AP,GP,HP, Sequences

Single Correct Answer Type

1. The 2008th term of the sequence $1, \underbrace{2,2,2}_{3}, \underbrace{3,3,3,3,3,3}_{6}, \underbrace{4,4,4,4,4,4,4,4,4,4}_{10}, \ldots$ where n occurs $\frac{n(n+1)}{2}$ times in the sequence, equals

(A) 24 (B) 23

(C) 22 (D) 21

Key. C

Sol.

		No. of terms
Group (1)	1	1
Group (2)	2, 2, 2	3
Group (3)	3, 3, ..., 3	6
Group (4)	4, 4, ..., 4	10
. . . . .		
. . . . .		
Group (r)	r, r, ..., r	$\frac{r^2+r}{2}$

Let 2008th term falls in rth group

$\Rightarrow 1 + 3 + 6 + 10 + \ldots + \frac{(r-1)^2+(r-1)}{2} < 2008 \le 1 + 3 + 6 + \ldots + \frac{r^2+r}{2}$

$\Rightarrow \frac{(r-1)r(r+1)}{6} < 2008 \le \frac{r(r+1)(r+2)}{6}$

$\Rightarrow r^3 - r < 12048 \le (r+1)^3 - (r+1)$ ... (i)

$\Rightarrow$ r will be nearer to cube root of 12048

Note: $22 < \sqrt[3]{12048} < 23$

for r = 22 inequality (i) holds

for r < 22 RHS of (1) is less than 12048

for r $\ge$ 23 LHS of (1) is greater than 12048

$\Rightarrow$ r = 22 is the required value $\Rightarrow$ 2008th term is 22

Ans. (C) 22.

1. If $a_k = \frac{1}{k(k+1)}, for\ k = 1,2,3.......n, then \left(\sum_{k=1}^{n} a_k\right)^2 =$

1) $\frac{n}{n+1}$ 2) $\frac{n^2}{(n+1)^2}$ 3) $\frac{n^4}{(n+1)^4}$ 4) $\frac{n^6}{(n+1)^6}$

Key. 2

Sol. $\left(\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+......+\frac{1}{n}-\frac{1}{n+n}\right)^2$

$\frac{n^2}{(n+1)^2}$

2. $\sum_{k=1}^{\infty}\frac{1}{k!}\left(\sum_{n=1}^{k}2^{n-1}\right)=$

1) e 2) e^2+e 3) e^2 4) e^2-e

Key. 4

Sol. $\sum_{k=1}^{\infty}\frac{1}{k!}\left(1+2+2^2+.......+2^{k-1}\right)$

$\sum_{k=1}^{\infty}\frac{2^k-1}{k!}=e^2-e$

3. Coefficient of x^{10} in the expansion of $(2+3x)e^{-x}$ is

1) $\frac{-26}{(10)!}$ 2) $\frac{-28}{(10)!}$ 3) $\frac{-30}{(10)!}$ 4) $\frac{-32}{(10)!}$

Key. 2

Sol. $(2+3x)\left(1-\frac{x}{1!}+\frac{x^2}{2!}-\frac{x^3}{3!}....+\frac{x^{10}}{10!}\right)$

$\frac{2}{10!}-\frac{3}{9!}=\frac{2-30}{10!}=\frac{-28}{10!}$

4. $\frac{1^2}{1!}+\frac{1^2+2^2}{2!}+\frac{1^2+2^2+3^2}{3!}+.....$

1) $\frac{17e}{6}$ B) $\frac{6e}{17}$ C) $\frac{11e}{7}$ D) $\frac{7e}{11}$

Key. 1

Sol. $\sum_{n=1}^{\infty}\frac{1^2+2^2+3^2+....+n^2}{n!}$

$=\frac{1}{6}\left(\sum_{n=1}^{\infty}\frac{2n^3}{n!}+\sum_{n=1}^{\infty}\frac{3n^2}{n!}+\sum_{n=1}^{\infty}\frac{n}{n!}\right)$

$=\frac{1}{6}(2\times5e+3\times2e+e)=\frac{17e}{6}$

5. $\sum_{n=1}^{\infty}\frac{2n^2+n+1}{n!}=$

1) $2e-1$ 2) $2e+1$ 3) $6e-1$ 4) $6e+1$

Key. 3

Sol. $\sum_{n=1}^{\infty}\frac{2n(n-1)+3n+1}{n!}=\sum_{n=1}^{\infty}\frac{2n(n-1)}{n!}+\sum_{n=1}^{\infty}\frac{3n}{n!}+\sum_{n=1}^{\infty}\frac{1}{n!}$

$2e+3e+e-1=6e-1$

6. The sum of the series $1+\frac{1}{4.2!}+\frac{1}{16.4!}+\frac{1}{64.6!}+.......$

1) $\frac{e-1}{\sqrt{e}}$ 2) $\frac{e+1}{\sqrt{e}}$ 3) $\frac{e-1}{\sqrt{e}}$ 4) $\frac{e+1}{2\sqrt{e}}$

Key. 4

Sol. $1+\frac{1}{4.2!}+\frac{1}{16.4!}+......$

$=\frac{e^{1/2}+e^{-1/2}}{2}=\frac{e+1}{2\sqrt{e}}$

8. If $|x|<1 \, and \; y=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+..........then\; x=$

1) $y+\frac{y^2}{2}+\frac{y^3}{3}+......$ 2) $y-\frac{y^2}{2}+\frac{y^3}{3}-\frac{y^4}{4}+......$

3) $y+\frac{y^2}{2!}+\frac{y^3}{3!}+......$ 4) $y-\frac{y^2}{2!}+\frac{y^3}{3!}-\frac{y^4}{4!}+......$

Key. 3

Sol. $y=x-x^2+\frac{x^3}{3}-\frac{x^4}{4}+....$

$y=\log_e(1+x)\Rightarrow 1+x=e^y$

$\Rightarrow x=y+\frac{y^2}{2!}+\frac{y^3}{3!}+.....$

9. In a sequence of (4n+1) terms, the first (2n+1) terms are in A.P., whose common difference is 2, and the last (2n+1) terms are in G.P whose common ratio is 0.5 if the middle terms of the A.P and G.P are equal then the middle term of the sequence is

A) $n2^{n-1}/2^n-1$ B) $n.2^{n+1}/\ 2^{2n}-1$ C) $n.2^n$ D) $n2^{n+1}/2^n-1$

Key. D

Sol. Let the first term is a , then first (2n+1) terms are $a, a+2, a+4, ...a+2.2n$. Clearly the middle term of the sequence of $4n+1$ term is $(2n+1)^{th}$ term, i.e . $a+4n$ also the middle term of the A.P of $(2n+1)$ term is $(n+1)^{th}$ term i.e., $a+2n$ .Again for the last $(2n+1)$ terms the first term will be $(2n+1)^{th}$ term of the A.P i.e. $a+4n$

$\therefore G.P\ is\ (a+4n),(1+4n)(0.5)^n$

Its middle term is $(a+4n)(0.5)^n$

According to the given condition,

$a+2n=(1+4n)(0.5)^n$

$\therefore\ a=\frac{2n-4n(0.5)^n}{(0.5)^n-1}$

$\therefore$ Required middle term = $a+4n=$

$$\frac{2n-4n(0.5)^n}{(0.5)^n-1}+4n=\frac{2n}{1-\left(\frac{1}{2}\right)^2}=\frac{n.2^{n-1}}{2^n-1}$$

10. The sum of the series $\frac{x}{1-x^2}+\frac{x^2}{1-x^4}+\frac{x^4}{1-x^8}+......$ to infinite terms, if $|x|<1$ is

A) $\frac{x}{1-x}$ B) $\frac{1}{1-x}$ C) $\frac{1+x}{1-x}$ D) 1

Key. A

Sol. The general term of the series is $t_n=\frac{x^{2^{n-1}}}{1-x^{2^n}}$

$$=\frac{1+x^{2^{n-1}}-1}{\left(1+x^{2^{n-1}}\right)\left(1-x^{2n-1}\right)}$$

$$\therefore l_n=\frac{1}{1-x^{2n-1}}-\frac{1}{1-x^{2^n}}$$

Now , $S_n=\sum_{n=1}^{n}t_n=\left[\left\{\frac{1}{1-x}-\frac{1}{1-x^2}\right\}\right]$

$$+\left\{\frac{1}{1-x^2}-\frac{1}{1-x^4}\right\}+....$$

$$+\left\{\frac{1}{1-x^{2^{n-1}}}-\frac{1}{1-x^{2^n}}\right\}\Bigg]=\frac{1}{1-x}-\frac{1}{1-x^{2^n}}$$

$\therefore$ The sum to infinite terms

$$=\lim_{n\to\infty}S_n=\frac{1}{1-x}-1=\frac{x}{1-x}$$

$$\left[\because \lim_{n\to\infty} x^{2^n}=0, as\ |x|<1\right]$$

11. If n arithmetic means are inserted between two sets of numbers a, 2b and 2a, b, where a,b $\in R$. Suppose that m^{th} arithmetic mean between these two sets of numbers is same, then the ratio a:b equals

A) $n-m+1:m$ B) $n-m+1:n$ C) $m:n-m+1$ D) $n:n-m+1$

Key. C

Sol. Let $A_1, A_2.....A_n$ be airthmetic means between a and 2b , then $A_m=a+m\left(\frac{2b-a}{n+1}\right)$

Again , let $B_1, B_2........B_n$ be arithmetic means

Between 2a and b then $B_m = 2a + m\left(\frac{b-2a}{n+1}\right)$

Now , $A_m = B_m \Rightarrow a + m\left(\frac{2b-a}{n+1}\right) = 2a +$

$m\left(\frac{b-2a}{n+1}\right) \Rightarrow m\left(\frac{b+a}{n+1}\right) = a \Rightarrow \frac{a}{b} = \frac{m}{n-m+1}$

12. The ratio of sum of first three terms of a G.P. to the sum of first six terms is $64:91$,the common ratio of G.P. is

1. $\frac{1}{4}$ 2. $\frac{3}{4}$ 3. $\frac{5}{4}$ 4. $\frac{7}{4}$

Key. 2

Sol. Given $\frac{S_3}{S_6} = \frac{64}{91} = \frac{a\left(r^3-1\right)}{a\left(r^6-1\right)}$

$\Rightarrow \frac{\left(r^3-1\right)}{\left(r^3+1\right)\left(r^3-1\right)} = \frac{64}{91}$

$\Rightarrow r^3 = \frac{27}{64} \quad \therefore \quad r = \frac{3}{4}$

13. Sum of the series $3+5+9+17+33+.....$to n terms is

1. $2^{n+1}-n-2$ 2. $2^{n+1}+n-2$ 3. 2^n+n-2 4. $2^{n+1}-n+2$

Key. 2

Sol. $S_n = 3+5+9+17+33+......$

$= (2+1)+\left(2^2+1\right)\left(2^3+1\right)+\left(2^4+1\right)+....$

$= \left(2+2^2+2^3+2^4+......n \text{ terms}\right)+n$

$= 2\left(2^n-1\right)+n = 2^{n+1}+n-2$

$= 2^{n+1}+n-2$

14. If one A.M. A and two G.M.s p and q be inserted between two numbers a and b, then which of the following is hold good

1. $a^3+b^3=2Apq$ 2. $p^3+q^3=2Apq$ 3. $a^3+b^3=2Aab$ 4. None of these.

Key. 2
Sol. Given $a+b=2A$

And $a, p, q, \quad b \in$ G.P.

$\therefore \quad p^2=aq$ and $q^2=pb$

$\Rightarrow \; p^3=apq$ and $q^3=bpq$

by adding we get

$p^3+q^3=apq+bpq$

$=pq(a+b)=2Apq$

15. If fourth term of a G.P. is 3, the product of the first seven terms is

1. 3^4 2. 3^7 3. 7^4 4. 4^7

Key. 2
Sol. As the number of terms are odd (7) let r, be the common ratio

So terms can be taken as $\frac{a}{r^3}, \frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2, ar^3$

$\therefore$ Product of the term $=a^7$

$=3^7$ as $(t_4=a=3)$

16. The number of divisors of 6912, 52480,32000 are in

1. A.P Only 2. G.P. Only 3. A.P. , G.P.& H.P. 4. None of these

Key. 3
Sol. If n is a + ve number.

$n=P_1^{k_1}.P_2^{k_2}...P_r^{k_r}$

(where $p_1, p_2, p_3, ... p_r$ are prime number) then number of divisors of n are

$= (k_1 + 1)(k_2 + 1)....(k_r + 1)$

$\therefore$ Number of prime factor of 6912 are $= 2^8.3^3$ so no. of divisors $= 9 \times 4 = 36$

Prime factors of 52,400 are $= 3^8 \times 2^3$

$\therefore$ No. of divisors $= 9 \times 4 = 36$

Prime factors of 32,000 are $= 5^3 \times 2^8$

$\therefore$ No. of divisors $= 9 \times 4 = 36$

Now each number having same number of divisors *i.e.*, 36,36,36

Each and every term is constant & constant sequence is always in A.P.& G.P. both as common difference is 0 and common ratio is 1.

17. If 1, $\log_{81}(3^x + 48)$, $\log_9\left(3^x - \frac{8}{3}\right)$ are in A.P., then the value of x equals

1. 9 2. 6 3. 2 4. 4

Key. 3

Sol. Given 1, $\log_9 2\left(3^x + 48\right), \log_9\left(3^x - 8/3\right), \in$ A.P.

$\Rightarrow \log_9 9,\ \frac{1}{2}\log_9\left(3^x + 48\right),\ \log_9\left(3^x - 8/3\right) \in$ A.P.

$\Rightarrow 9, \left(3^x + 48\right)^{1/2}, 3^x - 8/3 \in$ G.P. (By concept)

$\Rightarrow \log a, \log b, \log c \in$ A.P.

$\therefore a, b, c \in$ G.P. $\therefore 3^x + 48 = 9\left(3^x - 8/3\right)$

$8.3^x = 72$

$3^x = 9,\ 3^x = 3^2,\quad x = 2$.

18. If a,b,c are in H.P., then $\frac{a}{b+c},\frac{b}{c+a},\frac{c}{a+b}$ are in

1. A.P. 2. H.P. 3. G.P. 4. None of these

Key. 2

Sol. Given $a,b,c\in$ H.P.

So, $\frac{1}{a},\frac{1}{b},\frac{1}{c}\in$ A.P.

$\frac{a+b+c}{a},\frac{b+c+a}{b},\frac{c+a+b}{c}\in$ A.P.

By using concept if $a,b,c\in$ A.P.

Then their reciprocals are in H.P.

19. Between the numbers 2 and 20, 8 means are inserted, then their sum is

1. 88 2. 44 3. 176 4. None of these

Key. 1

Sol. Let $a, A_1, A_2....A_g, b\in A.P$

Where $a=2, b=20, n=8$

$\therefore$ sum of the means $=\frac{n}{2}(a+b)=\frac{8}{2}(2+20)=88$

20. In the expansion of $(1+x)^{70}$,the sum of coefficients of odd powers of x is

1. 0 2. 2^{69} 3. 2^{70} 4. 2^{71}

Key. 2

Sol. Fact. The sum of the coefficients of odd powers in the expansion of $(1+x)^n$ = sum of the coefficients of even powers in $(1+x)^n$

$=2^{n-1}$

$2^{70-1}=2^{69}$

21 If the arithmetic mean of two positive numbers $a\ \&\ b$ $(a>b)$ is twice their G.M., then

$a:b$ is

1. $6+\sqrt{7}:6-\sqrt{7}$ 2. $2+\sqrt{3}:2-\sqrt{3}$ 3. $5+\sqrt{6}:5-\sqrt{6}$ 4. None of these

Key. 2

Sol. $\frac{a+b}{2}=2\sqrt{ab}$

$a+b-4\sqrt{ab}=0$

$\frac{a}{b}+1-4\sqrt{\frac{a}{b}}=0$ (Dividing by b)

Or $\left(\sqrt{\frac{a}{b}}\right)^2-4\sqrt{\frac{a}{b}}+1=0$

$\therefore \sqrt{\frac{a}{b}}=\frac{4\pm 2\sqrt{3}}{2}=\left(2\pm\sqrt{3}\right)$

$\frac{a}{b}=\frac{2+\sqrt{3}}{2-\sqrt{3}}$

22 The number of terms common between the two series 2 + 5 + 8 + …up to 50 terms and the series 3 + 5 + 7 + 9 + ….up to 60 terms.

1. 24 2. 26 3. 25 4. None of these

Key. 4

Sol. Let m^{th} term of first A.P. be equal to the n^{th} term of the second A.P. then

2 , 5 , 8 ,….50 terms series 1

3 , 5 ,7 , …., 60 terms series 2

Common series 5 , 11, 17 , ….., 119

40^{th} term of series 1 = 59^{th} term of series 2 = 119 = last term of common series

$\Rightarrow a_n=5+(n-1)d \Rightarrow 119+1=6n \Rightarrow n=20.$

$\therefore$ Number of common terms is 20.

23 The sum of the series $1+\frac{9}{4}+\frac{36}{9}+\frac{100}{16}+....$ up to n terms if $n=16$ is

1. 446 2. 746 3. 546 4. 846

Key. 1

Sol. The given series can be written as $1^3+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+...$

$$t_n=\frac{1^3+2^3+3^3+.....n^3}{1+3+5+....+(2n-1)}$$

$$t_n=\frac{n^2(n+1)^2}{4n^2}=\frac{(n+1)^2}{4}$$

$$t_n=\frac{1}{4}(n+1)(n+1)$$

$$=\frac{1}{4}(n^2+2n+1)=\frac{1}{4}\left[\sum_{k=1}^{n}k^2+2\sum_{k=1}^{n}k+n\right]$$

$$\therefore S_n=\frac{1}{4}\left[\frac{n(n+1)(2n+2)}{6}+n(n+1)+n\right]$$

$$\therefore S_{16}=\frac{1}{4}\left[\frac{16.17.33}{6}+16.17+16\right]=\frac{1}{4}[88\times17+16\times8+16]=446$$

24 Sum of n terms of series

$ab+(a+1)(b+1)+(a+2)(b+2)+....+(a+(n-1))(b+(n-1))$

if $ab=\frac{1}{6} and (a+b)=\frac{1}{3}$, is

(A) $\frac{n}{6}(1-2n)^2$ (B) $\frac{n}{6}(1+n-2n^2)$ (C) $\frac{n}{6}(1-2n+2n^2)$ (D) none of these

Key. C

Sol. $s=ab+[ab+(a+b)+1]+[ab+2(a+b)+2^2]+....[ab+(n-1)(a+b)+(n-1)^2]$

$= nab+(a+b)\sum_{r=1}^{n-1}r+\sum_{r=1}^{n-1}r^2$

$= nab+(a+b)\frac{n(n-1)}{2}+\frac{(n-1))(n)(2n-1)}{6}$

$= \frac{n}{6}[1+(n-1)\{1+2n-1\}]$

$= \frac{n}{6}[1+2n(n-1)]=\frac{n}{6}(1-2n+2n^2)$

25 If log (a+c),log(a+b), log(b+c) are in A.P. and a,c,b are in H.P, then the value of a+b is (given a,b,c >0)

(A) 2c (B) 3c (C) 4c (D) 6c

Key. A

$\log(a+c)+\log(b+c)=2\log(a+b)$

$(a+c)(b+c)=(a+b)^2$

Sol. $\Rightarrow ab+c(a+b)+c^2=(a+b)^2$ (1)

$also, c=\frac{2ab}{a+b}\Rightarrow 2ab=c(a+b)$

$\Rightarrow 2ab+2c(a+b)+2c^2=2(a+b)^2$ ..... (2)

From (1) and(2),

$c(a+b)+2c(a+b)+2c^2=2(a+b)^2$

$2(a+b)^2-3c(a+b)-2c^2=0$

$\therefore a+b=\frac{3c\pm\sqrt{9c^2+16c^2}}{4}=\frac{3c\pm 5c}{4}=2c\ or -\frac{c}{2}$

$\therefore a+b=2c$ $(\because a,b,c>0)$

26 If $a_1,a_2,a_3,....,a_n$ are in A.P. with s_n as the sum of first 'n' terms $(s_0=0)$, then $\sum_{k=0}^{n} {}^nC_k s_k$ is equal to

(A) $2^{n-2}[na_1+s_n]$ (B) $2^n[a_1+s_n]$ (C) $2[na_1+s_n]$ (D) $2^{n-1}[a_1+s_n]$

Key. A

Sol. $\sum_{k=o}^{n} {}^nc_k s_k=\sum_{k=0}^{n} {}^nc_k\frac{k}{n}[2a+(k-1)d]$

$=[(a_1-\frac{d}{2})\sum_{k=0}^{n}k\,{}^nc_k+\frac{d}{2}\sum_{k=0}^{n}k^2\,{}^nc_k]$

$=\left(a_1-\frac{d}{2}\right)n.2^{n-1}+\frac{d}{2}[n.2^{n-1}+n(n-1)2^{n-2}]$

$=a_1.n.2^{n-1}+dn(n-1)2^{n-3}$

$= n.2^{n-3}[4a_1 + a_n - a_1] = n.2^{n-3}[3a_1 + a_n]$

$= 2^{n-3}[2na_1 + 2n\left(\frac{a_1 + a_n}{2}\right)]$

$= 2^{n-2}[na_1 + s_{n]}.$

27 The positive integral values of n such that $1.2^1 + 2.2^2 + 3.2^3 + 4.2^4 + 5.2^5 + ... + n.2^n = 2^{(n+10)} + 2$ is

(A) 313 (B) 513 (C) 413 (D) 613

Key. B

Sol.

