulus if

a) $|a|\le 2, |b|=1, \arg b=2\arg a$ b) $|a|\le 2, |b|=1, \arg b=\arg a$

c) $|a|\ge 2, |b|=2, \arg b=2\arg a$ d) $|a|\ge 2, |b|=2, \arg b=\arg a$

Key. A

Sol. Let $z_1=\cos\phi_1+i\sin\phi_1, \& z_2=\cos\phi_2+i\sin\phi_2$

Be the roots of $z^2+az+b=0$

$z_1 + z_2 = (-a)$ & $z_1 z_2 = b$

$$-2\cos\left(\frac{\phi_1 - \phi_2}{2}\right)\left[\cos\frac{\phi_1+\phi_2}{2} + i\sin\frac{\phi_1+\phi_2}{2}\right] = a$$

$$\Rightarrow \arg(a) = \frac{\phi_1+\phi_2}{2}$$

$\arg b = \phi_1 + \phi_2$

$\therefore \arg b = 2\arg a$

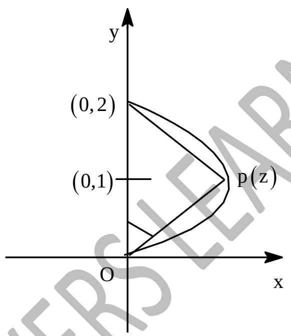
Also $|z_1 z_2| = |b| = 1$ and $|a| \le 2$

89. If $|z - i| = 1$ and $\arg(z) = \theta$ where $\theta \in \left(0, \frac{\pi}{2}\right)$, then $\cot\theta - \frac{2}{z}$ equals

a) $2i$ b) 3i c) i d) – i

Key. C

Sol. $< AOP = \frac{\pi}{2} - \theta$



$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta = \frac{|z|}{2}$$

Also $\frac{2i}{z} = \frac{OA}{OP}(\sin\theta + i\cos\theta) = \frac{2}{|z|}(\sin\theta + i\cos\theta)$

$= 1 + i\cot\theta$

$$\frac{2}{z} = -i + \cot\theta$$

$$\Rightarrow \cot\theta - \frac{2}{z} = i$$

90. Let $z = \frac{z_1 - z_2}{z_1\bar{z}_2 - 1}, z_1 \ne \frac{1}{\bar{z}_2}, 0 < |z_2| < 1$. If $|z| \le 1$ then

a) $|z_1| > 1$ b) $|z_1| \le 1$

c) $2 < |z_1| < 3$ d) $2 < |z_1| < 8$

Key. B

Sol. $z\bar{z} - 1 = \left(\frac{z_1 - z_2}{z_1\bar{z}_2 - 1}\right)\left(\frac{\bar{z}_1 - \bar{z}_2}{\bar{z}_1 z_2 - 1}\right) - 1$

$$=\frac{z_1\bar{z}_1+z_2\bar{z}_2-z_1z_2\bar{z}_1\bar{z}_2-1}{z_1z_2\bar{z}_1\bar{z}_2-z_1\bar{z}_2-\bar{z}_1z_2+1}$$

$$=\frac{\left(1-z_2\bar{z}_2\right)\left(z_1\bar{z}_1-1\right)}{\left(z_1\bar{z}_2-1\right)\left(\bar{z}_1z_2-1\right)}$$

$$\Rightarrow |z|^2-1=\frac{\left(1-|z_2|^2\right)\left(|z_1|^2-1\right)}{\left|z_1\bar{z}_2-1\right|^2}$$

$$\therefore |z_1|^2-1\le 0 \Rightarrow |z_1|\le 1$$

91. Let 'z' be a complex number satisfying $|z-2-i|\le 5$, Then $|z-14-6i|$ lies in

a) $[8,18]$ b) $(2,8)$

c) $[0,2]$ d) $[3,7]$

Key. A

Sol. $|z-14-6i|=|(z-2i)-(12+5i)|\le|z-2-i|+|12+5i|$

$\Rightarrow |z-14-6i|\le 5+13=18$

$\therefore$ Option a is correct

The complete solution can be obtained geometrically

92. If z_1,z_2 are complex numbers such that $z_1^3-3z_1z_2^2=2$ and $3z_1^2z_2-z_2^3=11$ then $\left|z_1^2+z_2^2\right|=$

A) 3 B) 4 C) 5 D) 6

Key. C

Sol. $z_1^3-3z_1z_2^2+3iz_1^2z_2-iz_2^3=2+11i \Rightarrow (z_1+iz_2)^3=2+11i$

Similarly, $(z_1-iz_2)^3=2-11i$

$\left|z_1^2+z_2^2\right|=\left|(z_1+iz_2)(z_1-iz_2)\right|=\left|(2+11i)^{1/3}(2-11i)^{1/3}\right|=5$

93. The circle $|z|=2$ intersects the curve whose equation is $z^2=\left(\bar{z}\right)^2+4i$ in the points $A,B,C,D.$ If z_1,z_2,z_3,z_4 represent the affixes of these points, then

A) $z_1z_2z_3z_4=1$

B) $\overline{z_1}+\overline{z_2}+\overline{z_3}+\overline{z_4}=0$

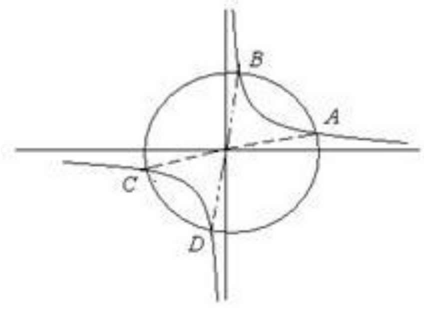
C) $z_1+z_2+z_3+z_4=2$

D) $\arg z_1 + \arg z_2 + \arg z_3 + \arg z_4 = 2k\pi,\ k = 0, 1\ or\ -1$

Key. B

Sol. $z^2 = (\bar{z})^2 + 4i \Rightarrow \left(\frac{z+\bar{z}}{2}\right)\left(\frac{z-\bar{z}}{2i}\right) = 1$ or $xy = 1$ (where $z = x + iy$)

The circle $x^2 + y^2 = 4$ intersects the rectangular hyperbola in four points, which are symmetrical about the origin in parts.



94. If $a_1, a_2, ..., a_n$ are real numbers with $a_n \neq 0$ and $\cos\alpha + i\sin\alpha$ is a root of $z^n + a_1 z^{n-1} + a_2 z^{n-2} + ... + a_{n-1} z + a_n = 0,$ then the sum $a_1\cos\alpha + a_2\cos 2\alpha + a_3\cos 3\alpha + ... + a_n\cos n\alpha$ is

A) 0 B) 1 C) -1 D) $\frac{1}{2}$

Key. C

Sol. $\cos\alpha + i\sin\alpha$ is a root of $a_n\left(\frac{1}{z}\right)^n + a_{n-1}\left(\frac{1}{z}\right)^{n-1} + ... + a_2\left(\frac{1}{z}\right)^2 + a_1\left(\frac{1}{z}\right) + 1 = 0.$

Equating real parts on both sides,

$a_n\cos n\alpha + a_{n-1}\cos(n-1)\alpha + ... + a_1\cos\alpha + 1 = 0$

95. .If $\left(\frac{3-z_1}{2-z_1}\right)\left(\frac{2-z_2}{3-z_2}\right) = k$, then points $A(z_1)$, $B(z_2)$, $C(3,0)$ and $D(2,0)$ (taken in clockwise sense) will

A) lie on a circle only for $k > 0$

B) lie on a circle only for $k < 0$

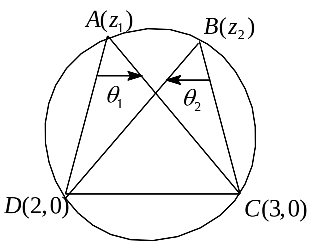
C) lie on a circle $\forall\ k \in R$

D) be vertices of a square $\forall\ k \in (0,1)$

Key: A

Sol : $\arg\left(\frac{3-z_1}{2-z_1}\right) + \arg\left(\frac{2-z_2}{3-z_2}\right)$

$= \arg\left(\frac{3-z_1}{2-z_1}\right)\left(\frac{2-z_2}{3-z_2}\right)$



If $k > 0$, its argument will be zero

So, θ_1 & θ_2 are equal in magnitude but opposite sign.

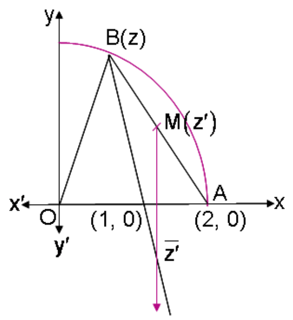
So DC subtends equal angle at A & B. So, points are concyclic if $k > 0$

96. If A(2,0) and B(z) are two points on the circle | z | = 2. $M(z')$ is the point on AB. If the point $\overline{z'}$ lies on the median of the triangle OAB where O is origin then $\arg(z')$ is

a) $\tan^{-1}\left(\frac{\sqrt{15}}{5}\right)$ b) $\tan^{-1}\left(\sqrt{15}\right)$ c) $\tan^{-1}\left(\frac{5}{\sqrt{15}}\right)$ d) $\frac{\pi}{2}$

Key: A

Sol: $M(z')$ is mid-point of AB, so $z' = \frac{z+2}{2}$



$\Rightarrow \overline{z'} = \frac{\bar{z}+2}{2}$

$\Rightarrow z, 1, \frac{\bar{z}}{2}+1$ are collinear

$$\Rightarrow \begin{vmatrix} z & \bar{z} & 1 \\ \frac{\bar{z}}{2}+1 & \frac{z}{2}+1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$\Rightarrow z\left(\frac{z}{2}+1-1\right) - \bar{z}\left(\frac{\bar{z}}{2}+1-1\right) + 1\left(\frac{\bar{z}}{2} - \frac{z}{2}\right) = 0$$

$$\Rightarrow \quad \frac{z^2}{2} - \frac{\bar{z}^2}{2} + \frac{(\bar{z}-z)}{2} = 0$$

$$\Rightarrow \quad (z-\bar{z})(z+\bar{z}-1) = 0$$

$$\Rightarrow \quad z-\bar{z} = 0 \text{ or } (z+\bar{z}-1) = 0$$

$$\Rightarrow \quad z+\bar{z} = 1 \text{ or } \operatorname{Re}(z) = \frac{1}{2}$$

$$|z| = 2 \Rightarrow \frac{1}{4} + \operatorname{Im}(z)^2 = 4$$

$\Rightarrow \qquad \text{Im}(z) = \frac{\sqrt{15}}{2}$

1. $z = \frac{1}{2} + \frac{i\sqrt{15}}{2}$

2. $\arg(z') = \tan^{-1}\left(\frac{\sqrt{15}}{5}\right)$

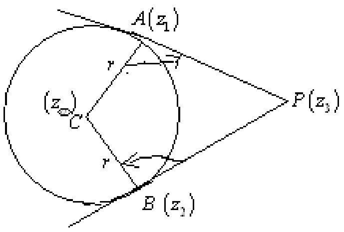
97. If the tangents at z_1, z_2 on the circle $|z - z_o| = r$ intersect at $z_3, then \ \frac{(z_3 - z_1)(z_o - z_2)}{(z_o - z_1)(z_3 - z_2)}$ equals

a) 1 b) -1 c) i d) – i

Key: B

Hint: $\frac{z_3 - z_1}{z_o - z_1} = \left(\frac{PA}{AC}\right) i \ and \ \frac{z_o - z_2}{z_3 - z_2} = \left(\frac{BC}{BP}\right)(i)$

$$\frac{(z_3 - z_1)(z_o - z_2)}{(z_o - z_1)(z_3 - z_2)} = \left(\frac{PA}{AC} \times \frac{BC}{PB}\right)(-1) = -1$$



98. If Z is a complex number then the number of complex numbers satisfying the equation $Z^{2009} = \overline{Z}$ is

A) 3 B) 2009 C) 2010 D) 2011

Key: D

Sol. $Z^{2009} = \overline{Z} \Rightarrow |Z| = 0$ or $|Z| = 1$

99. If $a_1, a_2, ..., a_n$ are real numbers with $a_n \neq 0$ *and* $\cos\alpha + i\sin\alpha$ *is a root of* $z^n + a_1 z^{n-1} + a_2 z^{n-2} + ... + a_{n-1} z + a_n = 0,$ then the sum $a_1 \cos\alpha + a_2 \cos 2\alpha + a_3 \cos 3\alpha + ... + a_n \cos n\alpha$ is

a) O b) 1 c) – 1 d) 1/2

Key: C

Sol. $\cos\alpha + i\sin\alpha$ is a root of $a_n\left(\frac{1}{z}\right)^n + a_{n-1}\left(\frac{1}{z}\right)^{n-1} + ... + a_2\left(\frac{1}{z}\right)^2 + a_1\left(\frac{1}{z}\right) + 1 = 0.$ Equating real parts on both sides , $a_n \cos n\alpha + a_{n-1}\cos(n-1)\alpha + ... + a_1 \cos\alpha + 1 = 0$.

100. If is a cube root of unity, then $\omega+\omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}+......\infty\right)}=$

a) i b) i^2 c) 0 d) i^3

Key: B

Sol. $\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+..........=\frac{1}{2}\left(1+\frac{3}{4}+\frac{9}{16}+..........\infty\right)$

$=\frac{1}{2}\left[\frac{1}{1-\frac{3}{4}}\right]=\frac{1}{2}\times 4=2.$

101. If z_1, z_2, z_3 are any three complex numbers on Argand plane then $z_1\left(\text{Im}\left(\bar{z}_2 z_3\right)\right)+z_2\left(\text{Im}\left(\bar{z}_3 z_1\right)\right)+z_3\left(\text{Im}\left(\bar{z}_1 z_2\right)\right)$ is equal to

(A) 0 (B) $z_1+z_2+z_3$

(C) $z_1 z_2 z_3$ (D) $\left(\frac{z_1+z_2+z_3}{z_1 z_2 z_3}\right)$

Key: A

Hint: $z_1\left(\frac{\bar{z}_2 z_3 - z_2\bar{z}_3}{2i}\right)+z_2\left(\frac{\bar{z}_3 z_1 - z_3\bar{z}_1}{2i}\right)+z_3\left(\frac{\bar{z}_1 z_2 - z_1\bar{z}_2}{2i}\right)=\frac{1}{2i}\times 0=0$

102. Let points P and Q correspond to the complex numbers α and β respectively in the complex plane. If $|\alpha|=4;$ and $4\alpha^2-2\alpha\beta+\beta^2=0,$ then the AREA OF THE $\Delta OPQ,$ O being the origin equals

A) $8\sqrt{3}$ B) $4\sqrt{3}$ C) $6\sqrt{3}$ D) $12\sqrt{3}$

Key: A

Hint: Conceptual

103. Suppose two complex numbers $z=a+ib; w=c+id$ satisfy the equation $\frac{z+w}{z}=\frac{w}{z+w}$ then

A) both a & c are zeros B) both b & d are zeros

C) both b & d must be non zeros D) at least one of b & d is non-zero

Key: D

Hint: $(z+w)^2=zw \Rightarrow z^2+zw+w^2=0$

Let $\frac{z}{w}=t \Rightarrow \frac{z}{w}=\frac{-1\pm\sqrt{3}i}{2}$

$\arg z-\arg w=\frac{2\pi}{3}$ or $\arg z-\arg w=-\frac{2\pi}{3}$

104. If x = a+ib is a complex number such that x^2 =3+4i and x^3=2+11i where i = $\sqrt{-1}$ then a+b =____________________________

1. 1 2. 2 3. 3 4. 4

Key. 3

Sol. $x=\frac{x^3}{x^2}=\frac{2+11i}{3+4i}=\frac{(2+11i)(3-4i)}{25}$

$\therefore a+ib=\frac{6+44+25i}{25}=2+i$

$\Rightarrow a=2, b=1 \Rightarrow a+b=3$

105. If the complex number Z satisfying $Z+|Z|=2+8i$ then value of $|Z|$ =

1. 8 2.17 3. 15 4. 24

Key. 2

Sol. Let $z=a+ib$

$\Rightarrow a+ib+\sqrt{a^2+b^2}=2+8i$

$\Rightarrow b=8,\ a+\sqrt{a^2+64}=2$

$a^2+64=a^2-4a+4$

$\Rightarrow -4a=60 \Rightarrow a=-15$

$\therefore |z|=\sqrt{a^2+b^2}=\sqrt{225+64}=\sqrt{289}=17$

106. If $|Z+2-i|=5$ then maximum value of $|3Z+9-7i|=$ ______________

1. 20 2.15 3. 5 4. 16

Key. 1

Sol. $|3Z+9-7i|=|3Z+6-3i+3-4i|$

$\leq |3(z+2-i)+|=|3-4i|$

$\leq 3|z+2-i|+\sqrt{3^2+4^2}$

$3(5)+5=20$

107. If Z-lies on the circle $|z-2i|=2\sqrt{2}$ *then value of* $\arg\left(\frac{Z-2}{Z+2}\right)$ *is*

1. $\frac{\pi}{3}$ 2. $\frac{\pi}{4}$ 3. $\frac{\pi}{6}$ 4. $\frac{\pi}{2}$

Key. 2

Sol. Circle with centre (0,2) cuts X-axis at A(-2,0) and B(2,0). Now AB subtends an angle 90^0 at the center C.

AB subtends an angle $\frac{\pi}{4}$ at any point z on the major arc circle arg $\left(\frac{z-2}{z+2}\right)=\frac{\pi}{4}$

108. If $\lambda \in R$ and non real roots of $2Z^2+2Z+\lambda=0$ and (0,0) forms vertices of an equilateral triangle then λ =

1. 1 2. $\frac{1}{2}$ 3. $\frac{1}{3}$ 4. $\frac{2}{3}$

Key. 4

Sol. Let $z_{1,}z_2$ be roots of $2z^2+2z+\lambda=0$

$z_1+z_2=-1 \quad z_1z_2=\frac{\lambda}{2}$

When origin , z_1z_2 forms equilateral Δ^{le}

We have $z_1^2+z_2^2=z_1z_2$

$(z_1+z_2)^2=3z_1z_2$

$1=\frac{3.\lambda}{2} \Rightarrow \lambda=\frac{2}{3}$

109. The greatest positive argument of z satisfying $|Z-4|=\mathrm{Re}(Z)$

1. $\frac{\pi}{3}$ 2. $\frac{2\pi}{3}$ 3. $\frac{\pi}{2}$ 4. $\frac{\pi}{4}$

Key. 4

Sol. $|x+iy-4|=x$

$(X-4)^2+y^2+x^2$

$y^2-8x+16=0$

z lies on the parabola with vertex (2,0) focus (4,0) and tangents from (0,0) ie a point on the directrix in x always include 90^0

$\therefore$ greatest arg(z) is $45^0=\frac{\pi}{4}$

110. If Z and W are two complex numbers such that $\bar{z}+i\,\overline{w}=0$ and arg (Zw)= π then arg (Z) = ______

1. $\frac{\pi}{4}$ 2. $\frac{\pi}{2}$ 3. $\frac{3\pi}{4}$ 4. $\frac{5\pi}{4}$

Key. 3

Sol. $\bar{z}+i\bar{w} \Rightarrow z-iw=0 \Rightarrow z=iw$

Arg $(zw)=\pi \Rightarrow \arg(z)+\arg(w)=\pi$

$\arg(iw)+\arg w=\pi$

$\arg i+2\arg w=\pi$

$\frac{\pi}{2}+2\arg w=\pi$

$2\arg w=\frac{\pi}{2}$

$\arg w=\frac{\pi}{4}\Rightarrow \arg(z)=\frac{3\pi}{4}$

111. If A (Z_1) B(Z_2) C(Z_3) are vertices of a triangle such that

$Z_3=\left(\frac{Z_2-iZ_1}{1-i}\right)$ and $|Z_1|=3, |Z_2|=4$ and $|Z_2+iZ_1|=|Z_1|+|Z_2|$ then area of triangle ABC is

1. $\frac{5}{2}$ 2. 0 3. $\frac{25}{2}$ 4. $\frac{25}{4}$

Key. 4

Sol. $|z_2+iz_1|=|z_1|+|z_2|\Rightarrow z_2, iz_1, 0$ are collinear.

$\therefore \arg(iz_1=\arg z_2)$

$\Rightarrow \arg i+\arg z_1=\arg z_2$

$\Rightarrow \arg z_2-\arg z_1=\frac{\pi}{2}$

$z_3=\frac{z_2-iz_1}{l-i}$

$(l-i)z_3=z_2-iz_1$

$z_3-z_2=i(z_3-z_1)$

$\frac{z_3-z_2}{z_3-z_1}=i\Rightarrow \arg\left(\frac{z_3-z_2}{z_3-z_1}\right)=\frac{\pi}{2}$ and $|z_3-z_2|=|z_3-z_1|$

$\therefore$ AB=BC, $\therefore AB^2=AC^2+BC^2$

$25=2AC^2$

$\Rightarrow AC=\frac{5}{\sqrt{2}}$

Required area= $\frac{1}{2}\times\frac{5}{\sqrt{2}}\times\frac{5}{\sqrt{2}}=\frac{25}{4}$ sq. units

112. The radius of the circle given by arg $\left(\frac{Z-5+4i}{Z+3-2i}\right)=\frac{\pi}{4}$

1. $5\sqrt{2}$ 2. 5 3. $\frac{5}{\sqrt{2}}$ 4. $\sqrt{2}$

Key. 1

Sol. A(5,-4) B(-3,2) subtends an angle $\frac{\pi}{4}$ at C(z) on the circle hence $\frac{\pi}{2}$ at centre

$$M \to M.dAB \therefore AM = \frac{AB}{2}$$

$$= \frac{\sqrt{64+36}}{2}\frac{10}{2} = 5$$

Radius $= \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$

113. f(x) = $2x^3 + 2x^2 - 7x + 72$ then $f\left(\frac{3-5i}{2}\right)$ = ____________

1. 1 2. 2 3.3 4.4

Key. 4

Sol. Let $x = \frac{3-5i}{2}$

$$2x = 3-5i$$

$$(2x-3)^2 = 5i$$

$$4x^2 - 12x + 9 = 25i^2$$

$$\Rightarrow 2x^2 - 12x + 34 = 0 \Rightarrow 2x^2 - 6x + 17 = 0$$

$$2x^2 - 6x + 17)2x^3 + 2x^2 = 7x + 72(x+4$$

$$2x^3 - 6x^2 + 17x$$

$$8x^2 - 24x + 72$$

$$8x^2 - 24x + 68$$

$$4$$

114. If $Cos\,\alpha + Cos\,\beta + Cos\gamma = 0 = Sin\,\alpha + Sin\,\beta + Sin\gamma$ then $Sin^2\,\alpha + Sin^2\,\beta + Sin^2\gamma$ = _______

1. $\frac{1}{2}$ 2. $\frac{3}{2}$ 3. 4 4. 1

Key. 2

Sol. Let $x = cis\alpha$ $y = cis\beta$ $z = cis\gamma$

Clearly $x + y + z = 0, \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

$$x^2 + y^2 + z^2 = (x+y+z)^2 - 2xyz\left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z}\right) = 0$$

$$= cis2\alpha + cis2\beta + cis2\gamma = 0$$

$$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = 0$$

$$1-2\sin^2\alpha+1-2\sin^2\beta+1-2\sin^2\gamma=0$$

$$\Rightarrow \sin^2\alpha+\sin^2\beta+\sin^2\gamma=\frac{3}{2}$$

115. If $Z_1 \, and \, Z_2$ are two complex numbers such that $Z_1{}^2+Z_2{}^2 \in R \; and \; Z_1(Z_1{}^2-3Z_2{}^2)=2$ $Z_2(3Z_1{}^2-Z_2{}^2)=11$ then $Z_1{}^2+Z_2{}^2=$

A) 5 2.125 3. 25 4. 15

Key. 1

Sol. $z_1\left(z_1^2-3z_2^2\right)=2$

$$z_1^2\left(z_1^4+9z_2^4-6z_1^2z_2^2\right)=4$$

$$\left(z_1^2\right)^3+9z_1^2z_2^4-6z_1^4z_2^2=4 \longrightarrow (1)$$

$$z_2^2\left(3z_1^2-z_2^2\right)^2=|121|$$

$$\Rightarrow\left(z_2^2\right)^3+9z_2^2z_1^4-6z_1^2z_2^4=121 \longrightarrow (2)$$

(1) + (2) $\Rightarrow \left(z_1^2+z_2^2\right)^3 125$

$$z_1^2+z_2^2=5$$

116. Let $z=\cos\theta+i\sin\theta$. Then, the value of $\sum_{m=1}^{15} lm\left(z^{2m-1}\right)$ at $\theta=2^0$ is

(A) $\frac{1}{2^0}$ (B) $\frac{1}{3\sin 2^0}$ (C) $\frac{1}{2\sin 2^0}$ (D) $\frac{1}{4\sin 2^0}$

Key. D

Sol. Given that $z=\cos\theta+isin\theta=e^{i\theta}$

$$\therefore \sum_{m=1}^{15}\left(z^{m-1}\right)=\sum_{m=1}^{15} lm\left(e^{i\theta}\right)^{2m-1}$$

$$=\sum_{m=1}^{15} lm\, e^{i(2m-1)\theta}$$

$$=\sin\theta+\sin 3\theta+\sin 5\theta+.......+\sin 29\theta$$

$$=\frac{\sin\left(\frac{\theta+29\theta}{2}\right)\sin\left(\frac{15\times 2\theta}{2}\right)}{\sin\left(\frac{2\theta}{2}\right)}$$

$$=\frac{\sin(15\theta)\sin(15\theta)}{\sin\theta}=\frac{1}{4\sin 2^0}$$

117. If z_1 is a root of the equation $a_0z^n+a_1z^{n-1}+.....+a_{n-1}z+a_n=3$, where $|a_i|<2$ for $i=0,1,.....n$. Then

(A) $|z_1|>\frac{1}{3}$ (B) $|z_1|<\frac{1}{4}$ (C) $|z_1|>\frac{1}{4}$ (D) $|z|<\frac{1}{3}$

Key. A

Sol. $a_0z^n + a_1z^{n-1} + a_2z^{n-2} + ... + a_{n-1}z + a_n = 3$

$|3| = |a_0z^n + a_1z^{n-1} + a_2z^{n-2} + ... + a_{n-1}z + a_n|$

$3 \le |a_0||z|^n + |a_1||z|^{n-1} + ... + a_{n-1}|z| + |a_n|$

$3 < 2\left(|z|^n + |z|^{n-1} + ... + |z| + 1\right)$

$\frac{3}{2} < 1 + |z| + |z|^2 + ... + |z|^n$

$\frac{1-|z|^{n+1}}{1-|z|} > \frac{3}{2}$

$2 - 2|z|^{n+1} < 3|z| - 1$

$3|z| - 1 > 0$

$|z| > \frac{1}{3}$

118. If $n \ge 3$ and $1, \alpha_1, \alpha_2, \alpha_3......\alpha_{n-1}$ are n roots of unity, then value of $\sum_{1\le i<j\le n-1} \alpha_i\alpha_j$ is

(a) 0 (b) 1 (c) -1 (d) $(-1)^n$

Key. B

Sol. $x^n - 1 = (x-1)(x-\alpha_1)(x-\alpha_2)............(x-\alpha_{n-1})$

$= x^n - x^{n-1}(1+\alpha_1+........+\alpha_{n-1}) + x^{n-2}\left(\sum_{i+j}\alpha i\,\alpha_j + \alpha_1 + \alpha_2.....+\alpha_{n-1}\right)$+..........-1 = 0

$\Rightarrow \sum_{i+j}\alpha i\,\alpha_j + \alpha_1 + \alpha_2.....+\alpha_n = 0$

$\sum_{i+j}\alpha i\,\alpha_j = 1$

119. If the equation $z^2 + z + \alpha = 0$ has a purely imaginary root and α lies on the circle |z| = 1 then the imaginary part of that root, is (are)

(A) $\pm\sqrt{2}$ (B) 0

(C) $\pm\sqrt{2-\sqrt{2}}$ (D) $\pm\sqrt{\frac{\sqrt{5}-1}{2}}$

Key. D

Sol. Let z = iβ (β∈R) be a root, then

$-\beta^2 + i\beta + \alpha = 0 \Rightarrow \alpha = \beta^2 - i\beta$

Now as |α| = 1

$\Rightarrow \beta^4 + \beta^2 = 1 \Rightarrow \beta^2 = \frac{-1+\sqrt{5}}{2}$

120. Let z (α, β) = cosα + $e^{i\beta}$ sinα (α, β∈R, i = $\sqrt{-1}$) then the exaustive set of values of modulus of z(θ, 2θ), as θ varies, is

(A) [0, 1] (B) [0, $\sqrt{2}$]

(C) [1, 2] (D) [$\sqrt{2}$, 2]

Key. B

Sol. $|z)\theta, 2\theta) = |\cos\theta + e^{i2\theta}\sin\theta|$

$= |\cos\theta + \sin\theta\cos 2\theta + i\sin\theta\sin 2\theta| = \sqrt{(\cos\theta + \sin\theta\cos 2\theta)^2 + \sin^2\theta\cos^2\theta}$

$= \sqrt{1+\sin 4\theta} \in [0, \sqrt{2}]$

121. If |z| = 1 and z ≠ ±1 then one of the possible values of arg(z) – arg (z + 1) – arg (z – 1), is

(A) –π/6 (B) π/3

(C) –π/2 (D) π/4

Key. C

Sol. $\arg(z) - \text{avg}\ |z+1) - \arg(z-1) = \arg\left(\frac{z}{z^2-1}\right) = \arg\left(\frac{z}{z^2 - z\bar{z}}\right)$

$= \arg\left(\frac{1}{z-\bar{z}}\right)$ = arg (purely imaginary no.)

122. If z_1, z_2, z_3 are three distinct complex numbers and a, b, c are three positive real numbers such that

$$\frac{a}{|z_2-z_3|} = \frac{b}{|z_3-z_1|} = \frac{c}{|z_1-z_2|} \text{ then } \frac{a^2}{z_2-z_3} + \frac{b^2}{z_3-z_1} + \frac{c^2}{z_1-z_2} \text{ is}$$

a) 3 abc b) $(abc)^3$ c) a + b + c d) 0

Key. D

Sol. $\frac{a}{|z_2-z_3|} = \lambda \Rightarrow \frac{a^2}{z_2-z_3} = \lambda^2(\bar{z}_2 - \bar{z}_3)$ *etc*

123. If $|z_1| = 2, |z_2| = 3, |z_3| = 4$ and $|2z_1 + 3z_2 + 4z_3| = 4$ then the absolute value of $8z_2z_3 + 27z_3z_1 + 64z_1z_2$ equals

(A) 24 (B) 48 (C) 72 (D) 96

Key. D

SOL. $|8z_2z_3 + 27z_3z_1 + 64z_1z_2|$

$= |z_1z_2z_3|\left|\frac{8}{z_1} + \frac{27}{z_2} + \frac{64}{z_3}\right|$

$= 24|2\overline{z_1} + 3\overline{z_2} + 4\overline{z_3}|$

$= 24 \times 4 = 96$

124. If ω is a cube root of unity, then $\omega + \omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}+....+\infty\right)}$ is

(A) i (B) i^2

(C) 0 (D) none of these

Key. B

Sol. $\frac{1}{2} + \frac{3}{8} + \frac{9}{32} + \frac{27}{128} + ... + \infty = \frac{1}{2}\left(1 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64} + ... + \infty\right)$

$$= \frac{1}{2}\left(\left(\frac{3}{4}\right)^0 + \left(\frac{3}{4}\right)^1 + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \ldots\infty\right) = \frac{1}{2}\left(\frac{1}{1-\frac{3}{4}}\right) = \frac{1}{2}.\frac{1}{\frac{1}{4}} = 2$$

So expression = $\omega + \omega^2 = -1 = i^2$.

125. Let P be a point on the circumcircle of the triangle whose vertices A, B, C (P, A, B, C are in order) are represented by the complex numbers ω^2, $2i\omega$ and -4 (ω is a non real cube root of unity) respectively such that PA.BC = PC. AB then the complex number associated with the mid–point of PB is

(A) $\omega - 1$ (B) 0

(C) $-i$ (D) $\omega - \omega^2$

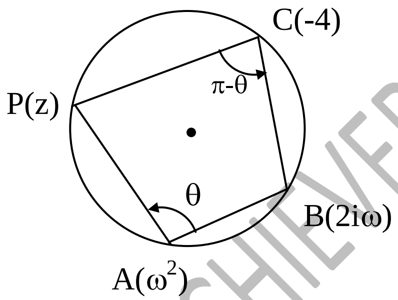
Key. B

Sol. Applying rotation formula at A and C,

$$\frac{z-\omega^2}{zi\omega-\omega^2} = \frac{PA}{AB}e^{i\theta}, \quad \frac{2i\omega+4}{z+4} = \frac{BC}{PC}e^{i(\pi-\theta)}$$

Multiplying we get, , $\frac{z-\omega^2}{2i\omega-\omega^2} \times \frac{2i\omega+4}{z+4} = -1$

$\Rightarrow z = -2i\omega$



126. The complex numbers satisfying (3Z+1) (4Z+1) (6Z+1)(12Z+1) = 2 is

a) $\frac{\sqrt{33}-5}{4}$ b) $\frac{\sqrt{33}+5}{24}$ c) $\frac{-i\sqrt{23}-5}{24}$ d) $\frac{-i\sqrt{23}+5}{24}$

Key. C

Sol. Given equation can be written as $(144z^2 + 60z+4)(144z^2+60z+6) = 48 \Rightarrow t(t+2) = 48$

Where $t = 144z^2 + 60z+4$

$\therefore$ t = 6 or -8 hence $z = \frac{-5\pm\sqrt{33}}{24}, \frac{-5\pm i\sqrt{23}}{24}$

127. If |z| = 1 and $z' = \frac{1+z^2}{z}$, then

(A) z′ lie on a line not passing through origin

(B) $|z'| = \sqrt{2}$

(C) Re (z′) = 0

(D) Im (z′) = 0

Key. D

Sol. z′ = $\frac{1+z^2}{z}.\frac{\overline{z}}{\overline{z}}$ = $\frac{\overline{z}+z}{z\overline{z}} = z+\overline{z}$ which is purely real.

$\Rightarrow$ Im (z′) = 0.

128. For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of

$|z_1 - z_2|$ is

a) 0 b) 7 c) 2 d) 17

Key. C

Sol. Conceptual

129. If $y_1 = \max\left||z-w|-|z-w^2|\right|$, where $|z| = 2$ and

$y_2 = \max\left||z-w|-|z-w^2|\right|$, where $|z| = \frac{1}{2}$ and $w \text{ and } w^2$

are complex cube roots of unity, then

a) $y_1 = \sqrt{3}; y_2 = \sqrt{3}$ b) $y_1 < \sqrt{3}; y_2 = \sqrt{3}$

c) $y_1 = \sqrt{3}; y_2 < \sqrt{3}$ d) $y_1 > \sqrt{3}; y_2 < \sqrt{3}$

Key. C

Sol. We have $\left||z_1|-|z_2|\right| \le |z_1 - z_2|$ and equality holds only when $\arg z_1 = \arg z_2$.

$\Rightarrow \left||z-w|-|z-w^2|\right| \le |w^2 - w| \le \sqrt{3}$ and equality can hold only when $|z| = 2$ and not when $|z| = \frac{1}{2}$

130. Let $f(x)$ be the remainder obtained on dividing $x^{2007} - 1$ by $(x^2+1)(x^2+x+1)$, then $f(x)$ is a polynomial of degree

a) 0 b) 1 c) 2 d) 3

Key. D

Sol. Let $x^{2007} - 1 = (x^2+1)(x^2+x+1)p(x) + f(x)$

Put $x = \pm i, w, w^2$ for get $f(x)$

131. If $\alpha \ne 1$ is any of 7[th] roots of unity then real part of $\alpha^{2009} + 3\alpha^{2010} + 5\alpha^{2011} + ----+$ up to 7 terms is

a) 7 b) 14 c) -7 d) -14

Key. C

Sol. Let $\alpha = cis\frac{2K\pi}{7}(K = 0\, to\, 6)$ $(\alpha^7 = 1)$ S $= \alpha^{2009} + 3\alpha^{2010} + 5\alpha^{2011} + ----- + 13\alpha^{2015}$

$$= 1+3\alpha+5\alpha^2+--------+13\alpha^6 (AGP)$$

$$= \frac{-14}{1-\alpha} = \frac{-14}{1-cis\frac{2K\pi}{7}}$$

$$= -7\left[1+i\cot\frac{K\pi}{7}\right]$$

132. All the complex numbers *z* that satisfy the equation $z^{10} = (1-z)^{10}$ lie on

a) $x = \frac{1}{2}$ b) $x = -\frac{1}{2}$ c) $y = \frac{1}{2}$ d) $y = -\frac{1}{2}$

Key. A

$$\frac{z^{10}}{(1-z)^{10}} = 1 \Rightarrow \frac{z}{1-z} = 1^{1/10} = cis\frac{2K\pi}{10} (K = 0\ to\ 9)$$

Sol.

$$\Rightarrow z = \frac{cis\frac{2K\pi}{10}}{1+cis\frac{2K\pi}{10}} = \frac{1}{2}+\frac{i}{2}\tan\frac{K\pi}{10}$$

133. If z is a complex number such that $|z-1|$ =1 then *arg* $\left(\frac{1}{z}-\frac{1}{2}\right)$ may be

a) $\frac{\pi}{6}$ b) $-\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) $-\frac{\pi}{4}$

Key. B

Sol. Since $|z-1| = 1 \Rightarrow z-1 = cis\theta \Rightarrow z = (1+\cos\theta)+i\sin\theta = 2\cos\frac{\theta}{2}cis\frac{\theta}{2}$

$$\therefore \frac{1}{z}-\frac{1}{2} = \frac{cis\left(-\frac{\theta}{2}\right)}{2\cos\frac{\theta}{2}}-\frac{1}{2} = -\frac{i}{2}\tan\frac{\theta}{2}$$ which is purely imaginary

134. $\theta \in [0, 2\pi]$ and z_1, z_2, z_3 are three complex numbers such that they are collinear and $(1+|\sin\theta|)z_1+(|\cos\theta|-1)z_2-\sqrt{2}\ z_3$ =0 .If at least one of the complex numbers z_1, z_2, z_3 is non- zero then number of possible values of θ is

a) Infinite b) 4 c) 2 d) 8

Key. B

Sol. If z_1, z_2, z_3 are collinear and $az_1+bz_2+cz_3 = 0$ then a+b+c =0 .Hence

$$1+|\sin\theta|+|\cos\theta|-1-\sqrt{2} = 0 \Rightarrow |\sin\theta|+|\cos\theta| = \sqrt{2}$$

$$\Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

135. If $\alpha_0, \alpha_1, \alpha_2, ...\alpha_{n-1}$ be the n, n^{th} roots of unity, then value of $\sum_{i=0}^{n-1}\frac{\alpha_i}{(3-\alpha_i)}$ is equal to

A) $\frac{n}{3^n-1}$ B) $\frac{n-1}{3^n-1}$ C) $\frac{n+1}{3^n-1}$ D) $\frac{n+2}{3^n-1}$

Key. A

Sol. Let $P=\sum_{i=0}^{n-1}\frac{\alpha_i}{3-\alpha_i}=-\sum_{i=0}^{n-1}\frac{(3-\alpha_i)-3}{(3-\alpha_i)}=3\sum_{i=0}^{n-1}\frac{1}{3-\alpha_i}-\sum_{i=0}^{n-1}1$ --- (i)

$Z^n-1=\prod_{i=0}^{n-1}(Z-\alpha_i)\ \log\left(Z^n-1\right)=\sum_{i=0}^{n-1}\ln(Z-\alpha_i)$

Diff. both sides w.r.t Z

$\frac{nZ^{n-1}}{Z^n-1}=\sum_{i=0}^{n-1}\frac{1}{z-\alpha_i}$ Put $Z=3$

$\Rightarrow\frac{n3^{n-1}}{3^n-1}=\sum_{i=0}^{n-1}\frac{1}{3-\alpha_i}$

$P=\frac{3n3^{n-1}}{3^n-1}-n=\frac{n3^n}{3^n-1}-n=\frac{n}{3^n-1}$

136. Let $\left|Z_1-1\right|=1,\left|Z_2+4\right|=2$ then maximum value of $\left|Z_1-Z_2\right|$ is

A) 8 B) 5 C) 4 D) 2

Key. A

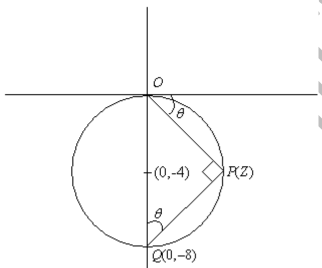
Sol. Max. distance between two curves lies along their common normal.

