CLASS : CC (Advanced) *APPLICATION OF DERIVATIVE* **TEST-19**

M.M.: 71 **Time: 60 Min.**

PART-A
[SINGLE CORRECT CHOICE TYPE]

Q.1 to Q.10 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct. **[10 × 3 = 30]**

Q.1 From the point (1, 1) tangents are drawn to the curve represented parametrically as $x = 2t - t^2$ and $y = t + t^2$. The distance between the points of contact, is

(A) $\frac{2\sqrt{43}}{9}$ (B) 2 (C) 3 (D) $\frac{2\sqrt{53}}{9}$

Q.2 Let f(x) be a function defined by $f(x) = \int_1^x t\,(t^2 - 3t + 2)\,dt$, $1 \le x \le 3$. Then the range of f(x) is

(A) [0, 2] (B) $\left[\frac{-1}{4}, 4\right]$ (C) $\left[\frac{-1}{4}, 2\right]$ (D) none of these

Q.3 If the eccentricity of the ellipse $3x^2 + 4y^2 = 12$ is changing at the rate 0.1/sec, then the time at which it will coincide the auxiliary circle, is

(A) 2 sec (B) 3 sec (C) 5 sec (D) 6 sec

Q.4 Let f and g be two continuous function such that gof is increasing and g is decreasing.
If $f\left(g(x^2 - 2kx + 3)\right) > f\left(g(k-3)\right) \ \forall\ x \in R$, then number of possible integral values of k is

(A) 3 (B) 4 (C) 5 (D) 6

Q.5 For the function $f(x) = \int_0^{x^2} e^{\frac{-t^2}{4}} (4 - t)\,dt$, which of the following statement is **incorrect**?

(A) a local maximum occurs at x = 2
(B) a local maximum occurs at x = –2
(C) a local maximum occurs at x = 0
(D) a local minimum occurs at x = 0

Q.6 Given $f'(1) = 1$ and $f(2x) = f(x)\ \forall\ x > 0$. If $f'(x)$ is differentiable, then there exists a number $c \in (2, 4)$ such that $f''(c)$ equals

(A) $\frac{1}{4}$ (B) $\frac{-1}{2}$ (C) $-\frac{1}{4}$ (D) $-\frac{1}{8}$

Q.7 If a variable tangent to the curve $x^2y = c^3$ makes intercepts a, b on x and y-axis, respectively, then the value of a^2b is

(A) $27c^3$ (B) $\frac{4}{27}c^3$ (C) $\frac{27}{4}c^3$ (D) $\frac{4}{9}c^3$

Q.8 Given that $f(x)$ is continuously differentiable on $a \le x \le b$, where $f(a) < 0$ and $f(b) > 0$. Which of the following statement is **not true**?

(A) $f(x)$ is bounded on $a \le x \le b$.

(B) The equation $f(x) = 0$ has at least one solution in $a < x < b$.

(C) The maximum and minimum values of $f(x)$ on $a \le x \le b$ occur at points where $f'(c) = 0$, $a < c < b$.

(D) There is at least one point c with $a < c < b$, where $f'(c) > 0$.

Q.9 The ordinates of the point on the curve $x^3 = y(x-a)^2, (a \ne 0)$ where the ordinate is minimum, is

(A) $\frac{27}{4}a$ (B) 8a (C) 3a (D) 2a

Q.10 The least integral value of k for $f(x) = \frac{e^{-x}}{\sqrt{k+8x^2}}$ is decreasing for all $x \in R$ is

(A) 1 (B) 2 (C) 3 (D) 4

[PARAGRAPH TYPE]

Q.11 to Q.15 has four choices (A), (B), (C), (D) out of which **ONLY ONE** is correct. **[5 × 3 = 15]**

Paragraph for question nos. 11 & 12

Let $f(x) = x^3 - 6\sin\theta \cdot x^2 + (2\sin^2\theta + q)x - 1$, $q, \theta \in R$ be a cubic polynomial in x. If exaclty one tangent can be drawn to the graph of $y = f(x)$ which is parallel to the line $y = x$, then

Q.11 the largest value of q is

(A) 7 (B) 9 (C) 11 (D) 13

Q.12 if q is smallest, then the value of $\left.\frac{d}{dx}\left(f^{-1}(x)\right)\right|_{x=9}$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{13}$ (C) $\frac{3}{17}$ (D) $\frac{2}{9}$

Paragraph for question nos. 13 to 15

Let $y = g(x)$ and $y = P(x)$ be two polynomial functions of degree two and four respectively such that

(i) $P(-x) = P(x)$ and $P(x) \geq 0 \ \forall \ x \in R$

(ii) $P(x)$ has two local minimum at $x = x_1, x_2$ such that $|x_1 - x_2| = 2$.