$2^1 + 2^2 + 2^3 + ... + 2^n = 2^{n+1} - 2$

$2^2 + 2^3 + ...2^n = 2^{n+1} - 2^2$

$2^3 + ... + 2^n = 2^{n+1} - 2^3$

... ...

$+2^n = 2^{n+1} - 2$

$= n\left(2^{n+1}\right) - \left(2^{n+1} - 2\right)$

$= 2^{n+1}(n-1) + 2$

Given that $2^{n+1}(n-1) + 2 = 2^{2+10} + 2$

$\Rightarrow (n-1)2^{n+1} = 2^{n+10}$

$\Rightarrow n - 1 = 2^9$

$\Rightarrow n = 2^9 + 1 = 513$

28 If a,b,c, are in A.P. and p, p' are respectively A.M. and G.M. between a and b while q, q' are respectively AM. And G.M. between b and c, then

(A) $p^2 + q^2 = p'^2 + q'^2$ (B) $pq = p'q'$

(C) $p^2 - q^2 = p'^2 - q'^2$ (D) $p^2 + p'^2 = q^2 + q'^2$

Key. C

Sol. We have $2b = a + c$ and a,p,b,q,c are in A.P

$\Rightarrow p = \frac{a+b}{2}, q = \frac{b+c}{2}$

Again , $p' = \sqrt{ab}$ and $q' = \sqrt{bc}$

$\therefore p^2 - q^2 = \frac{(a+b)^2 - (b+c)^2}{4}$

$= \frac{(a-c)(a+c+2b)}{4} = (a-c)b = p'^2 - q'^2$

29. The arithmetic mean of the nine numbers in the given set {9, 99, 999, ... 999999999} is a 9 digit number N, all whose digits are distinct. The number N does not contain the digit

(A) 0 (B) 2 (C) 5 (D) 9

Key. A

Sol. N = $\frac{1}{9}$ {9, 99, 999, ... 999999999} = 1 + 11 + 111 + ...+111111111

= 1 2 3 4 5 6 7 8 9 (A)

30. The minimum value of the expression $\frac{9x^2 \sin^2 x + 4}{x \sin x}$ for $x \in (0, \pi)$ is

(A) $\frac{16}{3}$ (B) 6 (C) 12 (D) $\frac{8}{3}$

Key. C

Sol. $E = 9x \sin x + \frac{4}{x \sin x}$ [note that $x \sin x > 0$ in $(0, \pi)$]

$E = \left(3\sqrt{x \sin x} - \frac{2}{\sqrt{x \sin x}}\right)^2 + 12$

∴ $E_{min} = 12$ which occurs when $3 x \sin x = 2$ ⇒ $x \sin x = 2/3$]

note that $x \sin x$ is continuous at $x = 0$ and attains the value $\pi/2$ which is greater than 2/3 at $x = \pi/2$, hence it must take the 2/3 in $(0, \pi/2)$]

31. There is a certain sequence of positive real numbers. Beginning from the third term, each term of the sequence is the sum of all the previous terms. The seventh term is equal to 1000 and the first term is equal to 1. The second term of this sequence is equal to

(A) 246 (B) $\frac{123}{2}$ (C) $\frac{123}{4}$ (D) 124

Key. B

Sol. sequence is $t_1 + t_2 + t_3 + t_4 + ...$

$t_3 = t_1 + t_2$; $t_7 = 1000$

$t_1 = 1$

but $t_7 = t_1 + t_2 + t_3 + t_4 + t_5 + t_6$

$1000 = 2(t_1 + t_2 + t_3 + t_4 + t_5)$

$= 4(t_1 + t_2 + t_3 + t_4)$

$= 8(t_1 + t_2 + t_3)$

$1000 = 16(t_1 + t_2)$

$t_1 + t_2 =$ ⇒ $t_2 = – 1 = – 1 =$

32. If $(1 + x + x^2)^{25} = a_0 + a_1x + a_2x^2 + ... + a_{50} . x^{50}$ then $a_0 + a_2 + a_4 + ... + a_{50}$ is :

(A) even (B) odd & of the form 3n

(C) odd & of the form (3n 1) (D) odd & of the form (3n + 1)

Key. A

Sol. putting x = 1 and 1 and adding

$a_0 + a_2 + ... + a_{50}$ = = ()

=

= = 2 [13 + $^{25}C_2$ + ... + $^{25}C_{25} . 2^{23}$]

 even

33. The sum of the series $(1^2 + 1).1! + (2^2 + 1).2! + (3^2 + 1). 3! + ... + (n^2 + 1). n!$ is :

(A) (n + 1). (n+2)! (B) n.(n+1)! (C) (n + 1). (n+1)! (D) none of these

Key. B

Sol. T_n = [n (n + 1) (n 1)] n! = n . (n+1)! (n 1). n!

Now put n = 1, 2, 3, ... , n and add

34. Find the sum of the infinite series $\frac{1}{9}+\frac{1}{18}+\frac{1}{30}+\frac{1}{45}+\frac{1}{63}+.....$

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{5}$ (D) $\frac{2}{3}$

Key. A

Sol. T_n = =

hence T_n using method of diff; T_n = =

 S_n = = Ans.

35. The sequence a_1, a_2, a_3, ... satisfies $a_1 = 19$, $a_9 = 99$, and for all n 3, a_n is the arithmetic mean of the first n – 1 terms. Then a_2 is equal to

(A) 179 (B) 99 (C) 79 (D) 59

Key. A

Sol. n 3, a_3 = ...(1)

a_4 = = $a_4 = a_3$

a_5 = = = a_4

$a_3 = a_4 = a_5 = ... = a_9 = 99$

put in equation (1)

99 = a_2 = 179 Ans.

36. If a, b, c are in G.P. then $\frac{1}{b-a}, \frac{1}{2b}, \frac{1}{b-c}$ are in

(A) A.P. (B) G.P. (C) H.P. (D) none

Key. A

Sol. Let a = x; b = xr; c = xr^2

hence the number are , ,

now, =

\+ = =

hence , , are in A.P.

37. Let d_1, d_2 , ..., d_k be all the distinct factors of a positive integer n including 1 and n. Suppose

$d_1 + d_2 + ... + d_k = 72$, then the value of $\frac{1}{d_1} + \frac{1}{d_2} + \ldots\ldots\ldots \frac{1}{d_k}$

(A) $\frac{72}{nk}$

(B) cannot be computed from the given information

(C) $\frac{72}{n}$

(D) None of these

Key. C

Sol. , ,..... are all distinct and each of these represents one of the number d_1, d_2,.........d_k.

 =

38. If *b* is the arithmetic mean between *a* and *x*; *b* is the geometric mean between '*a*' and y; '*b*' is the harmonic mean between a and *z*, (*a*, *b*, *x*, *y*, *z* > 0) then the value of *xyz* is

(A) a^3 (B) b^3 (C) $\frac{b^3(2a-b)}{2b-a}$ (D) $\frac{b^3(2b-a)}{2a-b}$

Key. D

39. The first term of an infinite geometric series is 2 and its sum be denoted by S. If | S – 2 | < 1/10 then the true set of the range of common ratio of the series is

(A) $\left(\frac{1}{10}, \frac{1}{5}\right)$ (B) $\left(-\frac{1}{2}, \frac{1}{2}\right) - \{0\}$

(C) $\left(-\frac{1}{19}, \frac{1}{20}\right) - \{0\}$ (D) $\left(-\frac{1}{19}, \frac{1}{21}\right) - \{0\}$

Key. D

40. The number of real values of the parameter $'k'$ for which $\left(\log_{16} x\right)^2 - \log_{16} x + \log_{16} k = 0$ will have unique solution

A) 2 B) 1 C) 4 D) 5

Key. B

Sol. For exactly one solution $4\log_{16} k = 1, k > 0 \Rightarrow k = 2$

41. If $3^{37} = 80\lambda + k,$ where $\lambda \in N,$ then $'k'$ is

A) 78 B) 3 C) 2 D) 9

Key. B

Sol. $3^{37}=3^{4\times 9}.3=3(81)^9=3(80+1)^9=3\left({}^9C_0 80^9+{}^9C_1 80^8+......+{}^9C_9\right)$.

Hence remainder is 3

42. The sum of first $'n'$ terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+.....$ is

A) 2^{n-1} B) $1-2^{-n}$ C) $2^{-n}-n+1$ D) $2^{-n}+n-1$

Key. D

Sol. $S=\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+........$ to $'n'$ terms

$S=\left(1-\frac{1}{2}\right)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{8}\right)+.........$ to $'n'$ terms

$$=(1+1+1+......)-\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+......\right)=n-\frac{1}{2}\frac{1-\left(\frac{1}{2}\right)^n}{2-\frac{1}{2}}=n-\left(1-\frac{1}{2^n}\right)=2^{-n}+n-1$$

43. The sum to n terms of the series

$\frac{1}{\lfloor 1}\left(\frac{1}{2}\right)+\frac{1.3}{\lfloor 2}\left(\frac{1}{2}\right)^2+\frac{1.3.5}{\lfloor 3}\left(\frac{1}{2}\right)^3+.....$ upto n terms is

(A) $\frac{1.3.5.....(2n-1)(2n+1)}{2^n\lfloor n}-1$ (B) $1-\frac{1.3.5.....(2n-1)}{\lfloor n\ \lfloor n}$

(C) $1-\frac{1.3.5.....(2n-3)}{2^{n-1}\lfloor n-1}$ (D) $\frac{1.3.5.....(2n-3)}{2^{n-1}\lfloor n-1}$

Key. A

Sol. $t_{n+1}=\frac{t_n\times(2n+1)}{(n+1)}\times\frac{1}{2}$

$(2n+2)t_{n+1}=(n+1)t_n$

$(2n+3)t_{n+1}-(2n+1)t_n=t_{n+1}$

Put n = 1

$5t_2-3t_1=t_2$

n = 2, $7t_3-5t_2=t_3$

$(2n+1)t_n-(2n-1)t_{n-1}=t_n$

$(2n+1)t_n-2t_1=S$

$$S=\frac{1.3.5.....(2n+1)}{\lfloor\underline{n}\times 2^{n}}-1$$

44. The sequence $\{x_1, x_2, ..., x_{50}\}$ has the property that for each k, x_k is k less than the sum of other 49 numbers. The value of $96x_{20}$ is

a) 300 b) 315 c) 1024 d) 0

Key : B

Sol : We have $x_k + k = S - x_k$ where $x_1 + x_2 + ... + x_k = s$

$\Rightarrow$ $2x_k + k = S$

$\Rightarrow$ $2(S)+\frac{50.51}{2}=50S$

$\Rightarrow$ $48(S)=25.51$

$\Rightarrow$ $x_{20}=\left(\frac{25.51}{48}-20\right)\frac{1}{2}=\frac{315}{96}.$

45. If the first and $(2n-1)^{th}$ terms of an A.P; a G.P. and H.P. are equal and their n^{th} terms are p,q and s respectively, then which of the following options is/are correct?

a) $p \ge q \ge s$ b) $p+s=q$ c) $ps=q^2$ d) $p=q=s$

KEY : C

HINT: Let the first term be a and $(2n-1)^{th}$ term be b then

$$p=a+(n-1)d=a+(n-1)\left(\frac{b-a}{2n-2}\right)=\frac{a+b}{2}$$

$$q=a.r^{n-1}=a\left(\frac{b}{a}\right)^{\frac{n-1}{2n-2}}=a\left(\frac{b}{a}\right)^{\frac{1}{2}}=\sqrt{ab}$$

$$\frac{1}{s}=\frac{1}{a}+(n-1)\left(\frac{\frac{1}{b}-\frac{1}{a}}{2n-2}\right)=\frac{\frac{1}{a}+\frac{1}{b}}{2}$$

p, q, r are the A.M, G.M, H.M of a, b.

$\therefore p \ge q \ge r$ and $ps = q^2$

46. If a_1, a_2, a_3, … a_{4001} are terms of an AP such that $\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+....+\frac{1}{a_{4000}a_{4001}}$ = 10 and $a_2 + a_{4000} = 50$ then $|a_1 - a_{4001}|$ is equal to

(A) 20 (B) 30

(C) 40 (D) 10

KEY : B

HINT: $\frac{4000}{a_1a_{4001}}=10 \Rightarrow a_1a_{4001}=400$

$a_1 + a_{4001} = 50$

$(a_1 - a_{4001})^2 = (a_1 + a_{4001})^2 - 4a_1 a_{4001}$

$\Rightarrow |a_1 - a_{4001}| = 30$

47. Statement-1 : The series for which the sum to n terms ($n \geq 1$), S_n is given by $S_n = 3n^2 + 4n + 5$ is an arithmetic progression (AP).

Statement-2 : The sum to n terms of an AP having non-zero common difference is a quadratic in n.

KEY : D

HINT: CONCEPTUAL

48. The fourth and fifth term of a sequence $\{t_n\}_{n\geq 1}$ are 4 and 5 respectively and the n[th] term is given as $t_n = 2t_{n-1} - t_{n-2},\ n \geq 3\ (n \in N)$. Then the sum to first 2009 terms is

(A) 2019045 (B) 2013021

(C) 2017036 (D) 2018040

KEY : A

HINT: $t_n = 2\,t_{n-1} - t_{n-2}$

$t_n - t_{n-1} = t_{n-1} - t_{n-2}$

$a_n = t_n - t_{n-1}, n \geq 3$

WE HAVE $a_n = a_{n-1}$

THUS $\{a_n\}$ IS A CONSTANT SEQUENCE

$a_5 = t_5 - t_4 = 1$

NOW $a_4 = t_4 - t_3 \Rightarrow 1 = 4 - t_3 \Rightarrow t_3 = 3$

SIMILARLY $t_2 = 2,\ t_1 = 1$

THUS $\{t_n\}$ IS AN A.P WITH $r = 1$ AND COMMON DIFFERENCE 1

$$\sum_{n=1}^{2009} t_n = \frac{2009 \times 2010}{2} = 2003 \times 1005 = 2019045$$

49. If $x^6 = 2x^3 - 1$ *and* x is not real then $\sum_{r=1}^{50} \left(x^r + x^{2r}\right)^3 =$

A) 100 B) 256 C) 76 D) 94

KEY : D

HINT : $x^3 = 1 \Rightarrow x = \omega, \omega^2$ $\quad x^r + x^{2r} = \begin{cases} 2 & if\ \ r\ \ is\, a\ \ multiple\ \ of\ \ 3 \\ -1 & if\ \ r\ \ is\ \ not\ \ a\ \ multiple\ \ of\ \ 3 \end{cases}$

50. If a, b, c are in A.P. b,c,d are in G.P. and c,d,e are in H.P. then a,c,e are in

(A) AP (B) GP

(C) HP (D) none

KEY : B

HINT : a, b, c are in AP $\Rightarrow\ a + c = 2b;\qquad (1)$

b, c, d are in GP $\Rightarrow c^2 = bd \qquad (2)$

c, d, e are in HP $\Rightarrow \frac{2ce}{c+e} = d \quad (3)$

$(1)\times(3) \Rightarrow \frac{(a+c)ce}{c+e} = bd = c^2$

$\therefore (a+c)e = c(c+e)$

$ae = c^2 \Rightarrow a, c, e$ are in G.P

51. If $a_n = \sum_{k=0}^{n} \frac{(\log_e 10)^n}{k!(n-k)!}$ for $n \geq 0$ then $a_0 + a_1 + a_2 + a_3 + \ldots$ upto ∞ equal is

(A) 10 (B) 10^2 (C) 10^3 (D) 10^4

Key : B

Hint : $a_n = \frac{(\log_e 10)^n}{n!} \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} = \frac{(\log_e 10)^n}{n!}\left[2^n\right] = \frac{(2\log_e 10)^n}{n!}$

Thus, $a_0 + a_1 + a_2 + \ldots$ upto infinity is

$= \sum_{n=0}^{\infty} \frac{(2\log_e 10)^n}{n!} = e^{2\log_e 10} = 100$

$\therefore$ (B) is the correct answer.

52. If a_1 is the greatest value of $f(x)$; where $f(x) = \left(\frac{1}{2+[\sin x]}\right)$ (where [.] denotes greatest integer function) and $a_{n+1} = \frac{(-1)^{n+2}}{(n+1)} + a_n$, then $\lim_{n\to\infty}(a_n)$ is

(A) 1 (B) e^2

(C) ln2 (D) ln3

Key: C

Hint: $a_1 = 1$

$\Rightarrow a_2 = 1 - \frac{1}{2}$

$\Rightarrow a_3 = 1 - \frac{1}{2} + \frac{1}{3}$

………..

………..

$a_\infty = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots\ldots\ldots$

= ln2

53. The sequence $\{x_k\}$ is defined by $x_{k+1} = x_k^2 + x_k$ and $x_1 = \frac{1}{2}$. Then $\left[\frac{1}{x_1+1} + \frac{1}{x_2+1} + \ldots + \frac{1}{x_{100}+1}\right]$ (where [.] denotes the greatest integer function) is equal to

(A) 0 (B) 2

(C) 4 (D) 1

Key: D

Hint: $\frac{1}{x_{k+1}} = \frac{1}{x_k(x_k+1)} = \frac{1}{x_k} - \frac{1}{x_k+1} \Rightarrow \frac{1}{x_k+1} = \frac{1}{x_k} - \frac{1}{x_{k-1}}$

$\therefore \frac{1}{x_1+1} = \frac{1}{x_2+1} + \ldots + \frac{1}{x_{100}+1} = \frac{1}{x_1} - \frac{1}{x_{101}}$

As $0 < \frac{1}{x_{101}} < 1$

$\therefore \left[\frac{1}{x_1+1} + \frac{1}{x_2+1} + \ldots + \frac{1}{x_{100}+1}\right] = 1$

54. If $S_n = \sum_{r=1}^{n} t_r = \frac{1}{6}n(2n^2+9n+13)$, then $\sum_{r=1}^{n}\sqrt{t_r}$ equals

(a) $\frac{1}{2}n(n+1)$ (b) $\frac{1}{2}n(n+2)$ (c) $\frac{1}{2}n(n+3)$ (d) $\frac{1}{2}n(n+5)$

Key: c

Hint: We have $t_n = S_n - S_{n-1}$ $\forall$ $n \geq 2$

$\therefore \quad t_n = \frac{1}{6}\left[2\left(n^3-(n-1)^3\right) + 9\left(n^2-(n-1)^2\right) + 13(n-n+1)\right]$

$= \frac{1}{6}\left[6n^2 - 6n + 2 + 9(2n-1) + 13\right]$

$= \frac{1}{6}(6n^2+12n+6) = (n+1)^2$

$\therefore \quad \sum_{r=1}^{n}\sqrt{t_r} = \sum_{r=1}^{n}(r+1) = \frac{1}{2}(n+1)(n+2) - 1 = \frac{1}{2}n(n+3)$

55. $\{a_n\}$ and $\{b_n\}$ be two sequences given by $a_n = (x)^{1/2^n} + (y)^{1/2^n}$ and $b_n = (x)^{1/2^n} - (y)^{1/2^n}$ for all $n \in N$, then $a_1\, a_2\, a_3 \ldots a_n$ is equal to

(A) $x-y$ (B) $\frac{x+y}{b_n}$

(C) $\frac{x-y}{b_n}$ (D) $\frac{xy}{b_n}$

Key : c

Sol : $a_1\, a_2 \ldots A_n = b_n \frac{a_1a_2\ldots a_n}{b_n}$

$= a_n\, b_n \frac{(a_1a_2\ldots a_{n-1})}{b_n}$

$= \left(x^{\frac{1}{2^{n-1}}} - y^{\frac{1}{2^{n-1}}}\right)\frac{(a_1\, a_2\ldots a_{n-1})}{b_n} = a_{n-1}\; b_{n-1}\; \frac{(a_1a_2\ldots a_{n-2})}{b_n}$

$= \frac{a_1b_1}{b_n} = \frac{(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})}{b_n} = \frac{x-y}{b_n}$