137. The complex number Z has argument θ, $-\frac{\pi}{2}<\theta<0$ and $\left|Z+4i\right|=4$, then $\cot\theta+\frac{8}{Z}=$

A) $1-i$ B) $1+i$ C) i D) $-i$

Key. C

Sol.



Applying rotation at P $\frac{Z+8i}{Z}=-i\cot\theta$

138. Let $\left|Z-(1+i)\right|<2$ then $\left|iZ+1+2i\right|$

A) < 7 B) < 9 C) < 5 D) < 10

Key. C

Sol. $\left|iZ+1+2i\right|=\left|i(z-(1+i)+3i\right|\le\left|Z-(1+i)\right|+3<2+3$

139. If $x^6=2x^3-1$ and *x* is not real then $\sum_{r=1}^{50}(x^r+x^{2r})^3=$

A) 0 B) 256 C) 76 D) 94

Key. D

Sol. $x=\omega,\omega^2$

$$\omega^r+\omega^{2r}=\begin{cases}2\text{ ,if } r \text{ is a multiple of 3}\\ -1\text{ ,if } r \text{ is not of a multiple of 3}\end{cases}$$

140. If $A(z_1),B(z_2),C(z_3)$ be the vertices of triangle ABC in which $\underline{|ABC}=\frac{\pi}{4}$ and $\frac{AB}{BC}=\sqrt{2}$ then z_2 is equal to

(a) $z_3+i(z_1+z_3)$ (b) $z_3-i(z_1-z_3)$ (c) $z_3+i(z_1-z_3)$ (d) $z_3-i(z_2-z_1)$

Key. B

Sol. $\frac{z_1-z_2}{z_3-z_2}=\sqrt{2}e^{\frac{i\pi}{4}}$

141. If $|z_1|=2$, $|z_2|=3$, $|z_3|=4$ and $|z_1+z_2+z_3|=5$ then $|4z_2z_3+9z_3z_1+16z_1z_2|$ =

a) 20 b) 24 c) 48 d) 120

Key. D

Sol. $|4z_2z_3+9z_3z_1+16z_1z_2|$

= $|z_1\overline{z_1}z_2z_3+\overline{z_2}z_2z_3z_1+\overline{z_3}z_3z_1z_2|$

= $||z_1||z_2||z_3|||z_1+z_2+z_3|=120$

142. If $\log_{\tan 30^\circ}\left(\frac{2|z|^2+2|z|-3}{|z|+1}\right)<-2$ then

a) $|z|<\frac{3}{2}$ b) $|z|>\frac{3}{2}$ c) $|z|>2$ d) $|z|<2$

Key. C

Sol. $\log_{\tan 30^\circ}\left(\frac{2|z^2|+2|z|-3}{|z+1|}\right)<-2$

$\Rightarrow \frac{2|z^2|+2|z|-3}{|z|+1}>3$

$\Rightarrow \left((|z|-2)(2|z|)+3\right)>0$

$\Rightarrow |z| > 2$

143. z_1 and z_2 be two complex numbers with α and β as their principal arguments, such that $\alpha+\beta>\pi$, then principal Arg$\left(z_1z_2\right)$ is

a) $\alpha+\beta+\pi$ b) $\alpha+\beta-\pi$ c) $\alpha+\beta-2\pi$ d) $\alpha+\beta$

Key. C

Sol. Take z_1 = i, z_2 = ω Arg z_1 = $\frac{\pi}{2}$, Arg z_2 = $\frac{2\pi}{3}$

$Argz_1 + Argz_2 = \frac{7\pi}{6}$ should be equivalent to $\frac{7\pi}{6} - 2\pi$

144. If the square root of $\frac{x^2}{y^2} + \frac{y^2}{x^2} + \frac{1}{2i}\left(\frac{x}{y} + \frac{y}{x}\right) + \frac{31}{16}$ is $\pm\left(\frac{x}{y} + \frac{y}{x} - \frac{i}{m}\right)$ then m is

a) 2 b) 3 c) 4 d) 5

Key. C

Sol. $\left(\frac{x}{y} + \frac{y}{x} - \frac{i}{m}\right)^2 = \frac{x^2}{y^2} + \frac{y^2}{x^2} + \frac{1}{2i}\left(\frac{x}{y} + \frac{y}{x}\right) + \frac{31}{16}$

$L.H.S =$

$= \left(\frac{x}{y} + \frac{y}{x}\right)^2 - \frac{2i}{m}\left(\frac{x}{y} + \frac{y}{x}\right) - \frac{1}{m^2}$

$= \frac{x^2}{y^2} + \frac{y^2}{x^2} + 2 + \frac{4}{m} \cdot \frac{1}{2i}\left(\frac{x}{y} + \frac{y}{x}\right) - \frac{1}{m^2}$

m = 4

145. If $\left|\frac{z_1 - 2z_2}{2 - z_1\overline{z_2}}\right|$ = 1 and $|z_2| \neq 1$, then value of $|z_1|$ =

a) 2 b) 1 c) 4 d) 5

Key. A

Sol. Conceptual

146. If $A_1(z_1)$, $A_2(\overline{z_1})$ are the adjacent vertices of a regular polygon. If $\frac{\operatorname{Im}(\overline{z_1})}{\operatorname{Re}(z_1)} = 1 - \sqrt{2}$ then number of sides of the polygon is equal to

a) 6 b) 8 c) 16 d)12

Key. B

Sol. Clearly origin is the centre of the polygon

Let $z_1 = re^{i\theta}$

$\overline{z_1} = re^{-i\theta}$

Re (z) = r$\cos\theta$

Im $(\overline{z_1})$ = – r$\sin\theta$

$\Rightarrow -\frac{\sin\theta}{\cos\theta}$ = 1 – $\sqrt{2}$ $\Rightarrow$ tan (θ) = $\sqrt{2} - 1$

$\Rightarrow \theta = \frac{\pi}{8}$ if 'n' be the no. of sides then θ = $\frac{\pi}{n}$

$\Rightarrow$ n = 8

147. If exactly one root of $z^2+az+b=0$ where a, b $\in C$ is purely imaginary, then

a) $\left(\bar{b}-b\right)^2=-\left(a\bar{b}+\bar{a}b\right)\left(a+\bar{a}\right)$ b) $\left(\bar{b}-b\right)^2=-\left(a\bar{b}+\bar{a}b\right)\left(a-\bar{a}\right)$

c) $\left(\bar{b}-b\right)^2=-\left(a\bar{b}-\bar{a}b\right)\left(a+\bar{a}\right)$ d) $\left(\bar{b}-b\right)^2=-\left(a\bar{b}-\bar{a}b\right)\left(a-\bar{a}\right)$

Key. A

Sol. $z^2+az+b=0$

Let z_0 is the purely imaginary root of the equation

Then $z_0{}^2+az_0+b=0$

$\Rightarrow z_0+\overline{z_0}$ = 0

$\Rightarrow \overline{z_0}=-z_0$

We have $\overline{\overline{z_0}^2+az_0+b}=0 \Rightarrow \overline{z_0}^2+\overline{az_0}+\bar{b}=0$

Now $\overline{z_0}^2+az_0+b$ and $z_0{}^2-\bar{a}z_0+\bar{b}=0$

We should have a common root. Find common root.

148. z_1 and z_2 are the roots of $z^2-az+b=0$, where $|z_1|=|z_2|=1$ and a, b are non-zero complex numbers, then

a) Arg(a) = 2 Arg(b) b) 2 Arg(a) = Arg(b)

c) Arg(a) = Arg(b) d) none of these

Key. B

Sol. $z_1+z_2=a \qquad z_1z_2=b$

Since $|z_1|=|z_2|=a$

$\Rightarrow$ Arg(a) = $\frac{1}{2}\left[Arg\left(z_1\right)+Arg\left(z_2\right)\right]$

Also Arg (b) = Arg $\left(z_1z_2\right)$

$\therefore$ Arg(a) = $\frac{1}{2}\left(Arg\left(b\right)\right) \Rightarrow 2Arg\left(a\right)=Arg\left(b\right)$

149. If $|z-2+2i|$ = 1, then the least value of $|z|$ is

a) $\sqrt{8}+1$ b) $\sqrt{6}+1$ c) $\sqrt{6}-1$ d) $\sqrt{8}-1$

Key. D

Sol. $|z-2+2i|=1 \Rightarrow$ z – 2 + 2i = cosθ + isinθ

z = (2 + cosθ) + i(sinθ – 2)

$|z|=\sqrt{4+4\cos\theta+\cos^2\theta+4-4\sin\theta+\sin^2\theta}$

= $\sqrt{9+4\left(\cos\theta-\sin\theta\right)}$

= $\sqrt{9+4\sqrt{2}\cos\left(\theta+\frac{\pi}{4}\right)}$

|z| is least if cos $\left(\theta+\frac{\pi}{4}\right)$ = −1 = $\sqrt{9-4\sqrt{2}}$

= $\sqrt{9-2\sqrt{8}}$ = $\sqrt{8}-1$

150. If the imaginary part of $\frac{2z+1}{iz+1}$ is -4, then the locus of the point representing z in the complex plane is

1) straight line 2) a parabola 3) a circle 4) an ellipse

Key. 3

Sol. Let $z=x+iy$

$\frac{2z+1}{iz+1}=\frac{2(x+iy)+1}{i(x+iy)+1}$

$=\frac{(2x+1)+2iy}{(1-y)+ix}$

$=\frac{[(2x+1)+2iy][(1-y)-ix]}{(1-y)^2+x^2}$

since $\text{Im}\left(\frac{2z+1}{iz+1}\right)=-4$, we get $\frac{2y(1-y)-x(2x+1)}{x^2+(1-y)^2}=-4$

$\Rightarrow 2x^2+2y^2+x-6y+4=0$

Which represents a circle.

151. If $Z_K=Cos\frac{K\pi}{10}+iSin\frac{K\pi}{10}$ then $z_1z_2z_3z_4$ is equal to

1) -1 2) 1 3) -2 4) 2

Key. 1

Sol. Let $z_K=w^K$ where $w=\cos\frac{\pi}{10}+i\sin\frac{\pi}{10}$

$\therefore z_1z_2z_3z_4=w.w^2.w^3.w^4$

$=w^{10}$

$=\cos\frac{10\pi}{10}+i\sin\frac{10\pi}{10}$ (∵ Demoviere's theorem)

$=\cos\pi+i\sin\pi$

$=-1$

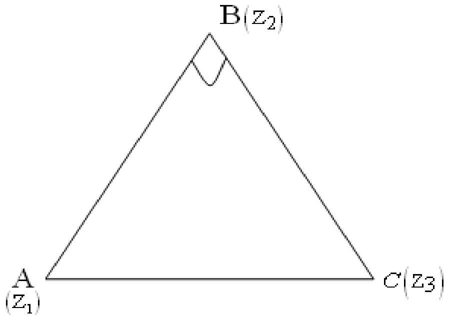
152. If z_1,z_2,z_3 are the vertices of an isosceles triangle, right angled at the vertex z_2, then value of $(z_1-z_2)^2+(z_2-z_3)^2$ is

1) -1 2) 0 3) $\left(z_1 - z_3\right)^2$ 4) None of these

Key. 2

Sol. Since $A\left(Z_1\right), B\left(Z_2\right), C\left(Z_3\right)$ is an Isosceless right angled triangle with right angle at B

$BA = BC$ and $\underline{|ABC} = 90^0$



$\Rightarrow \left|Z_1 - Z_2\right| = \left|Z_3 - Z_2\right|$ and $\arg\left(\dfrac{z_3 - z_2}{z_1 - z_2}\right) = \pi / 2$

$\therefore \dfrac{z_3 - z_2}{z_1 - z_2} = \dfrac{\left|z_3 - z_2\right|}{\left|z_1 - z_2\right|}\left(\cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}\right) = i$

$\left(z_3 - z_2\right)^2 = -\left(z_1 - z_2\right)^2$

$\Rightarrow \left(z_1 - z_2\right)^2 + \left(z_2 - z_3\right)^2 = 0$

153. If a, b, c, p, q, r are three non-zero complex numbers such that $\dfrac{p}{a} + \dfrac{q}{b} + \dfrac{r}{c} = 1 + i$ and $\dfrac{a}{p} + \dfrac{b}{q} + \dfrac{c}{r} = 0$ then value of $\dfrac{p^2}{a^2} + \dfrac{q^2}{b^2} + \dfrac{r^2}{c^2}$ is

1) 0 2) -1 3) 2i 4) -2i

Key. 3

Sol. We have $\left(1 + i\right)^2 = \left(\dfrac{p}{a} + \dfrac{q}{b} + \dfrac{r}{c}\right)^2$

$1 - 1 + 2i = \dfrac{p^2}{a^2} + \dfrac{q^2}{b^2} + \dfrac{r^2}{c^2} + 2\left(\dfrac{qr}{bc} + \dfrac{rp}{ca} + \dfrac{pq}{ab}\right)$

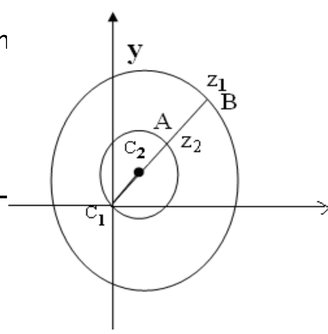
$2i = \dfrac{p^2}{a^2} + \dfrac{q^2}{b^2} + \dfrac{r^2}{c^2} + \dfrac{2abc}{pqr}\left(\dfrac{a}{p} + \dfrac{q}{b} + \dfrac{r}{c}\right)$

$= \dfrac{p^2}{a^2} + \dfrac{q^2}{b^2} + \dfrac{r^2}{c^2} + \dfrac{2abc}{pqr}(0)$

$= \dfrac{p^2}{a^2} + \dfrac{q^2}{b^2} + \dfrac{r^2}{c^2}$

154. For all complex numbers z_1, z_2 satisfying $\left|z_1\right| = 12$ and $\left|z_2 - 3 - 4i\right| = 5$, the minim

of $\left|z_1 - z_2\right|$ is



1) 0 2) 2 3) 7 4) 17

Key. 2

Sol. $|z_1| = 12 \Rightarrow z_1$ lies on circle with centre c_1 at origin and radius 12

$|z_2 - 3 - 4i| = 5 \Rightarrow z_2$ lies on the circle with centre $c_2\,(3+4i)$ and radius 5.

$\therefore |z_1 - z_2|$ will be minimum.

If z_1 and z_2 lies on the line joining c_1 and c_2 i.e on the line $z = 3+4i$

Minimum value of $|z_1 - z_2| = AB$

$= c_1B - c_1A$

$12 - 10 = 2$

155. If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$ then $|z_1 + z_2 + z_3|$ is

1) equal to 1 2) less than 1 3) greater than 3 4) equal to 3

Key. 1

Sol. $\because |z_1| = |z_2| = |z_3| = 1$ *we get* $z_1\bar{z}_1 = z_2\bar{z}_2 = z_3\bar{z}_3 = 1$

$\therefore 1 = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$ $\Rightarrow \bar{z}_1 = \frac{1}{z_1}, \bar{z}_2 = \frac{1}{z_2}, \bar{z}_3 = \frac{1}{z_3}$

$= |\bar{z}_1 + \bar{z}_2 + \bar{z}_3|$

$= |z_1 + z_2 + z_3|$

156. The complex numbers z_1, z_2 and z_3 satisfying $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$ are the vertices of a triangle which is

1) of area $\sqrt{3}$ 2) right angled and isosceles

3) equilateral 4) obtuse-angled and isosceles

Key. 3

Sol. $\left|\frac{z_1 - z_3}{z_2 - z_3}\right| = \left|\frac{1 - i\sqrt{3}}{2}\right| \quad \Rightarrow \frac{|z_1 - z_3|}{|z_2 - z_3|} = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$

$\Rightarrow |z_1 - z_3| = |z_2 - z_3|$

Again $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$

$\frac{z_1 - z_3}{z_2 - z_3} - 1 = \frac{1 - i\sqrt{3}}{2} - 1$

$$\frac{z_1 - z_2}{z_2 - z_3} = \frac{-1 - i\sqrt{3}}{2}$$

$$\left|\frac{z_1 - z_2}{z_2 - z_3}\right| = \left|\frac{-1}{2} \frac{-i\sqrt{3}}{2}\right|$$

$$\frac{|z_1 - z_2|}{|z_2 - z_3|} = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$$

$$\Rightarrow |z_1 - z_2| = |z_2 - z_3|$$

$\therefore$ z_1, z_2 and z_3 are the vertices of an equilateral triangle.

157. The vertices B and D of a parallelogram are $1-2i$ and $4+2i$ respectively. If the diagonals are at right angles and $|AC| = 2|BD|$, then the complex number representing A is

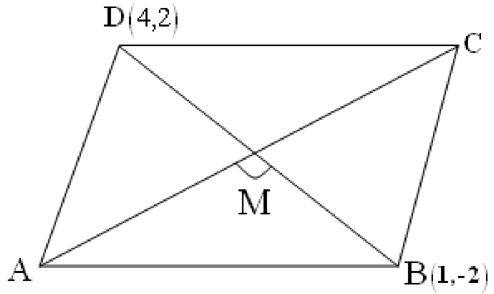
1) $\frac{3}{2}i + \frac{1}{2}$ 2) $3i - 4$ 3) $3i - \frac{3}{2}$ 4) $\frac{5}{2}$

Key. 3

Sol. Let affix of A be Z.
M = Mid point of BD

$$= \left(\frac{5}{2}, 0\right)$$

$$\lfloor \underline{AMB} = 90^0$$



$$\text{Arg}\left(\frac{1 - 2i - \frac{5}{2}}{z - \frac{5}{2}}\right) = \frac{\pi}{2}$$

$$\Rightarrow \frac{1 - 2i - \frac{5}{2}}{z - \frac{5}{2}} = \left|\frac{1 - 2i - \frac{5}{2}}{z - \frac{5}{2}}\right| cis\frac{\pi}{2}$$

$$= \frac{|BM|}{|AM|} i$$

$$= \frac{\left|\frac{BD}{2}\right|}{\left|\frac{Ac}{2}\right|} i$$

$$= \frac{|BD|}{|AC|} i$$

$$= \frac{1}{2} i \quad (\because |AC| = 2|BD|)$$

$$\therefore \left(\frac{-3}{2} \quad -2i\right)\frac{2}{i} = z - \frac{5}{2}$$

$$\Rightarrow z = \frac{5}{2} - \frac{3}{i} - c_1$$

$$= \frac{-3}{2} + 3i$$

Hence option(3)

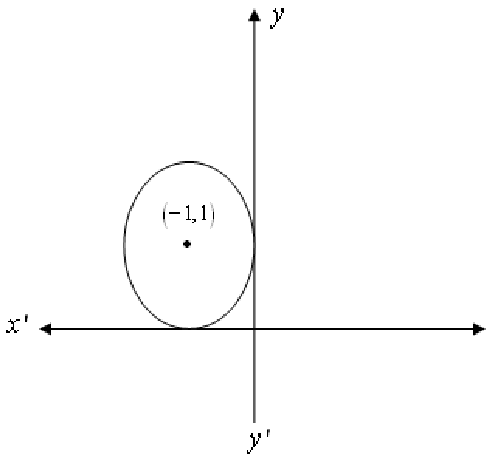
158. For all z satisfying $|z+1-i| = 1$ we have

1) $\frac{\pi}{2} \le Argz \le \pi$ 2) $-\pi \le Argz \le \frac{-\pi}{2}$ 3) $-\pi < Arg\ z \le \frac{\pi}{2}$ 4) None of these

Key. 1

Sol. $|z+1-i| = 1 \Rightarrow |z-(-1)+i| = 1$

$\therefore$ Locus of z is a circle whose centre is $(-1,1)$ and radius=1



The circle touches both the axes in the second quadrant

All points on this circle lie in the region $+ve$ Y-axis corresponding to Arg $z = \frac{\pi}{2}$

and -ve X-axis corresponds to Argz $= \pi$

$$\therefore \frac{\pi}{2} \le Arg\ Z \le \pi$$

Hence option (1)

159. The equation $|z-4i| + |z+4i| = 10$ represents

1) a circle 2) an ellipse 3) a line segment 4) None of these

Key. 2

Sol. $p(z)$, $A(4i), B(-4i)$ then

$$|AB| = |-4i - 4i|$$

$$= |-8i|$$

$$= 8$$

Now $|z-4i| + |z+4i| = 10$

$\Rightarrow |PA| + |PB| = 10$

$> |AB|$

$\therefore$ Locus of P is an ellipse

Hence option is (2).

160. If $z^5 = (z-1)^5$ then the roots are represented in the argand plane by the points that are

1) Collinear
2) Concylic
3) Vertices of a parallelogram
4) None of these

Key. 1

Sol. Let Z be a complex number satisfying

$Z^5 = (Z-1)^5$

$\Rightarrow |Z^5| = |(Z-1)^5|$

$\Rightarrow |Z|^5 = |Z-1|^5$

$\Rightarrow |Z| = |Z-1|$

Thus Z lies on the perpendicular bisector of the segment joining the

Origin and $A(1+i\,0)$ i.e Z lies on Re Z=$\frac{1}{2}$

Hence option (1)

161. Let | z – 5 + 12 *i* | $\le$ 1 and the least and greatest values of | z | are *m* and *n* and if *l* be the least positive value of $\frac{x^2+24x+1}{x}$ (x > 0), then *l* is

(A) $\frac{m+n}{2}$ (B) m + n (C) m (D) n

Key. 2

Sol. r^{th} term of given expression

$$= r(r+1-w)(r+1-w^2)$$

$$= (r+1-1)(r+1-w)(r+1-w^2)$$

$$= (r+1)^3 - 1 \left[\begin{array}{l} \therefore Let\ r+1 = x \\ (x-1)(x-w)(x-w^2) = x^3 - 1 \end{array} \right]$$

$\therefore$ Given expression value $= \sum_{r=1}^{n-1} r(r+1-w)(r+1-w^2)$

$$= \sum_{r=1}^{n} (r+1)^3 - 1$$

$$= 2^3 + 3^3 + ...n^3 - (n-1)$$

$$= (1^3 + 2^3 + 3^3 + ... + n^3) - n$$

$$= \frac{n^2(n+1)^2}{4} - n$$

Hence option (1).

162. If $x = 2+5i$ then the value of $x^3 - 5x^2 + 33x - 19$ is equal to

1) -5 2) -7 3) 7 4) 10

Key. 4

Sol. $x = 2+5i \quad \Rightarrow x - 2 = 5i$

$\Rightarrow (x-2)^2 = (5i)^2$

$x^2 - 4x + 4 = -25$

$\Rightarrow x^2 - 4x + 29 = 0 \rightarrow (i)$

Dividing $x^3 - 5x^2 + 33x - 19 \; by \; x^2 - 4x + 29$

$x^2 - 4x + 29)x^3 - 5x^2 + 33x - 19(x - 1$

$\underline{x^3 - 4x^2 + 29x}$

$-x^2 + 4x - 19$

$\underline{-x^2 + 4x - 29}$

10

$\therefore \; x^3 - 5x^2 + 33x - 19 = (x-1)(x^2 - 4x + 29) + 10$

$= (x-1)0 + 10 (\because \; from (1))$

$= 10$

Hence option (4).

163. The complex number z_1, z_2, z_3 are the vertices of an equilateral triangle. If z_0 is the circumcentre of the triangle then $z_1^{\;2} + z_2^{\;2} + z_3^{\;2} =$

1) $z_0^{\;2}$ 2) $3z_0^{\;2}$ 3) $z_0^{\;3}$ 4) $3z_0^{\;3}$

Key. 2

Sol. Since the triangle with Z_1, Z_2, Z_3 as vertices is an equilateral triangle, its circumcentre and centroid will coincide

$$\therefore Z_0 = \frac{Z_1 + Z_2 + Z_3}{3}$$

$$(3\,Z_0)^2 = Z_1^{\;2} + Z_2^{\;2} + Z_3^{\;2} + 2(Z_2 Z_3 + Z_3 Z_1 + Z_1 Z_2) \rightarrow (1)$$

Since the triangle is equilateral we have

$$Z_1^{\;2} + Z_2^{\;2} + Z_3^{\;2} = Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1 \rightarrow (2)$$

From (1) and (2) we get

$$9\,Z_0^{\;2} = Z_1^{\;2} + Z_2^{\;2} + Z_3^{\;2} + 2(Z_1^{\;2} + Z_2^{\;2} + Z_3^{\;2})$$

$$3\,Z_0^{\;2} = Z_1^{\;2} + Z_2^{\;2} + Z_3^{\;2}$$

Hence option (2).

164. The complex numbers $\sin x+i\cos 2x$ and $\cos x-i\sin 2x$ are conjugate to each other for

1) $x=n\pi$

2) $x=0$

3) $x=\left(n+\frac{1}{2}\right)\pi$

4) no value of x

Key. 4

Sol. Let $Z_1=\sin x+i\cos 2x,\ Z_2=\cos x-i\sin 2x$

$$\overline{Z_1}=Z_2$$

$$\sin x-i\cos 2x=\cos x-i\sin 2x$$

$$\Rightarrow \sin x=\cos x \ \ and \ \ \cos 2x=\sin 2x$$

$$\Rightarrow \tan x=1 \ \ and \ \ \tan 2x=1$$

$\Rightarrow x=\frac{\pi}{4}$ *and* $x=\frac{\pi}{8}$ which is not possible .Hence there is no value of x

Hence option (4).

165. Suppose z_1,z_2,z_3 are the vertices of an equilateral triangle inscribed in the circle $|z|=2$. If $z_1=1+i\sqrt{3}$ then z_2 may be

1) $1+i\sqrt{3},2$

2) $3-i\sqrt{2},4$

3) $1-i\sqrt{3},-2$

4) $2-i\sqrt{3},1$

Key. 3

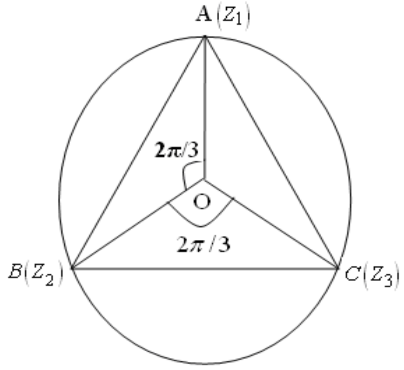
Sol. Let Z_1,Z_2,Z_3 are the vertices $\mathrm{A,B,C}$ of equilateral triangle ABC inscribed in a circle $|\mathrm{Z}|=2$ with centre $(0,0)$ and radius 2.

Given $Z_1=1+i\sqrt{3}$

Rotating OA about O by an angle $\frac{2\pi}{3}$ we have

$$\frac{Z-0}{1+i\sqrt{3}-0}=\frac{|Z-0|}{\left|+\left(\sqrt{3}-0\right)\right|}e^{\pm i\frac{2\pi}{3}}$$

$$Z=\left(1+i\sqrt{3}\right)\left(\cos\frac{2\pi}{3}\pm i\sin\frac{2\pi}{3}\right)$$



$$=\left(1+i\sqrt{3}\right)\left(\frac{-1}{2}\pm i\frac{\sqrt{3}}{2}\right)$$

$$=-\frac{\left(1+i\sqrt{3}\right)\left(1-i\sqrt{3}\right)}{2}\ (or)\ \frac{-\left(1+i\sqrt{3}\right)\left(1+i\sqrt{3}\right)}{2}$$

$$=\frac{-(1+3)}{2}\ or\ \frac{-\left(1-3+2i\sqrt{3}\right)}{2}$$

$$-2\ or\ 1-i\sqrt{3}$$

Hence option (3)

166. If $a=\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}$ then the quadratic equation whose roots are $\alpha=a+a^2+a^4$ and $\beta=a^3+a^5+a^6$ is

1) $x^2+x+2=0$
2) $x^2-5x+7=0$
3) $x^2-x+2=0$
4) $x^2+x-2=0$

Key. 1

Sol. $a=\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}$

$$a^7=\left(\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}\right)^7$$

$$=\cos 2\pi+i\sin 2\pi$$

$=1$ ....(1)

Sum of roots = $\alpha+\beta$

$$=a+a^2+a^4+a^3+a^5+a^6$$

$$=a+a^2+a^3+a^4+a^5+a^6$$

$$=\frac{a\left(1-a^6\right)}{1-a}$$

$$=\frac{a-a^7}{1-a}$$

$= \frac{a-1}{1-a}$

$= -1$....................(2)

Product of roots = $\alpha\beta$

$= (a+a^2+a^4)(a^3+a^5+a^6)$

$= a^4+a^5+a^7+a^6+a^7+a^9+a^7+a^8+a^{10}$

$= a^4+a^5+1+a^6+1+a^2+1+a+a^3 \quad (\because \textit{from}\,(1))$

$= 3+a+a^2+a^3+a^4+a^5+a^6$

$= 3+(-1) \quad (\because \textit{from}\,(2))$

$= 3-1=2$

Required equation is $x^2 - x(-1) + 2 = 0$

$x^2 + x + 2 = 0$

Hence option (1)

167. Let z and w be two non zero complex numbers such that $|z| = |w|$ and $\arg z + \arg w = \pi$. Then z

1) w 2) $\bar{w}$ 3) $-\bar{w}$ 4) $2w$

Key. 3

Sol. Let $\arg w = \theta$

$\therefore Arg\ z = \pi - \theta$

$w = |w|(\cos\theta + i\sin\theta)$ and $z = |z|[\cos(\pi-\theta) + i\sin(\pi-\theta)]$

$= |w|(-\cos\theta + i\sin\theta)$

$= -|w|(\cos\theta - i\sin\theta)$

$= -\bar{w}$

Hence option (3)

168. If $|z_1 - 1| \le 1, |z_2 - 2| \le 2, |z_3 - 3| \le 3$ then the greatest value of $|z_1 + z_2 + z_3|$ is

1) 6 2) 7 3) 9 4) 12

Key. 4

Sol. $\left|z_1+z_2+z_3\right|=\left|(z_1-1)+(z_2-2)+(z_3-3)+6\right|$

$\le\left|z_1-1\right|+\left|z_2-2\right|+\left|z_3-3\right|+6$

$\le 1+2+3+6$

≤ 12

Greatest value of $\left|z_1+z_2+z_3\right|=12$

Hence option (4)

169. The greatest and least value of $\left|z_1+z_2\right|$ if $z_1=24+7i$ and $\left|z_2\right|=6$

1) 31, 19 2) 25, 6 3) 31, 6 4) 19, 6

Key. 1

Sol. $\left|z_1+z_2\right|\le\left|z_1\right|+\left|z_2\right|$

$=\left|24+7i\right|+6$

$=\sqrt{(24)^2+7^2}+6=25+6=31$

Also $\left|z_1+z_2\right|=\left|z_1-(-z_2)\right|$

$\ge\left\|z_1\right|-\left|z_2\right\|=\left|25-6\right|=19$

$\therefore$ Least value = 19, Greatest value = 31

Hence option (1)

170. If $\frac{\pi}{2}<\alpha<\frac{3\pi}{2}$ then Modulus and argument of $(1+\cos 2\alpha)+i\sin 2\alpha$ is

1) $-2\sin\alpha,\frac{\pi}{6}$ 2) $-2\cos\alpha,\alpha-\pi$ 3) $-2\sin\alpha,\alpha-\pi$ 4) None of these

Key. 2

Sol. Let $z=(1+\cos 2\alpha)+i\sin 2\alpha$

$=2\cos^2\alpha+2i\sin\alpha\cos\alpha$

$=2\cos\alpha[\cos\alpha+i\sin\alpha]$

$= -2\cos\alpha\left[-\cos\alpha - i\sin\alpha\right]$

$= -2\cos\alpha\left[\cos(\alpha-\pi) + i\sin(\alpha-\pi)\right] \qquad \left[\because \frac{\pi}{2} < \alpha < \frac{3\pi}{2}\right]$

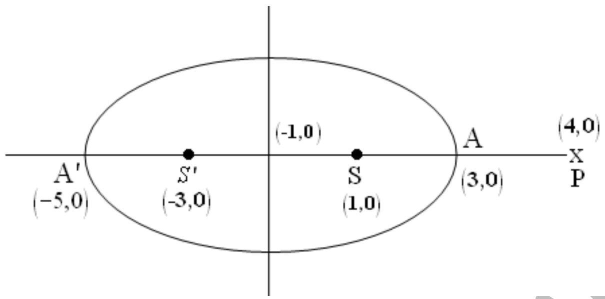
$\therefore |z| = -2\cos\alpha \;\; and \;\; \arg z = \alpha - \pi$

Hence option (2)

171. If $|z-1|+|z+3| \le 8$ then the minimum and maximum values of $|z-4|$ respectively is

1) 1, 9 2) 10, 15 3) 16, 22 4) 13, 18

Key. 1



Sol.

Given $|z-1|+|z+3| \le 8$

$\therefore$ z lies inside or on the ellipse whose foci are (1, 0) and (-3, 0) and vertices are (-5,0) and (3,0). Clearly the minimum and maximum values of $|z-4|$ are 1 and 9 respectively representing the distances PA and PA'.

$\therefore 1 \le |z-4| \le 9$

Hence option (1)

172. If $|z-25i| \le 15$ then |maximum argz – minimum argz| equals

1) $2\cos^{-1}\frac{3}{5}$ 2) $2\cos^{-1}\frac{4}{5}$

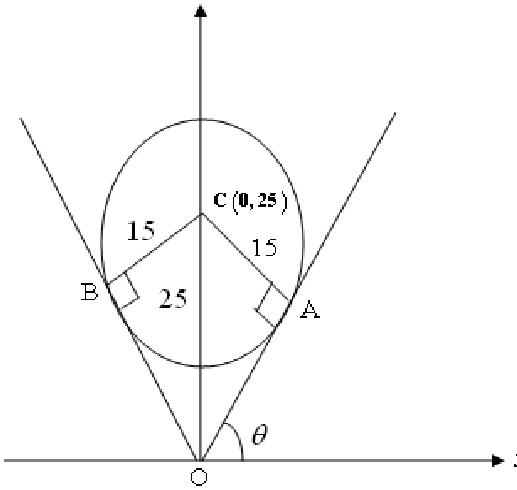
3) $\frac{\pi}{2}+\cos^{-1}\frac{3}{5}$ 4) $\sin^{-1}\frac{3}{5}-\cos^{-1}\frac{3}{5}$

Key. 2

Sol. If $|z-25i| \le 15$ then z lies either in the interior and or on the boundary of the circle with centre at $C(0,25)$ and radius equal to 15 .

The least argument is for point A and greatest argument is for point B from right

$\Delta OAC, \cos\left(\frac{\pi}{2}-\theta\right)=\frac{OA}{OC}=\frac{20}{25}=\frac{4}{5}$



$\frac{\pi}{2}-\theta=\cos^{-1}\left(\frac{4}{5}\right)$

Now for $|z-25i|\le 15$

|Maximum argz – Minimum argz| = |ArgB – ArgA|

$=\underline{|BOA}$

$=\underline{|BOX}-\underline{|AOX}=\frac{\pi}{2}+\frac{\pi}{2}-\theta-\theta$

$=\pi-2\theta$

$=2\cos^{-1}\frac{4}{5}$

Hence option (2)

173. Let z_1 and z_2 be two non-zero complex numbers such that $\frac{z_1}{z_2}+\frac{z_2}{z_1}=1$ then the origin and points represented by z_1 and z_2

1) lie on a straight line 2) form a right triangle

3) form an equilateral triangle 4) None of these

Key. 3

Sol. Let $\frac{z_1}{z_2}=z$ then $z+\frac{1}{z}=1$

$\Rightarrow z^2-z+1=0$

$\Rightarrow z=\frac{1\pm i\sqrt{3}}{2}$

$\therefore \frac{z_1}{z_2}=\frac{1\pm i\sqrt{3}}{2}$

If z_1 and z_2 are represented by A and B respectively and O be the origin, then

$\frac{OA}{OB}=\frac{|z_1|}{|z_2|}=\left|\frac{1\pm\sqrt{3}i}{2}\right|=\sqrt{\frac{1}{4}+\frac{3}{4}}=1$

$\Rightarrow OA = OB$

Also, $\dfrac{AB}{OB} = \dfrac{|z_2 - z_1|}{|z_2|} \qquad = \left|1 - \dfrac{z_1}{z_2}\right|$

$= \left|1 - \left(\dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2} i\right)\right|$

$\left|\dfrac{1}{2} \mp \dfrac{\sqrt{3}}{2} i\right| = \sqrt{\dfrac{1}{4} + \dfrac{3}{4}} = 1$

$\Rightarrow AB = OB$

Thus $OA = OB = AB$

$\therefore \Delta AOB$ is an equilateral triangle.