(iii) The line $y = 1$ touches the curve $y = P(x)$ at point of local maximum and encloses an area of $\frac{8\sqrt{2}}{15}$ sq. units with it

(iv) $\lim_{x \to 0} \frac{P(x) - g(x) - g(-x)}{x^2} = g'(-1)$.

Q.13 Minimum value of $P(x)$ equals

(A) 2 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Q.14 If $g(x)$ has maximum value at $x = -1$ then product of its roots is

(A) $\frac{-1}{2}$ (B) -1 (C) $\frac{1}{2}$ (D) 1

Q.15 $y = P(x)$ has inflexion points at

(A) $x = \pm\frac{1}{\sqrt{3}}$ (B) $x = \pm\frac{1}{2}$ (C) $x = \pm 2$ (D) $x = \pm\frac{1}{\sqrt{2}}$

[MULTIPLE CORRECT CHOICE TYPE]

Q.16 to Q.19 has four choices (A), (B), (C), (D) out of which **ONE OR MORE** may be correct. **[4 × 4 = 16]**

Q.16 Which of the following statements is(are) true for $f(x) = \begin{cases} \sqrt{x}, & x \geq 1 \\ x^3, & 0 \leq x < 1 \\ \frac{x^3}{3} - 4x, & x < 0 \end{cases}$?

(A) f(x) is monotonically increasing in $(-\infty, -2) \cup (0, \infty)$

(B) function attain its extremum value at x_1 and x_2 where $x_1 + x_2 = -2$

(C) function is not differentiable at two points

(D) function is discontinuous at one point

Q.17 Let $f: R \to R$ be a continuous and differentiable function such that $f(2)=3,\ f(6)=8$ and has local minimum at $x=2$. If $g(x)=\int_2^6 f(2x+t)\,dt$, then

(A) $g'(0)=5$
(B) $g'(0)=10$
(C) $\lim_{x\to 0}\frac{g\left(f(x+2)-3\right)-g(0)}{x}=0$
(D) $\lim_{x\to 1}\frac{f(6x)-f(6)}{x-1}=\lim_{x\to 0}\frac{6f(x+6)-16f(x+2)}{x}$

Q.18 Consider a polynomial function $f(x)=x^5+ax^4+bx^3+cx^2+dx+e$. If the line $y=2$ touches it at $x=1$ and $x=4$ where $x=4$ is also a point of inflection of $y=f(x)$, then

(A) at $x=\frac{11}{5}$, $f(x)$ has a local minimum.
(B) at $x=\frac{11}{5}$, $f(x)$ has a local maximum.
(C) the value of $f(2)+f(3)$ equals -8.
(D) the value of $\int_2^6 f(x)dx$ is $\frac{424}{5}$.

Q.19 If l and m are real number such that $x^3-lx^2+mx-27=0$ has its roots real and positive, then
(A) $l_{min}=3$
(B) $l_{min}=9$
(C) $m_{min}=8$
(D) $m_{min}=27$

PART-D
[INTEGER TYPE]

Q.1 & Q.2 are "Integer Type" questions. (The answer to each of the questions **are upto <u>4 digits</u>**) **[$2\times 5=10$]**

Q.1 If a differentiable function 'f' satisfies the relation

$$\int_0^{t^2} f^2(x)dx+\frac{1}{2}\int_0^{\pi/2} t^6\sin^3 x\,dx=\int_0^{t^2} 2xf(x)dx\ \ \forall t\in R,$$ then find the number of solution (s) of the equation $f(x)=\sqrt[3]{x}$.

Q.2 Let a cubic polynomial be $f(x)=\frac{x^3}{3}-x^2+ax+2$. If all the values of a for which $f(x)$ has a positive point of maximum also satisfy the inequality $3x^2-(b+1)x+b(b-2)<0$ and value of b lies in [p, q], then find the value of $(p+q)$.

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ANSWER KEY

PART-A

Q.1	D	Q.2	C	Q.3	C	Q.4	B	Q.5	C
Q.6	D	Q.7	C	Q.8	C	Q.9	A	Q.10	B
Q.11	C	Q.12	B	Q.13	C	Q.14	B	Q.15	A
Q.16	ABC	Q.17	BCD	Q.18	ACD	Q.19	BD		

PART-D

Q.1	2	Q.2	3