56. The sum of the series $\frac{9}{5^2.2.1}+\frac{13}{5^3.3.2}+\frac{17}{5^4.4.3}+---$ upto infinity

(A) 1 (B) $\frac{9}{5}$

(C) $\frac{1}{5}$ (D) $\frac{2}{5}$

Key. C

Sol. $Tr=\frac{4r+1}{5^r r(r-1)}, r\geq 2$

$$\frac{5r-(r-1)}{5^r r(r-1)}=\frac{1}{5^{r-1}(r-1)}-\frac{1}{5^r r}$$

$$\sum_{r=2}^{\infty}T_r=\left(\left(\frac{1}{5^1.1}-\frac{1}{5^2.2}\right)+\left(\frac{1}{5^2.2}-\frac{1}{5^3.3}\right)+\left(\frac{1}{5^3.3}-\frac{1}{5^4.4}\right)+---\infty\right)$$

$$=\frac{1}{5}$$

57. If a, b, c, d are distinct integers in AP such that $d=a^2+b^2+c^2$ then a + b + c + d is

(A) 0 (B) 1

(C) 2 (D) None

Key. C

Sol. $d=a^2+b^2+c^2 \Rightarrow a+3t=(a+t)^2+a^2+(a+2t)^2$

$$5t^2+3(2a-1)t+3a^2-a=0$$

$$D\geq 0 \Rightarrow 24a^2+16a-9\leq 0$$

$$\Rightarrow -\frac{1}{3}-\frac{\sqrt{70}}{2}<a<-\frac{1}{3}+\frac{\sqrt{70}}{2}$$

$$\Rightarrow a=-1,0$$

$$a=0, t=0,\frac{3}{5}$$

$$a=-1,t=1,\frac{4}{5}$$

$$\Rightarrow t=1$$

$$a+b+c+d=2$$

58. If $b+c, c+a, a+b$ are in H.P then show that a^2, b^2, c^2 are in

(a) A.P (b) G.P (c) H.P (d) A.G.P

Key. A

Sol. $$\frac{1}{c+a}-\frac{1}{b+c}=\frac{1}{a+b}-\frac{1}{c+a}$$

59. Sum of first n terms of a sequence is given by $3\,S_n = T_n^{\,2} + 3T_n - 2,\ (T_n > 0)$ where T_n is the nth term of sequence, then the value of T_2^2 is

A) $2-\sqrt{2}$ B) $2+\sqrt{2}$ C) $2+3\sqrt{2}$ D) $3+2\sqrt{2}$

Key. C

Sol. $S_1 = \dfrac{T_1^{\,2}+3T_1-2}{3} = T_1 \Rightarrow T_1^{\,2} = 2$

$$S_2 - S_1 = \frac{T_2^{\,2}-T_1^2+3(T_2-T_1)}{3} = T_2$$

$$T_2^2 - T_1^2 + 3(T_2 - T_1) = 3T_2$$

$$\Rightarrow T_2^2 = T_1^2 + 3T_1 = 2 + 3\sqrt{2}$$

60. If a, b, c are three distinct numbers such that a, b, c are in A.P. and b – a, c – b, a are in G.P., then a : b : c are in

(A) 2 : 3 : 4 (B) 3 : 4 : 5

(C) 1 : 3 : 5 (D) 1 : 2 : 3

Key. D

Sol. a, b, c are in A.P. $\Rightarrow 2b = a + c$ …..(1)

(b – a), (c – b), a are in G.P. $\Rightarrow (c-b)^2 = a(b-a)$

$\Rightarrow c - a = a(b - a)$ ….(2)

from (1) and (2)

$$\frac{a}{1} = \frac{b}{2} = \frac{c}{3}$$

61. The sum to n terms of the series

$\dfrac{1}{\lfloor 1}\left(\dfrac{1}{2}\right) + \dfrac{1.3}{\lfloor 2}\left(\dfrac{1}{2}\right)^2 + \dfrac{1.3.5}{\lfloor 3}\left(\dfrac{1}{2}\right)^3 + ...$ upto n terms is

(A) $\dfrac{1.3.5.....(2n-1)(2n+1)}{2^n \lfloor n} - 1$ (B) $1 - \dfrac{1.3.5.....(2n-1)}{\lfloor n\ \lfloor n}$

(C) $1 - \dfrac{1.3.5.....(2n-3)}{2^{n-1}\lfloor n-1}$ (D) $\dfrac{1.3.5.....(2n-3)}{2^{n-1}\lfloor n-1}$

Key. A

Sol. $t_{n+1} = \dfrac{t_n \times (2n+1)}{(n+1)} \times \dfrac{1}{2}$

$(2n+2)t_{n+1} = (n+1)t_n$

$(2n+3)t_{n+1} - (2n+1)t_n = t_{n+1}$

Put n = 1

$5t_2 - 3t_1 = t_2$

n = 2, $7t_3 - 5t_2 = t_3$

$(2n+1)t_n-(2n-1)t_{n-1}=t_n$

$(2n+1)t_n-2t_1=S$

$S=\dfrac{1.3.5.....(2n+1)}{\lfloor n\times 2^n}-1$

62. The sum to n terms of the series

$\dfrac{1}{\lfloor 1}\left(\dfrac{1}{2}\right)+\dfrac{1.3}{\lfloor 2}\left(\dfrac{1}{2}\right)^2+\dfrac{1.3.5}{\lfloor 3}\left(\dfrac{1}{2}\right)^3+.....$ upto n terms is

(A) $\dfrac{1.3.5.....(2n-1)(2n+1)}{2^n\lfloor n}-1$ (B) $1-\dfrac{1.3.5.....(2n-1)}{\lfloor n\ \ \lfloor n}$

(C) $1-\dfrac{1.3.5.....(2n-3)}{2^{n-1}\lfloor n-1}$ (D) $\dfrac{1.3.5.....(2n-3)}{2^{n-1}\lfloor n-1}$

Key. A

Sol. $t_{n+1}=\dfrac{t_n\times(2n+1)}{(n+1)}\times\dfrac{1}{2}$

$(2n+2)t_{n+1}=(n+1)t_n$

$(2n+3)t_{n+1}-(2n+1)t_n=t_{n+1}$

Put n = 1

$5t_2-3t_1=t_2$

n = 2, $7t_3-5t_2=t_3$

$(2n+1)t_n-(2n-1)t_{n-1}=t_n$

$(2n+1)t_n-2t_1=S$

$S=\dfrac{1.3.5.....(2n+1)}{\lfloor n\times 2^n}-1$

63. If the ratio of the sum to 'n' terms of two A.P's is (5n+3):(3n+4), then the ratio of their 17th terms is

a) 172:99 b)168:103 c)175:99 d)171:103

Key. B

Sol. $\dfrac{\frac{n}{2}\left[2a_1+(n-1)d_1\right]}{\frac{n}{2}\left[2a_1+(n-1)d_2\right]}=\dfrac{5n+3}{3n+4}\Rightarrow\dfrac{a_1+\left(\frac{n-1}{2}\right)d_1}{a_1+\left(\frac{n-1}{2}\right)d_2}=\dfrac{5n+3}{3n+4}$ put $\dfrac{n-1}{2}=16$

64. If x,y,z are in G..P and $a^x=b^y=c^z$, then

A) $log_b a=\log_a c$ b) $log_a^b=\log_a c$ C) $log_b a=\log_c b$ D) None

Key. C

Sol. $a^n=b^y=c^z=k.,\ y^2=xz\Rightarrow\left(\text{lo}\,l_b k\right)^2=\log_a k\ ,=\log_a k\Rightarrow\left(\log b\right)^2=\log a.\log c$

65. If the pth, qth, r th terms of an A.P are in G.P, then common ratio of G..P is

a) $\frac{pr}{q^2}$ b) $\frac{r}{p}$ c) $\frac{q+r}{p+q}$ d) $\frac{q-r}{p-q}$

Key. D

Sol.
$a+(p-1)d=k$ $\quad Find \frac{②-①}{③-②}$
$a+(q-1)d=kr$
$a+(r-1)d=kr^2$

66. If H_1, H_2 ...., H_{20} be 20 harmonic means between 2 and 3, then $\frac{H_1+2}{H_1-2}+\frac{H_{20}+3}{H_{20}-3}=$

a) 20 b) 21 c) 40 d) 38

Key. C

Sol. $H_1=\frac{63}{31},\ H_{20}=\frac{126}{43}$

67. If $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+$ ....to $\infty=\frac{\pi^2}{6}$, then $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+$....................=

a) $\frac{\pi^2}{8}$ b) $\frac{\pi^2}{12}$ c) $\frac{\pi^2}{3}$ d) $\frac{\pi^2}{2}$

Key. A

Sol. $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+.........=\frac{\pi^2}{6}-\frac{1}{2^2}\left(\frac{\pi^2}{6}\right)=\frac{\pi^2}{8}$

68. The 20th term of 2,9,20,35,54, ...is

a) 819 b) 820 c) 1009 d) 1010

Key. A

Sol. $t_n=2+\left(7+11+15......(n-1)\,terms\right)$

69. If x > 1, y > 1, z > 1 and x, y, z are in G.P. then $(\ln x^2)^{-1}$, $(\ln xy)^{-1}$, $(\ln xz)^{-1}$ are in
(A) A.P. (B) G.P.
(C) H.P. (D) none of these

Key. C

Sol. x > 1, y > 1, z > 1
x, y, z → G.P. ⇒ $l$nx, $l$ny, $l$nz are in A.P. 2$l$nx, $l$nxy, $l$nxz are in A.P.
$(l\text{n}x^2)^{-1}$, $(\text{in}xy)^{-1}$, $(l\text{n}xz)^{-1}$ are in H.P.

70. If $S_n=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+.....+\frac{1}{n}$ and n > 2, then S_n would always be

(A) more than $n(n+1)^{\frac{1}{n}}-n$ (B) less than $n(n+1)^{1/n}-n$

(C) equal to $n(n+1)^{\frac{1}{n}}-n$ (D) greater than or equal to $\frac{n(n+1)^{\frac{1}{n}}}{(n+5)}$

Key. A

Sol. $$\frac{(1+1)+\left(1+\frac{1}{2}\right)+\left(1+\frac{1}{3}\right)+\left(1+\frac{1}{4}\right)+.....+\left(1+\frac{1}{n}\right)}{n} > \left(2\times\frac{3}{2}\times\frac{4}{3}\times\frac{5}{4}\times.....\times\frac{n+1}{n}\right)^{1/n}$$

$$\Rightarrow \frac{n+S_n}{n} > (n+1)^{1/n} \;\Rightarrow\; S_n > n(n+1)^{1/n} - n$$

71. If a,b,c are in AP, then the sum of the coefficients of $\left\{1+\left(ax^2-2bx+c\right)^2\right\}^{1973}$ is

a) -2 b) -1 c) 0 d) 1

Key. D

Sol. $\because$ a, b, c are in A.P.

$\Rightarrow 2b = a+c$

$\Rightarrow a-2b+c=0$

Putting x=1

Required sum = $\left(1+a-2b+c\right)^{1973} = \left(1+0\right)^{1973} = 1$

72. $\{a_n\}$ and $\{b_n\}$ be two sequences given by $a_n = (x)^{1/2^n} + (y)^{1/2^n}$ and $b_n = (x)^{1/2^n} - (y)^{1/2^n}$ for all $n\in N$, then $a_1\, a_2\, a_3 \dots a_n$ is equal to

(A) $x - y$ (B) $\frac{x+y}{b_n}$

(C) $\frac{x-y}{b_n}$ (D) $\frac{xy}{b_n}$

Key. C

Sol. $a_1\, a_2 \dots a_n = b_n \dfrac{a_1 a_2 \dots a_n}{b_n}$

$= a_n\, b_n \dfrac{(a_1 a_2 \dots a_{n-1})}{b_n}$

$= \left(x^{\frac{1}{2^{n-1}}} - y^{\frac{1}{2^{n-1}}}\right)\dfrac{(a_1\, a_2 \dots a_{n-1})}{b_n} = a_{n-1}\; b_{n-1}\; \dfrac{(a_1 a_2 \dots a_{n-2})}{b_n}$

$= \dfrac{a_1 b_1}{b_n} = \dfrac{(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})}{b_n} = \dfrac{x-y}{b_n}$

73. $\frac{1}{2} - \frac{1}{2.2^2} + \frac{1}{3.2^3} - \frac{1}{4.2^4} + ... =$

1) $\frac{1}{4}$ 2) $\log_e\left(\frac{3}{4}\right)$ 3) $\log_e\left(\frac{3}{2}\right)$ 4) $\log_e\left(\frac{2}{3}\right)$

Key. 3

Sol. $\log_e\left(1+\frac{1}{2}\right) = \log_e \frac{3}{2}$

74. The ratio of sum of first three terms of a G.P. to the sum of first six terms is $64:91$,the common ratio of G.P. is

1. $\frac{1}{4}$ 2. $\frac{3}{4}$ 3. $\frac{5}{4}$ 4. $\frac{7}{4}$

Key. 2

Sol. Given $\frac{S_3}{S_6}=\frac{64}{91}=\frac{a\left(r^3-1\right)}{a\left(r^6-1\right)}$

$$\Rightarrow \frac{\left(r^3-1\right)}{\left(r^3+1\right)\left(r^3-1\right)}=\frac{64}{91}$$

$$\Rightarrow r^3=\frac{27}{64} \quad \therefore \quad r=\frac{3}{4}$$

75. Sum of the series $3+5+9+17+33+.....$to n terms is

1. $2^{n+1}-n-2$ 2. $2^{n+1}+n-2$ 3. 2^n+n-2 4. $2^{n+1}-n+2$

Key. 2

Sol. $S_n=3+5+9+17+33+......$

$$=\left(2+1\right)+\left(2^2+1\right)\left(2^3+1\right)+\left(2^4+1\right)+....$$

$$=\left(2+2^2+2^3+2^4+......n\text{ terms}\right)+n$$

$$=2\left(2^n-1\right)+n=2^{n+1}+n-2$$

$$=2^{n+1}+n-2$$

76. If one A.M. A and two G.M.s p and q be inserted between two numbers a and b, then which of the following is hold good

1. $a^3+b^3=2Apq$ 2. $p^3+q^3=2Apq$ 3. $a^3+b^3=2Aab$ 4. None of these.

Key. 2

Sol. Given $a+b=2A$

And $a,p,q,\quad b\in$ G.P.

$\therefore \quad p^2=aq$ and $q^2=pb$

$\Rightarrow p^3=apq$ and $q^3=bpq$

by adding we get

$$p^3 + q^3 = apq + bpq$$

$$= pq(a+b) = 2Apq$$

77. If fourth term of a G.P. is 3, the product of the first seven terms is

1. 3^4 2. 3^7 3. 7^4 4. 4^7

Key. 2

Sol. As the number of terms are odd (7) let r, be the common ratio

So terms can be taken as $\frac{a}{r^3}, \frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2, ar^3$

$\therefore$ Product of the term $= a^7$

$= 3^7$ as ($t_4 = a = 3$)

78. The number of divisors of 6912, 52480,32000 are in

1. A.P Only 2. G.P. Only 3. A.P. , G.P.& H.P. 4. None of these

Key. 3

Sol. If n is a + ve number.

$$n = P_1^{k_1}.P_2^{k_2}...P_r^{k_r}$$

(where $p_1, p_2, p_3, ...p_r$ are prime number) then number of divisors of n are

$$= (k_1+1)(k_2+1)....(k_r+1)$$

$\therefore$ Number of prime factor of 6912 are $= 2^8.3^3$ so no. of divisors $= 9 \times 4 = 36$

Prime factors of 52,400 are $= 3^8 \times 2^3$

$\therefore$ No. of divisors $= 9 \times 4 = 36$

Prime factors of 32,000 are $= 5^3 \times 2^8$

$\therefore$ No. of divisors $= 9 \times 4 = 36$

Now each number having same number of divisors *i.e.*, 36,36,36

Each and every term is constant & constant sequence is always in A.P.& G.P. both as common difference is 0 and common ratio is 1.

79. If 1, $\log_{81}(3^x + 48)$, $\log_9\left(3^x - \frac{8}{3}\right)$ are in A.P., then the value of x equals

1. 9 2. 6 3. 2 4. 4

Key. 3

Sol. Given 1, $\log_9 2\left(3^x + 48\right), \log_9\left(3^x - 8/3\right), \in$ A.P.

$\Rightarrow \log_9 9,\ \frac{1}{2}\log_9\left(3^x + 48\right),\ \log_9\left(3^x - 8/3\right) \in$ A.P.

$\Rightarrow 9, \left(3^x + 48\right)^{1/2}, 3^x - 8/3 \in$ G.P. (By concept)

$\Rightarrow \log a, \log b, \log c \in$ A.P.

$\therefore a, b, c \in$ G.P. $\therefore 3^x + 48 = 9\left(3^x - 8/3\right)$

$8.3^x = 72$

$3^x = 9,\ 3^x = 3^2,\ x = 2$.

80. If a, b, c are in H.P., then $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in

1. A.P. 2. H.P. 3. G.P. 4. None of these

Key. 2

Sol. Given $a, b, c \in$ H.P.

So, $\frac{1}{a}, \frac{1}{b}, \frac{1}{c} \in$ A.P.

$\frac{a+b+c}{a}, \frac{b+c+a}{b}, \frac{c+a+b}{c} \in$ A.P.

By using concept if $a, b, c \in$ A.P.

Then their reciprocals are in H.P.

81. Between the numbers 2 and 20, 8 means are inserted, then their sum is

1. 88 2. 44 3. 176 4. None of these

Key. 1

Sol. Let $a\ , A_1, A_2....A_g, b \in A.P$

Where $a = 2, b = 20, n = 8$

$\therefore$ sum of the means $= \frac{n}{2}(a+b) = \frac{8}{2}(2+20) = 88$

82 If the arithmetic mean of two positive numbers $a \& b\ (a > b)$ is twice their G.M., then $a:b$ is

1. $6+\sqrt{7}:6-\sqrt{7}$ 2. $2+\sqrt{3}:2-\sqrt{3}$ 3. $5+\sqrt{6}:5-\sqrt{6}$ 4. None of these

Key. 2

Sol. $\frac{a+b}{2} = 2\sqrt{ab}$

$a+b-4\sqrt{ab} = 0$

$\frac{a}{b}+1-4\sqrt{\frac{a}{b}} = 0$ (Dividing by b)

Or $\left(\sqrt{\frac{a}{b}}\right)^2 - 4\sqrt{\frac{a}{b}}+1 = 0$

$\therefore \sqrt{\frac{a}{b}} = \frac{4 \pm 2\sqrt{3}}{2} = \left(2 \pm \sqrt{3}\right)$

$\frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$

83 The number of terms common between the two series 2 + 5 + 8 + …up to 50 terms and the series 3 + 5 + 7 + 9 + ….up to 60 terms.

1. 24 2. 26 3. 25 4. None of these

Key. 4

Sol. Let m^{th} term of first A.P. be equal to the n^{th} term of the second A.P. then

2 , 5 , 8 ,….50 terms series 1

3 , 5 ,7 , …., 60 terms series 2

Common series 5 , 11, 17 , ….., 119

40^{th} term of series $1 =$ 59^{th} term of series 2 = 119 = last term of common series

$\Rightarrow\ a_n = 5+(n-1)d \Rightarrow 119+1 = 6n \Rightarrow n = 20.$

$\therefore$ Number of common terms is 20.