Hence option (3)

174. If the equation, $z^4 + a_1 z^3 + a_2 z^2 + a_3 z + a_4 = 0$, where a_1, a_2, a_3, a_4 are real coefficients different from zero has a pure imaginary root then the expression $\dfrac{a_3}{a_1 a_2} + \dfrac{a_1 a_4}{a_2 a_3}$ has the value equal to:

(A) 0 (B) 1 (C) – 2 (D) 2

Key. 2

Sol. we know that

$$\frac{1}{x-1} + \frac{1}{x-w} + \frac{1}{x-w^2} + ... + \frac{1}{x-w^{n-1}} = \frac{n\left(x^{n-1}\right)}{2^n - 1}$$

Put $x = 2$ we get

$$\frac{1}{2-1} + \frac{1}{2-w} + \frac{1}{2-w^2} + ... + \frac{1}{2-w^{n-1}} = \frac{n\,2^{n-1}}{2^n - 1}$$

$$\frac{1}{2-w} + \frac{1}{2-w^2} + ... + \frac{1}{2-w^{n-1}} = \frac{n.2^{n-1}}{2^n - 1} = \frac{n.2^{n-1} - 2^n + 1}{2^n - 1} = \frac{2^n(n-2) + 2}{2\left(2^n - 1\right)}$$

Hence option (4)

175. If α, β be the roots of the equation $u^2 - 2u + 2 = 0$ & if $\cot\theta = x + 1$, then $\dfrac{(x+\alpha)^n - (x+\beta)^n}{\alpha - \beta}$ is equal to

(A) $\frac{\sin n\theta}{\sin^n \theta}$ (B) $\frac{\cos n\theta}{\cos^n \theta}$ (C) $\frac{\sin n\theta}{\cos^n \theta}$ (D) $\frac{\cos n\theta}{\sin^n \theta}$

Key. 1

Sol. Let

$$S = \quad 1+3\alpha+5\alpha^{2\alpha^n}+(2n-3)^{n-2}\,4(2n-1)\alpha^{n-1}$$
$$\alpha S = \qquad \alpha+3\alpha^2+.................+(2n-3)\alpha^{n-1}+(2n-1)\alpha^n$$

$$S(1-\alpha)=1+2\alpha+2\alpha^2+...........2\alpha^{n-1}-(2n-1)\alpha^n$$
$$=1+2\alpha\left(1+\alpha+\alpha^2+..........\alpha^{n-2}\right)-(2n-1)\alpha^n$$
$$=1+\frac{2\alpha\left(1-\alpha^{n-1}\right)}{1-\alpha}-(2n-1)\alpha^n$$

$$=1+\frac{2\left(\alpha-\alpha^n\right)}{1-\alpha}-(2n-1)\alpha^n$$

$$=1+\frac{2(\alpha-1)}{1-\alpha}-(2n-1) \quad \left(\because \alpha^n=1\right)$$

$$=1-2-2n+1=-2n$$

$$S=\frac{-2n}{1-\alpha}=\frac{2n}{\alpha-1}$$

Hence option (4)

176. If $z=(\lambda+3)-i\sqrt{5-\lambda^2}$ then the locus of z is

1) ellipse 2) semi circle 3) parabola 4) straight line

Key. 2

Sol. Let $z=x+iy$ then $x=\lambda+3,\ y=-\sqrt{5-\lambda^2}$

$$\Rightarrow (x-3)^2=\lambda^2 \ and \ y^2=5-\lambda^2$$

(1) (2)

From (1) and (2) $(x-3)^2=5-y^2$

$$\Rightarrow (x-3)^2+y^2=5$$

Clearly it is a semi circle as $y<0$. Hence part of the circle lies below the x-axis.

Hence option (2)

177. If $x^2+x+1=0$ then the value of $\left(x+\frac{1}{x}\right)^2+\left(x^2+\frac{1}{x^2}\right)^2+.......\left(x^{27}+\frac{1}{x^{27}}\right)^2$ is

1) 27 2) 72 3) 45 4) 54

Key. 4

Sol. $x^2+x+1=0 \Rightarrow x=w \ or \ w^2$

Let $x=w$ then $x+\frac{1}{x}=w+\frac{1}{w}=w+w^2=-1$

$x^2+\frac{1}{x^2}=w^2+\frac{1}{w^2}=w^2+w=-1$

$x^3+\frac{1}{x^3}=w^3+\frac{1}{w^3}=1+1=2$

$x^4+\frac{1}{x^4}=w^4+\frac{1}{w^4}=w+\frac{1}{w}=-1$ etc.

$\therefore \left(x+\frac{1}{x}\right)^2+\left(x^2+\frac{1}{x^2}\right)^2+\left(x^3+\frac{1}{x^3}\right)^2+.......\left(x^{27}+\frac{1}{x^{27}}\right)^2$

$=18+9(2)^2=54$

Hence option (4)

178. If centre of a regular hexagon is at origin and one of the vertices on Argand diagram is $1+2i$, then its perimeter is

1) $2\sqrt{5}$ 2) $6\sqrt{2}$ 3) $4\sqrt{5}$ 4) $6\sqrt{5}$

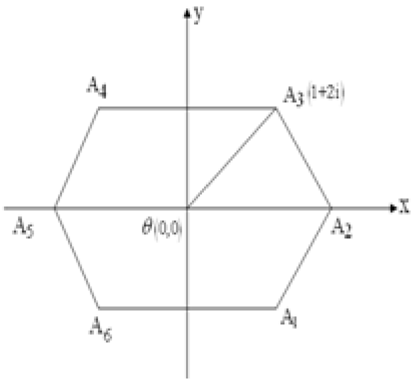
Key. 4

Sol. Let the vertices be $z_1, z_2, z_3, z_4, z_5, z_6$ w.r.t centre O at origin $|z_3|=\sqrt{5}$

Now $\Delta O A_2 A_3$ is equilateral $\Rightarrow OA_2=OA_3=A_2A_3=\sqrt{5}$

Perimeter = $6\sqrt{5}$

Hence option (4)

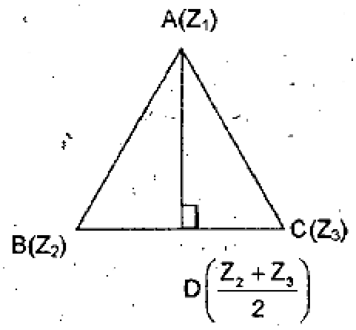


179. Let A(Z_1), B(Z_2), C(Z_3) be the vertices of an equilateral triangle ABC, then the value of $\arg\left(\frac{Z_2+Z_3-2Z_1}{Z_3-Z_2}\right)$ is equal to

a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{6}$

Ans. c

$$\arg\left(\frac{Z_2+Z_3-2Z_1}{Z_3-Z_2}\right)=\arg 2\left\{\frac{\left(\frac{Z_2+Z_3}{2}-Z_1\right)}{Z_3-Z_2}\right\}$$



$$=\arg\left\{\frac{\left(\frac{Z_2+Z_3}{2}-Z_1\right)}{Z_3-Z_2}\right\}=\frac{\pi}{2}$$

Clearly $AD\perp BC$

180. If $|z-1-i|=1$, then the locus of a point represented by the complex number 5(z – i) – 6 is

a) circle with centre (1, 0) and radius 3 b) circle with centre (-1, 0) and radius 5
c) line passing through origin d) line passing through (-1, 0)

Ans. b

Let w = 5(z – i) – 6

$\Rightarrow |w+1|=5|z-1-i|=5$

181. Let z be a complex number satisfying $\left|z^2+2z\cos\alpha\right|\le 1,\ (a\in R)$ then maximum value $\cot|z|$ must be

a) $\sqrt{2}+1$ b) $\sqrt{3}-1$ c) $\sqrt{3}+1$ d) $\sqrt{6}$

Ans. c

$\left|z^2+2z\cos\alpha\right|\le 1\Rightarrow |z||z+2\cos\alpha|\le 1$

$\Rightarrow |z+2\cos\alpha|\le |z|+|2\cos\alpha|$

$\Rightarrow |z|^2\left(|z|+|2\cos\alpha|\right)^2\le 1$

$\Rightarrow |z| \in \left[0, \sqrt{3}+1\right]$

182. Z_1 and Z_2 are the roots of $Z^2 - aZ + b = 0$ where $|Z_1| = |Z_2| = 1$ and $a, b \in C$, then

a) arg (a) = arg (b) b) arg(a) = 2 arg(b) c) 2arg(a) = arg(b) d) none of these

Ans. c

$Z_1 + Z_2 = a,\ Z_1 Z_2 = b\ and\ |Z_1| = |Z_2| = 1$

$\therefore\ \arg(a) = \frac{1}{2}\{\arg(Z_2) + \arg(Z_1)\}$ and $\arg(b) = \arg(Z_1 Z_2) = \arg(Z_1) + \arg(Z_2)$

$\therefore$ 2 arg(a) = arg (b)

183. If $|z-1| = 1$ and arg $z = \theta (z \neq 0)$ and $0 < \theta < \pi/2,$ then $1 - \frac{2}{z}$ is equal to

a) $\tan\theta$ b) $i\ \tan\theta$ c) $\tan\frac{\theta}{2}$ d) $i\ \tan\frac{\theta}{2}$

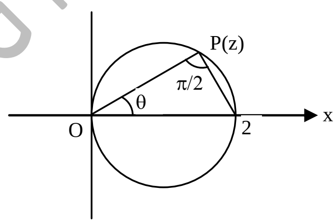
Ans. b

$\arg\left(\frac{2-z}{0-z}\right) = \frac{\pi}{2}$

$\Rightarrow \frac{z-2}{z} = \frac{AP}{OP} i$

$\tan\theta = \frac{AP}{QP}$

then $\frac{z-2}{z} = i \tan\theta$



184. If complex number z satisfies $|z - 6i| = Im(z),$ then range of $(\arg z - \arg \bar{z})$ will be

a) $\left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$ b) $\left[\frac{2\pi}{3}, \frac{4\pi}{2}\right]$ c) $\left[\frac{\pi}{3}, \frac{2\pi}{3}\right]$ d) $\left[\frac{3\pi}{4}, \frac{5\pi}{3}\right]$

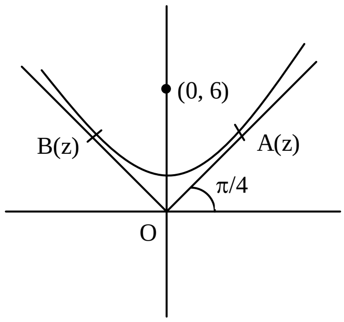
Ans. a

Clearly z lies on a parabola focus at (0, 6) and x-axis as directrix as $\arg \bar{z} = -\arg z$. Point of contact of the tangent drawn from origin to the parabola will corresponds to the maximum and minimum argument.

$(\arg z)_{\min} = \frac{\pi}{4}$

$(\arg z)_{\max} = \frac{3\pi}{4}$

$\frac{\pi}{2} \leq 2 \arg \leq \frac{3\pi}{2}$



185. If z_1, z_2, z_3 are three distinct complex numbers and a, b, c are three positive real numbers such that $\frac{a}{|z_2 - z_3|} = \frac{b}{|z_3 - z_1|} = \frac{c}{|z_1 - z_2|}$ then the value of $\frac{a^2}{z_2 - z_3} + \frac{b^2}{z_3 - z_1} + \frac{c^2}{z_1 - z_2}$ is

a) 0 b) 1 c) 2 d) 3

Ans. a

$$\left(\frac{a^2}{z_2-z_3}\right)+\left(\frac{b^2}{z_3-z_1}\right)+\left(\frac{c^2}{z_1-z_2}\right)=\lambda\left(\overline{z_2}-\overline{z_3}+\overline{z_5}-\overline{z_4}+\overline{z_1}-\overline{z_2}\right)=0$$

186. If $|z-1|+|z+3|\le 8$ then the range of values of $|z-4|$ is

a) [0, 7] b) [1, 8] c) [1, 9] d) [2, 5]

Ans. c

z lies inside or on the ellipse with foci (1, 0) and (-3, 0). Hence minimum and maximum values of $|z-4|$ are 1 and 9.

187. If x_1, x_2, x_3 …. x_n are the roots of $x^n + ax + b = 0$, then the value of $(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)$ ….. $(x_1 - x_n)$ is

a) $nx_1^n + a$ b) $nx_1^{n-1}+a$ c) $nx_1 + a^{n-1}$ d) $nx_1 + a^n$

Ans. b

$$x^n + ax + b = 0 = (x-x_1)(x-x_2)(x-x_3)....(x-x_n)$$

$$\Rightarrow (x-x_2)(x-x_3)......(x-x_n)=\frac{x^n+ax+b}{x-x_1}$$

$$\therefore (x_1-x_2)(x_1-x_3)(x_1-x_4)....(x_1-x_n)=\lim_{x\to x_1}\left(\frac{x^n+ax+b}{x-x_1}\right)=nx_1^{n-1}+a$$

188. Triangle ABC, $A(z_1), B(z_2)$ and $C(z_3)$ is inscribed in the circle | z | = 5. If $H(z_H)$ be the orthocenre of triangle ABC, then Z_H is equal to

(A) $\frac{2}{3}(z_1+z_2+z_3)$ (B) $\frac{4}{3}(z_1+z_2+z_3)$

(C) $(z_1+z_2+z_3)$ (D) $3(z_1+z_2+z_3)$

Key. C

Sol. Circumcentre of triangle ABC is origin. Let $G(Z_G)$ be it's centriod, then

$Z_G=\frac{1}{3}(z_1+z_2+z_3)$ the points O (0), $G(z_G), H(z_H)$ are collinear and OG : GH = 1 : 2

$$Z_G=\frac{2\times 0+1\times Z_H}{3}=Z_H=3Z_G=z_1+z_2+z_3$$

189. If tangents drawn to circle $|z|=2$ at $A(z_1)$ and $B(z_2)$ meet at $P(z_P)$, then

(A) $Z_P=\left(\frac{z_1+z_2}{2}\right)$ (B) $Z_P=\frac{2(z_1+z_2)}{\sqrt{z_1z_2}}$

(C) $Z_P=\frac{2z_1z_2}{z_1+z_2}$ (D) $Z_P^2=z_2z_1$

Key. C

Sol. Equation of tangentat $A(z_1)$ is

$$\frac{z}{z_1}+\frac{\bar{z}}{\bar{z}_1}=2\Rightarrow\frac{z}{z_1}+\frac{\bar{z}}{4}=2$$

$\Rightarrow \dfrac{z}{z_1^2}+\dfrac{\bar{z}}{4}=\dfrac{2}{z_1}$

Equation of tangent at B (z_2) is

$\dfrac{z}{z_2^2}+\dfrac{\bar{z}}{4}=\dfrac{2}{z_2}$

$\Rightarrow z\left(\dfrac{1}{z_1^2}-\dfrac{1}{z_2^2}\right)=2\left(\dfrac{1}{z_1}-\dfrac{1}{z_2}\right)$

$\Rightarrow z=\dfrac{2z_1z_2}{z_1+z_2}$

190. If $t^2+t+1=0$, then the value of $\left(t+\dfrac{1}{t}\right)^2+\left(t^2+\dfrac{1}{t^2}\right)^2+.....+\left(t^{27}+\dfrac{1}{t^{27}}\right)^2$ is

(A) 27 (B) 72
(C) 45 (D) 54

Key. D

Sol. $s=\left(t+\dfrac{1}{t}\right)^2+\left(t^2+\dfrac{1}{t^2}\right)^2+.....+\left(t^{27}+\dfrac{1}{t^{27}}\right)^2$

Let $t=\omega$ then

$S=\left\{(-1)^2+(-1)^2+.....18\,\text{terms}\right\}+\left\{(2)^2+.....9\text{ terms}\right\}$

$=18+9\times4=18+36=54$

191. Let n is of the form of 3P where P is an odd integer then,
${}^nc_0+{}^nc_3+{}^nc_6+{}^nc_9+.....+{}^nc_n$ equals

(A) $\dfrac{1}{3}\left(2^n-2\right)$ (B) $\dfrac{2}{3}\left(2^n-2\right)$

(C) $\dfrac{1}{3}\left(2^{n-1}-2\right)$ (D) $\dfrac{2}{3}(2^n+2)$

Key. A

Sol. $(1+x)^n=c_0+c_1x+c_2x^2+.....+c_nx^n$

$(1+\omega)^n=c_0+c_1\omega+c_2\omega^2+.....+c_n\omega^n$

$(1+\omega^2)^n=c_0+c_1\omega^2+c_2\omega^4+.....+c_n\omega^{2n}$

$2^n=c_0+c_1+c_2+.....+c_n$

$2^n+(-\omega)^n+\left(-\omega^2\right)^n=3c_0+3c_3+.....+3\,{}^nc_n$

$c_0+c_3+c_6+.....+c_n=\dfrac{1}{3}\left[2^n+(-1)^n\omega^n+(-1)^n\omega^{2n}\right]$

$=\dfrac{1}{3}\left[2^n+(-1)^{3P}\omega^{3P}+(-1)^{3P}\omega^{6P}\right]$

$=\dfrac{1}{3}\left[2^n-1-1\right]=\dfrac{1}{3}\left[2^n-2\right]$

192. Let z_1 and z_2 be two complex numbers with α and β as their principal arguments, such that $\alpha+\beta>\pi$, then principal arg(z_1z_2) is given by

(A) $\alpha+\beta+\pi$ (B) $\alpha+\beta-\pi$
(C) $\alpha+\beta-2\pi$ (D) $\alpha+\beta$

Key. C

Sol. arg (z_1z_2) = arg (z_1) + arg(z_2) + $2m\pi$, $m \in I$

$= \alpha + \beta - 2\pi$ which should be equivalent to negative angle $\frac{7\pi}{6} - 2\pi$

193. Let z and ω be two complex numbers, such that $|z|^2\omega - |\omega|^2 z = z - \omega$ and $z \neq \omega$, then

(A) $z = \bar{\omega}$ (B) $z\bar{\omega} = 1$
(C) $\bar{z}\omega = 2$ (D) $z\bar{\omega} = 2$

Key. B

Sol. $|z|^2\omega - |\omega|^2 z = z - \omega$

$\Rightarrow \omega\left[1+|z|^2\right] = z\left[1+|\omega|^2\right]$

$\omega\bar{\omega} = \frac{\left[1+|z|^2\right]}{\left[1+|\omega|^2\right]} = z\bar{\omega} = |\omega|^2\left\{\frac{1+|z|^2}{1+|\omega|^2}\right\}$

$\Rightarrow z\bar{\omega}$ is real number and therefore

$z\bar{\omega} = \omega\bar{z}$ …… (1)

$|z|^2\omega - |\omega|^2 z = z - \omega$

$z\bar{z}\omega - \omega\bar{\omega}z - z + \omega = 0$

$z(\bar{z}\omega - 1) - \omega(\bar{\omega}z - 1) = 0$ ….. (2)

From (1) and (2)

$z(z\bar{\omega} - 1) - \omega(z\bar{\omega} - 1) = 0$

$(z\bar{\omega} - 1)(z - \omega) = 0$

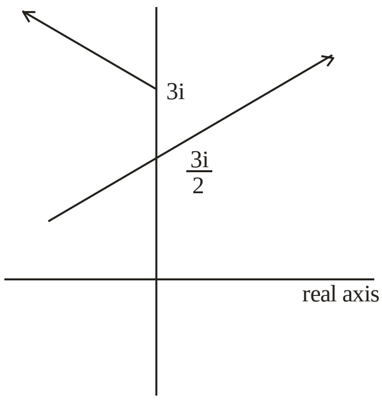
$z\bar{\omega} = 1 = \bar{z}\omega$ Since $z \neq \omega$

194. The point of intersection of the curves $\arg(z-3i) = \frac{3\pi}{4}$ and $\arg(2z+1-2i) = \frac{\pi}{4}$ is

(A) $\frac{3}{4} + i\frac{9}{4}$ (B) $1+3i$
(C) $1+i$ (D) no solution

Key. D

Sol.



$\arg(z-3i) = \frac{3\pi}{4}$ **…… (1)**

$\arg(2z+1-2i) = \frac{\pi}{4}$

$\Rightarrow \arg\left(z + \frac{1}{2} - i\right) + \arg(2) = \frac{\pi}{4}$

$\Rightarrow \arg\left[z - \left(-\frac{1}{2}\right) + i\right] = \frac{\pi}{4}$ **….. (2)**

No point of intersection of (1) and (2)

195. Let $\left|(z_1 - 2z_2)/(2 - z_1\overline{z_2})\right| = 1$ and $|z_2| \neq 1$ where z_1 and z_2 are complex numbers. The value of $|z_1|$ is

(A) 1 (B) 2
(C) cannot be obtained (D) none of these

Key. B

Sol. $|z_1 - 2z_2|^2 = |2 - z_1\overline{z_2}|^2$ i.e.,

$(z_1 - 2z_2)(\overline{z_1} - 2\overline{z_2}) = (2 - z_1\overline{z_2})(\overline{2 - z_1\overline{z_2}}) = (2 - z_1\overline{z_2})(2 - \overline{z_1}z_2)$

$\Rightarrow z_1\overline{z_1} - 2z_2\overline{z_1} - 2z_1\overline{z_2} + 4z_2\overline{z_2} = 4 - 2z_1\overline{z_2} - 2\overline{z_1}z_2 + |z_1|^2|z_2|^2$

$\Rightarrow (Z1)^2 - 4)(|z_2|^2 - 1) = 0$

Since $|z_2| \neq 1$,

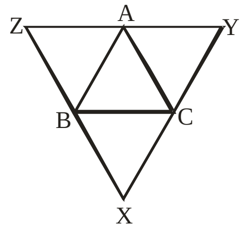
$\Rightarrow |z_1|^2 = 4 \Rightarrow |z_1| = 2$

196. Points $A(z_1)$, $B(z_2)$ and $C(z_3)$ form a triangle with centroid z_0. If triangles XCB, CYA and BAZ similar to triangle ABC are out wordly drawn on the sides of ΔABC, then centroid of ΔXYZ is

(A) $3z_0$ (B) $-z_0$
(C) z_0 (D) $-2z_0$

Key. C

Sol.



The quadrilateral abxc is a parallelogram. if z is the affix of x,

$$\frac{1}{2}(z_1 + z) = \frac{1}{2}(z_2 + z_3)$$

$z = z_2 + z_3 - z_1$

similarly affix of y is $z_1 + z_3 - z_2$ and that of z is $z_1 + z_2 - z_3$

centroid of ΔXYZ is

$$\frac{1}{3}(z_2 + z_3 - z_4 + z_1 + z_3 - z_2 + z_1 + z_3 - z_3)$$

$$= \frac{1}{3}(z_1 + z_2 + z_3) = z_0$$

197. The roots of $1 + z + z^3 + z^4 = 0$ are represented by the vertices of

(A) a square (B) an equilateral triangle
(C) a rhombus (D) a rectangle

Key. B

SOL. The given equation is $(1 + z)(1 + z^3) = 0$ the distinct roots being $-1, -\omega, -\omega^2$ which if be represented by points a, b and c in that order

$ab = |1 - \omega| = |\omega|\,|\omega^2 - 1| = |\omega^2 - 1|$

$bc = |\omega - \omega^2| = |\omega^2|\,|\omega^2 - 1| = |\omega^2 - 1|$

$ca = |\omega^2 - 1|$

THE THREE POINTS REPRESENT THE VERTICES OF AN EQUILATERAL TRIANGLE.

198. If ω is any complex number such that $z\omega = |z|^2$ and $|z-\bar{z}|+|\omega+\bar{\omega}|=4,$ then as ω varies, then the area bounded by the locus of z is
(A) 4 sq. Units (B) 8 sq. Units
(C) 16 sq. Units (D) 12 sq. Units

Key. B

Sol. $z\omega = |z|^2 \Rightarrow \omega = \bar{z}$

$|z+\bar{z}|+|z-\bar{z}|=4$

$|x| + |y| = 2$

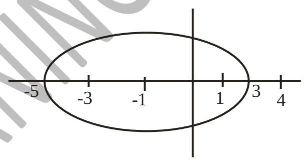
Which is a square $\therefore$ Area = 8 sq. Units

199. If $|z - 1| + |z + 3| \le 8$, then the range of values of $|z - 4|$ is,
(A) (0, 8) (B) [1, 9]
(C) [0, 8] (D) [5, 9]

Key. B

Sol.

z lies inside or on the ellipse. Clearly the minimum distance of z from the given point 4 is 1 and maximum distance is 9



200. The reflection of the complex number $\frac{6+10i}{(1+i)^2}$ in the straight line $i\bar{z}=z,$ is
(A) – 3 + 5i (B) – 3 - 5i
(C) 3 – 5i (D) 3 + 5i

Key. A

Sol. $\frac{6+10i}{(1+i)^2}=\frac{6+10i}{2i}=5-3i$

Put $z = x + iy$

$i(x - iy) = x + iy$

$ix + y = x + iy$

$\Rightarrow (x - y) - i(x - y) = 0$

$\Rightarrow x - y = 0$

Reflection is (–3 + 5i)

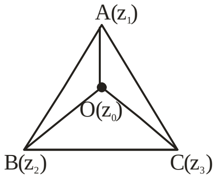
201. If z_1, z_2 and z_3 be the vertices of ΔABC, taken in anti-clock wise direction and z_0 be the circumcentre, then $\left(\frac{z_0-z_1}{z_0-z_2}\right)\frac{\sin 2A}{\sin 2B}+\left(\frac{z_0-z_3}{z_0-z_2}\right)\frac{\sin 2C}{\sin 2B}$ is equal to
(A) 0 (B) 1
(C) – 1 (D) 2

Key. C

Sol. Taking rotation at O

$$\frac{z_0-z_1}{z_0-z_2}=\cos 2C-i\sin 2C$$

$$\frac{z_0 - z_3}{z_0 - z_2} = \cos 2A + i\sin 2A$$



Now $\left(\frac{z_0 - z_1}{z_0 - z_2}\right)\frac{\sin 2A}{\sin 2B} + \left(\frac{z_0 - z_3}{z_0 - z_2}\right)\frac{\sin 2C}{\sin 2B}$

$$= \frac{\text{sin2A cos2C - isin2A sin2C + cos2Asin2C + isin2Asin2C}}{\sin 2B}$$

$$= \frac{\sin(2A + 2C)}{\sin 2B} = -1$$

202. If z = cosα + i sinα, 0 < α < π/6 then the argument of $\frac{z^4 - 1}{z^3 + 1}$ is

(A) $\frac{\pi}{2} + \frac{\alpha}{2}$ (B) $\frac{\pi}{2} - \frac{\alpha}{2}$

(C) $\frac{3\alpha}{2}$ (D) $2\alpha - \frac{\pi}{2}$

Key. A

Sol. $\arg\left(\frac{z^4 - 1}{z^3 + 1}\right) = arg(z^4 - 1) - arg(z^3 + 1) = \left(2\alpha - \frac{\pi}{2} + \pi\right) - \frac{3\alpha}{2} = \frac{\pi + \alpha}{2}$

203. If |z| = 1 and z′ = $\frac{1 + z^2}{z}$, then

(A) z′ lie on a line not passing through origin (B) |z′| = $\sqrt{2}$

(C) Re (z′) = 0 (D) Im (z′) = 0

Key. D

Sol. Conceptual

204. The number of complex numbers z satisfying $|z + \bar{z}| + |z - \bar{z}| = 4$ and |z + 2i| + |z – 2i| = 4 is/are

(A) 0 (B) 1

(C) 2 (D) 4

Key. C

SOL. ∴ |x| + |y| = 2 …..(i)

|z + 2i| + |z – 2i| = 4 …..(ii)

eq. (i) represent square & (ii) represent line segment solution are z = ± 2i.

205. If z_1, z_2 are complex numbers such that $z_1^3 - 3z_1z_2^2 = 2$ and $3z_1^2z_2 - z_2^3 = 11$ then $\left|z_1^2 + z_2^2\right|$ =

A) 3 B) 4 C) 5 D) 6

Key. C

Sol. $z_1^3 - 3z_1z_2^2 + 3iz_1^2z_2 - iz_2^3 = 2 + 11i \Rightarrow \left(z_1 + iz_2\right)^3 = 2 + 11i$

Similarly $\left(z_1 - iz_2\right)^3 = 2 - 11i$

$\left|z_1^2 + z_2^2\right| = \left|(z_1 + iz_2)(z_1 - iz_2)\right| = \left|(2 + 11i)^{1/3}(2 - 11i)^{1/3}\right| = 5$

Complex Numbers

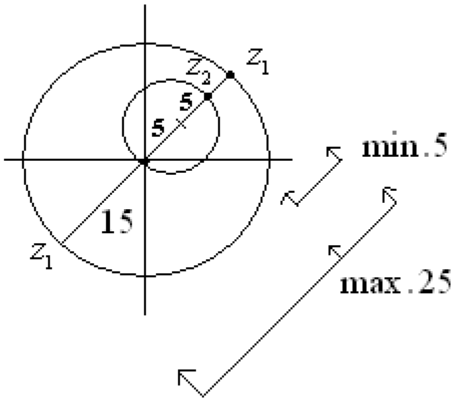
Multiple Correct Answer Type

1. If $|z_1| = 15$ and $|z_2 - 3 - 4i| = 5$, then

(A) $|z_1 - z_2|_{\min} = 5$ (B) $|z_1 - z_2|_{\min} = 10$

(C) $|z_1 - z_2|_{\max} = 20$ (D) $|z_1 - z_2|_{\max} = 25$

Key. A,D



Sol.

Min. value of $|z_1 - z_2| = 5$

Max. value of $|z_1 - z_2| = 25$

2. If Z_1 *and* Z_2 are two non-zero complex numbers such that $|Z_1 + Z_2|^2 = |Z_1|^2 + |Z_2|^2$, then

A) $Z_1\overline{Z_2}$ is purely imaginary

B) $\frac{Z_1}{Z_2}$ is purely imaginary

C) $Z_1\overline{Z_2} + \overline{Z_1}Z_2 = 0$

D) $0, Z_1, Z_2$ are the vertices of a right angled triangle which is right angled at origin

Key. A,B,C,D

Sol. $|Z_1 + Z_2|^2 = |Z_1|^2 + |Z_2|^2 \quad \Rightarrow (Z_1 + Z_2)(\overline{Z_1} + \overline{Z_2}) = |Z_1|^2 + |Z_2|^2$

$\Rightarrow \overline{Z_1}Z_2 + \overline{Z_2}Z_1 = 0 \Rightarrow \overline{Z_1}Z_2 = -Z_1\overline{Z_2} \Rightarrow Z_1\overline{Z_2}$ is purely imaginary

$\Rightarrow \frac{Z_1}{Z_2}$ is purely imaginary $\quad \Rightarrow \arg\left(\frac{Z_1}{Z_2}\right) = \frac{\pi}{2}$

thus $0, Z_1, Z_2$ form a right triangle

3. If $z_1 = a + ib$ and $z_2 = c + id$ are complex numbers such that $|z_1| = |z_2| = 1$ and $\text{Re}\left(z_1 \overline{z_2}\right) = 0$ then the pair of complex numbers $\omega_1 = a + ic$ and $\omega_2 = b + id$ satisfies

A) $|\omega_1| = 1$ B) $|\omega_2| = 1$ C) $\text{Re}\left(\omega_1 \overline{\omega_2}\right) = 0$ D) $\omega_1 \overline{\omega_2} = 0$

Key. A,B,C

Sol. $|z_1| = |z_2| = 1 \Rightarrow a^2 + b^2 = c^2 + d^2 = 1 \ldots(1)$

And $\text{Re}\left(z_1 \bar{z}_2\right) = 0 \Rightarrow \text{Re}\left\{(a+ib)(c-id)\right\} = 0 \Rightarrow ac + bd = 0 \ldots(2)$

Now from (1) and (2), $a^2 + b^2 = 1 \Rightarrow a^2 + \dfrac{a^2c^2}{d^2} = 1 \Rightarrow a^2 = d^2 \ldots(3)$

Also $c^2 + d^2 = 1 \Rightarrow c^2 + \dfrac{a^2c^2}{b^2} = 1 \Rightarrow b^2 = c^2 \ldots(4)$

$|\omega_1| = \sqrt{a^2 + c^2} = \sqrt{a^2 + b^2} = 1$ [From (1) and (4)]

And $|\omega_2| = \sqrt{b^2 + d^2} = \sqrt{c^2 + d^2} = 1$ [From (1) and (4)]

Further $\text{Re}\left(\omega_1 \overline{\omega_2}\right) = \text{Re}\left\{(a+ic)(b-id)\right\} = ab + cd = 0$ [From (2) & (4)]

Also $\text{Im}\left(\omega_1 \overline{\omega_2}\right) = bc - ad = \pm 1 \Rightarrow |\omega_1| = 1, |\omega_2| = 1$ and $\text{Re}\left(\omega_1 \bar{\omega}_2\right) = 0$

4. If points A and B are represented by the non-zero complex numbers z_1 and z_2 on the Argand plane such that $|z_1 + z_2| = |z_1 - z_2|$ and 0 is the origin, then

A) orthocentre of ΔOAB lies at O

B) circumcentre of ΔOAB is $\dfrac{z_1 + z_2}{2}$

C) $Arg\left(\dfrac{z_1}{z_2}\right) = \pm\dfrac{\pi}{2}$

D) ΔOAB is isosceles

Key. A,B,C

Sol. $|z_1 + z_2| = |z_1 - z_2| \quad \Rightarrow (z_1 + z_2)\left(\overline{z_1} + \overline{z_2}\right) = (z_1 - z_2)\left(\overline{z_1} - \overline{z_2}\right)$

$z_1\overline{z_2} + z_2\overline{z_1} = 0 \quad \Rightarrow \dfrac{z_1}{z_2} = -\left(\overline{\dfrac{z_1}{z_2}}\right) \Rightarrow \dfrac{z_1}{z_2}$ is purely imaginary

Also from (1) $|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2$

ΔOAB is a right angled triangle, we get right angle at O.

