CLASS : CC (Advanced) **Quadratic Equation & Logarithm** **TEST-23**

M.M.: 93 **Time: 60 Min**

PART-A
[SINGLE CORRECT CHOICE TYPE]

Q.1 to Q.15 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct. **[15 × 3 = 45]**

Q.1 If (x_0, y_0) is real solution of simultaneous equations $y = \log_x 2$ and $2\log_2 x^2 = \log_{x^2} 2^y$, then

(A) $y_0^2 = x_0$ (B) $y_0^{x_0} = 2$ (C) $x_0^{y_0} = \frac{1}{2}$ (D) $y_0 = x_0^2$

Q.2 A quadratic equation with integral coefficient has two different prime numbers as its roots if the sum of the coefficients of the equation is prime then the sum of the roots is

(A) 2 (B) 5 (C) 7 (D) 11

Q.3 If x_1, x_2, x_3, x_4 are roots of the equation $x^4 - x^3 \sin 2\beta + x^2 \cos 2\beta - x \cos \beta - \sin \beta = 0$ then $\sum_{i=1}^{4} \tan^{-1} x_i$ is equal to $\left(\text{where } \beta = \left(0, \frac{\pi}{2}\right) - \left\{\frac{\pi}{6}\right\} \right)$

(A) $\pi - \beta$ (B) $\pi - 2\beta$ (C) $\frac{\pi}{2} - \beta$ (D) $\frac{\pi}{2} - 2\beta$

Q.4 If a_i is positive or negative according as i is even or odd and equation $x^4 + a_1x^3 + a_2x^2 + a_3x + 5 = 0$ has four positive real roots then minimum value of a_1a_3 is equal to

(A) 20 (B) 40 (C) 80 (D) 160

Q.5 Let $A = \{-2, -1, 0, 1, 2\}$ and $f : A \to I$ where I denotes set of integers. If $f(x) = x^2 - 2x - 3$, then what is pre-image of (-3) ?

(A) 0 only (B) 2 only (C) 0, 2 (D) no value

Q.6 Consider a triangle ABC and let a, b and c be denote length's of the sides opposite to vertex A, B and C respectively. If a, b, c in order are arithmetic progression such that $x^2 + 3x + 5 = 0$ and $3x^2 + ax + c = 0$ have a common root then radius of incircle of triangle ABC is

(A) 3 (B) 4 (C) 5 (D) 6

Q.7 Number of values of a for which equation has $\frac{x^3-6x^2+11x-6}{x^3+x^2-10x+8}+\frac{a}{30}=0$ has no solutions, is

(A) 3 (B) 4 (C) 5 (D) infinite values

Q.8 The values of 'a' for which one root of the equation $x^2-(a+1)x+(a^2+a-8)=0$ exceeds 2 and the other is lesser than 2 are given by

(A) $3 < a < 10$ (B) $-2 < a < 3$ (C) $a \le -2$ (D) $a \ge 10$

Q.9 If p and q are non-zero real numbers and $\alpha^3+\beta^3=-p$, $\alpha\beta=q$, then a quadratic equation whose roots are $\frac{\alpha^2}{\beta},\frac{\beta^2}{\alpha}$ is

(A) $px^2+qx+p^2=0$ (B) $qx^2-px+q^2=0$ (C) $qx^2+px+q^2=0$ (D) $px^2-qx+p^2=0$

Q.10 The true solution set of $\log_{x^2}\left(\frac{x}{|x|}-x\right)\ge 0,$ is

(A) $(-\infty, 0)\cup(1, 2)$ (B) $(-\infty, 1)\cup(2, \infty)$ (C) $(-\infty, -1)\cup(0, 1)$ (D) $(-\infty, -2]\cup(0, 1)$

Q.11 Consider the equation $x^2+2x-n=0,$ where $n\in N$ and $n\in[5, 100]$. Total number of different values of n so that the given equation has integral roots is

(A) 4 (B) 8 (C) 3 (D) 6

Q.12 If $\cos^2\frac{\pi}{8}$ is a root of the equation $x^2+ax+b=0$ where $a, b\in Q$ then ordered pair (a, b) is equal to

(A) $\left(1, \frac{-1}{8}\right)$ (B) $\left(-1, \frac{-1}{8}\right)$ (C) $\left(-1, \frac{1}{8}\right)$ (D) $\left(1, \frac{1}{8}\right)$

Q.13 If the equations $x^2+b^2=1-2bx$ and $x^2+a^2=1-2ax$ have exactly one common root, than

(A) $a=b+2$ (B) $a=b^2+2$ (C) $a+b=2$ (D) $a+b+2=0$

Q.14 The minimum value of $x^2+2xy+3y^2-6x-2y,$ where $x, y\in R$, is equal to

(A) –9 (B) –11 (C) –12 (D) –10

Q.15 If α, β, γ are the roots of cubic equation $x^3-x^2=1,$ then the value of $\left(\frac{1+\alpha}{1-\alpha}\right)+\left(\frac{1+\beta}{1-\beta}\right)+\left(\frac{1+\gamma}{1-\gamma}\right)$ is equal to

(A) –5 (B) –6 (C) –7 (D) –2

[PARAGRAPH TYPE]

Q.16 to Q.19 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct. **[4 × 3 = 12]**

Paragraph for question nos. 16 & 17

Let $a x^2 + a^2 x + 2 = 0 \ (a \in R_0)$ be a quadratic equation and S be the set of values of a for which root of the equation are imaginary or equal.