84. If a,b,c are three positive numbers, then the minimum value of the expression $\dfrac{ab(a+b)+bc(b+c)+ca(c+a)}{bca}$

1. 3 2. 4 3. 6 4. 1

Key. 3

Sol. Given expression equal to

$$\frac{(a+b)}{c}+\frac{(b+c)}{a}+\frac{(c+a)}{b}$$

Or $\dfrac{a}{c}+\dfrac{b}{c}+\dfrac{b}{a}+\dfrac{c}{a}+\dfrac{c}{b}+\dfrac{a}{b}$

Using A.M. $\geq$ G.M. $\dfrac{\frac{a}{c}+\frac{b}{c}+\frac{b}{a}+\frac{c}{a}+\frac{c}{b}+\frac{a}{b}}{6} \geq \sqrt{\frac{a}{c}\frac{b}{c}\frac{b}{a}\frac{c}{a}\frac{c}{b}\frac{a}{b}}$

Or $\dfrac{a+b}{c}+\dfrac{b+c}{a}+\dfrac{c+a}{b} \geq 6$

85. Sum of n terms of series $ab+(a+1)(b+1)+(a+2)(b+2)+....+(a+(n-1))(b+(n-1))$ if $ab=\dfrac{1}{6}\, and\, (a+b)=\dfrac{1}{3}$, is

(A) $\dfrac{n}{6}(1-2n)^2$ (B) $\dfrac{n}{6}(1+n-2n^2)$ (C) $\dfrac{n}{6}(1-2n+2n^2)$ (D) none of these

Key. C

Sol. $s = ab + [ab + (a+b) + 1] + [ab + 2(a+b) + 2^2] + ...[ab + (n-1)(a+b) + (n-1)^2]$

$= nab + (a+b)\sum_{r=1}^{n-1} r + \sum_{r=1}^{n-1} r^2$

$= nab + (a+b)\dfrac{n(n-1)}{2} + \dfrac{(n-1))(n)(2n-1)}{6}$

$= \dfrac{n}{6}[1 + (n-1)\{1 + 2n - 1\}]$

$= \dfrac{n}{6}[1 + 2n(n-1)] = \dfrac{n}{6}(1 - 2n + 2n^2)$

86. If log (a+c),log(a+b), log(b+c) are in A.P. and a,c,b are in H.P, then the value of a+b is (given a,b,c >0)

(A) 2c (B) 3c (C) 4c (D) 6c

Key. A

$\log(a+c) + \log(b+c) = 2\log(a+b)$

$(a+c)(b+c) = (a+b)^2$

Sol. $\Rightarrow ab + c(a+b) + c^2 = (a+b)^2$ (1)

$also, c = \dfrac{2ab}{a+b} \Rightarrow 2ab = c(a+b)$

$\Rightarrow 2ab + 2c(a+b) + 2c^2 = 2(a+b)^2$ ..... (2)

From (1) and(2),

$c(a+b) + 2c(a+b) + 2c^2 = 2(a+b)^2$

$2(a+b)^2 - 3c(a+b) - 2c^2 = 0$

$\therefore a+b = \dfrac{3c \pm \sqrt{9c^2 + 16c^2}}{4} = \dfrac{3c \pm 5c}{4} = 2c \ or - \dfrac{c}{2}$

$\therefore a+b = 2c$ $(\because a,b,c > 0)$

87. If $a_1, a_2, a_3, ..., a_n$ are in A.P. with s_n as the sum of first 'n' terms $(s_0 = 0)$, then

$\sum_{k=0}^{n} {}^nC_k s_k$ is equal to

(A) $2^{n-2}[na_1 + s_n]$ (B) $2^n[a_1 + s_n]$ (C) $2[na_1 + s_n]$ (D) $2^{n-1}[a_1 + s_n]$

Key. A

Sol. $\sum_{k=o}^{n} {}^nc_k s_k = \sum_{k=0}^{n} {}^nc_k \dfrac{k}{n}[2a + (k-1)d]$

$= [(a_1 - \dfrac{d}{2})\sum_{k=0}^{n} k\,{}^nc_k + \dfrac{d}{2}\sum_{k=0}^{n} k^2\,{}^nc_k]$

$$= \left(a_1 - \frac{d}{2}\right) n.2^{n-1} + \frac{d}{2}[n.2^{n-1} + n(n-1)2^{n-2}]$$

$$= a_1.n.2^{n-1} + dn(n-1)2^{n-3}$$

$$= n.2^{n-3}[4a_1 + a_n - a_1] = n.2^{n-3}[3a_1 + a_n]$$

$$= 2^{n-3}[2na_1 + 2n\left(\frac{a_1 + a_n}{2}\right)]$$

$$= 2^{n-2}[na_1 + s_{n].}$$

88. If a,b,c, are in A.P. and p, p' are respectively A.M. and G.M. between a and b while q, q' are respectively AM. And G.M. between b and c, then

(A) $p^2 + q^2 = p'^2 + q'^2$ (B) $pq = p'q'$

(C) $p^2 - q^2 = p'^2 - q'^2$ (D) $p^2 + p'^2 = q^2 + q'^2$

Key. C

Sol. We have $2b = a + c$ and a,p,b,q,c are in A.P

$$\Rightarrow p = \frac{a+b}{2}, q = \frac{b+c}{2}$$

Again , $p' = \sqrt{ab}$ *and* $q' = \sqrt{bc}$

$$\therefore p^2 - q^2 = \frac{(a+b)^2 - (b+c)^2}{4}$$

$$= \frac{(a-c)(a+c+2b)}{4} = (a-c)b = p'^2 - q'^2$$

89. Through the centroid of an equilateral triangle a line parallel to the base is drawn. On this line, an arbitrary point p is taken inside the triangle. Let h denote the distance of p from the base of the triangle. Let h_1 and h_2 be the distance of p from the other two sides of the triangle, then

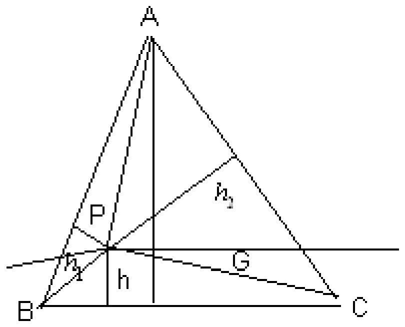
(A) h is the H.M. of h_1, h_2 (B) h is the G.M. of h_1, h_2

(C) h is the A.M, of h_1, h_2 (D) None of these

Key. C

Sol. $\Delta ABC = \Delta PBC + \Delta PAC + \Delta PAB$

$$\frac{1}{2}.a.3h = \frac{1}{2}a.h + \frac{1}{2}a\ h_1 + \frac{1}{2}a.h_2$$



$$h_1 + h_2 = 2h \Rightarrow h = \frac{h_1 + h_2}{2}$$

90. a,b,c are positive integers forming an increasing G.P. whose common ratio is a natural number, b – a is cube of a natural number and $\log_6 a + \log_6 b + \log_6 c = 6$, then a + b + c =

A) 100 B) 111 C) 122 D) 189

Key. D

Sol. $\log_6(abc) = 6 \Rightarrow (abc) = 6^6$

Let $a = \frac{b}{r}$ and c = br

$\Rightarrow$ b = 36 and $a = \frac{36}{r}$ $\Rightarrow$ r = 2, 3, 4, 6, 9, 12, 18

Also b – a = 36 $\left(1 - \frac{1}{r}\right)$ is a perfect cube. $\therefore$ r = 4

$\Rightarrow$ a + b + c = 36 + 9 + 144=189

91. If S,P and R are the sum, product and sum of the reciprocals of n terms of an increasing G.P. and $S^n = R^n . P^k$, then k is equal to

A) 1 B) 2 C) 3 D) none of these

Key. B

Sol. $S = \frac{a(1 - r^n)}{1 - r}, P = a^n . r^{\frac{n(n-1)}{2}}$

$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} ....$ to n terms $= \frac{1 - r^n}{a(1 - r)r^{n-1}}$

$S^n = R^n P^k \quad \Rightarrow \quad \left(\frac{S}{R}\right)^n = P^k$

$\Rightarrow \quad (a^2 r^{n-1})^n = P^k$

$\Rightarrow \quad P^2 = p^k \quad \Rightarrow \quad k = 2$

92. Sum of first hundred numbers common to the two A.P.'s 12, 15, 18, … and 17, 21, 25 …..

A) 56100 B) 65100 C) 61500 D) none of these

Key. C

Sol. AP I = 12, 15, 18, … (common difference $d_1 = 3$)

AP II = 17, 21, 25… (common difference $d_2 = 4$)

First term of the series of common numbers = 21

Here a = 21, common difference of the series of common numbers = L.C.M of d_1 and $d_2 = 12$

$\therefore$ Required sum of first hundred terms

$$= \frac{100}{2}\left[2\times 21+(100-1)12\right]=100[21+594]=61500$$

93. If 11 A.M. s are inserted between 28 and 10, then number of integral A.M's is

A) 5 B) 6 C) 7 D) 8

Key. A

Sol. Since $A_1, A_2, A_3, ..., A_{11}$ be 11 A.M. s between 28 and 10.

$\therefore$ 28, $A_1, A_2, ..., A_{11}$, 10 are in A.P.

Let 'd' be the common difference of A.P.

Also the number of terms = 13.

$10 = T_{13} = T_1 + 12d = 28 + 12d$

$\therefore \quad d = \frac{10-28}{12} = -\frac{18}{12} = -\frac{3}{2}$

$\therefore$ Number of integral A.M's is 5.

94. If a,b,c are in HP, then $\frac{1}{b-a}+\frac{1}{b-c}$ is equal to

A) $\frac{2}{b}$ B) $\frac{2}{a+c}$ C) $\frac{1}{a+c}$ D) none of these

Key. A

Sol. Q a,b,c are in H.P.

Q $b = \frac{2ac}{(a+c)}$

$\therefore \quad \frac{1}{b-a}+\frac{1}{b-c}$

$$\Rightarrow \frac{1}{\frac{2ac}{(a+c)}-a}+\frac{1}{\frac{2ac}{(a+c)}-c}$$

$$\Rightarrow (a+c)\left\{\frac{1}{a(c-a)}+\frac{1}{c(a-c)}\right\} \Rightarrow \frac{(a+c)}{(a-c)}\left\{-\frac{1}{a}+\frac{1}{c}\right\}$$

$$\Rightarrow \frac{(c+a)(a-c)}{ac(a-c)} \Rightarrow \frac{(a+c)}{ac}=\frac{2}{b}$$

95. Let a_n be the n^{th} term of an A.P. If $\sum_{r=1}^{100} a_{2r} = \alpha$ and $\sum_{r=1}^{100} a_{2r-1} = \beta$, then the common difference of the A.P. is

A) $\alpha-\beta$ B) $\beta-\alpha$ C) $\frac{\alpha-\beta}{2}$ D) none of these

Key. D

Sol. $a_1 + a_3 + a_5 + ... + a_{199} = \beta$

$a_2 + a_4 + a_6 + ... + a_{200} = \alpha$

$a_2 - a_1 + a_4 - a_3 + a_6 - a_5 ... a_{200} - a_{199} = \alpha - \beta$

$d + d + d \ldots\ldots\ldots d = \alpha - \beta$

$d = \frac{\alpha - \beta}{100}$

96. If a,b,c,d are in G.P., then $\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)$ equals to

A) a b + b c + c d B) $(a\,b + b\,c + c\,d)^2$ C) $(a\,b + b\,c + c\,d)^4$ D) none of these

Key. B

Sol. a,b,c,d are in G.P., let they are a, ar, ar^2, ar^3

$\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)$

$= a^2 \times a^2\left[1+r^2+r^4\right]\left[r^2+r^4+r^6\right]$

$= a^4r^2\left[1+r^2+r^4\right]^2$

$= \left[a^2r\left[1+r^2+r^4\right]\right]^2$

$= \left(ab+bc+cd\right)^2$

97. If a_1, a_2, a_3, a_4, a_5 are in H.P., then $a_1a_2 + a_2a_3 + a_3a_4 + a_4a_5$ is equal to

A) $2a_1a_5$ B) $3a_1a_5$ C) $4a_1a_5$ D) – 4

Key. C

Sol. a_1, a_2, a_3, a_4, a_5 are in H.P.

$\Rightarrow \quad a_2 = \frac{2a_1a_2}{a_1+a_3} \Rightarrow 2a_1a_3 = a_2a_1 + a_3a_2$

$a_4 = \frac{2a_3a_5}{a_3+a_5} \Rightarrow 2a_3a_5 = a_3a_4 + a_5a_4$

$\Rightarrow \quad a_1a_2 + a_2a_3 + a_3a_4 + a_4a_5 = 2a_1a_3 + 2a_3a_5$ …(i)

$a_3 = \frac{2\left(a_1a_5\right)}{a_1+a_5} \Rightarrow a_1a_3 + a_5a_3 = 2a_1a_5$ …(ii)

using (i) & (ii)

$a_1a_2 + a_2a_3 + a_3a_4 + a_4a_5 = 2\left(2a_1a_5\right) = 4a_1a_5$

98. If the sum to infinity of the series, $1+4x+7x^2+10x^3+........$, is $\frac{35}{16}$, where $|x| < 1$, then 'x' equals to

A) 19/7 B) 1/5 C) 1/4 D) none of these

Key. B

Sol. $S = 1+4x+7x^2+10x^3+............$

$x.S = x+4x^2+7x^3+.............$

Subtract

$S\left(1-x\right) = 1+3x+3x^2+3x^3+...........$

$S\left(1-x\right) = 1+3x\left(\frac{1}{1-x}\right) \qquad |x|<1$

$S = \frac{1+2x}{\left(1-x\right)^2}$

Given $\dfrac{1+2x}{(1-x)^2}=\dfrac{35}{16}$

$\Rightarrow \quad 16+32x=35+35x^2-70x \qquad \Rightarrow \quad 35x^2-102x+19=0$

$\Rightarrow \quad 35x^2-7x-95x+19=0 \qquad \Rightarrow \quad 7x(5x-1)-19(5x-1)=0$

$\Rightarrow \quad (5x-1)(7x-19)=0 \qquad \Rightarrow \quad x=\dfrac{1}{5},\dfrac{19}{7}$

But $|x|<1$ $\quad \therefore \quad x=\dfrac{1}{5}$

99. If a,b,c and d are four positive real numbers such that abcd = 1, the minimum value of (1 + a) (1 + b) (1 + c) (1 + d) is

A) 4 B) 1 C) 16 D) 18

Key. C

Sol. $1+a\ge 2\sqrt{a}$ $\quad \{AM\ge GM\}$

$1+b\ge 2\sqrt{b}$

$1+c\ge 2\sqrt{c}$

$1+d\ge 2\sqrt{d}$

$\therefore \quad (1+a)(1+b)(1+c)(1+d)\ge 16\sqrt{abcd}=16$

$\therefore$ min. value = 16 (for a = b = c = d = 1)

100. If the length of sides of a right triangle are in A.P., then the sines of the acute angle are

A) $\dfrac{3}{5},\dfrac{4}{5}$ B) $\sqrt{\dfrac{2}{3}},\sqrt{\dfrac{1}{3}}$

C) $\sqrt{\dfrac{\sqrt{5}-1}{2}},\sqrt{\dfrac{\sqrt{5}+1}{2}}$ D) $\sqrt{\dfrac{\sqrt{3}-1}{2}},\sqrt{\dfrac{\sqrt{3}+1}{2}}$

Key. A

Sol. Let the sides be a – d, a, a + d

Where a > d > 0

We have

$(a+d)^2=(a-d)^2+a^2$

$\Rightarrow \quad d=\dfrac{a}{4}$ we have $\sin\theta=\dfrac{a}{a+d} \quad \Rightarrow \quad \cos\theta=\dfrac{3}{5}, \qquad \sin\theta=\dfrac{4}{5}$

101. If $a_1,a_2,....a_n$ n distinct odd natural numbers not divisible by any prime greater than 5, then $\dfrac{1}{a_1}+\dfrac{1}{a_2}+....\dfrac{1}{a_n}$ is less than

A) $\dfrac{15}{8}$ B) $\dfrac{16}{8}$ C) $\dfrac{8}{15}$ D) $\dfrac{15}{4}$

Key. A

Sol. Since each a_1 is an odd number not divisible by a prime greater than 5, a_1 can be written as $a_1=3^r5^8$ where r, s are non-negative integers.

thus for all $n\in N$

$$\frac{1}{a_1}+\frac{1}{a_2}+...+\frac{1}{a_n}<\left(1+\frac{1}{3}+\frac{1}{3^2}+...+\right)\left(1+\frac{1}{5}+\frac{1}{5^2}+...+\right)=\frac{15}{8}$$

102. If the m^{th} term of the sequence defined by $t_n = \frac{\sqrt{n}}{n+2008}$ is the greatest term then $m =$

A) 2006 B) 2007 C) 2008 D) 2009

Key. C

Sol. Consider the function $f(x) = \frac{\sqrt{x}}{x+2008}, x \geq 1$

$$f^1(x) = \frac{(x+2008)\times\frac{1}{2\sqrt{x}} - \sqrt{x}}{(x+2008)^2}$$

$$= \frac{x+2008-2x}{2\sqrt{x}\,(x+2008)^2} = \frac{2008-x}{2\sqrt{x}(x+2008)^2}$$

$$f^1(x) = 0 \Rightarrow x = 2008$$

$$x \in (2008-\delta, 2008),\ f^1(x) > 0;\ x \in (2008, 2008+\delta),\ f^1(x) < 0$$

103. If a_1, a_2, a_3, ..., a_9 are in H.P. and $a_4 = 5$, $a_5 = 4$ then $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} =$

A) 31/15 B) 41/18 C) 50/21 D) 61/27

Key. C

Sol. Let $a_i = \frac{1}{a+(i-1)d}$, $i = 1,2,3,....,9$

$$a_4 = \frac{1}{a+3d} = 5 \Rightarrow a+3d = \frac{1}{5}$$

$$a_5 = \frac{1}{a+4d} = 4 \Rightarrow a+4d = \frac{1}{4}$$

$$\therefore a = d = \frac{1}{20} \Rightarrow a_i = \frac{20}{i}$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = (20)^3 \begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \end{vmatrix} = \frac{50}{21}$$

104. If $\log_{ax} x$, $\log_{bx} x$ $\log_{cx} x$ are in H.P. where a, b, c, x belong to (1, ∞), then a, b, c are in

(A) A.P. (B) G.P.

(C) H.P. (D) A.G.P.

Key. B

Sol. Since $\log_{ax}x$, $\log_{bx}x$, $\log_{cx}x$ are in H.P.

$\therefore$ $\log_x ax, \log_x bx, \log_x cx$ are in A.P.

$\Rightarrow$ $1+\log_x a, 1+\log_x b, 1+\log_x c$ are in A.P.

$\Rightarrow \frac{\log a}{\log x}, \frac{\log b}{\log x}, \frac{\log c}{\log x}$ are in A.P.

$\Rightarrow \log a, \log b, \log c$ are in A.P.

$\Rightarrow 2\log b = \log a + \log c = \log ac$

$\Rightarrow \log b^2 = \log ac \Rightarrow b^2 = ac$

$\Rightarrow$ a, b, c are in G.P.

$\therefore (b)$ holds.

AP, GP, HP, Sequences

Multiple Correct Answer Type

1. If a,b,c,d are four unequal positive number which are in A.P then

A) $\frac{1}{a}+\frac{1}{d}=\frac{1}{b}+\frac{1}{c}$ B) $\frac{1}{a}+\frac{1}{d}<\frac{1}{b}+\frac{1}{c}$ C) $\frac{1}{a}+\frac{1}{d}>\frac{1}{b}+\frac{1}{c}$ D) $\frac{1}{b}+\frac{1}{c}>\frac{4}{a+d}$

Key. C,D

Sol. Let $b=a+p, c=a+2p, d=a+3p$

$$\frac{\frac{1}{a}+\frac{1}{d}}{\frac{1}{b}+\frac{1}{c}}=\frac{\frac{1}{a}+\frac{1}{a+3p}}{\frac{1}{a+p}+\frac{1}{a+2p}}=\frac{(1+p)(1+2p)}{1(a+3p)}$$

$$=\frac{a^2+3ap+2p^2}{a^2+3ap}>1$$

$$\therefore \frac{1}{a}+\frac{1}{d}>\frac{1}{b}+\frac{1}{c}$$

$$\left(\frac{1}{b}+\frac{1}{c}\right)(a+d)\left(\frac{1}{a+p}+\frac{1}{a+2p}\right)(a+a+3p)$$

$$=\frac{(2a+3p)^2}{a^2+3ap+2p^2}=4\frac{p^2}{a^2+3ap+2p^2}>4$$

2. If the first and $(2n-1)^{th}$ terms of an A.P , G.P and H.P of positive terms are equal and their n^{th} terms are a,b,c respectively , then

A) a=b=c B) $a\ge b\ge c$ C) $b^2=ac$ D) $a+c=2b$

Key. B,C

Sol. Let A.P be $A, A+d, A+2d, ...$ Then

$t_{2n-1}=A+(2n-2)d=x(say)$, Then

$$(n-1)d=\frac{x-A}{2}$$

$$\therefore a=t_n=A+(n-1)d=a+\frac{x-A}{2}=\frac{A+x}{2}$$

Let G.P be A, AR, A R^2, ... Then

$$t_{2n-1}=A\,R^{2n-2}=x \Rightarrow R^{n-1}=\left(\frac{x}{A}\right)^{\frac{1}{2}}$$

$$\therefore b=t_n=A\,R^{n-1}=A\left(\frac{x}{A}\right)^{\frac{1}{2}} \Rightarrow \sqrt{Ax}$$

Let H.P be A $\frac{1}{\frac{1}{A}+D}, \frac{1}{\frac{1}{A}+2D}$ ....*then*

$t_{2n-1}=\frac{1}{\frac{1}{A}+(2n-2)}=x$ then

$(n-1)D=\frac{1}{2}\left(\frac{1}{x}-\frac{1}{A}\right)$

$\therefore c=t_n=\frac{1}{\frac{1}{A}+(n-1)D}=\frac{1}{\frac{1}{A}+\frac{1}{2}\left(\frac{1}{x}-\frac{1}{A}\right)}$

$=\frac{1}{\frac{1}{2}\left(\frac{1}{x}+\frac{1}{A}\right)}$

Clearly a,b, and c are A.M ., G.M and H.M between the numbers , x and A respectively

Hence $a\ge b\ge c$ also $b^2=ac$

3. In a G.P., the product of first four terms is 4 and the second term is the reciprocal of the fourth term. The sum of infinite terms of the G.P. can be

(A)-8 (B) $-\frac{8}{3}$ (C) $\frac{8}{3}$ (D) 8

Key. A,B,C,D

Sol. Let a, ar, ar^2, ar^3 are the first four terms of the G.P

$\therefore$ $a^4r^6=4 \,\&\, a^2r^4=1 \Rightarrow r^2=\frac{1}{4} \Rightarrow r=\pm\frac{1}{2} \,\&\, a=\pm 4$

$\therefore$ Sum of infinite G.P $=\frac{a}{1-r}=8,-8,\frac{8}{3},-\frac{8}{3}$

4. If 3 positive real number a,b,c are in A.P with abc=4 then [b] can be equal to (where [.] represents the integral part)

A) 1 B) 2 C) 3 D) 4

Key. A,B,C,D

Sol. $b\ge\sqrt{ac}\Rightarrow b^3\ge abc$

$\Rightarrow b^3\ge 4 \text{ or } b\ge(4)^{1/3}\Rightarrow[b]\ge 1$

5. If a, b, c are first three terms of a G.P. if the harmonic mean of a and b is 12 and arithmetic mean of b & c is 3, then

(A) no term of this G.P. is square of an integer

(B) arithmetic mean of a, b, c is 3

(C) b = ± 6

(D) common ratio of this G.P. is 2

Key. A,B

6. Suppose 'f' and 'g' are functions having second derivatives f'' and g'' everywhere, if $f(x).g(x)=1$ for all 'x' and f' and g' are never zero, then $\frac{f''(x)}{f'(x)}-\frac{g''(x)}{g'(x)}$ equals

A) $\frac{-2f'(x)}{f(x)}$ B) $\frac{-2g'(x)}{g(x)}$ C) $\frac{-f'(x)}{f(x)}$ D) $\frac{2f'(x)}{f(x)}$

Key. B,D

Sol. $g=\frac{1}{f} \Rightarrow g'=\frac{-1}{f^2}f'$

$$\Rightarrow g''=-\left[\frac{-2}{f^3}f'^2+\frac{1}{f^2}f''\right]=\frac{2}{f^3}f'^2-\frac{f''}{f^2}$$

$$\Rightarrow \frac{f''}{f'}-\frac{g''}{g}=\frac{f''}{f'}-\frac{\frac{2}{f^3}f'^2-\frac{f''}{f^2}}{\frac{-1}{f^2}f'}=\frac{f''}{f'}-\left(\frac{-2f'}{f}+\frac{f''}{f}\right)=\frac{2f'}{f}$$

In a similar manner, we can show that the same is equal to $\frac{2g'}{g}$.