So, circumcentre $= \dfrac{z_1 + z_2}{2}$

5. The adjacent vertices of a regular polygon of n sides are whose centre is at origin given by $\left(1+\sqrt{2},1\right),\left(1+\sqrt{2},-1\right)$.Then the value of n is

a) 8 b) 4 c) 12 d) 6

Key. A

Sol. Let $z=\left(1+\sqrt{2}\right)+i,\bar{z}=\left(1+\sqrt{2}\right)-i$ .For adjacent vertices

$$z=\bar{z}cis\frac{2\pi}{n}\Rightarrow cis\frac{2\pi}{n}=\frac{1+i\left(\sqrt{2}-1\right)}{1-i\left(\sqrt{2}-1\right)}=\frac{1+i\tan\frac{\pi}{8}}{1-i\tan\frac{\pi}{8}}=cis\frac{2\pi}{8}$$

$$\Rightarrow n=8$$

6. If α is a variable complex number such that $|\alpha|>1$and $z=\alpha+\frac{1}{\alpha}$ lies on a conic then

a) Eccentricity of the conic is $\frac{2|\alpha|}{1+|\alpha|^2}$

b) Distance between foci is 4

c) Length of latusrectum is $\frac{2\left(|\alpha|^2-1\right)}{|\alpha|^2+1}$

d) Distance between directrices is $\left(|\alpha|+\frac{1}{|\alpha|}\right)^2$

Key. A,B,D

Sol. Let $|\alpha|=r>1\ and\ \ \alpha=rcis\theta\ then\ \ z=x+iy=\alpha+\frac{1}{\alpha}=rcis\theta+\frac{cis(-\theta)}{r}$

$$\Rightarrow x=\left(r+\frac{1}{r}\right)\cos\theta\ and\ \ y=\left(r-\frac{1}{r}\right)\sin\theta$$

Eliminating θ gives $\frac{x^2}{\left(r+\frac{1}{r}\right)^2}+\frac{y^2}{\left(r-\frac{1}{r}\right)^2}=1$

Which is an ellipse . $a=r+\frac{1}{r}$, $b=r-\frac{1}{r}$ $\left(r=|\alpha|>1\Rightarrow a>b\right)$

$\therefore e\sqrt{1-\frac{b^2}{a^2}}=\frac{2}{r+\frac{1}{r}}$,distance between foci =2ae=4

distance between directrices = $\frac{2a}{e}$

7. The equations of two lines making on angle 45^0 with a given line $\bar{a}z + a\bar{z} + b = 0$ (where 'a' is a complex number and b is real) and passing through a given point C (c)(c is a complex number), is/are

A) $\frac{z+c}{a} + i\frac{\bar{z}-\bar{c}}{\bar{a}} = 0$ B) $\frac{z-c}{a} + i\frac{\bar{z}-\bar{c}}{\bar{a}} = 0$

C) $\frac{z-c}{a} - i\frac{\bar{z}-\bar{c}}{\bar{a}} = 0$ D) $\frac{z+c}{a} - i\frac{\bar{z}-\bar{c}}{\bar{a}} = 0$

Key. B,C

Sol. Let z_1, z_2 be two points on the given line then $\frac{z_1 - z_2}{\bar{z}_1 - \bar{z}_2} = -\frac{a}{\bar{a}}$ $-(1)$

Also $\frac{z_1 - z_2}{\bar{z}_1 - \bar{z}_2} = \pm i\frac{z-c}{\bar{z}-\bar{c}}$ $-(2)$

From (1) and (2) $\frac{z-c}{a} \pm i\frac{z-c}{a} = 0$

8. If $z = 1 + \cos\frac{6\pi}{5} + i\sin\frac{6\pi}{5}$ then

A) $|z| = 2\cos\frac{3\pi}{5}$ B) $|z| = 2\cos\frac{2\pi}{5}$ C) $\arg z = \frac{3\pi}{5}$ D) $\arg z = -\frac{2\pi}{5}$

Key. B,D

Sol. $z = 2\cos\frac{3\pi}{5}\left(\cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}\right)$

$= 2\cos\frac{2\pi}{5}\left(\cos\frac{2\pi}{5} - i\sin\frac{2\pi}{5}\right)$

9. Let $z_1, z_2, z_3, \ldots\ldots\ldots, z_n$ are the complex numbers such that $|z_1| = |z_2| = \ldots\ldots\ldots\ldots = |z_n|$ =1. If $z = \left(\sum_{k=1}^{n} z_k\right)\left(\sum_{k=1}^{n} \frac{1}{z_k}\right)$ then

A) z is purely imaginary

B) z is real

C) $0 < z \le n^2$

D) z is a complex number of the form $a + ib$

Key. B,C

Sol. $z=\left(z_1+z_2+...+z_n\right)\left(\frac{1}{z_1}+\frac{1}{z_2}+...+\frac{1}{z_n}\right)$

$\Rightarrow \left|z_1+z_2+.......+z_n\right|^2 \rightarrow$ which is real

$\le \left|z_1\right|^2+\left|z_2\right|^2+\left|z_3\right|^2+...+\left|z_n\right|^2=n^2$

10. If a,b,c are non – zero complex numbers of equal moduli and satisfy $az^2+bz+c=0$ then

A) $\min|z|=\frac{\sqrt{5}-1}{2}$

B) $\min|z|=0$

C) $\min|z|$ does not exist

D) $\max|z|=\frac{\sqrt{5}+1}{2}$

Key. A,D

Sol. $|a|=|b|=|c|=r$

$|c|=\left|-az^2+bz\right|\le r|z|^2+r|z|$

$\Rightarrow |z|^2+|z|-1\ge 0 \quad -(1)$

and $az^2=-(bz+c)$

$|a||z|^2 =\le r|z|+r$

$|z|^2-|z|-1\le 0 \qquad -(2)$

Solve (1) and (2)

11. Let x_1, x_2 are the roots of the quadratic equation $x^2+ax+b=0$ where a,b are complex numbers and y_1, y_2 are the roots of the quadratic equation $y^2+|a|y+|b|=0$. If $|x_1|=|x_2|=1$ then

A) $|y_1|=1$

B) $|y_2|=1$

C) $|y_1|\ne|y_2|$

D) $|y_1|=|y_2|=2$

Key. A,B

Sol. $x^2+ax+b=0 \Rightarrow |a|\le 2$ and $|b|=1$

$y=\frac{-|a|\pm\sqrt{|a|^2-4|b|}}{2}=\frac{-|a|\pm i\sqrt{4-|a|^2}}{2}$

$|y|=1$

12. If the equation $z^3+(3+i)z^2-3z-(m+i)=0$ where $m \in R$, has atleast one real root then 'm' can have the value equal to

a) 1 b) 2 c) 3 d) 5

Key. A,D

Sol. Let α is a real root then

$a^3+(3+i)\alpha^2-3\alpha=m+i$

$\Rightarrow \alpha^3+3\alpha^2-3\alpha-m=0 \ \& \alpha^2-1=0$

$\Rightarrow \alpha=1 \text{ or } -1$

$\Rightarrow m=1 \text{ or } 5$

13. If $|z-3|=\min\{|z-1|,|z-5|\}$ then $\operatorname{Re}(z)$ =____

a) 2 b) $\frac{5}{2}$ c) $\frac{7}{2}$ d) 4

Key. A,D

Sol. If $|z-1| \le |z-5|$

Then $|z^2-z-\bar{z}+1 \le |z|^2-5z+5\bar{z}+25 \ \& |z-3|=|z-1|$

$\Rightarrow 4(2+\bar{z}) \le 24$ $\Rightarrow z\bar{z}-3z=3\bar{z}+9$

$z+\bar{z} \le 6$ $=z\bar{z}-z-\bar{z}+1$

$\operatorname{Re}(z) \le 3$ $\Rightarrow z(z+\bar{z})=8$

$z+\bar{z}=4$

$\operatorname{Re}(z)=2$

14. If $z=x+iy$ then the equation $\left|\frac{2z-i}{z+1}\right|=m$ represents a circles when m can be

a) $\frac{1}{2}$ b) 1

c) 2 d) $3<m<2\sqrt{3}$

Key. A,B,D

Sol. $\left|z\frac{-i}{2}=\frac{m}{2}\right|z+1|$

$m \ne 2$

15. Let a,b,c be distinct complex numbers with $|a|=|b|=|c|=1$ and z_1,z_2 be the roots of the equation $az^2+bz+c=0$ with $|z_1|=1$. Let P and Q represent the complex numbers z_1 and z_2 in the argand plane with $\underline{|POQ}=\theta, 0^0<\theta<180^0$ (where O being the origin) then

A) $b^2 = ac;\ \theta = \frac{2\pi}{3}$ B) $\theta = \frac{2\pi}{3};\ PQ = \sqrt{3}$

C) $PQ = 2\sqrt{3};\ b^2 = ac$ D) $\theta = \frac{\pi}{3};\ b^2 = ac$

Key. A,B

Sol. $|z_1 + z_2| = \left|\frac{-b}{a}\right|;\ z_1 z_2 = \left|\frac{c}{a}\right|\ \therefore\ |z_1 + z_2|^2 = 1 \Rightarrow (z_1 + z_2)(\overline{z_1} + \overline{z_2}) = 1$

$\Rightarrow 2 + \overline{z_1} z_2 + z_2 \overline{z_1} = 1 \Rightarrow \frac{(z_1 + z_2)^2}{z_1 z_2} = 1 \Rightarrow \frac{b^2}{a^2} = \frac{c}{a} \Rightarrow b^2 = ac$

Now, $z_2 = z_1 e^{i\theta}$ then $|z_1 + z_2| = |z_1||1 + e^{i\theta}| \Rightarrow 2\cos\frac{\theta}{2} = 1\ \therefore\ \theta = \frac{2\pi}{3}$

$PQ = |z_2 - z_1| = \sqrt{3}$

16. The complex slope μ of a line containing the points z_1 and z_2 in the complex plane is defined as $\frac{z_1 - z_2}{\overline{z_1} - \overline{z_2}}$. *If* μ_1, μ_2 are the complex slopes of two lines L_1 and L_2, then

a) L_1 and L_2 are perpendicular *if* $\mu_1 + \mu_2 = 0$ b) L_1 and L_2 are parallel *if* $\mu_1 + \mu_2 = 0$

c) L_1 and L_2 are perpendicular *if* $\mu_1\mu_2 = -1$ d) L_1 and L_2 are parallel *if* $\mu_1 = \mu_2$

Key: A,D

Hint We observe that if z_o is a non – zero complex number and c is a real number , then the equation $\overline{z_o} z + z_o \overline{z} + c = 0$ represents a straight line with complex slope $\frac{-z_o}{\overline{z_o}}$

Let $L_1 : \overline{\alpha} z + \alpha \overline{z} + c = 0$ *and* $L_2 : \overline{\beta} z + \beta \overline{z} + d = 0$ *where* $\alpha = (a,b)$ *and* $\beta = (p,q)$ are non –zero complex numbers. Then their cartesian equations are

$ax + by + \frac{c}{2} = 0$ *and* $px + qy + \frac{d}{2} = 0$

$\therefore L_1 \perp L_2 \Leftrightarrow ap + bq = 0 \Leftrightarrow \alpha\overline{\beta} + \overline{\alpha}\beta = 0$

$\Leftrightarrow \frac{\alpha}{\overline{\alpha}} + \frac{\beta}{\overline{\beta}} = 0 \Leftrightarrow \mu_1 + \mu_2 = 0$ where $\mu_1 = -\frac{\alpha}{\overline{\alpha}}$ *and* $\mu_2 = \frac{-\beta}{\overline{\beta}}$ are the complex slopes of L_1 and L_2 respectively.

$L_1 \parallel L_2 \Leftrightarrow aq - bp = 0 \Leftrightarrow \alpha\overline{\beta} - \overline{\alpha}\beta = 0$

$\Leftrightarrow \frac{\alpha}{\overline{\alpha}} = \frac{\beta}{\overline{\beta}} \Leftrightarrow \mu_1 = \mu_2$.

17. Let z_1, z_2, z_3 in G.P. be roots of the equation $z^3 - bz^2 + 3z - 1 = 0$ then

(A) $z_2 = 1$ (B) $z_2 = 2$ (C) $b = 3$ (D) b can be -3

Key: A,C

Sol. $z_2^2 = z_1 z_3$

$\Rightarrow z_2^3 = 1$

$z_2 = 1, \omega, \omega^2$

1 - b + 3 - 1 = 0 $\Rightarrow$ b = 3

18. Let z_1, z_2, z_3 be the vertices of a triangle ABC. Then which of the following statements is correct?

(A) If $\frac{1}{z-z_1}+\frac{1}{z-z_2}+\frac{1}{z-z_3}=0$, where $z=\frac{z_1+z_2+z_3}{3}$, then ABC is an equilateral triangle.

(B) If ABC is an equilateral triangle then $\frac{1}{z-z_1}+\frac{1}{z-z_2}+\frac{1}{z-z_3}=0$, where $z=\frac{z_1+z_2+z_3}{3}$

(C) If $\begin{vmatrix} 1 & 1 & 1 \\ z_1 & z_2 & z_3 \\ z_2 & z_3 & z_1 \end{vmatrix}$=0, then the triangle ABC is equilateral

(D) If $|z_1|=|z_2|=|z_3|$ and $z_1+z_2+z_3=0$, then the triangle ABC is equilateral.

Key: A,B,C,D

Hint A necessary and sufficient condition for a triangle having vertices z_1, z_2 and z_3 to form an equilateral triangle is $z_1^2+z_2^2+z_3^2=z_1z_2+z_1z_3+z_2z_3$.

(A) and (B) will follow by performing some algebraic jugglery on the known condition given above.

To prove (D) note that $z_1+z_2+z_3=0$ can be changed to $\frac{1}{z_1}+\frac{1}{z_2}+\frac{1}{z_3}=0$

$\left(\because |z_1|=|z_2|=|z_3|\right)$

19. If z_1 and z_2 are two complex numbers such that $|z_1+z_2|^2=|z_1|^2+|z_2|^2$ then

(A) $\frac{z_1}{z_2}$ is purely real

(B) $\frac{z_1}{z_2}$ is purely imaginary

(C) $z_1\bar{z}_2+\bar{z}_1z_2=0$

(D) $\arg\left(\frac{z_1}{z_2}\right)=0$

Key: B,C

Hint: $|z_1+z_2|^2$

$=(z_1+z_2)(\bar{z}_1+\bar{z}_2)$

$=z_1\bar{z}_1+z_2\bar{z}_2+z_1\bar{z}_2+\bar{z}_1z_2$

$=|z_1|^2+|z_2|^2+z_1\bar{z}_2+\bar{z}_1z_2$

We have $z_1\bar{z}_2+\bar{z}_1z_2=0$

$\Rightarrow z_1\bar{z}_2 = -\bar{z}_1 z_2 \Rightarrow \frac{z_1}{z_2} = -\left(\frac{\bar{z}_1}{\bar{z}_2}\right)$

So, $\frac{z_1}{z_2}$ is purely imaginary.

20. Suppose three real numbers a, b, c are in G.P let $z = \frac{a+ib}{c-ib}$ then

(A) $Z = \frac{ib}{c}$ (B) $Z = \frac{ia}{b}$ (C) $Z = \frac{ia}{c}$ (D) $z = 0$

Key: A, B

Hint: Let r be common ratio of G.P. a, b, c we have

$$z = \frac{\frac{a}{b}+i}{\frac{c}{b}-1} = \frac{\frac{1}{r}+i}{r-1} = \frac{i}{r}$$

$$z = \frac{ib}{c} \text{ or } \frac{ia}{b}$$

21. If $A(z_1), B(z_2)$ and $C(z_3)$ are three points in argand plane where $|z_1 + z_2| = |z_1| - |z_2|$ and $|(1-i)z_1 + iz_3| = |z_1| + |z_3 - z_1|$, then

(A) A,B and C lie on a fixed circle with centre $\left(\frac{z_2 + z_3}{2}\right)$

(B) A, B, C form right angle triangle

(C) A, B, C from an equilateral triangle

(D) A, B, C form an obtuse angle triangle

Key: A, B

Hint: $\arg\frac{z_1}{z_2} = \pm\pi$ and $|z_1 + i(z_3 - z_1)| = |z_1| + |z_3 - z_1|$ iff arg $\frac{z_1}{z_3 - z_1} = \frac{\pi}{2}$

So centre of circle $= \left(\frac{z_2 + z_3}{2}\right)$ and ABC is right angle triangle.

22. The complex numbers satisfying the equation $(3z+1)(4z+1)(6z+1)(12z+1) = 2$ is /are

A) $\frac{\sqrt{33}-5}{24}$ B) $\frac{\sqrt{33}+5}{24}$ C) $\frac{-i\sqrt{23}-5}{24}$ D) $\frac{-i\sqrt{23}+5}{24}$

Key. A,C

Sol. $(3z+1)\ (4z+1)\ (6z+1)(12z+1) = 2$

$8\ (3z+1).\ 6\ (4z+1).4(6z+1)\ 2.(12z+1) = 2 \times 8 \times 6 \times 4 \times 2$

$(24z+8).\ (24z+6).\ (24z+4)\ (24z+2)\ =\ 768$

Let $24z+5=U$

$(U+3)(U+1)(U-1)(U-3)=768 \qquad \Rightarrow\ (U^2-9)(U^2-1)=768$

$\Rightarrow U^4-10U^2-759=0 \qquad \Rightarrow\ U^2=33\ or -23$

$\Rightarrow 24Z+5=\pm\sqrt{33}\ or \pm i\sqrt{23} \qquad z=\dfrac{\pm\sqrt{33}-5}{24}\ or\ \dfrac{\pm i\sqrt{23}-5}{24}$

23. If $|z_1|=15$ and $|z_2-3-4i|=5$, then

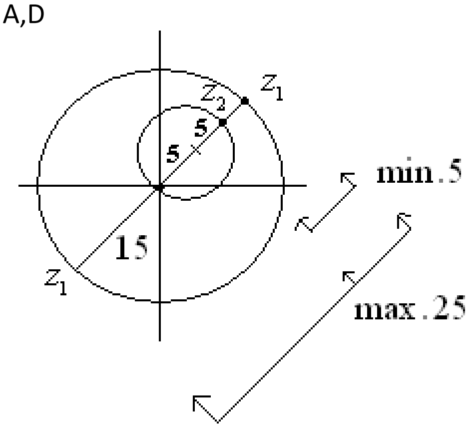
(A) $|z_1-z_2|_{\min}=5$

(B) $|z_1-z_2|_{\min}=10$

(C) $|z_1-z_2|_{\max}=20$

(D) $|z_1-z_2|_{\max}=25$

Key. A,D



Sol.

Min. value of $|z_1-z_2|=5$

Max. value of $|z_1-z_2|=25$

24. If $\dfrac{3iz_2}{5z_1}$ is purely real, then find $5\left|\dfrac{3z_1+7z_2}{3z_1-7z_2}\right|$.

Key. 5

Sol. Let $\dfrac{3iz_2}{5z_1}=K$ (real)

$\dfrac{z_2}{z_1}=\dfrac{5K}{3i}$

$5\left|\dfrac{3+7\dfrac{z_2}{z_1}}{3-7\dfrac{z_2}{z_1}}\right|=5\left|\dfrac{3+7\dfrac{35K}{3i}}{3-\dfrac{35K}{3i}}\right|$

$5\left|\dfrac{35K+9i}{35K-9i}\right|=5$

25. Let z_1 and z_2 be two distinct complex numbers and let $w = (1 - t)z_1 + tz_2$ for some real numbers t with $0 < t < 1$. If Arg (z) denotes the principal argument of a non–zero complex number z, then

(A) $|w - z_1| + |w - z_2| = |z_1 - z_2|$

(B) Arg $(w - z_1)$ = Arg $(w - z_2)$

(C) $\begin{vmatrix} w - z_1 & \bar{w} - \bar{z}_1 \\ z_2 - z_1 & \bar{z}_2 - \bar{z}_1 \end{vmatrix} = 0$

(D) Arg $(w - z_1)$ = Arg $(z_2 - z_1)$

Key. A,C,D

Sol. As, $w = \dfrac{(1-t)z_1 + tz_2}{(1-t)+t}$, lies on the line segment joining z_1 and z_2

26. Let z_2 be reflection of z_1 in $a\bar{z} + \bar{a}z + b = 0$. then

(A) Re $\{a(\bar{z}_1 + \bar{z}_2)\} = -b/2$

(B) Re $\{a(\bar{z}_1 + \bar{z}_2)\} = -2b$

(C) $\arg(z_2 - z_0) = -\arg(z_1 - z_0)$, where $a\bar{z}_0 + \bar{a}z_0 + b = 0$

(D) $\left(\dfrac{z_1 - z_2}{\bar{z}_1 - \bar{z}_2}\right) - \dfrac{a}{\bar{a}} = 0$

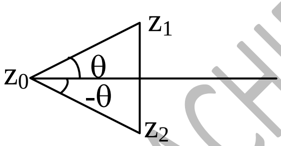
Key. D

Sol. $a\left(\dfrac{\bar{z}_1 + \bar{z}_2}{2}\right) + \bar{a}\left(\dfrac{\bar{z}_1 + \bar{z}_2}{2}\right) + b = 0$

$\Rightarrow a(\bar{z}_1 + \bar{z}_2) + \bar{a}(z_1 + z_2) + 2b = 0$

$\Rightarrow$ Re $a\left(\bar{z}_1 + \bar{z}_2\right) = -b$

$\dfrac{z_1 - z_2}{\bar{z}_1 - \bar{z}_2} - \dfrac{a}{\bar{a}} = 0$



27. Let z_1 and z_2 are non – zero (given) complex numbers and k be any positive real number. Consider the system of equations $|3z - z_1 - 2z_2| = |z_1 - z_2|$ and $\arg\left(\dfrac{z_1 - z_2}{z - kz_1 - (1-k)z_2}\right) = \pm\dfrac{\pi}{2}$. Then

(A) The system of equations has no solution if $k \in \left(\dfrac{2}{3}, \infty\right)$

(B) The system of equations have more than one solutions if $k \in \left(0, \dfrac{2}{3}\right)$

(C) The system of equations have no solution or if $k \in \left(0, \dfrac{2}{3}\right)$

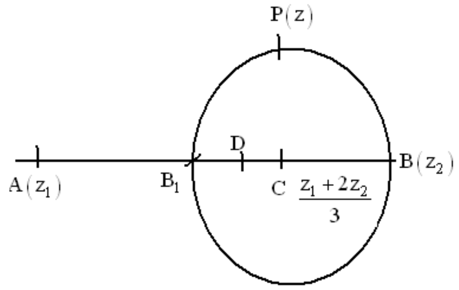
(D) The system of equations have more than one solution if $k \in \left(\dfrac{2}{3}, \infty\right)$

Key. A,B

Sol. $\left|z-\frac{z_1+2z_2}{3}\right|=\frac{1}{3}|z_1-z_2|$

And $\text{Arg}\left(\frac{z_1-z_2}{z-(kz_1+(1-k)z_2)}\right)=\pm\frac{\pi}{2}$

Angle between the line segment joining z_1 and z_2; z and $kz_1 + (1-k)z_2$ is $\frac{\pi}{2}$ $\frac{z_1+2z_2}{3}$ is a point on segment AB such that AC : CS = 2:1 and D $(kz_1+(1-k)z_2)$ is a point on segment AB such that AD:DB = $1-k : k \Rightarrow$ BD = $k|(z_1-z_2)|$



(a) for nosol $BD > BB_1 \Rightarrow k|z_1-z_2| > \frac{2}{3}|z_1-z_2| \Rightarrow k > \frac{2}{3}$

(b) for more than one sol. $0 < BD < BB_1$

$\Rightarrow 0 < k|z_1-z_2| < \frac{2}{3}|z_1-z_2|$

$0 < k < 2/3.$

28. If $\alpha_1, \alpha_2, \alpha_3 ------\alpha_{30}$ are 30th roots of unity then $\sum\sum_{1\le i<j\le 30}(\alpha_i\alpha_j)^3 =$

a) $\sum_{i=1}^{30}\alpha_i$ b) $10^{1/30}$ c) 1 d) 0

Key. A,D

Sol. $\sum\sum_{1\le i<j\le 30}\alpha_i^3\alpha_j^3 = \frac{(\alpha_1^3+\alpha_2^3+---+\alpha_{30}^3)(\alpha_1^6+\alpha_2^6+---+\alpha_{30}^6)}{2}$

$\alpha_1^3+\alpha_2^3+---+\alpha_{30}^3 = 1+(\alpha)^3+(\alpha^2)^3+------+(\alpha^{29})^3\left(\alpha = cis\frac{2\pi}{30}\right)$

$= \frac{1-(\alpha^3)^{30}}{1-\alpha^3} = 0$

Similarly $\alpha_1^6+\alpha_2^6+-----+\alpha_{30}^6 = 0$

29. If z_1, z_2, z_3 are vertices of a triangle then all complex numbers z which make the triangle in to a parallelogram are given by

a) $-(z_1+z_2+z_3)$ b) $z_1+z_2-z_3$ c) $z_2+z_3-z_1$ d) $z_3+z_1-z_2$

Key. B,C,D

Sol. Conceptual

30. The adjacent vertices of a regular polygon of n sides are whose centre is at origin given by $\left(1+\sqrt{2},1\right),\left(1+\sqrt{2},-1\right)$.Then the value of n is

a) 8 b) 4 c) 12 d) 6

Key. A

Sol. Let $z=\left(1+\sqrt{2}\right)+i,\bar{z}=\left(1+\sqrt{2}\right)-i$ .For adjacent vertices

$$z=\bar{z}cis\frac{2\pi}{n}\Rightarrow cis\frac{2\pi}{n}=\frac{1+i\left(\sqrt{2}-1\right)}{1-i\left(\sqrt{2}-1\right)}=\frac{1+i\tan\frac{\pi}{8}}{1-i\tan\frac{\pi}{8}}=cis\frac{2\pi}{8}$$

$$\Rightarrow n=8$$

31. If α is a variable complex number such that $|\alpha|>1$and $z=\alpha+\frac{1}{\alpha}$ lies on a conic then

a) Eccentricity of the conic is $\frac{2|\alpha|}{1+|\alpha|^2}$ b) Distance between foci is 4

c) Length of latusrectum is $\frac{2\left(|\alpha|^2-1\right)}{|\alpha|^2+1}$ d) Distance between directrices is $\left(|\alpha|+\frac{1}{|\alpha|}\right)^2$

Key. A,B,D

Sol. Let $|\alpha|=r>1$ *and* $\alpha=rcis\theta$ *then* $z=x+iy=\alpha+\frac{1}{\alpha}=rcis\theta+\frac{cis(-\theta)}{r}$

$$\Rightarrow x=\left(r+\frac{1}{r}\right)\cos\theta \text{ and } y=\left(r-\frac{1}{r}\right)\sin\theta$$

Eliminating θ gives $\frac{x^2}{\left(r+\frac{1}{r}\right)^2}+\frac{y^2}{\left(r-\frac{1}{r}\right)^2}=1$

Which is an ellipse . $a=r+\frac{1}{r}$, $b=r-\frac{1}{r}$ $\left(r=|\alpha|>1\Rightarrow a>b\right)$

$\therefore e\sqrt{1-\frac{b^2}{a^2}}=\frac{2}{r+\frac{1}{r}}$,distance between foci =2ae=4

distance between directrices= $\frac{2a}{e}$

32. If $Z\neq 0$ is a complex number then $Z,iZ,-Z,-iZ$ are the vertices of a

A) square B) rectangle C) rhombus D) parallelogram

Key. A,B,C,D

Sol. Conceptual

33. Let $Z_1 = x_1 + iy_1, Z_2 = x_2 + iy_2$ be complex numbers in fourth quadrant of argand plane and $|Z_1| = |Z_2| = 1$, $\text{Re}(Z_1 Z_2) = 0$. The complex numbers $Z_3 = x_1 + ix_2, Z_4 = y_1 + iy_2$, $Z_5 = x_1 + iy_2, Z_6 = x_2 + iy_1$ will always satisfy

A) $|Z_4| = 1$

B) $\arg(Z_3 Z_4) = -\pi/2$

C) $\dfrac{Z_5}{\cos(\arg Z_1)} + \dfrac{Z_6}{\sin(\arg Z_1)}$ is purely real

D) $Z_5^2 + (\bar{Z}_6)^2$ is purely imaginary

Key. A,B,C,D

Sol. $Z_1 = e^{i\theta_1}, Z_2 = e^{i\theta_2}, \text{Re}(Z_1 Z_2) = 0 \Rightarrow \theta_1 + \theta_2 = -\pi/2$

$Z_3 = e^{-i\theta_1}, Z_4 = -ie^{i\theta_1}, Z_5 = \cos\theta_1(1-i), Z_6 = \sin\theta_1(-1+i)$

34. Let Z satisfies $|Z + 2(1+i)| = \sqrt{2}$ then

A) max $(|Z|) = 4\sqrt{2}$

B) If $\arg(Z)$ is least $|Z| = \sqrt{6}$

C) $\max(\arg Z) = 13\pi/12$

D) $|\max(\arg Z) - \min(\arg Z)| = \pi/3$

Key. B,D

Sol. $\max(|Z|) = OP = 2\sqrt{2} + \sqrt{2} = 3\sqrt{2}$

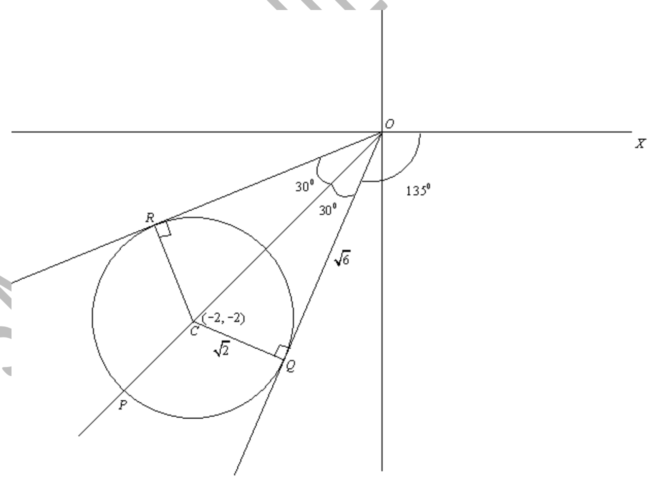
For Q $\arg Z$ is max

For R $\arg Z$ is min.

$OR = \sqrt{OC^2 - CR^2} = \sqrt{6}$ =15^0

$\max(\arg Z) = \angle XOQ = -(135^0 - 30^0) = -105^0$

$\min(\arg Z) = \angle XOR = -(135^0 + 30^0) = -165^0$



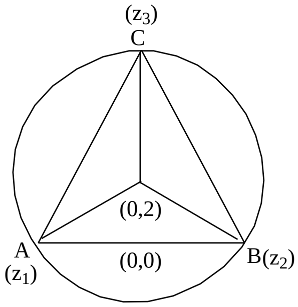
35. One vertex of the triangle of maximum area that can be inscribed in the curve $|z - 2i| = 2$ is $2 + 2i$, remaining vertices are

(a) $-1 + i(2+\sqrt{3})$ (b) $-1 - i(2+\sqrt{3})$ (c) $-1 + i(2-\sqrt{3})$ (d) $-1 - i(2-\sqrt{3})$

Key. A,C

Sol. $\frac{z_2 - z_0}{z_1 - z_0} = e^{i2\pi/3}$

$\frac{z_3 - z_0}{z_1 - z_0} = e^{-i\frac{2\pi}{3}}$



36. If all the three roots of $az^3 + bz^2 + cz + d = 0$ have negative real parts ($a, b, c \in R$) then
a) $ab > 0$ b) $bc > 0$ c) $ad > 0$ d) $bc - ad > 0$

Ans. a,b,c,d

Sol. Let $z_1 = x$, $z_2, z_3 = x_2 \pm iy_2$

$\Rightarrow z_1 + z_2 + z_3 = -\frac{b}{a} \Rightarrow x_1 + 2x_2 = -\frac{b}{a} < 0 \Rightarrow ab > 0$

Also, $z_1z_2z_3 = x_1\left[x_2^2 + y_2^2\right] = -\frac{d}{a} \Rightarrow ad > 0$

Also $-\frac{bc}{a^2} < x_1\left(x_2^2 + y_2^2\right)$

$\Rightarrow bc > ad$

37. ABCD is a rhombus its diagonal AC and BD intersect at the points M and satisfy BD=2AC. Its points D and M represent complex number 1 + i, 2 – i respectively then complex number represented by A
a) $3 - \frac{i}{2}$ b) $-3 + \frac{i}{2}$ c) $1 - \frac{3i}{2}$ d) $-1 + \frac{3i}{2}$

Ans. a, c

Sol. $\frac{z - (2-i)}{(1+i) - (2-i)} = \frac{1}{2} \pm i \Rightarrow z = 3 - \frac{i}{2}$ and $z = 1 - -\frac{3i}{2}$

38. If $\left|z + \frac{1}{z}\right| = 2$ then the true statements among the following are
A) Maximum value of $|z|$ is $\sqrt{2} + 1$ B) Minimum value of $|z|$ is $\sqrt{2} - 1$
C) Maximum value of $|z|$ is $\sqrt{5} + 2$ D) Minimum value of $|z|$ is $\sqrt{5} - 2$

Key. A,B

Sol. $|Z| = \left|Z + \frac{1}{Z} - \frac{1}{Z}\right| \le \left|Z + \frac{1}{Z}\right| + \left|\frac{1}{Z}\right|$

39. A complex number z satisfies the equation $|z^2 - 9| + |z^2| = 41$ then the true statements, among the following are
A) $|z+3| + |z-3| = 10$ B) $|z+3| + |z-3| = 8$
C) Maximum value $|z|$ is 5 D) Maximum value $|z|$ is 6

Key. A,C

Sol. $\left|Z^2-9\right|+\left|Z^2\right|=41 \Rightarrow \left|Z+3\right|+\left|Z-3\right|=10$

Complex Numbers

Assertion Reasoning Type

A) Statements - I is True, Statement -II is True & Statement-II is correct explanation for Statement-I

B) Statements - I is True, Statement -II is True & Statement-II is NOT explanation for Statement-I

C) Statements - I is True, Statement -II is False

D) Statements - I is False , Statement -II is True

1. Statement-I: Let $'Z'$ be a complex number satisfying

$$|Z-3|\le|Z-1|,\ |Z-3|\le|Z-5|,\ |Z-i|\le|Z+i|,\ |Z-i|\le|Z-5i|$$

then area of the region in which $'Z'$ lies is 12 sq. units

Statement-II: Area of trapezium

$$=\frac{1}{2}(\text{sum of parallel sides})\times(\text{distance between parallel sides})$$

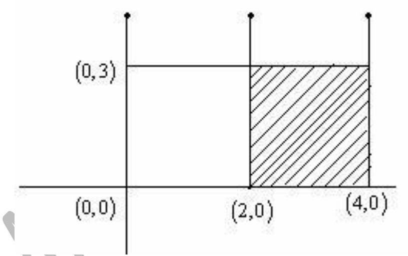
Key. D

Sol. Let $Z=x+iy$

We know that $Z+\overline{Z}=2\mathrm{Re}(Z)=2x,\ Z-\overline{Z}=2i\,\mathrm{Im}(Z)=2iy$

$|Z-3|\le|Z-1|\Rightarrow|Z-3|^2\le|Z-1|^2$

$\Rightarrow(Z-3)(\overline{Z}-3)\le(Z-1)(\overline{Z}-1)$



$Z\overline{Z}-3(Z+\overline{Z})+9\le Z\overline{Z}-1(Z+\overline{Z})+1$

$2(Z+\overline{Z})\ge 8\quad\Rightarrow x\ge 2$

Similarly $|Z-3|\le|Z-5|\Rightarrow\ x\le 4$

$|Z-i|\le|Z+i|\Rightarrow\ (Z-i)(\overline{Z}+i)\le(Z+i)(\overline{Z}-i)$

$\Rightarrow\ Z\overline{Z}+i(Z-\overline{Z})+1\le Z\overline{Z}+i(\overline{Z}-Z)+1$

$2i(Z-\overline{Z})\le 0\quad\Rightarrow 2i(2iy)\le 0\Rightarrow 4y\le 0\Rightarrow y\ge 0$

Similarly $|Z-i|\le|Z-5i|\Rightarrow y\le 3$

Hence $2\le x\le 4 \text{ and } 0\le y\le 3$

They form rectangle and its area $=6$

2. Let z_1, z_2 be two complex numbers satisfying $|z|=\sqrt{2}$ and $|z-3-3i|=2\sqrt{2}$. Then

Statement 1: $\min|z_1-z_2|=0$ and $\max|z_1-z_2|=6\sqrt{2}$

Statement 2: Two curves $|z|=\sqrt{2}$ and $|z-3-3i|=2\sqrt{2}$ touch each other externally.

Key. A

Sol. Distance between the centres $=3\sqrt{2}=$ sum of radii.

3. Statement-1: Two lines $a\bar{z}+\bar{a}z+b=0,\ a_1\bar{z}+\bar{a}_1z+b_1=0$

$\left(where\ \mathrm{a,a_1}\in C, a, a_1\ne 0 \text{ and } \mathrm{b,b_1}\in R\right)$ are parallel if and only if $\frac{a}{a_1}$ is purely real.

Statement-2: Two lines $a\bar{z}+\bar{a}z+b=0, a_1\bar{z}+\bar{a}_1z+b_1=0$

$\left(where\ \mathrm{a,a_1}\in C, a, a_1\ne 0 \text{ and } \mathrm{b,b_1}\in R\right)$ are perpendicular if and only if $\frac{a}{a_1}$ is purely imaginary.

Key. B

Sol. Let $a=\alpha+i\beta$ and $a_1=\alpha_1+i\beta_1$ Now, the two lines are given by

$2(\alpha x+\beta y)+b=0$ (1)

and $2(\alpha_1 x+\beta_1 y)+b_1=0$ (2)

The lines (1) and (2) are parallel if and only if

$-\frac{\alpha}{\beta}=-\frac{\alpha_1}{\beta_1}\Leftrightarrow\frac{\alpha}{i\beta}=\frac{\alpha_1}{i\beta}$

$\Leftrightarrow\frac{a}{a_1}=\overline{\left(\frac{a}{a_1}\right)}\Leftrightarrow\frac{a}{a_1}$ is real

Next,(1) and (2) are perpendicular to each other if and only if

$\left(-\frac{\alpha}{\beta}\right)\left(-\frac{\alpha_1}{\beta_1}\right)=-1$

$\Leftrightarrow\frac{a}{\bar{a}}=-\frac{a_1}{\bar{a}_1}\Leftrightarrow\frac{a}{a_1}$ is purely imaginary

4. Statement I : $\left|\frac{3z+i}{2z+3+4i}\right|=1.5$ represents a circle

Statement II: Perpendicular bisector of a line segment is a straight line

Key. D

Sol. $\left|\frac{3z+i}{2z+3+4i}\right|=1.5$

$\Rightarrow \frac{3\left|z+\frac{i}{3}\right|}{2\left|z+\frac{3}{2}+2i\right|}=\frac{3}{2} \quad \Rightarrow \left|z+\frac{i}{3}\right|=\left|z+\frac{3}{2}+2i\right|$

$\Rightarrow Z$ is equidistant from $A\left(-\frac{i}{3}\right)$ and $B\left(-\frac{3}{2}-2i\right)$

Thus, $'Z'$ lies on the perpendicular bisector of AB and hence it is straight line

5. $'z'$ is a unimodular complex number

Statement-1: $\arg\left(z^2+\bar{z}\right)=\arg z$

Statement-2: $\bar{z}=\cos\left(\arg z\right)-i\sin\left(\arg z\right)$

Key. D

Sol. $|z|=1 \Rightarrow z=\cos\theta+i\sin\theta$ where $\theta=\arg z$

$\bar{z}=\cos\theta-i\sin\theta$

Statement-2 is true

$\arg\left(z^2+\bar{z}\right)=\arg\{\cos 2\theta+i\sin 2\theta+\cos\theta-i\sin\theta\} =\tan^{-1}\frac{\sin 2\theta-\sin\theta}{\cos 2\theta+\cos\theta}$

$\Rightarrow \tan^{-1}\frac{2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}}{2\cos\frac{3\theta}{2}\cos\frac{\theta}{2}}=\tan^{-1}\tan\frac{\theta}{2}=\frac{\theta}{2}=\frac{1}{2}\arg z$

6. $\alpha=\cos\frac{2\pi}{5}+i\sin\frac{2\pi}{5}$ is a fifth root of unity $(\alpha\neq 1)$

Statement I : $a=\alpha+\alpha^4$ and $b=\alpha^2+\alpha^3$ are roots of $x^2+x-1=0$

Statement II : $1+\alpha+\alpha^2+\alpha^3+\alpha^4=0$

Key. A

Sol. Conceptual

7. z, w are two non zero complex numbers such that $\bar{z} + i\bar{w} = 0$ and $arg\,(zw) = \pi$

Statement I : $arg(z) = \frac{\pi}{2}$

Statement II : $|z| = |w|$

Key. D

Sol. Conceptual

8. z_1, z_2 are two distinct points in complex plane such that $2|z_1| = 3|z_2|$ and $z \in C$ be any point

Statement I : $z = \frac{2z_1}{3z_2} + \frac{3z_2}{2z_1}$ is a point such that $-2 \le \text{Re}(z) \le 2$

Statement II : If arg $(z_1) = \theta, \arg(z_2) = \theta + \alpha$ then $\frac{2z_1}{3z_2} + \frac{3z_2}{2z_1} = 2\cos\alpha$

Key. A

Sol. $z = \frac{2z_1}{3z_2} + \frac{3z_2}{2z_1} = \frac{2|z_1|cis\theta}{3|z_2|cis(\theta+\alpha)} + \frac{3|z_2|cis(\theta+\alpha)}{2|z_1|cis\theta}$

$= cis(-\alpha) + cis\alpha = 2\cos\alpha \in [-2, 2]$

9. z is a complex number such that $|z| = 1, z \neq \pm 1.$

Statement I : $\frac{z}{1-z^2}$ lies on imaginary axis

Statement II : $\frac{z}{1-z^2} + \frac{\bar{z}}{1-(\bar{z})^2} = 0$

Key. A

Sol. Conceptual

10. Statement – 1 : If $x + \frac{1}{x} = 1$ and $p = x^{4000} + \frac{1}{x^{4000}}$ and 'q' be the digit at unit place in the number $2^{2^n} + 1, n \in N$ and n > 1 then p + q = 8

Statement – 2 : w,w² are the roots of $x + \frac{1}{x} = -1, x^3 + \frac{1}{x^3} = 2$

Key. D

Sol. $x + \frac{1}{x} = 1 \Rightarrow x = -w$

$\therefore p = -1$

And $2^n = 4k$ for n > 1

$2^{2^n} = 2^{4k} =$ least digit 6

$\therefore$ p + q = 7 + - 1 = 6

11. Statement – 1 : If $|z| < \sqrt{2} - 1$ then $|z^2 + 2z\cos\alpha| < 1$

Statement – 2 : $|z_1 + z_2| \le |z_1| + |z_2|$ also $|\cos\alpha| \le 1$.

Key. A

Sol. $|z^2 + 2z\cos\alpha| \le |z|^2 + |2z\cos\alpha|$

$< \left(\sqrt{2} - 1\right)^2 + 2\left(\sqrt{2} - 1\right) < 1$

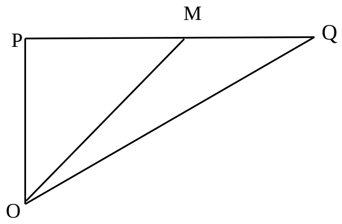
12. Statement 1 : If z_1, z_2 are complex numbers then $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2\left\{|z_1|^2 + |z_2|^2\right\}$

Statement 2 : If 'M' is the midpoint of the line segment PQ then

$OP^2 + OQ^2 = 2OM^2 + 2MP^2$ where 'O' is origin

Key. A

Sol. Let $OM = z_1, MP = z_2, MQ = -z_2$



Since $OP^2 + OQ^2 = 2MP^2 + 2OM^2$

$\therefore |z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + 2|z_2|^2$

13. Statement 1: If 'Z' is a complex number then the equation $|Z^2 - 1| = |Z|^2 - 1$ has infinitely many solutions.

Statement 2: If Z_1, Z_2 are complex numbers then $|Z_1 + Z_2| = |Z_1| + |Z_2|$ iff Z_1, Z_2 are positive real numbers.