[**Note :** R_0 denotes set of non-zero real numbers.]

Q.16 Number of integral values of 'a' in set S are

(A) 1 (B) 2 (C) 3 (D) 4

Q.17 If α and β are respectively sum and product of roots of the given equation, then $-\alpha\beta$ is

(A) a prime number (B) an odd integer

(C) an irrational number (D) dependent on value of a

Paragraph for question nos. 18 & 19

Let $f(n) = \sum_{r=2}^{n} \frac{r}{{}^{r}C_2 \, {}^{r+1}C_2}$, $a = \lim_{n\to\infty} f(n)$ and $x^2 - \left(2a - \frac{1}{2}\right)x + t = 0$ has two positive roots α and β.

Q.18 If $f(7) + f(8) = \frac{p}{q}$ where p and q are relatively prime, then the value of $(p - q)$ is

(A) 53 (B) 55 (C) 57 (D) 59

Q.19 Minimum value of $\left(\frac{4}{\alpha} + \frac{1}{\beta}\right)$ is

(A) 2 (B) 6 (C) 3 (D) 4

[MULTIPLE CORRECT CHOICE TYPE]

Q.20 has four choices (A), (B), (C), (D) out of which **ONE OR MORE** may be correct. **[1 × 4 = 4]**

Q.20 Let x_1 and x_2 be the roots of equation $x^2 - 3x + \alpha = 0$ and let x_3 and x_4 be the roots of the equation $x^2 - 12x + \beta = 0$. If the number x_1, x_2, x_3, x_4 (in that order) form an increasing geometric progression with common ratio 'r', then which of the following is/are **correct**?

(A) The value of r is equal to 2.

(B) The value of $(\alpha + \beta)$ equals 24.

(C) Geometric mean of α and β is 6.

(D) The sum of infinite geometric progression whose first term is 2 and common ratio $\frac{1}{r}$ is 4.

PART-B
[MATRIX TYPE]

[3 + 3 + 3 + 3 = 12]

Q.1 has **four** statements (A, B, C, D) given in **Column-I** and **four** statements (P, Q, R, S) given in **Column-II**. Any given statement in **Column-I** can have correct matching with one or more statement(s) given in **Column-II**.

Q.1	**Column-I**	**Column-II**
(A)	If α,β ($\alpha < \beta$) are the two roots of the equation $\frac{1-8(\log_{10} x)^2}{\log_{10} x - 2(\log_{10} x)^2} = 1$ then the value of $\frac{(\beta^3+1)}{\alpha^2\beta^3}$ is equal to	(P) 5
		(Q) 11
(B)	Number of solution(s) of equation $\log_2(x^2+3) = \frac{1}{2}\log_{1/3}\left(x+\frac{1}{x}\right), x>0$ is	(R) 0
(C)	Integers satisfying the equation $\lvert x\rvert + \left\lvert\frac{4-x^2}{x}\right\rvert = \left\lvert\frac{4}{x}\right\rvert$ is /are	
(D)	If $\log_c 2 \log_b 125 = \log_{10} 8 \log_c 10$ where $c>0, c\neq 1$, $b>1, b\neq 1$, then b is	(S) 1

PART-D
[INTEGER TYPE]

Q.1 to Q.4 are "Integer Type" questions. (The answer to each of the questions **are upto <u>4 digits</u>**) **[4 × 5 = 20]**

Q.1 Let r,s,t are roots of equation $8x^3 + 1001x + 2008 = 0$. Then absolute value of $(r+s)^3 + (s+t)^3 + (t+r)^3$ is 7k3 (where k is at ten's place). Find the value of k.

Q.2 Given that 9 is a root of the equation $\log_\pi(x^2+15a^2) - \log_\pi(a-2) = \log_\pi \frac{8ax}{a-2}$, $a>2$ then find the sum of the digits in other root.

Q.3 Let P(x) be a cubic polynomial with zeroes α, β, γ, if $\left(\frac{P\left(\frac{1}{2}\right)+P\left(-\frac{1}{2}\right)}{P(0)}\right) = 100$ find $\sqrt{\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}}$.

Q.4 The root of the equation $x^2 + 2(a-3)x + 9 = 0$ lie between –6 and 1 and $2, h_1, h_2, ...h_{20}, [a]$ are in H.P. and $2, a_1, a_2..... a_{20}, [a]$ are in A.P. where [a] denotes the integral part of a, then find the value of a_3h_{18}.

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ANSWER KEY

PART-A

Q.1	D	Q.2	B	Q.3	C	Q.4	C	Q.5	C
Q.6	A	Q.7	A	Q.8	B	Q.9	C	Q.10	D
Q.11	B	Q.12	C	Q.13	A	Q.14	B	Q.15	A
Q.16	B	Q.17	A	Q.18	D	Q.19	B	Q.20	AD

PART-B

Q.1 (A) Q; (B) R; (C) S; (D) P

PART-D

Q.1	0005	Q.2	0006	Q.3	0014	Q.4	0012