7. For a positive integer n, $let\ S(n)=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+.....+\frac{1}{2^n-1}$.Then,

a) $S_n\le n$ b) $S_n>n$

c) $S_{2n}\le n$ d) $S_{2n}>n$

KEY: A,D

HINT : $S(n)=1+\left(\frac{1}{2}+\frac{1}{3}\right)+\left(\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}\right)+...+\left(\frac{1}{2^{n-1}}+\frac{1}{2^{n-1}+1}+...+\frac{1}{2^n-1}\right)$

$\le 1+\left(\frac{1}{2}+\frac{1}{2}\right)+\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right)+...+\left(\frac{1}{2^{n-1}}+\frac{1}{2^{n-1}}+...+\frac{1}{2^{n-1}}\right)$

= 1 + 1 + 1 + ...+ 1 (n terms) = n

Also $S(n)\ge 1+\frac{1}{2}+\left(\frac{1}{3}+\frac{1}{4}\right)+\left(\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}\right)+...+\left(\frac{1}{2^{n-2}+1}+\frac{1}{2^{n-2}+2}+...+\frac{1}{2^{n-1}}\right)$

$>1+\frac{1}{2}+\frac{1}{2}+...+\frac{1}{2}=1+\left(\frac{n-1}{2}\right)=\frac{n+1}{2}$

$\therefore S(2n)>\frac{2n+1}{2}=n+\frac{1}{2}>n$

8. If $a_1, a_2, a_3.....a_n$ is sequence of +Ve numbers which are in AP with common difference 'd' & $a_1 + a_4 + a_7 + ... + a_{16} = 147$ then.

A) $a_1 + a_6 + a_{11} + a_{16} = 98$

B) $a_1 + a_{16} = 49$

C) $a_1 + a_4 + a_7 + ...a_{16} = 6a_1 + 45\,\text{d}$

D) Maximum value of $a_1 a_2 ....a_{16}$ is $\left(\frac{49}{2}\right)^{16}$

KEY : A,B,C,D

SOL : $a_1 + a_4 + a_7 + ... + a_{16} = 147$

$\Rightarrow 3(a_1 + a_{16}) = 147 \Rightarrow a_1 + a_{16} = 49$.

Again $a_1 + a_4 + a_7 + a_{10} + ... + a_{16}$

$= a_1 + a_1 + 3d + a_1 + 6d + ... + a_1 + 15d$

$= 6a_1 + 45d = 147$

$\Rightarrow 2a_1 + 15d = 49$

$a_1 + a_6 + a_{11} + a_{16} = a_1 + a_1 + 5d + a_1 + 10d + a_1 + 15d$

$= 4a_1 + 30d$

$= 2(2a_1 + 15d)$

$= 2(49) = 98$

Now using $AM \geq GM$

$$\frac{a_1 + a_2 + ... + a_{16}}{16} \geq (a_1 a_2 a_3 ... a_{16})^{\frac{1}{16}}$$

$$\frac{8(a_1 + a_{16})}{16} \geq (a_1 a_2 a_3 ... a_{16})^{\frac{1}{16}}$$

$$\left(\frac{49}{2}\right)^{16} \geq a_1 a_2 a_3 ... a_{16}$$

9. $\mathrm{T_r} = \frac{1}{\mathrm{r}\sqrt{\mathrm{r}+1} + (\mathrm{r}+1)\sqrt{\mathrm{r}}}$, then (here $\mathrm{r} \in \mathrm{N}$)

(A) $\mathrm{T_r} > \mathrm{T_{r+1}}$

(B) $\mathrm{T_r} < \mathrm{T_{r+1}}$

(C) $\sum_{\mathrm{r}=1}^{99} \mathrm{T_r} = \frac{9}{10}$

(D) $\sum_{\mathrm{r}=1}^{\mathrm{n}} \mathrm{T_r} < 1$

Key: A, C, D

Hint: $$\mathrm{T_r} = \frac{\mathrm{r}\left(\sqrt{\mathrm{r}+1}\right) - (\mathrm{r}+1)\sqrt{\mathrm{r}}}{\mathrm{r}^2(\mathrm{r}+1) - (\mathrm{r}+1)^2\mathrm{r}} = \frac{\mathrm{r}\sqrt{\mathrm{r}+1} - (\mathrm{r}+1)\sqrt{\mathrm{r}}}{-\mathrm{r}^2 - \mathrm{r}} = \frac{(\mathrm{r}+1)\sqrt{\mathrm{r}}}{\mathrm{r}(\mathrm{r}+1)} - \frac{\mathrm{r}\sqrt{\mathrm{r}+1}}{\mathrm{r}(\mathrm{r}+1)} = \frac{1}{\sqrt{\mathrm{r}}} - \frac{1}{\sqrt{\mathrm{r}+1}}$$

$$\Rightarrow \sum_{\mathrm{r}=1}^{99} \mathrm{T_r} = \frac{1}{1} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} ..... - \frac{1}{\sqrt{100}} = 1 - \frac{1}{\sqrt{100}} = \frac{9}{10}$$

Hence (a), (c) and (d) are correct.

10. If $S_{(n)} = 1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n},(n \in N)$, then $S_{(1)}+S_{(2)}+.......+S_{(n-1)}$ is equal to

(A) $nS_{(n)}-n$ (B) $nS_{(n)}-1$

(C) $(n-1)S_{(n-1)}-n$ (D) $nS_{(n-1)}-n+1$

Key: A, D

Hint: $S_{(1)}+S_{(2)}+.....+S_{(n-1)}$

$S_{(1)}$: 1

$S_{(2)} : 1+\frac{1}{2}$

$S_{(3)} : 1+\frac{1}{2}+\frac{1}{3}$

………..

…………..

$S_{(n-1)} : 1+\frac{1}{2}+\frac{1}{3}+......\frac{1}{n-1}$

Adding vertically :

$$=(n-1)+\frac{(n-2)}{2}+\frac{(n-3)}{3}+......\left(\frac{n-(n-1)}{(n-1)}\right)$$

$$=n\left[1+\frac{1}{2}+\frac{1}{3}+....\frac{1}{n-1}\right]-[1+1+1+.....1]=nS_{(n-1)}-(n-1)=nS_n-n$$

11. If $a_1 > 0$ for i = 1, 2, …, n and $a_1a_2....a_n = 1$, then $(2 + a_1)(2 + a_2)....(2 + a_n)$ is greater than

(a) $2^{n/2}$ (b) $2^{3n/2}$ (c) 2^{2n} (d) none of these

Key: a or b

Hint: We have

$$\frac{1}{2}(2+a_1) \ge \sqrt{2a_1}$$

$$\frac{1}{2}(2+a_2) \ge \sqrt{2a_2}$$

…………………..

$$\frac{1}{2}(2+a_n) \ge \sqrt{2a_n}$$

Multiplying above inequalities, we get

$$\frac{1}{2^n}(2+a_1)(2+a_2).....(2+a_n)$$

$$\ge 2^{n/2}\sqrt{a_1a_2.....a_n} = 2^{n/2}$$

$\Rightarrow$ $(2 + a_1)(2 + a_2) \ldots (2 + a_n) \geq 2^{3n/2}$

As all $a_i \neq 2$, thus we have strict inequality in the above inequality.

12. The pth term T_p of HP is q(p + q) and qth term T_q is p (p + q) when p > 1, q > 1, (p ≠ q) then

(A) $T_{p+q} = pq$ (B) $T_{pq} = p+q$

(C) $T_{p+q} > T_{pq}$ (D) $T_{pq} > T_{p+q}$

Key : A, B, C

Sol : T_p of $AP = \frac{1}{q(p+q)} = A+(p-1)D$ ... (i)

T_q of $AP = \frac{1}{P(p+q)} = A+(q-1)D$ ... (ii)

$$\frac{1}{T_{p+q}} = A+(p+q-1)D$$

and $$\frac{1}{T_{pq}} = A+(pq-1)D.$$

Now, solving Eqs. (i) and (ii), we get

$$A = D = \frac{1}{pq(p+q)}$$

$$\therefore \frac{1}{T_{p+q}} = A+(p+q-1)D = (p+q)D = \frac{1}{pq}$$

and $\frac{1}{T_{pq}} = A+f(p+q-1)D = pqD = \frac{1}{p+q}$

$\Rightarrow T_{p+q} = pq$ and $T_{pq} = p+q$

Also, $\because pq > p+q$

i.e, $T_{p+q} > T_{pq}$

13. If the arithmetic mean of two positive numbers a and b $(a > b)$ is twice their geometrical mean then a : b is

(A) $2+\sqrt{3}:2-\sqrt{3}$ (B) $7+4\sqrt{3}:1$

(C) $1:7-4\sqrt{3}$ (D) $2:\sqrt{3}$

Key. A,B,C

Sol. $\frac{a+b}{2} = 2\sqrt{ab} \Rightarrow \sqrt{\frac{a}{b}}+\sqrt{\frac{b}{a}} = 4 \quad \sqrt{\frac{a}{b}} = 2\pm\sqrt{3}$

$$\frac{\sqrt{\frac{a}{b}}}{\sqrt{\frac{b}{a}}} = \frac{2+\sqrt{3}}{2-\sqrt{3}} \Rightarrow \frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

$$\frac{a}{b} = 7+4\sqrt{3} \Rightarrow a:b = \left(7+4\sqrt{3}\right):1$$

$$\sqrt{\frac{b}{a}} = 2-\sqrt{3} \Rightarrow a:b = 1:7-4\sqrt{3}$$

14. If $b_1, b_2, b_3 \left(b_1 > 0\right)$ are three successive terms of a G.P. with common ratio r, the value of r for which the inequality $b_3 > 4b_2 - 3b_1$ holds is given by

(A) $r > 3$
(B) $1 < r < \frac{3}{2}$
(C) $r < 1$
(D) None of these

Key. A,C

Sol. $b_2 = b_1 r, b_3 = b_1 r^2$

$b_1 r^2 > 4b_1 r - 3b_1$

$\Rightarrow \quad r^2 > 4r - 3$

$\Rightarrow \quad r^2 - 4r + 3 > 0$

$\Rightarrow \quad (r-1)(r-3) > 0$

$r > 3$ or $r < 1$

15. If a, b, c, d are four unequal positive numbers which are in A.P., then

(A) $\frac{1}{a}+\frac{1}{d}=\frac{1}{b}+\frac{1}{c}$
(B) $\frac{1}{a}+\frac{1}{d}<\frac{1}{b}+\frac{1}{c}$
(C) $\frac{1}{a}+\frac{1}{d}>\frac{1}{b}+\frac{1}{c}$
(D) $\frac{1}{b}+\frac{1}{c}>\frac{4}{a+d}$

Key. C,D

Sol. Conceptual

16. If a, b, c are in H.P., then

(A) $\frac{a}{b+c-a}, \frac{b}{c+a-b}, \frac{c}{a+b-c}$ are in H.P.

(B) $\frac{2}{b}=\frac{1}{b-a}+\frac{1}{b-c}$

(C) $a-\frac{b}{2}, \frac{b}{2}, c-\frac{b}{2}$ are in G.P.

(D) $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in H.P.

Key. A,B,C,D

Sol. a, b, c are in H.P.

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

$\Rightarrow \frac{a+b+c}{a}, \frac{a+b+c}{b}, \frac{a+b+c}{c}$ are in A.P.

$\Rightarrow \frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P. [subtracting 1 from each term]

$\Rightarrow \frac{b+c}{a}-1, \frac{c+a}{b}, \frac{a+b-c}{c}$ are in A.P.

Thus $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in H.P.

And $\frac{a}{b+c-a}, \frac{b}{c+a-b}, \frac{c}{a+b-c}$ are also in H.P.

Also $b = 2ac/(a+c)$, so $\frac{1}{b-a}+\frac{1}{b-c}=\frac{2b-(a+c)}{(b-a)(b-c)}$

$=\frac{2b-(a+c)}{b^2-b(a+c)+ac}$

$=\frac{2b-2ac/b}{b^2-b(a+c)+ac}$

$=\frac{2}{b}.\frac{b^2-ac}{b^2-ac}=\frac{2}{b}$

Lastly, $\left(a-\frac{b}{2}\right)\left(c-\frac{b}{2}\right)=ac-\frac{b}{2}(a+c)+\frac{b^2}{4}$

$=ac-\frac{b}{2}.\frac{2ac}{b}+\frac{b^2}{4}=\frac{b^2}{4}$

$a-\frac{b}{2}, \frac{b}{2}, c-\frac{b}{2}$ are in G.P.

17. If in a ΔABC, a, b, c are in A.P. then it is necessary that

(A) $\frac{2}{3}<\frac{b}{c}<2$ (B) $\frac{1}{3}<\frac{b}{c}<\frac{2}{3}$

(C) $\frac{2}{3}<\frac{b}{a}<2$ (D) $\frac{1}{3}<\frac{b}{a}<\frac{2}{3}$

Key. A,C

Sol. a + c = 2b

a + b > c

b + c > a

a + b > c

3b > 2c

b + c > a

2c > b

$\Rightarrow \quad \frac{2}{3}<\frac{b}{c}<2$

Similarly for $\frac{b}{a}$

18. Let $S_1, S_2, -----, S_n$ be the sums of geometric series .Whose 1st terms are 1, 2, 3, -----,n

and common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ----, \frac{1}{n+1}$ respectively. Then

a) $S_1+S_2+---+S_n=\frac{n(n+3)}{2}$ b) $S_1.S_2----S_n=\lfloor n+1$

c) $\frac{1}{S_1S_2}+\frac{1}{S_2S_3}+----\frac{1}{S_{n-1}S_n}=\frac{n-1}{2(n+1)}$ d) $S_1^2.S_2^3.S_3^4-----S_n^{n+1}=1024/3$

Key. A,B,C

Sol. $S_r=r+r\left(\frac{1}{r+1}\right)+r\left(\frac{1}{r+1}\right)+---\infty=\frac{r}{1-\frac{1}{r+1}}=r+1$ verify a, b, c an correct and d is false.

19. If $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{c}+\frac{y}{d}=1$ intersect the axes at four concyclic points and $a^2+c^2=b^2+d^2$ then these lines can intersect at (a, b, c, d > 0)

(A) (1, 1) (B) (1, –1)

(C (2, –2) (D) (3, 3)

Key. A,B,C,D

Sol. a + c = 2b

a + b > c

b + c > a

a + b > c

3b > 2c

b + c > a

2c > b

$\Rightarrow \quad \frac{2}{3}<\frac{b}{c}<2$

Similarly for $\frac{b}{a}$

20. Three numbers in A.P. with common difference 'd' are removed from first n natural numbers and average of remaining number is found to be $\frac{43}{4}$ then ordered pair (n, d) can be

(A) (19, 5) (B) (19, 2)

(C) (23, 5) (D) (19, 8)

Key. A,B

SOL. LET REMOVED NUMBERS ARE A – D, A, A + D

SUM OF REMOVED NUMBERS = 3A

$\Rightarrow \quad 6\le 3A\le 3N-3$

$\Rightarrow \quad 2A\le A\le N-1$ …..(I)

ALSO $3a=\frac{n(n+1)}{2}-\frac{43}{4}(n-3)$

$a=\frac{2n^2-41n+129}{12}$ …..(II)

FROM (I) AND (II)

$17.5\le N\le 23.5$ $N\in N$

N = 18, 19, 20, 21, 22, 23

FOR $A\in N$, N MUST BE ODD

$\Rightarrow$ N MAY BE 19, 21, 23

WHEN N = 19, A = 6, D CAN BE 2 OR 5

WHEN N = 21 $A\notin N$ NOT POSSIBLE

when n = 23 $a\notin N$ not possible.

AP,GP,HP, Sequences

Assertion Reasoning Type

1. Statement 1 : 1,2,4,8,...... is a G.P., 4,8,16,32 is a G.P. and $1+4, 2+8, 4+16, 8+32, ...$ is also a G.P.

Statement 2 : Let general term of a G.P. with common ratio r be T_{k+1} and general term of another

G.P. with common ratio r be T'_{k+1} then the series whose general term $T''_{k+1} = T_{k+1} + T'_{k+1}$ is also a G.P. with common ratio r.

Key. A

2. Let S_k where $k \in N$ denotes sum of first 'K ' terms of A.P. If the sum of first '3n' terms of it is twice the sum of next ' n ' terms then

Statement I : The ratio of sum of first '2n' terms and the sum of next '2n' terms is 7 : 11

Statement II : S_n, S_{2n}, S_{3n} are in A.P.

KEY : C

HINT : $S_{3n} = 2(S_{4n} - S_{3n}) \Rightarrow 3S_{3n} = 2S_{4n}$

3. STATEMENT- 1

If a, b, c, d$\in$R$^+$ and $(a + b + c + d + 3)^5 = 9375\, abcd$, then a + b + c + d = 12

STATEMENT 2

If for +ve real numbers A.M. = G.M., then number are equal.

Key: A

4. Statement 1: One side of an equilateral triangle 24. The mid points of the sides are joined to form another triangle whose midpoints are in tern joined to form another triangle and continue the process infinite number times. Then sum of perimeters of all such triangles formed is 144.

Statement : If $\log_2(a+b) + \log_2(c+d) \geq 4$ then the minimum value of a+b+c+d is 8

Key. B

Sol. I) Sum of perimeters = $3(24+12+6+----) = 144$

II) $\log_2(a+b)(c+d) \geq 4$

$\Rightarrow (a+b)(c+d) \geq 2^4$

$\therefore \dfrac{a+b+c+d}{4} \geq \sqrt[4]{(a+b)(c+d)} \Rightarrow a+b+c+d \geq 8$

5. STATEMENT 1. : If p, q, r > 0 and (p + q) (p + r) (r + q) = 8p^3 then there must be p = q = r. because

STATEMENT 2.: If a_1, a_2, a_3 ..... a_n > 0 then $\dfrac{a_1 + a_2 + ...a_n}{n} \geq (a_1 a_2 a_3 ....a_n)^{1/n}$.

Key. A

Sol. If p = q = r and p, q, r > 0

then their A.M. $\geq$ G.M.

$$\frac{p+q}{2} = pq$$

$$\frac{q+p}{2} = qr$$

$$\frac{r+p}{2} = pr$$

$\Rightarrow$ (p + q) (q + r) (r + p) = 8pqr = $8p^3$.

6. Let 3 $a_1, b_2, c_3 \ldots a_{10}$ 6 be in AP and 3, $h_1, h_2, h_3 .. h_{10}$,6 be in HP then

Statement I: $a_2 h_9 + a_4 h_7 + a_6 h_5 + a_8 h_3 = 72$.

Statement II: product of the ith AM from left and ith HM from left of n AMS and n HMS inserted between two given numbers is independent of i

Key. C

Sol. Conceptual

7. STATEMENT-1: a, b, c are sides of ΔABC such that bc = λ^2 for some positive λ. Then $a \geq \lambda \sin\frac{A}{2}$

STATEMENT-2: A.M. of two given positive quantities $\geq$ G.M.