Key. C

Sol. Conceptual

14. Statement 1: If 'z' is a complex number $(z \ne 1)$ then $\left|\frac{z}{|z|} - 1\right| \le |\arg z|$

Statement 2: In a unit radius circle chord $(AP) \le arc(AP)$

Key. A

Sol. Chord $AP = \left|\frac{z}{|z|} - 1\right| \le arc(AP) = \alpha$; $\therefore \left|\frac{z}{|z|} - 1\right| \le |\arg z|$

15. If z_1, z_2, z_3 are the vertices of a triangle with z_0 as centroid, such that $|z_1 - z_0| = |z_2 - z_0| = |z_3 - z_0|$ then

STATEMENT-1 : $z_1^2 + z_2^2 + z_3^2 = 9z_0^2$

STATEMENT-2 : $z_1 + z_2 + z_3 = 3z_0$

Key: D

Sol. CONCEPTULE

16. STATEMENT 1: The graph $y = x^3 + ax^2 + bx + c$ has no extremum, if $a^2 < 3b, (a, b, c \in R)$

STTAEMENT 2: If f' (x) vanishes at x=a then f(x) has extremum at x=a

Key: C

$\frac{dy}{dx} = 3x^2 + 2ax + b$

Hint $\Delta = 4a^2 - 12b$

$= 4(a^2 - 3b) < 0$

$\Rightarrow 3x^2 + 2ax + b > 0 \forall x \in R$

17. Statement-I: a,b,c, are theree non-zero real numbers such that a+b+c=0 and z_1, z_2, z_3 are. three complex numbers such that $az_1 + bz_2 + cz_3 = 0$,then z_1, z_2 and z_3 are collinear.

Statement-II: If z_1, z_2, z_3 are collinear then $\begin{vmatrix} z_1 & \overline{z_1} & 1 \\ z_2 & \overline{z_2} & 1 \\ z_3 & \overline{z_3} & 1 \end{vmatrix} = 0$

KEY : B

HINT

CONCEPTUAL

18. Statement – 1 : Locus of 'z' given by $\left|z - (3 + 2i)\right| = \left|z \cos\left(\frac{\pi}{4} - \arg z\right)\right|$ represents a parabola

Statement – 2 : If distance of a variable point from a fixed point is equal to the distance from a fixed line, it represents a parabola

Key. C

Sol. If the point lies on the directrix, its pair of straight line

19. STATEMENT 1. :Let A (z_1), B(z_2) and C(z_3) be three points such that $az_1+bz_2+cz_3 = 0$, $a+b+c = 0$ for some a,b,c $\in R$ (at least one of a,b,c is non – zero). If $|z_1 - z_2| = |z_2 - z_3| = |z_2|$ then the area of ΔOAC is $|z_1 \ z_3|$, O being the origin .

because

STATEMENT 2.: Area of triangle $= \frac{1}{2}(\text{base} \times \text{height})$

Key. D

Sol. Let $c \neq 0$, now $a+b+c=0$

$c = -(a+b) \Rightarrow z_3 = \frac{az_1 + bz_2}{a+b}$

$\Rightarrow z_1, z_2, z_3$ are collinear

Area of $\Delta OAC = \frac{1}{2}OA \times OC = \frac{1}{2}|z_1||z_3|$

Taking B as centre and AB as radius draw a circle. It will pass through $O \Rightarrow OAC$ is right angled triangle right angled at O

20. Statement – 1 : If $|Z| < \sqrt{2} - 1,$ then $|z^2 + 2z.\cos\beta| < 1$

Because

Statement – 2 : $|z_1 + z_2| < |z_1| + |z_2| \& \cos\beta < 1$, then

Key. A

Sol. $|z^2 + 2z\cos\beta| < |z|^2 + |2z\cos\beta|$

$= 3 - 2\sqrt{2} + 2\sqrt{2} - 2 = 1$

21. Statement – 1 : Let z_1 and z_2 be two distinct points in an argand plane such that

$a|z_1| = b|z_2|$,

then $\frac{az_1}{bz_2} + \frac{bz_2}{az_1}$ is a point on the line segment [-2, 2] of the real axis

Statement – 2 : When arg $(z_1) = \theta$ and arg$(z_2) = \theta + \alpha$, then

$\frac{az_1}{bz_2} + \frac{bz_2}{az_1} = e^{i\alpha} + e^{-i\alpha} = 2\cos\alpha$

Key. A

Sol. $ar_1 = br_2$

$T = \frac{a}{b} \cdot \frac{r_1 e^{i\theta}}{r_2 e^{i(\theta+\alpha)}} T_2 = e^{i\alpha}$

$= 2\cos\alpha \in [-2, 2]$

22. $\alpha = \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}$ is a fifth root of unity $(\alpha \neq 1)$

Statement I : $a = \alpha + \alpha^4$ and $b = \alpha^2 + \alpha^3$ are roots of $x^2 + x - 1 = 0$

Statement II : $1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 = 0$

Key. A

Sol. Conceptual

23. z_1, z_2 are two distinct points in complex plane such that $2|z_1| = 3|z_2|$ and $z \in C$ be any point

Statement I : $z = \frac{2z_1}{3z_2} + \frac{3z_2}{2z_1}$ is a point such that $-2 \leq \text{Re}(z) \leq 2$

Statement II : If arg $(z_1) = \theta, \arg(z_2) = \theta + \alpha$ then $\frac{2z_1}{3z_2} + \frac{3z_2}{2z_1} = 2\cos\alpha$

Key. A

Sol. $z = \frac{2z_1}{3z_2} + \frac{3z_2}{2z_1} = \frac{2|z_1|cis\theta}{3|z_2|cis(\theta+\alpha)} + \frac{3|z_2|cis(\theta+\alpha)}{2|z_1|cis\theta}$

$$= cis(-\alpha) + cis\alpha = 2\cos\alpha \in [-2, 2]$$

24. z is a complex number such that $|z| = 1, z \neq \pm 1.$

Statement I : $\frac{z}{1-z^2}$ lies on imaginary axis

Statement II : $\frac{z}{1-z^2} + \frac{\overline{z}}{1-(\overline{z})^2} = 0$

Key. A

Sol. Conceptual

25. z, w are two non zero complex numbers such that $\overline{z} + i\overline{w} = 0$ and *arg* $(zw) = \pi$

Statement I : *arg*$(z) = \frac{\pi}{2}$

Statement II : $|z| = |w|$

Key. D

Sol. Conceptual

26. Statement - 1: Let $|Z| = 1$ and $W = \frac{Z^2}{Z^4 + 1}$ then W lies on real axis.

Statement - 2: If a complex number Z lies on imaginary axis then $\arg Z = \frac{\pi}{2}$ or $-\frac{\pi}{2}$

Key. B

Sol. Conceptual

27. Statement - 1: The sum of the roots of the equation $x^6 - 152x^3 + 3375 = 0$ having negative real parts is -8.

Statement - 2: Let $\alpha \in C$ then $(\alpha)^{1/n}$, $n \in N$ will have *n* values.

Key. B

Sol. $x = 5(1)^{1/3}, 3(1)^{1/3}$

28. Statement - 1: If Z lies on the circle having centre origin and radius unity then all the points represented by $\frac{Z^4}{1+Z^8}$ will lie on one of its diameters.

Statement - 2: If $|Z|=1$, $\arg\left(\frac{Z^4}{1+Z^8}\right)=\frac{\pi}{2}$.

Key. C

Sol. $|Z|=1,\ \omega=\frac{Z^4}{1+Z^8},\ \bar{\omega}=\omega\quad \omega$ lies on real axis

$$\therefore \arg\left(\frac{Z^4}{1+Z^8}\right)=0\ (or)\ \pi$$

29. Statement – I: If $\alpha=\cos\left(\frac{2\pi}{7}\right)+\sin\left(\frac{2\pi}{7}\right)$, $p=\alpha+\alpha^2+\alpha^4, q=\alpha^3+\alpha^5+\alpha^6$, then the equation where roots are p and q is x^2+x+2.

Statement – II : If α is a root of $Z^7=1$, then $1+\alpha+\alpha^2+.....+\alpha^6=0$

a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1

b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1

c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. a

Sol. α is 7[th] root of unity

$$\Rightarrow 1+\alpha+\alpha^2+....+\alpha^6=0,\ p+q=-1$$

$$pq=\alpha^4+\alpha^6+\alpha^5+\alpha^7+\alpha^8+\alpha^7+\alpha^9+\alpha^{10}=3+\left(\alpha+\alpha^2+\alpha^3+....+\alpha^6\right)=3+(1)=2$$

$$\Rightarrow x^2+x+2=0$$

Both I + II are true and II is the correct explanation

30. Statement – I: If $|z-z_1|^2+|z-z_2|^2=k$ represent a circle for all K > 0

Statement – II : $|z-z_1|^2+|z-z_2|^2=2\left|\frac{z_1+z_2}{2}\right|^2+\frac{1}{2}|z_1-z_2|^2$

a) Statement 1 is true, Statement – 2 is true; Statement 2 is a correct explanation for statement 1

b) Statement 1 is true, Statement 2 is true; Statement 2 is not a correct explanation for statement 1

c) Statement 1 is true; Statement 2 is false d) Statement 1 is false; Statement 2 is true

Ans. d

Sol. $|z-z_1|^2+|z-z_2|^2=k$ represents a circle, if

$$k\ge\frac{1}{2}|z_1-z_2|^2$$

31. STATEMENT-1: a, b, c are three non-zero real numbers such that a + b + c = 0 and z_1, z_2, z_3 are three complex numbers such that $az_1+bz_2+cz_3=0$, then z_1, z_2 and z_3 lie on a circle.

STATEMENT-2: If z_1, z_2 and z_3 are collinear then $\begin{vmatrix} z_1 & \overline{z_1} & 1 \\ z_2 & \overline{z_2} & 1 \\ z_3 & \overline{z_3} & 1 \end{vmatrix} = 0$.

Key. D

Sol. $a + b + c = 0$

$az_1 + bz_2 + cz_3 = 0$

$\Rightarrow az_1 + bz_2 - (a+b)z_3 = 0$

$\Rightarrow z_3 = \dfrac{az_1 + bz_2}{a+b}$

$\Rightarrow z_3$ divides the segment joining z_1 and z_2 in the ratio b : a

$\Rightarrow$ z_1, z_2 and z_3 are collinear.

32. STATEMENT-1: The number of complex numbers z satisfying $|z|^2 + a|z| + b = 0$ $(a, b \in R)$ is at the most 2.

STATEMENT-2: A quadratic equation in which all the coefficients are non-zero can have at most two roots.

Key. D

Sol. The statement-1 is false because $|z| = \dfrac{-a \pm \sqrt{a^2 - 4b}}{2}$

If $a^2 > 4b$ and $|z|$ is a positive number c, then | z | = C ⇒ | z | = c(cos θ + i sin θ)

$\Rightarrow$ infinite complex numbers satisfy the given equation.

Statement-2 is true. (A quadratic can have more than two roots if all the coefficients are zero).

33. z is a unimodular complex number

STATEMENT-1: $\arg\left(z^2+\bar{z}\right)=\arg z$

STATEMENT-2: $\bar{z}=\cos(\arg z)-i\sin(\arg z)$

Key. D

Sol. $|z|=1 \Rightarrow z=\cos\theta+i\sin\theta$ where $\theta=\arg z$

$\bar{z}=\cos\theta-i\sin\theta$

Statement-2 is true

$\arg\left(z^2+\bar{z}\right)=\arg\{\cos 2\theta+i\sin 2\theta+\cos\theta-i\sin\theta\}$

$=\tan^{-1}\dfrac{\sin 2\theta-\sin\theta}{\cos 2\theta+\cos\theta}$

$$\Rightarrow \tan^{-1}\frac{2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}}{2\cos\frac{3\theta}{2}\cos\frac{\theta}{2}}=\tan^{-1}\tan\frac{\theta}{2}=\frac{\theta}{2}=\frac{1}{2}\arg z$$

34. STATEMENT-1: Consider an ellipse having its foci at $A(z_1)$ and $B(z_2)$ in the argand plane. If the eccentricity of the ellipse be 'e' and it is known that origin is an interior point of ellipse, then $e\in\left[0,\dfrac{|z_1+z_2|}{|z_1|+|z_2|}\right]$.

STATEMENT-2 : If z_0 is the point interior to curve $|z-z_1|+|z-z_2|=\lambda$, where $\lambda > |z_1 - z_2|$ $\Rightarrow |z_0-z_1|+|z_0-z_2|<\lambda$

Key. D

Sol. If P(z) be any point on the ellipse. The equation of the ellipse is

$|z-z_1|+|z-z_2|=\dfrac{|z_1-z_2|}{e}$

for P(z) to lie in the ellipse, we have

$$|z-z_1|+|z-z_2|<\frac{|z_1-z_2|}{e}$$

$$e\in\left[0,\frac{|z_1-z_2|}{|z_1|+|z_2|}\right]$$

Statement-1 is false; Statement-2 is true.

35. STATEMENT -1: The locus of the centre of a circle which touches the circles $|z - z_1| = a$ and $|z - z_2| = b$ externally (z, z_1 and z_2 are complex numbers) will be hyperbola, where
$|z_1 - z_2| > |a - b|$.
STATEMENT-2: $||z - z_1| - |z - z_2|| < |z_1 - z_2| \Rightarrow z$ lies on hyperbola.
Key. C

Sol. Conceptual

36. Consider the equation $x^2 + ax + b = 0$
STATEMENT–1 : If one root of the above equation is $3 + 4i$, then another root is $3 - 4i$
STATEMENT–2: For a polynomial equation $f(x) = 0$ with real co-efficient, if imaginary roots exists, always they exist in pairs.
Key. D

Sol. Conceptual

37. Consider the equation $(z - 1)^n = (z + 1)^n$, $n \in N - \{1\}$, $z \in C$, the set of complex numbers.
STATEMENT-1 : All the roots of the given equation lie on the y-axis.
STATEMENT-2: The degree of the given equation is less than n.
Key. B

Sol. Conceptual

38. STATEMENT – 1: If $4z_1 - 5z_2 + z_3 = 0$ then z_1, z_2, z_3 are collinear
STATEMENT – 2: If $a, b, c \in R$ such that $az_1 + bz_2 + cz_3 = 0$ then z_1, z_2, z_3 are collinear
Key. C

Sol. Conceptual

Complex Numbers

Comprehension Type

Passage – 1

Let Z_1 and Z_2 be complex numbers such that $Z_1^2 - 4Z_2 = 16 + 20i.$ Also suppose that roots α and β of $t^2 + Z_1 t + Z_2 + m = 0$ for some complex number m satisfying $|\alpha - \beta| = 2\sqrt{7}$

1. The complex number m lies on

A) a square with side 7 and centre $(4,5)$

B) a circle with radius 7 and centre $(4,5)$

C) a circle with radius 7 and centre $(-4,5)$

D) a square with side 7 and centre $(-4,5)$

Key. A

2. The greatest value of $|m|$ is

A) $5+\sqrt{21}$ B) $5+\sqrt{23}$ C) $7+\sqrt{43}$ D) $7+\sqrt{41}$

Key. D

3. The least value of $|m|$ is

A) $7-\sqrt{41}$ B) $7-\sqrt{43}$ C) $5-\sqrt{23}$ D) $5+\sqrt{21}$

Key.

Sol. 1,2,3

$\alpha + \beta = -Z_1; \alpha\beta = Z_2 + m$

$(\alpha - \beta)^2 = Z_1^2 - 4Z_2 - 4m$

$= 16 + 20i - 4m$

$|\alpha - \beta|^2 = |16 + 20i - 4m|$

Q $|\alpha - \beta| = 2\sqrt{7}\, |m - 5i - 4| = 7 \Rightarrow |m - (4+5i)| = 7$

$\therefore$ m lies on a circle having centre $(4,5)$ and radius 7

Passage – 2

Consider the set of complex numbers A, B, C and S defined as

A = {z : | |z + 2| – |z – 2| | = 2}

B = {z : arg $\left(\frac{z-1}{z}\right) = \frac{\pi}{2}$ }

C = {z : arg (z – 1) = π}

S = {z : Re $\left(\frac{z-1}{z+1}\right) = 0$ }

4. A $\cap$B $\cap$ C =

(A) ϕ (B) (0, ∞)

(C) (0, 1) (D) (–1, 0)

5. If z_1 , z_2, $z_3 \in$ S, then minimum value of $|z_1 + z_2|^2 + |z_2 + z_3|^2 + |z_3 + z_1|^2$ is

(A) 5 (B) 3

(C) 2 (D) 9

6. If z lies on S, then arg $\left(\frac{2z+i-1}{2z+i+1}\right)$ equals

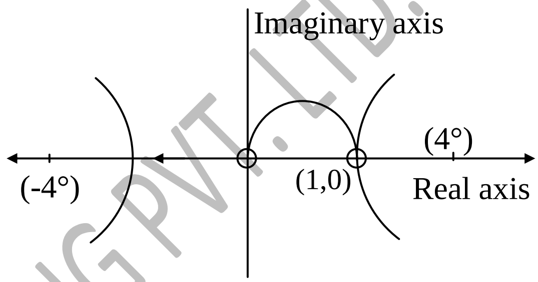
(A) $\pi/2$ (B) $\pi/4$

(C) $\pi/3$ (D) $-\pi/4$

Sol. 4. Ans. (a)

. A, B, C represented geometrically as

Clearly $A \cap B \cap C = \phi$



5. Ans. (b)

Clearly S represents the set of complex number lying on the circle $|z| = 1$, $z \neq -1$.

$|z_1 + z_2|^2 + |z_2 + z_3|^2 + |z_3 + z_1|^2 = 3 + (z_1 + z_2 + z_3)(\overline{z}_1 + \overline{z}_2 + \overline{z}_3)$

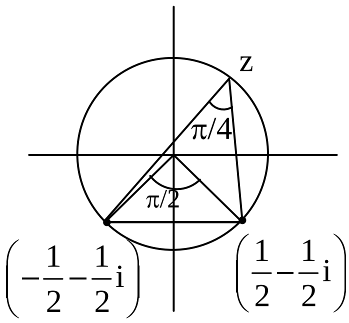
$= 3 + |z_1 + z_2 + z_3|^2 \geq 3.$

6. Ans. (b)

From the diagram

arg $\left(\frac{\frac{1}{2}-\frac{1}{2}i-z}{-\frac{1}{2}-\frac{1}{2}i-z}\right)=\frac{\pi}{4}$

$\Rightarrow$ arg $\left(\frac{2z+i-1}{2z+i+1}\right)=\frac{\pi}{4}$



Passage – 3

Let z_1 be a complex number of magnitude unity and z_2 be a complex number given by $z_2 = z_1^2 - z_1$. Answer the following questions.

7. If $\arg z_1 = \theta$ *then* $|z_2|$ is equal to

a) $2\left|\sin\frac{\theta}{2}\right|$ b) $2\left|\cos\frac{\theta}{2}\right|$ c) $\sqrt{2}\left|\sin\frac{\theta}{2}\right|$ d) $\sqrt{2}\left|\cos\frac{\theta}{2}\right|$

8. If $\arg z_1 = \theta$ and $4n\pi < \theta < (4n+2)\pi,$ (n is an integer), then $\arg z_2$ is equal to

a) $\frac{3\theta}{2}$ b) $\frac{\pi-3\theta}{2}$ c) $\frac{\pi+3\theta}{2}$ d) $\frac{\pi+\theta}{2}$

9. If $\arg z_1 = \theta \; and \; (4n+2)\pi < \theta < (4n+4)\pi,$ (n is an integer), then $\arg z_2$ is equal to

a) $\frac{\pi}{2}+3\theta$ b) $\frac{3\pi}{2}+\frac{3\theta}{2}$ c) $\frac{3\pi}{2}+3\theta$ d) $\frac{\pi}{2}+\frac{3\theta}{2}$

Sol. 7. (A) $z_1 = \cos\theta + i\sin\theta$

$$z_2 = \cos 2\theta + i\sin 2\theta - (\cos\theta + i\sin\theta)$$

$$= (\cos 2\theta - \cos\theta) + i(\sin 2\theta - \sin\theta)$$

$$|z_2|^2 = (\cos 2\theta - \cos\theta)^2 + (\sin 2\theta - \sin\theta)^2$$

$$2 - 2(\cos 2\theta + \sin 2\theta \sin\theta)$$

$$= 2 - 2\cos\theta$$

$$= \quad 4\sin^2\frac{\theta}{2}$$

8. (C) $z_2 = (\cos 2\theta - \cos\theta) + i(\sin 2\theta - \sin\theta)$

$$= -2\sin\frac{3\theta}{2}\sin\frac{\theta}{2} + i.2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}$$

$$= 2i\sin\frac{\theta}{2}\left(\cos\frac{3\theta}{2} + i\sin\frac{3\theta}{2}\right)$$

$$\arg z_2 = \arg\left(2i\sin\frac{\theta}{2}\right) + \arg\left(\cos\frac{3\theta}{2} + i\sin\frac{3\theta}{2}\right)$$

$$= \frac{\pi}{2} + \frac{3\theta}{2}$$

$$4n\pi < \theta < (4n+2)\pi \Rightarrow 2n\pi < \frac{\theta}{2} < (2n+1)\pi$$

$$\Rightarrow \sin\frac{\theta}{2} > 0$$

9. (B) $(4n+2)\pi < \theta < (4n+4)\pi$

$$\Rightarrow (2n+1)\pi < \frac{\theta}{2} < (2n+2)\pi \Rightarrow \sin\frac{\theta}{2} < 0$$

$$\therefore \arg z_2 = \frac{3\pi}{2} + \frac{3\theta}{2}$$

Passage – 4

Let $A_1, A_2, A_3, \ldots\ldots, A_n$ be a regular polygon of 'n' sides whose centre is origin O. let the complex numbers representing vertices $A_1, A_2, A_3, \ldots\ldots\ldots, A_n$ be $z_1, z_2, z_3, \ldots\ldots, z_n$ respectively. Let $OA_1 = OA_2 = \ldots\ldots = OA_n = 1$

10. The value of $|A_1A_2|^2+|A_1A_3|^2+.....+|A_1A_n|^2=$

A) n B) 2n C) $2(n-1)$ D) $2(n+1)$

Key. B

11. The distances $A_1A_j\ (j=2,3,.....,n)$ must be equal to

A) $\sin\frac{j\pi}{n}$ B) $2\sin\frac{(j-1)\pi}{n}$ C) $\cos\frac{j\pi}{n}$ D) $2\sin\frac{(j+1)\pi}{n}$

Key. B

12. The value of $|A_1A_2||A_1A_3|....|A_1A_n|$ must be equal to

A) 1 B) n C) $\sqrt{n}$ D) n^2

Key. B

Sol. 10. $LHS=\left(2-2\cos\frac{2\pi}{n}\right)+\left(2-2\cos\frac{4\pi}{n}\right)+.....+\left(2-2\cos(n-1)\frac{2\pi}{n}\right)$

$=2(n-1)-2\left(\cos\frac{2\pi}{n}+\cos\frac{4\pi}{n}\right)$

$=2(n-1)-2(-1)=2n$

11. Consider the triangle OA_1A_j

Let angle $\angle OA_1A_j=\alpha$

Then $(A_1A_j)^2=1^2+1^2-2(1)(1)\cos\alpha$

$=2(1-\cos\alpha)=2.2\sin^2\frac{\alpha}{2}$

$\Rightarrow A_1A_j=2\sin\frac{\alpha}{2}$ but $\alpha=\frac{2\pi}{n}(j-1)$

$\Rightarrow A_1A_j=2\sin\frac{(j-1)\pi}{n}$

12. $LHS=2^{n-1}\sin\frac{\pi}{n}\sin\frac{2\pi}{n}....\sin\frac{(n-1)\pi}{n}$

$=n$

Passage – 5

Consider a complex number $w = \frac{z-i}{2z+1}$ where $z = x + iy$, and $x, y \in R$

13. If the complex number $'w'$ is purely imaginary then locus of $'z'$ is

A) A straight line

B) a circle with centre $\left(-\frac{1}{4}, \frac{1}{2}\right)$ and radius $\frac{\sqrt{5}}{4}$

C) a circle with centre $\left(\frac{1}{4}, -\frac{1}{2}\right)$ and passing through origin

D) Neither a circle nor a straight line

Key. B

14. If the complex number $'w'$ is purely real then locus of $'z'$ is

A) A straight line passing through origin

B) a straight line with gradient 3 and y intercept (-1)

C) A straight line with gradient 2 and y intercept 1

D) A circle

Key. C

15. If $|w| = 1$, then locus of $'z'$ is

A) A point circle B) An imaginary circle

C) A real circle D) Not a circle

Key. C

Sol. 13. $w = \frac{z-i}{2z+1} = \frac{\left(2x^2 + x + 2y^2 - 2y\right) + i\left(y - 2x - 1\right)}{\left(2x+1\right)^2 + 4y^2}$

$\text{Re}(w) = 0 \Rightarrow x^2 + y^2 + \frac{x}{2} - y = 0$

14. $I_m(w) = 0 \Rightarrow y = 2x + 1$

15. $|w| = 1 \Rightarrow w\bar{w} = 1 \Rightarrow x^2 + y^2 + \frac{4x}{3} + \frac{2}{3}y = 0$

Passage – 6

$z = x+iy$ & (x,y) is a point represented by the complex number $'z'$ in the argand plane. $|z_1 - z_2|$ denotes distance between z_1 & z_2 in the argand plane.

16. The complex number $z = x+iy$ which satisfies the equation $\left|\frac{z-5i}{z+5i}\right| = 1$ lies on

A) the x- axis
B) The y axis
C) a circle with radius 5 and centre at origin
D) a line $y = 5$

Key. A

17. If $|z| = 5$ then the points representing the complex number $-i + \frac{15}{z}$ lie on the circle

A) with centre $(0,1)$ and radius $= 3$
B) with centre $(0,-1)$ and radius $= 3$
C) with centre $(1,0)$ and radius $= 5$
D) with centre $(-1,0)$ and radius $= 15$

Key. B

18. If $|z-i| \le 2$ & $z_1 = 3+4i$ then the maximum value of $|iz + z_1|$ is

A) $\sqrt{20} - 2$
B) 9
C) $\sqrt{20} + 2$
D) 8

Key. C

Sol. 16. Locus of $'z'$ is perpendicular bisector of the line segment joining $(0,5)$ and $(0,-5)$ its equation is $y = 0\,(i.e, x-axis)$

17. Let $w = -i + \frac{15}{z} \Rightarrow w + i = \frac{15}{z}$

$\Rightarrow |w+i| = \frac{15}{|z|} = 3$

$\therefore$ Locus of $'w'$ is a circle with centre $(0,-1)$ and radius 3

18. $|z-i| \le 2$ represents a disc with centre at $(0,1)$ and radius 2

$|iz + z_1| = |i||z - iz_1| = |z - i(3+4i)|$

$= |(z-i) + (4-2i)| \le |z-i| + |4-2i| \le 2 + \sqrt{20}$

Maximum value of $|iz + z_1| = \sqrt{20} + 2$

Passage – 7

Suppose z_1, z_2 and z_3 represent the vertices A, B and C of an equilateral triangle ABC respectively on the Argand plane. Then $AB = BC = CA \Rightarrow |z_2 - z_1| = |z_3 - z_2| = |z_1 - z_3|$

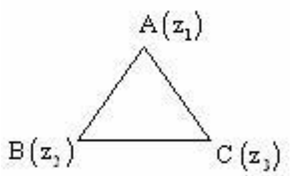
Also $\angle CAB = \frac{\pi}{3} \Rightarrow \arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right) = \pm\frac{\pi}{3}$

$$\therefore \frac{z_3 - z_1}{z_2 - z_1} = \left|\frac{z_3 - z_1}{z_2 - z_1}\right|\left\{\cos\left(\pm\frac{\pi}{3}\right) + i\sin\left(\pm\frac{\pi}{3}\right)\right\} = \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

$$\Rightarrow \frac{z_3 - z_1}{z_2 - z_1} - \frac{1}{2} = \pm\frac{\sqrt{3}}{2}i \Rightarrow \frac{2z_3 - z_1 - z_2}{2(z_2 - z_1)} = \pm\frac{\sqrt{3}}{2}i$$

Squaring, we get

$$\Rightarrow (2z_3 - z_1 - z_2)^2 = -3(z_2 - z_1)^2 \Rightarrow z_1^2 + z_2^2 + z_3^2 - z_1z_2 - z_2z_3 - z_3z_1 = 0$$



19. If the complex numbers z_1, z_2, z_3 represent the vertices of an equilateral triangle such that $|z_1| = |z_2| = |z_3|$ then $z_1 + z_2 + z_3 =$

A) 0 B) 3 C) ω D) ω^2

Key. A

20. If a and b are two real numbers lying between 0 and 1 such that $z_1 = a + i, z_2 = 1 + bi$ and $z_3 = 0$ form an equilateral triangle then

A) $a = 2 + \sqrt{3}$ B) $b = 4 - \sqrt{3}$ C) $a = b = 2 - \sqrt{3}$ D) $a = 2, b = \sqrt{3}$

Key. C

21. Let the complex numbers z_1, z_2 and z_3 be the vertices of an equilateral triangle. Let z_0 be the circumcentre of the triangle, then $z_1^2 + z_2^2 + z_3^2 =$

A) z_0^2 B) $3z_0^2$ C) $9z_0^2$ D) 0

Key. B

Sol. 19. $|z_1| = |z_2| = |z_3| \Rightarrow$ Origin O is the circumcentre, hence centroid

$$\therefore \frac{z_1 + z_2 + z_3}{3} = 0 \Rightarrow z_1 + z_2 + z_3 = 0$$

20. Using the result $z_1^2+z_2^2+z_3^2-z_1z_2-z_2z_3-z_3z_1=0$, we get

$a^2-1+2ai+1-b^2+2bi+0-a+b-i-abi=0$

$\therefore a^2-b^2-a+b=0$ and $2a+2b-ab-1=0$

$\Rightarrow a=b$ and $2a+2b-ab-1=0$ ($\because a+b=1$ does not give real solution)

$\therefore a=b$ and $a^2-4a+1=0$

$a=b=2-\sqrt{3}\ (\because a<1, b<1)$

21. $z_0=\dfrac{z_1+z_2+z_3}{3}\Rightarrow z_1^2+z_2^2+z_3^2+2z_1z_2+2z_2z_3+2z_3z_1=9z_0^2$

$\Rightarrow 3\left(z_1^2+z_2^2+z_3^2\right)=9z_0^2\Rightarrow z_1^2+z_2^2+z_3^2=3z_0^2$

$\left(\because z_1^2+z_2^2+z_3^2=z_1z_2+z_2z_3+z_3z_1\right)$

Passage – 8

The complex slope M of a line joining two points $z_1\ \&\ z_2$ in complex plane is defined as

$M=\dfrac{z_1-z_2}{\overline{z_1}-\overline{z_2}}$ .Its real slope m is $\tan\theta$, where θ is inclination of the line .Then

Answer the following

22. M =

a) $\dfrac{1+im}{1-im}$ b) $\dfrac{2m}{1+m^2}+i\left(\dfrac{1-m^2}{1+m^2}\right)$ c) $\dfrac{2m}{1+m^2}+i\left(\dfrac{1+m^2}{1-m^2}\right)$ d) $\dfrac{1-im}{m-i}$

Key. A

23. The inclination of a line whose complex slope is $-\omega^2$ (where ω is a non real cube root of unity) is

a) $\dfrac{2\pi}{3}$ b) $\dfrac{\pi}{3}$ c) $\dfrac{\pi}{6}$ d) $\dfrac{\pi}{12}$

Key. C

24. Which of the following is false?

a) $|M|=1$ b) M=m is never possible c) $m=i\left(\dfrac{M-1}{M+1}\right)$ d) $M=cis2\theta$

Key. C

Sol. 22,23,24

Given M = $\dfrac{z_1-z_2}{\overline{z_1}-\overline{z_2}}=\dfrac{(x_1-x_2)+i(y_1-y_2)}{(x_1-x_2)-i(y_1-y_2)}=\dfrac{1+i\tan\theta}{1-i\tan\theta}=\dfrac{1+im}{1-im}$

M = $i\left(\dfrac{2m}{1+m^2}\right)+\left(\dfrac{1-m^2}{1+m^2}\right)$

$\therefore M=\cos 2\theta+i\sin 2\theta=cis2\theta\Rightarrow|M|=1$

$-\omega^2 = \frac{1+i\sqrt{3}}{2} = cis2\theta \Rightarrow \theta = \frac{\pi}{6}$

Passage – 9

If α is any of 7th roots of unity , then $\alpha = \frac{cis2K\pi}{7}$ (K= 0 to 6) and $\sum_{i=1}^{6}\alpha^i = -1(\alpha \neq 1)$ and $\alpha^7 = 1$ Answer the following

25. The equation whose roots are $\alpha+\alpha^2+\alpha^4$ and $\alpha^3+\alpha^5+\alpha^6$ is $(\alpha \neq 1)$

a) $x^2+x-2=0$ b) $x^2+x+2=0$ c) $x^2-x+2=0$ d) $x^2-x-2=0$

Key. B

26. If $f(x)=1+2x+3x^2+4x^3+5x^4+6x^5+7x^6$ then for $\alpha \neq 1$,

$f(x)+f(\alpha x)+f(\alpha^2 x)+f(\alpha^3 x)+---+f(\alpha^6 x)=$

a) 42 b) 21 c) 14 d) 7

Key. D

27. $\sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7}=\sin\frac{4\pi}{7}\sin\frac{5\pi}{7}\sin\frac{6\pi}{7}=$

a) $\frac{\sqrt{7}}{16}$ b) $\frac{\sqrt{7}}{8}$ c) $\frac{\sqrt{7}}{32}$ d) $\frac{\sqrt{7}}{64}$

Key. B

Sol. 25,26,27

Let $\alpha = cis\frac{2\pi}{7}$ then $a=\alpha+\alpha^2+\alpha^4, b=\alpha^3+\alpha^5+\alpha^6$

$\Rightarrow a+b=\alpha+\alpha^2+\alpha^3+\alpha^4+\alpha^5+\alpha^6=-1$

& $ab=(\alpha+\alpha^2+\alpha^4)(\alpha^3+\alpha^5+\alpha^6)=2$

$\therefore$ Equation whose roots are a,b is $x^2-x(-1)+2=0$

$f(x)+f(\alpha x)+f(\alpha^2 x)+f(\alpha^3 x)+---+f(\alpha^6 x)$

= (1+1+1---+1) $+2x(\alpha+\alpha^2+\alpha^3+---+\alpha^6)+3x^2(1+\alpha^2+\alpha^4+---+\alpha^{12})+-----+$

$7(1+\alpha^6+\alpha^{12}+\alpha^{18}+---+\alpha^{36})7\,times$

=7

Also $\frac{x^7-1}{x-1} == (x-\alpha)(x-\alpha^2)(x-\alpha^3)(x-\alpha^4)(x-\alpha^5)(x-\alpha^6)$

Putting $\alpha = cis\frac{2\pi}{7}$ of applying $x \to 1$ gives

$7 = \left|\left(1-cis\frac{2\pi}{7}\right)\left(1-cis\frac{4\pi}{7}\right)-----\left(1-cis\frac{12\pi}{7}\right)\right|$

$7 = \left(2\sin\frac{\pi}{7}\right)\left(2\sin\frac{2\pi}{7}\right)-----\left(2\sin\frac{6\pi}{7}\right)$

$$\Rightarrow \sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7} = \sin\frac{4\pi}{7}\sin\frac{5\pi}{7}\sin\frac{6\pi}{7} = \sqrt{\frac{7}{64}} = \frac{\sqrt{7}}{8}$$

Passage – 10

If $\cos\alpha+\cos\beta+\cos\gamma = 0 = \sin\alpha+\sin\beta+\sin\gamma$ and $\delta = \frac{\alpha+\beta+\gamma}{3}$. The difference of any two of the angles α,β,γ is equivalent to $\frac{2\pi}{3}$. Then

28. The value of $\sin 6\alpha+\sin 6\beta+\sin 6\gamma$ interms of δ is

A) 3 sin2δ B) sin6δ C) 3 sin3δ D) 3 sin6δ

Key. D

29. If one of the angles α,β,γ is $\frac{\pi}{36}$ then the $\cos 12\alpha+\cos 12\beta+\cos 12\gamma =$

A) 0 B) 3 C) $\frac{3}{2}$ D) $\frac{2049}{2048}$

Key. C

30. If $\alpha = 10^0$ then $\cos^8\alpha+\cos^8\beta+\cos^8\gamma =$

A) $\frac{113}{128}$ B) $\frac{117}{128}$ C) $\frac{51}{128}$ D) none of these

Key. B

Sol. 28, 29, 30.

All questions works on

$\cos 3n\alpha + i\sin 3n\alpha + \cos 3n\beta + i\sin 3n\beta + \cos n\gamma + i\sin 3n\gamma = 3$

$\left(\cos n(\alpha+\beta+\gamma)+i\sin(\alpha+\beta+\gamma)\right)$

Passage – 11

If a complex number $z = a+ib$, $a>b$, $a,b\in R^+$ and complex number $q_k\ (1\le k\le n)$ denotes the nth root of unity, then all points P_k in the Argand plane, representing the complex number $\operatorname{Re}(z)\operatorname{Re}(q_k)+i\operatorname{Im}(z)\operatorname{Im}(q_k)$ lie on an ellipse. Further , if S is a focus on the positive axis of an ellipse then

31. $\sum_{k=1}^{n} P_k S =$

A) $(n-1)a$ B) $(n+1)a$ C) na D) $\frac{na}{2}$

Key. C

32. $\sum_{k=1}^{n} (P_k S)^2 =$

A) $n(3a^2 - b^2)$ B) $\frac{n}{2}(3a^2 - b^2)$ C) $\frac{1}{2}(3a^2 - b^2)$ D) $n(3a^2 + b^2)$

Key. B

Sol. 31, 32, Integer type questions

Passage – 12

Let z_1, z_2, z be three points A,B,P respectively in the Argand plane. Let P moves in the plane such that $\left|\frac{z - z_1}{z - z_2}\right| = \lambda \neq 1$, then locus of P is a circle. Let $z_1 = 2 + i, z_2 = -4 - 7i$.

33. If $\lambda = 2$, then the maximum area of the triangle ABP is

A) $\frac{100}{3} sq.units$ B) $\frac{200}{3} sq.units$ C) $34 sq.units$ D) $36 sq.units$

Key. A

34. If $\lambda = 3$, then the number of points P in the Argand plane such that area of $\Delta ABP = 19 sq.units$

A) 1 B) 2 C) 3 D) 0

Key. D

35. If $\lambda = \frac{2}{3}$, then the maximum distance of P from the line AB, is

A) 6 B) 12 C) 30 D) 10

Key. B

Sol. 1. The area is max. When height of triangle is radius

$$\therefore r = \frac{\lambda |z_1 - z_2|}{|\lambda^2 - 1|} = \frac{20}{3}$$

Area = $\frac{1}{2}\frac{20}{3}\times 10=\frac{100}{3}$

2. As max. area <19, so no pts

3. Max. distance = Radius $\therefore r=\frac{\lambda|z_1-z_2|}{\lambda^2-1}=12$

Passage – 13

$A(z_1), B(z_2), C(z_3)$ are the vertices of a triangle ABC inseribed in the cirble $|z|=2$. Internal angle bisector of the angle A meets the circumcircle again at $D(z_4)$.