Key. A

Sol. $\frac{b+c}{2} \geq \sqrt{bc} = \lambda$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = \frac{b+c}{\sin B + \sin C} \geq \frac{2\lambda}{2\cos\frac{A}{2}\cos\frac{B-C}{2}}$$

$$a \geq 2\lambda \sin\frac{A}{2}$$

8. Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1$, $b_2 = b_1 + a_2$, $b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.

STATEMENT-1: The numbers b_1, b_2, b_3, b_4 are neither in AP nor in GP and because

STATEMENT-2: The numbers b_1, b_2, b_3, b_4 are in H.P.

Key. C

Sol. Here $b_1 = a_1$, $b_2 = a_1 + a_2 = a_1(1 + r)$, $b_3 = a_1(1 + r) + a_1r^2 = a_1(1 + r + r^2)$

$b_4 = a_1(1 + r + r^2) + ar^3 = a_1(1 + r + r^2 + r^3)$,

r being the common ratio of the G.P.

Clearly, b_1, b_2, b_3, b_4 are neither in AP nor in GP nor in HP.

$\therefore$ STATEMENT-1 is true but STATEMENT-2 is false.

9. STATEMENT – 1 For $n \in N, 2^n > 1 + n\sqrt{\left(2^{n-1}\right)}$, $n \neq 1$

STATEMENT – 2 For two distinct positive real numbers, GM > HM and (AM)(HM) = $(GM)^2$

Key. B

Sol. $\because \frac{(AM)}{(GM)} = \frac{GM}{HM} > 1$

$AM > GM$

$$\frac{1+2+2^2+.....+2^{n-1}}{n}>\left(1.2.2^2.......2^{n-1}\right)^{1/n}$$

$$\Rightarrow \frac{\frac{1.\left(2^n-1\right)}{(2-1)}}{n}>\left\{2^{1+2+3+......+(n-1)}\right\}^{1/n}$$

10. Statement – 1: If a,b,c are non zero real numbers such that $3\left(a^2+b^2+c^2+1\right)=2\left(a+b+c+ab+bc+ca\right)$, then a,b,c are in A.P. as well as in G.P.

Statement – 2: A series is in A.P. as well as in G.P. if all the terms in the series are equal and non zero.

Key. A

Sol. $3\left(a^2+b^2+c^2+1\right)-2\left(a+b+c+ab+bc+ca\right)=0$

$\Rightarrow \quad (a-1)^2+(b-1)^2+(c-1)^2+(a-b)^2+(b-c)^2+(c-a)^2=0$

$\Rightarrow \quad a = b = c = 1$

11. Statement – 1: Equations $x^2-4x+1=0$ and $x^2-ax+b=0$, where a,b are rational numbers, have atleast one common root, then a = 4 and b = 1

Statement – 2; If two equations $ax^2+bx+c=0$ and $a_1x^2+b_1x+c_1=0$, where a,b,c, a_1,b_1,c_1 are non-zero rational numbers, have common irrational root, then $\frac{a}{a_1}=\frac{b}{b_1}=\frac{c}{c_1}$

Key. A

Sol. Obviously both the statements are true and statement – 2 explains statement – 1.

12. Statement – 1: 1,2,4,8,.... is a G.P., 4, 8, 16, 32 is a G.P. and 1 + 4, 2 + 8, 4 + 16, 8 + 32,...... is also a G.P.

Statement – 2: Let general term of a G.P. with common ratio r be T_{k+1} and general term of another G.P. with common ratio r be T'_{k+1}, then the series whose general term $T'_{k+1}=T_{k+1}+T'_{k+1}$ is also a G.P. with common ratio r.

Key. C

Sol. Taking 2, 4, 8, 16, ... and – 2, - 4, - 8, - 16

sum is 0, 0, 0, ... it is not in GP

13. Statement – 1: If one A.M. 'A' and two G.M.'s p and q be inserted between any two numbers, then $p^3+q^3=2Apq$

Statement – 2: If x,y,z are in G.P., then $y^2=xz$

Key. B

Sol. Statement – 1 a,A, b are in A.P. $\Rightarrow$ 2A = a + b ...(i)

a,p,q,b are in G.P. $\Rightarrow$ pq = ab ...(ii)

and let common ratio of G.P. be r

$\therefore \quad b=ar^3 \quad \Rightarrow \quad r=\left(\frac{b}{a}\right)^{\frac{1}{3}}$

p = ar $\quad \Rightarrow \quad p=a.\left(\frac{b}{a}\right)^{\frac{1}{3}} \quad \Rightarrow \quad p^3=a^2b$ ...(iii)

$q=ar^2 \quad \Rightarrow \quad q=a\left(\frac{b}{a}\right)^{\frac{2}{3}} \quad \Rightarrow \quad q^3=ab^2$ ...(iv)

From (i), (ii), (iii) & (iv)

$p^3 + q^2 = 2A\,pq$

Statement – 2 is obviously true

AP,GP,HP, Sequences

Comprehension Type

Paragraph – 1

If $x_1, x_2, x_3, \ldots.. x_n$ are n positive real numbers, then A.M. ≥ G.M.

$$\frac{x_1 + x_2 + \ldots.. + x_n}{n} \geq \sqrt[n]{x_1 x_2 \ldots.. x_n}$$

1. The minimum value of the function $4^{\sin^{-1}x} + 4^{\cos^{-1}x}$ $(-1 \leq x \leq 1)$ and the value of x, where it is attained is.

(A) $2.2^{\frac{\pi}{2}}$ at $x = \frac{1}{2}$
(B) $2.2^{\frac{\pi}{4}}$ at $x = \frac{1}{\sqrt{2}}$
(C) $2.2^{\frac{\pi}{2}}$ at $x = \frac{1}{\sqrt{2}}$
(D) $1 + 4^{\frac{\pi}{2}}$ at $x = 0$

Key. C

2. Which of the following inequalities is not true

(A) $\frac{x^2+3}{\sqrt{x^2+2}} \geq 2 \quad (x \in R)$
(B) $x^2 + y^2 + 1 \geq xy + x + y$ $(x, y \in R)$
(C) $\frac{x^3+x+2}{x} \geq 4$ $(x > 0)$
(D) $x^2 + \frac{1}{x^2} + 4 \leq 3\left|x + \frac{1}{x}\right|$ $(x \neq 0)$

Key. D

3. If the equation $x^4 - 4x^3 + ax^2 - bx + 1 = 0$ has four positive roots, then a + b is equal to

(A) 0
(B) 4
(C) 6
(D) 10

Key. D

Sol. 1. $4^{\sin^{-1}x} + 4^{\cos^{-1}x} \geq 2\sqrt{4^{\sin^{-1}x + \cos^{-1}x}} = 2.2^{\pi/2}$

equality holds if and only if $4^{\sin^{-1}x} = 4^{\cos^{-1}x}$

i.e. $x = \frac{1}{\sqrt{2}}$

2. options a, b, c are correct only d option is not correct

i.e. $x^2 + \frac{1}{x^2} + 4 - 3\left|x + \frac{1}{x}\right| = u^2 - 3u + 2 = (u-1)(u-2) \geq 0$

where $u = \left|x + \frac{1}{x}\right|$, Then $u \geq 2$

3. If x_1, x_2, x_3, x_4 are denote the roots of the given equation $\sum x_1 = 4,\ x_1x_2x_3x_4 = 1$

$\Rightarrow \quad \frac{\sum x_1}{4} = \sqrt[4]{x_1x_2x_3x_4}$

hence $x_1 = x_2 = x_3 = x_4 = 1$

$\Rightarrow$ $x^4 - 4x^3 + ax^2 - bx + 1 = (x-1)^4 = x^4 - 4x^3 + 6x^2 - 4x + 1$

Thus a = 6, b = 4.

Then a + b = 10.

Paragraph – 2

Sometimes we can find the sum of series by use of differentiation. If we know the sum of a series e.g. if $f(x) = f_1(x) + f_2(x) + \ldots\ldots$

$f'(x) = f_1'(x) + f_2'(x) + \ldots\ldots$

e.g. $(1 - x)^{-1} = 1 + x + x^2 + x^3 \ldots\ldots$ $|x| < 1$

Hence the sum of the AGP

$1 + 2x + 3x^2 + \ldots\ldots = (1 - x)^{-2}$ (By differentiation both the sides)

Now answer the question that follows

4. The sum of the series $\frac{2^2}{1!} + \frac{3^2}{2!} + \frac{4^2}{3!} + \ldots\ldots$ upto ∞ is

(A) 4e – 1 (B) 5e (C) 5e – 1 (D) 4e

Key. C

5. Sum of the series $1 - \frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \ldots\ldots$ upto ∞ is

(A) $\frac{1}{2}$ – n2 (B) 1 – n2 (C) ∞ (D) $\frac{3}{2}$ – n2

Key. D

6. Sum of the series $1 + 1 + \frac{3}{4} + \frac{4}{8} + \frac{5}{16} + \ldots\ldots$ upto infinite terms, is

(A) 4 (B) 2 (C) 1 (D) $\frac{1}{4}$

Key. A

Sol. 4. t_n = = = + +

= + + = e + 3e + e – 1 = 5e – 1

5. = 1 – + + ...

differentiating both sides with respect to x

= – + – + ...

put x = 1, – n 2 = – + – + ...

∴ – n 2 = 1 – + – + ...

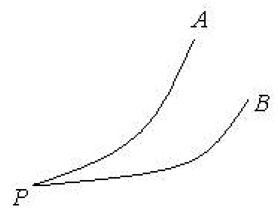
6. We know that , $1 + 2x + 3x^2 + \ldots\ldots = (1 - x)^{-2}$

put x = ,

we get 1 + 1 + + + + ... = = 4

Paragraph – 3

In the adjoining figure, we find two curves PA and PB through P. Clearly in the neighbourhood of P the curve PA is bending more rapidly than the curve PB. In other words curvature of PA is greater than that of PB. If PA and PB are regarded roughly as arcs of circles then clearly radius of PA is less than the radius of PB.

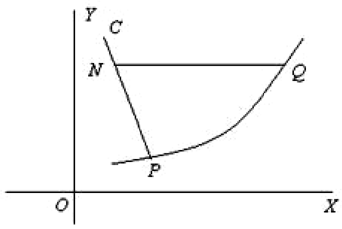


Let P be any point on a given curve and Q any other point on it. Let the normals at P and Q intersect in $'N'$. If $'N'$ tends to a definite position C as Q tends to P (from the right or from the left) then $'C'$ is called the centre of curvature of curve at P and distance CP is called the radius of curvature of P and is denoted by Greek letter ρ.

The reciprocal of the distance CP is called the curvature of the curve at P. The circle with its centre at C and radius CP is called the circle of curvature of the curve at P. Radius of curvature can be evaluated with the help of following formula;

$$\rho = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}}{\frac{d^2y}{dx^2}}$$

The formula does not hold good when the tangent at P is parallel to y - axis. Since the value of radius of curvature depends only on the curve and not on the axes. Therefore in such cases we interchange the axes of $'x'$ and $'y'$ and we have



$$\rho = \frac{\left\{1+\left(\frac{dx}{dy}\right)^2\right\}^{3/2}}{\frac{d^2x}{dy^2}}$$

7. Numerically radius of curvature of parabola $y^2 = 4ax$ at any point (x, y) is

A) $\frac{2(x+a)^{3/2}}{\sqrt{a}}$ B) $\frac{2(y+a)^{3/2}}{\sqrt{a}}$ C) $\frac{(x+a)^{3/2}}{\sqrt{a}}$ D) $\frac{(x+a)^2}{a^{3/2}}$

Key. A

Sol. Conceptual

8. Radius of curvature at any point of the curve $x = a(t+\sin t)$; $y = a(1-\cos t)$ is given by

A) $a\cos\frac{t}{2}$ B) $4a\cos\frac{t}{2}$ C) $4a\cos t$ D) $5a\cos t$

Key. B

Sol. Conceptual

9. Radius of curvature of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in terms of l, where l is the perpendicular distance from the centre upon the tangent at (x, y) is

A) $\frac{a^2 b^2}{l^3}$ B) $\frac{a^2 b}{l^2}$ C) $\frac{a^2 b^2}{l^2}$ D) $\frac{a^3 b^3}{l^3}$

Key. A

Sol. Conceptual

Paragraph – 4

If a sequence or series is not a direct form of an AP, GP, etc. Then its nth term can not be determined. In such cases, we use the following steps to find the nth term (T_n) of the given sequence.

Step – I : Find the differences between the successive terms of the given sequence. If these differences are in AP, then take $T_n = an^2 + bn + c$, where a,b,c are constants.

Step – II : If the successive differences found in step I are in GP with common ratio r, then take $T_n = a + bn + cr^{n-1}$, where a, b, c are constants.

Step – III : If the second successive differences (Differences of the differences) in step I are in AP, then take $T_n = an^3 + bn^2 + cn + d$, where a, b, c, d are constants.

Step – IV : If the second successive differences (Differences of the differences) in step I are in GP, then take $T_n = an^2 + bn + c + dr^{n-1}$, where a, b, c, d are constants.

Now let sequences :

A : 1, 6, 18, 40, 75, 126, ... B : 1, 1, 6, 26, 91, 291, ... C : ln 2 ln 4, ln 32, ln 1024 .....

10. If the nth term of the sequence *A* is $T_n = an^3 + bn^2 + cn + d$ then the value $6a + 2b - d$ is

(A) ln 2 (B) 2

(C) ln 8 (D) 4

Key. D

Sol. $T_n = an^3 + bn^2 + cn + d$

$T_1 = a + b + c + d = 1$

$T_2 = 8a + 4b + 2c + d = 6$

$6a + 2b - d = 4$

11. For the sequence 1, 1, 6, 26, 91, 291, …….. Find the S_{50} where $S_{50} = \sum_{r=1}^{50} T_r$

(A) $\frac{5}{8}(3^{50}-1)-3075$ (B) $\frac{5}{8}(3^{50}-1)-5075$

(C) $\frac{5}{8}\left(3^{50}-1\right)-1275$ (D) None of these

Key. A

Sol. $T_n=\frac{5}{4}3^{n-1}-\frac{5n}{2}+\frac{9}{4}$

$$S_{50}=\frac{5}{4}\left(1+3+...+3^{49}\right)-\frac{5}{2}\left(1+2+....+50\right)+50.\frac{9}{4}$$

$$=\frac{5}{4}\left(\frac{3^{50}-1}{2}\right)-\frac{5}{2}.\frac{50.51}{2}+\frac{450}{4}$$

$$=\frac{5}{8}\left(3^{50}-1\right)-\frac{125.51}{2}+\frac{450}{4}$$

$$=\frac{5}{8}\left(3^{50}-1\right)-3075$$

12. The sum of the series $1.n+2.(n-1)+3.(n-2)+....+n.1$

(A) $\frac{n(n+1)(n+2)}{6}$ (B) $\frac{n(n+1)(n+2)}{3}$

(C) $\frac{n(n+1)(2n+1)}{6}$ (D) $\frac{n(n+1)(2n+1)}{3}$

Key. A

Sol. $\sum_{r=1}^{n}r(n-r+1)=\sum_{r=1}^{n}(n+1)r-\sum_{r=1}^{n}r^2$

$$=(n+1)\sum n-\sum n^2$$

$$=\frac{(n+1)^2 n}{2}-\frac{n(n+1)(2n+1)}{6}$$

$$=\frac{n(n+1)}{6}(3n+3-2n-1)=\frac{n(n+1)(n+2)}{6}$$

Paragraph – 5

In a sequence of $(4n+1)$ terms the 1st $(2n+1)$ terms are in A.P. whose common difference is 2 and the last $(2n+1)$ terms are in G.P. whose common ratio is $\frac{1}{2}$. If the middle terms of the A.P. and G.P. are equal, then

13. Middle term of the sequence is

A) $\frac{n.2^{n+1}}{2^n-1}$ B) $\frac{n.2^{n+1}}{2^{2n}-1}$ C) $n2^n$ D) $(n+1)2^{n+1}$

14. First term of the sequence is

A) $\frac{4n+2n\ 2^n}{2^n-1}$ B) $\frac{4n-2n\ 2^n}{2^n-1}$ C) $\frac{2n-n\ 2^n}{2^n-1}$ D) $\frac{2n+n\ 2^n}{2^n-1}$

15. Middle term of the G.P. is

A) $\frac{2^n}{2^n-1}$ B) $\frac{n\,2^n}{2^n-1}$ C) $\frac{n}{2^n-1}$ D) $\frac{2n}{2^n-1}$

Key : 13-A, 14-B, 15-D

Sol: 13 – 15

1st $(2n+1)$ terms of A.P. are A, $A+2$, …., $A+4n$.

Last $(2n+1)$ terms of G.P. are $(A+4n)$, $(A+4n)\frac{1}{2}$, ……., $(A+4n)\frac{1}{2^{2n}}$

= $A+2n=\frac{A+4n}{2^n} \Rightarrow A=\frac{4n-2n\ 2^n}{2^n-1}$

Middle term of sequence = $T_{2n+1}=A+4n=\frac{n\,2^{n+1}}{2^n-1}$

Middle term of G.P. = $T_{n+1}=\frac{2n\ 2^n}{2^n-1}\times\frac{1}{2^2}=\frac{2n}{2^n-1}$

Paragraph – 6

Let $A_1, A_2, A_3, ..., A_m$ be arithmetic means between – 2 and 1027 and $G_1, G_2, G_3, ..., G_n$ be geometric means between 1 and 1024. Product of geometric means is 2^{45} and sum of arithmetic means is 1025 × 171.

16. The value of n is

A) 7 B) 9 C) 11 D) none of these

Key. B

Sol. $G_1G_2....G_n=\left(\sqrt{1\times1024}\right)^n=2^{5n}$

$\therefore \quad 2^{5n}=2^{45}$

$\therefore \quad n = 9$

17. The value of m is

A) 340 B) 342 C) 344 D) 346

Key. B

Sol. $A_1+A_2+A_3+...+A_{m-1}+A_m=1025\times171$

$\therefore \quad m\left(\frac{-2+1027}{2}\right)=1025\times171$

$\therefore \quad m = 342$

18. The value of $G_1+G_2+G_3+...G_n$ is

A) 1022 B) 2044 C) 512 D) none of these

Key. A

Sol. Since n = 9, $\therefore \quad =(1024)^{\frac{1}{9+1}}=2$

$\therefore \quad G_1=2, r=2$

$$G_1+G_2+...+G_n=\frac{2.\left(2^9-1\right)}{2-1}=1024-2=1022$$

19. The common difference of the progression $A_1, A_3, A_5, ..., A_{m-1}$ is

A) 6 B) 3 C) 2 D) 1

Key. A

Sol. Common difference of sequence $A_1, A_2, ..., A_m$ is $\frac{1027+2}{342+1} = 3$

$\therefore$ common difference of sequence $A_1, A_3, A_5, ..., A_{m-1}$ is 6

20. The numbers $2A_{171}, G_5^2 + 1, 2A_{172}$ are in

A) A.P. B) G.P. C) H.P. D) A.G.P.

Key. A

Sol. we have $A_{171} + A_{172} = -2 + 1027 = 1025$

$\therefore$ $\frac{2A_{171} + 2A_{172}}{2} = 1025$

Also $G_5 = 1 \times 2^5 = 32$

$\therefore$ $G_5^2 = 1024$ i.e. $G_5^2 + 1 = 1025$

$\therefore$ $2a_{171} G_5^2 + 1, 2A_{172}$ are in A.P.

Paragraph – 7

There are two sets A and B each of which consists of three numbers in A.P. whose sum is 15 and where D and d are the common differences such that D – d = 1. If $\frac{p}{q} = \frac{7}{8}$ where p and q are the product of the numbers respectively and d > 0, in the two sets

21. Value of p is

A) 100 B) 120 C) 105 D) 110

Key. C

22. Value of q is

A) 100 B) 120 C) 105 D) 110

Key. B

23. Value of D + d is

A) 1 B) 2 C) 3 D) 4

Key. C

Sol. Let numbers in set A be a – D, a, a + D, a, a + D and in set B be b – d, b, b + d

3a = 3b = 15 $\Rightarrow$ a = b = 5

Set A = {5 – D, 5, 5 + D}

Set B = {5 – d, 5, 5 + d}

Where D = d + 1

$$\frac{p}{q} = \frac{5(25 - D^2)}{5(25 - d^2)} = \frac{7}{8}$$

25(8 – 7) = 8 $(d+1)^2 - 7d^2$

$\Rightarrow$ $d = -17, 1$ but d > 0 $\Rightarrow$ d = 1

So numbers in Set A are 3, 5, 7

Number in Set B are 4, 5, 6

Now p = 3 × 5 × 7 = 105

q = 4 × 5 × 6 = 120

value of D + d = 3

Paragraph – 8

Four different integers form an increasing A.P. One of these numbers is equal to the sum of the squares of the other three numbers. Then

24. The smallest number is

A) – 2 B) 0 C) – 1 D) 2

Key. C

25. The common difference of the four numbers is

A) 2 B) 1 C) 3 D) 4

Key. B

26. The sum of all the four numbers is

A) 10 B) 8 C) 2 D) 6

Key. C

Sol. Let four integers be a – d, a, a + d and a + 2d

Where a and d are integers and d > 0.