36. Complex number representing point D is

a) $z_4=\frac{z_1z_2}{z_3}$ b) $z_4=\frac{z_2z_3}{z_1}$

c) $z_4=\frac{z_1z_2}{z_3}$ d) $\sqrt{z_2z_3}$

Key. D

37. $\text{Arg}\left(\frac{z_4}{z_2-z_3}\right)=$ ___

a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) $\frac{2\pi}{3}$

Key. C

38. $\text{Arg}(z_1z_2)$ is always

a) $\text{Arg}\, z_1+\text{Arg}\, z_2$

b) $\text{Arg}\, z_1-\text{Arg}\, z_2$

c) $\text{Arg}\, z_1+\text{Arg}\, z_2+n\pi, n=-1,0,1$

d) $\text{Arg}\, z_1+\text{Arg}\, z_2+2n\pi, n=-1,0,1$

Key. D

Sol. 36 to 38

$\frac{z_4}{z_2}=e^{iA}, \frac{z_3}{z_4}=e^{iA}$ and OD is the angular bisector of $\underline{|BOC}$ is isosceles

$\therefore OD\perp BC$

$\therefore \arg\left(\frac{z_4}{z_2-z_3}\right)=\frac{\pi}{2}$

Passage – 14

Consider triangle ABC in Argand plane. Let $A(0), B(1) \& C(i+i)$ be its vertices and M be the mid point of CA. let z be a variable complex number in the plane. Let us be another variable complex number defined as $u = z^2 + 1$

39. Locus of u, when z is on BM is

a) Circle b) Parabola

c) Ellipse d) Hyperbola

Key. B

40. Axis of locus of u, when z is on BM is

a) Real axis b) Imaginary axis

c) $z + \bar{z} = 2$ d) $z - \bar{z} = 2i$

Key. C

41. Direction of locus of z, when z is on BM is

a) Real axis b) Imaginary axis

c) $z + \bar{z} = 2$ d) $z - \bar{z} = 2i$

Key. D

Sol. 39 to 41

Binomial theorem: $y - 0 = -1(x-1)$

$x + y = 1$

$\therefore \sqrt{4-1} = t + i(1-t)$

$4 = 2t = 2i\ t(1-t)$

$x = 2t\ \& y = 2t(1-t)$

$\Rightarrow (x-1)^2 = -2\left(y - \frac{1}{2}\right)$

Axis x = 1, i.e. $z + \bar{z} = 2$

Direction y =1 i.e. $z + \bar{z} = 2i$

Passage – 15

Given $z^{-1} = (a+ib)^{-1} + (a+ic)^{-1}, z = x + iy, a, b, c$ being real with $a + ib, a + ic$ not zero.

42. The value of $x^2 + y^2$ equals

a) $\frac{(a^2-b^2)(a^2-c^2)}{4a^2+(b+c)^2}$ b) $\frac{(a^2+b^2)(a^2+c^2)}{4a^2+(b+c)^2}$

c) $\frac{(a^2+c^2)(a+b)}{a^2+b^2}$ d) $a^2 + b^2 + c^2$

Key. B

43. The value of $(x-a)^2+y^2$ equals

z) $\dfrac{(a^2-bc)^2}{4a^2+(b+c)^2}$ b) $\dfrac{a^2}{4a^2+c^2}$

c) $\dfrac{a^2+b^2+c^2}{a^2+b^2+4c^2}$ d) $\dfrac{(a^2+bc)^2}{4a^2+(b+c)^2}$

Key. D

44. The value of Re(z) i.e., 'x' equals

a) $\dfrac{2a^2+b^2+c^2}{4a^2+(b+c)^2}$ b) $\dfrac{a(2a^2+b^2+c^2)}{4a^2+b^2}$

c) $\dfrac{2(2a^2+b^2+c^2)}{a^2+b^2+c^2}$ d) $\dfrac{a(2a^2+b^2+c^2)}{4a^2+(b+c)^2}$

Key. D

Sol. 42. $\dfrac{1}{z}=\dfrac{2a+i(b+c)}{(a+ib)(a+ic)}$, i.e., $z=\dfrac{(a+ib)(a+ic)}{2a+i(b+c)}$

So that $x^2+y^2z\bar{z}=\dfrac{(a^2+b^2)(a^2+c^2)}{4a^2+(b+c)^2}$

43. $z-a=(x-a)+iy=-\dfrac{(a^2+bc)}{1a+i(b+c)}\Rightarrow(x-a)-iy=\dfrac{-(a^2+bc)}{2a-i(b+c)}$

$\Rightarrow(x-a)^2+y^2=(x-a+iy)(x-a-iy)$

$=\dfrac{(a^2+bc)^2}{4a^2+(b+c)^2}$

44. $(x^2+y^2)-\left[(x-a)^2+y^2\right]=2ax-a^2$

$\Rightarrow x=\dfrac{a(2a^2+b^2+c^2)}{4a^2+(b+c)^2}$

Passage – 16

If $z=r(\cos\theta+i\sin\theta)$ and n is a positive integer, then

$$z^{1/n}=r^{1/n}\left[\cos\frac{(2k\pi+\theta)}{n}+i\sin\frac{(2k\pi+\theta)}{n}\right]$$

Where k = 0, 1, 2, 3, …….. (n –1), given n, nth roots of the complex number z.

45. If $1, \omega, \omega^2$ ….. ω^{n-1} are the n, nth roots of unity and z_1 and z_2 are any two complex numbers, then

$$\sum_{k=0}^{n-1} |z_1 + \omega^k z_2|^2$$

(A) $n\left[|z_1|^2 + |z_2|^2\right]$ (B) $(n-1)\left[|z_1|^2 + |z_2|^2\right]$

(C) $(n+1)\left[|z_1|^2 + |z_2|^2\right]$ (D) $(n + 2)\ [|z_1|^2 + |z_2|^2]$

Key. A

46. If $1, \alpha, \alpha^2$, …… α^{n-1} are the nth roots of unity then $\sum_{i=1}^{n-1} \frac{1}{2-\alpha^i}$ is equal to

(A) $\frac{(n-2)2^{n-1}+1}{2^n-1}$ (B) $(n-2).2^n$

(C) $\frac{(n-2).2^{n-1}}{2^n-1}$ (D) $\frac{(n+2).2^{n-1}}{2^n-1}$

Key. A

47. If $1, \omega, \omega^2, ...\omega^{n-1}$ are the nth roots of unity then $(1-\omega)(1-\omega^2)....(1-\omega^{n-1})$ is equal to

(A) 0 (B) 1

(C) n (D) n^2

Key. C

Sol. 45. $\sum_{k=0}^{n-1}(z_1+\omega^k z_2)(\bar{z}_1+\bar{\omega}^k \bar{z}_2)$

$$= \sum_{k=0}^{n-1} z_1\bar{z}_1 + (\omega\bar{\omega})^k z_2\bar{z}_2 + z_1 z_2 \omega^k + z_1\bar{z}_2\bar{\omega}^k$$

$$= (|z_1|^2 + |z_2|^2)\sum_{k=0}^{n-1}(1) + \left(z_2\bar{z}_1\right)\frac{(1-\omega^n)}{1-\omega} + \left(z_1\bar{z}_2\right)\left(\frac{1-\bar{\omega}^n}{1-\bar{\omega}}\right)$$

$$= n(|z_1|^2 + |z_2|^2)$$

46. $x^n - 1 = (x-1)(x-\alpha)(x-\alpha^2).....(x-\alpha^{n-1})$

$$(x-\alpha)(x-\alpha^2)(x-\alpha^3).....(x-\alpha^{n-1}) = 1 + x + x^2 + ... + x^{n-1}$$

Taking log and then differentiating, we get

$$\frac{1}{x-\alpha} + \frac{1}{x-\alpha^2} + ... + \frac{1}{x-\alpha^{n-1}} = \frac{1+2x+.....+(n-1)x^{n-2}}{1+x+x^2+....+x^{n-1}}$$

$$\frac{1}{2-\alpha} + \frac{1}{2-\alpha^2} + ... + \frac{1}{2-\alpha^{n-1}} = \frac{1+2\times 2+3\times 2^2+......+(n-1)2^{n-2}}{1+2+2^2+.....+2^{n-1}}$$

47. $(x-\omega)(x-\omega^2)....(x-\omega^{n-1}) = 1 + x + x^2 + ... + x^{n-1}$

Put x = 1, we get

$(1-\omega)(1-\omega^2)...(1-\omega^{n-1}) = 1 + 1 + ...n \text{ times} = n$

Passage – 17

R′ is radius of circle passing through the centre of circumscribed circle of ΔABC and through the end point of base BC where B(1, 1), C(3, 5) and ∠BAC = 60º, R is radius of circle which satisfy z,

$\arg\left(\frac{z-(3+5i)}{z-(1+i)}\right) = \frac{\pi}{3}$.

48. Value of R

(A) $\frac{2\sqrt{5}}{\sqrt{3}}$ (B) $\frac{2\sqrt{3}}{\sqrt{5}}$ (C) $\frac{\sqrt{5}}{\sqrt{3}}$ (D) $\frac{\sqrt{3}}{\sqrt{5}}$

Key. A

49. Value of R′

(A) $\sqrt{3}$ (B) $\sqrt{13}$ (C) $\sqrt{11}$ (D) $\frac{2\sqrt{5}}{\sqrt{3}}$

Key. D

50. Co-ordinate of the centre of the circle having radius R is

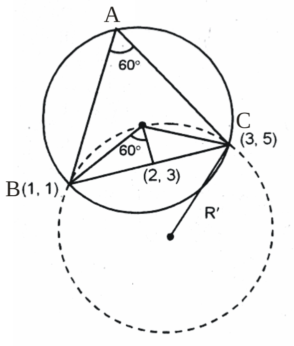
(A) $\left(2-\frac{1}{\sqrt{3}}, 3-\frac{1}{\sqrt{3}}\right)$ (B) $\left(2-\frac{2}{\sqrt{3}}, 3+\frac{1}{\sqrt{3}}\right)$

(C) $\left(1-\frac{2}{\sqrt{3}}, 2+\frac{1}{\sqrt{3}}\right)$ (D) $\left(3-\frac{2}{\sqrt{3}}, 2-\frac{3}{\sqrt{2}}\right)$

Key. B

Sol.

48, 49



$\frac{R}{\sqrt{5}} = \operatorname{cosec}60° = \frac{2}{\sqrt{3}}$

$R = \frac{2\sqrt{5}}{\sqrt{3}}$

$\frac{2\sqrt{5}}{\sin 120°} = 2R'$

$R' = 2\sqrt{\frac{5}{3}}$

50. Co-ordinate of the centre is (h + rcosθ, k + rsinθ)

$$=\left(2+\sqrt{\frac{5}{3}}\times-\frac{2}{\sqrt{5}},\ 3+\sqrt{\frac{5}{3}}\times\frac{1}{\sqrt{5}}\right)=\left(2-\frac{2}{\sqrt{3}},3+\frac{1}{\sqrt{3}}\right)$$

Passage – 18

A person walks $2\sqrt{2}$ units away from origin in south west direction $\left(S45^0 w\right)$ to reach A, then walks $\sqrt{2}$ units in south east direction $\left(S45^0 E\right)$ to reach B. From B he travels 4 units horizontally towards east to reach C, then he travels along a circular path with centre at origin through an angle of $\frac{2\pi}{3}$ in anti clockwise direction to reach his destination D.

51. Position of B in argand plane is

(A) $\sqrt{2}\, e^{-i\frac{3\pi}{4}}$ (B) $\sqrt{2}\left(2+i\right)e^{-i3\pi/4}$

(C) $\sqrt{2}\left(1+2i\right)e^{-i3\pi/4}$ (D) $-3+i$

52. Let the complex number Z represent C in argand plane then $\arg\left(Z\right)=$

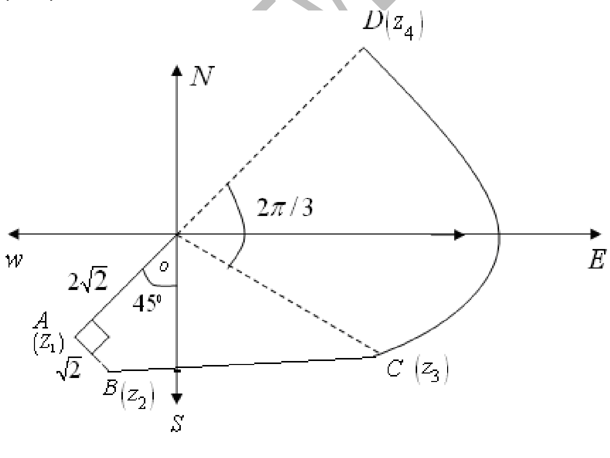
(A) $-\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $-\frac{\pi}{4}$ (D) $\frac{\pi}{3}$

53. Position of D in argand plane is (ω is an imaginary cube root of unity)

(A) $\left(3+i\right)\omega$ (B) $-\left(1+i\right)\omega^2$ (C) $3\left(1-i\right)\omega$ (D) $\left(1-3i\right)\omega$

Key: B-C-C

Hint: 51, 52, 53



. $Z_1=2\sqrt{2}\ \ e^{-i3\pi/4}=-2-2i$

$$\frac{Z_2-\left(-2-2i\right)}{O-\left(-2-2i\right)}=\frac{1}{2}\times e^{-i\pi/2}\left(rotation\ at\ \ A\right)$$

$\Rightarrow Z_2=-\left(1+i\right)\left(2+i\right)=-1-3i$

$\therefore\ Z_3=3-3i$

$\frac{Z_4 - 0}{3 - 3i - 0} = e^{i2\pi/3} \Rightarrow Z_4 = 3(1-i)\omega$

(rotation at O)

Passage – 19

Let Z_1 and Z_2 be complex numbers such that $Z_1^2 - 4Z_2 = 16 + 20i.$ Also suppose that roots α and β of $t^2 + Z_1 t + Z_2 + m = 0$ for some complex number m satisfying $|\alpha - \beta| = 2\sqrt{7}$

54. The complex number m lies on

A) a square with side 7 and centre $(4,5)$ B) a circle with radius 7 and centre $(4,5)$

C) a circle with radius 7 and centre $(-4,5)$ D) a square with side 7 and centre $(-4,5)$

Key: B

55. The greatest value of $|m|$ is

A) $5+\sqrt{21}$ B) $5+\sqrt{23}$ C) $7+\sqrt{43}$ D) $7+\sqrt{41}$

Key: D

56. The least value of $|m|$ is

A) $7-\sqrt{41}$ B) $7-\sqrt{43}$ C) $5-\sqrt{23}$ D) $5+\sqrt{21}$

Key: A

Hint: 54, 55, 56 $\alpha + \beta = -Z_1; \alpha\beta = Z_2 + m$

$(\alpha - \beta)^2 = Z_1^2 - 4Z_2 - 4m$

$= 16 + 20i - 4m$

$|\alpha - \beta|^2 = |16 + 20i - 4m|$

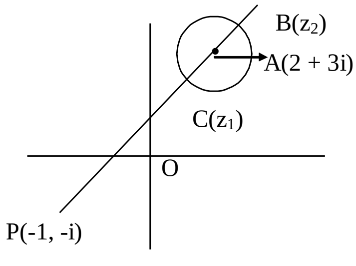
Q $|\alpha - \beta| = 2\sqrt{7} |m - 5i - 4| = 7 \Rightarrow |m - (4+5i)| = 7$

$\therefore$ m lies on a circle having centre $(4,5)$ and radius 7

Passage – 20

If |z - 2 - 3i| = λ and z_1 and z_2 be two complex number for which |z + 1 + i| is minimum and maximum respectively.

57. If arg $\left(\frac{z_1 + 1 + i}{z_2 - 2 - 3i}\right) = 0$, then range of λ will be



(A) (0, 5) (B) (2, 5)

(C) (0, $\sqrt{13}$) (D) (2, $\sqrt{13}$)

Key: A

Hint: for arg $\left(\frac{z_1 + 1 + i}{z_2 - 2 - 3i}\right) = 0$, point P lie out side the circle

$\Rightarrow$ AP > λ $\Rightarrow$ λ ∈ (0, 5)

58. If $\arg\left(\frac{z_1+1+i}{z_2-2-3i}\right)=\pi$, then range of λ will be

(A) $(0, \infty)$ (B) $(5, \infty)$ (C) $(\sqrt{13}, \infty)$ (D) none of these

Key: B

Hint: For $\arg\left(\frac{z_1+1+i}{z_2-2-3i}\right) = \pi$, point P must lie inside the circle

$\Rightarrow AP < \lambda \Rightarrow \lambda \in (5, \infty)$.

59. The range of λ for which maximum value of principal arg(z) exists

(A) $(0, \sqrt{13})$ (B) $(\sqrt{13}, \infty)$ (C) $(2, \sqrt{13})$ (D) $(2, \infty)$

Key: B

Hint: Maximum value of principal arg(z) is π and for this z must lie on the -ve x-axis, so origin must lie inside the circle

$\Rightarrow OA < \lambda$

$\Rightarrow \lambda \in (\sqrt{13}, \infty)$

Passage – 21

Whenever we have to find the sum of finite on infinite series of the form $a_0 \cos\alpha + a_1 \cos(\alpha + \beta) + \alpha_2 \cos(\alpha + 2\beta) + \ldots$ or $a_0 \sin\alpha + a_1 \sin(\alpha + \beta) + a_2 \sin(\alpha + 2\beta) + \ldots$ then we will use following method

Step – I : If the series whose sum is to be found in cosine, let this series be denoted by C. Then write another corresponding auxiliary series in sines and denote it by S and vice versa.

Step – II : Find C + iS, use $e^{i\theta} = \cos\theta + i\sin\theta$ and simplify.

C + iS series thus obtained, converts it to some standard series whose sum can be easily calculated.

Finally we convert this sum in A+ I B.

Step – III : Now equate real and imaginary parts from both sides to get the required result.

60. The sum of the series $\cos\alpha + x\cos(\alpha + \beta) + \frac{x^2}{2!}\cos(\alpha + 2\beta) + \ldots\ldots + \infty$

(A) $e^{x\cos\beta}\cdot\cos(\alpha + x\sin\beta)$ (B) $e^{x\cos\beta}\sin(\alpha + x\sin\beta)$

(C) $e^{x\sin\beta}\cdot\cos(\alpha + x\sin\beta)$ (D) $e^{x\sin\beta}\cdot\sin(\alpha + x\sin\beta)$

Key: A

Hint: Let, $C = \cos\alpha + x\cos(\alpha+\beta) + \frac{x^2}{2!}\cos(\alpha+2\beta) + \ldots. + \infty$

and $S = \sin\alpha + x\sin(\alpha+\beta) + \frac{x^2}{2!}\sin(\alpha+2\beta) + \ldots. + \infty$

$\therefore\ C + iS = (\cos\alpha + i\sin\alpha) + x(\cos(\alpha+\beta) + i\sin(\alpha+\beta))$

$+\frac{x^2}{2!}(\cos(\alpha+2\beta) + i\sin(\alpha+2\beta)) + \ldots\ldots + \infty$

$= e^{i\alpha} + x.e^{i(\alpha+\beta)} + \frac{x^2}{2!}e^{i(\alpha+2\beta)} + \ldots\ldots.. + \infty$

$$= e^{i\alpha}\left[1 + xe^{i\beta} + \frac{x^2}{2!}e^{i2\beta} + ... + \infty\right]$$

$$e^{i\alpha}e^{x.e^{i\beta}} = e^{i\alpha}.e^{x(\cos\beta + i\sin\beta)} = e^{x\cos\beta}\left[\cos(\alpha + x\sin\beta) + i\sin(\alpha + x\sin\beta)\right]$$

Equating real parts on both sides, we get

$$C = e^{x\cos\beta}\cos(\alpha + x\sin\beta)$$

61. The sum of the series $\sin\theta - \frac{\sin 2\theta}{2!} + \frac{\sin 3\theta}{3!}$ . . . to ∞

(A) $e^{-\cos\theta}\cdot\sin(\sin\theta)$ (B) $e^{-\sin\theta}\cdot\cos(\sin\theta)$

(C) $e^{-\cos\theta}\cdot\cos(\sin\theta)$ (D) $e^{-\sin\theta}\cdot\sin(\sin\theta)$

Key: A

Hint: Let $C = \cos\theta - \frac{\cos 2\theta}{2!} + \frac{\cos 3\theta}{3!} - ......\text{to}\infty$

$$S = \sin\theta - \frac{\sin 2\theta}{2!} + \frac{\sin 3\theta}{3!} - ......\text{to}\infty$$

Then $C + iS = e^{i\theta} - \frac{1}{2!}e^{i2\theta} + \frac{1}{3!}e^{i3\theta} - .....\text{to } \infty$

$\Rightarrow$ $C + iS = 1 - e^{-e^{i\theta}}$

$\Rightarrow$ $C + iS = 1 - e^{-\cos\theta}\left[\cos(\sin\theta) - i\sin(\sin\theta)\right]$

Now, evaluate imaginary parts on both sides, we get

$$S = e^{-\cos\theta}\sin(\sin\theta)$$

62. $\cos\frac{\pi}{3} + \frac{1}{2}\cos\frac{2\pi}{3} + \frac{1}{3}\cos\frac{3\pi}{3} + ... + \infty$ is equal to

(A) $\frac{\pi}{3}$ (B) 0

(C) 1 (D) $e^{i\pi/3}$

Key: B

Hint: Let $C = \cos\frac{\pi}{3} + \frac{1}{2}\cos\left(\frac{2\pi}{3}\right) + \frac{1}{3}\cos\left(\frac{3\pi}{3}\right) + ... + \infty$

And $S = \sin\frac{\pi}{3} + \frac{1}{2}\sin\left(\frac{2\pi}{3}\right) + \frac{1}{3}\sin\left(\frac{3\pi}{3}\right) + ... + \infty$

$$C + iS = e^{i\pi/3} + \frac{1}{2}e^{i(2\pi/3)} + \frac{1}{3}e^{i(3\pi/3)} + ... + \infty$$

$$= -\log_e\left(1 - e^{i\pi/3}\right) = -\log_e\left[1 - \left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)\right] = -\log_e\left[\frac{1}{2} - i\frac{\sqrt{3}}{2}\right] = 0 + i.\frac{\pi}{3}$$

Equating real parts from both sides, we get

$$C = 0$$

Passage – 22

If $z = r(\cos\theta + i\sin\theta)$ and n is a positive integer, then

$$z^{1/n} = r^{1/n}\left[\cos\frac{(2k\pi+\theta)}{n} + i\sin\frac{(2k\pi+\theta)}{n}\right]$$

Where k = 0, 1, 2, 3, …….. (n –1), given n, nth roots of the complex number z.

63. If $1, \omega, \omega^2$ ….. ω^{n-1} are the n, nth roots of unity and z_1 and z_2 are any two complex numbers, then

$$\sum_{k=0}^{n-1} |z_1 + \omega^k z_2|^2$$

(A) $n\left[|z_1|^2 + |z_2|^2\right]$ (B) $(n-1)\left[|z_1|^2 + |z_2|^2\right]$

(C) $(n+1)\left[|z_1|^2 + |z_2|^2\right]$ (D) $(n + 2)\ [|z_1|^2 + |z_2|^2]$

Key. A

Sol. $\displaystyle\sum_{k=0}^{n-1} (z_1 + \omega^k z_2)(\overline{z}_1 + \overline{\omega}^k \overline{z}_2)$

$= \displaystyle\sum_{k=0}^{n-1} z_1\overline{z}_1 + (\omega\overline{\omega})^k z_2\overline{z}_2 + z_1 z_2 \omega^k + z_1\overline{z}_2\overline{\omega}^k$

$= (|z_1|^2 + |z_2|^2)\displaystyle\sum_{k=0}^{n-1}(1) + \left(z_2\overline{z}_1\right)\frac{(1-\omega^n)}{1-\omega} + \left(z_1\overline{z}_2\right)\left(\frac{1-\overline{\omega}^n}{1-\overline{\omega}}\right)$

$= n(|z_1|^2 + |z_2|^2)$

64. If $1, \alpha, \alpha^2$, …… α^{n-1} are the nth roots of unity then $\displaystyle\sum_{i=1}^{n-1}\frac{1}{2-\alpha^i}$ is equal to

(A) $\dfrac{(n-2)2^{n-1}+1}{2^n-1}$ (B) $(n-2).2^n$

(C) $\dfrac{(n-2).2^{n-1}}{2^n-1}$ (D) $\dfrac{(n+2).2^{n-1}}{2^n-1}$

Key. A

Sol. $x^n - 1 = (x-1)(x-\alpha)(x-\alpha^2).....(x-\alpha^{n-1})$

$(x-\alpha)(x-\alpha^2)(x-\alpha^3).....(x-\alpha^{n-1}) = 1 + x + x^2 + ... + x^{n-1}$

Taking log and then differentiating, we get

$$\frac{1}{x-\alpha} + \frac{1}{x-\alpha^2} + ... + \frac{1}{x-\alpha^{n-1}} = \frac{1+2x+.....+(n-1)x^{n-2}}{1+x+x^2+....+x^{n-1}}$$

$$\frac{1}{2-\alpha} + \frac{1}{2-\alpha^2} + ... + \frac{1}{2-\alpha^{n-1}} = \frac{1+2\times2+3\times2^2+......+(n-1)2^{n-2}}{1+2+2^2+.....+2^{n-1}}$$

65. If $1, \omega, \omega^2, ...\omega^{n-1}$ are the nth roots of unity then $(1-\omega)(1-\omega^2)....(1-\omega^{n-1})$ is equal to

(A) 0 (B) 1

(C) n (D) n^2

Key. C

Sol. $(x-\omega)(x-\omega^2)....(x-\omega^{n-1}) = 1 + x + x^2 + ... + x^{n-1}$

Put x = 1, we get

$(1-\omega)(1-\omega^2)...(1-\omega^{n-1}) = 1 + 1 + ...n\text{ times} = n$

Passage – 23

R′ is radius of circle passing through the centre of circumscribed circle of ΔABC and through the end point of base BC where B(1, 1), C(3, 5) and ∠BAC = 60º, R is radius of circle which satisfy z,

$\arg\left(\dfrac{z-(3+5i)}{z-(1+i)}\right)=\dfrac{\pi}{3}$.

66. Value of R

(A) $\dfrac{2\sqrt{5}}{\sqrt{3}}$ (B) $\dfrac{2\sqrt{3}}{\sqrt{5}}$

(C) $\dfrac{\sqrt{5}}{\sqrt{3}}$ (D) $\dfrac{\sqrt{3}}{\sqrt{5}}$

Key. A

67. Value of R′

(A) $\sqrt{3}$ (B) $\sqrt{13}$

(C) $\sqrt{11}$ (D) $\dfrac{2\sqrt{5}}{\sqrt{3}}$

Key. D

68. Co-ordinate of the centre of the circle having radius R is

(A) $\left(2-\dfrac{1}{\sqrt{3}},\,3-\dfrac{1}{\sqrt{3}}\right)$ (B) $\left(2-\dfrac{2}{\sqrt{3}},\,3+\dfrac{1}{\sqrt{3}}\right)$

(C) $\left(1-\dfrac{2}{\sqrt{3}},\,2+\dfrac{1}{\sqrt{3}}\right)$ (D) $\left(3-\dfrac{2}{\sqrt{3}},\,2-\dfrac{3}{\sqrt{2}}\right)$

Key. B

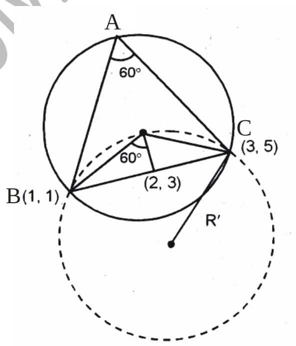
Sol. 66, 67

$\dfrac{R}{\sqrt{5}}=\operatorname{cosec}60^\circ=\dfrac{2}{\sqrt{3}}$

$R=\dfrac{2\sqrt{5}}{\sqrt{3}}$

$\dfrac{2\sqrt{5}}{\sin 120^\circ}=2R'$

$R'=2\sqrt{\dfrac{5}{3}}$



68. Co-ordinate of the centre is (h + rcosθ, k + rsinθ)

$$=\left(2+\sqrt{\frac{5}{3}}\times-\frac{2}{\sqrt{5}},\ 3+\sqrt{\frac{5}{3}}\times\frac{1}{\sqrt{5}}\right)=\left(2-\frac{2}{\sqrt{3}},3+\frac{1}{\sqrt{3}}\right)$$

Passage – 24

The points on the Argand plane corresponding to the nth roots of unity represent the vertices of a regular n–sided polygon inscribed in a unit circle centred at origin with one vertex of the polygon at (1, 0).

69. Let A be the point (0, 2) and points B_k $(0 \le k \le 9)$ the vertices of the above described polygon with n = 10 then the value of the product of distances $AB_3 . AB_4 . AB_5 . AB_6 . AB_7$ is

(A) $10\sqrt{21}$ (B) $20\sqrt{21}$

(C) $5\sqrt{41}$ (D) $15\sqrt{41}$

Key. C

70. The product of distances of any vertex from the remaining vertices of a nine–sided regular polygon inscribed in a circle of radius $\sqrt{2}$ units is

(A) 36 (B) 72

(C) 144 (D) 256

Key. C

71. The value of $\left|\cos\frac{\pi}{n}\cos\frac{2\pi}{n}\cos\frac{3\pi}{n}\ldots\cos\frac{(n-1)\pi}{n}\right|$ is

(A) $\frac{1+(-1)^n}{2^n}$ (B) $\frac{1+(-1)^n}{2^{n+1}}$

(C) $\frac{1-(-1)^n}{2^n}$ (D) $\frac{1-(-1)^n}{2^{n+1}}$

Key. C

Sol. 69. $z^{10} - 1 = (z-1)(z-\alpha)(z-\alpha^2)\ldots(z-\alpha^9)$ where $\alpha = e^{i\left(\frac{2\pi}{10}\right)}$

Putting z = 2i and taking modulus we get

$|(2i)^{10} - 1| = AB_0.AB_1.AB_2 \ldots AB_9$

$\Rightarrow (AB_3 . AB_4 \ldots AB_7)^2 = |-2^{10} - 1|$ (by symmetry)

$\Rightarrow AB_3 . AB_4 \ldots AB_7 = \sqrt{1025} = 5\sqrt{41}$.

70. The product of distances

$= |\sqrt{2}-\sqrt{2}\alpha||\sqrt{2}-\sqrt{2}\alpha^2|\ldots|\sqrt{2}-\sqrt{2}\alpha^8|$

$= (\sqrt{2})^8|z-\alpha||z-\alpha^2|\ldots|z-\alpha^8|$, at z = 1

$= 16 \times \frac{|z^9-1|}{|z-1|} = 16 \times |z^8 + z^7 + \ldots + z + 1|$

$= 16 \times 9 = 144$.

71. When z = –1, $|z-\alpha^r| = |-1-\cos\frac{2\pi r}{n} - i\sin\frac{2\pi r}{n}|$

$= |2\cos\frac{\pi r}{n}\left(\cos\frac{\pi r}{n}+i\sin\frac{\pi r}{n}\right)| = 2\left|\cos\frac{\pi r}{n}\right|$

So, $2^n\ \left|\cos\frac{\pi}{n}\cos\frac{2\pi}{n}\cos\frac{3\pi}{n}\ldots\cos\frac{(n-1)\pi}{n}\right| = |z-1|\ |z-\alpha|\ |z-\alpha^2| \ldots |z-\alpha^{n-1}|$

$\Rightarrow$ The required value = $\frac{|z^n - 1|}{2^n} = \frac{|(-1)^n - 1|}{2^n} = \frac{1-(-1)^n}{2^n}$

Passage – 25

Consider the set of complex numbers A,B,C and S defined as below.

$A = \{Z/|Z+2| - |Z-2| = 2\}$ $\quad B = \left\{Z/\arg\left(\frac{Z-1}{Z}\right) = \frac{\pi}{2}\right\}$

$C = \{Z/\arg(Z-1) = \pi\}$ $\quad S = \left\{Z/\text{Re}\left(\frac{Z-1}{Z+1}\right) = 0\right\}$ then

72. $A \cap B \cap C =$

a) φ b) $(0, \propto)$ c) $(0,1)$ d) $(-1,0)$

Key. A

73. If $Z_1, Z_2, Z_3 \in S$, then the minimum value of $|Z_1+Z_2|^2 + |Z_2+Z_3|^2 + |Z_3+Z_1|^2 =$

a)5 b) 3 c)2 d) 9

Key. B

74. If $Z \in S$, then arg $\left(\frac{2Z+i-1}{2Z+i+1}\right)$ can be

a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{-\pi}{3}$

Key. B

Sol. 72. clearly $A \cap B \cap C = \phi$

73. $z_1, z_2, z_3 \in s \Rightarrow |z_1|^2 = |z_2|^2 = |z_3|^2 = \frac{1}{2}$

$\therefore |z_1+z_2|^2 + |z_2+z_3|^2 + |z_3+z_1|^2 \geq 2\left(|z_1|^2 + |z_2|^2 + |z_3|^2\right) = 3$

74. $z \in s \Rightarrow 2z = \frac{1}{z}$ hence arg $\left(\frac{2z+i-1}{2z+i+1}\right) = \frac{\pi}{4}$

Passage – 26

Suppose z and w are two complex numbers such that |z| ≤ 1 and |w| ≤ 1 and |z + iw| = $|z - i\bar{w}|$ = 2 we know that $|z|^2 = z\bar{z}$ and |z + w| ≤ |z| + |w| then, answer the following questions.

76. Which of the following is correct

(A) $|z| = |w| = \frac{1}{2}$ (B) $|z| = \frac{1}{2}$ $|w| = \frac{3}{4}$

(C) $|z| = \frac{1}{4}$ $|w| = \frac{3}{4}$ (D) |z| = |w| = 1

Key. D

77. Number of complex number z satisfying above conditions are

(A) 1 (B) 2

(C) 4 (D) infinite

Key. B

78. Which of the following is true for z and w

(A) Re(z) = Re(w) (B) Im(z) = Im(w)

(C) Re(z) = Im(w) (D) Im(z) = Re(w)

Key. D

Sol. 76. $|z + iw| \le |z| + |w| \Rightarrow |z| + |w| \ge 2$

and $|z| + |w| \le 2$

$\Rightarrow$ $|z| + |w| = 2$

$\Rightarrow$ $|z| = |w| = 1$

$(z + iw)(\bar{z} - i\bar{w}) = 4$

$\Rightarrow$ $|z|^2 + |w|^2 + i(w\bar{z} - \bar{w}z) = 4$

$\Rightarrow$ $i(w\bar{z} - \bar{w}z) = 2$ …..(i)

Similarly

$(z - i\bar{w})(\bar{z} + iw) = 4$

$\Rightarrow$ $i(wz - \bar{z}\bar{w}) = 2$ …..(ii)

77. (i) + (ii)

$\Rightarrow$ $i\left[(w - \bar{w})(z + \bar{z})\right] = 4$

$\Rightarrow$ $i(2i\,\mathrm{Im}(w)) \times (2\,\mathrm{Re}(z)) = 4$

$\Rightarrow$ $I_m(w) \times \mathrm{Re}(z) = -1$

(i) – (ii)

$\Rightarrow$ $\left[(w + \bar{w})(z - \bar{z})\right] = 0$

$\Rightarrow$ Re(w) = 0 or Im(z) = 0

when Im(z) = 0

as $|z| = 1$

$\Rightarrow$ Re(z) = ± 1

$\Rightarrow$ z = ±1

when Re(w) = 0

$|w| = 1$

$\Rightarrow$ Im(w) = ± 1

$\Rightarrow$ w = ± i

Passage – 27

Let $A_1, A_2, A_3, \ldots\ldots\ldots A_n$ be the vertices of an n-sided regular polygon inscribed in a circle

79. $|A_1 A_2|^2 + |A_1 A_3|^2 + \ldots\ldots\ldots |A_1 A_n|^2 =$

a) n b) 2n c) 2n + 1 d) n / 2

Key. B

80. If the circle is of unit radius and if $\frac{1}{A_1 A_2} = \frac{1}{A_1 A_3} + \frac{1}{A_1 A_4}$, then n =

a) 5 b) 6 c) 7 d) 12

Key. C

81. $|A_1 A_2| . |A_1 A_3| \ldots\ldots\ldots |A_1 A_n| =$

a) n b) 2n c) 2n + 1 d) 4n

Key. A

Sol. 79. $|A_1 A_r|^2 = |z_1 - z_r|^2$

$$= 2 - 2\cos\frac{2(r-1)\pi}{n} = 2n$$

80. $$|A_1 A_r|^2 = |z_1|^2\left[2 - 2\cos^2\frac{(r-1)\pi}{n}\right]$$

$$n = 7$$

81. $$|A_1 A_r| = |z_1|\left|1 - e^{2(r-1)\sqrt{\pi/n}}\right|$$

$$\frac{z^n - 1}{z - 1} = \left(z - e^{i2\pi/n}\right)\left(z - e^{i4\pi/n}\right)$$

Passage – 28

Consider the set of complex numbers A, B, C and S defined as

A = {z : ||z + 2| – |z – 2|| = 2}

B = {z : arg $\left(\frac{z-1}{z}\right) = \frac{\pi}{2}$ }

C = {z : arg (z – 1) = π}

S = {z : Re $\left(\frac{z-1}{z+1}\right) = 0$ }

82. A ∩ B ∩ C =

(A) φ (B) (0, ∞)

(C) (0, 1) (D) (–1, 0)

Key. A

83. If z_1 , z_2, z_3 ∈ S, then minimum value of $|z_1 + z_2|^2 + |z_2 + z_3|^2 + |z_3 + z_1|^2$ is

(A) 5 (B) 3

(C) 2 (D) 9

Key. B

84. If z lies on S, then arg $\left(\frac{2z+i-1}{2z+i+1}\right)$ equals

(A) π/2 (B) π/4

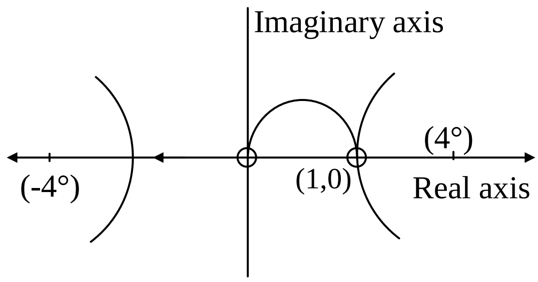
(C) π/3 (D) – π/4

Key. B

Sol.