$\because \quad a+2d=(a-d)^2+a^2+(a+d)^2$

$\Rightarrow \quad 2d^2-2d+3a^2-a=0$ …(i)

$\therefore \quad d=\frac{1}{2}\left[1\pm\sqrt{1+2a-6a^2}\right]$ …(ii)

Since d is positive integer

$\therefore \quad 1+2a-6a^2>0$

$6a^2-2a-1<0$

$\Rightarrow \quad \frac{1-\sqrt{7}}{6}<a<\frac{1+\sqrt{7}}{6}$ $\because$ a is an integer

$\therefore$ a = 0 Put in (ii)

$\therefore$ d = 1 or 0 but $\because$ d > 0

$\therefore$ d = 1

$\therefore$ The four numbers are : - 1, 0, 1, 2

Paragraph – 9

Let $n \in N$. The A.M, G.M, H.M respectively of the 'n' numbers $n+1$, $n+2$, $n+3$, ….. , $n+n$ are A_n, G_n, H_n

27. $\underset{n\to\infty}{Lt}\frac{A_n}{n}$ =

A) 1 B) $\frac{1}{2}$ C) $\frac{3}{2}$ D) 2

Key. C

28. $\underset{n\to\infty}{Lt}\frac{G_n}{n}$ =

A) $\frac{1}{e}$ B) $\frac{2}{e}$ C) $\frac{3}{e}$ D) $\frac{4}{e}$

Key. D

29. $\underset{n\to\infty}{Lt}\frac{H_n}{n}$ =

A) $\frac{1}{e}$ B) $\log_2 e$ C) $\frac{2}{e}$ D) $\log_4 e$

Key. B

Sol. **(27 – 29)**

$$A_n = \frac{(n+1)(n+2)+....+(n+n)}{n} = \frac{n^2+\frac{n(n+1)}{2}}{n} = n+\frac{n+1}{2}$$

$$= \frac{3n+1}{2}$$

$$G_n = \left[(n+1)(n+2)(n+3)......(n+n)\right]^{\frac{1}{n}}$$

$$\frac{1}{H_n} = \frac{1}{n}\left[\frac{1}{n+1}+\frac{1}{n+2}+.....+\frac{1}{n+n}\right]$$

$\underset{n\to\infty}{Lt}\frac{A_n}{n} = \frac{3}{2}$ Let $L = \underset{n\to\infty}{Lt}\frac{G_n}{n}$

$$\log_e L = \underset{n\to\infty}{Lt}\frac{1}{n}\sum_{r=1}^{n}\log\left(1+\frac{r}{n}\right) = \int_0^1 \log(1+x)dx$$

$$= \left[x\log(1+x)\right]_0^1 - \int_0^1 \frac{(1+x)-1}{1+x}dx = \ln 2 - [1-\ln(1+x)]_0^1$$

$$\underset{n\to\infty}{Lt}\frac{n}{H_n} = \underset{n\to\infty}{Lt}\frac{1}{n}\sum_{r=1}^{n}\frac{1}{1+\frac{r}{n}} = \int_0^1\frac{1}{1+x}dx = \log_e 2$$

$$\therefore \underset{n\to\infty}{Lt}\frac{H_n}{n} = \log_2 e$$

Paragraph – 10

$$\sum n = \frac{n(n+1)}{2}$$

$$\sum n^2 = \frac{n(n+1)(2n+1)}{6}, \sum n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

If $T_n = n(n+1)$, then

$$S_n = \left[n(n+1)\right]\frac{(n+2)}{3}$$

if $T_n = n(n+1)(n+2)$, then

$$S_n = \left[n(n+1)(n+2)\right]\frac{(n+3)}{4}.$$

Answer the following questions based upon above passage :

30. Sum of the series

$$\frac{1^3}{1}+\frac{1^3+2^3}{1+3}+\frac{1^3+2^3+3^3}{1+3+5}+....\text{to 16 terms is}$$

(A) 346 (B) 446

(C) 546 (D) 444

Key. B

31. $1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+....\text{to } \infty$ is

(A) $\frac{16}{35}$ (B) $\frac{11}{8}$

(C) $\frac{35}{16}$ (D) $\frac{7}{16}$

Key. C

32. The sum of the series

$$\frac{1}{3\times7}+\frac{1}{7\times11}+\frac{1}{11\times15}+....\text{to } \infty \text{ is}$$

(A) $\frac{1}{3}$ (B) $\frac{1}{6}$

(C) $\frac{1}{9}$ (D) $\frac{1}{12}$

Key. D

Sol. 30. Ans. (b)

$$T_n=\frac{\sum n^3}{\frac{n}{2}\left[2.1+(n-1).2\right]}$$

$$=\frac{1}{4}.\frac{n^2(n+1)^2}{n^2}=\frac{1}{4}\left(n^2+2n+1\right)$$

$$S_n=\frac{1}{4}\left[\sum n^2+2\sum n+\sum 1\right]$$

$$=\frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6}+2.\frac{(n+1)n}{2}+n\right]$$

Putting n = 16, we get

$S_{16}=446$

31. Ans. (c)

$$S=1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+....\infty$$

Then $\frac{1}{5}S=\frac{1}{5}+\frac{4}{5^2}+\frac{7}{5^3}+.....$

$$S\left(1-\frac{1}{5}\right)=1+3\left[\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+....\infty\right]$$

$$\frac{4}{5}S=1+3\left[\frac{1/5}{1-1/5}\right]=1+\frac{3}{4}=\frac{7}{4}$$

$\therefore$ S = $\frac{35}{16}$

Note : Your many use the formula

i.e. $S_\infty=\frac{ab}{1-r}+\frac{dbr}{(1-r)^2}$

where a = 1, d= 3, b = 1, r = 1/5

32. And. (d)

$$S=\frac{1}{4}\left[\left(\frac{1}{3}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{11}\right)+\left(\frac{1}{11}-\frac{1}{15}\right)+.....\infty\right]$$

$$S_{\infty}=\frac{1}{4}.\frac{1}{3}=\frac{1}{12}$$

AP,GP,HP, Sequences

Integer Answer Type

1. The value of $x.y.z = 55\ or\ \frac{343}{55}$ according as the series a,x,y,z,b form an A.P or H.P respectively , where a and b are positive natural numbers. Find the sum a+b

Key. 8

Sol. If a,x, y,z, b to are in A.P then the common difference d of the AP is given by

$$b = a + 4d \Rightarrow d = \frac{b-a}{4}$$

$$\therefore x = a + d = \frac{a+b-a}{4} = \frac{b+3a}{4}$$

$$y = a + 2d = \frac{a+b-a}{2} = \frac{a+b}{2}$$

$$z = a + 3d = a + 3\left(\frac{b-a}{4}\right) = \frac{a+3b}{4}$$

$$\therefore xyz = \frac{b+3a}{4} \times \frac{a+b}{2} \times \frac{a+3b}{4}$$

$$\Rightarrow 55 = \frac{(3a+b)(a+b)(a+3b)}{32}$$

$$\Rightarrow (3a+b)(a+b)(a+3b) = 55 \times 32$$

When a,x,y,z,b are in H.P . Then

$\frac{1}{a}, \frac{1}{x}, \frac{1}{y}, \frac{1}{b}$ are in AP

Let D be the common different of this A.P . Then

$$\frac{1}{b} = \frac{1}{a} + 4D \Rightarrow D = \frac{a-b}{4ab}$$

$$\therefore \frac{1}{x} = \frac{1}{a} + D = \frac{1}{a} + \frac{a-b}{4ab} = \frac{3b+a}{4ab}$$

$$\frac{1}{y} = \frac{1}{a} + 2D = \frac{1}{a} = \frac{a-b}{2ab} = \frac{a+b}{2ab}$$

$$\frac{1}{z} = \frac{1}{a} + 3D = \frac{1}{a} = \frac{3(a-b)}{4ab} = \frac{3a+b}{4ab}$$

$$\therefore \frac{1}{x}.\frac{1}{y}.\frac{1}{z} = \frac{(3a+b)(a+b)(3a+b)}{32a^3\ b^3}$$

$$\Rightarrow \frac{1}{xyz} = \frac{(3a+b)(a+b(a+3b))}{32a^3\ b^3}$$

$$\Rightarrow \frac{55}{343} = \frac{55 \times 32}{32\ a^3 b^3}$$

$$\Rightarrow (ab)^3 = 7^3$$

$\Rightarrow ab = 7$

$\Rightarrow a = a,\ b = 7,\ or\ a = 7,\ b = 1$

2. The largest positive term of the H.P whose 1st two terms are $\frac{2}{5}$ *and* $\frac{12}{23}$ is ____

Key. 6

Sol. First two terms of the corresponding A.P are $\frac{5}{2}$ *and* $\frac{23}{12}$

Let d be the common difference of the corresponding A.P , Then

$d = \frac{23}{12} - \frac{5}{2} = \frac{-7}{12}$

Let a_n be the nth term of the given H.P. Then ,

$$a_n = \frac{1}{\frac{5}{2} + (n-1)\left(\frac{-7}{12}\right)} = \frac{12}{30 - 7n + 7} = \frac{12}{37 - 7n}$$

Cleary , a_n will be greatest , if 37-7 n is least

$37 - 7n$ is least for n=5

Hence , $a_5 = \frac{12}{37-35} = 6$ is the largest positive term

3. If the sum of the n terms of the series $1^3 + 3.2^2 + 3^3 + 3.4^2 + 5^3 + 3.6^2 + ...$, where n is an even number , is given by $\frac{n}{k}\left(n^3 + an^2 + bn + c\right)$ then $b - a + c - k$ is

Key. 6

Sol. We have

$S = 1^3 + 3.2^2 + 3^3 + 3.4^2 + 5^3 + 3.6^2 + ...$

$S = \left(1^3 + 3^3 + 5^3 + ...\right) + 3.\left(2^2 + 4^2 + 6^2 + ...\right)$

$S = \left(1^3 + 3^3 + 5^3 + ...\right) + 12\left(1^2 + 2^2 + 3^2 + ...\right)$

Where $S_1 = 1^3 + 3^3 + 5^3 + ...$ *and* $S_2 = 1^2 + 2^2 + 3^2 + ...$

Now case arise

When n is , say even ,say n=2m $m \in N$

In this case S_1, *and* S_2 both contain m terms

$\therefore S_1 = 1^3 + 3^3 + 5^3 + ... + (2m-1)^3$

$$\sum_{r=1}^{M} (2r-1)^3$$

$$\sum_{r=1}^{m} \left(8r^3 - 12r^2 + 6r - 1\right)$$

$$=8\sum_{r=1}^{m} r^3 - 12\sum_{r=1}^{m} r^2 + 6\sum_{r=1}^{m} r - \sum_{r=1}^{m} 1$$

$$=8\left\{\frac{m(m+1)}{2}\right\}^2 - 12\left\{\frac{m(m+1)(2m+1)}{6}\right\} + \frac{6m(m+1)}{2} - m$$

$$=8\left\{\frac{n(n+2)}{8}\right\}^2 - \frac{12}{6}\left\{\frac{n}{2}\left(\frac{n+2}{2}\right)+(n+1)\right\} + 3\frac{n}{2}\left(\frac{n+2}{3}\right) - \frac{n}{2}$$

$$=\frac{n^2(n+2)^2}{8} - \frac{n(n+1)(n+2)}{2} + 3\frac{n(n+2)}{4} - \frac{n}{2}$$

$$S_2 = 1^2 + 2^2 + 3^2 + ... + m^2$$

$$=\frac{m(m+1)(2m+1)}{6}$$

$$=\frac{n(n+2)(n+1)}{24}$$

$$\therefore S = S_1 + 12S_2$$

$$=\frac{n^2(n+2)^2}{8} - \frac{n(n+1)(n+2)}{2} + \frac{3}{4}\, n\,(n+2) - \frac{n}{2} + \frac{n(n+1)(n+2)}{2}$$

$$=\frac{n^2(n+2)^2}{8} + \frac{3}{4}n(n+2) - \frac{n}{2}$$

$$=\frac{n}{8}\left(n^3 + 4n^2 + 10n -\right)$$

4. Find the natural number 'a' for which $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$ where $f(x) = 2^x$

Key. 3

Sol. $f(x) = 2^x$ *for all* $x \in N$

$$\therefore \sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$$

$$\Rightarrow \sum_{K=1}^{n} 2^{a+k} = 16(2^n - 1)$$

$$\Rightarrow \sum_{k=1}^{n} 2^a.2^k = 16(2^n - 1)$$

$$\Rightarrow 2^a\left(\sum_{k=1}^{n} 2^k\right) = 16(2^n - 1)$$

$$\Rightarrow 2^a\left(2 + 2^2 + ... + 2^n\right) = 16(2^n - 1)$$

$$\Rightarrow 2^a\left\{2\left(\frac{2^n - 1}{2-1}\right)\right\} = 16(2^n - 1)$$

$$\Rightarrow 2^{a+1}(2^n - 1) = 16(2^n - 1)$$

$\Rightarrow 2^{a+1} = 2^4$

$\Rightarrow a+1 = 4 \Rightarrow a = 3$

5. Let S = $\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+.....+\sqrt{1+\frac{1}{1999^2}+\frac{1}{2000^2}}$, then find |2000(S-2000)|.

ANS : 1

HINT: $t_r = \sqrt{1+\frac{1}{r^2}+\frac{1}{(r+1)^2}}$

$= \sqrt{\frac{r^2+(r+1)^2+r^2(r+1)^2}{r^2(r+1)^2}}$

$= \sqrt{\frac{2r^2+2r+1+r^2(r^2+2r+1)}{r^2(r+1)^2}}$

$= \sqrt{\frac{r^4+2r^3+3r^2+2r+1}{r^2(r+1)^2}}$

$= \frac{r^2+r+1}{r(r+1)} = \frac{1}{r(r+1)}+1$

$= 1+\frac{1}{r}-\frac{1}{r+1}$,

S = 2000 − $\frac{1}{2000}$, $|2000(S-2000)| = 1$.

6. A sequence is obtained by deleting all perfect squares from set of natural numbers. The remainder when the 2003^{rd} term of new sequence is divided by 2048 , is

Key: 0

Hint: Since $\left[\sqrt{2046}\right]=\left[\sqrt{2047}\right]=\left[\sqrt{2048}\right]=\left[\sqrt{2049}\right]=45$

$\therefore 2003^{rd}$ term is 2003 + 45 = 2048

Hence remainder is 0

7. If a and b are positive integers and a + 11b is divisible by 13 and a + 13b is divisible by 11. Then minimum value of a + b – 20 is

Key. 8

Sol. $a + 11b = 13I_1$

$a + 13b = 11I_2$

and proceed

8. Three numbers, the third of which is 4 from a decreasing G.P. If the last term is replaced by 3, the three numbers form an A.P, then the first number of the G.P. is

Key. 9

Sol. a, ar, ar^2

$2ar = a + 3 \Rightarrow a = \frac{3}{2r-1}$

$ar^2 = 4$

Solve, r = 2/3, a = 9

9. Find the greatest integer less than the number $\left(\frac{2011}{2010}\right)^{2010}$

Key. 2

Sol. $2 < \left(1+\frac{1}{n}\right)^n < 3 \;\; \forall \; n \geq 2, \; n \in N$

10. Find the natural number 'a' for which $\sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$ where $f(x) = 2^x$

Key. 3

Sol. $f(x) = 2^x \;\; for\, all \;\; x \in N$

$$\therefore \sum_{k=1}^{n} f(a+k) = 16(2^n - 1)$$

$$\Rightarrow \sum_{K=1}^{n} 2^{a+k} = 16(2^n - 1)$$

$$\Rightarrow \sum_{k=1}^{n} 2^a.2^k = 16(2^n - 1)$$

$$\Rightarrow 2^a\left(\sum_{k=1}^{n} 2^k\right) = 16(2^n - 1)$$

$$\Rightarrow 2^a\left(2 + 2^2 + ... + 2^n\right) = 16(2^n - 1)$$

$$\Rightarrow 2^a\left\{2\left(\frac{2^n - 1}{2-1}\right)\right\} = 16(2^n - 1)$$

$$\Rightarrow 2^{a+1}(2^n - 1) = 16(2^n - 1)$$

$$\Rightarrow 2^{a+1} = 2^4$$

$$\Rightarrow a + 1 = 4 \Rightarrow a = 3$$

11. Given a, b, c are positive integers forming an increasing G.P., b – a is a prefect square of a natural number, and $\log_6 a + \log_6 b + \log_6 c = 6$. Find the value of a + b + c

Ans. 111

Sol. a,b,c are in A.P.

$$b^2 = ac$$

$$\log_6 a + \log_6 b + \log_6 c = 6$$

$$a\ b\ c = 6^6$$

$$b^3 = 6^6$$

$$b = 6^2 = 36$$

$$ac = 36 \times 36 = 2^4 \times 3^4$$

$$b - a = N^2$$

$$36 - a = N^2$$

a is factor of $2^4 3^4$

a = 27 is possible value

36 – 27 = 9 = $(3)^2$

$\Rightarrow$ a = 27, b = 36, c = 48

A + b + c = 111 Ans.

12. Find the sum to infinity of a decreasing G.P. with the common ratio x such that |x| < 1; $x \neq 0$. The ratio of the fourth term to the second term is $\frac{1}{16}$ and the ratio of third term to the square of the second term is $\frac{1}{9}$.

Ans. 12

Sol. Let the series be a, ax, $ax^2, ax^3...$ given that |x| < 1 and $x \neq 0$

Also, $\frac{T_4}{T_2} = \frac{ax^3}{ax} = \frac{1}{16} \Rightarrow x^2 = \frac{1}{16}$

$\Rightarrow x = \pm\frac{1}{4}$

But since it is a decreasing G.P. $\Rightarrow x = \frac{1}{4}$

Also, $\frac{T_3}{T_2^2} = \frac{ax^2}{(ax)^2} = \frac{1}{9} \Rightarrow \frac{1}{a} = \frac{1}{9} \Rightarrow a = 9$

$S_\infty = \frac{a}{1-r} = \frac{9}{1-\frac{1}{4}} = \frac{9 \times 4}{3} = 12$ Ans.

13. If $\sum_{\alpha=4}^{n+3} 4(\alpha - 3) = An^2 + Bn + C$, then find the value of A + B – C

Ans. 4

Sol. $\sum_{\alpha=4}^{n+3} 4(\alpha - 3) = An^2 + Bn + C \Rightarrow \sum_{\alpha=1}^{n} 4\alpha = An^2 + Bn + C$

$\Rightarrow$ 2n(n + 1) $= An^2 + Bn + C$ $\Rightarrow$ A = 2, B = 2, C = 0

$\Rightarrow$ A + B + C = 4 Ans.

14. If $(1-P)(1+3x+9x^2+27x^3+81x^4+243x^5) = 1-P^6$, $P \neq 1$, then find the value of $\frac{P}{x}$

Ans. 3

Sol. $(1-P)(1+3x+9x^2+27x^3+81x^4+243x^5) = 1-P^5$

$\Rightarrow (1-P)\frac{1-(3x)^6}{1-3x} = 1-P^6$ which is possible only. If P = 3x

$\therefore \frac{P}{x} = 3$ ans.

15. If $(1^2 - a) + (2^2 - a_2) + (3^2 - a_3) + ... + (n^2 - a_n) = \frac{1}{3}n(n^2 - 1)$, then find the value of a_7.

Ans. 7

Sol. $(1^2 + 2^2 + ... + n^2) - (a_1 + a_2 + ... + a_n) = \frac{1}{3}n(n^2 - 1)$ …(i)

Replacing n by (n – 1), then

$$\left(1^2+2^2+...+(n-1)^2\right)-\left(a_1+a_2+...+a_{n-1}\right)=\frac{1}{3}(n-1)\left((n-1)^2-1\right) \quad ...(ii)$$

Subtracting (ii) from (i)

$n^2-a_n=n^2-n$

$\Rightarrow \quad a_n=n \Rightarrow \quad a_7=7$ Ans.

16. The sum of the terms of an infinitely decreasing GP is equal to the greatest value of the function $f(x)=x^3+3x-9$ on the interval [- 4, 3] and the difference between the first and second terms is $f'(0)$. Then find the value of 27r where is common ratio.

Ans. 18

Sol. f is increasing

so its greatest value is f(3) = 27.

Let the GP be a,ar, ar^2 … with, - | < r < |

$\frac{a}{1-r}=27$ and a – ar = 3 $\Rightarrow$ $r=\frac{4}{3}$ or $r=\frac{2}{3}$

but – 1 < r < 1 so $r=\frac{2}{3}$

$\Rightarrow$ 27r = 18 Ans.