82. A, B, C represented geometrically as

Clearly A ∩ B ∩ C = φ

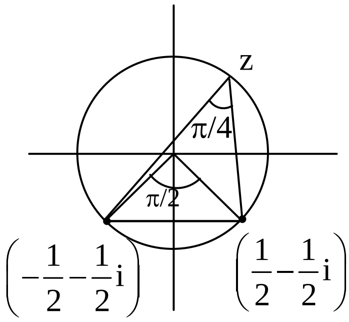


83. Clearly S represents the set of complex number lying on the circle |z| = 1, z ≠ –1.

$|z_1 + z_2|^2 + |z_2 + z_3|^2 + |z_3 + z_1|^2 = 3 + (z_1 + z_2 + z_3)\,(\overline{z}_1 + \overline{z}_2 + \overline{z}_3)$

$= 3 + |z_1 + z_2 + z_3|^2 \geq 3.$

84. From the diagram



arg $\left(\dfrac{\frac{1}{2}-\frac{1}{2}i-z}{-\frac{1}{2}-\frac{1}{2}i-z}\right)=\dfrac{\pi}{4}$

$\Rightarrow$ arg $\left(\dfrac{2z+i-1}{2z+i+1}\right)=\dfrac{\pi}{4}$

Passage – 29

If α is any of 7th roots of unity , then $\alpha = \dfrac{cis 2K\pi}{7}$ (K= 0 to 6) and $\sum_{i=1}^{6}\alpha^{i} = -1(\alpha \neq 1)$ and $\alpha^7 = 1$

Answer the following

85. The equation whose roots are $\alpha+\alpha^2+\alpha^4$ and $\alpha^3+\alpha^5+\alpha^6$ is $(\alpha \neq 1)$

a) $x^2+x-2=0$ b) $x^2+x+2=0$ c) $x^2-x+2=0$ d) $x^2-x-2=0$

Key. B

86. If $f(x)=1+2x+3x^2+4x^3+5x^4+6x^5+7x^6$ then for $\alpha \neq 1$,

$f(x)+f(\alpha x)+f(\alpha^2 x)+f(\alpha^3 x)+---+f(\alpha^6 x)=$

a) 42 b) 21 c) 14 d) 7

Key. D

87. $\sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7}=\sin\frac{4\pi}{7}\sin\frac{5\pi}{7}\sin\frac{6\pi}{7}=$

a) $\dfrac{\sqrt{7}}{16}$ b) $\dfrac{\sqrt{7}}{8}$ c) $\dfrac{\sqrt{7}}{32}$ d) $\dfrac{\sqrt{7}}{64}$

Key. B

Sol. 85,86,87

Let $\alpha = cis\dfrac{2\pi}{7}$ then $a=\alpha+\alpha^2+\alpha^4, b=\alpha^3+\alpha^5+\alpha^6$

$\Rightarrow a+b=\alpha+\alpha^2+\alpha^3+\alpha^4+\alpha^5+\alpha^6=-1$

& $ab=\left(\alpha+\alpha^2+\alpha^4\right)\left(\alpha^3+\alpha^5+\alpha^6\right)=2$

$\therefore$ Equation whose roots are a,b is $x^2-x(-1)+2=0$

$f(x)+f(\alpha x)+f(\alpha^2 x)+f(\alpha^3 x)+---+f(\alpha^6 x)$

= (1+1+1---+1) $+2x\left(\alpha+\alpha^2+\alpha^3+---+\alpha^6\right)+3x^2\left(1+\alpha^2+\alpha^4+---+\alpha^{12}\right)+----+7\left(1+\alpha^6+\alpha^{12}+\alpha^{18}+---+\alpha^{36}\right)$

7 *times*

=7

Also $\dfrac{x^7-1}{x-1}==(x-\alpha)(x-\alpha^2)(x-\alpha^3)(x-\alpha^4)(x-\alpha^5)(x-\alpha^6)$

Putting $\alpha = cis\frac{2\pi}{7}$ of applying $x \to 1$ gives

$$7 = \left|\left(1-cis\frac{2\pi}{7}\right)\left(1-cis\frac{4\pi}{7}\right)-----\left(1-cis\frac{12\pi}{7}\right)\right|$$

$$7 = \left(2\sin\frac{\pi}{7}\right)\left(2\sin\frac{2\pi}{7}\right)-----\left(2\sin\frac{6\pi}{7}\right)$$

$$\Rightarrow \sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7} = \sin\frac{4\pi}{7}\sin\frac{5\pi}{7}\sin\frac{6\pi}{7} = \sqrt{\frac{7}{64}} = \frac{\sqrt{7}}{8}$$

Passage – 30

The complex slope M of a line joining two points $z_1 \& z_2$ in complex plane is defined as $M = \frac{z_1 - z_2}{\bar{z}_1 - \bar{z}_2}$ .Its real slope m is $\tan\theta$, where θ is inclination of the line .Then

Answer the following

88. M =

a) $\frac{1+im}{1-im}$ b) $\frac{2m}{1+m^2} + i\left(\frac{1-m^2}{1+m^2}\right)$ c) $\frac{2m}{1+m^2} + i\left(\frac{1+m^2}{1-m^2}\right)$ d) $\frac{1-im}{m-i}$

Key. A

89. The inclination of a line whose complex slope is $-\omega^2$ (where ω is a non real cube root of unity) is

a) $\frac{2\pi}{3}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{12}$

Key. C

90. Which of the following is false?

a) $|M| = 1$ b) M=m is never possible c) $m = i\left(\frac{M-1}{M+1}\right)$ d) $M = cis2\theta$

Key. C

Sol. 88,89,90

Given M = $\frac{z_1 - z_2}{\bar{z}_1 - \bar{z}_2} = \frac{(x_1 - x_2) + i(y_1 - y_2)}{(x_1 - x_2) - i(y_1 - y_2)} = \frac{1 + i\tan\theta}{1 - i\tan\theta} = \frac{1+im}{1-im}$

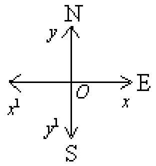
M = $i\left(\frac{2m}{1+m^2}\right) + \left(\frac{1-m^2}{1+m^2}\right)$

$$\therefore M = \cos 2\theta + i\sin 2\theta = cis2\theta \Rightarrow |M| = 1$$

$$-\omega^2 = \frac{1+i\sqrt{3}}{2} = cis2\theta \Rightarrow \theta = \frac{\pi}{6}$$

Passage – 31

A person walks $2\sqrt{2}$ units away from origin in south west direction $(S45^0W)$ to reach A then walks $\sqrt{2}$ units in south east direction $(S45^0E)$ to reach B. From B he travels 4 units horizontally towards east to reach C then he travels along a circular path through an angle of $2\pi/3$ in anti-clock wise direction to reach his destination D.

Notation:


91. Position of B in argand plane is

A) $\sqrt{2}e^{-i\frac{3\pi}{4}}$ B) $\sqrt{2}(2+i)e^{-i\frac{3\pi}{4}}$ C) $\sqrt{2}(1+2i)e^{-i\frac{3\pi}{4}}$ D) $-(3+i)$

Key. B

92. Position of D in argand plane is

A) $(3+i)\omega$ B) $-(1+i)\omega^2$ C) $(1-i)\omega$ D) 3(1-i) ω

Key. D

93. Let Z represent 'C' then $\arg(Z)=$

A) $-\pi/6$ B) $\pi/4$ C) $-\pi/4$ D) $-3\pi/4$

Key. C

Sol. 91 – 93

$A=2\sqrt{2}e^{-i\frac{3\pi}{4}}$

Applying rotation at A B(Z)

$$\Rightarrow \frac{Z-2\sqrt{2}e^{-i\frac{3\pi}{4}}}{0-2\sqrt{2}e^{-i\frac{3\pi}{4}}}=\frac{1}{2}e^{-i\frac{\pi}{2}}$$

$\Rightarrow B(Z)=-(1+3i)$

Passage – 32

The n^{th} roots of unity are $\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}$ where $k=0,1,2,......n-1$ so that n is odd

then $x^n-1=(x-1)\prod_{k=1}^{\frac{n-1}{2}}\left(x-\left(\cos\frac{2k\pi}{n}-i\sin\frac{2k\pi}{n}\right)\right)\left(x-\left(\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}\right)\right)$

94. The value of $\sin\frac{\pi}{7}\sin\frac{2\pi}{7}\sin\frac{3\pi}{7}$ is equal to

(a) $\frac{7}{8}$ (b) $\frac{\sqrt{7}}{4}$ (c) $\frac{\sqrt{7}}{8}$ (d) $\frac{7}{64}$

Key. C

95. The value of $\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{3\pi}{7}$ is equal to

(a) $-\frac{1}{8}$ (b) $\frac{1}{8}$ (c) $\frac{2}{7}$ (d) $-\frac{2}{7}$

Key. B

96. The value of $\cos\frac{\pi}{14}\cos\frac{3\pi}{14}\cos\frac{5\pi}{14}$ is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{16}$ (c) $\frac{\sqrt{7}}{8}$ (d) $\frac{1}{6}$

Key. C

Sol. 94 – 96.

Let $1,\alpha,\alpha^2,\alpha^3.......\alpha^6$ be the roots of $z^7=1$

Then $(z^7-1)=(z-1)(z-\alpha)(z-\bar{\alpha})(z-\alpha^2)(z-\bar{\alpha}^2)(z-\alpha^3)(z-\bar{\alpha}^3)$

$\Rightarrow z^6+z^5+z^4+z^3+z^2+z+1=(z-\alpha)(z-\bar{\alpha})(z-\alpha^2)(z-\bar{\alpha}^2)(z-\alpha^3)(z-\bar{\alpha}^3)$

$=(z^2-2\cos\alpha z+1)(z^2-2\cos 2\alpha z+1)(z^2-2\cos 3\alpha z+1) \quad \rightarrow (1)$

By putting $z=1,-1$ we get required values

Similarly we can do 20th question

Passage – 33

For a complex number z, $|z|^2=z\bar{z}$ when z = $\bar{z}$, it implies that 'z' is purely real. When z = – $\bar{z}$, it implies that 'z' is purely imaginary. For the argument, Arg$\left(\frac{z_1}{z_2}\right)=Arg\left(z_1\right)-Arg\left(z_2\right)$, $Arg\left(z_1z_2\right)=Arg\left(z_1\right)+Arg\left(z_2\right)$ also $|\sin\theta|\geq|\theta|$.

97. Let $z_1=4+6i$ and $z_2=10+6i$. If z is any complex number such that Arg$\left(\frac{z-z_2}{z-z_1}\right)$ = $\frac{\pi}{4}$, then $|z-7-9i|$ =

a) $2\sqrt{2}$ b) $4\sqrt{2}$ c) $\sqrt{2}$ d) 3 $\sqrt{2}$

Key. D

98. If $z_1=\frac{\left(\sqrt{3}-i\right)^2\left(1+\sqrt{3}i\right)}{1-i}$, $z_2=\frac{\left(1-\sqrt{3}i\right)^2\left(\sqrt{3}+i\right)}{1+i}$ then 3 $\left(Ampz_1\right)+Ampz_2$ is

a) 0 b) $\frac{\pi}{4}$ c) $\frac{3\pi}{4}$ d) $\frac{\pi}{2}$

Key. A

99. The value of amplitude of $\left(\sin\frac{8\pi}{5}+i\left(1+\cos\frac{8\pi}{5}\right)\right)$ =

a) $\frac{7\pi}{10}$ b) $\frac{-3\pi}{10}$ c) $\frac{2\pi}{7}$ d) $\frac{7\pi}{2}$

Key. A

Sol. 99. $\sin\frac{8\pi}{5}+I\left(1+COS\left(\frac{8\pi}{5}\right)\right)$

= 2cos $\left(\frac{4\pi}{5}\right)\left(\frac{4\pi}{5}\right)\left[\sin\frac{4\pi}{5}+i\cos\frac{4\pi}{5}\right]$

= –2cos $\frac{4\pi}{5}\left[-\sin\frac{4\pi}{5}-i\cos\frac{4\pi}{5}\right]\left(2cos\frac{4\pi}{5}<0\right)$

= –2cos $\frac{4\pi}{5}\left[\cos\frac{7\pi}{10}+i\sin\frac{7\pi}{10}\right]$

Ans : $\frac{7\pi}{10}$

Passage – 34

Given three equations in the complex plane

$iz+\bar{z}+1+I=0$ (1)

$(2-i)z=(2+i)\bar{z}$ (2)

$(2+i)z+(i-2)\bar{z}-4i=0$ (3)

100. If Z_1 denote the point of intersection of (2) and (3). Then Z_1 =

a) $\frac{1}{2}+i$ b) 1 + i c) $1+\frac{1}{2}i$ d) none of these

Ans. c

101. The distance between the point Z_1 and the line (1) is

a) $\frac{3}{\sqrt{2}}$ b) $3\sqrt{2}$ c) $\frac{3}{2\sqrt{2}}$ d) none of these

Ans. c

102. The equation of the circle which touches the line (1) and the lines (2) and (3) are normals to the circle is

a) $\left|Z-\left(\frac{1}{2}+i\right)\right|=\frac{3}{\sqrt{2}}$ b) $\left|Z-(1+i)\right|=3\sqrt{2}$ c) $\left|Z-\left(1+\frac{i}{2}\right)\right|=\frac{3}{2\sqrt{2}}$ d) none of these

Ans. c

Sol. $(2-i)Z=(2+i)\bar{Z}$ (1)

$\Rightarrow(2+i)+(i-2)\bar{Z}-4i=0$ (2)

Replace $\bar{Z}$ from above with the help of (1)

$(2+i)Z+\frac{(i-2)(2-i)}{2+i}Z-4i=0\Rightarrow Z=\frac{20i}{16i+8}=1+\frac{i}{2}$

Also $\frac{iZ}{1+i}+\frac{\bar{Z}}{1+i}+1=0\Rightarrow(1+i)Z+(1-i)\bar{Z}+2=0$

Radius of the circle = $\frac{\left|\frac{(1+i)(2+i)}{2}+\frac{(1-i)(2-i)}{2}+2\right|}{|1+i|+|1-i|}=\frac{3}{2\sqrt{2}}$

Equation of circle is $\left|Z-\left(1+\frac{i}{2}\right)\right|=\frac{3}{2\sqrt{2}}$

Passage – 35

In the equation $z^2+2\lambda z+1=0$, λ is a parameter which can take any real value, then

103. The roots of this equation lie on a certain circle, if
(A) $-1<\lambda<1$ (B) $\lambda>1$
(C) $\lambda<1$ (D) none of these

Key. A

104. One root lies inside the unit circle and one outside if
(A) $-1<\lambda<1$ (B) $\lambda>1$
(C) $\lambda<1$ (D) none of these

Key. B

105. For every large value of λ, the roots are approximately
(A) $-2\lambda, \frac{1}{\lambda}$ (B) $-\lambda, -\frac{1}{\lambda}$
(C) $-2\lambda, -\frac{1}{2\lambda}$ (D) none of these

Key. C

Sol. **103 - 105.**

$z^2+2\lambda z+1=0$

$\Rightarrow z=\frac{-2\lambda\pm\sqrt{4\lambda^2-4}}{2}=-\lambda\pm\sqrt{\lambda^2-1}$ ….. (1)

Case I : When $-1<\lambda<1$

If $-1<\lambda<1$ we have

$\lambda^2<1\Rightarrow\lambda^2-1<0\Rightarrow\lambda^2-1=-u^2$ where $u>0$

$\therefore \sqrt{\lambda^2-1}=\pm iu \ \therefore z=-\lambda\pm iu$

$\Rightarrow |z+\lambda|=u$

$\Rightarrow$ z lies on circle having centre $(-\lambda+0i)$ and radius u.

Case II : When $\lambda > 1$

If $\lambda^2-1>0$, let $\lambda^2-1=u^2$, where $u\in R$

$\Rightarrow \sqrt{\lambda^2-1}=\pm u$ Let $z_1=-\lambda+u$

and $z_2=-\lambda-u$. Then $z_1z_2=1\Rightarrow|z_1z_2|=1$

$|z_1||z_2|=1\Rightarrow$ either $(|z_1|<1$ and $|z_2|>1)$

or $(|z_1|>1$ and $|z_2|<1)$

Therefore, one root lies inside |z | < 1 and the other outside | z | < 1.

Case III : when λ is very large

In this case, roots of the equation are $z_1=-\lambda+u$ and $z_2=-\lambda-u$

Also z_1 and z_2 are real numbers.

$\lambda^2-1=u^2 : \sqrt{\lambda^2-1}=\pm u$

Since λ is very large $\Rightarrow$ u is also large and $u \simeq \lambda$.

Roots are $z_1=-\lambda+u=\frac{u^2-\lambda^2}{u+\lambda}=-\frac{1}{2\lambda}$

Passage – 36

If n is a natural number define polynomial $P_n(x)$ of degree n as follows :

$\cos n\theta = P_n(\cos\theta)$

For example, $P_2(x) = 2x^2 - 1$ and $P_3(x) = 4x^3 - 3x$

106. $\frac{1}{2x}\left[P_{n+1}(x) + P_{n-1}(x)\right]$ equals

(A) $P_{n+2}(x)$ (B) $P_{n-1}(x) + P_n(x)$

(C) $P_n(x)$ (D) $P_{n+1}(x) - P_n(x)$

Key. C

107. $\left(x + \sqrt{x^2 - 1}\right)^n + \left(x - \sqrt{x^2 - 1}\right)^n$ equals

(A) $P_n(x)$ (B) $P_{n+1}(x) + P_{n-1}(x)$

(C) $2P_n(x)$ (D) none of these

Key. C

108. $P_6(x)$ equals

(A) $36x^6 - 45x^4 + 18x^2 - 8$ (B) $32x^6 - 48x^4 + 18x^2 - 1$

(C) $36x^6 - 48x^4 + 18x^2 - 5$ (D) none of these

Key. B

Sol. 106. We have $P_1(x) = x$ and cos (n + 1) θ

$+\cos(n-1)\theta = 2\cos(n\theta)\cos\theta$

$\Rightarrow P_{n+1}(x) + P_{n-1}(x) = 2xP_n(x)$

$P_n(x) = \frac{1}{2x}\left[P_{n+1}(x) + P_{n-1}(x)\right]$

107. Put x = cos θ so that $\sqrt{x^2 - 1} = i\sin\theta$

$\left(x + \sqrt{x^2 - 1}\right)^n + \left(x - \sqrt{x^2 - 1}\right)^n$

$\Rightarrow \left[\cos\theta + i\sin\theta\right]^n + \left[\cos\theta - i\sin\theta\right]^n$

$\Rightarrow 2\cos n\theta$

Therefore $\left(x + \sqrt{x^2 - 1}\right)^n + \left(x - \sqrt{x^2 - 1}\right)^n = 2P_n(x)$

108. $2P_6(x) = \left(x + \sqrt{x^2 - 1}\right)^6 + \left(x - \sqrt{x^2 - 1}\right)^6$

$= 2\left[x^6 + 6c_2 x^4(x^2 - 1) + 6c_4 x^2(x^2 - 1)^2 + 6c_6(x^2 - 1)^3\right]$

$P_6(x) = 32x^6 - 48x^4 + 18x^2 - 1$

Passage – 37

Suppose z_1, z_2 and z_3 represent the vertices A, B and C of an equilateral triangle ABC on the Argand plane then $|z_3 - z_1| = |z_2 - z_1| = |z_3 - z_2|$

or $z_1^2 + z_2^2 + z_3^2 - z_1z_2 - z_2z_3 - z_3z_1 = 0$

109. If the complex numbers z_1, z_2, z_3 represent the vertices of an equilateral triangle such that $|z_1| = |z_2| = |z_3|$, then $z_1 + z_2 + z_3$ is

(A) 0 (B) ω

(C) ω^2 (D) 3

Key. A

110. The roots z_1, z_2, z_3 of the equation $x^3 + 3px^2 + 3qx + r = 0$, (p, q, r $\in$ £) form an equilateral triangle in the Argand plane if and only if

(A) $p^2 = q$ (B) $p = q^2$

(C) $p = \pm q$ (D) $|p| = |q|$

Key. A

111. If $|z| = 2$, the area of the triangle whose sides are $|z|$, $|\omega z|$ and $|z + \omega z|$ (whose ω is a complex cube root of unity) is

(A) $2\sqrt{3}$ (B) $\frac{3\sqrt{3}}{2}$

(C) 1 (D) $\sqrt{3}$

Key. D

Sol. **109.** $\frac{z_1 + z_2 + z_3}{3} = 0$

$\Rightarrow$ $\mathbf{Z_1 + Z_2 + Z_3 = 0}$

110. $\sum z_1 = -3p,\ \sum z_1z_2 = 3q,\ z_1z_2z_3 = -r$

$\Rightarrow$ $\mathbf{(Z_1 + Z_2 + Z_3)^2 = 3(Z_1Z_2 + Z_2Z_3 + Z_3Z_1)}$

$\Rightarrow$ $\mathbf{9P^2 = 9Q}$

$\Rightarrow$ $\mathbf{P^2 = Q}$

111. **WE HAVE $|Z| = 2$, $|WZ| = |W|\,|Z|$ AND $|Z + WZ| = |-W^2Z| = 2$**

area of triangle = $\frac{\sqrt{3}}{4}.2^2 = \sqrt{3}$

Passage – 38

If $\rho \neq 1$ is an nth roots of unity, then $1 + 5\rho + 9\rho^2 + \ldots.. + (4n-3)\rho^{n-1} = \frac{a.n}{\rho - 1}$ and Hence the values of the sums. $1 + 5\cos\left(\frac{2\pi}{n}\right) + 9\cos\left(\frac{4\pi}{n}\right) + \ldots.. + (4n-3)\cos\left(\frac{2(n-1)\pi}{n}\right) = b.n$ and

$5\sin\left(\frac{2\pi}{n}\right) + 9\sin\left(\frac{4\pi}{n}\right) + \ldots.. + (4n-3)\sin\left(\frac{2(n-1)\pi}{n}\right) = c\left(n\cot\frac{\pi}{n}\right)$ for $n \geq 2$.

112. Then a + b + c is equal to

(A) 0 (B) 8

(C) 2 (D) 4

Key. A

113. The argument of $z = a - i(b + c)$ is

(A) $\frac{3\pi}{4}$ (B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$ (D) 0

Key. C

114. If the triangle having the vertices $\{(a + b + c) + i(-b + c)\}$, $(a + b + c - ib)$, and $\{-c + i(a + b + c)\}$, then triangle is

(A) equilateral (B) obtuse angle

(C) right angled isosceles (D) acute angled

Key. C

Sol. **112. LET $S = 1 + 5\rho + 9\rho^2 + \dots (4N - 3)\rho^{N-1}$**

THEN $\rho S = \rho + 5\rho^2 + \dots + (4N - 7)\rho^{N-1} + (4N - 3)\rho^N$

SUBTRACTING

$$(1 - \rho)S = 1 + 4\rho + 4\rho^2 + \dots + 4\rho^{N-1} - 4N + 3$$

HENCE $S = \frac{4n}{\rho - 1}$

TAKING $\rho = e^{2\pi i/n}$

WE HAVE

$$1 + 5\rho^{2\pi i/n} + 9e^{4\pi i/n} + \dots + (4n-3)e^{2(n-1)i\pi/n} = \frac{4n}{e^{i2\pi/n} - 1}$$

$$= \frac{4n\left\{\left(\cos\frac{2\pi}{n}\right) - 1 - i\sin\frac{2\pi}{n}\right\}}{\left(\cos\frac{2\pi}{n} - 1\right)^2 + \sin^2\frac{2\pi}{n}} = \frac{n\left(\cos\frac{2\pi}{n} - 1 - i\sin\frac{2\pi}{n}\right)}{\sin^2\frac{\pi}{n}}$$

HENCE

$$1 + 5\cos\left(\frac{2\pi}{n}\right) + 9\cos\left(\frac{4\pi}{n}\right) + \dots + (4n-3)\cos\left(\frac{2(n-1)\pi}{n}\right) = -2n$$

$$5\sin\frac{2\pi}{n} + 9\sin\left(\frac{4\pi}{n}\right) + \dots + (4n-3)\sin\left(\frac{2(n-1)\pi}{n}\right) = -2n\cot\left(\frac{\pi}{n}\right)$$

$\therefore$ **A + B + C**

= 4 – 2 – 2 = 0

113. A = 4, B = –2, C = –2

Z = 4 + 4I

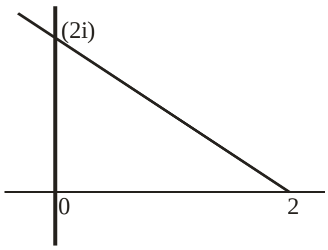
ARG(Z) = $\frac{\pi}{4}$

114. {A + B + C + I(–B + C)} = 0

{(A + B + C) – IB} = 2I

{–C + I(A + B + C)}= 2

the triangle is right angled isosceles.



Passage – 39

The circum circle of an acute angled triangle ABC is $|z|=r$. Complex numbers z_1, z_2, z_3 correspond to points A, B, C respectively. The tangents to the circum circle at the points B, C intersect at P. The altitude through vertex A meets the side BC at F. The internal angle bisector through vertex A meets the circum circle again at D.

115. The complex number representing point P is

A) $\frac{z_2z_3}{z_2+z_3}$ B) $\frac{2z_2z_3}{z_2+z_3}$ C) $\frac{z_2+z_3}{z_2z_3}$ D) $\frac{z_2+z_3}{2z_2z_3}$

Key. B

116. The complex number representing point F is

A) $\frac{(z_1+z_2)(z_1+z_3)}{z_2z_3}$ B) $\frac{(z_1+z_2)(z_1+z_3)}{2z_2z_3}$

C) $\frac{1}{2}\left(z_1+z_2+z_3-\frac{z_2z_3}{z_1}\right)$ D) $\frac{(z_1-z_2)(z_1-z_3)}{2z_1z_3}$

Key. C

117. The complex number representing point D is

A) One of the values of $\sqrt{z_2z_3}$ B) $\frac{z_2z_3}{z_2+z_3}$

C) $\frac{2z_2z_3}{z_2+z_3}$ D) z_2z_3

Key. A

Sol. **(115 – 117)**

The altitudes meet the circumcircle again at the points $\frac{-Z_2Z_3}{Z_1}, \frac{-Z_3Z_1}{Z_2}, \frac{-Z_4Z_2}{Z_3}$

Ortho centre of ΔABC is $Z_1+Z_2+Z_3$

Complex Numbers

Integer Answer Type

1. If $\frac{3iz_2}{5z_1}$ is purely real, then find $5\left|\frac{3z_1+7z_2}{3z_1-7z_2}\right|$.

Key. 5

Sol. Let $\frac{3iz_2}{5z_1}=K$ (real)

$\frac{z_2}{z_1}=\frac{5K}{3i}$

$5\left|\frac{3+7\frac{z_2}{z_1}}{3-7\frac{z_2}{z_1}}\right|=5\left|\frac{3+7\frac{35K}{3i}}{3-\frac{35K}{3i}}\right|$

$5\left|\frac{35K+9i}{35K-9i}\right|=5$

2. Let $1,w,w^2$ be the cube root of unity. The least possible degree of a polynomial with real coefficients having roots $2w,(2+3w),(2+3w^2),(2-w-w^2)$, is

Key. 5

Sol. Roots are $2w,(2+3w)(2+3w^2)(2-w-w^2)(2+3w)$ and $2+3w^2$ are conjugate each other 2w is complex root, then other root must be $2w^2$ (as conjugate root occur in conjugate pair)

$2-w-w^2=2-(-1)=3$ which is real.

Hence least degree of the polynomial : 5.

3. If a complex number z satisfies $|z-8-4i|+|z-14-4i|=10$, then the maximum value of $\arg(z)=\tan^{-1}\frac{11}{3k}$, find k.

Key. 4

Sol. (k = 12) locus of z is an ellipse $\frac{(x-11)^2}{25}+\frac{(y-4)^2}{16}=1$

Equation of tangent is $y-4=m(x-11)+c\Rightarrow c=11m-4$

As $c^2=a^2m^2+b^2$ for standard ellipse

$\Rightarrow(11m-4)^2=25m^2+16\Rightarrow m=0\ or\ m=\frac{11}{12}$

$\therefore\tan\theta=\frac{11}{12}\Rightarrow\theta=\tan^{-1}\frac{11}{12}$

4. If 'a' and 'b' are complex numbers. One of the roots of the equation $x^2+ax+b=0$ is purely real and the other is purely imaginary then $a^2-\bar{a}^{\,2} = kb$, find k

Key. 4

Sol. Let α and $i\beta$, $\alpha,\beta\in R$ are roots of

$x^2+ax+b=0 \Rightarrow \alpha+i\beta=-a, i\alpha\,\beta=b$

$\alpha-i\beta=-\bar{a}$

$\Rightarrow 2\alpha=-\left(a+\bar{a}\right)$ and $2i\beta$ =-$\left(a-\bar{a}\right)$

$\therefore 4i\alpha\,\beta=a^2-\bar{a}^{2} \Rightarrow 4b=a^2-\bar{a}^{2}$

5. There are two complex numbers z such that $|z-2-i|=1$ and $\arg z=\dfrac{\pi}{4}$. The product of modulus of these two complex number is k. find k

Key. 8

Sol. (K=8) Two points B and D

$\therefore B=|z_1| \Rightarrow |z_1|=2\sqrt{2}$

$z_1=2+2i$

and $D(z_2), z_2=1+1i \Rightarrow |z_2|=\sqrt{2}$

$\therefore |z_1z_2|=2\sqrt{2}\times\sqrt{2}=8$

6. The sum of the real parts of the complex numbers satisfying the equations $\left|\dfrac{z-4i}{z-2i}\right|=1$ and $\left|\dfrac{z-8+3i}{z+3i}\right|=\dfrac{3}{5}$ is $\dfrac{k}{5}$, find k.

Key. 5

Sol. $\left|\dfrac{z-4i}{z-2i}\right|=1 \Rightarrow z=x+3i$ using this in $\left|\dfrac{z-8+3i}{z+3i}\right|=\dfrac{3}{5} \Rightarrow 5|x-8+6i|=3|x+6i|$

$\Rightarrow x=8,17$

Two complex numbers 8+3i, 17+3i

Sum of real part = 8+ 17 =25

7. If the equation $z^4+a_1z^3+a_2z^2+a_3z+a_4=0$ where a_1,a_2,a_3,a_4 are real coefficient different from zero has purely imaginary roots then find the value of the expression $\dfrac{a_3}{a_1a_2}+\dfrac{a_1a_4}{a_2a_3}$.

Key. 1

Sol. Let $z=iy$

$\Rightarrow y^4-a_1y^3i-a_3y^2+ia_3y+a_4=0$

$\Rightarrow y^4-a_2y^2+a_4=0 \quad -(1)$ and $-a_1y^3+a_3y=0$

$\Rightarrow y=0$ or $y^2=\dfrac{a_3}{a_1} \quad -(2)$

From (1) and (2)

$\frac{a_3^2}{a_1^2} - a_2 \frac{a_3}{a_1} + a_4 = 0$

$\Rightarrow \frac{a_3}{a_1 a_2} + \frac{a_1 a_4}{a_2 a_3} = 1$

8. $\sum_{j=1}^{n-1} \frac{1}{1 - e^{\frac{2\pi ij}{n}}} = \frac{n-1}{k}$, find k. $\left(i = \sqrt{-1}\right)$

Key. 2

Sol. k = 2,

Let $e^{i\frac{2\pi}{n}} = \alpha$ then $\sum_{j=1}^{n-12} \frac{1}{1-e^j} = \frac{1}{1-\alpha} + \frac{1}{1-\alpha^2} + ... + \frac{1}{1-\alpha^{n-1}}$

Where α is a nth root of unity $\left(\alpha, \alpha^2, \alpha^3, ...\alpha^{n-1}\right)$ are the roots of

$\frac{x^n - 1}{x-1} = (x-\alpha)\left(x-\alpha^2\right).......\left(x-\alpha^{n-1}\right)$

Taking log on both side

$\log \frac{x^n - 1}{x-1} = \log(x-\alpha) + \log\left(x-\alpha^2\right) + ... + \log\left(x-\alpha^{n-1}\right)$

Diff w.r.t. x and use $\lim_{x \to 1}$

$\Rightarrow \frac{n-1}{2} = \frac{1}{1-\alpha} + \frac{1}{1-\alpha^2} + ... + \frac{1}{1-\alpha^{n-1}}$

9. Let A,B,C be equilateral triangle with $\frac{\sqrt{3}}{2} A = e^{i\pi/2}$, $\frac{\sqrt{3}}{2} B = e^{-i\pi/6}$, $\frac{\sqrt{3}}{2} C = e^{-i5\pi/6}$. Let P be any point on the incricle of ΔABC. Find the value of $PA^2 + PB^2 + PC^2$

Key. 5

Sol. Given triangle is an equilateral triangle

$\therefore$ incircle is $x^2 + y^2 = \frac{1}{3}$

Let point on the in circle is (x, y)

$\therefore PA^2 + PB^2 + PC^2 = x^2 + \left(y - \frac{2}{\sqrt{3}}\right)^2 + (x-1)^2 + \left(y + \frac{1}{\sqrt{3}}\right)^2 + (x+1)^2 + \left(y + \frac{1}{\sqrt{3}}\right)^2$

$= 3\left(x^2 + y^2\right) + 4$

$= 1 + 4 = 5$

10. Two lines $zi - \bar{z}i + 2 = 0$ and $z(1+i) + \bar{z}(1-i) + 2 = 0$ intersect at P. The complex numbers of points on the second line which are at a distance of 2 unit form the point P are $z = i \pm e^{i\frac{\pi}{k}}$, find k

Key. 4

Sol. k = 4

The lines are $zi-\bar{z}i+2=0$ and $z(1+i)+\bar{z}(1-i)+2=0$ at i.

Let point on the line be z then $|z-i|=2$

$\Rightarrow 2e^{i\theta}+i$ putting this in second equation $\Rightarrow \theta=\pi/4$

$\therefore$ points are $z=i\pm e^{i\pi/4}$

11. If $z_1, z_2, z_3, ..., z_n$ are in G.P with first term as unity such that $z_1+z_2+z_3+......+z_n=0$. Now if $z_1, z_2, z_3, ..., z_n$ represents the vertices of n-polygon, then the distance between incentre and circumcentre of the polygon is represented by 4k. Find k.

Key. 0

Sol. Let vertices be $1, \alpha, \alpha^2, ..., \alpha^{n-1}$.

Given $1+\alpha+\alpha^2+.........+\alpha^{n-1}=0 \Rightarrow a^n-1=0$

$\Rightarrow$ $z_1, z_2, z_3, ..., z_n$ are roots of $\alpha^n=1$

Which form regular polygon. So distance is zero.

12. Let λ, z_0 be two complex numbers. $A(z_1)$, $B(z_2)$, $C(z_3)$ be the vertices of a triangle such that $z_1=z_0+\lambda$, $z_2=z_0+\lambda e^{i\pi/4}$, $z_3=z_0+\lambda e^{i7\pi/11}$ and $\angle ABC=\frac{3k\pi}{22}$ then the value of k is

Key. 5

Sol. $|z_1-z_0|=|z_2-z_0|=|z_3-z_0|=|\lambda|$

$\frac{z_3-z_0}{z_2-z_0}=\frac{e^{i7\pi/11}}{e^{i\pi/4}}=e^{i17/44}$

$\Rightarrow \angle BSC=17\frac{\pi}{44} \Rightarrow \angle BAC=17\frac{\pi}{88}$

Similarly $\frac{z_2-z_0}{z_1-z_0}=e^{i\pi/4} \Rightarrow \angle ACB=\frac{\pi}{8}$

$\therefore \angle ABC=\pi-\frac{\pi}{8}-\frac{17\pi}{88}=\frac{15\pi}{22}$.

13. The roots of the equation $z^5+z^6+..........z^{10}=0$ where $z\neq 0,1$ are represented by vertices of a pentagon having longest side length is equal d. Find d^2.

Key. 3

Sol. $d^2=3$. Equation reduces to $z^6=1$

$$\Rightarrow z=\cos 2k\frac{\pi}{6}+i\sin 2k\frac{\pi}{6},\ k=1,2,3,4,5$$

The longest side = $\left|\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}-\left(\cos 5\frac{\pi}{3} i\sin 5\frac{\pi}{3}\right)\right|=\sqrt{3}$.

14. The complex number z satisfying $\left|z+2+i\right|+\left|z-2+i\right|=4$, $0\le \arg\left(z+2+2i\right)\le \frac{\pi}{4}$ and $3\frac{\pi}{4}\le \arg\left(z-2+2i\right)\le \pi$ will lie on a line segment of the length k. Find k.

Key. 2

Sol. $(k=2)$ length $AB=2$

15. If the argument of $(z-a)(\overline{z}-b)$ is equal to that of $\frac{(\sqrt{3}+i)(1+\sqrt{3}i)}{1+i}$, where a,b are real numbers. If locus of z is a circle with centre $\frac{3+i}{2}$ then find $(a+b)$.