17. Find the n^{th} term and the sum of n terms of the series 2, 5, 12, 31, 86,…. .

Ans. $t_n=3^{n-1}+n,\ S_n=\frac{3^n-1+n^2+n}{2}$

Sol. S = 2 + 5 + 12 + 31 + 86 + … + t_n

S = 2 + 5 + 12 + 31 + … + $t_{n-1}+t_n$

0 = 2 + 3 + 7 + 19 + 55 + … n terms $-t_n$

$\Rightarrow \quad t_n=2+3+7+19+55+...+t_n$

$\Rightarrow \quad t_n=2+3+7+19+...+t'_{n-1}+t'_n$

Subtract

0 = 2 + 1 + 4 + 12 + 36 + … n terms $-t'_n$

$\Rightarrow \quad t'_n=3$ + [4 + 12 + 36 + … (n – 2) terms]

$\Rightarrow \quad t'_n=3+\frac{4\left(3^{n-2}-1\right)}{3-1}$

$\Rightarrow \quad t'_n=2.3^{n-2}+1,\ (n\ge 2)$

Now $\quad t_n=\Sigma t'_n=2\sum_{n=2}^{n}3^{n-2}+\sum_{n=2}^{n}1+2$

$t_n=3^{n-1}+n$

Now $\quad S_n=\Sigma t_n$

$=\Sigma 3^{n-1}+\Sigma n$

$=\frac{3^n-1+n^2+n}{2}$ Ans. $t_n=3^{n-1}+n,\ S_n=\frac{3^n-1+n^2+n}{2}$

18. If S_n = 1.n + 2.(n–1) + 3.(n–2) + ……. + n.1 and $S_{25}=325\lambda$ then λ is

Key. 9

Sol. $T_r = r(n-r+1)$

$T_r = nr - r^2 + r$

$S_n = \sum_{r=1}^{n} T_r = n\sum_{r=1}^{n} r - \sum_{r=1}^{n} r^2 + \sum r = \frac{n \times n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$

$= \frac{n(n+1)}{2}\left[n - \frac{(2n+1)}{3} + 1\right] = \frac{n(n+1)}{2}\left[\frac{3n-2n-1+3}{3}\right] = \frac{n(n+1)(n+2)}{6}$

$S_n = \frac{n(n+1)(n+2)}{6}, S_{25} = \frac{25 \times 26 \times 27}{6} = 25 \times 13 \times 9$

$S_{25} = 325\lambda \Rightarrow \lambda = 9$

AP,GP,HP, Sequences

Matrix-Match Type

1. *Column-I* | *Column-II*

A) If p^{th}, q^{th}, r^{th} *and* S^{th} terms of an A.P are in G.P then $p-q, q-r, r-s$ — p) are all equal

B) If lnx, lny,lnz (x,y,z>1) are in G.P then 2x+ln (lnx), $3x+\ln(\ln y), 4x+\ln(\ln z)$ — q) are in A.P

C) If $n!, 3\times n!$ *and* $(n+1)!$ are in G.P then $n!, 5\times n!$ *and* $(n+1)!$ — r) are in G.P

D) If the arithmetic mean of $(b-c)^2, (c-a)^2$ *and* $(a-b)^2$ is same as that of $(b+c-2a)^2, (c+a-2b)^2, (a+b-2c)^2$ then a,b,c — s) are in H.P

Key. A-R;B-Q;C-Q;D-P

Sol. A) $A_p = a+(p-1)d$ ....(1)

$A_q = a+(q-1)d$ ....(2)

$A_r = a+(r-1)d$ .... (3)

$A_s = a+(s-1)d$ ....(4)

$A_q = kA_p$

$A_r = k^2 A_p$

$A_s = k^3 A_p$ $(\because A_p, A_q, A_r, A_s \ in\, G.P)$

$(p-q) = \frac{A_p - A_q}{d} = A_p \frac{(1-k)}{d}$ *from* (1) *and* (2)

$(q-r) = A_p K \frac{(1-k)}{d}$ from (2) and (3)

$(r-s) = A_p k^2 \frac{(1-k)}{d}$

$\Rightarrow p-q, q-r, r-s$ are in A.P

B) In x in y ln z are in G.P

$\Rightarrow \ln(\ln x), \ln(\ln y), \ln(\ln z)$ are in A.P

$\Rightarrow 2x+\ln(\ln x), 3x+\ln(\ln y), 4x+\ln(\ln z)$ are in A.

C) $n! 3\times n!$ *and* $(n+1)!$ are in G.P

$\Rightarrow 9(n!)^2 = n!(n+1)!$

$\Rightarrow (n+1) = 9 \Rightarrow n = 8$

$\therefore n! = 8!$

$5 \times n! = 5 \times 8!$

$(n+1)! = 9!$

$9! + 8! = 5 \times 9!$

$\Rightarrow n!, 5 \times n1 \; and \; (n+1)!$ are in A.P

D) $\dfrac{(b-c)^2 + (a-b)^2 + (c-a)^2}{3}$

$= \dfrac{(b-c-2a)^2 + (c+a-2b)^2 + (a+b-2c)^2}{3}$

$\Rightarrow (b+c-2a)^2 - (b-c)^2 + (c+a-2b)^2 - (c-a^2)$

$+(a+b-2c)^2 - (a-b)^2 = 0$

$\Rightarrow a = b = c$

2. Column-I
Column-II

(A)	The sequence a, b, 10, c, d is an arithmetic progression. The value of a + b + c + d	(P)	10
(B)	The sides of right triangle form a three term geometric sequence. The shortest side has length 2. The length of the hypotenuse is of the form where a ∈ N	(Q)	20
	and is a surd, then $a^2 + b^2$ equals	(R)	26
(C)	The sum of first three consecutive numbers of an infinite G..P. is 70, if the two extremes be multipled each by 4, and the mean by 5, the products are in A.P. The first term of the G.P. is	(S)	40
(D)	The diagonals of a parallelogram have a measure of 4 and 6 metres. They cut off forming an angle of 60°. If the perimeter of the parallelogram is where a, b ∈ N then (a + b) equals		

[Ans. (A) S; (B) R; (C) S; (D) R]

[Hint: (A) $b + c = a + d = 2 \cdot 10$

$\Rightarrow \quad a + b + c + d = 40$

(B) $(ar^2)^2 = a^2 + a^2r^2$ where a = 2

$\therefore \quad r^4 = 1 + r^2$

$r^4 - r^2 - 1 = 0$

let $r^2 = t$

$t^2 - t - 1 = 0$

$t = \quad \Rightarrow$ (reject)

$r^2 =$

$\therefore$ hypotenuse is $2 \times \ = 1 +$

comparing with

$a = 1,\ b = 5$

$\therefore$ $a^2 + b^2 = 1 + 25 = 26$ Ans.

(C) $a, ar, ar^2 \rightarrow$ G.P. $|\ r\ | < 1$

$a + ar + ar^2 = 70$

$\therefore$ $10ar = 4a + 4ar^2$

$10r = 4 + 4r^2$

$2r^2 - 5r + 2 = 0$

$2r^2 - 4r - r + 2 = 0$

$(2r - 1)(r - 2) = 0$

$\therefore$ $r = 2$ (reject) or $r = 1/2$

for $r = 1/2$

$a + \ + \ = 70$

$a + \ = 70$

$\Rightarrow \quad = 70$

$\Rightarrow \quad a = 40$

$\therefore$ series is 40, 20, 10

$\therefore$ first term of G.P. is 40 Ans.

(D) Using cosine rule

$a^2 = 9 + 4 - 2 \cdot 2 \cdot 3 \cdot = 13 + 6 = 19$

$a^2 = 19 \quad \Rightarrow \quad a =$

|||ly $b^2 = 9 + 4 - 2 \cdot 2 \cdot 3 \cdot$

$b^2 = 7 \quad \Rightarrow \quad b =$

$\therefore$ P = $\Rightarrow$ $a + b = 26$ Ans.]

3. Match the following:-

	Column I		Column II
A	The largest positive term of the H.P., whose first two terms are $\frac{2}{5}$ and $\frac{12}{23}$ is	P	2
B	If a, b, c are positive real number such that a+b+c=1, then minimum value of $\frac{(1+a)(1+b)(1+c)}{(1-a)(1-b)(1-c)}$ is	Q	4
C	The integers which belongs to the range of $f(x)=\frac{2x^2+2x+5}{x^2+x+1}$ can be	R	6
D	The values of x for which $\left(\frac{1}{3}\right)^{\frac{\lvert x+6\rvert}{1-\lvert x\rvert}}>9$ can be	S	7
		T	8

Key: A – R; B – T;
C – Q, R; D – P, Q, R, S

Hint:

$$(a) \text{Let the } H.P. \text{ be } \frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}+.....$$

$$\frac{1}{a}=\frac{2}{5}, \frac{1}{a+d}=\frac{12}{23} \Rightarrow a=\frac{5}{2}, d=\frac{-7}{12} \Rightarrow T_n=\frac{1}{a+(n-1)d}=\frac{12}{37-7n}, \text{ for } n=5,\ T_n \text{ is largest},\quad T_5=6$$

$$(b) 1+a=2-b-c=(1-b)+(1-c)\ge 2\sqrt{(1-b)(1-c)}$$

$$1+b\ge 2\sqrt{(1-a)(1-c)},\quad (1+c)\ge 2\sqrt{(1-a)(1-b)} \Rightarrow (1+a)(1+b)(1+c)\ge 8(1-a)(1-b)(1-c)$$

$$(c)\ y=\frac{2x^2+2x+5}{x^2+x+1} \Rightarrow (2-y)x^2+(2-y)x+(5-y)=0$$

$y=2$ does not satisify the eq.

$$\text{if } y\ne 2, x\in R \Rightarrow D\ge 0 \Rightarrow (2-y)^2-4(2-y)(5-y)\ge 0 \Rightarrow (2-y)(3y-18)\ge 0$$

$$\Rightarrow (y-2)(y-6)\le 0, y\ne 2 \Rightarrow y\in(2,6]$$

$(d)\left(\frac{1}{3}\right)^{\frac{\lvert x+6\rvert}{1-\lvert x\rvert}}>9$ In options all values are positive hence if x > 0

$$\left(\frac{1}{3}\right)^{\frac{x+6}{1-x}}>3^2 \Rightarrow 3^{-\left(\frac{x+6}{1-x}\right)}>3^2 \Rightarrow -\left(\frac{x+6}{1-x}\right)>2 \Rightarrow \frac{x+6}{x-1}>2$$

For x>1, x+6 > 2 x – 2, x < 8.

4. Match the following:-

	Column I		Column II
a	Number of divisor of $N = 2^3 3^2 5^5 7^4$ which leaves remainder 1 when divided by 4 is	p	16
b	If $a_1, a_2, \ldots\ldots a_{100}$ are in H.P. then the value of $\sum_{i=1}^{99} \frac{a_i\ a_{i+1}}{a_1\ a_{100}}$ is	q	48
c	The remainder when 3^{33} is divided by 75 is	r	126
d	The number of four digit number in which every digits exceeds the immediate preceding digit	s	36
		t	99

Key: A→Q; B→T;C→Q; D→R

Hint: a) Let X = $2^0 . 3^a . 5^b . 7^c$ is a divisor = $(4-1)^a (4+1)^b (8-1)^c$

$$0 \le a \le 2 \quad 0 \le b \le 5 \quad 0 \le c \le 4$$

$$4I + (-1)^a 1^b (-1)^c$$

a, c both odd, b takes any value OR a, c both even, b take any value

$$\Rightarrow If\ a = 0, b\ any, c = 0, 2, 4\ \Rightarrow 6 \times 3 = 18$$

$$if\ a = 1,\ b\ any, c = 1, 3 \Rightarrow 6 \times 2 = 12$$

$$if\ a = 2, b\ any, c = 0, 2, 4 = 6 \times 3 = 18$$

––––

48

B.

$$a_2 - a_1 = a_1 a_2 d$$

$$a_3 - a_2 = a_2 a_3 d$$

. .

. .

$$a_{100} - a_{99} = a_{99} a_{100} d$$

–––––––––––––

$$a_{100} - a_1 = d \sum_{i=1}^{99} a_i\ a_{i+1} = 99 a_1 a_{100} d$$

C.

$$3.3^{32} = 3(10-1)^{16} = 3\left[100I - 16_{C_{15}}.10 + 1\right]$$

$$= 3(100I - 160 + 1) = 3(100I^1 + 41)$$

$$= 300I^1 + 123 = 75I^{11} + 48$$

D.

Let four digits no is $x_1\ x_2\ x_3\ x_4$

$x_1 > x_2 > x_3 > x_4$

0 can not use at any place

Required no. = no. of ways of selecting 4 digit out of $9 = 9_{C_4} = 16$

5. Observe the following lists :

List – I	List – II
(A) If three unequal number a, b, c are A.P. and $b-c, c-b$, a are in GP., then $\frac{a^3+b^3+c^3}{3abc}$ is equal to	p) 4
(B) Let x be the arithmetic mean and y, z be two geometric means between any two positive numbers, then $\frac{y^3+z^3}{xyz}$ is equal to	q) 1
(C) If $a_1, a_2, a_3 -----, a_{50}$ are 50 distinct numbers in A.P and $a_1{}^2 - a_2{}^2 + a_3{}^2 ------- a_{50}{}^2 = \left(\frac{5}{7}\right)^n \left(a_1{}^2 - a_{50}{}^2\right)$, $(n \in N)$ then n =	r) 2
(D) $\lim_{n\to\infty} \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{2r^2}\right)\right\}$ is equal to	s) 3

Key. A – r , B – r , C – r , D – q

Sol. (A) $(b-a) = (c-b)$ and $(c-b)^2 = a(b-a)$

$\Rightarrow (b-a)^2 = a(b-a) \Rightarrow b = 2a, c = 3a$

$\therefore a:b:c = 1:2:3$

(B) $x = \frac{a+b}{2}, b = ar^3 \Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{3}}$

$$\frac{y^3+z^3}{myz} = \frac{a+b}{\frac{a+b}{2}} = 2$$

(C)

$$a_1{}^2 - a_2{}^2 + a_3{}^2 ------ a_{50}{}^2 = (a_1+a_2)(a_1-a_2) + (a_3+a_4)(a_3-a_4) + ------ + (a_{49}+a_{50})(a_{49}-a_{50})$$

$$= -d\left[a_1 + a_2 + ---- + a_{50}\right] = -\frac{25}{49}(a_{50}-a_1)(a_{50}+a_1)$$

$$= \left(\frac{25}{49}\right)\left(a_1{}^2 - a_{50}{}^2\right)$$

(D) $\tan^{-1}\left(\frac{1}{2r^2}\right) = \tan^{-1}\left(\frac{2}{4r^2}\right) = \tan^{-1}\left(\frac{2r+1-(2r-1)}{1+(2r+1)(2r-1)}\right)$

$= \tan^{-1}(2r+1) - \tan^{-1}(2r-1)$

6. Match the following

(A)	If $a_1, a_2, \ldots. a_{100}$ are in HP, then the value of $\sum_{i=1}^{99} \frac{a_i a_{i+1}}{a_1 a_{100}}$ is	(p)	5
(B)	Largest positive term of HP whose first two terms are $\frac{2}{5}$ and $\frac{12}{13}$ is	(q)	7
(C)	If *x* be probability that first row of 3 × 3 order matrix obtained by using elements {1, 2,…,9} without repetition, have number in decreasing order, then 36*x* equals	(r)	6
(D)	If *x* be probability that a randomly chosen 3 digit number has exactly 3 factors, then 900 *x* equals	(s)	– 99
		(t)	3

Key. (A-s), (B-r), (C-r), (D-q)

Sol. (A) Let d be C.D of AP $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3} \ldots. \frac{1}{a_n}$

$a_2 - a_1 = a_1 a_2 d$

$a_3 - a_2 = a_3 a_2 d$

$a_{100} - a_{99} = a_{99}\, a_{100}\, d$

Adding all these, we get

$a_{100} - a_1 = d\sum_{i=1}^{99} a_i a_{i+1}$

(B) $\frac{1}{a} = \frac{2}{5}, \frac{1}{a+d} = \frac{12}{23}$

$a = \frac{5}{2}, d = -\frac{7}{12}, T_n = \frac{12}{37-7n}$ for n = 5

T_n is largest positive

$T_5 = 6$

(C) Total no. of case 9!

no. of favourable cases ${}^9C_3$. 6!

(D) A number has exactly 3 factors if the number is square of a prime number. Squares of 11, 13, 17, 19, 23, 29, 31 are 3 digit number.

So required probability.

7. Match the following: -

Column – I		Column – II	
(A)	Suppose that $F(n+1) = \frac{2F(n)+1}{2}$ for n = 1, 2, 3, … and F(1) = 2. Then F(101) equals	(p)	42
(B)	If $a_1, a_2, a_3, \ldots.. a_{21}$ are in A.P. and $a_3 + a_5 + a_{11} + a_{17} + a_{19} = 10$ then the	(q)	1620

	value of $\sum_{i=1}^{21} a_i$ is		
(C)	10^{th} term of the sequence S = 1 + 5 + 13 + 29 + …., is	(r)	52
(D)	The sum of all two digit numbers which are not divisible by 2 or 3 is	(s)	2045
		(t)	2 + 4 + 6 + … + 12

Key. A $\to$ r; B $\to$ p,t; C $\to$ s; D $\to$ q

Sol. (A) $F(n+1) = \dfrac{2F(n)+1}{2} = F(n) + \dfrac{1}{2}$

$\therefore$ F(1), F(2), F(3), … is an AP with common difference $\dfrac{1}{2}$

(B) $a_1 + 2d + a_1 + 4d + a_1 + 10d + a_1 + 16d + a_1 + 18d = 5a_1 + 50d$

$= 5(a_1 + 10d) = 10$ i.e. $a_1 + 10d = 2$

Now, $\sum_{i=1}^{21} a_i = \dfrac{21}{2}[2a_1 + 20d] = 21(a_1 + 10d) = 42$

(C) S = 1 + 5 + 13 + 29 + … + t_{10}

S = 1 + 5 + 13 +… + $t_9 + t_{10}$

Subtrating

$t_{10} = 1 + 4 + 8 + 16 + ...$ up to 10 terms

= 1 + (4 + 8 + 16 + …….. up to 9 terms)

= 2045

(D) Sum of all two digit numbers $= \dfrac{90}{2}(10+99) = (45)(109)$

Sum of all two digit numbers is divisible by $2 = \dfrac{45}{2}(10+98) = (45)(54)$

Sum of all two digit numbers is divisible by $3 = \dfrac{30}{2}(12+99) = 15(54)$

Sum of all two digit numbers divisible by $6 = \dfrac{15}{2}(12+96) = 15(54)$

The required sum is 45(109) + 15(54) – (45) (54) – 15 (111) = 1620

8. Match the following: -

Column – I		Column – II	
(A)	The arithmetic mean of two positive numbers is 6 and their geometric mean G and harmonic mean H satisfy the relation $G^2 + 3H = 48$, then product of the two number is	(p)	$\dfrac{240}{77}$
(B)	The sum of the series $\dfrac{5}{1^2 4^2} + \dfrac{11}{4^2 7^2} + \dfrac{17}{7^2.10^2} + ...$ is	(q)	32
(C)	If the first two terms of a Harmonic Progression be $\dfrac{1}{2}$	(r)	$\dfrac{1}{3}$

	and $\frac{1}{3}$, then the Harmonic Mean of the first four terms is		
(D)	Geometric mean of – 4 and – 9	(s)	6
		(t)	- 6

Key. A $\to$ q; B $\to$ r; C $\to$ p; D $\to$ t

Sol. (A) a + b = 12

$$ab+\frac{6ab}{a+b}=48$$

$$ab+\frac{ab}{2}=48 \quad \therefore \quad ab = 32$$

(B) $S=\frac{5}{1^2}+\frac{11}{4^2.7^2}+\frac{11}{7^2.10^2}+.........$

$\Rightarrow$ $3S=\frac{3.5}{1^2.4^2}+\frac{3.11}{4^2.7^2}+\frac{3.17}{7^2.10^2}+..........$

$\Rightarrow$ $3S=\frac{(4-1).(4+1)}{1^2.4^2}+\frac{(7-4)(7+4)}{4^2.7^2}+\frac{(10-7)(10+7)}{7^2.10^2}+............$

$\Rightarrow$ $3S=\frac{4^2-1^2}{1^2.4^2}+\frac{7^2-4^2}{4^2.7^2}+\frac{10^2-7^2}{7^2.10^2}+............$

$\Rightarrow$ $3S=1-\frac{1}{4^2}+\frac{1}{4^2}-\frac{1}{7^2}+\frac{1}{7^2}-\frac{1}{10^2}+............$

$\Rightarrow$ $3S=1 \qquad S=\frac{1}{3}$

(C) H.M of $\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5}$ is $\frac{4}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}}=\frac{240}{77}$

(D) Since G.M. lies between the numbers $GM=-\sqrt{(-4)\times(-9)}=-6$