Key. 3

Sol. $\tan^{-1}\frac{(a-b)y}{x^2+y^2-(a+b)x+ab}=\frac{\pi}{4}$

$\Rightarrow x^2+y^2-(a+b)x-\left(a-b\right)y+ab=0$

Centre = $\frac{3+i}{2}\Rightarrow a+b=3$

16. If $Z=\frac{1}{2}\left(\sqrt{3}-i\right)$ then the least positive integral value of $'n'$ such that $\left(Z^{101}+i^{109}\right)^{106}=Z^n$ is $'k'$ then $\frac{2}{5}k=$

Key. 4

Sol. $Z=\frac{-1}{2}i\left(1+i\sqrt{3}\right)=i\,\omega^2$

$Z^{101}=i\,\omega$

$\left(Z^{101}+i^{109}\right)^{106}=-\left(i\,\omega^2\right)^{106}=-\omega^2$

$\therefore\ -\omega^2=\left(i\,\omega^2\right)^n=i^n\,\omega^{2n}$

$\omega^{2n-2}\,i^n=-1$

This is possible only when $N=4r+2$ and $2n-2$ is multiple of 3 i.e.,

$2\left(4r+2\right)-2$ is multiply of 3

i.e., $8r+2$ is multiple of $3\Rightarrow r=2$

$\therefore\ n=10\ \therefore\ \frac{2}{5}k=4$

17. If $\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5$ are the roots of the equation $x^5-1=0$, where $\alpha_k=\alpha^{k-1}$, $\alpha=e^{i2\pi/5}$ and $\lambda=\alpha_3^{1001},\mu=\alpha_4^{(669+1/3)}$, $\nu=\alpha_5^{(503+1/2)}$, then $[|\lambda^{2011}+\mu^{2011}+\nu^{2011}|]$ (where [.] denotes the greatest integer function) is

Key. 1

Sol. Clearly $\alpha_1 = 1$

$\alpha_2 = \alpha$

$\alpha_3 = \alpha^2$

$\alpha_4 = \alpha^3$

$\alpha_5 = \alpha^4$

where $\alpha = e^{i2\pi/5}$

$\therefore \lambda = \alpha_3^{1001} = (\alpha^2)^{1001} = \alpha^{2002} = \alpha^{5\times400+2} = \alpha^2$

$\mu = (\alpha_4)^{669+1/3} = (\alpha^3)^{(669+1/3)} = \alpha^{2008} = \alpha^3$

$\nu = (\alpha_5)^{503+1/2} = (\alpha^4)^{503+1/2}$

$= \alpha^{2014}. = \alpha^{5.402+4}$

$= \alpha^4$

Also sum of 2011th power of roots of unity is 0

So, $1 + \alpha^{2011} + \lambda^{2011} + \mu^{2011} + v^{2011} = 0$

$\lambda^{2011} + \mu^{2011} + v^{2011} = -(1 + \alpha^{2011})$

$\lambda^{2011} + \mu^{2011} + v^{2011} = -(1 + \alpha)$

$|\lambda^{2011} + \mu^{2011} + v^{2011}| = |-(1 + e^{i2\pi/5})|$

$= |1 + \cos 2\pi/5 + i \sin 2\pi/5| = |2\cos\pi/5(\cos\pi/5 + i \sin \pi/5)$

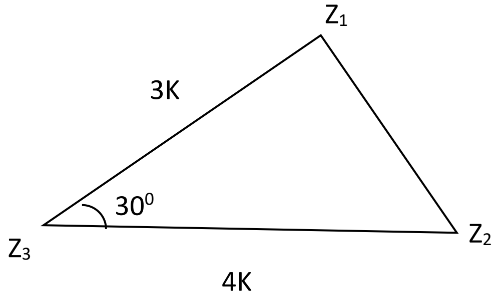
$= 2\frac{\sqrt{5}+1}{4} = \frac{\sqrt{5}+1}{2}$

$[|\lambda^{2011} + \mu^{2011} + v^{2011}|] = 1$

18. If $|Z_1 - Z_2| = \sqrt{25 - 12\sqrt{3}}$, and $\frac{Z_1 - Z_3}{Z_2 - Z_3} = \frac{3}{4} e^{i\frac{\pi}{6}}$, then area of triangle (in square units) whose vertices are represented by Z_1, Z_2, Z_3 is ………………

Key: 3

Hint:



$$\frac{|Z_1 - Z_3|}{|Z_2 - Z_3|} = \frac{3}{4}$$

$$let\,|Z_1 - Z_3| = 3k, |Z_2 - Z_3| = 4k$$

$$angle\,at\,Z_3 = \frac{\pi}{6}$$

$$cos\,30^0 = \frac{16k^2 + 9k^2 - 25 + 12\sqrt{3}}{2 \times 4k \times 3k} \Rightarrow k = 1$$

$$area = \frac{1}{2}.3.4\sin 30^0 = 3$$

19. Two lines $zi - \bar{z}i + 2 = 0$ and $z(1+i) + \bar{z}(1-i) + 2 = 0$ intersect at a point P. There is a complex number $\alpha = x + iy$ at a distance of 2 units from the point P which lies on line $z(1+i) + \bar{z}(1-i) + 2 = 0$. Find $[|x|]$ (where [.] represents greatest integer function).

Key: 1

Hint: Solving the equation of the lines we get $z = -\bar{z} \Rightarrow z = i$

$|\alpha - 1| = 2; \alpha = 2e^{i\theta} + i$, put it in the equation of the second line, we get

$$\cos\theta - \sin\theta = 0$$

$$\alpha = i \pm 2e^{\frac{i\pi}{4}}$$

$$\therefore x = \pm\sqrt{2}$$

$$\Rightarrow [|x|] = 1$$

20. If $\alpha = e^{i2\pi/7}$ and $f(x) = A_0 + \sum_{k=1}^{20} A_k x^k$ and the value of $f(x) + f(\alpha x) + f(\alpha^2 x) + ... + f(\alpha^6 x)$ is $k(A_0 + A_7 x^7 + A_{14} x^{14})$ then find the value of k.

Key: 7

Hint: $f(x) + f(\alpha x) + f(\alpha^2 x) + ... + f(\alpha^6 x) = 7A_0 + \sum_{k=1}^{20} A_k x^k (1 + \alpha^k + ... + \alpha^{6k})$

but when $k \neq 7$ and $k \neq 14$, then $1 + \alpha^k + \alpha^{2k} + ... + \alpha^{6k} = 0$

Hence

$$f(x) + f(\alpha x) + ... + f(\alpha^6 x) = 7A_0 + 7A_7 x^7 + 7A_{14} x^{14} = 7(A_0 + A_7 x^7 + A_{14} x^{14})$$

k = 7

21. If $Z_n = \left(\cos\left(\frac{\pi}{n(n+1)(n+2)}\right) + i\sin\left(\frac{\pi}{n(n+1)(n+2)}\right)\right)$ for n = 1,2,3…. and the principle argument value of $z = \lim_{n\to\infty}(z_1 z_2 ...... z_n)$ is $\frac{k\pi}{24}$, then find the value of k

Key: 6

Hint: $z_n = \frac{i\pi}{e^{n(n+1)(n+2)}}$

$\Rightarrow z = \lim_{n\to\infty} e^{\left(i\pi \sum_{n=1}^{n} \frac{1}{n(n+1)(n+2)}\right)}$

$\Rightarrow z = e^{\frac{i\pi}{4}} \Rightarrow \arg(z) = \frac{\pi}{4}$

22. Suppose that w is the imaginary $(2009)^{th}$ roots of unity. If $\left(2^{2009}-1\right)\sum_{r=1}^{2008}\frac{1}{2-w^r} = (a)\left(2^b\right)+c$ where a, b, c, $\in$ N, and the least value of (a + b + c) is (2008)K. The numerical value of K is

Key: 2

Hint: Let x be the $(2009)^{th}$ root of unity $\neq 1$, then

$x^{2009}-1 = (x-1)(x-w).....\left(x-w^{2008}\right)$

Taking log on both sides, we get

$\ln\left(x^{2009}-1\right) = \ln(x-1)+\ln(x-w)+\ln\left(x-w^2\right).....+\ln\left(x-w^{2008}\right)$

$\therefore$ On differentiate both the side w.r.t. x, we get

$\frac{(2009)x^{2008}}{x^{2009}-1} = \frac{1}{x-1}+\sum_{r=1}^{2008}\frac{1}{x-w^r}$ ....(2)

Putting x = 2 in equation (2) , we get

$\Rightarrow 1+\sum_{r=1}^{2008}\frac{1}{2-w^r} = \frac{2009\left(2^{2008}\right)}{2^{2009}-1}$

Multiplying both sides of above equation by $\left(2^{2009}-1\right)$, we get

$\therefore \left(2^{2009}-1\right)\sum_{r=1}^{2008}\frac{1}{2-w^r} = 2009.2^{2008}-2^{2009}+1$

$= 2^{2008}(2009-2)+1 = 2^{2008}.2007+1 = \left[(a)\left(2^b\right)+c\right]$

$\therefore a = 2007, b = 2008, c = 1$

Hence a + b + c = 4016

23. If $\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5$ are the roots of the equation $x^5-1=0$, where $\alpha_k = \alpha^{k-1}$, $\alpha = e^{i2\pi/5}$ and $\lambda = \alpha_3^{1001}, \mu = \alpha_4^{(669+1/3)}$, $\nu = \alpha_5^{(503+1/2)}$, then $[|\lambda^{2011}+\mu^{2011}+v^{2011}|]$ (where [.] denotes the greatest integer function) is

Key. 1

Sol. Clearly $\alpha_1 = 1$

$\alpha_2 = \alpha$

$\alpha_3 = \alpha^2$

$\alpha_4 = \alpha^3$

$\alpha_5 = \alpha^4$

where $\alpha = e^{i2\pi/5}$

$\therefore \lambda = \alpha_3^{1001} = (\alpha^2)^{1001} = \alpha^{2002} = \alpha^{5\times 400+2} = \alpha^2$

$\mu = (\alpha_4)^{669+1/3} = (\alpha^3)^{(669+1/3)} = \alpha^{2008} = \alpha^3$

$\nu = (\alpha_5)^{503+1/2} = (\alpha^4)^{503+1/2}$

$= \alpha^{2014}. = \alpha^{5.402+4}$

$= \alpha^4$

Also sum of 2011[th] power of roots of unity is 0

So, $1 + \alpha^{2011} + \lambda^{2011} + \mu^{2011} + v^{2011} = 0$

$\lambda^{2011} + \mu^{2011} + v^{2011} = -(1 + \alpha^{2011})$

$\lambda^{2011} + \mu^{2011} + v^{2011} = -(1 + \alpha)$

$|\lambda^{2011} + \mu^{2011} + v^{2011}| = |-(1 + e^{i2\pi/5})|$

$= |1 + \cos 2\pi/5 + i \sin 2\pi/5| = |2\cos\pi/5(\cos\pi/5 + i \sin \pi/5)$

$= 2\dfrac{\sqrt{5}+1}{4} = \dfrac{\sqrt{5}+1}{2}$

$[|\lambda^{2011} + \mu^{2011} + v^{2011}|] = 1$

24. Find the least +ve integral value of 'a' such that there is at least one complex number satisfying $\left|z+\sqrt{2}\right| < a^2 - 3a + 2$ and $\left|z + i\sqrt{2}\right| < a^2$

Key. 3

Sol. (a=3) Atleast one complex number z satisfy the required condition if the two circle intersect at two distinct points.

25. A triangle with vertices represented by z_1, z_2, z_3 has opposite sides of lengths in the ratio $2:\sqrt{19}:3$ respectively. Then the value of $4(z_1 - z_2)^2 + 6(z_1 - z_2)(z_3 - z_2) + 9(z_3 - z_2)^2$ is k. find k.

Key. 0

Sol. $(k=0)$ $\cos B = -\dfrac{1}{2}$ $B = \dfrac{2\pi}{3}$

By rotation $\dfrac{z_1 - z_2}{z_2 - z_2} = \left|\dfrac{z_1 - z_2}{z_3 - z_2}\right| e^{i\frac{2\pi}{3}}$

$$\Rightarrow 2(z_1 - z_2) + \frac{3}{2}(z_3 - z_2) = \left(z_3 - z_2\left(i\frac{3\sqrt{3}}{2}\right)\right)$$

Squaring to get the required result.

26. Let $1, w, w^2$ be the cube root of unity. The least possible degree of a polynomial with real coefficients having roots $2w, (2+3w), (2+3w^2), (2-w-w^2)$, is

Key. 5

Sol. Roots are $2w, (2+3w)(2+3w^2)(2-w-w^2)(2+3w)$ and $2+3w^2$ are conjugate each other 2w is complex root, then other root must be $2w^2$ (as conjugate root occur in conjugate pair)

$2-w-w^2 = 2-(-1) = 3$ which is real.

Hence least degree of the polynomial : 5.

27. If $2^7 \cos^3\theta \cdot \sin^5\theta = a\sin 8\theta 0 b \sin 6\theta + c\sin 4\theta + d\sin 2\theta$ and θ is real then the value of a + b + c + d must be equal to

Ans. 7

Sol. Let $z = e^{i\theta} \cdot 2\cos\theta = \left(z + \frac{1}{z}\right)$ and $2i\sin\theta = \left(z - \frac{1}{z}\right)$

Now : $(2\cos\theta)^3 (2i\sin\theta)^5 = \left(z+\frac{1}{z}\right)^3 \left(z - \frac{1}{z}\right)^5$

$= \left(z^8 - \frac{1}{z^8}\right) - 2\left(z^6 - \frac{1}{z^6}\right) - 2\left(z^4 - \frac{1}{z^4}\right) + 6\left(z^2 - \frac{1}{z^2}\right)$

Compare a = 1, b = 2, c = - 2, d = 6

a + b + c + d = 1 + 2 – 2 + 6 = 7

28. Let A_1, A_2 …. A_n be vertices of an n sided regular polygon such that $\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$.

Then find values of n

Ans. 7

Sol. Let $A_1(Z_1)$, $A_2(Z_2)$, $A_3(Z_3)$, $A_4(Z_4)$

$A_1A_2 = |Z_1| 2\sin\frac{\pi}{n}, A_1A_3 = |Z_1| 2\sin\frac{2\pi}{n}, A_1A_4 = |Z_1| 2\sin\frac{3\pi}{n}$

$\Rightarrow \frac{1}{\sin\frac{\pi}{n}} = \frac{1}{\sin\frac{2\pi}{n}} + \frac{1}{\sin\frac{3\pi}{n}} \Rightarrow n = 7$

29. Let $A_1(Z_1)$, $A_2(Z_2)$ be the adjacent vertices of a regular polygon. If $\frac{lm(\overline{Z_1})}{\mathrm{Re}(Z_1)} = 1-\sqrt{2}$, then the number of sides of the polygon are ________

Ans. 8

Sol. Let $z_1 = re^{i\theta}, \overline{Z_1}^{-i\theta} = re^{-i\theta}, \mathrm{Re}(z_1) = r\cos\theta, lm(\overline{Z}_1) = -r\sin\theta$

$\Rightarrow -\frac{\sin\theta}{\cos\theta} = 1-\sqrt{2} \Rightarrow \tan\theta = \sqrt{2}-1 \Rightarrow \theta = \frac{\pi}{8}$

If the number of sides be n, then $\theta = \frac{\pi}{n} \Rightarrow n = 8$

Complex Numbers

Matrix-Match Type

1. If $z = \cos\alpha + i\sin\alpha,\ 0 < \alpha < \frac{\pi}{6},$ then argument of

Column I		Column II	
(A)	$1+z^3$	(p)	$2\alpha - \frac{\pi}{2}$
(B)	$1-z^4$	(q)	$\frac{\pi}{2} - \frac{\alpha}{2}$
(C)	$\frac{z^3+1}{1-z^4}$	(r)	$\frac{3\alpha}{2}$
(D)	$\frac{z^4-1}{z^3+1}$	(s)	$\frac{\pi}{2} + \frac{\alpha}{2}$

Key. (A) $\to$(r); (B) $\to$(p); (C) $\to$(q); (D) $\to$(s)

Sol. A) $1+z^3 = 1+\cos 3\alpha + i\sin 3\alpha$

$$= 2\cos\frac{3\alpha}{2}\left[\cos\frac{3\alpha}{2} + i\sin\frac{3\alpha}{2}\right]$$

$$\therefore\ \arg\left(1+z^3\right) = \frac{3\alpha}{2}$$

B) $1-z^4 = 1-\cos 4\alpha - i\sin 4\alpha$

$$= 2\sin^2 2\alpha - 2i\sin 2\alpha\cos 2\alpha$$

$$= 2\sin 2\alpha\left[\cos\left(2\alpha - \frac{\pi}{2}\right) + i\sin\left(2\alpha - \frac{\pi}{2}\right)\right]$$

$$\therefore\ \arg\left(1-z^4\right) = 2\alpha - \frac{\pi}{2}$$

C) $\arg\left(\frac{1+z^3}{1-z^4}\right) = \arg\left(1+z^3\right) - \arg\left(1-z^4\right)$

$$= \frac{3\alpha}{2} - 2\alpha + \frac{\pi}{2} = \frac{\pi}{2} - \frac{\alpha}{2}$$

D) $\left(\frac{z^4-1}{z^3+1}\right) = \frac{\cos 4\alpha + i\sin 4\alpha - 1}{1+\cos 3\alpha + i\sin 3\alpha}$

$$= \frac{2i\sin 2\alpha\left[\cos 2\alpha + i\sin 2\alpha\right]}{2\cos\frac{3\alpha}{2}\left[\cos\frac{3\alpha}{2} + i\sin\frac{3\alpha}{2}\right]} \quad = \frac{i\sin 2\alpha}{\cos\frac{3\alpha}{2}}\left[\cos\left(\frac{\alpha}{2}\right) + i\sin\left(\frac{\alpha}{2}\right)\right]$$

$$\left(\because i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

$\therefore \arg\left(\frac{z^4-1}{z^3+1}\right)=\frac{\pi}{2}+\frac{\alpha}{2}$

2. Consider complex number $z=\cos\alpha+i\sin\alpha, 0<\alpha<\frac{\pi}{6}$. Then the argument of

Column I	Column II
a) $1+z^3$	p) $2\alpha-\frac{\pi}{2}$
b) $1-z^4$	q) $\frac{\pi}{2}+\frac{\alpha}{2}$
c) $\frac{1+z^3}{1-z^4}$	r) $\frac{3\alpha}{2}$
d) $\frac{z^4-1}{z^3+1}$	s) $\frac{\pi}{2}-\frac{\alpha}{2}$

Key. $a-r; b-p;$
$c-s; d-q$

Sol. a) $1+z^3=1+\cos 3\alpha+i\sin 3\alpha$

$=2\cos\left(\frac{3\alpha}{2}\right)\left[\cos\frac{3\alpha}{2}+i\sin\frac{3\alpha}{2}\right]$

$\Rightarrow \arg\left(1+z^3\right)=\frac{3\alpha}{2}$

b) $1-z^4=1-\cos 4\alpha-i\sin 4\alpha$

$=2\sin 2\alpha\left[\cos\left(2\alpha-\frac{\pi}{2}\right)+i\sin\left(2\alpha-\frac{\pi}{2}\right)\right]$

$\arg\left(1-z^4\right)=2\alpha-\frac{\pi}{2}$

c) $\frac{1+z^3}{1-z^4}=\frac{\cos\frac{3\alpha}{2}}{\sin 2\alpha}\left[\cos\left(\frac{\pi}{2}-\frac{\alpha}{2}\right)+i\sin\left(\frac{\pi}{2}-\frac{\alpha}{2}\right)\right]$

$\Rightarrow \arg\left(\frac{1+z^3}{1-z^4}\right)=\frac{\pi}{2}-\frac{\alpha}{2}$

d) $\frac{z^4-1}{z^3+1}=\frac{\sin 2\alpha}{\cos\left(\frac{3\alpha}{2}\right)}\left[\cos\left(\frac{\alpha}{2}+\frac{\pi}{2}\right)+i\sin\left(\frac{\alpha}{2}+\frac{\pi}{2}\right)\right]$

$\arg\left(\frac{z^4-1}{z^3+1}\right)=\frac{\pi}{2}+\frac{\alpha}{2}$

3. The complex numbers $z_1, z_2........z_n$ represent the vertices of a regular polygon of n sides, inscribed in a circle of unit radius and $z_3 + z_n = Az_1 + \bar{A}z_2$, $[x]$ be the greatest integer $\le x$. Then

	When n equals to		$[\lvert A\rvert]$ equals to
(A)	4	(p)	0
(B)	6	(q)	1
(C)	8	(r)	2
(D)	12	(s)	3

Key: $(a \to r; b \to r;$
$c \to q; d \to q)$

Sol. Let α be an interior angle of the polygon

$$z_n - z_1 = (z_2 - z_1)e^{i\alpha} \Rightarrow z_n = (1 - e^{i\alpha})z_1 + e^{i\alpha}z_2$$

$$z_3 - z_2 = (z_1 - z_2)e^{-i\alpha} \Rightarrow z_3 = e^{-i\alpha}z_1 + (1 - e^{-i\alpha})z_2$$

$$z_3 + z_n = (1 - 2i\sin\alpha)z_1 + (1 + 2i\sin\alpha)z_2$$

A $= 1 - 2i\sin\alpha$

$$|A| = \sqrt{3 - 2\cos 2\alpha} = \sqrt{3 - 2\cos\left(\frac{4\pi}{n}\right)}$$

When $n = 4 \Rightarrow |A| = \sqrt{5}$

$n = 6 \Rightarrow |A| = 2$

$n = 8 \Rightarrow |A| = \sqrt{3}$

$n = 12 \Rightarrow |A| = \sqrt{2}$

4.

	Column I		Column II
(A)	The maximum value of $\lvert\lvert z-\omega\rvert - \lvert z-\bar{\omega}\rvert\rvert$ (where $\lvert z\rvert = 5$ & ω, $\bar{\omega}$ complex cube root of unity) is	(p)	0
(B)	The triangle ABC where $A(z_1)$, $B(z_2)$, $C(z_3)$ has its circum centre at origin. If perpendicular from A to BC intersect the circumference at z_4 then $z_1z_4 + z_2z_3$ is	(q)	6
(C)	Tangent drawn to circle $(x - 1)^2 + (y - 1)^2 = 5$ at a point P meets the line $2x + y + 6 = 0$ at Q on the x-axis then the value of $\frac{(PQ)^2}{2}$ is	(r)	12

(D) Total number of common terms of 3, 7, 11, 15, 19 ... upto 60 terms and -3, -1, 1, 3 .... upto 26 terms is (s) $\sqrt{3}$

Key: (A-s), (B-p), (C-q), (D-r)

Hint (A) $||z-\omega| - |z-\bar{\omega}||_{max} = |\omega - \bar{\omega}| = \sqrt{3}$

(B) Applying rotation we get $z_4 = \frac{-z_2 z_3}{z_1}$ $\Rightarrow z_1 z_4 + z_2 z_3 = 0$

(C) $Q \equiv (-3, 0)$

$PQ = \sqrt{S_1} = \sqrt{12}$

(D) $T_{26} = -3 + (26 - 1) 2 = 47$

$T'_{60} = 3 + (60 - 1) 4 = 239$

Hence new series is 3, 7, 11 .....

$T_m = 3 + (m - 1). 4 \leq 47$

$\Rightarrow 4m - 1 \leq 47$

$4m \leq 48$

$m \leq 12.$

Hence number of common terms is 12.

5. Match the following

	Column I		Column II
(A)	The number of integral solutions of the equation $(1-i)^n = 2^n$	(p)	4
(B)	The number of common roots of the equations $x^3+2x^2+2x+1=0$ and $x^{2000}+x^{2002}+1=0$	(q)	3
(C)	The number of all non –zero complex numbers ‘z’ satisfying $\bar{z} = iz^2$	(r)	2
(D)	If Z is a complex number , then the number of solutions of $Z^2+\|Z\|=0$ are	(s)	1

Key: (A -s), (B -r), (C -q), (D –q)

Hint: (a) $|1-i|^n = 2^n \Rightarrow n/2 = 0 \Rightarrow n = 0$

(b) $x^3+2x^2+2x+1=0 \Rightarrow x=-1, \omega, \omega^2$

But $x=\omega,\omega^2$ will only satisfy $x^3+2x^2+2x+1=0 \Rightarrow x^{2000}+x^{2002}+1=0$

(c) $x+2xy = o \;\&\; x^2-y^2+y=0$

$\Rightarrow i, \frac{\sqrt{3}}{2}-\frac{i}{2}, -\frac{\sqrt{3}}{2}-\frac{i}{2}$

(d) $x^2-y^2+\sqrt{x^2+y^2}=0 \;\&\; 2xy=0 \Rightarrow z=0, i, -i$

6. Match the following

	Column–1		Column–2
(A)	If ω is a cube roots of unity, then $\left|\omega+\omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+........+\infty\right)}\right|$ is equal to	(P)	0
(B)	$\int_{-1}^{1}\left\{\sin^{-1}\left[x+\frac{3}{4}\right]\right\}dx=K\left(\frac{\pi}{4}\right)$ then K is equal to (where [.] denotes the greatest integer function)	(Q)	1
(C)	$\lim_{n\to\infty}\left[\frac{1}{n}+\frac{1}{n+2}+\frac{1}{n+4}+...+\frac{1}{3n}\right]=\frac{1}{2}\ln k,$ then the value of k is	(R)	3
(D)	z_1 and z_2 are two complex number satisfying $\lvert z_1+1\rvert+\lvert z_1-1\rvert=4$ and $\lvert z_2-2\rvert=1,$ then the maximum value of $\lvert z_1-z_2\rvert$ is	(S)	5

Key: (A) $\to$ Q, (B) $\to$Q, (C) $\to$ R, (D) $\to$ (S)

Hint: (A) $\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}+.....+\infty$ = $\frac{1}{2}\left[\left(\frac{3}{4}\right)^0+\left(\frac{3}{4}\right)^1+\left(\frac{3}{4}\right)^2+.....+\infty\right]=\frac{1}{2}\left[\frac{1}{1-\frac{3}{4}}\right]=2$

So therefore, $\left|\omega+\omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+........+\infty\right)}\right|=|\omega+\omega^2|=|-1|=1$

(B) $\int_{-1}^{1}\left\{\sin^{-1}\left[x+\frac{3}{4}\right]\right\}dx=K\left(\frac{\pi}{4}\right)$

$\Rightarrow \int_{-1}^{-3/4}-\frac{\pi}{2}dx+\int_{-3/4}^{1/4}0dx+\int_{1/4}^{1}\frac{\pi}{2}dx$

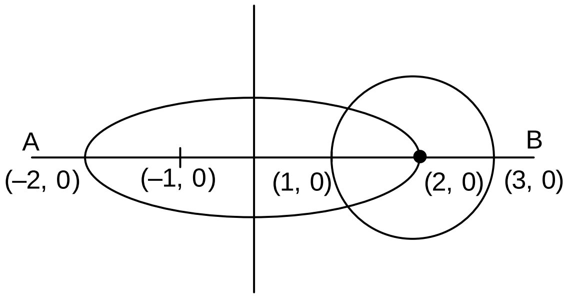
$\Rightarrow -\frac{\pi}{8}+\frac{\pi}{2}\times\frac{3}{4}$

$\Rightarrow \frac{\pi}{4}=K\left(\frac{\pi}{4}\right)\Rightarrow K=1$

(C) $\sum_{r=0}^{n}\frac{1}{n+2r}=\sum_{r=0}^{n}\frac{1}{n}.\frac{1}{1+2\left(\frac{r}{n}\right)}$

$\int_0^1\frac{dx}{1+2x}=\frac{[\ln(1+2x)]!}{2}=\frac{1}{2}[\ln 3-0]=\frac{\ln 3}{2}$

(D) Locus of z_1 is an ellipse having foci at (– 1 + 0.i) and (1 + 0.i)



Length of the major axis = 4

Locus of z_2 is a circle having centre at (2, 0) and radius 1.

Max. value of $|z_1 - z_2| = A - B = 5.$

7. Z_1, Z_2, Z_3 are vertices of a triangle. Match the condition in List-I with type of triangle in List-II

List-I		List-II	
A)	$Z_1^2 + Z_2^2 + Z_3^2 = Z_2Z_3 + Z_3Z_1 + Z_1Z_2$	p)	right angled
B)	$\text{Re}\left(\frac{Z_3 - Z_1}{Z_3 - Z_2}\right) = 0$	q)	obtuse angled
C)	$\text{Re}\left(\frac{Z_3 - Z_1}{Z_3 - Z_2}\right) < 0$	r)	isosceles and right angled
D)	$\frac{Z_3 - Z_1}{Z_3 - Z_2} = i$	s)	equilateral

KEY : A) s B) p C) q D) r

Sol. A) $\frac{z_3 - z_1}{z_2 - z_1} = \frac{z_1 - z_2}{z_3 - z_2} = e^{i\frac{\pi}{3}}$

B) $\text{Re}\left(\frac{z_3 - z_1}{z_2 - z_2}\right) = 0 \Rightarrow Arg\left(\frac{z_3 - z_1}{z_3 - z_2}\right) = \pm\frac{\pi}{2}$

C) $\text{Re}\left(\frac{z_3 - z_1}{z_2 - z_2}\right) < 0 \Rightarrow Arg\left(\frac{z_3 - z_1}{z_3 - z_2}\right) > \frac{\pi}{2}$

D) $\left(\frac{z_3 - z_1}{z_3 - z_2}\right) = i = e^{i\frac{\pi}{2}}$ & $|z_3 - z_1| = |z_3 - z_2|$

8.

	Column –I		Column –II
(A)	If $z = \frac{z_1 + i\,\overline{z}_2}{z_2 + i\,\overline{z}_1}$ then $\|z\|$ equals	(p)	0
(B)	If $z = \frac{z - 2i}{z + 2i}$ be purely imaginary then $\|z\|$ equals	(q)	1

(C)	If $\lvert z+6\rvert = \lvert 2z+3\rvert$ then $\lvert z\rvert$ equals	(r)	2
(D)	Let $\omega \neq 1$ be a cube root of unity and $z = \frac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} + \frac{a+b\omega+c\omega^2}{b+c\omega+a\omega^2}$ then $\lvert z\rvert$ equals	(s)	3
		(t)	4

Key. (A–q), (B–r), (C–s), (D–q)

Sol. (A) $|Z| = \frac{|z_1+i\bar{z}_2|}{|z_2+i\bar{z}_2|} = \frac{|i||-iz_1+\bar{z}_2|}{|\overline{z_2+iz_1}|} = \frac{1\times|\bar{z}_2-iz_1|}{|\bar{z}_2-iz_1|} = 1$

(B) Here arg $\left(\frac{z-2i}{z+2i}\right) = \frac{\pi}{2}$ or $-\frac{\pi}{2}$, So it represents a semicircle with diametric ends at points (0, 2) & (0, –2)

So, $|z|$ = radius = 2

(C) $(x + 6)^2 + y^2 = (2x + 3)^2 + (2y)^2 \Rightarrow x^2 + y^2 + 12x + 36 = 4x^2 + 4y^2 + 12x + 9$

$\Rightarrow 3x^2 + 3y^2 = 27 \Rightarrow x^2 + y^2 = 9$, $|z| = 3$

(D) $z = \frac{1}{w}\frac{aw+bw^2+cw^3}{c+aw+bw^2} + \frac{1}{w^2}\frac{aw^2+bw^3+cw^4}{b+cw+aw^2} = \frac{1}{w}\times 1 + \frac{1}{w^2}\times 1 = w^2 + w = -1$

$\Rightarrow |z| = 1$.

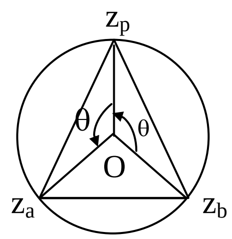
9. Let A and B be two points on complex plane representing the complex nos. Z_a and Z_b, are lying on a circle whose centre is at origin. Let $C(Z_c)$ be the mid–point of AB.

Column – I		Column – II	
(A)	If $P(Z_p)$ be the point on the circle such that PA = PB and $\angle APB < \pi/2$ then Z_p is	(p)	$Z_a Z_b$
(B)	If $P(Z_p)$ be the point on the circle such that PA = PB and $\angle APB > \pi/2$ then Z_p is	(q)	$-Z_a Z_b$
(C)	If $P(Z_p)$ and $Q(Z_q)$ be two points (on the either sides of AB) lying on the circle such that PQ is perpendicular to AB then $Z_p Z_q$ is	(r)	$\sqrt{Z_a Z_b}$
(D)	If $P(Z_p)$ be a point lying outside the circle such that AB is chord of contact of the point P w.r.t. the circle then $Z_p Z_c$ is	(s)	$-\sqrt{Z_a Z_b}$

Key. (A – s); (B – r); (C – q); (D – p)

Sol.

(A) $\frac{z_a}{z_p} = e^{i\theta} = \frac{z_p}{z_b} \Rightarrow z_p = \pm\sqrt{z_a z_b}$



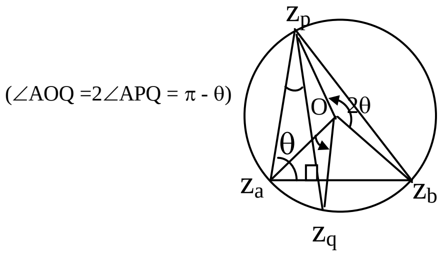
But when $\angle APB < \pi/2$, z_p and $\frac{z_a+z_b}{2}$ should lie on the opposite side of origin, so

$Z_p = -\sqrt{z_a z_b}$

(B) When $\angle APB > \pi/2$, Zp and $\frac{z_a+z_b}{2}$ lie on the same side of origin, so

$z_p = \sqrt{z_a z_b}$.

(C) $\frac{z_q}{z_a} = e^{i(\pi-\theta)}, \frac{z_p}{z_b} = e^{i2\theta} \Rightarrow \frac{z_p z_q}{z_a z_b} = e^{i\pi} = -1$

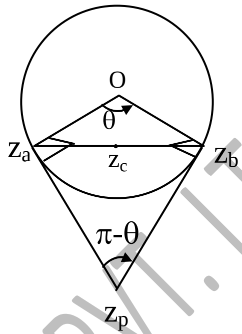


(D) $\frac{z_b}{z_a} = e^{i\theta}$, $\frac{z_a - z_p}{z_b - z_p} = e^{i(\pi-\theta)}$

$\Rightarrow \frac{z_b(z_a - z_p)}{z_a(z_b - z_p)} = -1$

$\Rightarrow$ zp = $\frac{2z_a z_b}{z_a + z_b}$

$\Rightarrow$ zp zc = $\frac{2z_a z_b}{z_a + z_b} \times \frac{z_a + z_b}{2}$ = zazb



10. Match the following: -

List – I	List - II
(A) If z_1, z_2 are conjugate complex numbers and z_3, z_4 are also conjugate then $Arg\left(\frac{z_3}{z_2}\right) = Arg$ of	p) $\frac{2}{3}$
(B) If $\frac{5z_2}{7z_1}$ is purely imaginary, then $\left\|\frac{2z_1+3z_2}{2z_1-3z_2}\right\|$ =	q) 1
(C) If $Arg\left(z^{1/3}\right) = \frac{1}{2}Arg\left(z^2 + \bar{z}z^{1/3}\right)$ then $\|z\|$	r) $\frac{z_1}{z_4}$
(D) One vertex of an equilateral triangle is at the origin and the other two vertices are given by $2z^2 + 2z + k = 0$, then k is	s) 2

Key. **A $\rightarrow$ r, B $\rightarrow$ q, C $\rightarrow$ q , D $\rightarrow$ p**

Sol. (D) $2z^2 + 2z + k = 0$

$z = \frac{-2 \pm \sqrt{4-8k}}{4}$

Since 'z' is a complex number

4 – 8k will be negative

$\Rightarrow$ k > ½

(0, 0), $\left(\frac{-1}{2}, \frac{\sqrt{2k-1}}{2}\right)$

$\left(\frac{-1}{2}, \frac{-1}{2}\sqrt{2k-1}\right)$

Since triangle is equilateral

$\therefore \ \frac{1}{4}(2k-1)+\frac{1}{4}=(2k-1)$

$\Rightarrow$ k = 2/3.

11. Match the following.

Column – I	Column – II
A) If 1, w, w^2 …. W^{n-1} are then n, nth roots of unity, then $(2-w)(2-w^2)\ldots(2-w^{n-1})$ equals	P) $2^n - 1$
B) If $z_1, z_2 \ldots z_n$ lie on a circle $\lvert z\rvert = 2$, then the value of $\lvert z_1+z_2+....+z_n\rvert - 4\left\lvert\frac{1}{z_1}+\frac{1}{z_2}+....+\frac{1}{z_n}\right\rvert$ is	Q) ${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + ...$
C) If p is a multiple of n, then sum of the pth power of nth roots of unity is	R) ${}^nC_{n-1} + {}^nC_{n-2} + {}^nC_{n-2} + ... + {}^nC_0$
D) Sum of all pairs of nth root of unity taken two at a time is	S) ${}^nC_1$
	T) $\left({}^{2n+1}C_0 + {}^{2n+1}C_1 + ...+ {}^{2n+1}C_n\right)^{1/2} - 1$

Ans. A – P,R,T ; B – Q ; C – S ; D – Q

Sol. A) $(z^n-1)=(z-1)(z-\omega)....(z-\omega^{n-1})$

Put z = 2

B) $4\left\lvert\frac{1}{z_1}+\frac{1}{z_2}+....\frac{1}{z_n}\right\rvert = \left\lvert\frac{\lvert z_1\rvert^2}{z_1}+\frac{\lvert z_2\rvert^2}{z_2}+....\frac{\lvert z_n\rvert^2}{z_n}\right\rvert = \left\lvert\overline{z_1+z_2+....+z_n}\right\rvert = \lvert z_1+z_2+...+z_n\rvert.$

Hence answer is 0.

C) Since p is the multiple of n, all will be 1 and sum = n

D) $\sum 1.\alpha = \frac{(1+\alpha+\alpha^2+....+\alpha^{n-1})^2-(1^2+\alpha^2+\alpha^4+....+\alpha^{2(n-1)})}{2}=0$

12. Number of solutions

	Column I		**Column II**
(A)	If z_1, z_2, z_3 are unimodular complex numbers such that $\lvert z_1 - z_2 + z_3\rvert = 4$ then $\left\lvert\frac{1}{z_1}-\frac{1}{z_2}+\frac{1}{z_3}\right\rvert$ is	(p)	3
(B)	If $\lvert z-2i\rvert \le 3$ then the maximum value of $\lvert iz+3\rvert$ is	(q)	2
(C)	If $\lvert z_1\rvert = 12$ and $\lvert z_2 - 3 - 4i\rvert = 5$ then the minimum value of $\lvert z_1 - z_2\rvert$ is	(r)	4
(D)	If z is a complex number satisfying $z\bar{z}-2(z+\bar{z})+3=0$ then the greatest value of $\lvert z\rvert$ is	(s)	5

Key. (A – r), (B – r), (C – q), (D – p)

Sol. Conceptual

13. If $(1+x)^{100} = \sum_{k=0}^{100} {}^{100}C_k x^k$, then value of

	COLUMN I		Column II
(A)	$c_0 - c_2 + c_4 - ...... + c_{100}$	(p)	0
(B)	$c_1 - c_3 + c_5 - ...... - c_{99}$	(q)	$2^{49}\left(2^{49}-1\right)$
(C)	$c_0 + c_4 + ... + c_{100}$	(r)	2^{98}
(D)	$c_1 + c_5 + ... + c_{97}$	(s)	-2^{50}

Key. (A – s), (B – p), (C – q), (D – r)

Sol. Put x = i

$$\sum_{k=0}^{100} C_k (i)^k = (1+i)^{100} = 2^{50}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{100} = 2^{50}(-1+0.i)$$

$\Rightarrow c_0 - c_2 + c_4 - ..... + c_{100} = -2^{50}$ ..... (1)

$c_1 - c_3 + c_5 - ..... - c_{99} = 0$ ..... (2)

$c_0 + c_2 + ... + c_{100} = 2^{99}$ ..... (3)

$c_1 + c_3 + ... + c_{99} = 2^{99}$ ..... (4)

From (1) and (2)

$c_0 + c_4 + ... + c_{100} = 2^{49}\left(2^{49}-1\right)$

From (2) and (4)

$c_1 + c_5 + ... + c_{97} = 2^{98}$

14. The value of

	Column I		Column II
(A)	$\arg\frac{z+1}{z-1} = \frac{\pi}{4}$	(p)	Parabola
(B)	$z = \frac{3i-t}{2+it}(t \in R)$	(q)	Part of a circle
(C)	$\arg z = \frac{\pi}{4}$	(r)	Full circle
(D)	$z = t + it^2 (t \in R)$	(s)	Ray

Key. (A – q), (B – r), (C – s), (D – p)

Sol. (D) $x + iy = t + it^2$

$x = t,\ y = t^2$

on eliminating t, we get

$y = x^2$ which is a parabola